Quantifying the Influence of Crosstalk-Crosstalk Beat Noise in Optical DPSK Systems

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***Abstract*— In-band crosstalk, due to multiple interferers, is one of the most severe physical impairments in optical transparent networks. Differential phase-shift keying (DPSK) has been identified as an attractive modulation scheme to be used in such environments due to its robustness to in-band crosstalk. At the output of the receiver photodiode, which is assumed to behave as a square law device, the in-band crosstalk interferes with the signal, resulting in the signal-crosstalk beat noise and the crosstalk-crosstalk beat noise. Usually this last noise contribution is neglected, but in this paper the impact of the crosstalk-crosstalk beating terms is considered and quantified. It is concluded that these terms have a growing influence as the optical signal-to-noise ratio (OSNR) value increases.**

Keywords- crosstalk; DPSK; receiver imperfections

I. INTRODUCTION

In-band crosstalk has been considered over the years as one of the most important physical impairments in the design of wavelength division multiplexing (WDM) networks [1]. Typically, this phenomena consists on multiple interfering signals that have the same nominal wavelength as the selected signal, and arises, mostly, due to imperfections of optical devices used to build networks elements, such as optical cross-connects and optical add-drop multiplexers (OADMs). At the receiver these crosstalk signals interfere with the desire signal originating the signal-crosstalk beat noise and also the crosstalk-crosstalk beat noise that can not be removed by filtering. In this way, in-band crosstalk becomes a serious source of system performance degradation.

In-band crosstalk has been analyzed with great persistence in the context of WDM networks based on on-off keying (OOK) modulation format, see [2] and references therein. However, in the recent years, some studies of this phenomena comprising other modulation formats, such as the differential phase-shift keying (DPSK) [3]-[6], have been also realized. In what concerns the DPSK format an improved tolerance to in-band crosstalk in relation to the traditional OOK has been obtained for the single interferer scenario [3]. The multiple interferer case has been also studied and it is concluded that the crosstalk tolerance is reduced when the number of interferers increases [4].

Common to almost all the studies that deal with in-band crosstalk is the assumption that the influence of the crosstalk-crosstalk beating terms can be neglected [2]. Nevertheless, there are some works in the OOK context that consider these terms, [7], [8]. In particular, in [8], it is concluded that these terms introduce some power penalty. In the context of DPSK systems [4] and [6] also consider the influence of the crosstalk-crosstalk beating terms, but neither studies give a quantitative analysis of the impact of these beating terms.

The purpose of this paper is to study the impact of the crosstalk-crosstalk beating terms in a direct detection DPSK system impaired by in-band crosstalk. In order to analyze this subject the formulation developed in [4] was used. This formulation permits to estimate the performance of optically pre-amplified DPSK receivers in the presence of in-band crosstalk due to an arbitrary number of interferers, considering the case of arbitrary optical and electrical filtering. The formulation uses an eigenfunction expansion technique to decompose the signal, the interference, and the amplified spontaneous emission (ASE) noise, at the optical filter input, in terms of a series of orthogonal functions and relies on the moment generating function (MGF) to describe the statistics of the decision variable.

The remainder of this paper is structured as follows. In Section II, we describe the model used to characterize the decision variable at the DPSK receiver output. In Section III, the MGF of the decision variable is derived. Numerical results showing the influence of the crosstalk-crosstalk beat noise are given in Section IV and some concluding remarks are provided in Section V.

II. EIGENFUNCTION EXPANSION OF THE DECISION VARIABLE

We consider a typical direct detection DPSK receiver using balanced detection, composed by an optical pre-amplifier with gain G, an optical filter, a delay interferometer with a differential delay equal to the bit period T, a balanced photodetector, and a post-detection electrical filter. The optical filter is assumed to have an arbitrary low-pass equivalent impulse response $h_o(t)$ and an optical bandwidth B_o, whereas the electrical filter is described by the impulse response $h_e(t)$ and by the electrical bandwidth B_e.

At the receiver input the incoming DPSK signal is impaired by in-band crosstalk due to N DPSK interferers, originated from N different sources with the same bit rate and nominal wavelength as the desired signal. The ASE noise originated from the optical pre-amplifier will also impair the signal. This noise is considered to be a zero mean white stationary Gaussian noise with a single-sided power spectral density in each polarization given by $N_o = h\nu_s(G-1)F/2$, where $h\nu_s$ is the photon energy at the signal wavelength, and F is the noise factor. The electrical field of the ASE noise at the optical filter output can be expressed in terms of in-phase $n_c(t)$, and quadrature $n_s(t)$ components, giving for the case of the ASE noise having the same polarization as the signal $\{[n_c(t)+jn_s(t)]\vec{r}\}*h_o(t)$, where $*$ denotes convolution and $\vec{r}$ the polarization unit vector. The complex envelope of the signal field and of the i-th interfering signal at the optical filter output during the interval $[0,T]$ can be represented, respectively, as

$$\vec{E}_s(t) = \{\sqrt{2GP_s}u(t)\exp[j\theta_s(t)]\vec{r}\} * h_o(t) \tag{1}$$

and

$$\vec{E}_{x,i}(t) = \{\sqrt{2GP_{x,i}}u(t)\exp[j\theta_{x,i}(t)+j\phi_{x,i}]\vec{r}\} * h_o(t). \tag{2}$$

In the above equations, P_s and $P_{x,i}$ are, respectively, the average signal power and the average crosstalk power incident at the amplifier input, $u(t)$ a rectangular pulse of unitary amplitude within the interval $[0,T]$ and zero elsewhere, $\phi_{x,i}$ is a random phase, and $\theta_s(t)$ and $\theta_{x,i}(t)$ are, respectively, the signal and crosstalk phases. These phases are given by $\theta_{s(x,i)}(t) = \theta_{s(x,i)}(t-T) + \pi(1-a_{s(x,i)})/2$, where $\theta_{s(x,i)}(t-T)$ is the phase in the previous time interval, and $a_{s(x,i)} = 1$ for symbol "one" and $a_{s(x,i)} = -1$ for the symbol "zero". Throughout this paper it is considered a worst case interference scenario reflected in the fact that all the interfering signals are assumed to be co-polarized and temporally aligned with the desired signal.

Assuming that $\vec{E}(t)$ is the field at the interferometer input that includes the signal, crosstalk and ASE noise, the electrical fields at the interferometer outputs are $(1/2)[\vec{E}(t)+\vec{E}(t-T)]$ for the constructive port and $(1/2)[\vec{E}(t)-\vec{E}(t-T)]$ for the destructive port. These fields are detected using a pair of identical photodiodes with unitary responsivities and the resulting currents are subtracted and filtered by the electrical filter. The decision variable v at the electrical filter output, defined at the decision time t_d, can then be written as

$$v(t_d) = v^+(t_d) - v^-(t_d) \tag{3}$$

where the random variable $v^+(t_d)$ results from the constructive port, and the random variable $v^-(t_d)$ from the destructive port. Applying the formalism developed in [4], that permits to write the random variables v^+ and v^- as a sum of independent random variables, and taken into account the ASE noise in the polarization orthogonal to the signal, v^+ and v^- can be written in the following form:

$$v^+ = \tfrac{1}{2}\sum_{k=0}^{\infty}\lambda_k[(y_{1c,k}+n_{1,k})^2 + (y_{1s,k}+n_{2,k})^2 + q_{1,k}^2 + q_{2,k}^2] \tag{4}$$

and

$$v^- = \tfrac{1}{2}\sum_{k=0}^{\infty}\lambda_k[(y_{2c,k}+n_{3,k})^2 + (y_{2s,k}+n_{4,k})^2 + q_{3,k}^2 + q_{4,k}^2] \tag{5}$$

where

$$y_{1c(2c),k} = \sqrt{2GP_s\alpha_s^{\pm}}u_k + \sum_{i=1}^{N}\sqrt{2GP_{x,i}\alpha_{x,i}^{\pm}}u_k\cos\Delta\theta_i \tag{6}$$

$$y_{1s(2s),k} = \sum_{i=1}^{N}\sqrt{2GP_{x,i}\alpha_{x,i}^{\pm}}u_k\sin\Delta\theta_i \tag{7}$$

$$n_{l,k} = \int_{-\infty}^{+\infty} n_l(t_d-\tau)\varphi_k(\tau)d\tau, \text{ for } l = 1,..,4 \tag{8}$$

with

$$u_k = \int_{-\infty}^{+\infty} u(t_d-\tau)\varphi_k(\tau)d\tau = \int_{-T+t_d}^{t_d}\varphi_k(\tau)d\tau, \tag{9}$$

$\alpha_s^{\pm} = (1\pm a_s)^2/4$, $\alpha_{x,i}^{\pm} = (1\pm a_{x,i})^2/4$ and the random process $\Delta\theta_i(t)$ represents the difference between the accumulated phases of the i-th interferer and the signal, and includes also the phase $\phi_{x,i}$. The noise terms $q_{l,k}$, for $l \in \{1,..,4\}$, presented in (4) and (5) are statistically distributed as the terms $n_{l,k}$, for $l \in \{1,..,4\}$, and are independent of them. In (8), the random coefficients $\{n_{l,k}\}$ are mutually independent zero-mean Gaussian random variables, with variances $< n_{l,k}^2 >= N_0/2$, for $l \in \{1,..,4\}$, and $n_{1(3)}(t) = (1/2)[n_c(t) \pm n_c(t-T)]$ and $n_{2(4)}(t) = (1/2)[n_s(t) \pm n_s(t-T)]$. In (4) and (5), λ_k is the kth eigenvalue, and in (8) and (9), $\varphi_k(t)$ is the corresponding eigenfunction of the integral equation [9]

$$\int_{-\infty}^{+\infty}\Lambda(t,\tau)\varphi_k(\tau)d\tau = \lambda_k\varphi_k(t) \tag{10}$$

where $\Lambda(t,\tau) = \int_{-\infty}^{+\infty} h_e(\varsigma)h_o(t-\varsigma)h_o(\tau-\varsigma)d\varsigma$.

III. MOMENT GENERATING FUNCTION DERIVATION

The statistics of the random variable v are described here using the MGF, which is defined as $M_v(s) =< e^{sv} >$. This function is computed by deriving, in the first place, the conditional MGF of v for a given realization of $\Delta\theta = [\Delta\theta_1, \Delta\theta_2, ..., \Delta\theta_N]$, denoted as $M_{v|\Delta\theta}(s)$, and then averaging over all the possible values of $\Delta\theta$. As a consequence, the unconditional MGF of v is given by [4]

$$M_v(s) = M_{v^+}(s)M_{v^-}(-s) \tag{11}$$

where

$$M_{v^{+(-)}}(s) = \frac{1}{\prod_{k=0}^{\infty}[1-s\lambda_k N_0/2]^2} M_{y_{1(2)}}\left[\sum_{k=0}^{\infty}\frac{s\lambda_k T\xi_k}{(1-s\lambda_k N_0/2)}\right] \tag{12}$$

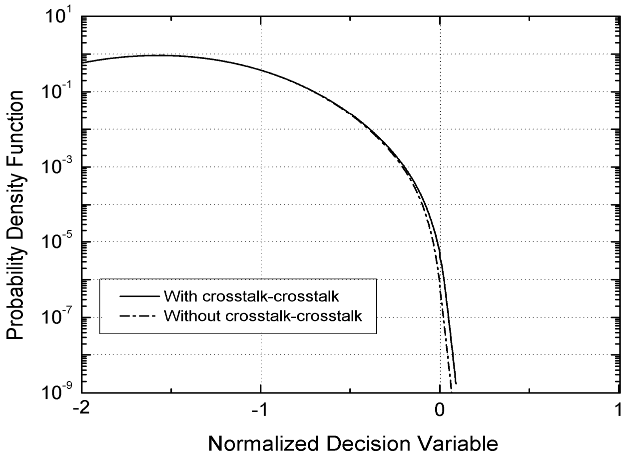
with

$$\begin{aligned} M_{y_{1(2)}}(s) &= \exp(sGP_s\alpha_s^{\pm}) \\ &\times \prod_{i=1}^{N}\exp(sGP_s\varepsilon_i\alpha_{x,i}^{\pm})I_0(s2GP_s\sqrt{\varepsilon_i\alpha_s^{\pm}\alpha_{x,i}^{\pm}}), \\ &\times \prod_{j=1}^{N-1}\prod_{i=j+1}^{N} I_0(s2GP_s\sqrt{\varepsilon_j\varepsilon_i\alpha_{x,j}^{\pm}\alpha_{x,i}^{\pm}}) \end{aligned} \tag{13}$$

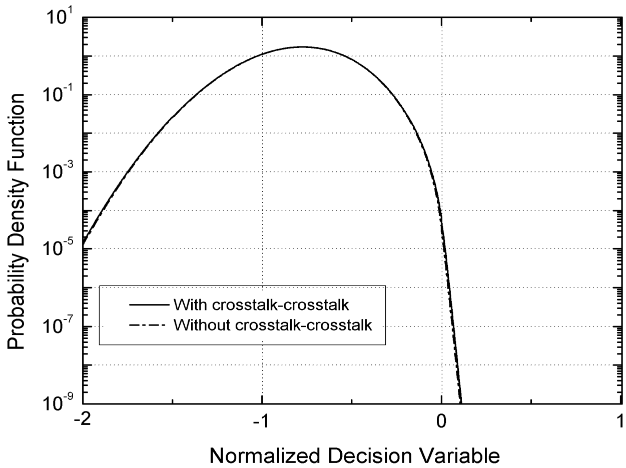
$I_0(.)$ denotes the modified Bessel function of the first kind of order zero, $\xi_k = u_k^2/T$, and ε_i is the crosstalk level of the i-th interferer defined as the ratio between the crosstalk power and the signal power ($\varepsilon_i = P_{x,i} / P_s$), whereas the total crosstalk level is given by $\varepsilon_T = \sum_{i=1}^{N} \varepsilon_i$. To simplify the calculation of $M_{y_{1(2)}}(s)$ we assume that the terms of the crosstalk-crosstalk contribution are mutually independent and uncorrelated from the terms of the signal-crosstalk contribution [4].

IV. RESULTS AND DISCUSSION

In order to quantify the impact of the crosstalk-crosstalk beating terms in optical direct detection DPSK systems the formulation developed previously is applied. In this way, this section presents some numerical results for the Gaussian receiver configuration, assuming a bit rate of 10 Gb/s and a Gaussian electrical filter with a 3-dB bandwidth such that $B_e T = 0.7$ [4]. Furthermore, the analysis takes into account the impact of the ASE noise from both polarizations. This noise is due to the optical pre-amplifier, which is characterized by $G = 30\,\text{dB}$ and $F = 5\,\text{dB}$.



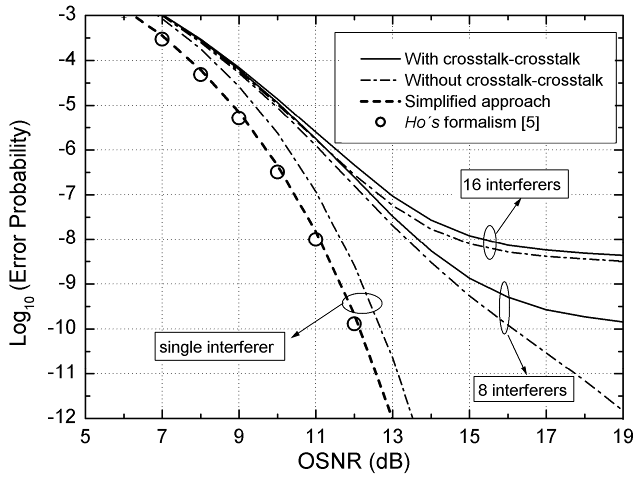
(a)



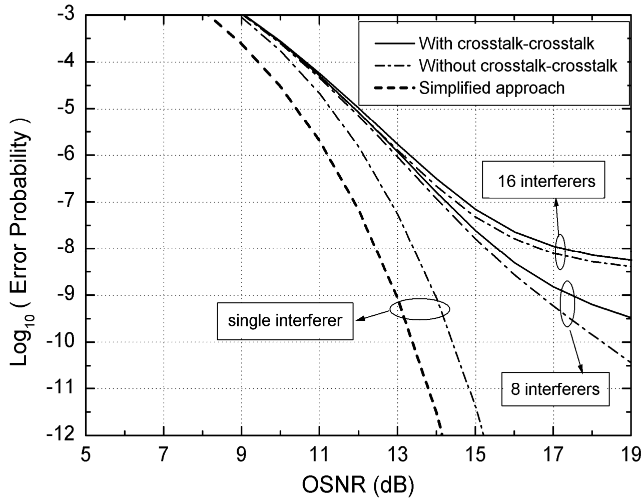
(b)

Figure 1. Probability density function of the normalized decision variable for symbol "zero", considering 8 interferers and the parameters: $B_o T = 5$ and $\varepsilon_T = -15\,\text{dB}$. (a) $P_s = -37\,\text{dBm}$; b) $P_s = -40\,\text{dBm}$.

We start our discussion with the evaluation of the PDF of the decision variable, since the precise knowledge of this function is crucial to gain insight into the influence of the crosstalk-crosstalk beating terms. This PDF is evaluated by using the inverse Laplace transform of the respective MGF given by (11). In these evaluations, it is assumed that the desired signal and the interferers are in the same symbol state ($a_s = a_{x,i}$, with $i \in \{1,..,N\}$). Fig. 1 shows the PDFs of the decision variable (normalized with respect to the signal power P_s) for symbol "zero" considering the case where the crosstalk-crosstalk beating terms are neglected and the case where these terms are included. It is assumed a total crosstalk level of -15 dB equally distributed among 8 interferers, and two values of the average optical signal power incident at the pre-amplifier, -37 dBm in Fig. 1(a) and -40 dBm in Fig. 1(b). As can be observed in Fig. 1(b) the presence of the crosstalk-crosstalk beating terms does not introduce any visible difference in the behavior of the PDFs. However, when the average power is increased to -37 dBm, Fig. 1(a), the inner tail of the PDF that include the crosstalk-crosstalk contribution is clearly above the one where these terms are neglected, which suggests that the error probability is increased when the crosstalk-crosstalk beatings terms are included in the analyses.



(a)



(b)

Figure 2. Error probability as a function of the OSNR for a total crosstalk level of -15 dB equally distributed among 8 and 16 interferers. (a) $B_o T = 1$; (b) $B_o T = 5$.

The effect of the crosstalk-crosstalk beating terms on the receiver performance is shown in Figs. 2(a) and 2(b) for $B_oT=1$ and $B_oT=5$, respectively, considering a total crosstalk level of -15 dB equally distributed among 8 and 16 interferers. The single interferer scenario is also represented for clearness. These figures plot the error probability as a function of the optical signal-to-noise ratio (OSNR) which is computed using the saddle point approximation method and assuming a binomial symbol conditioning on the interfering symbols [4]. The OSNR is defined in this work as the ratio of the average signal power before the optical filter to the ASE noise power in both polarizations evaluated using a noise bandwidth equal to the bit rate. As the first observation, it may be noted that the curve for the single interferer in Fig. 2(a), computed with the simplified approximation [4], based on a receiver with an optical filter with a large B_oT product and an integrate-and-dump electrical filter, is in excellent agreement with that obtained using [5, eq. (20)]. However, the corresponding curve obtained for the Gaussian receiver presents an OSNR penalty, at an error probability of 10^{-9}, of 0.6 and 1.1 dB, respectively, for $B_oT=1$ and $B_oT=5$ [4]. In what concerns the multiple interferer scenario, where the crosstalk-crosstalk beating terms can influence the system performance, several other observations can be made. In the first place, it is clear that the inclusion of the crosstalk-crosstalk beating terms give higher error probabilities especially for OSNR values greater than 15 dB, a trend that is already expected from the analysis of the PDF behavior. Second, in both figures the presence of an error floor is observable for both $N=8$ and $N=16$, and this phenomena becomes more noticeable when the crosstalk-crosstalk beating terms are included in the analyses. This floor is due to the in-band crosstalk, as has already been evidenced in [6], and in the case $N=16$ it even prevents to reach an error probability of 10^{-9}. Third, the crosstalk-crosstalk beating terms have more influence for $B_oT=1$ than for $B_oT=5$, which is justified by the fact that as the B_oT product increases the ASE noise becomes more influent on the system performance, and hence the in-band crosstalk becomes less important. The results for $B_oT=10$, not shown in the figures, were computed by the authors and confirm this trend.

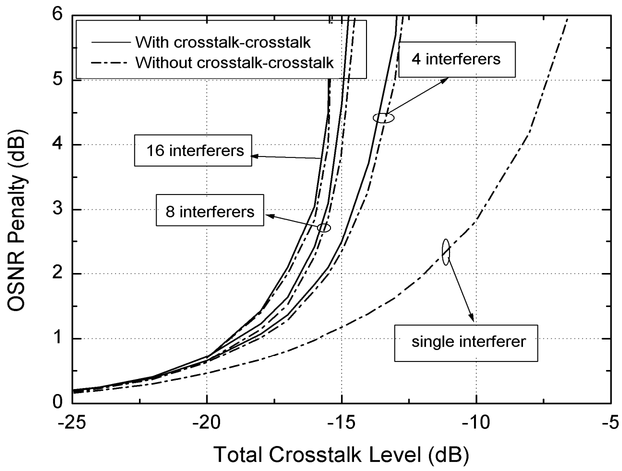


Figure 3. OSNR penalty versus the total crosstalk level for different numbers of interferers. The error probability is fixed at 10^{-9} and $B_oT=5$.

Next, we consider in Fig. 3 the results for the OSNR penalty due to in-band crosstalk as a function of the total crosstalk level ε_T, considering $B_oT=5$, and the two scenarios with respect to the crosstalk-crosstalk beating terms, with and without them. The OSNR penalty is defined in this work as the increment in decibels in the OSNR, required to maintain the error probability fixed at 10^{-9} in the presence of crosstalk. These results show that the crosstalk-crosstalk beating terms have little influence for OSNR penalties below 2 dB. Above this value we note some differences that were already predicted by the error probability computation in Fig. 2.

V. CONCLUSION

The impact of the crosstalk-crosstalk beating terms in an optically pre-amplified DPSK receiver is analyzed. The formulation relies on using an eigenfunction expansion technique and is sufficiently general to deal with the case of arbitrary optical and electrical filtering. Numerical results show that the inclusion of the crosstalk-crosstalk beating terms further increases the error probability, in particular for higher OSNR values and fasten the appearance of error floors, specially, for small B_oT products. However, for OSNR penalties below 2 dB, where the majority of the practical situations lie, the power penalty due to in-band crosstalk is only slightly affected by the inclusion of these terms.

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