	The Annals of Applied Statistics 0, Vol. 0, No. 00, 1–26	
1	https://doi.org/10.1214/23-AOAS1760 © Institute of Mathematical Statistics, 0	
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	A GENERAL FRAMEWORK FOR PENALIZED MIAED-EFFECTS	
	MULITIASK LEARNING WITH APPLICATIONS ON DNA METHYLATION	
	SURROGATE BIOMARKERS CREATION	
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	Recent evidence highlights the usefulness of DNA methylation (DNAm)	
	biomarkers as surrogates for exposure to risk factors for noncommunicable	
	diseases in epidemiological studies and randomized trials. DNAm variability	
	has been demonstrated to be tightly related to lifestyle behavior and expo-	
	sure to environmental risk factors, ultimately providing an unbiased proxy of	
	an individual state of health. At present, the creation of DNAm surrogates	
	relies on univariate penalized regression models, with elastic-net regularizer	
	being the gold standard when accomplishing the task. Nonetheless, more ad-	
	comes with a structured dependence pattern among the study samples. In this	
	work we propose a general framework for mixed-effects multitask learning	
	in presence of high-dimensional predictors to develop a multivariate DNAm	
	biomarker from a multicenter study. A penalized estimation scheme, based	
	on an expectation-maximization algorithm, is devised in which any penalty	
	criteria for fixed-effects models can be conveniently incorporated in the fit-	
	ting process. We apply the proposed methodology to create novel DNAm	
	surrogate biomarkers for multiple correlated risk factors for cardiovascular	
	diseases and comorbidities. We show that the proposed approach, modeling	
	multiple outcomes together, outperforms state-of-the-art alternatives both in	
	predictive power and biomolecular interpretation of the results.	
	1. Introduction. DNA methylation (DNAm) is an epigenetic process that regulates gene	
	expression, typically occurring in cytosine within CpG sites (CpGs) in the DNA sequence	
1	(Singal and Ginder (1999)). DNAm regulates gene expression in different manners. Specifi-	
	cally, high DNAm has been observed in bodies of highly transcribed genes, whereas DNAm	
	in gene promoters and first introns typically have an inverse correlation with gene expression	
	(Anastasiadi, Esteve-Codina and Piferrer (2018), Rauluseviciute, Drabløs and Rye (2020)).	
	Also, recent studies suggest that the relationship between genetic variation. DNAm and gene	
	expression is complex and tissue-specific highlighting that DNAm in non-CnG island re-	
	gions regulates the transcription of distal genes (van Fijk et al. (2012)). Advanced technol-	
	gives regulates the transcription of distal genes (van Eijk et al. (2012)). Advanced termor	
	ogy anows measuring whole-genome DIVAm for many samples at the same time. The most	
	continion ways for DIVAII measurements consist of whole-genome discipline sequencing	
	and DNAm microarray. The first commercial high-density microarray measuring genome-	
ľ	wide methylation was the HumanMethylation27 (27K CpGs) released by Illumina in 2009,	
	followed by the HumanMethylation450 (450K CpGs) and, more recently, by the Illumi-	
	naMethylation850 (850K CpGs, Campagna et al. (2021)). Since then, a tremendous amount	
	of associations between DNAm at individual CpG sites and different exposures, traits and	
	diseases have been identified in the so-called epigenome-wide association studies (EWAS,	
ŀ	Battram et al. (2022)). Concurrently, the development of surrogate scores, based on blood	
ŀ	Received April 2022: revised February 2023	
	<i>Key words and phrases.</i> Mixed-effects models, multitask learning. EM algorithm, penalized estimation, mul-	

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DNA methylation, has also received thriving attention in recent years: impressive epidemi-ological evidence has been established between DNAm and individual history of exposure to lifestyle and environmental risk factors (Zhong, Agha and Baccarelli (2016), Guida et al. (2015), Fiorito et al. (2018)). To this extent, multi-CpG DNAm biomarkers have been devised to predict patient-specific state of health indicators; and relevant examples include epigenetic clocks to measure "biological age" (Lu et al. (2019)), smoking habits (Guida et al. (2015)) and proxies for inflammatory proteins (Stevenson et al. (2020)). Remarkably, DNAm based scores have been demonstrated to outperform surveyed exposure measurements when pre-dicting diseases (Zhang et al. (2016), Conole et al. (2020)). A possible explanation for this somewhat counter-intuitive behavior being that DNA methylation intrinsically accounts for biases in self-reported exposure (e.g., underestimation of smoked cigarettes) as well as in-dividual responses to risk factors (e.g., the same amount of tobacco may produce different effects in dissimilar patients). From a modeling perspective, state-of-the-art methods for DNAm biomarkers creation generally rely on standard univariate penalized regression models, with elastic-net (Zou and Hastie (2005)) being the routinely employed technique when accomplishing the task. Indeed, the associated learning problem entirely falls within the "p bigger than N" framework: DNA methylation levels are measured at approximately a half million CpG sites for each sam-ple, with the dimension of the latter generally not exceeding the order of thousands in most studies. The afore-described procedure is shown to be widely effective in building DNAm biomarkers, with very recent contributions, including surrogate scores for short-term risk of cardiovascular events (Cappozzo et al. (2022)), cumulative lead exposure (Colicino et al. (2021)), DNAm surrogate for alcohol consumption, obesity indexes and blood measured in-flammatory proteins (Hillary and Marioni (2020)), and the identification of CpG sites associ-ated with clinical severity of COVID-19 disease (Castro de Moura et al. (2021)). Nonetheless, elastic-net penalties may be too restrictive when dealing with complex learning problems in-volving multivariate responses and distinctive dependence patterns across statistical units. The afore-said first layer of complexity is encountered when a multidimensional DNAm biomarker needs to be created to jointly model multiple risk factors and to coherently ac-count for the correlation structure among the response variables. Such a multivariate prob-lem, also known as multitask regression in the machine learning literature (Caruana (1997)), can be fruitfully untangled only if dedicated care is devoted in choosing the most appropriate penalty required for the analysis. For instance, one may opt for the incorporation of $\ell 1/\ell 2$ type of regularizers (Obozinski, Taskar and Jordan (2010), Obozinski, Wainwright and Jor-dan (2009), Li, Nan and Zhu (2015)) that extend the lasso (Tibshirani (1996)), group-lasso (Yuan and Lin (2006)) and sparse group-lasso (Laria, Carmen Aguilera-Morillo and Lillo (2019), Simon et al. (2013)) to the multiple response framework. Another option could con-template the inclusion, within the estimation procedure, of prior information related to the association structure among CpG sites: this is effectively achieved by means of graph-based penalties (Li and Li (2010), Kim, Pan and Shen (2013), Cheng et al. (2014), Dirmeier et al. (2018)). Furthermore, tree-based regularization methods have also been recently introduced in the literature to account for hierarchical structure over the responses in a single study (Kim and Xing (2012)) as well as when multiple data sources are at our disposal (Zhao and Zuck-nick (2020), Zhao et al. (2022)). For a thorough and up-to-date survey on the analysis of high-dimensional omics data via structured regularization, we refer the interested reader to Vinga (2021), while the monograph of Hastie, Tibshirani and Wainwright (2015) provides a general introduction to statistical learning with sparsity. A second layer of complexity is introduced when DNA samples and related blood mea-sured biomarkers are collected in a study comprising multiple cohorts. In such a situation, an unknown degree of heterogeneity may be included in the data, with patients coming from

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the same cohort sharing some degree of commonality. Observations in the dataset are thus no	
longer independent, and the cohortwise covariance structure needs to be properly estimated.	
Linear mixed-effects models (LMM) provide a convenient solution to this problem by adding	
a random component to the model specification (see, e.g., Pinheiro and Bates (2006), Gałecki	
and Burzykowski (2013), Demidenko (2013), for an introduction on the topic). While being	
able to capture unobserved heterogeneity, standard mixed models, very much like their fixed	
counterpart, cannot directly handle situations in which the number of predictors exceeds the	
sample size. In order to overcome this issue Schelldorfer, Bühlmann and van de Geer (2011)	
introduced a procedure for estimating high-dimensional LMM via an ℓ_1 -penalization. More	
recently. Rohart, San Cristobal and Laurent (2014) devised a general-purpose ECM algorithm	
(Meng and Rubin (1993)) for solving the same issue but achieving greater flexibility, as the	
proposed framework can be combined with any penalty structure previously developed for	
linear fixed-effects models.	
A multivariate mixed-effects model (MLMM) is an LMM in which multiple characteris-	
tics (response variables) are measured for the statistical units comprising the study. Despite	
being quite a long-established methodology (Reinsel (1984), Shah, Laird and Schoenfeld	
(1997)), its further development has not received much attention in the recent literature. Rel-	
evant exceptions include the computational strategies for handling missing values, proposed	
in Schafer and Yucel (2002), and the estimation theory based on hierarchical likelihood de-	
veloped in Chipperfield and Steel (2012). On this account and to the best of our knowledge,	
a unified approach for penalized MLMM estimation is still missing in the literature, and it	
could thus be a relevant contribution to the statistics and machine learning fields.	
Motivated by the problem of creating a DNAm biomarker for hypertension and hyper-	
lipidemia from a multicenter study, we propose in this article a general framework for high-	
dimensional multitask learning with random effects. Leveraging from the algorithm intro-	
duced in Rohart, San Cristobal and Laurent (2014) for the univariate response case, the esti-	
mation mechanism is effectively constructed to accommodate custom penalty types, building	
upon existing routines developed for regression with fixed-effects only.	
The remainder of the paper is structured as follows. Section 2 describes the EPIC Italy	
dataset, which gave the motivation for the development of the methodology proposed in this	
manuscript. In Section 3 we introduce the penalized mixed-effects model for multitask learn-	
ing, covering its formulation, inference and model selection. Section 4 presents a simulation	
study on synthetic data for three different scenarios. Section 5 outlines the results of the	
novel method applied to the EPIC Italy data for creating DNAm surrogates for cardiovascu-	
lar risk factors and comorbidities, comparing it with state-of-the-art alternatives. Section 6	
concludes the paper with a discussion and directions for future research. The R package	
em1mm implementing the proposed method accompanies the article, and it is freely available	
at https://github.com/AndreaCappozzo/emlmm.	
2. EPIC Italy data and study design. The considered dataset belongs to the Italian	
branch of the European Prospective Investigation into Cancer and Nutrition (EPIC) study,	
one of the largest cohort study in the world, with participants recruited across 10 European	
countries and followed for almost 15 years (Riboli et al. (2002)). For each participant lifestyle	
and personal history questionnaires were recorded, together with anthropomorphic measures	
and blood samples for DNA extraction. The EPIC Italy dataset is comprised of geographical	
subcohorts identified by the center of recruitment; particularly, we will consider the provinces	

of Ragusa and Varese and the cities of Turin and Naples. The latter center became associated with EPIC in later times through the Progetto ATENA study (Panico et al. (1992)). DNAm was measured with the HumanMethylation450 array, following standard laboratory proce-dures (see Fiorito et al. (2022), for a detailed description), while the preprocessing included

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remov	ing CpG sites and samples with a call rate lower than 95%, BMIQ method for reducing
technie	cal variability and bias introduced by type II probes and ComBat technique for batch
effect	adjustment (Marabita et al. (2013)).
Bv	profiting from the information recorded in the aforementioned subcohorts, we aim at
creatin	g a multidimensional DNAm biomarker for cardiovascular risk factors and comor-
biditie	s. To this extent, we consider a multivariate response comprised of $r = 5$ measures
nomely	s. To this extent, we consider a multivariate response comprised of $T = 5$ measures, we disstalic blood pressure (DBP) systelic blood pressure (SBP) high density lipopro-
tain (I	y, diastone blood pressure (DDF), systone blood pressure (SDF), high-density inpopio-
	IDL), low-density inpoprotein (LDL) and triggycendes (TG). These characteristics are
choser	as they represent the major risk factors for cardiovascular diseases (wu et al. (2015)).
In buil	ding a DNAm biomarker, the response variables are regressed on DNA methylation
values	for each CpG site, adjusted for sex and age. A total of $N = 5/4$ individuals in the
J = 4	cohorts showcase nonmissing values for every response variable: they comprise the
sample	e onto which all subsequent analyses will be performed. To reconstruct the process of
DNAn	n surrogates creation and validation, the EPIC Italy data is randomly split into two sets:
70% ($N_{\rm tr} = 401$) of it is employed for preprocessing and model fitting, while the remaining
30% (.	$N_{te} = 173$) acts as test set for assessing prediction accuracy. In addition, we will con-
sider s	amples from the EXPOsOMICS project (Fiorito et al. (2018)) as an external validation
datase	t to assess out of groups predictive performance. In details EXPOSOMICS is a case-
contro	l study on cardiovascular diseases (CVDs) nested in the EPIC Italy cohort, composed
by 276	5 volunteers (not overlapping with the main dataset). whose center of recruitment is
unkno	wn or different from the $J = 4$ observed in the learning phase
Cor	ning back to the data analysis pipeline, an enigenome-wide association study (FW Δ S)
Camp	(2021) is performed on the training set as a pre-screening procedure. In
dataila	log transformed DRD SRD HDL I DL and TG are separately regressed on each
ovoilol	log-transformed DDF, SDF, HDL, LDL and TO are separately regressed on each
	the cpG site, adjusting for sex and age. P-values are then confected and arranged in
increas	sing order. we then screen the set of predictors retaining, for each dimension of the
multiv	ariate response, the CpG sites whose p-values are smaller than the fifth percentile of
the res	sulting empirical distributions. The final set of covariates for the multitask learning
proble	m is achieved by taking the union of the resulting CpG sites separately preserved for
DBP, S	SBP, HDL, LDL and TG. In so doing, out of the whole initial set of 295,614 CpG sites,
62,128	3 DNA methylation features are retained for subsequent modeling. Together with sex
and ag	ge, this amounts to a total of $p = 62,130$ predictors and a <i>five</i> -dimensional response
for a t	raining sample size of $N_{\rm tr} = 401$. While variable screening in ultra-high feature space
is itse	f an ongoing research field (see, e.g., Fan and Lv (2008), Zhong, Wang and Chen
(2021)	, Fan, Samworth and Wu (2009), and references therein), we decided to rely on the
EWAS	technique, as it is the standard approach employed in epigenomics (Fazzari and Gre-
ally (2	010)).
As	previously mentioned, the considered training samples belong to four different centers
distrib	uted across Italy, with data for 91, 234, 44 and 32 volunteers, respectively, collected
in Turi	in. Varese, Ragusa and Naples provinces. The boxplots in Figure 1 emphasize the dif-
ference	es in the five response variables by center. To capture the centerwise variability and
to mai	ratio are response values of content to capture the content vise value intro and and an analytic sector $ratio ratio are are are are are are are are are are$
o norti	al pooling random intercent model shall be adopted. That is, $a = 1$ random effect
a parti	ar pooning random-intercept model shall be adopted. That is, a $q = 1$ random-effect
compc	ment is included in the model specification. Furthermore, the diomarkers comprising
the res	sponse vector snowcase some degree of relations, as displayed by the sample corre-
lation	matrix of Figure 2, so much so that it is sensible to regress them jointly to take ad-
vantag	e of their association structure in the model formulation. This challenging learning
task re	equires an ad hoc specification for a multivariate mixed-effects framework applicable
to high	n-dimensional predictors.

dataset.





FIG. 1. Boxplots of log-transformed diastolic blood pressure (DBP), high-density lipoprotein (HDL), low-den-sity lipoprotein (LDL), systolic blood pressure (SBP) and triglycerides (TG) for different Center, Italy EPIC training dataset.

3. Penalized mixed-effects model for multitask learning. In this section a novel approach for multivariate mixed-effects modeling based on penalized estimation is proposed.

3.1. Model definition. The multivariate linear mixed-effects model (Shah, Laird and Schoenfeld (1997)) expresses the $n_j \times r$ response matrix Y_j for the *j*th group as

$$Y_j = X_j \boldsymbol{B} + Z_j \boldsymbol{\Lambda}_j + \boldsymbol{E}_j$$

where, for each of the n_j samples in group j and $\sum_{j=1}^{J} n_j = N$, r response variables have been measured. The remainder terms define the following quantities:

• **B** is the $p \times r$ matrix of fixed-effects (including the intercept).

• Λ_i is the $q \times r$ matrix of random-effects.



• X_j is	the $n_j \times p$ fixed-effects design matrix.
• Z_i is	the $n_i \times q$ random-effects design matrix.
• $\vec{E_i}$ is	the $n_i \times r$ within-group error matrix.
• $j = 1$	\ldots, J , with J total number of groups.
By empl	oying the vec operator, we assume that
	$\operatorname{vec}(\mathbf{\Lambda}_j) \sim \mathcal{N}(0, \mathbf{\Psi}),$
where ¥ between distribut	' is a $qr \times qr$ positive semidefinite matrix, incorporating variations and covariations the <i>r</i> responses and the <i>q</i> random-effects. We further assume that the error term is ed as follows:
(2)	$\operatorname{vec}(\boldsymbol{E}_j) \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{\Sigma} \otimes \boldsymbol{I}_{n_j}),$
where Σ the ident between	is a $r \times r$ covariance matrix, capturing dependence among responses, and I_{n_j} is ity matrix of dimension $n_j \times n_j$. Formulation in (2) explicitly induces independence the row vectors of E_j . Therefore, the entire model can be rewritten in vec form,
	$\operatorname{vec}(\boldsymbol{Y}_j) \sim N((\boldsymbol{I}_r \otimes \boldsymbol{X}_j) \operatorname{vec}(\boldsymbol{B}), (\boldsymbol{I}_r \otimes \boldsymbol{Z}_j) \Psi(\boldsymbol{I}_r \otimes \boldsymbol{Z}_j)' + \boldsymbol{\Sigma} \otimes \boldsymbol{I}_{n_j}).$
Given a	sample of $N = \sum_{j=1}^{J} n_j$, the log-likelihood of model (1) reads
<i>ℓ</i> ($\boldsymbol{\theta}) = \sum_{i=1}^{n} -\frac{n_j}{2} \log 2\pi - \frac{1}{2} \log (\boldsymbol{I}_r \otimes \boldsymbol{Z}_j) \boldsymbol{\Psi} (\boldsymbol{I}_r \otimes \boldsymbol{Z}_j)' + \boldsymbol{\Sigma} \otimes \boldsymbol{I}_{n_j} $
(3)	<i>J</i> =1
(5)	$-\frac{1}{2} (\operatorname{vec}(\boldsymbol{Y}_j) - (\boldsymbol{I}_r \otimes \boldsymbol{X}_j) \operatorname{vec}(\boldsymbol{B}))' ((\boldsymbol{I}_r \otimes \boldsymbol{Z}_j) \boldsymbol{\Psi} (\boldsymbol{I}_r \otimes \boldsymbol{Z}_j)' + \boldsymbol{\Sigma} \otimes \boldsymbol{I}_{n_j})^{-1}$
	$\times (\operatorname{vec}(\boldsymbol{Y}_j) - (\boldsymbol{I}_r \otimes \boldsymbol{X}_j) \operatorname{vec}(\boldsymbol{B})),$
where θ in (1) is tainly, m (3) but r	$= \{B, \Sigma, \Psi\}$ is the set of parameters to be estimated. When the framework outlined employed for DNAm biomarker creation, the number of regressors p is, most cer- nuch larger than the sample size N. We are thus not directly interested in maximizing ather a penalized version of it, generically defined as follows:
(4)	$\ell_{\text{pen}}(\boldsymbol{\theta}) = \ell(\boldsymbol{\theta}) - p(\boldsymbol{B}; \lambda),$
with $p(x)$ of the co	B ; λ) being a penalty term employed to regularize the fixed-effects B as a function omplexity parameter $\lambda \ge 0$. Notice that, depending on the chosen penalty, more than plexity parameter could be involved in the definition of $p(\mathbf{B}; \lambda)$ (see Section 3.3 for laterals).
A ger subsection	neral-purpose algorithm for maximizing (4) can be devised, as described in the nex on.
3.2. $j = 1, .$ Rubin (1) quantitie	<i>Model estimation.</i> Direct maximization of (4) is unfeasible, as the terms $vec(\Lambda_j)$, <i>J</i> are unknown. We, therefore, devise an EM algorithm (Dempster, Laird and 1977)) in which the E-step computes the conditional expectations for the unobserved es, while a <i>complete penalized log-likelihood</i> is maximized in the M-step.
3.2.1. €(vec(Λ	<i>E-step.</i> The E-step requires the computation of $\mathbb{E}(\text{vec}(\Lambda_j) Y_j;\theta)$ and $\mathbf{A}_j) \text{vec}(\Lambda_j)' Y_j;\theta)$. This is achieved by noticing that the conditional density

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 $p(\text{vec}(\mathbf{\Lambda}_i)|\mathbf{Y}_i; \boldsymbol{\theta})$ is Normal. Updating formulae for the quantities of interest are thus derived as follows: $\hat{\boldsymbol{\Gamma}}_{i} = \mathbb{V}(\operatorname{vec}(\boldsymbol{\Lambda}_{i})|\boldsymbol{Y}_{i};\boldsymbol{\theta}) = [(\boldsymbol{I}_{r} \otimes \boldsymbol{Z}_{i})'(\boldsymbol{\Sigma} \otimes \boldsymbol{I}_{n_{i}})^{-1}(\boldsymbol{I}_{r} \otimes \boldsymbol{Z}_{i}) + \boldsymbol{\Psi}^{-1}]^{-1},$ (5) $\widehat{\operatorname{vec}(\Lambda_i)} = \mathbb{E}(\operatorname{vec}(\Lambda_i)|Y_i;\theta)$ (6) $= \hat{\Gamma}_i (I_r \otimes Z_i)' (\Sigma \otimes I_{n_i})^{-1} (\operatorname{vec}(Y_i) - (I_r \otimes X_i) \operatorname{vec}(B)).$ Consequently, the second moment $\hat{\boldsymbol{R}}_{i} = \mathbb{E}(\operatorname{vec}(\boldsymbol{\Lambda}_{i}) \operatorname{vec}(\boldsymbol{\Lambda}_{i})' | \boldsymbol{Y}_{i}; \boldsymbol{\theta})$ reads $\hat{\boldsymbol{R}}_{i} = \hat{\boldsymbol{\Gamma}}_{i} + \widehat{\operatorname{vec}(\boldsymbol{\Lambda}_{i})} \widehat{\operatorname{vec}(\boldsymbol{\Lambda}_{i})}^{\prime}.$ (7)At the *t*th iteration of the EM algorithm, the E-step requires the computation of (5)–(7), conditioning on the parameter values, estimated at iteration t - 1. Notice that we can directly define the conditional density of $Y_i | \Lambda_i$ by means of the matrix normal distribution $\boldsymbol{Y}_{i}|\boldsymbol{\Lambda}_{i} \sim m\mathcal{N}(\boldsymbol{X}_{i}\boldsymbol{B} + \boldsymbol{Z}_{i}\boldsymbol{\Lambda}_{i}, \boldsymbol{I}_{n_{i}}, \boldsymbol{\Sigma}),$ (8) where $X_j B + Z_j \Lambda_j$ is the $n_j \times r$ mean matrix, and I_{n_j} , Σ , respectively, identify the row and column covariance matrices (Dawid (1981)). Such a representation will be useful in specifying the update for **B** in the devised M-step: details are provided in the next subsection. 3.2.2. *M-step*. In the M-step we maximize the *complete penalized log-likelihood*,

$$\ell_{Cpen}(\boldsymbol{\theta}) = \sum_{j=1}^{J} \log(p(\operatorname{vec}(\boldsymbol{Y}_j) | \operatorname{vec}(\boldsymbol{\Lambda}_j); \boldsymbol{B}, \boldsymbol{\Sigma})) + \log(p(\operatorname{vec}(\boldsymbol{\Lambda}_j); \boldsymbol{\Psi})) - p(\boldsymbol{B}; \lambda)$$

$$\begin{array}{l} \begin{array}{c} 27\\ 28\\ 29 \end{array} \end{array} \left(9 \right) \qquad \qquad = \sum_{j=1}^{J} -\frac{n_j}{2} \log(2\pi) - \frac{1}{2} \log|\mathbf{\Sigma} \otimes \mathbf{I}_{n_j}| - \frac{1}{2} \mathbb{E} \left(\mathbf{e}'_j (\mathbf{\Sigma} \otimes \mathbf{I}_{n_j})^{-1} \mathbf{e}_j | \mathbf{Y}_j, \mathbf{\theta} \right) \end{array}$$

$$-\frac{n_j}{2}\log(2\pi) - \frac{1}{2}\log|\Psi| - \frac{1}{2}\mathbb{E}\left(\operatorname{vec}(\Lambda_j)'\Psi^{-1}\operatorname{vec}(\Lambda_j)|\boldsymbol{Y}_j,\boldsymbol{\theta}\right) - p(\boldsymbol{B};\lambda),$$

where $e_j = \operatorname{vec}(Y_j) - (I_r \otimes X_j) \operatorname{vec}(B) - (I_r \otimes Z_j) \operatorname{vec}(\Lambda_j)$ and the maximization is performed with respect to $\theta = \{B, \Sigma, \Psi\}$.

The updating formula for **B** clearly depends on the considered $p(B; \lambda)$ penalty. All the same, it is convenient to work with the matrix-variate representation defined in (8). In so doing, the objective function to be maximized wrt **B** reads

³⁸
₃₉
₄₀
(10)
$$Q_{B}(B) = -\frac{1}{2} \sum_{j=1}^{J} \operatorname{tr} (\boldsymbol{\Sigma}^{-1} (\tilde{\boldsymbol{Y}}_{j} - \boldsymbol{X}_{j} \boldsymbol{B})' (\tilde{\boldsymbol{Y}}_{j} - \boldsymbol{X}_{j} \boldsymbol{B})) - p(\boldsymbol{B}; \lambda),$$

where $\tilde{Y}_j = Y_j - Z_j \hat{\Lambda}_j$. $\hat{\Lambda}_j$ is recovered by applying the inverse of the vectorization operator to $\hat{Vec}(\Lambda_j)$, previously computed in the E-step. Simply put, the $\hat{Vec}(\Lambda_j)$ vector of length qris rearranged in a $q \times r$ matrix, obtaining $\hat{\Lambda}_j$. Start by noticing that, when no penalty is considered, maximization of (10) agrees with the generalized least squares (GLS) estimator assuming Σ and Ψ known (Shah, Laird and Schoenfeld (1997)). By exploiting properties of the trace operator, we can rewrite (10) defining the following minimization problem:

(1

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where $\|\cdot\|_{F}^{2}$ denotes the squared Frobenius norm and $\Sigma^{-1/2}$ is the symmetric positive definite square root of Σ^{-1} such that $\Sigma^{-1} = \Sigma^{-1/2} \Sigma^{-1/2}$. The representation in (11) allows to employ standard routines for multivariate penalized fixed-effects models for estimating **B**. In details for solving (11), a two-step updating scheme is devised. First, we compute

 (12)
$$\tilde{\boldsymbol{B}} = \operatorname*{arg\,min}_{\boldsymbol{B}} \frac{1}{2} \sum_{j=1}^{J} \|\boldsymbol{\Sigma}^{-1/2} \tilde{\boldsymbol{Y}}_{j} - \boldsymbol{X}_{j} \boldsymbol{B}\|_{F}^{2} + p(\boldsymbol{B}; \lambda),$$

that is, a fixed-effects penalized regression problem in which the response variable is $\Sigma^{-1/2}\tilde{Y}_j$, j = 1, ..., J; \tilde{B} is thus easily retrieved via fixed-effects routines for penalized estimation. Second, the solution to (11) is obtained postmultiplying \tilde{B} by $\Sigma^{1/2}$. Therefore, at each iteration of the EM-algorithm we first compute \tilde{B} , and then we set

$$\hat{\boldsymbol{B}} = \tilde{\boldsymbol{B}} \boldsymbol{\Sigma}^{1/2},$$

where B maximizes (10). This procedure stems from the rationale outlined, in Rohart, San Cristobal and Laurent (2014), where, contrarily to their original solution, in our context the updating steps are made more complex by the multidimensional nature of Y. The devised updating scheme allows to easily incorporate any $p(\mathbf{B}; \lambda)$ that has been previously defined for the fixed-effects framework and whose estimating routines are available. A list of possible penalties is proposed in Section 3.3.

Updating formulae for the covariance matrices Ψ and Σ agree with those of the unpenalized setting, namely,

$$\hat{\Psi} = \frac{1}{J} \sum_{j=1}^{J} \hat{R}_{j},$$
(14)
$$\hat{\Psi} = \frac{1}{J} \sum_{j=1}^{J} \hat{R}_{j},$$

and for the (h, k)th element of matrix Σ

$$\hat{\boldsymbol{\Sigma}}_{(h,k)} = \frac{1}{N} \sum_{j=1}^{J} \left[\mathbb{E}(\boldsymbol{E}_{jh} | \boldsymbol{Y}_j)' \mathbb{E}(\boldsymbol{E}_{jk} | \boldsymbol{Y}_j) \right]$$
(15)

+ tr[cov($\boldsymbol{E}_{jh}, \boldsymbol{E}_{jk} | \boldsymbol{Y}_{j}$)], $h, k = 1, \dots, r$,

where E_{jh} denotes the *h*th column of matrix $E_j = Y_j - Z_j \hat{\Lambda}_j - X_j B$, h = 1, ..., r.

3.3. Definition of $p(\mathbf{B}; \lambda)$. The EM algorithm devised in the previous section defines a general-purpose optimization strategy for penalized mixed-effects multitask learning. While any penalty type can, in principle, be defined, three notable examples, commonly used in this context, are the elastic net penalty (Zou and Hastie (2005)), the multivariate group-lasso penalty (Obozinski, Wainwright and Jordan (2011b)) and the netReg routines for Network-regularized linear models (Dirmeier et al. (2018)). Each of them is briefly described in the next subsections.

⁴³ 3.3.1. *Elastic-net penalty*. The first penalty type we consider is the renowned convex ⁴⁴ combination of lasso and ridge regularizers, whose magnitude of the former over the latter is ⁴⁵ controlled by the mixing parameter α , $0 \le \alpha \le 1$. In details the penalty expression reads

(16)
$$p(\boldsymbol{B}; \lambda, \alpha) = \lambda \left[(1-\alpha) \sum_{c=1}^{r} \sum_{l=2}^{p} b_{lc}^{2} + \alpha \sum_{c=1}^{r} \sum_{l=2}^{p} |b_{lc}| \right],$$

where b_{lc} denotes the element in the *l*th row and *c*th column of matrix **B**. Notice that the first row of **B** contains the *r* intercepts, and it is thus not penalized. Algorithmically, penalty

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(16) can be enforced employing standard and widely available routines for univariate penal-ized estimation, like the glmnet software (Tay, Narasimhan and Hastie (2021)). The only computational detail that shall be examined is how to prevent the default shrinkage of the rintercepts: the penalty, factor argument of the glmnet function effectively serves the purpose. The latter can also be employed in our framework to force coefficients that need not be penalized to enter the model specification.

3.3.2. Multivariate group-lasso penalty. This type of penalty imposes a group structure on the coefficients, forcing the same subset of predictors to be preserved across all r components of the response matrix. This feature is particularly desirable when building multivariate DNAm biomarkers, since it automatically identifies the CpG sites that are *jointly* related to the considered risk factors. Such a penalty is defined as follows:

(17)
$$p(\boldsymbol{B};\lambda,\alpha) = \lambda \left[(1-\alpha) \sum_{c=1}^{r} \sum_{l=2}^{p} b_{lc}^{2} + \alpha \sum_{l=2}^{p} \|\boldsymbol{b}_{l.}\|_{2} \right],$$

where b_{l} identifies the *l*th row of the matrix **B** such that each b_{l} , l = 2, ..., p is an rdimensional vector. Likewise, Section 3.3.1 summations over rows in (17) start at 2 since we do not penalize the vector of intercepts. This penalty behaves like the lasso but on the whole group of predictors for each of the r variables: they are either all zero, or else none are zero, but are shrunk by an amount depending on λ . Similarly to (16), the mixing parameter α controls the weight associated to ridge and group-lasso regularizers. The glmnet software, with family = "mgaussian" is again at our disposal for efficiently incorporating (17)in the framework outlined in the present paper.

3.3.3. Network-regularized penalty. The last penalty we consider allows for the inclusion of biological graph-prior knowledge in the estimation by accounting for the contribution of two nonnegative adjacency matrices, $G_X \in \mathbb{R}^{(p-1)\times(p-1)}_+$ and $G_Y \in \mathbb{R}^{r\times r}_+$, respectively, related to X and Y. In this case, $p(B; \lambda)$ assumes the following functional form:

$$p(\boldsymbol{B}; \lambda, \lambda_X, \lambda_Y) = \lambda \|\boldsymbol{B}_0\|_1 + \lambda_X \operatorname{tr} (\boldsymbol{B}'_0(\boldsymbol{D}_{G_X} - \boldsymbol{G}_X)\boldsymbol{B}_0) + \lambda_Y \operatorname{tr} (\boldsymbol{B}_0(\boldsymbol{D}_{G_Y} - \boldsymbol{G}_Y)\boldsymbol{B}'_0),$$

where B_0 is the $(p-1) \times r$ matrix of coefficients without the intercepts and D_{G_X} , D_{G_Y} indicate the degree matrices of G_X and G_Y , respectively (Chung and Graham (1997)). G_X and G_Y encode a biological similarity, forcing rows and columns of B_0 to be similar. The netReg R package (Dirmeier et al. (2018)) provides a convenient implementation of (18).

3.3.4. On the choice of $p(\mathbf{B}; \lambda)$. Leaving the flexibility attained by the methodology proposed in Section 3.1 aside, in practice, a functional form for $p(\mathbf{B}; \lambda)$ must be chosen when performing the analysis. Hereafter, we highlight pros and cons of the proposed approaches with respect to a mixed-effects multitask learning setting.

The elastic-net penalty in (16) does not take into account the multivariate nature of the problem in (4), as the shrinkage is applied directly to $vec(\mathbf{B})$. This behavior allows for cap-turing a wide variety of sparsity patterns that may be present in \boldsymbol{B} but does not impose any specific structure that could be desirable in a multivariate context. Differently, the multi-variate group-lasso of Section 3.3.2 defines a shrinkage term that forces the same subset of predictors to be preserved across all r components of the response Y. This can be seen as the generalization of the variable selection problem to the multivariate response setting, which is also known as *support union problem* or *row selection problem* in the literature (Obozinski, Wainwright and Jordan (2011b)). Lastly, the network-Regularized penalty in

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3.3.3 is p partially, (2014)).	particularly useful when the interaction among features and/or responses is, at least known such that it can be profited from within the learning mechanism (Cheng et al.
In rela the multi cases bet analysis	tion to the DNAm surrogate creation task motivating our methodological proposal, variate group-lasso is definitely the most appropriate penalty, as it not only show- ter prediction performances but it is also supported by biological reasons: a thorough for the EPIC dataset is reported in Section 5.
3.4. <i>F</i> presented	<i>Surther aspects.</i> Hereafter, we discuss some practical considerations related to the d methodology:
• Initial	<i>ization:</i> We start the algorithm with an M-step, setting $\hat{\theta}^{(0)} = \{\hat{B}^{(0)}, \hat{\Sigma}^{(0)}, \hat{\Psi}^{(0)}\}$. In
details	, both $\hat{\boldsymbol{\Sigma}}^{(0)}$ and $\hat{\boldsymbol{\Psi}}^{(0)}$ are initialized with identity matrices of dimension $r \times r$ and
$qr \times q$ randor ters.	$\mathbf{\hat{B}}^{(0)}$ is estimated from a penalized linear model (without the n-effects) employing the chosen penalty function with the associated hyperparame-
• Conve	rgence: The EM algorithm is considered to have converged once the relative differ-
ence in	n the objective function for two subsequent iterations is smaller than ε , for a given
$\varepsilon > 0,$	
	$ \ell_{\text{pen}}(\hat{\boldsymbol{\theta}}^{(t+1)}) - \ell_{\text{pen}}(\hat{\boldsymbol{\theta}}^{(t)}) $
	$\frac{1}{ \ell - \hat{\boldsymbol{\mu}}^{(t)} } < \varepsilon,$
	$ \mathcal{L}_{\text{pen}}(\mathbf{v}) $
where	$\hat{\theta}^{(t)} = \{\hat{B}^{(t)}, \hat{\Sigma}^{(t)}, \hat{\Psi}^{(t)}\}$ is the set of estimated values at the end of the <i>t</i> th iteration.
In our the cla	analyses ε is set equal to 10^{-6} . The procedure described in Section 3.2 falls within iss of expectation conditional maximization (ECM) algorithms, whose convergence ties have been proved in Mang and Rubin (1993) and in Section 5.2.3 of McI achieved
and K	rishnan (2008).
• Model	selection: A standard 10-fold cross validation (CV) strategy is employed for select-
(2014) Schwa	e tuning factors. Alternatively, as suggested in Rohart, San Cristobal and Laurent, one could employ a modified version of the Bayesian Information Criterion (BIC, rz (1978)).
(10)	$PLC = 2\ell(\hat{\theta}) = d_2 \log(N)$
(17)	$DIC = 2t(\sigma) - u_0 \log(N),$
where numbe would	$\ell(\boldsymbol{\theta})$ is the log-likelihood evaluated at $\boldsymbol{\theta}$, obtained maximizing (4), and d_0 is the er of nonzero parameters resulting from the penalized estimation. Another option be to rely on an interval search algorithm, like the efficient parameter selection via
global	optimization (Frohlich and Zell (2005)): an implementation is available in the $c060$
• Scalab	age (SIII et al. (2014)). addity: The devised methodology provides a framework for incorporating any penalty
in a hi	gh-dimensional mixed-effects multitask learning framework. To this extent, the data
dimen	sionality with which our procedure can cope as well as the overall computing time
very n	nuch depends on the scalability and efficiency associated to the chosen shrinkage
term.	Typically nevertheless, penalized likelihood approaches fail to be directly applied
to ultr	ahigh-dimensional problems (Fan, Samworth and Wu (2009)), and preprocessing
procec	sures, such as variable screening, are thus required prior to modeling. The epigenetic
(see Se	ection 2) but clearly other dimensionality reduction techniques could be considered
when	dealing with massive datasets. The interested reader is referred to Jordan (2013) for
a thou	ght-provoking investigation on the topic.

$b \ longlementation: Routines for fitting the penalized mixed-effects multitask learning methods have been implemented in R (R Core Team (2022)), and the source code is freely avail able in the Supplementary Material and at https://github.com/AndreaCappozzo/emlmm in the form of an R package. The three penalites, described in Section 3.3, are in cluded in the software and can be selected via the penalty_type argument of the ecm_mlmm_penalized function. As described in Section 3.3, the M-step heavily referred function. As described in Section 3.3, the M-step heavily reflects more first component. While in principle reasonable, it may happen in specific argument effects: Model in (1) assumes that each and every response requires a random-effects component. While in principle reasonable, it may happen in specific applications that only a subset of the r characteristics in Y enjoys group-dependent heterogeneity. The occurrence of such a scenario can be unveiled by looking at the r diago nal elements of dimension q in \hat{\Psi}: a response may be considered group-independent when the magnitude of the associated elements in diag(\hat{\Psi}) is significantly lower than the remain ing ones. Doing this way, the impact random-effects have on the different characteristic is retrieved as a by-product of the modeling procedure. 4. Simulation study. In this section we evaluate the model introduced in Section 3 or ynthetic data. The aim of the analyses reported hereafter is twofold. On the one hand, wo would like to validate the predictive power of the proposed procedure against its fixed-effect ounterpart when the random-effects way across dimensions in the multivariate response. One other hand, we assess the estimated model parameters and the recovery of the underlying parsity structure for different values of the shrinkage factor \lambda. 4.1. Experimental setup. We generate N = 600 data points according to model (1) with be following parameters: $								
able in the Supplementary Material and at https://github.com/AndreaCappozzo/emlma in the form of an R package. The three penalties, described in Section 3.3, are in cluded in the software and can be selected via the penalty_type argument of the ecm_mlmm_penalized function. As described in Section 3.3, the M-step heavily relies on previously developed fast and stable subroutines, while the E-step and the objective function evaluation have been implemented in c++ to reduce the overall computing time. <i>Response-specific random-effects</i> : Model in (1) assumes that each and every response requires a random-effects component. While in principle reasonable, it may happen in specific applications that only a subset of the <i>r</i> characteristics in <i>Y</i> enjoys group-dependen heterogeneity. The occurrence of such a scenario can be unveiled by looking at the <i>r</i> diago nal elements of dimension <i>q</i> in Ψ : a response may be considered group-independent where the magnitude of the associated elements in diag(Ψ) is significantly lower than the remain ing ones. Doing this way, the impact random-effects have on the different characteristic is retrieved as a by-product of the modeling procedure. 4. Simulation study . In this section we evaluate the model introduced in Section 3 or ynthetic data. The aim of the analyses reported hereafter is twofold. On the one hand, would like to validate the predictive power of the proposed procedure against its fixed-effect ounterpart when the random-effects vary across dimensions in the multivariate response. One ther hand, we assess the estimated model parameters and the recovery of the underlying parsity structure for different values of the shrinkage factor λ . 4. <i>Experimental setup</i> . We generate $N = 600$ data points according to model (1) with he following parameters: $\Psi = \begin{bmatrix} 50.00 & -1.59 & -0.60 & -0.22 & 2.38 \\ -1.59 & 40.00 & -0.96 & -0.91 & 0.37 \\ -0.60 & -0.96 & 30.00 & -0.43 & 0.50 \\ -0.22 & -0.91 & -0.43 & 20.00 & 0.80 \\ 2.38 & 0.37 & 0.50 & 0.80 & 0.16 \end{bmatrix}$. $\Sigma = \begin{bmatrix} 2.16 & 0.$	 Implementation have been impl 	: Routir emented	nes for fitt 1 in R (R	ting the p Core Te	enalized am (202	mixed-ef 2)), and	fects multitask le the source code	earning method is freely avail-
in the form of an R package. The three penalties, described in Section 3.3, are in cluded in the software and can be selected via the penalty_type argument of th ecm_mlmm_penalized function. As described in Section 3.3, the M-step heavily re lies on previously developed fast and stable subroutines, while the E-step and the objectiv function evaluation have been implemented in c++ to reduce the overall computing time. <i>Penspose-specific random-effects:</i> Model in (1) assumes that each and every response requires a random-effects component. While in principle reasonable, it may happen in specific applications that only a subset of the <i>r</i> characteristics in Y enjoys group-dependen heterogeneity. The occurrence of such a scenario can be unveiled by looking at the <i>r</i> diago nal elements of dimension <i>q</i> in $\hat{\Psi}$: a response may be considered group-independent whethe magnitude of the associated elements in diag($\hat{\Psi}$) is significantly lower than the remain ing ones. Doing this way, the impact random-effects have on the different characteristic is retrieved as a by-product of the modeling procedure. 4. Simulation study. In this section we evaluate the model introduced in Section 3 or ynthetic data. The aim of the analyses reported hereafter is twofold. On the one hand, we would like to validate the predictive power of the proposed procedure against its fixed-effect ounterpart when the random-effects vary across dimensions in the multivariate response. On the other hand, we assess the estimated model parameters and the recovery of the underlying parsity structure for different values of the shrinkage factor λ . 4.1. <i>Experimental setup.</i> We generate $N = 600$ data points according to model (1) with the following parameters: $\Psi = \begin{bmatrix} 50.00 & -1.59 & -0.60 & -0.22 & 2.38 \\ -1.59 & 40.00 & -0.96 & -0.91 & 0.37 \\ -0.60 & -0.95 & -0.03 & 2.16 & -0.03 \\ -0.22 & -0.91 & -0.43 & 20.00 & 0.80 \\ 2.38 & 0.37 & 0.55 & -0.10 \\ -0.80 & -0.33 & 2.16 & -0.03 & -0.13 \\ 0.91 & 0.55 & -0.03 & 2.16 & 0.02 \\ -0.26 & -0.10 & -0.1$	able in the Sur	plemen	tarv Mat	terial and	at https	://github	.com/AndreaCa	ppozzo/emlmm
cluded in the software and can be selected via the penalty_type argument of th ecm_mlmm_penalized function. As described in Section 3.3, the M-step heavily re lies on previously developed fast and stable subroutines, while the E-step and the objectivy function evaluation have been implemented in c++ to reduce the overall computing time. <i>Response-specific random-effects:</i> Model in (1) assumes that each and every response re quires a random-effects component. While in principle reasonable, it may happen in spe cific applications that only a subset of the <i>r</i> characteristics in <i>Y</i> enjoys group-dependen heterogeneity. The occurrence of such a scenario can be unveiled by looking at the <i>r</i> diago nal elements of dimension <i>q</i> in $\hat{\Psi}$: a response may be considered group-independent when the magnitude of the associated elements in diag($\hat{\Psi}$) is significantly lower than the remain ing ones. Doing this way, the impact random-effects have on the different characteristic is retrieved as a by-product of the modeling procedure. 4. Simulation study. In this section we evaluate the model introduced in Section 3 or ynthetic data. The aim of the analyses reported hereafter is twofold. On the one hand, we would like to validate the predictive power of the proposed procedure against its fixed-effect ounterpart when the random-effects vary across dimensions in the multivariate response. On he other hand, we assess the estimated model parameters and the recovery of the underlying parsity structure for different values of the shrinkage factor λ . 4.1. <i>Experimental setup.</i> We generate $N = 600$ data points according to model (1) with he following parameters: $\Psi = \begin{bmatrix} 50.00 & -1.59 & -0.60 & -0.22 & 2.38 \\ -1.59 & 40.00 & -0.96 & -0.91 & 0.37 \\ -0.60 & -0.96 & 30.00 & -0.43 & 0.50 \\ -0.22 & -0.91 & -0.43 & 20.00 & 0.80 \\ 2.38 & 0.37 & 0.50 & 0.80 & 0.16 \end{bmatrix}$ $\Sigma = \begin{bmatrix} 2.16 & 0.09 & -0.80 & 0.91 & -0.26 \\ 0.09 & 2.16 & -0.33 & 0.55 & -0.10 \\ -0.80 & -0.33 & 2.16 & -0.03 & -0.13 \\ 0.91 & 0.55 & -0.03 & 2.16 & 0.02 \\ -0.26 & -0.10 &$	in the form of	an R	nackage.	The thr	ee nenal	ties. des	cribed in Section	on 3.3 are in-
$\Psi = \begin{bmatrix} 50.00 & -1.59 & -0.60 & -0.22 & 2.38 \\ -1.59 & 40.00 & -0.96 & -0.91 & 0.37 \\ -0.60 & -0.96 & -0.91 & 0.37 \\ -0.60 & -0.96 & -0.91 & 0.37 \\ -0.60 & -0.96 & -0.91 & 0.37 \\ -0.60 & -0.33 & 2.16 & -0.33 \\ -0.22 & -0.91 & -0.43 & 20.00 & 0.80 \\ -0.26 & -0.10 & -0.13 & 0.02 & 2.16 \end{bmatrix}$	cluded in the s	oftware	and car	be sele	cted via	the pen	alty type a	rgument of the
It is on previously developed fast and stable subroutines, while the E-step and the objective function evaluation have been implemented in c++ to reduce the overall computing time. <i>Desponse-specific random-effects:</i> Model in (1) assumes that each and every response requires a random-effects component. While in principle reasonable, it may happen in specific applications that only a subset of the <i>r</i> characteristics in Y enjoys group-dependen heterogeneity. The occurrence of such a scenario can be unveiled by looking at the <i>r</i> diago nal elements of dimension <i>q</i> in $\hat{\Psi}$: a response may be considered group-independent when the magnitude of the associated elements in diag($\hat{\Psi}$) is significantly lower than the remain ing ones. Doing this way, the impact random-effects have on the different characteristic is retrieved as a by-product of the modeling procedure. 4. Simulation study. In this section we evaluate the model introduced in Section 3 or ynthetic data. The aim of the analyses reported hereafter is twofold. On the one hand, we would like to validate the predictive power of the proposed procedure against its fixed-effect ounterpart when the random-effects vary across dimensions in the multivariate response. On he other hand, we assess the estimated model parameters and the recovery of the underlying parsity structure for different values of the shrinkage factor λ . 4.1. <i>Experimental setup.</i> We generate $N = 600$ data points according to model (1) with the following parameters: $\Psi = \begin{bmatrix} 50.00 & -1.59 & -0.60 & -0.22 & 2.38 \\ -1.59 & 40.00 & -0.96 & -0.91 & 0.37 \\ -0.60 & -0.95 & 30.00 & -0.43 & 20.00 & 0.80 \\ 2.38 & 0.37 & 0.50 & 0.80 & 0.16 \end{bmatrix}$ $\Sigma = \begin{bmatrix} 2.16 & 0.09 & -0.80 & 0.91 & -0.26 \\ 0.09 & 2.16 & -0.33 & 2.16 & -0.03 \\ -0.26 & -0.10 & -0.13 & 0.02 & 2.16 \end{bmatrix}$ mplying that $r = 5$ and $q = 1$. Notice that Ψ is purposely constructed for the random-effect o differently affect the five dimensional response: while the first component showcases high ariance (first entry in the main diagonal), th	ecm mlmm pe	enaliz	zed func	tion. As	described	1 in Sect	ion 3.3. the M-s	step heavily re-
function evaluation have been implemented in <i>c</i> ++ to reduce the overall computing time. <i>Pesponse-specific random-effects:</i> Model in (1) assumes that each and every response re- quires a random-effects component. While in principle reasonable, it may happen in spe- cific applications that only a subset of the <i>r</i> characteristics in <i>Y</i> enjoys group-dependen heterogeneity. The occurrence of such a scenario can be unveiled by looking at the <i>r</i> diago- nal elements of dimension <i>q</i> in Ψ : a response may be considered group-independent when the magnitude of the associated elements in diag(Ψ) is significantly lower than the remain ing ones. Doing this way, the impact random-effects have on the different characteristic is retrieved as a by-product of the modeling procedure. 4. Simulation study. In this section we evaluate the model introduced in Section 3 or ynthetic data. The aim of the analyses reported hereafter is twofold. On the one hand, we vould like to validate the predictive power of the proposed procedure against its fixed-effect ounterpart when the random-effects vary across dimensions in the multivariate response. On he other hand, we assess the estimated model parameters and the recovery of the underlying parsity structure for different values of the shrinkage factor λ . 4.1. <i>Experimental setup</i> . We generate $N = 600$ data points according to model (1) with he following parameters: $\Psi = \begin{bmatrix} 50.00 & -1.59 & -0.60 & -0.22 & 2.38 \\ -0.59 & 40.00 & -0.96 & -0.91 & 0.37 \\ -0.60 & -0.33 & 0.55 & -0.10 \\ -0.80 & -0.33 & 0.55 & -0.10 \\ -0.80 & -0.33 & 0.55 & -0.10 \\ -0.80 & -0.33 & 0.55 & -0.10 \\ -0.80 & -0.33 & 0.16 & -0.02 \\ -0.26 & -0.10 & -0.13 & 0.02 & 2.16 \end{bmatrix}$, mplying that $r = 5$ and $q = 1$. Notice that Ψ is purposely constructed for the random-effect to differently affect the five dimensional response: while the first component showcases high ariance (first entry in the main diagonal), the last one is very small and close to 0. Further he error variances (diagonal element	lies on previous	ly devel	oped fast	t and stab	ole subrou	tines, w	hile the E-step ar	nd the objective
$ \begin{aligned} & \text{Response-specific random-effects:} Model in (1) assumes that each and every response requires a random-effects component. While in principle reasonable, it may happen in specific applications that only a subset of the r characteristics in Y enjoys group-dependen heterogeneity. The occurrence of such a scenario can be unveiled by looking at the r diagonal elements of dimension q in \Psi: a response may be considered group-independent when the magnitude of the associated elements in diag(\Psi) is significantly lower than the remain ing ones. Doing this way, the impact random-effects have on the different characteristic is retrieved as a by-product of the modeling procedure. $	function evaluat	tion hav	e been in	nplement	ted in c+	+ to redu	ice the overall co	omputing time.
quires a random-effects component. While in principle reasonable, it may happen in specific applications that only a subset of the <i>r</i> characteristics in <i>Y</i> enjoys group-dependen heterogeneity. The occurrence of such a scenario can be unveiled by looking at the <i>r</i> diago nal elements of dimension <i>q</i> in $\hat{\Psi}$: a response may be considered group-independent when the magnitude of the associated elements in diag($\hat{\Psi}$) is significantly lower than the remain ing ones. Doing this way, the impact random-effects have on the different characteristic is retrieved as a by-product of the modeling procedure. 4. Simulation study. In this section we evaluate the model introduced in Section 3 or ynthetic data. The aim of the analyses reported hereafter is twofold. On the one hand, we vould like to validate the predictive power of the proposed procedure against its fixed-effect ounterpart when the random-effects vary across dimensions in the multivariate response. On he other hand, we assess the estimated model parameters and the recovery of the underlying parsity structure for different values of the shrinkage factor λ . 4.1. <i>Experimental setup</i> . We generate $N = 600$ data points according to model (1) with he following parameters: $\Psi = \begin{bmatrix} 50.00 & -1.59 & -0.60 & -0.22 & 2.38 \\ -1.59 & 40.00 & -0.96 & -0.91 & 0.37 \\ -0.60 & -0.96 & 30.00 & -0.43 & 0.50 \\ -0.22 & -0.91 & -0.43 & 20.00 & 0.80 \\ 2.38 & 0.37 & 0.50 & 0.80 & 0.16 \end{bmatrix}$ $\Sigma = \begin{bmatrix} 2.16 & 0.09 & -0.80 & 0.91 & -0.26 \\ 0.09 & 2.16 & -0.03 & 0.16 \\ -0.80 & -0.33 & 0.55 & -0.10 \\ -0.80 & -0.33 & 2.16 & -0.03 & -0.13 \\ 0.91 & 0.55 & -0.03 & 2.16 \\ 0.92 & -0.26 & -0.10 & -0.13 & 0.02 & 2.16 \end{bmatrix}$ mplying that $r = 5$ and $q = 1$. Notice that Ψ is purposely constructed for the random-effect o differently affect the five dimensional response: while the first component showcases high ariance (first entry in the main diagonal), the last one is very small and close to 0. Further he error variances (diagonal elements of Σ) are held constant across dimensions to bette ing	• Response-speci	fic rand	om-effect	ts: Mode	l in (1) as	ssumes th	hat each and eve	ry response re-
cific applications that only a subset of the <i>r</i> characteristics in <i>Y</i> enjoys group-dependen heterogeneity. The occurrence of such a scenario can be unveiled by looking at the <i>r</i> diago nal elements of dimension <i>q</i> in $\hat{\Psi}$: a response may be considered group-independent when the magnitude of the associated elements in diag($\hat{\Psi}$) is significantly lower than the remain ing ones. Doing this way, the impact random-effects have on the different characteristic is retrieved as a by-product of the modeling procedure. 4. Simulation study. In this section we evaluate the model introduced in Section 3 or ynthetic data. The aim of the analyses reported hereafter is twofold. On the one hand, we would like to validate the predictive power of the proposed procedure against its fixed-effect ounterpart when the random-effects vary across dimensions in the multivariate response. On he other hand, we assess the estimated model parameters and the recovery of the underlying parsity structure for different values of the shrinkage factor λ . 4.1. <i>Experimental setup</i> . We generate $N = 600$ data points according to model (1) with he following parameters: $\Psi = \begin{bmatrix} 50.00 & -1.59 & -0.60 & -0.22 & 2.38 \\ -1.59 & 40.00 & -0.96 & -0.91 & 0.37 \\ -0.60 & -0.96 & 30.00 & -0.43 & 0.50 \\ .0.22 & -0.91 & -0.43 & 20.00 & 0.80 \\ 2.38 & 0.37 & 0.50 & 0.80 & 0.16 \end{bmatrix}$ $\Sigma = \begin{bmatrix} 2.16 & 0.09 & -0.80 & 0.91 & -0.26 \\ 0.09 & 2.16 & -0.33 & 0.55 & -0.10 \\ -0.26 & -0.10 & -0.13 & 0.02 & 2.16 \end{bmatrix}$, mplying that $r = 5$ and $q = 1$. Notice that Ψ is purposely constructed for the random-effect o differently affect the five dimensional response: while the first component showcases high ariance (first entry in the main diagonal), the last one is very small and close to 0. Further he error variances (diagonal elements of Σ) are held constant across dimensions to bette ighlight the impact the variability of the random-effect has on the models performance. The hatargenerating process assumes 10 equally-sized subpopulations, resulting in $J = 10$. The nattr	quires a randon	n-effects	s compor	ent. Whi	ile in prir	ciple rea	asonable, it may	happen in spe-
heterogeneity. The occurrence of such a scenario can be unveiled by looking at the <i>r</i> diago nal elements of dimension <i>q</i> in $\hat{\Psi}$: a response may be considered group-independent when the magnitude of the associated elements in diag($\hat{\Psi}$) is significantly lower than the remain ing ones. Doing this way, the impact random-effects have on the different characteristic is retrieved as a by-product of the modeling procedure. 4. Simulation study. In this section we evaluate the model introduced in Section 3 or ynthetic data. The aim of the analyses reported hereafter is twofold. On the one hand, we vould like to validate the predictive power of the proposed procedure against its fixed-effect ounterpart when the random-effects vary across dimensions in the multivariate response. On the other hand, we assess the estimated model parameters and the recovery of the underlying parsity structure for different values of the shrinkage factor λ . 4.1. <i>Experimental setup</i> . We generate $N = 600$ data points according to model (1) with the following parameters: $\Psi = \begin{bmatrix} 50.00 & -1.59 & -0.60 & -0.22 & 2.38\\ -1.59 & 40.00 & -0.96 & -0.91 & 0.37\\ -0.60 & -0.96 & 30.00 & -0.43 & 0.50\\ -0.22 & -0.91 & -0.43 & 20.00 & 0.80\\ 2.38 & 0.37 & 0.50 & 0.80 & 0.16 \end{bmatrix}$, $\Sigma = \begin{bmatrix} 2.16 & 0.09 & -0.80 & 0.91 & -0.26\\ 0.09 & 2.16 & -0.33 & 0.55 & -0.10\\ -0.80 & -0.33 & 2.16 & -0.03 & -0.13\\ 0.91 & 0.55 & -0.03 & 2.16 & 0.02\\ -0.26 & -0.10 & -0.13 & 0.02 & 2.16 \end{bmatrix}$, mplying that $r = 5$ and $q = 1$. Notice that Ψ is purposely constructed for the random-effect o differently affect the five dimensional response: while the first component showcases high ariance (first entry in the main diagonal), the last one is very small and close to 0. Further he error variances (diagonal elements of Σ) are held constant across dimensions to better ightly the impact the variability of the random-effect has on the models performance. The hat-generating process assumes 10 equally-sized subpopulations, resulting in $J = 10$. The natrix of fixed-effects B is	cific application	is that c	only a sul	bset of th	ne r chara	acteristic	s in Y enjoys gr	oup-dependent
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$\Psi = \begin{bmatrix} -1.59 & 40.00 & -0.96 & -0.91 & 0.37 \\ -0.60 & -0.96 & 30.00 & -0.43 & 0.50 \\ -0.22 & -0.91 & -0.43 & 20.00 & 0.80 \\ 2.38 & 0.37 & 0.50 & 0.80 & 0.16 \end{bmatrix},$ $\Sigma = \begin{bmatrix} 2.16 & 0.09 & -0.80 & 0.91 & -0.26 \\ 0.09 & 2.16 & -0.33 & 0.55 & -0.10 \\ -0.80 & -0.33 & 2.16 & -0.03 & -0.13 \\ 0.91 & 0.55 & -0.03 & 2.16 & 0.02 \\ -0.26 & -0.10 & -0.13 & 0.02 & 2.16 \end{bmatrix},$ mplying that $r = 5$ and $q = 1$. Notice that Ψ is purposely constructed for the random-effect to differently affect the five dimensional response: while the first component showcases high variance (first entry in the main diagonal), the last one is very small and close to 0. Further he error variances (diagonal elements of Σ) are held constant across dimensions to bette sighlight the impact the variability of the random-effects has on the models performance. The lata-generating process assumes 10 equally-sized subpopulations, resulting in $J = 10$. The natrix of fixed-effects B is of dimension $10,001 \times 5$, with distinct sparsity pattern according to the random sparsity pattern according the terms are responsed.			50.00	-1.59	-0.60	-0.22	2.38	
$\Psi = \begin{bmatrix} -0.60 & -0.96 & 30.00 & -0.43 & 0.50 \\ -0.22 & -0.91 & -0.43 & 20.00 & 0.80 \\ 2.38 & 0.37 & 0.50 & 0.80 & 0.16 \end{bmatrix},$ $\Sigma = \begin{bmatrix} 2.16 & 0.09 & -0.80 & 0.91 & -0.26 \\ 0.09 & 2.16 & -0.33 & 0.55 & -0.10 \\ -0.80 & -0.33 & 2.16 & -0.03 & -0.13 \\ 0.91 & 0.55 & -0.03 & 2.16 & 0.02 \\ -0.26 & -0.10 & -0.13 & 0.02 & 2.16 \end{bmatrix},$ mplying that $r = 5$ and $q = 1$. Notice that Ψ is purposely constructed for the random-effect to differently affect the five dimensional response: while the first component showcases high variance (first entry in the main diagonal), the last one is very small and close to 0. Further he error variances (diagonal elements of Σ) are held constant across dimensions to bette eighlight the impact the variability of the random-effects has on the models performance. The lata-generating process assumes 10 equally-sized subpopulations, resulting in $J = 10$. The natrix of fixed-effects B is of dimension $10,001 \times 5$, with distinct sparsity pattern according to the random set of the random set of Σ .		_	-1.59	40.00	-0.96	-0.91	0.37	
$\boldsymbol{\Sigma} = \begin{bmatrix} -0.22 & -0.91 & -0.43 & 20.00 & 0.80 \\ 2.38 & 0.37 & 0.50 & 0.80 & 0.16 \end{bmatrix}$ $\boldsymbol{\Sigma} = \begin{bmatrix} 2.16 & 0.09 & -0.80 & 0.91 & -0.26 \\ 0.09 & 2.16 & -0.33 & 0.55 & -0.10 \\ -0.80 & -0.33 & 2.16 & -0.03 & -0.13 \\ 0.91 & 0.55 & -0.03 & 2.16 & 0.02 \\ -0.26 & -0.10 & -0.13 & 0.02 & 2.16 \end{bmatrix},$ mplying that $r = 5$ and $q = 1$. Notice that $\boldsymbol{\Psi}$ is purposely constructed for the random-effect o differently affect the five dimensional response: while the first component showcases high variance (first entry in the main diagonal), the last one is very small and close to 0. Further he error variances (diagonal elements of $\boldsymbol{\Sigma}$) are held constant across dimensions to bette dighlight the impact the variability of the random-effects has on the models performance. The lata-generating process assumes 10 equally-sized subpopulations, resulting in $J = 10$. The natrix of fixed-effects \boldsymbol{B} is of dimension $10,001 \times 5$, with distinct sparsity pattern according a three commission.		$\Psi =$	-0.60	-0.96	30.00	-0.43	0.50 ,	
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$\boldsymbol{\Sigma} = \begin{bmatrix} 2.16 & 0.09 & -0.80 & 0.91 & -0.26 \\ 0.09 & 2.16 & -0.33 & 0.55 & -0.10 \\ -0.80 & -0.33 & 2.16 & -0.03 & -0.13 \\ 0.91 & 0.55 & -0.03 & 2.16 & 0.02 \\ -0.26 & -0.10 & -0.13 & 0.02 & 2.16 \end{bmatrix},$ mplying that $r = 5$ and $q = 1$. Notice that $\boldsymbol{\Psi}$ is purposely constructed for the random-effect o differently affect the five dimensional response: while the first component showcases high variance (first entry in the main diagonal), the last one is very small and close to 0. Further he error variances (diagonal elements of $\boldsymbol{\Sigma}$) are held constant across dimensions to bette highlight the impact the variability of the random-effects has on the models performance. The lata-generating process assumes 10 equally-sized subpopulations, resulting in $J = 10$. The natrix of fixed-effects \boldsymbol{B} is of dimension $10,001 \times 5$, with distinct sparsity pattern according the supervisor.			2.38	0.37	0.50	0.80	0.16	
$\Sigma = \begin{bmatrix} 0.09 & 2.16 & -0.33 & 0.55 & -0.10 \\ -0.80 & -0.33 & 2.16 & -0.03 & -0.13 \\ 0.91 & 0.55 & -0.03 & 2.16 & 0.02 \\ -0.26 & -0.10 & -0.13 & 0.02 & 2.16 \end{bmatrix},$ mplying that $r = 5$ and $q = 1$. Notice that Ψ is purposely constructed for the random-effect o differently affect the five dimensional response: while the first component showcases high variance (first entry in the main diagonal), the last one is very small and close to 0. Further he error variances (diagonal elements of Σ) are held constant across dimensions to bette dighlight the impact the variability of the random-effects has on the models performance. The lata-generating process assumes 10 equally-sized subpopulations, resulting in $J = 10$. The natrix of fixed-effects B is of dimension 10,001 × 5, with distinct sparsity pattern according to the supersonal supersona			2.16	0.09	-0.80	0.91	-0.26]	
$\Sigma = \begin{bmatrix} -0.80 & -0.33 & 2.16 & -0.03 & -0.13 \\ 0.91 & 0.55 & -0.03 & 2.16 & 0.02 \\ -0.26 & -0.10 & -0.13 & 0.02 & 2.16 \end{bmatrix},$ mplying that $r = 5$ and $q = 1$. Notice that Ψ is purposely constructed for the random-effect o differently affect the five dimensional response: while the first component showcases high variance (first entry in the main diagonal), the last one is very small and close to 0. Further the error variances (diagonal elements of Σ) are held constant across dimensions to bette highlight the impact the variability of the random-effects has on the models performance. The lata-generating process assumes 10 equally-sized subpopulations, resulting in $J = 10$. The natrix of fixed-effects B is of dimension 10,001 × 5, with distinct sparsity pattern according to the supervised supervised.			0.09	2.16	-0.33	0.55	-0.10	
$\begin{bmatrix} 0.91 & 0.55 & -0.03 & 2.16 & 0.02 \\ -0.26 & -0.10 & -0.13 & 0.02 & 2.16 \end{bmatrix}$ mplying that $r = 5$ and $q = 1$. Notice that Ψ is purposely constructed for the random-effect o differently affect the five dimensional response: while the first component showcases high variance (first entry in the main diagonal), the last one is very small and close to 0. Further he error variances (diagonal elements of Σ) are held constant across dimensions to bette highlight the impact the variability of the random-effects has on the models performance. The lata-generating process assumes 10 equally-sized subpopulations, resulting in $J = 10$. The natrix of fixed-effects B is of dimension 10,001 × 5, with distinct sparsity pattern according to the supervisor.		$\Sigma =$	-0.80	-0.33	2.16	-0.03	-0.13,	
$\begin{bmatrix} -0.26 & -0.10 & -0.13 & 0.02 & 2.16 \end{bmatrix}$ mplying that $r = 5$ and $q = 1$. Notice that Ψ is purposely constructed for the random-effect o differently affect the five dimensional response: while the first component showcases high variance (first entry in the main diagonal), the last one is very small and close to 0. Further he error variances (diagonal elements of Σ) are held constant across dimensions to bette highlight the impact the variability of the random-effects has on the models performance. The lata-generating process assumes 10 equally-sized subpopulations, resulting in $J = 10$. The natrix of fixed-effects B is of dimension 10,001 × 5, with distinct sparsity pattern according the three comprises.			0.91	0.55	-0.03	2.16	0.02	
mplying that $r = 5$ and $q = 1$. Notice that Ψ is purposely constructed for the random-effect o differently affect the five dimensional response: while the first component showcases high variance (first entry in the main diagonal), the last one is very small and close to 0. Further he error variances (diagonal elements of Σ) are held constant across dimensions to bette highlight the impact the variability of the random-effects has on the models performance. The lata-generating process assumes 10 equally-sized subpopulations, resulting in $J = 10$. The natrix of fixed-effects B is of dimension $10,001 \times 5$, with distinct sparsity pattern according the three converses.			-0.26	-0.10	-0.13	0.02	2.16	
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b differently affect the five differentiational response: while the first component showcases high variance (first entry in the main diagonal), the last one is very small and close to 0. Further he error variances (diagonal elements of Σ) are held constant across dimensions to bette highlight the impact the variability of the random-effects has on the models performance. The lata-generating process assumes 10 equally-sized subpopulations, resulting in $J = 10$. The natrix of fixed-effects B is of dimension 10,001 × 5, with distinct sparsity pattern according the three generation.	mplying that $r =$	5 and q	= 1. Not	ice that	Ψ is purp	osely col	Structed for the	random-effects
the error variances (diagonal elements of Σ) are held constant across dimensions to better he error variances (diagonal elements of Σ) are held constant across dimensions to better highlight the impact the variability of the random-effects has on the models performance. The lata-generating process assumes 10 equally-sized subpopulations, resulting in $J = 10$. The natrix of fixed-effects B is of dimension $10,001 \times 5$, with distinct sparsity pattern according the three generation.	o differently affect	t the five	e dimens	sional res	sponse: w	nile the	arst component s	snowcases nigr
highlight the impact the variability of the random-effects has on the models performance. The lata-generating process assumes 10 equally-sized subpopulations, resulting in $J = 10$. The natrix of fixed-effects B is of dimension $10,001 \times 5$, with distinct sparsity pattern according to the performance.	variance (first enti	y in the	main di	agonal),	(100 last 0)	ne is ver	y small and clos	e to U. Further
lata-generating process assumes 10 equally-sized subpopulations, resulting in $J = 10$. The natrix of fixed-effects B is of dimension $10,001 \times 5$, with distinct sparsity pattern according	ne error variance	s (uiago	mai elem	of the re-	∠) are ne	ata basa	and across dimer	isions to better
natrix of fixed-effects B is of dimension $10,001 \times 5$, with distinct sparsity pattern according	ligningni the impa	ci ine va	ullaDillty	0 or the rat	uom-em	teris nas (tions resulting	I = 10 The
naura of fixed-effects \boldsymbol{b} is of dimension 10,001 × 5, with distinct sparsity pattern according	nata-generating pr	ocess a	ssumes 1	o equally	y-sized si		tions, resulting	11 J = 10.1 Interms of 10.1 Interms of
	ланих ог пхел-ен	ects B 1	is of aime	ension IC	1,001 X J	, with dis	sunct sparsity pa	mern according

B rowwise sparse: *B* has entries equal to 0.5 for the first 100 rows, while all the other entries are equal to 0.

B sparse at random: B is equal to 0.5 for approximately 70% of its entries, while all the others are equal to 0.

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	B with dependence structure: B has entries whose magnitude agrees with the correlation
	b with dependence structure. B has childes whose magintude agrees with the contraction structure between the coveristed inducing coefficients to be similar when the absolute
	structure between the covariates, inducing coefficients to be similar when the absolute
	correlation between two predictors is high.
L	astly, \mathbf{Z}_i is an all-one column vector $\forall i = 1, \dots, 10$, while X_i has the first column equal
to	x_1 , meaning that the intercept is included in X_1 in our model specification. The remain-
ir	10,000 dimensions are generated according to a normal random vector with independent
m	parginals for the B rowwise sparse and B sparse at random scenarios while the $cor-$
m	at from triangle function from the fair package (DeBruine (2021)) has been used
to	$a = 110 \text{ m}_{eff} = 110 \text{ m}_{eff} = 100 $
ľ	Taking a cue from the Monte Carlo simulations of Li and Li (2010) for each replication
0	f our experiment the learning framework is structured as follows: we equally divide the
υ λ	I = 600 units in a training set an independent validation set and an independent test set
1	v = 000 units in a training set, an independent variation set and an independent test set,
16	entering a sample size of 200 for each. Nonce that, as to minine the process of DNAm $\frac{10000}{1000000000000000000000000000000$
SI	urrogates creation, the total number of variables ($p = 10,001$) is much larger than the sample
SI	ize. Seven different models, varying λ within a grid, are fitted on the training data:
•	Univariate elastic-net fixed-effects: Univariate elastic-net regression, obtained fitting inde-
	pendent models to each dimension of the multivariate response.
•	Elastic-net fixed-effects: A penalized multitask learning model with elastic-net regulariza-
	tion. The considered penalty is described in Section 3.3.1.
•	Group-lasso fixed-effects: A penalized multitask learning model with multivariate group-
	lasso regularization. The considered penalty is described in Section 3.3.2
•	<i>Network-regularized fixed-effects:</i> Graph-regularized multitask learning model with edge-
	based regularization. The considered penalty is described in Section 3.3.3.
	<i>Elastic-net random-effects:</i> The penalized MLMM methodology introduced in the paper
	with elastic-net regularization (Section 3.3.1).
	<i>Group-lasso random-effects</i> : The penalized MLMM methodology introduced in the paper
ſ	with group-lasso regularization (Section 3.3.2)
	Network-regularized random-effects: The penalized MLMM methodology introduced in
ľ	the paper with edge-based regularization (Section 3.3.3)
	the puper with edge bused regularization (Section 5.5.5).
S	uch an extensive comparison can be regarded as performing a within-scenario ablation study
ir	n which we start from a complex method, and we subsequently remove the random-effects
c	omponent and, finally, the borrow strength property of multivariate regression to be left with
u	nivariate elastic-net fixed-effects models. In this way we investigate the contribution of our
p	roposal to the overall system. The mixing parameter α was set equal to 0.5 for methods
W	vith elastic-net and group-lasso regularizers, while for the network-regularized penalty we
e	mploy <i>five</i> -fold CV to tune λ_X and λ_Y on the training set. For the latter penalty, the adja-
c	ency matrices G_X and G_Y are computed via a thresholding procedure on the correlation
m	natrices of X and Y , respectively, with a threshold equal to 0.1 (Langfelder and Horvath
(2	2008)). Subsequently, the validation dataset is used to select the best shrinkage parameter λ
m	ninimizing the RMSE for every model. The predictive performance is then evaluated on the
te	est set. Lastly, to assess out of groups prediction, models are further validated on 100 exter-
n	al samples, generated according to (1), coming from five extra subpopulations not observed
ir	the training set. The devised simulated experiment is replicated $MC = 100$ times: results
a	re reported in the next subsection.
	4.2. Simulation results. Figure 3 displays boxplots of the Root Mean Squared Error
(]	RMSE) computed for each component of the <i>five</i> -dimensional response on the test set. For
a	Il scenarios we observe that the componentwise predictive performance is heavily affected

by the magnitude of the related diagonal entry in the Ψ matrix. When the grouping effect

PENALIZED MIXED-EFFECTS MULTITASK LEARNING



FIG. 3. Boxplots of the Root Mean Squared Error (RMSE) for MC = 100 repetitions of the simulated exper-iment. RMSE is computed on 200 test points for different methods and three scenarios varying sparsity pattern for **B**.

is negligible (fifth dimension Y_5), all methods showcase comparable predictive performance. Contrarily, the RMSE deteriorates for fixed-effects models in those response components for which the grouping impact is more relevant. The same does not happen for the mixed-effects counterparts, as the random intercept effectively captures baseline differences across groups. Interestingly, the penalty type does not seem to influence the RMSE metric, with our proposal displaying excellent results irrespective of the chosen shrinkage functional for all scenarios. On the other hand, when it comes to perform out of groups prediction, the gain achieved by including random-effects decreases, and the outcome of models with fixed and mixed-effects are fairly similar. In details for the latter class of methods, the unconditional (population level) intercepts are employed when making predictions for unobserved groups. Notwithstanding, we recognize that results are no worse than those obtained with fixed-effects procedures, corroborating the generalizability of our proposals in external cohorts.

Figure 4 displays the analogue of the percentage of variation due to random effects (PVRE) metric, computed taking the ratio between the diagonal elements of $\hat{\Psi}$ and the sum of the diagonals of $\hat{\Psi}$ and $\hat{\Sigma}$. From the plot, it clearly emerges how the grouping impact differently affects the variability in the five components of the response.

Figure 5 reports boxplots of the Frobenius distance between true and estimated matrices of fixed-effects **B**. When looking at $\|\mathbf{B} - \mathbf{B}\|_F$ under the three scenarios, we observe some interesting facts. First off, it is immediately noticed that the univariate elastic-net fixed-effects model showcases the poorest performance, in particular, for the **B** with dependence structure experiment. This is due to the fact that the different components of the response vector are related in our simulated specification, and, therefore, fitting separate regression models re-sults in a loss of quality for the estimator. Second, we observe that for the **B** rowwise sparse scenario the group-lasso random-effects model is the best performing one among all the com-petitors, displaying the lowest median distance to B. This may be expected, as such method is precisely constructed to identify a matrix of fixed-effects with a rowwise sparsity pattern. Furthermore, notice that the performance of the group-lasso random-effects is slightly better





FIG. 4. Boxplots of the Percentage of Variation due to Random Effects (PVRE) for MC = 100 repetitions of the simulated experiment.

than its fixed-effects counterpart. For the remaining scenarios, the superiority of the mixed-effects procedures is not so apparent, and both fixed and random-effects models demonstrate a comparable performance. An only modest gain is showcased by the *network-regularized random-effects* method for which the inclusion of the adjacency matrices G_X and G_Y in the penalty specification helps in better recovering the **B** structure.

We now look at the ability of the competing procedures in identifying the true underlying sparsity patterns in the matrix of fixed-effects **B** under the different scenarios. In so doing we compute, for each replication of the simulated experiment, the F_1 score defined as follows:











FIG. 6. Boxplots of the F_1 score for MC = 100 repetitions of the simulated experiment for different methods and three scenarios varying sparsity pattern for B.

where with $t_{\rm D}$ we denote the number of zero entries in **B** correctly estimated as such, while fp and fn represent the number of nonzero entries wrongly shrunk to 0 and the number of zero entries not shrunk to 0, respectively. Figure 6 displays boxplots of such metric for differ-ent methods and scenarios. We notice that **B** rowwise sparse structure displays much higher F_1 score, irrespective of the considered method, than the other two cases. Intuitively, the for-mer scenario is less challenging since, while all penalty types can potentially accommodate a rowwise sparse **B**, group-lasso regularizers only force entire rows of **B** to be shrunk to 0. We further observe that the F_1 score is higher for the group-lasso random-effects than for its fixed-effects counterpart, highlighting that a penalized mixed-effects modeling strategy, in presence of grouped data and a rowwise sparse **B**, not only increases the predictive ac-curacy but also improves the recovery of the sparsity pattern in the fixed-effects matrix. The same does not happen in the remaining two scenarios for which all methods display a com-parable empirical distribution of the F_1 metric across simulations. The same behavior was already observed in high-dimensional linear mixed-effects modeling for univariate responses (Schelldorfer, Bühlmann and van de Geer (2011)).

As a last worthy note, we acknowledge that, as rightly underlined by an anonymous re-viewer, the present simulation study does not consider any violation in the distributional assumptions of the involved quantities. In this regard we replicated the experiment using both a multivariate skew-normal and multivariate skew-t distributions (Azzalini and Capi-tanio (2013)) as generative models for the error term, but we did not report the results in the paper since no dramatic changes were observed in model performances. While clearly more extreme scenarios could be considered, results in the literature have previously vali-dated the robustness of linear mixed-effects models to violations of distributional assump-tions (McCulloch and Neuhaus (2011)). For further details about the simulation study, the Supplementary Material (Cappozzo, Ieva and Fiorito (2023)) provides additional figures and a note on the overall computing times.

All in all, the good performances displayed by our proposal, particularly when coupled with a multivariate group-lasso penalty, encourage its usage in multivariate DNAm surrogates creation: promising results are reported in the next section.

5. DNAm biomarkers analysis for EPIC and EXPOSOMICS datasets. The method-ology described in Section 3 is employed to build a *five*-dimensional DNAm biomarker of

 hypertension and hyperlipidemia. As mentioned in the Introduction, DNAm surrogates possess extensive advantages over their blood-measured counterparts since: 1. DNAm biomarkers directly account for genetic susceptibility and subject specific response to risk factors. 2. DNAm biomarkers can immediately be computed whenever DNAm values are accessible. This is particularly useful when the risk factors of interest have not been directly measured 3. Further understanding of the biomolecular mechanisms associated with complex pheno types can be acquired through a pathway enrichment analysis (Reimand et al. (2019), al lowing to identify molecular pathways overrepresented among the regressors involved in the surrogate construction (i.e., the CpG sites whose associated parameters are not shrund to 0). We subsequently assess how well the so-devised surrogates perform, for both internal and external cohorts, in reconstructing the blood measured biomarkers (Section 5.1) and in predicting the clinical endpoint of interest, namely, the future presenc/absence of CVD events (Section 5.2). Lastly, we study from a biological perspective the CpG sites selection operated by the multivariate group-lasso penalty, comparing it with previous findings available in the literature (Section 5.3). 5.1. DNAm surrogates creation and validation. To construct multivariate DNAm surrogates, several penalized models are fitted to the EPIC Italy training set, varying shrinkage factors and considering both fixed and random-effects components. As mentioned in Section 2, the design matrix comprises of <i>p</i> = 62, 130 variables. Thus, redundancies are likely to occur as the feature space is constituted by the union of CpG sites prescreened by univariate enginenome-wide analyses. After having standardized the covariates, for each model the penalty term <i>i</i> is tund via 10-fold CV, while the mixing parameter <i>a</i> is kept fixed and equa to 0.5. Results on the internal cohort are summarized in Table 1, whe	16 A. CAPPOZZO, F. IEVA AND G. FIORITO
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	The employment of the multivariate group-lasso penalty within a mixed-effects multitask
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Vlodel	Penalty type	Response	DBP	HDL	LDL	SBP	TG
Random-effects	Group-lasso	Multivariate	0.1314	0.2384	0.2707	0.1412	0.4735
Random-effects	Elastic-net	Multivariate	0.1335	0.2504	0.2821	0.1450	0.4890
Fixed-effects	Group-lasso	Multivariate	0.1409	0.2750	0.2969	0.1574	0.5142
Fixed-effects	Elastic-net	Multivariate	0.1286	0.2479	0.2859	0.1359	0.5002
Fixed-effects	Elastic-net	Univariate	0.1331	0.2733	0.3136	0.1368	0.5251

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between DNAm and the exposure variable) the same set of CpG sites (Tyler, Crawford and Pendergrass (2013), Richard et al. (2017)).

Internal validation results, obtained for the EPIC Italy test set, highlight the benefits of mixed-effects modeling while constructing DNAm surrogates for data possessing a grouping-structure. The random intercept allows to account for centerwise variability that is induced by geographic genetical variation as well as by samples collection and storage that are likely to differ across centers. Notice that, in general, if not properly modeled, the unexplained heterogeneity in multicenter studies could be taken care of with dedicated batch-effect re-moval procedures (see, e.g., Johnson, Li and Rabinovic (2007)). Nonetheless, when devel-oping DNAm biomarkers, it is of interest to devise study-invariant surrogates to be readily computed also for samples not belonging to the learning cohort. To this aim, we validate the performance of the models estimated on the EPIC training set in constructing surrogates for the external EXPOsOMICS cohort (see Section 2). In this context the grouping informa-tion (i.e., the center of recruitment) cannot be considered when performing predictions with mixed-effects models, and the unconditional (population-level) intercepts are thus utilized. RMSE between estimated and blood-measured biomarkers for the EXPOSOMICS valida-tion cohort are reported in Table 2. Likewise for the EPIC Italy test set, the lowest RMSEs for all but SBP biomarker are retained employing a penalized random-intercept model with multivariate group-lasso penalty. Interestingly, the predictive outcomes, obtained in the EX-POSOMICS validation dataset, are comparable with the ones reported in Table 1 with slightly worse performances, as it may be expected for those dimensions of the response displaying higher PVRE indexes. All in all, the proposed approach exhibits promising results when it comes to multivariate DNAm biomarker creation, outperforming the current employed pro-cedure in both internal and external validation cohorts.

5.2. Association of DNAm surrogates with CVD risk. DNAm surrogates creation is not a stand-alone regression problem, as its primary aim is to provide reliable covariates for dis-eases prediction models (Fernández-Sanlés et al. (2021), Odintsova et al. (2021), Hidalgo et al. (2021)). We, therefore, validate whether employing the estimated DNAm surrogates acts as a superior proxy of blood measured biomarkers in association analyses. In details within the cohort of patients in the EPIC Italy test set, we build logistic regression models to predict the probability of cardiovascular risk, using as regressors either the blood measured biomarkers or the two best performing DNAm surrogates devised in the previous section, ad-justing for sex and age. The receiver operating characteristic (ROC) curves and the associated area under the curve (AUC) metrics for the considered methods are displayed in Figure 7. As expected, in light of the RMSE results reported in Table 1, we notice that classification perfor-mances are similar among the competing models. Nonetheless, the logistic curves regressed on the DNAm based surrogates seem to outperform the blood measured counterparts. Inter-estingly, all surrogates in Table 1 define logistic regression models whose AUCs are higher



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FIG. 7. Receiver operating characteristic curves and area under the curve metrics for the association analyses of DNAm surrogates with CVD risk, Italy EPIC test set.

than those retrieved by the blood-measured biomarkers, with the best performance attained by the penalized random-intercept model with multivariate group-lasso penalty.

We further assess the association of DNAm surrogates with CVD risk in the external EX-POsOMICS study. For this dataset values of the blood-based biomarkers are available only for a subset of volunteers; we thus construct the logistic regression models by means of the surrogates only. Such a situation is quite common in validation data and in line with the prin-ciple DNAm surrogates that were devised in the first place. Also, in this context the predictive performance of our novel proposal, coupled with a multivariate group lasso penalty, is higher with respect to state-of-the-art surrogates created via elastic-net fixed-effects models. The associated receiver operating characteristic curves as well as additional figures related to the DNAm biomarkers analysis are reported in the Supplementary Material (Cappozzo, Ieva and Fiorito (2023)).

The association analyses described in this section cast light on the applicability of the devised DNAm surrogates as an enriched and patient-specific proxy of their blood measured counterparts, in both internal and external cohorts. These favorable outcomes indicate that using models based on DNAm surrogates could be more appropriate for prediction tasks, such as CVD prevention, since they can possibly incorporate individual characteristics not directly recorded in the blood-measured biomarkers.

5.3. CpG sites selection and gene set enrichment analyses of inflammatory pathways. In the previous sections the newly devised random-intercept model for multitask learning has demonstrated superior predictive performance when it comes to DNAm surrogates creation and CVD prediction. Hereafter, we examine the epidemiological rationale of the multivariate group-lasso penalty, compared to univariate elastic-nets, investigating the biological reliabil-ity of the selected features (CpG sites). The univariate elastic-nets extract 178, 504, 518, 79,

A. CAPPOZZO, F. IEVA AND G. FIORITO 497 CpG sites for diastolic blood pressure, HDL cholesterol, LDL cholesterol, systolic blood pressure and triglycerides, respectively. As reported in Table 1, the total number of unique З CpGs is 1712. However, despite the high degree of correlation among the multivariate out-comes, no CpGs were in common in the five sets, and only a minor percentage of CpGs was shared among two or more responses. Instead, as previously described, our MLMM proce-dure regularized with a multivariate group-lasso penalty extracts 417 features that are asso-ciated with the five outcomes at the same time, a biological mechanism known as pleiotropy (Atchley and Hall (1991a, 1991b)). We are further interested in assessing whether the selected CpG sites are associated with specific biological pathways. To do so, gene set enrichment analyses (Subramanian et al. (2005)) are performed on the features retained by penalized MLMM and univariate elastic-net models. Specifically, given that the number of CpGs extracted for each method is small (hundreds of CpGs selected from an initial set of 295,614 features), an analysis based on all of the biomolecular pathways described in the canonical datasets, that is, KEGG (Yi et al. (2020)), GO (Gene Ontology Consortium (2004)) and Reactome (Fabregat et al. (2018)), would be underpowered. Therefore, to overcome this limitation and with inflammation being the main mechanism involved in the onset of the majority of chronic diseases, we focus our analyses on the 17 inflammatory pathways described in Loza et al. (2007). Enrichment analy-ses results are summarized in Table 3. For each list of CpGs, we test for overrepresentation of features in inflammatory pathways using the method implemented in the missMethyl R package (Phipson, Maksimovic and Oshlack (2016)). Considering the CpGs extracted by our multivariate approach, we find significant enrichment for CpGs in four inflammatory path-ways. These results agree with previous literature suggesting that hypertension and hyperlip-idaemia are associated with the dysregulation of molecular pathways regulating apoptosis, oxidative stress and the immune system (Senoner and Dichtl (2019), Dong et al. (2020)). In-stead, modeling the outcomes one by one, using univariate models, leads to a less consistent

TABLE 3

Empirical p-values of the enrichment analyses computed using a permutation procedure via the gometh function in the missMethyl R package. Empirical p-values lower than 0.05, highlighted in bold, indicate significant overrepresentation

		Uni	variate E	lastic-net	Fixed-e	nects
Inflammatory pathway	Group-lasso Random-effects	DBP	HDL	LDL	SBP	TG
Leukocyte signaling	0.006	0.14	0.35	0.01	1	1
ROS/Glutathione/Cytotoxic granules	0.009	0.06	0.15	1	0.03	0.15
Apoptosis Signaling	0.01	1	1	0.17	1	0.07
Natural Killer Cell Signaling	0.01	0.36	0.10	1	1	1
PI3K/AKT Signaling	0.18	1	0.26	0.03	1	0.22
Innate pathogen detection	0.26	0.12	0.30	0.31	1	1
Cytokine signaling	0.36	1	1	0.0003	1	0.10
Adhesion-Extravasation-Migration	1	0.18	0.003	0.02	0.09	0.10
Calcium Signaling	1	1	1	0.08	1	1
Complement Cascase	1	1	1	1	1	0.16
Glucocorticoid/PPAR signaling	1	1	0.10	1	0.17	0.10
G-Protein Coupled Receptor Signaling	1	1	1	1	0.05	0.27
MAPK signaling	1	1	0.14	0.49	1	0.005
NF-kB signaling	1	1	0.16	0.16	1	0.15
Phagocytosis-Ag presentation	1	1	1	0.26	1	1
Eicosanoid Signaling	1	1	1	1	1	1
TNF Superfamily Signaling	1	1	0.01	0.14	1	1

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pattern of associations. In fact, we find only one (and always different) significant pathway
 per analysis.

All in all, the results reported in this section support the advantages of modeling mul tiple correlated outcomes not only from a prediction perspective, for both blood measured
 biomarkers and endpoint of interest but also considering the biological reliability of the ex tracted features.

6. Discussion and further work. In the present paper, we have proposed a novel frame-work for mixed-effects multitask learning suitable for high-dimensional data. The ubiquitous presence in modern applications of "p bigger than N" problems asks for the development of ad hoc statistical tools able to cope with such scenarios. By resorting to penalized likelihood estimation, we have devised a general purpose EM algorithm capable of accommodating any penalty type that has been previously defined for fixed-effects models. We have examined three functional forms for the penalty term, discussing pros and cons of each and providing convenient routines for model fitting. The proposal has been accompanied by some consider-ations on distinguishing features, like how to quantify response specific random-effects, and other more general issues concerning initialization, convergence and model selection.

The work has been motivated by the problem of developing a multivariate DNAm biomarker of cardiovascular and high blood pressure comorbidities from a multicenter study. The EPIC Italy dataset has been analyzed using diastolic blood pressure, systolic blood pressure, high-density lipoprotein, low-density lipoprotein and triglycerides as response vari-ables, regressing them on 62, 128 CpG sites and accounting for between-center heterogeneity. Our modeling framework, coupled with a multivariate group-lasso penalty, has demonstrated to outperform the state-of-the-art alternative, both in terms of predictive power and biomed-ical interpretation. Remarkably, the number of CpG sites deemed as relevant in the multidi-mensional surrogate creation was found to be lower than those identified by separately fitting penalized models for each risk factor. Decreasing the amount of relevant CpG sites is crucial to reduce sequencing costs for future studies, with the final aim of querying only a limited number of targeted genomic regions. Such a result may thereupon favor the adoption of our methodological approach for building DNAm surrogates.

The devised pipeline also possesses some limitations. The EWAS results are adjusted for clinical covariates external to the analysis, and this may thus affect the preprocessing out-come. Moreover, the level of strictness in the screening process is influenced by the chosen threshold on the p-values. On this wise two different, yet both sensible, strategies can be adopted. On the one hand, one may rely on the "Occam's razor" principle, preferring to use a stricter threshold being it the simplest and fastest option. On the other hand, one can pos-itively include many redundant variables in the design matrix, relying on the model ability to shrink coefficients of irrelevant features to zero. Concurrently, further insights about the associations between DNA methylation and blood-measured biomarkers may be unraveled by means of sparse multiple canonical correlation analysis (Rodosthenous, Shahrezaei and Evangelou (2020), Witten, Tibshirani and Hastie (2009), Witten and Tibshirani (2009)), while other modeling approaches, such as deep-learning (Nguyen et al. (2022), Yuan et al. (2022)), Bayesian methods (Zhao et al. (2021a, 2021b)) and boosting machines (Sigrist (2022)), could be profitably adapted to build DNAm multidimensional surrogates.

A direction for future research concerns promoting the application of the proposed pro-cedure in creating additional multidimensional DNAm biomarkers, conveniently embedding mixed-effects and customized penalty types. In this regard and of particular interest may be the definition of a shrinkage term for which the grouping in B is introduced from both responses and predictors: the former is induced by the multivariate nature of Y (i.e., the r re-sponses), while the latter can stem from any structure present in X (e.g., CpG islands). Such

	22 A. CAPPOZZO, F. IEVA AND G. FIORITO	
1	a problem could be solved by extending the multivariate sparse group lasso, proposed by Li,	1
2	Nan and Zhu (2015), to the mixed-effects framework.	2
3	In addition, having assumed random intercepts for each and every component in a low-	3
4	dimensional response framework was only motivated by the application at hand, and it may	4
5	not be valid in general. Thus, a two-fold methodological development naturally arises: a first	5
6	one concerning the definition of response-specific random-effects in multitask learning and	6
7	another accounting for the inclusion of custom penalties when dealing with high-dimensional	7
8	response variables. Furthermore, the latter may also possess a mixed-type structure, with	8
9	components simultaneously being nominal, ordinal, discrete and/or continuous. Some pro-	9
10	posals are currently under study, and they will be the object of future work.	10
11		11
12	Acknowledgments. The authors would like to thank the three anonymous reviewers, the	12
13	Editor, and the Associate Editor for their thorough examination of the manuscript and their	13
14	valuable, constructive comments. Their input has greatly enhanced the quality of the work.	14
15	The authors also thank the EPIC Italy research group (Carlotta Sacerdote, Vittorio Krogh,	15
16	Domenico Palli, Salvatore Panico, Rosario Tumino, Paolo Vineis and their collaborators) for	16
17	giving access to the data used in this study.	17
10		10
19	SUPPLEMENTARY MATERIAL	20
20	Additional figures (DOI: 10 1214/23-AOAS1760SUPPA: pdf) It contains additional fig-	20
22	ures for both the simulation study and the real data analysis reported in Sections 4 and 5.	22
23	Code (DOI: 10.1214/23-AOAS1760SUPPB: .zip). It contains the R package em]mm im-	23
24 25	plementing the method proposed in the manuscript.	24 25
26	DEEDENCES	26
27	KEFEKENCES	27
28 29	ANASTASIADI, D., ESTEVE-CODINA, A. and PIFERRER, F. (2018). Consistent inverse correlation between DNA methylation of the first intron and gene expression across tissues and species. <i>Epigenet. Chromatin</i> 11 37. https://doi.org/10.1186/s12072.018.0205_1	28 29
30 31	ATCHLEY, W. R. and HALL, B. K. (1991a). A model for development and evolution of complex morphological structures. <i>Biol. Pay. Camb. Philos. Soc.</i> 66 101, 157	30 31
32 33	ATCHLEY, W. R. and HALL, B. K. (1991b). A model for development and evolution of complex morphological structures. <i>Biol. Rev.</i> 66 101–157.	32 33
34	AZZALINI, A. and CAPITANIO, A. (2013). The Skew-Normal and Related Families 3. Cambridge Univ. Press,	34
35	Cambridge.	35
36	BATTRAM, T., YOUSEFI, P., CRAWFORD, G., PRINCE, C., BABAEI, M. S., SHARP, G., HATCHER, C., VEGA-	36
37	SALAS, M. J., KHODABAKHSH, S. et al. (2022). The EWAS catalog: A database of epigenome-wide associ- ation studies. <i>Wellcome Open Res</i> 7	37
38	CAMPAGNA, M. P., XAVIER, A., LECHNER-SCOTT, J., MALTBY, V., SCOTT, R. J., BUTZKUEVEN, H.,	38
39	JOKUBAITIS, V. G. and LEA, R. A. (2021). Epigenome-wide association studies: Current knowledge, strate-	39
40	gies and recommendations. Clin. Epigenet. 13 214. https://doi.org/10.1186/s13148-021-01200-8	40
41	CAPPOZZO, A., IEVA, F. and FIORITO, G. (2023). Supplement to "A general framework for penalized	41
42	https://doi.org/10.1214/23-AOAS1760SUPPA https://doi.org/10.1214/23-AOAS1760SUPPR	42
43	CAPPOZZO, A., MCCRORY, C., ROBINSON, O., FRENI STERRANTINO, A., SACERDOTE, C., KROGH, V.,	43
44	PANICO, S., TUMINO, R., IACOVIELLO, L. et al. (2022). A blood DNA methylation biomarker for predicting	44
45	short-term risk of cardiovascular events. <i>Clin. Epigenet.</i> 14 121.	45
46	UARUANA, R. (1997). Multitask learning. Mach. Learn. 28 41–75.	46
47	AGUILERA-ALBESA, S., TROYA, J., VALENCIA-RAMOS, J. et al. (2021). Enigenome-wide association study	47
48	of Covid-19 severity with respiratory failure. <i>eBioMedicine</i> 66 103339.	48
49	CHENG, W., ZHANG, X., GUO, Z., SHI, Y. and WANG, W. (2014). Graph-regularized dual Lasso for robust	49
50	eQTL mapping. <i>Bioinformatics</i> 30 139–148.	50
01		51

PENALIZED MIXED-EFFECTS MULTITASK LEARNING CHIPPERFIELD, J. O. and STEEL, D. G. (2012). Multivariate random effect models with complete and incomplete data. J. Multivariate Anal. 109 146-155. MR2922860 https://doi.org/10.1016/j.jmva.2012.02.014 CHUNG, F. R. K. and GRAHAM, F. C. (1997). Spectral Graph Theory 92. Am. Math. Soc., Providence. Colicino, E., Just, A., Kioumourtzoglou, M.-A., Vokonas, P., Cardenas, A., Sparrow, D., Weis-SKOPF, M., NIE, L. H., HU, H. et al. (2021). Blood DNA methylation biomarkers of cumulative lead exposure in adults. J. Expo. Sci. Environ. Epidemiol. 31 108-116. Conole, E. L. S., Stevenson, A. J., Green, C., Harris, S. E., Maniega, S. M., Valdés-HERNÁNDEZ, M. D. C., HARRIS, M. A., BASTIN, M. E., WARDLAW, J. M. et al. (2020). An epigenetic proxy of chronic inflammation outperforms serum levels as a biomarker of brain ageing. MedRxiv 2020.10.08.20205245. GENE ONTOLOGY CONSORTIUM (2004). The Gene Ontology (GO) database and informatics resource. Nucleic Acids Res. 32 258D-261. DAWID, A. P. (1981). Some matrix-variate distribution theory: Notational considerations and a Bayesian application. Biometrika 68 265-274. MR0614963 https://doi.org/10.1093/biomet/68.1.265 DEBRUINE, L. (2021). faux: Simulation for Factorial Designs. DEMIDENKO, E. (2013). Mixed Models: Theory and Applications with R, 2nd ed. Wiley Series in Probability and Statistics. Wiley, Hoboken, NJ. MR3235905 DEMPSTER, A. P., LAIRD, N. M. and RUBIN, D. B. (1977). Maximum likelihood from incomplete data via the EM algorithm. J. Roy. Statist. Soc. Ser. B 39 1-38. MR0501537 DIRMEIER, S., FUCHS, C., MUELLER, N. S. and THEIS, F. J. (2018). netReg: Network-regularized linear models for biological association studies. Bioinformatics 34 896-898. https://doi.org/10.1093/bioinformatics/ btx677 DONG, W., CHEN, H., WANG, L., CAO, X., BU, X., PENG, Y., DONG, A., YING, M., CHEN, X. et al. (2020). Exploring the shared genes of hypertension, diabetes and hyperlipidemia based on microarray. Braz. J. Pharm. Sci. 56 1–12. Fabregat, A., Jupe, S., Matthews, L., Sidiropoulos, K., Gillespie, M., Garapati, P., Haw, R. JASSAL, B., KORNINGER, F. et al. (2018). The reactome pathway knowledgebase. Nucleic Acids Res. 46 D649-D655. FAN, J. and LV, J. (2008). Sure independence screening for ultrahigh dimensional feature space. J. R. Stat. Soc. Ser. B. Stat. Methodol. 70 849-911. MR2530322 https://doi.org/10.1111/j.1467-9868.2008.00674.x FAN, J., SAMWORTH, R. and WU, Y. (2009). Ultrahigh dimensional feature selection: Beyond the linear model. J. Mach. Learn. Res. 10 2013-2038. MR2550099 FAZZARI, M. J. and GREALLY, J. M. (2010). Introduction to Epigenomics and Epigenome-Wide Analysis. In Statistical Methods in Molecular Biology 243-265. Humana Press, Totowa, NJ. FERNÁNDEZ-SANLÉS, A., SAYOLS-BAIXERAS, S., SUBIRANA, I., SENTÍ, M., PÉREZ-FERNÁNDEZ, S., DE CASTRO MOURA, M., ESTELLER, M., MARRUGAT, J. and ELOSUA, R. (2021). DNA methylation biomarkers of myocardial infarction and cardiovascular disease. Clin. Epigenet. 13 86. https://doi.org/10.1186/ s13148-021-01078-6 FIORITO, G., PEDRON, S., OCHOA-ROSALES, C., MCCRORY, C., POLIDORO, S., ZHANG, Y., DUGUÉ, P.-A. RATLIFF, S., ZHAO, W. N. et al. (2022). The role of epigenetic clocks in explaining educational inequalities in mortality: A multicohort study and meta-analysis. J. Gerontol., Ser. A 77 1750-1759. FIORITO, G., VLAANDEREN, J., POLIDORO, S., GULLIVER, J., GALASSI, C., RANZI, A., KROGH, V., GRI-ONI, S., AGNOLI, C. et al. (2018). Oxidative stress and inflammation mediate the effect of air pollution on cardio- and cerebrovascular disease: A prospective study in nonsmokers. Environ. Mol. Mutagen. 59 234-246.

- https://doi.org/10.1002/em.22153 FROHLICH, H. and ZELL, A. (2005). Efficient parameter selection for support vector machines in classification and regression via model-based global optimization. In Proceedings. 2005 IEEE International Joint Confer-ence on Neural Networks, 2005. 3 1431-1436. IEEE, Los Alamitos. GAŁECKI, A. and BURZYKOWSKI, T. (2013). Linear Mixed-Effects Models Using R. Springer Texts in Statistics.
- Springer, New York. MR3024843 https://doi.org/10.1007/978-1-4614-3900-4 Guida, F., Sandanger, T. M., Castagné, R., Campanella, G., Polidoro, S., Palli, D., Krogh, V.
- TUMINO, R., SACERDOTE, C. et al. (2015). Dynamics of smoking-induced genome-wide methylation changes with time since smoking cessation. Hum. Mol. Genet. 24 2349-2359. https://doi.org/10.1093/hmg/ ddu751 HASTIE, T., TIBSHIRANI, R. and WAINWRIGHT, M. (2015). Statistical Learning with Sparsity. Monographs on
- Statistics and Applied Probability 143. CRC Press, Boca Raton, FL. MR3616141 HIDALGO, B. A., MINNIEFIELD, B., PATKI, A., TANNER, R., BAGHERI, M., TIWARI, H. K., ARNETT, D. K.
- and IRVIN, M. R. (2021). A 6-CpG validated methylation risk score model for metabolic syndrome: The HyperGEN and GOLDN studies. PLoS ONE 16 e0259836. https://doi.org/10.1371/journal.pone.0259836

З

A. CAPPOZZO, F. IEVA AND G. FIORITO HILLARY, R. F. and MARIONI, R. E. (2020). MethylDetectR: A software for methylation-based health profiling. Wellcome Open Res. 5 283. https://doi.org/10.12688/wellcomeopenres.16458.2 JOHNSON, W. E., LI, C. and RABINOVIC, A. (2007). Adjusting batch effects in microarray expression data using empirical Bayes methods. Biostatistics 8 118-127. JORDAN, M. I. (2013). On statistics, computation and scalability. Bernoulli 19 1378–1390. MR3102908 https://doi.org/10.3150/12-BEJSP17 KIM, S., PAN, W. and SHEN, X. (2013). Network-based penalized regression with application to genomic data. Biometrics 69 582-593. MR3106586 https://doi.org/10.1111/biom.12035 KIM, S. and XING, E. P. (2012). Tree-guided group lasso for multi-response regression with structured sparsity, with an application to EQTL mapping. Ann. Appl. Stat. 6 1095-1117. MR3012522 https://doi.org/10.1214/ 12-AOAS549 LANGFELDER, P. and HORVATH, S. (2008). WGCNA: An R package for weighted correlation network analysis. BMC Bioinform. 9 559. LARIA, J. C., CARMEN AGUILERA-MORILLO, M. and LILLO, R. E. (2019). An iterative sparse-group lasso. J. Comput. Graph. Statist. 28 722-731. MR4007753 https://doi.org/10.1080/10618600.2019.1573687 LI, C. and LI, H. (2010). Variable selection and regression analysis for graph-structured covariates with an application to genomics. Ann. Appl. Stat. 4 1498-1516. MR2758338 https://doi.org/10.1214/10-AOAS332 LI, Y., NAN, B. and ZHU, J. (2015). Multivariate sparse group lasso for the multivariate multiple linear regression with an arbitrary group structure. Biometrics 71 354–363. MR3366240 https://doi.org/10.1111/biom.12292 LOZA, M. J., MCCALL, C. E., LI, L., ISAACS, W. B., XU, J. and CHANG, B.-L. (2007). Assembly of inflammation-related genes for pathway-focused genetic analysis. PLoS ONE 2 e1035. LU, A. T., QUACH, A., WILSON, J. G., REINER, A. P., AVIV, A., RAJ, K., HOU, L., BACCARELLI, A. A., LI, Y. et al. (2019). DNA methylation GrimAge strongly predicts lifespan and healthspan. Aging 11 303–327. Marabita, F., Almgren, M., Lindholm, M. E., Ruhrmann, S., Fagerström-Billai, F., Jagodic, M., SUNDBERG, C. J., EKSTRÖM, T. J., TESCHENDORFF, A. E. et al. (2013). An evaluation of analysis pipelines for DNA methylation profiling using the illumina HumanMethylation450 BeadChip platform. Epigenetics 8 333-346. https://doi.org/10.4161/epi.24008 MCCULLOCH, C. E. and NEUHAUS, J. M. (2011). Misspecifying the shape of a random effects distribution: Why getting it wrong may not matter. Statist. Sci. 26 388-402. MR2917962 https://doi.org/10.1214/11-STS361 MCLACHLAN, G. J. and KRISHNAN, T. (2008). The EM Algorithm and Extensions, 2nd ed. Wiley Series in Probability and Statistics. Wiley, Hoboken, NJ. MR2392878 https://doi.org/10.1002/9780470191613 MENG, X.-L. and RUBIN, D. B. (1993). Maximum likelihood estimation via the ECM algorithm: A general framework. Biometrika 80 267-278. MR1243503 https://doi.org/10.1093/biomet/80.2.267 NGUYEN, T. M., LE, H. L., HWANG, K.-B., HONG, Y.-C. and KIM, J. H. (2022). Predicting high blood pressure using DNA methylome-based machine learning models. Biomedicines 10 1406. OBOZINSKI, G., TASKAR, B. and JORDAN, M. I. (2010). Joint covariate selection and joint subspace selec-tion for multiple classification problems. Stat. Comput. 20 231-252. MR2610775 https://doi.org/10.1007/ s11222-008-9111-x OBOZINSKI, G., WAINWRIGHT, M. J. and JORDAN, M. I. (2009). High-dimensional support union recovery in multivariate regression. In Advances in Neural Information Processing Systems 21-Proceedings of the 2008 Conference 1217–1224. OBOZINSKI, G., WAINWRIGHT, M. J. and JORDAN, M. I. (2011b). Support union recovery in high-dimensional multivariate regression. Ann. Statist. 39 1-47. MR2797839 https://doi.org/10.1214/09-AOS776 ODINTSOVA, V. V., REBATTU, V., HAGENBEEK, F. A., POOL, R., BECK, J. J., EHLI, E. A., VAN BEIJSTER-VELDT, C. E. M., LIGTHART, L., WILLEMSEN, G. et al. (2021). Predicting complex traits and exposures from polygenic scores and blood and buccal DNA methylation profiles. Front. Psychiatr. 12 1–17. PANICO, S., DELLO IACOVO, R., CELENTANO, E., GALASSO, R., MUTI, P., SALVATORE, M. and MANCINI, M. (1992). Progetto ATENA, a study on the etiology of major chronic diseases in women: De-sign, rationale and objectives. Eur. J. Epidemiol. 8 601-608. PHIPSON, B., MAKSIMOVIC, J. and OSHLACK, A. (2016). missMethyl: An R package for analyzing data from Illumina's HumanMethylation450 platform. Bioinformatics 32 286-288. PINHEIRO, J. and BATES, D. (2006). Mixed-Effects Models in S and S-PLUS. Springer, Berlin. RAULUSEVICIUTE, I., DRABLØS, F. and RYE, M. B. (2020). DNA hypermethylation associated with upregu-lated gene expression in prostate cancer demonstrates the diversity of epigenetic regulation. BMC Med. Genom. 13 6. REIMAND, J., ISSERLIN, R., VOISIN, V., KUCERA, M., TANNUS-LOPES, C., ROSTAMIANFAR, A., WADI, L., MEYER, M., WONG, J. et al. (2019). Pathway enrichment analysis and visualization of omics data using g:Profiler, GSEA, Cytoscape and EnrichmentMap. Nat. Protoc. 14 482-517. REINSEL, G. (1984). Estimation and prediction in a multivariate random effects generalized linear model. J. Amer. Statist. Assoc. 79 406-414. MR0755095

	PENALIZED MIXED-EFFECTS MULTITASK LEARNING 25	
1	RIROLL E. HUNT K. SLIMANI, N. FERRARI, P. NORAT T. FAHEY, M. CHARRONDIÈRE, I. HÉMON, B.	1
2	CASAGRANDE, C. et al. (2002). European Prospective Investigation into Cancer and Nutrition (EPIC): Study	2
3	populations and data collection. <i>Public Health Nutr.</i> 5 1113–1124.	3
4	RICHARD, M. A., HUAN, T., LIGTHART, S., GONDALIA, R., JHUN, M. A., BRODY, J. A., IRVIN, M. R.,	1
5	MARIONI, R., SHEN, J. et al. (2017). DNA methylation analysis identifies loci for blood pressure regulation.	5
6	Am. J. Hum. Genet. 101 888–902.	6
7	KODOSTHENOUS, I., SHAHREZAEI, V. and EVANGELOU, M. (2020). Integrating multi-OMICS data through sparse canonical correlation analysis for the prediction of complex traits: A comparison study. <i>Bioinformatics</i>	7
,	36 4616–4625. https://doi.org/10.1093/bioinformatics/btaa530	,
8	ROHART, F., SAN CRISTOBAL, M. and LAURENT, B. (2014). Selection of fixed effects in high dimensional	8
9	linear mixed models using a multicycle ECM algorithm. Comput. Statist. Data Anal. 80 209-222. MR3240488	9
10	https://doi.org/10.1016/j.csda.2014.06.022	10
11	SCHAFER, J. L. and YUCEL, R. M. (2002). Computational strategies for multivariate linear mixed-effects	11
12	models with missing values. J. Comput. Graph. Statist. 11 437–457. MR 1938143 https://doi.org/10.1198/	12
13	SCHELLDORFER, L. BÜHLMANN, P. and VAN DE GEER, S. (2011). Estimation for high-dimensional linear	13
14	mixed-effects models using ℓ_1 -penalization. Scand. J. Stat. 38 197–214. MR2829596 https://doi.org/10.1111/	14
15	j.1467-9469.2011.00740.x	15
16	SCHWARZ, G. (1978). Estimating the dimension of a model. Ann. Statist. 6 461-464. MR0468014	16
17	SENONER, T. and DICHTL, W. (2019). Oxidative stress in cardiovascular diseases: Still a therapeutic target?	17
18	Nutrients 11. SHAH A LAIRD N and SCHOENEELD D (1907) A random effects model for multiple characteristics with	18
19	possibly missing data J. Amer. Statist. Assoc. 92 775–779. MR1467867 https://doi.org/10.2307/2965726	19
20	SIGRIST, F. (2022). Latent Gaussian model boosting. <i>IEEE Trans. Pattern Anal. Mach. Intell.</i> 1–1.	20
21	SILL, M., HIELSCHER, T., BECKER, N. and ZUCKNICK, M. (2014). c060: Extended inference with lasso and	21
22	elastic-net regularized Cox and generalized linear models. J. Stat. Softw. 62.	22
23	SIMON, N., FRIEDMAN, J., HASTIE, T. and TIBSHIRANI, R. (2013). A sparse-group lasso. J. Comput. Graph.	23
24	Statist. 22 251–245. MRS175712 nups://doi.org/10.1080/10018000.2012.081250 SINGAL R and GINDER G D (1999) DNA methylation $Blood 03 4050 4070$	24
25	STEVENSON, A. J., MCCARTNEY, D. L., HILLARY, R. F., CAMPBELL, A., MORRIS, S. W., BERMING-	25
26	HAM, M. L., WALKER, R. M., EVANS, K. L., BOUTIN, T. S. et al. (2020). Characterisation of an	26
27	inflammation-related epigenetic score and its association with cognitive ability. Clin. Epigenet. 12 113.	27
28	SUBRAMANIAN, A., TAMAYO, P., MOOTHA, V. K., MUKHERJEE, S., EBERT, B. L., GILLETTE, M. A.,	28
29	PAULOVICH, A., POMEROY, S. L., GOLUB, T. R. et al. (2005). Gene set enrichment analysis: A knowledge-	29
30	based approach for interpreting genome-wide expression profiles. <i>Proc. Natl. Acaa. Sci. USA</i> 102 15545–	30
31	TAY, J. K., NARASIMHAN, B. and HASTIE, T. (2021). Elastic net regularization paths for all generalized linear	31
32	models.	32
33	R CORE TEAM (2022). R: A Language and Environment for Statistical Computing. R Foundation for Statistical	33
34	Computing, Vienna, Austria.	34
35	TIBSHIRANI, R. (1996). Regression shrinkage and selection via the lasso. J. Roy. Statist. Soc. Ser. B 58 267–288.	35
36	MR13/9242 TYLER A. L. CRAWFORD D. C. and PENDERGRASS, S. A. (2013) Detecting and characterizing pleiotropy:	36
37	New methods for uncovering the connection between the complexity of genomic architecture and multiple	37
38	phenotypes. In <i>Biocomputing</i> 2014 183–187. World Scientific, Singapore.	38
30	VAN EIJK, K. R., DE JONG, S., BOKS, M. P. M., LANGEVELD, T., COLAS, F., VELDINK, J. H., DE	30
40	KOVEL, C. G. F., JANSON, E., STRENGMAN, E. et al. (2012). Genetic analysis of DNA methylation and	40
41	gene expression levels in whole blood of healthy human subjects. <i>BMC Genomics</i> 13 636.	40
40	v INGA, S. (2021). Structured sparsity regularization for analyzing high-dimensional onlics data. <i>Brief. Bioinform.</i> 22, 77–87. https://doi.org/10.1093/bib/bbaa122	41
42	WITTEN, D. M., TIBSHIRANI, R. and HASTIE, T. (2009). A penalized matrix decomposition, with applications	42
43	to sparse principal components and canonical correlation analysis. <i>Biostatistics</i> 10 515–534.	43
44	WITTEN, D. M. and TIBSHIRANI, R. J. (2009). Extensions of sparse canonical correlation analysis with applica-	44
45	tions to genomic data. Stat. Appl. Genet. Mol. Biol. 8 28. MR2533636 https://doi.org/10.2202/1544-6115.1470	45
46	WU, CY., HU, HY., CHOU, YJ., HUANG, N., CHOU, YC. and LI, CP. (2015). High blood pressure and	46
47	an-cause and cardiovascular disease mortanices in community-dwelling older addits. <i>Medicine</i> 94 e2100. YE Y FANG Y WILK LIII Y and ZHANG W (2020) Comprehensive gene and pathway analysis of	47
48	cervical cancer progression. Oncol. Lett. 19 3316–3332.	48
49	YUAN, M. and LIN, Y. (2006). Model selection and estimation in regression with grouped variables. J. R. Stat.	49
50	Soc. Ser. B. Stat. Methodol. 68 49-67. MR2212574 https://doi.org/10.1111/j.1467-9868.2005.00532.x	50
51		51

2	6 A. CAPPOZZO, F. IEVA AND G. FIORITO	
Y	YUAN, T., EDELMANN, D., FAN, Z., ALWERS, E., KATHER, J. N., BRENNER, H. and HOFFMEISTER, M. (2022). Machine learning in the identification of prognostic DNA methylation biomarkers among patients	S
7	 with cancer: A systematic review of epigenome-wide studies. <i>MedRxiv</i>. ZHANG, Y., ELGIZOULI, M., SCHÖTTKER, B., HOLLECZEK, B., NIETERS, A. and BRENNER, H. (2016) Smoking-associated DNA methylation markers predict lung cancer incidence. <i>Clin. Epigenet.</i> 8 1–12. 	
Ζ	HAO, Z., BANTERLE, M., BOTTOLO, L., RICHARDSON, S., LEWIN, A. and ZUCKNICK, M. (2021a).	
	BayesSUR: An R package for high-dimensional multivariate Bayesian variable and covariance selection in linear regression. <i>J. Stat. Softw.</i> 100 .	ı
Z	CHAO, Z., BANTERLE, M., LEWIN, A. and ZUCKNICK, M. (2021b). Structured Bayesian variable selec- tion for multiple related response variables and high-dimensional predictors. ArXiv Preprint. Available at arXiv:2101.05200, 1, 22	t
z	THAO, Z., WANG, S., ZUCKNICK, M. and AITTOKALLIO, T. (2022). Tissue-specific identification of multi-	
	omics features for pan-cancer drug response prediction. <i>iScience</i> 25 104767.	
Z	CHAO, Z. and ZUCKNICK, M. (2020). Structured penalized regression for drug sensitivity prediction. J. R. Stat. Soc. Ser. C. Appl. Stat. 69 525–545. MR4098960	•
Z	HONG, J., AGHA, G. and BACCARELLI, A. A. (2016). The role of DNA methylation in cardiovascular risk and	ł
	disease: Methodological aspects, study design, and data analysis for epidemiological studies. <i>Circ. Res.</i> 118 119–131. https://doi.org/10.1161/CIRCRESAHA.115.305206	3
Ζ	CHONG, W., WANG, J. and CHEN, X. (2021). Censored mean variance sure independence screening for ultrahigh	ı
	aimensionai survivai data. Comput. Statist. Data Anal. 159 10/206. MR4233350 https://doi.org/10.1016/j. csda 2021 107206	•
z	COU. H. and HASTIE. T. (2005). Regularization and variable selection via the elastic net. J. R. Stat. Soc. Ser. B.	
Γ	Stat. Methodol. 67 768–768.	

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The list of entries below corresponds to the original Reference section of your article. The bibliography section on previous page was retrieved from MathSciNet applying an automated procedure.
Please check both lists and indicate those entries which lead to mistaken sources in automatically generated
Reference list.
ANASTASIADI, D., ESTEVE-CODINA, A. and PIFERRER, F. (2018). Consistent inverse correlation between DNA
The methylation of the first infrom and gene expression across ussues and species. <i>Epigenetics & Chromatin</i> 11 57.
structures. <i>Riological Reviews of the Cambridge Philosophical Society</i> 66 101–157
ATCHLEY, W. R. and HALL, B. K. (1991b). A model for development and evolution of complex morphological
structures. Biological Reviews 66 101–157.
AZZALINI, A. and CAPITANIO, A. (2013). The Skew-Normal and Related Families 3. Cambridge University
Press.
BATTRAM, T., YOUSEFI, P., CRAWFORD, G., PRINCE, C., BABAEI, M. S., SHARP, G., HATCHER, C., VEGA-
SALAS, M. J., KHODABAKHSH, S., WHITEHURST, O. and OTHERS (2022). The EWAS Catalog: a database of enjgenome-wide association studies. <i>Wellcome onen research</i> 7
CAMPAGNA, M. P., XAVIER, A., LECHNER-SCOTT, J. MALTRY, V., SCOTT, R. J., BUTZKUEVEN, H.
JOKUBAITIS, V. G. and LEA, R. A. (2021). Epigenome-wide association studies: current knowledge, strate-
gies and recommendations. Clinical Epigenetics 13 214.
CAPPOZZO, A., IEVA, F. and FIORITO, G. (2023). Supplement to "A general framework for penalized mixed-
effects multitask learning with applications on DNA methylation surrogate biomarkers creation".
CAPPOZZO, A., MCCRORY, C., ROBINSON, O., FRENI STERRANTINO, A., SACERDOTE, C., KROGH, V.,
PANICO, S., IUMINO, K., IACOVIELLO, L., KICCERI, F., SIERI, S., CHIODINI, P., MCKAY, G. J., MCKNICHT A. I. KEE, F. YOUNG, I. S. MCGUINNESS, B. CHIMMINS, F. M. APPAWONG, T. F.
KENNY, R. A., O'HALLORAN, A., POLIDORO, S., SOLINAS, G., VINEIS, P., IEVA, F. and FIORITO, G.
(2022). A blood DNA methylation biomarker for predicting short-term risk of cardiovascular events. <i>Clinical</i>
Epigenetics 14 121.
CARUANA, R. (1997). Multitask learning. Machine learning 28 41–75.
CASTRO DE MOURA, M., DAVALOS, V., PLANAS-SERRA, L., ALVAREZ-ERRICO, D., ARRIBAS, C., RUIZ, M.,
AGUILERA-ALBESA, S., IROYA, J., VALENCIA-KAMOS, J., VELEZ-SANIAMARIA, V., KODRIGUEZ- PALMERO A VILLAR-GARCIA I HORCAIADA I P ALRI S CASASNOVAS C RULLA RE
VERTE, L., DIETL, B., DALMAIL, D., ARRANZ, M. J., LLUCIÀ-CAROL, L., PLANAS, A. M., PÉREZ-
TUR, J., FERNANDEZ-CADENAS, I., VILLARES, P., TENORIO, J., COLOBRAN, R., MARTIN-NALDA, A.,
SOLER-PALACIN, P., VIDAL, F., PUJOL, A. and ESTELLER, M. (2021). Epigenome-wide association study
of COVID-19 severity with respiratory failure. EBioMedicine 66 103339.
CHENG, W., ZHANG, X., GUO, Z., SHI, Y. and WANG, W. (2014). Graph-regularized dual Lasso for robust
eQ1L mapping. Bioinformatics 30 139–148.
plete data <i>Journal of Multivariate Analysis</i> 109 146–155
CHUNG, F. R. K. and GRAHAM, F. C. (1997). Spectral graph theory 92 . American Mathematical Soc.
Colicino, E., Just, A., Kioumourtzoglou, MA., Vokonas, P., Cardenas, A., Sparrow, D., Weis-
SKOPF, M., NIE, L. H., HU, H., SCHWARTZ, J. D., WRIGHT, R. O. and BACCARELLI, A. A. (2021). Blood
DNA methylation biomarkers of cumulative lead exposure in adults. Journal of Exposure Science & Environ-
mental Epidemiology 31 108–116.
UUNULE, E. L. S., STEVENSUN, A. J., UREEN, U., HARRIS, S. E., MANIEGA, S. M., VALDES- Hernández M. D. C. Harris, M. A. Rastin, M. F. Warddiaw, I. M. Deady, I. J. Muron, V. F.
WHALLEY, H. C., MARIONI, R. E. and Cox, S. R. (2020). An enigenetic proxy of chronic inflammation
outperforms serum levels as a biomarker of brain ageing. <i>medRxiv</i> 2020.10.08.20205245.
GENE ONTOLOGY CONSORTIUM (2004). The Gene Ontology (GO) database and informatics resource. Nucleic
<i>Acids Research</i> 32 258D–261.
DAWID, A. P. (1981). Some Matrix-Variate Distribution Theory: Notational Considerations and a Bayesian Ap-
plication. <i>Biometrika</i> 08 205.
DEBIGINE, E. (2021). IAUX. Simulation for Factorial Designs. DEMIDENKO, E. (2013). Mixed models: theory and applications with R. John Wiley & Sons
DEMPSTER, A. P., LAIRD, N. M. and RUBIN, D. B. (1977). Maximum Likelihood from Incomplete Data Via
the EM Algorithm. Journal of the Royal Statistical Society: Series B (Methodological) 39 1–22.
DIRMEIER, S., FUCHS, C., MUELLER, N. S. and THEIS, F. J. (2018). NetReg: Network-regularized linear
models for biological association studies. <i>Bioinformatics</i> 34 896–898.

1	Dong, W., Chen, H., Wang, L., Cao, X., Bu, X., Peng, Y., Dong, A., Ying, M., Chen, X., Zhang, X.	1
2	and YAO, L. (2020). Exploring the shared genes of hypertension, diabetes and hyperlipidemia based on mi- croarray. <i>Brazilian Journal of Pharmaceutical Sciences</i> 56 1–12	2
3	FABREGAT, A., JUPE, S., MATTHEWS, L., SIDIROPOULOS, K., GILLESPIE, M., GARAPATI, P., HAW, R.,	3
4	JASSAL, B., KORNINGER, F., MAY, B., MILACIC, M., ROCA, C. D., ROTHFELS, K., SEVILLA, C.,	4
5	SHAMOVSKY, V., SHORSER, S., VARUSAI, T., VITERI, G., WEISER, J., WU, G., STEIN, L., HERM-	5
6	JAKOB, H. and D'EUSTACHIO, P. (2018). The Reactome Pathway Knowledgebase. <i>Nucleic Acids Research</i>	6
7	46 D649–D655.	7
8	FAN, J. and LV, J. (2008). Sure independence screening for ultranign dimensional feature space. <i>Journal of the</i> <i>Royal Statistical Society: Series B (Statistical Methodology)</i> 70 849, 911	8
9	FAN, J., SAMWORTH, R. and WU, Y. (2009). Ultrahigh Dimensional Feature Selection: Beyond the Linear	9
10	Model. Journal of Machine Learning Research 10 2013–2038.	10
11	FAZZARI, M. J. and GREALLY, J. M. (2010). Introduction to Epigenomics and Epigenome-Wide Analysis In	11
12	Statistical Methods in Molecular Biology 243–265. Humana Press, Totowa, NJ.	12
13	FERNÁNDEZ-SANLÉS, A., SAYOLS-BAIXERAS, S., SUBIRANA, I., SENTÍ, M., PÉREZ-FERNÁNDEZ, S., DE	13
14	CASTRO MOURA, M., ESTELLER, M., MARRUGAT, J. and ELOSUA, R. (2021). DNA methylation biomark-	14
15	FIORITO, G., PEDRON, S., OCHOA-ROSALES, C., MCCRORY, C., POLIDORO, S., ZHANG, Y., DUGUÉ, PA.,	15
16	RATLIFF, S., ZHAO, W. N., MCKAY, G. J., COSTA, G., SOLINAS, M. G., HARRIS, K. M., TUMINO, R.,	16
17	GRIONI, S., RICCERI, F., PANICO, S., BRENNER, H., SCHWETTMANN, L., WALDENBERGER, M.,	17
18	MATIAS-GARCIA, P. R., PETERS, A., HODGE, A., GILES, G. G., SCHMITZ, L. L., LEVINE, M.,	18
19	SMITH, J. A., LIU, Y., KEE, F., YOUNG, I. S., MCGUINNESS, B., MCKNIGHT, A. J., VAN MEURS, J.,	19
20	VOORTMAN, T., KENNY, R. A., VINEIS, P. and CARMELI, C. (2022). The Role of Epigenetic Clocks in	20
21	Explaining Educational inequalities in Mortanty: A Multiconort Study and Meta-analysis. The Journals of Gerontology: Series 4 77 1750–1759	21
22	FIORITO, G., VLAANDEREN, J., POLIDORO, S., GULLIVER, J., GALASSI, C., RANZI, A., KROGH, V., GRI-	22
23	ONI, S., AGNOLI, C., SACERDOTE, C., PANICO, S., TSAI, MY., PROBST-HENSCH, N., HOEK, G.,	23
24	HERCEG, Z., VERMEULEN, R., GHANTOUS, A., VINEIS, P. and NACCARATI, A. (2018). Oxidative stress	24
25	and inflammation mediate the effect of air pollution on cardio- and cerebrovascular disease: A prospective	24
20	study in nonsmokers. Environmental and Molecular Mutagenesis 59 234–246.	20
20	FROHLICH, H. and ZELL, A. (2005). Efficient parameter selection for support vector machines in classification and regression via model based global entimization. In <i>Proceedings</i> , 2005, <i>IEEE International Joint Confer</i>	20
27	ence on Neural Networks 2005 3 1431–1436 IEEE	27
28	GAŁECKI, A. and BURZYKOWSKI, T. (2013). Linear Mixed-Effects Models Using R. Springer Texts in Statistics.	28
29	Springer New York, New York, NY.	29
30	GUIDA, F., SANDANGER, T. M., CASTAGNÉ, R., CAMPANELLA, G., POLIDORO, S., PALLI, D., KROGH, V.,	30
31	TUMINO, R., SACERDOTE, C., PANICO, S., SEVERI, G., KYRTOPOULOS, S. A., GEORGIADIS, P., VER-	31
32	MEULEN, R. C. H., LUND, E., VINEIS, P. and CHADEAU-HYAM, M. (2015). Dynamics of smoking-induced	32
33	2359	33
34	HASTIE, T., TIBSHIRANI, R. and WAINWRIGHT, M. (2015). <i>Statistical Learning with Sparsity</i> . Chapman and	34
35	Hall/CRC.	35
36	HIDALGO, B. A., MINNIEFIELD, B., PATKI, A., TANNER, R., BAGHERI, M., TIWARI, H. K., ARNETT, D. K.	36
37	and IRVIN, M. R. (2021). A 6-CpG validated methylation risk score model for metabolic syndrome: The	37
38	HyperGEN and GOLDN studies. <i>PLOS ONE</i> 16 e0259836.	38
39	Wellcome Open Research 5 283	39
40	JOHNSON, W. E., LI, C. and RABINOVIC, A. (2007). Adjusting batch effects in microarray expression data using	40
41	empirical Bayes methods. <i>Biostatistics</i> 8 118–127.	41
42	JORDAN, M. I. (2013). On statistics, computation and scalability. Bernoulli 19 1378–1390.	42
43	KIM, S., PAN, W. and SHEN, X. (2013). Network-Based Penalized Regression With Application to Genomic	43
44	Data. Biometrics 69 582–593.	44
45	with an application to eOTL mapping. The Annals of Applied Statistics 6 1095–1117	45
46	LANGFELDER, P. and HORVATH, S. (2008). WGCNA: an R package for weighted correlation network analysis.	46
47	BMC Bioinformatics 9 559.	47
48	LARIA, J. C., CARMEN AGUILERA-MORILLO, M. and LILLO, R. E. (2019). An Iterative Sparse-Group Lasso.	48
49	Journal of Computational and Graphical Statistics 28 722–731.	49
50	LI, C. and LI, H. (2010). Variable selection and regression analysis for graph-structured covariates with an application to genomics. The Annals of Applied Statistics 4 1408, 1516	50
51	appreation to genomics. The Annuis of Applieu situisites 4 1496–1510.	51

1	LI, Y., NAN, B. and ZHU, J. (2015). Multivariate sparse group lasso for the multivariate multiple linear regression	1
2	with an arbitrary group structure. <i>Biometrics</i> 71 354–363.	2
3	LOZA, M. J., MCCALL, C. E., LI, L., ISAACS, W. D., AU, J. and CHANG, DL. (2007). Assembly of Inflammation-Related Genes for Pathway-Focused Genetic Analysis. <i>PLoS ONE</i> 2 e1035	3
4	LU, A. T., OUACH, A., WILSON, J. G., REINER, A. P., AVIV, A., RAJ, K., HOU, L., BACCARELLI, A. A.,	4
5	LI, Y., STEWART, J. D., WHITSEL, E. A., ASSIMES, T. L., FERRUCCI, L. and HORVATH, S. (2019). DNA	5
6	methylation GrimAge strongly predicts lifespan and healthspan. Aging 11 303–327.	6
7	MARABITA, F., ALMGREN, M., LINDHOLM, M. E., RUHRMANN, S., FAGERSTRÖM-BILLAI, F., JAGODIC, M.,	7
8	SUNDBERG, C. J., EKSTRÖM, T. J., TESCHENDORFF, A. E., TEGNÉR, J. and GOMEZ-CABRERO, D. (2013).	8
9	An evaluation of analysis pipelines for DNA methylation profiling using the Illumina HumanMethylation450	9
10	BeadChip platform. <i>Epigenetics</i> 8 333–346.	10
11	MCCULLOCH, C. E. and NEUHAUS, J. M. (2011). Misspecifying the Shape of a Random Effects Distribution:	11
12	Why Getting It wrong May Not Matter. Statistical Science 20 388–402. MCLACHIAN G. L. and KRISHNAN, T. (2008). The FM Algorithm and Extensions. 2F. Wiley Series in Proba	12
13	bility and Statistics 54 John Wiley & Sons Inc. Hoboken NI JISA	13
14	MENG, XL. and RUBIN, D. B. (1993). Maximum Likelihood Estimation via the ECM Algorithm: A General	14
14	Framework. <i>Biometrika</i> 80 267.	14
15	NGUYEN, T. M., LE, H. L., HWANG, KB., HONG, YC. and KIM, J. H. (2022). Predicting High Blood	15
16	Pressure Using DNA Methylome-Based Machine Learning Models. Biomedicines 10 1406.	16
17	OBOZINSKI, G., TASKAR, B. and JORDAN, M. I. (2010). Joint covariate selection and joint subspace selection	17
18	for multiple classification problems. <i>Statistics and Computing</i> 20 231–252.	18
19	OBOZINSKI, G., WAINWRIGHT, M. J. and JORDAN, M. I. (2009). High-dimensional support union recovery	19
20	in multivariate regression. Advances in Neural Information Processing Systems 21—Proceedings of the 2008	20
21	Conjerence 1217–1224. Orozinski G. WAINWRIGHT M. L and IORDAN M. I. (2011). Support union recovery in high-dimensional	21
22	multivariate regression. Annals of Statistics 39 1–47.	22
23	Odintsova, V. V., Rebattu, V., Hagenbeek, F. A., Pool, R., Beck, J. J., Ehli, E. A., van Beijster-	23
24	VELDT, C. E. M., LIGTHART, L., WILLEMSEN, G., DE GEUS, E. J. C., HOTTENGA, JJ., BOOMSMA, D. I.	24
25	and VAN DONGEN, J. (2021). Predicting Complex Traits and Exposures From Polygenic Scores and Blood	25
26	and Buccal DNA Methylation Profiles. Frontiers in Psychiatry 12 1–17.	26
27	PANICO, S., DELLO IACOVO, R., CELENTANO, E., GALASSO, R., MUTI, P., SALVATORE, M. and	27
28	MANCINI, M. (1992). Progetto ATENA, A study on the etiology of major chronic diseases in women: Design,	28
20	rationale and objectives. European Journal of Epidemiology 8 601–608. PUIDSON B. MAKSIMONIC I and OSILLACK A (2016) missMathyli an P package for analyzing data from	20
20	Illumina's HumanMethylation450 platform <i>Bioinformatics</i> 32 286–288	20
30	PINHEIRO, J. and BATES, D. (2006). <i>Mixed-effects models in S and S-PLUS</i> . Springer science & business media.	30
31	RAULUSEVICIUTE, I., DRABLØS, F. and RYE, M. B. (2020). DNA hypermethylation associated with upreg-	31
32	ulated gene expression in prostate cancer demonstrates the diversity of epigenetic regulation. BMC Medical	32
33	Genomics 13 6.	33
34	REIMAND, J., ISSERLIN, R., VOISIN, V., KUCERA, M., TANNUS-LOPES, C., ROSTAMIANFAR, A., WADI, L.,	34
35	MEYER, M., WONG, J., XU, C., MERICO, D. and BADER, G. D. (2019). Pathway enrichment analysis	35
36	and visualization of omics data using g:Profiler, GSEA, Cytoscape and EnrichmentMap. <i>Nature Protocols</i> 14	36
37	402-517. REINSEL G (1984) Estimation and Prediction in a Multivariate Random Effects Generalized Linear Model	37
38	Journal of the American Statistical Association 79 406–414	38
39	RIBOLI, E., HUNT, K., SLIMANI, N., FERRARI, P., NORAT, T., FAHEY, M., CHARRONDIÈRE, U., HÉ-	39
40	MON, B., CASAGRANDE, C., VIGNAT, J., OVERVAD, K., TJØNNELAND, A., CLAVEL-CHAPELON, F.,	40
41	THIÉBAUT, A., WAHRENDORF, J., BOEING, H., TRICHOPOULOS, D., TRICHOPOULOU, A., VINEIS, P.,	41
42	Palli, D., Bueno-de Mesquita, H., Peeters, P., Lund, E., Engeset, D., González, C., Barri-	42
43	CARTE, A., BERGLUND, G., HALLMANS, G., DAY, N., KEY, T., KAAKS, R. and SARACCI, R. (2002).	43
44	European Prospective Investigation into Cancer and Nutrition (EPIC): study populations and data collection.	44
45	Puolic Healin Ivalfalon 5 1115–1124. Richard M. A. Huan, T. Ligthart S. Gondalia, R. Ihun, M. A. Brody, I. A. Irvin, M. R.	45
46	MARIONI, R., SHEN, J., TSAI, PC., MONTASSER, M. E., JIA, Y., SYME, C., SALFATI, F. L., BOFRWIN-	46
47	KLE, E., GUAN, W., MOSLEY, T. H., BRESSLER, J., MORRISON, A. C., LIU, C., MENDELSON, M. M.	⊿7
48	UITTERLINDEN, A. G., VAN MEURS, J. B., FRANCO, O. H., ZHANG, G., LI, Y., STEWART, J. D.,	יד ۵۸
40	BIS, J. C., PSATY, B. M., CHEN, YD. I., KARDIA, S. L. R., ZHAO, W., TURNER, S. T., ABSHER, D.,	40
49 50	ASLIBEKYAN, S., STARR, J. M., MCRAE, A. F., HOU, L., JUST, A. C., SCHWARTZ, J. D., VOKONAS, P. S.,	49
50	MENNI, C., SPECTOR, T. D., SHULDINER, A., DAMCOTT, C. M., ROTTER, J. I., PALMAS, W., LIU, Y.,	50
51		51

1	PAUS, T., HORVATH, S., O'CONNELL, J. R., GUO, X., PAUSOVA, Z., ASSIMES, T. L., SOTOODEHNIA, N.,	1
2	SMITH, J. A., ARNETT, D. K., DEARY, I. J., BACCARELLI, A. A., BELL, J. T., WHITSEL, E., DE-	2
3	HGHAN, A., LEVY, D., FORNAGE, M., HEIJMANS, B. T., 'T HOEN, P. A. C., VAN MEURS, J., ISAACS, A.,	3
4	JANSEN, R., FRANKE, L., BOOMSMA, D. I., POOL, R., VAN DONGEN, J., HOTTENGA, J. J., VAN	4
5	WIMENGA C ZHERNAKOVA A TIGCHELAAR E E SLAGBOOM P E BEEKMAN M DEELEN L	5
6	VAN HEEMST, D., VELDINK, J. H., VAN DEN BERG, L. H., VAN DIJIIN, C. M., HOFMAN, A., UITTERLIN-	6
7	DEN, A. G., JHAMAI, P. M., VERBIEST, M., SUCHIMAN, H. E. D., VERKERK, M., VAN DER BREGGEN, R.,	7
8	VAN ROOIJ, J., LAKENBERG, N., MEI, H., VAN ITERSON, M., VAN GALEN, M., BOT, J., VAN'T HOF, P.,	8
9	DEELEN, P., NOOREN, I., MOED, M., VERMAAT, M., ZHERNAKOVA, D. V., LUIJK, R., BONDER, M. J.,	9
10	VAN DIJK, F., ARINDRARTO, W., KIELBASA, S. M., SWERTZ, M. A. and VAN ZWET, E. W. (2017). DNA	10
11	Methylation Analysis Identifies Loci for Blood Pressure Regulation. <i>The American Journal of Human Genetics</i>	11
10	101 888-902. RODOSTHENOUS T. SHAHREZAEL V and EVANCELOU M. (2020). Integrating multi-OMICS data through	10
12	sparse canonical correlation analysis for the prediction of complex traits: a comparison study. <i>Bioinformatics</i>	12
13	36 4616–4625.	13
14	ROHART, F., SAN CRISTOBAL, M. and LAURENT, B. (2014). Selection of fixed effects in high dimensional linear	14
15	mixed models using a multicycle ECM algorithm. Computational Statistics & Data Analysis 80 209-222.	15
16	SCHAFER, J. L. and YUCEL, R. M. (2002). Computational Strategies for Multivariate Linear Mixed-Effects	16
17	Models With Missing Values. Journal of Computational and Graphical Statistics 11 437–457.	17
18	SCHELLDORFER, J., BUHLMANN, P. and DE GEER, S. V. (2011). Estimation for High-Dimensional Linear Mixed Effects Models Using (1 Penalization Scandingwign Journal of Statistics 38 107, 214	18
19	SCHWARZ, G. (1978). Estimating the dimension of a model. <i>The Annals of Statistics</i> 6 461–464.	19
20	SENONER, T. and DICHTL, W. (2019). Oxidative stress in cardiovascular diseases: Still a therapeutic target?	20
21	Nutrients 11.	21
22	SHAH, A., LAIRD, N. and SCHOENFELD, D. (1997). A Random-Effects Model for Multiple Characteristics with	22
23	Possibly Missing Data. Journal of the American Statistical Association 92 775–779.	23
24	SIGRIST, F. (2022). Latent Gaussian Model Boosting. <i>IEEE Transactions on Pattern Analysis and Machine Intel-</i>	24
25	ligence 1-1.	25
26	Elastic-Net Regularized Cox and Generalized Linear Models. <i>Journal of Statistical Software</i> 62	26
27	SIMON, N., FRIEDMAN, J., HASTIE, T. and TIBSHIRANI, R. (2013). A Sparse-Group Lasso. Journal of Com-	27
28	putational and Graphical Statistics 22 231–245.	28
29	SINGAL, R. and GINDER, G. D. (1999). DNA Methylation. Blood 93 4059-4070.	29
20	STEVENSON, A. J., MCCARTNEY, D. L., HILLARY, R. F., CAMPBELL, A., MORRIS, S. W., BERMING-	20
21	HAM, M. L., WALKER, R. M., EVANS, K. L., BOUTIN, T. S., HAYWARD, C., MCRAE, A. F., MC-	21
20	COLL, B. W., SPIRES-JONES, I. L., MCINTOSH, A. M., DEARY, I. J. and MARIONI, K. E. (2020). Char- acterisation of an inflammation-related enigenetic score and its association with cognitive ability. <i>Clinical</i>	20
32	Enjoenetics 12 113.	32
33	SUBRAMANIAN, A., TAMAYO, P., MOOTHA, V. K., MUKHERJEE, S., EBERT, B. L., GILLETTE, M. A.,	33
34	PAULOVICH, A., POMEROY, S. L., GOLUB, T. R., LANDER, E. S. and MESIROV, J. P. (2005). Gene set	34
35	enrichment analysis: A knowledge-based approach for interpreting genome-wide expression profiles. Pro-	35
36	ceedings of the National Academy of Sciences 102 15545–15550.	36
37	IAY, J. K., NARASIMHAN, B. and HASTIE, T. (2021). Elastic Net Regularization Paths for All Generalized	37
38	LINEAR MODELS. R CORE TEAM (2022) R : A Language and Environment for Statistical Computing R Foundation for Statistical	38
39	Computing Vienna Austria	39
40	TIBSHIRANI, R. (1996). Regression Shrinkage and Selection Via the Lasso. Journal of the Royal Statistical	40
41	Society: Series B (Methodological) 58 267–288.	41
42	TYLER, A. L., CRAWFORD, D. C. and PENDERGRASS, S. A. (2013). Detecting and characterizing pleiotropy:	42
43	new methods for uncovering the connection between the complexity of genomic architecture and multiple	43
44	phenotypes. In <i>Biocomputing</i> 2014 183–187. WORLD SCIENTIFIC.	44
45	VAN EIJK, K. K., DE JUNG, S., BUKS, M. P. M., LANGEVELD, I., COLAS, F., VELDINK, J. H., DE KOVEL C G F JANSON F STRENGMAN F JANGEELDER D KAUN P S VAN DEN REDG J H	45
46	HORVATH, S. and OPHOFF, R. A. (2012). Genetic analysis of DNA methylation and gene expression levels	46
47	in whole blood of healthy human subjects. <i>BMC Genomics</i> 13 636.	47
48	VINGA, S. (2021). Structured sparsity regularization for analyzing high-dimensional omics data. Briefings in	48
10	Bioinformatics 22 77–87.	-10
49 50	WITTEN, D. M., TIBSHIRANI, R. and HASTIE, T. (2009). A penalized matrix decomposition, with applications	49 50
50	to sparse principal components and canonical correlation analysis. <i>Biostatistics</i> 10 515–534.	50

1	WITTEN, D. M. and TIBSHIRANI, R. J. (2009). Extensions of Sparse Canonical Correlation Analysis with Ap-	1
2	plications to Genomic Data. Statistical Applications in Genetics and Molecular Biology 8 1-27.	2
3	WU, CY., HU, HY., CHOU, YJ., HUANG, N., CHOU, YC. and LI, CP. (2015). High Blood Pressure and	3
4	All-Cause and Cardiovascular Disease Mortalities in Community-Dwelling Older Adults. <i>Medicine</i> 94 e2160.	4
5	YI, Y., FANG, Y., WU, K., LIU, Y. and ZHANG, W. (2020). Comprehensive gene and pathway analysis of	5
6	cervical cancer progression. Oncology Letters 19 3316–3332.	6
7	Y UAN, M. and LIN, Y. (2006). Model selection and estimation in regression with grouped variables. <i>Journal of</i>	7
<i>'</i>	THE ROYAL STATISTICAL SOCIETY. SETTES D (STATISTICAL METHODOLOGY) 00 49-07.	,
8	(2022) Machine learning in the identification of prognostic DNA methylation biomarkers among natients	8
9	with cancer: a systematic review of epigenome-wide studies, <i>medRxiv</i> .	9
10	ZHANG, Y., ELGIZOULI, M., SCHÖTTKER, B., HOLLECZEK, B., NIETERS, A. and BRENNER, H. (2016).	10
11	Smoking-associated DNA methylation markers predict lung cancer incidence. <i>Clinical Epigenetics</i> 8 1–12.	11
12	ZHAO, Z., BANTERLE, M., BOTTOLO, L., RICHARDSON, S., LEWIN, A. and ZUCKNICK, M. (2021a).	12
13	BayesSUR: An R Package for High-Dimensional Multivariate Bayesian Variable and Covariance Selection	13
14	in Linear Regression . Journal of Statistical Software 100 .	14
15	ZHAO, Z., BANTERLE, M., LEWIN, A. and ZUCKNICK, M. (2021b). Structured Bayesian variable selection for	15
16	multiple related response variables and nigh-dimensional predictors. arXiv preprint arXiv:2101.05899 1–55. ZHAO, Z., WANG, S., ZHCKNICK, M. and AHTTOKALLIO, T. (2022). Tissue specific identification of multi-	16
17	omics features for pan-cancer drug response prediction <i>iScience</i> 25 104767	17
18	ZHAO, Z. and ZUCKNICK, M. (2020). Structured penalized regression for drug sensitivity prediction. <i>Journal of</i>	18
19	the Royal Statistical Society: Series C (Applied Statistics) 69 525–545.	19
20	ZHONG, J., AGHA, G. and BACCARELLI, A. A. (2016). The Role of DNA Methylation in Cardiovascular Risk	20
21	and Disease. Circulation Research 118 119–131.	21
22	ZHONG, W., WANG, J. and CHEN, X. (2021). Censored mean variance sure independence screening for ultrahigh	22
22	dimensional survival data. Computational Statistics & Data Analysis 159 107206.	22
23	200, H. and HASTIE, I. (2005). Regularization and variable selection via the elastic net. <i>Journal of the Royal</i> Statistical Society: Sarias B (Statistical Methodology) 67 ,768,768	23
24	Statistical Society. Series B (Statistical Methodology) 07 708–708.	24
25		25
26		26
27		27
28		28
29		29
30		30
31		31
32		32
33		33
34		34
35		35
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200 https://orcid.org/0000-0003-0165-1983 [2:pp.1,1] OK	24
200 https://orcid.org/0000-0002-7651-5452 [2:pp.1,1] OK	25
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200 https://github.com/AndreaCappozzo/emlmm [4:pp.3,3,11,11] OK	20
404 https://doi.org/10.1214/23-AOAS1/60SUPPA [4:pp.22,22,22,22] Not Found	29
404 https://doi.org/10.1214/23-A0AS1760SUPPB [4:pp.22,22,22,22] Not Found	30
Sol http://arxiv.org/abs/arxiv:2101.05899 [2:pp.26,26] Moved Permanentry	31
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