ADVANCES IN CAPITAL REPLACEMENT MODELLING WITH APPLICATIONS

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ABSTRACT

This thesis addresses the capital replacement modelling problems associated with a mixed, or inhomogeneous, fleet and also takes account of the fleet size problem. Applications considered relate to a fleet of buses and a fleet of medical equipment. The initial chapters introduce the notion of capital replacement modelling and review previous work in the field, as well as reviewing the fleet size problem. Replacement policies are also put in the context of the fleet rather than the context of a 'typical plant'. In the third chapter, we present our first attempt to model capital replacement with variable fleet size over a finite planning horizon. A two cycle model is developed in which the notion of penalty cost for breakdown is introduced. This cost is incurred when demand is not met. To take account of the cost of unmet demand, a simple failure model for plant is proposed. The replacement model is applied to a fleet of ventilators in an intensive care unit of a hospital. In the fourth chapter we develop various models for the case of replacement of a sub-fleet within a mixed fleet. These models themselves have variable finite planning horizon of variable length and build on developments described earlier in the thesis. Other aspects such as the increased cost of sub-optimal policy due to delayed replacement, smaller replacement sub-fleet etc. are also considered. The models developed in chapter 4 are applied, in the following chapter, to a fleet of buses operated by a Malaysian inter-city bus company. Sensitivity analysis on different factors is also carried out. Finally the sensitivity of optimal decision policy to the choice of the replacement model is described in the context of the bus application.

CHAPTER I

Introduction to Capital Replacement Modelling

1.1 Introduction

Capital replacement has long been, and will remain, a topic of interest. This is because it is a part of the strategic planning of capital expenditure. Indeed capital expenditure is the process of deciding whether or not to commit resources to projects - replacement of plant, say - whose costs are spread over several time periods (Bierman and Dyckman, 1976). The strategy is then to spend the capital in a reasonable manner with the objective of minimising (maximising) expenses (profit). Replacement policies in general deal with all sort of items, from a small component of a device to a fleet of aeroplanes. The approach, however, is different when dealing with a component than with a plant. In the component replacement case, the factors of interest are generally the distribution of the time to failure, the cost of preventive replacement and cost of failure, and the long run cost per unit time is minimised. For large expensive plant on the other hand, economic factors such as discount factor, rate of inflation, interest rate and tax parameters are considered, with purchase, operation, maintenance and resale costs also considered. The implication is that capital expenditure is planned over a certain specified period, the planning horizon. The planning horizon is expressed in months or years and may have finite or infinite length. In this thesis we deal only with capital replacement over a finite planning horizon.

For plant replacement, when to replace a current plant, a fleet or a part of it, is one of the main concerns in decision-making. Another concern for the decision-maker or manager is the choice of the new plant to purchase. This of course is an important issue for the decision maker, but it is often out of control of the modeller when the choice is fixed in advance (e.g. political decision). A good policy can lead to large savings in the total cost of operating a plant or a fleet of plant. To achieve this goal different approaches are used, which can be based on either the experience of the operator or a modelling approach or a combination of both. Replacement is often motivated by a declining performance of the plant or higher cost of maintaining/operating it or simply technical obsolescence. All these factors should be taken into account in the replacement decision.

Our thesis is concerned with the modelling side of the replacement decision. We have attempted to develop a number of capital replacement models in order to deal more realistically with the situation under consideration. We have described models which attempt to reflect the actual replacement problem. We have developed models for the replacement of plant within a fleet where plant are of different types and ages. This represents the corner-stone of our contribution in the field of capital replacement policy. We have also considered the fleet size problem which we believe is a part of the replacement problem, and therefore is incorporated in the models we consider. The fleet size problem is concerned with when equipment in a fleet cannot meet a certain demand at a given time, or when some are not used for a long period of time as a result of low demand. Therefore an optimum size of the fleet is sought in order to balance between these two situations. The concept of penalty cost of unavailability is introduced in the models we develop in order to consider the fleet size problem within the context of capital replacement.

The structure of this thesis is as follows. In chapter 2 we present a review of capital replacement modelling as well as of the fleet size problem. Early capital replacement models, known as economic life models, are discussed. We have attempted to establish a classification of capital replacement models according to the replacement policy which is carried out by the operator/manager of the plant. This will help the operator/manager to choose the suitable model for his application.

In chapter 3 we develop a model related to the case of the replacement of an entire fleet in which the fleet size problem is introduced. In order to model the fleet size simultaneously with the time to replacement, we consider the concept of penalty cost of unavailability due to breakdown. This is considered using a simple model. The replacement model is then applied to a fleet of 10 ventilators used in an intensive care unit of a hospital. Sensitivity analysis is also carried out.

In chapter 4 we consider a fleet which is comprised of sub-fleets which differ from each other either by their ages or model of equipment or both. Various models are proposed which take into account: the number of cycles in the planning horizon; the fleet size which is either kept fixed at its current size or is allowed to vary; the nature of the last cycle which is either an 'operating' cycle only - that is no replacement is undertaken at its end - or an 'operate-buy-and-sell' cycle. To deal with each situation, a mathematical model is developed. Costs of sub-optimal policies are also assessed.

In chapter 5 the models we have developed in the previous chapter are applied to a case study relating to the fleet of a large Malaysian inter-city bus company. The fleet is mixed and comprises 5 sub-fleets of different types and ages. Data on maintenance cost were collected and models for costs as well as failures are fitted. Since the accuracy of the data is often questionable, sensitivity analysis is carried out accordingly. This sensitivity analysis concerns all the parameters involved in the models namely: maintenance cost; resale value; penalty cost and discount factor.

Finally, in chapter 6 we consider sensitivity analysis of the optimal policy to the choice of the replacement model using the class of models developed in chapter 4. This sensitivity analysis is carried out using the data related to the buses.

We should, however, emphasise that such modelling in general can really only support decision makers and guide policy. We do not claim that such modelling can replace the role of the experienced fleet manager/decision-maker. The modeller and decision-maker must work together if such models are to be adopted in practice.

CHAPTER II

Capital Replacement Modelling and Fleet Size Problem

2.1 Introduction

A large amount of published work related to capital replacement has appeared up to date. This fact shows that researchers and managers (decision-makers) are aware of the importance of choosing the right decision when it comes to replacing an existing plant. For that purpose many models have been developed. Christer and Waller (1987b), described the basis on which capital replacement decisions were taken within 19 British organisations surveyed by the authors. The authors noticed that performance tended to dominate decisions concerning computers, whereas, for certain types of vehicles, cost considerations dominated. The authors defined different sorts of relationships between factors using the Spearman's rank correlation technique. In the Christer and Waller survey (1987b), most organisations would have qualified as having a policy for replacement decision-making. It also appeared that for the more costly items modelling is undertaken but has little influence on the final decision, whereas for less costly items, if modelling was undertaken it would influence the decision process. This survey seems to contradict the findings of the survey conducted in the USA by Hsu (1988) who surveyed 200 randomly selected Fortune 500 industrial firms. In this survey it was reported that 89% of the firms have definite replacement policies for equipment, but it did not state what percentage of those among the 89% really use modelling. Hsu found that the proportion of firms having a replacement policy tends to increase as the capital of the firm increases. Unfortunately, both papers agreed that using modelling for capital replacement within an organisation is a 'luxury' that only large organisations can afford.

2.2 A simple example

For the finite planning horizon, the optimum age of replacement exists as an economic compromise between increasing and decreasing cost functions. The decreasing cost function corresponds to the spreading of the capital cost over a longer time, resulting in a lower average cost per unit time. In contrast, the increasing cost function is that of decreasing efficiency due to age and wear. The total cost is obtained by summing both terms. In Figure 2.1 we consider an example for a plant whose capital cost is R=500 units, maintenance cost $M(t)$ and resale $S(t)$ value are respectively expressed as

$$M(t) = 7.731t^{1.50}$$

and

$$S(t) = R \times 0.613 \times 0.811^{t}.$$

Through this example, see Figure 2.1, we may show graphically that the optimal solution, that is the value t^* of t (the age at replacement) which makes the total cost minimal, exists and is finite. It is not always possible to determine analytically the optimum solution, and therefore numerical methods are often used. Graphical methods have also been used to determine the optimal solution (Walker, 1994).

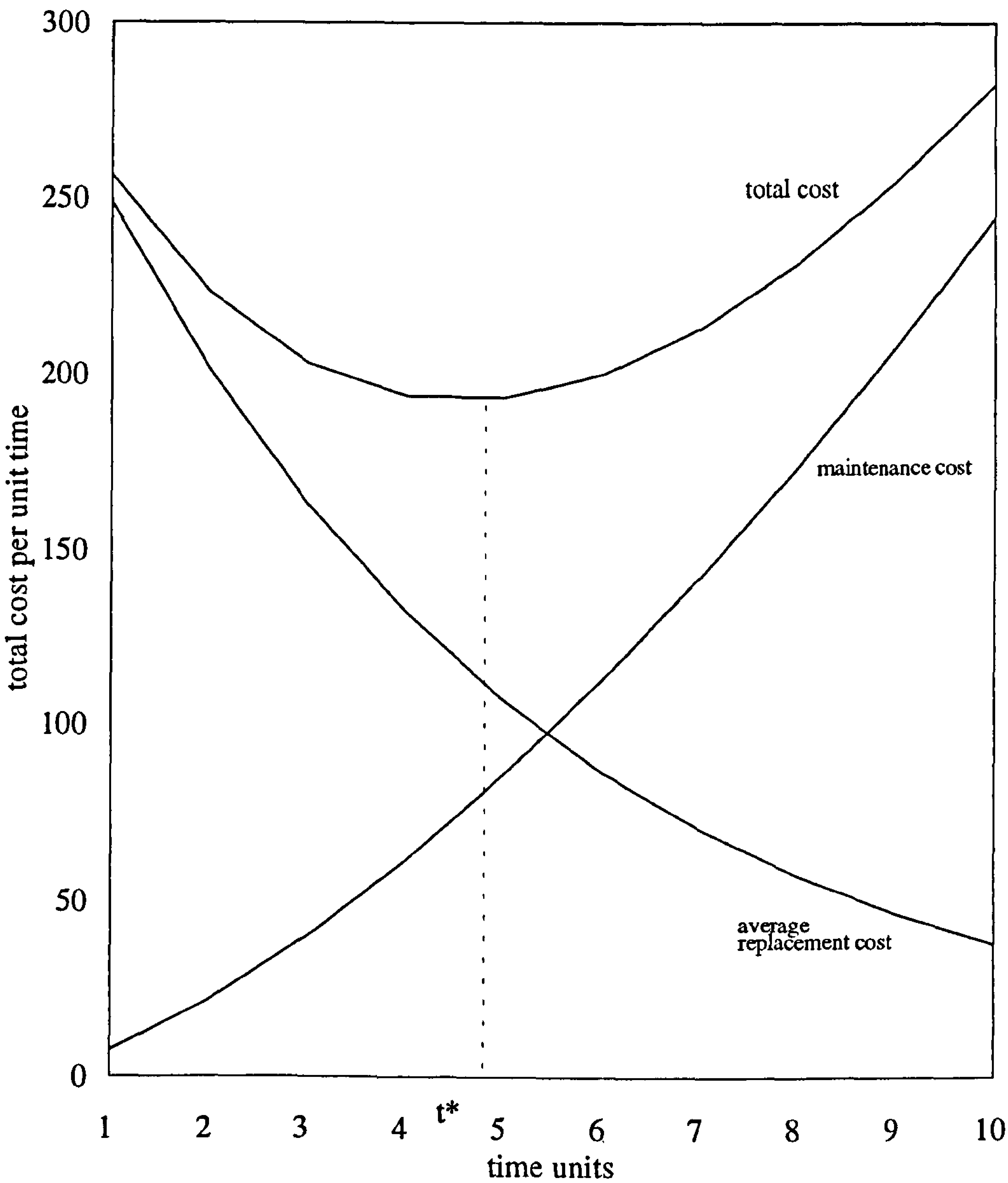


Figure 2.1. Minimum cost and optimal time of replacement.

2.3 Early capital replacement models (economic life models)

Early economic life models over one replacement cycle were basically all expressed as

$$C_1(T)=\frac{1}{T}\left\{P+\int_0^T f(t)dt-S(T)\right\} \tag{2.1}$$

where P, $f(t)$ and $S(.)$ are respectively the capital cost, the operating cost per unit time and the resale value. Equation (2.1) represents the cost per unit time over the cycle of length T. The optimum value of T is the economic life which minimises equation (2.1). Another approach for the determination of the economic life considers an infinite planning horizon and a discount factor r in order to consider costs at net present value. The notation for equation (2.2) is the same as equation (2.1). Equation (2.2) represents the total discounted cost over all time and T is selected to minimise

$$C_2(T)=\frac{\left\{P+\int_0^T f(t)r^t dt-S(T)\right\}}{(1-r^T)}. \tag{2.2}$$

$C_2(.)$ becomes invalid if $r \geq 1$, which makes the criterion inapplicable in this case. In this model, technological change is not taken into account. Despite these weaknesses, such models have been used extensively by many authors.

Preinreich (1940) for instance proposed criteria and an approach to determine the economic life for a typical plant using the type of models described above with some variations. The author considered replacement for industrial machines, where factors such as rate of production, market price of the product, demand for product and goodwill are considered. The author extended the scope of the then current practice of replacing a machine every T units of time by an identical one and operating it under the same economic conditions. The economic life T^* was obtained by minimising the unknown lowest unit cost of a product denoted ω where costs were discounted to present value. The objective function to be minimised was expressed as

$$\omega = \frac{B - S(T)e^{-iT} + \int_0^T E(t)e^{-it}dt}{\int_0^T Q(t)e^{-it}dt}, \tag{2.3}$$

where: B is the original cost of a single machine; T is the variable length of a cycle; $Q(t)$ is the rate of production; $E(t)$ is the rate of all expenses; $i(t)$ is the continuous compounded interest rate, and $S(T)$ is the resale value. Equation (2.3) is analogous to equation (2.1), where cost per unit time becomes unit cost of a product. The author then considered a slightly different formulation of the objective function, which became a maximisation of the capital value V which was expressed as

$$V = \int_0^T [zQ(t) - E(t)]e^{-it}dt + Se^{-iT}, \tag{2.4}$$

where z is the market price of the product, the other notations are identical to those of the equation (2.3). The rest of the paper was a development of issues such as single machine replacement, finite and infinite chain of replacement. Dynamic state, that is when economic conditions are variable, and technical obsolescence were also considered.

The Eilon *et al* (1966) paper is referred to as a landmark for research on capital replacement. This case study was concerned with replacement policy for a fleet of 140 fork lift trucks. For the study only a sample of 10 trucks were considered. Two approaches were described. In the first one, an average maintenance cost for the sample was determined with all the trucks on the fleet treated as average trucks, and therefore an uniform replacement policy was used for the whole fleet. In the second approach, the authors considered separately each truck in the sample of size 10. With the first policy the age of replacement was 7 years, while for the second the ages varied between 5 and 12 years. Two economic life models related to replacement of fork lift trucks were presented. The two models were denoted model 1 and model 2 and represented the average annual cost over a single replacement cycle and the total discounted cost over an infinite replacement cycle respectively. Model 1 was defined as follows:

$$T = \frac{A - S - C\gamma}{n} + \frac{1}{n}\int_0^n f(t)dt, \qquad (2.5)$$

where A is the acquisition cost, S the resale value, $f(t)$ the maintenance cost of a truck t years old, n the age of truck when replaced, C the capital allowances and γ the rate of taxation. The annual amount of tax payable is reduced by an annual depreciation charge or allowance. This study was done on the eve of the introduction of new tax legislation. The authors used a corporation tax which was assumed fixed at 40%, although not officially implemented, in replacement of the then existing income and profit taxes. Changes in tax laws often occur, therefore updating of the models should be carried out as soon as a new tax law is introduced. The second model which was developed and denoted model 2 was

$$V = \frac{A - Sr^n - C_p\gamma + \int_0^n f(t)r^t dt}{1 - r^n}, \qquad (2.6)$$

where $r=1/(1+i)$, the discount rate with i as the rate of interest; C_p the present value of capital allowances. The authors assumed that the replacement cycle was repeated indefinitely under the same conditions. Although this assumption was mathematically valid, it was absolutely not in practice. In terms of results, the authors showed that in the presence of the capital allowances the economic life was dramatically reduced with respect to the absence of capital allowances.

These models in general deal with the replacement of a typical plant within a fleet and did not attempt to solve the actual replacement problem; namely, how long to retain the current plant? What model or type of plant should be purchased? Economic life models of this type were also not able to cope with the case of a large fleet in which plant are of different ages and types.

2.4 Finite horizon models

In real world applications, finite horizon models are desired and accepted, especially for cost prediction, as well as for economic factors such as inflation rate or discount factor. The length of the planning horizon may be either fixed or variable. In this thesis we have considered both cases, although the latter is considered in the main. Christer and

Goodbody (1980) developed a two cycle model with variable length of the planning horizon, and the function to be minimised was expressed as

$$C(N;K,L)=\frac{\left\{\int_0^K f(N+t)r^t dt+r^K\left(P+\int_0^L f(t)r^t dt+r^L P\right)\right\}}{(K+L)}, \forall r>0, \qquad (2.7)$$

which represents the total discounted cost per unit time of operating a plant currently N years old for a further K years, replacing it with a possibly different equipment model, and then operating for a further L years before replacing again with an equipment model of the same type. Notations are common to those of equations (2.1) and (2.2). We can notice in equation (2.7) that the resale value of the plant was not considered but could be incorporated without any difficulty. This formulation had the advantage of coping with the situation in which the discount factor $r \geq 1$. A variable discount factor was also formulated in the model.

Technological development was discussed in detail in Christer (1988), and was applied to the problem of non-like-with-like replacement in which prediction for operating cost of new plant was made using historical data and a predictive ratio method. This was applied to a large fleet of vehicles which comprised Ford Transits and Ford A0609s, which were to be replaced by Bedford CF250s and Dodge 556s respectively. This method consisted of collecting data on the operating cost of the old plant and more limited data on that of the new plant, and then the ratio of the average cumulative cost of the new plant to the old was determined. The estimate for the jth quarter for the new vehicle was obtained using a sample of 8 vehicles from each type and was formulated as follows

$$\text{Dodge}(j)=\text{y}(j) \times \text{Ford A0609}(j) \qquad (2.8)$$

where $\text{y}(j)$ is the quadratic fit of the ratio of the operating cost of new vehicle to the old in the jth quarter, Dodge(j) and Ford A0609(j) are respectively the quarterly estimate of the operating cost of the new and the operating cost of the old vehicle in the jth quarter.

The cost estimation was obtained for a period of 8 years. For years 1 and 2 historical data were used since data for the new Dodge were available for a period up to 20 months. Equation (2.8) was used to predict costs for years 3, 4 etc. Elton and Gruber (1976) also considered technological improvements but assumed linear trend in technological factors.

Another cost criterion, namely the total discounted rent criterion was introduced in Christer (1984). This criterion stated simply that if a plant is rented and an equal amount of money, *R*, is paid at the end of each year over a period of n years say, and if each year the value of the rent is discounted to its net present value at a rate *r*, then, at the end of the planning horizon, the total rent must equate with the total discounted cost (TDC) of operating, selling and buying the plant. This is expressed mathematically as

$$Rr + Rr^2 + ... + Rr^n = TDC,$$

where *r* is the discount rate. Therefore the value of the equivalent rent is

$$R = \frac{TDC}{\sum_{i=1}^{n} r^i}.$$

The model described in equation (2.7) was refined by Christer and Waller (1987a) who introduced tax parameters. The authors found that tax did influence total replacement cost but did not influence the optimal replacement decision, namely the optimal replacement age. This is due to the introduction of the new 1984 Finance Act. Under this new tax legislation, the capital allowance depends upon the period in which the capital purchases are made, and the corporation tax rate also depended upon the company profit. The introduction of this new legislation made the capital allowance scheme much simpler than the then existing one. The authors used an analysis of variance to investigate the effect of the input parameters on the output parameters. The input parameters which showed an important effect on the optimal age of replacement for the two cycle rent model were the operating cost and the current age of the first vehicle. The tax factors showed very little influence.

Later Christer and Scarf (1994) introduced a penalty cost for 'serious' failure' in the two-cycle model of Christer and Goodbody (1980) in a case study related to medical equipment. The cost criterion used was the total discounted cost per unit time, where the total discounted cost is expressed as

$$C(N,K,L)=\sum_{t=1}^{K}(C_o(N+t)+p_o(N+t))r^{t-1}+$$
$$r^K\left\{R_n-S_o(N+K)+(\sum_{t=1}^{L}(C_n(t)+p_n(t))r^{t-1}+(R_n-S_n(L))r^L)\right\} \quad (2.9)$$

The optimum values of K and L are obtained by minimising

$$\frac{C(N,K,L)}{K+L},$$

or alternatively by minimising the equivalent rent. $C_i(t)$, $p_i(t)$ and $S_i(t)$ are respectively the maintenance cost, the penalty cost and the resale value of the plant i of age t, the subscripts o and n stand for old and new plant respectively and r is the discount factor.

These models address real replacement problems only if considering a single plant. If the fleet contains many plant then these are likely to be inhomogeneous in age and type and so these models again only model replacement of typical plant of a certain current age. However, these models can be extended easily to consider entire replacement of inhomogeneous fleet simply by summing age (and plant specification) related operating cost and resale values over the entire fleet. The models are also appropriate for replacement of a homogeneous fleet.

2.5 Practical considerations

In this section we will define briefly some of the concepts underlying the models we have mentioned.

1. *Maintenance cost model*: In order to model maintenance cost for a plant, data for the old and the new plant need to be, if available, as reliable as possible, because most of the uncertainty in maintenance and replacement decision problems lies with the adequacy of

data relating to maintenance history. Typically simple linear, exponential or power law type regression models are fitted to the data and used for simple prediction.

2. *Resale value function*: Resale values are often obtained from some specialised guide or directly from the second hand market. For example in the UK, we find the Glass's guide and the CAP Red Book (Kobbacy and Nicol, 1994) for commercial vehicles. From the prices given in these guides one can easily model the resale value function again using regression techniques. The prices of old plant are also influenced by the introduction of new models in the market (Scarf, 1994).

3. *Discounting:* The question of discounted cash flow (French, 1988) is raised when future costs are expressed in terms of present values. When money is loaned for a certain period, there is always a risk that it may not be returned. Interest rate is introduced to make this risk acceptable (Holland *et al*, 1983). It is clear that the future worth of an amount of money is greater than its present worth. This is due to the combination of inflation and the interest rate or any other rate of return on investment. Thus a scaling of all future costs to their present value using a discount factor is necessary. Therefore discounting should be taken into account in any capital replacement decision.

4. *Tax laws:* Tax rates are determined by a tax authority and are subject to change. Laws on taxation change for different reasons which are either political or economical, therefore modellers should be aware of tax factors, especially when a new tax legislation is introduced.

5. *Penalty cost*: When breakdowns occur and provoke a stoppage of production or service, this can lead to a financial problem for the manager/operator. This cost of inconvenience or lost of opportunity may be modelled in a subjective way by the operator, who can allocate a penalty cost for each breakdown. Optimal replacement

policy can then be determined for a range of values for the penalty cost. This idea was used in Christer and Scarf (1994).

6. *Rent model versus total discounted cost per unit time model (tdc)*: These two criteria are often used in capital replacement modelling. Rent models are a useful class of replacement models which can be used to study the replacement of capital equipment over an infinite series of cycles. Indeed if we consider an infinite series of identical cycles under both criteria we find that when the discount factor approaches 1 the value of the *tdc* is infinite whereas the value of the equivalent rent is expressed exactly as equation (2.1) (Christer 1984, Scarf 1994). In this case it would be recommended to use the rent model. For a replacement policy over a finite period both criteria can be used in a similar way without distinction, provided that usage is at least reasonably constant.

7. *Planning horizon*: In capital replacement policy, the planning horizon may be either finite or infinite, fixed or variable. The infinite planning horizon implies that if replacement has to be made on the basis of non-like-with-like, then new model of equipment, as well as economic factors and failure costs need to be predicted in an objective fashion. This, of course, is rather difficult, if not impossible to realise in practice. On the other hand for the finite planning horizon, prediction of the costs of the model of equipment is relatively easy to consider. In this case the length of the planning horizon is either variable or fixed. For a variable planning horizon, replacement decision can in certain circumstances be the realisation of assets, which can impose replacement when it is not needed (*end-of-horizon-effects*) and also the sale of the 'best' plant at the end of the planning horizon. With a fixed planning horizon, the choice for the length of the planning horizon should be made adequately in order not to impose a poor replacement schedule. The optimum policy may be determined using a range of values for the length of the horizon.

2.6 Developments

2.6.1 Nature of the replacement problem

The models of section 2.3 consider 'typical' plant replacement and this approach in practice works as follows: once economic life has been determined, those plant/equipment/items identified as nearing their economic life become candidates for replacement. Such policy assumes data is representative of 'typical' plant, replacement is like-with-like, and is not dependent on whether the fleet is large or just consisting of a single item.

Christer and Goodbody (1980), in the variable finite planning horizon case, attempted to address the actual replacement problem, that is, given current age, when to replace, while taking account of type of new plant, that is non like-with-like replacement. This approach considers the replacement of a 'typical' item/plant when replacement is non-like-with-like. It is argued here that in order to model the actual replacement problem however, we need to put the replacement in the context of the fleet. Therefore we suggest that capital replacement models can be classified into four categories, denoted case 1, case 2, case 3 and case 4. These four situations are identified and can be defined as

1• single plant replacement of a unique plant,

2• entire fleet replacement (homogeneous and inhomogeneous fleet),

3• sub-fleet replacement,

4• single plant replacement in a large fleet.

Such a classification is however not exclusive, indeed case 3 could easily be used to solve the replacement problem related to case 2, by considering the number of sub-fleet equal to one. By the same artifice case 1 could be solved using case 2 by considering the size of the fleet equal to one.

2.6 2 Case 1: Single plant replacement

The models considered so far here deal with this question. But as soon as the plant is a part of a large fleet, these models become questionable. This is because essentially if you replace a plant within a fleet, the replacement of that plant has implications for the whole fleet, not just for that plant and its replacement plant. For example, if a fleet of computers comprises personal computers (PCs) of the 40386's processor type say, and if only a single PC is replaced with a more technologically advanced PC, a Pentium type say, it is obvious that the usage pattern of the fleet will change because most of the users want to use the new computer. Therefore the replacement model would not be applicable for the rest of the fleet and might need to be revised. For a fleet of vehicles this effect, although perhaps less exaggerated, will still be present.

2.6 3 Case 2: Model for replacing the entire fleet

Some situations require the replacement of the entire fleet (e.g. computers within an organisation). The models of section 2.3 can deal with this case simply by summing age (and plant specification) related operating cost and resale values over the entire fleet. The models are also appropriate for replacement of a homogeneous fleet. Entire fleet replacement however, is not always desirable, especially for inhomogeneous fleet (different age and plant type) and above all when capital cost is subject to budget limitations. Extension to early models described in sections 2.3 and 2.4 are discussed in this thesis in chapters 3 and 4.

2.6.4 Case 3: Sub-fleet replacement

Here a fleet is considered as comprising a number, say, r of sub-fleets. This classification into sub-fleets may be either done according the type of plant/equipment, or by class of ages, or both, or any other classification such as condition (good/bad). This classification would be defined by the operator/manager and certain sub-fleets would be considered as candidates for replacement, rather than individual plant. The problem can then be formulated as which sub-fleet to replace first and when to replace it. The choice of the equipment model type for purchase is again defined in advance by the operator/manager of the fleet. Some refinement concerning the fleet size can be introduced in the model, that is to define the size of the replacement sub-fleet.

This case is developed in detail throughout this thesis in chapters 4 and 5 and represents its main contribution to the area.

2.6.5 Case 4: Repair limit approach

This approach considers the case 4 mentioned in section 2.5.1. It assumes that replacement is done singly, and independently of the fleet. This approach can be useful when sufficient data on individual plant are available and also provided the fleet is a large one. When a plant requires an expensive repair before its economic life is reached, and if this is expensive enough in comparison with a repair limit, it may be worthwhile replacing the machine early. When the repair-or-replace decision arises for an individual plant, the most appropriate model is the repair limit model. This can of course be done if we assume that extensive data on operating and maintenance cost are available. Drinkwater and Hastings (1967), then Hastings (1969) addressed this problem. The former considered a case study of a fleet of British Army vehicle Land Rovers, to which the repair limit model was applied. The repair decision has two alternatives: repair the vehicle or scrap the vehicle and substitute a new one. The repair limit equation was derived and was expressed as

$$r_0(t) = \theta g(t) - m(t),$$

where $r_0(t)$, θ, $g(t)$ and $m(t)$ are respectively the repair limit at time t, the expected future cost per vehicle-year up to t, the expected remaining life and the expected total cost of the future repairs. If at time t a vehicle required a repair, the rule which led to the decision was based on the comparison of the value of θ with the cost of this repair added to the expected cost for future repairs per unit time over the remaining life of the vehicle. If the value of this sum exceeded the value of θ the vehicle is then replaced, otherwise it is repaired. It was observed that savings of 7 percent could be made over "economic life" replacement policy. Hastings (1969) used dynamic programming to determine the optimum repair limit. The author also introduced discounting and discussed technological change.

Jardine *et al* (1976) extended the work of Hastings (1969) by using annual maintenance cost limits (AMCL), in which tax allowance and discounting were introduced. The AMCL was used to determine whether or not a vehicle of a specified age should be kept in a fleet or replaced. The limits were determined in order to minimise the total discounted cost of maintaining and replacing a vehicle over a finite planning horizon. The rule for policy is based on comparing the immediate and the estimated future cost of maintenance with the AMCL. If the estimated maintenance cost of a vehicle for the next year exceeded the AMCL of a vehicle, then the decision of replacing it was taken, otherwise it was kept for a further year of operation. The improvement of this approach in comparison with the repair limit approach of Hastings (1969) lies in the fact that in the latter the decision is taken upon the last repair while for the former the decision is based on the estimated annual maintenance cost for the next year.

Simms *et al* (1984) addressed the issue of optimum age based mixed for a fleet of buses over a fixed finite horizon. The authors sought the best policy of buying, selling and operating buses using the total discounted cost criterion. It was assumed that the number of buses required each year over the planning horizon was satisfied. The problem was to determine the number of buses to buy and sell each year. It was therefore not an economic life problem as such. The authors attempted to consider a repair limit type approach in the context of a fleet of buses which was operated over a finite fixed planning horizon. Maintenance cost data by age for each bus were extracted from record on work orders. Then, dynamic programming was used in which each year in the planning horizon represented a stage where a decision to either keep, buy or sell (scrap) a batch of 50 buses (B-bus) was made.

Hensher and Zhu (1994) used a similar approach to the repair limit, based on the comparison of the Annualised Equivalent Cost (AEC) for replacement with the AEC for repair called the AEC for rebuild. A 0-1 integer programming model is formulated and the solution is proposed using a heuristic algorithm to replace the whole vehicle fleet. The idea of the algorithm is to replace the plant with the largest negative gap between the AECs of the replacement and rebuilds, and to repair the plant with the highest positive gap between the replacement and rebuilds AECs subject to the age and budget constraints. This method also requires extensive data on individual plant for the whole fleet.

Note that where the replacement is not-like-with-like such repair limit type policies can lead to an increasing number of vehicle types in the fleet, and therefore an increasing technical burden on the maintenance facility.

2.7 Fleet size problem

Kirby (1959) made the first attempt to tackle the problem of fleet size optimisation. The fleet in question was wagons in a small railway system, for which the author attempted to strike a balance between the low utilisation of the owned fleet and the hiring of extra wagons at increased cost. A model was developed in order to determine the number of owned and hired wagons that would minimise cost. The author assumed that the cost of an owned wagon is 1 unit per day, the hired costs k units per day, $p(x)$ is the probability that x wagons were needed on any day. The cost was then expressed as

$$C = n + k\int_{n}^{\infty}(x-n)p(x)dx,$$

where n^* minimises C. A discrete formulation would have been adequate.

Wyatt (1961) extended the Kirby's model by introducing a variable cost when equipment was in use, in addition to owning and hiring cost. The model was applied to a fleet of barges for which demand was periodic.

A more recent paper which addressed the fleet size problem related to replacement schedule was written by Vemuganti *et al* (1989). The authors used a network model, precisely the minimum cost-flow model, to determine the optimal replacement schedule for a fleet of vehicles of various types and ages over a finite planning horizon. The reason for using the minimum cost-flow model is that the solution is all integer. The length of the planning horizon was fixed and known. Variation in fleet size was also considered. The aim of the model was to determine the number of vehicles required at the beginning of each new period over the planning horizon, where account should be taken on budget limitations. The approach using network models assumes linearity which is not always valid in real world application. With the network models, if additional requirements need to be implemented, the network structure of the models would be destroyed.

The problem of fleet size has often been modelled as a separate issue from capital replacement problems, although being an integral part of it. An inadequate fleet size, would lead, if oversized, to a high replacement cost. On the other hand, if undersized it would lead to high operating cost due to overuse, and failure to meet demand, and therefore would lead to early replacement. In order to balance between these two situations we seek to model the fleet size and the age of replacement in a simultaneous manner.

CHAPTER III

A Simple Capital Replacement Model with Variable Fleet Size.

3.1 Introduction

In this chapter we have made our first attempt in modelling capital replacement with variable fleet size, in the case of a replacement of the entire fleet. We have developed replacement models relating to either homogeneous fleet or inhomogeneous fleet. By homogeneous fleet, we mean, a fleet which consists of items that are all identical in model and age. On the other hand, an inhomogeneous fleet is a fleet which comprises items of different age, or models or both. For our case study, we consider that the fleet is homogeneous. This makes the task of modelling replacement easier than in the case of a mixed fleet which will be addressed in the next chapter of this thesis.

"At what time should a currently operating plant or fleet of plant be replaced?". This put simply, is the standard replacement problem (Christer and Goodbody, 1980). "How many items of plant are required to maintain a certain level of availability?". This is the standard fleet size problem (Kirby, 1959). Both of these questions can be addressed from an economic point of view: that is, solutions can be sought on the basis of minimising the long term cost. In the literature, many authors have considered these problems as separate issues and for more details we can refer to chapter 2.

In this chapter we present a robust approach to solve the fleet replacement problem in which the fleet size is allowed to vary at replacement, and a penalty cost is necessarily incurred when demand is not met. In this way we do not impose the restriction that some given demand is always met (e.g. Simms *et al.*, 1984), but instead seek to model the cost consequences of not meeting demand, as part of the replacement problem. This approach enables us to consider an unconstrained objective function. Also we do not impose the restriction that replacement is like with like. Particularly with the influence of technological development of plant, it would be sensible to consider whether new plant purchased at replacement is sufficient to meet demand. It may not be that demand itself is changing, but that the reliability, and hence availability, is changing with the purchase of new plant. In these circumstances, it would be sensible to consider age at replacement and fleet size, both of which relate to capital expenditure, in the replacement decision process.

In order to take into account technological development, we have considered a finite horizon. For this purpose a simple two-cycle model which consists of two successive 'operate-buy-and-sell' cycles is developed in which the size and the age at replacement of the fleet represent the principal decision variables. The cost parameters considered in the model are the maintenance cost, the penalty cost of unavailability, the replacement cost and the resale cost. In order to model the penalty cost of unmet demand, we require a model for the number of plant in the fleet which are unavailable due to failure at time t. We propose to model this unavailability as a birth-and-death process, with failures considered as births arriving at some rate dependent on age, and repairs considered as deaths. Taylor and Jackson (1954) used a similar process to model provision of spare engines for a fleet of aircraft. Figure 4.1 illustrates the failure and repair process to which the fleet is subjected. We assume that demand is constant over time. Demand could however, be treated as a stochastic process occurring in parallel with the failure process, but this approach has not been considered. We will however, carry out sensitivity analysis by allocating different values for the demand. Note that the current fleet is of size N, where U out of N items are in use and N-U are considered as spares. Problems of unavailability occur when the number of failed plant is greater than the number of spares. We, also describe a much simpler failure and repair model, as difficulties remain with the birth-and-death process model approach of Taylor and Jackson (1954). This simple model is used in the case study. We may however, consider the birth-and-death approach for future work.

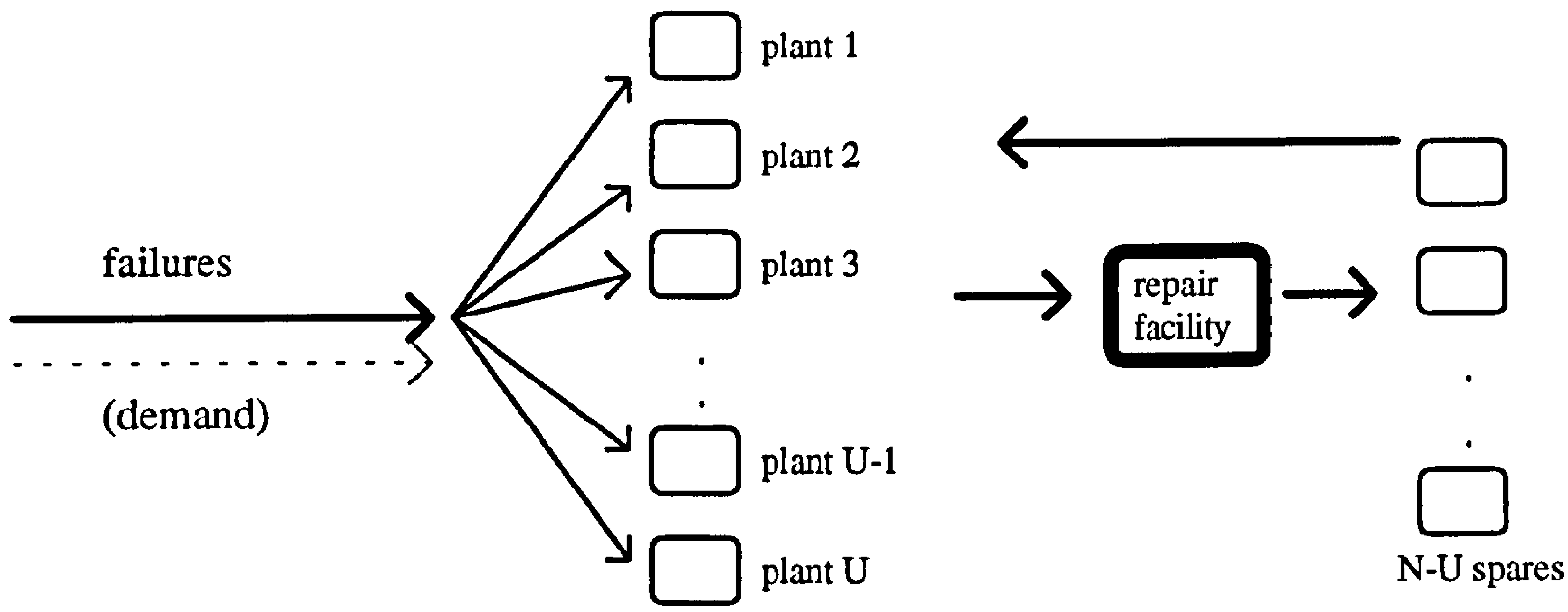


Figure 3.1. Failure and repair process

Our motivation towards this problem is that often equipment is either under utilised or over utilised. The former situation will lead to higher replacement cost while the latter will lead to high maintenance cost due to frequent breakdowns. A balance between these two situations needs to be set by determining the optimal fleet size and the optimal age of replacement by minimising either the total discounted cost per unit time or alternatively the equivalent rent (Christer, 1984). As an illustration of our approach we will consider an application to data relating to medical equipment (Christer and Scarf, 1994), namely ventilators in the operating department of a large hospital. This equipment is essential to carry out operations, and if for some reason the equipment was not available, the operation would be delayed and the consequences might be dramatic for a patient. This situation may also incur a high cost for the hospital in terms of compensation for a lack of care for a patient in an emergency. The decision-maker can however prevent this situation by taking into account in his replacement decision the penalty cost for unavailability due to breakdown or unavailability.

To carry out the modelling and estimation of different costs factors such as maintenance and operating costs as well as failure model, we need to assume that sufficient data are available. For the application of the model we have used data from Christer and Scarf (1994), where information on equipment covering the period 1978-1989 were extracted from the medical equipment management system at the Liverpool Royal Hospital. As far as modelling is concerned, the different costs of interest are the maintenance cost, the penalty cost of unavailability, the resale value and the purchase cost. Note that this chapter forms the basis of a research paper (Scarf and Bouamra, 1993).

3.2 The Models

Our replacement strategy consists of two 'operate-buy-and-sell' cycles, where the decision variables are the length of the first and the second cycle and the size of the replacement fleet at the end of the first cycle. It is formulated as follows: retain the current fleet of size N for K units time, then replace it by a fleet of size N_K, which is kept for L further units time. This is illustrated in Figure 3.2. It is clear that only the decision at the end of the first cycle is of importance for the operator, that is the decision on the current fleet. It would be expected that the analysis would be repeated for new plant, thus using a rolling horizon approach (Dekker *et al* 1993)

Note in Figure 3.2 that the size of the replacement fleet at the end of the second cycle is equal to the size of the replacement fleet at the end of the first cycle. The reason behind this choice is due to the fact that if the size of the fleet at the end of the second cycle was allowed to vary, the model would suggest not to buy any equipment, because there is no further operating cycle to be considered.

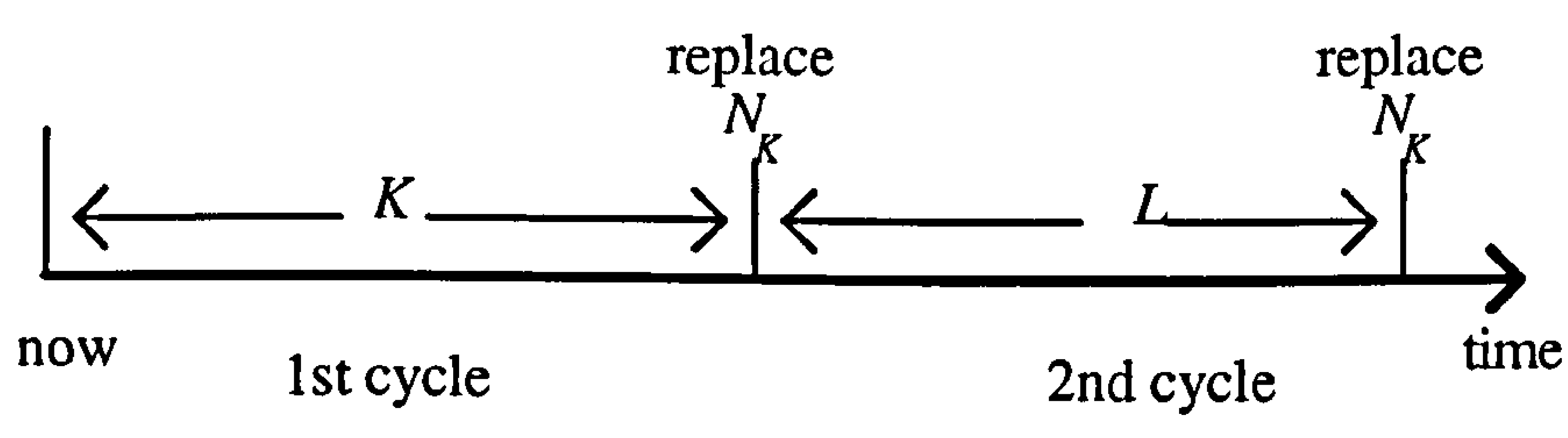


Figure 3.2. Two cycle replacement strategy

3.2.1 Notation

Before presenting the replacement model, let us define some notations as follows;

N - the size of the current fleet,

U - the demand, which is assumed constant over time,

τ - age of the current fleet,

$\lambda_t^{(i)}$ - mean number of failures of a machine in cycle i, (i=1,2), which is age (time) dependent,

μ - rate of repair which is assumed constant, $1/\mu$ is the mean repair time.

ν - discount factor defined as $(100+\iota)/(100+\eta)$, where ι and η are the inflation rate and internal rate of return respectively,

K - length of the first cycle expressed in years,

L - length of the second cycle,

N_K - size of the replacement fleet at the end of the first and second cycle.

3.2.2 Maintenance cost

In any area of research, the collection of reliable data has always been a difficult task. There is therefore no exception for the area of maintenance. For most situations data are available even in quantity, but the quality needs often be controlled. To control these uncertainties, sensitivity analysis is carried out in association with tools for data analysis and modelling such as the GLIM package (McCullagh and Nelder, 1990). To carry out our modelling on maintenance cost and failures, we need data on maintenance cost and number of failures by age and type in the fleet. This requirement on data availability enables us to develop our replacement model for all situations relating to the nature of the fleet, which might be either homogeneous, that is a fleet where all the plant are identical in terms of age and model of equipment, or inhomogeneous, that is a fleet which comprises plant which differ on age or model of equipment or both.

3.2.3 Penalty cost of unmet demand

The following direction was explored for modelling the penalty cost incurred when demand is not met. This cost has been introduced to enable us to determine the optimal fleet size at replacement. If N is the size of the current fleet, U the demand, $N-U$ will then be defined as the number of spare plant. Let j be the number of failed plant at a point in time. If $j>N-U$, a shortage in plant arises and suppose then a penalty cost is incurred. When a plant fails, it is sent to the repair facility, and a spare plant takes over. A repaired plant will join the fleet of spares, or the operating fleet if required.

Suppose that, for the whole fleet, failures are considered as births arriving with rate Λ_t (t age), while, for an individual plant, a repair is considered as a death with rate μ, where $1/\mu$ is the mean service time (mean time to complete repair). Assume that plant fail independently. If one considers a system made up of plant having a mean number of failures at age t, $\lambda_{m,t}$, then the mean number of failures of the whole fleet denoted Λ_t can be written as the sum of the individual mean number of failures at age t if all plant fail independently, that is $\Lambda_t = \sum_m \lambda_{m,t}$, where m indexes each of the (operating) plant in the fleet. Assume that the presence of other plant does not affect the characteristics of any particular plant. For a homogeneous fleet $\lambda_{m,t} = \lambda_t$ (all m).

We model the failure and repair process as a birth-and-death process, and we first need to give some definitions related to this stochastic process. Let X(t) represent the size of the population of failures at time t. Under the assumptions of our model, the birth-and-death process {X(t)} has infinitesimal transitions probabilities

$$P_{x,y}(t) = \Pr\{X(t+s) = y | X(s) = x\},$$

(Karlin and Taylor, 1975) which satisfies the following equations:

$$P_{j,j+1}(h) = \lambda_j h, \qquad j \geq 0, \tag{3.1}$$

$$P_{j,j-1}(h) = \mu_j h \qquad j \geq 1 \tag{3.2}$$

$$P_{j,j}(h) = 1 - (\lambda_j + \mu_j)h, \qquad j \geq 1 \tag{3.3}$$

$$\lambda_j = (N - j)\lambda_t, \tag{3.4}$$

$$\mu_j = j\mu, \tag{3.5}$$

provided the fleet is homogeneous, that is each plant is the same age and model. If the fleet is almost homogeneous in that, perhaps models and ages differ only slightly, then equations (3.4) and (3.5) will be approximately true. Note that equations (3.1), (3.2) and (3.3) imply, respectively, a failure (birth), a repair (death) and no event in a time interval of length h. Equation (3.4) is appropriate since if j plant are currently failed and under repair then N-j are at risk, while equation (3.5) implies that the rate of repair is proportional to the number of plant failed. If $\mu_j = \mu$ $(j>0)$, then the model would imply that the repair facility could only repair one plant at time t. This option has not been considered in this thesis but might be considered for future work. The probability that there are j failed plant in the fleet at time t, $P_j(t)$, is defined through the set of differential equations as follows:

$$\left.\begin{aligned}
&P_0'(t) = -N\lambda_t P_0(t) + \mu P_1(t), && j=0 \\
&P_j(t) = -(N-j+1)\lambda_t P_{j-1}(t) - ((N-j)\lambda_t + j\mu)P_j(t) && \\
&\qquad + (j+1)\mu P_{j+1}(t), && j=1,\ldots,N-1 \\
&P_N'(t) = \lambda_t P_{N-1}(t) - N\mu P_N(t), && j=N
\end{aligned}\right\} \quad (3.6)$$

Here $P'(t)$ denotes $dP(t)/dt$. This system of N+1 equations with N+1 unknown functions $P_j(t)$ (j=0,1,...,N) is called an Erlang system. For this system, the steady state is always obtained according to Markov's theorem because the number of states is finite, Khintchine (1960). Under the steady state condition the solution to the set of differential equations (3.6) is then given by the recurrence equation (see appendix I)

$$P_{j+1} = (N-j)\rho_t P_j/(j+1),$$

where $\rho_t (= \lambda_t/\mu)$ and represents the proportion of time the average plant is unavailable. The probability that there are j failed plant in the fleet at time t is given by

$$P_j = \binom{N}{j}\rho_t^j P_0 \quad \text{for } j=0,\ldots,N, \qquad (3.7)$$

where $\binom{N}{j} = \frac{N!}{j!(N-j)!}$, and n!=n(n-1)....3.2.1 for all n. Using the normalising condition $\sum_{j=0}^{N} P_j(t) = 1$, it follows immediately that

$$P_0 = \left[\sum_{j=0}^{N}\binom{N}{j}\rho_t^j\right]^{-1} = (1+\rho_t)^{-N}.$$

We assume that demand, U (U>*0*), is fixed over time. This assumption is justified for our application which is concerned with medical equipment when the care of patients is based on appointment.

Using the model we have formulated above, calculation of the total unmet demand over some interval [0,T] is not straightforward since we only know the distribution of the instantaneous unmet demand from equation (3.7). The non-stationarity nature of the model also makes the problem more difficult. We therefore propose to consider a simpler model. This simple approach may be justified since other uncertainties such as that of the maintenance cost will have a much greater bearing on results of the replacement modelling.

3.2.4 A simple model for penalty cost of unmet demand

In order to formulate the penalty cost of unavailability (unmet demand) we consider the failure process as a discrete one. We also consider a very simple model in which

(i) failures of plant aged τ occur according to a Poisson distribution with mean $\lambda(\tau)$ per unit time interval. (The convenient unit time interval for our application is the day).

(ii) Any plant failed on day i is unavailable for that day, but repaired and available for the next.

(iii) A penalty cost p is incurred for each plant unavailability (unmet demand).

(iv) Failures of plant occur independently of all other plant. Thus on day t of cycle c the number of failures for the whole fleet, $Z_c(t)$, will have a Poisson distribution with some mean $\Lambda_c(t)$, and

$$\Pr(Z_c(t) = z) = \frac{[\Lambda_c(t)]^z e^{-\Lambda_c(t)}}{z!}.$$

In cycle c, the number of spare equipment will be $X = N_c - U$, and a penalty cost will be incurred when the number of failures exceeds the number of spares or the demand exceeds the size of the fleet N_c. Note that X needs not be positive and denote $X'=\max(0,X)$ and $X''=\max(0,U-N_c)$. It follows that the expected penalty cost of unavailability on day t will be

$$P_c(t)=p[X''+\sum_{z=X'+1}^{N_c}(z-X')\Pr\{Z_c(t)=z\}]. \tag{3.8}$$

The first term in equation (3.8) will be zero if the fleet size exceeds the demand, which would be the usual case. The second term is a summation over all possible outcomes in which the number of failures exceeds the number of spares and thus incurring a penalty for unavailability.

In the health service industry the values of p may be large, especially if patient care is seriously affected. In another situation, if a bus operator has to cancel a trip, then the penalty cost may represent the loss of revenue and customer goodwill resulting. We should emphasise that we are not seeking here to suggest appropriate values for the penalty cost, p, but merely to indicate to management that such notion needs to be taken into account in replacement decisions, and further to indicate, through modelling, the sensitivity of the fleet replacement decision to the magnitude of p. The choice of p is left to the appraisal of the decision-maker who will choose its most appropriate and acceptable value.

3.2.5 Resale value

Resale value is the second hand cost of the plant. It is a time dependent cost factor which generally decreases in a very fast manner except in some situations where an unsteady and unstable economy prevails. In the absence of a 'second hand' market, the resale value for some equipment is set to zero (scrap value) or the equipment is kept as a spare if it is not technically obsolete.

3.2.6 Cost model

We will consider models for the replacement of the entire fleet for the following 3 cases:

(i) an homogeneous fleet (same age, same model of equipment),

(ii) an inhomogeneous fleet of same model of equipment but different ages,

(iii) an inhomogeneous fleet of different ages and model types of equipment.

We now define the notation of the cost factors used in the entire fleet replacement models.

$M_c(t)$ - maintenance cost per unit time for a plant of age t for cycle c;

$P_c(t)$ - expected penalty cost per unit time for fleet of age t for cycle c;

$S_c(t)$ - resale value of a plant of age t at the end of cycle c;

R_c - replacement cost per plant at the end of cycle c.

The cost criteria considered are the equivalent rent and the total discounted cost per unit time. We first define the total discounted cost (TDC) for the case (i) as:

$$TDC_i(K,N_K,L) = \sum_{t=1}^{K} N[M_1(\tau+t)+P_1(\tau+t)]v^{t-\frac{1}{2}} + v^K\{[N_K R_1 - NS_1(\tau+K)] + \sum_{t=1}^{L} N_K[(M_2(t)+P_2(t)]v^{t-\frac{1}{2}} + v^L[N_K(R_2 - S_2(L))]\}. \quad (3.9a)$$

For the replacement of a 'typical' plant (Christer and Scarf, 1994) we just set $N = N_K = 1$ in equation (3.9a). The total discounted cost for case (ii) is expressed as:

$$TDC_{ii}(K,L,N_K) = \sum_{t=1}^{K}\sum_{m=1}^{N}[M_1(\tau_m+t)+P_1(\tau_m+t)]v^{t-\frac{1}{2}} + v^K\{N_K R_1 - NS_1(\tau_m+K) + \sum_{t=1}^{L} N_K(M_2(t)+P_2(t))v^{t-\frac{1}{2}} + v^L N_K[R_2 - S_2(L)]\}. \quad (3.9b)$$

where τ_M is the current age of the plant M. Again the total discounted cost for case (iii) is expressed as

$$TDC_{iii}(K,L,N_K) = \sum_{t=1}^{K}\sum_{j=1}^{r}\sum_{m=1}^{n_j}[M_1^j(\tau_{j,m}+t)+P_1(\tau_{j,m}+t)]v^{t-\frac{1}{2}} + v^K\{N_K R_1 - S_1(K)\} + \sum_{t=1}^{L} N_K[M_2(t)+P_2(t)]v^{t-\frac{1}{2}} + v^L N_K[R_2 - S_2(L)]\}. \quad (3.9c)$$

where r is the number of model types, n_r number of plant of model r, M_1^j maintenance cost for model type j in cycle 1, $\tau_{j,m}$ is the current age of the mth plant of model type j and

$$S_1(K) = \sum_{j=1}^{r} \sum_{m=1}^{n_r} s_1^j(\tau_{j,m} + K),$$

where s_1^j is the resale value for model type j for cycle 1. M_2, S_2 and P_2 are expressed in a same fashion as in case (i) and (ii).

In practice the discrete formulation is a necessary requirement, and appropriate units for K and L should be used. All costs are based on present values and are discounted to mid-year, that is all the operating costs are assumed to occur in the middle of the respective time period. We then consider for case (i), say

$$TDC_i(K,L,N_K) \Big/ \sum_{t=1}^{K+L} v^{t-\frac{1}{2}}, \tag{3.10a}$$

which represents the equivalent rent value for the fleet, and determine those K, L, N_K for which equation (3.10a) is a minimum. An alternative objective function would be the total discounted cost per unit of time for the same case (i), that is

$$TDC_i(K,L,N_K) \Big/ (K+L). \tag{3.10b}$$

Both of the above objective functions are appropriate when usage is assumed approximately constant.

3.2.7 Further generalisations

A model when usage is no longer constant may be considered. In this case when the objective function can be described as the total discounted cost per unit of usage. Therefore, if the usage is time dependent, this can be expressed as (for case (i) above for example)

$$Z = \frac{TDC_i(K,N_K,L)}{\sum_{t=1}^{K} \sum_{j=1}^{N} U_{1j}(\tau+t) + \sum_{t=1}^{L} \sum_{j=1}^{N_K} U_{2j}(t)}, \tag{3.11}$$

given that

$$U_1 \propto \frac{1}{N}$$

and

$$U_2 \propto \frac{1}{N_K},$$

where $U_{cj}(t)$ is the usage function of the jth plant at time t in cycle c (c=1,2). We can also model the situation of non constant usage for cases (i) and (iii) without any difficulty. Thus the case when usage is time dependent requires information on usage of individual plant in the fleet. For example, in car replacement policy the cost functions are modelled as a function of both mileage and age (Scarf, 1994).

We could also model the case when the fleet is not sold or scrapped but kept as spares for one more cycle. Again for the case (i), the total discounted cost would be expressed as

$$TDC_i^{\Psi}(K,N_K,L) = \sum_{t=1}^{K+L} N[M_1(\tau+t)+P_1(\tau+t)]\nu^{t-\frac{1}{2}} + \nu^K\{N_K R_1$$
$$+\sum_{t=1}^{L} N_K[(M_2(t)+P_2(t)]\nu^{t-\frac{1}{2}} + \nu^L[N_K(R_2 - S_2(L)) - NS_1(\tau+K+L)]\}. \quad (3.12)$$

In equation 3.12, we assume that the current fleet is used in the second cycle under the same conditions as in the first cycle, that is to say that the maintenance cost function is kept the same. In practice, therefore, this assumption is not always true because the old fleet is usually used only in time of peak demand (Simms *et al* 1984).

3.3 Case study

In their paper, Christer and Scarf (1994) presented maintenance data for a number of ventilators used in the operating department at Liverpool Royal Hospital (LRH). This department consisted of 10 theatres, each equipped with the same model of ventilator (Servovent) which costs £2700, with approximately the same age. This latter characteristic was due to the fact that the hospital was a relatively new building (12 years old at the time of the study) and all theatres had been equipped in a homogeneous manner from new. The data presented by the authors relate to a sample of differing models of ventilators within the whole hospital. For the purpose of the modelling here, we use the published data on the sample of six Servovents and assume these are representative of the ten which were currently being used in the operating department. There was no explicit data available on the demand. However operations were scheduled by appointment and the number of operations performed did not vary greatly from one working day to the next. Thus it was reasonable to consider demand as a constant denoted U. Some usage data was collected from the Operating Department records of the Liverpool Royal Hospital (LRH) and it was shown that the average number of operations performed was approximately constant -see C.O.R.A.S's technical report, 1990).

Since the current fleet of ventilators are approximately the same age we use the simple approach as in Christer and Scarf (1994). It would be no problem to extend the application to case (ii) or (iii). For the purpose of this illustration, we ask the following question: given the current fleet of ventilators (Servovents), of which we assume the data are representative, how much longer should we retain this fleet and, at replacement, how many new ventilators should be purchased? We shall ask this question for various: penalty cost, p; demand, U; and current ages, τ. It should be noted that at the time the data were collected the Servovents were 12 years old and candidates for immediate replacement. Consequently the differing values of τ are considered to indicate the behaviour of the model, rather than address the then immediate replacement problem at LRH. In the first instance we shall consider replacement of like with like, that is the new fleet are also Servovents. Then we shall consider the case in which the Servovents are replaced with another model, the BIR MK8, for which a limited amount of maintenance history data were available.

3.3.1 The Data

In table 3.1 the failure history of the sample of 12 ventilators along with the inventory number at LRH over the period 1978-1989 is given. We are interested in the data on the subset of 6 Servovents ventilators and data related to these 6 machines are summarised in table 3.2. Figure 3.3 illustrates the cumulative number of failures per machine per year for the sample of 6 ventilators Servovent. The graph shows an increasing mean number of failures for the first eight years and a sudden drop in the number of failures in the last four years. This could possibly be explained by changing maintenance practice or a discontinuity in the quality of recording failures (Christer and Scarf, 1994). For the case study we have not considered resale value for the simple reason that there is little or no second hand market for that equipment in the UK.

Table 3.1. Number of failures per year for sample of 12 ventilators at LRH.

Item /Inv. No.	Year											
	78	79	80	81	82	83	84	85	86	87	88	89
ERICA R1137						5	1	0	2	1	2	2
ERICA R1144						1	2	3	3	2	0	0
Servovent R1061	1	0	0	0	2	1	0	1	0	0	0	0
Servovent R1103	0	0	0	0	0	1	2	1	0	0	0	0
PC3P R1115			0	0	0	0	1	0	0	0	0	0
Servovent R1099		0	0	0	0	1	2	2	0	0	0	0
BIR MK8 R1077			0	0	0	0	0	0	0	1	0	0
BIR MK8 R1075			1	0	0	0	0	0	1	0	0	0
Servovent R1063	0	0	0	0	0	1	0	0	0	1	0	0
Servovent R1062	0	0	1	0	1	3	0	0	0	0	0	0
Blease MP2 R1057	0	0	0	0	0	0	0	0	0	0	0	0
Servovent R1060	0	0	1	0	0	1	1	1	0	0	0	1

Table 3.2. Failure data for the sample of 6 Servovents at LRH.

	Age (years)											
	1	2	3	4	5	6	7	8	9	10	11	12
Number of failures	1	0	2	0	4	9	5	3	0	1	1	1
Total labour time to repair failures/hrs	6	0	8	0	16	46	25	28	0	6	3	6
Downtime/days	0	0	0	0	9	6	8	0	0	0	0	0

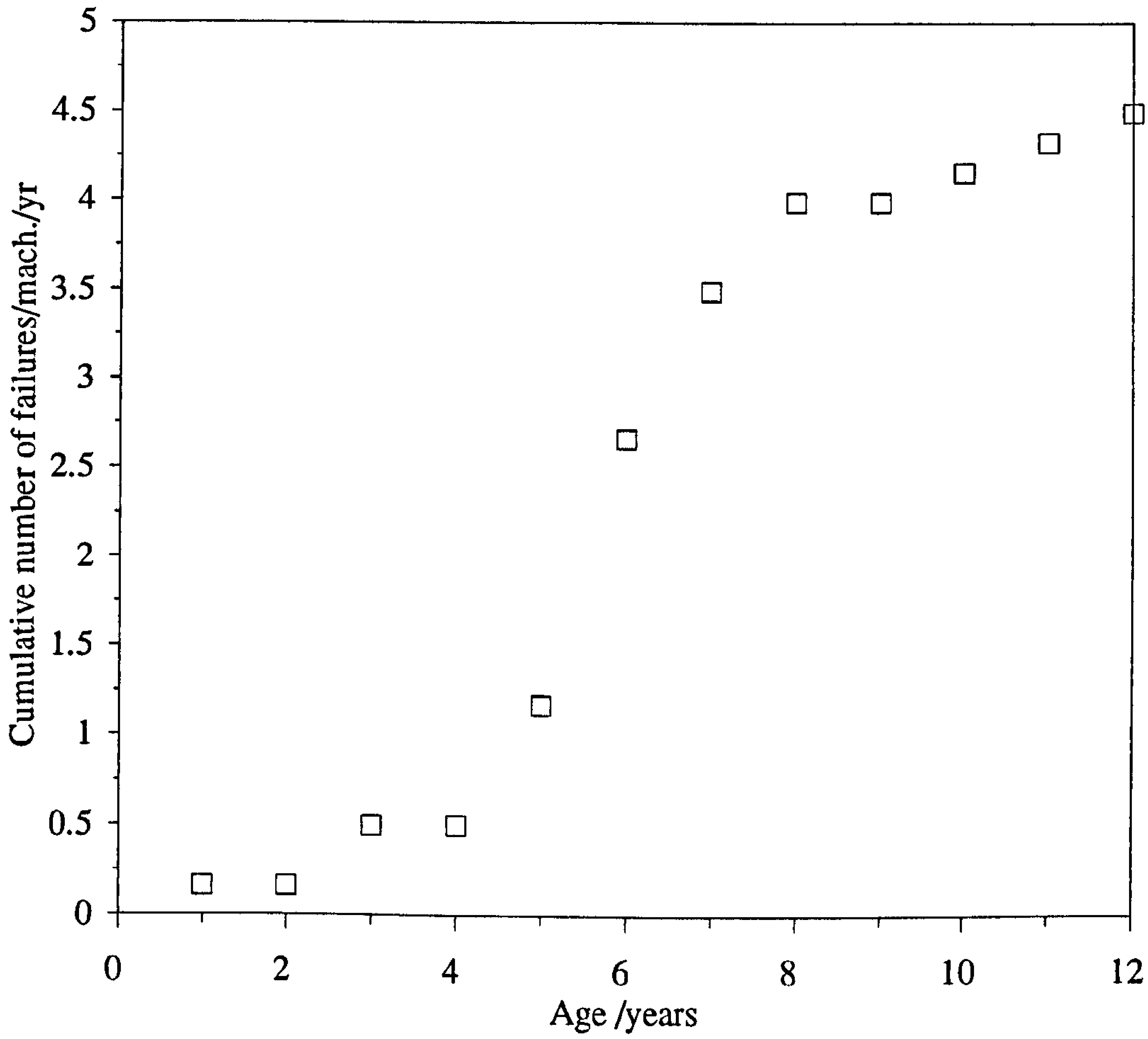


Figure 3.3. Cumulative Failure curve for the sample of 6 ventilators

3.3.2 The failure model

For the purpose of our work, maintenance cost has been modelled according to the data which have been made available by the department of health to the group of the Centre for Operational Research and Applied Statistics (C.O.R.A.S, 1990) of Salford University for the pilot study on medical equipment. These data have also been used by Christer and Scarf (1994). The maintenance cost is the sum of the service and failure cost and has been estimated as follows.

(i) Service cost per year = (average time for service) × (number of service per year) × (labour cost per hour),

(ii) Failure cost per year = (cost per failure) × (expected number of failures), where the cost of failure was broken down as follows:

(a) (average labour time to repair failure) × (labour cost per hour);

(b) downtime cost = (average downtime) × (rent per day), that is downtime is costed in terms of rental figure;

(c) spare cost = $\frac{1}{2}$ × labour cost;

(d) wasted nursing time cost.

The mean number of failures in the first cycle for the Servovents, with purchase cost £2770, denoted $\lambda_t^{(1)}$, is modelled using a log-linear model as reported by Christer and Scarf (1994) and defined as $\log(\lambda_t^{(1)}) = -2.2334 + 0.304t$ (t age in years). We note that only the first eight years were considered in the modelling. In the case where the replacement policy is not like-with-like, the new replacement fleet is assumed to comprise the BIR MK8 with replacement cost, £3840. The log-linear failure model, based on limited data, is given by $\log(\lambda_t^{(2)}) = -4.08 + 0.271t$. Maintenance cost has been estimated through service and failure cost, where the costs in pounds sterling (£) were estimated as follows. Service cost per year per plant: average time for service, 3 hours; number of service per year, 4; labour cost per hour, £15; which gives a service cost of £180 per year per machine. Failure cost per year: average labour time to repair failure, 5 hours; labour cost per hour, £15. Downtime cost: average downtime, 1 day; rent per day, £20; spare cost≈ £40; wasted nursing time, £30; giving a total of £165.

The second major cost involved in the model is the penalty cost incurred when a failure occurs and a needed machine is not available. This may lead to a hazardous situation for the patients. This cost has been associated with a serious failure in Christer and Scarf (1994), where such a failure was defined as a failure occurring during, for example, an operation. Here we associate the penalty cost with the case in which the number of failures exceeds the number of spares available. This is considered using the simple model described in section 3.2.4, with the mean number of failures per day calculated using the log-linear models defined above.

3.4 Results of 'like-with-like' replacement

We will consider the two optimisation criteria defined by equations 3.9 and 3.10, namely the equivalent rent per year and the total discounted cost per year respectively.

3.4.1 The equivalent rent model

Results from the equivalent rent model are presented for the case of fixed fleet size that is when the size of the fleet is frozen at its current size, and the case where the size of the replacement fleet is allowed to vary.

3.4.1.1 Fixed fleet size

Table 3.3 shows the optimum K and L along with the minimum cost incurred for various ages of the current equipment, demands and penalty costs per machine-day. We consider the size of the replacement fleet as fixed at size 10. We can note that there is some influence of the value of the demand on the values of K^* and L^*. The age of the current fleet influences the values of K^*. A reduction of 2 years is observed on K^* when the current age increases from 6 to 8 years. There is however no influence of the age of the current fleet on L^*. This is expected since the replacement fleet in the second cycle is new and does not depend on the age of the current fleet.

Table 3.3. Results (K^*, L^* and $TDC^* / \sum_{i=1}^{K^*+L^*} v^{i-\frac{1}{2}}$) for 'like-with-like' replacement, fixed fleet size and constant demand.

	Demand	demand=8		demand=9		demand=10	
age (years)	Penalty per machine day (£)	K*	L*	K*	L*	K*	L*
		(min cost in £)		(min cost in £)		(min cost in £)	
6	10	6	12	5	12	5	11
		(8272)		(8350)		(9088)	
	50	6	12	5	11	3	9
		(8290)		(8576)		(11381)	
	100	6	12	5	11	3	8
		(8306)		(8829)		(13332)	
	1000	5	11	3	9	1	4
		(8529)		(10977)		(29636)	
8	10	4	12	4	12	3	11
		(9041)		(9126)		(9921)	
	50	4	12	3	12	2	9
		(9062)		(9441)		(12313)	
	100	4	12	3	11	2	8
		(9087)		(9698)		(14441)	
	1000	3	11	1	9	1	5
		(9410)		(12115)		(33162)	

3.4.1.2 Variable fleet size

Table 3.4 relates to the case where the size of the replacement fleet, N_K is allowed to vary. Table 3.4 shows the influence of the demand on the optimum values of K and N_K. Figures 3.4-3.7 present some of those results graphically. We can observe in Figures 3.5 and 3.7 for which the penalty cost is set to £1000 per machine day, that if the decision-maker preferred to keep the same size for the replacement fleet, the extra costs incurred approximately would be respectively £2000 and £2150 per year for a period 11 years for both situations described in those two figures.

Table 3.4. Results (K^*, N_K^*, L^* and $TDC^* / \sum_{i=1}^{K^*+L^*} v^{i-\frac{1}{2}}$) for 'like with like' replacement, variable fleet size and constant demand.

	Demand	demand=8			demand=9			demand=10		
age (years)	Penalty per machine day (£)	K*	N_K^*	L*	K*	N_K^*	L*	K*	N_K^*	L*
			(min cost in £)			(min cost in £)			(min cost in £)	
6	10	6	10 (8272)	12	5	10 (8350)	12	5	10 (9088)	11
	50	6	10 (8290)	12	5	10 (8576)	11	3	11 (10296)	11
	100	6	10 (8306)	12	5	10 (8829)	11	2	11 (10948)	11
	1000	5	10 (8529)	11	3	11 (10357)	11	1	12 (13652)	11
8	10	4	10 (9041)	12	4	10 (9126)	12	3	10 (9921)	11
	50	4	10 (9062)	12	3	10 (9441)	12	2	11 (10982)	12
	100	4	10 (9087)	12	3	10 (9698)	11	1	11 (11516)	11
	1000	3	10 (9410)	11	1	11 (11206)	11	1	12 (15345)	12

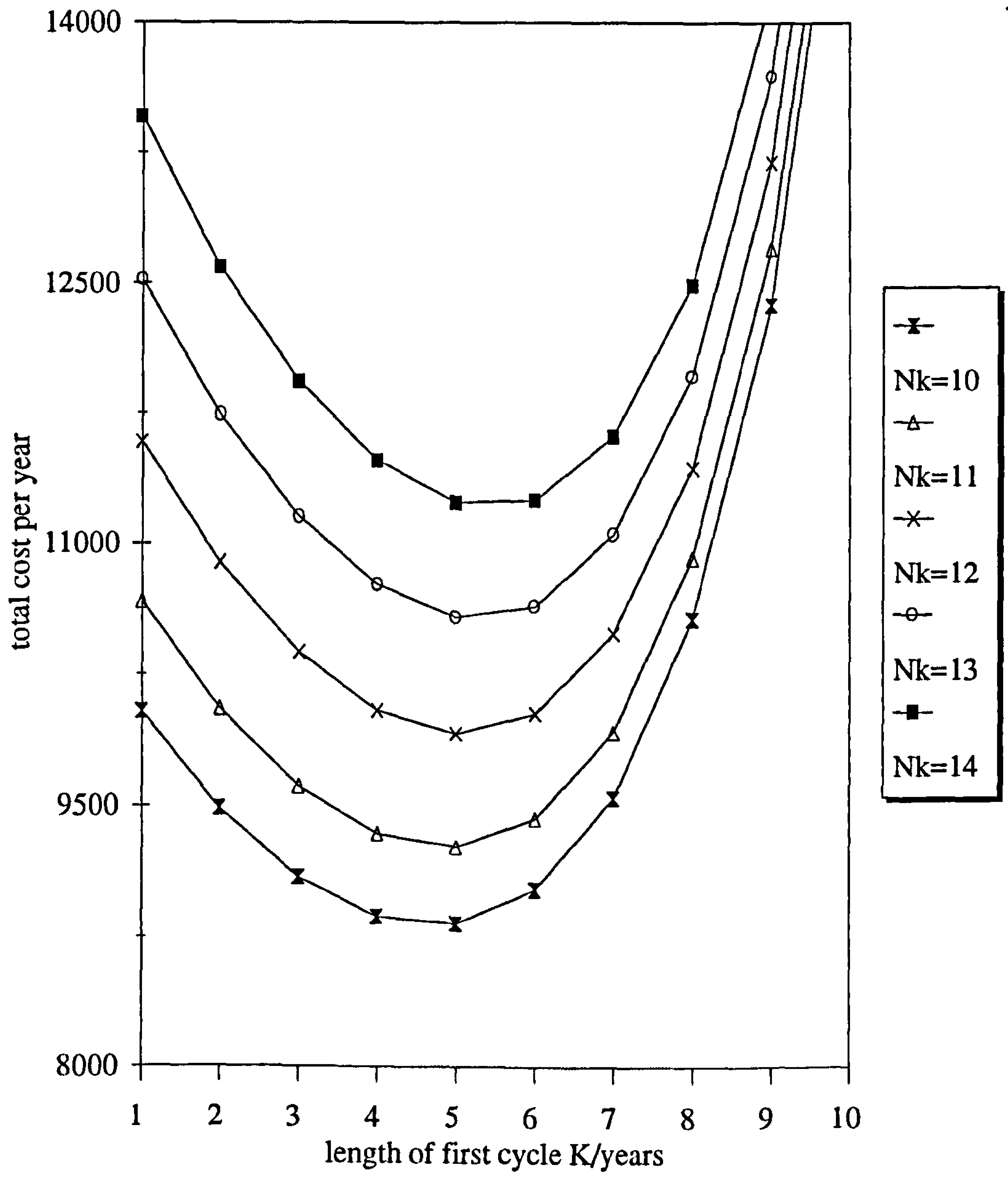


Figure 3.4 Cost per year of the equivalent rent against length of first cycle for various size of new fleet, replacement 'like with like' and demand constant (current age=6 years, demand=9 and p=£100 per machine per day).

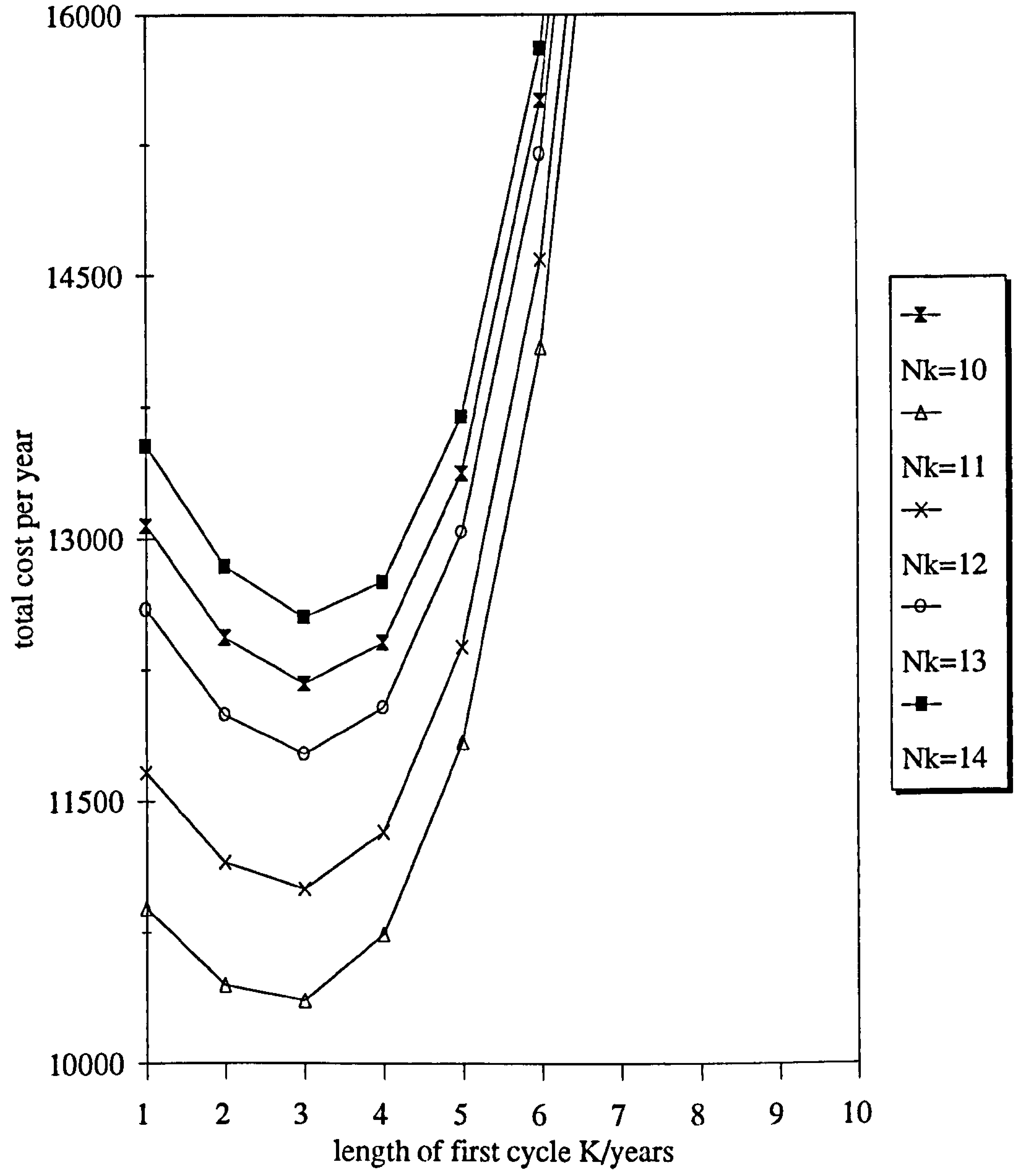


Figure 3.5 Cost per year of the equivalent rent against length of first cycle for various sizes of new fleet, replacement like with like and demand constant (current age=6 years, demand=9 and p=£1000 per machine per day).

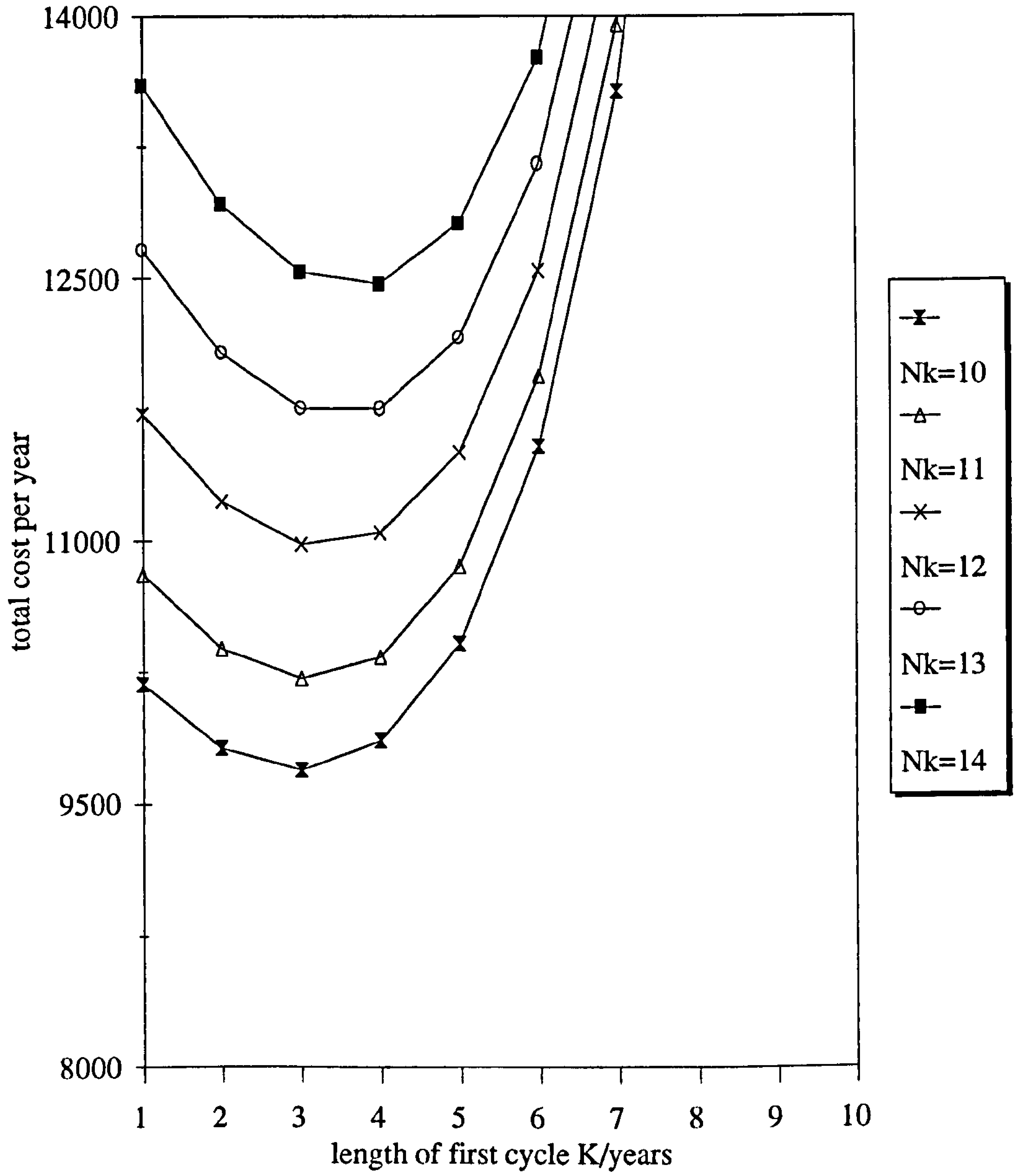


Figure 3.6 Cost per year of the equivalent rent against length of first cycle for various sizes of new fleet, replacement like with like and demand constant (current age=8 years, demand=9 and p=£100 per machine per day).

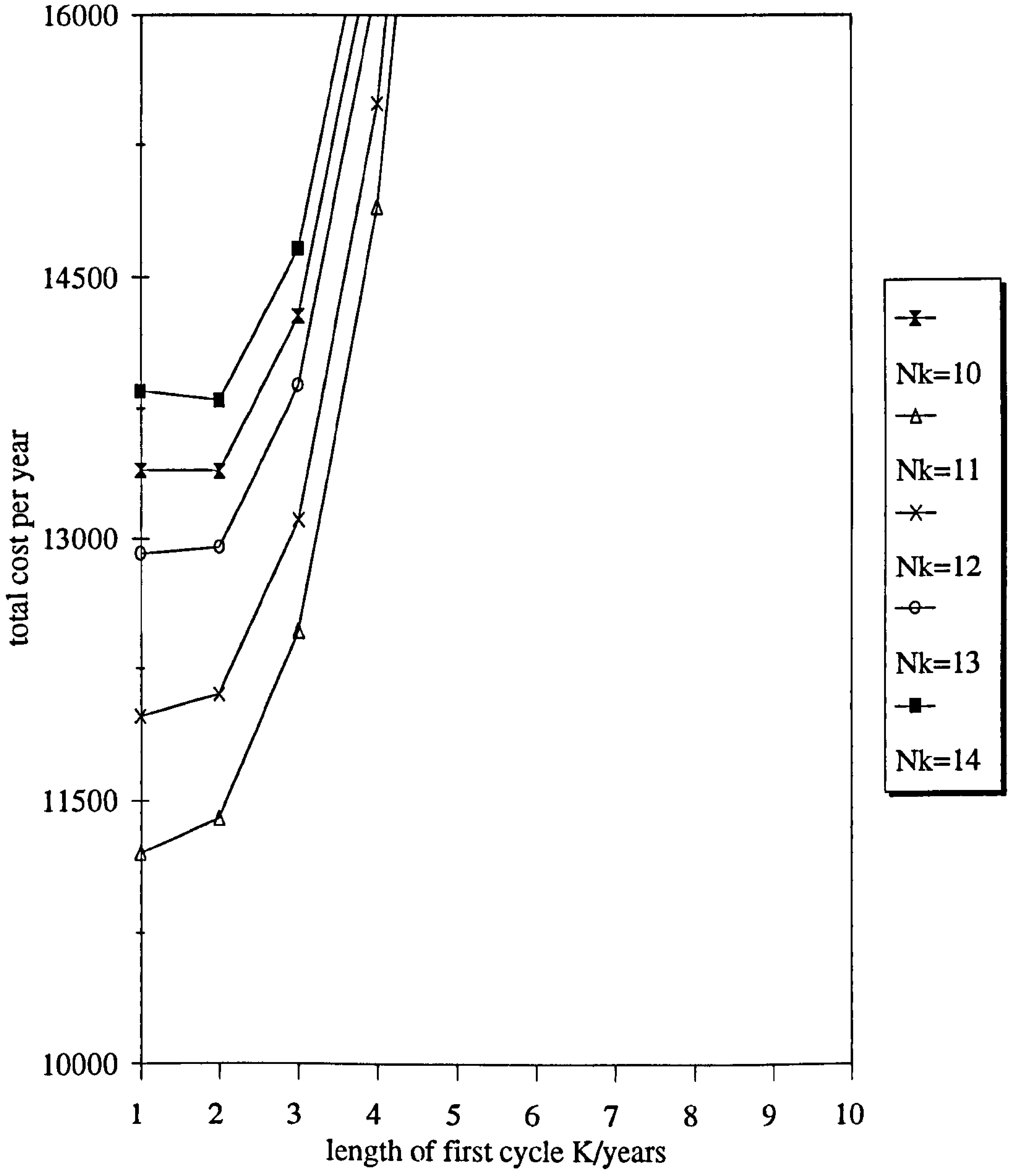


Figure 3.7 Cost per year of the equivalent rent against length of first cycle for various sizes of new fleet, replacement like with like and demand constant (current age=8 years, demand=9 and p=£1000 per machine per day).

3.4.2 The total discounted cost model

Table 3.5 shows results from the replacement model where the optimal values of K^* and L^* were obtained by minimising the total discounted cost (*TDC*) per unit time (equation. (3.10)). We can observe that the value of the minimum cost are relatively smaller than those obtained from the rent model but this is a result of the formulation of the model when the discount factor is smaller than one. On the other hand the values of K^* and L^* are closer. It is however shown in Scarf (1994) that if we consider an infinite series of 'operate-sell- and-buy' cycles and if the discount factor $\nu \rightarrow 1$, the total cost discounted to present value tends to infinity and therefore has no explicit interpretation, whereas the rent model, in the same circumstances, tends to the value of a special case, namely the average annual cost. We can observe that the values obtained in table 3.5 are very close to those obtained by Christer and Scarf (1994) for the same cost criterion, namely the total discounted cost per unit time for the replacement of a single plant. A little difference is noticed on the minimum cost which is mainly due to our formulation of the cost of unmet demand. The latter cost has been formulated differently in Christer and Scarf (1994), who consider penalty cost only for 'serious' failure, that is a failure occurring for example during an operation. We have only considered the case of fixed fleet size for this criterion.

Table 3.5. Results (K^*, L^* and $TDC^*/(K^*+L^*)$) for 'like with like' replacement, fixed fleet size and constant demand.

	Demand	demand=8		demand=9		demand=10	
age (years)	Penalty per machine day (£)	K*	L*	K*	L*	K*	L*
		(min cost in £)		(min cost in £)		(min cost in £)	
6	10	6	13	6	13	6	12
		(6329)		(6396)		(7074)	
	50	6	13	6	12	4	10
		(6346)		(6652)		(9221)	
	100	6	13	5	12	3	9
		(6368)		(6923)		(11151)	
	1000	6	12	3	9	1	5
		(6664)		(9110)		(26995)	
8	10	5	13	4	13	4	12
		(7069)		(7161)		(7864)	
	50	5	13	4	12	3	10
		(7097)		(7461)		(10166)	
	100	4	13	4	12	2	9
		(7133)		(7771)		(12155)	
	1000	4	12	2	9	1	5
		(7494)		(10263)		(29643)	

3.5 Result of 'not-like-with-like' replacement

Now, we consider the case when the replacement fleet is no longer the same as the current fleet, that is we introduce the notion of technological change in the modelling. In terms of results, if we want to compare them with those of the previous case that is the 'like-with-like' replacement, care should be made with the new fleet, BIR MK8, since the data available were limited and its operating conditions somewhat unknown. We have introduced this analysis purely to illustrate that such a replacement scenario may be investigated. Of course in practice it is quite likely that replacement will be not like-with-like and only limited maintenance history data for new plant will exist.

3.5.1 The equivalent rent model

As we did for the case of 'like-with-like' replacement, we investigate both cases, the fixed fleet and the variable fleet size case.

3.5.1.1 Fixed fleet size

Table 3.6 presents results in the same fashion that of table 3.3. We notice that there is very little or no difference on the values of K^* for both tables, but the difference is more sensitive for L^*, which is explained as a result of the apparent reliability of the BIR MK8. Since the replacement fleet has a smaller mean number of failures, the model would suggest to keep it longer.

3.5.1.2 Variable fleet size

In table 3.7 we can note that the values of K^* and N_K^* have not changed from the values of table 3.4, we note however a change for the values of L^* for the same reason that of the case of fixed fleet size in the previous section 3.5.1.1. We have, however made the assumption that no decrease in fleet size with respect to the current fleet size is allowed. This assumption is reasonable, since demand on machine is expected to increase. Figures 3.8-3.11 illustrate the cost of the equivalent rent for various sizes of the replacement fleet. From Figure 3.11 we can evaluate the extra cost which would be incurred if the operator kept the size of the replacement fleet as the size of the current fleet, $N_K = 10$, instead of the recommended size $N_K^* = 11$. This extra cost is approximately £1000 per year for a period of 19 years (L^*=19).

Table 3.6. Results (K^*, L^* and $TDC \cdot / \sum_{i=1}^{K^*+L^*} v^{i-\frac{1}{2}}$) for 'not-like-with-like' replacement, fixed fleet size and constant demand.

Demand		demand=8		demand=9		demand=10	
age (years)	penalty per machine day (£)	K* (min cost in £)	L*	K* (min cost in £)	L*	K* (min cost in £)	L*
	10	4	19	4	19	4	18
		(6246)		(6269)		(6685)	
	50	4	19	4	18	2	15
		(6249)		(6365)		(7857)	
6	100	4	19	4	18	2	14
		(6253)		(6359)		(8802)	
	1000	4	19	2	16	1	9
		(6327)		(7279)		(16347)	
	10	2	19	2	19	2	18
		(6605)		(6626)		(7021)	
	50	2	19	2	19	1	16
		(6608)		(6716)		(8168)	
8	100	2	19	2	18	1	14
		(6612)		(6811)		(9097)	
	1000	2	19	1	16	1	19
		(6685)		(7606)		(18190)	

Table 3.7. Results (K^*, N_K^*, L^* and $TDC \cdot / \sum_{i=1}^{K^*+L^*} v^{i-\frac{1}{2}}$) for 'not-like-with-like' replacement, variable fleet size and constant demand.

Demand		demand=8			demand=9			demand=10		
age (years)	Penalty per machine day (£)	K*	N_K^* (min cost in £)	L*	K*	N_K^* (min cost in £)	L*	K*	N_K^* (min cost in £)	L*
	10	4	10	19	4	10	19	4	10	18
			(6246)			(6269)			(6685)	
	100	4	10	19	4	10	18	1	11	18
			(6253)			(6459)			(7542)	
6	500	4	10	19	3	10	17	1	11	17
			(6286)			(6500)			(8450)	
	1000	4	10	19	2	10	16	1	12	19
			(6327)			(7279)			(9163)	
	10	2	10	19	2	10	19	2	10	18
			(6605)			(6626)			(7021)	
	100	2	10	19	2	10	18	1	11	18
			(6612)			(6811)			(7728)	
8	500	2	10	19	1	10	17	1	11	17
			(6645)			(7266)			(9068)	
	1000	2	10	19	1	11	19	1	12	19
			(6685)			(7569)			(10223)	

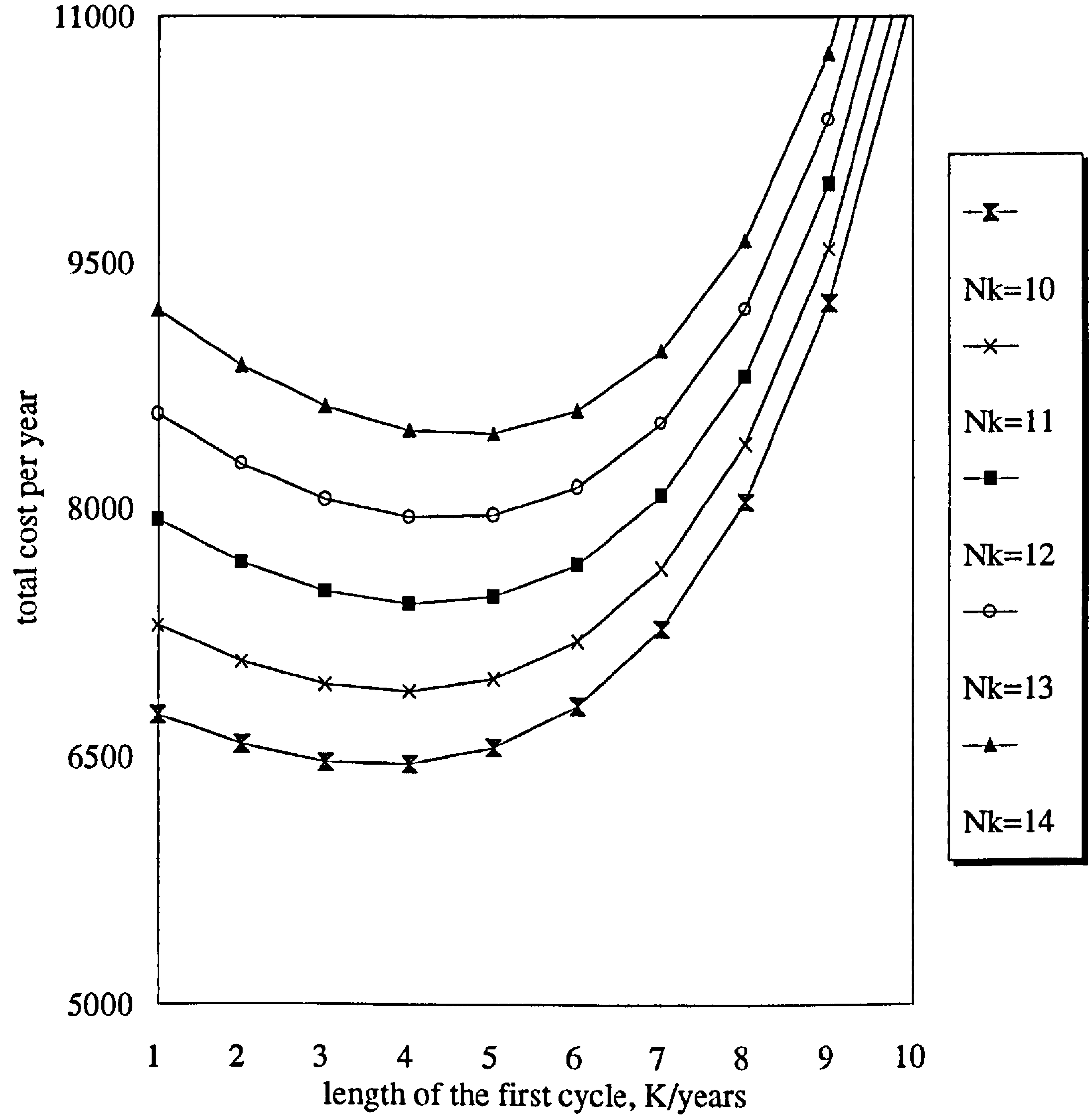


Figure 3.8 Cost per year of the equivalent rent against length of first cycle for various sizes of new fleet, replacement 'not-like-with-like' and demand constant (current age=6 years, demand=9 and p=£100 per machine per day).

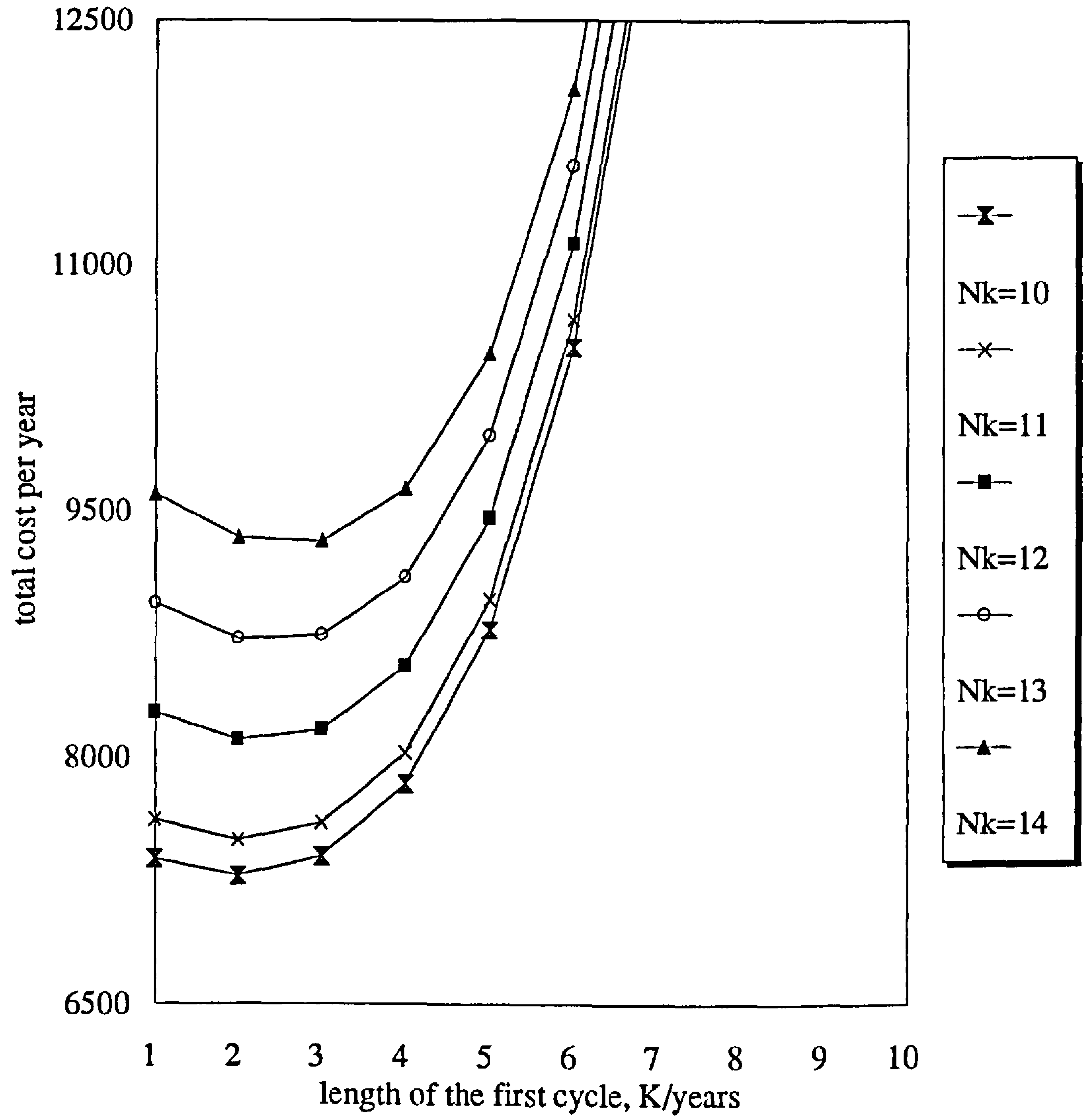


Figure 3.9 Cost per year of the equivalent rent against length of first cycle for various sizes of new fleet, replacement 'not-like-like-with-like' and demand constant (current age=6 years, demand=9 and p=£1000 per machine per day).

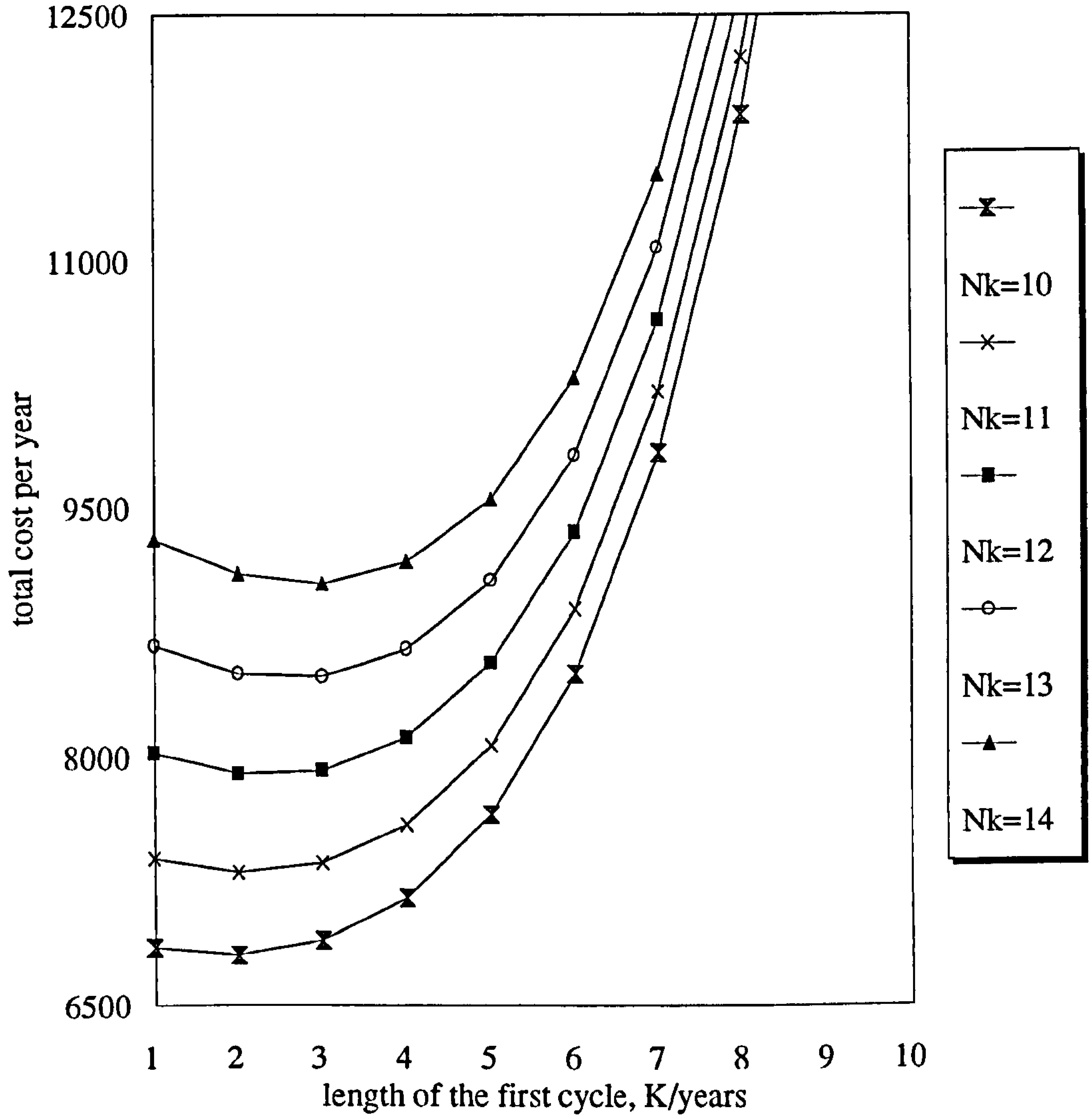


Figure 3.10 Cost per year of the equivalent rent against length of first cycle for various sizes of new fleet, replacement 'not-like-with-like' and demand constant (current age=8 years, demand=9 and p=£100 per machine per day).

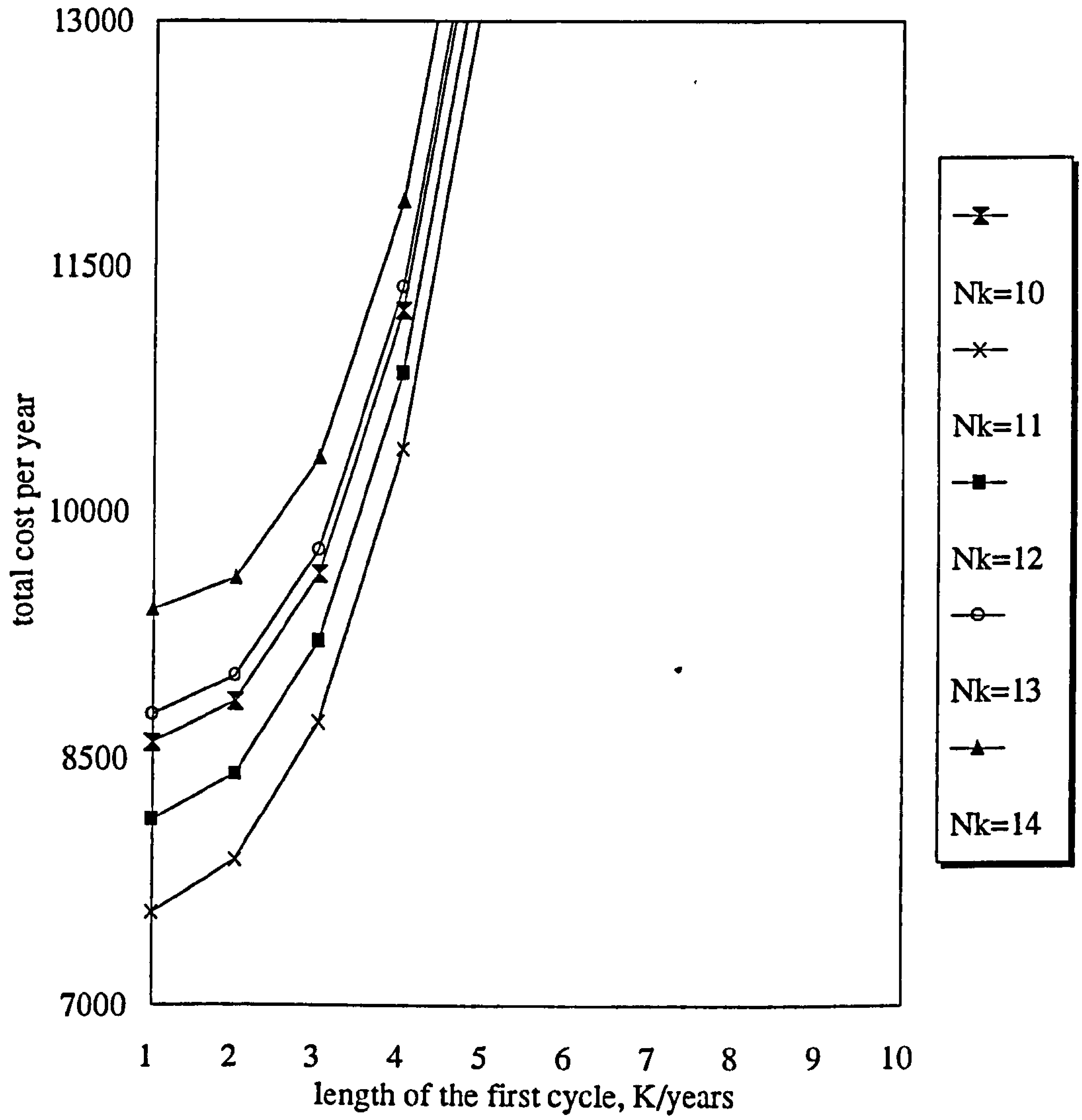


Figure 3.11 Cost per year of the equivalent rent against length of first cycle for various sizes of new fleet, replacement 'not-like-with-like' and demand constant (current age=8 years, demand=9 and p=£1000 per machine per day).

3.5.2 Total discounted cost per unit time

In the same way as section 3.4.2, we only consider the case of fixed fleet size. The results are presented in table 3.8. We can observe a high value for L^* which is due to the fact that the replacement fleet is more reliable therefore, it is kept much longer.

Table 3.8 Results (K^*, L^* and $TDC^*/(K^*+L^*)$) for 'not like-with-like' replacement, fixed fleet size and constant demand.

	Demand	demand=8		demand=9		demand=10	
age (years)	Penalty per machine day (£)	K*	L*	K*	L*	K*	L*
		(min cost in £)		(min cost in £)		(min cost in £)	
6	10	5	21	5	21	5	20
		(4386)		(4419)		(4826)	
	50	5	21	5	20	3	17
		(4393)		(4534)		(5974)	
	100	5	21	4	19	2	15
		(4402)		(4662)		(6926)	
	1000	5	20	2	16	1	9
		(4516)		(5603)		(14092)	
8	10	3	21	3	21	3	20
		(4745)		(4780)		(5177)	
	50	3	21	3	20	2	17
		(4755)		(4889)		(6386)	
	100	3	21	3	19	1	15
		(4767)		(5037)		(7251)	
	1000	3	20	1	17	1	10
		(4889)		(5930)		(15642)	

3.6 Sensitivity analysis to the discount factor ν

In real world situation, for some economical reasons the discount factor may vary over time (Christer and Goodbody, 1980). In our study, we consider the discount factor ν constant. However, we will carry out a sensitivity analysis to this factor in order to observe any change on the optimal decision with respect to the value of the discount factor. For this we consider a range of values of ν, which we apply to the replacement model related only to case (i) (equation 3.9a), that is for the equivalent rent criterion. This also can be done for the other models without any difficulty. Table 3.9 shows no sensitive change on the optimal values of the decision variables K^* and L^* . We can observe a slight decrease of the values of K^* and L^* when the discount factor approaches the value 1 in the case of 'like-with-like' replacement. In the case of not like-with-like replacement there is a decrease for K^* for the value 0.92 of the discount factor and then stays constant. There is an overall increase on the cost except for the first three values of the discount factor in the case of 'not like-with-like' replacement.

Table 3.9. Sensitivity to discount factor, using the equivalent rent criterion, $(TDC^* / \sum_{i=1}^{K^*+L^*} v^{i-1})$ for the case of 'like-with-like' and 'not like-with-like' replacement (age=6, demand=8, penalty=£100).

Discount factor	like-with-like			not like-with-like		
	K*	L*	£(pounds)	K*	L*	£(pounds)
0.92	6	12	7761	5	20	6233
0.93	6	12	7858	4	20	6218
0.94	6	12	7961	4	20	6208
0.95	6	12	8069	4	19	6211
0.96	6	12	8184	4	19	6224
0.97	6	12	8306	4	19	6253
0.98	6	12	8436	4	19	6301
0.99	5	12	8571	4	19	6368
1	5	12	8708	4	19	6444
1.01	5	12	8845	4	19	6536

3.7 Conclusions for case study

For the case of 'like-with-like' replacement we have observed that there is a strong influence of the age of the current fleet on the value of K^* for both cases; fixed and variable fleet size. For example for the case of fixed fleet size (table 3.3), when the age of the current fleet $\tau = 6$ years and the demand is 8, K^*=6 years for values of the penalty from 10 to £100. When $\tau = 8$ years, K^* decreases to 4 years for the same values of the demand and the penalty cost. When demand=9, the values of K^* vary from 5 to 3 years when the current fleet is 6 years old and from 4 to 1 year when the current fleet is 8 years old. When demand is 10 the values of K^* are decreasing from 5 to 1 year when $\tau = 6$ years, and from 3 to 1 year when $\tau = 8$ years. For the case of variable fleet size (table 3.4) we can observe an almost identical behaviour of K^*. We can also observe that the value of N_K^* is kept the same as its current size, that is 10 for low values of the penalty cost (10, 50, 100) and low demand (8, 9), but when the demand is high (10) and the penalty cost is medium (£100, £500) or high (£1000) the values of N_K^* increase respectively by one (11) or two (12) units from their current size.

For the case of 'not-like-with-like' replacement we have also observed that there is a strong influence of the age of the current fleet on the value of K^* for both cases; fixed and variable fleet size. For example for the case of fixed fleet size (table 3.6), when the age of the current fleet $\tau = 6$ years and the demand is 8, K^*=4 years . But when $\tau = 8$ years, K^* decreases to 2 years for the same values of the demand. When demand=9, the values of K^* are respectively 4 and 2 years when the current fleet is 6 and 8 years old, but when the penalty cost is high (£1000) the values of K^* are 2 and 1 respectively. When demand is 10 the values of K^* are decreasing from 4 to 1 year when $\tau = 6$ years, and from 2 to 1 year when $\tau = 8$ years. For the case of variable fleet size (table 3.7) we can observe an almost identical behaviour of K^*. We can also observe that the value of N_K^* is kept the same as its current size, that is 10 for low values of the penalty cost (10, 50, 100) and low demand (8, 9), but when the demand is high (10) and the penalty cost is medium (£100, £500) or high (£1000) the values of N_K^* increase respectively by one (11) or two (12) units from their current size.

For the total discounted cost the values of K^* are the same as those obtained with the rent criterion for both replacement policies, the 'like-with-like' and the 'not-like-with-like' replacement.

The sensitivity analysis to the discount factor shows that there is a very little influence of this factor on the values of K^* for both replacement policies, namely the 'like-with-like' and the 'not-like-with-like' replacement. Table 3.9 shows that the values of K^* vary from 6 to 5 years for the former policy and from 5 to 4 years for the latter when the discount factor varies from 0.92 to 1.01.

Finally, both criteria, namely the rent model and the total discounted cost per unit time show a similar behaviour of the optimal decision variables K^* with respect to the values of the age of the current fleet, the demand and the penalty cost. It should be noted however, that the minimum cost is smaller in the case of the total discounted cost per unit time criterion. This is due to the formulation of both criteria and when the discount factor is smaller than 1 (see equation (310.a) and (3.10b)).

3.8 Discussion

The model reported here is an attempt to improve the current practice of modelling replacement and optimum fleet size decisions. There is now the option of readily allowing for a variable fleet size at scheduled replacements. The value of such an option will be apparent when the nature of demand for the plant is changing, and also when the nature of technological improvement itself implies a change in the reliability of plant. Thus the expectation of technological development is accepted, with the task of modelling it simplified because of the limited duration of the time over which forecasting is required, and also because the nature of the replacement plant may already be known. Various methods exist for cost modelling under these circumstances (Christer, 1988).

The role of the penalty cost of unmet demand in the decision making process cannot be overemphasised. In this thesis, we do not attempt to estimate this penalty cost, but merely to present optimal fleet replacement decisions for a range of penalty costs. In this way, the penalty cost may be used to influence a decision, and to illustrate the cost consequences of making alternative decisions. Thus the proposed decision-making procedure does not attempt to remove the requirement for valued judgements, but merely to provide a cost measure associated with these judgements.

Application of the model also requires some attempt to estimate the demand for particular plant. Such data may not be available in the normal course of events and may need to be collected by sample survey, with appropriate analysis following. It will still, however, be possible to consider replacement decisions in the light of only limited availability of demand data by considering demand in a similar fashion to that of penalty cost above (as done in tables 3.4-3.7). Although demand may be considered in some deterministic time-varying fashion, the case of stochastic demand is a more difficult problem and is not considered here.

The two cost criteria, namely the total discounted cost and the discounted equivalent rent have both been used in the case study. We have observed that the optimum of the decision variables are almost constant same. The former criterion, however incurs a smaller cost with respect to the latter but this is only due to the formulation of the objective function. We should bear in mind that these models are only guideline for the operator, the decision-makers however should decide which is appropriate to them according their experience and the budget available.

In this chapter we have also described generalisations of the capital replacement model when the usage is not constant, that is the objective function is formulated as the total discounted cost per unit of usage. In the case where data on usage are available there is no difficulty to carry out the same work we have done with either the total discounted cost per unit time or the discounted cost of the equivalent rent. As a generalisation, we have also described the model when the current fleet is kept as spares but fully used. It is interesting to look in the future at the case where the retired fleet is kept as spares only for peak demand, this of course in the perspective that data on demand are available.

In the next chapter we will consider this variable fleet size replacement model in the context in which it is not desirable to replace the whole fleet. In real world applications, the fleet is often a mix of sub-fleets of different models of plant, and also varying current age composition within sub-fleets. This presents itself as a more difficult task than the modelling considered here. The replacement problem may then not be to determine how much longer to retain the whole fleet and the size of the new fleet, but to determine which sub-fleet(s) to replace first, when to replace it (them), and how many and of what model should the new sub-fleet(s) comprise.

CHAPTER IV

Capital Replacement Models for a Mixed Fleet

4.1 Introduction

In this chapter several replacement models are described for a fleet which is inhomogeneous in both age and "model" of equipment. Although the definition of the term "model" might be considered generally, in the application considered, equipments are individual vehicles in a fleet of buses and the "model" describes the vehicle specification. The fleet itself is considered as comprising a number of "sub-fleets" which would usually be homogeneous in model and age, for example, with replacements made to individual sub-fleets rather than to the fleet as a whole. Such an approach resembles more closely real replacement modelling problems, in which the operator has a mixed or complex fleet and is concerned with such issues as: Which sub-fleet should be sold at the next replacement? Which model should be chosen for the new sub-fleet? (See for example Christer, 1988; Bean *et al.*, 1994). How many vehicles of the chosen model should be purchased?

For this purpose, various models which deal with different situations are presented. The models have variable, but finite horizons which consist of one, two or three cycles. We have considered the time horizon as short in order to make possible the prediction of economic factors for new equipment (Christer, 1984) as well as for the new model of equipment. The way the replacement policy is modelled means we could consider an infinite number of cycles. Indeed, when the existing sub-fleets are all replaced, the first replacement sub-fleet is itself replaced, and so is the second replacement sub-fleet, etc. We, however do not consider a number of cycles greater than three for computational reasons and also to avoid heavy notation. In fact, the introduction of a second cycle in the modelling is only made to influence the first cycle (Christer, 1984), that is the current cycle. If a third cycle is introduced, the influence of the first cycle and importance of the optimal decision for this cycle may be diluted.

Therefore, any increase in the number of cycles may make the decision of the first cycle questionable. A balance has to be struck. Each model is related to a specific situation, which depends either on the number of cycles, the number of decision variables, the length of each cycle or the nature of the fleet size which is either fixed or variable.

Each of these models leads to particular optimal values for the decision variables. We need to compare all the models in order to evaluate to what extent the optimal solutions are affected from one model to another and to notice also any behaviour of a model.

Some of the replacement models we describe are applied to real data relating to the fleet of a Malaysian inter-city bus company operating a mixed fleet of buses. The size of the fleet is one hundred and twenty five and comprises five sub-fleets of different makes, ages and sizes. Data on maintenance costs and breakdowns were obtained from the company. Models for maintenance cost and number of breakdowns were fitted using the statistical package GLIM. This is considered in chapter 5.

The models also allow for a changing fleet size, which would reflect demand for the fleet and perhaps changing reliability as a result of the use of newer technology. The models may be simplified to the case in which the fleet size is fixed over replacements with consequently only one principal decision variable, the time to replacement of the first sub-fleet. Optimal values for the decision variables are based on minimising a cost function such as the equivalent rent (Christer, 1984) or total discounted cost per unit time, or unit of usage when data on usage are available. The choices of which sub-fleet to be sold and the model for the new sub-fleet, at first replacement, might be made by minimising the chosen cost function over all possible combinations of sub-fleet to be sold and model to be purchased. The optimal replacement decision is chosen among those combinations incurring minimal cost. In practice it is expected that the operator would indicate the range of possibility for such choices, thus providing useful input and a level of control of the modelling process which is highly desirable in practice (Russell, 1982). Furthermore, the assignment of individual equipment to sub-fleets would be within the control of the operator. Also it is assumed that equipment is bought new: in principle it is a simple matter to extend the models to the case in which used equipment might be purchased (Scarf, 1994).

It is assumed that data relating to maintenance, and in particular failures, is available and that these are sufficient for modelling purposes. In order to consider simultaneously the time to replacement and the size of the new sub-fleet, the cost of unavailability (unmet demand) must be formulated. This is done through the concept of a penalty cost (see Christer and Scarf, 1994). For a fleet operated under simple conditions, that is in which all "trips" are of constant duration and fixed in number, this formulation is straightforward.

In the application described in chapter 5 we consider how the models might provide the operator with meaningful decision support for a number of scenarios (fixed fleet size; variable fleet size) and for a range of penalty costs of unavailability. Emphasis should also be placed on considering also the additional cost of sub-optimal decisions. These cost considerations are also explored in the current chapter. Such sub-optimal decisions may be:

1)- delayed replacement; the replacement decision is often subject to availability of cash flow, but also by innovative change in technology (see Kusaka et *al*, 1990). If the cash flow is not available, or the innovative equipment is not yet available in the market, the decision of replacement might be delayed, and an extra cost incurred.

2)- Alternative equipment model choice for the replacement sub-fleets; the mathematical model determines all possible scenarios of sub-fleet replacement and recommends the optimal, but for some reasons, say, marketing, political decision or even customer's preference, the operator may make another choice for the replacement sub-fleet. This of course leads to an extra cost that can be quantified.

3)- Smaller replacement sub-fleet size: the operator, for reason of budgeting constraints, say, may be unable to buy the number of items recommended by the model, and he then replaces the current sub-fleet with a sub-fleet of smaller size. This will incur extra cost mainly through the penalty cost when demand is not met.

The infinite horizon has often been used by authors in the past (Eilon et *al*, 1966) but its weakness lies in the fact that technological development is not taken into account (replacement is assumed like-with-like), or if taken into account, it is not clear how it will be modelled. An another reason is the prohibitive number of decision variables (infinite) involved, as well as the long range forecast which is not of first importance to the operator. Technology is evolving fast and it will be difficult to forecast accurately models for new equipment with all the costs involved (purchase, resale, maintenance costs and so forth.). Technological improvement might imply a change in the reliability of plant and also an improvement in quality for the operator and customer. Increased revenue associated with such changes might be incorporated into the modelling process. Of course, such a change in reliability might not be for the better, e.g. for the case of sophisticated equipment; if the staff is not sufficiently trained, breakdowns might occur more frequently than expected because of misuse or poor maintenance practice. For a finite horizon, the task of modelling technological development is simplified because of the limited duration of the time over which forecasting is required, and also because the nature of the replacement plant might already be known, whereas in the case of infinite horizon technological improvement cannot be modelled in an objective way, because assumptions for, say, cost factors and failures processes for new equipment are often made in a simplistic manner, such as linear growth over time for costs (Elton and Gruber, 1976).

Other authors have looked at the optimum mix for fleets: Simms *et al.* (1984) sought an optimum age-based mix over a fixed finite horizon; Gould (1969) considered an optimum mix of models for the fleet on the basis of load and demand constraints. In the work considered here, the current mix of the fleet is of interest only in as much as this has bearing on the current replacement problem. In another words, as far as all the items in the fleet have the same duties (usage), the nature of the fleet mix is not relevant for our replacement strategy. This however, is not always true (Simms et al, 1984), in some situations the purchase of a new sub-fleet will affect the fleet mixes and will have an impact on the future decisions.

Some of the models and in particular the application considered in the next chapter formed the basis of a recently published paper (Scarf and Bouamra, 1995).

4.2 Preliminary considerations

4.2.1 Common notation

In this chapter, we present several replacement models for a mixed fleet which is assumed to be composed of r sub-fleets. Variable finite horizon replacement models are formulated with either one, two or three cycles, with a single sub-fleet replaced with a new sub-fleet at the end of each cycle. We define the notation which is common to all models. For convenience, we call the sub-fleet which is replaced at the end of the first cycle sub-fleet 1, and denote its size n_1. The principal decision variables are the time from now (time $t=0$) to first replacement, denoted by K. Other notation is as follows:

N_c- size of the fleet in cycle c, so that $N_1 = \sum_{i=1}^{r} n_i$, where n_i represents the size of sub-fleet i, $i = 1,\ldots,r$;

τ_{ij} - the current age (at $t=0$) of equipment j in sub-fleet i;

$M_i(\tau)$ - maintenance cost (expected) per unit time for equipment (each) aged τ in sub-fleet i,

R' - current cost new of each equipment in first replacement sub-fleet;

p - penalty cost of unavailability per equipment per unit time;

U - demand for the fleet ($U>0$);

ν - discount factor.

4.2.2. Maintenance costs

The greatest uncertainty in many maintenance and replacement decision problems lies with the adequacy of data relating to maintenance history. Hsu (1988) in his survey for UK based companies, showed that 97% of the companies that have definite policies for equipment replacement take into account maintenance expenditure for their replacement policies. Maintenance strategies such as planned maintenance and condition monitoring are adopted by many companies in order to keep the equipment on-line and run efficiently (Kobbacy and Nicol, 1994). Maintenance activities incur a large amount of expenditure within an organisation. In order to establish an efficient maintenance policy, reliable and complete data need to be collected. Subsequently, models for the maintenance cost are fitted using regression techniques. Sensitivity analysis may also be conducted to observe any effect on the replacement decision. Maintenance cost is in general worked out from the cost of parts, labour and lubricants. Other operating costs would include costs of fuel for example. For our application which is presented in chapter 5, data of the maintenance cost which have been made available to us by the company is based on cost of parts, labour, lubricant and tyres for each equipment per year over a period of four years.

4.2.3 Penalty Cost

The notion of penalty appears to be readily recognised and accepted, but difficult to quantify. Its presence is currently used in replacement decision processes, though not in an objective or quantitative manner. Penalty cost is incurred mainly when equipment fails. Although penalty cost might not be known, it can be assumed as a parameter p, and the sensitivity of a decision to this parameter might be explored for an equipment and the extent to which penalty should influence a decision can be established. The penalty cost model is considered as in section 3.2.4. Appropriate choice of p may be made by the operator.

4.2.4 Resale costs

Resale value is used in the context of equipment replacement. If an equipment becomes obsolete, aged or incurs high operating cost, the operator would consider the decision to replace it. Scrapping the existent equipment is an option, if it provides a profitable disposable value; otherwise the equipment might be kept as a spare in the fleet. Such depreciation cost is an important cost factor of running an equipment and can be estimated on the basis of time or usage. Estimates of resale values are available from accountancy and/or specialist guides (Walker, 1994). We might cite the Glass's guide for resale values (Waller, 1985). In Christer and Waller (1987a) and in Walker (1994) the depreciation cost or resale value was modelled as $s(\tau) = R\sigma\theta^{\tau}$, $0 \le \sigma \le 1$, $0 \le \theta \le 1$, where R represents the cost new of the equipment, τ its current age, σ is the antilog of the intercept of the regression and represents the very early depreciation after purchase and finally θ is the antilog of the slope of the regression and represents the periodic depreciation. It is, however, not the only formulation for the resale cost. Lake and Muhlemann (1979) considered a replacement problem of a wrapping machine for biscuits. Different models for the resale value were considered such as :

-i) a constant value, that is the resale value was equal to its scrap value regardless of its age, that is $S(t)=k$,

-ii) a linear decline of the resale value with the age of the machine, that is $S(t) = A(1-bt)$,

-iii) a monthly depreciation at constant rate d, that is $S(t) = A(1-d)^t$.

The authors carried out sensitivity analysis in order to compare the impact on optimal decision of these formulations and indicated a significant difference on the value of the optimal age of replacement with respect to the formulation used. Eilon *et al* (1966) and Scarf (1994) considered an exponential age dependent model for resale value.

4.2.5 Discount factor

If a capital sum is retained for a number of years before purchasing, the values of the sum would be different from its values if purchase was made immediately. This is due to the combination of inflation undermining the value of money and the return on investment (Scarf, 1994). All the costs in the models are based upon current day values using a discount factor. The discount factor denoted ν is defined as in Christer and Goodbody (1980), that is $(100+\iota\ /100+\eta\ )$ where ι and η are the inflation and the internal rate of return respectively . Note that, although the discount factor ν is assumed constant, it corresponds, in fact, to different inflating and discounting situations. It is known that in practice, the discount factor is time dependent and does not remain constant for any length of time (Kobbacy and Nicol, 1994), since inflation, interest rate or any internal rate of return vary constantly. In that case the discount factor in any year *t*, $\nu(t)$ might be expressed as (Christer and Waller, 1987a)

$$\nu(t) = {(100+\iota_t)}/{(100+\eta_t)},$$

where ι_t and η_t are the inflation rate and any rate of return (e.g. interest rate) in year *t*, respectively. The discount factor over *n* years, taken to end of the year, R_n, is given by

$$R_n = \prod_{t=1}^{n} \nu(t),$$

or if considered at the midpoint of the year *n*, it is denoted H_n and is given by

$$H_n = \prod_{t=1}^{n-1} \nu(t)\sqrt{\nu(n)}.$$

This can easily be introduced into the models (Christer and Goodbody, 1980). Kobbacy and Nicol, (1994) used the same discount factor for a study related to replacement of commercial vehicles (tractors unit) in the UK. The actual values of the discount factor were collected over 30 years, from 1960 to 1990 on a monthly basis, and then were compared with simulated data. The results of the simulation shows the same trend as the observed data. The authors carried out a simulation using a Markov chain approach. The transition probabilities from state (month) to state are estimated through the simulation using the observed data of the discount factor and the probability distribution of change from month to month.

4.2.6 Tax considerations

Considerations such as tax have not been taken into account in the modelling, although it may be sometimes necessary to include. Eilon *et al* (1966) did include tax into the replacement model, and it had appeared to have an influence upon the optimal decision. Christer and Waller (1987a) also introduced tax into replacement models, but it appeared that following the 1984 finance act, which simplified the tax allowance scheme, no major effect had been noticed on the optimal decision. The effect of adjusting for tax allowances came to the same thing that to premultiply the objective function by a constant factor (1-u), where u is the corporation tax. This of course leaves the optimum values of the decision variables unaffected (Christer and Goodbody, 1980). It is worth pointing out that maintenance costs are considered as expenses against profit, therefore tax is not paid on them (Eilon *et al*, 1966).

4.3 The Models

In this chapter, we describe various replacement models with finite planning horizons which consist of two or three cycles. The length of a cycle, which is a decision variable is generally continuous. However, for the sake of simplicity of computation, we use discrete models. When a finite horizon is considered, technological improvement is taken into account in the modelling because the new equipment is either already operational within the organisation or elsewhere, and therefore any data such as costs and failures are either known or the way to approach the task might be known. If the planning horizon is infinite, input factors such as costs or output for new equipment need to have some assumptions such as a linear growth, and the replacement strategy is uniform all over the cycles (Elton and Gruber, 1976) in that case technological improvement is not taken into consideration in an objective way. Also, assuming a uniform replacement strategy is debatable, since maintenance and inspection policies are constantly improved within organisation.

The models described consider the rolling replacement of sub-fleet (see Figure 4.1). In practice, for the first cycle, a sub-fleet for replacement is identified, along with equipment model for its replacement sub-fleet (the new plant) and the replacement is made at the end of the cycle. Similarly for the second cycle, a sub-fleet for replacement and the new plant are identified, and the replacement made, and so on. In this way, the fleet is evolving through time. The models we consider differ in the extent to which they take account of future costs. For example, for the second model (model IIa) we consider, costs include a second cycle which ends with a replacement. Note that r' and r'' are the first and the second replacement sub-fleet at the end of the first and the second cycle respectively.

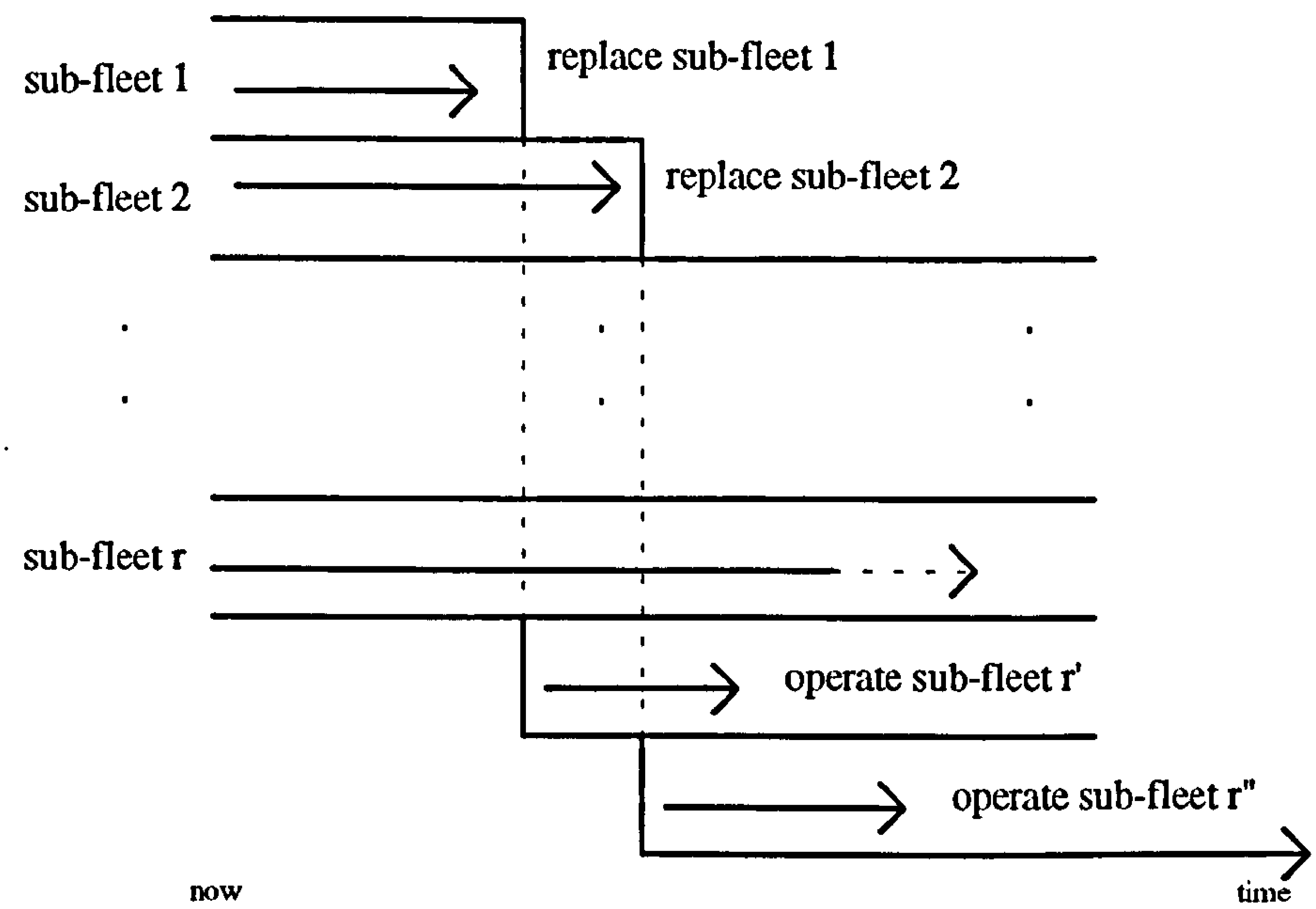


Figure 4.1. Replacement schedule for the whole fleet.

The finite horizon replacement models have been denoted model I, model IIa, model IIb, model IIc, model IId, model IIIa, model IIIb, model IV and model V which correspond to respectively a one cycle model with fixed fleet size, a two cycle model, that is an 'operate-sell-and-buy' cycle with fixed fleet size; a two cycle model with variable size of the first replacement sub-fleet and fixed size for the second replacement sub-fleet; a two cycle model with variable fleet size for both cycle, a two cycle model with variable fleet size for the first cycle, the second cycle is an operate cycle only with fixed fleet size, a three cycle model with fixed length of the third cycle which is an 'operate' cycle only, where the size of the first and the second replacement sub-fleets are variable; a three cycle model where the length of the third cycle is a decision variable besides those of the first and second cycle; the two cycle model IIb with two sub-fleets only, and finally a fixed finite horizon model (see Figures 4.2.a - 4.2.h).

4.3.1 One cycle model (model I)

Initially we consider a default model, with one cycle, denoted model I. This is introduced for comparison purposes considered later (chapter 6). Figure 4.2.a shows the replacement strategy.

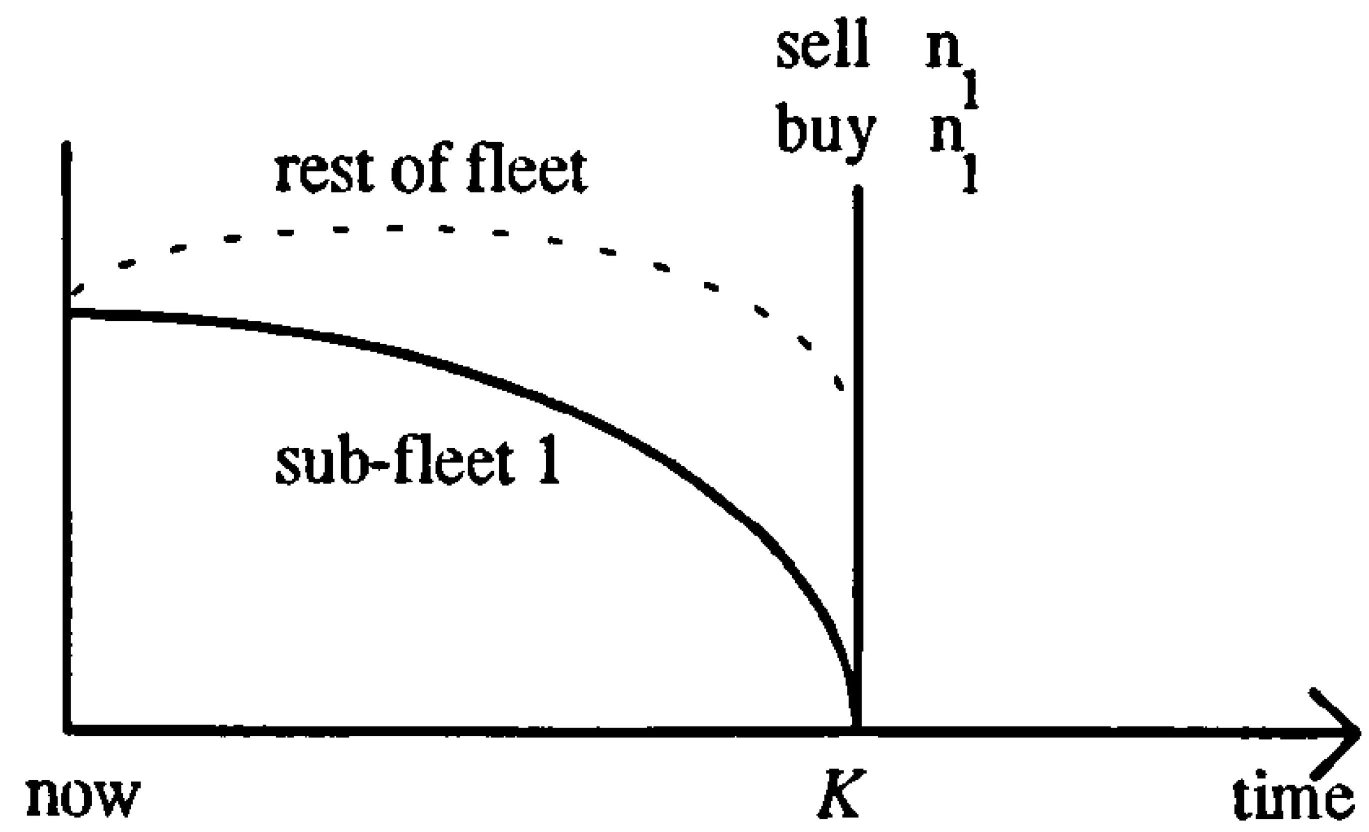


Figure 4.2.a. One cycle sub-fleet replacement model, with fixed sub-fleet size.

The total discounted cost is denoted TDC_I and is expressed as

$$TDC_I(K) = \sum_{t=1}^{K} C_1(t)v^t + v^K\{n_1R' - S_1(K)\} \tag{4.1}$$

where R' is the cost of each item in the the new replacement sub-fleet at the end of the first cycle. $S_1(.)$ is the resale value for sub-fleet 1 and is expressed as,

$$S_1(K) = \sum_{j=1}^{n_1} s_1(\tau_{1j} + K)$$

where $s_1(\tau_{1j})$ is the resale value of the jth item aged τ of sub-fleet 1. The total cost is expressed by

$$C_1(t) = \sum_{i=1}^{r}\sum_{j=1}^{n_i} M_i(\tau_{ij} + t) + P_1(t),$$

where $M_i(t)$ is the maintenance cost of each equipment in the ith sub-fleet and the penalty costs of unavailability $P_1(t)$ in the time interval t in the first cycle is then expressed as

$$P_1(t) = p'[X'' + \sum_{z=X'+1}^{N_1}(z - X')\Pr\{Z_1(t) = z\}].$$

where X' and X'' are defined as in section 3.2.4 of chapter 3.

The objective function is either the total discounted cost per unit time

$$TDC_I(K) \Big/ (K),$$

or alternatively, the equivalent rent

$$TDC_I(K) \Big/ \{\sum_{i=1}^{K} v^i\}.$$

4.3.2 Two cycle model, fixed fleet size for all cycles (model IIa)

This case represents the replacement strategy where the size of each replacement sub-fleet is kept fixed to its current value. The decision to operate and buy the same size of the sub-fleet is mainly taken for budgeting constraints, especially for organisation where the allocation of budget is centralised (e.g. state owned company). In this case the only decision variables would be the length K and L of the first and the second cycle respectively. Figure 4.2.b presents the replacement strategy. Thus this model considers the operation and replacement of two sub-fleets and the operation of the rest of the fleet. The cost criterion is the total discounted cost, denoted TDC_{IIa} and is expressed as follows

$$TDC_{IIa}(K,L) = \sum_{t=1}^{K} C_1(t)v^t + \sum_{t=K+1}^{K+L} C_2(t)v^t$$
$$+ v^K\{n_1R' - S_1(K) + v^L[n_2R'' - S_2(K+L)]\}. \qquad (4.2)$$

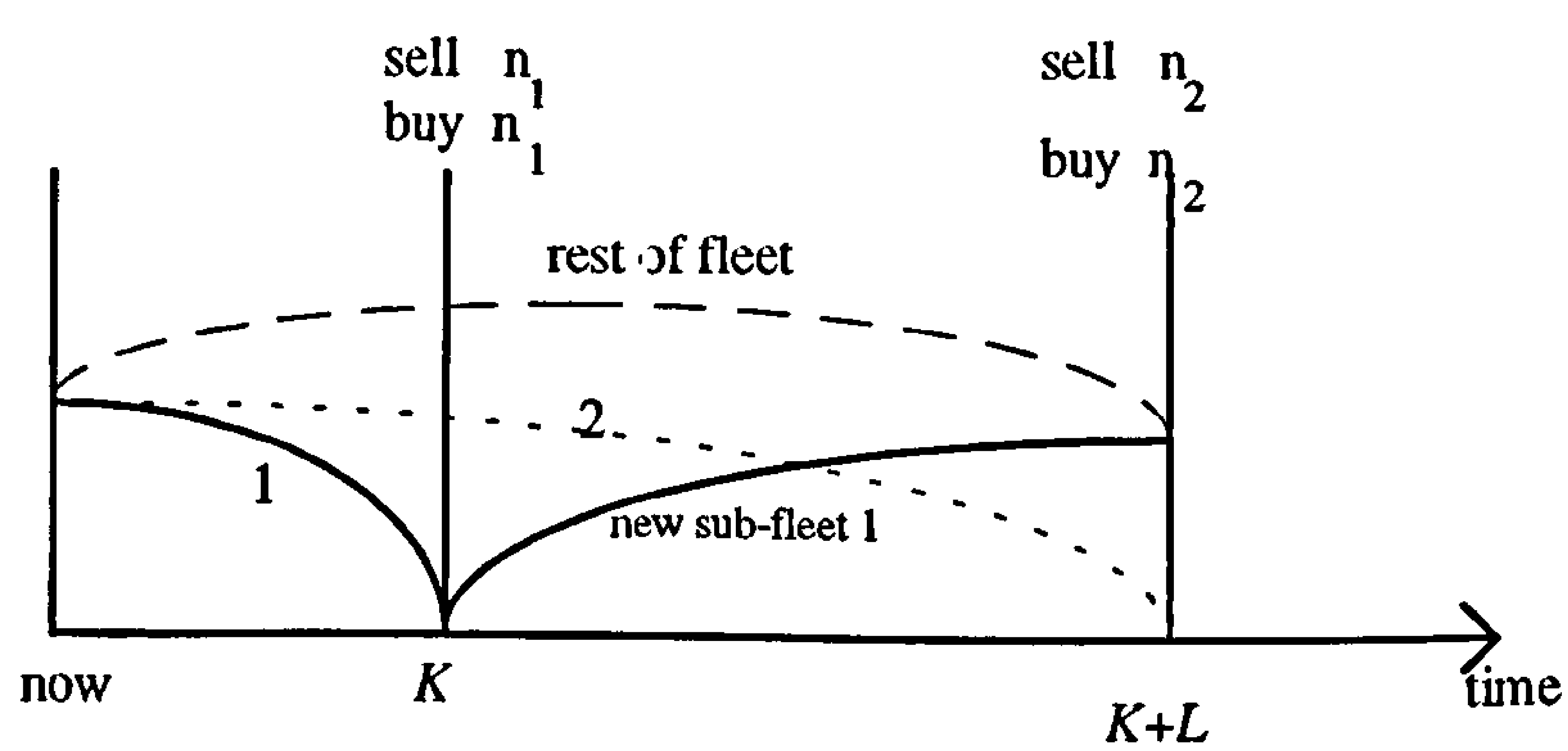


Figure 4.2.b. Two cycle sub-fleet replacement model, with fixed sub-fleet size. —— represents respectively 'operate-and-sell' the first sub-fleet and 'buy-and operate' the replacement sub-fleet; -------- represents 'operate-and-sell sub-fleet 2 and 'buy' replacement sub-fleet; — — — —represents 'operate' the rest of the fleet.

R'' is the current cost new of each equipment in second replacement sub-fleet. $S_1(.)$ and $S_2(.)$ are the resale value of sub-fleets 1 and 2 respectively and are expressed as follows

$$S_1(K) = \sum_{j=1}^{n_1} s_1(\tau_{1j} + K) \tag{4.3}$$

$$S_2(K+L) = \sum_{j=1}^{n_2} s_2(\tau_{2j} + K + L) \tag{4.4}$$

where $s_1(\tau_{1j})$ and $s_2(\tau_{2j})$ are the resale value of the jth item aged τ of sub-fleet 1 and 2 respectively. The costs $C_1(t)$ and $C_2(t)$ are the sum of the maintenance and the penalty cost per unit time in the first and second cycle respectively.

$$C_1(t) = \sum_{i=1}^{r} \sum_{j=1}^{n_i} M_i(\tau_{ij} + t) + P_1(t), \qquad (t = 1, \ldots, K), \tag{4.5}$$

$$C_2(t) = n_1 M'(t-K) + \sum_{i=2}^{r} \sum_{j=1}^{n_i} M_i(\tau_{ij} + t) + P_2(t), \qquad (t = K+1, \ldots, K+L), \tag{4.6}$$

where $M_i(t)$ and $M'(t)$ are the maintenance cost of each equipment in the ith sub-fleet and each equipment in the replacement sub-fleet respectively. The mean number of failures for the whole fleet in time interval t for both cycles 1 and 2, denoted $\Lambda_c(t)$, $c = 1,2$, is expressed as follows

$$\Lambda_1(t) = \sum_{i=1}^{r} \sum_{j=1}^{n_i} \lambda_i(\tau_{ij} + t), \qquad (t = 1, \ldots, K), \tag{4.7}$$

$$\Lambda_2(t) = n_1 \lambda'(t-K) + \sum_{i=2}^{r} \sum_{j=1}^{n_i} \lambda_i(\tau_{ij} + t), \qquad (t = K+1, \ldots, K+L). \tag{4.8}$$

where $\lambda'(t)$ is the mean number of failures for a single equipment in the replacement sub-fleet which is assumed equal for each item and $\lambda_i(t)$ is the mean number of failures for a single equipment in the sub-fleet i in the time interval t. The penalty costs of unavailability $P_1(t)$ and $P_2(t)$ in the time interval t in first and second cycle respectively are then expressed as

$$P_1(t) = p'[X'' + \sum_{z=X'+1}^{N_1} (z - X') \Pr\{Z_1(t) = z\}], \tag{4.9}$$

$$P_2(t) = p'[X'' + \sum_{z=X'+1}^{N_1} (z - X') \Pr\{Z_2(t) = z\}], \tag{4.10}$$

where $p' = p\Delta$, p is the cost of unavailability per equipment per day; and Δ is the number of days in the unit time interval (e.g. the month or year).

Note that the total size of the fleet, N_1 is kept constant in both cycles. The objective function to minimise is either the equivalent rent which is expressed as

$$TDC_{IIa}(K,L) \Big/ \{\sum_{i=1}^{K+L} v^i\}, \tag{4.11}$$

or the total discounted cost per unit time

$$TDC_{IIa}(K,L) \Big/ (K+L) \tag{4.12}$$

and the optimal values of the decision variables are denoted K^* and L^*. When the equivalent rent or the total discounted cost per unit time, equations (4.11) or (4.12), is optimised, the sub-fleet to be replaced at the end of the first cycle, sub-fleet 1, has to be chosen in advance. Also sub-fleet 2 and the nature of the replacement sub-fleets need to be determined in advance. These decisions might themselves be optimised by minimising the chosen cost function over all possible combinations of sub-fleets to be sold and models to be purchased. In practice the range of possibility for such choices might be narrowed greatly by the experience of the operator.

4.3.3 Two cycle model, variable size for only the first replacement sub-fleet, (model IIb)

Here a finite horizon replacement model is considered and formulated with principal decision variables: time to first replacement K , size of the new sub-fleet at the first replacement, N_K. The second cycle is introduced to influence the first cycle and to model the on-going requirement for the fleet with time to replacement of the second sub-fleet as decision variable L. However the size of the second replacement sub-fleet is frozen at its current value, which is convenient for the current purpose (see Figure

4.2.c). Equation 4.13 shows indeed, that the second replacement sub-fleet, of size n_2, is not operated for any further cycle, therefore if it was allowed to vary the model would suggest not to buy any equipment. Our cost criterion is the total discounted cost (TDC_{IIb}) over the two cycles 1 and 2 which is expressed as follows

$$TDC_{IIb}(K,L) = \sum_{t=1}^{K} C_1(t)v^t + \sum_{t=K+1}^{K+L} C_2(t)v^t + v^K\{N_K R' - S_1(K) + v^L[n_2 R'' - S_2(K+L)]\}, \tag{4.13}$$

where $C_c(t)$ is the sum of the maintenance and penalty costs per unit time in cycle c (c=1,2). Resale values are expressed as in the section 4.3.2. The costs $C_c(t)$, c=1, 2, can be expressed as

$$C_1(t) = \sum_{i=1}^{r}\sum_{j=1}^{n_i} M_i(\tau_{ij}+t) + P_1(t), \qquad (t = 1,\dots,K), \tag{4.14}$$

$$C_2(t) = n_K M'(t-K) + \sum_{i=2}^{r}\sum_{j=1}^{n_i} M_i(\tau_{ij}+t) + P_2(t), \qquad (t = K+1,\dots,K+L), \tag{4.15}$$

where $M_i(t)$ and $M'(t)$ are defined with the same way as in the section 4.3.1. The mean number of failures for both cycles 1 and 2, denoted $\Lambda_c(t)$, c=1,2, is expressed as follows

$$\Lambda_1(t) = \sum_{i=1}^{r}\sum_{j=1}^{n_i} \lambda_i(\tau_{ij}+t), \qquad (t = 1,\dots,K), \tag{4.16}$$

$$\Lambda_2(t) = N_K \lambda'(t-K) + \sum_{i=2}^{r}\sum_{j=1}^{n_i} \lambda_i(\tau_{ij}+t), \qquad (t = K+1,\dots,K+L). \tag{4.17}$$

where $\lambda'(t)$ and $\lambda_i(t)$ are defined as in section 4.3.1. The penalty cost of unavailability $P_1(t)$ and $P_2(t)$ in first and second cycle respectively are expressed as

$$P_1(t) = p'[X'' + \sum_{z=X'+1}^{N_1}(z - X')\Pr\{Z_1(t) = z\}], \tag{4.18}$$

$$P_2(t) = p'[X'' + \sum_{z=X'+1}^{N_2}(z - X')\Pr\{Z_2(t) = z\}], \tag{4.19}$$

where the size of the whole fleet becomes $N_2 = N_K + \sum_{i=2}^{r} n_i$, N_2 is the size of the whole fleet in the second cycle. The replacement strategy is presented in Figure 4.2.c.

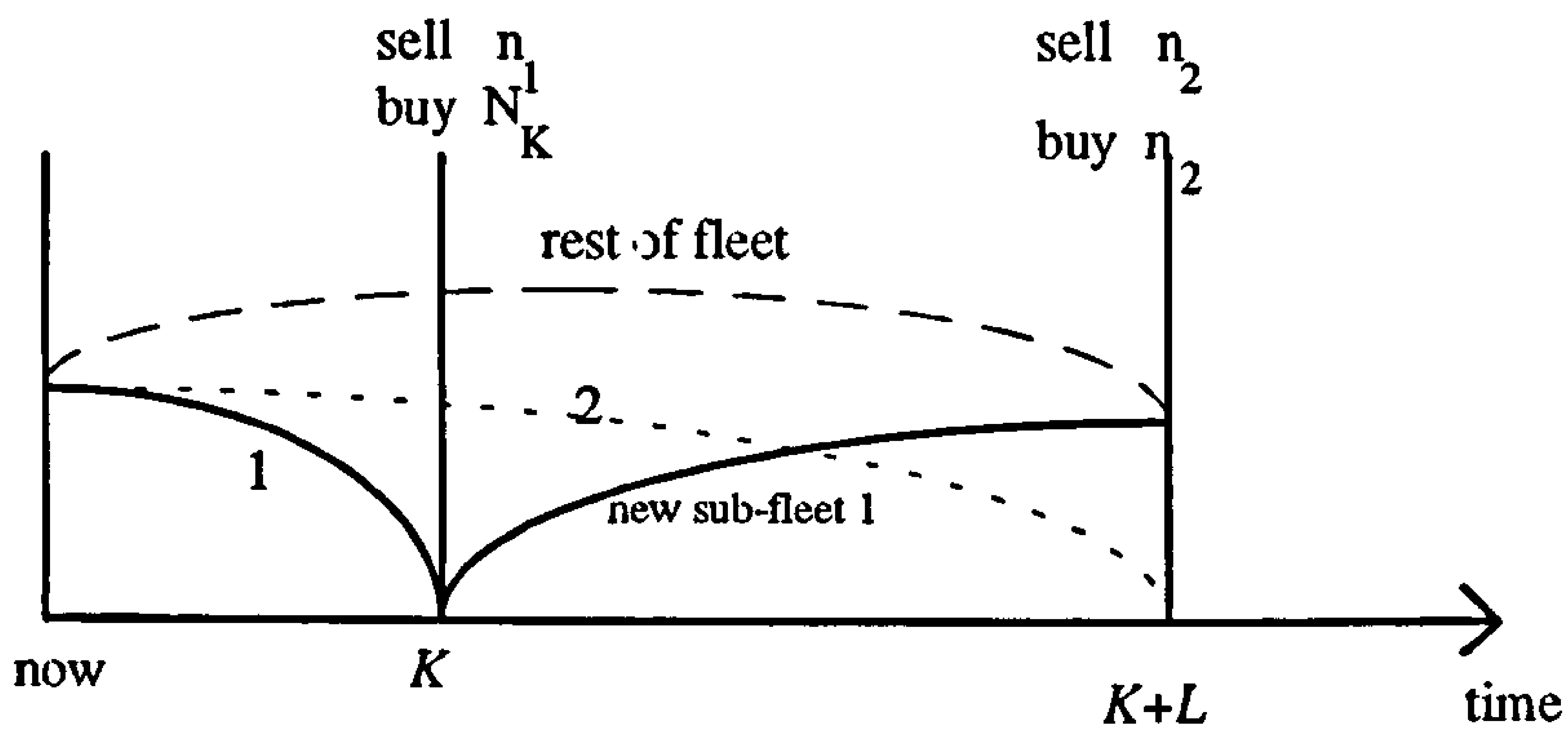


Figure 4.2.c. Two cycle sub-fleet replacement model, with variable fleet size at first replacement. —— represents respectively 'operate-and-sell' the first sub-fleet and 'buy-and operate' the replacement sub-fleet; .... represents 'operate-and-sell' sub-fleet 2 and 'buy' replacement sub-fleet; – – – –represents 'operate' the rest of the fleet.

Again the equivalent rent is expressed as

$$TDC_{IIb}(K, N_K, L) \Big/ \{\sum_{i=1}^{K+L} v^i\} \tag{4.20}$$

and the discounted cost per unit time

$$TDC_{IIb}(K, N_K, L) \Big/ (K + L) \tag{4.21}$$

The optimal values of decision variables are denoted K^*, L^* and N_K^*.

4.3.4 Two cycle model, variable fleet size for both cycle (model IIc)

A variable sub-fleet size at the second replacement, denoted N_L, might be considered. It however, does not influence the optimality of the replacement decision-making, but it does influence the values of the decision variables. This is due to the fact that the size of the second replacement sub-fleet, N_L, appears only at the replacement action and is not considered for a further operating cycle (see equation (4.22)). This means that, the model will suggest not to buy any item at the end of the second cycle, that is $N_L^* = 0$, which is not sensible in practice. In this case the total discounted cost is expressed as

$$TDC_{IIc}(K,N_K,L,N_L) = \sum_{t=1}^{K} C_1(t)v^t + \sum_{t=K+1}^{K+L} C_2(t)v^t$$
$$+v^K\{[N_K R' - S_1(K)] + v^L[N_L R'' - S_2(K+L)]\}, \quad (4.22)$$

where all the costs involved are defined in the same way as in section 4.3.3. The replacement strategy is presented in Figure 4.2.d

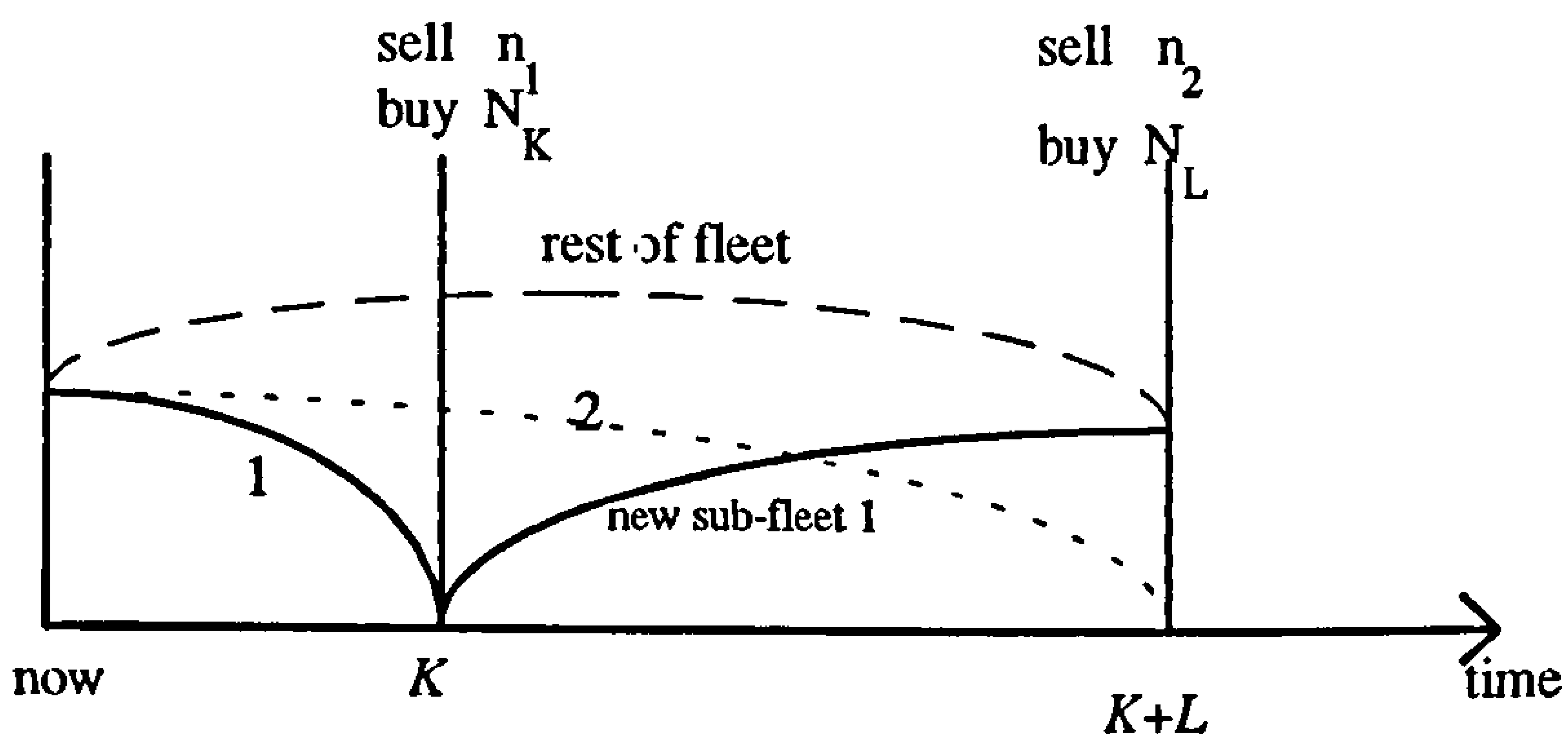


Figure 4.2.d. Two cycle sub-fleet replacement model, with variable fleet size for both cycles. ——— represents respectively 'operate-and-sell' the first sub-fleet and 'buy-and operate' the replacement sub-fleet; .... represents 'operate-and-sell sub-fleet 2 and 'buy' replacement sub-fleet; — ———represents 'operate' the rest of the fleet.

The objective function to minimise is either the equivalent rent which is expressed as

$$TDC_{IIc}(K,N_K,L,N_L) \Big/ \{\sum_{i=1}^{K+L} v^i\} \quad (4.23)$$

or the total discounted cost per unit time

$$TDC_{IIc}(K,N_K,L,N_L) \Big/ (K+L) \quad (4.24)$$

4.3.5 Two cycle model, variable fleet size for the first cycle, the second cycle is an 'operate' cycle only, with fixed fleet size (model IId)

The other way to use the two cycle model consists of considering only an 'operate-sell-and-buy' cycle for the first cycle, the second cycle is an 'operate' cycle only; that is no replacement is carried out at the end of the second cycle. It is a question of how much of the future cost the model takes account of, and the significance of end of horizon effect on the immediate replacement issue: when to replace the first sub-fleet. It should be noted that l, the length of the second cycle is not a decision variable but is fixed in advance. The value of l will influence the first cycle, therefore the operator should make a choice based on his experience or he might consider a range of values for l and then choose the most appropriate. The total discounted cost is expressed as follows:

$$TDC_{IId}(K,N_K)=\sum_{t=1}^{K} C_1(t)v^t + \sum_{t=K+1}^{K+l} C_2(t)v^t + v^K\{N_K R' - S_1(K)\}. \qquad (4.25)$$

where again costs are defined as in section 4.3.2. The optimal decision variables, K^* and N_K^* are obtained by minimising either the equivalent rent:

$$TDC_{IId}(K,N_K)\Big/\{\sum_{t=1}^{K+l} v^t\}, \qquad (4.26)$$

or the total discounted cost per unit time

$$TDC_{IId}(K,N_K)\Big/(K+l). \qquad (4.27)$$

The replacement strategy is presented in Figure 4.2.e.

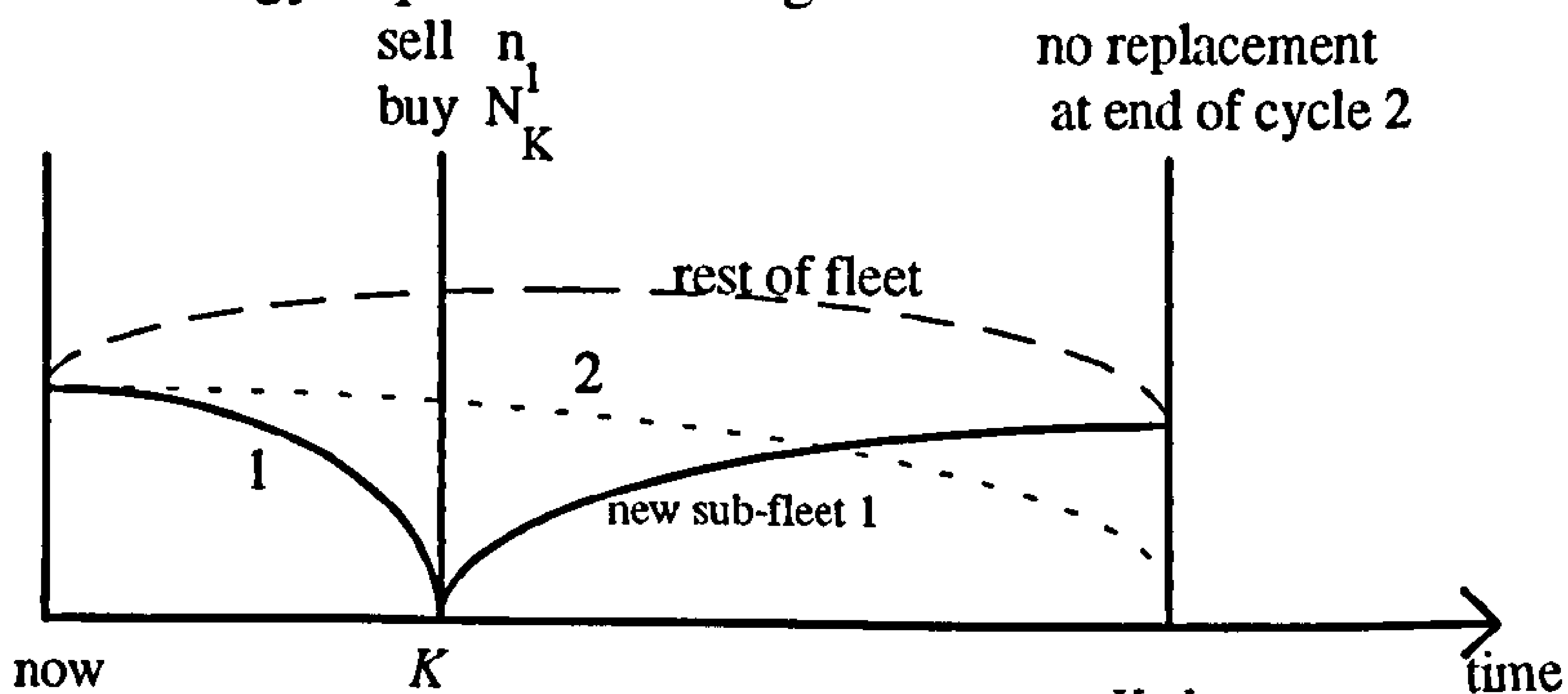


Figure 4.2.e. Two cycle sub-fleet replacement with no replacement at the end of the second cycle. Fixed length of the second cycle. ——— represents respectively 'operate-and-sell' the first sub-fleet and 'buy-and operate' replacement sub-fleet; .... represents 'operate' only sub-fleet 2; — — — — represents 'operate' rest of the fleet.

4.3.6 Three cycle model, variable fleet size, 'operate' only for the third cycle (model IIIa)

In order to consider a variable sub-fleet size for the second replacement sub-fleet, we have to introduce a third cycle of length m. This could be either an 'operate' only or an 'operate-buy-and-sell' cycle. We consider first, the case when the third cycle is an operating cycle only, that is no purchase or resale of equipment are performed at the end of it. The decision variables are the time of replacement K and L of the first and the second cycle respectively, as well as the size N_K and N_L of the replacement sub-fleet at the end of the first and the second cycle respectively. Here also the value of m will influence the second cycle, therefore the operator should make a choice based on his experience or he might consider a range of values for m and then choose the most appropriate. The total discounted cost can be expressed as follows

$$TDC_{IIIa}(K,N_K,L,N_L,m)=\sum_{t=1}^{K} C_1(t)v^t + \sum_{t=K+1}^{K+L} C_2(t)v^t + \sum_{t=K+L+1}^{K+L+m} C_3(t)v^t$$
$$+v^K\{[N_K R' - S_1(K)] + v^L[N_L R'' - S_2(K+L)]\} \quad (4.28)$$

Note that C_1, C_2, S_1, S_2, R' and R'' have already been defined in section 4.3.2. $C_3(t)$ is the maintenance cost per unit time in cycle 3 and is expressed as follows.

$$C_3(t) = N_L M''(t-K-L) + N_K M'(t-K) + \sum_{i=3}^{r}\sum_{j=1}^{n_i} M_i(\tau_{ij}+t) + P_3(t),$$
$$(t = K+L+1,\ldots,K+L+m), \quad (4.29)$$

where N_L is the size of the second replacement sub-fleet, $M''(t)$ is the maintenance cost for each equipment of the second replacement sub-fleet which is assumed identical for all items in the new sub-fleet. $P_3(t)$ is the penalty cost for unavailability in the third cycle and is expressed as

$$P_3(t) = p'[X'' + \sum_{z=X'+1}^{N_3}(z-X')\Pr\{Z_3(t)=z\}], \quad (4.30)$$

where the size of the whole fleet becomes $N_3 = N_K + N_L + \sum_{i=3}^{r} n_i$ and the mean number of failures for the whole fleet in cycle 3, denoted $\Lambda_3(t)$ is defined as

$$\Lambda_3(t) = N_L\lambda''(t-K-L)+N_K\lambda'(t-K)+\sum_{i=3}^{r}\sum_{j=1}^{n_i}\lambda_i(\tau_{ij}+t),$$

$$(t = K+L+1,....,K+L+m), \quad (4.31)$$

Here again $\lambda''(t)$ is the mean number of failures for each equipment of the second replacement sub-fleet which is assumed identical for all items in the new sub-fleet. The equivalent rent will be then expressed as

$$TDC_{IIIa}(K,N_K,L,N_L) \Big/ \{\sum_{t=1}^{K+L+m} v^t\}, \quad (4.32)$$

and the total discounted cost per unit time will be

$$TDC_{IIIa}(K,N_K,L,N_L) \Big/ (K+L+m) \quad (4.33)$$

The replacement strategy is presented in Figure 4.2.f

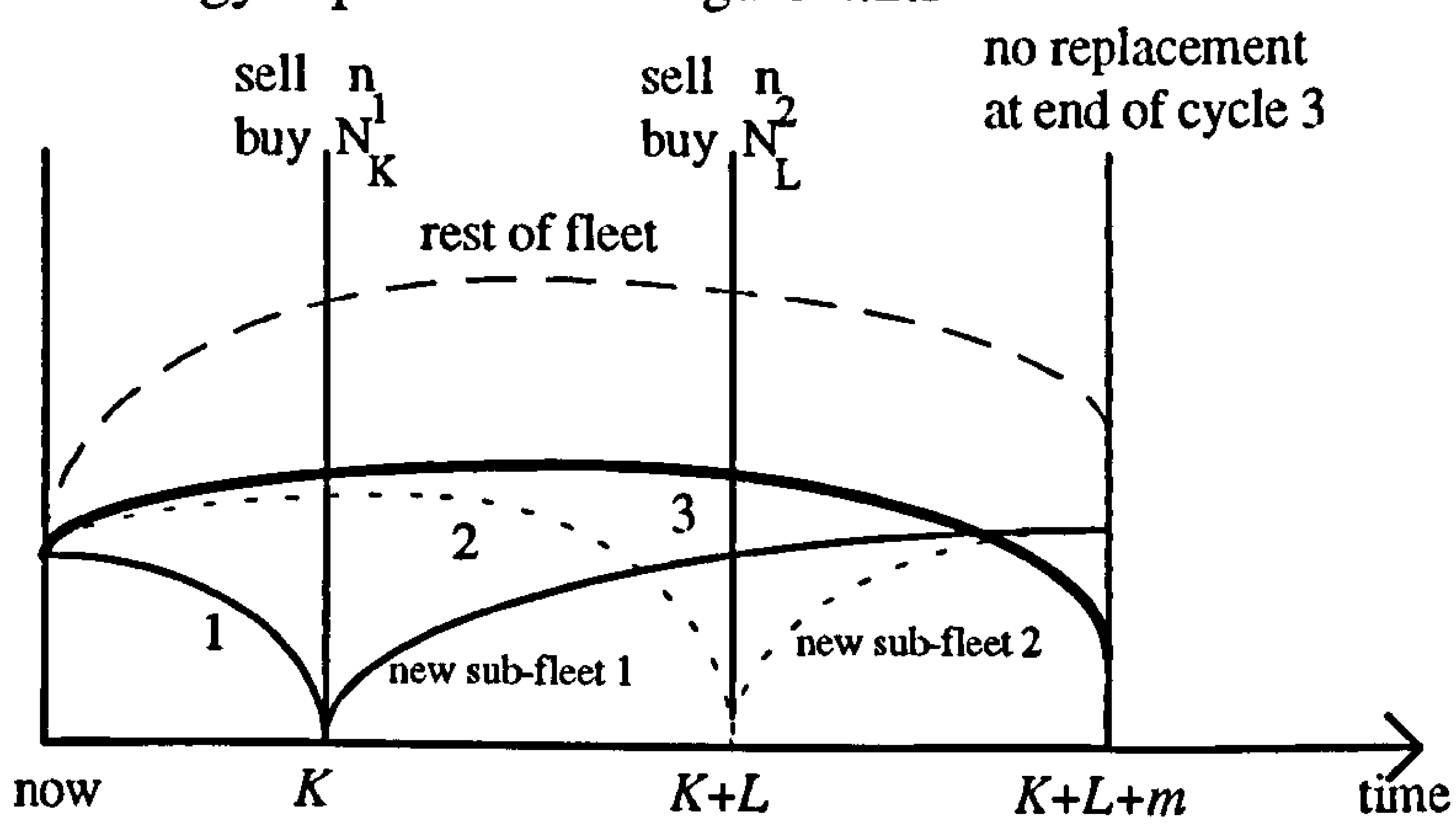


Figure 4.2.f. Three cycle sub-fleet replacement model with variable fleet size and fixed length of third cycle. ——— represents respectively 'operate-and-sell' sub-fleet 1 and 'buy-and operate' replacement sub-fleet; .... represents 'operate-and-sell' sub-fleet 2 and 'buy and-operate' replacement sub-fleet; ━━━ represents 'operate' sub-fleet 3; – – – – 'operate' rest of the fleet.

4.3.7 Three cycle model with variable length of the third cycle (model IIIb)

This model is an extension of the model III-a where we consider the length of the third cycle as a decision variable M. This is in addition to the time of replacement at the first and the second cycle as well as the size of the sub-fleet replacement at the end of the first and the second replacement, the third replacement sub-fleet is kept equal to its current value (see section 4.3.5). The introduction of the third cycle is only made to influence the second cycle. This, however increases the computational complexity.

The total discounted cost can be expressed as follows

$$TDC_{IIIb}(K,N_K,L,N_L,M)=\sum_{t=1}^{K}C_1(t)v^t+\sum_{t=K+1}^{K+L}C_2(t)v^t+\sum_{t=K+L+1}^{K+L+M}C_3(t)v^t$$
$$+v^K\{[N_KR'-S_1(K)]+v^L\{[N_LR''-S_2(K+L)]$$
$$+v^M[n_3R'''-S_3(K+L+M)]\}\}. \qquad (4.34)$$

Where the cost for the first and the second cycle, C_1, C_2, S_1, S_2, R' and R'' are defined in the fashion as in section 4.3.2. R''' is the current replacement cost of each equipment at the end of the third cycle. $S_3(\tau)$ and $C_3(t)$ are the resale and maintenance cost in the third cycle and are expressed as

$$S_3(K+L+M)=\sum_{j=1}^{n_3}s_3(\tau_{3_j}+K+L+M) \qquad (4.35)$$

$$C_3(t)=N_LM''(t-K-L)+N_KM'(t-K)+\sum_{i=3}^{r}\sum_{j=1}^{n_i}M_i(\tau_{ij}+t)+P_3(t),$$
$$(t=K+L+1,....,K+L+M), \qquad (4.36)$$

where $P_3(t)$ is defined as in equation (4.30). The equivalent rent will be then expressed as

$$TDC_{IIIb}(K,N_K,L,N_L,M)\Big/\{\sum_{t=1}^{K+L+M}v^t\}, \qquad (4.37)$$

and the total discounted cost per unit time will be

$$TDC_{IIIb}(K,N_K,L,N_L,M)\Big/(K+L+M) \qquad (4.38)$$

The replacement strategy is presented in Figure 4.2.*g*.

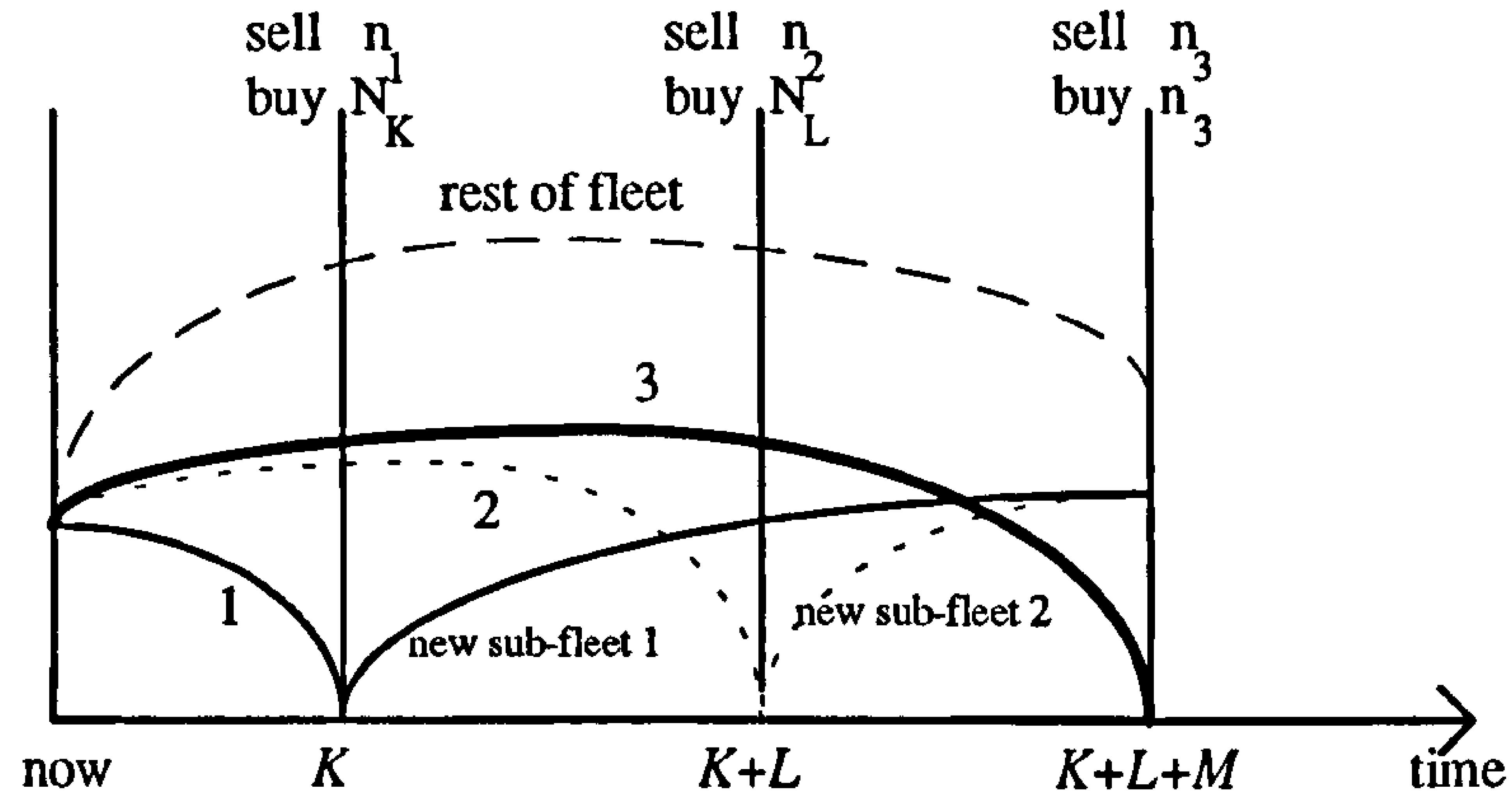


Figure 4.2.g. Three cycle sub-fleet replacement model with variable fleet size and variable length of the third cycle. ——— represents respectively 'operate-and-sell' sub-fleet 1 and 'buy-and operate' replacement sub-fleet; .... represents 'operate-and-sell' sub-fleet 2 and 'buy and-operate' replacement sub-fleet; ▬▬ represents 'operate-and sell' sub-fleet 3 and 'buy' replacement sub-fleet; – – – – 'operate' rest of the fleet.

4.3.8 Two sub-fleets model (model IV)

The model might easily be formulated when there are only two sub-fleets, this can then be expressed as follows using the two cycle model (model IIb) defined in equation (4.2).

$$TDC_{IV}(K,N_K,L)=\sum_{t=1}^{K}C_1(t)v^t+\sum_{t=K+1}^{K+L}C_2(t)v^t$$
$$+v^K\{[N_KR'-S_1(K)]+v^L[n_2R''-S_2(K+L)]\},\quad (4.39)$$

where the costs are defined as

$$C_1(t)=\sum_{j=1}^{n_1}M_1(\tau_{1j}+t)+\sum_{j=1}^{n_2}M_2(\tau_{2j}+t)+P_1(t)\qquad (t=1,...K),\qquad (4.40)$$

$$C_2(t)=N_KM'(t-K)+\sum_{j=1}^{n_2}M_2(\tau_{2j}+t)+P_2(t)\qquad t=K+1,...,K+L,\quad (4.41)$$

where $M_1(t)$, $M_2(t)$ and $M''(t)$ are the maintenance cost for each equipment in the first, the second and the replacement sub-fleet respectively. We assume that the items of the replacement sub-fleet have the same maintenance cost. $P_1(t)$ and $P_2(t)$ are the penalty cost of unavailability incurred in the first and the second cycle respectively and are defined as follows,

$$P_1(t)=p'[X''+\sum_{z=X'+1}^{n_1+n_2}(z-X')\Pr\{Z_1(t)=z\}],\qquad (4.42)$$

$$P_2(t) = p'[X'' + \sum_{z=X'+1}^{N_K+n_2} (z - X')\Pr\{Z_2(t) = z\}]. \quad (4.43)$$

When there is only one sub-fleet the problem is much easier, and then the approach described in chapter 3 might be considered (entire fleet replacement).

4.3.9 Fixed finite horizon model (model V)

The previous models consider a fixed number of cycles (2 or 3) within the horizon. An interesting problem to look at for future work will be the case when the length of the planning horizon is fixed. The length of the horizon is denoted H and the number of cycles which is denoted N becomes a decision variable as well as the length of each cycle K_i,, $i = 1,\ldots,N$. For convenience we introduce the notation $K_0 = 0$. The decision variables are constrained so that $H_i = \sum_{j=0}^{i} K_j$, $H = H_N = \sum_{0}^{N} K_j$. The size of the current sub-fleets is denoted n_i, $i = 1,\ldots,r$ and the replacement sub-fleet sizes are denoted n_{K_i}, $i = 1,\ldots,N$, with $n_{K_n} = n_N$. The difference between this approach and the models defined in the previous sections lies in the fact that the horizon is fixed but the number of cycle is allowed to vary, while in the other models the number of cycles is fixed but the length of the horizon varies. We denote $C_i(t)$ as the total cost in cycle *i*, R_j and S_j as the cost new of the replacement sub-fleet at the end of the *jth* cycle and the resale value of the *jth* sub-fleet, respectively. For the sake of simplicity, we assume that replacement of sub-fleet is made in chronological order, e.g. sub-fleet 1 replaced first, sub-fleet 2 replaced second, and so forth. The choice of *H*, when fixed in advance, should be made very carefully, because optimal values of the decision variables are highly correlated to it (de Sousa and Guimaraes, 1992, Scarf and Christer 1995). The replacement strategy is presented in Figure 4.2.h.

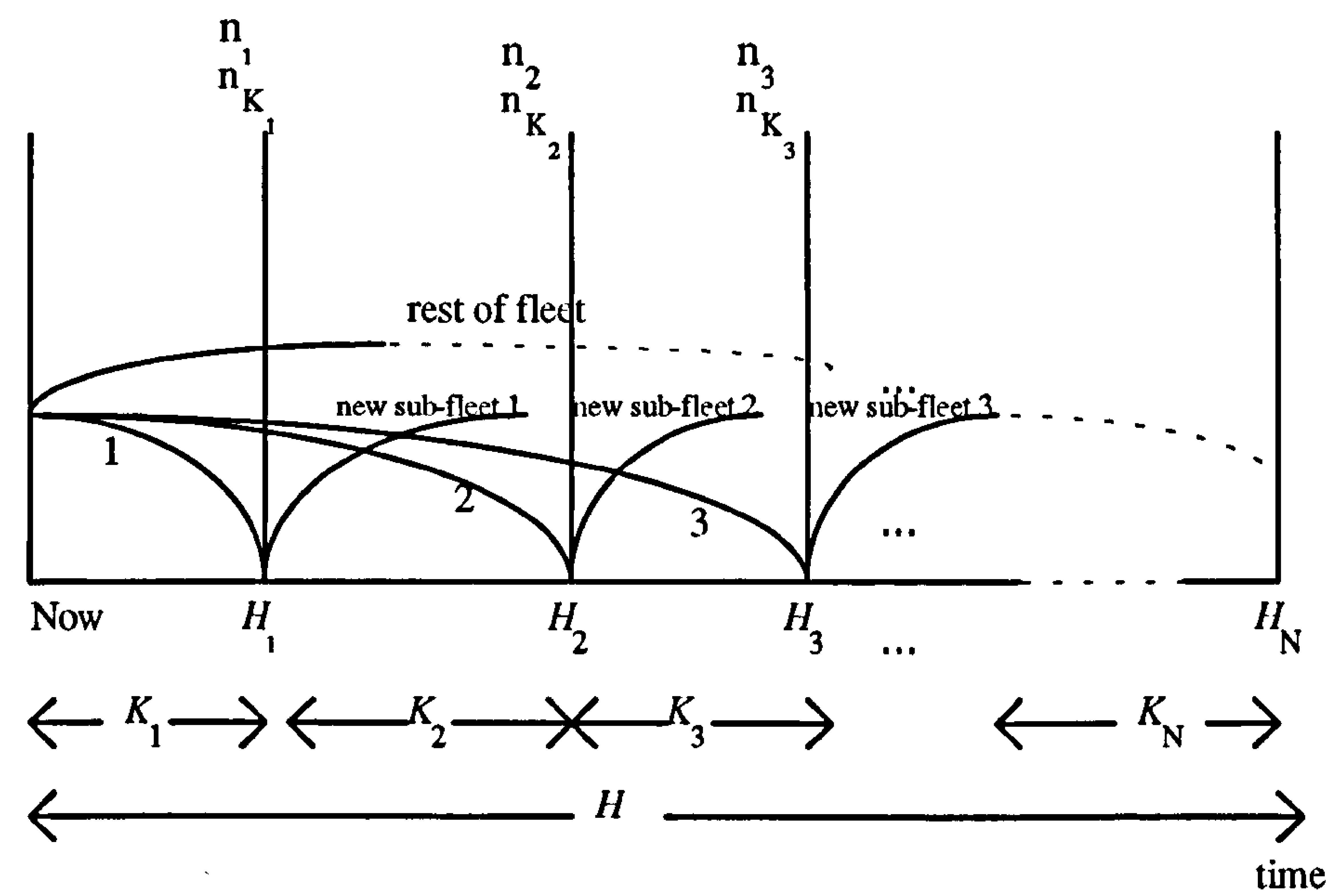


Figure 4.2.h. Fixed finite planning horizon for sub-fleet replacement.

This approach depicts the real world replacement situation, where an operator fixes an horizon, large enough, of length H and then is interested in how many replacement cycles N are needed to cover the whole horizon, how long is each replacement cycle K_i and how large should be the size of each replacement sub-fleet. This approach also minimises spurious end of horizon effect that variable length horizon models may produce. Although there are a large number of decision variables, we can however, express the objective function in similar way to previous sections. The total discounted cost can be expressed as

$$TDC_V(N,K_1,\dots,K_N,n_{K_1},\dots,n_{K_N}) = \sum_{i=1}^{N}\{ \sum_{t=H_{i-1}+1}^{H_i} C_i(t)v^t + v^{H_i}\{\sum_{j=1}^{i}[n_{K_j}R_j - n_jS_j(H_j)]\}\}, \tag{4.44}$$

where

$$C_1(t) = \sum_{j=i}^{r}\sum_{k=1}^{n_j} M_j(\tau_{jk}+t) + P_1(t), \quad t=1,\dots,K_1, \tag{4.45}$$

and for any cycle i, i=2,...,N

$$C_i(t)=\sum_{j=1}^{i-1} n_{K_j} M_j'(t-H_j)+\sum_{j=i}^{r}\sum_{k=1}^{n_j} M_j(\tau_{jk}+t)+P_i(t) \qquad t=H_{i-1}+1,\ldots,H_i, \quad (4.46)$$

where $M_i(\tau)$ and $M_j'(\tau)$ are respectively the maintenance cost of an equipment of age τ in cycle j and the maintenance cost of the new replacement equipment. We assume that the replacement sub-fleet is the same in each cycle, but this assumption can be relaxed without any difficulty. $P_j(t)$ is the penalty cost in cycle j. The mean number of failures per day in time interval t in cycle 1, is given by

$$\Lambda_1(t)=\sum_{j=1}^{r}\sum_{k=1}^{n_j}\lambda_1(\tau_{jk}+t) \qquad t=1,\ldots,K_1 \quad ,$$

and for any cycle i, i=2,...,N

$$\Lambda_i(t)=\sum_{j=1}^{i-1} n_{K_j}\lambda_j'(t-H_j)+\sum_{j=i}^{r}\sum_{k=1}^{n_j}\lambda_i(\tau_{jk}+t), \qquad t=H_{i-1}+1,\ldots,H_i$$

where $\lambda_j'(t)$ if the mean number of failures for the new replacement equipment in cycle j. The penalty cost in cycle 1 at time t is then expressed as

$$P_1(t)=p'\{X''+\sum_{z=X'+1}^{N_1}(z-X')\Pr[Z_1(t)=z)]\}, \qquad (4.47)$$

and for any cycle i, i=2,...,N the penalty is expressed as

$$P_i(t)=p'\{X''+\sum_{z=X'+1}^{N_i}(z-X')\Pr[Z_i(t)=z]\}, \qquad (4.48)$$

where the size of the whole fleet becomes $N_i=\sum_{j=1}^{i} N_{K_j}+\sum_{j=N-i}^{r} n_j$.

For this model, of course, we have made the same assumptions concerning the maintenance cost and the mean number of failures for the replacement sub-fleets as those made in the previous sections. In other words, we assume that each vehicle in the replacement sub-fleets for a given cycle has the same mean number of failures and the same maintenance cost per unit time. Again the rent or the total discounted cost per unit time may be minimised subject to constraint of course. Optimum policy would be considered for a range of values of H and a policy chosen which is not too sensitive to

H. We could also fix the fleet size, either throughout or at all but first replacement.

4.4 Usage consideration

Broadly speaking, usage is the measure of time that equipment is actually used in terms of serving customers. This measure depends on the nature of the equipment and might then be defined in various ways according to the type of equipment. In the case of, on one hand a fire extinguisher or a medical equipment in an emergency unit, usage will be the actual time because the availability of such equipment is vital. On the other hand, for equipment such as buses, which are in use only when sent for trips, usage in that case will be the cumulative number of bus-hours of trips. Christer and Scarf (1994) expresed usage in terms of cumulative number of machine-hours of dialysis. Note that all the objective functions we defined are appropriate when usage is assumed approximately constant within sub-fleets, that is assuming that usage is measured by time (age). In other words, equipment usage is constant for each item within the sub-fleet, that is

$$U_{1ij}(\tau) \propto 1 \Big/ \sum_{i=1}^{r} n_i,$$
$$U_{2ij}(\tau) \propto 1 \Big/ (\sum_{i=2}^{r} n_i + N_K),$$
$$U'(\tau) \propto 1 \Big/ (\sum_{i=2}^{r} n_i + N_K),$$

where $U_{1ij}(\tau)$ and $U_{2ij}(\tau)$ represent the usage function of the jth item, of age τ, in the sub-fleet i in the first and the second cycle respectively, and $U'(\tau)$ is the usage function of the replacement sub-fleet which, for simplicity, is assumed identical for each item in the replacement sub-fleet. In order to take into account the factor usage, given sufficient data of course, we need to establish the pattern of usage over time. Here it is important to look at whether individual items of a model of equipment are equally likely to be used. It is also important to check whether usage has changed over time with the age of the equipment. The level of usage is important as it can show whether there seems to be a shortage or excess of items (over or under utilisation of equipment). We need to compare usage to other parameters such as failure, maintenance cost to see if relationships exist and hence if some of the parameters can be described as functions of usage. In that case for example equation 4.21 becomes the total discounted cost per unit of usage:

$$\frac{TDC_{IIa}(K,N_K,L)}{\sum_{t=1}^{K}[\sum_{i=1}^{r}\sum_{j=1}^{n_i}U_{1ij}(\tau_{ij}+t)]+\sum_{t=1}^{L}[\sum_{i=2}^{r}\sum_{j=1}^{n_i}U_{2ij}(\tau_{ij}+K+t)+N_K U'(t)]} \qquad (4.49)$$

Thus provided sufficient data exists for this purpose, the model is able to cope with differing usage across the fleet. That is the maintenance and operating cost might be expressed by usage instead of age, which ultimately, is the most desirable way for modelling replacement. This, indeed, will remove the ambiguity which exists when replacing automatically an equipment given information based only on its age rather than on its usage. It often happens in practice, e.g. in the case of computers, that if a new generation of computer joins a fleet of relatively old computers, the users are more attracted to use the new ones instead of the existing equipment. This will lead to a higher usage for the new equipment, therefore more risk of breakdowns than the old equipment and therefore an increasing maintenance cost and ultimately an earlier replacement. The capital replacement models based on age become invalid in this case.

4.5 Retirement of sub-fleet as spares (model VI)

We might also consider the case of retirement of sub-fleets as occasional spares (or even the case when sub-fleets are retained and fully used). This can, in principle, also be modelled using the above approach. The number of sub-fleets would simply increase by one at each replacement, with the costs associated with the retired sub-fleet added. Predicting the mean number of failures and maintenance costs for a retired sub-fleet would be difficult however, and it is likely that no data would be available for this, because such items would be used only occasionally (period of peak demand), and then it would certainly not have the same mean number of failures or maintenance cost. We can however, simplify the model for this situation with the assumption that if the retired sub-fleet was retained for a further L units of time, it would have the same mean number of failures and maintenance cost as if it were in normal usage. The total discounted cost, using the two cycle model with variable fleet size (model IIb) can then be expressed as

$$TDC_{VI}(K,N_K,L)=\sum_{t=1}^{r} C_1(t)v^t + \sum_{t=K+1}^{K+L} C_2(t)v^t - S_1(K+L)$$

$$+v^K N_K R' + v^{K+L}\{n_2 R'\}, \quad (4.50)$$

where $C_1(t)$ and $C_2(t)$ the maintenance cost per unit time for the first and second cycle are defined as follows

$$C_1(t)=\sum_{i=1}^{r}\sum_{j=1}^{n_i} M_i(\tau_{ij}+t)+P_1(t) \qquad (t=1,\ldots,K), \qquad (4.51)$$

$$C_2(t)=N_K M'(t-K)+\sum_{i=1}^{r}\sum_{j=1}^{n_i} M_i(\tau_{ij}+t)+P_2(t), \qquad (t=K+1,\ldots,K+L). \qquad (4.52)$$

The penalty cost of unavailability incurred in the first and the second cycle, $P_1(t)$ and $P_2(t)$, respectively are defined as follows,

$$P_1(t)=p[X''+\sum_{z=X'+1}^{N_1}(z-X')\Pr\{Z_1(t)=z\}, \qquad (4.53)$$

$$P_2(t)=p[X''+\sum_{z=X'+1}^{N_2}(z-X')\Pr\{Z_2(t)=z\} \qquad (4.54)$$

where the size of the whole fleet becomes $N_2=N_K+\sum_{i=1}^{r} n_i$.

Note that the index i in the summation of the second equation (4.52) starts from 1, in other words, the sub-fleet 1 is also included in the second cycle. In this model we have assumed that, at the end of the first cycle, the sub-fleet 1 is kept for a further L units of time, and is then sold (scrapped). However, the purchase of a new replacement sub-fleet at the end of each cycle is made as usual. $S_1(.)$ is the resale value for sub-fleet

1, it is considered only at the end of the second cycle because the sub-fleet 1 has been retained for the second cycle. It can be defined as follows:

$$S_1(K+L)=\sum_{j=1}^{n_1} s_1(\tau_{1j}+K+L). \tag{4.55}$$

4.6 Cost of sub-optimal decisions

For various reasons, operators are sometimes constrained to rely on sub-optimal decisions, rather than optimal ones. This might occur when, say, budgeting constraints prevent the operator from carrying out replacement on due date (model's recommendation), leading to delayed replacement. We may also consider the case of alternative choice of replacement sub-fleet, which occurs sometimes for customer's preference, marketing or even political reasons. The case of considering a smaller replacement sub-fleet size rather than the optimal sub-fleet size, that is, say , $N_K < N_K^*$, is also investigated. This situation occurs mainly for reason of capital budgeting.

4.6.1 Delayed replacement

In the case of delayed replacement, the consequence of this postponement incurs an extra cost which needs to be quantified by the operator (Christer and Scarf, 1994). If the replacement at the end of the first cycle is delayed for, say, one unit of time from the optimal time of replacement K^*, that is a delay from K^* to K^*+1, the extra cost incurred for one unit time beyond the optimal value of the decision variables is expressed as follows: (using the total discounted cost per unit time criterion for two cycle model with variable fleet size at the first replacement, which we denoted model IIb in section 4.3.2)

$$TDC_{IIb}(K^*+1,N_K^*,L^*)-TDC_{IIb}(K^*,N_K^*,L^*). \tag{4.56}$$

This is because the average discounted cost per unit time under the optimal strategy is

$$TDC_{IIb}(K^*,N_K^*,L^*)\Big/_{(K^*+L^*)},$$

and so a measure of the marginal discounted cost of delaying the replacement decision by one unit of time is given by

$$[TDC_{IIb}(K^*+1,N_K^*,L^*)-TDC_{IIb}(K^*,N_K^*,L^*)]-\frac{TDC_{IIb}(K^*,N_K^*,L^*)}{K^*+L^*}. \qquad (4.57)$$

If the replacement is delayed for a further k time units beyond the optimal K^*, the marginal increased is expressed as

$$[TDC_{IIb}(K^*+k,N_K^*,L^*)-TDC_{IIb}(K^*,N_K^*,L^*)]-\frac{k\,TDC_{IIb}(K^*,N_K^*,L^*)}{K^*+L^*}. \qquad (4.58)$$

4.6.2 Alternative choice of sub-fleet to be replaced

If for some reason the operator did not comply with the model's recommendation for the sub-fleet to be replaced, he would then opt for a different choice of a sub-fleet to be replaced. This decision will, of course incur an extra cost which can be evaluated from the model. In the case of a fleet of buses which comprises r sub-fleets, denoted 1, 2,...,r, if the model recommends for example to replace first the sub-fleet 1 and then the sub-fleet 2, while the operator for some reason, say, comfort for passengers, decides to replace sub-fleet 2 and then sub-fleet 1, the extra cost incurred will be the difference of the cost incurred under the former optimal strategy (model) to the cost incurred under the latter optimal strategy (operator's choice). The total discounted cost over two cycles defined in equation (4.2) and the subsequent equations become

$$TDC_{alt}(K,N_K,L)=\sum_{t=1}^{K}C_1(t)v^t+\sum_{t=K+1}^{K+L}C_2(t)v^t$$
$$+v^K\{N_KR'-S_1(K)+v^L[n_1R''-S_2(K+L)]\}, \qquad (4.59)$$

The cost incurred when such a choice is made is obtained by

$$TDC_{alt}(K^{alt},N_K^{alt},L^{alt})/(K^{alt}+L^{alt})-TDC_{IIb}(K^*,N_K^*,L^*)/(K^*+L^*), \qquad (4.60)$$

where K^{alt}, N_K^{alt} and L^{alt} are the optimum decision variables under the second choice, that is replacing sub-fleet 2 first and then sub-fleet 1.

4.6.3 Smaller replacement sub-fleet size

We consider the case of smaller replacement sub-fleet size, when the size of the replacement sub-fleet is smaller than the optimal sub-fleet size recommended by the model, in other words, when $N_K < N_K^*$. This happens mainly for reason of budgeting constraints. In this case the operator may face problems of under-capacity, leading therefore to greater unmet demand. This may also imply an increasing maintenance cost. The overall extra cost can be quantified as follows. If the size of the replacement sub-fleet N_K, is reduced by, say, one unit with respect to the optimal replacement sub-fleet N_K^* we might determine the extra cost using, for instance, the total discounted cost criterion for model IIb, which is the two cycle model with variable fleet size at the first replacement, as

$$TDC_{IIb}(K^*,N_K^*-1,L^*)/(K^*+L^*)-TDC_{IIb}(K^*,N_K^*,L^*)/(K^*+L^*), \qquad (4.61)$$

4.7 Sensitivity analysis

It is useful to determine the change in optimal decision policy produced by a given change in input, for various factors such as maintenance costs, resale cost, discount factor or any other factor. This procedure which determines the sensitivity of the optimal decision to changes in each of the number of factors is known as informal sensitivity analysis (Helton, 1993). The purpose of sensitivity analysis is to determine which factors the optimal decision is most sensitive to. Sensitivity analysis should be performed to observe the effect of variation in costs on optimal decision policy. As we have said in section 4.2.2, maintenance cost is subject to uncertainty due to input data, therefore an uncertainty analysis on model prediction might be carried out beside a sensitivity analysis on parameters. Christer and Waller (1987a) used analysis of variance to investigate the degree to which variation in each of the input parameters affected the resulting output parameters. The influence in question concerned the age of replacement and a penalty measure for sub-optimal choice. Kobbacy and Nicol (1994) on their part use statistical simulation of economic parameters such as rate of inflation, interest rate, discount factor using a Markov chain approach. They also investigate changes on cost parameters such as maintenance, resale and purchase cost. The equipment under study is the same as the one studied by Christer and Waller (1987a), namely a fleet of trucks. The results of the simulation of Kobbacy and Nicol (1994) although different in magnitude, are in agreement with those obtained by Christer and Waller (1987a) who used analysis of variance.

4.7.1 Sensitivity to maintenance cost

Uncertainty in maintenance input data makes any fitted model subject to judgmental estimates of quantities about which we might be unsure. One can estimate these errors through statistical techniques such as estimation by confidence intervals, analysis of variance, tests of hypothesis, and so forth. Since the fitted function depends on parameters, it would be interesting, as a start, to carry out a sensitivity analysis by considering a range of different values for those parameters. This will enable us to evaluate the effect on optimal decision policy.

4.7.2 Sensitivity to discount factor

The uncertainty in inflation, interest or any other economic rate make the discount factor a key factor in any sensitivity analysis. As stated in Scarf (1994), discounting has little influence on the optimum decision. The same conclusion was drawn by Kobbacy and Nicol (1994). For our case, we want to see whether discounting influences the optimality of the replacement schedule. We might consider, for example, whether the optimal replacement strategy, which consists of replacing the sub-fleet 1 first and then sub-fleet 2, remains unchanged by varying the values of v.

4.7.3 Sensitivity to resale cost

In order to evaluate the influence of resale values on the optimal policy, we might conduct a sensitivity analysis by considering different models for the resale values. In the literature, exponentially (Scarf, 1994) or linearly depreciating (Lake and Muhlemann, 1979) costs are the most common models for resale values. We may also consider sensitivity to resale values by scaling resale values, that is to consider the resale value for different values of a multiplicative factor α, where α may take values like 0, 0.5, 1.5 or 2.

4.7.4 Sensitivity to purchase price

The cost of a replacement sub-fleet may increase in the future as a result of inflation, technological improvement or many other factors which control world economy. Therefore the effect of this increase needs to be investigated.

4.8 Discussion

The timeliness of our approach lies in the fact that we have modelled capital replacement policy for a fleet for which a particular sub-fleet interacts with other sub-fleets operating within the fleet. We have also addressed issues such as optimal fleet size as well as economic life. The important decision variables here are of course the choice of sub-fleet 1, for first replacement, the choice of new equipment model and time to first replacement. Other decision variables are more speculative, and exist purely to influence the first cycle. Modelling and hence the values of decision variables should be updated periodically.

We have described several replacement models based on either a variable or a fixed finite horizon. The number of cycles within the horizon is either fixed and consists of two or three cycles in the case of a variable horizon, or is variable in the case of a fixed planning horizon. An infinite horizon has not been considered here because of its non applicability in the context of a variable fleet size. We also discuss issues relating to the estimation and formulation of costs such as maintenance, penalty, resale and purchase costs which contribute to the problem.

We have also considered how we might carry out sensitivity analysis due to uncertainty in the input data for maintenance cost, the resale values (alternative models, e.g. exponential or linear model), the discount factor and replacement cost.

The penalty cost of unavailability has not been estimated through modelling, because of the subjectivity of its concept. We suggest, however, that modellers overcome this problem by considering a range of values of the penalty cost. This enables the operator to be informed to make his choice for replacement in knowledge of the financial consequences he might face.

Costs incurred, when sub-optimal decisions are chosen, such as delayed replacement, smaller replacement sub-fleet size and alternative choice of sub-fleet to be replaced have also been formulated. These would provide information to the operator about the financial consequences he may face in choosing alternative decision policies.

Note that the formulation as presented in section 4.2 allows the possibility for a sub-fleet to be composed of a single equipment. Of course the draw back of such apparent flexibility in the modelling is the complexity of the computational problem. In the model, the mixed fleet is assumed to contain at least three sub-fleets, but the model can cope with two or even one sub-fleet. Furthermore, in order to consider costs at present value, costs are discounted to the end of the time interval, but there it is a simple matter to discount to the beginning or even the mid-point of the time interval.

For penalty cost consideration, some assumptions related to demand, service time and repair time have been made in order to simplify the problem. We have assumed that demand, service time and repair time were constant. We should, however bear in mind that, in some circumstances, these assumptions might not be applicable. We do not consider these cases here, but they may be addressed in future work.

CHAPTER V

Application of Capital Replacement Models for a Mixed Fleet

5.1 Introduction

The state owned company Ekspres Nasional Berhad operates inter-city bus services in Malaysia, with a fixed number of scheduled trips each day which result in a demand of one hundred and twenty-four buses. The composition of the fleet was mixed, with, at the time of the study (late 1992) five models of varying ages as illustrated in table 5.1 and Figure 5.1. This diversity in models and partners is due to commercial and marketing reasons; Isuzu and Mitsubishi are made in Japan, Mercedes in Germany and Cummin in the U.S.A. The prices new (age zero) and resale values for these models are shown in Figure 5.4. Aggregate data on maintenance were obtained for each bus, for a period of nearly 4 years (1990-early 1993) in the second stage of our work. Note that the Isuzu CSA was made in Japan and cost M$750k new, the Mercedes made in Germany and cost M$230k new, the Cummin made in U.S.A and assembled in Thailand which cost M$500k new and finally the Mitsubishi made in Japan and cost M$800k new. The currency exchange rate at the time of the study was $M4.00=£1.00.

The model earmarked for the purchase for future replacements was the Isuzu CJR, is assembled in Malaysia and costing M$300k new. The choice of the new model for replacement took into account technological change, that is the replacement is not-like-with-like (Christer and Scarf, 1994). The Isuzu CJR has a cost when new which is relatively low in comparison with that of models in the other existing sub-fleets.

In this chapter we will investigate the problem of capital replacement for the fleet of buses described above. First we present an initial analysis in which we consider the first set of data on maintenance cost that we obtained from the company. These data are based on the annual average cost for each sub-fleet. Model fitting was carried out (Figure 5.2.b) and some results are presented. The final analysis is carried out using the full set of data which we obtained later from the company, which consists of the annual maintenance cost for each bus in each sub-fleet. All the results were obtained using the two cycle models denoted model IIa and IIb for fixed and variable replacement sub-fleet size respectively. These are described in chapter 4. Both cost criteria were considered, namely the cost of the equivalent rent and the total discounted cost per unit time.

Costs of sub-optimal decisions such as delayed replacement, smaller replacement sub-fleet size and alternative model choices for replacement sub-fleet are also investigated. Finally sensitivity analysis is carried out. The sensitivity analysis is concerned with all the cost factors including: the maintenance cost, the resale value; and parameters such as the discount factor v and demand.

We should emphasise the fact that the lengths of the cycles, K and L, are expressed in months. That is the unit time interval is one month. This is mainly for computation reasons, but this also gives optima which are precise enough for decision-makers. All the replacement models which we refer to are those described in chapter 4.

Table 5.1. Distribution of the buses per make and model of equipment (1992).

Make	Model	Number
Isuzu CSA	650 V10 101 401	37
Mitsubishi	MS 715	30
Cummin	LTA 10	16
Mercedes Benz	OF 1113 OF 1413 OF 1313	34
Isuzu CJR	580	8

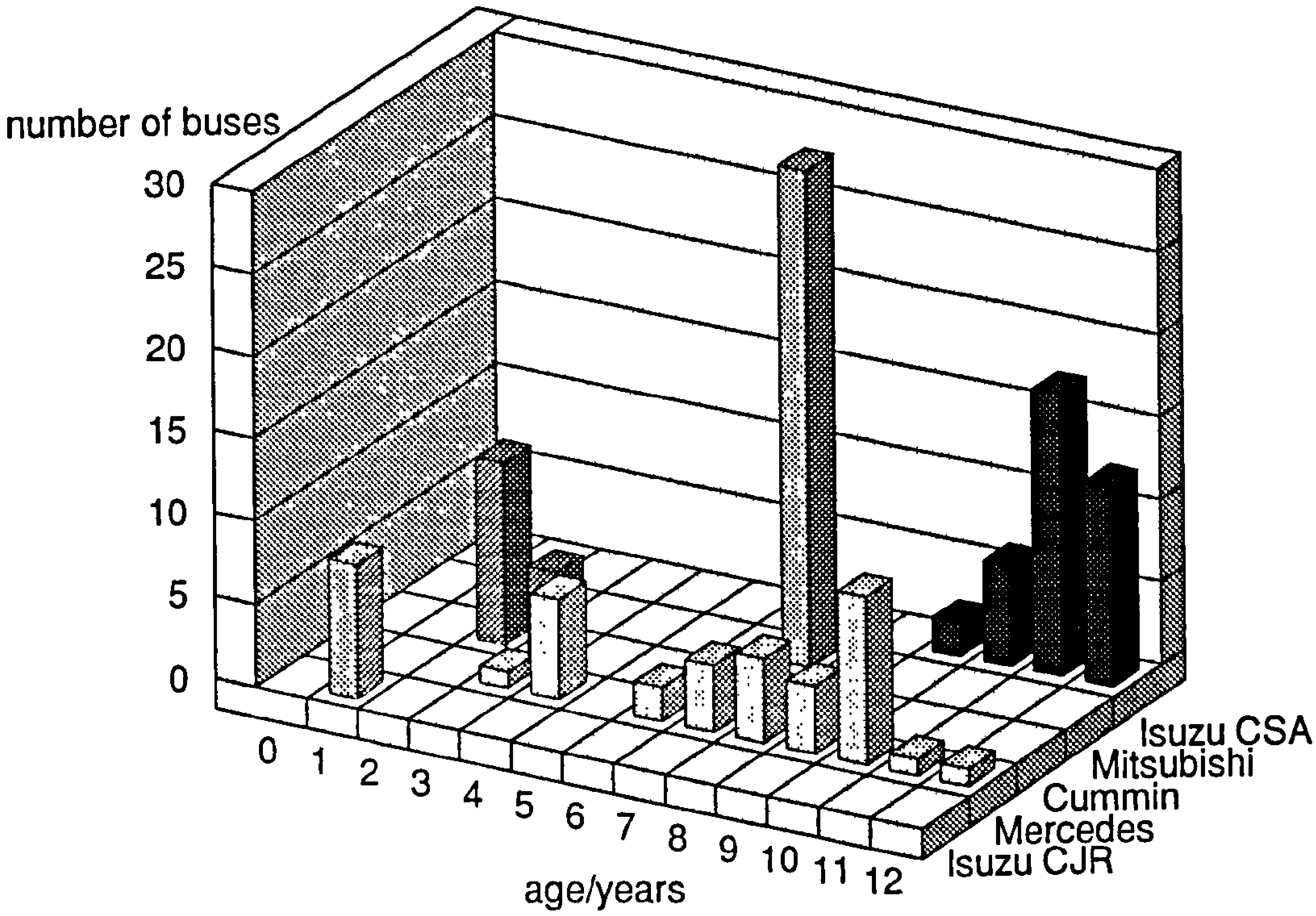


Figure 5.1. Composition of the fleet by model and age in 1992.

5.2 Preliminary considerations

5.2.1 Maintenance cost

In the first stage of this work, the data we used were essentially based on the overall annual average maintenance cost per bus by sub-fleet. The observed and fitted values of the annual average maintenance cost are illustrated in Figures 5.2.a and 5.2.b. The fitted model was assumed log-log-linear and is expressed as a power law function by

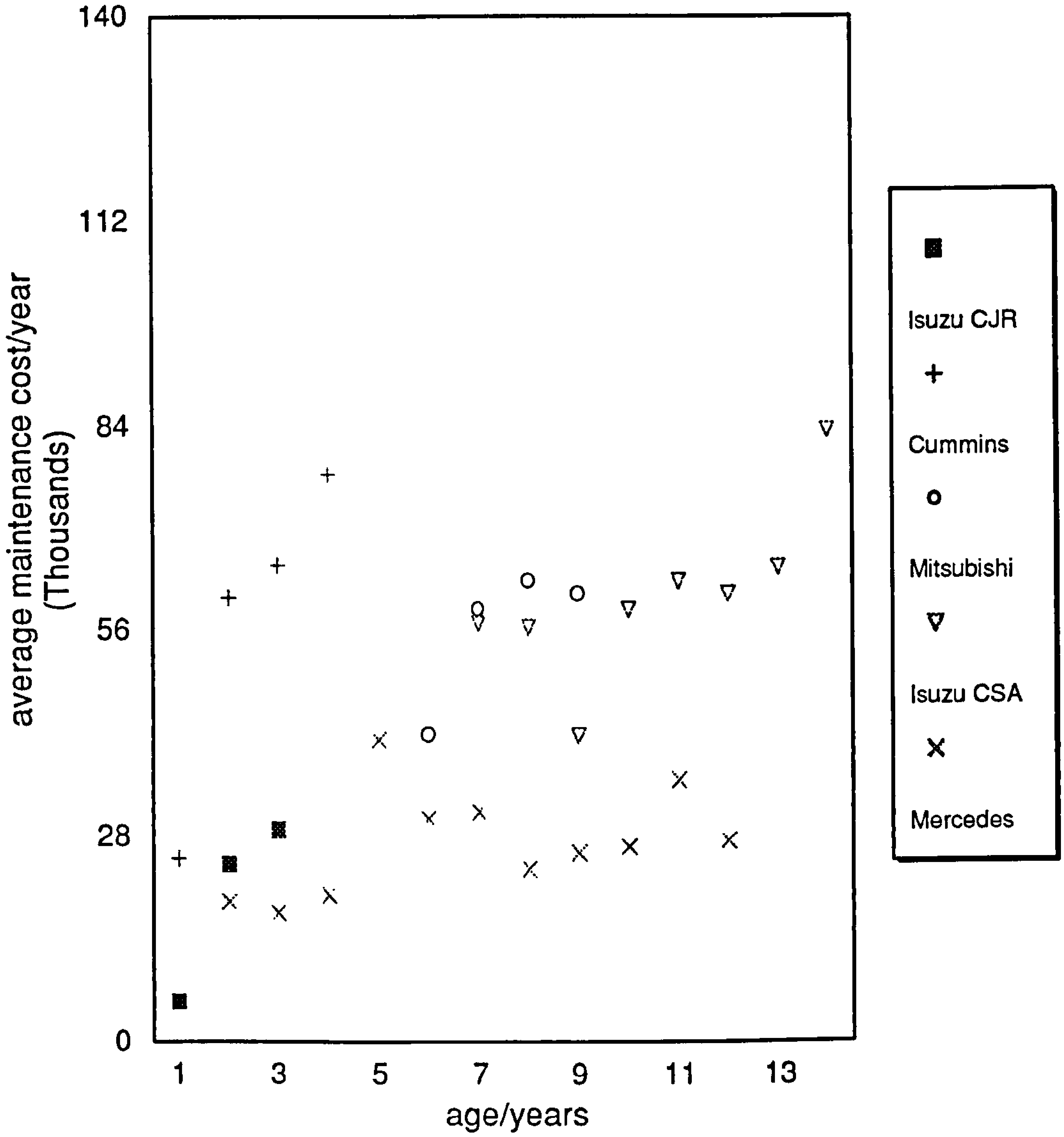
$$M(t)=\alpha t^{\beta},$$

or can be expressed linearly as

$$\log(M(t)) = \log(\alpha)+\beta\log(t) \qquad (5.1)$$

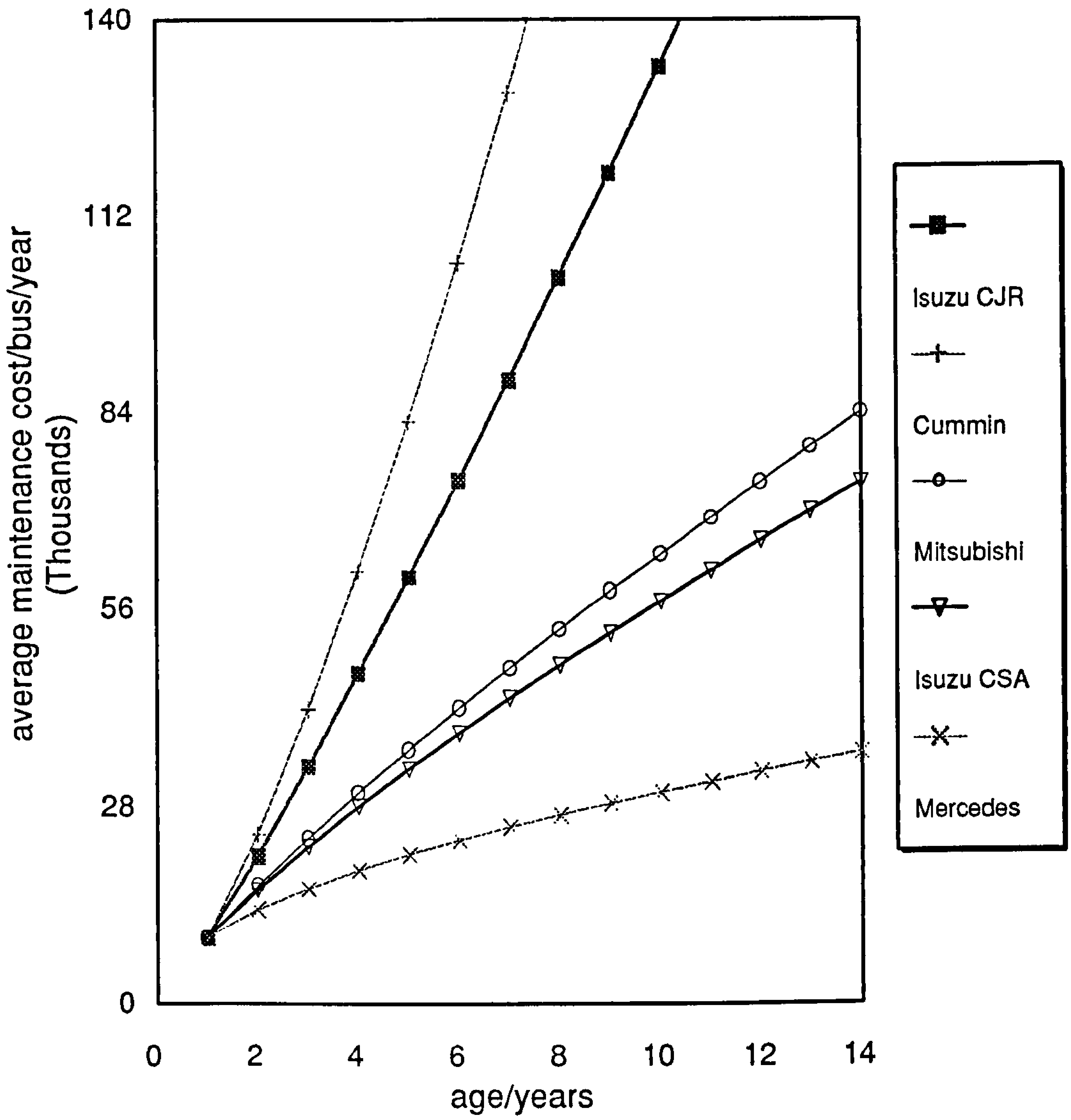
where α and β are estimated using the statistical package GLIM (McCullagh and Nelder, 1990). The parameter β is the slope (log-log scale) of the regression and represents the increase in the age-dependent maintenance cost per unit time. The coefficient α is the intercept (log-log scale) of the regression and represents the constant component of the age-dependent maintenance cost (cost in time unit 1). To model the maintenance cost we assume that the annual average maintenance cost for each sub-fleet is allocated to each bus in the corresponding sub-fleet (according to age). Thus we assume that every bus of the same age in a particular sub-fleet has the same (annual average) maintenance cost. A first attempt to determine the optimal policy is made using this set of data and results are presented in tables 5.2-5.4 and Figures 5.5.a-5.7.

The model with common intercept was the best fit and parameters estimates are given with Figure 5.2.b.



(a)

Figure 5.2.a. Observed annual average maintenance cost per sub-fleet per bus against age



(b)

Figure 5.2.b. Fitted maintenance cost per year for each bus in the fleet for each sub-fleet. Mercedes, $M(\tau) = 9681\tau^{0.50}$; Cummin, $M(\tau) = 9681\tau^{1.33}$; Isuzu CSA, $M(\tau) = 9681\tau^{0.77}$; Mitsubishi, $M(\tau) = 9681\tau^{0.82}$; Isuzu CJR, $M(\tau) = 9681\tau^{1.14}$

5.2.2 Penalty cost

Only data on breakdowns on the road were available, therefore we have considered only breakdowns occurring during a trip, that is for an on-service period. The penalty cost model developed in chapter 4 is still appropriate in these circumstances. The occurrence of a breakdown on the road, especially in the remote locations, can incur a high cost for the operator. In this situation if a spare bus was available from the nearest depot, it would replace the failed one otherwise the company should carry the passengers of the bus to their ultimate destination by either using another bus or even taxis. By taking into account and accepting the notion of penalty cost, the operator may reduce the risk of paying a high price when such events occur, either by operating newer sub-fleets, and/or by increasing the sub-fleet size. We have not considered breakdown off-service, that is when a bus has stopped at the end of the day after a trip for routine inspection (daily inspection policy within the company) or before departure because the data were not available. We have defined the penalty cost, in chapter 4 section 4.2.3, as the cost incurred for unavailability of a vehicle, or for unmet demand. The modelling of the penalty cost is not an easy task, because of the subjectivity of this cost factor (Christer and Scarf, 1994). We can however overcome this difficulty in considering a wide range of acceptable values of the penalty cost per breakdown for the operator. This will enable us to establish the influence of this parameter on the decision variables through sensitivity analysis. Christer and Scarf (1994) showed the strong influence of the penalty cost on the decision variables. The data were extracted from a survey based on the average number of breakdowns per month over a period of 4 years (1990-early 1993) (see appendix II). We have used a log-linear model for the number of failures using the statistical package GLIM (McCullagh and Nelder, 1990). This implies that in general the mean number of failures per month is given by $\lambda(t) = \exp(\delta + \gamma t)$, where τ is the age of the vehicle in months, δ and γ are estimated by GLIM. Figure 5.3 presents the fitted mean number of breakdowns.

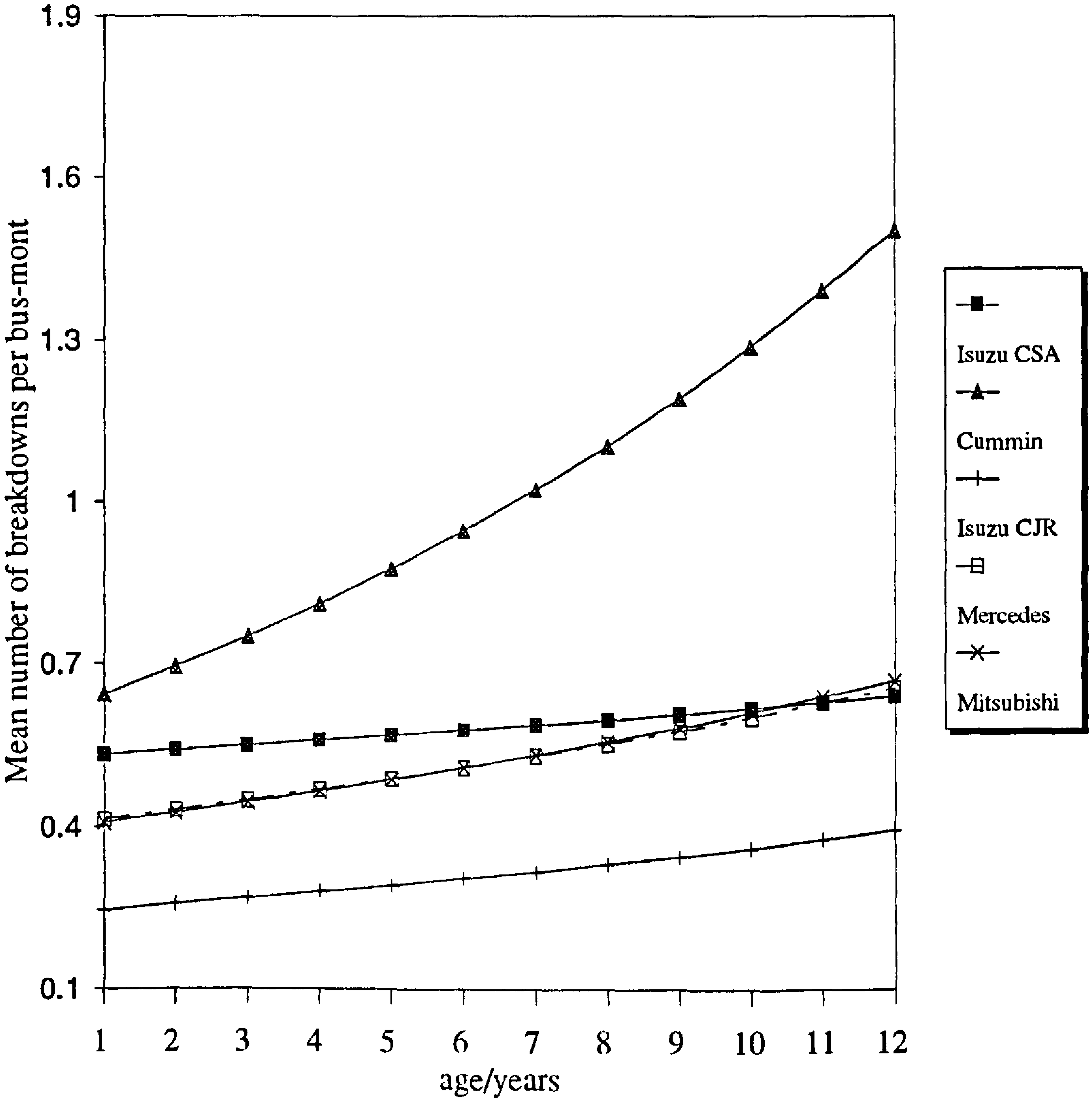


Figure 5.3. Fitted age-related mean number of breakdowns on the road for each sub-fleet. Mercedes, $\lambda(t) = \exp(-0.92 + 0.041t)$; Cummin, $\lambda(t) = \exp(-0.52 + 0.077t)$; Isuzu CSA, $\lambda(t) = \exp(-0.64 + 0.016t)$; Mitsubishi, $\lambda(t)= \exp(-0.94+0.045t)$; Isuzu CJR; $\lambda(t) = \exp(-1.44 + 0.042t)$.

Note that the penalty cost formulation (equation (4.2)) requires that $\lambda(\tau)$ be rescaled to the mean number of failures per day. This is not a log-linear process, but a log-linear model for the number of failures per month which is assumed to have a Poisson distribution with mean dependent on age. Note that no breakdown data were available for the Mitsubishi sub-fleet, so the average over different models was used.

5.2.3 Resale model

The resale value of an item of equipment depends on its age, mileage and condition and of course the state of the market of supply and demand. The latter point is not relevant in industrialised countries, but in some developing countries where demand is far higher than supply. Figure 5.4 emphasises this fact, indeed if we look at the Mitsubishi sub-fleet we can observe that the cost of a three year old Mitsubishi bus is almost the same as a new Mercedes bus. This is due to the high price of a new Mitsubishi ($M800k) and the relatively low price of a new Mercedes ($M230k). In the absence of historical data on resale values we choose the model of Christer and Waller (1987a) for resale values, which is expressed as

$$s(\tau) = R\sigma\theta^{\tau}.$$

This resale value model leads to estimates of σ and θ of 0.613 and 0.811, respectively (Figure 5.4). It is, of course not the only representation of the resale value model. Many other models might be considered, such as, a linear or an exponential model. A sensitivity analysis to resale values is carried out.

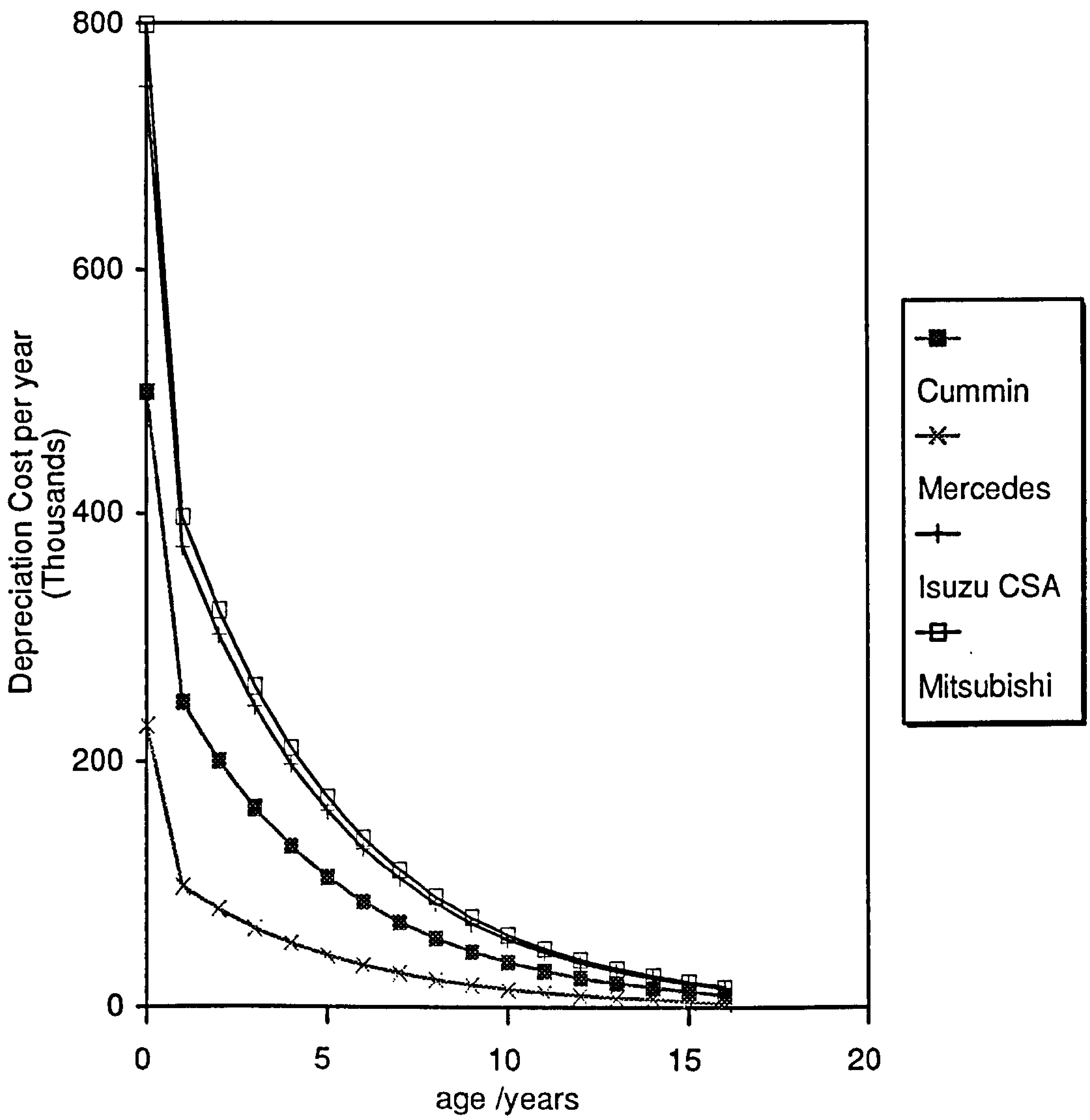


Figure 5.4. Prices new (age zero) and resale values for models comprising each sub-fleet, $s(\tau) = R \times 0.613 \times 0.811^{\tau}$

5.2.4 Demand model

We have considered demand as constant since the company operates a fixed number of trips per day. This demand might, however, increase for some reason, say, an increase in the number of passengers or the opening of new routes. We will consider these changes in demand by simply considering a range of values for the demand, and then carrying out sensitivity analysis. The case of random demand is not considered in this thesis, but it could be considered in future work.

5.3 Initial Analysis.

In this section we consider some results obtained from the first set of data we obtained, which consists of the overall annual average maintenance cost per bus by sub-fleet (Figure 5.2.a). This is inserted to illustrate how optimal policy can change as the fleet develops and more data become available. These results were published in the paper by Scarf and Bouamra (1995). The fitted values are illustrated in Figure 5.2.b. We can observe that the graphs show widely different maintenance costs for the different sub-fleets. We carried out our replacement policy using the equivalent rent criterion, using the two cycle models, model IIa and IIb, for fixed and variable fleet size respectively. The full analysis is considered in the next section.

The first set of results is illustrated in tables 5.2 and 5.3, related to model IIa. Table 5.3 shows that the optimum policy, that is the policy which incurs the minimum cost of the equivalent rent, is to replace the Cummin first and then the Mitsubishi second, which we denote as policy I. However, the company was at the time considering the replacement of the Mercedes sub-fleet first and the Cummin second. This is denoted as policy II, and appears from table 5.3 to be a much more expensive option. If the Mercedes sub-fleet is to be replaced in the short-term then the Cummin-Mercedes schedule is near-optimal. We can for instance evaluate the saving which could be made by choosing policy I instead of policy II. We can see that if policy I is chosen the minimum cost is $M697k over a period of 55 months (K^*=7, L^*=48), whereas for policy II, the minimum cost is $M806k over a period of 65 months (K^*=47, L^*=18). The extra cost incurred for the company by choosing policy II is around $M110k for a period of approximately 5 years.

Table 5.4 illustrates results from model IIb. The optimal decision stays unchanged, that is to replace the Cummin sub-fleet first and then the Mitsubishi. Figures 5.5.a-5.5.c illustrate the total cost of the equivalent rent against time of first replacement for various lengths of the second cycle for the policy Mercedes first Cummin second, Cummin first Mercedes second and Mitsubishi first Cummin second respectively. Figure 5.6 illustrates the cost of the equivalent rent against time of first replacement for various choice of sub-fleet to be sold at the first and the second replacement. The lowest curve represents the policy, with minimum cost.

Table 5.2. Cost per month (for whole fleet) of the minimum equivalent rent and optimum values of decisions variables for various choices of sub-fleets to be sold at first and second replacement (two cycle model, fixed fleet size, model IIa).

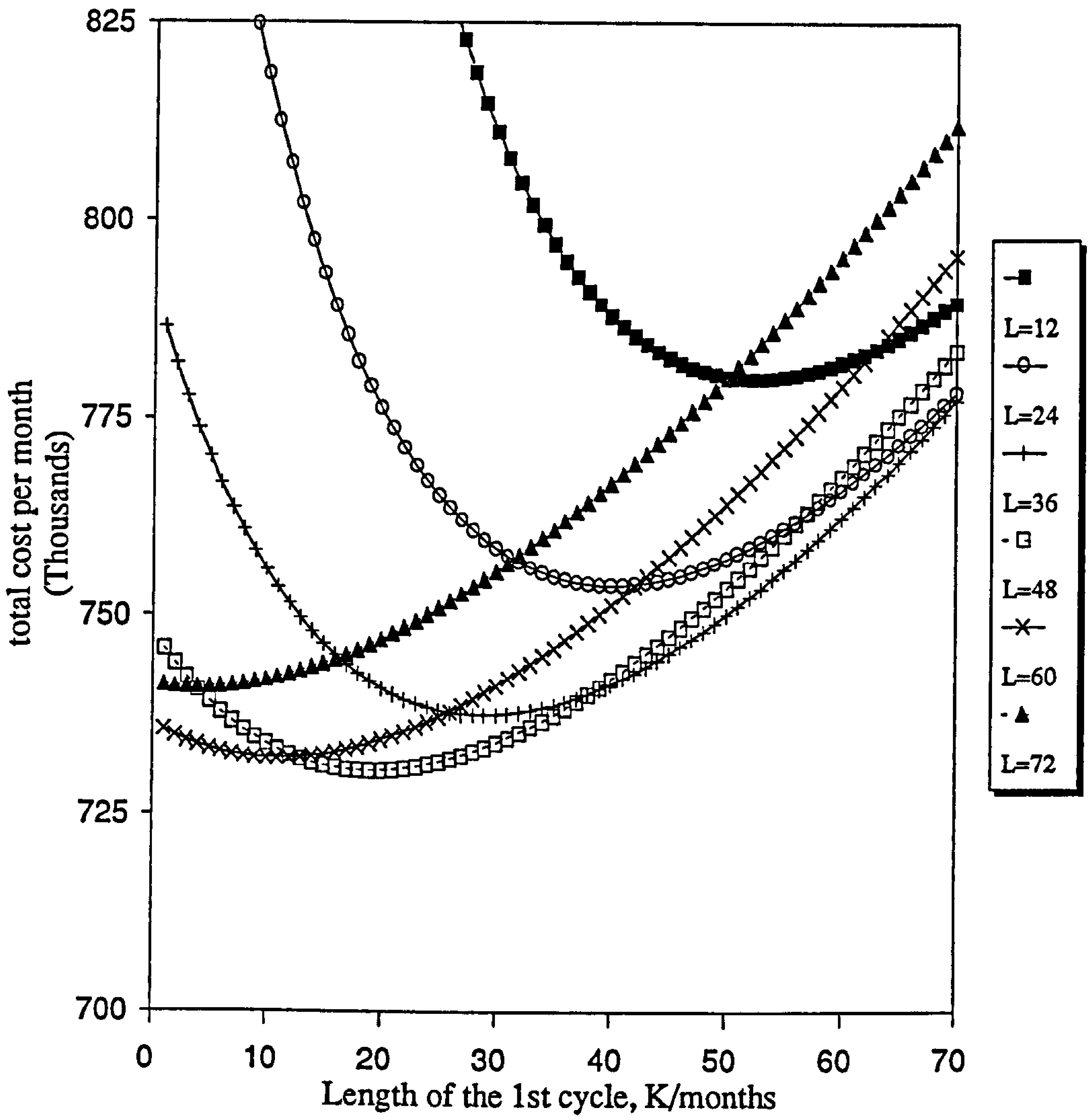
Replacement schedule 1st repl. - 2nd repl.	K* months	L* months	Costs (M$000)
C - Me	**16**	**52**	**730**
C - Mit	**7**	**48**	**697**
C - Is	18	52	738
Me - C	47	18	806
Me - Mit	62	12	856
Me - Is	67	15	886
Mit - C	**2**	**36**	**705**
Mit - Is	17	48	818
Mit - Me	16	48	809
Is - C	2	50	734
Is - Mit	15	48	792
Is - Me	23	49	821

Table 5.3. Minimum cost per month of the equivalent rent for various choices of sub-fleets to be sold at first and second replacement. No penalty cost incurred, discount factor $v = 0.97$. (two cycle model, fixed fleet size, model IIa)

2nd repl. / 1st repl.	C	Me	Mit	Is
C	*	**730**	**697**	738
Me	806	*	856	886
Mit	**705**	809	*	818
Is	734	821	792	*

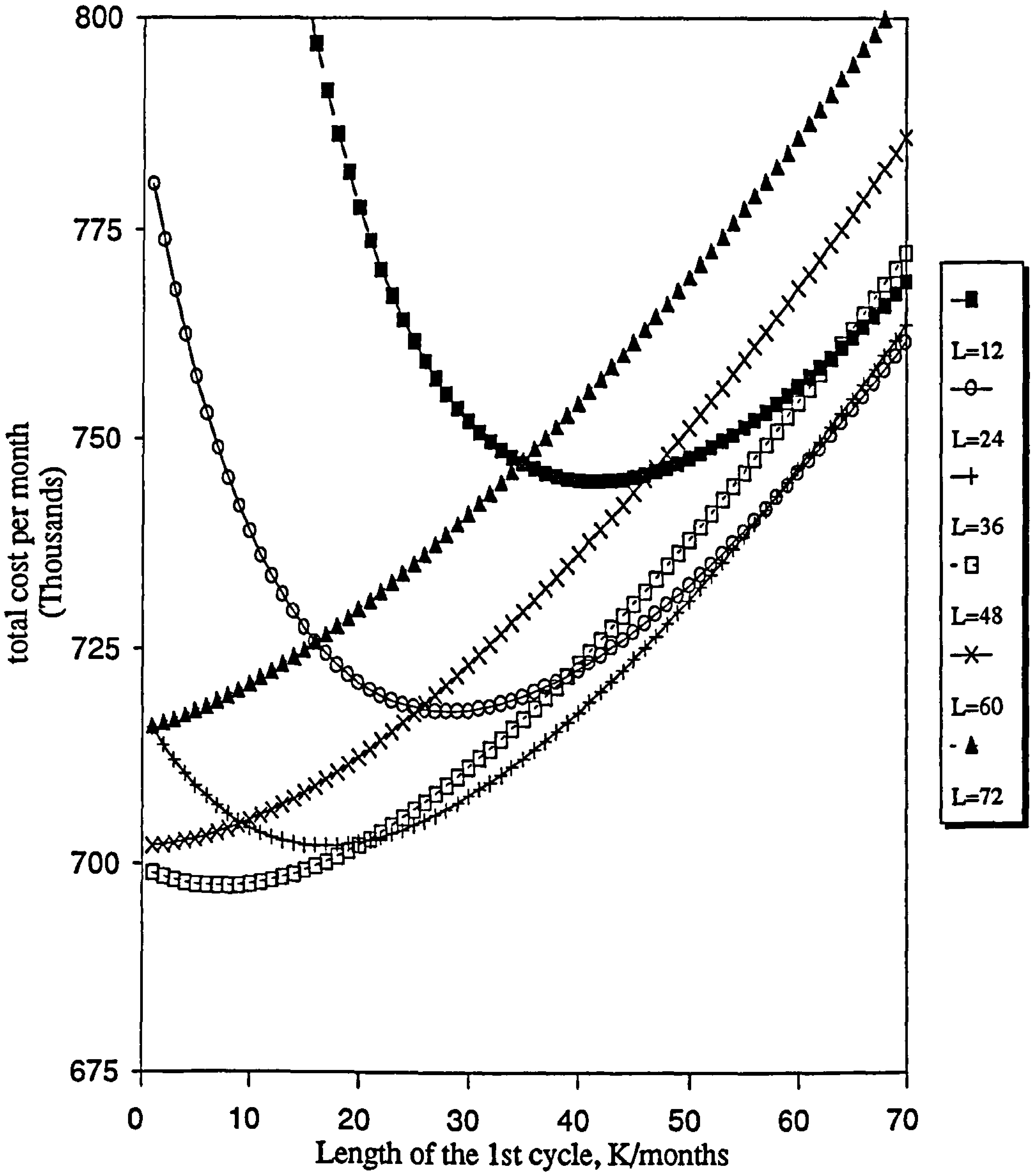
Table 5.4. Minimum cost per month of the equivalent rent for various choices of sub-fleets to be sold at first and second replacement. Penalty cost of unavailability p'=$M1000 (≈£250), discount factor $v = 0.97$. (two cycle model, variable fleet size, model IIb)

2nd repl. / 1st repl.	C	Me	Mit	Is
C	*	**756**	**722**	765
Me	855	*	897	928
Mit	**730**	838	*	846
Is	**756**	848	792	*



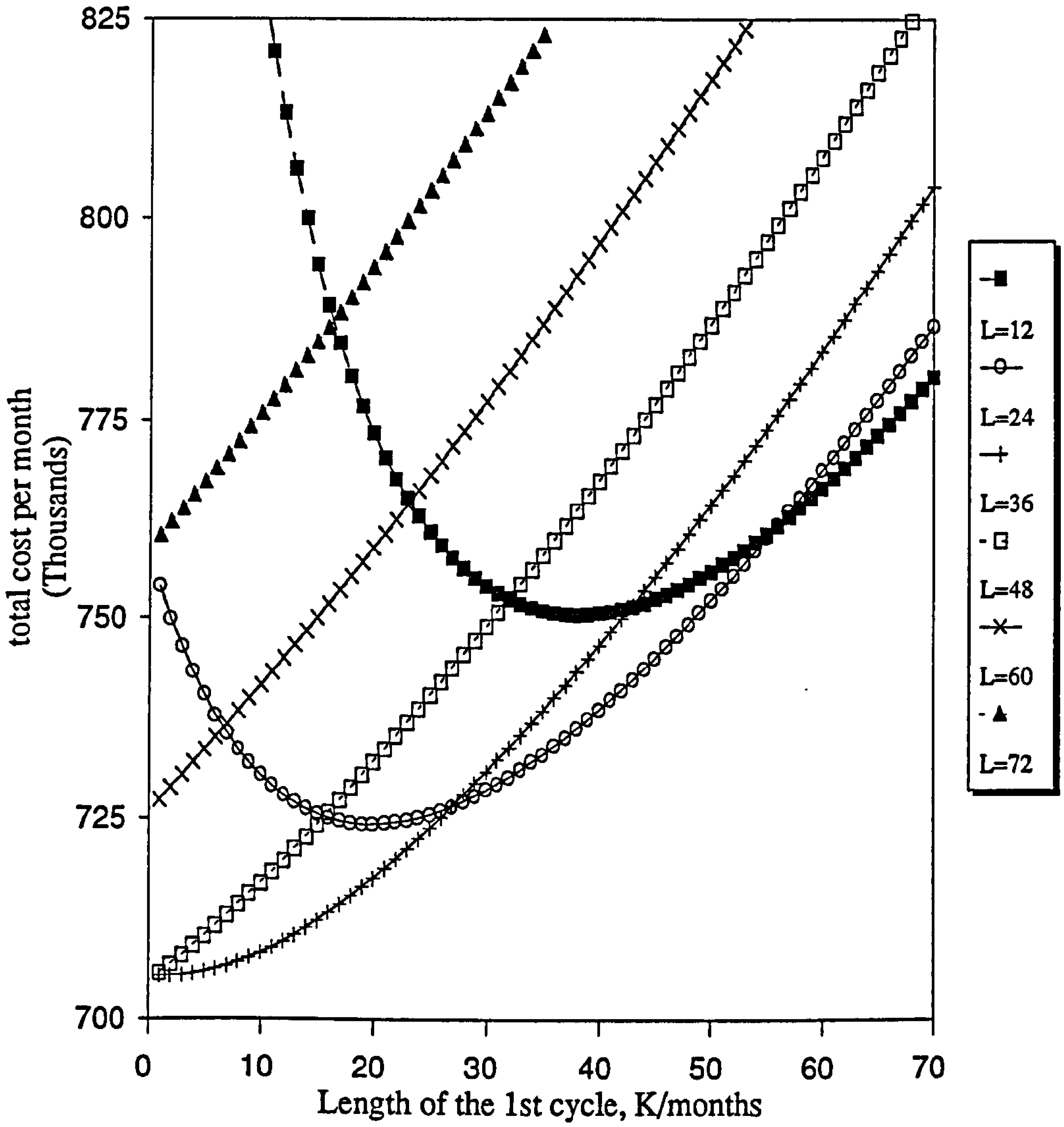
(a)

Figure 5.5.a. Cost of equivalent rent per month (for whole fleet) Vs. length of first cycle K, for various lengths of second cycle L, Cummin replaced first, Mercedes second. Two cycle model with fixed fleet size, no penalty inccured, (model IIa).



(b)

Figure 5.5.b. Cost of equivalent rent per month (for whole fleet) Vs. length of first cycle K, for various lengths of second cycle L, Cummin replaced first, Mitsubishi second. Two cycle model with fixed fleet size, no penalty inccured, (model IIa).



(c)

Figure 5.5.c. Cost of equivalent rent per month (for whole fleet) Vs. length of first cycle K, for various lengths of second cycle L, Mitsubishi replaced first, Cummin second. Two cycle model with fixed fleet size, no penalty inccured, (model IIa).

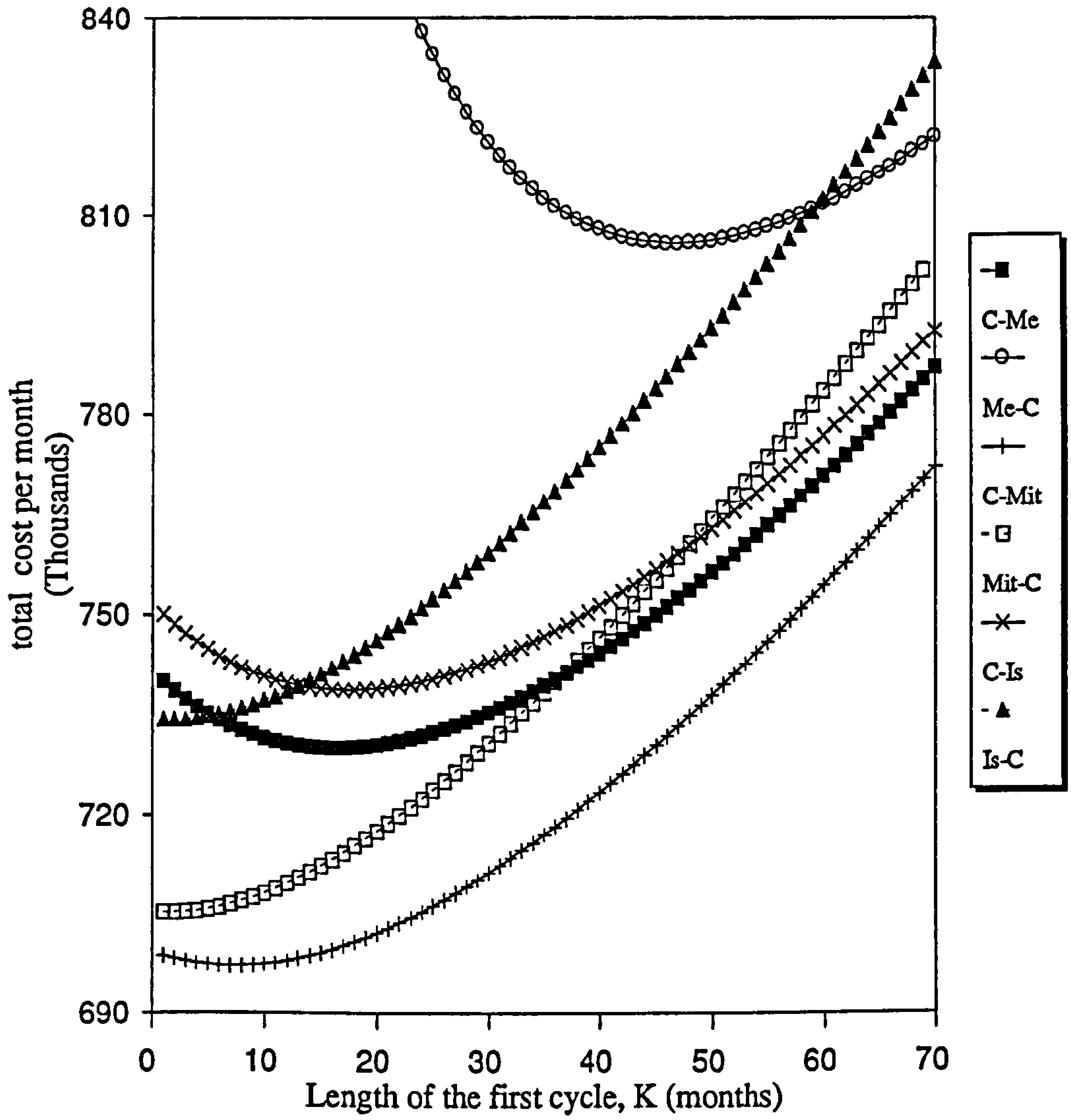


Figure 5.6. Cost of equivalent rent per month (for whole fleet) Vs. length of first cycle K, for various choices of sub-fleets to be sold at the first and second replacement. C-Me:Cummin replaced first, Mercedes second, etc. (Isuzu CJR purchased, L=L*). Two cycle model with fixed fleet size, model IIa.

5.4 Final Analysis

5.4.1 Introduction

In this section we are using the data for maintenance cost which is illustrated in Figures 5.8.a and 5.8.b. With the arrival of new and updated data on maintenance cost for each bus in the fleet, we are now in a position to carry out a final analysis (see appendix III). Meanwhile, we can compare the results on optimal policy with those obtained previously with the initial analysis and then draw some conclusions. The main concern of the company is to operate a policy of buying and selling equal numbers of buses, thus maintaining a fixed fleet size. In this case, the question of interest is which sub-fleet to replace first, what model to purchase and over what time scale should the replacement take place. The company wished to replace the Mercedes sub-fleet, due to the lack of comfort of these buses (seats, air conditioning). Consideration of the maintenance costs suggested to them that the Cummins sub-fleet, although relatively new, was also a candidate for replacement. The Isuzu CJR model was purchased at each replacement. Note that the new Isuzu CJR sub-fleet was not considered as candidate for replacement. We will, however go beyond the company's concern by investigating also the case of variable fleet size.

Optimal replacement policy for this fleet of buses is determined using the two cycle models, denoted model IIa and IIb in chapter 4 section 4.2.3. For this purpose a program written in FORTRAN 77 enables us to find numerical solutions (see appendix IV). Figure 5.7 presents the replacement strategy for the two cycle model, denoted IIb, on which we based all our results related to the variable fleet size. In the case of fixed fleet size, the figure is identical except that the value of the size of the first sub-fleet replacement is n_1 instead of N_K. The total size of the current fleet is one hundred and twenty five (125), we have considered that the value of demand is fixed and equals one hundred and twenty four (124), that is the fleet operates with one spare bus. We will however carry out a sensitivity analysis to demand by varying its value over a range of values.

Figure 5.7 gives an overview of the replacement strategy which is carried out over two cycles.

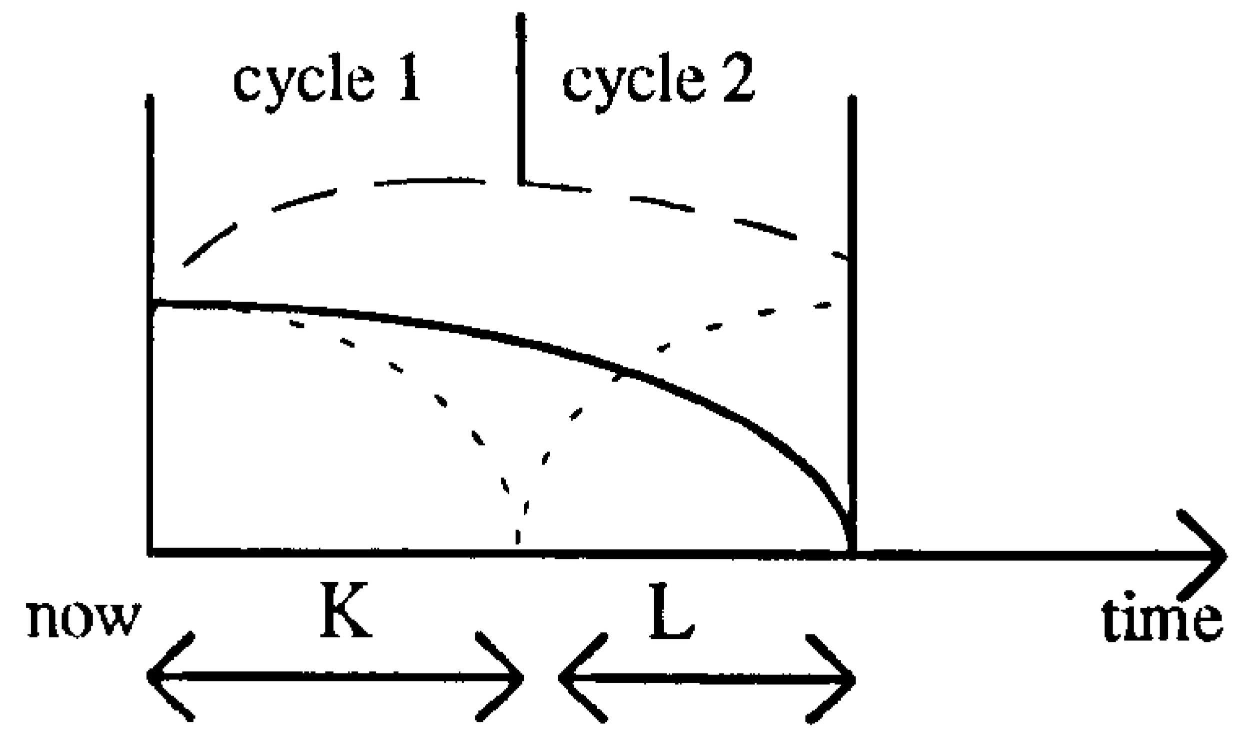


Figure 5.7. An example of replacement strategy using the two cycle model: —— represents respectively 'operate-and-sell' the first sub-fleet and 'buy-and-operate' the replacement sub-fleet; -------represents 'operate-and-sell' sub-fleet 2; — — — —represents 'operate' the rest of the fleet.

Thus, as new data became available, an updated maintenance cost model was considered. The penalty cost, resale value and demand models are as described in section 5.2.

5.4.2 Updated maintenance cost model

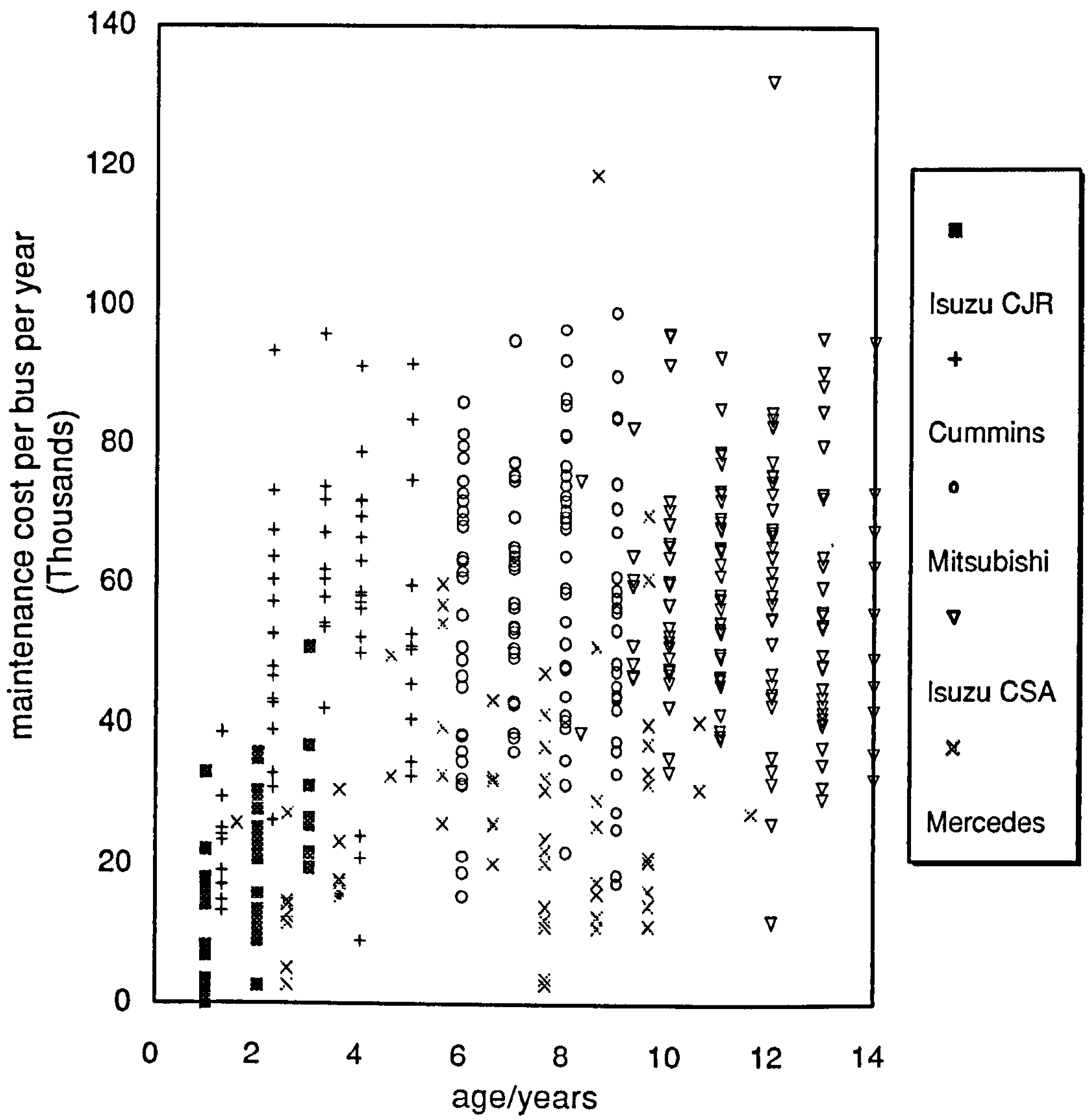
The maintenance costs which have been made available to us by the operator are based on cost of parts, labour, lubricant and tyres for each vehicle cumulated over the year for a period of two to four years. The cost of fuel, although very important for bus companies, has not been incorporated into the maintenance cost because it was not available. It therefore has to be considered as a fixed cost as the road tax and the insurance. We summarise the period which is covered by the data for each sub-fleet in table 5.5.

Table 5.5. Period over which data were observed.

sub-fleet model	period covered	number of years
Cummin	1990: up to september 1991: whole year 1992: whole year 1993: up to april	4
Mercedes	1990: up to september 1991: up to august	2
Mitsubishi	1990: up to september 1991: whole year 1992: whole year 1993: whole year	4
Isuzu CSA	1990: up to september 1991: whole year 1992: whole year 1993: whole year	4
Isuzu CJR	1991: whole year 1992: whole year 1993: up to april	3

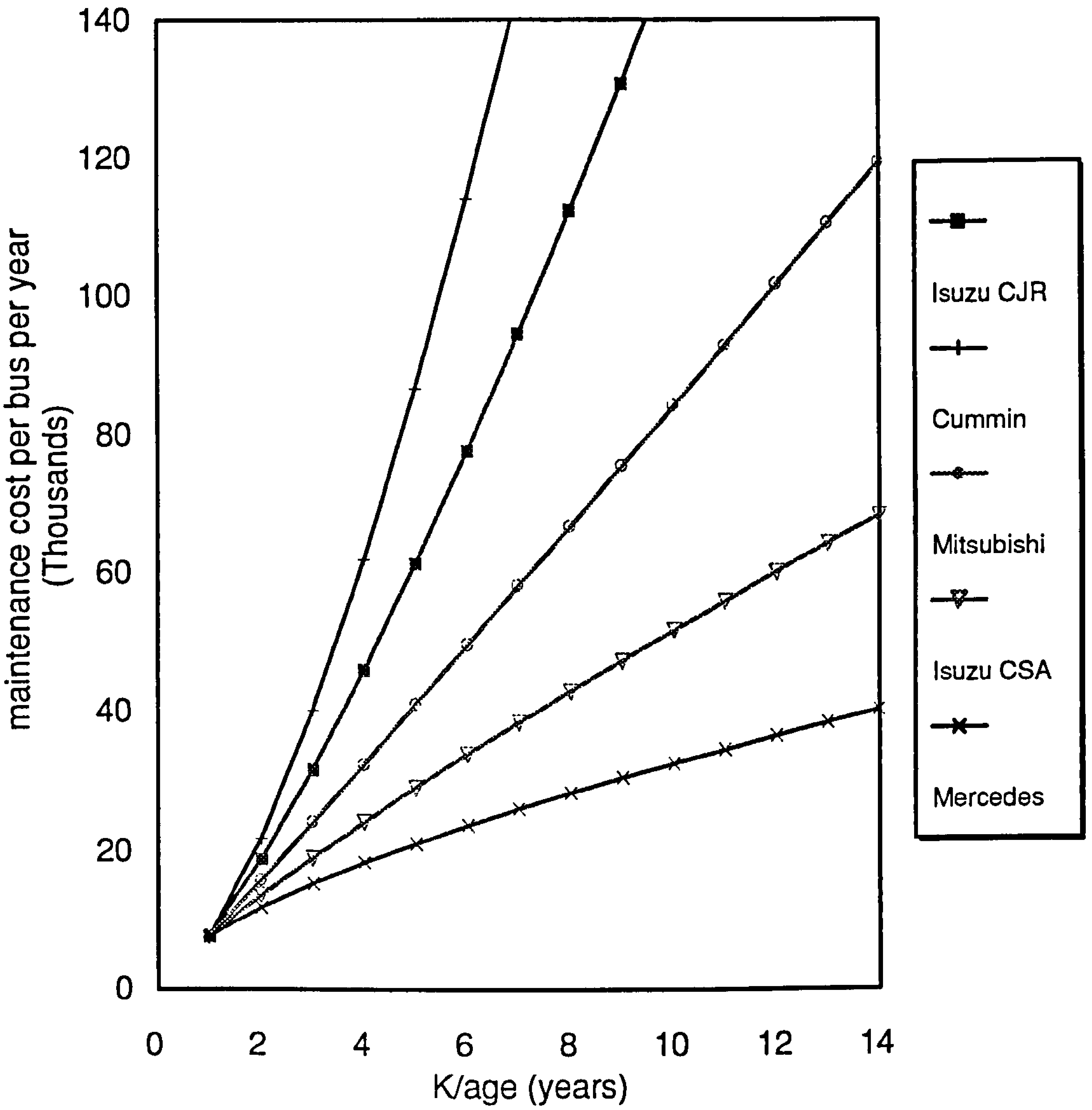
This record of maintenance costs data enabled us to fit a regression model using the statistical package GLIM (McCullagh and Nelder, 1990). We fitted the maintenance cost model, equation (5.1) with the slope β varying across sub-fleet and with common intercept. It was decided that the earlier assumption was a fair one because new equipment is partially covered by the guarantee. This is also consistent with the model initially fitted in section 5.2.1. Furthermore, with the separate intercept model the rate of increase of maintenance costs for the new Isuzu (CJR) is unrealistically high (see Figure 5.15). Later in this chapter we will carry out sensitivity analysis on maintenance cost considering a maintenance model in which the intercepts were different for different sub-fleets.

We can observe in Figure 5.8.b that the fitted maintenance costs for the Cummins and the Isuzus CJR are relatively higher than the cost of other sub-fleets, although they are the newest sub-fleets, (2-4 years of age on average). This is probably due to the fact that the models predicted for these two sub-fleets were based only on data from early ages of the sub-fleets, that is when the vehicles were between 1 and 4 years old, whereas for the rest of the sub-fleets the modelling was based upon data covering a wider range of age, e.g. for the Isuzu CSA, the range was from 8 to 14 years. We should, however expect that the current maintenance policy has to be improved in order to reduce the high cost incurred for the two relatively new sub-fleets, namely the Isuzus CJR and the Cummins. We can, in the future, update the modelling in the light of new data following a new and improved maintenance policy. For that purpose, we might point out that a study concerning inspection and maintenance policy is presently undertaken as a part of a research project for the ENB company, using the delay time technique (Christer and Desa, 1992).



(a)

Figure 5.8.a Observed maintenance cost per year for each bus in the fleet against age.



(b)

Figure 5.8.b. Fitted maintenance cost per year for each bus in the fleet for each sub-fleet. Mercedes, $M(\tau) = 7731\tau^{0.63}$; Cummin, $M(\tau) = 7731\tau^{1.50}$; Isuzu CSA, $M(\tau) = 7731\tau^{0.83}$; Mitsubishi, $M(\tau) = 7731\tau^{1.04}$; Isuzu CJR, $M(\tau) = 7731\tau^{1.29}$

5.4.3 Results for fixed fleet size

We consider the option of fixed fleet size. Firstly, to comply with the company's policy which is based on operating a fleet with fixed size and secondly, to evaluate the extra cost incurred when variable fleet size option is considered, that is replacing with smaller sub-fleet size rather than with the optimal one. In other words we evaluate how much it costs the company to keep the same fleet size at replacement instead of increasing the replacement sub-fleet size as suggested by the model. Results using both objective functions, namely, the equivalent rent and the total discounted cost per unit time, based on the two cycle models (model IIa) described in chapter 4, section 4.3.2, are presented. We present separately our results for each cost criterion starting with the equivalent rent model.

5.4.3.1 Equivalent rent model

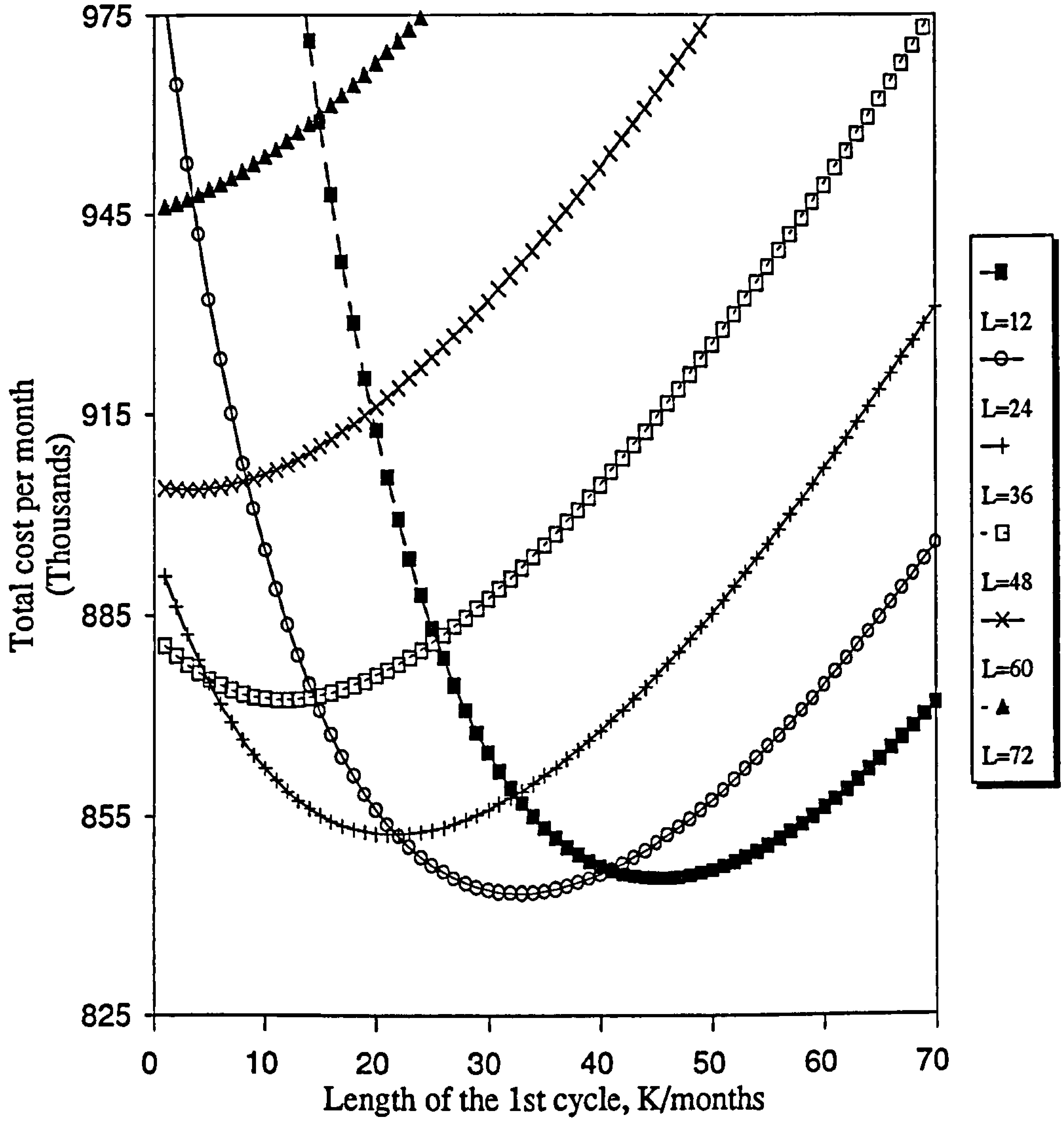
The choice of the equivalent rent criterion to conduct most of the work is motivated by the fact that in the case of infinite horizon, Scarf (1994) has observed that, if the value of the discount factor approached the value 1, the total discounted cost increases to infinity whereas the equivalent rent takes a limit value equal to the average cost per unit time. This makes the equivalent rent model more attractive.

Table 5.6 presents the optimum values of K* and L* as well as the minimum cost of the equivalent rent per month. Note that all the results were obtained from model IIa by minimising equation (4.15). Table 5.6 summarises the optimal decision for $L=L^*$ and shows the strategy which incurs the minimum cost is to replace the Mitsubishi sub-fleet first and the Cummin sub-fleet second. The second best policy in terms of minimum cost, is to replace the Cummin first then the Mitsubishi second. The next best policy replaces the Cummin first and Mercedes second. As the company were interested in replacement of the Mercedes sub-fleet in the short-term, the policy with Mercedes replaced first and then the Cummin sub-fleet is highlighted. In this work, we would like to present the cost of all possible decisions and then the ultimate decision can be taken by the operator. Figures

5.9.a-5.9.h show the cost (for the whole fleet) against the age of first replacement for various lengths of the second cycle *L*, for different replacement schedules, Me-C; C-Me; etc. Figure 5.10 illustrates a summary for the optimum age of first replacement for each strategy with the corresponding optimum value for the length of the second cycle *L**. In table 5.6, we can observe that the replacement decisions with the Mitsubishi suggest either an immediate (1 month for Cummin-Mitsubishi) replacement or a relatively short length of time (7 months for Mitsubishi-Isuzu CSA).

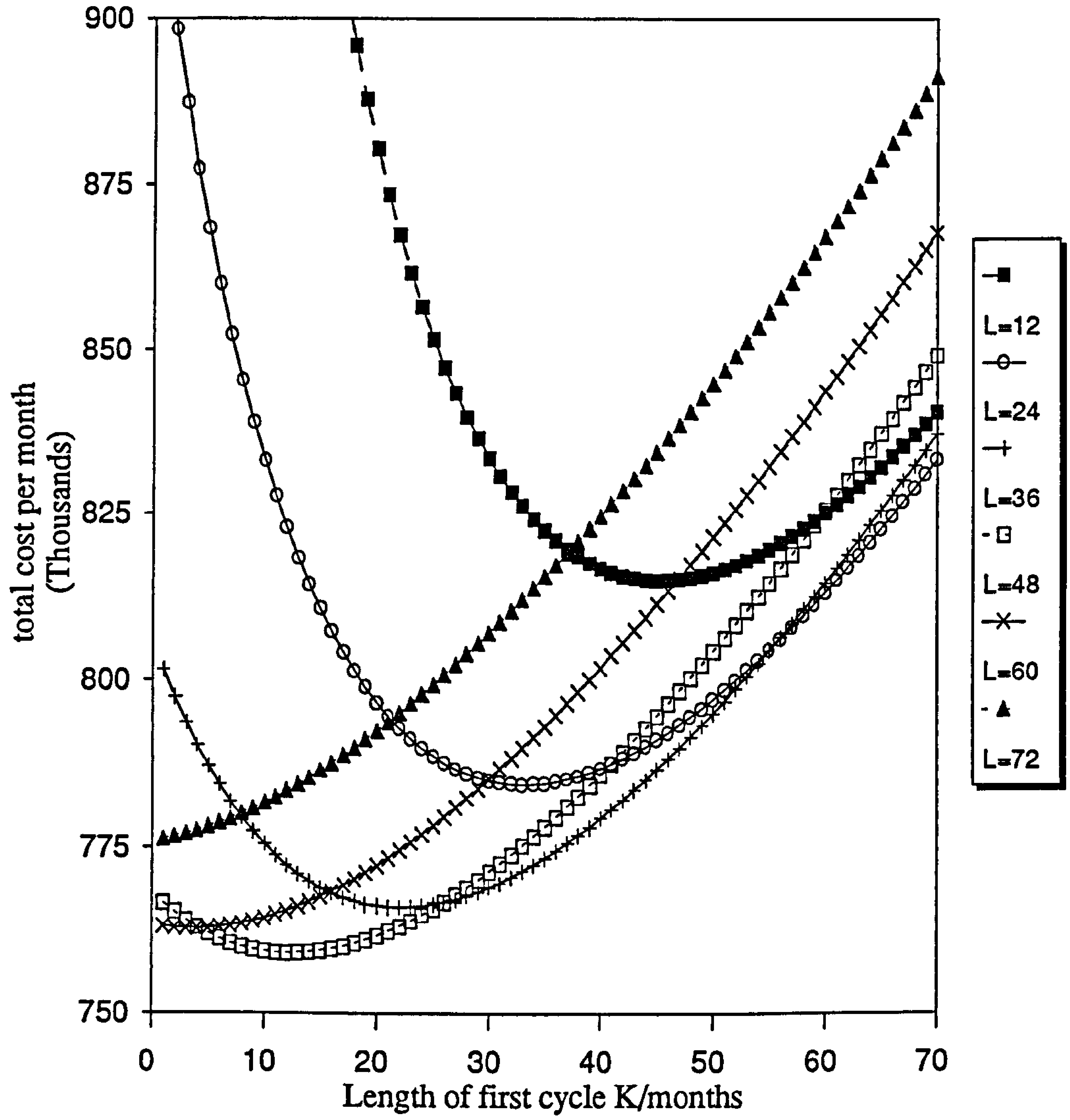
Table 5.6. Cost per month (for whole fleet) of the minimum equivalent rent and optimum values of decisions variables for various choices of sub-fleets to be sold at first and second replacement. (two cycle model, fixed fleet size, model IIa)

Replacement schedule 1st repl. - 2nd repl.	K* months	L* months	min. cost $M000's
C - Me	**12**	**48**	**759**
C - Mit	**1**	**48**	**719**
C - Is	13	48	768
Me - C	33	24	843
Me - Mit	40	24	899
Me - Is	47	24	936
Mit - C	**1**	**36**	**693**
Mit - Is	7	48	821
Mit - Me	6	48	812
Is - C	2	45	782
Is - Mit	11	45	845
Is - Me	20	44	880



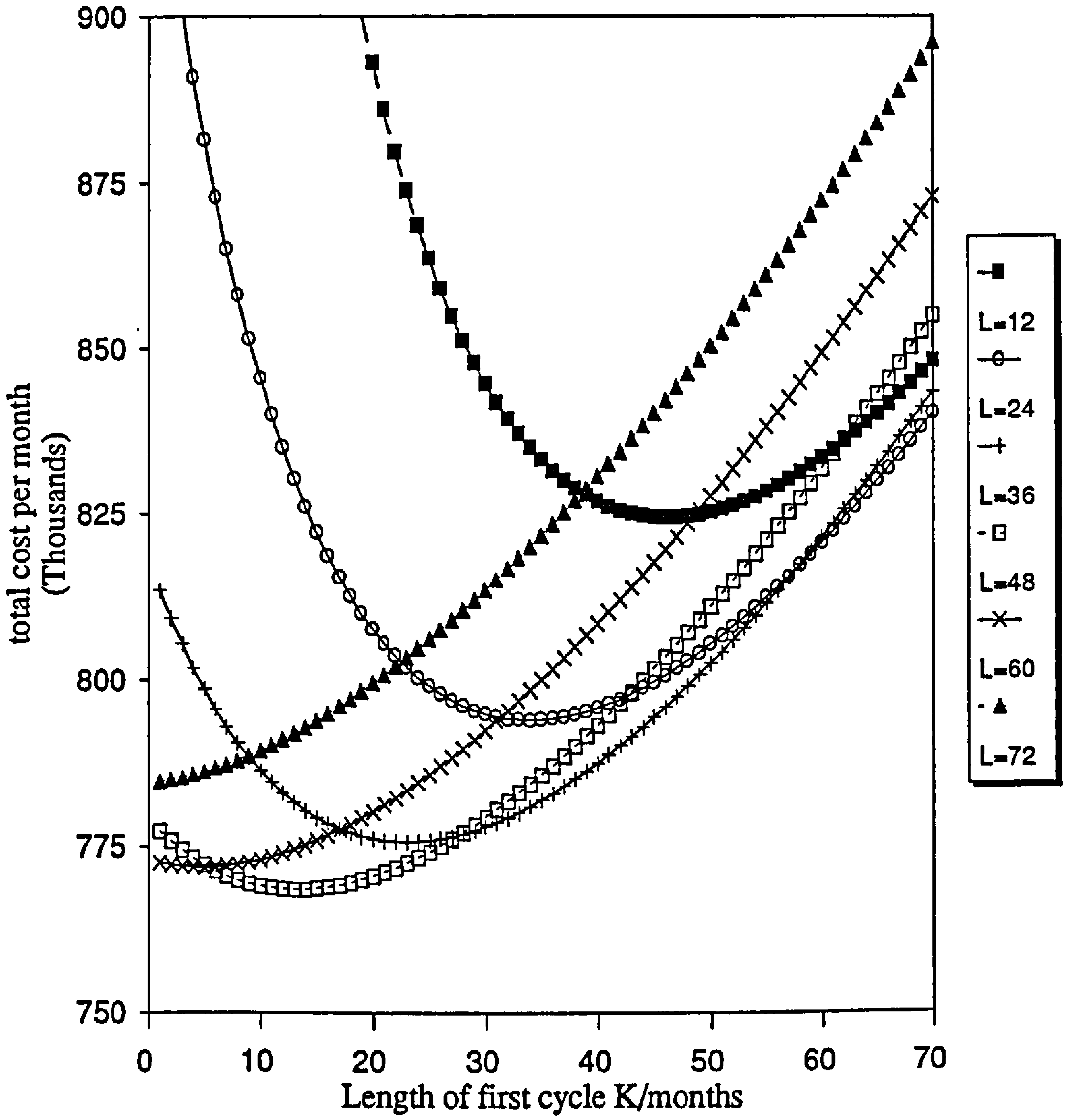
(a)

Figure 5.9.a. Cost of equivalent rent per month (for whole fleet) vs. lenght of first cycle, K, for various lengths of second cycle, Mercedes sub-fleet replaced first, Cummin second. Two cycle model, fixed fleet size (model IIa).



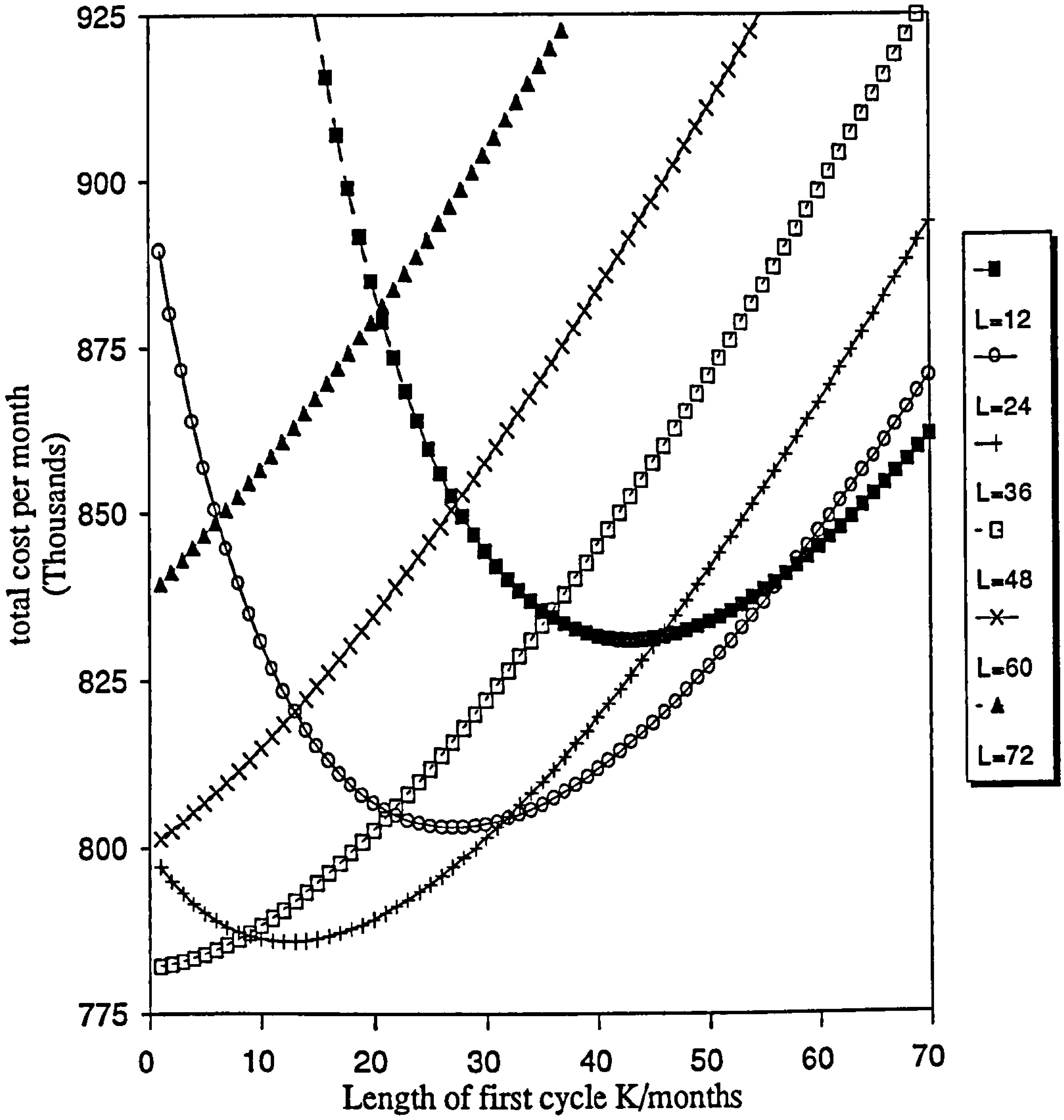
(b)

Figure 5.9.b. Cost of equivalent rent per month (for whole fleet) vs. length of first cycle K, for various lengths of second cycle, Cummin sub-fleet replaced first, Mercedes second. Two cycle model, fixed fleet size (model IIa).



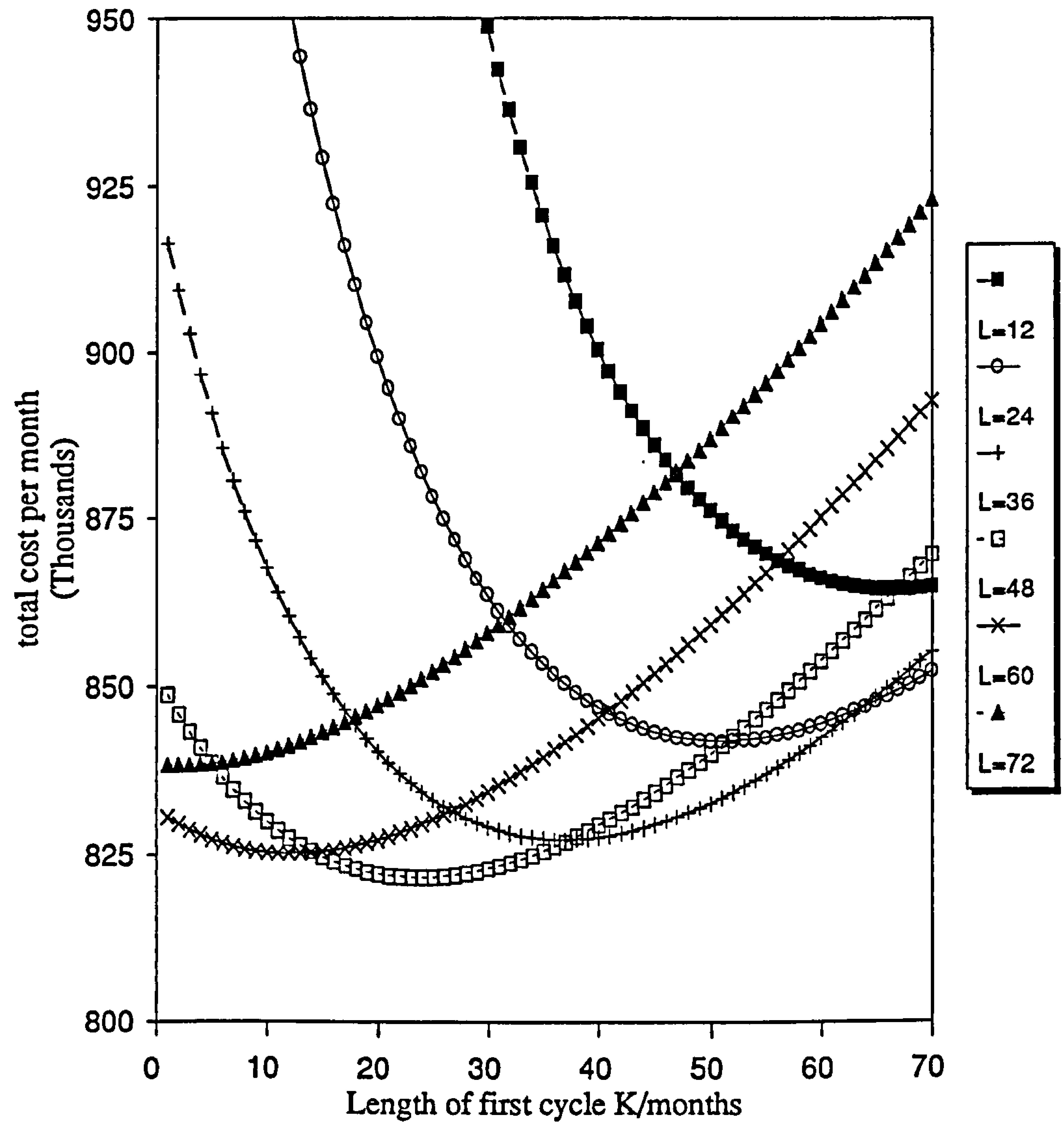
(c)

Figure 5.9.c. Cost of equivalent rent per month (for whole fleet) vs. length of first cycle K, for various lengths of second cycle, Cummin sub-fleet replaced first, Isuzu CSA second. Two cycle model, fixed fleet size (model IIa).



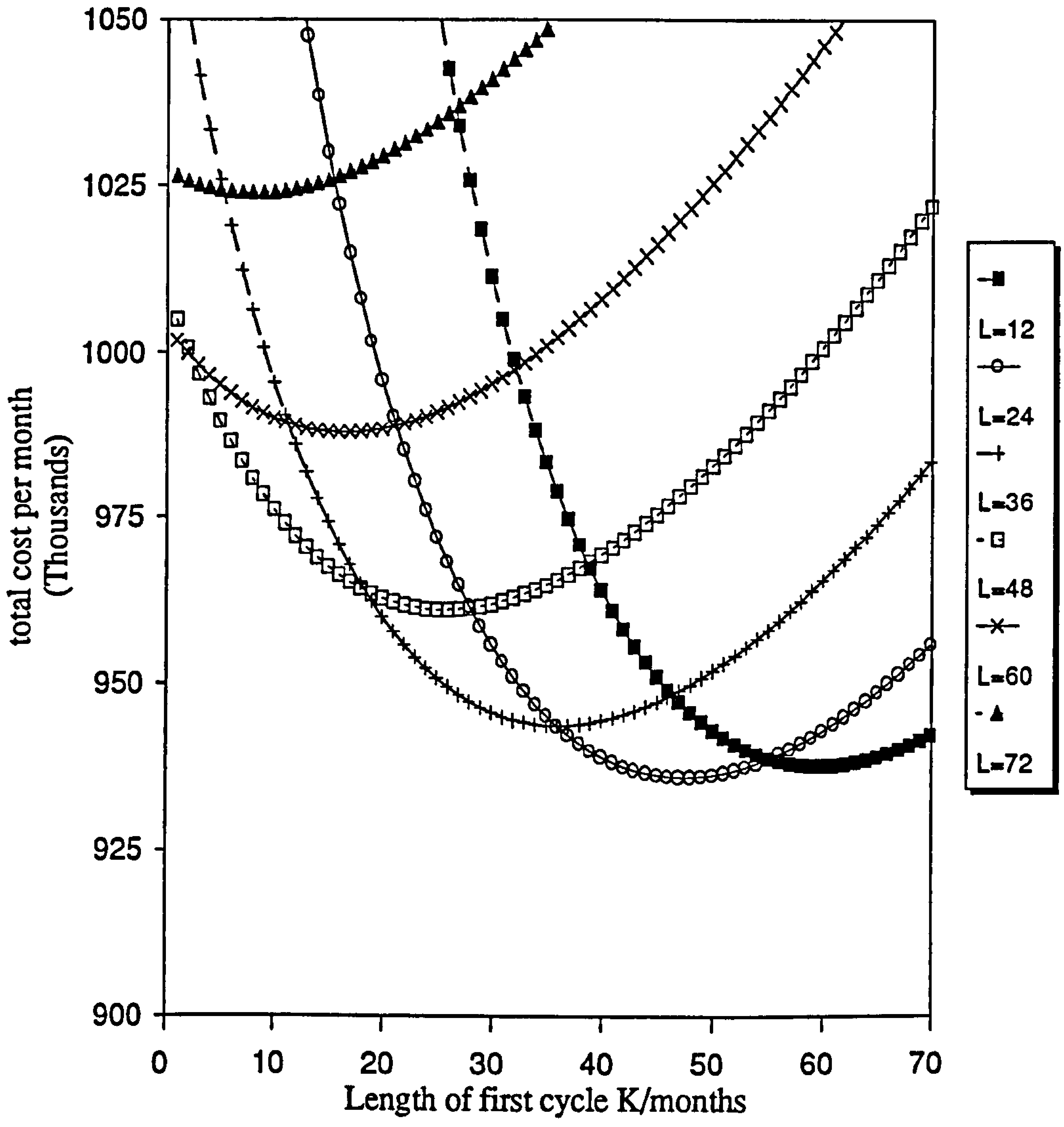
(d)

Figure 5.9.d. Cost of equivalent rent per month (for whole fleet) vs. length of first cycle K, for various lengths of second cycle, Isuzu CSA sub-fleet replaced first, Cummin second. Two cycle model, fixed fleet size (model IIa).



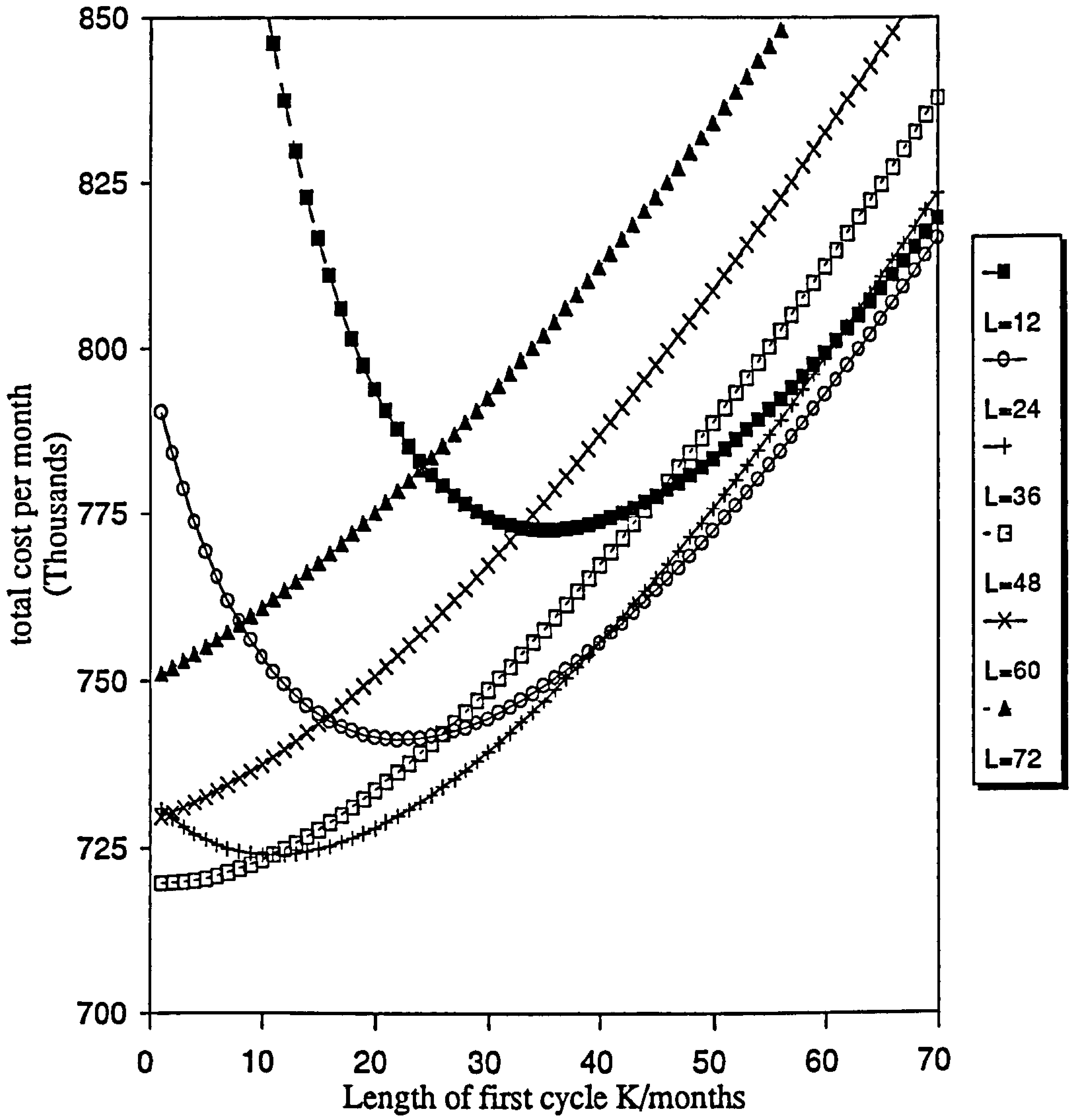
(e)

Figure 5.9.e. Cost of equivalent rent per month (for whole fleet) vs. length of first cycle K, for various lengths of second cycle, Isuzu CSA sub-fleet replaced first, Mercedes second. Two cycle model, fixed fleet size (model IIa).



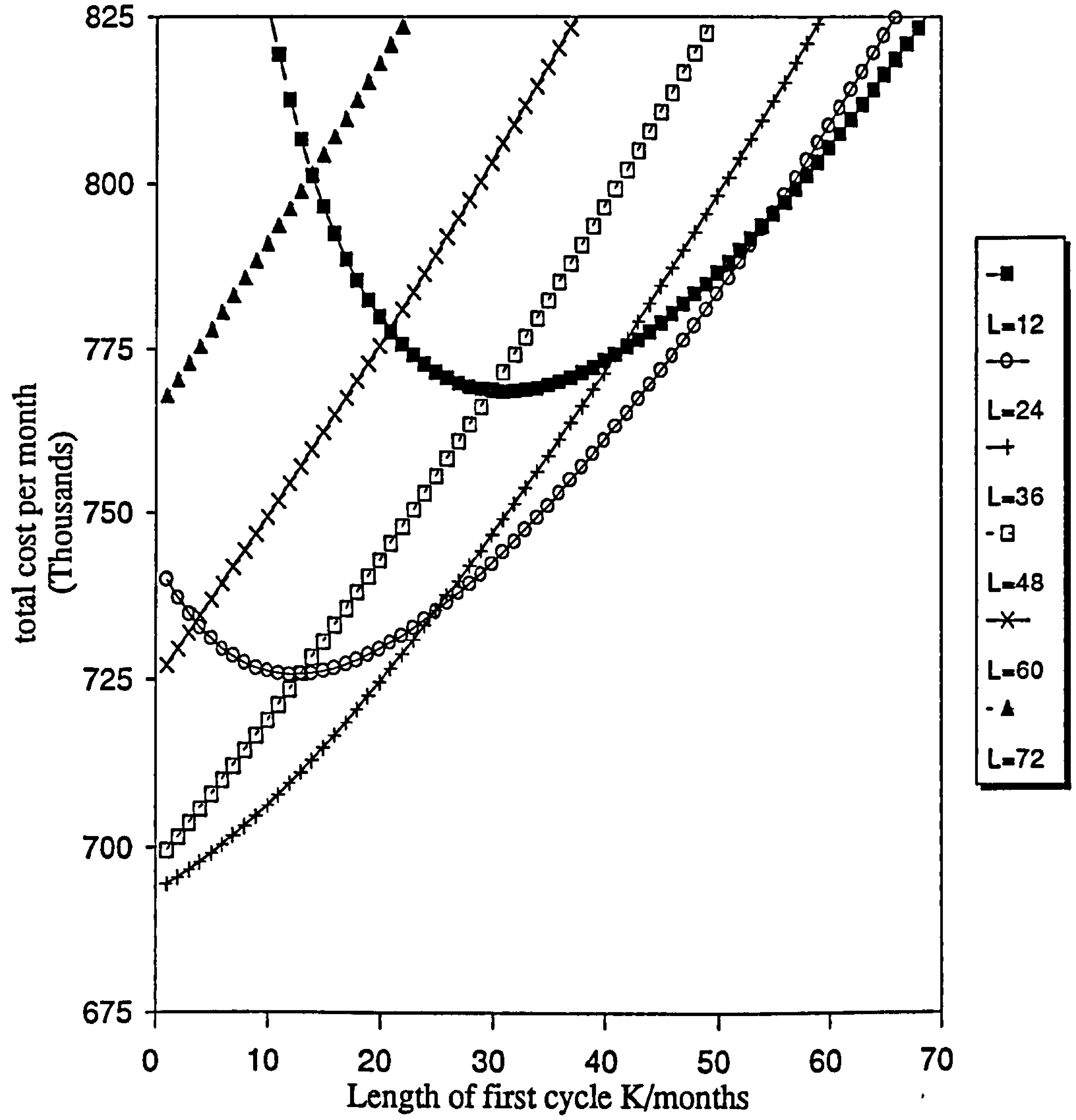
(f)

Figure 5.9.f. Cost of equivalent rent per month (for whole fleet) vs. length of first cycle K, for various lengths of second cycle, Mercedes replaced first, Isuzu CSA second. Two cycle model, fixed fleet size (model IIa).



(g)

Figure 5.9.g. Cost of equivalent rent per month (for whole fleet) vs. length of first cycle K, for various lengths of second cycle, Cummin replaced first, Mitsubishi second. Two cycle model, fixed fleet size (model IIa).



(h)

Figure 5.9.h. Cost of equivalent rent per month (for whole fleet) vs. length of first cycle K, for various lengths of second cycle, Mitsubishi replaced first, Cummin second. Two cycle model, fixed fleet size (model IIa).

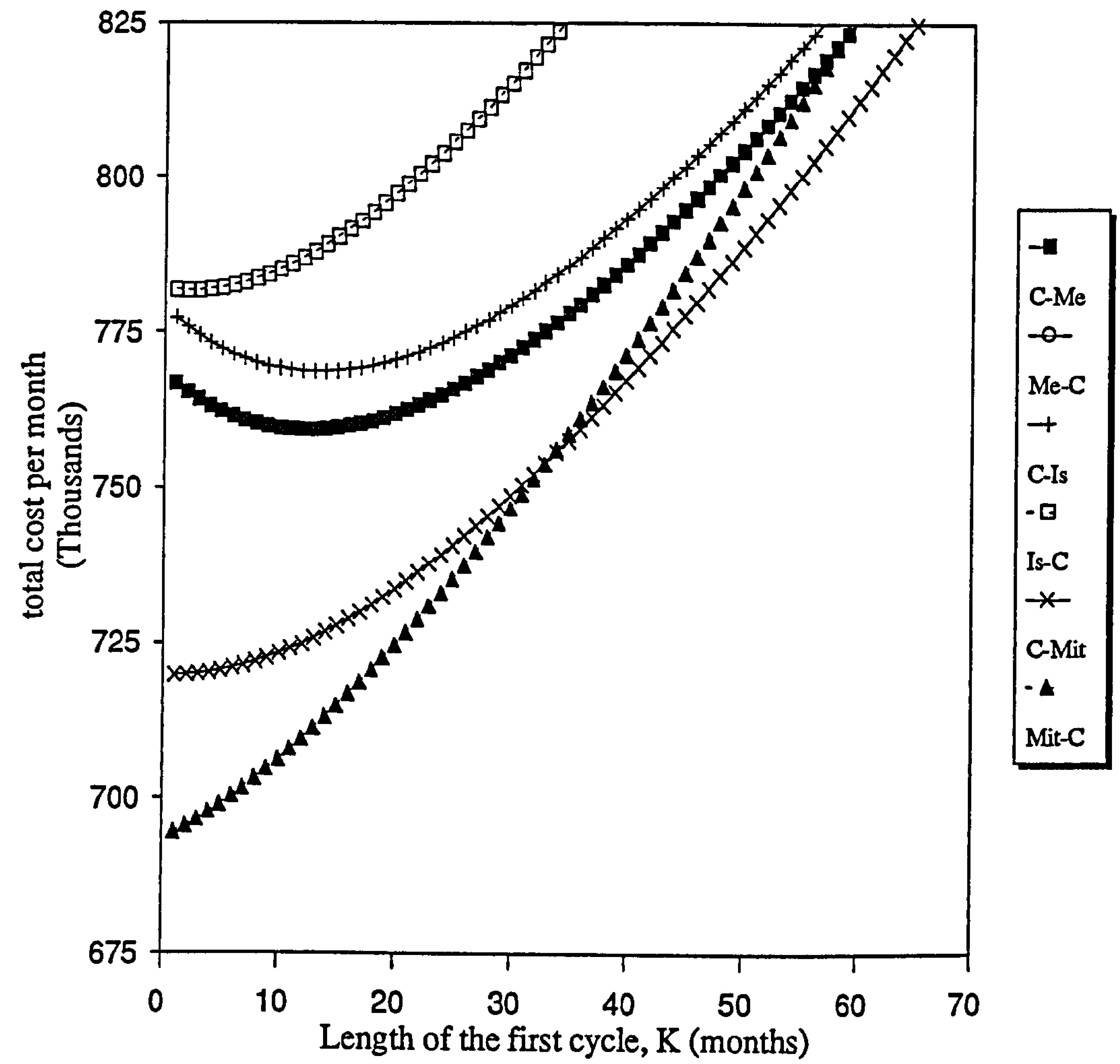


Figure 5.10. Cost of equivalent rent per month (for whole fleet) vs. length of first cycle, K,, for various choices of sub-fleets to be sold at the first and second replacement. C-Me: Cummin replaced first, Mercedes second, etc. (Isuzu CJR purchased; $L=L^*$) Two cycle model, fixed fleet size (model IIa).

5.4.2.2 Total discounted cost per unit time

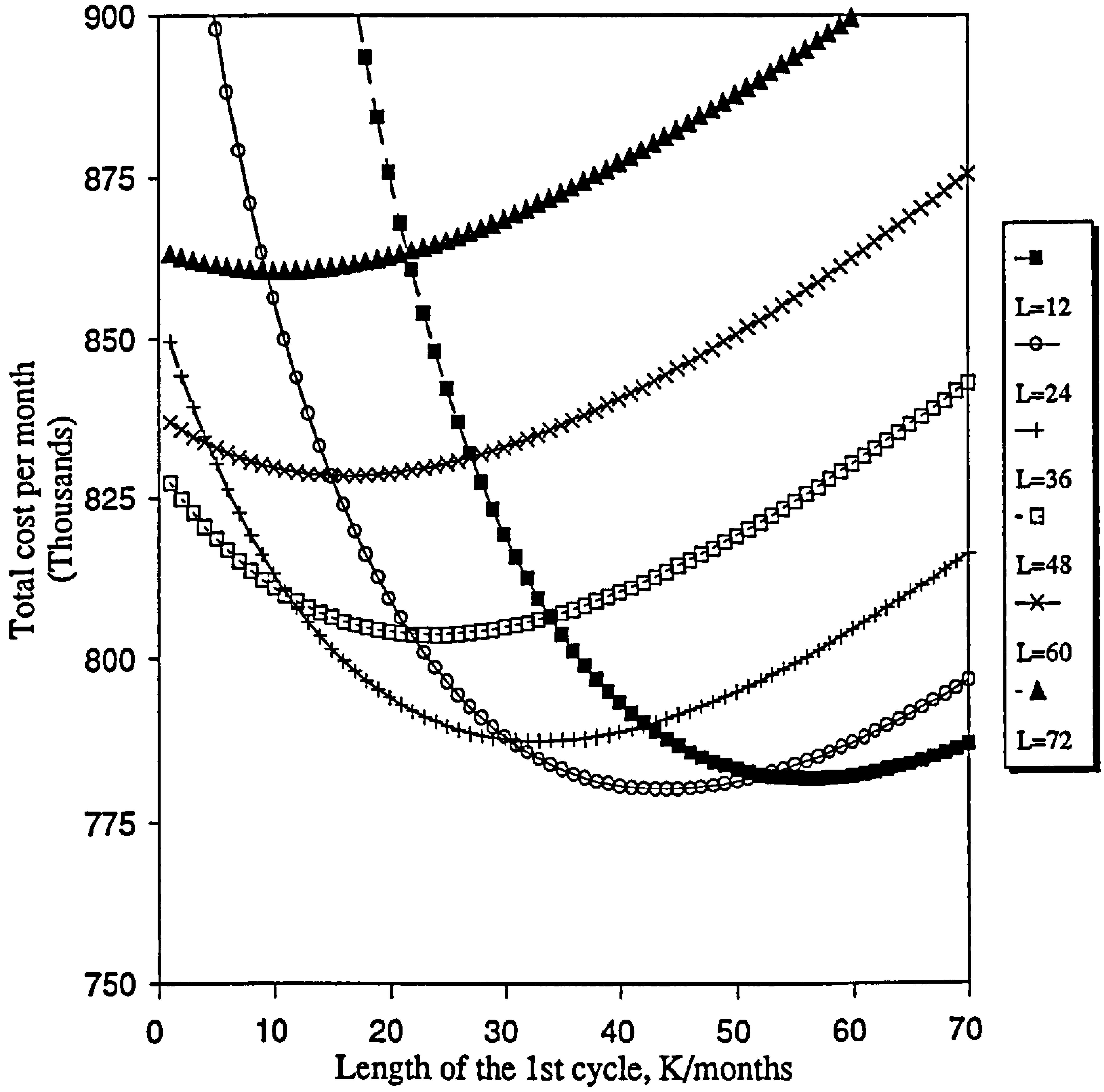
The same analysis in the case of the equivalent rent model is conducted, it is based this time on the minimisation of the total discounted cost per unit time using model IIa (equation 4.16). Table 5.7 presents the optimum values for various choices of sub-fleets to be sold at first and second replacement. It appears from table 5.7 that the optimal time for replacement in the first cycle, that is the values of K^* are longer in comparison with the rent model, but the values of L^* are closer. However the costs incurred are smaller. This is because

$$K + L > \sum_{i=1}^{K+L} v^i ,$$

for any value of $v < 1$, where these two quantities represent the denominators of the same cost function, namely the total discounted cost, in equations (4.20) and (4.21) respectively. Figures 5.11.a-5.11.h illustrate the total discounted cost per month for a range of values of the length of the second cycle L to the closest 12 months, for various replacement schedules such as replacing the Cummin sub-fleet first and then the Mercedes sub-fleet (C-Me), etc. Figure 5.12 illustrates the optimal strategy over different scenarios of replacement. We might notice that the optimal strategy (which sub-fleet to replace first) has not changed by using either the equivalent rent or the total discounted cost per unit time. We should view the total discounted cost per unit time as an alternative criterion.

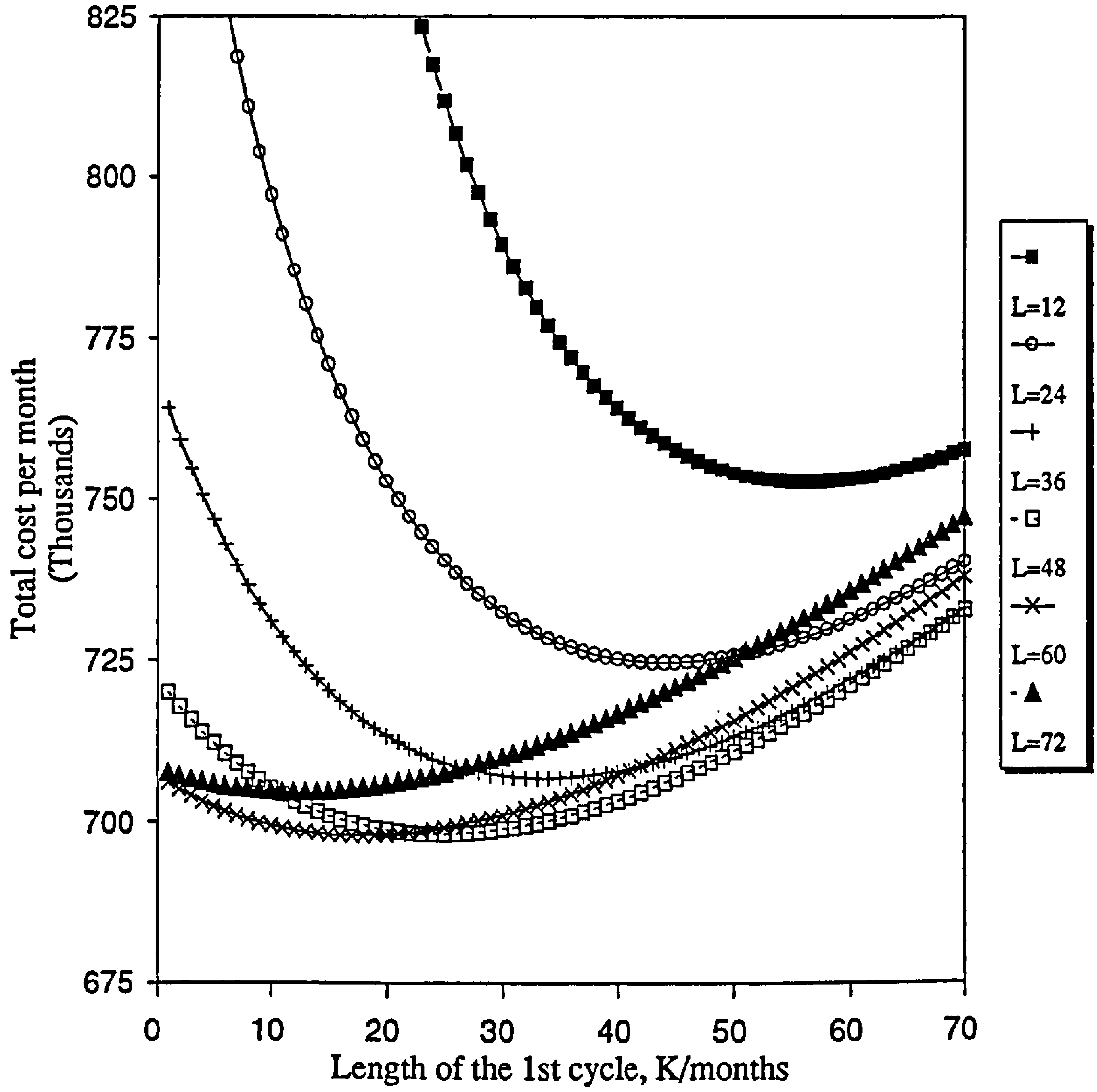
Table 5.7. Total discounted cost per month (for whole fleet) and optimum values of decision variables for various choices of sub-fleets to be sold at first and second replacement.

Replacement schedule 1st repl. - 2nd repl.	K* months	L* months	min. cost $M000's
C - Me	**21**	**54**	**697**
C - Mit	**11**	**50**	**671**
C - Is	22	54	704
Me - C	48	20	779
Me - Mit	58	20	822
Me - Is	66	20	848
Mit - C	**1**	**44**	**656**
Mit - Is	15	52	761
Mit - Me	13	52	753
Is - C	12	45	732
Is - Mit	24	44	782
Is - Me	33	44	805



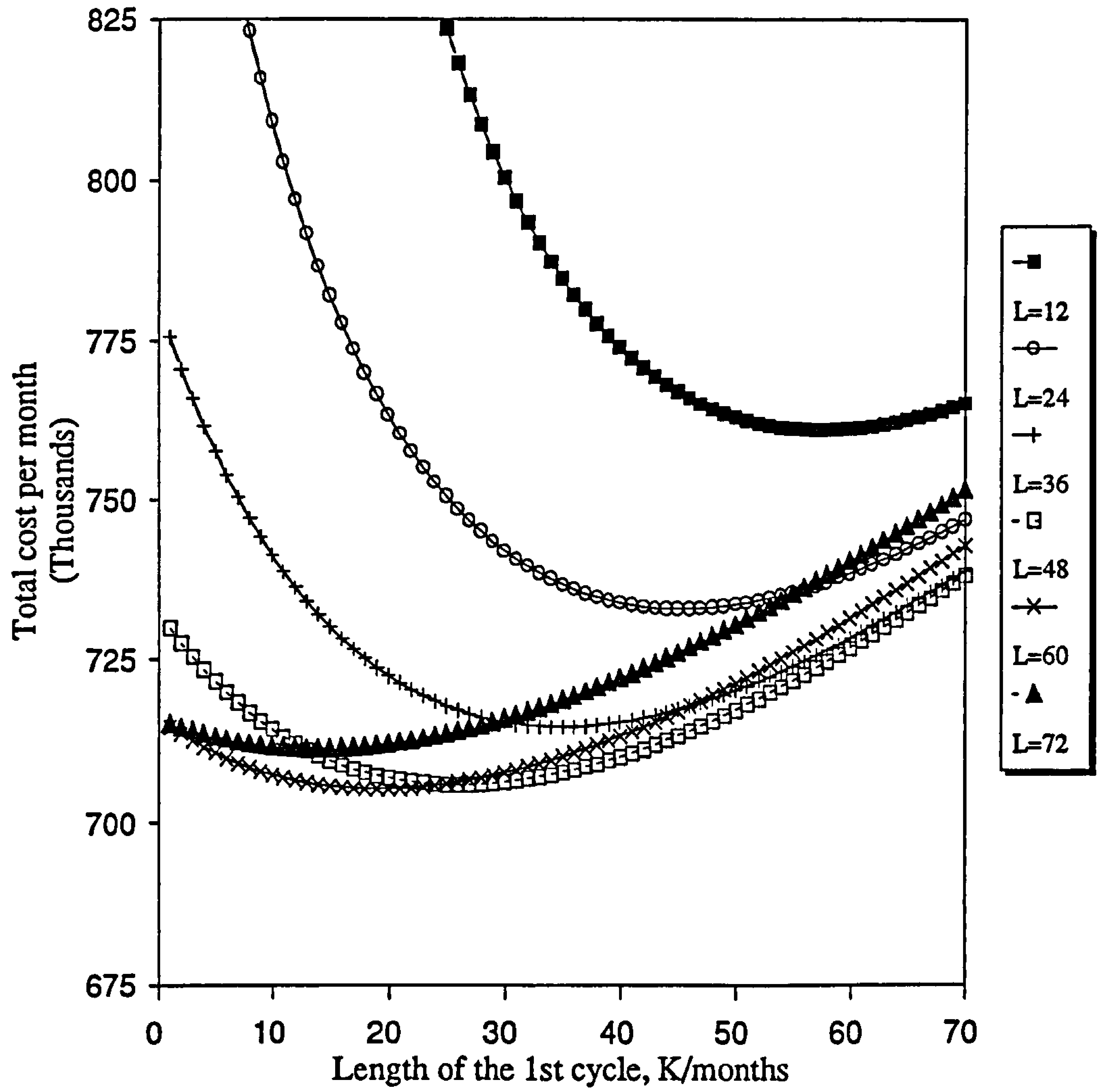
(a)

Figure 5.11.a. Total discounted cost per month and optimal values of *K* for different values of *L*; Mercedes sub-fleet replaced first, Cummin second. Two cycle model with fixed fleet size (model IIa)



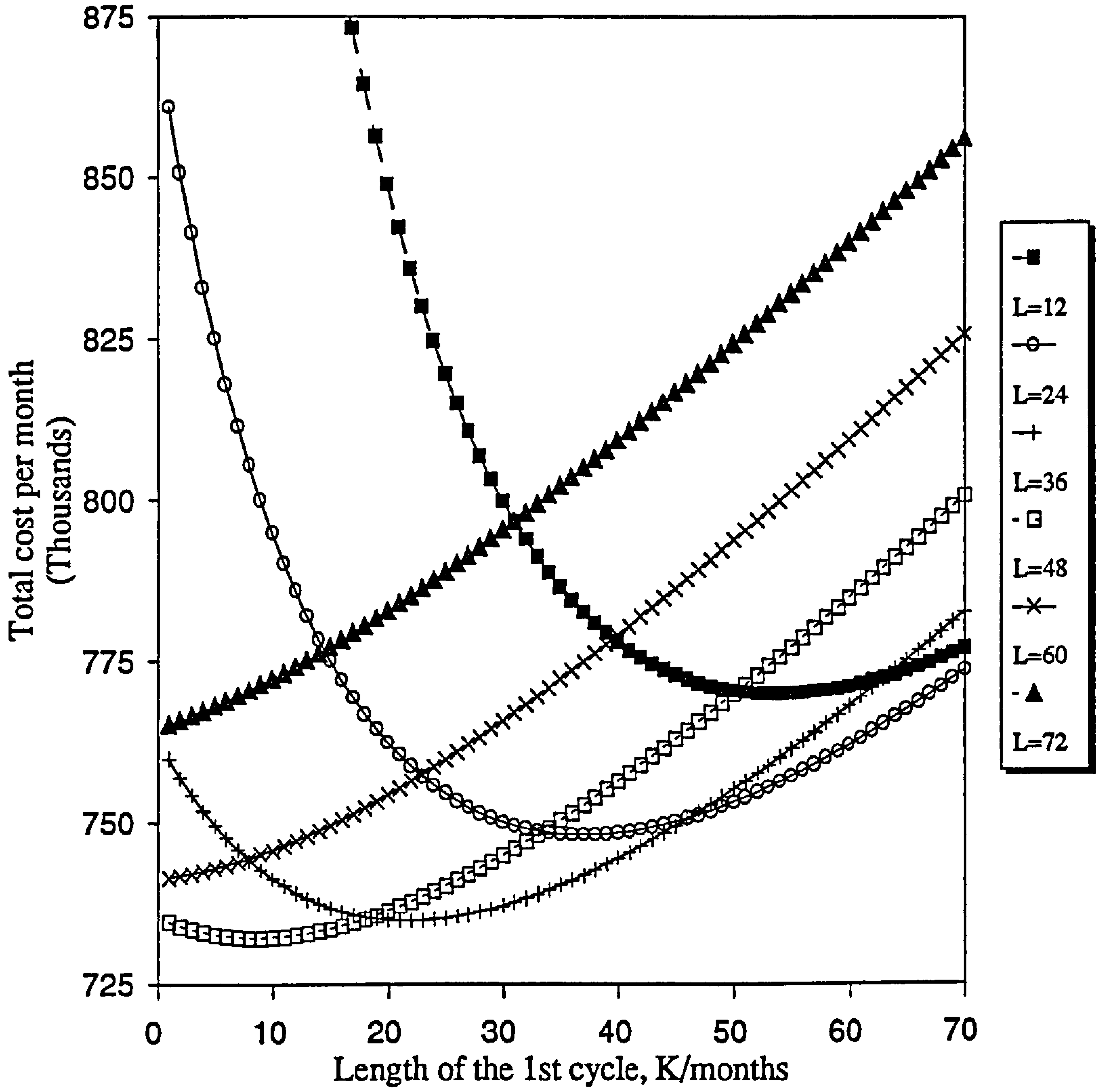
(b)

Figure 5.11.b. Total discounted cost per month and optimal values of *K* for different values of *L*; Cummin sub-fleet replaced first, Mercedes second. Two cycle model with fixed fleet size (model IIa)



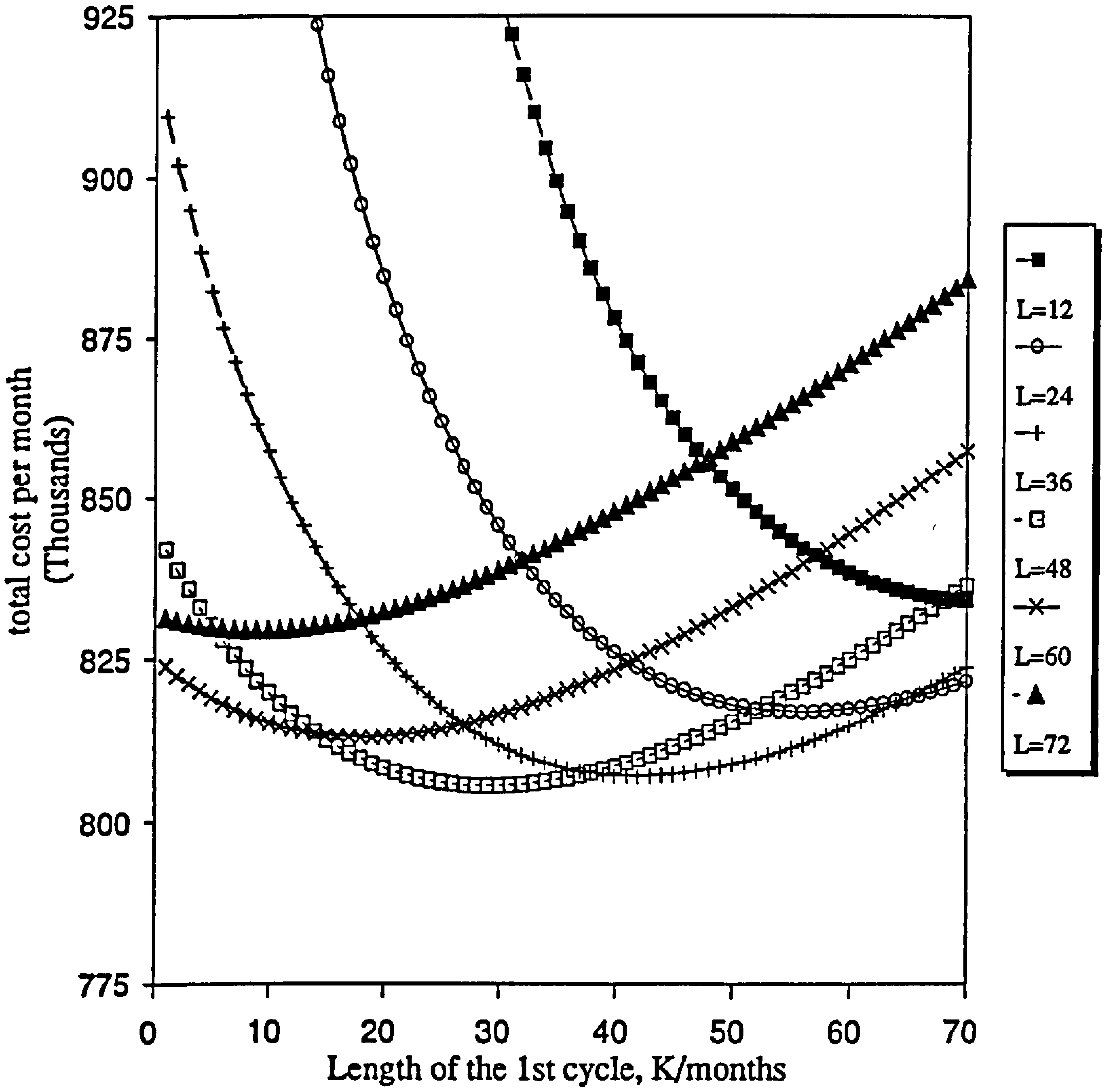
(c)

Figure 5.11.c. Total discounted cost per month and optimal values of *K* for different values of *L*; Cummin sub-fleet replaced first, Isuzu CSA second. Two cycle model with fixed fleet size (model IIa).



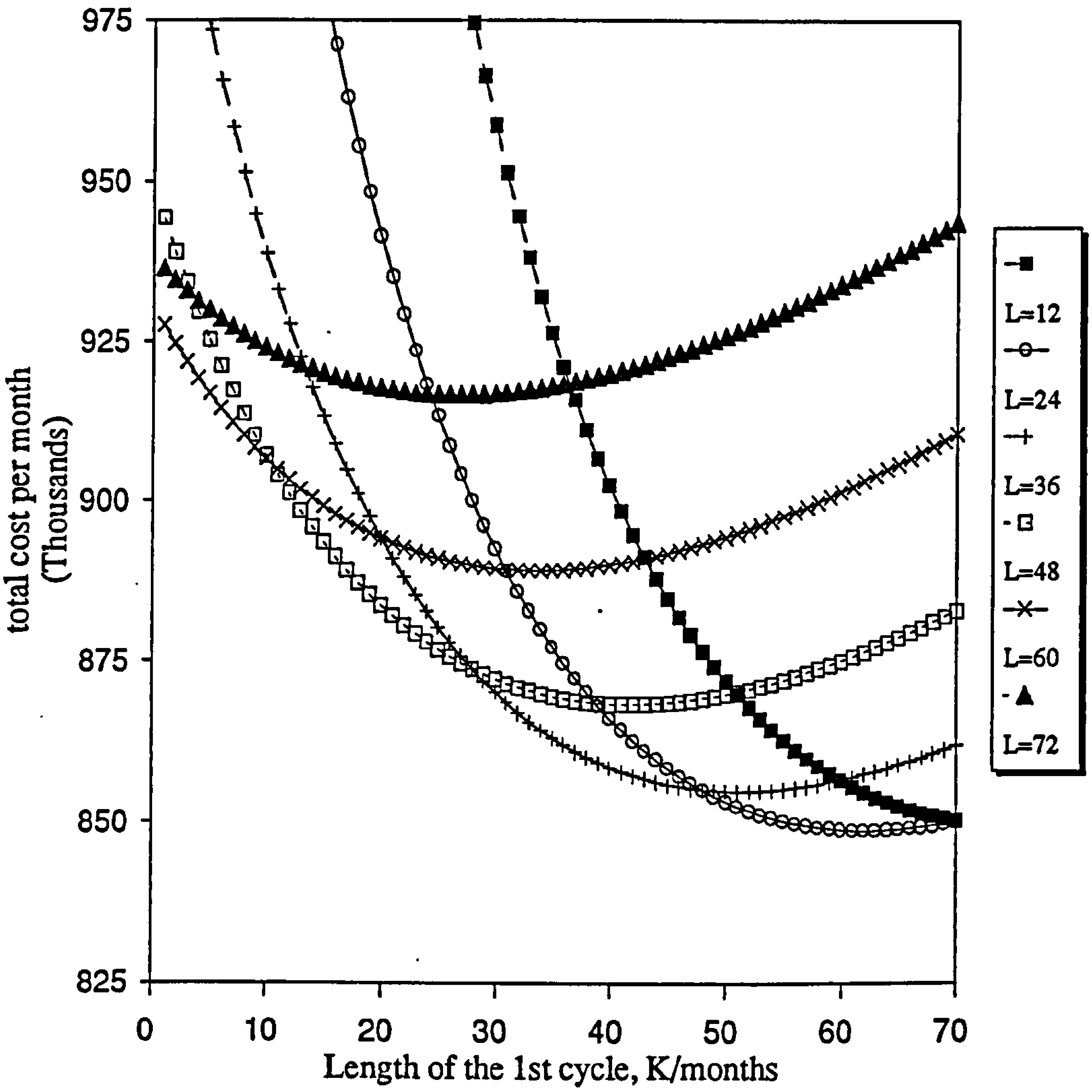
(d)

Figure 5.11.d. Total discounted cost per month and optimal values of K for different values of L; Isuzu CSA sub-fleet replaced first, Cummin second. Two cycle model with fixed fleet size (model IIa)



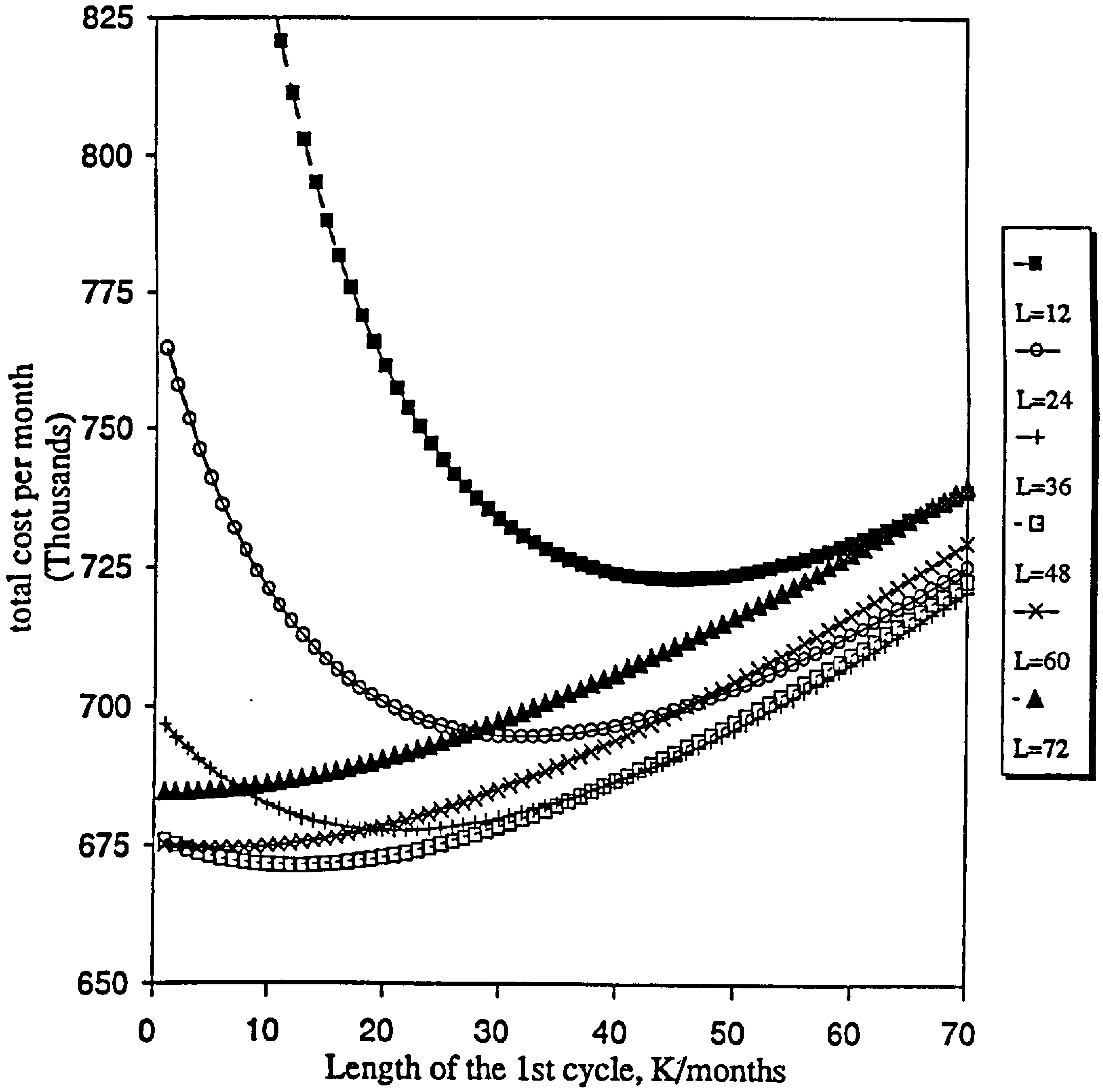
(e)

Figure 5.11.e. Total discounted cost per month and optimal values of K for different values of L; Isuzu CSA sub-fleet replaced first, Mercedes second. Two cycle model with fixed fleet size (model IIa)



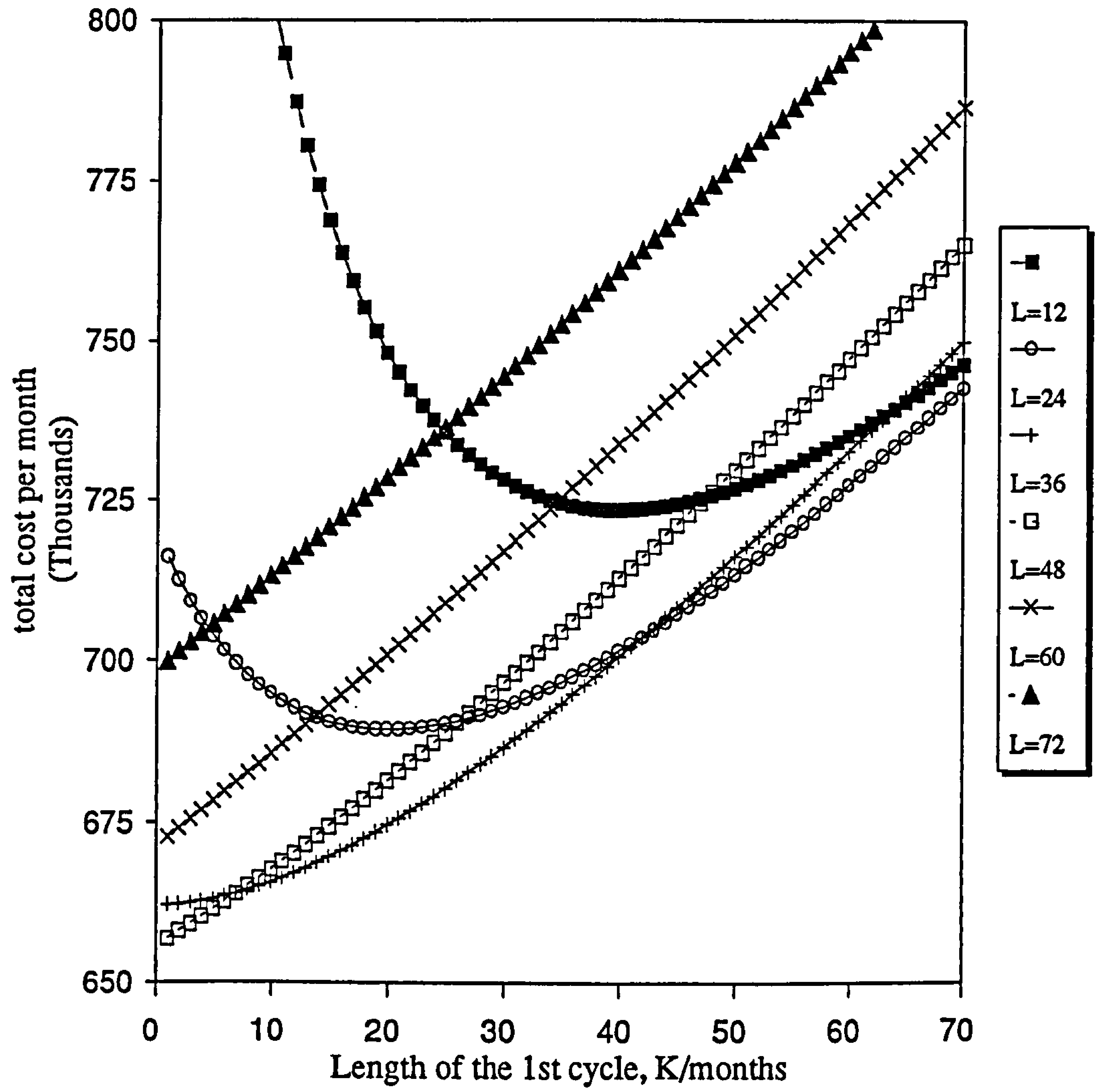
(f)

Figure 5.11.f. Total discounted cost per month and optimal values of *K* for different values of *L*;Mercedes sub-fleet replaced first, Isuzu CSA second. Two cycle model with fixed fleet size (model IIa)



(g)

Figure 5.11.g. Total discounted cost per month and optimal values of *K* for different values of *L*;Cummin sub-fleet replaced first, Mitsubishi second. Two cycle model with fixed fleet size (model IIa)



(h)

Figure 5.11.h. Total discounted cost per month and optimal values of K for different values of L:Mitsubishi sub-fleet replaced first, Cummin second. Two cycle model with fixed fleet size (model IIa)

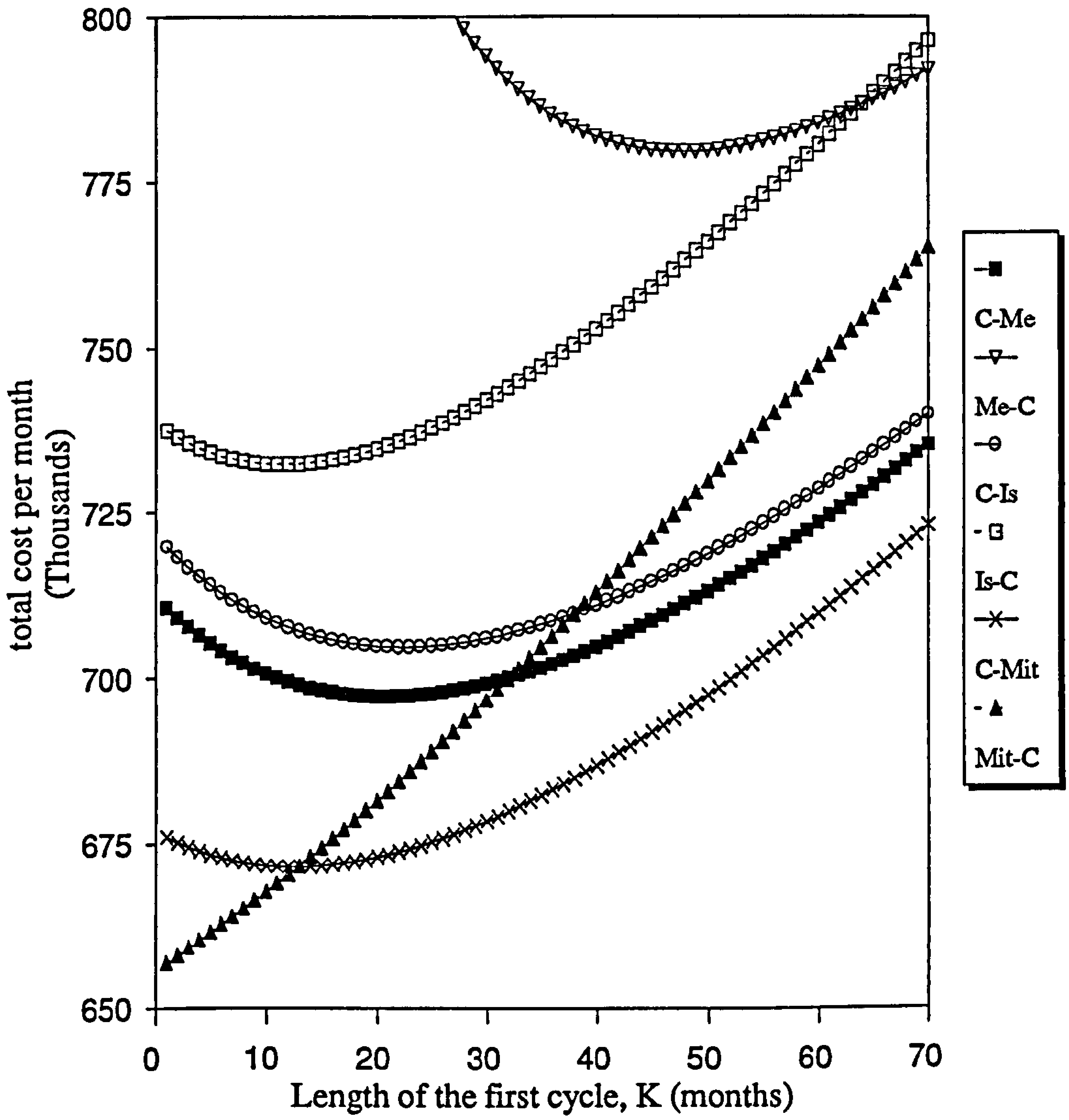


Figure 5.12. Total discounted cost per month (whole fleet) vs.length of first cycle, K, for various choices of sub-fleets to be sold at the first and second replacment, (L=L*). Two cycle model, fixed fleet size (model IIa).

5.4.4 Result for Variable Fleet Size

Here we consider the additional question for both objective function; the equivalent rent per month and the total discounted cost per unit time: How many new buses should be purchased at the replacement of the first sub-fleet? We will conduct the same analysis as in section 5.4.3, that is using the two cycle model (model IIb) presented in chapter 4, section 4.3.2.

5.4.4.1 *Equivalent rent model*

The corresponding cost of the minimum equivalent rent and optimum values of decision variables are presented in tables 5.8-5.13 for a range of penalty costs for various replacement schedules. Table 5.8 considers the replacement strategy Cummin-Mercedes, that is replacing first the Cummin sub-fleet and the Mercedes sub-fleet in second. Table 5.9 considers the replacement strategy Mercedes-Cummin; table 5.10 the strategy Cummin-Isuzu CSA; table 5.11 the strategy Isuzu CSA-Cummin; table 5.11, Cummin-Mitsubishi; and finally table 5.13, the strategy Mitsubishi-Cummin. We can observe a strong influence of the penalty cost on the values of K^*, N_K^* and L^*. This feature has also been observed in Christer and Scarf, (1994) for the values of K^* and L^*. We can also observe in tables 5.8-5.13 that the optimal replacement schedule does not change with penalty cost with respect to the case of fixed fleet size. The best policy is still Mitsubishi sub-fleet replaced first, then the Cummin sub-fleet second for all the values of the penalty cost.

Table 5.8. Cost per month (for whole fleet) of the minimum equivalent rent and optimal values of decision variables for various ranges of penalty costs. Cummins replaced first, Mercedes second Two cycle model, variable fleet size (model IIb).

Penalty cost per breakdown	K* (months)	N_K^*	L* (months)	Cost $M000's
300	12	15	48	770
340	10	16	50	772
500	9	17	52	777
1000	5	18	54	784
2000	1	19	54	789

Table 5.9. Cost per month (for whole fleet) of the minimum equivalent rent and optimum values of decision variables for various ranges of penalty costs. Mercedes replaced first, Cummins second. Two cycle model, variable fleet size (model IIb).

Penalty cost per breakdown	K* (months)	N_K^*	L* (months)	Cost $M000's
700	34	34	22	870
1000	34	35	22	881
1500	33	36	22	896
2000	32	36	22	910

Table 5.10. Cost per month (for whole fleet) of the minimum equivalent rent and optimum values of decision variables for various ranges of penalty costs. Cummin replaced first, Isuzu CSA second. Two cycle model, variable fleet size (model IIb).

Penalty cost per breakdown	K* (months)	N_K^*	L* (months)	Cost $M000's
350	11	16	50	781
500	10	17	52	786
1000	6	18	54	793
2000	1	19	58	799

Table 5.11. Cost per month (for whole fleet) of the minimum equivalent rent and optimum values of decision variables for various ranges of penalty costs. Isuzu CSA replaced first, Cummin second. Two cycle model, variable fleet size (model IIb).

Penalty cost per breakdown	K* (months)	N_K^*	L* (months)	Cost $M000's
350	1	37	46	793
500	1	38	46	796
1000	1	39	46	804
2000	1	39	46	809

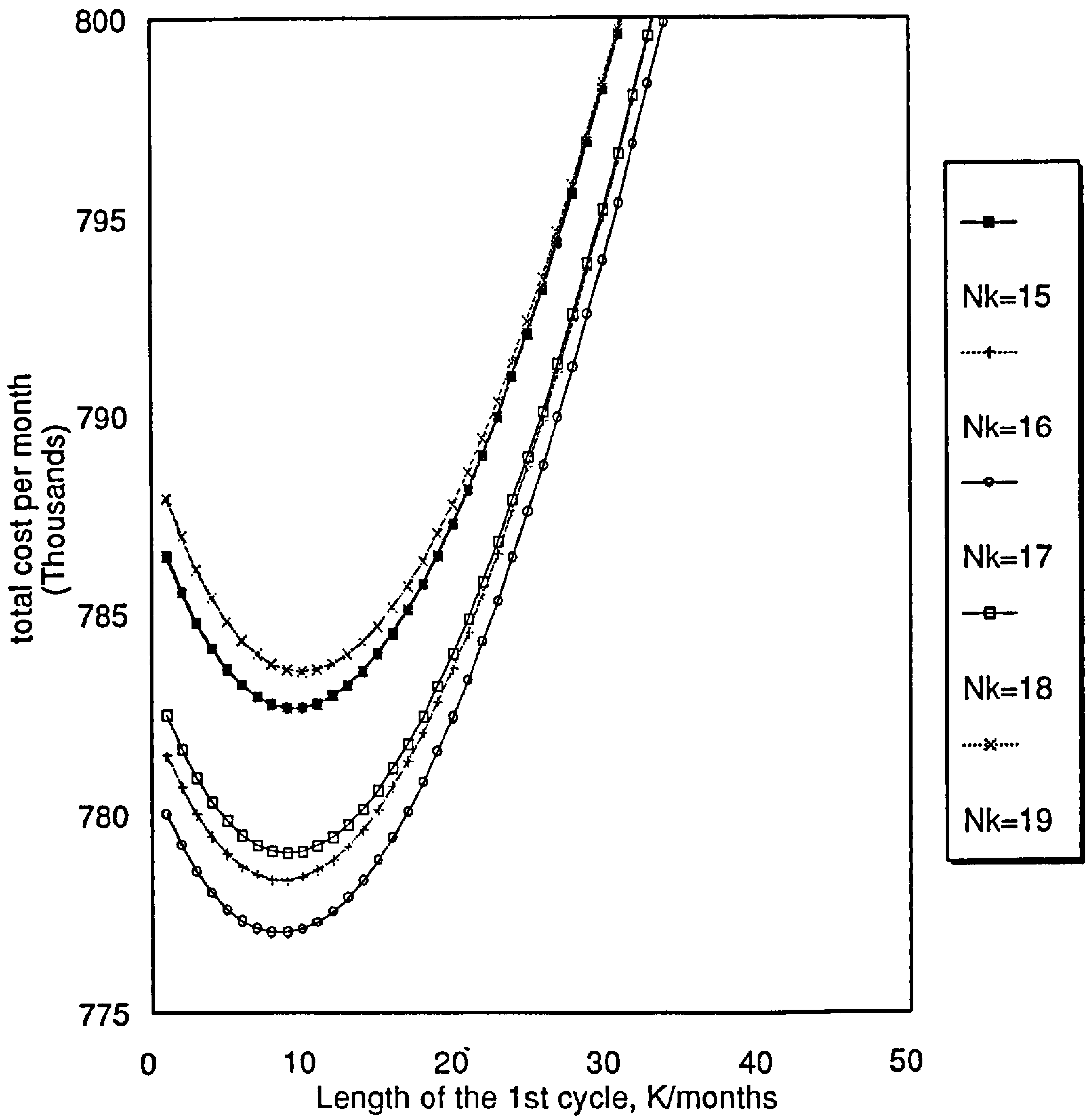
Table 5.12. Cost per month (for whole fleet) of the minimum equivalent rent and optimum values of decision variables for various ranges of penalty costs. Cummin first, Mitsubishi second. Two cycle model, variable fleet size (model IIb).

Penalty cost per breakdown	K* (months)	N_K^*	L* (months)	Cost $M000's
350	2	16	46	732
500	1	17	47	737
1000	1	18	48	743
2000	1	19	49	749

Table 5.13. Cost per month (for whole fleet) of the minimum equivalent rent and optimum values of decision variables for various ranges of penalty costs. Mitsubishi first, Cummin second. Two cycle model, variable fleet size (model IIb).

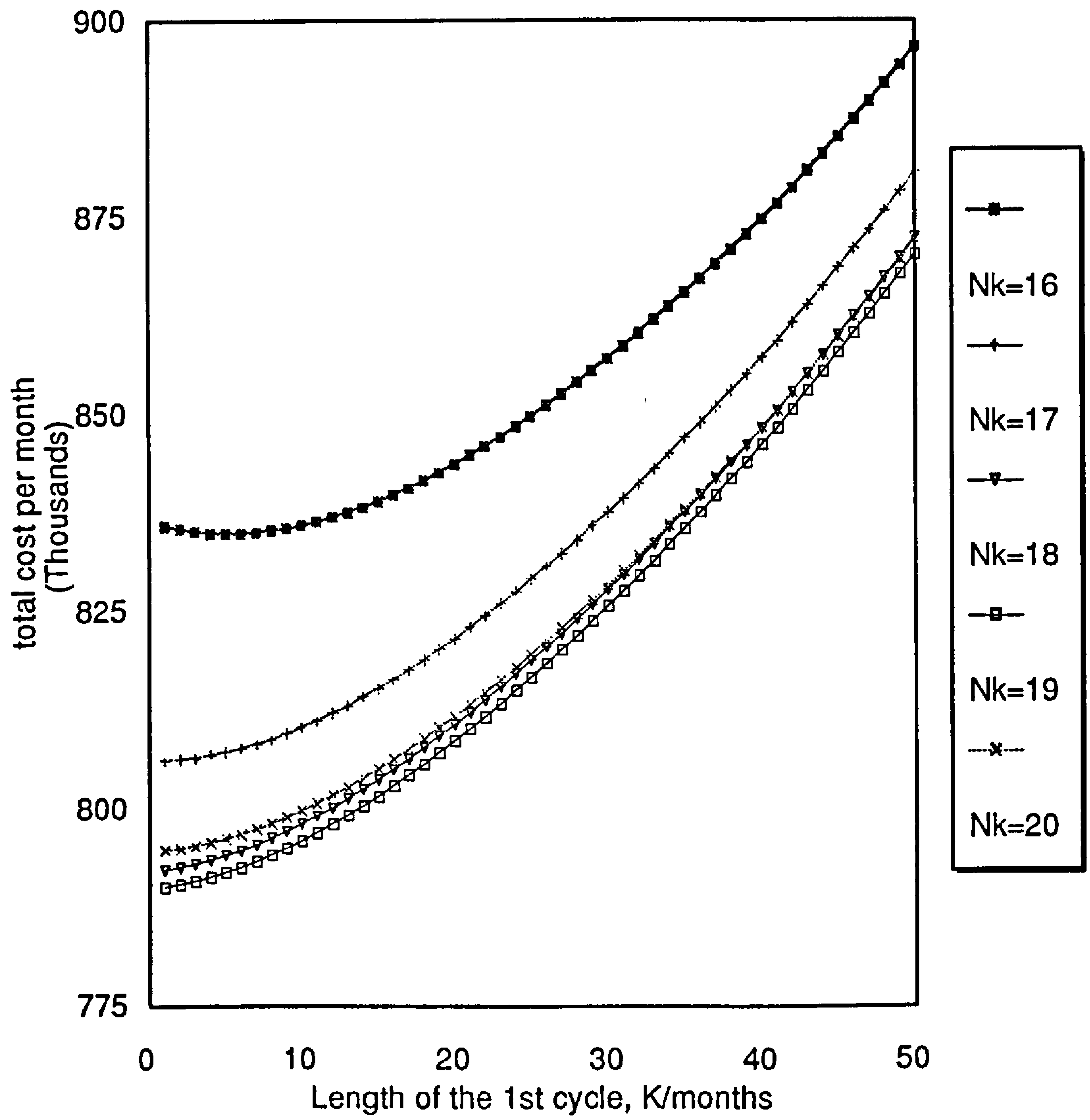
Penalty cost per breakdown	K* (months)	N_K^*	L* (months)	Cost $M000's
400	1	30	40	706
500	1	31	40	710
1000	1	32	40	717
2000	1	33	42	724

The Figures 5.13.a and 5.13.b illustrate total cost of the equivalent rent against time to replace for a value of the penalty cost equals to $M500 and $M2000 respectively, for the strategy which consists of replacing Cummin first and Mercedes second. In Figure 5.13.a we can observe that there is no sensitive difference between costs when the size of the replacement sub-fleet is kept equal to its current size ($N_K^* = 16$) instead of increasing it by one unit as recommended by the model. But if we increase the penalty up to $M2000, we can observe in Figure 5.13.b that keeping the current size ($N_K^* = 16$) for the replacement sub-fleet instead of increasing the size by 3 units, will incur an extra cost of $M30k per month for a period of 5 years. On the other hand, in Figures 5.13.c and 5.13.d we consider the replacement schedule Mitsubishi first, Cummin second for the same cost criterion, namely the equivalent rent. Two values of the penalty cost are considered, $M500 and $M2000. For the first value of the penalty cost, that is $M500, the difference in cost with respect to the current sub-fleet size is insignificant ($M69 per month for a period of approximately 3 years). For a penalty cost of $M2000 however, keeping the current sub-fleet size ($N_K^*=30$) for the replacement sub-fleet would cost an extra $M40k per month for a period of 3 years. We can notice that the extra cost incurred for both replacement schedules is nearly the same. Figures 5.14 and 5.15 illustrate respectively the cost of the equivalent rent against time to first replacement for a range of penalty cost with corresponding optimum sub-fleet sizes for the strategies Cummin replaced first, Mercedes second and Mitsubishi replaced first, Cummin second.



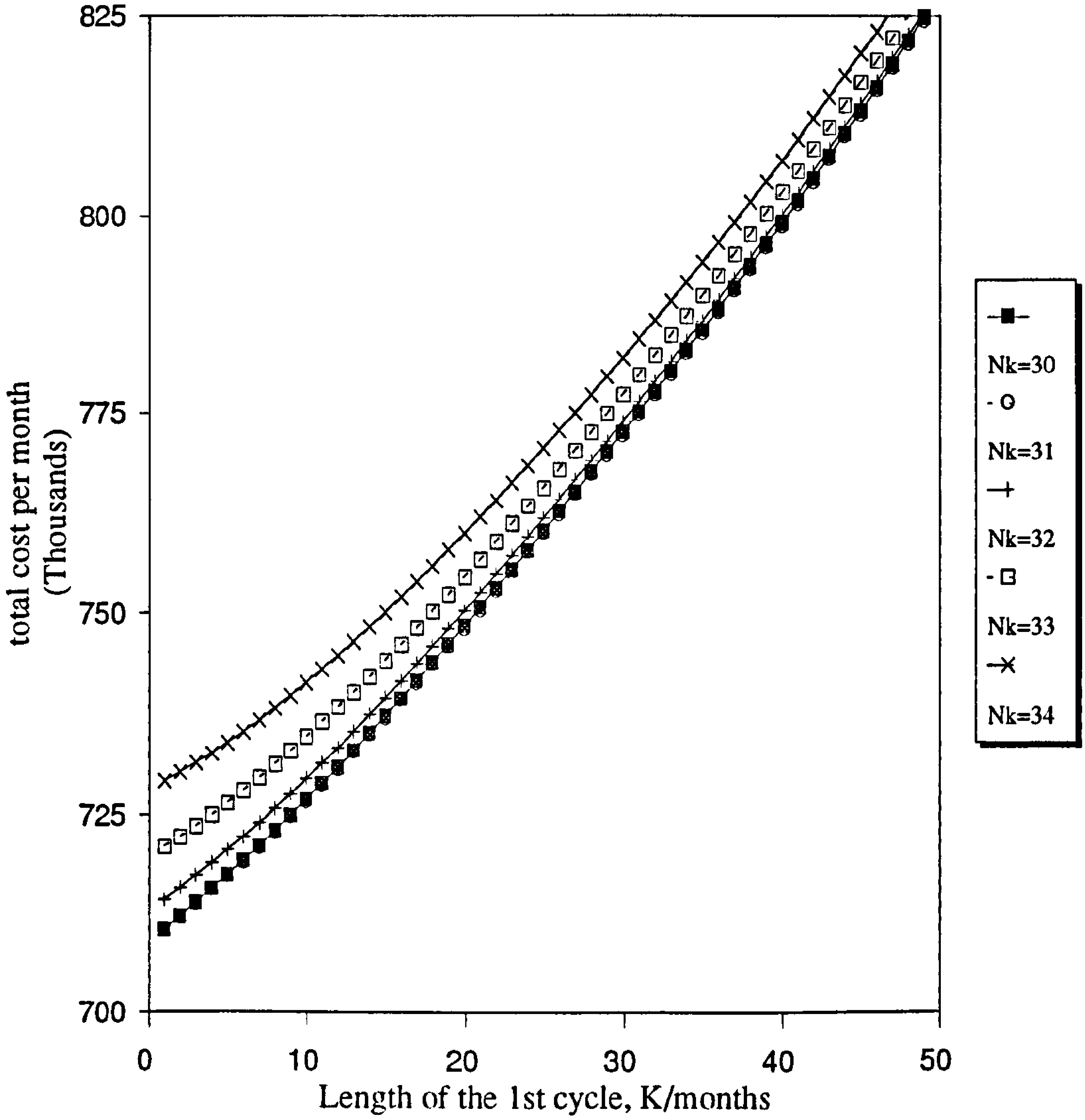
(a)

Figure 5.13.a. Cost of the equivalent rent per month (for whole fleet) vs. time to first replacement, K, for a range of values of N_K. Penalty=$M500. Cummin replaced first, Mercedes second. Two cycle model, variable fleet size (model IIb), (L=L*).



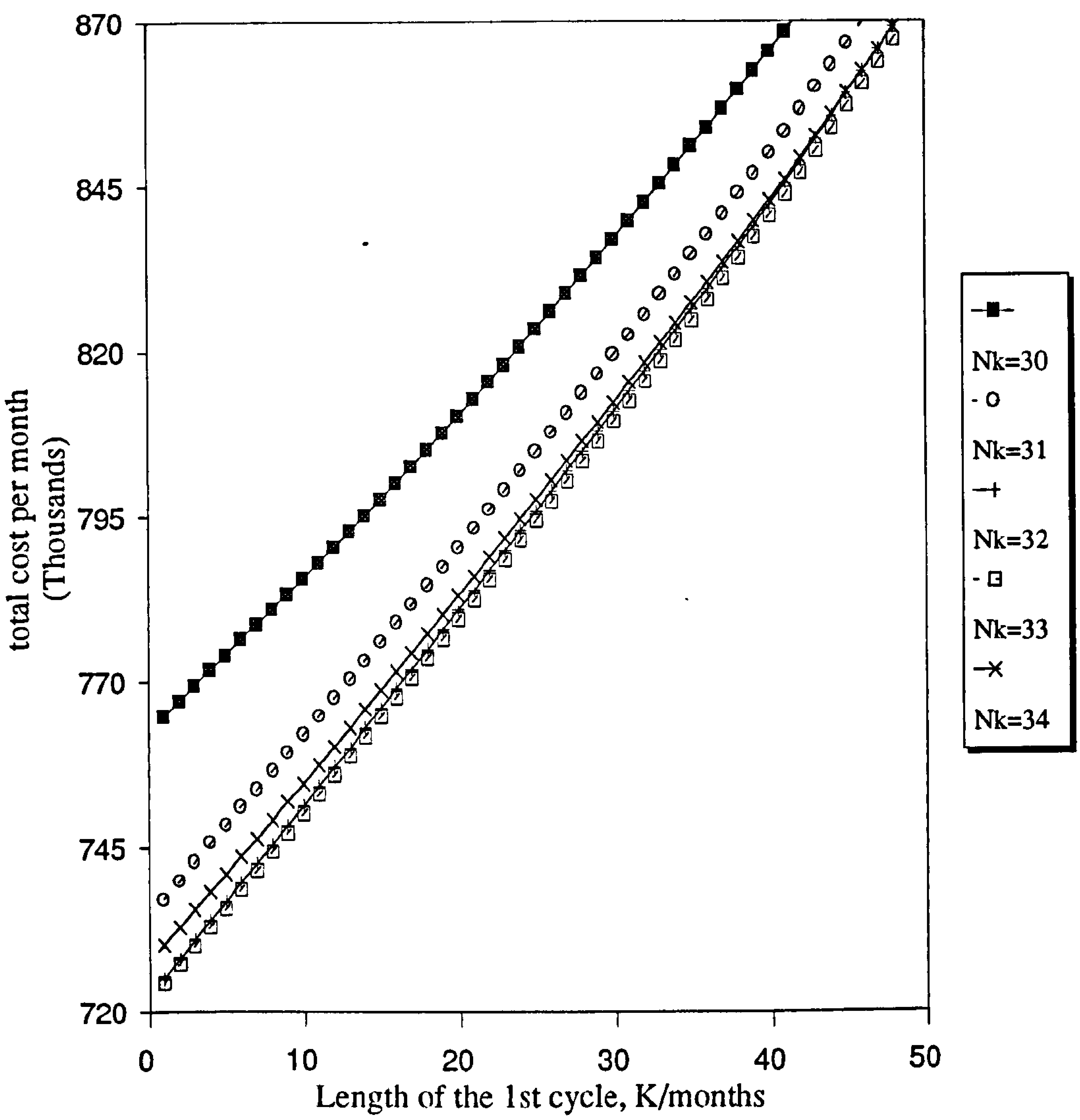
(b)

Figure 5.13.b. Cost of the equivalent rent per month (for whole fleet) vs. time to first replacement, K, for a range of values of N_K. Penalty=$M2000. Cummin replaced first, Mercedes second. Two cycle model, variable fleet size (model IIb), (L=L*).



(c)

Figure 5.13.c. Cost of the equivalent rent per month (for whole fleet) vs. time to first replacement, K, for a range of values of N_K. Penalty=$M500. Mitsubishi replaced first, Cummin second. Two cycle model, variable fleet size (model IIb), *(L=L*)*.



(d)

Figure 5.13.d. Cost of the equivalent rent per month (for whole fleet) vs. time to first replacement, K, for a range of values of N_K. Penalty=$M2000. Mitsubishi replaced first, Cummin second. Two cycle model, variable fleet size (model IIb), (L=L*).

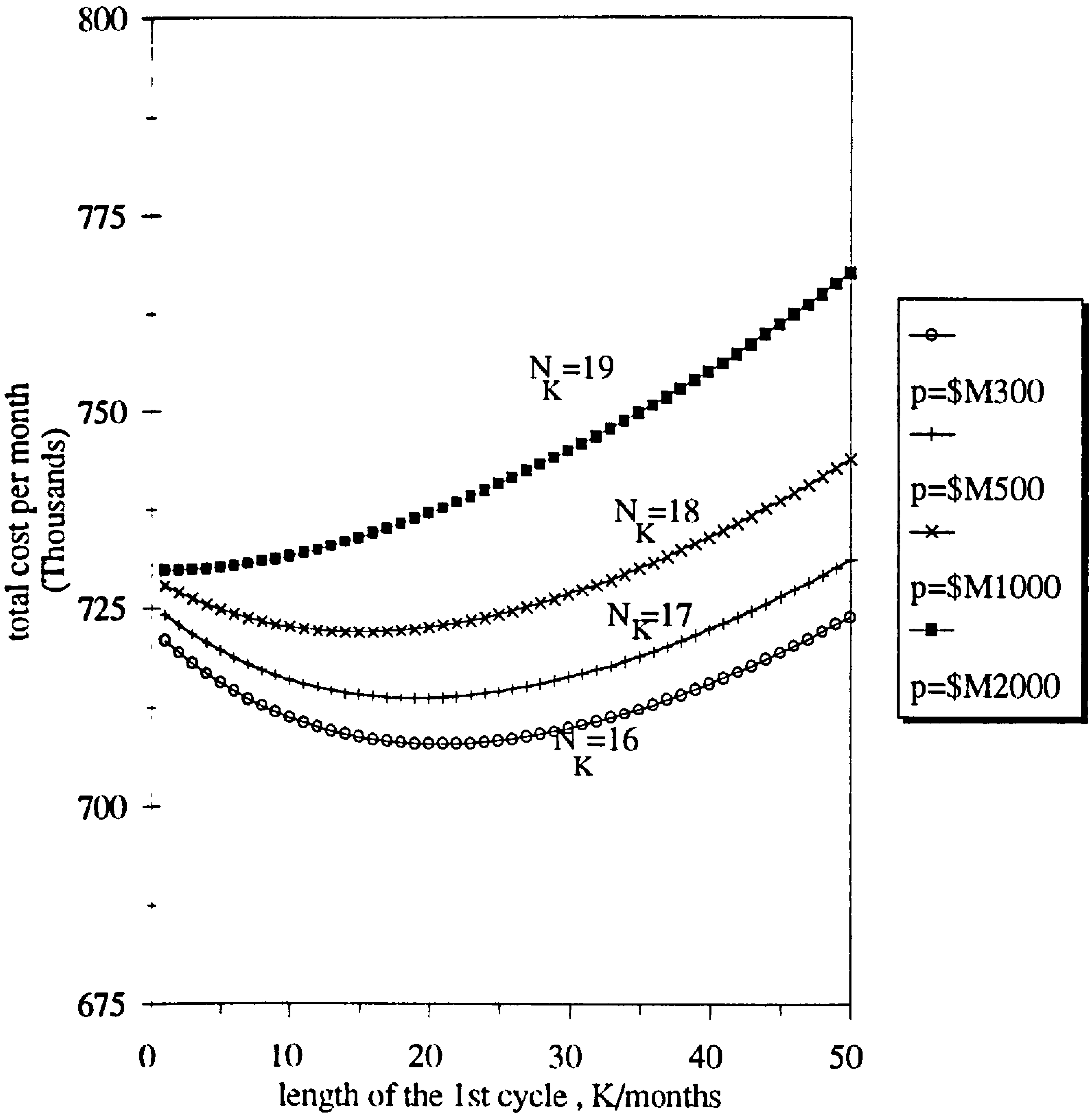


Figure 5.14. Cost of the equivalent rent per month (for whole fleet) vs. time to first replacement, K, for range of penalty costs. Corresponding optimal values of N_K shown. Cummin replaced first, Mercedes second. Two cycle model, variable fleet size (model IIb), ($L=L^*$).

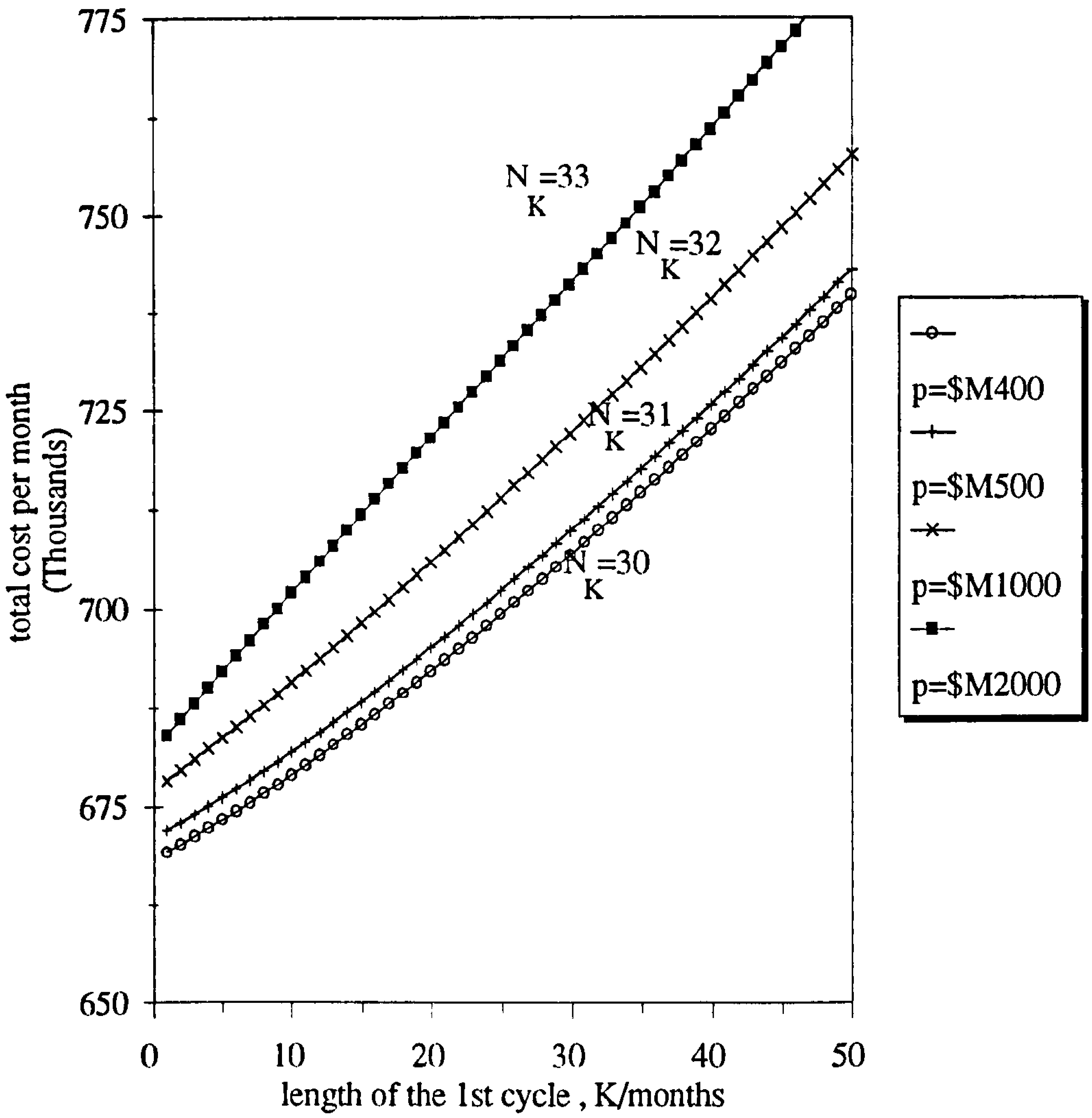


Figure 5.15. Cost of the equivalent rent per month (for whole fleet) vs. time to first replacement, K, for range of penalty costs. Corresponding optimal values of N_K shown. Mitsubishi replaced first, Cummin second. Two cycle model, variable fleet size (model IIb), (L=L^*).

5.4.4.2 *Total discounted cost per unit time*

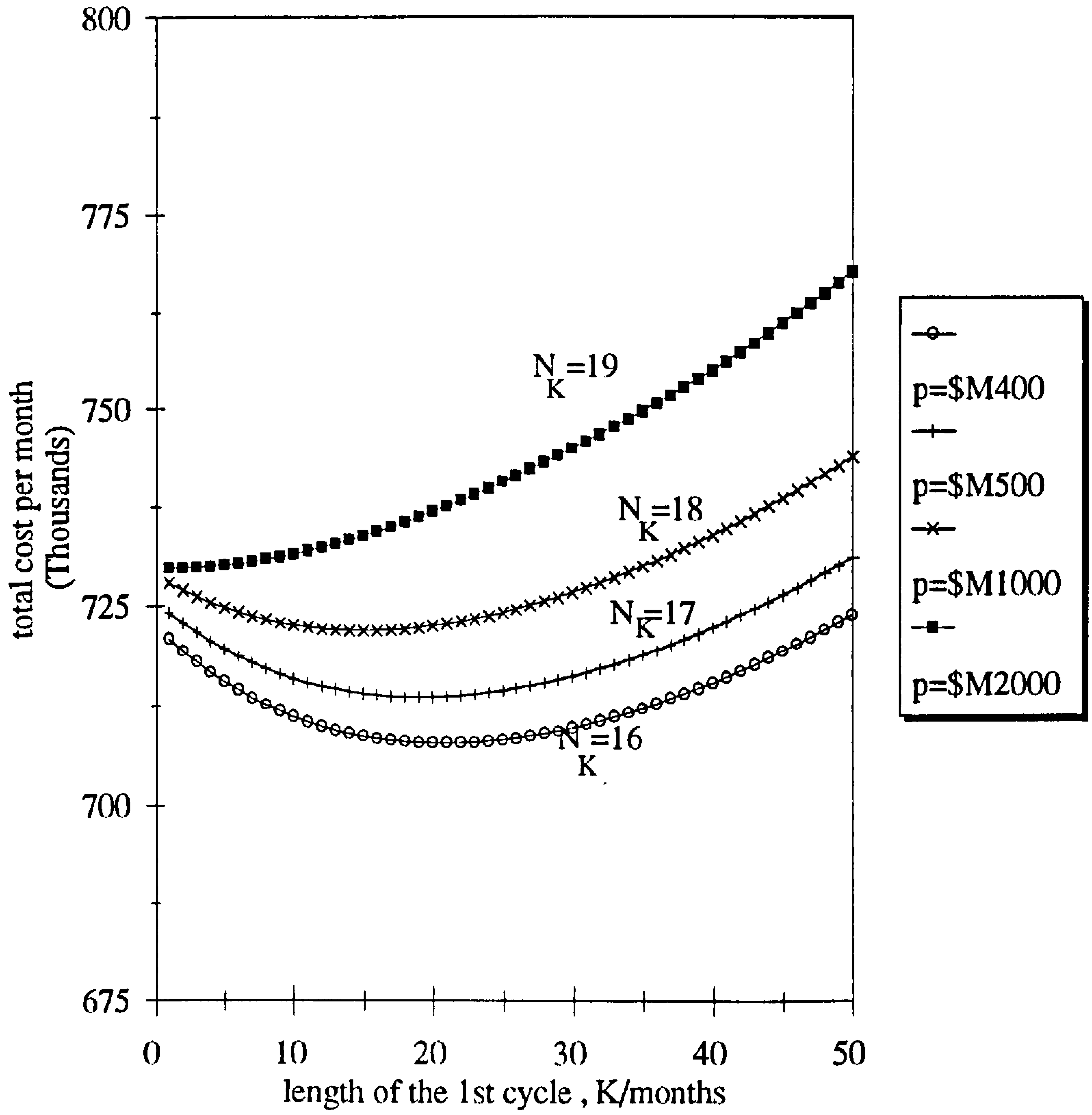
We have conducted the same work as for the previous section by using the total discounted cost per unit time as the objective function. The results we have obtained are summarised in tables 5.14 and 5.15, which represent the total discounted cost per unit time and the optimum values of the decision variables, namely age of replacement and sub-fleet size, for various ranges of penalty cost. We have considered the replacement strategies Cummin replaced first, Mercedes second and Mitsubishi replaced first, Cummin second. In table 5.14 we can observe that the values of K^* and L^* are greater than those obtained with the equivalent rent criterion (table 5.8), whereas the values of N_K^* are similar, but the costs are smaller for both tables. On the other hand in table 5.15 we can observe that the values of K^* and N_K^* are similar to those obtained in table 5.13 (rent criterion), however a slight difference is observed on the values of L^* which are larger than those of table 5.13. The results of tables 5.14 and 5.15 are illustrated in Figures 5.16.a and 5.16.b

Table 5.14. Minimum total discounted cost per month and optimum values of decision variables for various ranges of penalty cost. Cummin replaced first, Mercedes second. Two cycle model, variable fleet size ,(model IIb).

penalty cost per breakdown	K* (months)	N_K^*	L* (months)	TDC/month $M000's
300	20	16	54	709
500	20	17	56	713
1000	15	18	58	722
2000	1	19	66	729

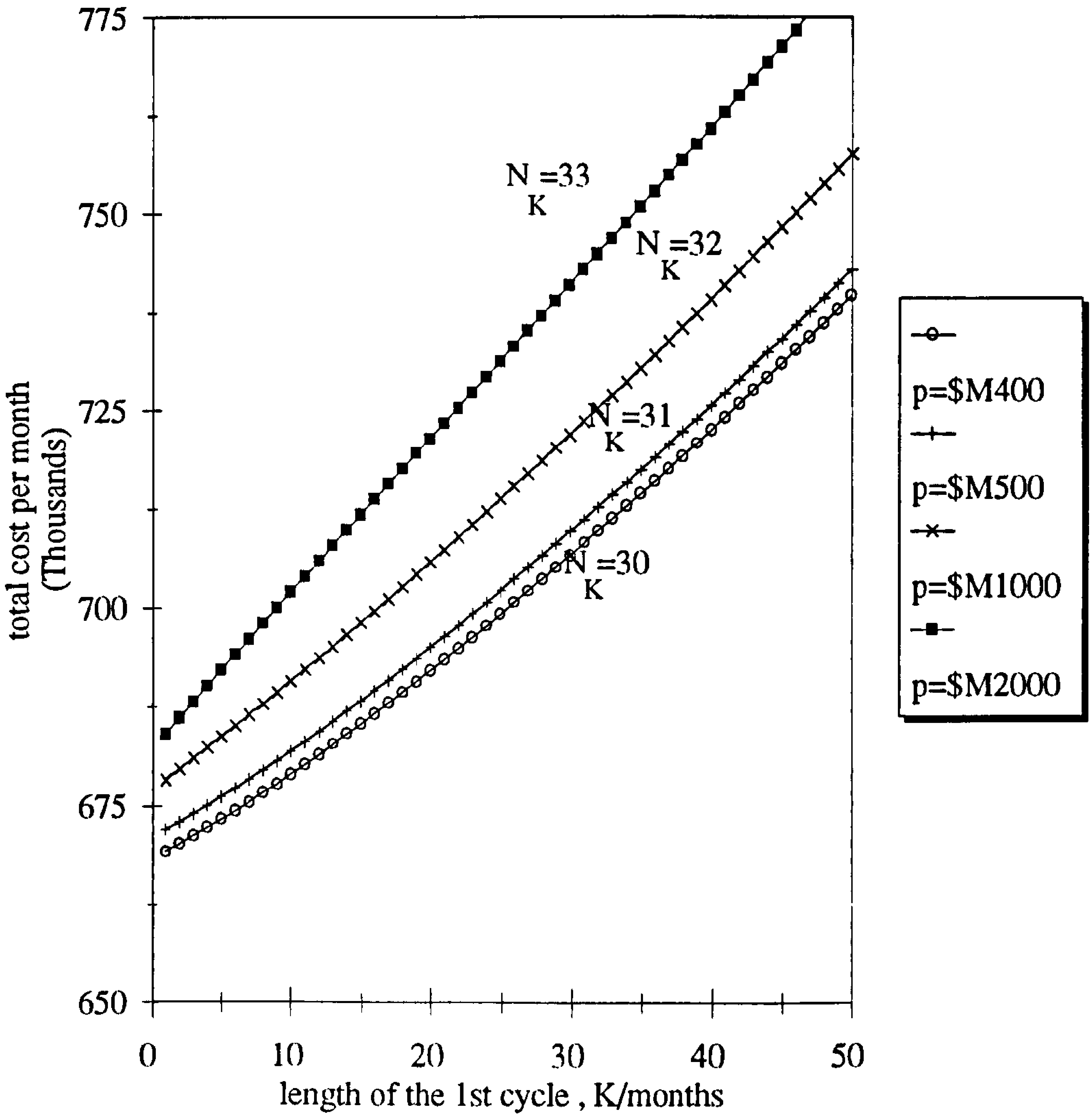
Table 5.15. Minimum total discounted cost per month and optimum values of decision variables for various ranges of penalty cost. Mitsubishi replaced first, Cummin second. Two cycle model, variable fleet size ,(model IIb).

penalty cost per breakdown	K* (months)	N_K^*	L* (months)	TDC/month $M000's
400	1	30	44	669
500	1	31	44	672
1000	1	32	45	678
2000	1	33	46	684



(a)

Figure 5.16.a . Cost of the total discounted cost per month (for whole fleet) vs. time to first replacement, K, for range of penalty costs. Corresponding optimal values of N_K shown. Cummin replaced first, Mercedes second. Two cycle model, variable fleet size (model IIb), ($L=L^*$).



(b)

Figure 5.16.b. Cost of the total discounted cost per month (for whole fleet) vs. time to first replacement, K, for range of penalty costs. Corresponding optimal values of N_K shown. Mitsubishi replaced first, Cummin second. Two cycle model, variable fleet size (model IIb), ($L=L^*$).

5.5 Cost of sub-optimal decisions

For various reasons and in many situations, it is not always possible for an operator to implement the optimal decision policy. The operator may rely on implementing some sub-optimal decision which, of course, incurs some extra cost. We consider here the costs of optimal decisions, such as those due to, delayed replacement, alternative replacement scheduled (choice of sub-fleet to be replaced) and smaller replacement sub-fleet size.

5.5.1 Delayed replacement

The marginal increased costs (equation (4.48), chapter 4, section 4.6.1) for a delayed replacement for a period of 12 and 24 months using the total discounted cost per month criterion, for both fixed fleet size (model IIa, equation (4.8)) and variable fleet size (model IIb, equation (4.15)), are presented in tables 5.16 and 5.17 respectively. For the variable fleet size case, we have considered both strategies Cummin-Mercedes and Mitsubishi-Cummin. With model IIa, we can observe in table 5.16, that a delay of 12 months incurs generally an increased cost of about $M300k, that is an equivalent cost of one Isuzu CJR (replacement sub-fleet by default). There is however exception for the replacement schedule Mitsubishi-Cummin for which the increased cost is $M731k. On the other hand for a delay of 24 months the increased cost is in general about $M1200k, that is the cost of four new Isuzu CJR buses. For the replacement schedule Mitsubishi-Cummin the increased cost incurred is $M1998k, that is the cost of 6 new Isuzu CJR. For the variable fleet size case, that is model IIb, we have considered a range of values of the penalty cost between $M300 and $M2000 for the replacement strategies Cummin-Mercedes and Mitsubishi-Cummin. The results are respectively presented in tables 5.17 and 5.18. In table 5.17 we can observe that the increased cost decreases as the penalty cost increases from $M500 to $M2000 for both 12 months and 24 months delay. This is due the fact that for this replacement schedule the curves of the total discounted cost per unit time are flat in the neighbourhood of the optimum value (Figure 5.15.a). On the other hand, in table 5.18, that is for the replacement schedule Mitsubishi-Cummin, we can observe that the increased cost of delayed replacement increases as the penalty cost increases. This is due to the fact that the value of the rate of increase of the total discounted cost per unit time is high. Figure 5.16.b gives a clear picture of that increase.

Table 5.16 Marginal increased discounted cost for delayed replacement for fixed fleet size for various replacement schedules. Two cycle model, fixed fleet size (model IIa) for the total discounted cost criteria, ($L=L^*$).

Repl. Schedule 1st repl.-2nd repl.	increased cost of delayed replacement (M$000's)	
	12 months delay	24 months delay
Cummin - Mercedes	277	1117
Cummin - Mitsubishi	234	847
Cummin - Isuzu CSA	251	1059
Mercedes - Cummin	333	1305
Mercedes - Mitsubishi	325	1265
Mercedes - Isuzu CSA	297	1217
Mitsubishi - Cummin	731	1998
Mitsubishi - Mercedes	265	1108
Mitsubishi - Isuzu CSA	286	1143
Isuzu CSA - Cummin	331	1286
Isuzu CSA - Mercedes	304	1240
Isuzu CSA - Mitsubishi	289	1184

Table 5.17. Marginal increased discounted cost for delayed replacement, for various ranges of penalty cost for variable fleet size. Two cycle model, variable fleet size for the total discounted cost criteria (model IIb). Cummin sub-fleet replaced first, Mercedes sub-fleet second.

Penalty cost per breakdown	increased cost of delayed replacement (M$000's)	
	12 months delay	24 months delay
300	244	1054
500	314	1195
1000	268	1088
2000	238	989

Table 5.18. Marginal increased discounted cost for delayed replacement, for various ranges of penalty cost for variable fleet size. Two cycle model, variable fleet size for the total discounted cost criteria (model IIb). Mistubishi sub-fleet replaced first, Cummin sub-fleet second.

Penalty cost per breakdown	increased cost of delayed replacement (M$000's)	
	12 months delay	24 months delay
400	772	2073
500	777	2086
1000	975	2486
2000	1410	3361

5.5.2 Alternative replacement schedules

Tables 5.6. and 5.7 in section 5.4.2 show the total cost incurred per month for each choice of replacement schedule decision for both criteria, the equivalent rent and the total discounted cost per unit time respectively. We might notice that when a choice for replacing a sub-fleet is made, sometimes the cost incurred is relatively high. In the case presented in table 5.6, that is under the rent model, replacing for example the Cummin sub-fleet first and then the Mercedes sub-fleet will cost $M759k per month, while replacing Mercedes first and then the Cummin will cost $M843k per month, that is a difference of $M84k per month for a period of 3 years if the operator choose the second option. On the other hand, if we consider the replacement schedule Mitsubishi first and then the Cummin, the cost per month will be $M693k. But if the company prefer to replace the Mercedes first and then the Cummin, the extra cost incurred would be $M150k per month over a period of 3 years. For the total discounted cost criterion, results from table 5.7 show that for the replacement schedule Cummin first Mercedes second versus the schedule Mercedes Cummin, a difference of $M82k over a period of 4 years is incurred, whereas the schedule Mitsubishi-Cummin versus Mercedes-Cummin incurred $M123k over a period of 4 years. We can notice that, for both criteria, the extra cost incurred and the period over which it is incurred are identical for the Cummin-Mercedes schedule, it is however a little bit lower for the schedule Mitsubishi-Cummin ($M123k per month).

5.5.3 Smaller replacement sub-fleet size

The sensitive effect of using a smaller replacement sub-fleet size can be observed when the penalty cost of unavailability is set to $M2000, for example, by considering Figure 5.12.b. If, instead of replacing by a sub-fleet of size 19, as is suggested by the model, the operator keeps the current size for the replacement sub-fleet (Isuzu CJR), that is 16, an extra cost of $M30K per month would be incurred for a period of 5 years. If we consider, however the purchase of three extra buses which cost $M300k each, the operator has to spend $M900k over a period of 5 years, which represents $M16k per month, which is half the cost incurred if the operator kept the fleet at its current size. Therefore it is recommended for the operator to buy three extra buses. Table 5.19 shows the cost incurred when the replacement sub-fleet size is smaller than the optimal size obtained through the model. We assume that the operator keeps the size of the fleet at its current size, but there is no difficulty to consider another size for the replacement sub-fleet.

Table 5.19. Cost incurred when replacement sub-fleet size is smaller than the optimal size for various values of the penalty cost.

Replacement schedule	Penalty M$	Nk=N* (cost M$000)	Nk= N1 (cost M$000)
	500	31 (710)	30 (711)
Mitsubishi - Cummin	1000	32 (717)	30 (728)
	2000	33 (724)	30 (764)
	500	17 (777)	16 (778)
Cummin - Mercedes	1000	18 (784)	16 (797)
	2000	19 (789)	16 (835)

5.6 Sensitivity Analysis

5.6.1 Introduction

In this section, we carry on sensitivity analysis on maintenance cost, discount factor, resale value, purchase cost and demand.

5.6.2 Sensitivity analysis to maintenance cost

For the maintenance cost, we consider the case when the *y*-intercepts of the maintenance cost function for each bus are non identical, that is when the cost function is expressed as

$$M_i(t) = \alpha_i t^{\beta_i},$$

where i indexes the sub-fleet number. We also consider sensitivity on the value of β by varying its value.

First we consider sensitivity to fitting. The maintenance cost is fitted as a log-log linear function using the statistical package GLIM (McCullagh and Nelder, 1990), where the parameters α and β are defined for each sub-fleet. Figure 5.17 shows the function of the maintenance cost for each sub-fleet. For the Isuzu CJR, we can observe that the predicted cost from the age three, is relatively high. From this fitting, we want to observe the effect on the optimal decision. We will consider the case of fixed fleet size, that is to use the two cycle model denoted model IIa, and the results are presented in table 5.20. This table shows an increase for the value of the optimum age of the first replacement in comparison with table 5.6. This increase in the value K^* is explained by the fact that the replacement sub-fleet (Isuzu CJR) has now a very high maintenance cost, therefore the operator would rather keep the current fleet as long as possible. We also, can observe that the optimum policy, in regard to the minimum cost, has not changed. Indeed, the strategy Mitsubishi replaced first, Cummin second incurs the smallest cost, but recommends immediate replacement. The near optimal strategy has slightly changed regarding the third best policy which is no longer Cummin first, Mercedes second but rather Isuzu CSA first, Mitsubishi second. This stability in the optimum decision shows a good behaviour of the mathematical model we have used.

Table 5.20. Minimum cost of the equivalent rent optimum values of the decision variables K and L for various replacement schedule. Two cycle model, with fixed fleet size (model IIa)

Replacement schedule 1st repl. - 2nd repl.	K* months	L* months	min. cost $M000's
C - Me	20	36	780
C - Mit	9	36	735
C - Is	20	36	789
Me - C	35	24	804
Me - Mit	42	24	858
Me - Is	48	24	895
Mit - C	1	36	669
Mit - Is	22	36	799
Mit - Me	22	36	790
Is - C	13	36	756
Is - Mit	21	36	819
Is - Me	28	36	852

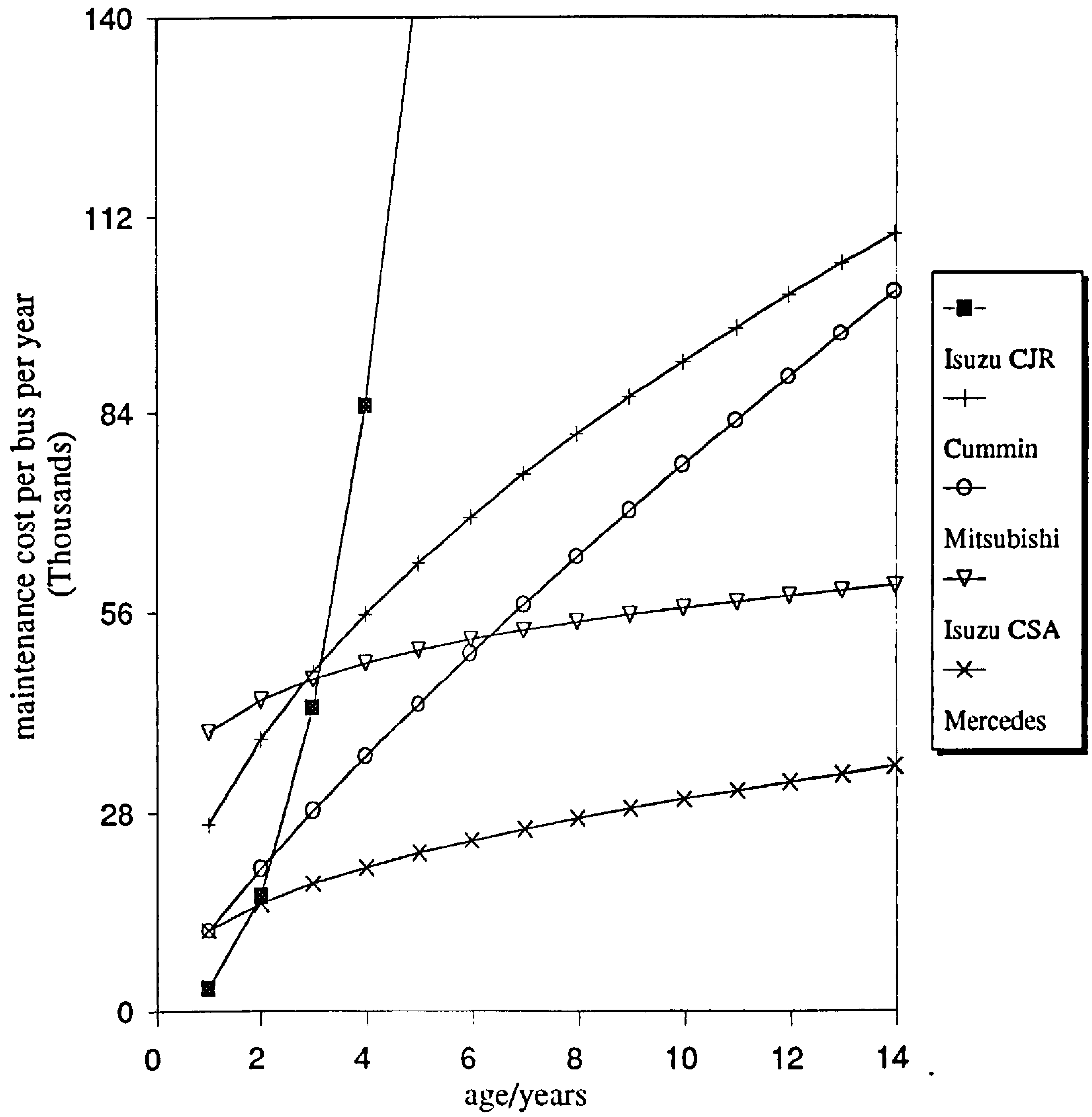


Figure 5.17. Fitted maintenance cost per year for each bus for each sub-fleet. Mercedes, $M(\tau) = 11384\tau^{0.42}$; Cummin, $M(\tau) = 26370l\tau^{0.54}$; Isuzu CSA, $M(\tau) = 39222\tau^{0.16}$, Mitsubishi, $M(\tau) = 11350\tau^{0.83}$; Isuzu CJR, $M(\tau) = 3093\tau^{2.39}$.

The other way to look at the sensitivity analysis on the maintenance cost is to vary the value of β and observe the effect on the replacement decision. As mentioned previously, the company's policy was to replace the Mercedes sub-fleet first and then the Cummins. The maintenance cost is fitted as a power law function (log-log linear), that is the cost function is expressed as

$$M(t) = \alpha t^{\beta}$$

We will try to determine how much the value of β for the Cummins, i.e. the rate of increase of the maintenance cost per unit time (on log-log scale), should vary in order to comply with the company's policy, that is how much the current maintenance practice of the Cummin need to be improved. Madu, (1994) discusses how the Total Productive Management (or Maintenance) (TPM) might improve the effectiveness of any maintenance operation. TPM is a Japanese maintenance philosophy which consists of maximising equipment effectiveness by involving maintenance staff at any level of decision (Pintelon and Gelders, 1992). Planned maintenance models are used to optimise inspection time in order to reduce downtime due to breakdowns. This is an important factor towards improving maintenance practice which was used by Christer and Waller (1984) through the delay time technique.

In absence of information about any new maintenance practice, we have carried out a sensitivity analysis on maintenance cost by varying the parameter β through a range of values using the two cycle model for fixed fleet size (model IIa). For that purpose, and as an illustration we consider both replacement schedules; Cummin replaced first, Mercedes second (C-Me), and Mercedes replaced first, Cummin second (Me-C). In table 5.6, we can observe that the replacement schedule C-Me incurs a lower cost than the replacement schedule Me-C. Using a range of different values of β for the maintenance cost function of the Cummin sub-fleet we can reverse the decision Cummin-Mercedes to Mercedes-Cummin. Table 5.21 shows that for a value of $\beta \leq 0.7$, the replacement schedule based on minimal cost which is suggested recommends to replace the Mercedes sub-fleet first and then the Cummins. By decreasing the value of β for the Cummin sub-fleet by at least 53%($=\frac{1.50-0.7}{1.50}\times 100$), the equivalent rent per month for the whole fleet for the policy Me-C, is reduced by about 15%($=\frac{843000-717000}{843000}\times 100$). This shows the amount of effort the company should make to improve the quality of their equipment. Figure 5.18 illustrates the point at which the policy changes from the replacement schedule Cummin-Mercedes to Mercedes-Cummin. This of course, if necessary, can be done for all different replacement schedules without difficulty.

Table 5.21. Minimum cost per month of the equivalent rent, for different values of the slope β (log-log scale) for two different replacement strategies. C-Me; Me-C. Two cycle model, fixed fleet size (model IIa)

ß	Cummin- Mercedes ($M000's)	Mercedes Cummin ($M000's)
0.5	716	705
0.7	724	717
0.9	734	735
1.1	744	760
1.2	749	776
1.3	753	795
1.5	759	843

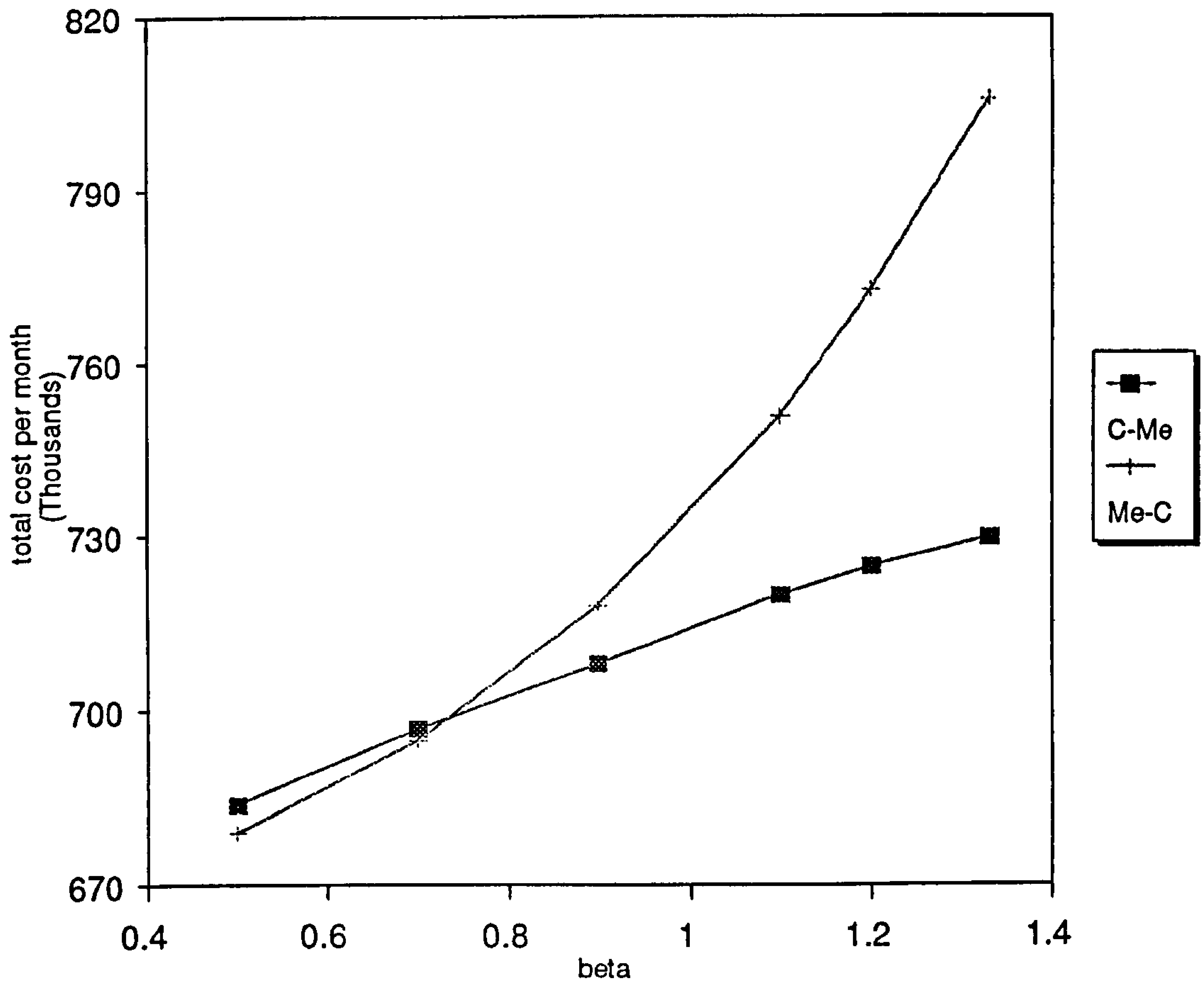


Figure 5.18. Effect of β on optimal cost.

5.6.3 Sensitivity analysis for discount factor

As an illustration again, we consider the replacement schedule Cummin-Mercedes against Mercedes-Cummin. A wide range of values of the discount factor have been considered starting from 0.92 to 1.04, using model IIa, that is the two cycle model with fixed fleet size, for both criteria, namely the equivalent rent and the total discounted cost per unit time. A value of the discount factor greater than one means a higher rate of inflation compared to the rate of return, this will obviously lead to earlier replacement. Tables 5.22 and 5.24 show the effect of the discount factor on the optimal values of the decision variables and on the minimum cost of the equivalent rent and the total discounted cost per month respectively. Tables 5.22 and 5.23 show that the replacement policy Cummin-Mercedes has not been affected by the variation of the discount factor with respect to the replacement policy Mercedes-Cummin. The replacement strategy Cummin-Mercedes appears to be always cheaper than the replacement strategy Mercedes-Cummin chosen by the company. This conclusion, of course, is based only on this example, but it shows that the decision variables are affected by the variation of the discount factor but not the replacement decision, that is what sub-fleet to replace first. We might notice however, that, as the value of the discount factor ν decreases, that is the rate of return η is greater than the interest rate ι, the optimal value of the length of the first cycle K^*, increases, whereas the length of the second cycle is getting shorter. This is an expected result because as far as the plant produces a much higher rate of return it is recommendable to keep the operating plant much longer. We might also notice that, as the length K^*, of the first cycle increases, the length L^*, of the second cycle decreases as a result of the ageing effect of the second sub-fleet due to interaction between sub-fleets. In other words, as long as the first sub-fleet is kept, the second sub-fleet has its age increasing by the same length of time K^*, besides its current age, therefore its replacement is expected to be sooner.

For the total discounted cost criterion, table 5.24 shows that when ν is greater than 1, the minimum cost is greater than the minimum cost under the equivalent rent criterion, this is obvious since the denominator of the equivalent rent becomes greater than the denominator of the total discounted cost. Figure 5.19 shows the cost of the equivalent rent against age of replacement for various values of the discount factor using the two cycle model for fixed fleet size (model IIa). Figures 5.20 and 5.22 illustrate the effect of the discount factor on the values of the optimum *K*, *L* and minimum cost of the equivalent rent and the total discounted cost per month respectively, for the replacement strategy Cummin - Mercedes, for the case of fixed fleet size (model IIa). Figure 5.21 illustrates the effect of the discount factor on the values of K^*, L^* and the minimum cost of the equivalent rent for the replacement strategy Mercedes - Cummin. In all those figures we can observe that the values of K^*+L^* is roughly constant.

Table 5.22. Cost per month (for whole fleet) of the minimum equivalent rent and optimal values of decision variables for various ranges of discount factor, two cycle model, fixed fleet size (model IIa). Cummin first, Mercedes second.

Discount	Cummin-Mercedes		
factor	K*	L*	M$000
0.92	31	32	769
0.93	30	32	769
0.94	24	38	769
0.95	23	38	767
0.96	17	44	764
0.97	11	50	759
0.98	5	56	752
0.99	1	62	743
1	1	62	734
1.01	1	68	724
1.02	1	68	713
1.03	1	74	703
1.04	1	74	692

Table 5.23. Cost per month (for whole fleet) of the minimum equivalent rent and optimal values of decision variables for various ranges of discount factor, two cycle model, fixed fleet size (model IIa). Mercedes first, Cummin second.

Discount	Mercedes-Cummin		
factor	K*	L*	M$000
0.92	56	6	820
0.93	55	6	826
0.94	48	12	832
0.95	47	12	837
0.96	40	18	841
0.97	39	18	843
0.98	31	24	844
0.99	24	30	843
1	17	36	838
1.01	4	48	830
1.02	2	48	819
1.03	1	54	807
1.04	1	54	794

Table 5.24. Cost per month (for whole fleet) of the total discounted cost and optimal values of decision variables for various ranges of discount factor, two cycle model, fixed fleet size (model IIa). Cummin first, Mercedes second.

Discount	Cummin-Mercedes		
factor	K*	L*	M$000
0.92	48	56	583
0.93	48	56	607
0.94	48	50	631
0.95	44	50	656
0.96	32	50	687
0.97	20	56	697
0.98	12	56	712
0.99	2	62	724
1	1	62	734
1.01	1	62	744
1.02	1	62	754
1.03	1	62	765
1.04	1	62	777

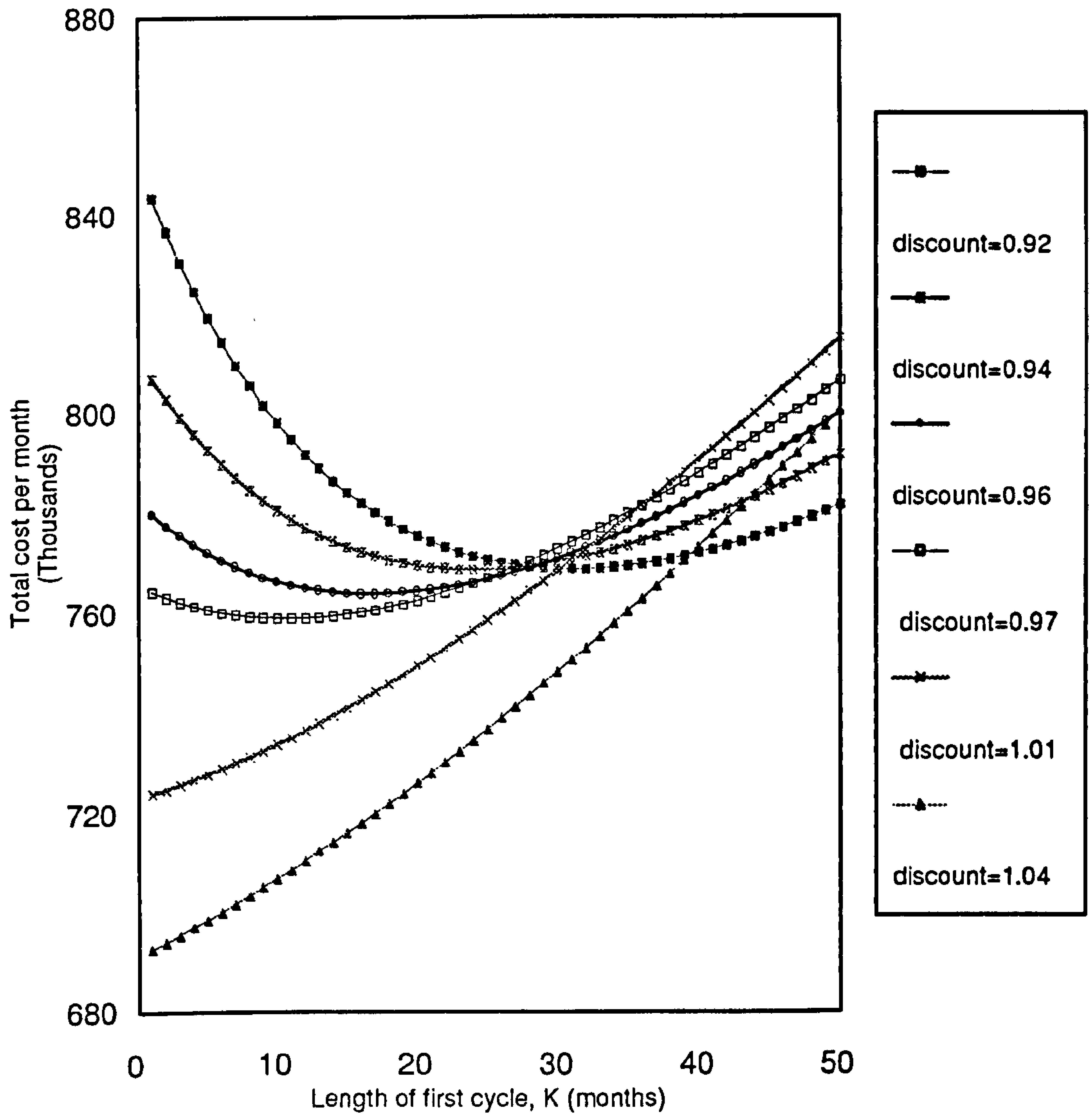


Figure 5.19. Cost of the equivalent rent per month vs. time of first replacement, K, for a range of values of discount factor using rent model (model IIa) for replacement strategy C-me.

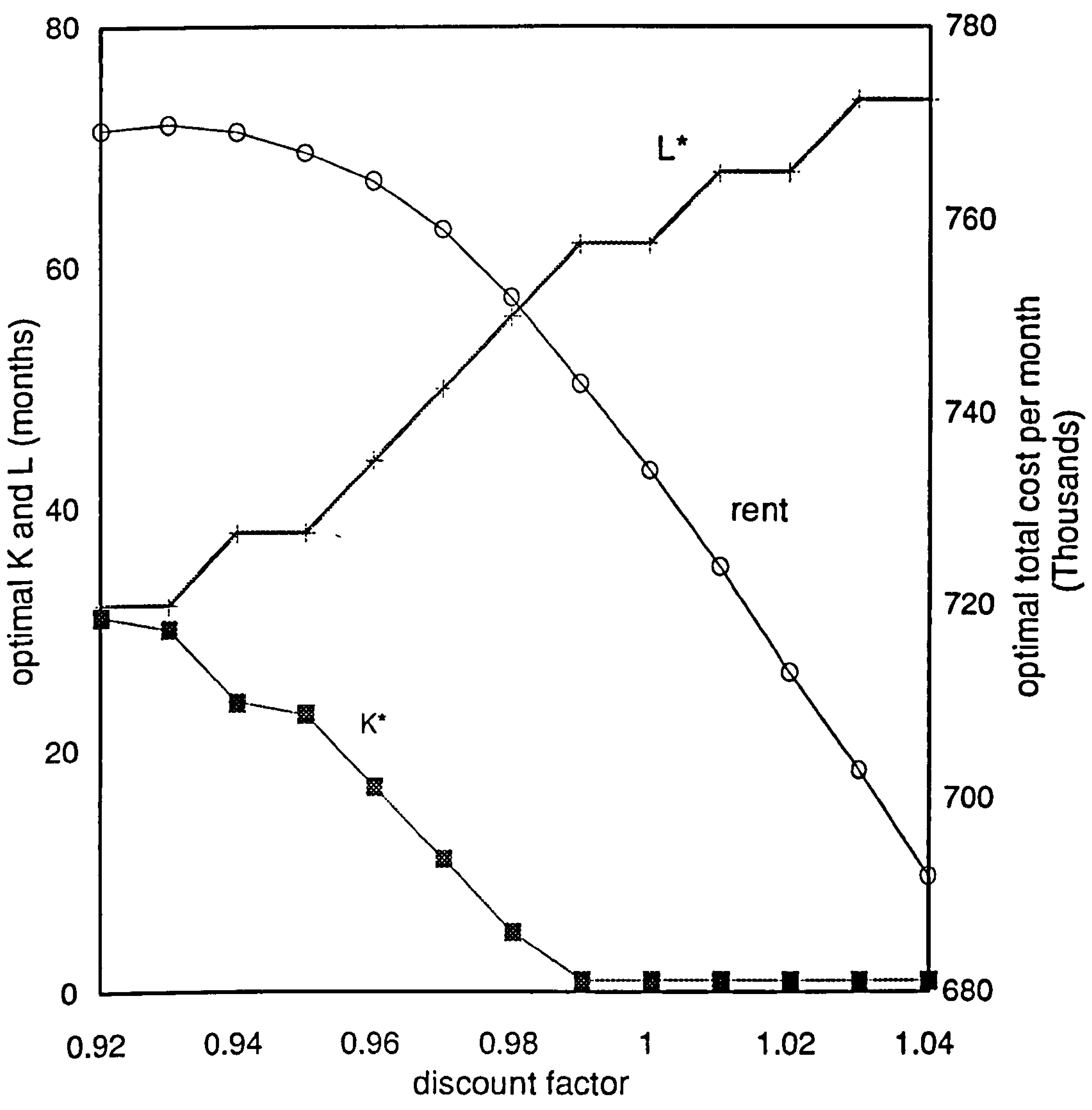


Figure 5.20. Effect of discount factor on optimal age of replacement of 1st and 2nd cycle and optimal equivalent rent, for replacement strategy C-Me, (model IIa).

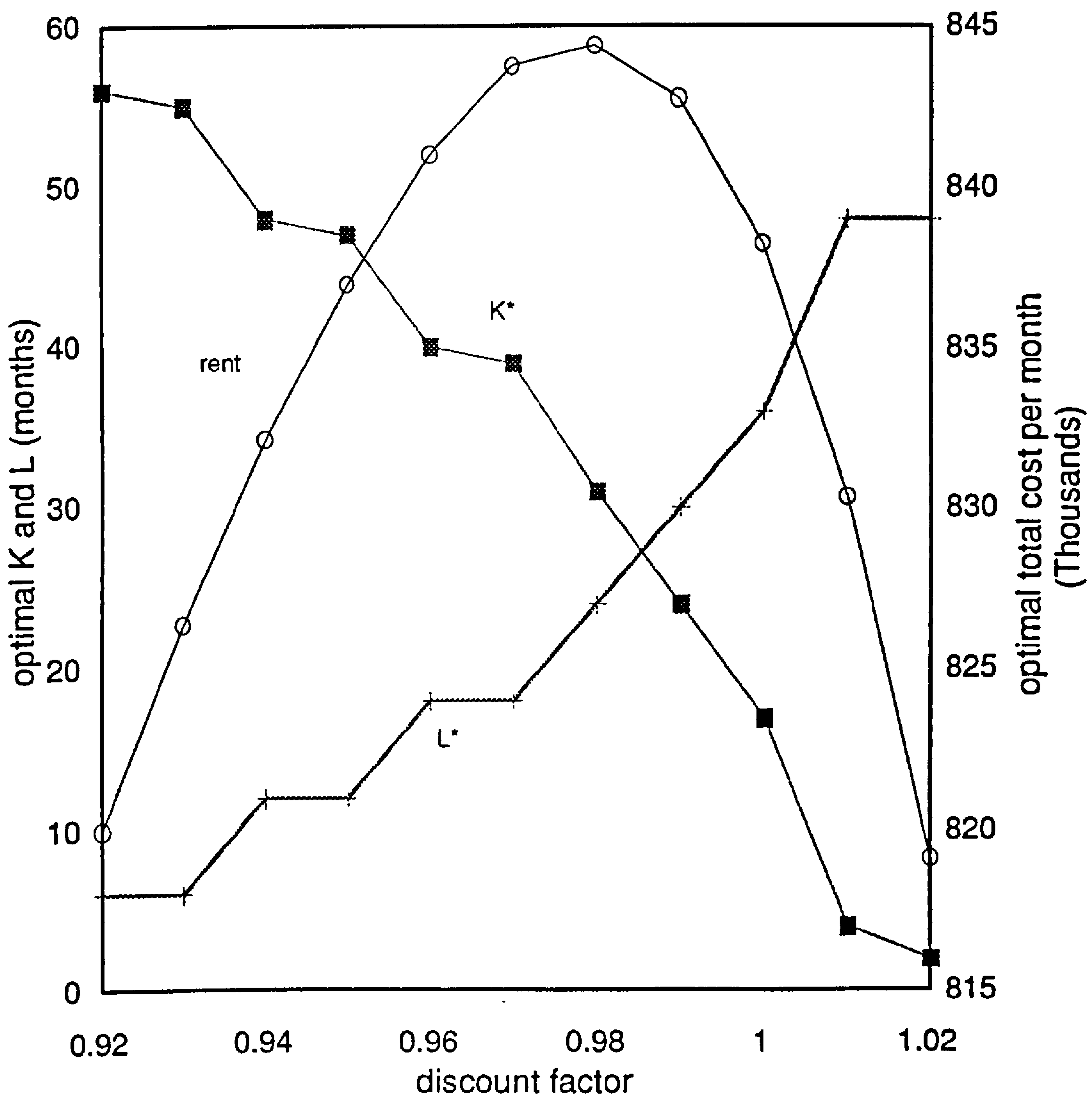


Figure 5.21. Effect of discount factor on optimal age of replacement of 1st and 2nd cycle and optimal equivalent rent, for replacement strategy Me-C, (model IIa).

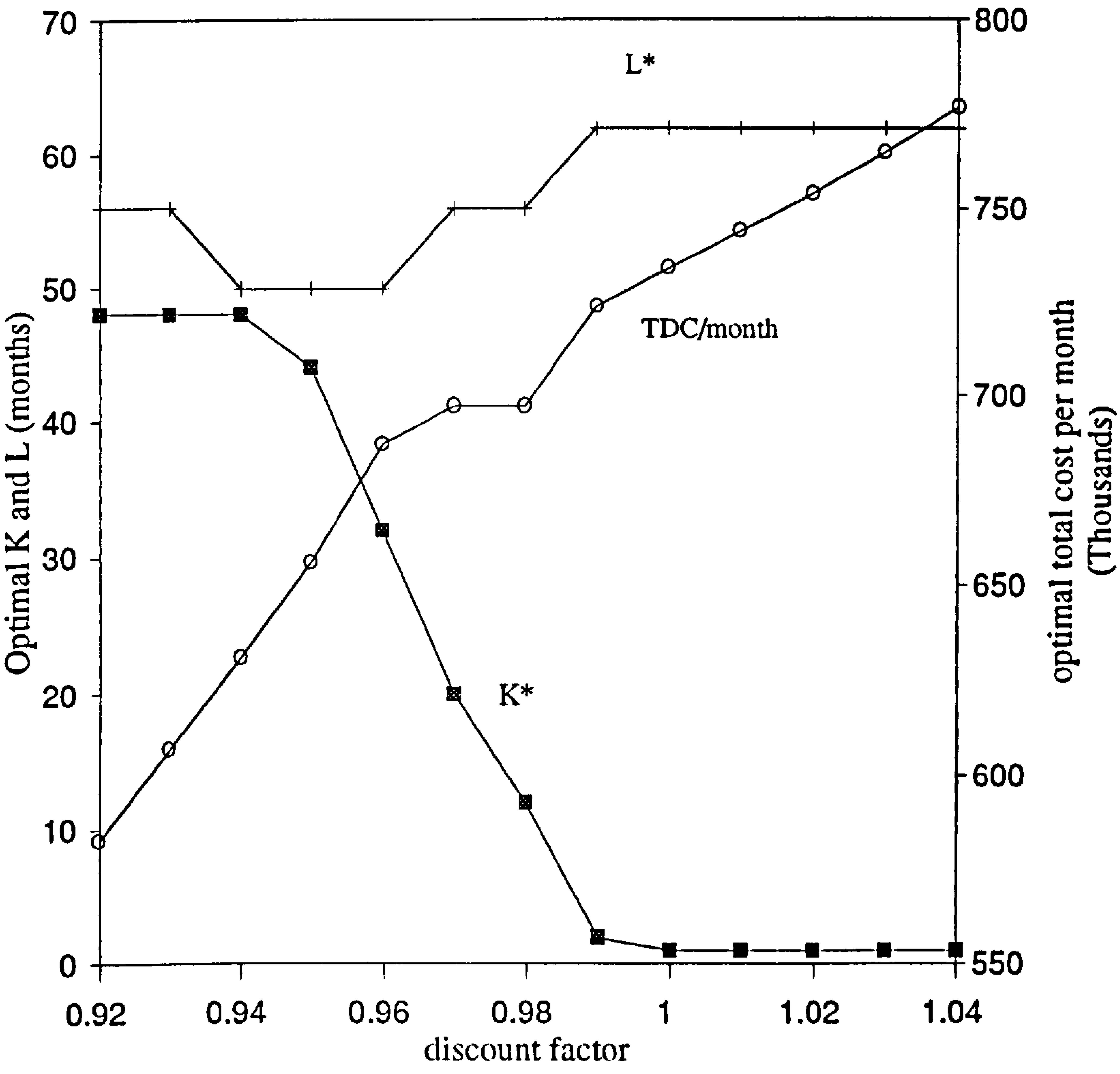


Figure 5.22. Effect of discount factor on optimal age of replacement of 1st and 2nd cycle and optimal total discounted cost per month, for replacement strategy C-Me, (model IIa).

5.6.4 Sensitivity analysis to resale costs

By considering a resale value factor, defined as the ratio of the assumed resale price to the current resale value (Kobbacy and Nicol, 1994), that is

$$\text{resale value factor} = \frac{\text{assumed resale value}}{\text{current resale value}},$$

we carry out sensitivity analysis on resale value to show the effect of a variation of the resale value factor on the optimal age of replacement K^* and the optimal equivalent rent per month.

We have first considered the two cycle model using the rent criterion with fixed fleet size (model IIa). Table 5.25 and Figure 5.23 shows for an increase up to 50% above the current resale value a decrease of the optimum age of replacement, K^*, and the equivalent rent per month are observed. It is however unrealistic, that resale values are so high especially in an environment where economy is stable and inflation under control, but this is plausible in most of the developing countries. At the opposite case, that is decreasing the resale value by 50% or more below its current value, increases in the age of replacement as well as the equivalent rent per month can be observed.

Table 5.25. Effect of resale value factor on K^* and on minimum equivalent rent, Cummin replaced first , Mercedes second, two cycle model, fixed fleet size (model IIa).

resale value factor	K*	Cost ($M000's)
0	24	793
0.5	19	778
1	11	759
1.5	1	734
2	1	703

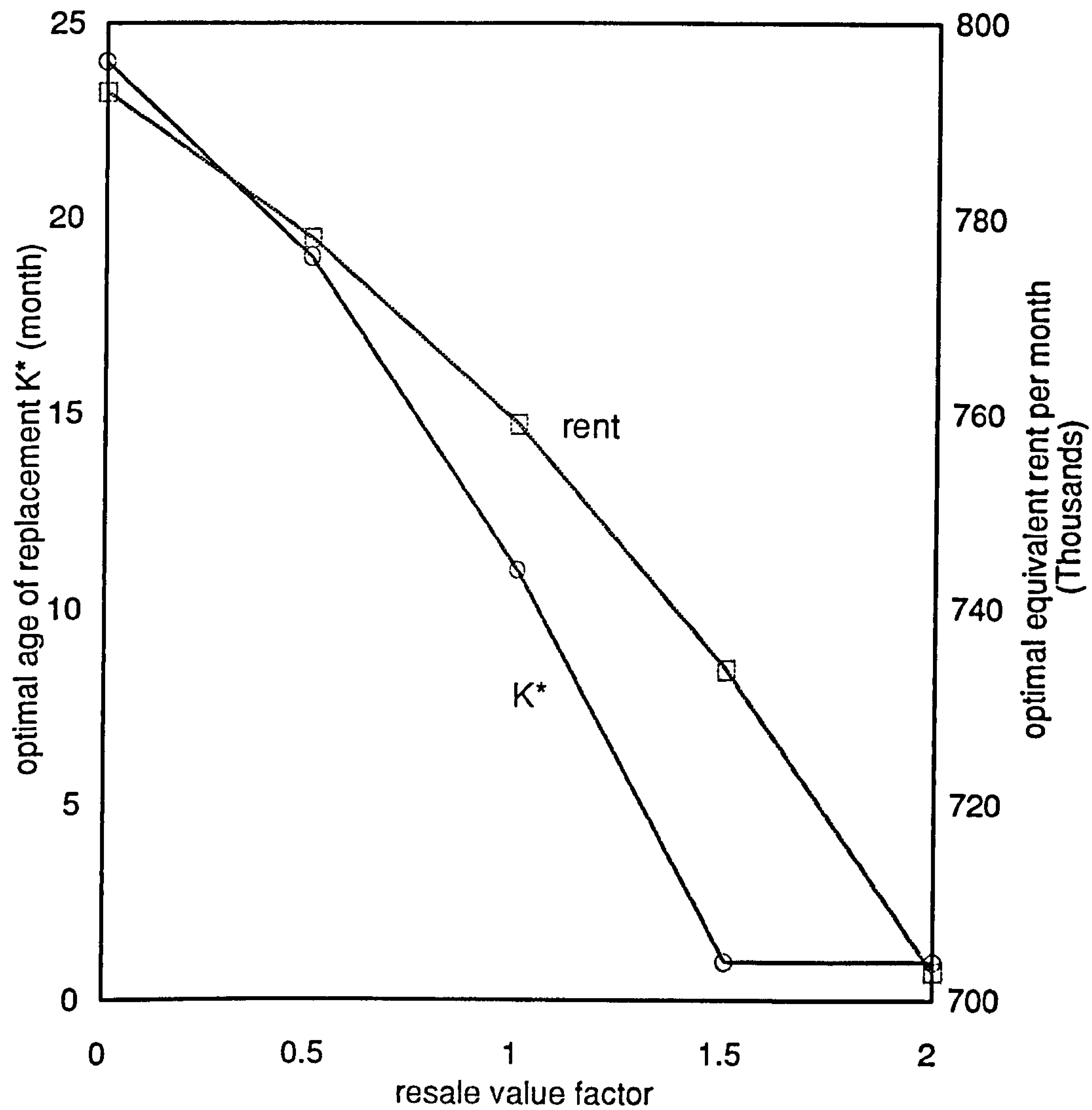


Figure 5.23. Effect of resale value on optimal age of replacement and costs, for replacement strategy C-Me.,(rent model IIa)

5.6.5 Sensitivity analysis to purchase price of new model

We should expect that in the future, because of the inflation and many other factors the purchase cost of new of buses will increase. We have also considered this eventuality to observe the effect of cost increase on optimal decision variables. The two cycle model for fixed fleet size has been used and the results are presented in table 5.26 and Figure 5.24. In table 5.26 we can observe an increase for both values of K^* and L^*. Figure 5.24 shows that an increase of 16% above the current purchase cost leads to an increase of 9 months in the length of the horizon. This is explain by the fact that if the cost of the replacement increases dramatically the operator prefers to keep the current equipment as long as possible. The increase for the value of L^* is due to the fact that we assume that the second replacement sub-fleet is of the same type as the one purchased at the end of the first cycle, namely the Isuzu CJR. This assumption is fair since the length of the planning horizon is short. For the case of a variable fleet size, we have used again the two cycle model which is related to variable fleet size case (model IIb) for the replacement strategy Cummin-Mercedes. The value of the penalty cost of unavailability due to breakdown is set to $M1000. In table 5.27 we have not observed variation on the replacement sub-fleet size but the values of both K^* and L^* have increased for the same reasons as previously.

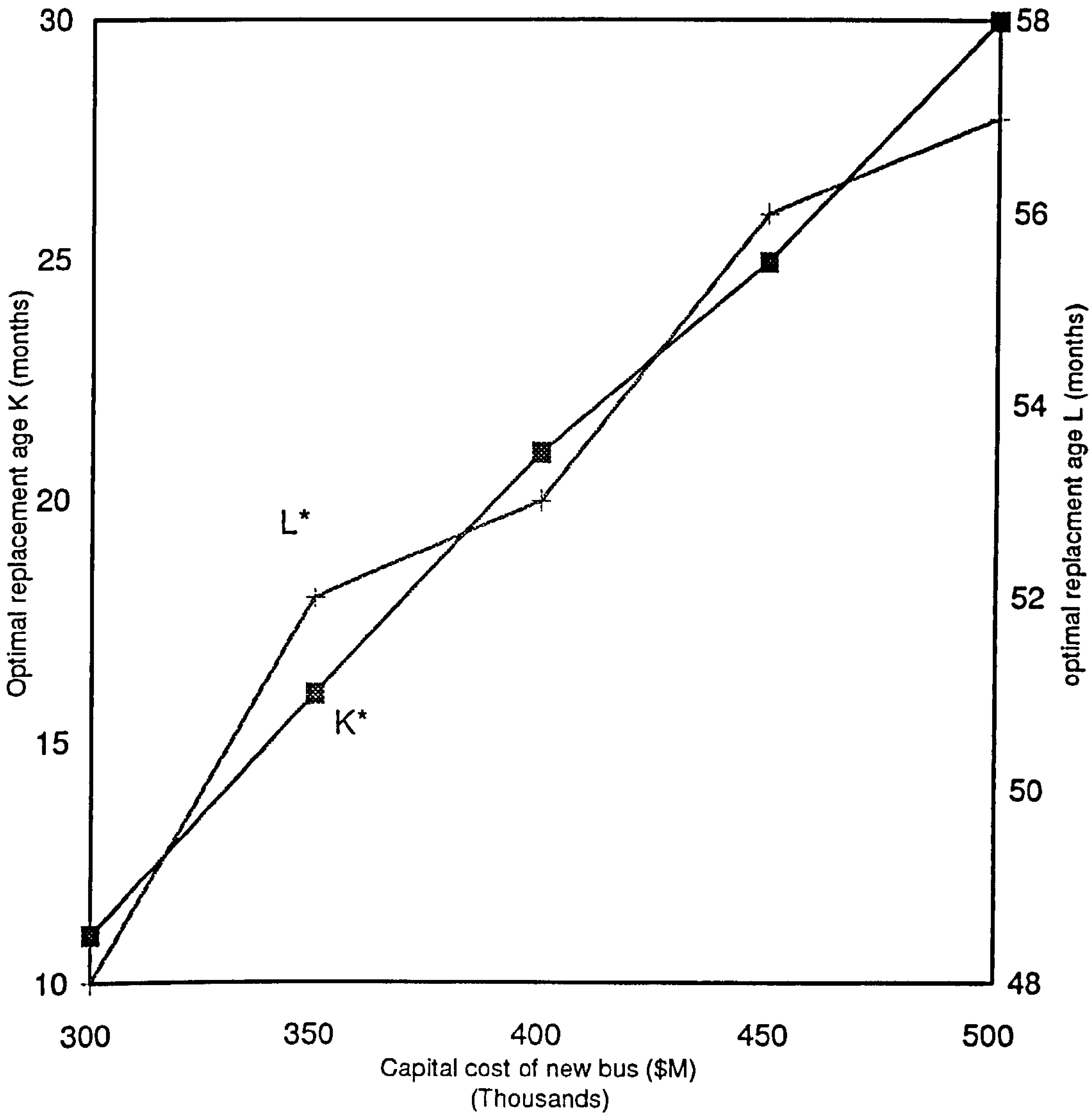


Figure 5.24. Effect of purchase cost of new bus on optimal age of replacement for first and second cycle, replacement strategy C-Me, (rent model IIa)

Table 5.26. Effect of purchase cost of new bus, Cummin replaced first, Mercedes second,(two cycle model, fixed fleet size, rent model IIa).

Purchase cost $M000's	K^* (months)	L^* (months)
300,000	11	48
350,000	16	52
400,000	21	53
450,000	25	56
500,000	30	57

Table 5.27. Effect of purchase cost of new bus, Cummin replaced first, Mercedes second,(two cycle model, variable fleet size, rent model IIb). Penalty cost =$M1000.

Purchase cost $M000's	K^* (months)	N_K^*	L^* (months)
300,000	5	18	54
350,000	11	18	56
400,000	16	18	58
450,000	20	18	60
500,000	25	18	60

5.6.6 Sensitivity analysis to demand

As we stated in section 5.2.4 an increase in demand might occur for some reasons, say opening of new routes or an increase in the number of passengers. This increase might provoke an increase in usage and therefore an overutilisation of the buses. This may lead to more breakdowns and an increase in maintenance expenses. In order to quantify all these consequences of an increase of demand, we will use the two cycle model with variable fleet size (model IIb) for two values of the penalty cost of unavailability, $M350 and $M500. We have considered the replacement strategy Cummin-Mercedes for illustration only. We have obtained the results presented in tables 5.28 and 5.29 and Figures 5.25 and 5.26. These tables and figures show the effect of increasing demand on the optimal values of the decision variables. Note that the size of the fleet is one hundred and twenty five. A range of values of demand from 123 to 130 have been considered. Table 5.28, for a penalty cost p=\$M350, shows that an increase in demand by n units of the current demand would cause an increase of n+1 units in the size of the sub-fleet replacement, while the optimal age of replacement decreases. On the other hand table 5.29, for a penalty cost p=\$M500, shows that an increase of the demand by n units would cause an increase of the size of the replacement sub-fleet by n+2 units. This shows that when demand is combined with the penalty cost of unavailability there is a strong effect on the size of the replacement sub-fleet as well as on the optimal age of replacement. We can also observe an increase in the optimal age of replacement of the second sub-fleet as well as in the minimum cost. We can also notice that the Figures 5.25 and 5.26 show the same pattern for the minimum rent and the optimal sub-fleet size, there is however a slight difference for K^* which decreases faster for the higher value of the penalty cost, that is \$M500.

Table 5.28. Effect of demand on optimal age of replacement, optimal fleet size and optimal costs for a penalty cost per breakdown p=$M350. Cummin first, Mercedes second, (two cycle model , variable fleet size, rent model IIb)

demand	K* months	L* months	N_K^*	Cost $M000s
123	10	50	15	764
124	10	50	16	772
125	10	50	17	781
126	8	52	18	789
127	6	54	19	798
128	5	55	20	806
129	4	56	21	814
130	3	56	22	822

Table 5.29. Effect of demand on optimal age of replacement, optimal fleet size and optimal costs for a penalty cost per breakdown p=$M500. Cummin first, Mercedes second, (two cycle model, variable fleet size, rent model IIb)

demand	K* months	L* months	N_K^*	Cost ($M000's)
123	10	50	16	768
124	9	52	17	777
125	8	52	18	785
126	5	54	19	794
127	2	56	20	802
128	1	58	21	810
129	1	58	22	818
130	1	58	23	826

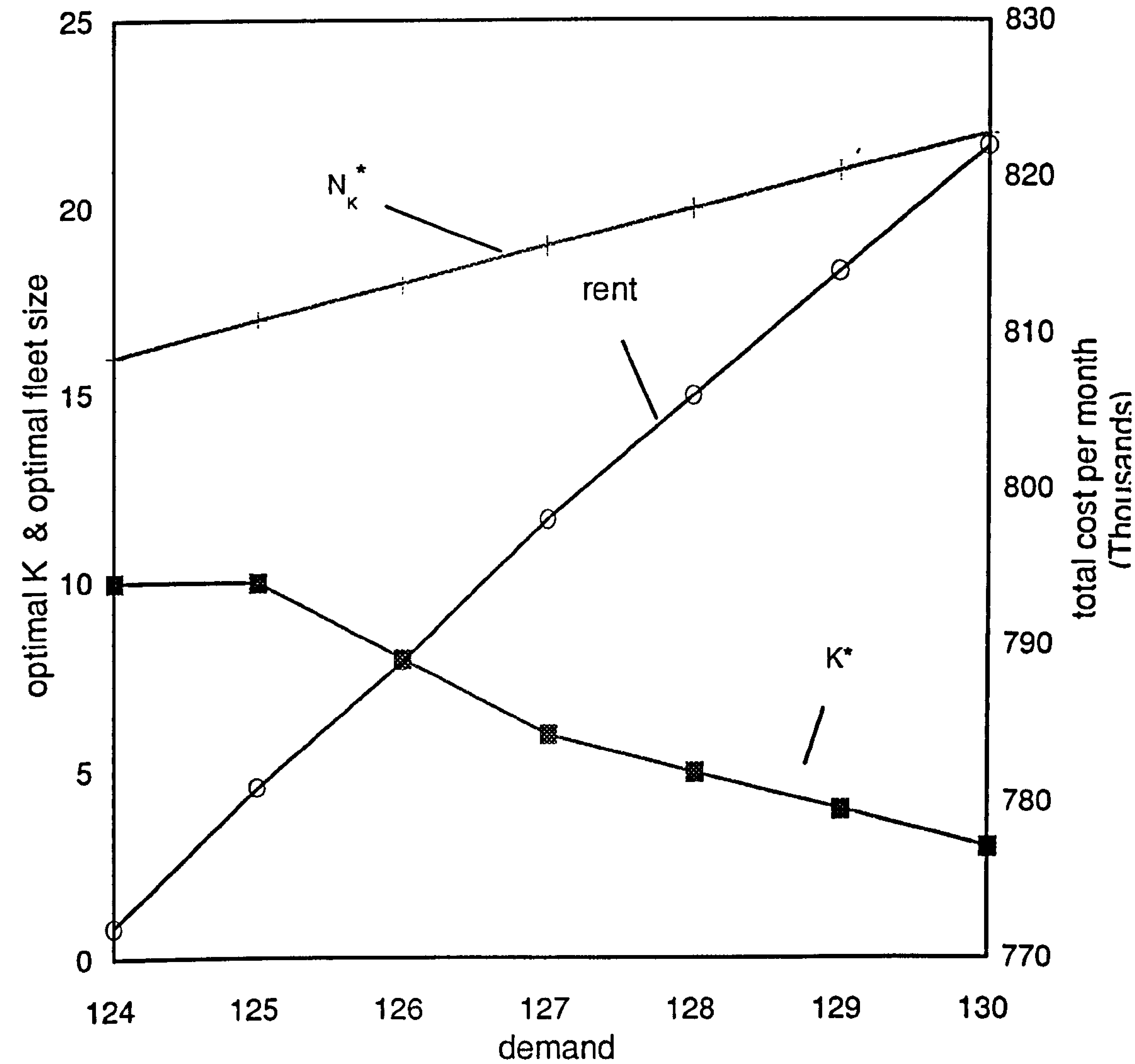


Figure 5.25. Effect of demand on optimal age of replacement, optimal fleet size and optimal equivalent rent per month for penalty cost of unavailability p=$M350. Cummin replaced first, Mercedes second, rent model IIb

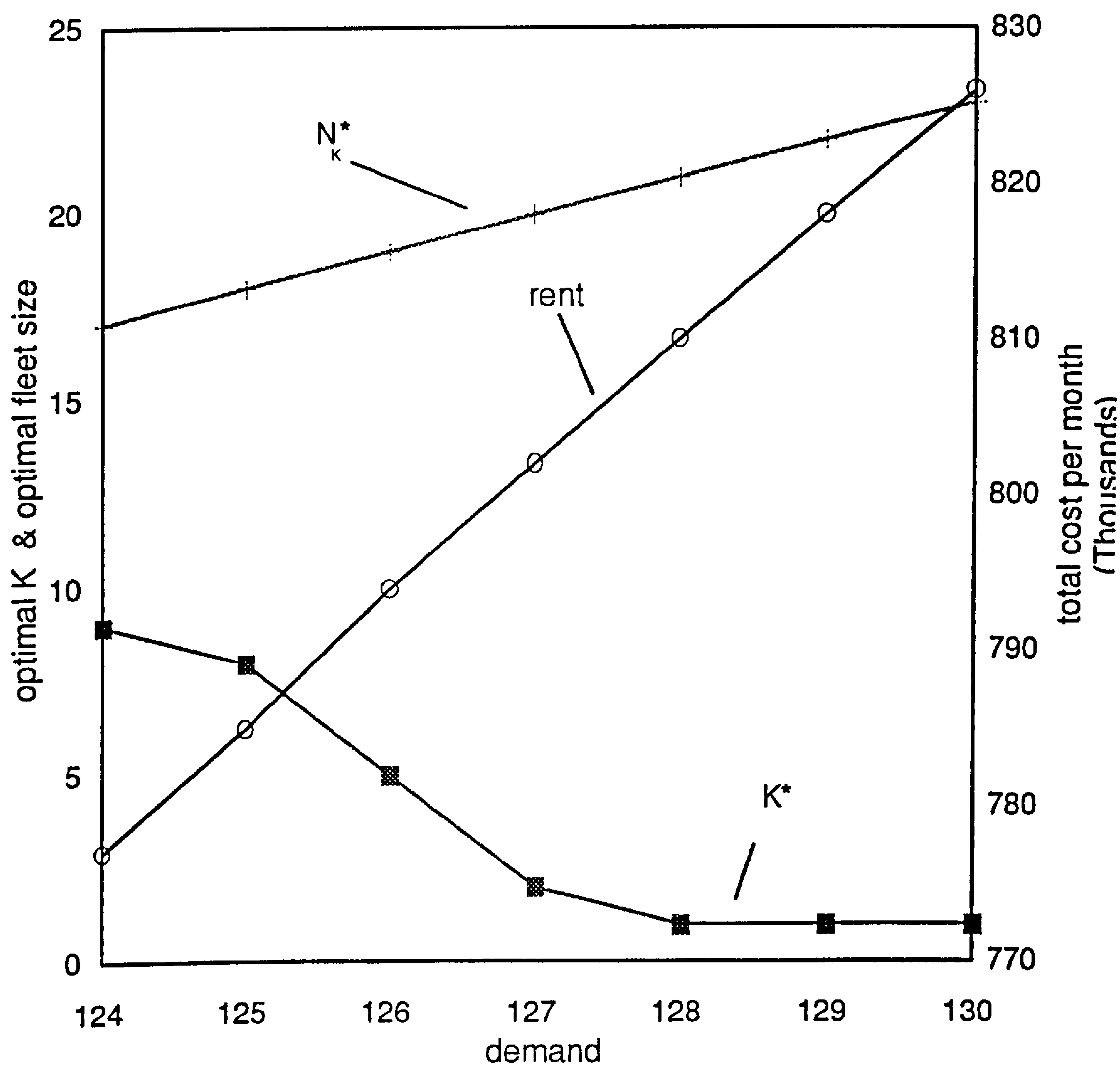


Figure 5.26. Effect of demand on the optimal age of replacement, optimal fleet size and equivalent rent per month for penalty cost of unavailability p=$M500. Cummin replaced first, Mercedes second, rent model IIb.

5.7 Discussion

Regardless of whether the fleet size is held fixed or not, the optimum policy is to replace the Mitsubishi sub-fleet first, as soon as possible, and then the Cummin sub-fleet after three years. Resale values for the Mitsubishi are perhaps artificially high, however, due to the high purchase cost of these vehicles ($M800k). Given that the company wished to replace the Mercedes sub-fleet first, the increased cost of this policy over the optimal is approximately M$150k per month (for a period of 3 years), when considering fixed fleet size and a zero penalty cost of unavailability. Earlier conclusions based on the preliminary analysis indicated that the Cummin be replaced within 1 to 2 years, depending on whether the Mitsubishi or Mercedes were replaced second. Discussion with management revealed that the company use a "rule of thumb" policy, which says "Do not replace vehicle younger than 7 years". It would have therefore been politically, difficult to justify replacing the Cummins within 1 or 2 years.

From tables 5.8-5.13 in section 5.4.4.1, for example management would need to justify a penalty cost (of unavailability due to a breakdown on the road) of only M$1000 (≈£250) for an increase in the fleet size by 2 to become optimal. Note however, the difference in cost between policies with differing sizes for the new sub-fleet is small in comparison to that for policies with differing sub-fleet replaced first. This is perhaps due to the simplicity of the failure model used in the application and the relatively low cost of the replacement sub-fleet (Isuzu CJR). There is much scope for extending this in future modelling work with the company.

Note that the optimal policy derived throughout assumes operation of vehicles is independent of replacement made, and that the nature of maintenance practice will continue as previous. Some studies are undertaken within the company in order to improve the current maintenance practice such as minimising downtime and optimising time of inspection, by using delay time technique (Christer and Desa 1992). As soon as the new maintenance policy is set up and data are available we might update this work in the future.

The models also assume uniform usage given that no usage information was available. This is not a desirable situation in general.

For the application considered the model provides meaningful decision support for the operator for a number of scenarios. If the fleet size is held fixed then the costs of alternative choices of sub-fleets to be sold at replacements might be calculated, and the optimum time scale for these replacement determined. When the fleet size is allowed to vary, optimum fleet sizes are determined for a range of penalty costs of unavailability. The increased cost of policies with a smaller than optimal replacement sub-fleet size can easily be determined , as well as the increased cost of delayed replacement which has been determined in both cases (fixed and variable fleet size) (tables 5.16-5.18, section 5.5.1). We do not attempt to estimate the penalty cost of unavailability, but merely to present optimal sub-fleet replacement decisions for a range of penalty costs. In this way, the penalty cost might be used to influence a decision, and to illustrate the cost consequences of making alternative decisions.

Several replacement models with either two or three cycles have been described and formulated in chapter 4. The two cycle models denoted model IIa and model IIb appear to be more suitable for our purpose in the sense that the horizon is not large. This allows the operator to consider technological change since the replacement equipment is often already known and operational within or outside the organisation. These models also require less time for computation when compared with the three cycle models denoted model III-a or model III-b. These latter models require a computational time equal to $\zeta \times \eta \times \mu$, where ζ is the computational time required to run the two cycle model program and η and μ are the number of values in the loop N_L (size of the second replacement sub-fleet) and the loop M (length of the third cycle) of the computer program used respectively. The two cycle model with length of each cycle as a decision variable, has been used extensively in the literature (Christer, 1984; Christer and Scarf, 1994, Kobbacy and Nicol, 1994)) for modelling replacement when the nature of and requirement for plant is changing, and for this reason was adopted here.

Technological improvement might imply a change in the reliability of plant and also, an improvement in quality for the operator and customer. Increased revenue associated with such changes might be incorporated into the modelling process. Of course, such a change in reliability might not be for the better, e.g. for the case of sophisticated equipment; if the staff is not sufficiently trained, breakdowns might occur more frequently than expected because of misuse.

The models applied here are an attempt to improve the current practice of modelling replacement and optimum fleet size decisions, by considering real replacement problems. The value of the approach is that it looks at the replacement problem in its entirety, with account taken of: differing current ages and specification of equipment; technological change; and the nature of demand of the plant. Two cost criteria have been used, namely the equivalent rent per unit time and the total discounted cost per unit time. Both criteria came out with the same result in optimal schedule, that is to replace the Mitsubishi sub-fleet first and then the Cummin sub-fleet. We noticed however a differences on minimum costs per month which are higher for the equivalent rent, but the optimal time to replacement of the first cycle is larger for the total discounted cost per month. The value of the optimal fleet size is still, however, unchanged with two cost criteria for each value of the penalty cost of unavailability.

In the models we have presented in chapter 5, we have considered a variable finite planning horizon. The finite fixed horizon model (chapter 4 section 4.3.9) has not been applied to the bus data. This is left for future work.

5.8 Further work

In order to consider optimal replacement sub-fleet size, a model of the demand for equipment is required. For the example considered in this chapter, demand for fleet was a known constant due to the fact that the company operates a fixed number of scheduled routes each day. Modelling the more general setting, in which demand is random, would be more difficult however. The approach here could be used as a first order approximation (based on expected values) to this more difficult problem. Further, data for modelling demand might not be readily available. It would still, however, be possible to consider replacement decisions in the light of only limited availability of demand data, by considering demand in a similar fashion to that of the penalty cost above with replacement decisions presented for a number of different demand model scenarios. We can also consider for future work the possibility for the demand, the service time and repair time to be random.

Finally application of the replacement model with a fixed planning horizon, which has been presented in chapter 4 section 4.3.9 could be considered. This model would have the advantage of being able to compare different replacement schedules (which sub-fleet to replace) over the same length of the planning horizon, which is not the case for the models applied here.

CHAPTER VI

Sensitivity of Optimal Replacement Policy to Model Choice

6.1 Introduction

This chapter considers various capital replacement models considered in chapter 4 . These are applied to the fleet described in chapter 5. This fleet comprises a mix of sub-fleets of different makes (models) and age. The models themselves all have a finite planning horizon of variable length. Typically, the models are formulated either with one 'operate-sell-and buy' cycle or with two or three 'operate-sell-and-buy' cycles or with one or two 'operate-sell-and-buy' cycle followed by an 'operate' cycle. These models are presented in chapter 4, section 4.3.

In the face of diverse problems which occur in the real life, models are developed accordingly to cope with all sort of situations. As modelling should deal with real life situations, it seems natural to consider a finite planning horizon. This makes the costing of technological change possible as discussed earlier. The issue which is raised is concerned with to what extent future costs are taken into account, either through the length of the horizon or the number of replacements to undertake (number of cycles). We consider various replacement models for which we investigate the behaviour of the optimal solution with respect to the model choice. Optimal replacement decisions are obtained from each model, and this enables us to show the sensitivity of the optimal solution to the model choice. The length of the horizon is variable and depends on the number and the length of the cycles. Only the decision variables relating to the immediate decision, that is the length of the first cycle and the size of the first replacement sub-fleet, are of real interest. It is expected that such models would be re-fitted for new equipment, thus implying a rolling horizon approach (Dekker *et al*, 1993). However, as the number of cycles increases the importance of the first cycle is diluted at the expense of the other cycles. The nature of the last cycle for a two or a three cycle model, that is either it is an 'operate' cycle only or an 'operate-buy-and-sell' cycle, may also be important to consider.

The finite horizon replacement models which we have described in chapter 4 correspond respectively to:

1- a one cycle model, that is one 'operate-sell- and buy' cycle where the fleet size is fixed which is denoted model I;

2- a two cycle model; two 'operate-sell-and-buy' cycles with fixed fleet size which is denoted model IIa;

3- a two cycle model with variable size of the first replacement sub-fleet and fixed size for the second replacement sub-fleet which is denoted model IIb;

4- a two cycle model where the second cycle is an 'operate' cycle only, that is no replacement is carried out at the end of the second cycle. In the first cycle, we consider a variable fleet size (model IId).

5- A three cycle model with three 'operate-buy-and-sell cycles, with fixed fleet size for all the cycle (model IIIa).

6- A three cycle model where the third cycle is an 'operate' cycle only, that is no replacement is carried out at the end of the third cycle. For the first and the second cycle we have considered variable fleet size (model IIIb).

7- A three cycle model with three 'operate-sell-and-buy' cycles. We consider variable fleet size for the first and second cycle (model IIIc).

6.2 Results

The present section begins with a presentation of the results obtained from each model for the optimal solution (obtained using FORTRAN77 program) in the context of the case study presented in chapter 5. The final section discusses and compares the sensitivity of the optimal solution with respect to the choice of the replacement model. Note here that the replacement model (mathematical) should not be confused with the equipment model. The (mathematical) models are compared on the basis of the behaviour of the optimal decision with respect to the (mathematical) model choice. It is important to note that the conclusions which are drawn are specific to the current example study. In other words, if we consider any other application the behaviour of the optimal decision with respect to model choice may be different. We should also clarify the point that the results obtained in this chapter consider that the Mitsubishi sub-fleet is a candidate for replacement. This assumption is only made to illustrate the purpose of the work. The company were in fact more concerned with the replacement of the Mercedes or Cummin in the short term. The work we did in chapter 5 was only an approach to solve the company's replacement problem.

However, as the case is of primary interest, it is important to make this comparison between models. To make the models comparable, a penalty cost of unavailability of M$1000 (≈£250 at the time of the study) is used for each model. Our case study concerns the fleet whose composition, based on the latest information we have, is illustrated in Figure 6.1. This figure shows that from early 1993 a new replacement sub-fleet (MAN) has been purchased by the company, whereas in our study (chapter 5) we have assumed that the sub-fleet replacement is always represented by the Isuzu CJR sub-fleet. This is still a valid assumption since at the period of the study (early 1992) the MAN sub-fleet was not present and the intention to purchase this model of equipment was unknown. Note that the company did in fact replace the Mercedes and Cummin sub-fleets. There is however no difficulty to update the model with new data. A continual updating may be carried out as far as the company will provide us with new data.

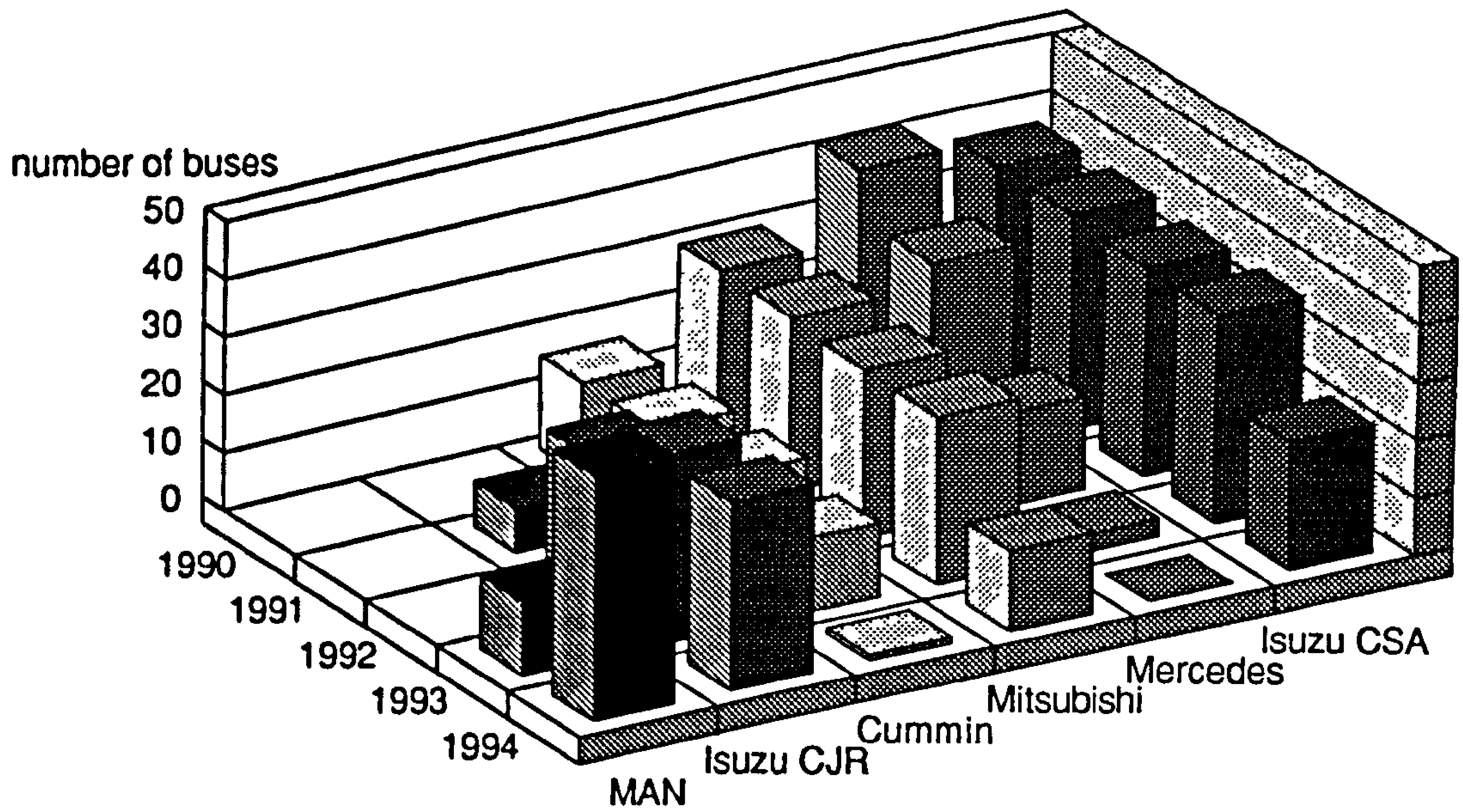


Figure 6.1. Composition of the fleet (1990-1994)

6.2.1 Sensitivity to model choice of the sub-fleet to be replaced first

In tables 6.1-6.7, along with table 6.8 which summarises the optimal decision policies, we can observe that the choice of the first sub-fleet to be replaced is the Cummin sub-fleet in model IId, IIIa, IIIb and IIIc. Models IIa and IIb suggest however to replace the Mitsubishi sub-fleet first. But if we observe the next best replacement decision for model IIa and IIb we find that the replacement of the Cummin sub-fleet is the second best option with a very small difference on the value of the minimum cost. We note that the Cummin sub-fleet, although relatively new in the fleet has a high maintenance cost and a high mean number of failures (Figures 5.8.b and 5.3 in chapter 5) in comparison with the Mitsubishi sub-fleet. In the two cycle models, model IIa and IIb, the decision to replace the Mitsubishi first may be due to the end-of-horizon effect which imposes to sell the best asset. The cost new of the Mitsubishi is relatively high and this tends to inflate its second hand value. The introduction of a third cycle can make the decision at the end of the first cycle more objective, because the fleet has to be operated for two more cycles.

Figure 6.1 illustrates the composition of the fleet from 1990 to 1994, and it shows that all the Cummins but one were sold by the end of 1994.

Table 6.1. Minimum cost per month of the equivalent rent and optimum age of first replacement for various choices of sub-fleets to be sold at first replacement. Penalty cost of unavailability p=M$1000 (≈£250), discount factor $\nu = 0.97$. One cycle model with fixed fleet size (model I)

Replacement Schedule	K* months	Cost M$000's
Cummin	27	**687**
Mitsubishi	39	782
Mercedes	50	835
Isuzu CSA	51	846

Table 6.2. Minimum cost per month of the equivalent rent for various choices of sub-fleets to be sold at first and second replacement. Penalty cost of unavailability p=M$1000 (≈£250), discount factor $\nu = 0.97$. Two 'operate-buy-and-sell' cycle model with fixed fleet size (model IIa)

2nd repl. 1st repl.	C	Me	Mit	Is
C	*	797	**756**	806
Me	883	*	940	978
Mit	**730**	850	*	860
Is	815	839	890	*

Table 6.3. Minimum cost per month of the equivalent rent for various choices of sub-fleets to be sold at first and second replacement. Penalty cost of unavailability p=M$1000 (≈£250), .Two 'operate-buy-and-sell' cycle model with variable fleet size in the first cycle (Model IIb)

2nd repl. 1st repl.	C	Me	Mit	Is
C	*	784	**743**	794
Me	882	*	937	976
Mit	**721**	834	*	844
Is	804	907	870	*

Table 6.4. Minimum cost per month of the equivalent rent for various choices of sub-fleets to be sold at first and second replacement. Penalty cost of unavailability p=M$1000 (≈£250), .Two cycle model with variable fleet size in the first cycle, the second cycle is an 'operate' cycle only (Model IId)

sub-fleet to replace	Costs $M000's
C	**575**
Me	813
Mit	631
Is	729

Table 6.5. Minimum cost per month of the equivalent rent for various choices of sub-fleets to be sold at first and second replacement. Penalty cost of unavailability p=M$1000 (≈£250), discount factor $\nu = 0.97$. Three 'operate-buy-and-sell' cycle model with fixed fleet size (model IIIa)

2nd repl. 1st repl.	C	Me	Mit	Is
C	*	884	**820**	856
Me	907	*	1011	1017
Mit	**824**	996	*	880
Is	843	946	959	*

Table 6.6. Minimum cost per month of the equivalent rent for various choices of sub-fleets to be sold at first and second replacement. Penalty cost of unavailability p=M$1000 (≈£250), discount factor $\nu = 0.97$. Three cycle model with variable fleet size for the two first cycles, the third cycle is an 'operate' cycle only (model IIIb)

2nd repl. 1st repl.	C	Me	Mit	Is
C	*	782	**699**	734
Me	806	*	882	918
Mit	**701**	861	*	816
Is	738	816	898	*

Table 6.7. Minimum cost per month of the equivalent rent for various choices of sub-fleets to be sold at first and second replacement. Penalty cost of unavailability p=M$1000 (≈£250), discount factor $v = 0.97$. Three 'operate-buy-and-sell' cycle model with variable fleet size for the first two cycles (model IIIc)

2nd repl. / 1st repl.	C	Me	Mit	Is
C	*	873	**806**	845
Me	927	*	934	1013
Mit	**814**	985	*	934
Is	835	870	994	*

Table 6.8. Optimal age and replacement decision with minimum cost of the equivalent rent per month for different replacement models. Penalty cost of unavailability p=M$1000 (≈£250), discount factor $v = 0.97$.

Models	K* months	1st replacement	L* months	2nd replacement	M* months	Min cost M$000's
model I	27	Cummin	*	*	*	687
model IIa	1	Mitsubishi	48	Cummin	*	730
model IIb	1	Mitsubishi	46	Cummin	*	721
model IId	1	Cummin	28	*	*	575
model IIIa	20	Cummin	20	Mitsubishi	54	820
model IIIb	5	Cummin	18	Mitsubishi	46	699
model IIIc	22	Cummin	8	Mitsubishi	62	806

6.2.2 Sensitivity of K^* to model choice

Table 6.8 along with tables 6.9-6.14 show some difference for the values of K^* between the two cycle models and the three cycle models. We can observe in table 6.8 that the values of K^* for the two cycle models are all equal to one, which means immediate replacement. In tables 6.9 and 6.10, the values of K^* are all small except those related to the policy which consists of replacing the Mercedes sub-fleet first, because the Mercedes, although old, appears very reliable and to incur low maintenance expenses. For the three cycle models, model IIIa and IIIc, tables 6.12-6.14 show that the values of K^* are relatively large. The values of K^* for model IIIb are relatively smaller than those of model IIIa and IIIb. We can notice that the value of K^* for the one cycle model (model

I) is closer to the values obtained with model IIIa and IIIc. This can be explained by the fact that the introduction of a third cycle dilutes the influence of L^* over K^*. We should also point out that the value of the penalty cost has an influence on the value of K^* (see e.g. table 5.8 , chapter 5); the greater the value of the penalty, the smaller the value of K^*.

Table 6.9. Optimum age of replacement for various choices of sub-fleets to be replaced at first and second replacement. Penalty cost of unavailability p=M$1000, two 'operate-buy-and-sell' cycle model, fixed fleet size for all cycles (model IIa)

Replacement schedule	K* months	L* months	Cost $M000's
C - Me	11	48	797
C - Mit	1	48	756
C - Is	12	48	806
Me - C	32	24	884
Me - Mit	39	24	940
Me - Is	46	24	978
Mit - C	1	36	730
Mit - Is	5	48	860
Mit - Me	3	48	850
Is - C	1	48	815
Is - Mit	6	48	890
Is - Me	21	36	839

Table 6.10. Optimum age of replacement for various choices of sub-fleets to be replaced at first and second replacement. Penalty cost of unavailability p=M$1000, two 'operate-buy-and-sell' cycle model, variable fleet size in the first cycle (model IIb).

replacement schedule	K* months	N_k^*	L* months	Cost $M000's
C - Me	5	18	54	784
C - Mit	1	18	48	743
C - Is	6	18	54	794
Me - C	35	35	22	882
Me - Mit	37	35	26	937
Me- Is	43	35	26	976
Mit - C	1	32	46	721
Mit - Me	1	32	54	834
Mit - Is	1	32	54	844
Is - C	1	39	48	804
Is - Mit	1	39	52	870
Is - Me	11	39	50	907

Table 6.11. Minimum cost per month of the equivalent rent and optimum age of first replacement for various choices of sub-fleets to be sold at first replacement. Penalty cost of unavailability p=M$1000 (≈£250). discount factor $v = 0.97$. Two cycle model with 'operate-buy-and-sell' and variable fleet size for the first cycle, second cycle is an 'operate' cycle only (model IId)

replacement schedule	K* months	N_k^*	l* months	Cost M$000's
Cummin	1	17	28	575
Mitsubishi	1	32	24	631
Mercedes	24	35	34	813
Isuzu CSA	1	39	42	729

Table 6.12. Optimum age and size of replacement sub-fleets for various choices of sub-fleets to be replaced at first and second replacement. Penalty cost of unavailability p=M$1000, three 'operate-buy-and-sell' cycle model, fixed fleet size (model IIIa)

replacement schedule	K* months	L* months	M* months	Cost $M000's
C - Me	22	32	34	884
C - Mit	20	20	54	820
C - Is	19	23	50	856
Me - C	40	1	44	907
Me - Mit	44	1	40	1011
Me- Is	42	1	38	1017
Mit - C	25	6	60	824
Mit - Me	30	26	30	996
Mit - Is	18	12	44	880
Is - C	22	5	56	843
Is - Mit	24	10	48	946
Is - Me	22	25	28	959

Table 6.13. Optimum age and size of replacement sub-fleets for various choices of sub-fleets to be replaced at first and second replacement. Penalty cost of unavailability p=M$1000, three cycle model, 'operate-buy-and-sell' and variable fleet size for the first and second cycle. The third cycle is an 'operate' cycle only (model IIIb)

replacement schedule	K* months	N_K^*	L* months	N_L^*	m* months	Costs $M000's
C - Me	12	19	30	32	28	782
C - Mit	5	19	18	28	46	699
C - Is	7	19	16	36	46	734
Me - C	26	37	1	14	42	806
Me - Mit	28	37	1	29	40	882
Me- Is	28	37	1	35	40	918
Mit - C	10	33	8	15	50	701
Mit - Me	15	33	28	32	26	861
Mit - Is	10	33	14	36	44	816
Is - C	7	40	10	14	48	738
Is - Mit	7	40	14	29	44	816
Is - Me	14	40	28	32	26	898

Table 6.14. Optimum age and size of replacement sub-fleets for various choices of sub-fleets to be replaced at first and second replacement. Penalty cost of unavailability p=M$1000, three 'operate-buy-and-sell' cycle model, variable fleet size for the first and second cycle (model IIIc)

replacement schedule	K* months	N_K^*	L months	N_L^*	M* months	Costs $M000's
C - Me	18	19	36	32	32	873
C - Mit	16	19	22	29	56	806
C - Is	16	19	25	36	51	845
Me - C	38	37	1	15	50	927
Me - Mit	32	37	1	29	42	934
Me- Is	40	37	1	36	40	1013
Mit - C	22	33	8	15	62	814
Mit - Me	26	19	31	32	29	985
Mit - Is	20	33	18	36	46	934
Is - C	18	40	6	15	56	835
Is - Mit	13	40	14	29	46	870
Is - Me	24	40	28	32	28	994

6.2.3 Two cycle models versus three cycle models

In table 6.8 we can observe that, first the values of K^* for the two cycle models are far smaller than those of the three cycles model. On the other hand, tables 6.10, 6.12 and 6.13 show that the size of the first replacement sub-fleet, N_K^*, increases by three units from its current size for the three cycle model while it increases by two units for the two cycle models when the penalty cost of unavailability is set to M$1000. We note also in tables 6.12 and 6.13 that a decrease of two units occurs for the size of the second replacement sub-fleet N_L^*. Table 6.8 shows also that the optimal replacement schedule for the two cycle models is Mitsubishi first Cummin second whereas for the three cycle models it is Cummin first Mitsubishi second. Figures 6.2 and 6.3 illustrates the difference on costs and time of first replacement for the best policy (optimal) and the next best policy (sub-optimal) for model IIa versus model IIIa and model IIb versus model IIIc respectively.

We can notice that the difference between the models (model IIa vs. model IIIa, and model IIb vs. model IIIc) is large but the difference within the models (optimal vs. sub-optimal) is very small. Thus although the effect on cost is large, effect on optimal schedule is smaller. However again K^* is smaller for the two cycle models, again suggesting that assets are being realised at the end of the horizon.

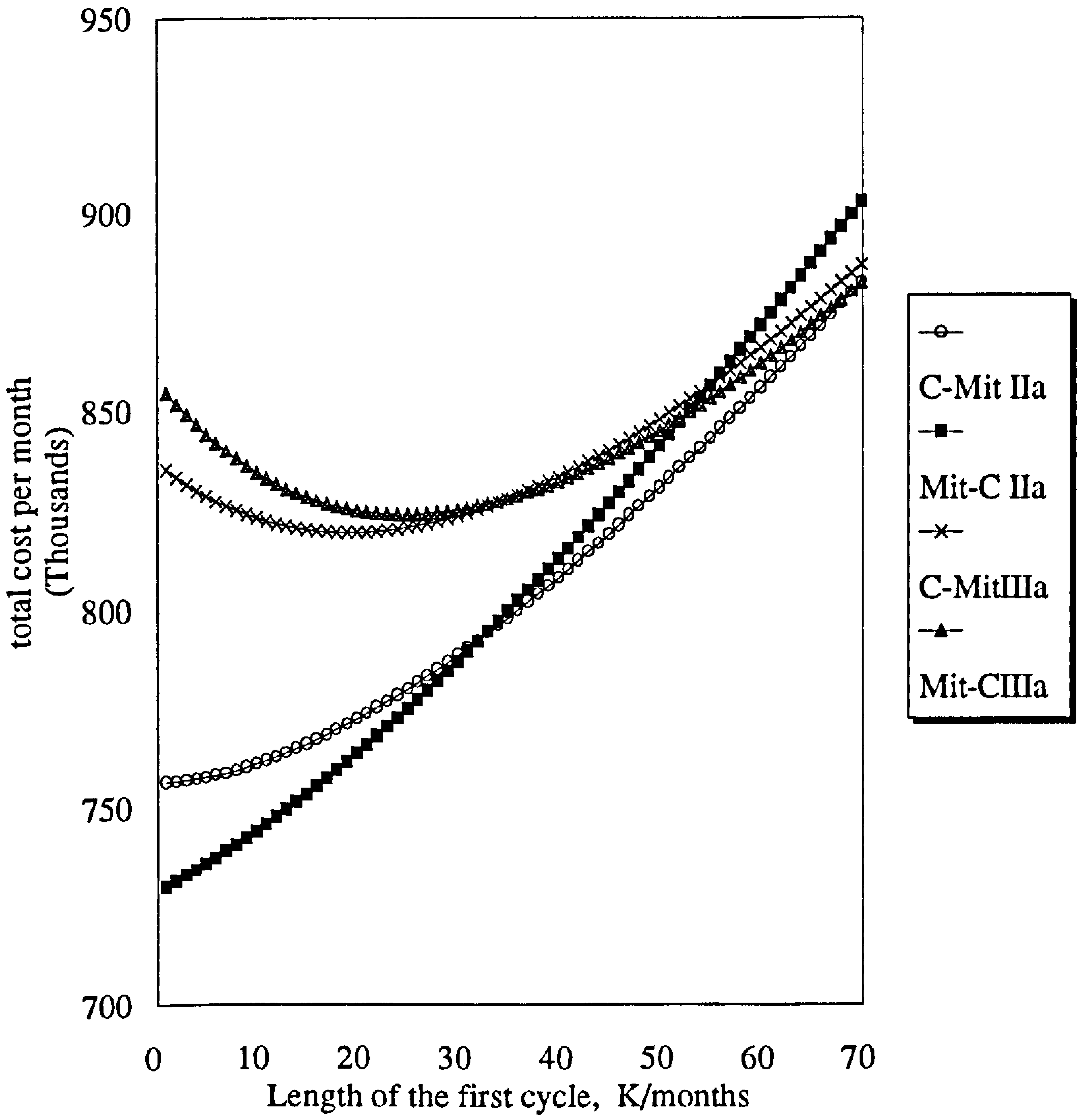


Figure 6.2. Comparison of the cost of the equivalent rent for the best policy (minimum cost) and the next best policy under the two cycle model with fixed fleet size (model IIa) vs. the three cycle model with fixed fleet size (model IIIa).

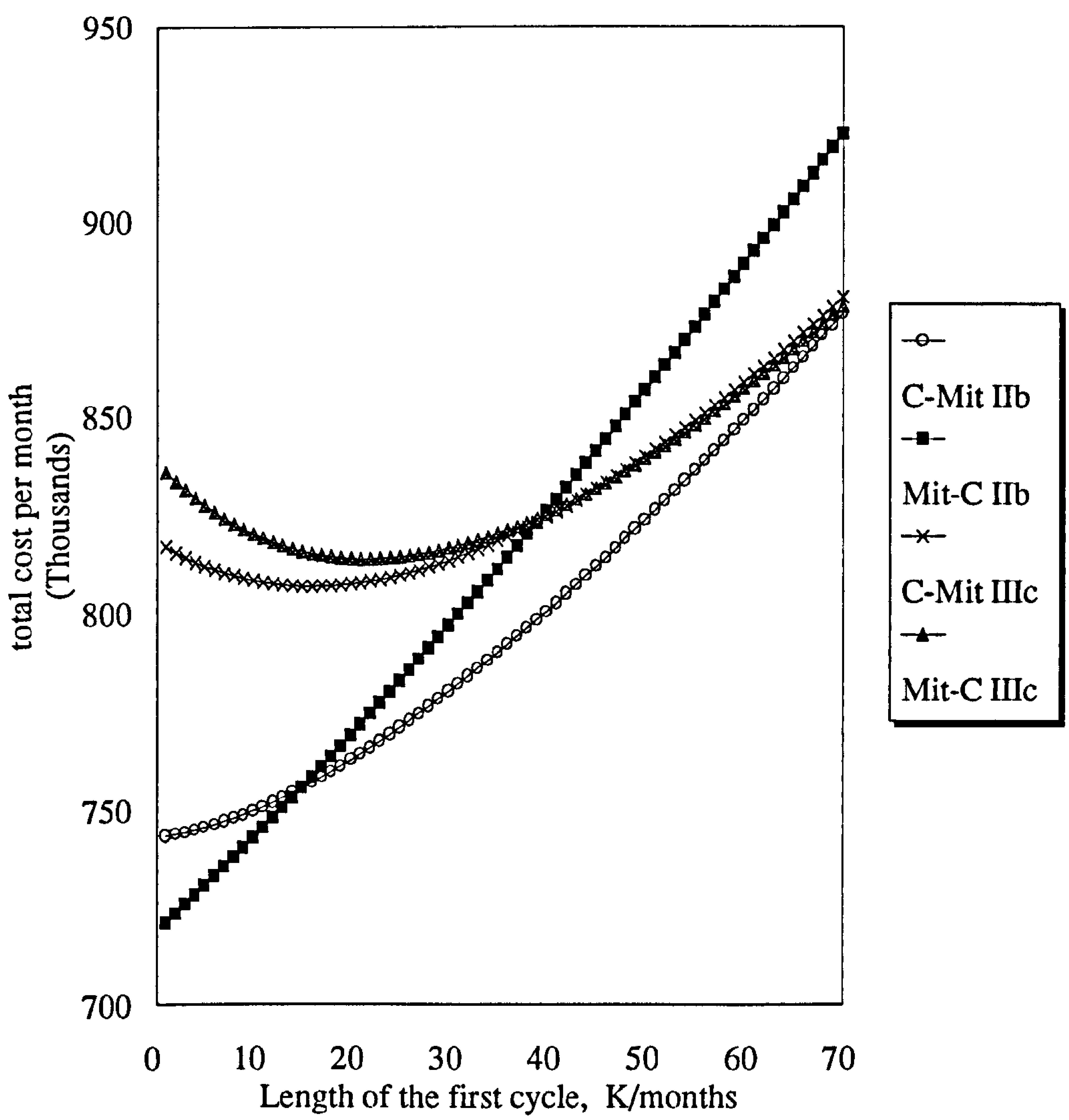


Figure 6.3. Comparison of the cost of the equivalent rent for the best policy (minimum cost) and the next best policy under the two cycle model with variable fleet size (model IIb) vs. the three cycle model with variable fleet size (model IIIc).

6.2.4 Sensitivity of the model choice to the nature of the last cycle

The nature of the last cycle is either considered as an 'operate' only cycle, which is a cycle where no replacement is considered at its end, or an 'operate-buy-and-sell' cycle. The type of equipment for the second and the third replacement sub-fleet are irrelevant for the models in which the last cycle is an 'operate' cycle only, namely model IId and model IIIb respectively. As far as the optimal replacement decision is concerned, all the models suggest to replace the Cummin sub-fleet first, except model IIa and IIb, which suggest to replace the Mitsubishi first, but Figures 6.2-6.5 show that there is little difference in cost between options such as Cummin-Mitsubishi or Mitsubishi-Cummin. Figure 6.4 (two cycle models) shows a minimum for the objective function at $K^{*}=1$, but in Figure 6.5 (three cycle models) the minimum has $K^{*}>>1$ either for model IIIc (last cycle is an 'operate-buy-and-sell') or IIIb (last cycle is an 'operate' only cycle). The immediate replacement ($K^{*}=1$) suggested by the two cycle models is mainly due to the value of the penalty cost, which has apparently a stronger influence on the two cycle models than the three cycle models. Again the realisation of assets also is a major factor for replacement.

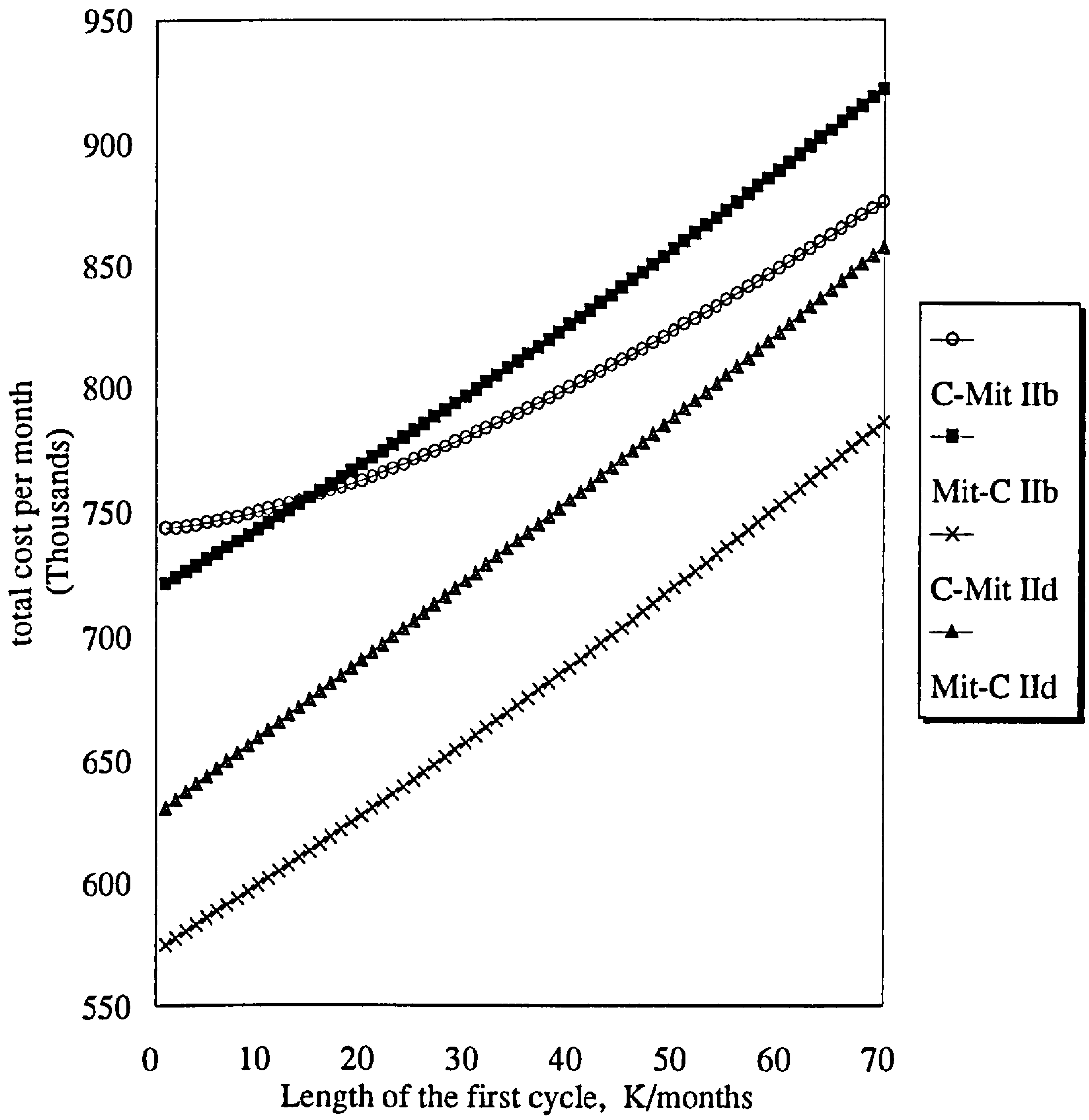


Figure 6.4 Comparison of the cost of the equivalent rent for the two cycle models, last cycle 'operate' only vs. 'operate-buy-and-sell'.

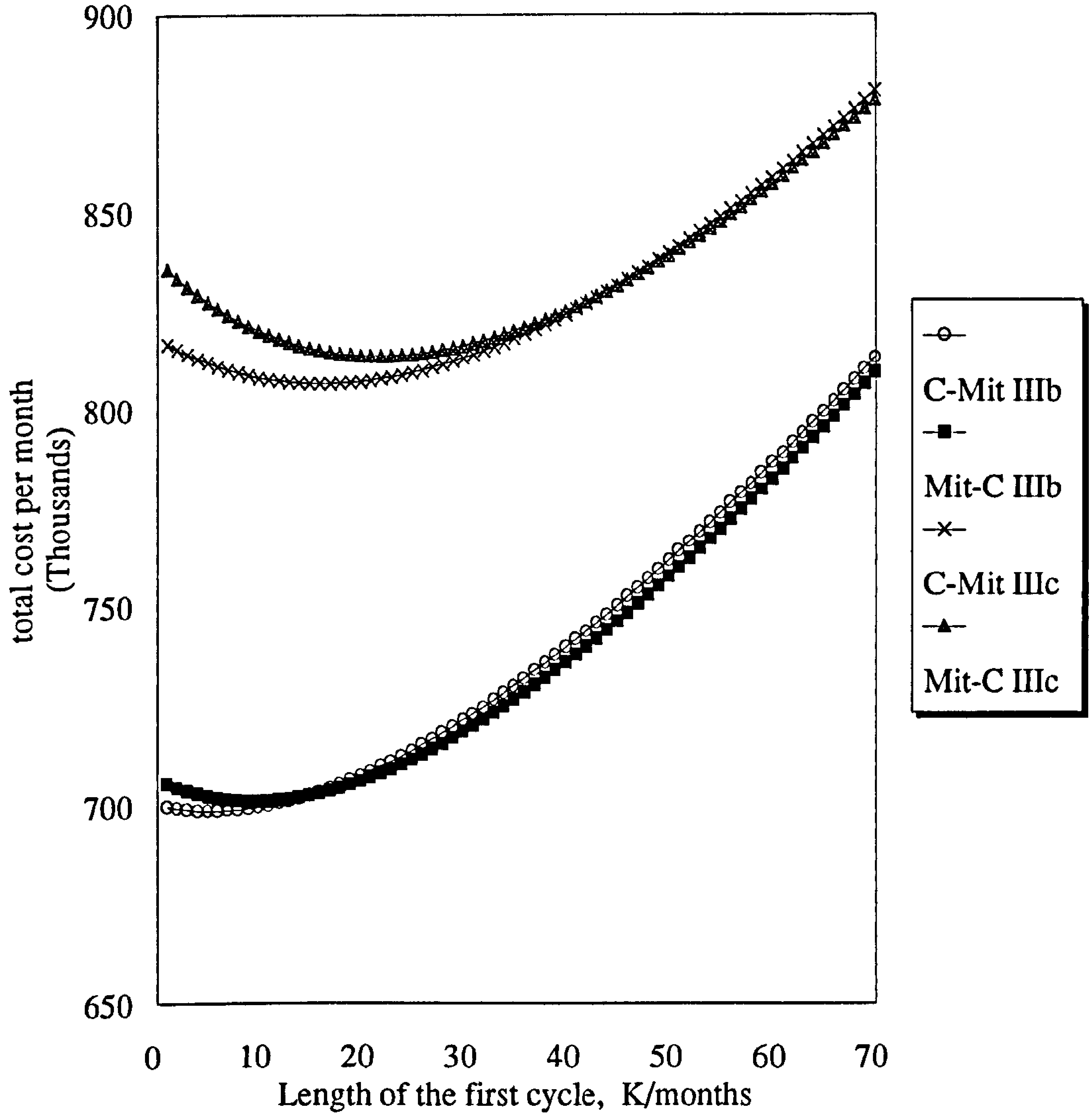


Figure 6.5 Comparison of the cost of the equivalent rent for the three cycle models, last cycle 'operate' only vs. 'operate-buy-and-sell'.

6.2.5 Sensitivity to variable fleet size with respect to fixed fleet size

We consider the case when the size of the replacement sub-fleet is allowed to vary versus the case when it is kept fixed. In Figure 6.8 we can observe that the extra cost incurred is approximately M$12k when the size of the replacement sub-fleet is kept equal to its current size ($N_K = 16$) instead of the recommended size ($N_K = 18$) for the two cycle models, namely model IIa and IIb. The penalty cost of unavailability is set to M$1000 for the policy which consists of replacing first the Cummin sub-fleet and then the Mitsubishi sub-fleet. Figures 6.6 and 6.7 illustrates the best and the next best replacement policy for the variable fleet size versus the fixed fleet size case for the two and the three cycle models respectively. There appears to be little effect on K^*.

6.3 Discussion

In this chapter we discussed sensitivity with respect to first replacement sub-fleet choice, the number of cycles, the nature of the last cycle which is either an 'operate' or an 'operate-buy-and -sell' cycle and the variable fleet size versus the fixed fleet size. It appears from this work that, on one hand, the optimal schedule, that is what sub-fleet to replace first, has not been affected dramatically by differing models. The age of replacement, on the other hand has been affected as well as the replacement sub-fleet size. The results are summarised in Figure 6.9. We have not discussed results from the one cycle model, because it would always choose a policy which sells the best asset first. The two cycle models are the most tractable both computationally and from the point of view of prediction of costs. In the two cycle model, the second cycle exists to influence the first cycle which is the cycle of interest. In the three cycle model, the importance of the first cycle is diluted. But care must be taken with such models to ensure that realisation of assets should not dominate the on-going requirement for the function of the sub-fleet (Scarf, 1994).

In order to avoid the subjective choice of the number of cycles, it would be interesting to consider a horizon with fixed length, say H, where the number of cycles becomes a decision variable (Scarf, 1994; de Sousa and Guimaraes, 1992) as well as the length of the cycle and the fleet size. The choice of the length of the horizon should be made carefully because the optimum policy is highly dependent on it (Scarf, 1994).

This sensitivity analysis has been carried out in the context of an actual application. it would be interesting in future to consider in general the effect of model choice on 'optimal policy'. The approach to this does not appear straightforward however.

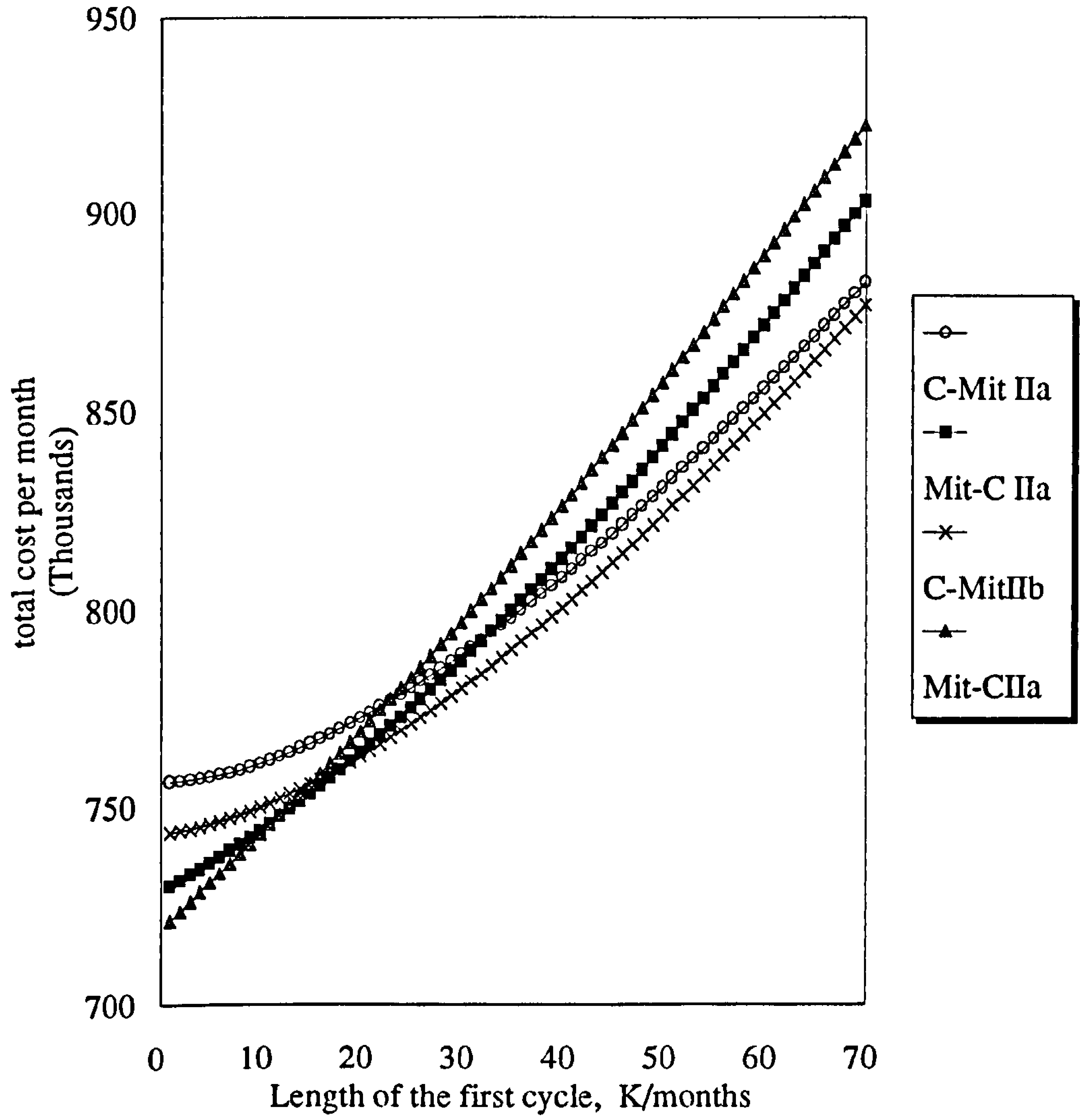


Figure 6.6. Variable fleet size vs. fixed fleet size; two cycle model with variable fleet size at first replacement (model IIb) vs. the two cycle model with fixed fleet size (model IIa). Cummin replaced first, Mitsubishi second. Penalty cost of unavailability p=M$1000.

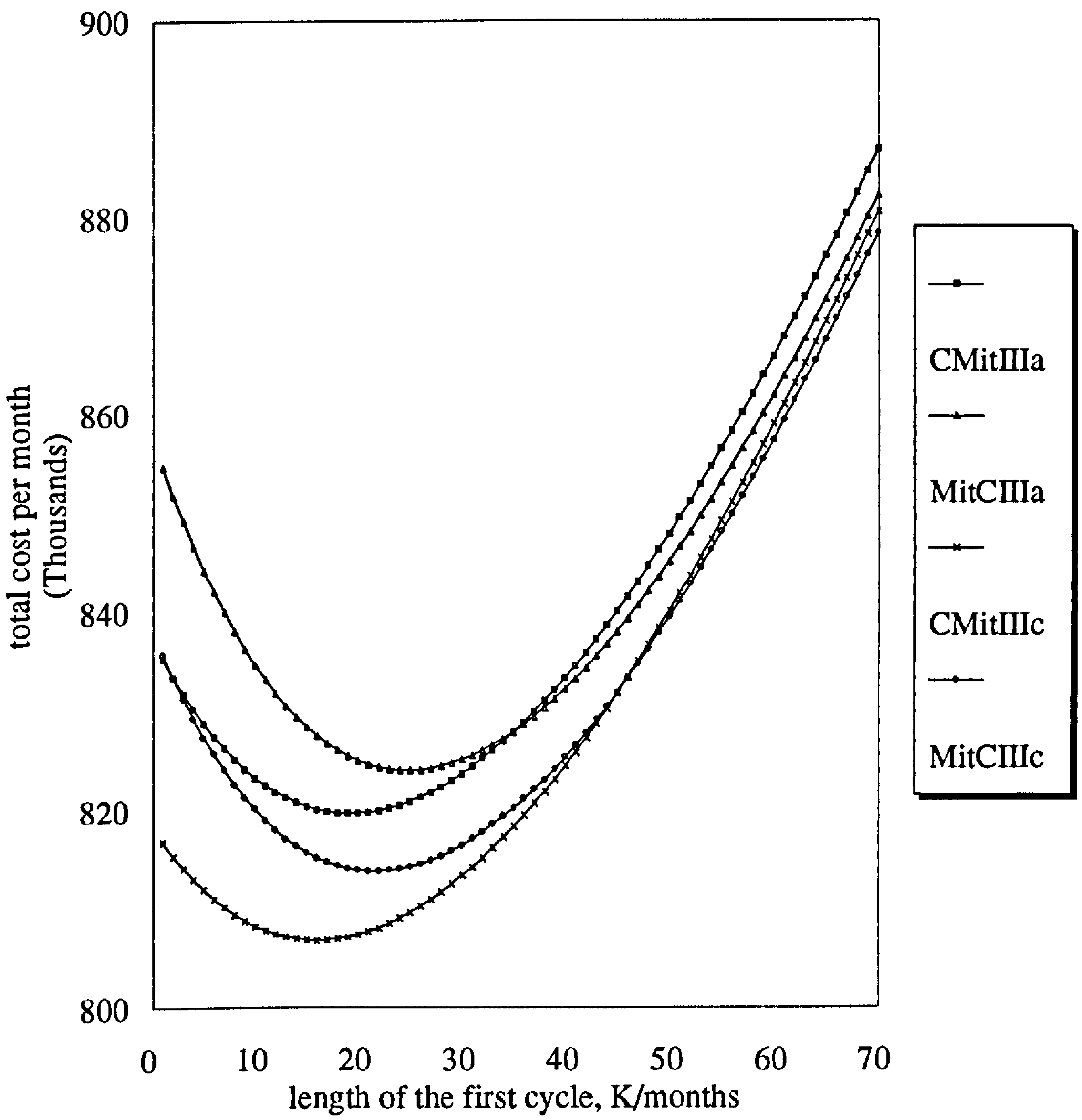


Figure 6.7 Variable fleet size vs. fixed fleet size; three cycle model with variable fleet size at first replacement (model IIIc) vs. the three cycle model with fixed fleet size (model IIIa). Cummin replaced first, Mitsubishi second. Penalty cost of unavailability p=M$1000.

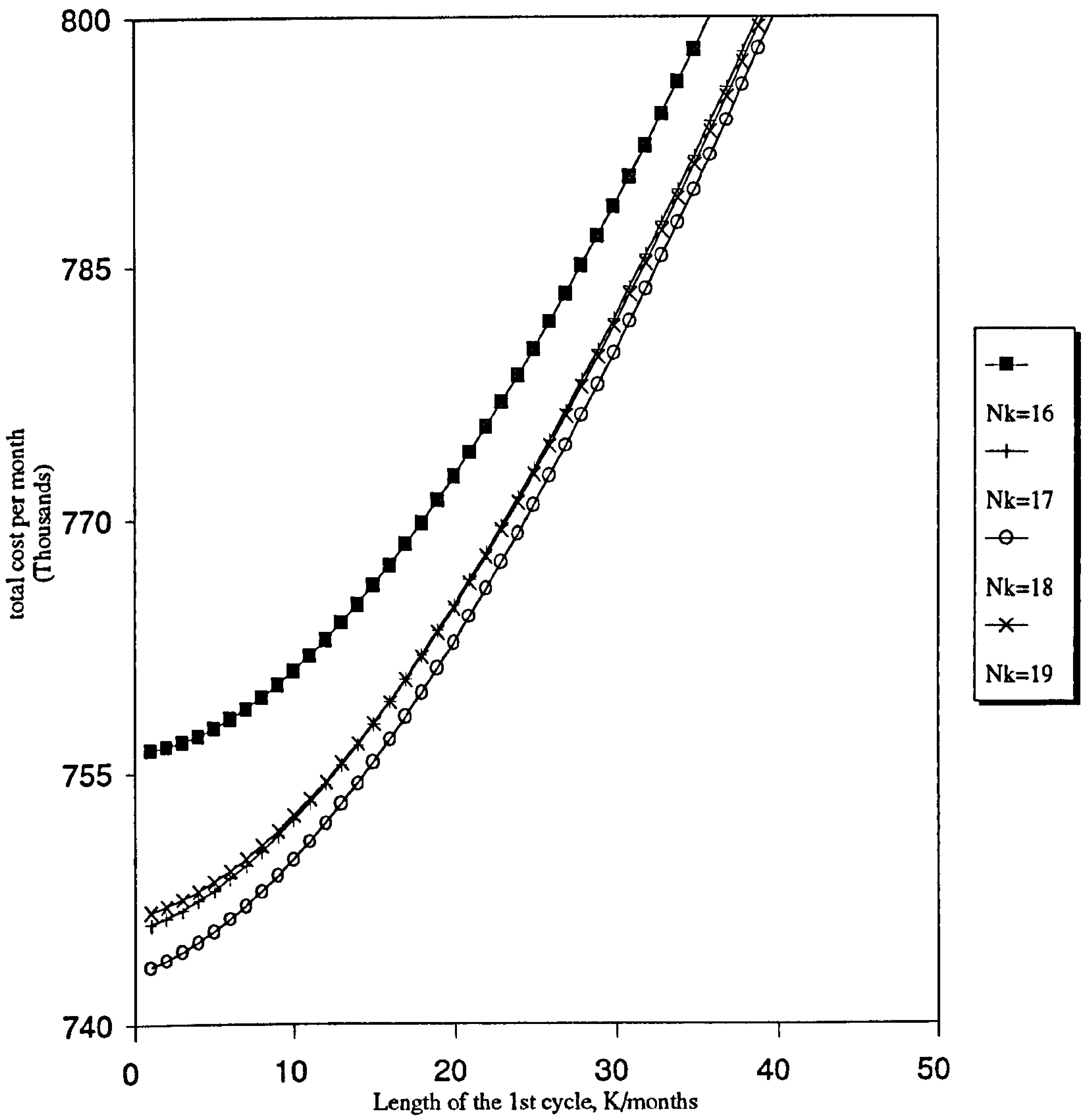


Figure 6.8. Cost per month of the equivalent rent vs. the length of the first cycle for various values of the replacement sub-fleet size . Cummin is replaced first, Mitsubishi second. Penalty cost of unavailability p=M$1000. Two cycle model with variable fleet size (model IIb). (L=L*).

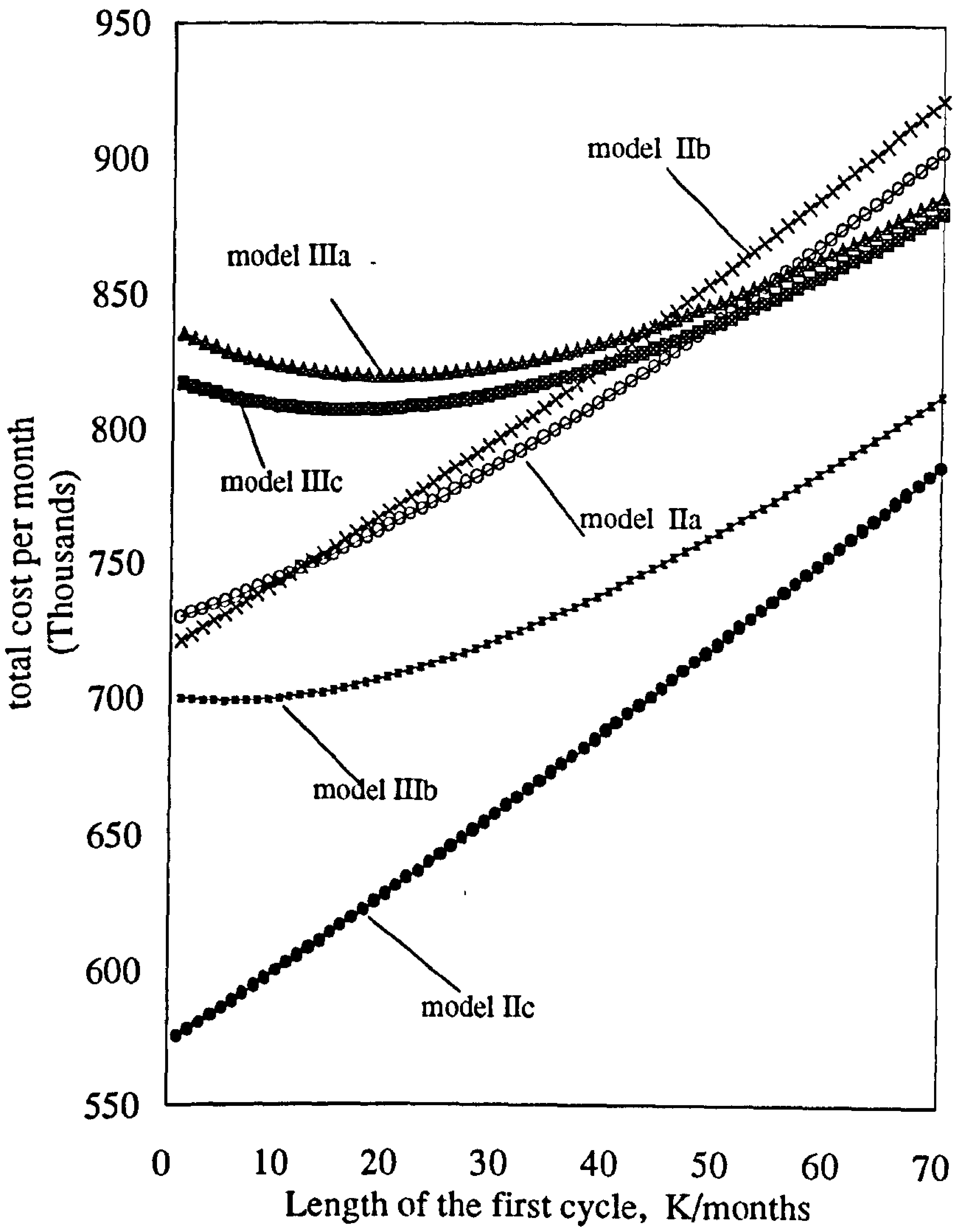


Figure 6.9. Cost per month of the equivalent rent vs. the length of the first cycle for various replacement models, based on results of table 6.8. ($L=L^*$, $M=M^*$, $N_K = N_K^*$). Penalty of unavailability p=M$1000.

APPENDIX I

We assume that, for i=0,...,N $\lambda_{i,t}=(N-i)\lambda_t$ and $\mu_0=0$, $\mu_i=i\mu$ where $\lambda_{i,t}$, λ_t, μ_i and μ are defined in chapter 3. Let j be the number of failure, j=0,....N.

For j=0

$$P_0(t+dt)=P_0(t)(1-\lambda_{0,t}dt)+P_1(t)\mu_1 dt,$$

$$P_0(t+dt)=P_0(t)-\lambda_{0,t}P_0(t)dt+P_1(t)\mu_1 dt,$$

$$P_0(t+dt)-P_0(t)=-\lambda_{0,t}P_0(t)dt+P_1(t)\mu_1 dt,$$

$$\frac{P_0(t+dt)-P_0(t)}{dt}=-\lambda_{0,t}P_0(t)+P_1(t)\mu_1, \qquad \text{(A1)}$$

as $dt\to 0$ equation (A1) becomes

$$P_0'(t)=\frac{dP_0(t)}{dt}=-\lambda_{0,t}P_0(t)+P_1(t)\mu_1=-N\lambda_t P_0(t)+\mu P_1(t)$$

$$P_0'(t)=-N\lambda_t P_0(t)+\mu P_1(t). \qquad \text{(A2)}$$

For j=1,...,N-1

$$P_j(t+dt)=P_{j-1}(t)\lambda_{j-1,t}+P_j(t)(1-(\lambda_{j,t}+\mu_j)dt)+P_{j+1}(t)\mu_{j+1}dt,$$

$$P_j(t+dt)=P_{j-1}(t)\lambda_{j-1,t}+P_j(t)-(\lambda_{j,t}+\mu_j)P_j(t)dt+P_{j+1}(t)\mu_{j+1}dt,$$

$$P_j(t+dt)-P_j(t)=P_{j-1}(t)(N-j+1)\lambda_t dt-((N-j)\lambda_t+j\mu)P_j(t)dt+P_{j+1}(t)(j+1)\mu dt,$$

$$\frac{P_j(t+dt)-P_j(t)}{dt}=P_{j-1}(t)(N-j+1)\lambda_t-((N-j)\lambda_t+j\mu)P_j(t)+P_{j+1}(t)(j+1)\mu, \qquad \text{(A3)}$$

as $dt\to 0$ equation (A3) becomes

$$P_j'(t)=\frac{dP_j(t)}{dt}=(N-j+1)\lambda_t P_{j-1}(t)-((N-j)\lambda_t+j\mu)P_j(t)+P_{j+1}(t)(j+1)\mu. \qquad \text{(A4)}$$

For $j=N$

$$P_N(t+dt)=P_{N-1}(t)\lambda_{N-1,t}dt+P_N(t)(1-\mu_N)dt,$$

$$P_N(t+dt)=P_{N-1}(t)\lambda_t+P_N(t)dt-N\mu P_N(t),$$

$$P_N(t+dt)-P_N(t)=\lambda_t P_{N-1}(t)dt-N\mu P_N(t)dt,$$

$$\frac{P_N(t+dt)-P_N(t)}{dt}=\lambda_t P_{N-1}(t)-N\mu P_N(t), \qquad \text{(A5)}$$

as $dt\to 0$ equation (A5) becomes

$$P_N'(t)=\frac{dP_N(t)}{dt}=\lambda_t P_{N-1}(t)-N\mu P_N(t). \qquad \text{(A6)}$$

At the steady state equation (A2) becomes

$$P_0'(t)=0=-N\lambda_t P_0+\mu P_1,$$

$$P_1 = N\rho_t P_0 \ , \qquad \text{(A7)}$$

where $\rho_t = \frac{\lambda_t}{\mu}$. Equation (A6) will also become

$$P_N'(t)=\lambda_t P_{N-1}-N\mu P_N=0,$$

$$P_N = \frac{1}{N}\rho_t P_{N-1} \qquad \text{(A8)}$$

Equation (A4) becomes

$$P_N'(t)=(N-j+1)\lambda_t P_{j-1}-((N-j)\lambda_t+j\mu)P_j+(j+1)\mu P_{j+1}=0,$$

Let consider

$$z_i = \left.\begin{matrix} -(N-j)\lambda_t P_{j-1}+(j+1)\mu P_{j+1} & \text{if } \ i=j \\ -(N-j+1)\lambda_t P_{j-1}+j\mu P_j & \text{if } \ i=j-1 \end{matrix}\right\} \qquad \text{(A9)}$$

From equation (A9) we can notice that $z_j - z_{j-1}=0$. This implies that $z_j=0$ for all j=1,...,N-1. We then obtain the recurrent relationship expressed as

$$P_{j+1} = \frac{N-j}{j+1}\rho_t P_j. \qquad \text{(A10)}$$

Equation (A10) gives by mathematical induction

$$P_{j+1} = \frac{(N-j)(N-j+1)(N-j+2)....N}{(j+1)j(j-1).......1}\rho_t^{j+1}P_0,$$

$$P_{j+1} = \frac{N!}{(N-j-1)!(j+1)!}\rho_t^{j+1}P_0,$$

$$P_j = \binom{N}{j}\rho_t^j P_0. \qquad \text{(A11)}$$

P_0 can be obtained by the normalising condition $\sum_j P_j = 1$, that is

$$P_0 = \left[\sum_{j=0}^{N}\binom{N}{j}\rho_t^j\right]^{-1} = (1+\rho_t)^{-N}.$$

APPENDIX II

Maintenance costs

Isuzu CJR bus number	cost 1991 (M$)	cost 1992 (M$)	cost 1993 (up to April)
1300	3297	23765	8522
3885	3661	36011	10436
202	4912	35949	12332
535	3540	22893	7265
565	1207	25262	6540
575	2294	22313	6584
656	2856	30137	8905
676	1848	30540	17012
6208		33227	10004
6281		16395	11709
2000		22219	9978
2020		17933	7895
272		18143	7069
292		7282	3401
343		6984	4512
353		14887	8264
400		8407	8103
500		263	3002
600		3435	11765
800		2880	6974
161		3407	9321
8016		1515	4051
8017		482	3384
6001		111	870
6002		3	5298
6003			1152
1001			4772
3910			296
3915			975
150			385

Isuzu CSA

bus number	cost 1990 (M$) (up to September)	cost 1991 (M$)	cost 1992 (M$)	cost 1993 (M$) (up to April)
709	34841	55365	59610	20894
710	40947	69492	90564	13986
711	45996	57436	72444	16468
714	48634	83781	88637	22574
715	51000	67545	84978	31654
716	34326	35600	48298	11983
6567	40299	51692	48505	18672
6568	28656	63990	53854	10755
6569	42533	44082	49962	76857
6571	69552	64131	45328	15222
6573	59108	12093	64009	70959
6574	48745	33660	36861	24427
4860	25029	65203	45895	10407
4861	39589	77548	75728	26679
4862	36029	79162	69997	24337
4863	44975	63002	75617	11483
4864	68703	57985	58484	18606
4865	38384	73551	47430	14673
4866	47902	73150	82869	18090
4867	26531	6017	4533	70
4868	52898	38605	60309	20937
4869	31970	69571	77732	9850
4870	49504	49663	31782	98735
6171	35502	68145	61919	31826
6172	40443	41636	84781	13683
6173	71804	54744	67122	14292
6174	34642	65439	67498	13915
6175	51520	72105	73162	13392
6176	35129	57859	70851	18721
5226	48083	49646	46728	8678
5227	45528	71902	69039	14293
5228	61830	51954	39439	44120
5231	38471	57097	46985	24941
5301	35377	60360	85339	14862
5302	29232	65508	53210	18383
4709	56138	59819	96055	16729
4710	12169	48725	47787	19507

Mitsubishi bus number	cost 1990 (M$) (up to September)	cost 1991 (M$)	cost 1992 (M$)	cost 1993 (M$) (up to April)
1155	46191	77187	69414	28010
1166	52638	75370	63910	16214
1177	58491	75358	48288	60046
1221	51122	56586	81115	19329
1331	38203	63933	76724	15880
1551	61054	53633	75574	29936
8484	47387	53584	96455	27921
9250	23424	53941	47956	24790
9252	53770	62643	67860	22468
9253	56030	77321	44053	14743
9255	14200	36159	81392	12133
9256	51847	57231	92135	17818
9257	34008	50790	21869	11044
9258	24252	49342	71680	51932
9259	41654	94998	41327	20312
9260	47678	38827	59338	18683
9261	26051	62051	51609	23617
9262	28974	38193	40576	15188
2275	11721	53057	85771	6213
2276	15879	75467	31633	19693
2703	54672	50190	72570	14729
2704	45644	43261	86692	14600
2705	93951	64571	74098	12557
2706	64589	50814	70366	23627
2707	28786	74741	54611	9222
2708	59818	63118	35013	33001
2834	23644	65163	58662	8417
2835	35156	38807	68752	14072
2836	27094	69508	39604	18907
2839	36867	42908	178457	5833

Cummin bus number	cost 1990 (M$) (up to September)	cost 1991 (M$)	cost 1992 (M$)	cost 1993 (M$) (up to April)
98	19846	54370	42311	26286
5353	19754	61962	67351	16685
5533	23360	58094	60591	3073
6611	24864	73946	95873	8048
2525	32280	71911	53833	7042
8400	12972	39357	56491	15276
8401	18933	57394	91258	10904
8402	18228	63769	69522	17047
8403	22332	43562	63276	16891
9697	17694	93490	57372	13581
1033	14399	60446	66599	27871
1073	29225	73237	52387	30551
1075	11203	46844	58287	19898
1077	12988	52806	71905	24965
1103	10094	48178	58691	17614
1187	12900	67602	71755	11582

Mercedes bus number	cost 1990 (M$) (up to September)	cost 1991 (M$) (up to August)
6572	23239	18381
8967	10757	27036
2952	19262	26809
3068	9517	46629
9506	13336	13739
9507	8379	29941
9663	22076	14231
9664	38378	40474
8642	89062	10939
8793	11944	7674
2684	26504	22190
3580	8527	24683
4266	22051	21158
5000	9181	1930
8000	10632	2537
7474	35587	27726
4545	15279	13600
4646	24071	21712
650	19368	15931
2500	32525	20580
2501	24470	14757
1650	19513	18403
1651	42764	20330
1652	24665	17292
2004	44939	29034
6660	24479	26549
6661	37330	36207
4235	2029	10709
4794	9571	11683
5293	3790	11896
7845	11083	20484
809	8773	10224
810	10690	15593
7168	19495	18294

APPENDIX III

Breakdowns in the road.

	Reg year/ total buses		Average number of breakdowns per month			
			1990 record	Survey I 18/09/91 15/10/91	Survey II 01/03/92 29/04/92	Survey III 27/08/93 17/09/93
Cummin	1989	5	5.1	2.2	2	3
	1990	11	5.6	13.3	8.5	9
Isuzu CSA	1980	12	6.7	7.8	7.5	7.5
	1981	17	11.8	3.3	11	15
	1982	6	3.9	2.2	4	6
	1983	2	0.7	5.6	0	0
Mercedes	1980	1	0.7	0	0.5	-
	1981	1	0.3	0	1	-
	1982	10	5.6	8.9	8	-
	1983	4	1.6	1.1	0.5	-
	1984	5	3	0	2.5	-
	1985	4	2.6	1.1	2	-
	1986	2	1.6	2.2	1	-
	1988	6	1.9	0	1	-
Isuzu CJR	1991	8	-	-	2.5	3
	1992	17	-	-	1.5	9
	1993	1	-	-	0	9

APPENDIX IV

This is the FORTRAN program used for the results given in chapter 5 and 6.

```
C*************************************************************
C This program considers the two cycle model with variable length
C of the horizon and variable fleet size. It is however flexible
C and can consider fixed fleet size. It either considers the
C equivalent rent criterion or the total dicounted cost/ unit time.
C The time unit is the month.
C Sensitivity analysis on purchase cost, discount factor resale
C value and penalty cost are also considered.  Costs of sub-optimal
C decision such as delayed replacement can also be considered if needed.
C*************************************************************
    PROGRAM  SUB_FLEET SIZE
    DOUBLE PRECISION SOM1,SOM2,UNMETD1,UNMETD2
    DIMENSION COSTNEW(10),AGE(10,120),N(10),ALPHA(10),BETA(10)
   1 ,GAMMA(10),DELTA(10),RESALE1(10),RESALE2(10),penarray(1)
    DOUBLE PRECISION X1,X2,S,SS,SS1,TDC,TC1,TC2, XX2,S1,SSS
    DATA PENARRAY/000./
C*************************************************************
C** File 'quattro.dat' contains results of the computation ready for
C** quattropro spreadsheet.
    OPEN (7,FILE='quattro.dat')
    OPEN (8,FILE='RESULT')
C**  NK is the size of the 1st replacement sub-fleet.
C**  NL          "          2nd          "            .
C*************************************************************
C Loop for sensitivity analysis on purchase cost (if needed).
    do 98 nr=10,10
    r=nr/10.
C*************************************************************
C Loop for sensitivity on discount factor (if needed)
    do 99 ndisc=97,97
    discount=real(ndisc)/100.
C*************************************************************
C Loop for penalty cost (if needed)
    DO 303 IP=1,1
```

```
      PCOST=PENARRAY(IP)
      VAL_MIN=1.0E+09
C*************************************************************
C Reading data about sub-fleets and replacement cost and maintenance
C costs. Alpha and Beta relate to the maintenance cost power law function
C Cost/bus/month=Alpha *(Age/month)**Beta.
C Gamma and Delta are parameters of resale model. Lastly the parameters of
C the replacement sub-fleet are read.
C*************************************************************
      OPEN (9,file='fleet1.dat')
      READ(9,*) NSUB,NDEMAND
      DO 999 IR=1,NSUB
      READ (9,*) N(IR),(AGE(IR,JR),JR=1,N(IR)),COSTNEW(IR),
     1 ALPHA(IR),BETA(IR),GAMMA(IR),DELTA(IR)
  999 CONTINUE
      READ (9,*) COSTNEW(IR+1),ALPHA(IR+1),BETA(IR+1),
     1 GAMMA(IR+1),DELTA(IR+1)
      READ (9,*) COSTNEW(IR+2),ALPHA(IR+2),BETA(IR+2),
     1 GAMMA(IR+2),DELTA(IR+2)
      close (9)
C*************************************************************
C Decision variables loops,
      DO 300 NL=N(2),N(2)
         DO 306 NK=N(1),N(1)
          DO 100 L=1,70
           DO 150 K=12,72,6
C*************************************************************
C FIRST CYCLE
C*************************************************************
            SOM1=0.0
             DO 2000 jj=1,k
             NTOTAL1=0
             X1=0.
             S1=0.0
C*************************************************************
C Loop for resale value, maintenance cost and mean number of failure
                DO 111 NR1=1,NSUB
                RESALE1(NR1)=0.0
```

```
         DO 110 NS1=1,N(NR1)
         S1=S1+(1./12.)*(EXP(GAMMA(NR1))*(real(JJ)/12.
   1     +AGE(NR1,Ns1))**DELTA(NR1))*DISCOUNT**
   2     (real(JJ)/12.)
         X1=X1+(1./30.42)*EXP(ALPHA(NR1))*EXP(BETA(NR1)*
   1      (REAL(JJ)/12.+AGE(NR1,Ns1)))
         RESALE1(NR1)=RESALE1(NR1)+r*COSTNEW(NR1)
   1     *0.613*(0.81)**(REAL(K)/12.+AGE(NR1,NS1))
110      CONTINUE
        NTOTAL1=NTOTAL1+N(NR1)
111    CONTINUE
         NZ1=INT(X1-2*SQRT(X1))
         NT1=NTOTAL1-NDEMAND+1
         NZ2=INT(X1+2*SQRT(X1))
         NT2=NTOTAL1
         UNMETD1=0.0
         NXS1=MAX(0,NDEMAND-NTOTAL1)
C*****************************************************************
C Computation of the unmet demand and the penalty cost
         DO 1110 NNZ1=MAX(NT1,NZ1),MIN(NT2,NZ2)
         PROB1=EXP(-X1)*(X1**NNZ1)/FACT1(NNZ1)
         UNMETD1=UNMETD1+REAL(NNZ1-MAX(NT1-1,NZ1))*
   1     PROB1
1110     CONTINUE
     PENCOST1=(365./12.)*PCOST*(real(NXS1)+UNMETD1)*DISCOUNT**
   1  (real(JJ)/12.)
       SOM1=SOM+(S1+PENCOST1)
2000   CONTINUE
C*****************************************************************

TC1=SOM1+(DISCOUNT**(REAL(K)/12.))*(REAL(NK)*COSTNEW(Nr1+1)-
   1  RESALE1(1))
C*****************************************************************
```

```
C*****************************************************************
C SECOND CYCLE
C*****************************************************************
      ss1=0.0
      SOM2=0.0D0
            DO 1000 ii=1,L
            SS=0.0D0
            NTOT2=0
            XX2=0.0D0
             DO 222 NR2=2,NSUB
             RESALE2(NR2)=0.0D0
              DO 220 NS2=1,N(NR2)
              SS=SS+(1./12.)*EXP(GAMMA(NR2))*
     1         (REAL(II+K)/12.+AGE(NR2,Ns2))**DELTA(NR2)
              RESALE2(NR2)=RESALE2(NR2)+
     1         r*COSTNEW(NR2)*0.613*(0.81)**(REAL(L+K)/12.
     2         +AGE(NR2,NS2))
              XX2=XX2+(1/30.42)*exp(ALPHA(NR2)+BETA(NR2)*
     1         (REAL(II+K)/12.+AGE(NR2,Ns2)))
 220           CONTINUE
           NTOT2=NTOT2+N(NR2)
 222          CONTINUE

            X2=REAL(NK)*(1/30.42)*EXP(ALPHA(Nr2+1)+BETA(Nr2+1)
     1        *real(II)/12.)+XX2

            SS1=SS+(1./12.)*(REAL(NK)*EXP(GAMMA(Nr2+1))*
     1        (real(II)/12.)**DELTA(Nr2+1))*DISCOUNT**
     2        (real(II)/12.)
             NTOTAL2=NTOT2+NK
            NZZ1=INT(X2-2*SQRT(X2))
            NTT1=NTOTAL2-NDEMAND+1
            NZZ2=INT(X2+2*SQRT(X2))
            NTT2=NTOTAL2
            UNMETD2=0.0D0
            NXS2=MAX(0,NDEMAND-NTOTAL2)
```

```
              DO 2220 NNZ2=MAX(NTT1,NZZ1),MIN(NTT2,NZZ2)
              PROB2=EXP(-X2)*(X2**NNZ2)/fact1(NNZ2)
              UNMETD2=UNMETD2+REAL(NNZ2-MAX(NTT1-1,NZZ1))*
   1           PROB2
2220          CONTINUE

      PENCOST2=(365./12.)*PCOST*(real(nxs2)+UNMETD2)*DISCOUNT**
   1        (REAL(II)/12.)
            SOM2=SOM2+(SS1+PENCOST2)
 1000       CONTINUE
C*****************************************************************
      TC2= (DISCOUNT**(REAL(K)/12.))*(SOM2+ (DISCOUNT**(real(L)/12.))
   1   *(real(NL)*COSTNEW(Nr2+1)-RESALE2(2)))
C*****************************************************************
C Rent criterion
       SUM_DISC=0.0
              DO 60 II=1,K+L
              SUM_DISC=SUM_DISC+DISCOUNT**(real(II)/12.)
  60          CONTINUE
              TDC=(TC1+TC2)/SUM_DISC
C*****************************************************************
C Total discounted cost per unit time (if needed)
C             TDC=(TC1+TC2)/(K+L)
C*****************************************************************
         WRITE(*,80) K,NK,L,NL,TDC,pcost,discount
         write(7,86) K,TDC
C*****************************************************************
C Decides if the current cost for decsion variables is minimum.
         IF (TDC.LT.VAL_MIN) THEN
         VAL_MIN=TDC
         KOPT=K
         LOPT=L
         NKOPT=NK
         NLOPT=NL
         ENDIF
C*****************************************************************
150    CONTINUE
   PRINT *, 'MINCOST=',VAL_MIN, ' Kopt= ',KOPT,' NKopt=',NKOPT
```

```
      PRINT *, 'Lopt=',LOPT,' NLopt=',NLOPT
100      CONTINUE
306     CONTINUE
300    CONTINUE
        write (8,82)  kopt,lopt,nkopt,nlopt,pcost,val_min
303    CONTINUE
99     CONTINUE
98     CONTINUE
80   FORMAT(1X,'TDC(',i3,i3,i3,i3,')=',f18.2,'pcost=',f8.2,
    1 'discount=',f4.2)
82   FORMAT(1x,i3,1x,i3,1x,i3,1x,i3,1x,'penalty=',f8.2,'
    1 mincost=',f18.2,/)
86   FORMAT(i2,2X,F15.2)
      CLOSE (7)
      CLOSE (8)
      END
C*******************************************************************
C**   THIS PROGRAM CALCULATES THE VALUE OF N!
      REAL FUNCTION FACT1(NF)
      INTEGER NF
      FACT=1
       DO 13 I=1,NF
       FACT=FACT*I
 13  CONTINUE
      FACT1=FACT
      RETURN
      END
C*******************************************************************
```

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