Chapter

THE PORTUGUESE TOURISM MARKET: A MODEL

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ABSTRACT

In Portugal, tourism is one of the most important economic sectors. Recently, with the development of the global crisis, particularly reaching the southern European Countries, this sector has strengthened and reinforced its importance due to the important rents got by Portuguese regional and local agents. Consequently, this topic is now particularly relevant. In this study, factors explaining demand, supply and prices are discussed. At macroeconomic level, it is seen how they contribute to model the Portuguese tourism market. A relationship among the variables is analyzed and its modeling is represented mathematically. The model gives

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an important contribution to show the importance of these variables and relationships in the determination of macroeconomic aggregates in the Portuguese tourism market.

INTRODUCTION

The discussion of tourism problems is classical. Many recent developments have brought tourism models to the actual debate. However, nowadays there is a painful global crisis existing in many economies, as it is the case of Portugal or even Greece, for example. Accordingly, this topic results now particularly relevant.

The economy of tourism gathers a wide range of very diverse studies highlighting the importance of tourism as an economic activity and as an economic sector with high social and even cultural implications in communities' forms of organization. Tourism has become a strong economic force in many countries and regions around the world, bringing important development and a wide diversity of tourism activities to regions and countries. Strengthening these activities, a set of many significant transformations resulted as a consequence as happened many considerable social, cultural, economic, political and environmental changes in societies, causing significant changes in people's job opportunities and working conditions of populations, in the patterns of life and in the distribution of income among individuals, groups and regions.

In the international context of Tourism, an ever increasing number of destinations worldwide have opened up to, and invested in tourism, turning tourism into a key driver of socio-economic progress through export revenues, the creation of jobs and enterprises, and infrastructure development.

As can be seen in the UNWTO (2014a), over the past six decades, tourism has experienced continued expansion and diversification, becoming one of the largest and fastest-growing economic sectors in the world. Many new destinations have emerged in addition to the traditional favorites of Europe and North America. Despite occasional shocks, international tourist arrivals have shown virtually uninterrupted growth – from 25 million in 1950 to 278 million in 1980, 528 million in 1995, and 1087 million in 2013.

In 2014, international tourism set a new record with over 1.1 thousand million international tourists travelling the world (an increase of about 5% according to UNWTO World Tourism - UNWTO 2014b).

Tourism represents (UNWTO 2014a):

- 9% of GDP (direct, indirect and induced impact).
- 1 in 11 jobs.
- 6% of the world exports.

The tourism in Portugal is very important in terms of macroeconomic aggregates. Very particularly, there are several Portuguese regions in which tourism is the main contributor sector for Gross National Product.

In terms of the World Economic Forum (WEF) - Travel & Tourism Competitiveness Index (2011), Portugal is one of the 20th most competitive tourism destinations in the world ranked in the 18th place in the WEF Index (2011), what shows how important tourism is as economic activity in Portugal. According to the Report of World Tourism Organization (UNWTO 2008), Portugal already had more than 12 million people visiting Portugal, more than the Portuguese population. In 2008, tourism has generated about 5% of GVA of Economy, which is approximately 7.3 billion euro. According to the Report on Competitiveness of Travel and Tourism, 2008 (World Economic Forum 2008), Portugal occupied the 15th place that year, in a list of 130 countries in the ranking of competitiveness of the tourism sector. Overall Portugal climbed seven positions compared to 2007 (22nd in 2007) and four positions in all 27 EU countries. Portugal is ranked in the 8th place in overnight stays and in the 11th place in tourism revenue, in terms of the UE 27 Ranking (Eurostat 2011).

Amador and Cabral (2009) presented a detailed analysis of the evolution of the services sector in Portugal and showed that this favorable trend was being verified in this sector in general and in particular that Portugal was revealing a comparative advantage in the sector of Travel and Tourism.

In 2011, in Portugal tourism represented:

- About 12 million international tourists.
- 40 million overnight stays.
- 8.1 thousand million euro tourism revenue.
- A positive "tourism balance" of 5.1 thousand million euro.
- 43% of total exports of services.

- 14% of total exports of goods and services.

Source: INE (2012), Banco de Portugal (2012).

These facts explain why a model like the one developed in this study for tourism market in Portugal is so relevant. The model implemented considers a set of variables and relationships that will be explained in the next chapter.

MAIN MODEL FEATURES AND THEORETICAL CONCEPTS

In the Tourism Business Literature there are very different approaches for modelling tourism markets. Many are based on marketing area features, intending to develop models based for example on motivations or aspects related to destination *versus* origin countries (for instance Valle et al. (2006) explore the relationship between tourists satisfaction, measured by overall satisfaction in terms of holiday experience, destination attributes and met expectations, and destination loyalty intentions), others use the importance and the updated concern about sustainability (see for example Butowski 2012), others yet use macroeconomic aspects to create dynamic general equilibrium models, studying for instance macroeconomic effects resulting from the increase in tourism demand due for example to an exogenous increase in foreigner's income - Schubert and Brida (2009) state for example that by using a dynamic general equilibrium model it is possible to show that an increase in tourism demand leads to an increase in relative price of domestically produced tourism services and rises tourism production. Tribe (2011) for example explains the importance of microeconomic issues, as the actions of individuals (demand) and firms (supply), interacting to determine prices in specific markets; this author also highlights the importance of macroeconomic issues, by considering the economy as a whole and dealing with aggregates (concepts as consumers expenditure, aggregate demand, national output or product are used). Based on the interaction resulting from the aggregates, general price level and the rate of inflation may be got rather than prices in individual markets.

For Portuguese market it was considered important to develop a model based on macroeconomic and microeconomics aspects, allowing to obtain equilibrium prices for the market as a whole and also for specific markets. So, a set of variables were defined in order to develop models (see Silva et al. 2011a, b).

Tourism economic analysis needs variables measurement such as individuals (demand) and firms (supply) actions. It needs also interactions

among these actions to determine prices. At a macroeconomic level, national product measurement is important for models working these variables (Tribe 2011).

The present model main features for the Portuguese tourism market (based on Silva 1991) have in consideration the following aspects:

- The study of the market of the tour operator (travel agency);
- A strong support on microeconomic theory;
- It is supported on 3 items: demand, supply and prices;
- It permits to obtain results associated to exogenous variables and to tourism rents.

In fact, the macroeconomic building of a model like this one is supported on a set of suppositions around the tourism product, economic agents and the several different types of existing markets.

Tourism Product

Tourism Product is the trip plan of a tourist, self-made or not, being the overnight staying/revenue per bed the variable to be quantified. If a tourist is not national, this product is obtained in the market buying a "package" supplied by the tour operator. Taking into account that plans and programs are well diversified, it is considered that the variable "overnight staying" is good enough to be an approximation to the Homogeneous Product.

Economic Agents

In the Portuguese tourism market it is relevant to consider three economic agents:

- The tour operator/travel agency
- The accommodation company/hosting company (hotels, hostels, etc)
- The tourist.

Tour Operator

Headquartered in the emitters, the tour operator is responsible for a big part of tourists that come to Portugal from abroad.

The tour operator is the main agent on this market. Tour operators are near the tourist and have lots of possible destinations available to send tourists to. Besides, several tour agents have a significant part of market share and this is quite relevant to be considered in the analysis.

Their positioning in the market and their economic capacity allow them to be very accurately vigilant to the features of the markets and to be aware of changes in the habits and economic conditions of the tourists.

This shows how important is their great bargaining power among firms that offer services in the country, especially in companies for accommodation (hotels and similar).

It is particularly impressive the power of intervention on prices and the possibility of withdrawal of accommodation services if they do not have enough demand for the programs they offer. The costs of this operator will thus be closely linked to the acquisition of services to include in the package.

Company for Accommodation/Hosting Company

The hosting company emerges as the main supporting infrastructure for tourists. Its short-term, goals are to increase occupancy and to improve the revenue per bed. The tour operator is its main interlocutor, that makes it a price taker (can' not enforce the rack rate, which is the price that would maximize its utility function).

The company has significant fixed costs. The "variable costs" are the variable that has a direct and proportional connection with "overnight stays."

The supply of accommodation is in turn a function of the stock of fixed capital, which combined with the staff, ensures the service to be provided to tourists.

Tourist

Moving from his/her place of habitual residence, whether abroad or in Portugal, for a period exceeding 24 hours, the tourist accommodation is the main support infrastructure in place he/she is visiting.

The tourist is a consumer who seeks the tourism product because he needs it and because he has financial capacity to acquire it.

This "good" or tourism product results from a concrete plan that tourists draw up, or results from one plan accepted by the tourist that is drawn up by the

operator. It includes the location or destination (possibly several), and the activities he wants to accomplish and yet the cost travel.

It is considered that tourists from abroad primarily use the services of tour operators ('packages' or holiday programs). National tourists, in turn, project (produce) their holiday program, which leads them to directly contact the housing companies.

Tour operator, hosting company and tourist represent the major economic players in the Portuguese tourism market. Each one of them wants to maximize its respective utility function and will have its own restrictions. The tour operator may not sell more "packages" than the acquired overnights. The hosting company has the housing capacity as the main obstacle to an increased supply of overnight stays. Tourists will naturally be dependent on either their needs or their ability to purchase travel.

THE DESIGN OF THE MODEL
AND THEORETICAL DISCUSSION

Some Blocks for the Model

In this model of the tourism market in Portugal there are three blocks: the demand, supply and prices.

Demand
It is considered that tourism demand is explained by exogenous variables, by policy and by prices.

Demand functions are developed for national tourists, for foreign tourists by emitter countries, and a global demand function.

Demand Function for Foreign Visitors by Country
For emitter countries of foreign tourists in Portugal (Spain, France, Holland, Germany and UK are the main tourists' suppliers for the Portuguese market), the main determinants of overnights (variable to be explained) relate to the following variables:

- Non-essential consumption of households in the emitters;
- Purchasing power of the currency of the emitters;

- Price of accommodation in Portugal corrected by the exchange rate of the emitter country, if it is the case;
- Price of goods and services in Portugal weighted by competition from other countries of destination;
- Overnights spent in the previous year,
- Time (trend effect).

For each of these variables the following explaining factors are listed:

a)   Purchasing power of the currency of the emitters ($PCME$);

This variable relies on the assumption that tourists are sensitive to changes in exchange rates and price increases in tourist destinations, compared with prices at the place of residence or in the emitter country. Its calculation is derived from the knowledge of inflation rates in the country of destination adjusted by the existing ones in the country of origin. This will deflate the exchange rates, showing how much the holiday in a tourist destination costs in real terms, compared with its cost at the place of residence.

b)   Non-essential consumption of households in the emitters ($CPNE$)

For establishing the values of this variable was considered as non-essential consumption the one that does not include expenditure on (statistical aggregates):

- Food, beverages and tobacco;
- Clothing and Footwear;
- Housing;
- Medicines and medical care.

These items appear in the National Accounts as part of final consumption of households on the economic territory.
This represents the hypothesis that the families will travel in tourism after the guaranteed expenditure on goods and basic services.

c)   The price of accommodation in Portugal adjusted by the exchange rate of the emitter country ($PHTC$)

This variable is considered as associated with the average price of "packages" sold in the UK, weighted by the exchange rate of each emitter country (if considering just the main non-euro emitter country).

    d) Price of goods and services (IPC) in Portugal weighted by the prices of goods and services (IPC) of competing countries.

The purpose of this variable is not only to measure the relationship between the prices of goods and services between sender and receiver country weighted by their exchange rate, but also to include the competitiveness of other destinations comparatively to Portugal, via weights which result from the market share for these countries (destinations) to the main flows of tourists travelling to Portugal.

    e) Overnight stays in the previous year (D_{-1})

The purpose of a visit that took place recently, either to repeat or to inform others, influences the flow of tourists away.

    f) Trend effect (T)

Broadening the scope of the previous variable, this effect has structural characteristics that, in tourism, in many situations can' not be neglected (habits, knowledge to be extended or the pleasure of seeing landscapes and climates can be explained by the trend).

Mathematically tourism demand for the emitter country can be expressed as follows:

$$DEST = D_1 + D_2 = f(CPNE, PCME, PHTC, PR, DEST(-1), T, DU)$$

Being:

$DEST$ - a measure of demand for Portuguese tourism services, here represented by the number of overnight stays of foreigners (for the emitter country) annually considered;

D_1 - Foreigners in group;

D_2 - Foreigners individually;

$PCME$ - Purchasing power of the currency of the emitters;

CPNE - Non-essential consumption of households in the emitters;

PHTC - The price of accommodation in Portugal adjusted by the exchange rate of the emitter country;

PR - The price of goods and services in Portugal weighted by the prices of goods and services by major competitors in tourism;

DEST(−1) - A measure of demand: represented by the number of overnight stays of foreigners in the previous period;

T - The tendency or effect "trend-time";

DU - The "DUMMY" variable.

It is expected that variables *PCME*, *CPNE*, *DEST*(−1) and *T* are directly related with dependent variable *DEST*, this is $\frac{\partial(DEST)}{\partial(PCME)} > 0$, $\frac{\partial(DEST)}{\partial(CPNE)} > 0$, $\frac{\partial(DEST)}{\partial(DEST(-1))} > 0$ and $\frac{\partial(DEST)}{\partial(T)} > 0$. Otherwise, it is expected that variables *PHTC* and *PR* are inversely related to dependent variable *DEST*, as follows: $\frac{\partial(DEST)}{\partial(PHTC)} < 0$ and $\frac{\partial(DEST)}{\partial(PR)} < 0$.

To estimate the impact of changes in the variables in tourism demand, the coefficients associated with this impact need to be estimated, and the previous equation be expressed in the following way:

$$DEST = a_0 CPNE^{a_1} PCME^{a_2} PHTC^{a_3} PR^{a_4} [DEST(-1)]^{a_5} T^{a_6} E$$

Using the method of least squares regression, and after a logarithm, this equation can be estimated as presented below, introducing DUMMY variable (just now for reasons of mathematical operating of the model):

$$\ln DEST = \ln a_o + a_1 \ln CPNE + a_2 \ln PCME + a_3 \ln PHTC + a_4 \ln PR + a_5 \ln DEST(-1) + a_6 \ln T + \lambda DU + \ln E$$

So it is allowed the coefficients to be interpreted as the elasticity of the dependent variable for each of the explanatory variables.

Demand Global Function for Foreign Visitors

The explanatory variables in global demand for foreign visitors presented in this model are:

- Private consumption in EU countries;

- Housing Prices in Euro;
- Purchasing power of the Euro (once United Kingdom is an important issuing country - one of the five most important - it is used the relationship Euro *versus* British Pound); and just considered the exchange rate Euro/Pound).

The approach made to the variables used in the demand function by the issuing country has some limitations, which are related to the failure to achieve overall series for those variables and not considering appropriate to proceed to a mere sum of the grades obtained for each issuing country.

In its algebraic presentation, this function comes with the following form:

$$DEG = f(CP, PHEURO, PCEURO)$$

where

DEG -Measure of demand for Portuguese tourism services, represented by the number of overnight stays of foreigners;

CP - Private consumption in EU countries.

$PHEURO$ - Housing price in euros, running the euro as reference currency for all emitting countries (excluding UK: it will be considered its exchange rate with the pound sterling).

$PCEURO$ - The purchasing power of the Euro in relation to the pound.

It will be used the same method as in the previous equation.

Demand Function for National Tourists

To explain the nights spent by Portuguese Tourists in national hotels (D_p), the variable private consumption (CP) and trend (T) are considered as the major determinants.

Algebraically this function will be:

$$D_p = f(CP, T)$$

It will be used the same method of the previous functions.

Supply

Function of Tourism Production

The tourism production consists of the nights spent, considering the maximum capacity of companies. The potential production will result from considering the effective overnight bed occupancy.

This potential production is correlated with investment which changes will be reflected in the stock of capital, and with staff admitted in a situation of rigidity.

Moreover, it is admitted the existence of constant returns on scale, with a Cobb-Douglas function, and it will be established a relationship between the average productivity of labour and the coefficient of capitalistic intensity, emerging the capital as an explanatory variable.

Thus

$$X = f(L, K)$$

where

X - The potential tourism production that is represented by tourists' overnights;

L - The labour factor, given by the average of workers in service during the year;

K - The effective capital stock.

So, not forgetting the trend,

$$X = AL^{1-a} K^a e^{T^t}$$

or

$$\frac{X}{L} = A(\frac{K}{L})^a e^{T^t}$$

after the application of logarithms, it comes

$$\ln(\frac{K}{L}) = \ln A + a \ln(\frac{K}{L}) + T^t$$

In this model, tourism production can be compared to the tourism demand.

Prices
It is considered that the price will influence demand, through supply side.

At first, we have the price of the "package" offered by tour operator (linked to exogenous variables such as inflation in the emitters) to establish the international demand for tour "packages." This price also influences the housing supply.

In turn, the hosting company submit a higher price (the price of "counter") for taking up excess capacity. For the definition of this price, this company will carry out a costs analysis.

An estimate of the cost-function in the hosting company will assume the dependent variable (variable costs) will be explained by overnights and by the prices of goods and services purchased by this company (intermediate consumption).

Formalization

The Demand

Be

$$D = D_1 + D_2 + D_3$$

where

D - The total number of overnight stays in housing companies;

D_1 - The number of nights spent by foreigners, in group;

D_2 - The number of nights spent by foreigners, individually;

D_3 - The number of nights spent by nationals;

$D_1 + D_2$ - The total number of nights spent by foreigners in the accommodation companies.

$$D_1 + D_2 = DEST = f(Y_1, Y_2, Y_3, Y_4, Y_5, Y_6)$$

and

Y_1 - Nonessential consumption of foreign tourists

Y_2 - Purchasing power of the currency of the country of residence of foreign tourist

Y_3 - Price of the "package" for Portugal weighted by the exchange rate

Y_4 - Price of goods and services in Portugal weighted by the price of goods and services from competing countries of Portugal

Y_5 - Nights of the previous year

Y_6 - Trend

$$D_3 = f(Y_7)$$

Being

Y_7 the private consumption in Portugal.

Supply

Be

$$X = f(L, K)$$

In which

L - The number of people working in hosting companies;

K - The fixed capital stock for hosting companies.

and

$$C = f(D_1 + D_2 + D_3)$$

Being C the variable hosting companies total costs.

The Market and the Determination of Prices

Tour Operator and Its Utility Maximization (Profit)

Be

$$L = p_v q_1(p_v) - p_c q_2(p_c)$$

In which

L - The profit of the foreigner tour operator

p_v - The selling price of touristic product (D) to the tourist, sold by tour operator;

p_c - The buying price of the touristic product (D) by the tour operator to the hosting company;

q_1 - The touristic product (D) sold by the tour operator to the foreigner tourists – the quantities are measured in terms of overnights;

q_2 - The touristic product (D) bought by foreigner tour operator to the hosting company.

For the maximization of profit the following model is considered:

$$MaxL(p_v, p_c) = p_v q_1(p_v) - p_c q_2(p_c)$$
Sub. To

$$q_1(p_v) - q_2(p_c) \leq 0,$$

$$p_v, p_c \geq 0.$$

The Lagrange function is

$$Z(p_v, p_c, s, \lambda) = p_v q_1(p_v) - p_c q_2(p_c) + \lambda(-q_1(p_v) + q_2(p_c) - s)$$

Call s an auxiliary variable and λ the Lagrange multiplier.

After Kuhn-Tucker conditions, with $\lambda = 0$:

$$q_1 = A\frac{1}{p_v}, A \in R$$

$$q_2 = B\frac{1}{p_c}, B \in R$$

Once

$$q_1(p_v) + p_v \frac{\partial q_1}{\partial p_v} = 0 \,,$$

$$q_2(p_c) + p_c \frac{\partial q_{12}}{\partial p_c} = 0 \text{ and}$$

$-q_1(p_v) + q_2(p_c) - s = 0$, being $s \geq 0$ the auxiliary variable.

So,

$$-A\frac{1}{p_v} + B\frac{1}{p_c} - s = 0 \text{ and so}$$

$$-A\frac{1}{p_v} + B\frac{1}{p_c} \geq 0 \,.$$

Finally,

$$A\frac{1}{p_v} \leq B\frac{1}{p_c} \text{ or } p_c \leq \frac{B}{A}p_v \,.$$

Let $\frac{B}{A} = D$, considering $0 < D < 1$,

$p_c = Dp_v$ and consequently

$p_c < p_v$

Now, if $s = 0$ (the case in which the restriction equals 0),

$$Z(p_v, p_c, \lambda) = p_v q_1(p_v) - p_c q_2(p_c) - \lambda(q_1(p_v) - q_2(p_c))$$

After first order conditions,

$$\lambda = \frac{q_1(p_v)}{\frac{\partial q_1}{\partial p_v}} + p_v$$

$$\lambda = \frac{q_2(p_c)}{\frac{\partial q_2}{\partial p_c}} + p_c$$

and so,

$$\frac{q_1(p_v)}{\frac{\partial q_1}{\partial p_v}} + p_v = \frac{q_2(p_c)}{\frac{\partial q_2}{\partial p_c}} + p_c,$$

Resulting, after some operations,

$$\frac{p_v}{p_c} = \frac{1 + \frac{1}{e_{q_2}}}{1 + \frac{1}{e_{q_1}}}$$

If p_v is considered constant, p_c will depend on the demand functions.

With the conditions of saturation $(q_1 = q_2)$, and $p_v > 0$ and $p_c > 0$, the respective elasticity will be studied through

$$\frac{p_v}{p_c} = \frac{1 + \frac{1}{e_{q_2}}}{1 + \frac{1}{e_{q_1}}} > 0$$

Five scenarios are possible:

- $e_{q_2} < -1$ and $e_{q_1} < -1$

- $e_{q_2} < -1$ and $e_{q_1} > 0$
- $e_{q_2} > 0$ and $e_{q_1} < -1$
- $e_{q_2} > 0$ and $e_{q_1} > 0$
- $-1 < e_{q_2} < 0$ and $-1 < e_{q_1} < 0$

Considering the first scenario $(e_{q_2} < -1$ and $e_{q_1} < -1)$, $p_c < p_v$ and conditions of saturation $q_1 = q_2$,

$$\frac{\partial q_1}{\partial p_v} > \frac{\partial q_2}{\partial p_c}.$$

For the model purposes, this result shows a bigger variation rate in the market of products (overnights, bought by tourists to the tour operators after prices' changes) than in the market of factors (overnights bought by tour operators to the hosting houses).

An example can be found when $q_1 = \frac{A}{p_v} + B$. On this situation there is an inverse relationship between price (p_v) and overnights (q_1) but also an important representative variable of, for instance, the fidelity rate in relation to a destination region.

And now for e_{q_1} (the same procedure for e_{q_2}):

$$\frac{\partial q_1}{\partial p_v}\frac{p_v}{q_1} = -\frac{A}{(p_v)^2}\cdot\frac{p_v}{\dfrac{A}{p_v}+B} = -\frac{A}{A+p_vB} < -1.$$

Considering a lesser elasticity for overnights demanded by tourists and values for each elasticity, for example:

$$e_{q_1} = -1.2 \text{ and } e_{q_2} = -1.7,$$

$$\frac{p_v}{p_c} = \frac{1+\dfrac{1}{e_{q_2}}}{1+\dfrac{1}{e_{q_1}}} = \frac{0.41}{0.17} \cong 2.41$$

This means that there is a relationship between prices of about 2.41 independently of the value of each one.

Hosting Company and Its Utility Maximization (Profit)

After fixing p_v by tour operator and being p_c determined, it is now just necessary that hosting company use this price to the maximization of its profit. So,

$$\Pi = \overline{P_1}\,\overline{D_1} + P_2 D_2 + \overline{P_3}\,\overline{D_3} - C(\overline{D_1} + D_2 + \overline{D_3}).$$

Being

$$P_3 = P_2 \text{ and } \overline{D_1} + D_2 = f(P_2), \text{ and so } D_2 = f(P_2) - \overline{D_1}, \text{ in which}$$

P_1 = Price of foreigners' overnights, in group. It is given to the company, once it is determined by tour operator.

P_2 = Price of foreigners' overnights, individual

P_3 = Price of nationals' overnights

So,

$$\Pi = P_2(f(P_2) - \overline{D_1}) + \overline{P_3}\,\overline{D_3} + \overline{P_1}\,\overline{D_1} - C(\overline{D_1} + D_2 + \overline{D_3})$$

The hosting company will maximize its profits and deplete its offer capacity, having by restriction the installed capacity.

$$Max\Pi = P_2(f(P_2) - \overline{D_1} + \overline{D_3}) + \overline{P_1}\,\overline{D_1} - C(f(P_2) + \overline{D_3})$$

Subj. to

$$\overline{D_1} + D_2 + \overline{D_3} = \overline{X}$$

(Either $D_2 = \overline{X} - \overline{D_1} - \overline{D_3}$ or $\overline{D_1} + D_2 + \overline{D_3} - \overline{X} = 0$).

Considering the Lagrange function and using conditions for maximization, then

$$P_2 = -\frac{\overline{X}}{\dfrac{\partial f}{\partial P_2}} + \frac{\overline{D_1}}{\dfrac{\partial f}{\partial P_2}} + \frac{\partial C}{\partial D}$$

$$P_2 = -\frac{\overline{X} - \overline{D_1}}{\dfrac{\partial f}{\partial P_2}} + \frac{\partial C}{\partial D}$$

Finally,

$$P_2 = -\frac{1}{\dfrac{\partial f}{\partial P_2}}(\overline{X} - \overline{D_1}) + \frac{\partial C}{\partial D} \text{ , being } -\frac{1}{\dfrac{\partial f}{\partial P_2}} = K \text{ , } K > 0.$$

This means that a hypothesis for resolution can be presented having in account a first order linear differential equation with constant coefficients (see Ferreira and Amaral 1988)[1].

Besides,

Either

$$P_2 = -\frac{\partial P_2}{\partial D_2}(\overline{X} - \overline{D_1}) + \frac{\partial C}{\partial D}$$

or

$$\frac{\partial P_2}{\partial D_2}(\overline{X} - \overline{D_1}) + P_2 = C'$$

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[1] The solution is not presented in this chapter, because it is very difficult to interpret it in the considered practical situation.

Being

$$\frac{\partial C}{\partial D} = C'.$$

Solving the homogeneous equation $\frac{\partial P_2}{\partial D_2}(\overline{X} - \overline{D_1}) + P_2 = 0$, which solution can give us some clues about the stability of the complete solution, it is obtained:

$$P_2 = Ce^{\frac{D_2}{(\overline{X} - \overline{D_1})}}$$

A particular solution is:

$P_2 = K$. With $K = C'$ will be

$P_2 = C'$.

The general solution:

$$P_2 = \overline{P_2} + P_2^* = Ce^{\frac{D_2}{(\overline{X} - \overline{D_1})}} + C'$$

and

$\overline{X} = \overline{D_1} + D_2 + \overline{D_3} + D^*$, and D^* is the non-used capacity

If $D^* = 0$ and $\overline{D_3} = 0$, it comes $\dfrac{D_2}{(\overline{X} - \overline{D_1})} = 1$

If $\overline{D_3} \neq 0$, $\dfrac{D_2}{(\overline{X} - \overline{D_1})} < 1$

If $D^* \neq 0 \wedge \overline{D_3} \neq 0$, it comes $\dfrac{D_2}{(\overline{X} - \overline{D_1})} < 1$

So, we have $0 < \dfrac{D_2}{(\overline{X} - \overline{D_1})} \leq 1$

CONCLUSION

Along this study a model for Portuguese tourism market was presented. Factors explaining demand, supply and prices were discussed. At macroeconomic level, it may be seen how these factors contribute to model the Portuguese tourism market. A relationship among the variables is analyzed and its modelling is represented mathematically. The model allows to conclude about the contribution of this kind of model to show the importance of these variables and relationships to the determination of macroeconomic aggregates in the Portuguese tourism market. The study of the influence of relevant variables and their relationships appears as a crucial investigation on the perception and improvement of the Portuguese tourism market operators. This study provides a model that allows to work the quantitative data and obtain the economic impacts of agents in the tourism sector. The mathematical formulation allows also to study tour operators market (market's main agents). It also permits to analyze quantitative economic impacts of hosting companies' total rents (which are the main supportive infrastructures).

The proposal of this model is workable by using the total rents of hosting companies, aiming yet to analyze the quantitative economic impact of these rents in the Portuguese National Product.

Such a model like the one developed represents a tool for tourism companies to improve their results and the services they provide.

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