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Original Article

Wind Speed Modelling using Some Modified Gamma Distribution

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Abstract *- Wind speed as a renewable energy source, therefore wind speed data is needed to produce statistical modeling, especially in determining the best probability density function. for this purpose, several modified Gamma distributions will be used and tested to determine the best model to describe wind speed in Pekanbaru. the main goal of this study is to find the best fitting distribution to the daily wind speed measured over Pekanbaru region for the years 1999-2020 by using the four modified Gamma distributions. the maximum likelihood method will be used to get the estimated parameter value from the distribution used in this study. the distributions will be selected based on graphical inspection probability density function (pdf), cumulative distribution function (cdf) and numerical criteria (Akaike's information criterion (AIC). in most the cases, graphical inspection gave the same result but their AIC result differed. the best fit result was chosen as the distribution with the lowest values of AIC. in general, the Sujatha distribution has been selected as the best model.*

Keywords – *Mixture distribution, Modified gamma, Renewable energy, Wind speed.*

1. Introduction

Wind speed is one of the renewable energy sources. Wind energy is now one of the most cost-effective and never ending natural resource, therefore wind energy becomes one of the most efficient sources of renewable energy. the nature of the wind speed that is generated by nature and never runs out will produce a lot of wind speed data. Wind speed data in the form of fast and slow wind speeds will be very useful for research, especially in determining the best probability model in describing wind speed patterns in an area. It is well-known that wind energy potential can be estimated by using the distribution of the wind speed. Over the past years, research activity in the area of wind-speed distribution modelling has increased considerably. for the prediction of wind-speed distributions statistical models, several statistical distributions have been used for the description of the wind speed distribution. Thus, finding suitable wind speed distribution is one of the most important steps for the accurate estimation of wind energy potential of a specific region. A number of previous studies compared statistical distributions with measurements in order to examine how well the probability density function (pdf) describe the statistical properties of the measured wind speed. An overview of recent studies is presented in Table 1. Table 1 shows that a large number of different pdf were previously compared with wind speed data.

Table 1. Overview of recent studies that use different statistical distributions to assess wind speed occurrence probabilities

Author	Distributions
Carta, J. A. and Ramirez, P. 2007	Two components mixture Weibull [1]
Almalki, J. S. and Yuan, J. 2013	A new modified Weibull distribution [2]
Carta et al., 2008	mixture of von Mises distributions [3]
Kollu R et al., 2012	Mixture gamma and Weibull distribution gamma and Weibull distributions, mixture normal distribution, mixture of two-component truncated normal distribution [4]
Sayed, A.IA and Sabri, S.R.S. 2022	Transformed modified internal rate of return on Gamma Distribution for Long Term Stock Investment Modelling [5]
Toure, S.	Investigations into some simple expressions of the Gamma Function in wind power by the wiebull distribution [6]
Al Buhairi, M.H	A statistical Analysis of wind speed data and an assessment of wind energy potential in taiz-yemen [7]
Ivana P. et al. 2017	Application of four probability distributions for wind speed modeling [8]

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the objective of this study is to propose four modified Gamma distributions namely mixture of Gamma (1, β) and Gamma(2, β) with their mixing weighted $\frac{\beta}{\beta+1}$ and $\frac{1}{\beta+1}$ respectively, mixture of Gamma (1, β) and Gamma (2, β) with their weighted $\frac{\beta^2}{\beta^2+1}$ and $\frac{1}{\beta^2+1}$ respectively, mixture of Gamma (1, β) and gamma(3, β) with their mixing weighted $\frac{\beta^2}{\beta^2+2}$ and $\frac{2}{\beta^2+2}$ respectively and mixture of Gamma (1, β), Gamma (2, β) and Gamma (3, β) with their mixing weighted $\frac{\beta^2}{\beta^2+\beta+2}$, $\frac{\beta}{\beta^2+\beta+2}$, $\frac{2}{\beta^2+\beta+2}$. Comparison of the proposed mixture distributions with existing distribution functions is done to demonstrate their suitability in describing wind speed characteristics.

Modelling daily wind speed data by using the two parameter Weibull distribution for ten selected weather stations in Sri Lanka is presented. the objective of this study is to model the daily wind speed distributions and determinethe two parameters, *k* and *c*, of a Weibull function that can describe the wind speed distributions in a few selected weather stations scattered around the island. It was seen thatthe daily wind speed distribution can be modelled with a reasonable accuracy, using thetwo parameter Weibull distribution for the stations considered in this work [9]. Wind speed modelling is of increasing interest, both for basic research and for applications, as, e.g., for wind turbine development and strategies to construct large wind power plants. Generally, such modelling is hampered by the non-stationary features of wind speed data that, to a large extent, reflect the turbulent dynamics in the atmosphere. We study how these features can be captured by nested ARIMA models. in this approach, wind speed fluctuations in given time windows are modelled by one stochastic process, and the parameter variation between successive windows by another one. for deriving the wind speed model, we use 20 months of data collected at the FINO1 platform at the North Sea and use a variable transformation that best maps the wind speed onto a Gaussian random variable. We find that wind speed increments can be well reproduced for up to four standard deviations [10].

This paper [11] proposes a stochastic wind power model based on an autoregressive integrated moving average (ARIMA) process. the model takes into account the nonstationarity and physical limits of stochastic wind power generation. the model is constructed based on wind power measurement of one year from the Nysted offshore wind farm in Denmark. the proposed limited-ARIMA (LARIMA) model introduces a limiter and characterizes the stochastic wind power generation by mean level, temporal correlation and driving noise. the model is validated against the measurement in terms of temporal correlation and probability distribution. the LARIMA model outperforms a first-order transition matrix based discrete Markov model in terms of temporal correlation, probability distribution and model parameter number. the proposed LARIMA model is further extended to include the monthly variation of the stochastic wind power generation.

One of the most crucial prerequisites for effective wind power planning and operation in power systems is precise wind speed forecasting. Highly random fluctuations of wind influenced by the conditions of the atmosphere, weather and terrain result in difficulties of forecasting regardless of whether it is short-term or long-term. the current study has developed a method to model wind speed data predictions with dependence on seasonal wind variations over a particular time frame, usually a year, in the form of a Weibull distribution model with an artificial neural network (ANN). As a result, the essential dependencies between the wind speed and seasonal weather variation are exploited. the proposed model utilizes the ANN to predict the wind speed data, which has similar chronological and seasonal characteristics to the actual wind data. This model was applied to wind speed databases from selected sites in Malaysia, namely Mersing, Kudat, and Kuala Terengganu, to validate the proposed model. the results indicate that the proposed hybrid artificial neural network (HANN) model is capable of depicting the fluctuating wind speed during different seasons of the year at different locations [12]. in this study [13], five numerical Weibull distribution methods, namely, the maximum likelihood method, the modified maximum likelihood method (MLM), the energy pattern factor method (EPF), the graphical method (GM), and the empirical method (EM) were explored using hourly synoptic data collected from 1985 to 2013 in the district of Maroua in Cameroon. the performance analysis revealed that the MLM was the most accurate model followed by the EPF and the GM. Furthermore, the comparison between the wind speed standard deviation predicted by the proposed models and the measured data showed that the MLM has a smaller relative error of -3.33% on average compared to -11.67% on average for the EPF and -8.86% on average for the GM. As a result, the MLM was precisely recommended to estimate the scale and shape parameters for an accurate and efficient wind energy potential evaluation [13]. in this work [14], thereliability of wind-based electric microgrids was evaluated using a Markov model, taking the intermittent nature of wind speedinto account. the effects of different wind speed modelling techniques based on the auto-regressive moving average method,Markov model, and probability distribution function on the reliability analysis of electric microgrids were assessed. the Roy Billinton test system was used to illustrate the analysis and evaluate the different load and system indices. in the paper [15], wind speed potential analysis realized using

Inverse Weibull Distribution (IWD) for Bilecik, Turkey. Maximum likelihood method for parameter estimation used for wind speed modelling analysis. All analysis is carried out by Matrix Laboratory (MATLAB) programming language. Monthly and yearly wind speeds are modeled by Inverse Weibulldistribution. Accuracy of the modelling is evaluated in terms of Root Mean Square Error (RMSE).

In this study [16] weconsider four different probability distributions: the 2-parameter Weibull, the 3-parameter Weibull, the 2-parameter Gamma and the 2-parameter Lognormal. All of them are applied on the wind speed data recorded at the airport in Dolný Hričov. Parameters of the density distributions are estimated by the maximum likelihood method. in order to select the best fitting distribution there are used the χ^2 test, the Kolmogorov-Smirnov test, the Akaike information criterion, the Bayesian information criterion, the coefficient of determination and the root mean square error. Based on the results the 3-parameter Weibull performs as the best and the 2-parameter Weibull distribution performs as the second best. This research reviews and compares the popular parametric and non-parametric models for wind speed probability distribution and the estimation methods for these models' parameters (the widely used methods and stochastic heuristic optimization algorithm) [17]. in this study [18], information from several goodness-of-fit criteria, e.g., the R2 coefficient, Kolmogorov–Smirnov statistic, Akaike's information criterion, and deviation in skewness/kurtosis were integrated for the conclusive selection of the best-fit distribution model of wind-speed data. the gamma distribution is one of the continuous distributions; the distributions are very versatile and give useful presentations of many physical situations. They [19] are perhaps the most applied statistical distribution in the area of reliability. in this paper, we present the study of properties and applications of gamma distribution to real life situations such as fitting the gamma distribution into data, burn-out time of electrical devices and reliability theory. the study employs the moment generating function approach and the special case of gamma distribution to show that the gamma distribution is a legitimate continuous probability distribution showing its characteristics.

This study [20] proposed a hybrid computational model by incorporating Simulated Annealing algorithm (SA) in the maximum likelihood in estimation the parameters of Generalized Gamma Distribution (GGD). the purpose is to improve the searching capacity of max imum likelihood estimator for Generalized Gamma Distribution (GGD). A simulated Annealing algorithm (SA) is one of the global search heuristics computational approach inspired by the metallurgical process, in which metal is rapidly heated to a high temper ature, then cooled slowly until it reaches the lowest-energy state, increasing its strength and making it easier to work with. It has been applied in approximating global optimization in a large search space for various optimization problems. This study employs a Simulated Annealing algorithm (SA) to improve the global search capacity of maximum likelihood methods (MLE) in estimating the parameters of the Generalized Gamma Distribution (GGD). the Generalized Gamma Distribution (GGD) constitutes an extensive family that contains nearly all of the most commonly used distributions including the exponential, Weibull, and lognormal distribu tion. the performances of the proposed estimation method are computed based on their biases and mean square errors through a simulation study. the study reveals that the Simulated Annealing algorithm (SA) performs better than the classical estimators in estimating the parameters of Generalized Gamma Distribution (GGD) [20].

The volume extinction coefficients at 1.03 μm, 3.70 μm, and 10.38 μm, normalized to that at 0.50- μm wavelength, are calculated as a function of the shape parameters of the modified gamma size distribution using parameter ranges appropriate for haze and fog droplet polydispersions. Based on the sensitivity of the normalized volume extinction coefficients on the shape parameters, different procedures are proposed for utilizing the extinction features in giving form to the size distribution corresponding to the various evolutionary stages of the water droplet population. Such a methodology presents applicability in the field of fog forecast [21]. the MGD model is therefore more fundamentally relevant to size distributions of nonspherical particles than is often appreciated. the central purpose of this paper is to serve as a concise single-source reference for the mathematical properties of, and conversions between, atmospheric particle size distributions that can expressed as MGDs, including exponential and gamma distributions as special cases. [22].

For the first time, a new generalization of generalized gamma distribution called the modified generalized gamma distribution has been introduced to provide greater flexibility in modeling data from a practical viewpoint. the new distribution generalizes some recently introduced generalizations of the gamma distribution. Various properties of the proposed distribution, including explicit expressions for the moments, quantiles, mode, moment generating function, mean deviation, mean residual lifetime and expression of the entropies are derived. the distribution is capable of monotonically increasing, decreasing, bathtub-shaped, and upside-down bathtub-shaped hazard rates. the maximum likelihood estimators of unknown parameters cannot be obtained in explicit forms, and they have to be obtained by solving non-linear equations only. Two real data sets have been analyzed to show how the proposed models work in practice [23]. the generalized Gamma model has been applied in a variety of research fields, including reliability engineering and lifetime analysis. Indeed, we know that, from the above, it is unbounded. Data have a bounded service area in a variety of applications. A new five-parameter bounded generalized Gamma model, the bounded Weibull model with four parameters, the bounded Gamma model with four parameters, the bounded generalized Gaussian model with three parameters, the bounded exponential model with three parameters, and the bounded Rayleigh model with two parameters, is presented in this paper as a special case. This approach to the problem, which utilizes a bounded support area, allows for a great deal of versatility in fitting various shapes of observed data. Numerous properties of the proposed distribution have been deduced, including explicit expressions for the moments, quantiles, mode, moment generating function, mean variance, mean residual lifespan, and entropies, skewness, kurtosis, hazard function, survival function, r th order statistic, and median distributions. the delivery has hazard frequencies that are monotonically increasing or declining, bathtub-shaped, or upside-down bathtub-shaped. We use the Newton Raphson approach to approximate model parameters that increase the log-likelihood function and some of the parameters have a closed iterative structure. Six actual data sets and six simulated data sets were tested to demonstrate how the proposed model works in reality. We illustrate why the Model is more stable and less affected by sample size. Additionally, the suggested model for wavelet histogram fitting of images and sounds is very accurate [24].

Yemen possesses a very good potential of renewable energy, such as solar and wind energy. Wind energy is an alternative clean energy source compared to fossil fuel, which pollute the lower layer of the atmosphere. in this study, statistical methods are used to analyze the wind speed data of Taiz in the southwest of Yemen. Wind speed is the most important parameter in the design and study of wind energy conversion systems. the wind speed data were obtained from the National Water Resources Information Center in Taiz (TaizNWRIC) over a four year period, 1999 to 2002. in the present study, the wind energy potential of the location is statistically analyzed based on wind speed data, measured over a period of four years. the probability distributions are derived from the wind data and their distributional parameters are identified. Two probability density functions are fitted to the measured probability distributions on a yearly basis. the wind energy potential of the location is studied based on the Weibull and the Rayleigh models [7].

The Weibull distribution is a probability density function (PDF) which is widely used in the study of meteorological data. the statistical analysis of the wind speed v by using the Weibull distribution leads to the estimate of the mean wind speed $< v >$, the variance of v around $< v >$ and the mean power density in the wind. the gamma function Γ is involved in those calculations, particularly Γ (1+1/k), Γ (1+2/k) and Γ (1+3/k). the paper reports the use of the Weibull PDF $f(v)$ to estimate the gamma function. the study was performed by looking for the wind speeds related to the maximum values of $f(v)$, $v^2 f(v)$ and $v^3 f(v)$. As a result, some approximate relationships were obtained for Γ (1+1/k), Γ (1+2/k) and Γ (1+3/k), that use some fitting polynomial functions. Very good agreements were found between the exact and the estimated values of Γ (1+n/k) that can be used for the estimation of the mean wind speed $< v >$, the variance σ^2 of the wind speed v; around the mean speed and the average wind power density [6]. the volume extinction coefficients at 1.03 μm, 3.70 μm, and 10.38 μm, normalized to that at 0.50-μm wave length, are calculated as a function of the shape parameters of the modified gamma size distribution using parameter ranges appropriate for haze and fog droplet polydispersions. Based on the sensitivity of the nor malized volume extinction coefficients on the shape parameters, different procedures are proposed for utiliz ing the extinction features in giving form to the size distribution corresponding to the various evolutionary stages of the water droplet population. Such a methodology presents applicability in the field of fog fore cast [25].

The wind energy is one of the most significant alternative clean energy source and rapidly developing renewable energy sources in the world. for the evaluation of wind energy potential, probability density functions (pdfs) are usually used to model wind speed distributions. the selection of the appropriate pdf reduces the wind power estimation error and also allow to achieve characteristics. in the literature, different pdfs used to model wind speed data for wind energy applications. in this study, we propose a new probability distribution to model the wind speed data. Firstly, we defined the new probability distribution named Poisson-Gamma (PG) distribution and we analyzed a wind speed data sets which are about five pressure degree for the station. We obtained the data sets from Turkish State Meteorological Service. Then, we modelled the data sets with Exponential, Weibull, Lomax, 3 parameters Burr, Gumbel, Gamma, Rayleigh which are used to model wind speed data, and PG distributions. Finally, we compared the distribution, to select the best fitted model and demonstrated that PG distribution modeled the data sets better [26].

2. Data and Study Area

Pekanbaru City is an industrial city located in Riau Province. A city with a tropical climate that has wind speeds that vary from 0.2 m/s to 15.8 m/s. Initial information on wind speed in the city of Pekanbaru can be seen in the descriptive statistics for daily wind speed are presented in Table 2. the variations of wind speed data that are not so large (1,514 m/s) indicate that this wind speed is quite stable and very good as a source of electrical energy for household purposes. the average wind speed which is not too high (2.088) has added to the belief that the wind speed in the Pekanbaru area cannot be used as a source of electrical energy for industrial purposes.the original data consisted of wind speed records from 1999 to 2020, which were provided by the meteorological, climatological, and geophysical agency of Pekanbaru city, Indonesia. the data and the histogram or characteristic wind speed are presented in Figure 1.

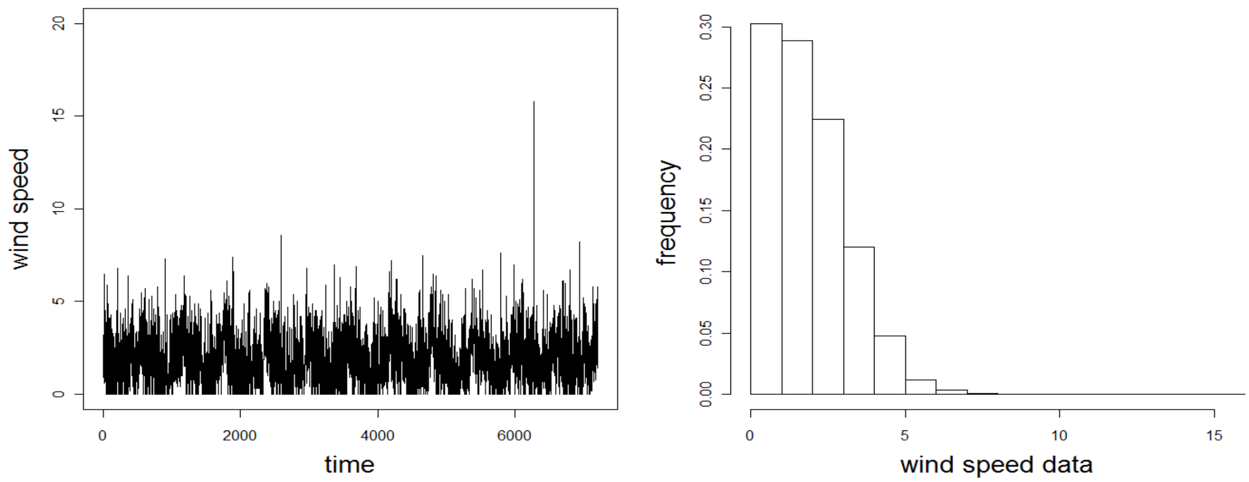

Fig. 1 Plot and histogram wind speed data on Pekanbaru respectively

Table 2. the descriptive statistics for daily wind speed

Statistics	Mean	Variation	Minimum	Maximum
	2.088	1.51451	0.200	15.800

3. Methods

Wind speed distribution modeling requires analysis of wind speed data over a number of years. the primary tools to describe wind speed characteristics are probability density functions. Four modified probability density function of Gamma distributions associated with modeling wind speed, are considered in this paper. the probability density function and distribution function for each distribution that we consider are as given in Table 3, where *y* denote the observed values of the random variable representing the event of interest, β is the parameter

Table 3. Probability Density Function (pdf) and distribution function (cdf) of some modified Gamma distribution

	Gamma modified distribution	Probability density function (pdf) dan distributin function (cdf)
1	mixture of Gamma(1, β) and Gamma(2, β) with their mixing weighted $\frac{\beta}{\beta+1}$ and $\frac{1}{\beta+1}$ respectively *)	$f(y,\beta)=\frac{\beta^2}{\beta+1}(1+y)e^{-\beta y}, \quad y>0$ $F(y)=1-\left(1+\frac{y\beta}{\beta+1}\right)e^{-\theta y}$
2	mixture of Gamma (1, β) and Gamma (2, β) with their weighted $\frac{\beta^2}{\beta^2+1}$ and $\frac{1}{\beta^2+1}$ respectively**)	$f(y,\beta)=\frac{\beta^2}{\beta^2+1}(\beta+y)e^{-\beta y}, \quad y>0$ $F(y)=1-\left(1+\frac{\beta y}{\beta^2+1}\right)e^{-y\beta}$
3	mixture of Gamma (1, β) and gamma(3, β) with their mixing weighted $\frac{\beta^2}{\beta^2+2}$ and $\frac{2}{\beta^2+2}$ respectively ***)	$f(y,\beta)=\frac{\beta^3}{\beta^2+2}(1+y^2)e^{-\beta y}, \quad y>0$ $F(y)=1-\left(1+\frac{\beta y(\beta y+2)}{\beta^2+2}\right)$
4	mixture of Gamma(1, β), Gamma(2, β) and Gamma(3, β) with their mixing weighted $\frac{\beta^2}{\beta^2+\beta+2}$, $\frac{\beta}{\beta^2+\beta+2}$, $\frac{2}{\beta^2+\beta+2}$****)	$f(y,\beta)=\frac{\beta^3}{\beta^2+\beta+2}(1+y+y^2)e^{-\beta y}, y>0$ $F(y)=1-\left(1+\frac{\beta y(\beta y+\beta+2)}{\beta^2+\beta+2}\right)e^{-\beta y}$

the Gamma Modified distribution have known a Lindley distribution*), Shanker distribution**), Akash distribution***), and Sujatha Distribution****) [27] [28] [29]

For selecting the best fit model, choice of the model definition, parameter estimation tools are important. the parameter estimation of the distribution function are calculated using maximum likelihood method. the procedure of goodness of fit tests for model selection, both numerically and graphically, is discussed.

4. Results and Discussion

The fitting of daily wind speed collected across Pekanbaru part of Riau Province was considered using data from the period between 1999 and 2020. the data used for are presented and also the wind speed data histogram are presented on Figure 1. for the purpose of modelling the wind speed, various distributions have been used, such as Lindley distribution, Shanker distribution, Akash distribution, and Sujatha distribution. Behavior of the pdf for varying values of parameters β shown in Figures 2. It is clearly seen from the graphs of pdf it is obvious that four simple mixture distribution is monotonically decreasing. As the value of parameter β increases.

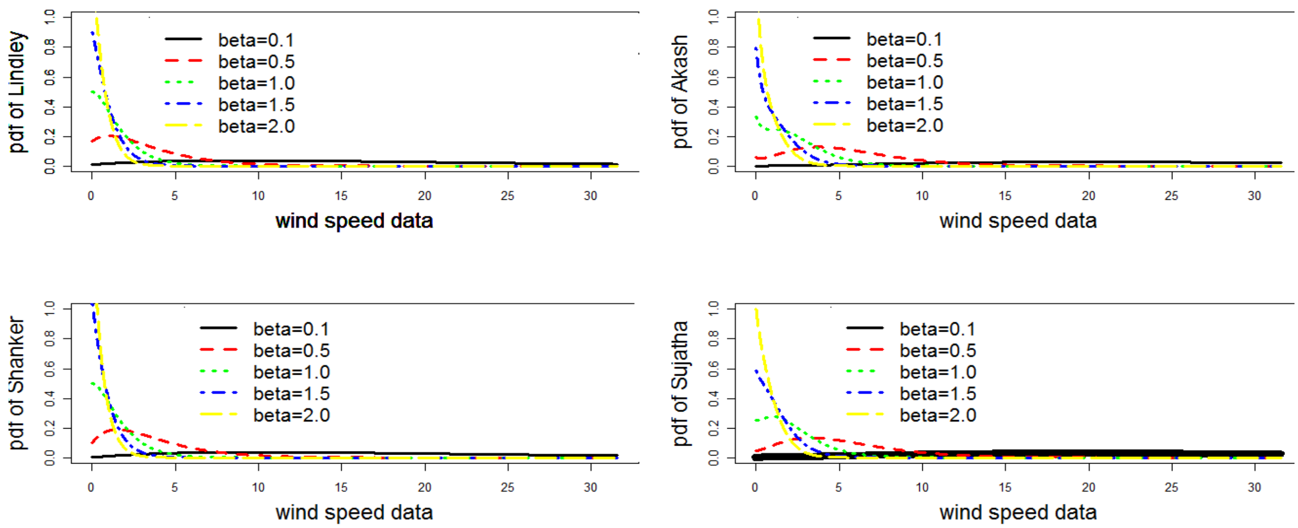

Fig. 2 Graph of pdf some modified Gamma Distributions (the Lindley, Akash, Shanker, and Sujatha) for different values of the parameter beta (β).

Tables 4 shows the parameter estimates and the statistical parameters for fitness evaluation of pdf namely AIC currently analyzed for the four modified Gamma distribution at the Pekanbaru station. From Table 4, by comparing each model, it is clear that the Akash have the highest AIC values, implying that pdf is not a good model for wind speed in Pekanbaru. However, as the number of components mixture increase for Sujatha distribution, the AIC values decrease, which implies that the use of more components in the modified Gamma distribution models provides a model that more adequately fits the data.

Table 4. the estimated parameters and AIC values for wind speed data

	Lindley	Akash	Shanker	Sujatha
Parameters (beta)	0.8196445	0.5289609	0.8359775	1.141017
AIC	22994.07	33709.43	22964.63	22595.84

Figure 3 and 4 shows the fitted four modified Gamma distribution, based on pdf and cdf respectively. From this figure, Lindley, Shanker and Sujatha distribution model is able to provide a good result for wind speed data. However, instead of graphical evaluation, Table 2 provides a more meaningful comparison using AIC values. Because the Sujatha distribution has been determined to be a good model for the wind speed data,

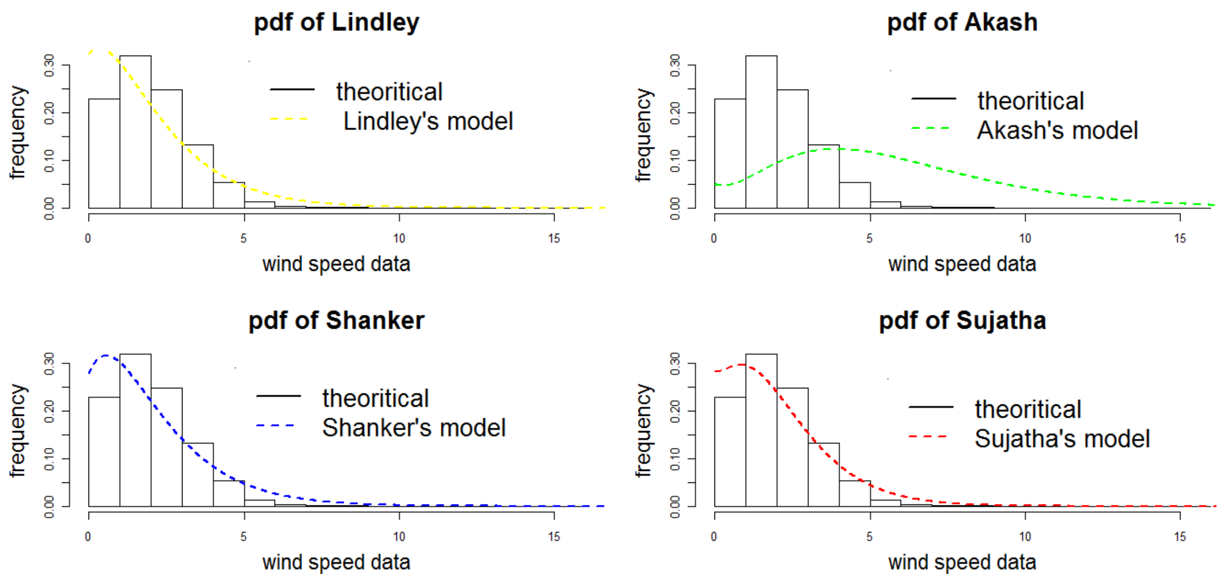

Fig. 3 Fitted pdf plots of some modified Gamma distributions (Lindley, Akash, Shanker and Sujatha) for the given dataset.

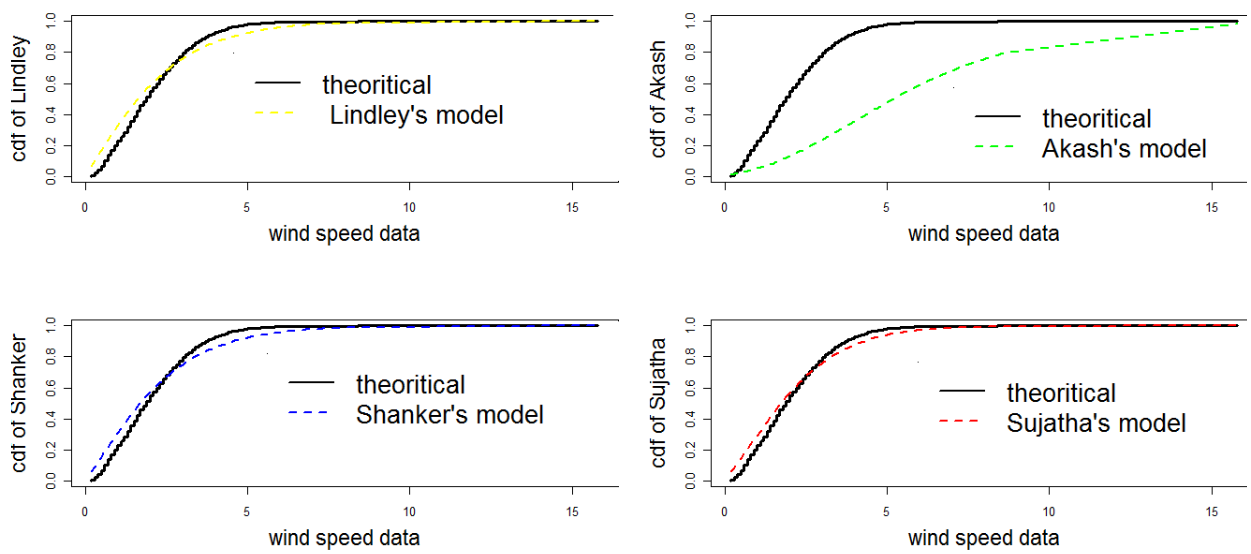

Fig. 4 Fitted cdf plots of some modified Gamma distributions (Lindley, Akash, Shanker and Sujatha) for the given dataset.

5. Conclusion

in this research focused on determining the best statistical model wind speed in Pekanbaru. the Four modified Gamma distribution namely Lindley, Shanker, Akash and Sujatha distribution. There of were fit to the data. the results obtained based on graphical and AIC values indicated that Sujatha distribution with three components adequately modelled the wind speed distribution in Pekanbaru. Additionally,From the pdf curve Sujatha distribution can be seen that the most probable wind speed occurs at a speed of 0.5-1 m/s with the probability of 32%.

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