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Judul : Analysis of student ability in functional thinking through The function table: the case of square function



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## ANALYSIS OF STUDENT ABILITY IN FUNCTIONAL THINKING THROUGH THE FUNCTION TABLE: THE CASE OF SQUARE FUNCTION

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### Abstrak

Salah satu cara untuk menelusuri kemampuan mahasiswa dalam memahami konsep fungsi dapat dilakukan dengan cara menganalisis kemampuan berpikir fungsionalnnya. Penelitian ini bertujuan untuk mengidentifikasi dan mengetahui kemampuan berpikir fungsional mahasiswa melalui tabel fungsi. Penelitian ini merupakan penelitian kualitatif yang bersifat eksploratif. Subjek penelitian adalah 2 dari 56 mahasiswa semester 5 Jurusan Pendidikan Matematika Universitas Islam Negeri Suska Riau. Subjek penelitian dipilih secara purposive yaitu mahasiswa yang menyelesaikan secara benar lembar tes dan memiliki kemampuan komunikasi yang lancar. Data dikumpulkan melalui tes dan wawancara. Hasil lembar jawaban tes dianalisis berdasarkan kerangka kerja berpikir fungsional dan hasil wawancara dianalisis untuk mendalami dan mengklarifikasi berpikir fungsional mahasiswa yang belum terungkap pada lembar jawaban. Hasil analisis data menunjukan bahwa ada dua tahapan penyelesaian yaitu tahap pertama, menentukan pola bilangan dan berpikir tentang pola bilangan sampai ke-n, menentukan beda dari  $X_n$ , menentukan beda dari  $Y_n$ , menentukan perubahan nilai antara  $X_n$  dan  $Y_n$ , menggeneralisasikan barisan  $X_n$ , dan menggeneralisasikan barisan  $Y_n$ . Sedangkan tahap kedua, menentukan perubahan nilai antara  $X_n$  dan  $Y_n$ , menggeneralisasikan barisan  $X_n$ , menggeneralisasikan barisan  $Y_n$ , dan menggeneralisasikan hubungan antara  $X_n$  dan  $Y_n$ . Berdasarkan hasil penelitian menunjukkan bahwa mahasiswa mampu memenuhi semua kerangka kerja berpikir fungsional dalam menyelesaikan masalah melalui tabel fungsi.

Kata kunci: berpikir fungsional, fungsi kuadrat, pemecahan masalah, tabel fungsi

#### Abstract

One way to track students' ability to understand the concept of function can be done by analyzing their functional thinking skills. This study aims to identify and determine students' functional thinking skills through function tables. This research is an exploratory qualitative research. The research subjects were 2 out of 56 5th semester students of the Department of Mathematics Education, Suska Riau State Islamic University. The research subjects were selected purposively, namely students who correctly completed the test sheet and had fluent communication skills. Data were collected through tests and interviews. The results of the test answer sheets were analyzed based on the functional thinking framework and the results of the interviews were analyzed to explore and clarify students' functional thinking that had not been revealed on the answer sheets. The results of data analysis show that there are two stages of completion, namely the first stage, determining the number pattern and thinking about the number pattern up to the nth, determining the difference from  $X_n$  determining the difference from  $Y_n$ , determining  $Y_n$  sequence. While the second stage, determine the change in value between  $X_n$  and  $Y_n$ . Based on the results of the study, it showed that students were able to fulfill all the frameworks for solving problems through the function table

Keywords: functional thinking, function table, mathematical problem solving, quadratic function

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There are five functional thinking frameworks according to Blanton, dkk,

(2015), first, generalizing linear data and

organizing it in a function table; second,

identify recursive patterns and describe

in words, using patterns to predict

approximate data; third, identify

covariational relationships and describe

in words; fourth, identify the function

rules and describe in words and

variables; and fifth use function rules to

predict function values broadly.

Meanwhile, according to Blanton &

Kaput (2011) dan Tanis (2011) there are

three functional thinking frameworks,

namely: first, recursive patterning which

means looking for variations or patterns

of variation in a series of values for

variables so that certain values can be

obtained based on previous values;

second, covariational thinking is focused

on analyzing two variations of quantities

simultaneously and understanding that

change is an explicit and dynamic part of

the function description (e.g., "when x

increases by 1, y increases by 3"), and

third, correspondence relationships are

based on identification of correlations.

between variables (for example, "y is 3

times x plus 2"). This study refers to the

functional thinking framework of Smith and Tanish which is used as an indicator

of functional thinking. The indicators

used to analyze functional thinking are

outlined in Table 1.

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## INTRODUCTION

At the level of junior high school education up to college the function material is taught. However, understanding the function is not a concept that is easily understood by students. Many students experience errors in representing and interpreting functional forms. These difficulties and misunderstandings will have an impact on student learning outcomes, if a solution is not found. So that teachers/lecturers need to provide practice questions related to function material and familiarize students with functional thinking from an early age. Functional thinking itself is a mental activity in generalizing the relationship between covariant quantities that can be represented through words, algebra, tables and graphs. This is in line with the opinion of Markworth (2010) dan Blanton, dkk (2015) stating that functional thinking is a generalization of the relationship between covariant quantities and can be represented through words, algebraic notation, tables and graphs. So functional thinking ability is the ability of students to make a generalization of the relationship between two (or more) quantity variations that can be represented through words, algebra, tables and graphs.

Table 1. Indicators for Analyzing Functional Thinking

Functional Thinking	Description	Indicators for Analyzing		Code
Framework			Functional Thinking	
Define a recursive	Observing objects	≻	Determine the number pattern	PL1
pattern	in tabular form and	of the given object and think		
	thinking about the		about the number pattern up to	
	next unknown		the nth.	
	object	≻	Determine the difference of $X_n$	PL2

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Functional Thinking Framework	Description	Indicators for Analyzing Functional Thinking	Code
		Determine the difference from	PL3
		$Y_n$	
Determine the	Determine the	Determines the value change	PN1
covariational	change in the value	in the sequence $X_n$	
relationship	of the relationship	Determines the value change	PN2
	between variations	in the sequence $Y_n$	
	in quantity in a	Determines the change in	PN3
	given problem	value between $X_n$ and $Y_n$	
Determine	Generalizing the	$\triangleright$ Generalizing the sequence $X_n$	GB1
correspondence	relationship	$\triangleright$ Generalizing the sequence $Y_n$	GB2
	between quantity	Generalizing the relationship	GB3
	variations on a	between $X_n$ and $Y_n$	
	given problem		

Many researches on functional thinking have been carried out, one of the experts who conducted research on functional thinking is Mceldoon (2010) who conducted research on the assessment of elementary school students in functional thinking. Blanton et al. (2016) who found that there were eight levels of functional thinking of students, namely the first level of prestructure, at this level students did not describe a recursive pattern in a single data value sequence or any relationship between two covariates; both special recursive levels, at this level students have not developed the underlying recursive pattern as a generalization; the three general recursive levels, at this level students begin to be able to think about recursive patterns; four specific functional, at this level students show special functional level thinking that conceptualizes functional relationships as certain relationships between certain appropriate values; fifth is primitivegeneral functional, at this level students conceptualize the general can relationship between two quantities even though their representation has primitive characteristics; the sixth appears

functional-general, at this level begins the emergence of generalizations of functional relationships even though their representation is incomplete; the seven general functionals are summarized, at this level students show thinking at the general-functional level which is conceptualized as a general function of the relationship between two quantities that is explicitly recorded; the last eight functions as objects, at this level students perceive boundaries related to the general form of the relationship between quantities. Blanton & Kaput (2005); Doorman, dkk (2012); Stephens (2017); Stephens, dkk (2017); Warren, dkk (2006); Wilkie (2004); Wilkie (2015); Wilkie & Clarke (2015); Wilkie & Clarke (2016) design and develop learning for teachers so as to improve students' functional thinking. Research Yuniati et al. (2019) found some representations of students in functional thinking, but in general the representations that appear are algebraic representations. Furthermore Yuniati et al (2020); Yuniati et al. (2020); Yuniati & Suparjono (2021); Yuniati & Suparjono (2019) found that students were able to think partially functionally Commented [A3]: Sesuaikan dengan template

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and use arithmetic sequence formulas. On the other hand Stephens et al. (2017) dan Stephens et al. (2017) found that through intervention can improve students' ability to determine recursive patterns and covariance relationships between variables. In this study, looking for student responses that are sometimes different from certain tasks (tasks given in the form of linear functions and quadratic functions). Then Tanisli (2011) found that through function tables students can think functionally, that is, they can determine recursive patterns, determine covariance relationships and be able to generalize covariance relationships with symbolic representations. But from several previous studies, no one has conducted research that uses function tables to determine the functional thinking ability of students who can generalize the form of quadratic functions

The quadratic function is a mathematical material that must be studied by high school (SMA)/Madrasah Aliyah (MA) students even up to college. The material for quadratic functions has many applications in everyday life and is a prerequisite for studying other mathematics, such as derivatives. integrals, linear programming and geometry. Given the importance of students understanding the material quadratic function, it is necessary to explore the extent to which students' ability to understand the material of quadratic functions. Therefore, the purpose of this study is to explore students' abilities in functional thinking through function tables involving quadratic functions.

## **RESEARCH METHODS**

This research is a qualitative research that is exploratory. There were 56 students who participated in this study in the fifth semester of Mathematics Education at the State Islamic University of Suska Riau. The student is given a test via google meet and the time given is 60 minutes. The tests given are non-routine mathematical task. Based on the results of the answer sheet analysis, two groups of different answers were obtained. Then from each group one student's answer was chosen, so there were two student answers that were used as research subjects. The research subjects were selected purposively, namely students who had completed correctly and had fluent communication skills.

Data collection uses tests and interview guidelines to identify students' functional thinking abilities. The test was adopted from Tanişli (2011) which uses a function table to explore students' functional thinking. In solving test questions, students are expected to be able to: 1) determine recursive patterns, 2) determine covariational changes, and 3) generalize correspondence. The indicators/framework can be seen in table 1. The interviews used were unstructured interviews. Because the questions in the interview guide used are still very likely to develop according to the conditions or characteristics of the respondents. This activity was documented with an audiovisual recording device and interviews were conducted via chat via WhatsApp. The next step is triangulation which is used to test the validity of the data. Creswell (2012) states that triangulation is the process of corroborating evidence based on different individuals, types of data (observation records and interviews), or data collection methods (documents and interviews) in descriptions and themes in research. qualitative Researchers examine each source of information and find evidence to support a theme. This ensures that the research is accurate

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because the information refers to an information source, individual, or process. The triangulation method used in this research is to compare student answer sheets and interviews.

In general, the process of data analysis in qualitative research includes: data reduction, data categorization, synthesis, and ends with the preparation of working hypotheses (Creswell 2012). Qualitative data analysis is carried out and interactively takes place continuously until complete, so that the data is saturated. The size of the data saturation is indicated by the absence of new data or information. The stages of data analysis in this qualitative study were modified from Creswell (2012) as follows: 1) transcribing interview data, scanning student answer sheets, and arranging the data into certain types based on the characteristics of the data; 2) reducing data, namely removing unnecessary data and organizing raw data obtained from the field; 3) analyze the data in more detail by coding or categorizing the data. At this stage, the coding of these categories with special terms can be seen in table 1; 4) drawing the structure of students' functional thinking in solving problems based on data categorization; and 5) drawing conclusions based on the results of data analysis, both obtained by using tests and interviews.

## **RESULTS AND DISCUSSION**

Based on the research subject's answer sheet S1, the first step is to understand the given function table by reading and looking at the test sheet. Then determine the number pattern on the *x* variable, where the number pattern is the addition of 1 and the number pattern on the *y* variable is a multilevel number, namely +2, +3, +4, +5, and so on. While the subject of S2 did not

determine the number pattern on the variable x and variable y. The answer sheets for S1 and S2 subjects in determining number patterns can be seen in Figure 1.



Figure 1. Answer sheets for subjects S1 and S2 determine the number pattern

In determining the covariational relationship, the subject of S1 stated that the relationship between many numbers x and many numbers y is if x = 2 then y = 1, if x = 3 then y = 3, if x = 4then y = 6, if x = 5 then y = 10, and so on. While the subject of S2 stated that there is a relationship between many numbers x and y numbers, namely to determine y in the second row and so on is the sum of the numbers x and y in the previous row or  $x = x_1$ ,  $y = y_1$ ,  $y_2 =$  $x_1 + y_1, y_3 = x_2 + y_2, y_4 = x_3 + y_3,$  $y_5 = x_4 + y_4, \ y_6 = x_5 + y_5, y_7 = x_6 + y_5$  $y_6$ , and so on. The answer sheets for subjects S1 and S2 in determining the covariational relationship can be seen in Figure 2.

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JIKA X=2, MAKGY=1	ada kubungunnya autors henuek bil x den bil.
J1Ka X=3, maka Y≥3	W(4) 3: 7, 2 5: 4, 1
JING X=4 I MARA Y=6	32× X1+3 = 2+1×3
JIKG X = 5, Maka Y = 10	<u>49</u> • <del>1</del> 2 • <del>1</del> <u>3</u> • <u>3</u> + <u>3</u> • <u>6</u> <u>48</u> • <del>1</del> 3 + <u>4</u> • <u>6</u> • <u>6</u> = 10
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dan seterusnya	116 × 75 + 46 + 5 + 15 + 21 47 = 76 + 46 + 7 + 71 = 28
	Ab



Generalizing the relationship between covariance variations (correspondence), subjects S1 and S2 generalize many numbers x using the arithmetic sequence formula  $U_n = a + (n - 1)b$  so that the formula for many numbers x is  $U_n = n + 1$ . The answer sheets for subjects S1 and S2 in determining the correspondence of many x numbers can be seen in Figure 3.



Figure 3. Answer sheets for subjects S1 and S2 determine the correspondence of many numbers x

Next, subject S1 generalizes many numbers y using the formula  $U_n = an^2 + bn + c$ , while subject S2 generalizes many numbers y using the formula  $U_n = \frac{a}{0!} + \frac{(n-1)b}{1!} + \frac{(n-1)(n-2)c}{2!}$ . So the result of generalizing many numbers y is  $U_n = \frac{1}{2}n^2 + \frac{1}{2}n$ . The answer sheets for subjects S1 and S2 in determining the correspondence of many y numbers can be seen in Figure 4.



Figure 4. Answer sheets for subjects S1 and S2 determine the correspondence of many numbers y

Then the subject of S2 generalizes many numbers x and many numbers y by trial and error so that the formula  $y_n = \frac{x_n^2 - x_n}{2}$  is obtained. The answer sheet for the subject of S2 in determining the correspondence of many x numbers and many y numbers can be seen in Figure 5.

yn =	$\frac{\chi n^2 - \chi n}{2}$
y2 =	$\frac{3^2-3}{2}$
3	9-3 = 6 = 3

Figure 5. The answer sheet for subject S2 determines the correspondence of many numbers x and y

The results show that the stages of the S1 subject in solving the problem can be described as follows: 1) The subject determines the number pattern of the given object and thinks about the number pattern up to the nth, 2) The subject determines the difference from  $X_n$ , 3) The subject determines the difference

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from  $Y_n$ , 4) The subject determines the change in value between  $X_n$  and  $Y_n$ , 5) The subject generalizes the  $X_n$  sequence, and 6) The subject generalizes the  $Y_n$  sequence. While the solution stages of the S2 subject in solving the problem through the function table can be described as follows: 1) The subject determines the change in value between  $X_n$  and  $Y_n$ , 2) The subject generalizes the  $X_n$  sequence, 3) The subject generalizes the  $Y_n$  sequence, and 4) The subject generalizes the relationship between  $X_n$ and  $Y_n$ . The scheme of the two subjects in solving the problem can be seen in Figure 6.





Empirical data shows that students' functional thinking in solving problems through function tables starts from determining the number pattern, namely the variable number pattern x, then the variable number pattern y. This was done by the subject of S1, while the subject of S2 in solving problems through a function table without looking for number patterns. This number pattern is usually called a recursive pattern which is described in the research findings of Lannin, et al. (2006) and Warren et al.

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(2006). This approach is used by students to see patterns in the dependent variable in the function table without considering the independent variable. Then otherwise determine the pattern of independent variables in the function table without considering the dependent variable. In this case, the recursive pattern is done partially, namely determining the recursive pattern on the variable x and determining the recursive pattern of the variable y. Next, subjects S1 and S2 both determine the change in value (covariance) between  $X_n$  and  $Y_n$ . Changes in value are usually referred to as the relationship between two quantities or a covariational relationship Tanisli (2011). The relationship between the two quantities in this study there are two ways, namely first, the relationship between the independent and dependent variables using the implication of "if...then...". Second, using mathematical examples and operations (ie, +). This is in line with research findings Tanişli (2011) that the relationship between two quantities is explained in a semiform, using familiar symbolic mathematical symbols for numbers (1, 2,3, etc.) and mathematical operations (+, -, x).

In generalizing the relationship between variations in the quantity of S1 subjects solved separately, so did S2 subjects. However, subject S2 can generalize the relationship between  $X_n$ and  $Y_n$ . Generalizing the relationship between quantity variations is usually called the Smith correspondence (Pinto and Cañadas, 2012) and Tanişli (2011). In this case, students have difficulty in generalizing the relationship between  $X_n$ and  $Y_n$ , namely  $y_n = \frac{x_n^2 - x_n}{2}$ . This is in accordance with the findings of Tanişli (2011), namely students have difficulty in completing the general form y = 2x - a

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and y = 3x - a. Thus, it can be concluded that students have difficulty in generalizing the relationship between quantities in the general form of functions in subtraction operations. Another finding, students prefer numerical representations in determining recursive patterns and use algebraic representations in determining relationships between covariations and generalizing correspondences. This is in line with the research of Yuniati et al. (2019) which states that most students use algebraic representations in functional thinking.

# CONCLUSION AND SUGGESTIONS

This study analyzes students' ability in functional thinking through a table of functions involving quadratic functions. We found two stages of solution in solving the problem. The first stage, 1) students determine the number pattern of the given object and think about the pattern of numbers up to the nth, 2) students determine the recursive pattern, 3) students determine the covariational relationship between  $X_n$ and  $Y_n$ , 5) students generalize the sequence  $X_n$ , and 6) students generalize the sequence  $Y_n$ . While the second stage, 1) students determine the covariational relationship between  $X_n$  and  $Y_n$ , 2) students generalize the  $X_n$  sequence, 3) students generalize the  $Y_n$  sequence, and 4) students generalize the relationship between  $X_n$  and  $Y_n$ . From the results of this study, it can provide knowledge for lecturers that by providing routine and non-routine practice questions, students can explore functional thinking. In addition, lecturers can also determine students' abilities in representing and interpreting functional forms. However, this study has limitations, namely the provision of tests via google meet and interviews conducted via whatsapp, due

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to the covid-19 pandemic so that it is not optimal.

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## ANALYSIS OF STUDENT ABILITY IN FUNCTIONAL THINKING THROUGH THE FUNCTION TABLE: THE CASE OF SQUARE FUNCTION

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### Abstrak

Salah satu cara untuk menelusuri kemampuan mahasiswa dalam memahami konsep fungsi dapat dilakukan dengan cara menganalisis kemampuan berpikir fungsionalnnya. Penelitian ini bertujuan untuk mengidentifikasi dan mengetahui kemampuan berpikir fungsional mahasiswa melalui tabel fungsi. Penelitian ini merupakan penelitian kualitatif yang bersifat eksploratif. Subjek penelitian adalah 2 dari 56 mahasiswa semester 5 Jurusan Pendidikan Matematika Universitas Islam Negeri Suska Riau. Subjek penelitian dipilih secara purposive yaitu mahasiswa yang menyelesaikan secara benar lembar tes dan memiliki kemampuan komunikasi yang lancar. Data dikumpulkan melalui tes dan wawancara. Hasil lembar jawaban tes dianalisis berdasarkan kerangka kerja berpikir fungsional dan hasil wawancara dianalisis untuk mendalami dan mengklarifikasi berpikir fungsional mahasiswa yang belum terungkap pada lembar jawaban. Hasil analisis data menunjukan bahwa ada dua tahapan penyelesaian yaitu tahap pertama, menentukan pola bilangan dan berpikir tentang pola bilangan sampai ke-n, menentukan beda dari  $X_n$ , menentukan beda dari  $Y_n$ , menentukan perubahan nilai antara  $X_n$  dan  $Y_n$ , menggeneralisasikan barisan  $X_n$ , dan menggeneralisasikan barisan  $Y_n$ . Sedangkan tahap kedua, menentukan perubahan nilai antara  $X_n$  dan  $Y_n$ , menggeneralisasikan barisan  $X_n$ , menggeneralisasikan barisan  $Y_n$ , dan menggeneralisasikan hubungan antara  $X_n$  dan  $Y_n$ . Berdasarkan hasil penelitian menunjukkan bahwa mahasiswa mampu memenuhi semua kerangka kerja berpikir fungsional dalam menyelesaikan masalah melalui tabel fungsi.

Kata kunci: berpikir fungsional, fungsi kuadrat, pemecahan masalah, tabel fungsi

#### Abstract

One way to track students' ability to understand the concept of function can be done by analyzing their functional thinking skills. This study aims to identify and determine students' functional thinking skills through function tables. This research is an exploratory qualitative research. The research subjects were 2 out of 56 5th semester students of the Department of Mathematics Education, Suska Riau State Islamic University. The research subjects were selected purposively, namely students who correctly completed the test sheet and had fluent communication skills. Data were collected through tests and interviews. The results of the test answer sheets were analyzed based on the functional thinking framework and the results of the interviews were analyzed to explore and clarify students' functional thinking that had not been revealed on the answer sheets. The results of data analysis show that there are two stages of completion, namely the first stage, determining the number pattern and thinking about the number pattern up to the nth, determining the difference from  $X_n$  determining the difference from  $Y_n$ , determining  $Y_n$  sequence. While the second stage, determine the change in value between  $X_n$  and  $Y_n$ . Based on the results of the study, it showed that students were able to fulfill all the frameworks for solving problems through the function table

Keywords: functional thinking, function table, mathematical problem solving, quadratic function

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There are five functional thinking

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## INTRODUCTION

At the level of junior high school education up to college the function material is taught. However, understanding the function is not a concept that is easily understood by students. Many students experience errors in representing and interpreting functional forms. These difficulties and misunderstandings will have an impact on student learning outcomes, if a solution is not found. So that teachers/lecturers need to provide practice questions related to function material and familiarize students with functional thinking from an early age. Functional thinking itself is a mental activity in generalizing the relationship between covariant quantities that can be represented through words, algebra, tables and graphs. This is in line with the opinion of Markworth (2010) dan Blanton, dkk (2015) stating that functional thinking is a generalization of the relationship between covariant quantities and can be represented through words, algebraic notation, tables and graphs. So functional thinking ability is the ability of students to make a generalization of the relationship between two (or more) quantity variations that can be represented through words, algebra, tables and graphs.

frameworks according to Blanton, dkk, (2015), first, generalizing linear data and organizing it in a function table; second, identify recursive patterns and describe in words, using patterns to predict approximate data; third, identify covariational relationships and describe in words; fourth, identify the function rules and describe in words and variables; and fifth use function rules to predict function values broadly. Meanwhile, according to Blanton & Kaput (2011) dan Tanis (2011) there are three functional thinking frameworks, namely: first, recursive patterning which means looking for variations or patterns of variation in a series of values for variables so that certain values can be obtained based on previous values; second, covariational thinking is focused on analyzing two variations of quantities simultaneously and understanding that change is an explicit and dynamic part of the function description (e.g., "when x increases by 1, y increases by 3"), and third, correspondence relationships are based on identification of correlations. between variables (for example, "y is 3 times x plus 2"). This study refers to the functional thinking framework of Smith and Tanish which is used as an indicator of functional thinking. The indicators used to analyze functional thinking are outlined in Table 1.

Table 1. Indicators for Analyzing Functional Thinking

Functional Thinking	Description	Indicators for Analyzing Co	de
Framework		Functional Thinking	
Define a recursive	Observing objects	Determine the number pattern PI	_1
pattern	in tabular form and	of the given object and think	
	thinking about the	about the number pattern up to	
	next unknown	the nth.	
	object	$\triangleright$ Determine the difference of $X_n$ PI	.2

Commented [P1]: Pendahuluan +- 1 halaman saja

- Lihat Template, urutan yang harus ditulis dalam PENDAHULUAN. 1.Perlu sedikit latar belakang umum kajian yang berkaitan dengan judul.
  - State of the art (kajian review literatur singkat) penelitianpenelitian sebelumnya (yang mirip) untuk menjustifikasi novelty (Kebaruan) artikel ini (harus ada rujukan ke jurnal 10 tahun terakhir):

3. Gap analysis atau Pernyataan kesenjangan (orisinalitas) atau kebaruan (novelty) penelitian ini dengan penelitian2 sebelumnya yang relevan (nirit) atau berdasarkan state of the art.
4. Uraikan Permasalahan berdasarkan fakta dan/atau hipotesis (jika ada).

Solusi untuk menyelesaikan masalah tesebut.

6.hasil yang diharapkan atau tujuan penelitian dalam artikel ini.

Contoh :

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#### A. PENDAHULUAN

 Era teknologi saat ini pembelajaran sudah sepatutnya berbasis digital. Salah satu yg sangat dibutuhkan yaitu e-book.
 Beberapa penelitian yg berkaitan dengan e-book menyatakan bahwa e-book sangat praktis dan efektif untuk proses pembelajaran

(Si A (2018); Si B (2019); Si C (2020))

3. Namun dari penelitian-penelitian tsb, belum ada yang

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<b>Functional Thinking</b>	Description	Indicators for Analyzing	Code
Framework		Functional Thinking	
		Determine the difference from	PL3
		$Y_n$	
Determine the	Determine the	Determines the value change	PN1
covariational	change in the value	in the sequence $X_n$	
relationship	of the relationship	Determines the value change	PN2
	between variations	in the sequence $Y_n$	
	in quantity in a	Determines the change in	PN3
	given problem	value between $X_n$ and $Y_n$	
Determine	Generalizing the	$\triangleright$ Generalizing the sequence $X_n$	GB1
correspondence	relationship	$\triangleright$ Generalizing the sequence $Y_n$	GB2
	between quantity	Generalizing the relationship	GB3
	variations on a	between $X_n$ and $Y_n$	
	given problem		

Many researches on functional thinking have been carried out, one of the experts who conducted research on functional thinking is Mceldoon (2010) who conducted research on the assessment of elementary school students in functional thinking. Blanton et al. (2016) who found that there were eight levels of functional thinking of students, namely the first level of prestructure, at this level students did not describe a recursive pattern in a single data value sequence or any relationship between two covariates; both special recursive levels, at this level students have not developed the underlying recursive pattern as a generalization; the three general recursive levels, at this level students begin to be able to think about recursive patterns; four specific functional, at this level students show special functional level thinking that conceptualizes functional relationships as certain relationships between certain appropriate values; fifth is primitivegeneral functional, at this level students can conceptualize the general relationship between two quantities even though their representation has primitive characteristics; the sixth appears

functional-general, at this level begins the emergence of generalizations of functional relationships even though their representation is incomplete; the seven general functionals are summarized, at this level students show thinking at the general-functional level which is conceptualized as a general function of the relationship between two quantities that is explicitly recorded; the last eight functions as objects, at this level students perceive boundaries related to the general form of the relationship between quantities. Blanton & Kaput (2005); Doorman, dkk (2012); Stephens (2017); Stephens, dkk (2017); Warren, dkk (2006); Wilkie (2004); Wilkie (2015); Wilkie & Clarke (2015); Wilkie & Clarke (2016) design and develop learning for teachers so as to improve students' functional thinking. Research Yuniati et al. (2019) found some representations of students in functional thinking, but in general the representations that appear are algebraic representations. Furthermore Yuniati et al (2020); Yuniati et al. (2020); Yuniati & Suparjono (2021); Yuniati & Suparjono (2019) found that students were able to think partially functionally

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and use arithmetic sequence formulas. On the other hand Stephens et al. (2017) dan Stephens et al. (2017) found that through intervention can improve students' ability to determine recursive patterns and covariance relationships between variables. In this study, looking for student responses that are sometimes different from certain tasks (tasks given in the form of linear functions and quadratic functions). Then Tanisli (2011) found that through function tables students can think functionally, that is, they can determine recursive patterns, determine covariance relationships and be able to generalize covariance relationships with symbolic representations. But from several previous studies, no one has conducted research that uses function tables to determine the functional thinking ability of students who can generalize the form of quadratic functions

The quadratic function is a mathematical material that must be studied by high school (SMA)/Madrasah Aliyah (MA) students even up to college. The material for quadratic functions has many applications in everyday life and is a prerequisite for studying other mathematics, such as derivatives. integrals, linear programming and geometry. Given the importance of students understanding the material quadratic function, it is necessary to explore the extent to which students' ability to understand the material of quadratic functions. Therefore, the purpose of this study is to explore students' abilities in functional thinking through function tables involving quadratic functions.

## **RESEARCH METHODS**

This research is a qualitative research that is exploratory. There were 56 students who participated in this study in the fifth semester of Mathematics Education at the State Islamic University of Suska Riau. The student is given a test via google meet and the time given is 60 minutes. The tests given are non-routine mathematical task. Based on the results of the answer sheet analysis, two groups of different answers were obtained. Then from each group one student's answer was chosen, so there were two student answers that were used as research subjects. The research subjects were selected purposively, namely students who had completed correctly and had fluent communication skills.

Data collection uses tests and interview guidelines to identify students' functional thinking abilities. The test was adopted from Tanişli (2011) which uses a function table to explore students' functional thinking. In solving test questions, students are expected to be able to: 1) determine recursive patterns, 2) determine covariational changes, and 3) generalize correspondence. The indicators/framework can be seen in table 1. The interviews used were unstructured interviews. Because the questions in the interview guide used are still very likely to develop according to the conditions or characteristics of the This activity respondents. was documented with an audiovisual recording device and interviews were conducted via chat via WhatsApp. The next step is triangulation which is used to test the validity of the data. Creswell (2012) states that triangulation is the process of corroborating evidence based on different individuals, types of data (observation records and interviews), or data collection methods (documents and interviews) in descriptions and themes in research qualitative Researchers examine each source of information and find evidence to support a theme. This ensures that the research is accurate

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because the information refers to an information source, individual, or process. The triangulation method used in this research is to compare student answer sheets and interviews.

In general, the process of data analysis in qualitative research includes: data reduction, data categorization, synthesis, and ends with the preparation of working hypotheses (Creswell 2012). Qualitative data analysis is carried out and interactively takes place continuously until complete, so that the data is saturated. The size of the data saturation is indicated by the absence of new data or information. The stages of data analysis in this qualitative study were modified from Creswell (2012) as follows: 1) transcribing interview data, scanning student answer sheets, and arranging the data into certain types based on the characteristics of the data; 2) reducing data, namely removing unnecessary data and organizing raw data obtained from the field; 3) analyze the data in more detail by coding or categorizing the data. At this stage, the coding of these categories with special terms can be seen in table 1; 4) drawing the structure of students' functional thinking in solving problems based on data categorization; and 5) drawing conclusions based on the results of data analysis, both obtained by using tests and interviews.

## **RESULTS AND DISCUSSION**

Based on the research subject's answer sheet S1, the first step is to understand the given function table by reading and looking at the test sheet. Then determine the number pattern on the *x* variable, where the number pattern is the addition of 1 and the number pattern on the *y* variable is a multilevel number, namely +2, +3, +4, +5, and so on. While the subject of S2 did not

determine the number pattern on the variable x and variable y. The answer sheets for S1 and S2 subjects in determining number patterns can be seen in Figure 1.



Figure 1. Answer sheets for subjects S1 and S2 determine the number pattern

In determining the covariational relationship, the subject of S1 stated that the relationship between many numbers x and many numbers y is if x = 2 then y = 1, if x = 3 then y = 3, if x = 4then y = 6, if x = 5 then y = 10, and so on. While the subject of S2 stated that there is a relationship between many numbers x and y numbers, namely to determine y in the second row and so on is the sum of the numbers x and y in the previous row or  $x = x_1$ ,  $y = y_1$ ,  $y_2 =$  $x_1 + y_1$ ,  $y_3 = x_2 + y_2$ ,  $y_4 = x_3 + y_3$ ,  $y_5 = x_4 + y_4, \ y_6 = x_5 + y_5, y_7 = x_6 + y_5$  $y_6$ , and so on. The answer sheets for subjects S1 and S2 in determining the covariational relationship can be seen in Figure 2.

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JIKA X=2, MAKGY=1	ada kubungunnya autors henuek bil x den bil.
J1Ka X=3, maka Y≥3	W(4) 3: 7, 2 5: 4, 1
JING X=4 I MARA Y=6	32× X1+3 = 2+1×3
JIKG X = 5, Maka Y = 10	<u>49</u> • <del>1</del> 2 • <del>1</del> <u>3</u> • <u>3</u> + <u>3</u> • <u>6</u> <u>48</u> • <del>1</del> 3 + <u>4</u> • <u>6</u> • <u>6</u> = 10
	45. Ky + 44 = 5 + 10 = 15
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Generalizing the relationship between covariance variations (correspondence), subjects S1 and S2 generalize many numbers x using the arithmetic sequence formula  $U_n = a + (n - 1)b$  so that the formula for many numbers x is  $U_n = n + 1$ . The answer sheets for subjects S1 and S2 in determining the correspondence of many x numbers can be seen in Figure 3.



Figure 3. Answer sheets for subjects S1 and S2 determine the correspondence of many numbers x

Next, subject S1 generalizes many numbers y using the formula  $U_n = an^2 + bn + c$ , while subject S2 generalizes many numbers y using the formula  $U_n = \frac{a}{0!} + \frac{(n-1)b}{1!} + \frac{(n-1)(n-2)c}{2!}$ . So the result of generalizing many numbers y is  $U_n = \frac{1}{2}n^2 + \frac{1}{2}n$ . The answer sheets for subjects S1 and S2 in determining the correspondence of many y numbers can be seen in Figure 4.



Figure 4. Answer sheets for subjects S1 and S2 determine the correspondence of many numbers y

Then the subject of S2 generalizes many numbers x and many numbers y by trial and error so that the formula  $y_n = \frac{x_n^2 - x_n}{2}$  is obtained. The answer sheet for the subject of S2 in determining the correspondence of many x numbers and many y numbers can be seen in Figure 5.

yn =	$\frac{\chi n^2 - \chi n}{2}$
y2 =	$\frac{3^2-3}{2}$
3	9-3 = 6 = 3

Figure 5. The answer sheet for subject S2 determines the correspondence of many numbers x and y

The results show that the stages of the S1 subject in solving the problem can be described as follows: 1) The subject determines the number pattern of the given object and thinks about the number pattern up to the nth, 2) The subject determines the difference from  $X_n$ , 3) The subject determines the difference

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from  $Y_n$ , 4) The subject determines the change in value between  $X_n$  and  $Y_n$ , 5) The subject generalizes the  $X_n$  sequence, and 6) The subject generalizes the  $Y_n$  sequence. While the solution stages of the S2 subject in solving the problem through the function table can be described as follows: 1) The subject determines the change in value between  $X_n$  and  $Y_n$ , 2) The subject generalizes the  $X_n$  sequence, 3) The subject generalizes the  $Y_n$  sequence, and 4) The subject generalizes the relationship between  $X_n$ and  $Y_n$ . The scheme of the two subjects in solving the problem can be seen in Figure 6.





Empirical data shows that students' functional thinking in solving problems through function tables starts from determining the number pattern, namely the variable number pattern x, then the variable number pattern y. This was done by the subject of S1, while the subject of S2 in solving problems through a function table without looking for number patterns. This number pattern is usually called a recursive pattern which is described in the research findings of Lannin, et al. (2006) and Warren et al.

(2006). This approach is used by students to see patterns in the dependent variable in the function table without considering the independent variable. Then otherwise determine the pattern of independent variables in the function table without considering the dependent variable. In this case, the recursive pattern is done partially, namely determining the recursive pattern on the variable x and determining the recursive pattern of the variable y. Next, subjects S1 and S2 both determine the change in value (covariance) between  $X_n$  and  $Y_n$ . Changes in value are usually referred to as the relationship between two quantities or a covariational relationship Tanisli (2011). The relationship between the two quantities in this study there are two ways, namely first, the relationship between the independent and dependent variables using the implication of "if...then...". Second, using mathematical examples and operations (ie, +). This is in line with research findings Tanişli (2011) that the relationship between two quantities is explained in a semiform, using familiar symbolic mathematical symbols for numbers (1, 2,3, etc.) and mathematical operations (+, -, x).

In generalizing the relationship between variations in the quantity of S1 subjects solved separately, so did S2 subjects. However, subject S2 can generalize the relationship between  $X_n$ and  $Y_n$ . Generalizing the relationship between quantity variations is usually called the Smith correspondence (Pinto and Cañadas, 2012) and Tanişli (2011). In this case, students have difficulty in generalizing the relationship between  $X_n$ and  $Y_n$ , namely  $y_n = \frac{x_n^2 - x_n}{2}$ . This is in accordance with the findings of Tanişli (2011), namely students have difficulty in completing the general form y = 2x - a

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and y = 3x - a. Thus, it can be concluded that students have difficulty in generalizing the relationship between quantities in the general form of functions in subtraction operations. Another finding, students prefer numerical representations in determining recursive patterns and use algebraic representations in determining relationships between covariations and generalizing correspondences. This is in line with the research of Yuniati et al. (2019) which states that most students use algebraic representations in functional thinking.

# CONCLUSION AND SUGGESTIONS

This study analyzes students' ability in functional thinking through a table of functions involving quadratic functions. We found two stages of solution in solving the problem. The first stage, 1) students determine the number pattern of the given object and think about the pattern of numbers up to the nth, 2) students determine the recursive pattern, 3) students determine the covariational relationship between  $X_n$ and  $Y_n$ , 5) students generalize the sequence  $X_n$ , and 6) students generalize the sequence  $Y_n$ . While the second stage, 1) students determine the covariational relationship between  $X_n$  and  $Y_n$ , 2) students generalize the  $X_n$  sequence, 3) students generalize the  $Y_n$  sequence, and 4) students generalize the relationship between  $X_n$  and  $Y_n$ . From the results of this study, it can provide knowledge for lecturers that by providing routine and non-routine practice questions, students can explore functional thinking. In addition, lecturers can also determine students' abilities in representing and interpreting functional forms. However, this study has limitations, namely the provision of tests via google meet and interviews conducted via whatsapp, due to the covid-19 pandemic so that it is not optimal.

## ACKNOWLEDGMENT

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sebelumnya (**dari state of the art**). Misal penelitian ini sejalan dengan penelitian si A (Tahun), si B (Tahun), si C (Tahun), dst. 5. Tambahkan implikasi/dampak/kontribusi hasil penelitian baik teoritis maupun penerapan

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Commented [P7]: Belum APA 6th edition

## SELF REVIEW INSTRUMENT

## AKSIOMA: JURNAL PROGRAM STUDI PENDIDIKAN MATEMATIKA

## Panduan: isi nilai 1 jika ya/ada, isi 0 jika tidak/tidak ada

No	Indikator	Ya (1) / Tidak (0)	Keterangan
1. Judul d	an Abstrak		
1.1	Judul kurang dari 15 kata	1	memenuhi
	Judul ditulis dengan lugas dan		
1.2	menggambarkan isi artikel	1	memenuhi
	Nama-nama penulis ditulis dengan benar		
	sesuai template, disertai afiliasi dan alamat		
1.3	email korespondensi	1	memenuhi
	Abstrak memuat Latar belakang masalah,		
1.4	tujuan, metode, hasil dan pembahasan	1	memenuhi
1.5	Abstrak kurang dari 200 kata	1	memenuhi
	Terdapat keyword 3-6 kata dan		
1.6	menggambarkan kata kunci dari artikel	1	memenuhi
2. Pendah	uluan/Latar Belakang		
2.1	Pendahuluan singkat terkait riset yang		
	dilakukan, maksimal 1 paragraf	1	memenuhi
2.2	State of the art - kajian penelitian-penelitian		
	sebelumnya (yang mirip) untuk menjustifikasi		
	novelty (Kebaruan) artikel ini (harus ada		
	rujukan ke jurnal 10 tahun terakhir, 5 Tahun		
	lerakhir lebih baik);	1	memenuhi
2.3	Gap Analysis - Terdapat uraian <u>Gap analysis</u>		
	(Pernyataan kesenjangan (Orisinalitas) atau		
	dengan populitiona coholumnya yang reloyan		
	berdasarkan state of the art	1	memenuhi
2.4	Permasalahan berdasarkan fakta dan/atau	1	
2.7	hipotesis (iika ada). Masalah harus obiektif		
	(dipermasalahkan oleh jurnal-jurnal		
	bereputasi atau lembaga riset bereputasi.		
	bukan dipermasalahkan berdasarakan		
	pendapat author)	1	memenuhi
2.5	Solusi untuk menyelesaikan masalah tesebut	1	memenuhi
2.6	<b>Objective</b> - Terdapat tujuan penelitian	1	memenuhi
		•	•
3. Metod	2		
3.1	Rancangan/Prosedur/Tahapan penelitian		
	diuraikan secara operasional	1	memenuhi
3.2	Subjek, lokasi, dan/atau sampel spesifik dan		
	jelas jumlahnya	1	memenuhi
3.3	Instrumen Penelitian dijelaskan dan teknis		
	pengumpulan datanya diuraikan secara		
	operasional/detail	1	memenuhi
3.4	Ada penjelasan cara validasi data	1	memenuhi

3.5	Teknis analisis data diuraikan secara jelas		
	(masing-masing instrumen harus ada		
	penielasan bagaimana cara menganalisis		
	datanya). Rumus-rumus yang umum tidak		
	perlu disajikan dalam artikel	1	memenuhi
4. Hasil da	an Pembahasan		
Penyajian	Data Hasil Penelitian		
4.1	a. Penyaijan hasil penelitian: Jika dalam		
	bentuk tabel, data dalam tabel sudah		
	disaiikan dengan benar (datanya sudah diolah		
	atau direkap, bukan data mentah).		
	menggunakan satuan internasional, hanya		
	garis horizontal yang mengapit header tabel		
	dan garis penutup tabel, serta tabel sudah		
	diberikan nomor dan judul tabel.	1	memenuhi
	b. Jika dalam bentuk gambar, gambar sudah		
	disajikan dengan benar, semua komponen		
	penting diberikan keterangan. Jika berupa		
	grafik, pastikan ada legenda, axis title sudah		
	benar yang disertai dengan unit dan dalam		
	skala yang proporsional. Garis-garis dalam		
	grafik terbaca dengan jelas dengan ketebalan		
	yang sesuai (kurva harus lebih jelas daripada		
	gridline-nya). Gambar sudah dilengkapi		
	dengan nomor dan judul gambar.	1	memenuhi
	c. Jika dalam bentuk deskripsi (kualitatif),		
	pastikan bahwa klaim yang dideskripsikan		
	mencerminkan keadaan yang tidak bisa		
	diungkap tanpa analisis (bukan pengetahuan		
	umum yang diketahui tanpa penelitian).		
	Pastikan tidak ada sitasi dalam klaim hasil		
	penelitian ini.	1	memenuhi
4.2	Untuk 4.1.a dan 4.1.b, ada paragraf pengantar		
	yang menjelaskan temuan penting dalam		
	tabel dan gambar tersebut. Kemudian,		
	gambar atau tabel telah disebutkan dalam		
	paragraf ini (Gambar dan Tabel adalah		
	instrumen pendukung klaim hasil penelitian).	1	memenuhi
4.3	Data hasil penelitian sudah diolah (bukan data		
	mentah). Dituangkan dalam tabel atau		
	gambar.	1	memenuhi
4.3	Penyajian data hasil penelitian yang sama		
	harus salah satu saja, dalam bentuk gambar		
	atau tabel. Tidak boleh keduanya. Jangan		
	sampai ada gambar atau tabel yang double		
	atau tumpang tindih, misalnya satu data		
	disajikan dalam bentuk baik tabel maupun		
	gambar, harus pilih salah satu.	1	memenuhi

4.4	Resolusi gambar tinggi/jelas, tidak		
	blur/buram, tidak ada bayangan, tidak ada		
	border luar pada gambar. Gambar		
	diutamakan menggunakan scanner, jika		
	menggunakan kamera HP pastikan gambar		
	tidak miring	1	memenuhi
4.5	Ada nama dan nomor tabel, rata kanan kiri		
	(justify)	1	memenuhi
4.6	Ada nama dan nomor gambar, rata tengah		
	(center)	1	memenuhi
4.7	Setiap tabel/gambar harus ada kalimat yang		
	merujuk pada tabel/gambar tersebut. Misal		
	berdasarkan Tabel 1 ditemukan hasil yaitu	1	memenuhi
4.8	Setiap table/gambar harus ada penjelasan apa		
	maksud/makna dari table/gambar tsb, bukan		
	hanya menjelaskan angka yang ada pada		
	table/gambar.	1	memenuhi
4.9	Setiap Equation ada penomorannya	1	memenuhi
Pembaha	san		
4.10	Authors memberikan argumen terhadap hasil		
	penelitian yang telah diklaim, ada penjelasan		
	sebab-akibat yang logis dan dirangkai dalam		
	bentuk 'Cerita baru' mengunakan kalimat		
	sendiri	1	memenuhi
4.11	Authors menguraikan temuan penelitian	1	memenuhi
4.12	Ada uraian mengenai kelebihan dan/atau		
	kekurangan hasil penelitian	1	memenuhi
4.13	Ada uraian yang membandingkan hasil		
	penelitian dengan penelitian relevan yang		
	telah diuraikan dalam <b>state of the art</b> pada		
	pendahuluan	1	memenuhi
4.14	Ada uraian kontribusi atau dampak dari hasil		
	penelitian	1	memenuhi
5. Kesimp	oulan dan Saran		
	Kesimpulan menjawab tujuan penelitian, dengan		
	ditambahkan simpulan atau temuan secara		
	singkat/garis besarnya saja. Bukan mengulang		
5.1	pembahasan lagi.	1	memenuhi
	Ada saran terkait penelitian atau saran dari		
	penelitian yang belum terselesaikan atau		
5.2	saran untuk penelitian selanjutnya.	1	memenuhi
6. Refere	nsi		
6.1	Jumlah referensi minimal 15, lebih dari 80%		
	referensi harus berasal dari sumber primer		
	(primer > 80% atau >= 81%, sumber primer		
	yaitu jurnal penelitian, prosiding, buku hasil		
	penelitian, skripsi/thesis/disertasi). Lebih		
	utama dari Jurnal semua. Kutipan dari buku		
	teks/teori maksimal 10% dari total referensi	1	memenuhi
6.2	Referensi 10 Tahun terakhir (70% = 3 tahun		
	terakhir, 30% nya 10 tahun terakhir). Lebih		
	utama jika referensi 5 tahun terakhir	1	memenuhi

6.3	Sitasi dan Daftar pustaka wajib menggunakan		
	reference manager seperti mendeley, zotero,		
	dan sejenisnya	1	memenuhi
6.4	Format daftar pustaka menggunakan APA		
	Style 6th edition	1	memenuhi
	Total Skor (%)	100%	
	Keputusan Penilaian		Accepted

lsi 1 jika penyajian hasil penelitian hanya salah satu (Tabel, gambar/grafil, atau deskriptif)

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