A Voltage Unbalance Mitigation Technique for Low-voltage Applications with Large Single-phase Loads

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Abstract—**In this paper a voltage unbalance mitigation technique for low-voltage microgrids or feeders in presence of large single-phase loads is introduced. In order to take maximum advantage of the existing hardware, the proposed solution consists of a sequence-based decentralized voltage control to be embedded in three-phase VSC connecting distributed generation to the considered system. Furthermore, a centralized controller is proposed to define optimal negative and zero sequence voltage reference. Control effectiveness is numerically verified considering a low-voltage feeder case study.**

Index Terms— **Power Quality, Smart Grid, Voltage Unbalance Mitigation, Distributed Generation.**

I. INTRODUCTION

Voltage unbalance is a traditional power system issue with well-known causes and established mitigation techniques [1]. Furthermore, specific normative requirements are included in technical IEC normative [2]. However, due to the increasing penetration of single-phase loads and generators, including EV (Electric Vehicle) stations, heat pumps and smaller photovoltaic (PV) generators, new voltage balance issues may rise in weaker grids, such as microgrids or longer low-voltage (LV) feeders [3].

Different studies recognize the issue and propose some interesting solutions. In order to address the voltage unbalance issue, it is firstly necessary to formulate power converter control algorithms whose effectiveness is not affected by voltage unbalance [4]-[7]. Then, a first possible approach is to design a control algorithm to mitigate voltage unbalance at the Point of Common Coupling (PCC) [8]-[11], which is clearly beneficial for grid operation. However, this solution may require additional storage devices when unbalance issues are due to unbalanced generation on different phases [12] or specific converters capable of withstanding the power oscillations related to unbalanced voltages/currents [13]. Further development in voltage unbalance mitigation include centralized controller to coordinate single-phase PV converters [14], active filters [15] and load side demand strategies [16] to mitigate voltage unbalance issues at the PCC. However, one could argue that the most of the aforementioned approaches, even though effective on their specific target, requires additional hardware. Furthermore, their effectiveness is limited to the microgrid PCC or one specific converter PCC, which is not necessarily the most beneficial condition for loads.

In order to overcome these issues, this paper proposes a centralized control technique to mitigate voltage unbalance. In fact, the diffusion of distributed generation and information technologies in electric networks, as well as the adoption of advanced converter control techniques, suggests considering a different approach to mitigate voltage unbalance. Consequently, a sequence-based control is proposed for distributed VSCs such that standard controls can be implemented for the positive sequence, while introducing specific voltage controls for negative and zero sequence. Considering that effectively controlled power converters can operate almost unaffected by voltage unbalance issues, it is then possible to design a centralized controller to compute optimal negative and zero sequence voltage references to minimize voltage unbalance experienced by loads.

The paper is organized as follows: section II presents some preliminary considerations and the definition of the considered performance indexes, while in section III a general formulation for the problem is provided and a control proposal is developed. Section IV presents the considered case study, while simulation results are exposed and discussed in section V. Further discussion on the applicability of the proposed control design is presented in Section VI, while final conclusions are reported in section VII.

II. PRELIMINARY CONSIDERATIONS

For the sake of clarity, consider a three-phase, four-wire, low-voltage network with N voltage-controlled nodes and M load nodes. In such a system, voltage unbalance is mostly originated by single-phase loads, which introduce negative and zero sequence currents and, consequently, negative and zero sequence voltages originated by voltage drops.

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In order to address this issue, a possible solution is to use distributed generators VSCs to regulate not only positive sequence quantities, but also negative and zero sequence, which usually requires only minor hardware modification. The control of positive sequence for distributed generators connected to the public grid is well-established in literature and will not be discussed in this paper; however, one should notice that, usually, only positive sequence current is controlled, resulting in an open circuit for negative and zero sequences. However, a four-wire system allows for three independent currents, which implies that it is possible to introduce high-performance control algorithm separately on each sequence current, for instance, but not limited to, by means of the control synthesis procedure described in [17]. This approach exhibits some significant advantages, in that it allows taking care of the different parameters and behaviour of sequence equivalent circuits [18], [19] to obtain optimal performances. Furthermore, while phase-quantities-based control algorithm should consider mutual coupling among phases, sequence-quantities-based control algorithms are naturally decoupled under the common assumption of constructively symmetric components. However, the definition of optimal negative and zero sequence voltage references is not trivial.

Firstly, in this paper it is assumed that load voltages are foremost considered for voltage unbalance mitigation, while it is assumed that controlled generators do not suffer from voltage unbalance. In fact, there is no reason why a static converter should suffer from negative and zero sequence voltage components as long as it is properly controlled and its capabilities are respected. Consequently, in the following a technique to define the negative and zero sequence voltage references to minimize (or cancel, when possible) voltage unbalance at loads terminals is proposed. For these purposes, the performance indexes considered to evaluate control effectiveness is the quadratic voltage sum, which, for negative and zero sequences, respectively, results in

$$\begin{aligned} \varepsilon^- &= \sum_{i=1}^{M} g_i^- \left|V_i^-\right|^2 = \sum_{i=1}^{M} g_i^- V_i^{-*} \cdot V_i^- \\ \varepsilon^o &= \sum_{i=1}^{M} g_i^o \left|V_i^o\right|^2 = \sum_{i=1}^{M} g_i^o V_i^{o*} \cdot V_i^o \end{aligned} \tag{1}$$

where V_i^- and V_i^o represent, respectively, the negative and zero sequence voltage phasors of the i-th node, apex * indicates the respective quantity conjugate, and g_i^-, g_i^o are real-valued weight coefficients, with $0 \leq g_i^-, g_i^o \leq 1$. Furthermore, it will be highlighted that, neglecting degenerate networks, the N degree of freedom represented by the controlled generators allow the cancellation of M negative and zero nodal voltages, such that the indexes ε^-, ε^o (1) can be forced to zero only if $N \geq M$. Otherwise, it is only possible to find an optimal solution which minimizes the aforementioned indexes. From here on, it is assumed $N \leq$ M in that, usually, it is reasonable to assume that voltage-controlled nodes are less numerous than load nodes.

III. General Control Design

For the sake of compactness, a general approach is developed in this Section. This can be applied to minimize negative sequence alone in case of three-wire, three-phase networks or to negative and zero sequences, separately, in case of three-phase, four-wire networks.

A. Problem Formulation

For the purposes of this paper, some assumptions are introduced. Firstly, the system is considered in quasi-stationary conditions and the load positive-sequence power/current absorption is assumed to be independent from the control action on negative and zero sequences. This assumption clearly introduces some extent of approximation, which, however, is negligible in case of moderate voltage unbalance. Furthermore, one should recognize that, in order to define the optimal negative and zero sequence voltage references, the complete information regarding the system is needed, including, in principle, network topology, impedances and loads. The considerations reported in the following apply, separately, for negative and zero sequences.

Under the previous assumptions, the cost function (1) can be considered in general form as

$$\varepsilon = \mathbf{V}^{T*}\mathbf{G}\mathbf{V} \tag{2}$$

in which $\mathbf{V}$ is the nodal voltages vector and $\mathbf{G}$ is an $M \times M$ diagonal matrix including the weight coefficients introduced in (1), namely:

$$\mathbf{V} = \begin{bmatrix} V_1 & \cdots & V_M \end{bmatrix}^T \tag{3}$$

$$\mathbf{G} = diag\begin{bmatrix} g_1 & \cdots & g_M \end{bmatrix} \tag{4}$$

Under the hypothesis of system linearity, the nodal voltages vector can be formulated as

$$\mathbf{V} = \mathbf{V}_0 + \mathbf{A}\mathbf{E} \tag{5}$$

with

$$\begin{aligned} &\mathbf{V}_0 = \begin{bmatrix} V_{0,1} & \cdots & V_{0,M} \end{bmatrix}^T, \quad \mathbf{E} = \begin{bmatrix} E_1 & \cdots & E_N \end{bmatrix}^T \\ &\mathbf{A} = \begin{bmatrix} \alpha_{1,1} & \cdots & \alpha_{1,N} \\ \vdots & \ddots & \vdots \\ \alpha_{M,1} & \cdots & \alpha_{M,N} \end{bmatrix}_{M \times N} \end{aligned} \tag{6}$$

where E_j represents the voltage phasor imposed by the j-th controlled generator and $V_{0,i}$ represents the voltage phasor at the i-th node terminals under the condition $\mathbf{E}$=$\mathbf{0}$. The complex elements of $\mathbf{A}$ are defined as

$$\alpha_{i,j} = \left.\frac{V_i}{E_j}\right|_{E_{h,\forall h \neq j}=0,\ \mathbf{V}_0=\mathbf{0}} \tag{7}$$

Note that (5) only requires the hypothesis of system linearity, which is usually non-restrictive. However, in case of nonlinear systems, (5) can be applied to a linearized system model: in this case, the minimum condition is not exactly reached due to approximations, but the proposed approach still allows reducing voltage unbalance significantly.

Summarizing, in (5) each nodal voltage is formulated as the sum of two terms: the first one depends only on unbalanced loads, possible non-controllable generators and line impedances, while the second term depends only on the sum of controllable generators voltages, weighted by means of coefficients depending on line and load impedances. Note that, according to (7), $\alpha_{i,j}$ are complex constants resulting from line and load impedances ratios. However, since the load equivalent admittances are generally much smaller than line admittances, in the following the former will be considered negligible with respect to the latter. This implies that only grid topology and line parameters, possibly including the fourth wire, are to be known in order to evaluate $\alpha_{i,j}$ for negative and zero sequence equivalent circuits, separately.

Even though the minimum problem defined by the cost function (2) with respect to the control variable vector **E** can be solved numerically, in the following an alternative vector approach based on a perturbation method is presented. Thus, consider the incremented generator voltages

$$\mathbf{E}' = \mathbf{E} + \mathbf{e} \tag{8}$$

where the apex ‘ indicates incremented variables and **e** represent the controlled voltage increment. According to (8), (5) is now reformulated in terms of incremented variables as

$$\mathbf{V}' = \mathbf{V} + \mathbf{A}\mathbf{e} \tag{9}$$

Applying (9) in (2) the incremented cost function results in

$$\begin{aligned} \varepsilon' &= \left[\mathbf{V}'\right]^{T*}\mathbf{G}\mathbf{V}' = \\ &= \varepsilon + \mathbf{e}^{T*}\mathbf{B}\mathbf{e} + \mathbf{V}^{T*}\mathbf{G}\mathbf{A}\mathbf{e} + \mathbf{e}^{T*}\mathbf{A}^{T*}\mathbf{G}\mathbf{V} \end{aligned} \tag{10}$$

where

$$\begin{aligned} &\mathbf{B} = \mathbf{A}^{T*}\mathbf{G}\mathbf{A} = \\ &= \begin{bmatrix} \sum_{i=1}^{M} g_i \alpha_{i,1}^{*}\alpha_{i,1} & \cdots & \sum_{i=1}^{M} g_i \alpha_{i,1}^{*}\alpha_{i,N} \\ \vdots & \ddots & \vdots \\ \sum_{i=1}^{M} g_i \alpha_{i,N}^{*}\alpha_{i,1} & \cdots & \sum_{i=1}^{M} g_i \alpha_{i,N}^{*}\alpha_{i,N} \end{bmatrix}_{N\times N} \end{aligned} \tag{11}$$

Matrix **B** is a Hermitian form, such that **B** is positive-defined under the hypothesis of **A** being a full-rank matrix, which is granted under non-restrictive hypothesis. Consequently, one can note that the first and second terms of (10) are non-negative quantities. Consider now the following conditions

$$\begin{cases} \mathbf{A}^{T*}\mathbf{G}\mathbf{V} = 0 \\ \mathbf{V}^{T*}\mathbf{G}\mathbf{A} = 0 \end{cases} \tag{12}$$

over the third and fourth terms of (10). Note that the first of (12) also implies the second condition, being its transposed conjugate, to be null. Under conditions (12), according to (10), $\varepsilon' > \varepsilon$ for any non-trivial increment set **e.** Therefore (12) represents the minimum condition of ε. Furthermore, the incremental method leading to (12) allows the definition of a minimum condition depending only on actual nodal voltages **V** and on grid parameters and not on $\mathbf{V}_0$, which is not directly measurable. Consequently, (12) allows the minimization of voltage unbalance from available measurements and justifies the proposed incremental approach.

B. Control Design

The centralized controller should be designed to provide suitable negative and zero sequence voltage references for voltage-controlled nodes to comply with condition (12). In order to obtain the desired control laws, consider (5) and, by left multiplication for $\mathbf{A}^{T*}\mathbf{G}$, the controlled system is obtained as

$$\mathbf{A}^{T*}\mathbf{G}\mathbf{V} = \mathbf{A}^{T*}\mathbf{G}\mathbf{V}_0 + \mathbf{B}\mathbf{E} \tag{13}$$

where $\mathbf{B} = \mathbf{A}^{T*}\mathbf{G}\mathbf{A}$ (11). From (13), it is theoretically possible to obtain an analytic solution for the minimum problem; however, $\mathbf{V}_0$ is generally not measurable, so that a different approach is needed. Furthermore, since **B** is a non-diagonal matrix, (13) highlights that the system is intrinsically coupled. Considering (13), a complex control law would generally be required; however, being **B** Hermitian, which implies that its diagonal terms are real constants , it is possible to define a control law based on a real function. Assuming, for the sake of simplicity, equal proportional and integral gains for each control equation, the proposed control results in

$$\mathbf{E} = -\left(k_P + \frac{k_I}{s}\right)\mathbf{B}^{-1}\mathbf{A}^{T*}\mathbf{G}\mathbf{V} \tag{14}$$

in which the control inputs have been decoupled. Note that **B** is required to be non-singular to ensure system controllability. Since (14) enforces condition (12), N linear relations on nodal voltages are forced to zero; in particular, if $N = M$, not only the minimum condition is reached, but also all voltages are forced to zero. This can be proved considering that, when $N = M$, **A** is a square, full-rank, constant matrix, such that the only solution for (12) is $\mathbf{V} = 0$.

IV. CASE STUDY

The grid considered as a case study is reported in Figure 1. It consists of a radial three-phase, four-wire, 400 V feeder, which represents the common practice in LV distribution systems. The connection to the main grid is performed by means of a Δ-y, 20 kV - 400 V transformer, characterized by a rated power of 250 kVA and a short-circuit voltage $v_{sh\%} = 8\%$. All lines are supposed to be realized with the same cable type and section, in particular with LV, unipolar, 240 mm^2 cables for both the phase and neutral wires; line lengths are reported in Table I. Considering the notation used in Section III, connected to grid there are:

- $N = 2$ controllable generators (PV 1 and PV 2)

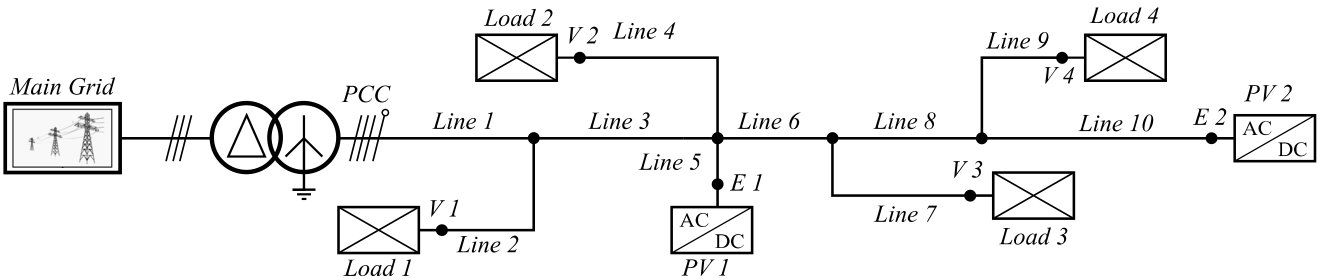

Figure 1. Considered grid.

- $M = 4$ loads

Loads and generators powers are reported in Table II. The regulator coefficients used in (14) are set $k_P = 0.01$ and $k_I = 1$. The weight coefficients, introduced in (1), are set equal to one for the four load voltages and equal to zero for the two controlled voltages and for the PCC. This implies that only load voltages will benefit directly from the proposed control action. The PCC voltage will also be improved by reflection, but it is not considered a priority in this case. Obviously, different arrangements are possible and equally valid depending on the specific case.

The loads are constituted of a mix of three-phase and single-phase units, as usual for domestic/residential and small industrial applications; in particular, load 1 is constituted by a globally balanced three-phase load while loads 2, 3 and 4 present significant single-phase loads. Load 3 includes 6 x 7.3 kW single-phase loads, which can be representative of mode 2 EV charging stations (single-phase, 230 V, 32 A, largest standard single-phase solution for domestic users). Loads 2 and 4 include 3 x 15 kW single-phase loads, which can be representative of larger domestic heat pumps (i.e. [20] for a commercial example). As usual, single-phase loads are equally divided among the three phases, such that the load seen by the main grid is globally balanced. It is worth noting that single-phase loads are usually represented by many small ones, such that the effect of the connection or disconnection of some single-phase loads produces little or no effect on voltage balance. However, when single-phase loads are constituted by few, larger loads, as in this case, it is not unlikely to experience quite a severe load unbalance when only some loads are connected, since their operation is not deterministic and, even though somehow predictable by means of statistical inferences, largely randomised. Consequently, a worst-case scenery has been considered in the simulations presented in the following Section.

TABLE I. LINE LENGTHS

Line #	Length [km]	Line #	Length [km]
1	0.2	6	0.35
2	0.5	7	0.4
3	0.4	8	0.25
4	0.5	9	0.4
5	0.6	10	0.5

TABLE II. GENERATORS AND LOADS

Generator/Load	Power [kW]
Load 1	50, 3- phase, cosφ = 0.95
Load 2	20, 3-phase + 3 x 15, 1-phase, cosφ = 0.95
Load 3	20, 3-phase + 6 x 7.3, 1-phase, cosφ = 0.95
Load 4	20, 3-phase + 3 x 15, 1-phase, cosφ = 0.95
PV 1	50, 3-phase
PV 2	75, 3-phase

V. SIMULATION RESULTS

Simulations have been performed to prove control effectiveness in mitigating voltage unbalance issues. As a worst-case scenario, all three-phase loads are connected to the grid, while only the single-phase loads connected to phase 1 are connected. Even though this may seem an unlikely situation, it is worth of consideration since [2] states that under normal operating conditions, during each period of one week, 95% of the 10-minute mean r.m.s. values of negative sequence voltage should not exceed 2% of positive sequence voltage.

The results in terms of positive, negative and zero sequence voltages, expressed in pu with respect to rated positive sequence voltage, are reported in Table III in case no mitigating action is performed by controlled generators, while Table IV reports the same quantities in case the voltage unbalance mitigating technique proposed in Section III is applied to the system. Additionally, the last line of each table reports the quadratic sum of negative and zero sequence voltages, selected as performance indexes according to (1). One can notice that, as expected, positive sequence voltage is substantially unaffected by the proposed control technique. The minimal variations on positive sequence voltage may be ascribed to the slightly different power flow due to converter action, since the transformer at PCC, as usual, has a fixed voltage tap. On the contrary, when no mitigating action is performed by controllable generators, significant negative and zero sequence voltages are present both at converters and loads connection points. While power converters can be insensitive to voltage unbalance when correctly controlled, it is usually considered undesirable to provide unbalanced voltage to loads. Furthermore, negative sequence voltage seen by loads 2, 3 and 4 clearly exceeds the normative limits defined in [2]. On the contrary, one can notice that, when the voltage unbalance mitigating technique proposed in Section III is applied, voltage unbalance is greatly reduced. Furthermore, all loads experience a negative voltage well below 1 %, while only controlled generators experience a negative sequence voltage around 1.6 %; however, this is

TABLE III. SEQUENCE VOLTAGES WITHOUT VOLTAGE UNBALANCE MITIGATING CONVERTER ACTION

Load/Generator	Positive Sequence [pu]	Negative Sequence [pu]	Zero Sequence [pu]
PCC	1.02	$1.24 \cdot 10^{-2}$	$4.18 \cdot 10^{-2}$
Load 1	0.99	$1.71 \cdot 10^{-2}$	$5.42 \cdot 10^{-2}$
Load 2	1.00	$3.35 \cdot 10^{-2}$	$1.16 \cdot 10^{-1}$
PV 1	1.03	$2.86 \cdot 10^{-2}$	$9.89 \cdot 10^{-2}$
Load 3	1.00	$3.89 \cdot 10^{-2}$	$1.34 \cdot 10^{-1}$
Load 4	1.00	$4.14 \cdot 10^{-2}$	$1.43 \cdot 10^{-1}$
PV 2	1.04	$3.76 \cdot 10^{-2}$	$1.31 \cdot 10^{-1}$
ε^-, ε^o	/	$4.64 \cdot 10^{-3}$	$5.48 \cdot 10^{-2}$

TABLE IV. SEQUENCE VOLTAGES WITH VOLTAGE UNBALANCE MITIGATING CONVERTER ACTION

Load/Generator	Positive Sequence [pu]	Negative Sequence [pu]	Zero Sequence [pu]
PCC	1.02	$2.04 \cdot 10^{-3}$	$7.84 \cdot 10^{-3}$
Load 1	0.99	$2.80 \cdot 10^{-3}$	$1.02 \cdot 10^{-2}$
Load 2	0.99	$1.41 \cdot 10^{-3}$	$5.86 \cdot 10^{-3}$
PV 1	1.03	$1.68 \cdot 10^{-2}$	$6.66 \cdot 10^{-2}$
Load 3	0.99	$1.22 \cdot 10^{-3}$	$4.90 \cdot 10^{-3}$
Load 4	1.00	$7.15 \cdot 10^{-4}$	$2.86 \cdot 10^{-3}$
PV 2	1.03	$1.62 \cdot 10^{-2}$	$6.49 \cdot 10^{-2}$
ε^-, ε^o	/	$1.19 \cdot 10^{-5}$	$1.71 \cdot 10^{-4}$

acceptable under the assumptions of this paper. If one would judge the negative or zero sequence voltage seen by controlled generators excessive, he should consider that it is possible to modify the proposed control approach to include controlled generators voltage in the performance index to be minimized by setting their weight coefficients to a value greater than zero. However, this would lead to slightly larger negative and zero sequence voltages experienced by loads. Note also that, in this paper, the PCC has intentionally not been considered in the optimization procedure by setting its weighting coefficient equal to zero, since the target was to minimize the voltage unbalance at load terminals; however, from the simulation it is evident that, as expected, the proposed method has an indirect positive effect also on voltage unbalance at PCC. Furthermore, the latter can be included in the optimization procedure, if necessary, by setting its weighting coefficient to a value larger than zero.

VI. DISCUSSION ON CONTROL APPLICABILITY

Considering the results presented in Section V, it is of interest to draw some additional considerations on the applicability of the proposed control design. In order to apply the proposed control to a real feeder, the first issues to be addressed are measurement and network parameters availability. In fact, the implementation of the proposed control design would require a three-phase voltage measure at each node, in order to evaluate amplitude and phase of each sequence voltage. Measures do not need to be fast, since the control works in quasi-stationary conditions, but it needs to be accurate enough to detect negative and zero sequence voltages in the order of 1% of the grid voltage. Additionally, measures need to be synchronized, since the relative phase of measures performed at different nodes needs to be maintained. A possible low-cost solution to comply with these requirements can be found in [21]. Furthermore, grid topology and parameters are required to determine the elements of matrix A (6), which are necessary to realize the proposed control design. This data may be available from the distribution system operator or estimated. An example of estimation technique can be found in [22], along with additional references on this topic.

Secondly, the possible impact of the proposed control design on inverter performance, such as efficiency, and sizing is to be considered. Indeed, additional requirements for inverters may be necessary in order to apply the proposed control design, which, however, can still be considered favourable with respect to the installation of additional converters often proposed in literature. When the proposed control is considered, the maximum inverter current should be evaluated considering negative sequence current too, which may require a slight oversizing of the converter. However, quantification of the required oversizing cannot be generally assessed, in that it depends on the specific network parameters and loads. Further discussion on this issue lies outside the purposes of this paper, but specific studies are foreseen to address this problem. Additionally, inverter power capability and dynamics should be reconsidered. In fact, single-phase produce 100/120 Hz power oscillations, which need to be compensated by converters to mitigate voltage unbalance. This requires the converters to be able to provide a pulsating power, which is not typical of three phase systems. However, converters able to work under these conditions are known in literature [13] and, in general, this dynamic is not so fast to be considered a real problem for power converters. Considering now converter and grid efficiency, they may be affected by the proposed control design, in that it alters the negative sequence power flow significantly. However, this effect is strongly dependent on the specific system topology and parameters, which does not allow to draw general considerations. In particular, the relative position of single-phase loads and inverters seems to be the most significant cause of better or worse efficiency. Again, further discussion on this issue lies outside the purposes of this paper, but specific studies are foreseen to address this problem.

Lastly, it is reasonable to consider what can be done in case where the distributed generators are largely single-phase. Indeed, a technique to coordinate single-phase converters to mitigate voltage unbalance is proposed in [14]. However, it is worth considering that single-phase generators are usually significantly smaller than three-phase generators, hence it is reasonable to assume that the action of three-phase converters would be dominant with respect to the one of single-phase converters. On the basis of this consideration, [14] can be considered a solution when most power is provided by single-phase generators. On the contrary, when three-phase generators are available, the present control design should be considered. Only in case of grids where power provided by three-phase and single-phase generators are comparable, it may be of interest to extend the proposed control design to include single-phase generators too. However, this is considered a minority case and developments in these regards lie outside the purposes of this paper.

VII. Conclusions

In this paper a voltage unbalance mitigation technique for low-voltage microgrids or feeders in presence of large single-phase loads and distributed generators is introduced. Negative and zero sequence voltage controllers are integrated in distributed generators VSC controls, such that a centralized controller can be introduced for a general grid topology. The proposed control allows defining optimal negative and zero sequence voltage references as functions of nodal voltages and grid impedances for a general grid with non-predetermined number of loads, controllable and uncontrollable generators. A low-voltage feeder case study is considered to evaluate control effectiveness by means of numerical simulations, the results of which prove control effectiveness in strongly mitigating negative and zero sequence voltages experienced by loads. Even though the presented theoretical aspects and simulation results prove that the proposed control technique is suitable to reduce voltage unbalance, this approach is not actually foreseen by the present technical normative. However, it could be considered as a future extension of the ancillary services required from active users connected to public networks.

Future developments of this study will include an in-depth analysis of power converter control, including both voltage-controlled and current-controlled converters. Additionally, a detailed analysis of the control dynamics will be performed to assess the effect of transient due to changes in single-phase loads.

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