# Essays on Asset Pricing and Financial Regulation

by

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Submitted to the Department of Finance

in partial fulfilment of the requirements for the degree of

Doctor of Philosophy

at

Imperial College London

December 2020

# Declaration of Originality

I certify that this thesis and the research to which it refers, are the product of my own work, and that any ideas or quotations from the work of other people, published or otherwise, are fully acknowledged in accordance with the standard referencing practices of the discipline.

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# Acknowledgement

I have been extremely privileged to be a student of Professor Alexander Michaelides whose constant supports guided me throughout my doctoral studies and this thesis. I am forever grateful to Alex for his leadership, motivation, enthusiasm, patience and immense knowledge over the past five years at Imperial College London that made this scholarly work possible. I wish to express my profound gratitude to Professor Franklin Allen for his continuous supports and dedicated involvement in every step throughout the development of this research during the past five years.

I wish to express my sincere appreciation to my committee members Professors Harjoat Bhamra and Savitar Sundaresan for their immense contributions that instilled great confidence in both myself and my work, their inspirations, their incredible feedback and insightful input over the many years. I am grateful to Professors Patrick Bolton and Dimitrios Tsomocos who warmly contributed to further development of this scholarly work with their insightful inputs on the many aspects that have collectively made this thesis an impactful research.

I am thankful to Andrea Buraschi, John Campbell, Gilles Chemla, Pasquale Della Corte, Douglas Gale, Chryssi Giannitsarou, Christopher Hansman, Marcin Kacperczyk, Paymon Khorrami, Robin Lumsdaine, Zaici Li, Maria-Teresa Marchica, David Miles, Ramana Nanda, Monika Piazzesi, Tarun Ramadorai, Ricardo Reis, Yun Him Adrian Lam and Paolo Zaffaroni for their valuable feedback and numerous helpful conversations. I would like to thank Salim Baz, Cláudia Custódio, Adam Denny, Marco Francischello, Can Gao, Rajkamal Iyer, Lu Liu, Nikolaos Rapanos, Oren Schneorson, Yifan Yang, Yi Zhang, my fellow graduate students and many other seminar participants for their feedback and comments.

I wish to express my gratitude to Imperial College Business School, Catherine Lester, Katherine Campbell, Nicola Hann, Pamela Monteith, Moira Rankin, Louise Rowland, Jason Murray and Tracy Andrew for their many supports towards my doctoral studies.

I have been privileged to be a member of Imperial College London and would like to express my gratitude to all of Imperial's communities for providing an outstanding environment of support, collaborative work and academic excellence that nurtures impactful research. A special thanks to Imperial College London Presidential Scholarship that supported me throughout this research.

Last but not least, a very special thanks to four very special people in life, my parents Farzaneh, Keyvan, my sister Haleh for their unconditional supports and above all to my wife, Atiyeh for her unceasing encouragement, her expertise as a young economist throughout this thesis and my doctoral studies, whose endless supports made the journey possible.

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## Abstract

I show that when the banking sector's assets comprise large excess reserves and loans, jointly determined capital regulation and interest-on-excess-reserves (IOER) policies provide welfare gains. In general equilibrium, falling IOER is associated with a proportional fall in deposit rate only when IOER is above the zero bound. This leads to a faster fall in the bank's interest expenses than its interest incomes. Given any lending level, lower net interest expenses enhance bank solvency. Nonetheless, the risk-weighted capital regulation remains unchanged and hence becomes socially costly. I show that jointly determined policies achieve welfare gains by loosening the capital requirement and lowering IOER to expand the credit flow, while bank failure likelihood remains constant. Conversely, lowering IOER below the zero bound is associated with a nonresponsive deposit rate that leads to growing net interest expenses and worsening bank solvency. In that case, I show that a stricter capital constraint together with a lower IOER provide social value.

The aftermath of the financial crisis inherited heightened economic uncertainty and low productivity. These features prompted the banking sectors across the developed economies to rely heavily on excess reserves offered by the central banks despite the negative nominal IOER policy rate. Nonetheless, the negative relationship between the overall interest expenses of the banking sector with the IOER around the zero lower bound further exacerbates the over-reliance on excess reserves particularly when rates are negative. Financial regulator faces a trade-off between the costly failure of an under-capitalized banking system and costs generated by interconnections between interest expenses on oversized excess reserves and government guarantees to depositors. I show that first, the risk-weighted optimal capital regulation exhibits a negative correlation with the IOER policy rate, and second present a socially optimal financial regulation that balances the social gains of negative IOER rate, generated by reduced over-reliance on idle reserves, against its social costs, generated by the increased default likelihood of the banking institutions.

# Contents

# List of Figures

# List of Tables

# Chapter 1

# Micro-Foundations of Macroeconomic Policy: Financial and Monetary Regimes

## 1.1 Introduction

Financial and monetary regimes form integral components of macroeconomic environments. From the society's perspective, understanding the functions and the interworking mechanisms between these components provides a foundation to deliver welfare gains to financial institutions and ultimately the households.

While financial and monetary regimes often differ across economies, their basic characteristics remain universal irrespective of political or industrial structures. In general, financial and monetary regimes across the majority of economies are implemented by the following institutions: the *financial system*; *government financial regulation and supervision*; and the *central bank*. First, the financial system consists of individual or institutional participants that interact via markets. The banking institutions are among the most common examples of financial institutions that act as a conduit of funds. The primary medium of transaction is often dealt in each economy's domestic currency, and capital markets instruments such as bonds, equities, commercial papers, and government bonds. The financial system provides broad services via institutionalization of savings-borrowings processes that are intended to offer an efficient transfer of resources across the economic segments. The financial system also offers an intertemporal wealth transfer flexibility to those with different preferences towards consumption or saving. Lastly, aside from serving as a conduit between the ultimate lenders and borrowers, the financial system provides a platform for the conduct of financial and monetary policies through its entrenched spread across the macroeconomic landscape.

Government financial regulation is intended to monitor and ensure the stability and soundness of the financial system through a wide range of guidelines and policy initiatives. While governments serve numerous functions within the macroeconomic landscape, their focus on designing and implementing an effective financial regulation has been evergrowing over the recent decades. Regulatory interventions have been evolving over time in response to innovations and growth throughout the financial systems to ensure the safety and transparency of the services delivered to the society. The central bank is often referred to a special government institution that maintains its independence. Similar to the governments growing roles, the objectives of the central banks have been extended to monitor and regulate the financial system through policies that influence credit, reserves, interest rates and the overall level of economic activity. Aside from regulatory services, central banks provide national payments systems such as conducting clearances amongst depository institutions. For instance, in the United States, the Federal Reserve System provides a wide range of services such as financial regulation and supervision, whereas, in Japan the central bank plays a relatively limited role.

## 1.2 Institutional Setting

Well-functioning financial and monetary regimes are amongst the necessary conditions that lead to an increased standard of living. More specifically, growth in household wealth while maintaining economic stability broadly forms the objective of financial and monetary policy initiatives.

From a macroeconomic-finance perspective, the uncertainty[1] associated with future outcomes is an important driver of current economic performance. Uncertain economic outcomes or shocks to demand and supply are autonomous or unexpected exogenous surprises that lead to changes in investment and spending decisions. Shocks to the supply side of the economy are often the result of unexpected changes in the supply of real commodities, disruptions in operational processes, and technological changes that impact the productivity of capital or labour inputs. Similarly, demand shocks are the result of expected changes in preferences such as behavioural drivers that lead to real economic or financial implications.[2]

[1]The notion of uncertainty primarily refers to decision making under unknown probability measures, whereas risk refers to a sub-case of uncertainty when the probability is measurable. Within the context of this study, probabilities are always assumed to be measurable and evaluated based on von-Neumann Morgenstern expected utility framework. Section (1.3.4) formally introduces an institutional definition for risk and its sources.

[2]**?** shows that independent demand-driven shocks have causal short- to medium-term effects on future

Financial and monetary regimes provide welfare gains to society by enhancing the efficiency of the financial system. Providing an efficient flow of funds between the lenders and borrowers leads to further economic activity while policies with prudential goals ensure that the ultimate macroeconomic performance remains sustainable. Regulatory and supervisory initiatives become particularly necessary in order to provide welfare gains when the objectives of the financial system deviate from those of the households and the society. In the economic theory context, financial decisions made by the representative household sector who is ultimately the owner of the economy is characterized by preferences that exhibit aversion towards uncertain economic outcomes. This attitude towards uncertain outcomes that establishes the relationship between risk-reward is the foundation for the valuation of current and future wealth. Financial system whose decisions are a major driver of the eventual allocation of resources throughout the macroeconomic landscape exhibit preferences that appear to be less risk-averse than those of the households.[3] The financial system often seeks to deliver higher rewards to the society at the expense of committing to a higher risk profile. Nonetheless, from the perspective of the society or the households, high-risk exposures lead to a lower valuation of rewards. The financial and monetary regimes envisage the differences of preferences and deliver higher valuation of wealth to the society through policies that regulate the relationship between risk and reward within the financial system.

The inherent risk-reward trade-offs, embedded in the decisions made by the financial institutions, lead to real economic implications. Many institutions within the financial system benefit from the limited liability condition that prevents their valuation from falling negative even though the value of their total assets falls below their total liabilities. Particularly for those institutions that rely highly on debt-financing, the limited liability leads to decisions that disregard the risk and weigh even more heavily on return. Financial and monetary regimes provide welfare gains to societies by designing and implementing policies that regulate the liabilities structure of the financial institution to limit decisions that lead to a socially undesirable outcome.

economic performance. Investors expectations about future asset prices that is partly driven by variations in investor's rate of time preference is an essential driver of valuation risk (**?**). The implication of demand-side shocks on economic fluctuations remains an open question, however, **?** provides evidence that the persistence of shocks to the demand side is a key driver of the future level of aggregate macroeconomic variables. Other studies within non-expected utility framework, such as **?** examine the implications of consumer's perceptions of probabilities for determining economic fundamentals in a dynamic setting.

[3] In fact, financial intermediaries are amongst the institutions within the financial system that exhibit risk-neutral or risk-seeking behaviour (**?** and **?**). Financial decisions make by near-default intermediary institutions that benefit from limited liability often is characterized by risk-loving attitudes.

**Flow of Funds** Financial system provides a platform where the transfer of funds between ultimate lenders and borrowers is channelled through direct and indirect (intermediated) finance. Investment funds that are channelled through markets constitute direct finance accounting for thirty per cent of the flow of funds in the United States. Given the nature of financial decisions via direct interactions of the lenders and borrowers, this channel often serves the investment and financing needs of large enterprises and governments. In particular, larger enterprises benefit from the ability to monitor and evaluate the creditworthiness of their borrowers. Conducting costly assessments, both the forms of acquiring and processing information, creates frictions for smaller individuals or institutional investors who lack such infrastructures. Table (1.1) illustrates the most common money market financial instruments used in direct financing in the United States.[4]

| Financial Instrument | Description | Default Risk (Secondary Market) |
|---|---|---|
| Treasury Bills | Issued by the federal government in maturities up to one year; they pay no interest coupon and are sold at a discount. | None (Active) |
| Certificates of Deposits | Issued by large banks in denominations of $100,000 or larger and are legally negotiable; that is, they can be sold to another entity before maturity and issued in a range of maturities. | Balances above $250,000 (Modest) |
| Commercial Paper | Short-term debt issued by banks and non-financial business entities with maturities up to nine months. | Yes (Modest) |
| Repurchase Agreements | Banks, non-financial entities or any entity that holds securities used as collateral for the repo. The issuer uses securities such as T-bills for collateral. The issuer sells the repo with a promise to repurchase it in a short period of time, usually overnight or less than two weeks. | Determined by default risk of securities (None) |
| Federal Funds | Overnight loans of reserve balances held by a bank at the Federal Reserve to another bank with a deposit account at the Federal Reserve. Federal funds are not loans by the federal government or Federal Reserve. The name comes from the fact that funds are transferred from the lending bank to the borrowing bank via the Federal Reserve's wire transfer facility. The interest rate on federal funds, called the federal funds rate, is a key variable in the conduct of monetary policy. | Depends on default risk of bank (None) |

**Table 1.1:** List of common money market financial instruments used in the U.S. financial markets within the direct finance approach. The instrument above have maturities up to one year. Each instrument is associated with default risk indicated in the last column and access to secondary markets.

In the United States and many other economies, intermediated finance or indirect finance accounts for the majority of funds that are transferred between the lenders and borrower through the financial institutions. Table (1.2) illustrates the most common capital market financial instruments used in direct financing in the United States. Depository institutions,

[4]Financial instruments with maturities shorter or longer than one year are referred to as money market instruments and capital market instruments, respectively

pension funds, credit unions and S&Ls are among the main institutions that transform investments into liquidity to the rest of the business and production sectors. An important feature of this channel is its ability to provide services for a wider range of investors and financiers such as households, and small- to medium-sized enterprises. This feature provides increased access to the financial system and introduces major changes to the fundamental agreements between the lenders and borrowers. Particularly, in the direct financing, the lender evaluates and assumes the risk whereas, in indirect financing, the intermediary often assumes part of the risk or offers further services that lower the risk at certain costs and mitigates the difficulties that smaller market participants face in a direct interaction.

Within the banking institutions context, depositors in the U.S. benefit from up to $250,000 deposit insurance by the government, which essentially eliminates the lenders' concerns associated with the default risk for balances below the coverage limit. The deposit protection limit is applicable to each person's total eligible deposits. Balances above the cut-off threshold bear default risk as the difference falls outside the deposit insurance agency's coverage.[5] Nevertheless, when the person-level coverage indicates that each account holder is protected up to the deposit protection limit, protections on joint accounts add up by the multiples of account holders. In the U.K. deposits held in banks, building societies and credit unions (including in Northern Ireland) that are authorised by the Prudential Regulation Authority, are protected up to £85,000. This includes, for example, eligible deposits in current accounts, savings accounts, cash ISAs (held with a deposit taker) or savings bonds.[6]

The default risk of a financial institution is driven by the probability that the issuer will fail to honour promised repayments (principal or its interests) at maturity. For instance, the riskiness of government-issued debt varies depending on the underlying soundness of the economy. In August 2011, the U.S. government creditworthiness was downgraded from AAA to AA+ by Standard & Poor. Despite access to a wide range of resources, government defaults remain a consideration for investors.

Financial institutions are divided into depository and nondepository institutions based on their liabilities structure. A financial institution that is legally permitted to accept de-

[5] A depository institutions that are covered by the government guarantees may own several legal names or brands. This indicates that depositors who have deposits in more than one account under a single legal depository institution entity, or multiple accounts under different brands owned by a single depository institution, are only protected up to a total of a single coverage limit (e.g. $250,000 in the U.S. or £85,000 in the UK) across all these accounts. Depositors with eligible deposits that add up to more than the deposit protection limit are able to keep their deposits fully protected by splitting their deposits across different authorised intuitions.

[6] In the U.K., the deposit protection offers a temporary coverage for up to 6 months above the £85,000 limit (Protection will be up to £1million in most cases) for certain types of deposits classified as temporary high balances, such as the proceeds from private property sales.

| Financial Instrument | Description | Default Risk (Secondary Market) |
|---|---|---|
| Equities or Stock | Issued by financial and non-financial corporations (Maturities: none) | Yes (Active) |
| Corporate Bonds | Issued by financial and non-financial corporations (Maturities range of maturities up to 20 and 30 years) | Yes (Active) |
| Treasury Notes | Federal Government issued (Maturities less than 10 years) | No (Active) |
| Treasury Bonds | Issued by the federal government (Maturities greater than ten years and up to 30 years) | No (Active) |
| Municipals | Bonds issued by local, regional and state governments (Maturities up to 30 years in maturity) | Yes (Moderate) |
| Mortgage-backed Bonds, Mortgages | Mortgages are long-term loans issued by banks, S&Ls, savings banks, credit unions and mortgage brokers. The mortgages are then sold, in many cases to mortgage-related GSEs, which hold the mortgages or bundle them into a mortgage-backed bond to sell in the capital market. | Yes (Active) |

**Table 1.2:** List of common capital market financial instruments used in the U.S. financial markets within the direct finance approach. The instrument above have maturities longer than one year. Each instrument is associated with default risk indicated in the last column and access to secondary markets.

posits from the general public is a depository institution consisting of banks, S&Ls, credit unions and saving banks.[7] Although nonbank depository institutions differed in terms of services that they offered such as accepting only time-deposits and saving accounts, their functions are now have become very similar to other banking institutions that are the largest in terms of balance sheet size. In the U.S. there are about 820 S&Ls and savings banks; and 7,400 credit unions, whereas there are about 6,500 banks. In terms of assets structure, banks are amongst the most well-diversified institutions whose operations involve a large number of lenders and borrowers. Irrespective of the political and industrial landscapes, banks maintain a central role in the financial system, remain amongst the highly regulated institutions and are an essential consideration for the transmission of macroeconomic policies.

Nondepository institutions remain subject to less stringent regulation relative to depository institutions, since they legally cannot accept deposits from the society. They raise funds in the form of contributions from a large range of investors and provide funding in the form of credit to borrowers or investment in production sectors. Nondepository institutions can further be divided into investments institutions and contractual saving institutions. Table (1.3) describes the major financial institutions that operate within the intermediated finan-

[7] In the United States, depository institutions are required to obtain a banking charter from the federal government or state-level bank regulatory authority and are generally mandated to insure their deposits by the Federal Deposit Insurance Corporation.

| Institutions | Liabilities | Assets |
|---|---|---|
| **Depository Institutions** | | |
| S&Ls, Saving Banks, Credit Unions | Checking, Saving & Time-Deposits | Mortgage Loans, Consumer Loans |
| Commercial Banks | Checking, Saving & Time-Deposits, Certificates of Deposits | Consumer and business loans; mortgages; Treasury securities and municipal securities |
| **Nondepository Institutions** | | |
| Pension Funds | Retirement Policies | Corporate Bonds, equity |
| Mutual Funds | Shares | Bonds and equities |
| Money Market Funds | Shares | Money Market instruments |
| Finance Companies | Bonds and equities | Consumer and Business Loans |
| Life Insurances | Insurance Policies | Corporate Bonds, Equity, Mortgages |

**Table 1.3:** Borrowing approaches or sources of funds is the primary determinant to divide financial institutions into two types. The tables list common depository and nondepository financial institutions, and their balance sheet components.

cial system. Direct and intermediated finance overlap as a result of innovations within the financial intermediaries who are able to issue their own securities by combining other instruments such as mortgages and loans. Securities generated within intermediaries sectors are often sold in open markets to investors through direct finance. In the U.S., both direct and intermediated financing serve major roles to provide funding to the private sector. Specifically, non-financial businesses acquire 59% of their external financing from depository and non-financial institutions, 29% through issuing bonds and 12% by issuing equity. Outside the U.S., the intermediated financing is even more prevalent, for instance in Japan and Germany, 86% of the acquired funding is provided by the depository and nondepository institutions, whereas issuing bonds and equity account for 6% and 8%, respectively.

**Interest Rates and Prices** Interest rates or prices of securities are determined by financial transactions between the market participants who are involved in demanding and supplying funds. The underlying agreement between the two counterparties is based on borrower's promise to pay (but not guarantee), lender's acceptance, risks and other conditions embedded in the transactions. Lender's claim is a financial asset and borrower's liability. For example, banking institutions reserves deposited to the central bank's reserves deposit facility form a liability in the central bank's balance sheet that is remunerated at by the interest on reserves which can be positive, zero or negative.

Financial markets that operate through direct finance determine the interest rates (or price of securities) based on market mechanisms and vary more frequently, in contrast to interest rates on securities traded through intermediated finance markets. Within the context of financial and monetary regimes, interest rates have become increasingly more important tools for macroeconomic policy. In particular, the main monetary policy, such as the policy rate or the federal funds rate is decided by the monetary authorities and vary quarterly. This feature enables the policymakers to adjust the rates according to the ongoing financial and monetary environment on a more frequent basis. In contrast, the majority of financial regulatory tools, such as bank capital regulation rarely varies within a decade.

Each security is priced according to its risk-reward trade-offs and also depending on demand and supply by market participants. Financial and monetary authorities are also market participants who determine the interest rates on the securities issued by the platforms. Interest on reserves and interest on excess reserves that are determined by the central banks, or similar monetary authorities, have become key financial policy tools to regulate the size of (excess) reserves across the banking institutions in developing economies during the aftermath of the 2008 financial crisis. These tools impact the financial system or major sectors within the financial system and lead to real economic implications across the macroeconomy. This indicates that policy decision made by various financial and monetary authorities lead to an interrelated impact on financial system, households and enterprises and therefore developing an understanding of their interworking is a basis to welfare analysis.

Another important characteristic of interest rates is their implications for cashflows and earning within enterprises. Ultimately, interest rates are driven by macroeconomic fundamentals, preferences and information about future trends that form current valuations of financial assets and ultimately real economic indicators such as consumption. Aside from driving the cashflows, interest rates are an essential component of discount rates used in the valuation of future cashflows. Lastly, debt-liabilities often pay interest rates to claim holders who reserve the right to force the borrower into bankruptcy, should the borrower become unable to honour the interest payments. This indicates that when interest rates rise, interest expenses that form an ongoing cost for the borrowers increase. Interest expenses are among the key drivers that determine whether borrowers remain able to repay their debt-liability, in particular, when adverse shocks to borrowers limit their resources. This indicates that aside from discounted cashflows, interest rates determine the valuation of future cashflows through the solvency likelihood of the underlying issuers. From the society's perspective, a well-diversified investor or a representative household considers the impact of interest rates through cashflows, discount rates and the solvency channels simultaneously to form their valuation.

**The Supply of Financing and Demand for Financing** The relationship between the interest rate and quantity of funds supplied to a financial market is referred to as the supply of financing. This schedule illustrates the minimum interest rate at which the lender is willing to provide funds. The interest rate and the quantity of funds exhibit a positive relationship since the lender is willing to lend further funds at higher interest rates. This supply function is alternatively be presented as a downward-sloping relationship when considering the quantity of funds against the current prices. When funds are provided by the households as the ultimate investors, changes in their income, wealth or their perception over uncertainty lead to shifts in the relationship between the quantity and price of funds supplied to the markets. Financial asset purchases such as bonds and equities by investors lead to supplies of funds to borrowers who raise financing. Alternatively, aside from quantities and prices, funds supplied by the financial institutions depend on technological and institutional changes, perception of uncertainty and macroeconomic policy.

The relationship between the interest rate and quantity of fund demanded from financial markets is referred to as the demand for financing. This schedule represents the maximum interest rate at which the borrower is willing to raise funds. The interest rate and quantity of financing required are negatively related since the borrower is willing to raise more funds at lower interest rates. The demand for funding is alternatively illustrated as an upward-sloping curve when the quantity of funds is considered against the price of funds. This schedule is shifted by a wide range of determinants such as preferences for liquidity or early consumption when the borrower is a household, or because of institutional or technological changes when the borrower is an institution.

## 1.3 Financial and Monetary Regulation

Providing welfare gains through output expansion and macroeconomic stability are common goals of financial and monetary regimes. Financial regulation is more specifically focused on the stability and efficiency of the financial system, protection of investors and delivering an uninterrupted flow of fund to the borrowers. In comparison, monetary tools focus on broader objectives including the welfare of households, enterprises, and even government fiscal considerations. Nevertheless, from the perspective of the financial system, the scope of monetary and financial regimes overlap: the banking institutions interact directly with the central banks via many channels such as mandatory reserve requirements or voluntary excess reserves. Changes in the interest rates on these deposit facilities that are determined by the central banks lead to substantial implications for the asset structure and risk profiles of the banking institutions that concern the financial regulation. The following section discusses three regulatory interventions of the banking institutions that remain within the overlapping scopes of financial and monetary regimes.

### 1.3.1 Market Failure versus Intervention Failure

Market failure mitigation establishes the fundamental rationale behind the macroeconomic policy. From a conservative perspective, government interventions amount to reallocation of resources that lead to undesirable welfare implications when markets are considered well-functioning. However, the case for government involvement through macroeconomic policy is warranted when markets fail to internalize welfare costs. Policymakers consider the welfare gains associated with policies against welfare losses that often arise due to distortions resulted from the interventions. In particular, even with minimal government involvement, financial and monetary policies lead to numerous setbacks and are often entwined with elongated debates posed by pro-regulation (proponents) versus pro-deregulation (opponents) perspectives.[8]

Even at the minimal level of interventions, financial and monetary policies are associated with drawbacks. In this context, policies that provide guarantees or assurances provide financial stability by lowering abrupt responses of the society to undesirable outcomes, however, they also alter human behaviour and lead to moral hazard.[9] This, however, remains an open question for the regulatory agencies and academic research to design and implement effective policies that address deficiencies of the financial system while leading to minimal unwanted distortions to institutions and market participants.[10]

[8]Another important characteristic of wide government interventions, particularly across the financial system arises due to the incentives of the private sector to establish relationships with regulatory agencies. This has led to extensive lobbying between the lawmakers, politicians and the institutions seek to obtain favourable resolutions.

[9]Such regulatory mandates incentivize banks to participate in riskier investments. The increased risk-taking behaviour, at relatively low expected payoffs, may give high profits to shareholders. However, if investments fail, financial institution goes bankrupt and losses need to be absorbed by the government. The savings and loan (S&L) crisis during the 1980s and 1990s is an example of such adverse setback of safety net, when about one out of three savings and loan associations in the United States failed. In particular, S&Ls institutions overly relied on the Federal Savings and Loan Insurance Corporation (FSLIC) and as a result, FSLIC was unable to cover losses in many cases.

[10]An alternative approach used by governments to mitigate negative externalities of the banking system is to act as the lender of last resort. Central bank is able to provides funding to failing or near-default institutions in the form of loans. Loans are primarily intended to provide mitigation when an institution is solvent but illiquid at a certain time interval. This approach reduces bank panics but does not eliminate them, for example, as described by Friedman and Schwartz (1963), during the Great Depression of the 1930s, the Federal Reserve System had the ability to provide funding to mitigate panics.

### 1.3.2 Deposit Insurance

Deposit insurance is an important scheme that provides a guarantee to bank depositors and rules out self-fulfilling bank runs that are unwarranted by the fundamentals (**?**). In the U.S., deposit insurance was initially introduced during the Great Contraction in 1932 and authorized by the Banking Act of 1933. The modern deposit insurance scheme is implemented by the Federal Deposit Insurance Corporation (FDIC). This was a unique feature of the U.S. financial systems until 1974 when 12 other economies had incorporated the deposit insurance into their institutional settings. In 1999, deposit insurance had become an integral part of the global financial system with legal presences in 71 economies (**?**).

Nonetheless, the welfare gains associated with this intervention have to be considered against the potential drawbacks. The moral hazard problem distorts the relationship between the risk profile of an underlying depository institution that provides deposit-insured accounts and its cost of debt. In the absence of the deposit insurance scheme, each depository institution must take into account the marginal changes in its cost of debt given marginal changes in its risk profile. Well-informed depositors consider the possibility of their debt-holders default and require a higher compensation to supply funding in the form of deposits. This provides a market-driven mechanism that leads to lower risk-taking behaviour by the depository institutions. However, when the deposit insurance provides a guarantee to the supplier of funds, the cost of debt falls, leading to over-reliance of borrowers on cheap debt. This has posed a challenge for policymakers to balance the benefits of providing confidence to the depositors against the negative welfare implications of highly-levered depository institutions. Various approaches were suggested to counter-balance the moral hazard problem such as providing a limited insurance coverage to benefit only the smaller depositors who particularly lack knowledge of the financial system. This, however, was not practical in reality due to coverage applicable to accounts rather than individuals. Another approach that similarly was proved to be difficult to implement relied on risk-based deposit insurance premium.

Funds deposited to banks form a significant segment of the liabilities structure of the banking institutions. The instantaneous or short-term maturities on this type of debt becomes a concern for the banking institutions and more broadly financial regulatory agencies when banks transform demand deposits into illiquid long-term assets, such as loans and securities. The first underlying agreement behind deposit accounts indicates that investors are promised their entire balance of their checking accounts at the instance when they demand them. In practice, banks are able to meet deposit withdrawals instantaneously or on a short-notice basis, however, if a bank fails to honour depositor's request for withdrawal on demand, it can be enforced into bankruptcy.

Second, the agreement between depositors and banks indicates that withdrawals from deposit accounts are made on a sequential basis. Specifically, depositors are promised immediate withdrawals on a first-come, first-served sequence. This condition bears significant importance when depositors anticipate that the bank is likely to be unable to honour its entire demand deposits due to insufficient funds. The underlying rationale behind such adverse anticipation stems from depositors' understanding of poor performance of the bank, even if unwarranted by economic fundamentals.[11] When the bank has limited ability to meet large withdrawal requests, depositors who attempt to withdraw their funds earlier are more likely to obtain their entire balances in full. If the bank becomes unable to meet the entire sequence of withdrawals, depositors whose attempts are placed later in the sequence are likely to reach the bank after a bankruptcy event. In such cases, deposit claims are met only partially on a pro-rata basis against available resources of the borrowing bank.[12]

Bank runs stem from (rational) behavioural panics but also warranted expectations of poor future economic performance. The notion of bank runs and its implication for government guarantees follows the existence of two equilibria: the welfare-maximizing equilibrium is implemented when depositors maintain their confidence in the ability of the borrowers (banks) to honour debt-liabilities, against the socially undesirable equilibrium in which panic-based withdrawals trigger a self-fulfilling failure of the borrowers, leading to lower returns and welfare losses (**?**, **?**, **?** and **?**). Economic downturns are associated with the fall of asset values including loans, securities and other real assets that form the value of the banking institutions. When the anticipation of an economic downturn drives down the value of assets that banks hold to the limit that their net worth becomes negative or close to zero, widespread failures across banking institutions become more frequent.

In a competitive market-based environment, bankruptcies are considered natural. Bankruptcies are only considered costly from the society's perspective when liquidations are associ-

[11]While most depositors are unable to reliably assess the financial health of the banking institutions to initiate a panic-based run, they often tend to accelerate an already ongoing panic because they remain unable to warrant the underlying reason behind the panic.

[12]The asset recovery rate on failed bank's available resources is almost certainly less than one. Specifically, a bankruptcy event is associated with additional deadweight losses such as legal and liquidation cost, leading to further falls in the total asset value of a bankrupt institution. **?** estimates that a bankruptcy process is associated with 30% loss of bank's total assets due to legal and liquidation proceedings. Similarly, **?** and **?** show that bankruptcy cost can vary between 10% to 23% of total assets within non-financial firms and between 15% to 30% of total assets for financial firms. **?**; **?** and **?** provide comprehensive studies that examine bankruptcy cost according to several measurements and show that in some cases these costs can account for more than 30 cents on the dollar.

ated with deadweight losses. Nevertheless, within the financial intermediation sector, and particularly the banking sector, bankruptcies are associated with further negative externalities. Failure of one bank is likely to lead to failure of other intermediaries such as banks or institutions that provide a core role within the macroeconomic landscape. Banks maintain a high degree of interconnectedness amongst themselves and to the rest of the economy when compared to other enterprises, making their failures excessively disruptive to the society. In particular, in the United States, interbank lending accounts for over 4% of bank total assets which in absolute value is comparable to 50% of a representative bank's capital and almost 90% of its tier one capital value.

### 1.3.3 Reserve Requirements

Fractional reserves system indicates that depository institutions only hold a fraction of their acquired funds from the deposit accounts in reserves. Depository institutions that are federally insured are mandated to maintain reserve requirements as stipulated by the Federal Reserves. Reserves requirements are only applicable to demand deposits whereas time deposits and saving account are exempted and are not subject to reserve requirements. Specifically, depository institutions satisfy this requirement by holding funds in the form of vault cash and should the vault cash falls below the requirement, the balance must be satisfied in the form of reserve deposits at the Federal Reserves or at another depository institution on a pass-through basis. In 2008, the Federal Reserves implemented interest on reserves payment to depository institutions. This interest rate was initially set at 0.25% on both required and excess reserves which are any voluntary funds deposited in the form of reserves in addition to the required limit. These interest rates are decided by the Federal Reserve, or similar monetary authority, and serve important regulatory purposes. First, since the two interest rates are set by the monetary authority and are applicable to a class of liquid and risk-free assets, they remain very close to the main monetary policy or the federal funds rate. Second, these interest rates compensate reserves providers for the opportunity cost of funds. Third, although the interest rates remain close to each other and share strong co-movements, they may differ to implement policy objectives.

During the aftermath of the 2008 financial crisis as well as the 2020 pandemic, interest rate fell to their historical lower bounds. The prolonged fall of the interest rates is partially attributable to low productivity and heightened uncertainty. This downward trend has limited the ability of the monetary and financial regulatory authorities to use lowered interest rates as the main stimulus for economic expansion. Aside from the wider macroeconomic

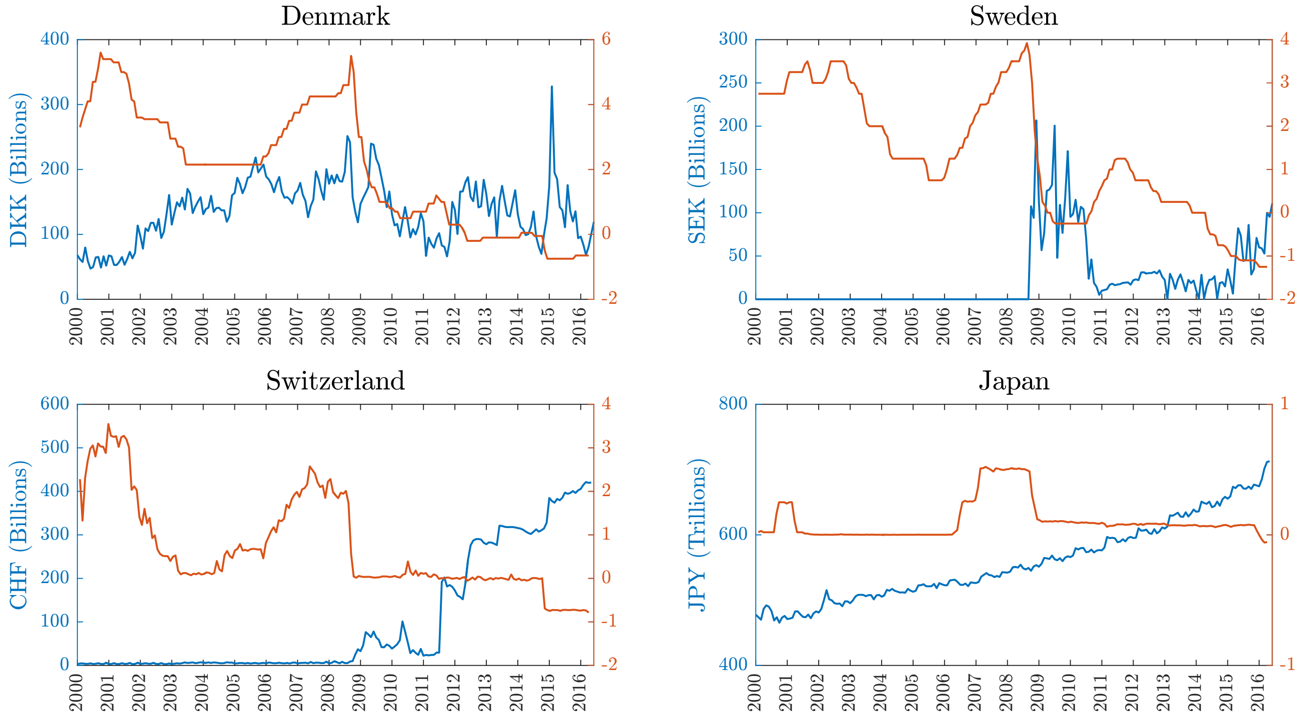


**Figure 1.1:** Excess reserves balances of the depository institutions in Denmark, Sweden, Switzerland and Japan, are illustrated on the left axis, and interest-on-excess-reserve paid by the central banks on the right axis.

implications, the lowered interest rates indicate that both interest on reserves and interest on excess reserves (IOER) are bound within a tight domain around zero. In particular, because the required reserves are non-voluntary requirements, interest on reserves is less flexible to fall particularly to the negative territory. However, IOER is a compensation on voluntary reserves which enables the policymaker to set its level within the negative territory to achieve further objectives.

The issue of negative interest rates was first introduced into the policy debates in 2015. Several central banks announced negative IOER on balances that had dramatically grown during 2009-2015. In mid-2016 the European Central Bank (ECB) and the central banks of Switzerland, Japan and Sweden began to pay negative interest rates in the close neighbourhood of $-0.40\%$. Aside from the prolonged fall in the interest rates, central banks decisions to further drop the rates was motivated by the initiative to dissuade the depository institutions from holding substantial quantities of funds in reserves and thereby encourage lending expansions to borrowers.

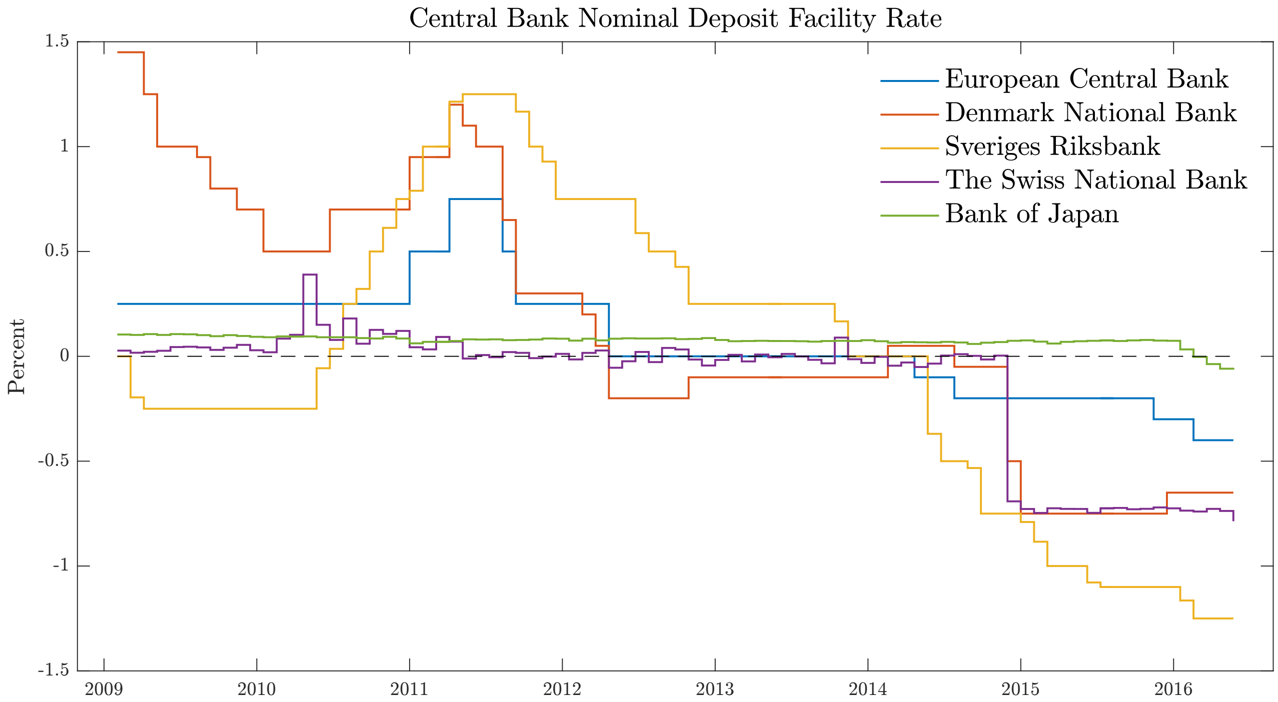


**Figure 1.2:** Interest payments on excess reserves by the European, Switzerland, Sweden and Denmark central banks have consistently fallen into the negative negative territory since 2015-2016, with magnitude falling to three quarters of percentage point for Sweden and Switzerland.

### 1.3.4 Capital Requirement

The Basel Committee on Banking Supervision (henceforth the Basel Committee), began its operations to investigate the financial health of the banking institutions and recommend guidelines to mitigate vulnerabilities on a global scale during 1980s. Despite its initial informal presence that lacked constitution, or legal existence, since its Accord of 1988 commonly known as Basel I, and the later successors (Basel II, 2004 and Basel III, 2010) it has become the predominant power in global banking regulation.

The rules determined by the Basel Committee on banking institutions' capital and liquidity are formally followed by over thirty economies and also used as guidelines for much wider economies.[13] These rules serve as a baseline that describes the minimum requirement to maintain the safety and stability of the banking institutions and often further demanding regulations at national levels are implemented to complement the baseline rules. The Basel regimes comprise three pillars: first, capital regulation of the banking institutions that was

[13]The rules determined by the Basel Committee are not legally binding at domestic levels even for the member states, nonetheless, the compliance is implemented through a system of peer pressure organized by the committee. The accords are particularly intended for the internationally active banks. For example, the United States and the EU, apply them to all banks under their jurisdictions.

the focus of the initial accords and recently, the rules are extended to incorporate liquidity requirements. The second pillar provides recommendations for the national or regional agencies that act as supervisors to monitor and measure the performance of the banking institutions. The supervisory agencies evaluate the risk profile of the banks, administer stress tests and assess compliance of the banking institutions with the guidelines provided.[14] The third pillar is concerned with examining how the banks approach capital adequacy rules through disclosures requirements. These requirements are focused on securitization exposures and sponsorship of off-balance sheet vehicles which are used to review how a bank calculates its regulatory capital ratios.

The fundamental rationale behind capital rules for the banking institutions is that the shareholders' equity should provide a sufficient contribution to form the value of the bank's total assets. This contribution increases the ability of the institution to absorb potential adverse shocks to its assets side and maintain its solvency. Given the central role of the banking institutions within the macroeconomies that provide the flow of funds and their tendency to disregard the cheap cost of debt discussed in the aforementioned moral hazard problem, imposing a capital requirement is a well-justified and widely-implemented policy. The amount of capital that a bank is required to maintain varies across institutions even if two banks have an equal balance sheet size but differ in their risk profiles.

**Institutional Definition of Risk** While the definition of risk and its measurement varies for each bank, the Basel Accord provides an institutionalized definition for risk. The main risk that the banking institutions must hold capital against is the credit risk. Unexpected changes in the ability or willingness of borrowers who have acquired funding from a bank, form the core source of risk within the Basel Accords. This indicator was first introduced and incorporated into Basel I, however, with the evolution of the banking institutions since 1988, the definition of risk has been amended. Specifically, commercial banks now have moved significant amounts of credit intermediation to market intermediaries that resemble investment banking operations which prompted the Basel Committee to introduce trading risk as an additional important source of risk. The trading risk captures variations in the value of securities that banks hold particularly for the purpose of market-making or proprietary-trading. This amendment was implemented in 1996, between Basel I and Basel II, and is

[14] The goal of the second pillar considers supervisory measures to address firm-wide governance and risk management; capturing the risk of off-balance sheet exposures and securitization activities; managing risk concentrations; providing incentives for banks to better manage risk and returns over the long term; sound compensation practices; valuation practices; stress testing; accounting standards for financial instruments.

referred to as market risk amendment.

This modification provided an effective approach to measure the risk profile of the banking institutions from 1996 to 2006. During this period, the composition of the assets side of banks included loans and non-loan assets. The latter consisted principally of positions in debt- or equity-based securities traded on liquid markets. The high liquidity of the non-loan assets indicated that in the event of facing a decline in the value of non-loan assets, banks had an ability of recover from the fall in the value of their total assets within a short-term time interval. Nevertheless, during 2006-2007 large banks had begun to engage in more complex financial instruments. Specifically, in the years leading up to the 2008 financial crisis banks held sizeable positions in credit instruments that did not trade on liquid markets such as mortgage trading, asset-backed securities, credit derivative, and securitization warehousing.[15] Large positions in illiquid assets imply that a short-term recovery from major declines in asset values became a remote possibility. The development of the new asset classes prompted the Basel Committee to revisit its definition of risk assessment. After the financial crisis, the revised Basel Accord that is referred to as an interim Basel 2.5, addressed the implications of the heightened risk profiles by requiring banks to hold additional capital that effectively increased bank capital holding to three to four times.[16]

The Basel Accord provides a quantitative measure to associate the sources of risk with capital holding. Similar to the identification of risk sources, risk calibration has also been a controversial debate among the banking institutions and policymakers. Capital requirement is described by the ratio of bank capital to its total assets. The absolute capital requirement increases when the total assets increases, or when the risk profile of the total assets increases. The notion of risk profile is based on assigning weights to different types of assets that the banking institutions hold. The Basel Accord amendment of 1996 instructed that subject supervisory approval, the historical data within each bank can be used to implement an internal risk assessment and risk-weighting. Currently, the guidelines provided by the Basel Accord indicate that banks can either follow the standardized approach provided by the Basel Committee, or implement an internal ratings-based approach based on their own historical data.

Banks widespread adoption of the internal risk-weighting approach is partly explained

[15] Bank of International Settlement Reports, Fundamental Review of the Trading Book (2012), Annex 1, Figures 1 and 2.

[16] In addition to the credit risk and the market risk, the Basel risk model incorporates the operational risk that identifies further sources of risk and triggers capital requirement levels. In particular, this category captures the implications of complexity and opacity of the assets held by the banking institutions on the ability of the banking institutions to meet their debt-liabilities.

by the flexibilities within the risk assessment models that often enables the banks to understate risk profiles. The flexibility in developing internal risk assessment models also depends on the quality of the internally available data. Specifically, the data tends to overstate the importance of economic indicators during expansionary periods against distress periods which happen after capital levels are relaxed. The external validity of the internal risk assessment approach is examined through stress tests administered by international and national authorities. These involve hypothetical scenarios that include various shocks to the bank balance sheets to investigate the survival of the banking institutions given their capital ratios. The supervisory stress test serves as a major qualification for the internal risk assessment approach as it introduces a unifying assessment basis across the banking sector.

The internal risk assessment approach is further investigated against the non-risk-weighted capital requirement, which is referred to as a leverage ratio. The leverage ratio establishes a floor that the risk-weighted requirement must satisfy. This secondary examination provides two advantages, first, it ensures that even though the internal risk assessment overweighs historical data during benign economic episodes, the capital holdings remain above a minimum standardized level. Second, the non-risk-based approach eliminates the possibility of model risk in the implementation of financial stabilization policies.

Lastly, the definition of capital remains amongst the most controversial debates. The shareholders' equity constitutes the first component of bank capital which refers to the funds invested in an institution through ordinary share subscriptions and the retained undistributed profits. The Basel nomenclature refers to this component as the tier 1 capital. The second component that forms bank capital is formed by 'additional tier 1' or tier 2 capital which refer to funds that can be acquired via shareholders' equity or supplied by subordinated debt. In particular, funds in the form subordinated debt are considered into the tier 2 class only if they are convertible into equity. This class of bank liability is divided into two types based on the maturity of the debt, where for the 'additional tier 1', the hybrid debt must be perpetual and for the tier 2, debt must have a minimum maturity of five years.[17]

The Basel III approach introduces additional capital buffers that lead to further tier 1 capital holding. Specifically, failure to meet the minimum capital requirement is faced with stringent responses by the regulatory and supervisory agencies that could lead to bank closures. Failure to satisfy capital buffers, however, leads to restriction in the profit distribution

[17] Within the context of Basel III, the contribution of this component was increased to 4.5% from the previous Accord in which only 2% of the contribution had to be financed through shareholders' investments. The additional tier 1 accounts for 1.5% and tier 2 accounts for 2.0% which overall construct the Basel-8% capital requirement.

to shareholders and the managerial board in the form of bonuses with the goal of rebuilding the capital. Capital conservation buffer is the first class of buffers that applies to all banks irrespective of time. This buffer is primarily focused on mitigating the impact of economic cycles. Specifically, maintaining the capital requirement is possible by keeping the capital side unchanged, while reducing the asset side through granting fewer loans. While this response keeps a bank in compliance with the regulatory requirements, it also leads to a significant fall in lending to the real economy which then adversely impact a wider macroeconomic landscape. The Basel III 2.5% capital conservation buffer is intended to ensure banks capital buffers beyond the baseline requirement is capable of absorbing temporary losses and continue providing lending to market participants. The 'global systemically important bank' capital buffer is an additional tier 1 requirement that only applies to banks which serve core roles. This additional capital holding is similarly concerned with the safety of the financial system rather than the safety of individual institutions. The systemic importance describes that the role of banks within the financial system is an important driver of risk, in particular, the likelihood of facing adverse economic outcomes as well as the severity of the outcome. The definition of systemically important banks is based on their size, interconnectedness, cross-border operational scale, and internal complexity. For instance, in the United States, under the Dodd-Frank Act, the Federal Reserve applies enhanced prudential supervision and requires higher capital standards on banks with total assets in excess on $50bn.

The multi-layered approaches by the Basel and local regulatory agencies provide a minimum set of standards to ensure the safety of the banking sector. These standards, first, counter-balance the subsidies provided by the deposit insurance to the banking institutions and second minimize the negative externalities of costly bank failure to the wider macroeconomic landscape. Nonetheless, substantial evidence by practitioners and academic literature suggests that banks remain under-capitalized. The notion of under-capitalization captures the ideal amount of capital that the banks should hold to be able to provide lending to the real economy that ensures sustainable growth and long-term expansion of household wealth and their welfare. Chapter (2) provides a welfare-maximizing approach to determine a risk-weighted capital requirement policy within a general equilibrium context. This essentially establishes a benchmark to assign a welfare value to the bank capital level, which then serves as a basis for the policymaker to deliver welfare gains to the society.

Another often understated service provided by the banking institutions to the society is its ability to accept deposits. This is of particular importance within the discussion of capital regulation because the households have preferences for investment in risk-free assets.

Although banks remain under-capitalized, requiring too much capital also leads to negative welfare implications. Specifically, the subsequent section discusses that households ability to invest their saving in deposits leads to welfare gains through the portfolio rebalancing channels.

## 1.4 Micro-founded Financial Intermediation Model

This section provides a static two-dates $t \in \{1, 2\}$ model with perfectly competitive credit market. Investors, firms and a bank maximize their corresponding objectives. There is a financial regulatory authority who sets three requirements on the banking sector. Prices, returns and interest rates are determined in general equilibrium discussed in section (3.1.5).

**Investors** Consider an economy with a large number of identical risk-averse investors and an aggregate endowment $w$. The representative investor maximizes the following expected value of lifetime utility $u(.)$ subject to each period's budget constraint:

$$\max_{\{c_1, d, e\}} \quad u(c_1) + \beta \mathbb{E}_1 u(c_2) \tag{1.4.1}$$

$$\text{s.t.} \quad p_1 c_1 + d + e \leq w \tag{1.4.2}$$

$$p_2 c_2 \leq (1 + i_d)d + (1 + \widetilde{r}_e)e + \Pi^F \tag{1.4.3}$$

$$c_t, \ d, \ e \geq 0 \tag{1.4.4}$$

where $\beta \in (0, 1)$ is the subjective discount factor, $\mathbb{E}_1[.]$ is the mathematical conditional expectation operator at date-1 with respect to stochastic properties of date-2 income that is driven by the equity return outcome, $c_t$ is the period consumption, $d$ is deposits that receive a net risk-free rate $i_d$ and $e$ is risky bank equity introduced later in the financial intermediation section which receives a stochastic net return $\widetilde{r}$ and $\Pi^F$ is firm's profit. A representative investor solves this problem to derive each period optimal consumption as well as financial decisions $d^*(\beta, w, p_t, i_d, \widetilde{r}_e)$ and $e^*(\beta, w, p_t, i_d, \widetilde{r}_e)$. The investor's utility function is twice-differentiable, increasing $u_c(.) > 0$ and concave $u_{cc}(.) < 0$ in each period's consumption indicating that constraints (1.4.2) and (1.4.3) bind. Setting up the Lagrangian to derive the first-order-conditions and re-arranging gives the following intertemporal optimality trade-offs:

$$u_c(c_1) = \beta(1+i^d)\mathbb{E}_1\left[u_c(c_2)\left(\frac{p_1}{p_2}\right)\right] \tag{1.4.5}$$

$$u_c(c_1) = \beta\mathbb{E}_1\left[(1+\widetilde{r}_e)u_c(c_2)\left(\frac{p_1}{p_2}\right)\right] \tag{1.4.6}$$

Noting that deposit interests are risk-free since deposit balances are guaranteed by deposit protection. Equation (1.4.5) determines how much deposits to hold since it balances the marginal cost of withholding a unit away from date-1 consumption to invest at the net deposit rate and consuming it at date-2. Similarly, equation (1.4.6) states that bank capital investments depends on its expected return, evaluated in marginal utility terms. There are four sectors in this general equilibrium model and in order to derive closed-form solutions, I assume the following two simplifications. First, suppose that the investor derives utility from consumption with log form and, second, that investments in bank equity returns $r_e > i_d$ with probability $\lambda \in (0,1)$ and nothing otherwise, such that $\mathbb{E}_1\widetilde{r}_e > i_d$. This indicates that bank capital investors expect to receive a high return on their investment if the bank is solvent or receive nothing when it goes bankrupt in which case they rely only on their deposit income to finance their date-2 consumption.The investor's intertemporal optimality conditions may be re-rewritten as:

$$1 = \beta(1+i_d)\left(\frac{c_2}{c_1}\right)^{-1}\left(\frac{1}{1+\pi_c}\right) \tag{1.4.7}$$

$$1 = \beta\lambda(1+r_e)\left(\frac{c_2}{c_1}\right)^{-1}\left(\frac{1}{1+\pi_c}\right) + \beta(1-\lambda)\left(\frac{c_2}{c_1}\right)^{-1}\left(\frac{1}{1+\pi_c}\right) \tag{1.4.8}$$

where $\pi_c$ is the net growth in consumption good price. Solving equations (1.4.2) and (1.4.3) together with the budget constraints (1.4.7) and (1.4.8) determine the following consumption and investment decisions,[18]

$$c_1^*(\beta, w, p_t) = \frac{w}{1+\beta} \tag{1.4.9}$$

$$d^*(\beta, w, i_d, r_e, \lambda) = \left(\frac{\beta}{1+\beta}\right)\left(\frac{r_e - r_e\lambda}{r_e - i_d}\right)w \tag{1.4.10}$$

$$e^*(\beta, w, i_d, r_e, \lambda) = \left(\frac{\beta}{1+\beta}\right)\left(\frac{\lambda r_e - i_d}{r_e - i_d}\right)w \tag{1.4.11}$$

[18] Appendix (1.7)

where $p_1 = 1$. Equations (1.4.9)-(1.4.11) determine optimal levels of consumption, deposit and equity given good's price and investment returns. Given the unique properties of the logarithmic utility function, we observe that the consumption decision is independent of deposit and equity rates. Equation (1.4.10) is the supply of deposits to the bank which has the following partial derivatives:

$$d^*_\beta > 0,\ d^*_{i_d} > 0,\ d^*_{r_e} < 0,\ d^*_\lambda < 0,\ d^*_w > 0 \tag{1.4.12}$$

The first partial change $d^*_\beta$ implies that when investors become more patient, their supply of deposits or investments increase. This is because a more patient investor is willing to postpone consumption to future translates to lower date-2 marginal utility of consumption in equation (**??**) and therefore higher date-2 consumption. The second expression $d^*_{i_d}$ indicates that supply of deposits to the bank increases at higher deposit rates, which is consistent with properties of supply. The third expression $d^*_{r_e}$ reflect the substitution effect between deposits and equity. In particular, this partial derivative is negative because an increase in the equity rate drives up the resources invest in equity leaving less resources available as deposits. The fourth expression $d^*_\lambda$ has similar implications as $d^*_{r_e}$ since increases in the probability of repayment translates into higher expected return, which make investing in equity more favourable. The last expression is trivial indicating that the overall size of deposit supplies to the bank increases as the overall endowments in the economy grows. Equation (**??**) determines the equity decision with the following partials,

$$e^*_\beta > 0,\ e^*_{i_d} < 0,\ e^*_{r_e} > 0,\ e^*_\lambda > 0,\ e^*_w > 0 \tag{1.4.13}$$

which share the same economic intuition as those of deposit.

**Firms** Consider a representative firm with a single-factor Cobb-Douglas production technology $f(k) = k^\alpha$ that produces the consumption good. The firm is all externally funded and acquires its working capital $k$ from the bank at date-1 in the form of a loan at a net rate $r_L$ which it has to repay at date-2. The production is subject to an exogenous shock which indicates that the firm succeeds with probability $p$ or fails with probability $1 - p$. The firm maximizes the profit,

$$\max_k \Pi^F \equiv (1 + \pi_c)k^\alpha - (1 + r_L)k \tag{1.4.14}$$

where $\alpha$ is the output elasticity of capital. The firm takes the loan rate $r_L$ and consumption good price as given and chooses the following optimal amount of capital:

$$k^*(\alpha, r_L, \pi_c) = \left[\frac{\alpha(1+\pi_c)}{1+r_L}\right]^{\frac{1}{1-\alpha}} \tag{1.4.15}$$

Equation (1.4.15) is the demand for credit which is decreasing in loan rate, $k^*_{r_L} < 0$ and increasing in consumption good price, $k^*_{\pi_c} > 0$. Substituting (1.4.15) into the production and profit functions (1.4.14) gives:

$$f(k^*) = \left[\frac{\alpha(1+\pi_c)}{1+r_L}\right]^{\frac{\alpha}{1-\alpha}} \tag{1.4.16}$$

$$\Pi^F = (1-\alpha)(1+\pi_c)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{1+r_L}\right)^{\frac{\alpha}{1-\alpha}} \tag{1.4.17}$$

The profit is distributed to investors.

**Financial Intermediation** Suppose a risk-neutral bank acts as an intermediary between investors and firms. On the liability side, bank collects deposits $d^*$ and raises capital $e^*$ from the investors, and on the assets side, it provides finances for firm's production activities. The bank is subject to regulatory constraints on both sides of its balance sheet. In particular, the bank must comply with a minimum capital to debt ratio on the assets side and maintain a minimum required reserves with the regulator. The balance sheet of the bank is:

| **Assets** | | | | | = | **Liabilities** | + | **Capital** |
|---|---|---|---|---|---|---|---|---|
| Required Reserves | + | Excess Reserves | + | Loanable Funds | = | Deposits | + | Equity |

**Figure 1.3**

The bank must satisfy the minimum reserves requirements which receives no interest and has the option to deposit additional funds above the required reserves to the regulator. These funds are the excess reserves that are subject to net interest rate $i_b$. The bank operates in a perfect competition market and maximizes the profit function,

$$\max_{e,B} \quad \mathbb{E}_1\left[(1+r_L)L+(1+i_B)B+\psi(B)\right]-(1+i_d)d-(1+r_e)e \tag{1.4.18}$$

$$\text{s.t.} \quad R+B+L=d+e \tag{1.4.19}$$

$$\eta d \leq e \tag{1.4.20}$$

$$L \geq 0 \tag{1.4.21}$$

where the first term in equation (1.4.18) is the expected revenue of bank from loans to firm. This term is stochastic which reflects credits risk that arises from the firm failing to repay the loan. The second and third terms are expected revenue and liquidity services provided by excess reserves $B$ receiving net interest $i_B$, and $\psi(B)$, respectively.

One of the important implication of the 2008 crisis was that short-term debt market, such as commercial papers became excessively limited. Commercial papers and similar instruments serve key role in providing short-term liquidity to banks and other institutions. However, after the crisis, bank began to look for alternatives that fulfil the same purpose. This provides accounts for increases in the excess reserve since banks prefer to hold additional resources for their short-term uses. In equation (1.4.18), the term $\psi(.)$ represents liquidity services provided by holding excess reserves as a short-term source of funding. I assume that such benefits are increasing ($\psi_B > 0$) but concave ($\psi_{BB} < 0$) in excess reserves implying that the benefit of one additional reserves unit decreases as the total amount increases. One implication of this functional assumption is that the bank is always a lender in the interbank market which may be counterintuitive, however, this offers simplification for the purpose of our study where the key focus is on the current empirical evidence with massive excess reserves held by representative bank. I provide further economic intuition on this specific functional form in section 4.

The final two terms are costs associated with raising deposits and equity. Equation (1.4.19) is bank's balance sheet that specifies how bank's resources split into required reserves $R$, excess reserves $B$ and corporate loanable funds $L$. In particular, suppose that the regulator sets required reserves to be a constant fraction $\overline{\omega}$ of total deposits,

$$R = \overline{\omega} d \tag{1.4.22}$$

which indicates that the bank needs to determine how much excess reserves to hold with the regulator and that the rest becomes the available funds in the credit market for the firm.

Equation (1.4.20) is the capital requirement constraints where $\eta \in (0, 1)$ determines that banks needs to keep $\eta$ units of equity per each unit of deposit that it collects. Equation (1.4.21) is the feasibility constraints that requires bank's investments in the corporate loan market to be non-negative.

Given the perfectly competitive deposit, equity and loan markets, the bank has no power to set these rates, which are determined in general equilibrium. Nonetheless, the bank decides on its capital structure and that how it allocates its resources between excess reserves and loanable funds. Given the fact that raising equity funds is more expensive than deposits, we can re-write inequality in (1.4.20) in binding form. This implies that the bank always chooses to hold the minimum amount of equity possible that is set by the regulator. The intuition behind this result is that bank capital investors require a premium that compensates them for the risk of losing their investment in the default state.

Re-writing bank's objective function using the regulatory constraints gives:

$$\max_{B} \quad \mathbb{E}_1 \left[ (1 + r_L)L + (1 + i_B)B + \psi(B) \right] - (1 + i_d)d - (1 + r_e)e \tag{1.4.23}$$

$$\text{s.t.} \quad B + L = (1 - \overline{\omega} - \eta)d \tag{1.4.24}$$

where $(1 - \overline{\omega} - \eta)$ in equation (1.4.24) is the total amount of funds available to firm after meeting the required reserves constraint. This implies that the total funds available can be split between excess reserves and loans to firms. Empirical evidence suggest that banks always choose to hold non-zero excess reserves, where in our model, $\psi(B)$ captures this feature. In particular, bank receives large benefits to maintain a low amount of funds as excess reserves $(\lim_{B \to 0} \psi_B(B)) = \infty$.

Assuming $\psi(B) = \kappa \ln(B)$ and deriving the first-order-conditions for the constrained bank problem gives:

$$B^* = \frac{\kappa}{r_L - i_B} \tag{1.4.25}$$

which is the optimal excess reserves of the bank that it supplies to the interbank market with $i_B$ set by the regulator. The importance of liquidity services offered by excess reserves relative to interest revenue is captured by the term $\kappa$. The bank considers spending one unit of funds on this market as long as the market interest rate in addition to liquidity services offset spending one unit of funds in the corporate loan market. This allocation is inversely proportional to the gap between two market rates and decreases when this gap widens. Lastly, interbank market investment proportionally increases with increases in the liquidity importance coefficient $\kappa$.

**Regulation** Financial intermediaries are subject to various regulations. In this model, a financial regulator is in charge of imposing plausible constraints on the banking sector to maximize the social welfare of the economy. In particular, there are three regulatory requirements that the representative bank needs to comply with. First, equation (1.4.20) illustrates the capital requirement on banks liabilities. This requirement ensures that the bank has a minimum amount of equity-to-deposit ratio on its balance sheet. The main function of this constraints is to reduce exposure to deposit runs. In particular, in the case of bank default, banks is liquidated and its proceeds are distributed to the depositors pro rata[19]. This offers partial insurance to the depositors and reduces the probability of runs.

Second, equation (1.4.22) sets minimum reserves requirements that the bank needs to meet. This reduces the overall available funds to the corporate and interbank markets. This offers a source of funds to the regulator to establish deposit insurance[20] above capital requirement limit in the case of bank default. Third, the term $i_B$ in equation (1.4.25) is the final regulatory instrument and is controlled by the regulator. This term in the interbank bank interest rate and may be positive, zero or negative and affects the willingness to invest in excess reserves by the commercial bank.

The regulatory requirements are:

$$e \geq \eta d, \; R = \overline{\omega} d, \; B^* = \frac{\kappa}{r_L - i_B}$$

where the required and excess reserves are regulator's liabilities to the commercial bank. In particular, the balance sheet of the regulator is:

| Assets | | Liabilities and Capital |
|---|---|---|
| Liquidity Reserves + Deposit Insurance Funds | = | Required Reserves + Excess Reserves |

### Market Equilibrium

The general equilibrium model incorporates investor, firms, bank and regulatory sectors. The economy spans two dates with five separate markets, deposit, equity, loan, interbank and goods, that need to clear.

[19] In this model there is no bankruptcy costs.

[20] Alternatively, a government or regulatory sector may directly subtract funds from investors and reserve them as a deposit insurance balance in preparation of liquidity needs in a bankruptcy state.

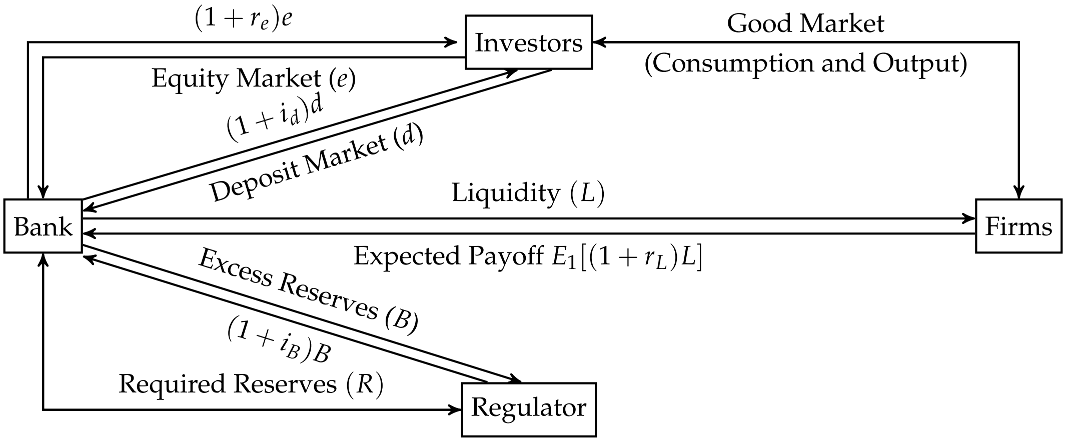


**Figure 1.4:** The figure shows the structure of the economy, where the regulator controls the net interbank rate ($i_B$) which determines the optimal balances in the excess reserves. The required reserves $(R)$ is also determined by the regulator which is set to be a constant fraction of bank deposits ($\overline{\omega}d$).

The deposits market is the interaction between supply of deposits by investors and demand for deposits by the bank as a liability. The deposit rate $i_d$ equilibrates this market such that both counterparts maximize their objectives subject to their constraints. Similarly, $r_e$ clears the equity market. The regulator's capital requirement has important indications in these two markets. In particular, in the unregulated equilibrium case $\eta = 0$, the bank prefers to hold least amount of equity on its balance sheet as it is the last source to raise funds. However, capital requirements prevents achieving this first-best in this market as the bank is forced to hold more equity and less deposits when $\eta \neq 0$. An alternative way to examine this implication is that $e = \eta d$ binds, suggesting that the corresponding Lagrange multiplier is non-zero and therefore there is shadow gains in relaxing this constraints. However, from the regulator's perspective this welfare cost is offset by the gains in offering insurance to the depositors and therefore the overall welfare improves. This regulation is a market imperfect and has important implications on the deposit and equity rates, particularly, with $\eta = 0$ the spread $r_e - i_d$ is zero.

The corporate loans market gives firms access to credit to finance their working capital needs. Since firms are entirely externally funded, their demand for capital is their demand for liquidity which the bank provides[21]. First, the available funds on the bank's balance sheet forms the supply of liquidity to this market. When there is no excess reserves, bank supplies all of its funds to the firm which it expects to receive at the end of the period in addition to remunerations from the loan rate $r_L$. In this case the supply of funds from the

[21] In general, firms may interact with investors directly and raise part of the liquidity need. In this case, bank's loan may be a complementary source of finance if there is shortage in the investors-firms market.

bank side is constant and inelastic, and that $r_L$ clears the market. Second, when bank has access to the excess reserves market, it chooses to optimally allocate its funds to both excess reserves and corporate loans markets. In this case, the supply of credit to the corporate loan market is no longer constant, but increasing in $r_L$. The intuition behind this result is that when $i_B$ or $\kappa$ increases (decreases) in the interbank market, the bank is more willing to increase (decrease) its investments in this market. This leaves less resources available to the corporate loan market, and thus the corporate loan rate must increase (decrease) to compensate for this gain (loss).

Third, note that the firm's production is risky. In particular, the firm succeeds to repay its liabilities in the good state and goes bankrupt in the default state. On this account, the bank maximizes its expected profit and raises $r_L$ to compensate for the losses in the default state. For instance, suppose that the production sector is a measure one continuum with infinitesimally small firms of whom 20% are expected to go bankrupt. In this case, the bank raises $r_L$ by a factor of $^1/_{80\%} = 25\%$. This additional increase in the loan rate is the finance premium that bank imposes ex ante on a borrower to cover losses driven by the bankrupt firms.

The interbank market is the interaction between the regulator and the commercial bank where $i_B$ clears the market. In fact, depending on regulatory objective, this rate makes excess reserves a favorable or unfavorable. In particular, $i_B > 0$ encourages the commercial bank to invest further fund in this market, but $i_B < 0$ may also occur. In the case of negative rate, the bank is encouraged to withdraw its funds from the interbank market and invest them as corporate loans to the firm. This is an important policy instrument that alters allocation of resources in the economy.

The last market is the consumption good market where firms interact with the investors. The price of consumption good equilibrates this market such that firms output clear. The ratio of consumption good prices between two dates forms the gross inflation rate.

In this model, bank has two decisions, first the optimal capital structure on its liabilities side, and second optimal split between corporate loans and excess reserves on its assets side. Beginning with the liabilities and noting that the regulatory constraint binds, the bank sets its optimal equity-to-deposit ratio equal to $\eta$, therefore,

$$L + B \;=\; \frac{\beta}{1+\beta}(1-\eta)(1-\overline{\omega}+\eta)w$$

where the RHS is the optimal amount of deposits and equity raised in those markets minus the required reserves fraction $\overline{\omega}$.

**Proposition 1.4.1** (Static Equilibrium Characterization)**.** General equilibrium is a set of optimal allocations $\{c_1, c_2, d, e, k, L, B\}$ and equilibrating interest rates, remunerations and ratio of consumption good prices $\{i_d, r_e, r_L, i_B, \pi_c\}$ such that investors, firms, bank and regulator maximize their objectives subject to their constraints and all markets clear.

The optimal interbank, corporate loan, deposit and equity rates are:

$$
\begin{aligned}
i_B^* &= \bar{i} && (1.4.26)\\
r_L^* &= \frac{\alpha_1 - \frac{\kappa}{i_B} - \frac{\beta}{1+\beta}(1-\eta)(1-\overline{\omega}+\eta)w}{\alpha_2 + \frac{\kappa}{i_B^2}} && (1.4.27)\\
i_d^* &= \left[\frac{\beta}{1+\beta}(1-\eta)w\frac{\lambda-\eta^2}{\lambda-\eta}\right]^{-1}\left[(1+r_L)L + \frac{1+i_B}{r_L - i_B}\right] && (1.4.28)\\
r_e^* &= \frac{1}{\lambda} i_d && (1.4.29)
\end{aligned}
$$

where $\alpha_1 = \alpha^{\frac{1}{1-\alpha}}$ and $\alpha_2 = \frac{\alpha_1}{1-\alpha}$ are monotonic transformations of $\alpha$. Equation (1.4.27) shows the equilibrium corporate loan rate that equilibrates demand for liquidity (capital) by firm and supply of liquidity by the bank. The partial derivatives of this equilibrium rate are,

$$
\frac{\partial r_L}{\partial \eta}, \frac{\partial r_L}{\partial i_B}, \frac{\partial r_L}{\partial \overline{\omega}}, \frac{\partial r_L}{\partial \alpha} > 0 \qquad (1.4.30)
$$

where the first partial derivative is positive indicating that when required equity-to-loan increases (decrease), the loan rate increases (decreases). The intuition behind this result is that higher $\eta$ indicates that the bank needs to choose a capital structure with more equity that is more expensive than deposits, which overall, increase the cost of financing for the firms. The second partial indicates that when the interbank rate increases, the loan rate increases. The economic interpretation of this finding is that higher $i_B$ encourages the bank to channel more resources into excess reserves and as a result there is less funds available to loan market and supply shrinks, and therefore corporate finance rate increases. The third expression indicates that when reserves requirements increases, the loan rate increases. This relation has similar mechanism as $i_B$ since increasing the reserve requirements reduces the supply of funds and therefore loans become more expensive. The last expression implies that when firms become more productive then their demand for capital increases and as a result the demand curve for capital moves upward, thus increasing the loans and the loan rate.

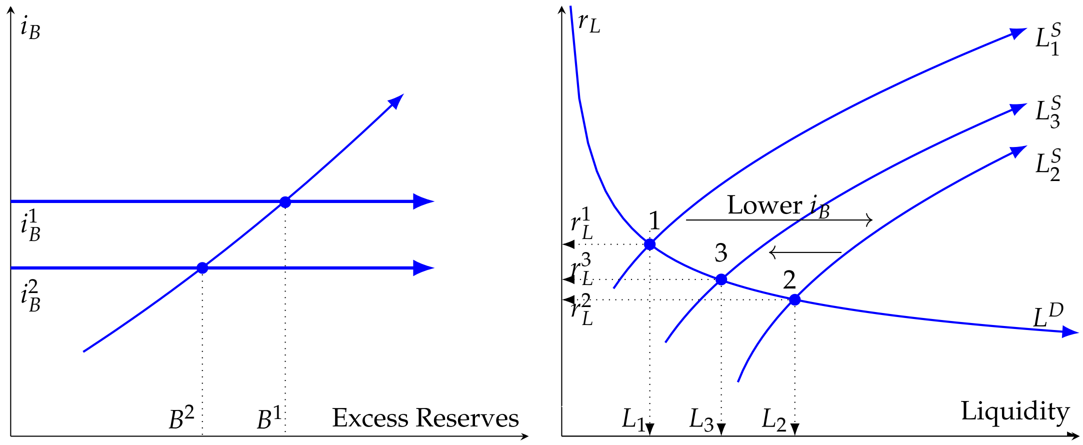


**Figure 1.5:** This figure illustrates that the supply of liquidity increase when the interbank rate decreases and supply moves to $L_2^S$ from $L_1^S$. Given that the demand remains the same, reduces to $r_L^2$. However, reduction in $r_L$ increase the demand for excess reserves which then leaves less resources for the corporate loan market and therefore supply of liquidity shrink to $L_3^S$ from $L_2^S$.

## 1.5 Implications of Negative Nominal Rates

Central banks of Japan, Denmark, Norway, Switzerland and the European Central Bank have adopted negative interest rate policy. This policy indicates that commercial banks need to pay charges on their excess reserves held at their central banks. In response, commercial banks invest their funds in alternative short-term assets, which then drives down the yields. First, this fall can result in negative yields on short-term asset as well, and second, an investor aims to partially switch to long-term assets such as mortgage and corporate bonds, thereby reducing yields on those assets as well but expected to have positive yields mainly due to credit risk compensation and longer investment horizon[22].

In fact, this mechanism is similar to conventional monetary policies such as policies implemented by the Federal Open Market Committee that cut the short-term interest rate by 6.8, 5.5 and 5.1 percentage points during the 1990-91, 2001 and 2008 recessions, respectively. In particular, real interest rate, the gap between the nominal interest rate and the inflation, has been negative many times prior to the 2008 crisis. Figure 2 shows the effective Federal Funds rate since 1950 to 2016 within which negative real interest rate is fairly often, especially during the recession marked as shaded bars. This coincidence is actually part of the monetary objectives which aim at lowering both real and nominal rates to stimulate eco-

[22]Central banks can use quantitative easing (QE) to directly target lowering long-term interest rates to incentivize borrowing and spending. This drives the prices up and lowers yields on the target assets which then encourages investors to sell their assets and switch to corporate equities or bonds. This instrument was commonly used by central banks from late 2008 until October 2014 which prevented price deflation and stimulated economic recovery. However, QE is difficult to calibrate and communicate and thus central banks may consider to used it for limited periods.

nomic activity. However, one difference in the monetary policies used during this period

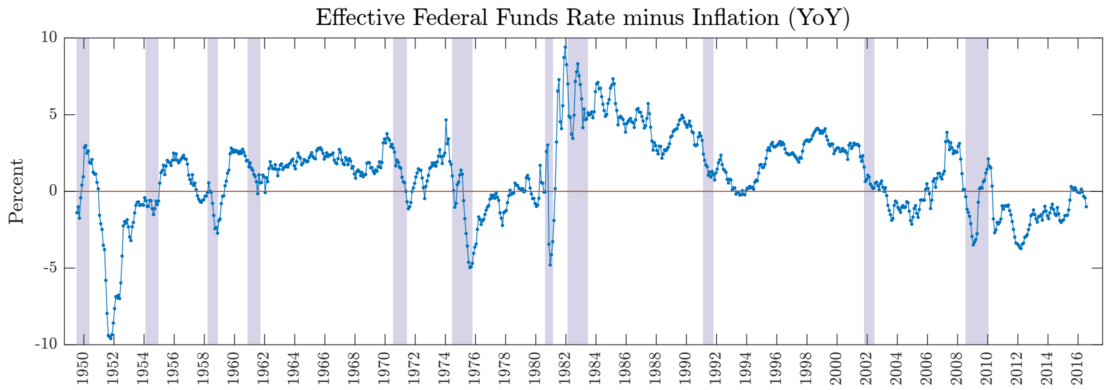


**Figure 1.6:** Inflation adjusted effective Federal funds rate illustrate the real interest rate in the US. The figure shows that the real rate has indeed been negative for many instances, mainly during recessions.

is that until 2008 the nominal funds rate has always been above zero. Since late 2008, this rate remained marginally above zero, between a fourth to half percent, which prevented the possibility of further lowering without using the negative nominal rate.

Implementing nominal negative interest rate is not without drawbacks, in particular, there are legal and operational constraints that may prevent or limit implementability of this policy. First, the US law states that the Federal Reserves is allowed to pay interest on its reserves but it is unclear whether this interest can be negative. However, one solution to this is that the Fed can charge a fee for accepting reserves. This already is in place as the Fed has been charging commercial banks to cover actual costs of providing services as well as additional supervisory and regulatory costs. (add safety and value costs)

Second, an immediate implication of this policy is that investors may choose to hold currency which pays zero interest but incurs storage, transaction and security costs. These costs are lower for commercial banks since they already have sufficient facilities to hold additional amounts of currency in their vaults. Based on these operational costs, the Fed concluded in 2010 that the nominal interest rate can be about -0.35 percent. Many central banks in Europe implemented negative policy rates below this limit with minimal currency hoarding effects. For instance, Switzerland and Sweden set the policy rate at -0.75 and -0.50 percent, respectively, and in the longer-term asset markets, German government bonds have marginal negative rates, suggesting that the negative rate policy may indeed be effective. One particular difference between successful implementation of this policy in, say Switzerland and not the US, is that market participants need to believe that the policy remain prevalent for a long time to have a full effect on long-term rates.

The main concern about negative rate policy is that commercial banks[23] may be unable or unwilling to pass these rates on to their depositors. In this case, it indicates that driving down the interest rate on reserves and possibly other forms of short-term asset shrinks banking profit and may adversely affect the amount of their lending. However, on the liability side, the largest sources of funding of commercial banks are institutional depositors, wholesale funding markets and foreign depositors, who incur significant hoarding cost. This suggests that, apart from small depositors, a large fraction of the liability side is fairly inelastic to modestly negative rate policy.

The asset side of commercial banks is more difficult to predict. First, lower average funding rate, in a competitive credit market, drives down the loan rate. This reduces the financing rate for production firms and is expected to stimulate higher economic activity. Second, assuming that negative policy rate is mostly resorted to during recessions, there is a considerable possibility that credit borrowers are more likely to be unable to repay the loans. This translates into higher loan rates charged by commercial bank in order to offset expected losses, due to firm defaults. Overall, this suggests that the equilibrium loan rate lowers but may still remain in the positive territory. The negative interest rate policy appears to have advantages together with manageable cost. Essentially, a modest negative rate does not pose a major discontinuity in the financial markets, and as evidence in the Europe suggests, rates between -1 to 0 percent implemented without triggering massive currency hoarding.

Excess reserves are the cash funds above the requirements deposited at the central bank. Empirical evidence suggests that since the financial crisis, commercial banks in the US, the UK and so forth have increased their excess reserves. For example, in the US excess reserves grew from $1.9B to $2.6T from August 2008 to January 2015.

This is of particular importance because the Federal Reserve and other central banks mandate set required reserve mandates which already lowers banks' available loanable funds. For instance, the Federal Reserves' Board of Governors in 2015 requires, a 3 and 10 percent reserves ratios on net transaction accounts over $14.5M and then over $103.6M, respectively. In such cases, a holding above these requirements is an excess reserve. Clearly banks encounter holding costs, in terms of forgone interest, which they to meet their liquidity needs which may also be costly if reserves fall below instantaneous need such as deposit

[23] Apart from commercial banks, the negative rate policy may induce significant disruptions to the functioning of some other financial institutions and markets. Money market funds (MMF), institutions that promise at least full amount of investment, are more prevalent in the US comparing to Europe or Japan. These institutions are important providers of short-term funding for commercial banks and other financial institutions who may have to shut down in presence of negative interest policy since they highly rely on low positive rate to cover their operational and management fees.

withdrawals or asset purchases that require immediate action. This indicates that banks need to actively balance opportunity cost of holding reserves and that of switching to an alternative investment. Empirical evidence on active reserves management suggests that the optimal excess reserves is non-zero.

Federal funds market is an interbank market where a commercial bank can borrow overnight loans to obtain reserves. The spread between lending and borrowing in this market gives a simple measure to represent opportunity cost of raising additional funds. From 1959 to October 2008, the reserves in the banking system grew at a stable annual average rate of 3.0 percent, which was the same as the rate that deposits grew. Excess reserves also maintained a stable 5.0 percent share of total reserve within the same timespan with reasonable rises during periods of economic distress such as September 2001. However, after October 2008, unprecedented increases in the amount of reserves together with introduction of unconventional monetary policies altered the trade-off between the costs associated with excess reserves management. In particular, holding reserves is less costly than prior to the financial crisis period which resulted in significant increases in excess reserves in the banking system. In the height of the 2008 crisis, default risk in the overnight loan market increases, however, since then this risk has lessened and stabilized since early 2013 which has made this market a relatively favorable investment for comparing to holding short-term liquid assets that are subject to inflation risk.

Since mid-2014, four European central banks began to adopt negative interest rate policy. This implementation has important implications for commercial banks since the excess reserves option has abruptly become more expensive, which makes alternative short-term investments more favorable.

An important aspect of negative nominal interest rate is that it aims at encouraging commercial banks to withdraw their reserves from the interbank market and spend them on other types of investments, such as corporate loans, that actually stimulates the economy. As discussed in the micro-founded model, negative interest rate may still not be enough to encourage banks to maintain low or zero excess reserves. However, another implication of this could be that banks find it profitable to borrow in the interbank market, $B < 0$. In fact this indicates that the interbank market pays the banks to borrow. This scenario is, nonetheless, unlikely because of the following reasons. First, for this to happen, the interbank rate needs to drop massively to incentives such borrowing strategy. In fact, one of the underlying reasons that a central bank is able to implement negative interest rate policy is that it implicitly claims that it charges a fee to safely hold funds in a high risk economic

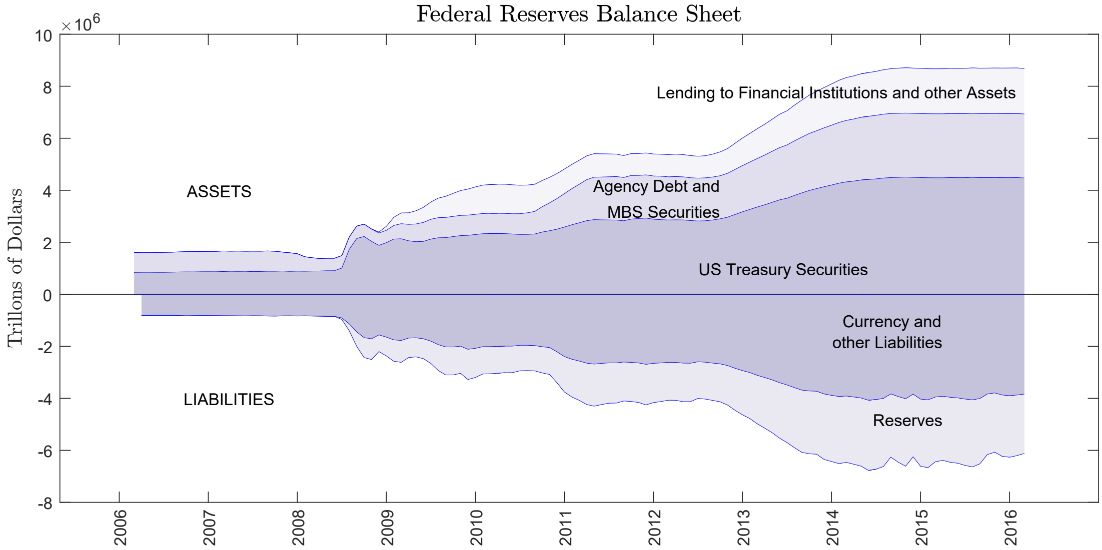


**Figure 1.7:** This figure illustrate that the balance sheet of the Federal Reserve has changes after 2008. Essentially, before early 2008, the balance item was very small relative to other items in the figure because substantial reserve holding was not justified. After the 2008 crisis, we observe that on the assets side, US Treasury securities and Agency Debt and MBS Securities, as well as, bank reserves on the liabilities side have substantially increased.

environment. However, commercial banks are significantly less trustworthy than a central banks, suggesting that lending rates to them need to include a finance premium, which in marginally negative rate environment could result in a total zero or positive rate. This established a self-fulfilling mechanism to push lending and borrowing incentives toward a desirable equilibrium. Second, maintaining a positive excess reserve balance offers liquidity services to banks which, similarly, in a negative interest rate environment, may be much more desirable to banks rather than having negative balances.

## 1.6 Concluding Remarks

This study examines how implementing negative nominal interest rate alters a representative bank's optimal asset allocations between excess reserves in the interbank market and available funds in the corporate loan market. The results of the micro-founded model show that when the interbank interest rate falls into negative territory, bank reduces its excess reserves balances, which then provides more funds in the corporate loans market. Given that the demand remains unchanged, this reduces the loan rate and increases the availability of credit to production firms. However, there is a secondary effect that may partially reverse this process. In particular, when loan rate decreases, the spread between loan rate and interbank rate also decreases which accordingly makes holding excess reserves more desirable. The overall effect of this policy in the interbank market depends on the relative importance of holding excess reserves as a source of liquidity, to loss of funds due to negative nominal interest rate. For significantly large decreases in the interbank rate, the corporate loan rate drops and the overall supply of credit increases.

## 1.7 Appendix: Optimal Deposit and Equity Derivations

The investors' Lagrangian problem is increasing in consumption in each period, ensuring that periods' budget constraints bind. Together with the concavity of the Lagrangian in consumption, the first-order-conditions and constraints fully characterize the solution to the investor's problem. Using the first-order-conditions in equations (**??**) and (**??**) form investor's problem and substituting the value of consumption in periods one and two, $c_1 = w - d - e$, and $c_2 = (1+i_d)d + (1+r_e)e$, we have:

$$\frac{1}{w-d-e} = \frac{\beta\lambda(1+i_d)}{(1+i_d)d+(1+r_e)e} + \frac{\beta(1-\lambda)}{d} \tag{1.7.1}$$

$$\frac{1}{w-d-e} = \frac{\beta(1+r_e)}{(1+i_d)d+(1+r_e)e} \tag{1.7.2}$$

Using equation (1.7.2), we have:

$$e = \frac{\beta\lambda(1+r_e)w - (i_d+\beta\lambda(1+r_e))d}{(1+r_e)+\beta\lambda(1+r_e)} \tag{1.7.3}$$

Substituting for $e$ in equation (1.7.1) gives.

$$\frac{1}{w-d-\frac{\beta\lambda(1+r_e)w-(i+\beta\lambda(1+r_e))d}{(1+r_e)+\beta\lambda(1+r_e)}} = \frac{\beta\lambda(1+i_d)}{(1+i_d)d+(1+r_e)\frac{\beta\lambda rw-(i+\beta\lambda r)d}{(1+r_e)+\beta\lambda(1+r_e)}} + \frac{\beta(1-\lambda)}{d}$$

Simplifying gives,

$$\beta(1-\lambda)rw = [1+\beta\lambda+(1-\lambda)\beta](r_e-i_d)d \tag{1.7.4}$$

Solving for $d$ gives the optimal deposit investment:

$$d^*(\beta,w,i_d,r_e,\lambda) = \frac{\beta}{1+\beta}\frac{(1-\lambda)r_e}{r_e-i_d}w \tag{1.7.5}$$

Substituting in equation (1.7.3) gives the optimal equity investment:

$$e^*(\beta,w,i_d,r_e,\lambda) = \frac{\beta}{1+\beta}\frac{i_d-\lambda r_e}{r_e-i_d}w \tag{1.7.6}$$

The investor takes the deposit and equity investment rates as given and allocates endowment to $d^*(\beta,w,i_d,r_e,\lambda)$ and $e^*(\beta,w,i_d,r_e,\lambda)$.

# Chapter 2

# Risk-Based Capital Regulation under Negative Interest Rate

## 2.1 Introduction

Regulatory interventions have always been ensued by heated debates. In the years after the financial crisis reached its darkest moment, academic literature and high legislative chambers were inundated by discussions related to risk-based capital requirements. Opponents often have expressed dissatisfaction against the intervention arguing that capital holding above the laissez-faire outcome is expensive to the banking institutions leading to lower lending and ultimately suppressed economic growth. Whereas the proponents of the regulation have argued that fragility of the banking sector, that is associated with a significant economic cost to the society, rationalizes the intervention.

Despite prolonged arguments presented by the opposing sides, these debates rarely reached to an agreeable conclusion. An important but often ignored reason to the disagreement was that the arguments emanated from incomparable bases. More specifically, the opponents' view presented by the banking institutions weighed heavily on the cost of equity as the central reason to rail against capital holdings above the laissez-faire outcome.[1] Ignoring the underlying merits behind their argument for the moment, their perspective focused on the role of the asset prices as the main reason to advocate for capital deregulation. This stance, however, was not readily reconcilable with the proponents social perspective whose arguments mainly built on a welfare analysis that is concerned with the negative externalities associated with costly bank failure.[2]

[1] **?**, **?**, **?**, **?**, **?**.
[2] **?**, **?**.

Much of the discussions among the academic and legislative literature on this context has understandably been devoted to the welfare implications of bank failure.[3] Nonetheless, the opponents' view on the cost of capital has remained a consistent defence that has languished further arguments to increase capital holding above Basel III. Recent empirical studies provide evidence that even in the presence of capital buffers in addition to the risk-based capital requirement, the banking institutions remain significantly undercapitalized (**?**).

Lack of a rigorous quantitative basis to evaluate the cost of capital for the banking institutions at an aggregate level is among the core reasons why the proponents have failed to discredit the merits behind capital deregulation. The methodological framework in this study, first develops a foundation to establish a realistic valuation of the bank capital in a general equilibrium under aggregate uncertainty setting. This framework simultaneously integrates the asset pricing and banking regulation disciplines to provide a mapping between the cost of capital and welfare implications of bank failure. This salient connection serves as a solution to reconcile the two counterarguments in favour and against bank capital holding.

A comprehensive capital regulation that enhances the welfare considers three simple components: (i) how is the bank funded? (ii) what is the risk profile of the bank's assets? (iii) what is the valuation of bank net worth? Existing studies focusing on bank funding show that government-guarantees provide welfare gains by preventing self-fulfilling runs on bank debt, even if not originally justified by fundamentals (**?**). Nonetheless, government-guarantees break the link between the cost of debt and borrower's default risk and lead to the under-capitalization of the banking system. This gives rise to an alternative distortion generated by more frequent bank failure and motivates capital regulation which provides gains by lowering socially undesirable defaults (**?**, **?**).[4] However, studies that concentrate on the liabilities provide limited predictions about the importance of bank assets composition. I show that the effectiveness of optimal capital regulation depends on the assets side of the bank balance sheet, particularly when the monetary policy targets reserves management. A large strand of literature focusing on the assets side of the bank balance sheet shows that conditioning the risk profile to capital provides welfare gains (**?**, **?**). However, this literature considers that households as the ultimate providers of financing, in the form of debt or equity, play a limited role or that the supply of financing is fixed. I show that households' optimal consumption-saving behaviour has important implications for the equilibrium cost of

[3] In particular, **?** provides a comprehensive survey on loss given default across financial and non-financial sectors showing that the ex-post asset recovery rate may fall to 70% per dollar.

[4] For further reference see **?**, **?**, **?**, **?**, **?**, **?**, **?**, **?**.

debt that is a determinant of the banking sector's default risk. This equilibrium mechanism predicts that as the cost of debt falls, capital constraint becomes effectively overburdening and hence socially costly.

These shortcomings provide motivation to raise the following two questions: First, what is the optimal capital regulation of the banking system in an environment where the cost of financing (in the form of debt or equity) and risk profile of the asset side arise endogenously? Second, how does the effectiveness of this optimal capital regulation depend on the IOER that is decided separately by the monetary authority? I address these questions by developing a general equilibrium model in which banks finance themselves by accepting deposits and raising equity from households, and invest their funds in excess reserves and loans subject to non-diversifiable risk.

The analysis in this chapter takes IOER as a given policy and shows that the optimal risk-weighted capital requirement offers welfare gains by lowering the likelihood of bank failure and its associated distortions that are ultimately borne by society (**?**). Nonetheless, the general equilibrium provides an additional important prediction. When the bank is required to raise more capital to satisfy the capital constraint, its demand for debt financing falls. This channel leads to a lower equilibrium deposit rate. Given any lending level, lower interest expenses expand the bank's ability to meet its debt liabilities and enhance the bank's solvency. The optimal risk-weighted capital regulation, even in general equilibrium, fails to consider this effect and hence becomes socially costly.

I show that when IOER is above the zero bound, a marginal decrease in this rate is accompanied by a proportional decrease in the equilibrium deposit rate.[5] Because the proportion of deposits in liabilities always exceeds that of the reserves on the asset side of the balance sheet, lower IOER leads to a faster fall in interest expenses than interest incomes.[6] As a result, the social cost of the optimal capital constraint, which is decided in isolation of the IOER policy, increases as IOER falls towards the zero bound. This finding is an important motivation for a jointly decided capital regulation and IOER. Particularly, a lower IOER that is accompanied by a looser capital constraint is able to expand the credit flow to the real economy, while the bank's default likelihood remains constant.

This general equilibrium framework provides a secondary prediction: the relationship between the optimal capital regulation and IOER reverses when IOER becomes very low or falls below zero. This finding is important to effective policy analysis in the current era

[5] This result is consistent with an empirical finding in (**?**).

[6] This notion is described in an ongoing work by **?** and is referred to as the '*reversal rate*'.

with low or negative interest rate environment. I show that any further reduction in this territory is accompanied by a nonresponsive equilibrium deposit rate because depositors always require strictly positive compensations for their time preference to forgo consumption. This nonproportional transmission mechanism from IOER to deposit rate indicates that the bank's interest incomes from reserves fall faster than its interest expenses on deposits. Given any lending level, the bank's solvency worsens, nonetheless, the capital regulation fails to consider this effect. An interactive policy initiative provides social value when a falling IOER, below zero bound, is accompanied by a stricter capital constraint.

This paper is organized to provide a brief overview of existing and ongoing studies that examine interconnections between capital regulation and IOER in Section (2.2). I develop a dynamic general equilibrium model in Sections (2.3)-(2.5.2) to study the implications of financial regulation on welfare, real economy, and fragility of the banking sector. Section (2.6) provides a numerical solution and discusses welfare and asset pricing implications. Section (2.8) concludes.

## 2.2 Background

Financial regulation provides social value by addressing distortions that intermediaries fail to internalize. Banking systems serve a significant role across the macroeconomies and policymakers are often highly concerned to ensure the stability of this sector through an array of regulatory initiatives. **?** show that bank deposit insurance provides social value by preventing self-confirming runs on bank debt, especially when panic-based runs are not originally justified by fundamentals. However, deposit insurance increases the bank's willingness to over-rely on debt financing because a bank's heightened default risk as a result of undercapitalization is no longer captured by the cost of debt.

Bank capital requirement regulation has formed an integral component of global financial regulatory architecture. Regulators' wider economic outlook is conveyed to the banking system through partnerships with banks to ensure their capital structure meets certain standards[7] in relation to the risk profile of their assets. A bank with higher exposure to riskier borrowers is required to hold more capital with the intention of increasing bank's ability to meet its debt liabilities should the borrowers become unable to repay their liabilities to the bank.[8] **?** provide a comprehensive study to recommend capital requirement policies that

[7] The primary source of regulatory guidance for such regulation has been the voluntary set of measures suggested by the Basel Committee on Banking Supervision (**?**).

[8] These standards have drawbacks, for example, their heavy reliance on privately provided credit ratings

set forth stricter risk-weighted capital structures to increase bank's stake in risk-taking and ultimately decrease the socially undesirable bank failures.[9]

Bank failures have substantial implications for welfare consideration since bankruptcies in the banking system is associated with realized losses that are estimated to be about 30% of total ex-post assets. These losses include expenses that arise only when bankruptcy is triggered which involves lengthy legal processes, costly liquidations and sale of assets, and lost charters.[10] **?** estimates that a bankruptcy process is associated with around 30% loss of bank's total assets due to legal and liquidation proceedings. Similarly, **?** and **?** show that bankruptcy cost can vary between 10% to 23% of the total assets within non-financial firms and between 15% to 30% of the total assets for financial firms. **?**; **?** and **?** provide comprehensive studies that examine bankruptcy cost according to several measurements and show that in some cases these costs are even larger and can account for more than 30 cents on the dollar.

In this context, financial regulation internalizes bankruptcy costs, that are otherwise ignored by individual banks, and sets a minimum risk-weighted capital requirement policy to lower the possibility of bank failure and by this means its associated deadweight loss. Academic and professional literature has studied the impact of capital requirement on banks within macroeconomic settings (**?**; **?**, **?**; **?**; **?**) to show how the financial accelerator effect slows down when bank's capital is subject to less fluctuations due to the introduction of capital requirements. The key mechanism that motivates setting capital requirement works through the bank's decision that fails to internalize the negative impacts of their over-borrowing on the financial stability across the sector.

The first contribution of this paper is to extend the finding of existing literature with a general equilibrium approach in which the banking system is exposed to uninsurable uncertainty through loans to borrowers. The introduction of aggregate uncertainty is a key ingredient as it creates a close resemblance to an economy that faces the potential loss of productivity and financial crises due to inability of the borrower to raise further funding at the sector level to meet debt contracts. General equilibrium framework has important implications to incorporate interrelated feedback between lenders whose decision to provide

leads to inaccuracies and creates distortions. Credit ratings are less accurate than credit spreads and the standards neither distinguish among issues within a particular rating category nor among issues with different spreads.

[9]Such policies are yet to be adopted by the regulators into the financial system. Kern Alexander (2015) addresses international efforts to regulate bank capital requirements and leverage which are negated by factors such as asymmetric lobbying against stricter rules (**?**, **?**).

[10]Losses on assets that occur prior to the bank's failure but are not reported on the bank's balance sheet at the time of the failure.

financing to the banking system is dependent on the profitability of equity investment and dividend payouts under defaults and solvencies, and borrowers whose valuation of future cashflow incorporates their shareholder's preferences.

Another consideration that factors into the regulator's decision to set capital requirement is to take into account the lender's ability and willingness to participate in equity investment of the banking system. Setting stricter capital requirement implies that households, as the ultimate provider of financing, need to take a smaller position in risk-free investments such as deposits, and larger position in risky investment which eventually forms bank's capital. When households are reluctant to participate in stock market or purchase risky equities, increasing capital requirements amounts to additional resistance by the banking sector because the marginal price of capital has to increase significantly to convince holders of risk-free assets to rebalance their portfolios which leads to falling risk-free rate (deposit rate) and widening equity premium.

The last ingredient that the regulator takes into account when deciding on capital regulation is the efficiency of the financial market that intermediates funds from investors to equity borrowers. Although deposit investment is costless in most economies[11], equity investment requires services from financial intermediaries such as investment banks and brokers. These costs include underwriting fees, broker's bid-ask spreads, etc. that are charged to lenders or borrowers throughout an intermediation process which lower the ultimate equity investment's return to lenders or dampens raised capital that reaches borrowers. In this context, the regulator considers such costs through intermediation process as a secondary deadweight loss that is socially undesirable when a decision on capital regulation is evaluated. Specifically, when financial markets are perfectly efficient and intermediation fees are zero, the regulator is only concerned with recommending a sufficiently high level of capital that eliminates bank failures. This sets an upper bound on capital regulation, however, as intermediation fees increase, regulator considers this deadweight loss against costly bankruptcies and recommends a capital regulation policy that balances welfare gains of higher bank capital associated with less frequent failures versus gains associated with lower funds channelled through costly intermediation.

Bank asset holding includes cash or its equivalents, reserves, Treasuries and other risk-free investments and loans to borrower. Stricker risk-weighted capital regulation requires more equity per unit of risky investment that limits the share of risky asset holding and

[11] Fees, minimum deposit limits, and many transaction costs are in place but in general deposit investment is a more accessible financial investment relative to equity purchases that incurs intermediation fees

increases the share of assets in risk-free investments. In this context, reserves deposit facility that is available to the banking system serves as a risk-free investment that increases when the bank's capital regulation amounts to lower loans. Since 2008, excess reserves[12] held by the banking system with the central banks dramatically increased in the U.S. banking system from $45 billions in September 2008 to nearly $1T by January 2009. **?** and **?** show that part of such changes in holding reserves is explained by the implications of heightened uncertainty and low productivity that lead the banking system to seek out a safe investment to avoid bankruptcies that rose during the 2008 financial crisis. Unlike required reserves that are mandatory deposits, excess reserves are voluntary deposits that receive IOER paid by the central bank to the banking system which can be positive, zero or negative. As IOER is decreased, excess reserves become a less attractive investment which are substituted for by loans to the business sector. However, this portfolio rebalancing due to IOER is interrelated with risk-weighted capital requirement across the bank's balance sheet which leads to a tighter capital requirement.

The dependencies between IOER and the risk-weighted capital requirement bear welfare implications that calls for a joint response by the monetary authority in charge of IOER and the financial regulator in charge of capital requirement. From a welfare perspective, capital requirement is a policy tool that is able to counter the deficiency caused by the bankruptcy cost in the states in which the banking sector fails. Without further deficiencies, any IOER that results in changes in capital requirement is irrelevant to welfare. However, a joint policy tool that includes IOER needs to consider that although reserves provide financial stability, they are an unproductive investment and interest payments on reserves has to be funded from taxes. The choice of taxation is an important consideration as it ultimately determines whether deposit insurance can provide full compensation in any default state (**?**, **?** and **?**). In particular, when tax resources are equivalent to outstanding deposits less the reserves, then depositors are guaranteed to receive their funds even if the bank fails due to an adverse shock to its borrowers who become unable to repay their liabilities to the bank. As taxes fall short of the amount deposits less reserves, the deposit insurance is able to offer only partial insurance to depositors in real terms.

This deficiency arises due to the choice of taxation in relation to the capital structure and asset allocation of the banking system. A joint financial regulatory policy that compensates reserves by IOER has to take into account that interest expenses are a further force that lower

[12]Excess reserves are funds that are deposited to the central banks in addition to the required reserves, that are mandated to be held as a proportion of the total assets for legal requirements. While required reserves grew modestly over the past decade, excess reserves have grown at an unprecedented rate.

taxation which limit the ability of deposit insurance to compensate deposit holders. Consequently, the interaction between IOER and other policy tools is associated with a welfare implication which has to be considered when deciding an optimal IOER that has interactions with capital requirement regulation. The social value of bank equity and social value of reserves are two consideration to policymakers that counter bankruptcy cost and deposit insurance's ability and have to be decided jointly.

## 2.3 The Model

Assume time is discrete, with dates $t = 0, 1, 2, \ldots$. The economy consists of three sectors: a representative household, a representative bank (commercial bank or intermediary) and a financial regulator. The methodology is organized with the following set-up. Section (2.4) presents the optimal behaviour of the households and the banking sector without the regulator's minimum capital requirement intervention and discusses general equilibrium implications. Second, the regulator's problem to set the minimum capital requirement is presented together with bank's problem subject to the regulatory constraint in Section (2.5), followed by general equilibrium implications, given an exogenous interest-on-excess-reserves rate. Section (2.6) illustrates a calibration exercise with minimum capital requirement policy and discusses the implications. Deposit insurance service is provided by the regulator across Sections (2.4)-(2.6).

### 2.3.1 Preferences

The household is an infinitely-lived dynasty that lives off financial wealth. At each date-$t$, the household chooses optimal consumption-saving and portfolio allocation to two investment opportunities, deposits and equity. The deposit is a risk-free investment compensated at gross interest rate $R_{D,t+1}$ by the banking sector and benefits from deposit insurance guarantee. The equity is a risky investment that is subject to stochastic return $\mathbb{E}_t[R_{E,t+1}] > R_{D,t+1}$ and is protected by limited liability such that in any default state, equity investor is only responsible up to the original investments. The maximization problem of the household is described by the following recursive utility preferences[13]

$$\{C_t^*, D_{t+1}^*, E_{t+1}^*\}_{t=0}^{\infty} \in \underset{\{D_{t+1}, E_{t+1}\}}{\arg\max}\ \mathbb{E}_0\left[U(C_t, \mathbb{E}_t U_{t+1})\right] \tag{2.3.1}$$

$$U(C_t, \mathbb{E}_t U_{t+1}) = \left\{(1-\beta)C_t^{1-\frac{1}{\psi}} + \beta\left(\mathbb{E}_t U_{t+1}^{1-\gamma}\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right\}^{\frac{1}{1-\frac{1}{\psi}}} \tag{2.3.2}$$

where at each date-$t$, the household decides on optimal consumption and portfolio allocation subject to the intertemporal budget constraint described below, receives utility from real consumption $C_t$, $\beta \in (0,1)$ is the subjective discount factor, $\gamma$ is the coefficient of relative risk aversion, and $\psi$ is the elasticity of intertemporal substitution (EIS). The household's attitude towards static risk is separated from intertemporal substitution of consumption with preferences for early resolution of uncertainty such that $\gamma > \frac{1}{\psi}$ throughout the model. The conditional expectation operator $\mathbb{E}_t[.]$ evaluates household's probabilistic assessment of outcomes over solvency and default.

The investment environment includes risk-free deposit investment $D_{t+1}$ that is chosen at date-$t$ backed by deposit insurance. Deposits receive gross deposit interest $R_{t+1}^D$, and equity $E_{t+1}$ investments receive a stochastic gross return that is protected by limited liability when the underlying issuer defaults,

$$R_{E,t+1}^{+} = \max\left\{\frac{P_{E,t+1} + div_{t+1}}{P_{E,t}}, 0\right\} \tag{2.3.3}$$

where $P_{E,t}$ and $div_t$ are the price of equity and dividend, respectively. Equity investment is assumed to be subject to a linear cost $\kappa \in (0,1)$. The intertemporal budget constraint is,

$$P_{C,t}C_t + \underbrace{D_{t+1} + E_{t+1}}_{\text{Saving}} = \underbrace{(1-\tau_{t+1})}_{\text{Premium}}\Bigg(\underbrace{\overline{R_{D,t}D_t}}_{\text{Deposit Insured}} + \underbrace{R_{E,t}^{+}(1-\kappa)E_t}_{\text{Limited Liability}}\Bigg) + \underbrace{Tr_{t+1}}_{\text{Transfer}} \tag{2.3.4}$$

where $\tau_{t+1}$ is a fraction of household income that is taxed and $Tr_t \geq 0$ is a transfer that the household receives from the regulator described in section (2.3.3). The right-hand-side of equation (2.3.4) describes household's wealth $W_t$ which evolves at rate $R_{W,t+1}$ between two consecutive dates $t$ and $t+1$ according to:

$$R_{W,t+1} = (1-\tau_{t+1})(1-\theta_{t+1})R_{D,t+1} + (1-\tau_{t+1})\theta_{t+1}(1-\kappa)R_{E,t+1} + \frac{Tr_{t+1}}{W_t} \tag{2.3.5}$$

[13]**?** and **?**.

where $\theta_{t+1}$ is the portfolio weight on risky asset. The household's value function is,

$$V_t = \left\{ (1-\beta) \left( \frac{C_t}{W_t} \right)^{1-\frac{1}{\psi}} + \beta \left( 1 - \frac{C_t}{W_t} \right)^{1-\frac{1}{\psi}} \left( \mathbb{E}_t \left[ V_{t+1}^{1-\gamma} \left( R_{W,t+1}^{1-\gamma} \right) \right] \right)^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\psi}} \quad (2.3.6)$$

When the elasticity of intertemporal substitution approaches one, the household becomes infinitely indifferent to substitute consumption over time and the value function approaches $1-\beta$. In this case, the household's overall investment becomes independent of the return on wealth which leads to a fixed size of the financial sector. The break-down of this fixed investment among deposits and equity varies depending on the deposit rate and the price of equity.

### 2.3.2 The Bank

The representative banking sector is in charge of intermediating funds[14] from the households to borrowers by accepting deposits and issuing equity to raise capital. The bank invests its financings in two purposes: issues a commercial loan portfolio that earns stochastic return $R_{L,t+1}$ per each unit of investment, or invests in reserves in the deposit facility provided by the regulator to earn a risk-free rate determined by the IOER $(R_{X,t+1})$. At the end of each period, bank's liabilities consist of deposits plus interest which must be honoured for the bank to remain solvent, in which case earnings from loans and reserves are transferred to deposit holders, and then equity investors. The bank, however, is able to declare bankruptcy when it is unable to meet its liabilities in which case deposit holders are compensated partially by the bank and equity value is zero.[15]

Let $\mathcal{D}_{t+1}$ denote bank's finances from accepting deposits and $\mathcal{E}_{t+1}$ denote finances from issuing equity. At each date-$t$ the bank decides how to finance its operations by choosing an optimal capital structure and a portfolio allocation to maximize the present value[16] of the following cashflow described by:

$$div_{t+1} = \max \left\{ \underbrace{\underbrace{R_{X,t+1} X_{t+1}}_{\text{Reserve Income}} + \underbrace{R_{L,t+1} L_{t+1}}_{\text{Loan Portfolio}}}_{\text{Total Reveune}} - \underbrace{R_{D,t+1} \mathcal{D}_{t+1}}_{\text{Deposit Cost}}, 0 \right\} - \underbrace{\frac{1}{P_{E,t}} \mathcal{E}_{t+1}}_{\substack{\text{Shareholder} \\ \text{Value}}} \quad (2.3.7)$$

[14]The intermediation service reduces potential costs which lenders and borrowers would have incurred throughout a dis-intermediated economy. Banking sector offers a welfare gain by minimizing searching and monitoring costs across sectors. Although such associated costs are only assumed implicitly in this study, the welfare-improving implications form the basis for the presence of a banking sector.

[15]Deposit insurance compensates depositors for any remaining uncovered parts of their deposits.

[16]The present valuation is determined by a unique stochastic discount factor described in Section (2.4.1).

where $X_{t+1}$ and $L_{t+1}$ denote investments made by the bank in the reserves deposit facility and loans, each receiving gross interest-on–excess-reserves and a stochastic loan rate, respectively. Equation (2.3.7) is the total dividend value that bank is able to generate after paying out its deposits and its interest and the original investment value. Table (2.1) characterizes the bank balance sheet at each date, consisting of debt $\mathcal{D}_{t+1}$, capital $\mathcal{E}_{t+1}$, reserves $X_{t+1}$ and loans $L_{t+1}$ such that,

$$L_{t+1} + X_{t+1} = \mathcal{D}_{t+1} + \mathcal{E}_{t+1} \tag{2.3.8}$$

Let $\eta_{t+1}$ and $\omega_{t+1}$ denote equity-to-assets and loan-to-assets ratios derived from banks balance sheet at each period, respectively,[17] such that $(\eta_{t+1}, \omega_{t+1}) \in [0,1] \times [0,1]$. The risk-neutral bank maximizes economic profit over the solvency region ($\Delta_s$) according to,

| Assets | | Liabilities | |
|---|---|---|---|
| Reserves $(1-\omega_{t+1})$ | $X_{t+1}$ | Deposits $(1-\eta_{t+1})$ | $D_{t+1}$ |
| Loans $(\omega_{t+1})$ | $L_{t+1}$ | Shareholder Value $(\eta_{t+1})$ | $E_{t+1}$ |
| Balance Sheet Size | $A_{t+1}$ | | |

**Table 2.1:** The table describes bank's balance sheet with deposits and equity forming the liabilities side and reserves and loans forming the assets side.

$$\max_{\eta_{t+1}, A_{t+1}, \omega_{t+1}} \int_{\Delta_s} M_{t,t+1} div_{t+1} dF(z) \tag{2.3.9}$$

Subject to,

$$X_{t+1} + L_{t+1} = \mathcal{D}_{t+1} + \mathcal{E}_{t+1} \tag{2.3.10}$$

$$\eta_{t+1} \geq \overline{\eta}_{t+1} \tag{2.3.11}$$

$$(\eta_{t+1}, \omega_{t+1}) \in [0,1] \times [0,1] \tag{2.3.12}$$

where $A_{t+1}$ is the total balance sheet size and $M_{t,t+1}$ is the stochastic discount factor[18] of the households who own bank's equity. The bank discounts expected economic profit at future dates with respect to the probability space $(\Omega, \mathscr{F}, F)$ to choose decisions given the price of equity, the deposit rate and IOER. Equation (2.3.10) is bank's balance sheet constraint

[17] As Section (2.4.3) shows, in equilibrium the optimal capital structure decision $\eta^*_{t+1}$ and portfolio allocation by the bank $\omega^*_{t+1}$ take interior solutions. This implies that in equilibrium bank's capital structure includes both debt and capital and its portfolio allocation includes both reserves and loans.

[18] Section (2.4.1) characterizes the stochastic discount factor given by and intertemporal capital asset pricing framework.

where $X_{t+1}$, $L_{t+1}$, $\mathcal{D}_{t+1}$ and $\mathcal{E}_{t+1}$ are reserves, loans, deposits and equity components of the balance sheet, respectively. The bank chooses total balance sheet size, and the following two fractions, equity-to-asset and loan-to-assets ratios, over the solvency region. Equation (2.3.11) is the minimum capital requirement constraint that stipulates for any balance sheet size, the bank must finance at least a certain fraction $\overline{\eta}_{t+1}$ of its total liabilities through equity.

**Defaults —** The bank is only concerned with the solvency region defined by $\Delta_s$. The solvency region is specified by an interval over the loan rate state space where the bank remains solvent. The end-of-period ex-post loan rate that breaks even between revenues and outstanding liabilities formulated according to the following condition specifies the minimum loan rate outcome that has to realise such that the bank is able to meet its debt liabilities:

$$\underbrace{R_{p,t+1}A_{t+1}}_{\text{Total Revenues plus Interest Income/Expense}} = \underbrace{R_{D,t+1}\mathcal{D}_{t+1}}_{\text{Total Liabilities plus interest payment}} \tag{2.3.13}$$

where $R_{p,t+1} = (1-\omega_{t+1})R_{X,t+1} + \omega_{t+1}R_{L,t+1}$ denotes the gross return on bank portfolio. Given bank's decisions $\eta_{t+1}$, $A_{t+1}$ and $\omega_{t+1}$, equation (2.3.13) pins down a unique gross loan rate $R_{b,t+1}$ in the state space that makes the bank just-solvent to pay off its debt-holders. At loan rate $R_{b,t+1}$, the bank is collecting only a fraction of its outstanding loans, which together with reserves, enable the bank to remain solvent. Nonetheless, this implies that the value of its shareholders is equal to zero:

$$R_{E,t+1} = \begin{cases} \dfrac{R_{p,t+1}A_{t+1} - R_{D,t+1}\mathcal{D}_{t+1}}{\mathcal{E}_{t+1}} & \text{if } R_{L,t+1} > R_{b,t+1} \\ 0 & \text{if } R_{L,t+1} \leq R_{b,t+1} \end{cases} \tag{2.3.14}$$

Assuming a strictly positive beginning-of-period equity value $\mathcal{E}_{t+1} > 0$, then condition (2.3.13) implies that because $A_{t+1} > \mathcal{D}_{t+1}$ then $R_{p,t+1} < R_{D,t+1}$. The threshold loan rate is given by:

$$R_{b,t+1}(\eta_{t+1}, \omega_{t+1}; R_{X,t+1}, R_{D,t+1}) = \max\left\{\frac{1-\eta_{t+1}}{\omega_{t+1}}R_{D,t+1} - \frac{1-\omega_{t+1}}{\omega_{t+1}}R_{X,t+1}, 0\right\} \tag{2.3.15}$$

The threshold loan rate that separate solvency from default is endogenously driven by the ability of the bank to withstand adverse shocks. Henceforth the shorthand just-solvent loan rate $R_{b,t+1}$, specifies the default and solvency regions, respectively, over the possible loan outcome in the state space:

$$\Delta_f \ := \ [0, R_{b,t+1}) \tag{2.3.16}$$

$$\Delta_s \ := \ [R_{b,t+1}, \infty) \tag{2.3.17}$$

Given bank's decision on capital structure and portfolio composition, the default threshold is known at date-$t$. A higher equity-to-asset ratio (ceteris paribus) enables the bank to withstand a greater adverse shock driven by encountering a higher number of non-performing loans, and remain solvent.

The threshold loan rate $R_{b,t+1}$ is weakly decreasing in $\eta_{t+1}$. In particular, in an extreme case, when the bank is over-capitalized[19] such that it is able to cover its exposure to risky loans with capital alone ($\omega_{t+1} < \eta_{t+1}$), then $R_{b,t+1}$ is equal to zero and is constant in $\eta_{t+1}$. Conversely, a higher loan-to-asset ratio (ceteris paribus) worsens bank's ability to withstand adverse outcomes and therefore $R_{b,t+1}$ is weakly increasing in $\omega_{t+1}$. Similarly, in an extreme case when the bank is over-capitalized then $R_{b,t+1}$ is equal to zero for any $\omega_{t+1} < \eta_{t+1}$.

The threshold loan rate $R_{b,t+1}$ is increasing in deposit rate since a higher deposit rate increases interest payments to bank's debt holders and increases the likelihood of a default outcome. Conversely, $R_{b,t+1}$ is decreasing in IOER because a higher IOER contributes as an interest income to bank and extends its ability to meet its liabilities. Interestingly, $R_{b,t+1}$ is independent of bank's balance sheet size $A_{t+1}$ in a special case when the return on bank lending exhibits a constant return to scale (CRS).[20] This implies that the bank may choose any balance sheet size but the key driver of its default depends on $\eta_{t+1}$, $\omega_{t+1}$, $R_{X,t+1}$ and $R_{D,t+1}$ only, because the compositions inside the balance sheet determines ability to withstand adverse outcomes for any arbitrary balance sheet size.

The bank faces bankruptcy when $R_{p,t+1}A_{t+1}$ is strictly less than its outstanding liabilities $R_{D,t+1}\mathcal{D}_{t+1}$. The probability of default depends on the properties of the aggregate shock to bank's borrowers who repay their own liabilities to the bank:

$$\mathbb{P}\left(\text{Default}_{t+1}\right) \ = \ 1 - \mathbb{P}\left(R_{p,t+1}A_{t+1} \geq R_{D,t+1}\mathcal{D}_{t+1}\right) \tag{2.3.18}$$

In a default state, realized loan rate is strictly less than the threshold[21] $R_{b,t+1}$ and subse-

[19] Over-capitalization with respect a profit maximizing approach such that additional equity holding leads to lower economic profit at the end of the period.

[20] When, however, the loan section exhibits a decreasing return to scale such that larger scale is associated with lower effective return per unit, the balance sheet size matters to $R_{b,t+1}$ because larger $A_{t+1}$ implies lower return on loan section which accordingly limits bank's ability to meet its liabilities for a given adverse outcome.

[21] For instance, a default threshold $R_{b,t+1} = 0.75$ indicates that the (minimum) net loan rate that a bank can

quently the bank is forced into bankruptcy and its proceeds are distributed to the debt holders on pro rata basis[22]. Limited liability condition prevents equity investors to internalize losses beyond their initial equity investments which indicates that in any default state, the bank is subsequently unable to fully compensate its debtors and the risk is partially passable to deposit accounts. This introduces the possibility of Diamond-Dybvig financial panic where depositors may start to withdraw their funds in anticipation of a potential default even though it is unwarranted by the fundamentals. Deposit insurance offered by the regulator rules out this specific financial panic by promising depositors a guarantee on their risk-free investments.

The bank solves the problem in (2.3.9) by choosing first, total balance sheet size ($A_{t+1}$) and funding composition $\eta_{t+1}$ given the price of equity and deposit rate.[23] The solution to the bank problem on the funding side are two demand functions or 'twin demands' for capital that are jointly determined by the price of equity, deposit rate, and also asset allocation choice $\omega_{t+1}$ from the assets side of the bank balance sheet. The bank trades with the households to pin down equilibrium capital structure and their prices, given any $\omega_{t+1}$. Third the bank considers IOER and the expected loan rate to pin down its portfolio allocation which overall solve the bank problem.

**Bank Borrowers and Production —** For tractability, I assume that the bank grants loans to borrowers who have no alternative access to financing and engage in production activities in a non-financial sector. This assumption maintains bank's central role to act as an intermediary between households and the ultimate borrowers, however, this also indicates that the non-financial sector is all-externally financed.[24] First, the non-financial sector is subject to aggregate uncertainty and engages in a static production process which requires financing at the beginning of each period and pays off a stochastic outcome at the end of each period. Second, the aggregate uncertainty assumption implies that bank's lending to borrowers is non-diversifiable across the non-financial sector. The underlying loan contract between the

withstand to remain solvent is $R_{b,t+1} - 1 = -25\%$. When the bank realizes an ex-post loan rate $R_{L,t+1} < R_{b,t+1}$ then its total assets value falls below total liabilities and has to declare bankruptcy.

[22]The bankruptcy definition is stipulated by the debt contract between the bank and its debt-holders. Particularly when the bank denies repaying debt-holders in full, it can be forced into bankruptcy. The term liquidation within this context refers to enforcing debt contract to seize bank's (end-of-period) assets in the event of bankruptcy. In the discrete-time model presented in this section, after bankruptcy a new bank is set up and continues its service by raising debt and capital.

[23]Subject to boundary constraints to ensure interior or corner solution.

[24]**?** and **?** provide a basis to study the role of the production sector's equity through extended frameworks in which a lower operating cost of an intermediary relative to a financial market that intermediates funds directly between the borrowers and lender determines the equilibrium cost of capital.

bank and its borrowers stipulates that a loan is considered non-performing when the borrower fails to repay the original borrowed amount plus interest that is decided between two counter-parties ex-ante. In any default state, the bank is allowed to seize borrower's total assets which together with the non-diversifiable risk profile implies the bank's loan section is non-performing in that default state.

In this context, because the non-financial sector is unable to raise financing directly from the households, it is also unable to redistribute dividends (if any) to households in a solvency state, and the banks also receives any dividend from the non-financial sector which effectively implies the bank serves as the owner of the non-financial sector. The outcome of the non-financial sector is the real economic output that is consumed in the goods market by the households.

**Technology —** The bank faces a log-normally distributed shock per unit of investment in the loan section with the following Cobb-Douglas production technology that is subject to an exogenous aggregate shock $z_{t+1}$,

$$h(L_{t+1}, z_{t+1}) \;=\; z_{t+1} L_{t+1}^{\alpha} \tag{2.3.19}$$

where $\log z_t = \mu_z + \sigma_z \epsilon_{t+1}$, $\epsilon_t \sim \mathcal{N}(0,1)$ and $\alpha \in (0,1]$.

### 2.3.3 Financial Regulator

The financial regulator provides the following three services: offers deposit insurance, sets the minimum risk-weighted capital requirement, and accepts reserve deposits from the banking sector described in Figure (2.1). Deposit insurance is a guarantee that compensates depositors in full in default states. The minimum risk-weighted capital requirement considers a welfare maximizing objective that internalizes costly bankruptcies that both the households and banking sectors fail to internalize.[25] Lastly, accepting deposits from the banking system is a form of reserves deposit facility.

The banking sector described in the previous section is only concerned with the solvency region, however, bank's capital structure includes funding that is raised through debt contracts which allows debt holders to force the bank into bankruptcy due to inability to honor

[25] This set-up assumes complete information. More precisely, the bank's objective is set up to only consider solvency region. However, the households are concerned about social costs of the bankruptcy. Because the regulator shares the same objective function as that of the households, therefore the households does not need to be concerned about bankruptcy and its associated social costs.

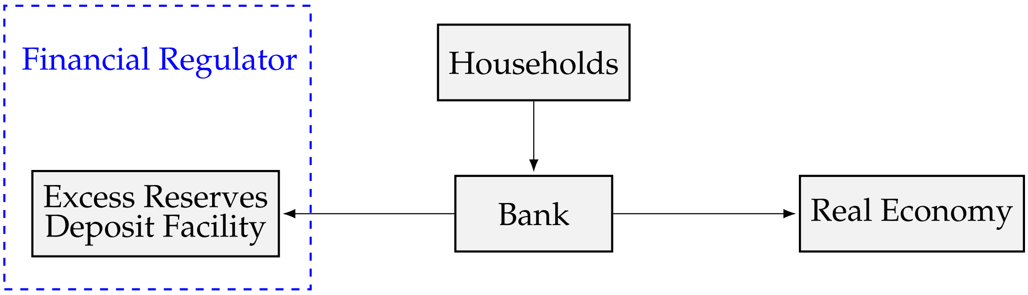


**Figure 2.1:** The diagram illustrates sectors in the economy. The households invest in bank equity, and deposit their funds into bank deposit facility as a risk-free investment. The bank is given two investment opportunities: channel funds to the real economy as loans, and hold a share of its funds in the excess reserves deposit facility provided by the financial regulator.

debt contracts in full. **?** estimates that a bankruptcy process is associated with 30% loss of bank's total assets due to legal and liquidation proceedings. Similarly, **?** and **?** show that bankruptcy cost can vary between 10% to 23% of total assets within non-financial firms and between 15% to 30% of total assets for financial firms. **?**; **?** and **?** provide comprehensive studies that examine bankruptcy cost according to several measurements and show that in some cases these costs can account for more than 30 cents on the dollar.

In this context, bankruptcy cost per unit of wealth is denoted by $\chi \in (0,1)$ that characterizes a proportional fraction of banking sector's total assets that is lost due to bankruptcy process when a default occurs. The financial regulator who is concerned with both solvency and default outcomes, considers such costs and sets a minimum (risk-weighted) capital requirement to maximize the following social welfare function.

$$\max_{Q_{X,t},\overline{\eta}_{t+1}} \mathbb{E}_0 \left[ U(C_t, \mathbb{E}_t U_{t+1}) \right] \tag{2.3.20}$$

subject to,

$$P_{C,t}C_t + D_{t+1} + E_{t+1} = (1-\tau_{t+1}) \left( \frac{1-\overline{\eta}_{t+1}}{Q_{D,t}} + \overline{\eta}_{t+1}(1-\kappa)R_{E,t} \right) A_t + Tr_{t+1} \tag{2.3.21}$$

where $Q_{D,t}$ is the price of deposits and $\overline{\eta}_{t+1} \in [0,1]$ where the transfer function is,

$$\frac{\mathrm{Tr}_{t+1}}{W_t} = \begin{cases} \tau_{t+1} - (1-\tau_{t+1})(1-\omega_{t+1})r_{X,t+1} & \text{if } z_{b,t+1} \leq z_{t+1} \text{ (non-default)} \\ \tau_{t+1} - (1-\tau_{t+1})(1-\omega_{t+1})r_{X,t+1} - \Lambda_{t+1} & \text{if } z_{s,t+1} \leq z_{t+1} < z_{b,t+1} \text{ (default)} \\ 0 & \text{if } z_{t+1} < z_{s,t+1} \text{ (inadequate deposit insuranc} \end{cases}$$

the term $\Lambda_{t+1}$ denotes uncovered share of debt contracts (uncompensated deposits in relation to the whole deposits plus promised interests) from the banking sector,

$$\Lambda_{t+1} \;=\; (1-\tau_{t+1}).\left(\frac{1-\eta_{t+1}}{Q_{D,t}} - \chi.R_{p,t+1}\right)A_{t+1}$$

where $r_{X,t+1} \equiv R_{X,t+1} - 1 = 1/Q_{X,t} - 1$ and that $r_{X,t+1} \lesseqgtr 0$ is the net IOER offered on reserves deposit facility offered by the regulator to the banking sector, and $\overline{\eta}_{t+1}$ is the minimum (risk-weighted) capital requirement set on the banking sector.

First, social welfare function in (2.3.20) is identical to the utility function of the households which regulator maximizes considering regulatory tools available in this context. Equation (2.3.21) characterizes regulators resource constraint that internalizes transfers to households.

Second, the regulator raises funds through a proportional taxation[26] $\tau_{t+1}$ from the households. These funds are available to the regulator to offer deposit insurance[27] in a default state and to cover interest expenses on reserves when IOER is positive. When IOER is negative, then reserves deposit facility provides an interest remuneration to the regulator since the proportion of reserves $1-\omega_{t+1}$, scaled by after tax resources $1-\tau_{t+1}$ earns interest income when $r_{X,t+1} < 0$.

Third, the regulator considers three possible outcome intervals when considering the transfer. The non-default region is characterized by the aggregate shock outcome $z_{b,t+1} \leq z_{t+1}$ specifying that the banking sector remains solvent. The default region is characterized by $z_{s,t+1} \leq z_{t+1} < z_{b,t+1}$ specifying that due to realizing a large adverse shock, the banking sector's total assets falls below its debt liabilities. In this case, the bank defaults and its post bankruptcy proceeds are described by the recovered assets $(1-\chi).R_{p,t+1}A_{t+1}$. In this case, the regulator compensates depositors out of its available resources which implies that although deposits are risk-free, households receive a smaller transfer. From a welfare perspective, the regulator considers fraction $\chi.R_{p,t+1}A_{t+1}$ as a deadweight loss that is socially undesirable to the economy.

Fourth, the choice of taxation is taken as given and the solution methodology considers the following two possible cases. When taxation is sufficiently large enough to provide full insurance on deposits. This case requires taxes to be equal to deposits plus promised interest, less the reserves (plus its interests) such that any resulting uncovered deposits within the

[26]The notion of taxation in this context is to simplify the analysis by setting up the regulator to use taxation for the purpose of deposits insurance in a default state, which indicates that taxes serves as a ex-ante premium.

[27]Deposit insurance fee can be charged directly from the banking sector, however, in this context with a representative households sector and banking sector, applying the charges directly to household offers tractability.

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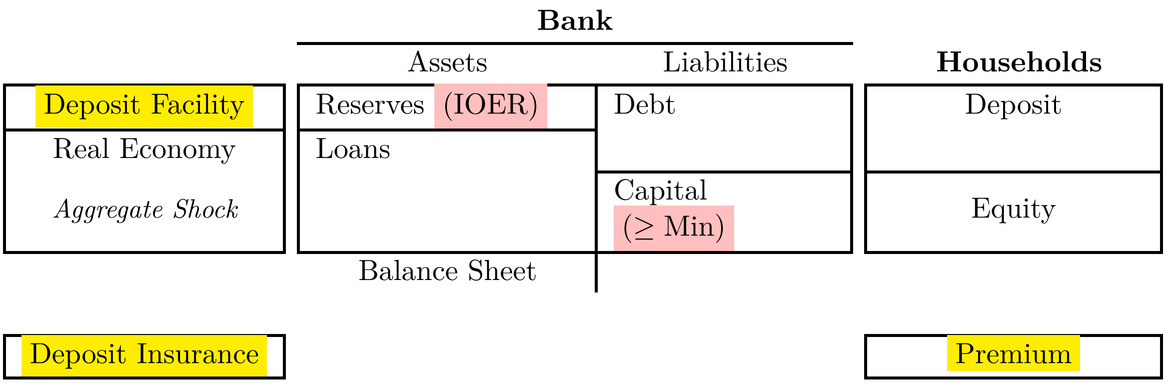



**Table 2:** The diagram describes flow of funds from households (deposit and equity) to the banking sector's liabilities (debt and capital) which subsequently is channelled to the real economy (lending) and excess reserves deposit facility. Policymakers services in charge of financial regulation and interest-on-excess-reserves is highlighted in the diagram. Premium is the taxation that policymaker raises at the beginning of each period, in anticipation of any defaults in the banking sector, to provide government guarantees to depositors. This resource also serves as a fund to pay any positive (negative) interest expenses on excess reserves deposit facility.

**Figure 2.2:** The diagram describes the flow of funds from households (deposit and equity) to the banking sector's liabilities (debt and capital) which subsequently is channelled to the real economy (lending) and excess reserves deposit facility. Services provided by the policymaker (financial regulation and interest-on-excess-reserves) are highlighted in the diagram. Premium is the taxation that policymaker raises at the beginning of each period, in anticipation of any defaults in the banking sector, to provide government guarantees to depositors. This resource also serves as a fund to pay any positive (negative) interest expenses on excess reserves deposit facility.

Fourth, the choice of taxation is taken as given and the solution section considers the following two possible cases. When taxation is sufficiently large enough to provide full insurance on deposits. This case requires taxes to be equal to deposits (plus promised interest) less the reserves (plus interests) such that any resulting uncovered deposits within the banking sector can be covered by the taxes and reserves, for example, in an extreme case when the entire loan section of the banking sector is eliminated due to an adverse large shock. However, when taxation is insufficient to cover deposits in real terms, the regulator can offer only partial insurance on deposits.

Figure (??) illustrates the role of the financial sector within the financial system. The premium (taxes) are raised from the households to finance deposit insurance. The regulator also is in charge of two policies across banking system's balance sheet that interact with each other. The following section describes the relationship between the risk-weighted capital regulation and IOER in details.



banking sector can be covered by the taxes and reserves, for example, in an extreme case when the entire loan section of the banking sector is eliminated due to an adverse large shock. However, when taxation is insufficient to cover deposits in real terms, the regulator can offer only partial insurance on deposits.[28] Figure (2.2) illustrates the role of the financial sector within the financial system. The premium (taxes) are raised from the households to finance deposit insurance. The regulator also is in charge of two policies across banking system's balance sheet that interact with each other. The following section describes the relationship between the risk-weighted capital regulation and IOER in details.

## 2.4 Laissez-faire Intermediation

A welfare analysis based on the laissez-faire allocations provides a framework to measure social the costs of distortions. Section (2.4.1) describes the optimal behaviour of the households who are the providers of financing to the financial sector, and Section (2.4.2) discusses the optimal decisions of the banking sector to raise funds from the households and channelling them the ultimate borrowers.

[28]The model in this context assumes that deposit insurance is financed endogenously from the taxation. Nonetheless, taxes are taken as an exogenous policy. The size of optimal deposit insurance matters for welfare analysis since an under-funded deposit insurance is unable to provide compensation for depositors in full when failures are widespread. Nonetheless, putting aside large quantities of funds in anticipation of unlikely widespread failures is socially costly. Recent studies (**?**, **?**) review this question and provide an optimal deposit insurance funding level that is socially desirable.

### 2.4.1 Supply of Financing

The households maximize the expected utility of future consumption stream subject to the intertemporal budget constraint. At each date-$t$, the households choose the optimal consumption and portfolio choice. The first order condition with respect to consumption yields the following Euler equation,

$$1 = \mathbb{E}_t \left[ M_{t,t+1} R_{W,t+1} \right] \tag{2.4.1}$$

where

$$M_{t,t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\rho \frac{\gamma-1}{1-\rho}} \frac{V_{t+1}}{[\mathbb{E} V_{t+1}^{1-\theta}]^{\frac{1}{1-\theta}}} \tag{2.4.2}$$

denotes household's stochastic discount factor. Return on household's wealth includes both the equity and deposit returns in the solvency state and the deposit income only in the default state. Consumption-saving policy function is constant over time when the stochastic process governing equity return is i.i.d. therefore I conjecture that the consumption policy function $C_t = (1 - \varphi) R_{W,t} W_t$ solves the intertemporal problem as a special case without history-dependence[29] where $\varphi$ is the marginal propensity to save (MPS). Solving for the value of MPS gives the following investment-to-wealth ratio in logarithmic units:

$$\log(\mathrm{MPS}_{t+1}) = \psi \log(\beta) + \frac{1 - \psi^{-1}}{\psi^{-1}} \left[ \mathbb{E}_t r_{W,t+1}(\theta_{t+1}) + \frac{1}{2}(1 - \gamma)\sigma_{r_W}^2 \right] \tag{2.4.3}$$

This ratio is positively related to investor's subjective discount factor or impatience parameter $\beta$, such that higher patience implies higher saving if $\psi < 1$ and $\gamma > 1$. The first order condition with respect to portfolio choice $\theta_{t+1}$ is given by:

$$\theta_{t+1}^* = \underbrace{\frac{\mathbb{E}_t \log R_{E,t+1} - \log R_{D,t+1} + \sigma_E^2/2}{\gamma \sigma_E^2}}_{\text{Merton's myopic demand}} + \underbrace{\frac{1}{\gamma \sigma_E^2} \log \Phi(R_{E,t+1} > 0)}_{\text{default disincentive}} \tag{2.4.4}$$

The first term on the right-hand-side describes Merton's (rational) myopic allocation to risky asset.[30] The second term denoted by $\Phi(.)$ characterizes the role of endogenous defaults and is measured by the probability of solvency of the underlying risky asset issuer that

[29] The conjecture follows the results in **?** and **?**.

[30] Investment in risky asset does not depend on elasticity of intertemporal substitution when the shock to economy follows an i.i.d. process.

the household holds which appears in logarithmic units. This term is a negative factor to lower household's investment when defaults are possible. However, as the likelihood of solvency increases, the demand for the risky asset increases. In the limiting case when the underlying issuer is solvent in all states, the solvency probability ($\Phi(.)$) approaches one, and demand attributed by the default disincentive ($\log \Phi(.)$) approaches zero simplifying the household's demand to that of the Merton's model when a default is ruled out.

Given the deposit rate and the price of the equity, the optimal total investment (2.4.3) together with (2.4.4) fully characterize household's decisions to supply funds to the banking sector in the form of deposit and equity,

$$\begin{aligned} D_{t+1}(Q_{D,t}, P_{E,t}) &= \left[ \text{MPS}_{t+1}(\theta^*_{t+1}) \times (W_t - C^*_t) \right] \times (1 - \theta^*_{t+1}) \\ E_{t+1}(Q_{D,t}, P_{E,t}) &= \left[ \text{MPS}_{t+1}(\theta^*_{t+1}) \times (W_t - C^*_t) \right] \times \theta^*_{t+1} \end{aligned}$$

where the supply of funds to the deposit market increases in deposit rate but decreases in equity return. Conversely, equity investment falls as the price of equity increases or when the deposit rate increases.

### 2.4.2 Demands for Financing

The risk-neutral expected present value problem in (2.3.9) indicates that bank's funding and asset allocation decisions affect the following two channels.[31] First, bank considers the cost of capital when raising funds from the capital markets in order to maximize its profit. Second, allocation of funds to loans increases bank's cashflow since the expected loan rate is above the IOER. However, a high loan-to-assets ratio or a low equity-to-assets ratio decrease the possibility of remaining solvent which lower bank's profit through the expectation channel. Approximating the problem in (2.3.9) to separate the expectation (probability) channel from the (discounted) dividend channel gives,[32]

$$\max_{\theta_{t+1}, A_{t+1}, \omega_{t+1}} \underbrace{\Phi\left[\lambda\left(R_{b,t+1}\right)\right]}_{\text{Probability Channel}} \times \underbrace{\mathbb{E}_t\left[M_{t,t+1} div_{t+1}\right]}_{\text{Discounted Dividend Channel}} \tag{2.4.5}$$

[31] Subject to balance sheet constraint (2.3.10) and boundary constraints (2.3.12).
[32] Appendix 2.9.1

where the first term quantifies the explicit probability of solvency and the second term quantifies the discounted dividend.[33] Because the aggregate shock to the bank borrowers is log-normally distributed, the quantiles over the loan rate is re-arranged as the following such that probability channel in expression (2.4.2) is a standard Normal accumulative distribution function,

$$\lambda(R_{b,t+1}) \quad = \quad \frac{\mu_z + \sigma_z^2 - \log(R_{b,t+1})}{\sigma_z} \tag{2.4.6}$$

henceforth $\lambda_{t+1}$, is associated with log-normally distributed loan rate threshold $R_{b,t+1}$. First, because $R_{b,t+1}$ is weakly decreasing in $\eta_{t+1}$ (ceteris paribus), then $\Phi(\lambda_{t+1})$ is weakly increasing in $\eta_{t+1}$ indicating that a higher equity-to-assets ratio increases the probability of solvency. This is because a higher equity-to-assets ratio lowers the solvency threshold $R_{b,t+1}$ which corresponds to a lower standardized quantile $\lambda(.)$.[34] Second, because $R_{b,t+1}$ is weakly increasing in $\omega_{t+1}$ (ceteris paribus), then $\Phi(\lambda_{t+1})$ is weakly decreasing in $\omega_{t+1}$ indicating higher loan-to-assets ratio lowers the probability of solvency.

The second term in (2.4.2) can be re-arranged as,

$$M_{t,t+1}div_{t+1} \quad = \quad M_{t,t+1}\Bigg[\underbrace{\frac{1-\omega_{t+1}}{Q_{X,t}}A_{t+1}}_{\text{Reserves plus IOR}} + \underbrace{\omega_{t+1}z_{t+1}A_{t+1}}_{\text{Loan plus interest}} - \underbrace{\frac{1-\eta_{t+1}}{Q_{D,t}}A_{t+1}}_{\text{Deposit Financing}} - \underbrace{\frac{\eta_{t+1}}{P_{E,t}}A_{t+1}}_{\text{Equity Investment}}\Bigg] \tag{2.4.7}$$

where $Q_{D,t} = 1/R_{D,t+1}$ and $Q_{X,t} = 1/R_{X,t+1}$ are the prices of deposits and reserves, respectively. The constant return to scale technology implies that $A_{t+1}$ does not affect the probability channel and the risk-neural property implies that balance sheet size is linear in the dividend value.

**First-Order-Condition (Balance Sheet Size)** The first order condition of bank problem with respect to $A_{t+1}$ is given by,

$$0 \quad = \quad \left[\frac{\partial}{\partial A_{t+1}}\Phi(\lambda_{t+1})\right].\mathbb{E}_t\left[M_{t,t+1}div_{t+1}\right] + \Phi(\lambda_{t+1}).\frac{\partial}{\partial A_{t+1}}\mathbb{E}_t\left[M_{t,t+1}div_{t+1}\right] \tag{2.4.8}$$

decomposition in (2.4.2) results in the product rule in the first order condition above that shows marginal changes in the balance sheet size leads to marginal changes in the present

[33]The separation between the probability and cashflow channels simplifies the first-order conditions discussed the sections below as the dividend value is always non-negative over the solvency region, more specifically, a max[.] operator that ensures that dividend value remains non-negative can be removed to obtain derivatives that are continuous in bank's decisions.

[34]Both the CDF and quantiles functions, $\Phi(.)$ and $\lambda(.)$, are strictly monotonic in their arguments.

value of dividend, keeping probability of solvency constant, and marginal changes in probability of solvency while keeping the dividend channel constant. Re-arranging (2.4.2) in logarithmic units gives:

$$\frac{\partial}{\partial A_{t+1}} \log \Phi\left(\lambda_{t+1}\right) = \frac{\partial}{\partial A_{t+1}} \log \mathbb{E}_t \left[ M_{t,t+1} div_{t+1} \right] \tag{2.4.9}$$

an optimal balance sheet size decision $A^*(\eta_{t+1}, \omega_{t+1}, P_{E,t}, Q_{D,t}; Q_{X,t})$ by bank that solves problem (2.4.2) trades off percentage changes in probability of solvency[35] $\%\Delta_A \Phi(.)$, against percentage change in expected present value of dividend $\%\Delta_A \mathbb{E}_t[M_{t,t+1} div_{t+1}]$.

First, the probability channel always motivates the bank to choose a smaller balance sheet size due to decreasing return to scale feature of the loan section. This is indicated by the sign of the term $\%\Delta_A \Phi(.)$ that is always negative for any balance sheet size. As the bank increases its balance sheet size, the marginal loan rate falls which reduces its ability to meet deposit expenses. Further, $\%\Delta_A \Phi(.)$ is increasing in price of deposit and price of equity because higher funding prices lower cost of financing, for example, when the bank is able to raise debt through deposits at a lower deposit rate then it faces a higher $\%\Delta_A \Phi(.)$ which indicates that the balance sheet size can increase on the margin.

Similarly, when the degree of decreasing return to scale ($\alpha$) falls, the probability channel become a stronger motivation to decrease balance sheet size because a lower $\alpha$ reduces marginal loan rate. As a special case when $\alpha = 1$ the probability channel becomes irrelevant to bank's decision making because the choice of balance sheet is independent of marginal return from loan section. In this special case the first order condition with respect to size only interacts with the dividend channel and the probability of solvency remains constant for any choice of size. This indicates that the solvency is only driven by the composition of components inside the balance sheet and not the size itself and therefore any size is therefore optimal. More formally, the expectation operator[36] on the right hand size of equation (2.5.2) does not depend on endogenous variables and that the bank optimal decisions takes $M_{t,t+1}$ as given then,

$$0 = \Phi\left[\lambda\left(R_{b,t+1}\right)\right] \mathbb{E}_t \left[ M_{t,t+1} \frac{\partial}{\partial A_{t+1}} div_{t+1} \right] \tag{2.4.10}$$

Since probability of solvency is always strictly positive because for any equity-to-assets and

[35] A positive (negative) but constant $\%\Delta_A \Phi(.)$ indicates that probability of solvency increases (decreases) at a fixed rate, and when $\%\Delta_A \Phi(.)$ is zero then probability of solvency remains fixed.

[36] Integral boundaries are the support for random variable over $[0, \infty)$.

loan-to-assets ratios the bank can always remain solvent for an arbitrarily large loan rate outcome, then:

$$0 = \mathbb{E}_t\left[M_{t,t+1}\frac{\partial}{\partial A_{t+1}}div_{t+1}\right] \tag{2.4.11}$$

which results in the following first order condition that indicates, on the margin, the expected present value of cost of financing should be equal to the expected present value of one unit of investment return from bank's portfolio,

$$\mathbb{E}_t\left[M_{t+1}\left(\frac{1-\omega_{t+1}}{Q_{X,t}} + \omega_{t+1}z_{t+1}\right)\right] = \mathbb{E}_t\left[M_{t+1}\left(\frac{1-\eta_{t+1}}{Q_{D,t}} + \frac{\eta_{t+1}}{P_{E,t}}\right)\right] \tag{2.4.12}$$

the balance sheet size is always at its optimum when PV of financing cost equals PV of portfolio return. When, however, the PV of financing cost is greater than that of the portfolio return, the bank chooses a balance sheet size equal to zero and when the PV of financing cost is lower than that of the portfolio return the bank chooses a size that grows without bounds. Equilibrium mechanism, however, specifies that equation (2.4.12) must hold with equality which then establishes a condition between the prices of deposits, equity and reserves (and moments of loan).

In a more general case when $\alpha \in (0,1)$, right-hand-side of equation (2.5.2) summarizes the effect of dividend channel with the term $\%\Delta_A\mathbb{E}_t[M_{t,t+1}div_{t+1}]$. Specifically, this term is monogenically decreasing in size because as the balance sheet grows (absent probability channel) lower marginal rate from loan section reduces the expected value of profit in resent value terms. The term $\%\Delta_A\mathbb{E}_t[M_{t,t+1}div_{t+1}]$ is very large when size is small and begins to fall as the size increases. When the marginal loan rate, together with bank's income from reserves become equal to cost of financing then $\%\Delta_A\mathbb{E}_t[M_{t,t+1}div_{t+1}]$ is zero which corresponds to the maximum present value of bank profit. Any further increase in the size beyond this limit amounts to a negative expected profit.

Further, the bank faces lower cost of financing when price of deposit and equity increase which accordingly enable the bank to increase the balance sheet size that is associated with a lower marginal loan rate. In a special case, when $\alpha = 1$ the dividend value becomes linear in size which implies that the bank faces an indeterminate choice with respect to size. In this case, the expected return on bank portfolio must be equal to the expect cost of financing, otherwise the optimal size increases without bound when investing in portfolio is always marginally more profitable than marginal cost of financing, or the size is zero when expected portfolio return is lower than cost of financing.

The solution to the first-order-condition (2.5.2) is a unique choice of balance sheet that equates percentage change in probability of solvency and percentage change in expected dividend value. When $\%\Delta_A \Phi(.) < \%\Delta_A \mathbb{E}_t[M_{t,t+1} div_{t+1}]$ bank is able to increase the size to obtain more profit at the expense of lowering the probability of solvency. When $\%\Delta_A \Phi(.) > \%\Delta_A \mathbb{E}_t[M_{t,t+1} div_{t+1}]$ then the balance sheet must shrink such that the solvency increases at the expense of lower dividend. Since $\%\Delta_A \Phi(.)$ is always negative and $\%\Delta_A \mathbb{E}_t[M_{t,t+1} div_{t+1}]$ is monotonically decreasing in size, the optimal balance sheet size in a general case when $\alpha \in (0,1)$ is always smaller than the case when $\alpha = 1$.

Before discussing the optimal capital structure choice it is worth examining the relationship between optimal size and any funding composition on the liabilities side. Higher choice of equity-to-asset ratio $\eta_{t+1}$ increases bank's ability to withstand more adverse shock outcomes thus $\%\Delta_A \Phi(.)$ is increasing in $\eta_{t+1}$ which indicates that the bank can increase its balance sheet size when its equity-to-assets ratio increases (ceteris paribus).

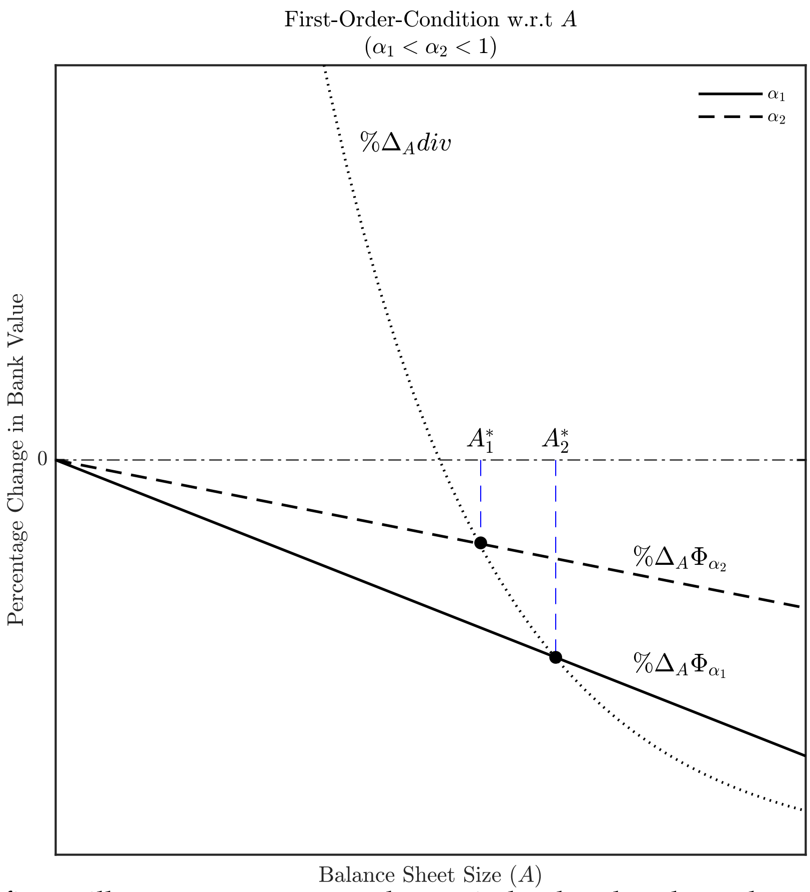



**Figure 2.3:** This figure illustrates percentage change in bank value through cashflow and solvency components when balance sheet size changes. The dotted lines show that as balance sheet size grows, bank value increases at a decreasing rate when $\%\Delta_A div > 0$. When $\%\Delta_A div = 0$ increasing balance sheet size amounts to no changes in cashflow channel. The solid line shows the solvency effect of increasing bank balance sheet size on its value

**First-Order-Condition (Capital Structure)** The first order condition with respect to capital structure is given by,

$$0 = \left[\frac{\partial}{\partial \eta_{t+1}} \Phi\left(\lambda_{t+1}\right)\right] . \mathbb{E}_t \left[M_{t,t+1} div_{t+1}\right] + \Phi\left(\lambda_{t+1}\right) . \frac{\partial}{\partial \eta_{t+1}} \mathbb{E}_t \left[M_{t,t+1} div_{t+1}\right] \quad (2.4.13)$$

using the decomposition in (2.4.2), the expression above is re-arranged to percentage changes as the following,

$$\frac{\partial}{\partial \eta_{t+1}} \log \Phi\left(\lambda_{t+1}\right) = \frac{\partial}{\partial \eta_{t+1}} \log \mathbb{E}_t \left[M_{t,t+1} div_{t+1}\right] \quad (2.4.14)$$

where similar to the previous part left-hand-side summarizes percentage change in probability channel due to changes in equity-to-asset ratio $\%\Delta_\eta \Phi(.)$. As the bank increases $\eta_{t+1}$, probability of solvency increases because higher equity-to-assets ratio increases bank's ability to withstand adverse shock outcomes. Formally, this effect is captured by the sign of the term $\%\Delta_\eta \Phi(.)$ that is always positive for any choice of $\eta_{t+1}$. Further, increasing equity-to-assets ratio monotonically improves the chance of solvency, however, when the bank is over-capitalised[37] the marginal gain in probability of solvency is very small and defaults are very rare. This is reflected by the slope of $\%\Delta_\eta \Phi(.)$ which is decreasing in $\eta_{t+1}$, specifically, when $\eta_{t+1}$ is very small, the percentage change in probability of solvency is large because each additional unit of equity can considerably lower defaults.

As $\eta_{t+1}$ increases, $\%\Delta_\eta \Phi(.)$ decreases upto the point at which $\%\Delta_\eta \Phi(.)$ becomes very close to zero showing that the probability of solvency approaches one.[38] Increasing equity-to-assets ratio beyond this limit has no impact on the solvency channel and as a result $\%\Delta_\eta \Phi(.)$ is weakly decreasing in $\eta_{t+1}$. Furthermore, $\%\Delta_\eta \Phi(.)$ is highly dependant on the price of deposits as the end-of-period interest expenses is an important determinant whether the bank remains solvent. Thus $\%\Delta_\eta \Phi(.)$ is decreasing in the price of deposit because the bank is able to withstand relatively more adverse shock when deposit interest expenses fall. Interestingly, the term $\%\Delta_\eta \Phi(.)$ is independent of the price of equity because defaults is only driven by debt contracts.

The right-hand-side of equation (2.4.14) summarizes the effect of capital structure choice

[37] Over-capitalization is a relative term with respect to loan-to-assets ratio discussed in the next subsection. However, it is innocuous in to assume a bank is over-capitalised when its equity-to-assets and loan-to-assets ratios are close to each other reflecting a bank that hold enough equity to withstand a very large adverse shock to loans and remain solvent.

[38] In this case, the break-even threshold $R_{b,t+1}$ is equal to zero indicating that there is no possible loan outcome that sets bank's total assets below its total liabilities. Note that when $R_{b,t+1}$ becomes very small (net ex-post loan rate is $-100\%$ i.e. all of bank loan section disappears in the extreme case) then $\lim_{R_{b,t+1} \to 0} \lambda(R_{b,t+1}) \to \infty$ and the associated CDF is equal to one in the limit.

on expected present value of bank profit. In particular, $\%\Delta_\eta \mathbb{E}_t[M_{t,t+1} div_{t+1}]$ is negative and monotonically decreasing[39] in $\eta_{t+1}$ when $P_{E,t} < Q_{D,t}$ as the bank considers equity more expensive relative to deposits due to its riskiness.

When $\%\Delta_\eta \Phi(.) < \%\Delta_\eta \mathbb{E}_t[M_{t,t+1} div_{t+1}]$ is a driver to increase $\eta_{t+1}$ which results in lower expected present value of dividend but increases the probability of solvency. When $\%\Delta_\eta \Phi(.) = \%\Delta_\eta \mathbb{E}_t[M_{t,t+1} div_{t+1}]$ the bank balances the marginal contribution of equity to solvency against expected dividend value. The marginal contribution of equity to expected economic profit through probability channel is a factor that bids up equity price against deposits price from bank's perspective. As the equity becomes mores scarce, the bank is willing to accept lower price today since equity's marginal probability contribution is very high. Solving equation (2.4.14) for $\eta^*_{t+1}$ gives,

$$\eta^*_{t+1} \quad = \quad \eta^*(A_{t+1}, \omega_{t+1}, P_{E,t}, Q_{D,t}; Q_{X,t}) \tag{2.4.15}$$

which is bank's optimal capital structure for any $Q_{D,t}$ and $P_{E,t}$. Particularly, $\eta^*_{t+1}$ specifies that when the price of equity at date-$t$ increases (ceteris paribus), bank increases its demand for equity financing because it is able to raise more funding per share. Conversely, when the price of deposit increases (ceteris paribus) the bank lowers $\eta^*_{t+1}$ as equity becomes relatively more expensive than deposit financing and bank shifts its liabilities towards more debt.[40] Bank's funding decision is fully characterized by equations (2.5.2) and (2.4.14) which are solved for in equilibrium for deposit and equity prices in the following subsection.

**First-Order-Condition (Asset Allocation)** The first order condition with respect to capital structure is given by,

$$0 \quad = \quad \left[\frac{\partial}{\partial \omega_{t+1}} \Phi\left(\lambda_{t+1}\right)\right] . \, \mathbb{E}_t \left[M_{t,t+1} div_{t+1}\right] + \Phi\left(\lambda_{t+1}\right) . \frac{\partial}{\partial \omega_{t+1}} \mathbb{E}_t \left[M_{t,t+1} div_{t+1}\right] \tag{2.4.16}$$

using the decomposition in (2.4.2), the expression above is re-arranged as the following,

$$\frac{\partial}{\partial \omega_{t+1}} \log \Phi\left(\lambda_{t+1}\right) \quad = \quad \frac{\partial}{\partial \omega_{t+1}} \log \mathbb{E}_t \left[M_{t,t+1} div_{t+1}\right] \tag{2.4.17}$$

[39]The assumption $P_{E,t} < Q_{D,t}$ relies on equilibrium outcome discussed in the subsequent sections but since suppliers of funding are risk-averse, then it is reasonable to restrict the discussion to cases in which price of equity is always below the price of deposits.

[40]In an extreme case when the deposits (equity) price is very high, the bank finds it optimal to raise more debt (equity) even outside $\eta^*_{t+1} \in [0, 1]$ interval. These cases are discussed later and eliminated as the funding composition can include zero equity at the very least.

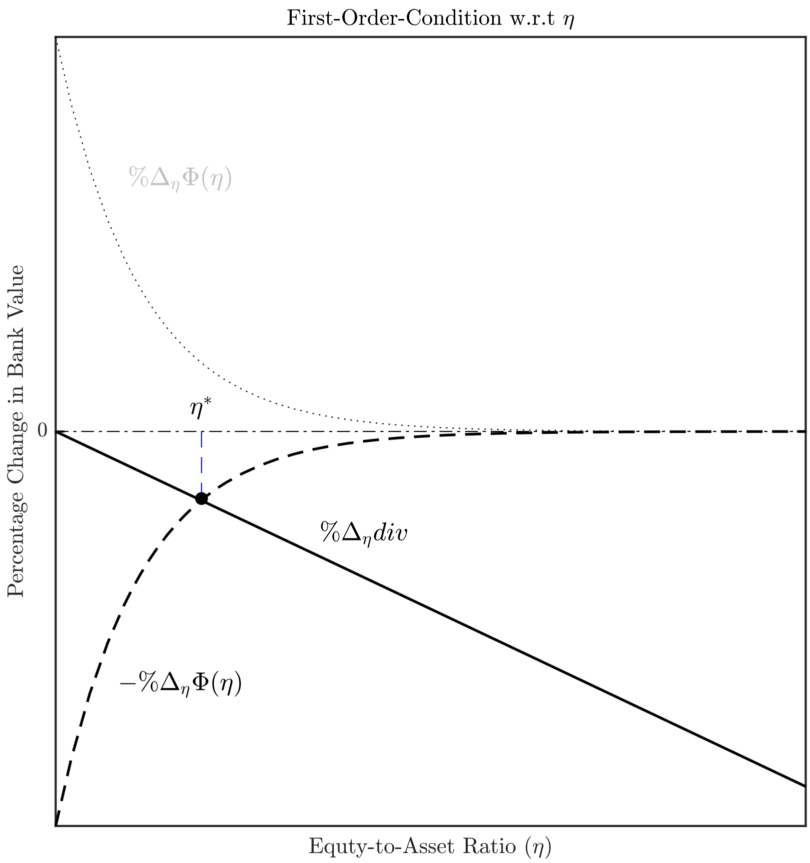



**Figure 2.4:** The figure illustrates percentage change in bank's value through cashflow and probability components. The solid line illustrates that bank's value falls when bank raises further financing through equity when equity is more costly than debt financing. The dashed line illustrates (times a negative to depicts first-order-condition) that bank's value increases through higher likelihood of solvency but at a decreasing rate because each additional unit of equity provide lower marginal contribution to solvency likelihood.

the left-hand-side summarizes percentage change in probability channel due to changes in loan-to-asset ratio $\%\Delta_\omega \Phi(.)$. As the bank increases $\omega_{t+1}$ probability of solvency decreases because higher loan-to-assets ratio increases exposure to shock outcomes and lowers bank's ability to withstand adverse shock outcomes. Formally, this effect is captured by the sign of term $\%\Delta_\omega \Phi(.)$ that is always negative for any choice of $\omega_{t+1}$. Further, increasing loan-to-asset ratio monotonically worsens chance of solvency however, when the bank is over-capitalised the marginal gain in probability of solvency is very small and defaults are very rare. This is reflected by $\%\Delta_\omega \Phi(.) = 0$ when $\omega_{t+1} \leq \eta_{t+1}$. As $\omega_{t+1}$ increases, $\%\Delta_\omega \Phi(.)$ increases monotonically reflecting growing chance of default due further exposure to aggregate shock.

The right-hand-side of equation (2.4.17) summarizes the effect of asset allocation on expected present value of bank profit. When $\%\Delta_\omega \Phi(.) < \%\Delta_\omega \mathbb{E}_t[M_{t,t+1} div_{t+1}]$ the bank increases $\omega_{t+1}$ which results in lower expect present value of dividend at the expense of increasing the probability of default. When $\%\Delta_\omega \Phi(.) = \%\Delta_\omega \mathbb{E}_t[M_{t,t+1} div_{t+1}]$ the bank balances the marginal contribution of asset allocation (to loan) to solvency against expected dividend. Solving equation (2.4.17) for $\omega^*_{t+1}$ gives,

$$\omega^*_{t+1} \quad = \quad \omega^*(A_{t+1}, \eta_{t+1}, P_{E,t}, Q_{D,t}; Q_{X,t}) \tag{2.4.18}$$

which is bank's optimal asset allocation choice for any $Q_{D,t}$ and $P_{E,t}$.

### 2.4.3 Laissez-faire Equilibrium

Market clearing conditions on the deposits and equity markets establish the following equilibrium conditions:

$$\underbrace{D_{t+1}(Q_{D,t}, P_{E,t}; Q_{X,t}, \omega_{t+1})}_{\text{Supply of Capital (household deposits)}} \quad = \quad \underbrace{\mathcal{D}_{t+1}(Q_{D,t}, P_{E,t}; Q_{X,t}, \omega_{t+1})}_{\text{Demand for Capital (bank debt)}} \tag{2.4.19}$$

$$\underbrace{E_{t+1}(Q_{D,t}, P_{E,t}; Q_{X,t}, \omega_{t+1})}_{\text{Supply of Capital (household equity)}} \quad = \quad \underbrace{\mathcal{E}_{t+1}(Q_{D,t}, P_{E,t}; Q_{X,t}, \omega_{t+1})}_{\text{Demand for equity (bank capital)}} \tag{2.4.20}$$

This first condition (2.4.20) clears the deposits market for a specific deposit prices,[41] give any price of equity, $Q_{D,t}(P_{E,t})$. Second, the resulting market clearing deposit price $Q_{D,t}(P_{E,t})$ is solved for jointly with the equity market clearing condition for a specific price of equity, given other variables that are determined outside the funding markets.

Because the household's valuation of deposit and equity arises endogenously, the equilibrium price of equity is determined by households' preferences for earning from bank dividend against its default risk. As bank extends lending, on the one hand its share price is bid up due to higher embedded cashflow but on the other hand, increased exposure to aggregate uncertainty lowers its expected share price through default risk. When the bank is highly leveraged, each additional unit of equity provides a sizable contribution to its net worth because default risk is relatively a more important driver of its share price. As bank's capital structure comprises further equity relative to total assets, marginal contribution of equity to reduce default risk diminishes and equity's higher cost relative to debt becomes a more important consideration for its net worth.

The general equilibrium framework in this section shows that equity premium compensation to risk-averse investor falls as the equity-to-assets ratio in bank capital structure in-

[41] Equation (2.4.12) from bank's first order condition with respect to balance sheet size determines a relationship between the price of deposits in terms of price of equity.

creases. When the bank raises capital through the equity market, first, its share price falls due to a higher demand for capital because the equity investor requires compensation to forgo consumption. However, a fall in share price is less steep because a risk-averse equity investor prices lower riskiness of their investment.

## 2.5 Intermediation with Capital Regulation

The financial regulator maximizes social welfare function in (2.3.20) with respect to minimum capital requirement choice that has a bearing on liabilities of the banking sector. This regulatory policy takes bank's asset allocation decision as given to find the optimal capital requirement conditional on $\omega_{t+1}$, or henceforth the risk-weighted capital requirement $\overline{\eta}^{*}_{t+1}(\omega_{t+1})$.

**First-Order-Condition (RW-Capital Requirement)** Marginal changes in $\overline{\eta}_{t+1}(\omega_{t+1})$ gives the following FOC over the default and solvency regions, respectively:

$$0 = \underbrace{\int_{0}^{z_{b,t+1}} M_{t,t+1}\left[\frac{d\mathrm{Tr}_{t+1}}{d\overline{\eta}_{t+1}} - \frac{1}{Q_{D,t}}\right] dF}_{\text{Default Region}} + \underbrace{\int_{z_{b,t+1}}^{\infty} M_{t,t+1}\left[(1-\kappa)R_{E,t+1} - \frac{1}{Q_{D,t}}\right] dF}_{\text{Solvency Region}} + \mathcal{U} \tag{2.5.1}$$

where the first term shows the present value of marginal changes in $\overline{\eta}_{t+1}(\omega_{t+1})$ over the default region where the realization of shock is low $z_{t+1} < z_{b,t+1}$ such that banking sector's total assets valuation falls below its debt liabilities. The regulator considers that equity income to households is zero and deposits plus interest is the only financial income households earn.

Further, over the default region, regulator evaluates changes in the transfer value because bankruptcy requires the deposit insurance service to remain capable to compensate depositors for any uncovered fraction of their deposit investments which is funded from regulator's resources. Once the bank defaults, its ex-post total assets value falls further below its total liabilities due to bankruptcy cost that incurs in any default state which increases the amount that deposit insurance needs to pay to depositors to guarantee their investments in full.

The second term on the right-hand-side of equation (2.5.1) shows the present value of marginal changes due $\overline{\eta}_{t+1}(\omega_{t+1})$ over the solvency region where the households are able to receive financial income from equity and deposit investments. The transfer function remains unchanged over solvency because the bank honors its debt contracts and deposit insurance need not to intervene. The last term in equation (2.5.1) summarizes the direct welfare effect associated with bankruptcy cost exactly at the default threshold by comparing just-solvency against just-defaults outcome. Re-arranging equation (2.5.1) using the decomposition lemma discussed above gives,

$$0 = \underbrace{(1 - \overline{\eta}_{t+1} + \kappa \overline{\eta}_{t+1})}_{\text{after-purchase investment}} \int_0^{\infty} M_{t,t+1} \Big\{ \chi.(1 - \Phi(\lambda)) + \Phi(\lambda) \Big\} R_{p,t+1} dF$$

where the first term is less than one when purchasing bank equity incurs fee $1 - \kappa$ per unit of wealth, leading to lower savings. When equity purchasing is costless $\kappa = 1$, then the first term has no interaction with $\overline{\eta}_{t+1}$. The second component on the right-hand-side shows the marginal effect of capital regulation on probability of solvency through $\Phi(\lambda)$ which increases as $\overline{\eta}_{t+1}$ increases. Let $\varpi(\chi) = \chi.(1 - \Phi(\lambda)) + \Phi(\lambda)$ denote the probability effect where $\chi$ shows the ex-post liquidation proceeds ($1 - \chi$ is the proportional bankruptcy cost) that occurs over the default region. When $\chi = 1$ then $\varpi(\chi) = 1$ showing that probability component $\varpi(\chi)$ is irrelevant to regulator's decision because there is no deadweight loss associated with defaults therefore the likelihood of default region $1 - \Phi(\lambda)$ is immaterial to welfare.

As $\chi$ decreases (proportional bankruptcy cost increases) $\varpi(\chi)$ becomes smaller showing the welfare loss in regulator's value function due to deadweight loss through probability channel. When $\chi \in (0, 1)$ the regulator always is concerned with costly bankruptcies because increasing $\overline{\eta}_{t+1}$ amounts to increasing the probability of solvency that lowers its associated ex-ante distortion. As $\overline{\eta}_{t+1}$ monotonically (weakly) increases $\varpi(\chi)$, the regulator recommends higher $\overline{\eta}_{t+1}$, and in an extreme case when equity purchase is costless ($\kappa = 1$), optimal capital requirement[42] is 100%. The optimal capital RW-capital requirement trades-off social costs of equity purchase fee against social benefits of less bank failure and its associated bankruptcy cost and is given by,

$$\overline{\eta}^*_{t+1} = 1 + \frac{1 - \omega_{t+1}}{Q_{X,t}} + \varphi_0(\mu_L, \sigma).B. \left( \varphi_1(\mu_L, \sigma) - \log\left( \frac{1 - \kappa}{1 - \chi}.B \right) \right)$$

where $B = \frac{Q_{D,t}\omega^{\alpha}_{t+1}}{A^{1-\alpha}}$, $\varphi_0(\mu_L, \sigma) < 0$ and $\varphi_1(\mu_L, \sigma) > \frac{1-\kappa}{1-\chi} > 0$. The solution specifies that when equity purchase fee increases (lower $\kappa$) then $\overline{\eta}^*_{t+1}$ decreases leading to lower dead-

[42]When $\overline{\eta}_{t+1} > \omega_{t+1}$ the bank is always solvent as it owns more equity than its loans therefore increasing the capital requirement beyond this limit leads to no further welfare gains as any optimal capital requirement is associate with $\Phi(\lambda) = 1$ and $\overline{\eta}^*_{t+1}$ has multiple solutions.

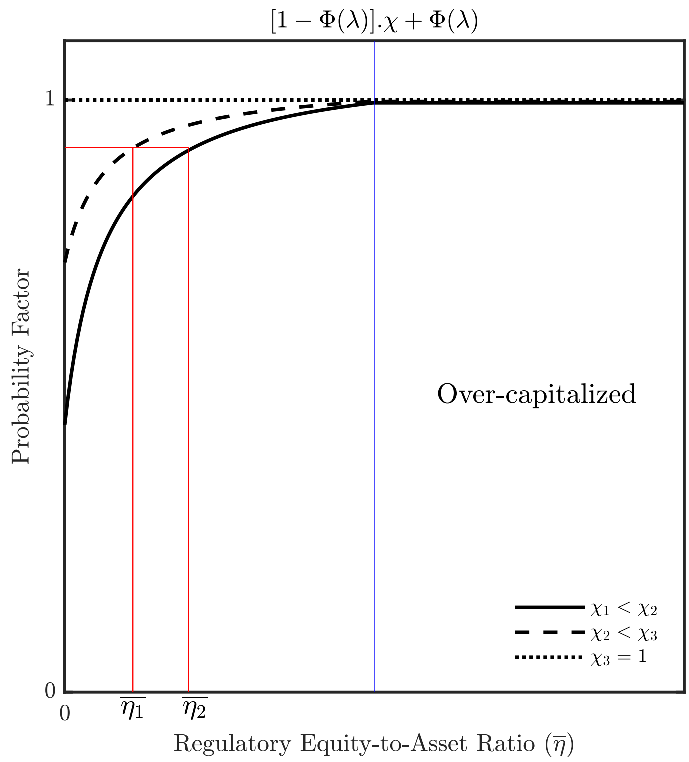



**Figure 2.5:** The probability factor $\varpi(\chi) = \chi.(1-\Phi(\lambda)) + \Phi(\lambda)$ increases when bank's capital structure includes more equity (ceteris paribus). As bankruptcy becomes more costly ($\chi < 1$) then $\varpi(\chi)$ increases more sharply when capital structure includes more equity. When bankruptcy is costless ($\chi = 1$) then changes in capital structure leads to no welfare gain through probability factor $\varpi(\chi)$.

weight loss. When bankruptcy cost increases (lower $\chi$) then $\overline{\eta}^*_{t+1}$ increases to lower the probability of default where distortion lowers the welfare. When the price of deposits $Q_{D,t}$ increase $\overline{\eta}^*_{t+1}$ decreases because the bank needs to pay lower interest payments to depositors. When the balance sheet size increases, the optimal capital requirement increases because of the decreasing return to scale on bank's loan section. When the price of reserves increases, the capital requirement increases because the bank earns lower interest income from its reserves investments. This effect becomes smaller as loan-to-assets ratio increases which lowers bank's exposure to interest income from or expenses due to reserves.

Figure (2.6) illustrates changes in optimal capital requirement for given loan allocation by bank for three different bankruptcy cost parameter values ($\chi_1 < \chi_2 < \chi_3 < 1$) and equity purchase parameter values ($\kappa_1 < \kappa_2 < \kappa_3 < 1$). In particular, the slope of curves describe the ratio $\overline{\eta}_{t+1}/\omega_{t+1}$ which is the RW-capital requirement. As bankruptcy cost decreases, slope of curves in the left diagram become steeper which show lower equity requirement per unit of loan because the regulator is less concerned with costly defaults. As the fee associated with equity purchases decreases, the slopes of curves in the right diagram become flatter showing higher equity requirement per unit of loan because the regulator is less concerned

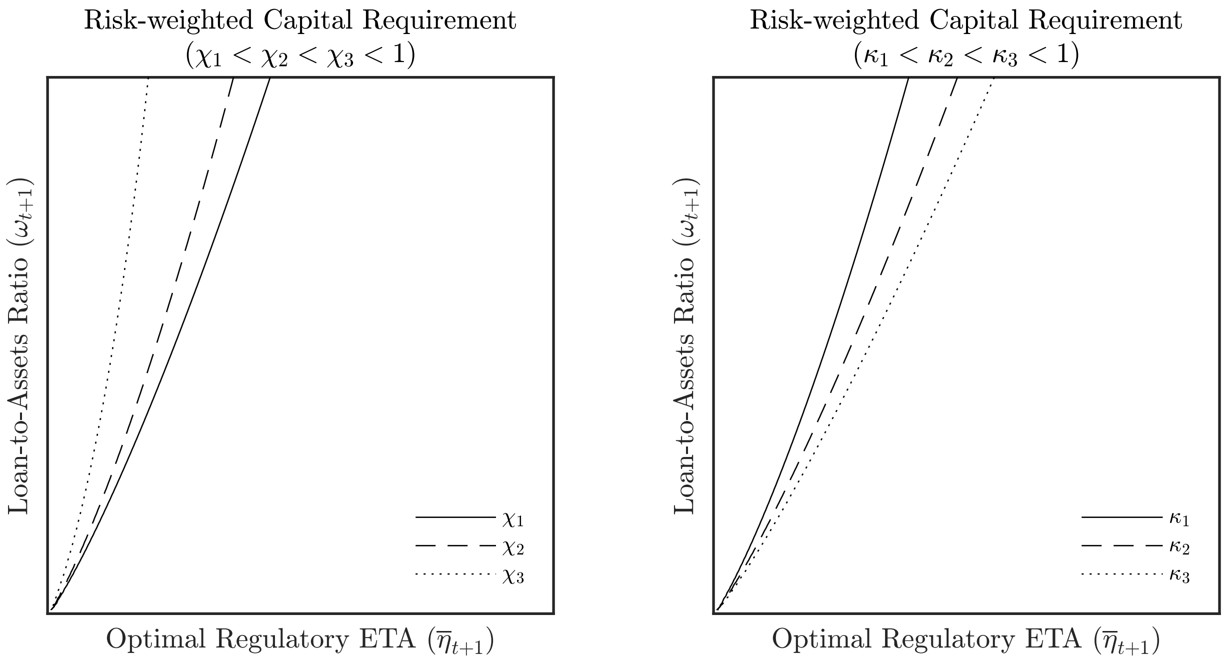



**Figure 2.6:** The slope of curves are RW-capital requirement in the space $(\overline{\eta}_{t+1}, \omega_{t+1})$. The graph illustrates changes in RW-capital requirement when bankruptcy cost parameter value changes (left) and when equity purchase fee changes (right).

with deadweight loss during equity purchases.

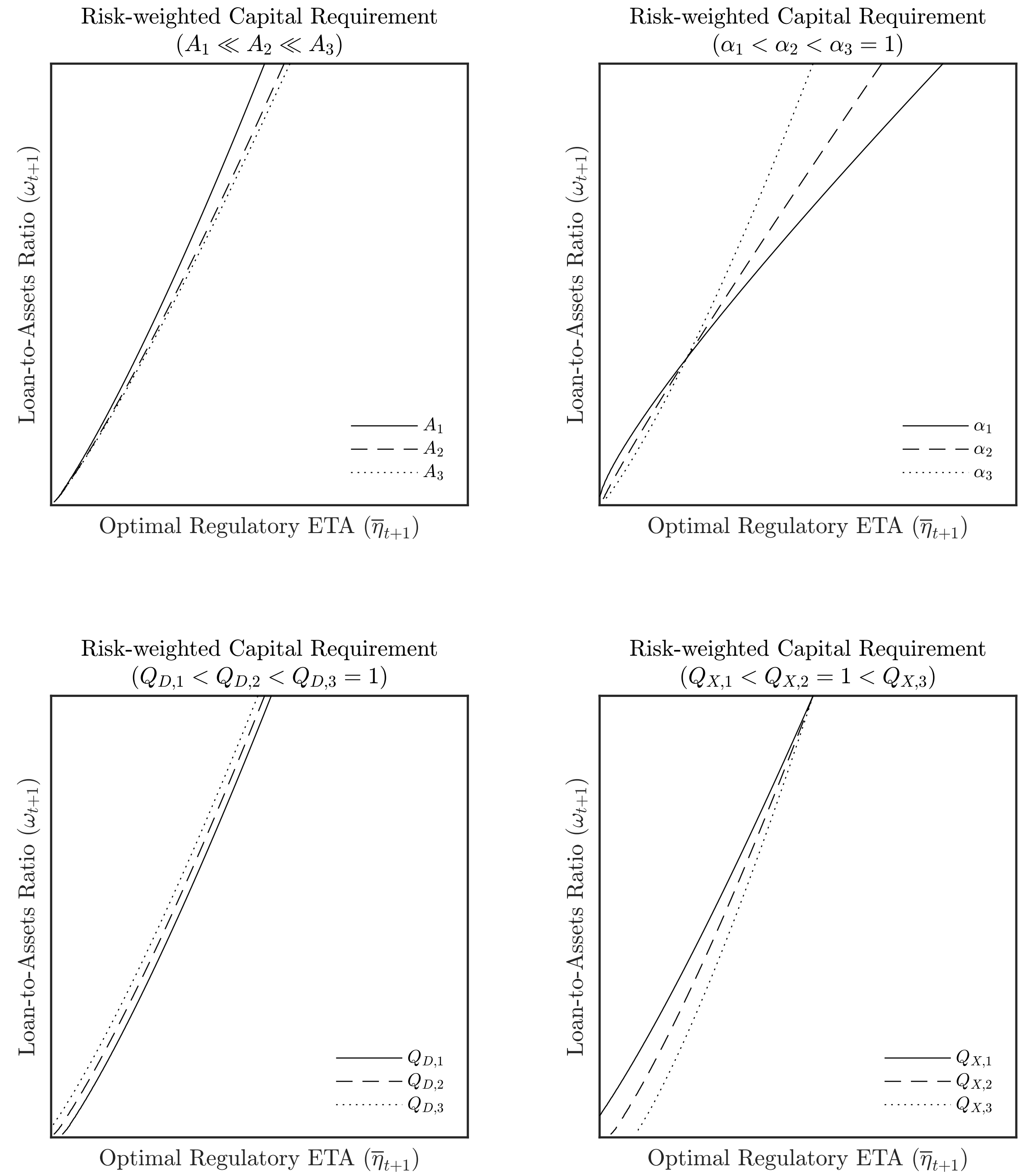
The price of equity is irrelevant to the optimal capital requirement because regulator's consideration is focused on distortions related to defaults that are determined by debtholder's contracts and not shareholders. The regulator is concerned with the welfare of the economy that includes both the debtholders and shareholders, however, the welfare improvement is achieved by reducing distortions so that households obtain higher consumption due to minimal deadweight losses.

### 2.5.1 Demands for Financing under Capital Regulation

**First-Order-Condition (Balance Sheet Size under Capital Regulation)** Given the capital requirement, choosing the balance sheet size also determines the capital structure on funding side. Substituting $\overline{\eta}^*_{t+1}$ into the dividend function and probability of solvency gives the following first order condition:

$$\frac{d}{dA_{t+1}} \log \Phi\left(\lambda_{t+1}\right) = \frac{d}{dA_{t+1}} \log \mathbb{E}_t \left[ M_{t,t+1} div_{t+1} \right] \tag{2.5.2}$$

the first order condition shows the trade offs between marginal gain in dividend against lowering probability of solvency due to lower marginal return from the loan section that is subject to decreasing return to scale. This first order condition is the same as the case




without capital regulation, however, the decision $\overline{\eta}^*_{t+1}$ is predetermined.

### 2.5.2 Equilibrium with Capital Regulation

Capital structure of the bank complies with RW-capital requirement for any balance sheet size. First, in order for the deposits and equity market to clear, the bank raises funds by choosing its balance sheet size considering the capital regulation $A^*_{t+1}(\overline{\eta}_{t+1})$ through total savings by the households $S^*_{t+1}$:

$$A^*_{t+1}(\overline{\eta}_{t+1}) \quad = \quad S^*_{t+1} \tag{2.5.3}$$

Second, in equilibrium, the portfolio choice of the households including deposits and equity must be equal to the capital structure of the bank that is predicated by RW-capital requirement,

$$\overline{\eta}_{t+1} \quad = \quad \theta^*_{t+1} \tag{2.5.4}$$

As a special case with logarithmic utility, the first market clearing condition simplifies to:

$$A^*_{t+1}(\overline{\eta}_{t+1}) \quad = \quad \underbrace{(1-\tau_{t+1}).(1-\theta^*_{t+1}+\kappa.\theta^*_{t+1}).(1-\beta).W_t}_{\text{supply of funds}} \tag{2.5.5}$$

where the first term on the right-hand-side shows the effect of equity purchase fee on lowering the total supply fund to the economy. When $\kappa = 1$ then equity purchase is costless and the supply of funds is fixed. The term $1 - \tau_{t+1}$ show household's disposable income after paying proportional taxation to the regulator.

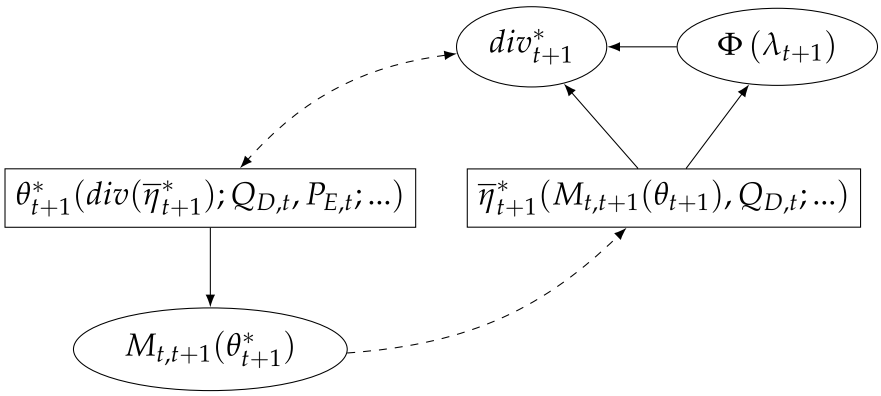



**Figure 2.7:** The diagram describes optimal schedules of the household and the bank under capital regulation where the regulator's decision to on minimum equity-to-assets ratio derives the default probability and dividend flow. Bank's decision on balance sheet size together with the regulatory ratio fully characterize bank's optimal behaviour.

## 2.6 Calibration

In the previous section, results relied on the assumption that investors have a unit elasticity of intertemporal substitution (EIS). This section extends the implications of the model to more general cases when households are only assumed to have an early resolution of uncertainty with $\gamma > 1$ and $\gamma\psi > 1$.

As households become more risk-averse, their preference to hold a larger fraction of their wealth in deposits grows. Under equilibrium with binding capital regulation, the equity-to-assets ratio of the banking sector is equal to the household's portfolio share in risky asset which remains intact as households become more risk-averse leading to the following two effects. First, the equilibrium mechanism indicates in order to clear the markets deposit rate must fall, leading to greater equity premium as a result of more risk-averse investors. Second, lower deposit rate is associated with lower interest expenses which incentivizes the bank to extend lending until the marginal reduction in its net worth due to added risk to its asset side wears off increased gain in its net worth due to enhanced solvency. However, higher lending requires the bank to support its riskier portfolio with additional equity by raising further capital from the equity market leading to equity prices to fall, further expanding the equity premium. This effect is partially dampened because as the bank becomes more capitalized, its exposure to default falls leading to lower compensation to its investors due to lower default risk.

| **Structural Parameterization** | | |
|---|---|---|
| Description | Notation | Value |
| Household Subjective Discount Factor | $\beta$ | 0.99 |
| Household Coefficient of Relative Risk-aversion | $\gamma$ | 1.00 |
| Household Elasticity of Intertemporal Substitution | $\psi$ | 1.00 |
| Bankruptcy Cost Parameter (proportional cost: $1-\chi$) | $\chi$ | 70.00% |
| Intermediation Cost Parameter (proportional cost: $1-\kappa$) | $\kappa$ | 98.50% |
| Lending Decreasing Return to Scale | $\alpha$ | 0.95 |
| Aggregate Shock (Lending) Mean Parameter | $\mu_L$ | 0.085 |
| Aggregate Shock (Lending) S.D. Parameter | $\sigma_L$ | 11.75% |
| Aggregate Shock (Lending) Expectation | $e^{\mu_L+\frac{1}{2}\sigma_L^2}$ | 0.0963 |
| Aggregate Shock (Lending) Variance | $e^{2\mu_L+\sigma_L^2}(e^{\sigma_L^2}-1)$ | 0.017 |

**Table 2.2:** Calibration Parameterization

This result describes that changes in household's risk-aversion has important implications for equilibrium prices, however, equilibrium allocations remain less affected due to frictions of capital constraints. Extended lending to the real sector is an expansionary effect, only when IOER remains above zero bound, which increases the total expected income to households. This wealth effect lowers asset prices because the stochastic discount factor of the households is negatively correlated with the aggregate wealth. Expectations of higher incomes (through their equity investment) lower the marginal utility of each unit of consumption at future dates and leads to lower valuations of bank net worth. The effect on bank equity price reverses when IOER is below zero bound because the equilibrium deposit rate is very close to zero and any further increase in risk-aversion only leads to marginal fall in deposit rate. As a result, the bank's solvency due to higher next interest expenses is followed by lower credit flow to the real sector in order for the bank to maintain its net worth valuation. This contractionary effect amounts to lower expected wealth which indicates that the marginal utility of consumption at future dates increases leading to higher bank equity price today. This mechanism provides accounts to explain equity premium that is more consistent with empirical observations despite low degrees of risk aversion.

The framework in Section (2.4) shows the aggregate saving is driven by preferences towards substitution of consumption over time. More precisely, as the EIS approaches one, households become infinitely indifferent to transfer consumption across time and hence they consume a fixed fraction of their wealth equal to $1-\beta$. When the policymaker lowers IOER, the banking sector initially extends lending which leads to expansions of the real sector. Although the equilibrium deposit rate falls as IOER decreases, expansion in the real economy indicates that the expected wealth of the households grows because their financial income from investing in bank net worth grows. This mechanism increases return on wealth, however, overall savings by the households with unit EIS remain unchanged. This arises as a special case result because of the household's infinite reluctance to substitute intertemporally which leads them to consume the annuity of value of their wealth each period.[43]

Conversely, when the household's preference to substitute consumption over time is characterized by an EIS other than unit, the total supply of saving in financial assets varies with return on wealth. When EIS is lower than the unit, households prefer to increase their consumption-wealth ratio because any additional increase in return on wealth leads to income effect to dominate the substitution effect. Subsequently, households increase their consumption today and save less which leads to the equilibrium size of the financial sector to shrink. In this context, the bank's balance sheet size is negatively affected because the overall funding through deposit and equity investments by their investors is reduced. Section (2.4.2) shows that lower balance sheet size is associated with a higher marginal return on lending to the real sector. Specifically, the decreasing return to scale assumption implies that the banking sector is able to generate a higher return on each unit of lending when the economy scales down.

[43] This result is consistent with the predictions of the recursive preferences with an intertemporal-CAPM framework (**?**, **?** and **?**.

Lowering IOER when supply of investment is driven by households who are reluctant to substitute consumption over time has the following implications. First, the equilibrium excess reserves unambiguously falls because the spread between IOER and the expected lending rate expands specifically due to the decreasing return to scale effect. This mechanism leads the banking sector to lower the share of its assets in excess reserves. On the aggregate level, the relative size of excess reserves is further reduced because of the overall lower savings in the financial sector. Second, the implications to the real sector is characterized by an expansion of extensive margin against shrinkage of the size of the financial sector. Bank's decision to increase lending trades off the loss of net worth valuation due to higher default risk against gain in valuation due to higher cashflow. However, on the margin the lending rate higher when the economy is scaled down, the default risk channel is dampened by the bank's ability to meet its debt liabilities as a result of a higher lending rate. The optimal capital requirement factors higher marginal productivity, associated with a lower size of the real sector, into account and prescribes a looser minimum equity-to-assets ratio. In equilibrium, this leads to a higher equilibrium deposit rate and lower risky asset price because bank's looser capital constraint drives the demand for debt financing upwards until the marginal gain from raising funds from deposits equates marginal loss of net worth valuation due to heightened default risk.

As households become more risk-averse, when their EIS is less than unit, their preference to hold a larger share of their financial investment in deposits grows. However, capital requirement constraint indicates that increased aversion towards risk leads to a lower equilibrium deposit rate. Higher coefficient of risk aversion, keeping EIS below one, implies that bank's cost of debt falls and lending grows because the bank is able to afford further risk on its asset side. This result contrasts the finding of theoretical models based on partial equilibrium. More precisely, in a partial equilibrium model, higher risk aversion leads to lowera return on wealth because the household's portfolio includes a larger share of risk-free asset. However, in general equilibrium, higher risk-averse investors accept a lower equilibrium deposit rate which together with capital constraints, lower the cost of debt for the bank and lead to higher return on wealth as a result of extended lending to the real sector. This indicates that although consumption-wealth ratio increases in return on wealth, increasing the attitude towards risk is associated with higher return on wealth and subsequently, aggregate saving falls when EIS is lower than unity.

Before proceeding to the numerical illustration, it is worth mentioning that when EIS is above one, the aggregate saving is driven by the substitution effect that exceeds the income effect. Particularly, in this case, higher return on wealth is followed by higher saving which expands the size of the financial sector. This result indicates that when households are willing to substitute consumption over time, higher return on wealth increases the marginal utility of consumption today as consumption becomes more expensive relative to future dates. Subsequently, aggregate saving increases and in equilibrium, bank's balance sheet expands leading to the following two implications: first, the extensive margin on excess reserves grows because the marginal loan rate falls when the financial sector expands. This result leads to extenuating implications for over-reliance on excess reserves. Alternatively, the decreasing return to scale implies that the bank is concerned with its solvency channel because, on the margin, it is less able to meet its liabilities at the end of the period and rebalances its portfolio away from lending to the real sector. Second, the extended size of the financial sector prompts the regulator to tighten the capital regulation leading to a lower equilibrium deposit rate and lower risky asset price. The social welfare function is increas-

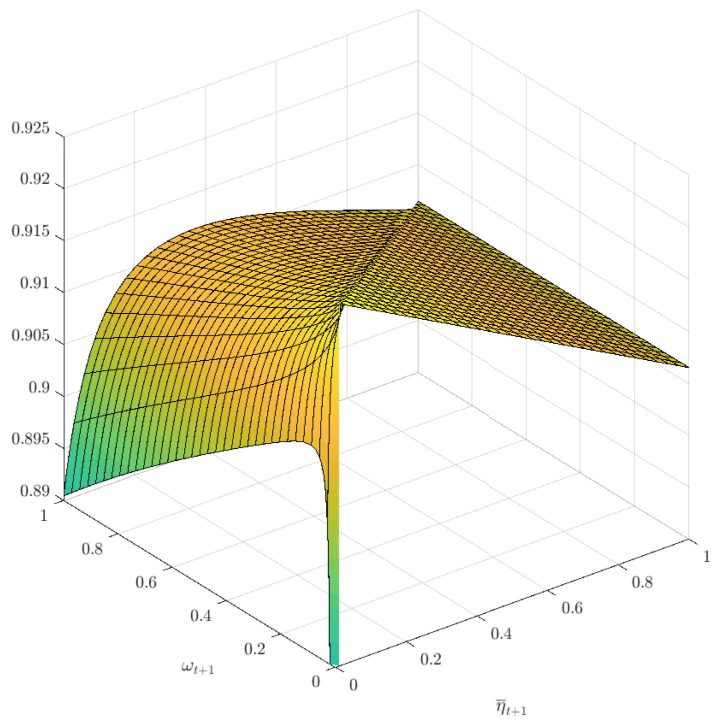



**Figure 2.8:** The surface illustrates the social welfare value over capital regulation ($\overline{\eta}_{t+1}$) and asset allocation ($\omega_{t+1}$) space. A kink along the diagonal is generated by the solvency condition at which the bank's net worth remains at zero regardless of severity of a default. The welfare is linear in capital regulation decision and asset allocation when $\overline{\eta}_{t+1} \geq \omega_{t+1}$ because when bank's exposure to risk is fully coverable by its equity then higher capital structure provides no further welfare gain and the variations in welfare is driven by equity intermediation cost.

ing in capital regulation decision ($\overline{\eta}_{t+1}$) when the marginal social gains in higher bank capitalization exceeds marginal costs of equity intermediation. Particularly, the proportional cost associated with raising capital through equity market due to costly intermediation increases in size when the regulator sets a higher capital constraint. Given each choice of asset allocation by the bank $\omega_{t+1}$, regulator's optimal capital regulation maximizes the social welfare $\overline{\eta}^{*}_{t+1}(\omega_{t+1})$. Figure (2.8) illustrate the social welfare function with the following parameterization,[44] $\{\beta, \gamma, \psi, \kappa, \chi, \alpha, \mu_L, \sigma_L\} = \{0.99, 1, 1, 0.70, 0.985, 0.95, 0.085, 0.1175\}$. The social welfare surface exhibits a linear characterization over $(\overline{\eta}_{t+1}, \omega_{t+1})$ when $\overline{\eta}_{t+1} \geq \omega_{t+1}$ because the banking sector is able to cover any adverse negative outcome to its borrowers and remain solvent as it is overcapitalized. As a result, the regulator's ability to require-

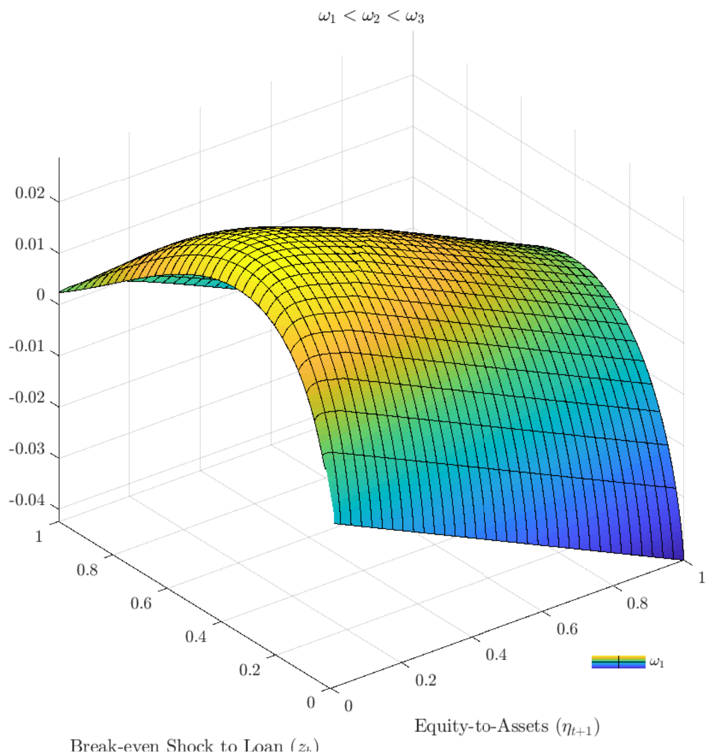



**Figure 2.9:** The surface shows bank net worth valuation over $(\eta_{t+1}, \omega_{t+1})$ space. Over the capital structure choice, the bank faces trade-off between cost of equity against gains in valuation due to more capitalization. Over the portfolio choice dimension, the bank faces trade-offs between higher return from lending against exposure to higher risk which negatively affects its net worth by its risk-averse shareholders.

ment higher equity-to-assets ratio on the liabilities of the banking sector is inconsequential to welfare as it provides no further gain. Any higher capital requirement constraint, however, incurs equity intermediation cost which leads to a proportional cost that lower social welfare along $\overline{\eta}_{t+1}$ for any given asset allocation decision. The risk-neutral bank maximizes the present value of expected future cashflow by financing its operations through deposits

[44] Parameters refer to, subjective discount factor, coefficient of relative risk aversion, elasticity of intertemporal substitution, after-purchase equity intermediation (cost: $1-\kappa$), ex-post liquidation proceeds (bankruptcy cost: $1-\chi$), and the degree of decreasing return to scale, respectively. The last two parameters $\mu_l$ and $\sigma_L$ described the log-normal distribution of the loan sector, with expected mean and variance equal to $e^{\mu_L + \frac{1}{2}\sigma_L^2}$ and $e^{2\mu_L + \sigma_L^2} \times (e^{\sigma_L^2} - 1)$, respectively.

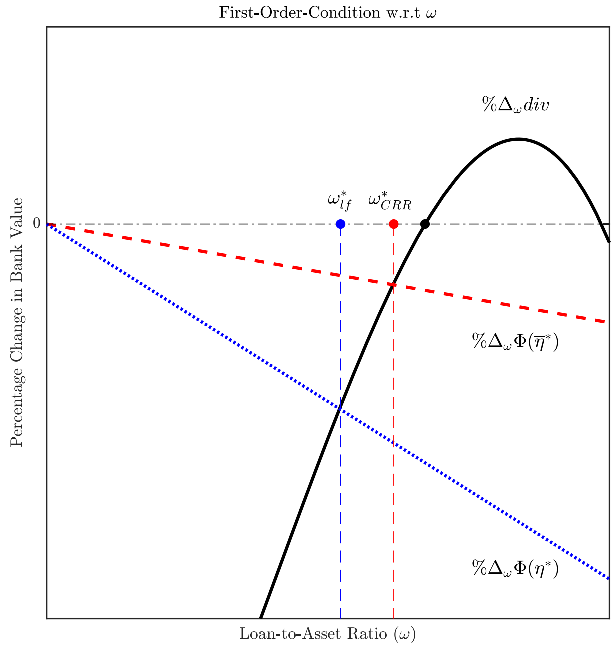
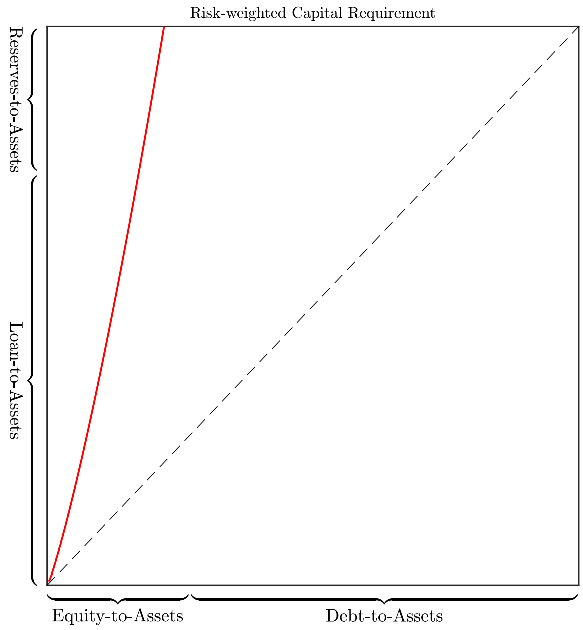



**Figure 2.10:** The figure illustrates bank's ability to extend loans when capital regulation requires the bank to hold additional equity per unit of loan. The solid line shows percentage change in bank's value function given a unit change in allocation to loan, the dotted line shows percentage change in bank's solvency when its portfolio holding of loan increases, and dashed line shows percentage change in solvency when bank complies with capital regulation.

and equity. Bank's decision on raising funding considers the implications of capital structure on its net worth valuation by its risk-averse investors who simultaneously considers cash flow and solvency. Because equity contributes to reduce bank default risk and enhance its valuation, the objective function of the bank over capital structure exhibits a concave characteristic. Specifically, the trade-off between the cost of equity against gains in valuation comprises two opposing considerations that the bank balances in order to determine its optimal capital structure. The present value problem of the bank also considers the benefits of investing a higher share of funds in loans.

However, because the stochastic discount factor is negatively correlated with the variance of bank portfolio, over-investment in loan leads to lower bank valuations through the shareholder's valuation that arises endogenously. As a result, bank decision on optimal portfolio considers the trade-offs between higher lending return against volatility of asset side that is ultimately characterized by a concave value function over portfolio choice decision. The optimal risk-weighted capital regulation evaluates social costs associated with bank failure which are not internalized by the bank. This schedule serves as a constraint that conditions the bank's lending to its capital structure.



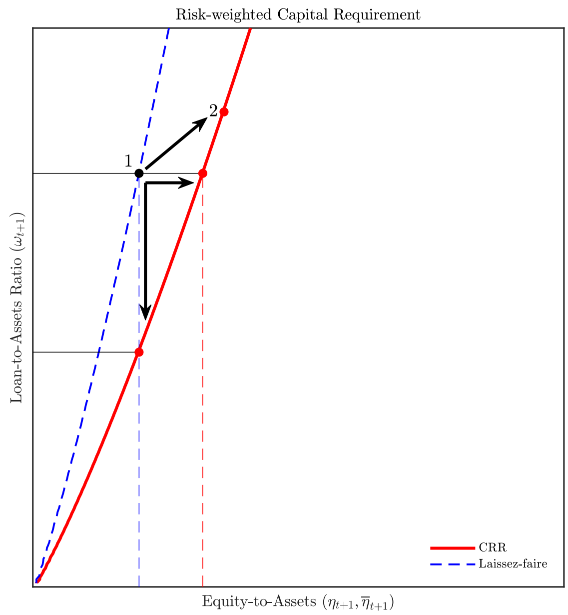




**Figure 2.11:** The left figure illustrates equilibrium deposit rate in response to given interest-on-reserves rate. Variations within higher IOERs rate is associated with changes in equilibrium deposit rate in the same direction and comparable magnitude, however, as interest-on-reserves fall, equilibrium deposit rate becomes less responsive to changes in interest-on-reserves and remains strictly positive. The figure on the right illustrates bank's portfolio rebalancing when RW-capital regulation requires the bank to hold higher equity per loans. The solid dashed line described bank's laissez-faire loan-to-equity schedule and the solid line describes regulated loan-to-assets schedule which is always toward the outer right side of unregulated schedule.

**Optimal Risk-weighted Capital Requirement (% of total assets)**

Panel A: Low Loss Given Default ($1 - \chi = 10\%$)

| $\omega_{t+1}$ | Interest-on-Excess-Reserves $(r_{X,t+1})$ in percentage points | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1.75 | 1.50 | 1.25 | 1.00 | 0.75 | 0.50 | 0.25 | 0.00 | $-0.25$ | $-0.50$ | $-0.75$ |
| 0% | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 10% | 2.32 | 2.13 | 1.95 | 1.76 | 1.57 | 1.39 | 1.20 | 1.01 | 1.19 | 1.37 | 1.57 |
| 25% | 4.64 | 4.26 | 3.89 | 3.52 | 3.14 | 2.77 | 2.40 | 2.02 | 2.38 | 2.74 | 3.14 |
| 50% | 6.96 | 6.40 | 5.84 | 5.28 | 4.72 | 4.16 | 3.60 | 3.04 | 3.57 | 4.11 | 4.72 |
| 75% | 9.27 | 8.53 | 7.78 | 7.03 | 6.29 | 5.54 | 4.79 | 4.05 | 4.76 | 5.47 | 6.29 |
| 90% | 11.59 | 10.66 | 9.73 | 8.79 | 7.86 | 6.93 | 5.99 | 5.06 | 5.95 | 6.84 | 7.86 |
| 100% | 13.91 | 12.79 | 11.67 | 10.55 | 9.43 | 8.31 | 7.19 | 6.07 | 7.14 | 8.21 | 9.43 |

Panel B: Medium Loss Given Default ($1 - \chi = 20\%$)

| $\omega_{t+1}$ | Interest-on-Excess-Reserves $(r_{X,t+1})$ in percentage points | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1.75 | 1.50 | 1.25 | 1.00 | 0.75 | 0.50 | 0.25 | 0.00 | $-0.25$ | $-0.50$ | $-0.75$ |
| 0% | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 10% | 2.51 | 2.33 | 2.15 | 1.97 | 1.79 | 1.61 | 1.44 | 1.26 | 1.46 | 1.65 | 1.83 |
| 25% | 5.01 | 4.65 | 4.30 | 3.94 | 3.58 | 3.23 | 2.87 | 2.51 | 2.93 | 3.30 | 3.66 |
| 50% | 7.52 | 6.98 | 6.45 | 5.91 | 5.38 | 4.84 | 4.31 | 3.77 | 4.39 | 4.95 | 5.49 |
| 75% | 10.02 | 9.31 | 8.59 | 7.88 | 7.17 | 6.45 | 5.74 | 5.03 | 5.85 | 6.59 | 7.31 |
| 90% | 12.53 | 11.63 | 10.74 | 9.85 | 8.96 | 8.07 | 7.18 | 6.28 | 7.32 | 8.24 | 9.14 |
| 100% | 15.03 | 13.96 | 12.89 | 11.82 | 10.75 | 9.68 | 8.61 | 7.54 | 8.78 | 9.89 | 10.97 |

Panel C: High Loss Given Default ($1 - \chi = 30\%$)

| $\omega_{t+1}$ | Interest-on-Excess-Reserves $(r_{X,t+1})$ in percentage points | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1.75 | 1.50 | 1.25 | 1.00 | 0.75 | 0.50 | 0.25 | 0.00 | $-0.25$ | $-0.50$ | $-0.75$ |
| 0% | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 10% | 2.80 | 2.61 | 2.43 | 2.24 | 2.06 | 1.87 | 1.69 | 1.51 | 1.69 | 1.88 | 2.07 |
| 25% | 5.59 | 5.22 | 4.85 | 4.48 | 4.11 | 3.74 | 3.37 | 3.01 | 3.39 | 3.76 | 4.14 |
| 50% | 8.39 | 7.83 | 7.28 | 6.72 | 6.17 | 5.61 | 5.06 | 4.52 | 5.08 | 5.65 | 6.21 |
| 75% | 11.18 | 10.44 | 9.70 | 8.96 | 8.22 | 7.48 | 6.74 | 6.02 | 6.77 | 7.53 | 8.28 |
| 90% | 13.98 | 13.05 | 12.13 | 11.20 | 10.28 | 9.35 | 8.43 | 7.53 | 8.47 | 9.41 | 10.35 |
| 100% | 16.77 | 15.66 | 14.55 | 13.44 | 12.33 | 11.22 | 10.11 | 9.03 | 10.16 | 11.29 | 12.42 |

**Table 2.3:** The table illustrates the optimal risk-weighted capital regulation set by the policymaker on the banking sector to condition its asset allocation to its capital structure. Panel (A) shows the required capital as a percentage of total assets and various IOER where the required capital ubiquitously increases when share of total asset invested in lending ($\omega_{t+1}$) increases. As IOER is lowered, the corresponding required capital decreases for any given level of asset allocation when IOER is above zero. As IOER reaches zero and marginally negative, the policymaker requires the liabilities to includes lower capital per unit of loan. Panels (B) and (C) replicate the same quantities when the bankruptcy cost and its associated deadweight losses increases leading to higher minimum capital requirement given IOER and share of funds invested in risky asset.

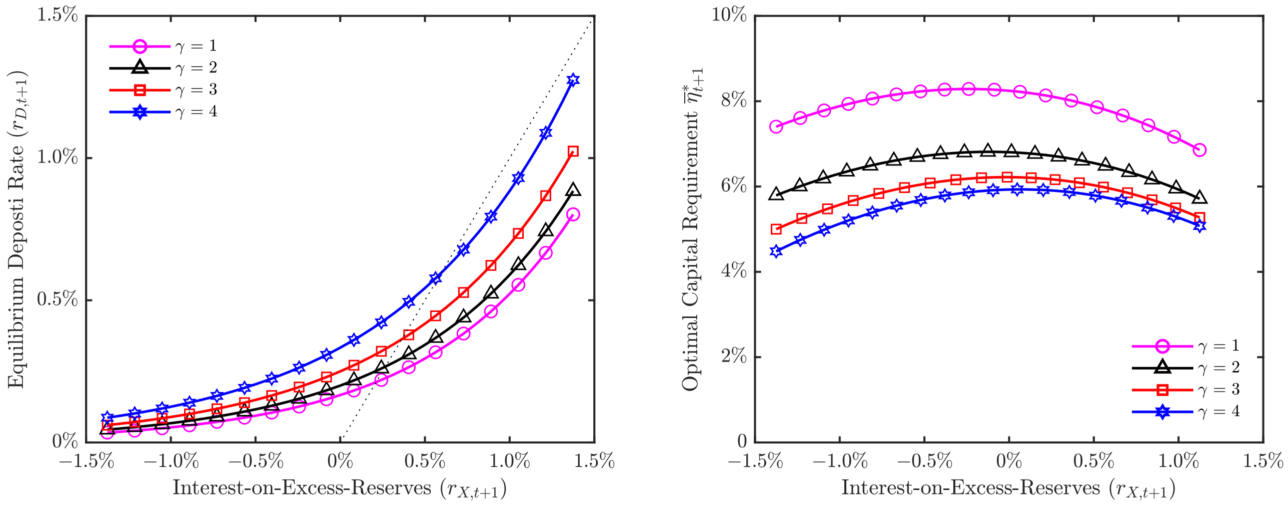



**Figure 2.12:** The figure on the left depicts equilibrium deposit rate variations given exogenous changes in IOER. When household's risk-aversion increases, for any given value of IOER, equilibrium deposit rate falls particularly when IOER is above the zero bound. The figure on the right shows optimal capital regulation in response to equilibrium deposit rate.

## 2.7 Discussion

The 2007-2008 financial crisis and its aftermath prompted policymakers to re-evaluate regulatory instruments that were intended to address the banking system's negative externalities to society. The model in this paper considered a financial regulatory policy together with a monetary policy tool that are available to policymaker to address distortions in banking system generated by costly bankruptcies and overreliance on interest-bearing reserves as safe assets that strain credit flow to the real economy. The calibration exercises shows that when IOER is taken as given, capital regulation is able to lower the likelihood of bank failure by requiring the bank to maintain a higher equity per loan ratio. From bank's perspective, this implies that in order to comply with the regulation, further capital per unit of loan must be raised through the equity market which is more expensive, in terms of price per each unit of fund, relative to debt. The general equilibrium implications indicate that as the bank seeks to raise financing from the equity market, the price of equity falls which further increases the cost of meeting the capital regulation. This sheds light on three channels that entail important implications when considering costly equity financing.

First, raising funds through the equity market comes at a more expensive price at purchase which ultimately narrows cashflow generated by the difference between bank's revenues less its costs, but on the margin, each additional unit of equity also increases the likelihood of bank solvency which is priced by bank investors. The decomposition in section (2.4.2) shows bank's expected profit is determined by the product of cashflow component and solvency component indicating that although bank's equity return falls due to higher cost of funding, the fall is partly offset by marginal contribution of each additional unit of equity to solvency and therefore bank value. Results in the solution methodology show that the contribution of the solvency component to increase expected profit is substantial when equity is scarce and fades as bank's equity-to-assets increases because it becomes less likely for the bank to declare bankruptcy due to delinquencies among borrower.

Although the bank is risk-neutral, it is still concerned about pecuniary implications of holding equity for (expected) profitability and therefore never chooses to finance all of its funds from debt. This is because the marginal contribution of equity to expected profit is higher than that of the cashflow when bank's capital structure includes limited equity. In laissez-faire equilibrium, marginal contribution of equity to solvency and cashflow are equal, but in equilibrium with capital regulation, bank equity has a lower marginal contribution to solvency channel than it has to cashflow channel which indicates capital regulation is always binding.

The third channel furthers this equilibrium analysis and shows that higher bank capital leads to lower riskiness of bank equity and lowers the risk compensation that bank has to pay to raise funds from risk-averse households. When capital regulation is levied, the bank considers that its market share price is bound to fall because of increased demand for capital but as each equity unit is added to its capital structure, lower risk compensation bids up the share price which dampens the increasing cost of capital as further equity is raised to comply with capital regulation.[45]

Households internalize higher non-financial income through the transfers that they receive from the regulator when defaults are less likely. However, the household's financial income comprises the present value of deposit income in default and the present value of excess return in solvency which increases when solvency becomes more likely. This effect lowers the stochastic discount factor which has two effects. First, this implies that the deposit rate increases because the household marginal utility of consumption becomes flatter with added income and requires higher risk-free[46] compensation to invest in the deposits,

[45] Transitioning from laissez-faire to equilibrium with capital regulation.

[46] Although section (2.6) takes the choice of taxation as given, the level of taxation is still an important decision for the equilibrium asset prices. First, it is important for the regulator to raise an adequate level of taxation to be able to provide guarantees on deposits so that deposit insurance eliminates the possibility of bank runs. Any value of taxation higher than the difference between outstanding loans plus interest less the reserves plus interest is irrelevant to bank runs specifically because deposits are always guaranteed in real terms. However, as taxation falls below this certain limit, there exists some states of the world in which extremely adverse negative shock to bank borrowers can bankrupt the bank such that the deposit insurance fund becomes unable to cover the depositors in full. This study does not examine the welfare implications of taxation and assumes that the deposit insurance is provided in real terms by taxing the economy in anticipation of worst-case scenario

and second, higher stochastic discount factor is associated with lower bank valuation which subsequently lower's households demand for bank equity. As a result, the equity premium narrows under equilibrium with capital regulation. However, this effect is dampened because, on the margin, the bank holds more debt which bids down its share price due to higher required compensation for additional default risk.

When the regulator exogenously lowers interest-on-reserves, the equilibrium deposit rate falls. First, lower interest-on-reserves implies that, because the spread between the expected loan and reserves widens, the bank substitutes reserves with loans on its asset side. This re-allocation must be accompanied by higher equity on the liabilities side to satisfy regulator's risk-weighted capital requirement which is ensued by a lower equity price, and accordingly, a lower deposit rate because the bank demand for debt financing is reduced. This transmission mechanism across bank assets-liabilities implies that exogenous changes in interest-on-reserves moves the equilibrium deposit rate in the same direction, however, as falling interest-on-reserves nears zero, or possibly below zero, the equilibrium deposit rate become less responsive. This is because households are endogenously forming their valuations about investments and as long as they require a minimal compensation for time preference, they always require a strictly positive deposit rate and subsequently, falling interest-on-reserves is associated with an increasingly flatter response by equilibrium deposit rate particularly when interest-on-reserves is very low or negative.

When considering extensive margins, the bank's funding from deposits is always larger than bank's investment in reserves. As equilibrium deposit rate falls, bank's interest expenses on deposits fall faster than reduced interest incomes from reserves due to higher relative extensive margins in deposits than reserves. This mechanism indicates that bank's default risk falls thereby, first extending its ability to meet debt liabilities at the end of the period and, second, the bank is able to increase lending to its borrowers until the marginal gain from loan revenues become equal to increased default risk due to increased loans.

However, flattening response of deposit rate to falling interest-on-reserves narrows the difference between interest expenses and interest incomes that allows the bank to extend its lending. When interest-on-excess-reserves is close to zero, falling deposit rate offers limited reduction in bank interest expenses which together with sharper drop in interest income from reserves, amounts to a net decrease in interest incomes that leads to higher bank default risk. The bank optimally reacts to added default risk by lowering its lending which then lowers the real output. The underlying hump-shaped relationship between interest-

shock outcome.

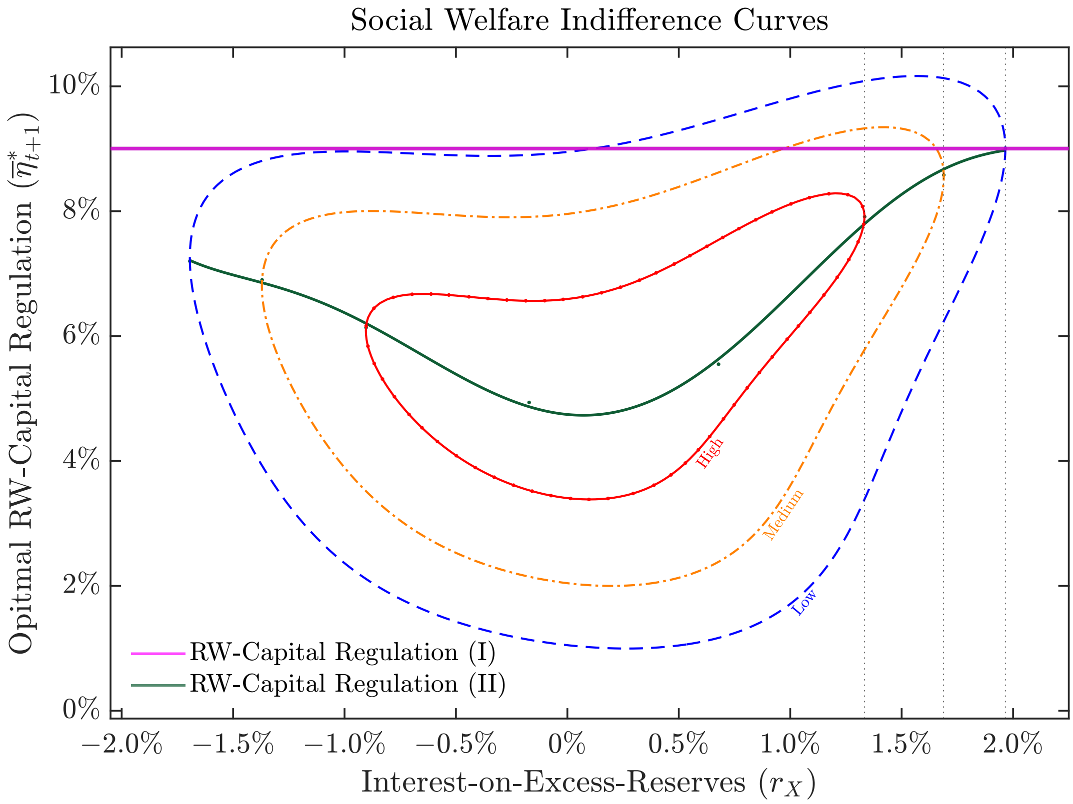



**Figure 2.13:** The figure illustrates the social welfare function level curves. The social welfare increases towards the inner contour curves depicted by dashed (blue), dashed-dotted (orange) and solid (red) contours. The horizontal solid line describes the RW-capital regulation (I) that is decided in isolation of interest rate policy which is always associated with a lower welfare, relative to RW-capital regulation (II). Regulatory schedule (II) considers the welfare implications of lower IOER and is less strict than (I) and is able to achieve higher welfare relative to (I).

on-reserves specifies that RW-capital regulation needs to tighten as interest-on-reserves falls from a positive level to close to zero and the needs to loosen if interest-on-reserves falls further to zero or below zero. Optimal capital regulation in response to any interest-on-reserves value considers welfare benefits of higher equity per loan, relative to laissez-faire allocation.

This result shows capital regulation addresses distortions associated with costly bankruptcy at the expense of strains on credit flow to the real sector when interest-on-reserves is very low. The regulator considers the non-monotonic interaction between two policies to choose an optimal interest-on-reserves rate that provides social value by expanding credit while capital regulation is at its optimum. First, high interest-on-reserves, given an optimal capital regulation, is associated with high remuneration of reserves that has to be paid from regulator's resources to the banking sector. Regulator's resources are financed from taxation of the economy to cover interest expenses but also are intended to compensate depositors in any default state as a part of government guarantee provided by the deposit insurance service. The equilibrium analysis in this section shows that over-reliance on excess reserves together with high interest burdens the regulator's resources and therefore taxes must increase to maintain guarantees in real term, otherwise, depositors' loss of confidence in given guarantees, even if not originally justified by fundamentals, will tend to be self-confirming. Second, an optimal interest-on-reserves policy considers credit flow to output sector against added default risk due to exposure of the banking sector to extended lending when interest-on-reserves is above zero. Conversely, very low or negative interest-on-excess-reserves trades off social costs of lower lending against lower default risk within the banking sector.

## 2.8 Conclusion

Regulation of the banking system through the risk-weighted capital requirements has been among the forefront macroprudential policies. Capital regulation serves an important function to narrow the gap between the amount of capital that the banking institutions hold, and the amount that it *should* hold. The first driver of this difference is given by the inherent gap between the risk attitudes of the banking institutions and their ultimate investors towards risk. This (rational) behavioural difference amongst the two is associated with important welfare implications since the banking institutions' decision in the allocation of funds leads to risk-reward profiles that are suboptimal to the ultimate owners of funds.

The context in this study provides an asset pricing and financial regulatory framework to assess the welfare valuations of owners of funds over such *intermediated* risk-reward profiles. The lower aversion towards riskier allocations of funds often entails lending expansions that translate to lower risk-adjusted rewards to ultimate investors, despite higher expected returns. The laissez-faire equilibrium analysis in Section (2.4.3) shows that the welfare is significantly lessened when a representative banking sector engages with a representative household sector to raise funds because the gap between the optimal versus eventual allocation of funds to the ultimate borrowers is extensive. The analysis in Section (2.5) then develops a general equilibrium approach to determine a risk-weighted capital requirement that regulates the capital structure of the banking institutions when according to the riskiness of its assets side.

The novelty of this general equilibrium approach provides a realistic approach for the policymakers to incorporate the following key components, that remained an open area of research in the literature, into regulatory decisions: first, the ICAPM-EZW valuation over risk and reward profiles that arises endogenously in this context incorporates the behaviour of the households as a part of the policymaker's decisions. This is a quintessential feature since financial policies are applicable at the sector level and are inherently expected to affect asset prices across the macroeconomy. The endogenous valuation by the households considers the price of the bank equity and its expected dividend that are determined by the banking institutions performance. The banking institutions decisions that drive performance are driven by the households valuation of their net worth today that is determined by discounting future expected cashflows by the households stochastic discount factor. The fixed-point analysis in this study simultaneously interacts the bank and the investors valuations and provides a theoretical explanation to capture how market forces translate to asset prices and ultimately the capital structure of the banking institutions. The financial regulator's capital requirement in the context of general equilibrium in this study captures the riskiness of a bank equity through the perspective of the ultimate owners of capital or the households. More specifically, the model predicts that when the households become more risk-averse, their preference to hold a larger share of their savings in deposits increases. This leads to a lower equilibrium deposit rate which incentivises the bank to finance a larger share of its operations through debt. The optimal capital regulation, however, instructs the bank to lower the share of its assets invested in lending to ensure that its risk-reward profile remains in line with that of the social perspective.

Another driver of the differences between the way banking institution finances its operations is determined by the limited liability should the bank defaults. In particular, this becomes a consequential consideration for the providers of financing, and subsequently the regulators, when defaults are associated with deadweight costs. The analysis in this study provides a novel approach to separate the solvency and cashflow channels when the underlying issuer of the equity is subject to the limited liability. This separation provides a framework for the policymakers to evaluate the welfare implications of the banking institutions fragility against the welfare gains associated with extended lending.

## 2.9 Appendix

### 2.9.1 Cashflow and Solvency Channels

Let $g(k)$ denote expectations function over a subset of $x$ support:

$$g(k) = \int_k^\infty x f(x) dx \tag{2.9.1}$$

where $f(x)$ is the lognormal distribution, therefore:

$$g(k) = \int_k^\infty \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln x - \mu)^2}{2\sigma^2}\right) dx \tag{2.9.2}$$

Using a change of variables $y = \frac{\ln x - \mu}{\sigma}$, $dx = \sigma \exp(\sigma y + \mu) dy$ gives:

$$\int_{y=(\ln k-\mu)/\sigma}^\infty \frac{1}{\sigma\sqrt{2\pi}} \exp(-\frac{1}{2}y^2)\sigma \exp(\sigma y + \mu) dy \tag{2.9.3}$$

Completing the square

$$= \int \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2}y^2 + \sigma y + \mu) dy \; = \int \frac{1}{\sqrt{2\pi}} \exp[-\frac{1}{2}(y-\sigma)^2 + (\mu + \frac{1}{2}\sigma^2)] dy \tag{2.9.4}$$

$$= \; = \exp(\mu + \frac{\sigma^2}{2}) \frac{1}{\sqrt{2\pi}} \int_{y=(\ln K-\mu)/\sigma}^\infty \exp(-\frac{1}{2}(y-\sigma)^2) dy \tag{2.9.5}$$

Apply the change of variable $v = y - \sigma$ and $dy = dv$ to re-write the original expectation as:

$$\begin{aligned} g(k) &= \exp(\mu + \frac{\sigma^2}{2}) \frac{1}{\sqrt{2\pi}} \int_{v=(\ln k-\mu)/\sigma-\sigma}^\infty \exp(-\frac{1}{2}v^2) dv \qquad (2.9.6)\\ &= \exp\left(\mu + \frac{\sigma^2}{2}\right) \left[1 - \Phi(\frac{\ln k - \mu - \sigma^2}{\sigma})\right] \\ &= \exp\left(\mu + \frac{\sigma^2}{2}\right) \Phi\left(\frac{-\ln k + \mu + \sigma^2}{\sigma}\right) \end{aligned}$$

where $1 - \Phi(x) = \Phi(-x)$.

### 2.9.2 Stochastic Discount Factor and the Equity Premium

The stochastic discount factor is ( i.i.d. return):

$$\begin{aligned}
\log M_{t,t+1} &= -\gamma \log R_{h,t+1} - \log \mathbb{E}_t[R_{h,t+1}^{1-\gamma}] && (2.9.7)\\
&= -\gamma \log R_{h,t+1} - \mathbb{E}_t \log R_{h,t+1}^{1-\gamma} - \frac{1}{2}\mathbb{V}_t \log R_{h,t+1}^{1-\gamma} && (2.9.8)\\
&= -\gamma \log R_{h,t+1} - (1-\gamma)\mathbb{E}_t \log R_{h,t+1} - \frac{(1-\gamma)^2}{2}\mathbb{V}_t \log R_{h,t+1} && (2.9.9)
\end{aligned}$$

the (logarithmic) moments are:

$$\begin{aligned}
\mathbb{E}_t \log M_{t,t+1} &= -\mathbb{E}_t \log R_{h,t+1} - \frac{(1-\gamma)^2}{2}\mathbb{V}_t \log R_{h,t+1} && (2.9.10)\\
\mathbb{V}_t \log M_{t,t+1} &= \gamma^2 \mathbb{V}_t \log R_{h,t+1} && (2.9.11)
\end{aligned}$$

using the Euler equation with respect to deposit investment, $1 = R_{D,t+1}\mathbb{E}_t M_{t,t+1}$, the deposit rate (household's risk-free rate) is:

$$\begin{aligned}
0 &= \log R_{D,t+1} + \log \mathbb{E}_t M_{t,t+1} && (2.9.12)\\
&= \log R_{D,t+1} + \mathbb{E}_t \log M_{t,t+1} + \frac{1}{2}\mathbb{V}_t \log M_{t,t+1} && (2.9.13)\\
&= \log R_{D,t+1} - \mathbb{E}_t \log R_{h,t+1} - \frac{(1-\gamma)^2}{2}\mathbb{V}_t \log R_{h,t+1} + \frac{\gamma^2}{2}\mathbb{V}_t \log R_{h,t+1} && (2.9.14)\\
&= r_{D,t+1} - \mathbb{E}_t r_{h,t+1} + \frac{2\gamma-1}{2}\mathbb{V}_t r_{h,t+1} && (2.9.15)\\
&= r_{D,t+1} - (1-\pi_{t+1}) r_{D,t+1} - \pi_{t+1}\mathbb{E}_t r_{E,t+1} + \frac{2\gamma-1}{2}\mathbb{V}_t r_{h,t+1} && (2.9.16)\\
&= -\pi_{t+1}(\mathbb{E}_t r_{E,t+1} - r_{D,t+1}) + \frac{2\gamma-1}{2}\mathbb{V}_t r_{h,t+1} && (2.9.17)
\end{aligned}$$

The equity premium is: $\mathbb{E}_t r_{E,t+1} - r_{D,t+1} = \left(\gamma - \frac{1}{2}\right)\mathbb{V}_t r_{E,t+1}$

### 2.9.3 Optimal Capital Regulation

The social welfare function is evaluated by household's utility function given regulator's resources,

$$g = \begin{cases} (1-\tau).\left[\frac{1-\overline{\eta}}{Q_{D,t}} + (1-\kappa).\overline{\eta}.R_{E,t+1}\right] + \left[\tau - (1-\tau).(1-\omega_{t+1}).r_x\right] & \text{in solvency} \\ (1-\tau).\frac{1-\overline{\eta}}{Q_{D,t}} + \left[\tau - (1-\tau).\left[\left(\frac{1-\overline{\eta}}{Q_{D,t}} - \chi.R_{p,t+1}\right) - (1-\omega_{t+1}).r_x\right]\right] & \text{in default} \end{cases}$$

The first derivative of w.r.t. capital regulation choice $\overline{\eta}_{t+1}$ is

$$0 = \frac{\chi + \Phi(\lambda_{t+1}).(1-\chi)}{1-\chi} - \frac{1-\overline{\eta}_{t+1}(1-\kappa)}{1-\kappa}\left(-\frac{1}{\sigma}\frac{\partial\Phi(\lambda_{t+1})}{\partial\tau_{t+1}}\frac{\partial\lambda_{t+1}}{\partial z_{b,t+1}}\right)\frac{\partial z_{b,t+1}}{\partial\overline{\eta}_{t+1}}$$

Approximating the term $-\frac{1}{\sigma}\frac{\partial\Phi(\lambda_{t+1})}{\partial\tau_{t+1}}\frac{\partial\lambda_{t+1}}{\partial z_{b,t+1}}$ with the following exponential affine function, $e^{a_0+a_1 z_b}$ where $a_0$ and $a_1$ are functions of $\mu$ and $\sigma$. Table (2.4) shows the optimal capital regulation given the IOER within $[-0.75\%, 1.75\%]$ when the bankruptcy cost is low (Panel A), medium (Panel B) and high (Panel C), respectively, and equity intermediation cost is 1%. Table (2.5), replicates the same exercise when the intermediation cost associated with the equity market fall to 0.5%.

| $\omega_{t+1}$ | **Optimal Risk-weighted Capital Requirement (% of total assets)** | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Panel A: Low Loss Given Default ($1-\chi = 10\%, \kappa = 99\%$) | | | | | | | | | | |
| | Interest-on-Excess-Reserves ($r_{X,t+1}$) in percentage points | | | | | | | | | | |
| | 1.75 | 1.50 | 1.25 | 1.00 | 0.75 | 0.50 | 0.25 | 0.00 | $-0.25$ | $-0.50$ | $-0.75$ |
| 0% | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 10% | 2.36 | 2.22 | 2.09 | 1.95 | 1.81 | 1.67 | 1.53 | 1.39 | 1.55 | 1.69 | 1.84 |
| 25% | 4.72 | 4.44 | 4.17 | 3.89 | 3.61 | 3.34 | 3.06 | 2.78 | 3.10 | 3.38 | 3.67 |
| 50% | 7.08 | 6.67 | 6.26 | 5.84 | 5.42 | 5.01 | 4.59 | 4.18 | 4.65 | 5.07 | 5.51 |
| 75% | 9.44 | 8.89 | 8.34 | 7.78 | 7.23 | 6.67 | 6.12 | 5.57 | 6.19 | 6.76 | 7.35 |
| 90% | 11.80 | 11.11 | 10.43 | 9.73 | 9.03 | 8.34 | 7.65 | 6.96 | 7.74 | 8.45 | 9.18 |
| 100% | 14.16 | 13.33 | 12.51 | 11.67 | 10.84 | 10.01 | 9.18 | 8.35 | 9.29 | 10.14 | 11.02 |
| | Panel B: Medium Loss Given Default ($1-\chi = 20\%, \kappa = 99\%$) | | | | | | | | | | |
| $\omega_{t+1}$ | Interest-on-Excess-Reserves ($r_{X,t+1}$) in percentage points | | | | | | | | | | |
| | 1.75 | 1.50 | 1.25 | 1.00 | 0.75 | 0.50 | 0.25 | 0.00 | $-0.25$ | $-0.50$ | $-0.75$ |
| 0% | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 10% | 2.51 | 2.38 | 2.26 | 2.14 | 2.01 | 1.89 | 1.77 | 1.64 | 1.77 | 1.88 | 2.00 |
| 25% | 5.01 | 4.76 | 4.52 | 4.27 | 4.02 | 3.78 | 3.53 | 3.28 | 3.54 | 3.76 | 4.01 |
| 50% | 7.52 | 7.15 | 6.78 | 6.41 | 6.04 | 5.67 | 5.30 | 4.93 | 5.32 | 5.65 | 6.01 |
| 75% | 10.02 | 9.53 | 9.03 | 8.54 | 8.05 | 7.55 | 7.06 | 6.57 | 7.09 | 7.53 | 8.01 |
| 90% | 12.53 | 11.91 | 11.29 | 10.68 | 10.06 | 9.44 | 8.83 | 8.21 | 8.86 | 9.41 | 10.02 |
| 100% | 15.03 | 14.29 | 13.55 | 12.81 | 12.07 | 11.33 | 10.59 | 9.85 | 10.63 | 11.29 | 12.02 |
| | Panel C: High Loss Given Default ($1-\chi = 30\%, \kappa = 99\%$) | | | | | | | | | | |
| $\omega_{t+1}$ | Interest-on-Excess-Reserves ($r_{X,t+1}$) in percentage points | | | | | | | | | | |
| | 1.75 | 1.50 | 1.25 | 1.00 | 0.75 | 0.50 | 0.25 | 0.00 | $-0.25$ | $-0.50$ | $-0.75$ |
| 0% | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 10% | 2.88 | 2.73 | 2.58 | 2.42 | 2.27 | 2.12 | 1.96 | 1.81 | 1.95 | 2.09 | 2.26 |
| 25% | 5.76 | 5.46 | 5.15 | 4.84 | 4.54 | 4.23 | 3.92 | 3.62 | 3.89 | 4.17 | 4.52 |
| 50% | 8.65 | 8.19 | 7.73 | 7.27 | 6.81 | 6.35 | 5.89 | 5.43 | 5.84 | 6.26 | 6.79 |
| 75% | 11.53 | 10.91 | 10.30 | 9.69 | 9.07 | 8.46 | 7.85 | 7.23 | 7.79 | 8.34 | 9.05 |
| 90% | 14.41 | 13.64 | 12.88 | 12.11 | 11.34 | 10.58 | 9.81 | 9.04 | 9.73 | 10.43 | 11.31 |
| 100% | 17.29 | 16.37 | 15.45 | 14.53 | 13.61 | 12.69 | 11.77 | 10.85 | 11.68 | 12.51 | 13.57 |

**Table 2.4:** The table illustrates the optimal risk-weighted capital regulation set by the policymaker on the banking sector to condition its asset allocation to its capital structure. Panel (A) shows the required capital as a percentage of total assets and various IOER where the required capital ubiquitously increases when share of total asset invested in lending ($\omega_{t+1}$) increases. As IOER is lowered, the corresponding required capital decreases for any given level of asset allocation when IOER is above zero. As IOER reaches zero and marginally negative, the policymaker requires the liabilities to includes lower capital per unit of loan. Panels (B) and (C) replicate the same quantities when the bankruptcy cost and its associated deadweight losses increases leading to higher minimum capital requirement given IOER and share of funds invested in risky asset.

| $\omega_{t+1}$ | **Optimal Risk-weighted Capital Requirement (% of total assets)** Panel A: Low Loss Given Default ($1-\chi = 10\%, \kappa = 99.5\%$) Interest-on-Excess-Reserves ($r_{X,t+1}$) in percentage points | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1.75 | 1.50 | 1.25 | 1.00 | 0.75 | 0.50 | 0.25 | 0.00 | −0.25 | −0.50 | −0.75 |
| 0% | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 10% | 2.43 | 2.30 | 2.17 | 2.04 | 1.91 | 1.78 | 1.65 | 1.52 | 1.65 | 1.79 | 1.92 |
| 25% | 4.86 | 4.60 | 4.34 | 4.08 | 3.82 | 3.56 | 3.30 | 3.04 | 3.30 | 3.57 | 3.85 |
| 50% | 7.29 | 6.90 | 6.51 | 6.12 | 5.73 | 5.34 | 4.95 | 4.56 | 4.96 | 5.36 | 5.77 |
| 75% | 9.71 | 9.19 | 8.67 | 8.15 | 7.63 | 7.11 | 6.59 | 6.08 | 6.61 | 7.14 | 7.69 |
| 90% | 12.14 | 11.49 | 10.84 | 10.19 | 9.54 | 8.89 | 8.24 | 7.60 | 8.26 | 8.93 | 9.62 |
| 100% | 14.57 | 13.79 | 13.01 | 12.23 | 11.45 | 10.67 | 9.89 | 9.12 | 9.91 | 10.71 | 11.54 |
| | Panel B: Medium Loss Given Default ($1-\chi = 20\%, \kappa = 99.5\%$) | | | | | | | | | | |
| $\omega_{t+1}$ | Interest-on-Excess-Reserves ($r_{X,t+1}$) in percentage points | | | | | | | | | | |
| | 1.75 | 1.50 | 1.25 | 1.00 | 0.75 | 0.50 | 0.25 | 0.00 | −0.25 | −0.50 | −0.75 |
| 0% | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 10% | 2.57 | 2.45 | 2.33 | 2.21 | 2.09 | 1.97 | 1.85 | 1.73 | 1.85 | 1.97 | 2.06 |
| 25% | 5.13 | 4.89 | 4.65 | 4.41 | 4.17 | 3.93 | 3.69 | 3.45 | 3.71 | 3.94 | 4.11 |
| 50% | 7.70 | 7.34 | 6.98 | 6.62 | 6.26 | 5.90 | 5.54 | 5.18 | 5.56 | 5.92 | 6.17 |
| 75% | 10.26 | 9.78 | 9.30 | 8.82 | 8.34 | 7.86 | 7.38 | 6.90 | 7.41 | 7.89 | 8.23 |
| 90% | 12.83 | 12.23 | 11.63 | 11.03 | 10.43 | 9.83 | 9.23 | 8.63 | 9.27 | 9.86 | 10.28 |
| 100% | 15.39 | 14.67 | 13.95 | 13.23 | 12.51 | 11.79 | 11.07 | 10.35 | 11.12 | 11.83 | 12.34 |
| | Panel C: High Loss Given Default ($1-\chi = 30\%, \kappa = 99.5\%$) | | | | | | | | | | |
| $\omega_{t+1}$ | Interest-on-Excess-Reserves ($r_{X,t+1}$) in percentage points | | | | | | | | | | |
| | 1.75 | 1.50 | 1.25 | 1.00 | 0.75 | 0.50 | 0.25 | 0.00 | −0.25 | −0.50 | −0.75 |
| 0% | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 10% | 2.97 | 2.86 | 2.76 | 2.66 | 2.55 | 2.45 | 2.35 | 2.24 | 2.36 | 2.46 | 2.55 |
| 25% | 5.93 | 5.72 | 5.52 | 5.31 | 5.10 | 4.90 | 4.69 | 4.48 | 4.72 | 4.91 | 5.10 |
| 50% | 8.90 | 8.59 | 8.28 | 7.97 | 7.66 | 7.35 | 7.04 | 6.73 | 7.09 | 7.37 | 7.65 |
| 75% | 11.86 | 11.45 | 11.03 | 10.62 | 10.21 | 9.79 | 9.38 | 8.97 | 9.45 | 9.82 | 10.19 |
| 90% | 14.83 | 14.31 | 13.79 | 13.28 | 12.76 | 12.24 | 11.73 | 11.21 | 11.81 | 12.28 | 12.74 |
| 100% | 17.79 | 17.17 | 16.55 | 15.93 | 15.31 | 14.69 | 14.07 | 13.45 | 14.17 | 14.73 | 15.29 |

**Table 2.5:** The table illustrates the optimal risk-weighted capital regulation set by the policymaker on the banking sector to condition its asset allocation to its capital structure. Panel (A) shows the required capital as a percentage of total assets and various IOER where the required capital ubiquitously increases when share of total asset invested in lending ($\omega_{t+1}$) increases. As IOER is lowered, the corresponding required capital decreases for any given level of asset allocation when IOER is above zero. As IOER reaches zero and marginally negative, the policymaker requires the liabilities to includes lower capital per unit of loan. Panels (B) and (C) replicate the same quantities when the bankruptcy cost and its associated deadweight losses increases leading to higher minimum capital requirement given IOER and share of funds invested in risky asset.

### 2.9.4 Solvency Condition

Given the bank's decisions determining capital structure and asset allocation, he solvency condition is given by $R_{b,t+1}(\eta_{t+1}, \omega_{t+1}; R_{X,t+1}, R_{D,t+1})$:

$$R_{b,t+1}(\eta_{t+1}, \omega_{t+1}; R_{X,t+1}, R_{D,t+1}) = \max\left\{\frac{1-\eta_{t+1}}{\omega_{t+1}} R_{D,t+1} - \frac{1-\omega_{t+1}}{\omega_{t+1}} R_{X,t+1}, 0\right\} \tag{2.9.18}$$

The threshold is (weakly) decreasing in the capital structure decision ($\eta_{t+1}$). This is because more deposits, relative to total assets size, increases bank's reliance on a higher loan outcome at the end of each period. The probability of bank solvency is driven by the default threshold, particularly because the CDF is a monotonic transformation of $R_{b,t+1}(\eta_{t+1}, \omega_{t+1}; R_{X,t+1}, R_{D,t+1})$, the relationship between the capital structure and asset allocation of the bank transfers to the probability of default:

$$\Phi\left(\lambda\left[R_{b,t+1}(\eta_{t+1}, \omega_{t+1}; R_{X,t+1}, R_{D,t+1})\right]\right) = \mathbb{P}\left(R_{p,t+1}A_{t+1} < R_{D,t+1}\mathcal{D}_{t+1}\right) \tag{2.9.19}$$

where the transformation $\lambda(R_{b,t+1}) = \frac{\mu_z + \sigma_z^2 - \log(R_{b,t+1})}{\sigma_z}$ translates log-normally distributed shock outcome quantiles to the standard Normal quantiles.

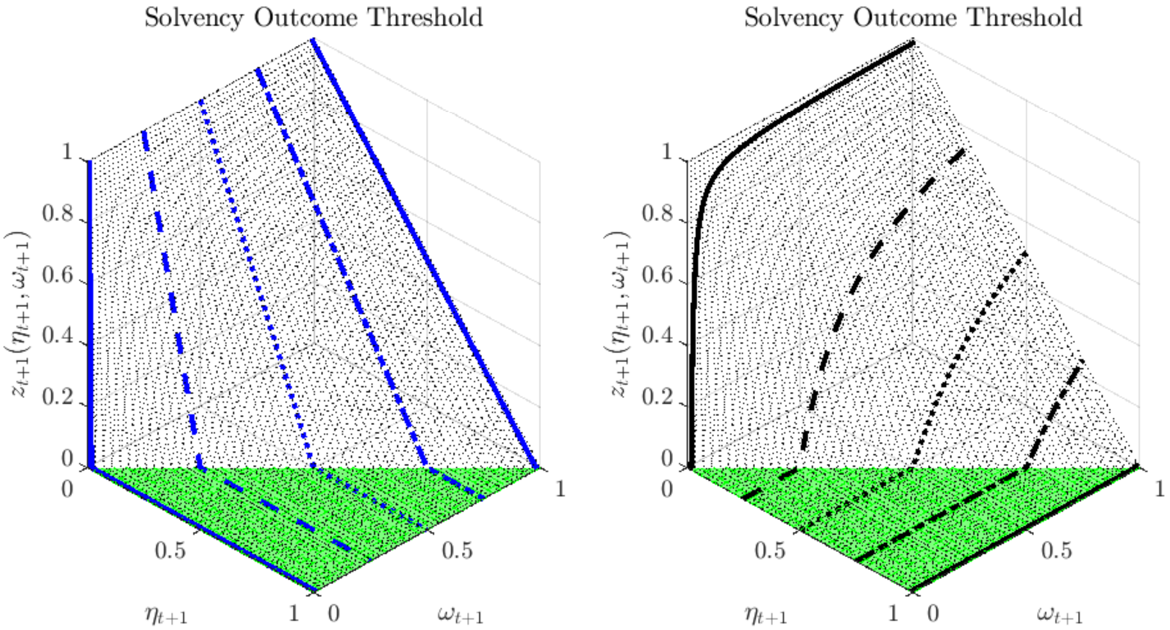



**Figure 2.14:** The surfaces illustrate the loan default threshold that arises endogenously. The default threshold depends on bank's capital structure ($\eta_{t+1}$) and its asset allocation decisions ($\omega_{t+1}$). Diagram on the left describes that as the bank lowers the share of deposits in its capital structure, the default threshold falls, initially, linearly and then becomes constant at zero. This mechanism holds for any asset allocation decision. As the allocation of funds to risky loans increases, the default threshold increases. The diagram on the right illustrates the default threshold particularly for asset allocation decision. As allocation to loans increases, relative to the size of the bank's balance sheet, the default threshold increases. This mechanism shows that when the bank holds more equity on its capital structure, the default threshold falls.

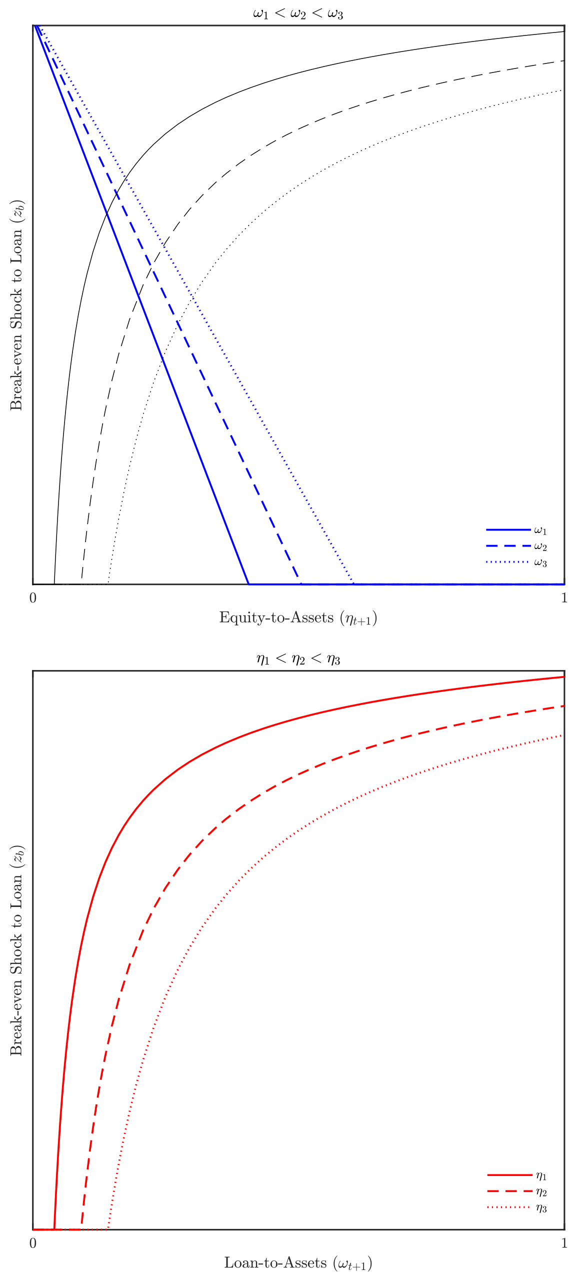



**Figure 2.15:** Figure on the left depicts the relationship between $R_{b,t+1}$ and capital structure, given low (solid line in blue), medium (dashed line in blue) and high (dotted line in blue) levels of risky loans on the assets side. Figure on the right repeats the same exercise between $R_{b,t+1}$ and asset allocation decision, taking the capital structure choice as given.

# Chapter 3

# Optimal Negative Interest Rate: Monetary and Financial Regulatory Synergies

## 3.1 Introduction

The negative interest rate has been among the frontier policies to counter the recent economic downturns. The 2020 Pandemic resurfaced the policy's role that was originally deployed to assuage prolonged slowdowns associated with the aftermath of the 2008 Financial Crisis. While lowering the cost of financing is a well-established policy initiative in response to adverse economic outcomes, the effective pass-through implications of the negative interest rate through the financial intermediaries remains an open question.

This study examines how the negative interest rate policy leads to real economic implications through the banking institutions. The interest rate policy is primarily a monetary lever, nonetheless, its tight relationship with the interest-on-excess-reserves (IOER), paid on oversized excess reserves held by the banking system, generates substantial impacts on the banking institutions overall performance. This provides the motivation to first, examine how the negative interest rate policy is translated to an ultimate lending rate for the real sector through the banking institutions. Second, the tight relationship between the main monetary policy and the IOER, provides motivation to examine how the interaction amongst policy initiatives by the monetary and financial regulatory authorities leads to welfare implications

Over the past decade, oversized excess reserves of the banking system has comprised over one-third of the total assets of major central banks in charge of 40% of the world economy.[1] Figures (3.1)-(3.2) show that in October 2019, excess reserves of the depository institutions accounted for nearly 30% of the total assets of the Federal Reserves and the ECB. Interest-on-excess-reserves (IOER) is one of the policies used by the monetary authority to regulate reserves of the banking system. The cross-dependency between IOER and capital regulation of the banking system is an important consideration with welfare implications because conflicting effects among the two policies may lead to over-regulation of the banking sector and disruptions in credit flow to the real sector. Alternatively, two policies may lead to under-regulation and re-expose the banking system to heightened default risk and possibly failures with socially undesirable outcomes.

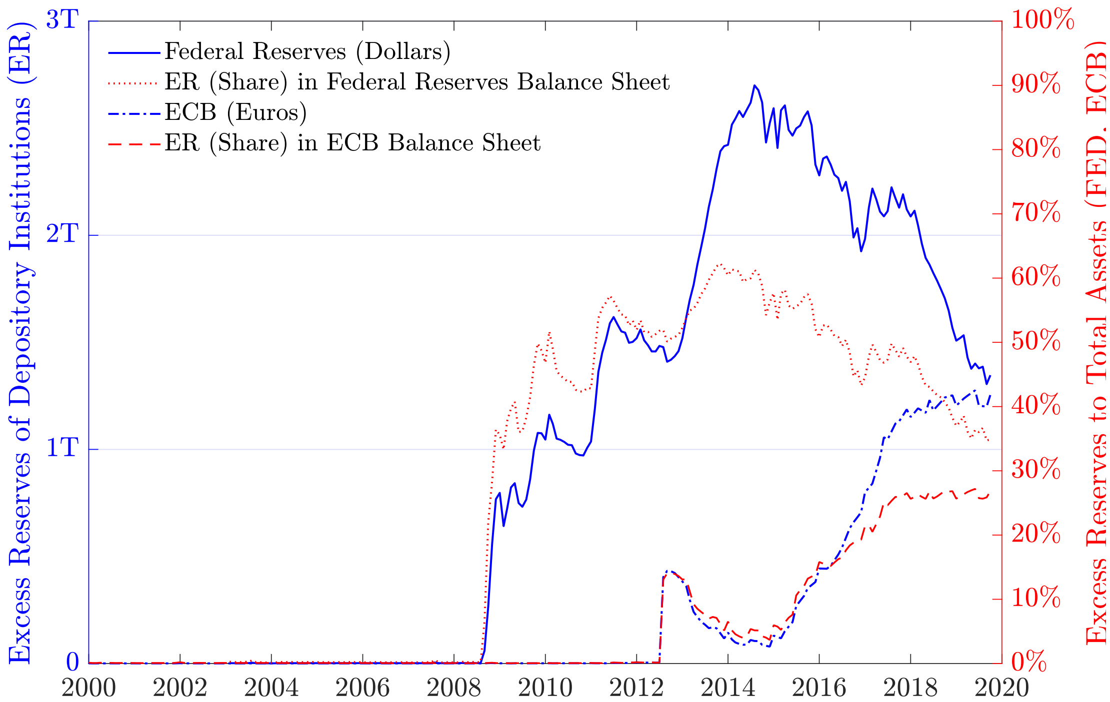



**Figure 3.1:** The figure illustrates excess reserves balances of the depository institutions in the U.S. on the right axis, and interest-on-excess-reserve paid by the Federal Reserves on the left axis.

The aftermath of the 2008 financial crisis highlighted the lack of analytical frameworks to integrate multiple policies and assess their welfare implications. Policymakers constantly address distortions associated with each aspect of the economy with individual policies. Nonetheless, the policymaker's ability to provide welfare gains through a broad range of levers is limited by the understanding of the interconnecting channels among policies. A

[1]Between January 2019 to October 2019, depository institutions in the United States held $1.41T of funds in excess reserves that accounts for over 40% of the total balance sheet size of the Federal Reserves. Over the same period, the ECB held over €1.9T in excess reserves forming a slightly smaller share relative to the consolidated balance sheet of the Eurosystem. The similar pattern holds for the Danish National Bank, the Swiss National Bank, the Sveriges Riksbank, and the Bank of Japan.

quintessential feature of IOER is its dual-role. This policy is decided by the monetary authority and historically, it has been heavily correlated with the main monetary policy.[2] When the monetary authority targets reserves management, IOER simultaneously affects bank balance sheet to a great extent which strengthens the connections between the main monetary policy and the capital regulation.

Existing studies in macro-finance and banking literature investigating the implications of the negative interest rate policy often focus on the interconnections between the policy and the assets side of the banking institutions. This strand of the literature provides a limited prediction about how the negative interest rate policy is passed through the real sector because when rates are negative, the exceedingly steep marginal utility of consumption of the depositors limits the banks ability to pass the negative rates to its depositors. An alternative strand of literature has tackled this shortcoming through partial equilibrium approaches and shows that given exogenous deposit holdings, the negative interest rate policy leads to lower cost of borrowing for the real sector. Nonetheless, such approaches fail to consider the downsides of the negative interest rate policy as the rate may fall indeterminately.

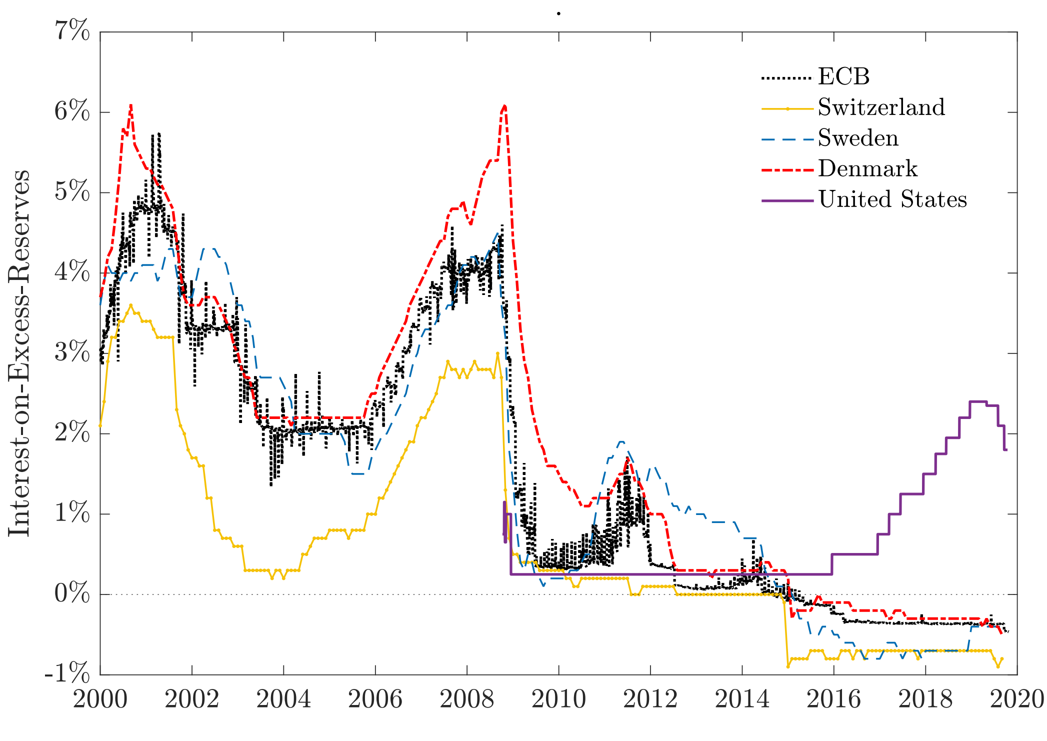



**Figure 3.2:** The figure shows the base rates paid on excess reserves by the European, Switzerland, Sweden and Denmark central banks have been consistently negative since 2015, with magnitude falling to three quarters of percentage point for Sweden and Switzerland.

[2]In the United States, the Federal funds rate and IOER are heavily correlated. This stylized fact holds among other advanced economies ranging between 0.94–0.99.

These shortcomings provide the motivation to incorporate both sides of the banking institutions balance sheet into the policy initiative to determine an interest rate that arises endogenously. More importantly, aside from an endogenous interest rate that is passed-through the bank financiers and borrowers, the responses by the extensive margins on bank assets and liabilities are incorporated through a general equilibrium approach. In particular, this methodology provides a framework in which trade-offs associated with the welfare implications of the interest rate policy determine an optimal interest rate policy.

Second, I provide a framework to assess the welfare implications of the optimal joint policy that simultaneously minimizes distortions associated with costly bank failure while reducing distortions associated with idle oversized excess reserves. I show that a joint regulation, including a positively correlated capital regulation and IOER provides social value when IOER remains above zero bound. Conversely, when IOER is below zero, the policymaker is able to provide social benefits by a joint policy that is characterized by the negatively correlated financial regulatory and monetary instruments. This non-monotonic relationship provides motivation for an integration between IOER and capital regulation.[3] Each lever addresses one distortion to provide welfare gains, whereas a joint policy that considers the interconnections between both levers is able to provide further benefits. Particularly, an optimal IOER policy addresses overreliance on idle excess reserves while capital regulation addresses inefficiencies of costly bank failure.

An optimal IOER considers interest expenses, or alternatively interest incomes[4], associated with oversized excess reserves. A narrower spread between the lending rate and IOER is an incentive for the banks to invest further funds in reserves. However, large quantities of interest payments are ultimately financed from taxation which strains government funds that are intended to serve multiple purposes. In this paper, deposit insurance is a tax-financed service that provides a guarantee for deposits held at the banks by deposit investors when banks default. I show that as IOER increases, first, the policymaker increases taxation in order to finance interest expenses which leads to a lower size of the financial sector and lower real economic activity. Second, credit flow by the banking sector to the real

[3] In the U.S, The Federal Open Market Committee (FOMC) is in charge of monetary policy that includes setting IOER, whereas capital regulation is implemented by Financial Supervision Committee, in the United Kingdom, interest rate policy is decided by the Bank of England while bank regulation is implemented by the Financial Services Authority (FSA).

[4] During 2018:Q3-2019:Q3, excess reserves balances of depository institutions in the U.S. received nearly \$2.43B in net interest incomes given an average IOER of 1.85% which is equivalent to approximately 10% to total excess reserves balance in 2008:Q3. A central bank's interest earnings ordinarily are transferred as tax revenues to the Treasury, by the Federal Reserves or other major central banks, whereas interest expenses on reserves need to be financed from the Treasury.

economy is further decreased because, on the margin, risky lending becomes less attractive relative to reserves. When IOER is below zero, reserves provide interest incomes for the policymaker leading to lower taxation because part of funds intended for deposit insurance is financed from paying negative interests. This mechanism increases the size of the financial sector but leads to a lower output because credit flow to the real sector is substituted with further reserves investment.

This result relies on the assumption that banks are unable to hold cash and therefore find it optimal to store large quantities of their funds in reserves even if the interest rate on this investment falls below zero. Figure (3.1) shows the quantities of funds held in excess reserve deposit facilities at the Federal Reserve and the ECB during the past twenty years. This provides one explanation in support of the argument why the banking system did not rely heavily on storing funds in the form of cash hoarding. Instead, the balances held in excess reserve dramatically increased in over 40% of the world economy during the aftermath of the 2008 crisis even though the interest on reserves remained very low and even negative for many advanced economies. Irving Fisher argued that when a commodity can be stored costlessly over time, then the lower bound in terms of units of that commodity will always remain positive or at least zero.[5] However, the generalization of this result to this context is less straightforward because the storage of large quantities of funds is costly, even for the banking sector.

This paper is organized to provide a brief overview of existing and ongoing studies that examine interconnections between capital regulation and IOER in Section (3.2). I develop a dynamic general equilibrium model in Sections (3.3)-(3.5) to study the implications of financial regulation on welfare, the real economy, and fragility of the banking sector. Section (3.6) provides a numerical solution and discusses welfare and asset pricing implications. Section (3.7) concludes.

## 3.2 Background

The low or negative interest rate environment has been a prevalent feature of the aftermath era of the financial crisis across the advanced economies.[6] In particular, the advent of un-

[5]The Theory of Interest, Irving Fisher, pp52.

[6]The the historical origin of the negative interest rate theory stems from a seminal work of Silvio Gesell (1916). The set up in the present studies assume that the households are unable to hold cash for several reasons such as storage and safety cost, or transactional conveniences. Although cash holding with a fixed fee is still considered across the class of partial equilibrium models, it is not sensible to assume that households can hold cash in a general equilibrium model with a fixed cost. In particular, the marginal cost of storing

conventional monetary policies deployed to re-stabilize the economies together with the prolonged period of heightened economic uncertainty and low productivity, convinced the five central banks in charge of 25% of the world economy to implement the negative nominal interest rate policy since mid-2014.[7]

The first strand of the literature studying the negative interest rate from a policy perspective provides evidence that the low interest rate increases the credit supply to the borrowers through the bank lending channel because the falling rate increases the opportunity cost of reserves holding for the banking system. On the assets side of the banking institutions, the marginal trade-off between a unit of voluntary reserves holding against a unit of lending is driven by the IOER rate and the characteristics of the commercial loan portfolio. The first comparative static analysis suggests that a lower interest rate policy is expected to stimulate the output through an expansion of lending to the production sector. The evidence provided by the studies focusing on the transmission of the interest rate policy to the lending channels finds a heterogenous response amongst banking institutions. More specifically, the empirical evidence finds that the interest rate changes explains variations in portfolio rebalancing only among low-deposit banks. This finding has motivated the literature to incorporate the heterogeneity in banks' capital structures to shed light on the effective transmission of the monetary policy. **?** and **?** argue that high-deposit banks take on more risk when the interest rate are low to generate higher return on assets and maintain their profitability (**?**, **?**, **?** and **?**).

The analytical literature on this frontier develop a foundation to consider banks' net interest margins, a key driver of bank profitability, to explain why the transmission of the policy weakens when banks over-rely on deposit financing.[8]. This study contributes to the existing literature by developing a foundation in which the households valuation of bank net worth arises endogenously and prices the equity of the bank. Particularly, the theoretical studies focusing on bank regulation and asset pricing have often considered the lending or funding sides of the bank balance sheet in isolation of each other. The valuation foundation in the Section (x) captures the uncertainty associated with the bank asset side and provide one explanation on how the transmission of the monetary policy leads to real economic implications when the pass-through effects of the policy to the lending rate and deposit rate arise endogenously. Another strand of the literature document the dampened pass-through

cash increases significantly at a macroeconomic level leading to even more substantial inconveniences when hoarding is considered against paying negative nominal interest rates (**?** and **?**).

[7] The Danish National Bank, the Swiss National Bank, the Sveriges Riksbank, and the Bank of Japan.

[8] **?**, **?** and **?**

effects of the interest rate policy on deposit rates but provide limited explanations on how the effect impact the economic welfare (**?** and **?** and **?**). An important feature of the study in this chapter is its ability to identify the trade-offs associated with the changes in the interest rate policy and present a welfare-maximizing approach to determine an optimal policy.

Another important consideration in this context is to examine the interactions between the interest rate policy and bank capital regulation. The study in the previous chapter is among the frontier frameworks that investigates the bank capital regulation in general equilibrium under aggregate uncertainty. **?** and **?** provide a dynamic equilibrium model in which capital regulation is determined over the financial cycles. However, one limitation of this study is the assumption to set an exogenous stochastic discount factor which poses as a limitation in the current context. The framework in the subsequent section extends the finding of the previous chapter in which the bank capital regulation is determined in a general equilibrium setting when the households form their valuations over asset prices endogenously. This foundation enables the policymaker to evaluate the welfare gains associated with capital regulation together with the trade-offs associated with the interest rate policy and enhance the output across the production sector while regulating the fragility of the banking sector through the capital requirement that is determined jointly. The presence of an endogenous valuation of bank net worth provides a more realistic foundation at macroeconomic level to capture how the households, the banking institutions and the regulators interact simultaneously and enable the policymakers to propose a jointly determined policy toolset that delivers welfare gains to the society.

## 3.3 The Model

The framework in this section develops a three-sector economy where the investments by the ultimate savers are intermediated to borrowers in a production section by a representative bank. The model in this section extends the discrete-time set up developed in the previous chapter, with dates $t = 0, 1, 2, \ldots$. There are three sectors in the economy: a representative household, a representative bank (commercial bank or intermediary) and a financial regulator. Section (3.4) formulates the optimal behaviour of the households and the banking sector given an exogenous minimum capital requirement and discusses its general equilibrium implications. Second, the policymaker's problem to set the interest-on-excess-reserves policy rate is presented together with bank's problem subject to the regulatory constraint in Section (3.4). Section (3.5) presents a general equilibrium model with the optimal IOER policy and minimum capital requirement policies and discusses the interactions between the policies. Deposit insurance service is provided by the regulator across Sections (3.3)-(3.4).

### 3.3.1 Preferences

The household is an infinitely-lived dynasty that lives off financial wealth. At each date-$t$, the household chooses optimal consumption-saving and portfolio allocation to two investment opportunities, deposits and equity. The deposit is a risk-free investment compensated at gross interest rate $R_{D,t+1}$ by the banking sector and benefits from deposit insurance guarantee. The equity is a risky investment that is subject to stochastic return $\mathbb{E}_t[R_{E,t+1}] > R_{D,t+1}$ and is protected by limited liability such that in any default state, equity investor is only responsible up to the original investments. The maximization problem of the household is described by the following recursive utility preferences[9]

$$\{C_t^*, D_{t+1}^*, E_{t+1}^*\}_{t=0}^{\infty} \in \underset{\{D_{t+1},E_{t+1}\}}{\arg\max}\ \mathbb{E}_0\left[U(C_t, \mathbb{E}_t U_{t+1})\right] \tag{3.3.1}$$

$$U(C_t, \mathbb{E}_t U_{t+1}) = \left\{(1-\beta)C_t^{1-\frac{1}{\psi}} + \beta\left(\mathbb{E}_t U_{t+1}^{1-\gamma}\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right\}^{\frac{1}{1-\frac{1}{\psi}}} \tag{3.3.2}$$

where at each date-$t$, the household decides on optimal consumption and portfolio allocation subject to the intertemporal budget constraint described below, receives utility from real consumption $C_t$, $\beta \in (0,1)$ is the subjective discount factor, $\gamma$ is the coefficient of relative risk aversion, and $\psi$ is the elasticity of intertemporal substitution (EIS). The household's attitude towards static risk is separated from intertemporal substitution of consumption with preferences for early resolution of uncertainty such that $\gamma > \frac{1}{\psi}$ throughout the model. The conditional expectation operator $\mathbb{E}_t[.]$ evaluates household's probabilistic assessment of outcomes over solvency and default.

The investment environment includes risk-free deposit investment $D_{t+1}$ that is chosen at date-$t$ backed by deposit insurance. Deposits receive gross deposit interest $R_{t+1}^D$, and equity $E_{t+1}$ with receives a stochastic gross return that is protected by limited liability when the underlying issuer defaults,

$$R_{E,t+1}^{+} = \max\left\{\frac{P_{E,t+1} + div_{t+1}}{P_{E,t}}, 0\right\} \tag{3.3.3}$$

[9] **?** and **?**.

where $P_{E,t}$ and $div_t$ are the price of equity and dividend, respectively. Equity investment is assumed to be subject to a linear cost $\kappa \in (0,1)$. The intertemporal budget constraint is,

$$P_{C,t}C_t + \underbrace{D_{t+1} + E_{t+1}}_{\text{Saving}} = \underbrace{(1-\tau_{t+1})}_{\text{Premium}} \Bigg( \underbrace{\overline{R_{D,t}D_t}}_{\text{Deposit Insured}} + \underbrace{R^{+}_{E,t}(1-\kappa)E_t}_{\text{Limited Liability}} \Bigg) + \underbrace{Tr_{t+1}}_{\text{Transfer}} \quad (3.3.4)$$

where $\tau_{t+1}$ is a fraction of household income that is taxed and $Tr_t \geq 0$ is a transfer that the household receives from the regulator described in Section (3.3.3). The right-hand-side of equation (3.3.4) describes household's wealth $W_t$ which evolves at rate $R_{W,t+1}$ between two consecutive dates $t$ and $t+1$ according to:

$$R_{W,t+1} = (1-\tau_{t+1})(1-\theta_{t+1})R_{D,t+1} + (1-\tau_{t+1})\theta_{t+1}(1-\kappa)R_{E,t+1} + \frac{Tr_{t+1}}{W_t} \quad (3.3.5)$$

where $\theta_{t+1}$ is the portfolio weight on risky asset. The household's value function is,

$$V_t = \left\{ (1-\beta)\left(\frac{C_t}{W_t}\right)^{1-\frac{1}{\psi}} + \beta\left(1-\frac{C_t}{W_t}\right)^{1-\frac{1}{\psi}} \left(\mathbb{E}_t\left[V_{t+1}^{1-\gamma} R_{W,t+1}^{1-\gamma}\right]\right)^{1-\frac{1}{\psi}} \right\}^{\frac{1}{1-\psi}} \quad (3.3.6)$$

When the elasticity of intertemporal substitution approach one, the household become infinitely indifferent to substitute consumption over time and the value function approaches $1-\beta$. In this case, the household's overall investment becomes independent of the return on wealth which leads to a fixed size of the financial sector. The break-down of this fixed investment among deposits and equity varies depending on the deposit rate and the price of equity.

### 3.3.2 The Bank

The representative banking sector is in charge of intermediating funds.[10] from the households to borrowers by accepting deposits and issuing equity to raise capital. The bank invests its financings in two purposes: issues a commercial loan portfolio that earns stochastic return $R_{L,t+1}$ per each unit of investment, or invests in reserves in the deposit facility provided by the regulator to earn risk-free IOER $(R_{X,t+1})$. At the end of each period, Bank's liabilities consist of deposits plus interest which must be honored for the bank to remain sol-

[10]The intermediation process is a key service that reduces potential costs which lenders and borrowers would have faced throughout a dis-intermediated economy. A banking sector offers an important welfare improving implication by minimizing searching and monitoring costs across sectors. Although such associated costs are only assumed implicitly in this study, the welfare improving implications form the basis for the presence of a banking sector.

vent, in which case earnings from loans and reserves are transferred to deposit holders, and then equity investors. The bank, however, is able to declare bankruptcy when it is unable to meet its liabilities in which case deposit holders are compensated partially[11] by the bank and equity value is zero.

Let $\mathcal{D}_{t+1}$ denote bank's finances from accepting deposits and $\mathcal{E}_{t+1}$ denote finances from issuing equity. At each date-$t$ the bank decides how to finance its operations by choosing an optimal capital structure and a portfolio allocation to maximize the present value of the following cashflow described by:

$$div_{t+1} = \max\Big\{ \underbrace{\underbrace{R_{X,t+1}X_{t+1}}_{\text{Reserve Income}} + \underbrace{R_{L,t+1}L_{t+1}}_{\text{Loan Portfolio}}}_{\text{Total Reveune}} - \underbrace{R_{D,t+1}\mathcal{D}_{t+1}}_{\text{Deposit Cost}}, 0 \Big\} - \underbrace{\frac{1}{P_{E,t}}\mathcal{E}_{t+1}}_{\text{Shareholder Value}} \tag{3.3.7}$$

where $X_{t+1}$ and $L_{t+1}$ denote investments made by the bank in the reserves deposit facility and loans, each receiving gross interest-on-reserves and stochastic loan rate, respectively. Equation (3.3.7) is the total dividend value that bank is able to generate after paying out its deposits and its interest and the original investment value. Table (3.1) characterizes the bank balance sheet at each date, consisting of debt $\mathcal{D}_{t+1}$, capital $\mathcal{E}_{t+1}$, reserves $X_{t+1}$ and loans $L_{t+1}$ such that,

$$L_{t+1} + X_{t+1} = \mathcal{D}_{t+1} + \mathcal{E}_{t+1} \tag{3.3.8}$$

Let $\eta_{t+1}$ and $\omega_{t+1}$ denote equity-to-assets and loan-to-assets ratios derived from banks balance sheet at each period, respectively,[12] such that $(\eta_{t+1}, \omega_{t+1}) \in [0,1] \times [0,1]$. The risk-neutral bank maximizes economic profit over the solvency region $(\Delta_h)$ according to,

$$\max_{\eta_{t+1}, A_{t+1}, \omega_{t+1}} \int_{\Delta_h} M_{t,t+1} div_{t+1} dF(z) \tag{3.3.9}$$

[11] The deposit insurance compensates depositors for any remaining uncovered parts of their deposits.

[12] As section (3.4.3) shows, in equilibrium the optimal capital structure decision $\eta^*_{t+1}$ and portfolio allocation by the bank $\omega^*_{t+1}$ take interior solutions. This implies that in equilibrium bank's capital structure includes both debt and capital and its portfolio allocation includes both reserves and loans.

Subject to,

$$X_{t+1} + L_{t+1} = \mathcal{D}_{t+1} + \mathcal{E}_{t+1} \tag{3.3.10}$$

$$\eta_{t+1} \geq \overline{\eta}_{t+1} \tag{3.3.11}$$

$$(\eta_{t+1}, \omega_{t+1}) \in [0,1] \times [0,1] \tag{3.3.12}$$

where $A_{t+1}$ is the total balance sheet size and $M_{t,t+1}$ is the stochastic discount factor[13] of the households who own bank's equity. The bank discounts expected economic profit at date-$t+1$ with respect to probability space $(\Omega, \mathscr{F}, F)$ to choose decisions given the price of equity, the deposit rate and IOER. Equation (3.3.10) is bank's balance sheet constraint where $X_{t+1}$, $L_{t+1}$, $\mathcal{D}_{t+1}$ and $\mathcal{E}_{t+1}$ are reserves, loans, deposits and equity components of the balance sheet, respectively.

| Assets | | Liabilities | |
|---|---|---|---|
| Reserves $(1-\omega_{t+1})$ | $X_{t+1}$ | Deposits $(1-\eta_{t+1})$ | $D_{t+1}$ |
| Loans $(\omega_{t+1})$ | $L_{t+1}$ | Shareholder Value $(\eta_{t+1})$ | $E_{t+1}$ |
| Balance Sheet Size | $A_{t+1}$ | | |

**Table 3.1:** The table describes bank's balance sheet with deposits and equity forming the liabilities side and reserves and loans forming the assets side.

The bank chooses total balance sheet size, and the following two fractions, equity-to-asset and loan-to-assets ratios, over the solvency region. Equation (3.3.11) is the minimum capital requirement constraint that stipulates for any balance sheet size, the bank must finance at least a certain fraction $\overline{\eta}_{t+1}$ of its total liabilities through equity.

**Defaults —** The bank is only concerned with the solvency region defined by $\Delta_h$. The solvency region is determined by the end-of-period ex-post loan rate that breaks even between revenues and outstanding liabilities formulated according to the following condition,

$$\underbrace{R_{p,t+1}A_{t+1}}_{\text{Total Revenues plus Interest Income/Expense}} = \underbrace{R_{D,t+1}\mathcal{D}_{t+1}}_{\text{Total Liabilities plus interest payment}} \tag{3.3.13}$$

where $R_{p,t+1} = (1-\omega_{t+1})R_{X,t+1} + \omega_{t+1}R_{L,t+1}$ denotes the gross return on bank portfolio. Given bank's decisions $\eta_{t+1}$, $A_{t+1}$ and $\omega_{t+1}$, equation (3.3.13) pins down a unique gross loan

[13] Section (3.4.1) characterizes the stochastic discount factor given by and intertemporal capital asset pricing framework.

rate $R_{b,t+1}$ in the state space that makes the bank just-solvent to pay off its debt-holders. At loan rate $R_{b,t+1}$, the bank is collecting only a fraction of its outstanding loans which together with reserves enable the bank to remain solvent. Nonetheless, this implies that the value of its shareholders is equal to zero:

$$R_{E,t+1} = \frac{R_{p,t+1}A_{t+1} - R_{D,t+1}\mathcal{D}_{t+1}}{\mathcal{E}_{t+1}} = 0, \text{ if } R_{L,t+1} = R_{b,t+1} \tag{3.3.14}$$

Assuming a strictly positive beginning-of-period equity value $\mathcal{E}_{t+1} > 0$, then condition (3.3.13) implies that because $A_{t+1} > \mathcal{D}_{t+1}$ then $R_{p,t+1} < R_{D,t+1}$. The threshold loan rate is given by:

$$R_{b,t+1}(\eta_{t+1}, \omega_{t+1}; R_{X,t+1}, R_{D,t+1}) = \max\left\{\frac{1-\eta_{t+1}}{\omega_{t+1}} R_{D,t+1} - \frac{1-\omega_{t+1}}{\omega_{t+1}} R_{X,t+1}, 0\right\} \tag{3.3.15}$$

Henceforth the shorthand just-solvent loan rate $R_{b,t+1}$, specifies the default and solvency regions, respectively, over the possible loan outcome in the state space:

$$\Delta_f := [0, R_{b,t+1}) \tag{3.3.16}$$

$$\Delta_s := [R_{b,t+1}, \infty) \tag{3.3.17}$$

Given bank's decision on capital structure and portfolio composition, the default threshold is known at date-$t$. A higher equity-to-asset ratio (ceteris paribus) enables the bank to withstand a greater adverse shock, for example a higher number of non-performing loans, and remain solvent. As a result, $R_{b,t+1}$ is weakly decreasing in $\eta_{t+1}$. In an extreme case, when the bank is over-capitalized such that it is able to cover its exposure to risky loans with capital alone ($\omega_{t+1} < \eta_{t+1}$), then $R_{b,t+1}$ is equal to zero and is constant in $\eta_{t+1}$. Conversely, a higher loan-to-asset ratio (ceteris paribus) worsens bank's ability to withstand adverse outcomes and therefore $R_{b,t+1}$ is weakly increasing in $\omega_{t+1}$. Similarly, in an extreme case when the bank is over-capitalized then $R_{b,t+1}$ is equal to zero for any $\omega_{t+1} < \eta_{t+1}$.

The threshold loan rate $R_{b,t+1}$ is increasing in deposit rate because a higher deposit rate increases interest payments to bank's debt holders and hence increases the likelihood of ending up in a default outcome. Conversely, $R_{b,t+1}$ is decreasing in IOER because a higher IOER contributes as an interest income to bank and extends its ability to meet its liabilities. Interestingly, $R_{b,t+1}$ is independent of bank's balance sheet size $A_{t+1}$ in a special case when the return on bank lending exhibits a constant return to scale (CRS).[14] Intuitively, this implies

[14]When, however, the loan section exhibits a decreasing return to scale such that larger scale is associated

that the bank may choose any balance sheet size but the key driver of its default depends on $\eta_{t+1}$, $\omega_{t+1}$, $R_{X,t+1}$ and $R_{D,t+1}$ only, because the compositions inside the balance sheet determines ability to withstand adverse outcomes for any arbitrary balance sheet size.

The bank faces bankruptcy when its end-of-period revenues $R_{p,t+1}A_{t+1}$ is strictly less than its outstanding liabilities $R_{D,t+1}\mathcal{D}_{t+1}$. The probability of default depends on the properties of aggregate shock to bank's borrowers who repay their own liabilities to the bank:

$$\mathbb{P}\left(\text{Default}_{t+1}\right) = 1 - \mathbb{P}\left(R_{p,t+1}A_{t+1} \geq R_{D,t+1}\mathcal{D}_{t+1}\right) \tag{3.3.18}$$

In a default state, realized loan rate is strictly less than the threshold[15] $R_{b,t+1}$ and subsequently the bank is forced into bankruptcy and its proceeds are distributed to the debt holders on pro rata basis[16]. Limited liability condition prevents equity investors to internalize losses beyond their initial equity investments which indicates that in any default state, the bank is subsequently unable to fully compensate its debtors and the risk is partially passable to deposit accounts. This introduces the possibility of Diamond-Dybvig financial panic where depositors may start to withdraw their funds in anticipation of a potential default. Deposit insurance offered by the regulator rules out this specific financial panic by promising depositors a guarantee on their risk-free investments.

The bank solves the problem in (3.3.9) by choosing first, total balance sheet size ($A_{t+1}$) and funding composition $\eta_{t+1}$ given the price of equity and deposit rate.[17] The solution to the bank problem on the funding side thus are two demand functions or 'twin demands' for capital that are jointly determined by the price of equity, deposit rate, and also asset allocation choice $\omega_{t+1}$ from the assets side of the bank balance sheet. The bank trades with the households to pin down equilibrium capital structure and their prices, given any $\omega_{t+1}$. Third the bank considers IOER and the expected loan rate to pin down its portfolio allocation which overall solve the bank problem.

For tractability, I assume that the bank grants loans to borrowers who have no alternative access to financing and engage in production activities in a non-financial sector. This

with lower effective return per unit, the balance sheet size matters to $R_{b,t+1}$ because larger $A_{t+1}$ implies lower return on loan section which accordingly limits bank's ability to meet its liabilities for a given adverse outcome.

[15]For instance, a default threshold $R_{b,t+1} = 0.75$ indicates that the (minimum) net loan rate that a bank can withstand to remain solvent is $R_{b,t+1} - 1 = -25\%$. When the bank realizes an ex-post loan rate $R_{L,t+1} < R_{b,t+1}$ then its total assets value falls below total liabilities and has to declare bankruptcy.

[16]The bankruptcy definition is stipulated by the debt contract between the bank and its debt-holders. Particularly when the bank denies repaying debt-holders in full, it can be forced into bankruptcy. The term liquidation within this context refers to enforcing debt contract to seize bank's (end-of-period) assets in the event of bankruptcy. It is worth mentioning that in the discrete-time model presented in this section, after bankruptcy a new bank is set up and continues its service by raising debt and capital.

[17]Subject to boundary constraints to ensure interior or corner solution.

assumption maintains bank's central role to act as an intermediary between households and the ultimate borrowers, however, this also indicates that the non-financial sector is all-externally financed. First, the non-financial sector is subject to aggregate uncertainty and engages in a static production process which requires financing at the beginning of each period and pays off a stochastic outcome at the end of each period. Second, the aggregate uncertainty assumption implies that bank's lending to borrowers is non-diversifiable across the non-financial sector. The underlying loan contract between the bank and its borrowers stipulates that a loan is considered non-performing when the borrower fails to repay the original borrowed amount plus interest that is decided between two counter-parties ex-ante. In any default state, the bank is allowed to seize borrower's total assets which together with the non-diversifiable risk profile implies the bank's loan section is non-performing in that default state.

In this context, because the non-financial sector in unable to raise financing directly from the households, it is also unable to redistribute dividends (if any) to households in a solvency state, and the banks also receives any dividend from the non-financial sector which effectively implies the bank serves as the owner of the non-financial sector. The outcome of the non-financial sector is the real economic output that is consumed in the goods market by the households.

The bank faces a log-normally distributed shock per unit of investment in the loan section with the following Cobb-Douglas production technology that is subject to an exogenous aggregate shock $z_{t+1}$,

$$h(L_{t+1}, z_{t+1}) = z_{t+1} L_{t+1}^{\alpha} \tag{3.3.19}$$

where $\log z_t = \mu_z + \sigma_z \epsilon_{t+1}$, $\epsilon_t \sim \mathcal{N}(0,1)$ and $\alpha \in (0,1]$.

### 3.3.3 Financial Regulator

The financial regulator provides the following services: determines a welfare-maximizing IOER, offers deposit insurance, sets an exogenous minimum risk-weighted capital requirement, and accepts reserve deposits from the banking sector described in Figure (3.3). Deposit insurance is a guarantee that compensates depositors in full in default states. The minimum risk-weighted capital requirement considers a welfare maximizing objective that internalizes costly bankruptcy that both the household and banking sectors fail to internalize.[18] Lastly, accepting deposits from the banking system is a form of reserves deposit facility.

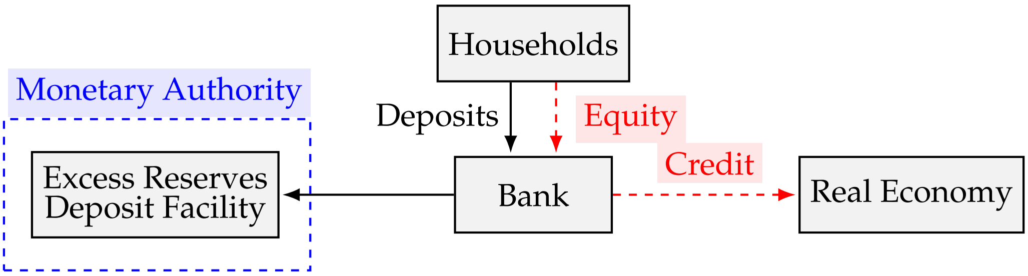



**Figure 3.3:** The diagram illustrates sectors in the economy. The households invest in bank equity and deposit their funds into bank deposit facility as a risk-free investment. The bank is given two investment opportunities: channel funds to the real economy as a loan, and hold a share of its funds in the excess reserves deposit facility provided by the financial regulator.

The banking sector described in the previous section is only concerned with the solvency region. However, bank's capital structure includes funding that is raised through debt contracts which allows debt holders to force the bank into bankruptcy due to inability to honor debt contracts in full. **?** estimates that a bankruptcy process is associated with 30% loss of bank's total assets due to legal and liquidation proceedings. Similarly, **?** and **?** show that bankruptcy cost can vary between 10% to 23% of total assets within non-financial firms and between 15% to 30% of total assets for financial firms. **?**; **?** and **?** provide comprehensive studies that examine bankruptcy cost according to several measurements and show that in some cases these costs can account for more than 30 cents on the dollar.

In this context, bankruptcy cost is denoted by $\chi \in (0,1)$ that characterizes a proportional fraction of banking sector's total assets that is lost due to bankruptcy process when a default occurs, alternatively, the fraction $1-\chi$ is characterises bank's ex-post asset recovery rate. The financial regulator is concerned with balancing the welfare gains of channelling maximal funds to the real economy against the welfare cost of defaults within the banking sector. The policymaker's objective is to determine the optimal IOER that solves the following social welfare problem:

$$\max_{Q_{X,t}} \mathbb{E}_0 \left[ U(C_t, \mathbb{E}_t U_{t+1}) \right] \tag{3.3.20}$$

[18]This set-up assumes complete information. The bank's objective is set up to only consider solvency region. However, the households are concerned about social costs of the bankruptcy. Because the regulator shares the same objective function as that of the households, therefore the households does not need to be concerned about bankruptcy and its associated social costs.

subject to,

$$P_{C,t}C_t + D_{t+1} + E_{t+1} = (1-\tau_{t+1})\left(\frac{1-\overline{\eta}_{t+1}}{Q_{D,t}} + \overline{\eta}_{t+1}(1-\kappa)R_{E,t}\right) + Tr_{t+1} \tag{3.3.21}$$

and $\overline{\eta}_{t+1} \in [0,1]$ where the transfer function is,

$$\frac{\mathrm{Tr}_{t+1}}{W_t} = \begin{cases} \tau_{t+1} - (1-\tau_{t+1})(1-\omega_{t+1})r_{X,t+1} & \text{if } z_{b,t+1} \leq z_{t+1} \text{ (non-default)} \\ \tau_{t+1} - (1-\tau_{t+1})(1-\omega_{t+1})r_{X,t+1} - \Lambda_{t+1} & \text{if } z_{s,t+1} \leq z_{t+1} < z_{b,t+1} \text{ (default)} \\ 0 & \text{if } z_{t+1} < z_{s,t+1} \text{ (inadequate deposit insurance} \end{cases}$$

where the term $\Lambda_{t+1}$ denotes uncovered share of debt contracts (uncompensated deposits in relation to the whole deposits plus promised interests) from the banking sector,

$$\Lambda_{t+1} = (1-\tau_{t+1}).\left(\frac{1-\eta_{t+1}}{Q_{D,t}} - \chi.R_{p,t+1}A_{t+1}\right)$$

where $r_{X,t+1} \equiv R_{X,t+1} - 1 = 1/Q_{X,t} - 1$ and that $r_{X,t+1} \lesseqgtr 0$ is the net IOER offered on reserves deposit facility offered by the regulator to the banking sector, and $\overline{\eta}_{t+1}$ is the minimum (risk-weighted) capital requirement set on the banking sector.

First, social welfare function in (3.3.20) is identical to present value of the utility function of the households which regulator maximizes considering regulatory tools available in this context. Equation (3.3.21) characterizes regulators resource constraint that internalizes transfers to households.

Second, the regulator raises funds through a proportional taxation[19] $\tau_{t+1}$ from the households. These funds are available to the regulator to offer deposit insurance[20] in a default state and to cover interest expenses on reserves when IOER are positive. The transfer function has no interaction with interest-on-reserves when IOER is zero. When IOER is negative, then reserves deposit facility provide an interest income to the regulator since the proportion of reserves $1-\omega_{t+1}$, scaled by after tax resources $1-\tau_{t+1}$ earns interest income when $r_{X,t+1} < 0$.

Third, the regulator considers three possible outcome intervals when considering the transfer. The non-default region is characterized by the aggregate shock outcome $z_{b,t+1} \leq$

[19]The notion of taxation in this context is to simplify the analysis since regulator uses these funds for the purpose of deposits insurance, which indicates that taxes serves as a ex-ante premium, and also to pay interest payments.

[20]Deposit insurance fee can be charged directly from the banking sector, however, in this context with a representative households sector and banking sector, applying the charges directly to household offers tractability.

$z_{t+1}$ specifying that the banking sector remains solvent. The default region is characterized by $z_{s,t+1} \leq z_{t+1} < z_{b,t+1}$ specifying that due to realizing an large adverse shock, the banking sector's total assets falls below its debt liabilities. In this case, the bank defaults and its post bankruptcy proceeds are described by $\chi.R_{p,t+1}A_{t+1}$. The regulator compensates depositors out of its available resources which implies that although deposits are risk-free, households receive a smaller transfer. From a welfare perspective, the regulator considers fraction $(1 - \chi).R_{p,t+1}A_{t+1}$ as a deadweight loss that is socially undesirable to the economy.

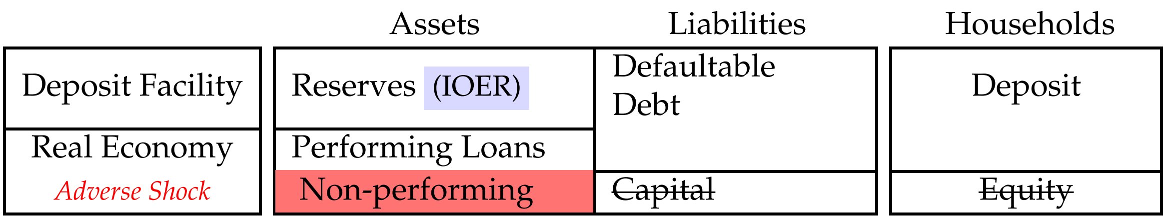
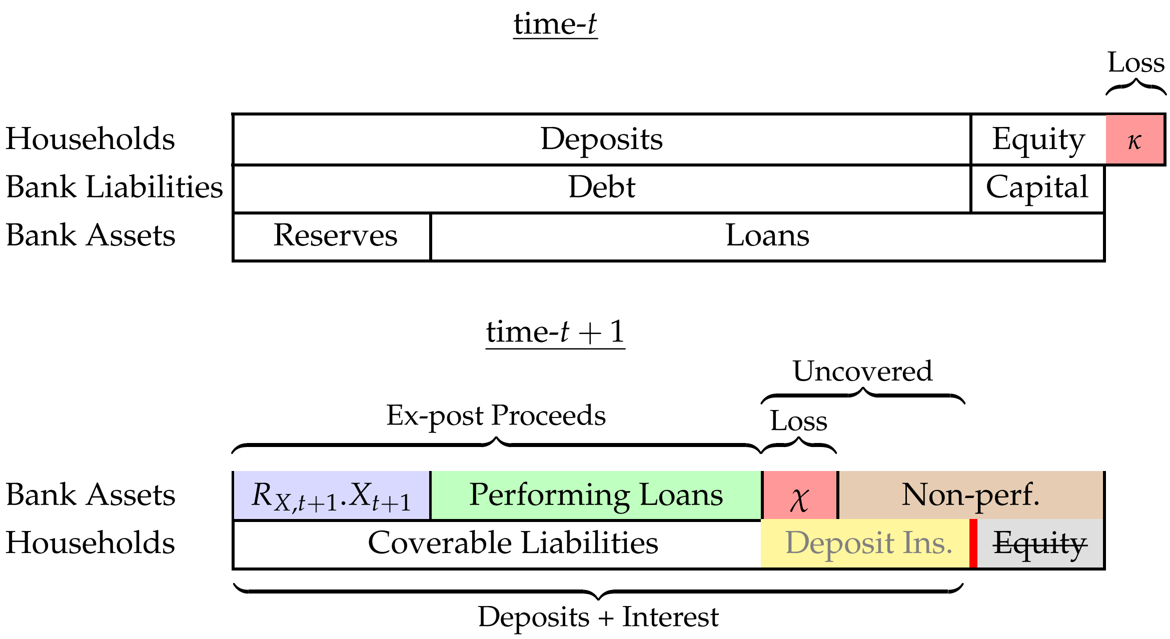



**Figure 3.4:** The diagram describes flow of funds from households (deposit and equity) to the banking sector's liabilities (debt and capital) which subsequently is channelled to the real economy (lending) and excess reserves deposit facility. Policymakers services in charge of financial regulation and interest-on-excess-reserves is highlighted in the diagram. Premium is the taxation that policymaker raises at the beginning of each period, in anticipation of any defaults in the banking sector, to provide government guarantees to depositors. This resource also serves as a fund to pay any positive (negative) interest expenses on excess reserves deposit facility.

Fourth, the choice of taxation is taken as given and the solution section considers the following two possible cases. When taxation is sufficiently large enough to provide full insurance on deposits. This case requires taxes to be equal to deposits (plus promised interest) less the reserves (plus interests) such that any resulting uncovered deposits within the banking sector can be covered by the taxes and reserves, for example, in an extreme case when the entire loan section of the banking sector is eliminated due to an adverse large shock. However, when taxation is insufficient to cover deposits in real terms, the regulator can offer only partial insurance on deposits.[21] Figure (3.4) illustrates the role of the financial sector within the financial system. The premium (taxes) are raised from the households to finance deposit insurance. The regulator also is in charge of two policies across banking system's

[21]The model in this context assumes that deposit insurance is financed endogenously from the taxation. Nonetheless, taxes are taken as an exogenous policy. The size of optimal deposit insurance matters for welfare analysis since because an under-funded deposit insurance is unable to provide compensation for depositors in full when failures are widespread. Nonetheless, putting aside large quantities of funds in anticipation of unlikely widespread failures is socially costly. The seminal work by **?** followed by recent studies within the context of government guarantees (**?**, **?**) review this question and provide an optimal deposit insurance funding level that is socially desirable.




balance sheet that interact with each other. The following section describes the relationship between the risk-weighted capital regulation and IOER in details.

## 3.4 Laissez-faire Intermediation

A welfare analysis based on the laissez-faire allocations provides a framework to measure social costs associated with the each distortion. The optimal behaviour of the households who are the providers of financing to the financial sector and ultimately the real economy are described in the following sub-section, followed by the optimal decision of the banking sector to raise financing in sub-section (3.4.2).

### 3.4.1 Supply of Financing

The household maximizes expected utility of future consumption stream subject to the intertemporal budget constraint. At each date-$t$, the household chooses optimal consumption and portfolio choice. The first order condition with respect to consumption yields the following Euler equation,

$$1 = \mathbb{E}_t \left[ M_{t,t+1} R_{W,t+1} \right] \tag{3.4.1}$$

where

$$M_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\rho\frac{\gamma-1}{1-\rho}} \frac{V_{t+1}}{[\mathbb{E}V_{t+1}^{1-\theta}]^{\frac{1}{1-\theta}}} \tag{3.4.2}$$

denotes household's stochastic discount factor. Return on household's wealth includes both the equity and deposit returns in the solvency state and the deposit income only in the default state. Consumption-saving policy function is constant over time when the stochastic process governing equity return is i.i.d. therefore I conjecture that the consumption policy function $C_t = (1-\varphi)R_{W,t}W_t$ solves the intertemporal problem as a special case with i.i.d. uncertainty[22] where $\varphi$ is the marginal propensity to save (MPS). Solving for the value of MPS gives the following investment-to-wealth ratio in logarithmic units:

$$\log(\text{MPS}_{t+1}) = \psi \log(\beta) + \frac{1-\psi^{-1}}{\psi^{-1}}\left[\mathbb{E}_t r_{W,t+1}(\theta_{t+1}) + \frac{1}{2}(1-\gamma)\sigma^2_{r_W}\right] \tag{3.4.3}$$

This ratio is positively related to investor's subjective discount factor or impatience parameter $\beta$, such that higher patience implies higher saving if $\psi < 1$ and $\gamma > 1$. The first order condition with respect to portfolio choice $\theta_{t+1}$ is given by:

$$\theta^*_{t+1} = \underbrace{\frac{\mathbb{E}_t \log R_{E,t+1} - \log R_{D,t+1} + \sigma^2_E/2}{\gamma\sigma^2_E}}_{\text{Merton's myopic demand}} + \underbrace{\frac{1}{\gamma\sigma^2_E}\log\Phi(R_{E,t+1} > 0)}_{\text{default disincentive}} \tag{3.4.4}$$

The first term on the right-hand-side describes Merton's (rational) myopic allocation to risky asset.[23] The second term denoted by $\Phi(.)$ characterizes the role of endogenous defaults and is measured by the probability of solvency of the underlying risky asset issuer that the household holds which appears in logarithmic units. This term is a negative factor to lower household's investment when defaults are possible.

However, as the likelihood of solvency increases, the demand for the risky asset increases. In the limiting case when the underlying issuer is solvent in all states, $\log\Phi(.)$ is zero showing that household's demand simplifies to that of the Merton's model when a default is ruled out. The optimal total investment (3.4.3) together with (3.4.4) fully characterize household's decisions to supply financing to the banking sector in the form of deposit

[22]The conjecture follows the results in **?** and **?**.
[23]Investment in risky asset does not depend on elasticity of intertemporal substitution when the shock to economy follows an i.i.d. process.

and equity, given the deposit rate and the price of the equity:

$$
\begin{aligned}
D_{t+1}(Q_{D,t}, P_{E,t}) &= \left[\mathrm{MPS}_{t+1}(\theta^*_{t+1}) \times (W_t - C^*_t)\right] \times (1-\theta^*_{t+1}) \\
E_{t+1}(Q_{D,t}, P_{E,t}) &= \left[\mathrm{MPS}_{t+1}(\theta^*_{t+1}) \times (W_t - C^*_t)\right] \times \theta^*_{t+1}
\end{aligned}
$$

where the supply of funds to the deposit market increases in deposit rate but decreases in equity return. Conversely, equity investment falls as the price of equity increases or the deposit rate increases.

### 3.4.2 Demands for Financing

The risk-neutral expected present value problem in (3.3.9) indicates that bank's funding and asset allocation decisions affect the following two channels:[24] first, the bank considers cost of capital when raising funds from the capital markets in order to maximize its profit. Second, extended allocation of funds to loans increases bank's cashflow. However, a high loan-to-assets ratio or a low equity-to-assets ratio decrease the possibility of remaining solvent which lower bank's profit through the expectation channel. Approximating the problem in (3.3.9) to separate the expectation (probability) channel from the (discounted) dividend channel gives:

$$
\max_{\theta_{t+1}, A_{t+1}, \omega_{t+1}} \quad \underbrace{\Phi\left[\lambda\left(R_{b,t+1}\right)\right]}_{\text{Probability Channel}} \quad \times \quad \underbrace{\mathbb{E}_t\left[M_{t,t+1} div_{t+1}\right]}_{\text{Discounted Dividend Channel}} \tag{3.4.5}
$$

where the first term quantifies the explicit probability of solvency and the second term quantifies the discounted dividend.[25] The logarithmic quantile[26]

$$
\lambda(R_{b,t+1}) = \frac{\mu_z + \sigma_z^2 - \log(R_{b,t+1})}{\sigma_z} \tag{3.4.6}
$$

henceforth $\lambda_{t+1}$, is associated with log-normally distributed loan rate threshold $R_{b,t+1}$. First, because $R_{b,t+1}$ is weakly decreasing in $\eta_{t+1}$ (ceteris paribus), then $\Phi\left(\lambda_{t+1}\right)$ is weakly increasing in $\eta_{t+1}$ indicating that a higher equity-to-assets ratio increases the probability of

[24] Subject to balance sheet constraint (3.3.10) and boundary constraints (3.3.12).

[25] Doing so implies that (first) dividend is treated as a random variable without max[.] operator and can be both positive or negative, (second) $\mathbb{E}_t[.]$ is over both solvency and default regions. As an illustration consider that the kink on $\max[div, 0]$ function over the state space is the exact break-even loan rate

[26] Because the loan is log-normally distributed, the problem can be written in terms of a standard Normal cumulative distribution function with standardized logarithmic quantiles.

solvency. This is because a higher equity-to-assets ratio lowers break-even threshold $R_{b,t+1}$ which corresponds to a lower standardized quantile $\lambda(.)$. Note that both functions $\Phi(.)$ and $\lambda(.)$ are strictly monotonic in their arguments.

Second, because $R_{b,t+1}$ is weakly increasing in $\omega_{t+1}$ (ceteris paribus), then $\Phi(\lambda_{t+1})$ is weakly decreasing in $\omega_{t+1}$ indicating higher loan-to-assets ratio lowers the probability of solvency. The second term in (3.4.2) can be written as,

$$M_{t,t+1}div_{t+1} = M_{t,t+1}\Bigg[\underbrace{\frac{1-\omega_{t+1}}{Q_{X,t}}A_{t+1}}_{\text{Reserves plus IOR}} + \underbrace{\omega_{t+1}z_{t+1}A_{t+1}}_{\text{Loan plus interest}} - \underbrace{\frac{1-\eta_{t+1}}{Q_{D,t}}A_{t+1}}_{\text{Deposit Financing}} - \underbrace{\frac{\eta_{t+1}}{P_{E,t}}A_{t+1}}_{\text{Equity Investment}}\Bigg] \quad (3.4.7)$$

where $Q_{D,t} = 1/R_{D,t+1}$ and $Q_{X,t} = 1/R_{X,t+1}$ are the prices of deposits and reserves, respectively. The constant return to scale technology implies that $A_{t+1}$ does not affect the probability channel and the risk-neural property implies that balance sheet size is linear in dividend channel.

**First-Order-Condition (Balance Sheet Size)** The first order condition of bank problem with respect to $A_{t+1}$ is given by,

$$0 = \left[\frac{\partial}{\partial A_{t+1}}\Phi(\lambda_{t+1})\right].\mathbb{E}_t\left[M_{t,t+1}div_{t+1}\right] + \Phi(\lambda_{t+1}).\frac{\partial}{\partial A_{t+1}}\mathbb{E}_t\left[M_{t,t+1}div_{t+1}\right] \quad (3.4.8)$$

decomposition in (3.4.2) results in the product rule in the first order condition above that tracks in impact of balance sheet size on marginal changes in present value of dividend, keeping probability of solvency constant, and marginal changes in probability of solvency while keeping the dividend channel constant. Re-arranging (3.4.16) gives:

$$\frac{\partial}{\partial A_{t+1}}\log\Phi(\lambda_{t+1}) = \frac{\partial}{\partial A_{t+1}}\log\mathbb{E}_t\left[M_{t,t+1}div_{t+1}\right] \quad (3.4.9)$$

an optimal balance sheet size decision $A^*(\eta_{t+1}, \omega_{t+1}, P_{E,t}, Q_{D,t}; Q_{X,t})$ by bank that solves problem (3.4.2) trades off percentage change in probability of solvency[27] $\%\Delta_A\Phi(.)$, against percentage change in expected present value of dividend $\%\Delta_A\mathbb{E}_t[M_{t,t+1}div_{t+1}]$.

First, the probability channel always motivates the bank to choose a smaller balance sheet size due to decreasing return to scale feature of the loan section. This is indicated by the sign of the term $\%\Delta_A\Phi(.)$ that is always negative for any balance sheet size. As the bank increases

[27] A positive (negative) but constant $\%\Delta_A\Phi(.)$ indicates that probability of solvency increases (decreases) at a fixed rate, and when $\%\Delta_A\Phi(.)$ is zero then probability of solvency remains fixed.

its balance sheet size, the marginal loan rate falls which reduces its ability to meet deposit expenses. Further, $\%\Delta_A \Phi(.)$ is increasing in price of deposit and price of equity because higher funding prices lower cost of financing, for example, when the bank is able to raise debt through deposits at a lower deposit rate then it faces a higher $\%\Delta_A \Phi(.)$ which indicates that the balance sheet size can increase on the margin.

Similarly, when the degree of decreasing return to scale ($\alpha$) falls, the probability channel become a stronger motivation to decrease balance sheet size because a lower $\alpha$ reduces marginal loan rate. As a special case when $\alpha = 1$ the probability channel become irrelevant to bank's decision making because the choice of balance sheet is independent of marginal return from loan section. In this special case the first order condition with respect to size only interacts with the dividend channel and the probability of solvency remains constant for any choice of size. Intuitively, this case indicates that the solvency is only driven by the composition of components inside the balance sheet and not the size itself and therefore any size is therefore optimal. More formally, the expectation operator[28] on the right hand size of equation (3.4.9) does not depend on endogenous variables and that the bank optimal decisions takes $M_{t,t+1}$ as given then,

$$0 = \Phi\Big[\lambda\big(R_{b,t+1}\big)\Big]\mathbb{E}_t\left[M_{t,t+1}\frac{\partial}{\partial A_{t+1}}div_{t+1}\right] \tag{3.4.10}$$

Since probability of solvency is always strictly positive because for any equity-to-assets and loan-to-asset ratios the bank can always remain solvent for an arbitrarily large loan rate outcome, then:

$$0 = \mathbb{E}_t\left[M_{t,t+1}\frac{\partial}{\partial A_{t+1}}div_{t+1}\right] \tag{3.4.11}$$

which results in the following first order condition that indicates, on the margin, the expected present value of cost of financing should be equal to the expected present value of one unit of investment return from bank's portfolio,

$$\mathbb{E}_t\left[M_{t+1}\left(\frac{1-\omega_{t+1}}{Q_{X,t}}+\omega_{t+1}z_{t+1}\right)\right] = \mathbb{E}_t\left[M_{t+1}\left(\frac{1-\eta_{t+1}}{Q_{D,t}}+\frac{\eta_{t+1}}{P_{E,t}}\right)\right] \tag{3.4.12}$$

the balance sheet size is always at its optimum when PV of financing cost equal PV of portfolio return. When, however, the PV of financing cost is greater than that of the portfolio return, the bank chooses balance sheet size equal to zero and when the PV of financing cost

[28] Integral boundaries are the support for random variable over $[0, \infty)$.

is lower than that of the portfolio return the bank choose a size that grows without bounds. Equilibrium mechanism, however, specifies that equation (3.4.12) must hold with equality which then establishes a condition between the prices of deposits, equity and reserves (and moments of loan).

In a more general case when $\alpha \in (0, 1)$, right-hand-side of equation (3.4.9) summarizes the effect of dividend channel with the term $\%\Delta_A \mathbb{E}_t[M_{t,t+1} div_{t+1}]$. Specifically, this term is monogenically decreasing in size because as the balance sheet grows (absent probability channel) lower marginal rate from loan section reduces the expected value of profit in resent value terms. The term $\%\Delta_A \mathbb{E}_t[M_{t,t+1} div_{t+1}]$ is very large when size is small and begins to fall as the size increases. When the marginal loan rate, together with bank's income from reserves become equal to cost of financing then $\%\Delta_A \mathbb{E}_t[M_{t,t+1} div_{t+1}]$ is zero which corresponds to the maximum present value of bank profit. Any further increase in the size beyond this limit amounts to a negative expected profit.

Further, the bank faces lower cost of financing when price of deposit and equity increase which accordingly enable the bank to increase the balance sheet size that is associated with a lower marginal loan rate. In a special case, when $\alpha = 1$ the dividend become linear in size which implies that the bank faces an indeterminate choice with respect to size. In this case, the expected return on bank portfolio must be equal to expect cost of financing, otherwise the optimal size increases without bound when investing in portfolio is always marginally more profitable than marginal cost of financing, or the size is zero when expected portfolio return is lower than cost of financing.

The solution to first-order-condition (3.4.9) is a unique choice of balance sheet that equates percentage change in probability of solvency and percentage change in expected dividend channel. When $\%\Delta_A \Phi(.) < \%\Delta_A \mathbb{E}_t[M_{t,t+1} div_{t+1}]$ bank is able to increase the size to obtain more profit at the expense of lowering the probability of solvency. When $\%\Delta_A \Phi(.) > \%\Delta_A \mathbb{E}_t[M_{t,t+1} div_{t+1}]$ then the balance sheet must shrink such that the solvency increases at the expense of lower dividend. Since $\%\Delta_A \Phi(.)$ is always negative and $\%\Delta_A \mathbb{E}_t[M_{t,t+1} div_{t+1}]$ is monotonically decreasing in size, the optimal balance sheet size in a general case when $\alpha \in (0, 1)$ is always smaller than the case when $\alpha = 1$.

Before discussing the optimal capital structure choice it is worth examining the relationship between optimal size and any funding composition on the liabilities side. Higher choice of equity-to-asset ration $\eta_{t+1}$ increases bank's ability to withstand more adverse shock outcomes thus $\%\Delta_A \Phi(.)$ is increasing in $\eta_{t+1}$ which indicates that the bank can increase its balance sheet size when its equity-to-assets ratio increases (ceteris paribus).

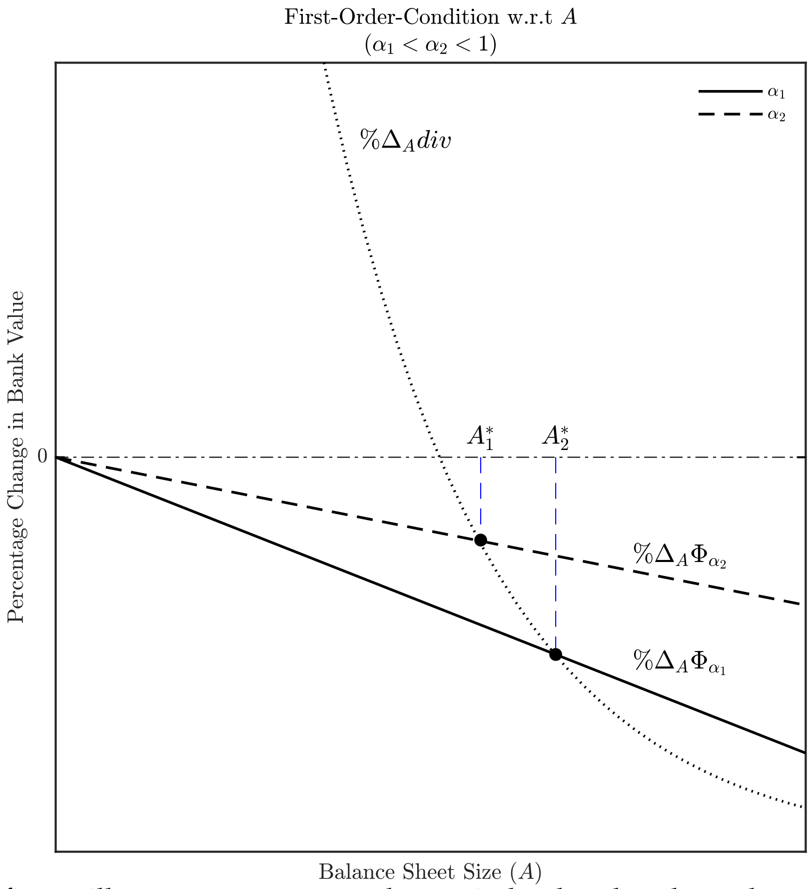



**Figure 3.5:** This figure illustrates percentage change in bank value through cashflow and solvency components when balance sheet size changes. The dotted lines shows that as balance sheet size grows, bank value increases at a decreasing rate when $\%\Delta_A div > 0$. When $\%\Delta_A div = 0$ increasing balance sheet size amount to no changes in cashflow channel. The solid line shows the solvency effect of increasing bank balance sheet size on its value

**First-Order-Condition (Capital Structure)** The first order condition with respect to capital structure is given by,

$$0 = \left[\frac{\partial}{\partial \eta_{t+1}} \Phi\left(\lambda_{t+1}\right)\right] . \mathbb{E}_t\left[M_{t,t+1} div_{t+1}\right] + \Phi\left(\lambda_{t+1}\right) . \frac{\partial}{\partial \eta_{t+1}} \mathbb{E}_t\left[M_{t,t+1} div_{t+1}\right] \tag{3.4.13}$$

using the decomposition in (3.4.2), the expression above is re-arranged as the following,

$$\frac{\partial}{\partial \eta_{t+1}} \log \Phi\left(\lambda_{t+1}\right) = \frac{\partial}{\partial \eta_{t+1}} \log \mathbb{E}_t\left[M_{t,t+1} div_{t+1}\right] \tag{3.4.14}$$

where similar to the previous part left-hand-side summarizes percentage change in probability channel due to changes in equity-to-asset ratio $\%\Delta_\eta \Phi(.)$. As the bank increases $\eta_{t+1}$ probability of solvency increases because higher equity-to-asset ratio increases bank's ability to withstand adverse shock outcomes. Formally, this effect is captured by the sign of the term $\%\Delta_\eta \Phi(.)$ that is always positive for any choice of $\eta_{t+1}$. Further, increasing equity-to-asset ratio monotonically improves the chance of solvency however, when the bank is overcapitalised[29] the marginal gain in probability of solvency is very small and defaults are very rare. This is reflected by the slope of $\%\Delta_\eta \Phi(.)$ which is decreasing in $\eta_{t+1}$, specifically, when $\eta_{t+1}$ is very small, the percentage change in probability of solvency is large because each additional unit of equity can considerably lower defaults.

As $\eta_{t+1}$ increases, $\%\Delta_\eta \Phi(.)$ decreases upto the point at which $\%\Delta_\eta \Phi(.)$ become very close to zero showing that the probability of solvency is reaching one[30] . Increasing equity-to-asset ratio beyond this limit has to impact on the solvency channel and as a result $\%\Delta_\eta \Phi(.)$ is weakly decreasing in $\eta_{t+1}$. Furthermore, $\%\Delta_\eta \Phi(.)$ is highly dependant on the price of deposits as the end-of-period interest expenses is an important determinant whether the bank remains solvent. Thus $\%\Delta_\eta \Phi(.)$ is decreasing in the price of deposit because the bank is able to withstand relatively more adverse shock when deposit interest expenses fall. Interestingly, the term $\%\Delta_\eta \Phi(.)$ is independent of the price of equity because defaults is only driven by debt contracts.

The right-hand-side of equation (3.4.14) summarizes the effect of capital structure choice on expected present value of bank profit. In particular, $\%\Delta_\eta \mathbb{E}_t[M_{t,t+1} div_{t+1}]$ is negative and monotonically decreasing[31] in $\eta_{t+1}$ when $P_{E,t} < Q_{D,t}$ as the bank considers equity more expensive relative to deposits due to its riskiness. When $\%\Delta_\eta \Phi(.) < \%\Delta_\eta \mathbb{E}_t[M_{t,t+1} div_{t+1}]$ is a driver to increase $\eta_{t+1}$ which results in lower expect present value of dividend but increases the probability of solvency. When $\%\Delta_\eta \Phi(.) = \%\Delta_\eta \mathbb{E}_t[M_{t,t+1} div_{t+1}]$ the bank balances the marginal contribution of equity to solvency against expected dividend.

The marginal contribution of equity to expected economic profit through probability channel is a factor that bids up equity price against deposits price from bank's perspective. As the equity become mores scarce, the bank is willing to accept lower price today as equity's marginal probability contribution is very high. Solving equation (3.4.14) for $\eta^*_{t+1}$

[29] Over-capitalization is a relative term with respect to loan-to-asset ratio discussed in the next subsection. However, it is innocuous in to assume a bank is overcapitalised when its equity-to-asset and loan-to-asset ratios are close to each other which reflect a bank that hold enough equity to withstand a very large adverse shock to loans and remain solvent.

[30] In this case, the break-even threshold $R_{b,t+1}$ is equal to zero indicating that there is no possible loan outcome that set bank's total assets below its total liabilities. Note that when $R_{b,t+1} = 0$ (net ex-post loan rate is $-100\%$ i.e. all of bank loan section disappears in the extreme case) then $\lim \lambda(R_{b,t+1}) = \infty$ and the associated CDF is equal to one in the limit.

[31] The assumption $P_{E,t} < Q_{D,t}$ relies on equilibrium outcome discussed in the subsequent sections but since suppliers of funding are risk-averse, then it is reasonable to restrict the discussion to cases in which price of equity is always below the price of deposits.

gives,

$$\eta^*_{t+1} \quad = \quad \eta^*(A_{t+1}, \omega_{t+1}, P_{E,t}, Q_{D,t}; Q_{X,t}) \tag{3.4.15}$$

which is bank's optimal capital structure for any $Q_{D,t}$ and $P_{E,t}$. Particularly, $\eta^*_{t+1}$ specifies that when the price of equity at date-$t$ increases (ceteris paribus), bank increases its demand for equity financing because it is able to raise more funding per share. Conversely, when the price of deposit increases (ceteris paribus) the bank lowers $\eta^*_{t+1}$ as equity becomes relatively more expensive than deposit financing and bank shifts its liabilities towards more debt[32]. Bank's funding decision is fully characterized by equations (3.4.9) and (3.4.14) which are solved for in equilibrium for deposit and equity prices in the following subsection.

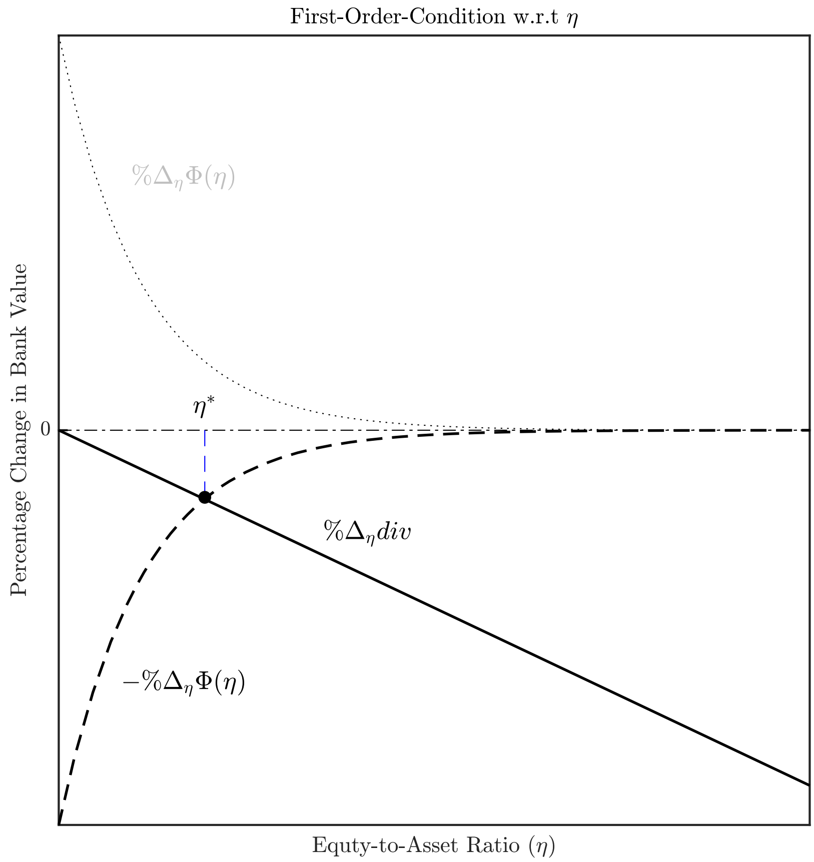



**Figure 3.6:** The figure illustrates percentage change in bank's value through cashflow and probability components. The solid line illustrates that bank's value falls when bank raises further financing through equity when equity is more costly than debt financing. The dashed line illustrates (times a negative to depicts first-order-condition) that bank's value increases through higher likelihood of solvency but at a decreasing rate because each additional unit of equity provide lower marginal contribution to solvency likelihood.

[32]In an extreme case when the deposits (equity) price is very high, the bank finds optimal to raise more debt (equity) even outside $\eta^*_{t+1} \in [0, 1]$ interval. These cases are discussed later and eliminated as the funding composition can include zero equity at the very least.

**First-Order-Condition (Asset Allocation)** The first order condition with respect to capital structure is given by,

$$0 = \left[\frac{\partial}{\partial \omega_{t+1}} \Phi\left(\lambda_{t+1}\right)\right] . \mathbb{E}_t\left[M_{t,t+1} div_{t+1}\right] + \Phi\left(\lambda_{t+1}\right) . \frac{\partial}{\partial \omega_{t+1}} \mathbb{E}_t\left[M_{t,t+1} div_{t+1}\right] \quad (3.4.16)$$

using the decomposition in (3.4.2), the expression above is re-arranged as the following,

$$\frac{\partial}{\partial \omega_{t+1}} \log \Phi\left(\lambda_{t+1}\right) = \frac{\partial}{\partial \omega_{t+1}} \log \mathbb{E}_t\left[M_{t,t+1} div_{t+1}\right] \quad (3.4.17)$$

the left-hand-side summarizes percentage change in probability channel due to changes in loan-to-asset ratio $\%\Delta_\omega \Phi(.)$. As the bank increases $\omega t + 1$ probability of solvency decreases because higher loan-to-asset ratio increases exposure to shock outcomes and lowers bank's ability to withstand adverse shock outcomes. Formally, this effect is captured by the sign of term $\%\Delta_\omega \Phi(.)$ that is always negative for any choice of $\omega t + 1$. Further, increasing loan-to-asset ratio monotonically worsen chance of solvency however, when the bank is overcapitalised the marginal gain in probability of solvency is very small and defaults are very rare. This is reflected by $\%\Delta_\omega \Phi(.) = 0$ when $\omega_{t+1} \leq \eta_{t+1}$. As $\omega_{t+1}$ increases, $\%\Delta_\omega \Phi(.)$ increases monotonically reflecting growing chance of default due further exposure to aggregate shock.

The right-hand-side of equation (3.4.17) summarizes the effect of asset allocation on expected present value of bank profit. When $\%\Delta_\omega \Phi(.) < \%\Delta_\omega \mathbb{E}_t[M_{t,t+1} div_{t+1}]$ the bank increases $\omega t + 1$ which results in lower expect present value of dividend at the expense of increasing the probability of default. When $\%\Delta_\omega \Phi(.) = \%\Delta_\omega \mathbb{E}_t[M_{t,t+1} div_{t+1}]$ the bank balances the marginal contribution of asset allocation (to loan) to solvency against expected dividend. Solving equation (3.4.17) for $\omega^*_{t+1}$ gives, $\omega^*_{t+1} = \omega^*(A_{t+1}, \eta_{t+1}, P_{E,t}, Q_{D,t}; Q_{X,t})$ which is bank's optimal asset allocation choice for any $Q_{D,t}$ and $P_{E,t}$.

### 3.4.3 Laissez-faire Equilibrium

Market clearing conditions on the deposits and equity markets establish the following equilibrium conditions:

$$\underbrace{D_{t+1}(Q_{D,t}, P_{E,t}; Q_{X,t}, \omega_{t+1})}_{\text{Supply of Capital (household deposits)}} = \underbrace{\mathcal{D}_{t+1}(Q_{D,t}, P_{E,t}; Q_{X,t}, \omega_{t+1})}_{\text{Demand for Capital (bank debt)}} \quad (3.4.18)$$

$$\underbrace{E_{t+1}(Q_{D,t}, P_{E,t}; Q_{X,t}, \omega_{t+1})}_{\text{Supply of Capital (household equity)}} = \underbrace{\mathcal{E}_{t+1}(Q_{D,t}, P_{E,t}; Q_{X,t}, \omega_{t+1})}_{\text{Demand for equity (bank capital)}} \quad (3.4.19)$$

This first condition (3.4.19) clears the deposits market for a specific deposit prices,[33] give any price of equity, $Q_{D,t}(P_{E,t})$. Second, the resulting market clearing deposit price $Q_{D,t}(P_{E,t})$ is solved for jointly with the equity market clearing condition for a specific price of equity, given other variables that are determined outside the funding markets.

Because the household's valuation of deposit and equity arises endogenously, the equilibrium price of equity is determined by households' preferences for earning from bank dividend against its default risk. As bank extends lending, on the one hand its share price is bid up due to higher embedded cashflow but on the other hand, increased exposure to aggregate uncertainty lowers its expected share price through default risk. When the bank is highly leveraged, each additional unit of equity provides a sizable contribution to its net worth because default risk is relatively a more important driver of its share price. As bank's capital structure comprises further equity relative to total assets, marginal contribution of equity to reduce default risk diminishes and equity's higher cost relative to debt becomes a more important consideration for its net worth.

The general equilibrium framework in this section shows that equity premium compensation to risk-averse investor falls as the equity-to-asset ratio in bank capital structure increases. When the bank raises capital through the equity market, first, its share price falls due to a higher demand for capital because the equity investor requires compensation to forgo consumption. However, a fall in share price is less steep because a risk-averse equity investor prices lower riskiness of their investment.

## 3.5 Intermediation with Regulated Interest Rate

The interaction between the capital constraint and IOER has important welfare implications. The policymaker considers the welfare gains, in terms of consumption units, associated with change of IOER over the default region, against gains over the solvency region. The first order condition with respect to this interest rate instrument is given by:

$$0 = \frac{\partial}{\partial r_{X,t+1}} \left\{ \int_0^{z_{b,t+1}} \underbrace{M_{t,t+1}\left[ r_{X,t+1}(1-\omega_{t+1}) - \Lambda_{t+1} \right]}_{\substack{\text{discounted transfer value} \\ \text{in default}}} dF + \int_{z_{b,t+1}}^{\infty} \underbrace{M_{t,t+1}\left[ r_{X,t+1}(1-\omega_{t+1}) \right]}_{\substack{\text{discounted transfer value} \\ \text{in Solvency}}} dF \right\} \tag{3.5.1}$$

[33] Equation (3.4.12) from bank's first order condition with respect to balance sheet size determines a relationship between the price of deposits in terms of price of equity.

where the first term on the right-hand-side shows the (marginal) social value of a unit of transfer given changes in IOER in default. Regulator considers the following trade off: in any default state, higher IOER increases ex-post liquidation proceeds within the banking sector which implies a lower deadweight loss in that state. However, IOER is financed from taxation indicating that transfer to households falls as IOER rate paid on excess reserves increases. The regulator considers the aforementioned opposable effects to determine trade-offs over the default region. In solvency, the regulator is only concerned with lowering interest expenses associated with excess reserves because the banking sector is able to meet its deposit liabilities and losses are equal to zero. Re-writing the condition given in (3.5.1) using lemma (x) yields the following result:

$$0 = \frac{\partial}{\partial r_{X,t+1}} \left\{ \mathbb{E}_t M_{t,t+1} \left[ r_{X,t+1}(1-\omega_{t+1}) - \underbrace{(1-\Phi(z_{b,t+1}))}_{\substack{\text{endogenous} \\ \text{default probability}}} \Lambda_{t+1} \right] \right\} \tag{3.5.2}$$

where an explicit default probability term $1-\Phi(z_{b,t+1})$ shows the impact of IOER decision on likelihood of default. This term captures a non-symmetric role of loss term $\Lambda_{t+1}$ which arises only in default. As the regulator lowers IOER, defaults become more likely because equilibrium deposit rate decreases in IOER but at a diminishing rate leading to a growing net interest expenses in the banking sector.

### 3.5.1 Equilibrium with Regulated Interest Rate

Capital structure of the bank complies with RW-capital requirement for any balance sheet size. First, in order for the deposits and equity market to clear, the bank raises funds by choosing its balance sheet size considering the capital regulation $A^*_{t+1}(\overline{\eta}_{t+1})$ through total savings by the households $S^*_{t+1}$:

$$A^*_{t+1}(\overline{\eta}_{t+1}) = S^*_{t+1} \tag{3.5.3}$$

Second, in equilibrium, the portfolio choice of the households including deposits and equity must be equal to the capital structure of the bank that is given by the following fixed-point,

$$\overline{\eta}^*_{t+1}(M_{t,t+1}(\theta_{t+1}), Q_{D,t}; ...) = \theta^*_{t+1}(div(\overline{\eta}^*_{t+1}); Q_{D,t}, P_{E,t}; ...) \tag{3.5.4}$$

the optimal portfolio choice of the household depends on the capital structure of the bank, and similarly, the capital structure of the bank is determined in part by the portfolio of the households. While quantities $\overline{\eta}^*_{t+1}(\theta_{t+1})$ and $\theta^*_{t+1}(\overline{\eta}_{t+1})$ are notoriously challenging to pin down in general, $\overline{\eta}^*_{t+1}(\theta^*_{t+1})$ and $\theta^*_{t+1}(\overline{\eta}^*_{t+1})$ are pinned down in equilibrium following the iterative substitution given by:

$$\overline{\eta}^*_{t+1}\Big(\theta^*_{t+1}(\overline{\eta}^*_{t+1}(...))\Big) \quad = \quad \theta^*_{t+1}\Big(\overline{\eta}^*_{t+1}(\theta^*_{t+1}(...))\Big) \tag{3.5.5}$$

As a special case with logarithmic utility, the first market clearing condition simplifies to:

$$A^*_{t+1}(\overline{\eta}_{t+1}) \quad = \quad \underbrace{(1-\tau_{t+1}).(1-\theta^*_{t+1}+\kappa.\theta^*_{t+1}).(1-\beta).W_t}_{\text{supply of funds}} \tag{3.5.6}$$

where the first term on the right-hand-side shows the effect of equity purchase fee on lowering the total supply fund to the economy. When $\kappa = 1$ then equity purchase is costless and the supply of funds is fixed. The term $1-\tau_{t+1}$ show household's disposable income after paying proportional taxation to the regulator.

A welfare analysis based on the laissez-faire allocations incorporates the optimal behaviour of counterparties under the following frictions: first, the deposit insurance guarantees bank debt, giving deposit investor the confidence that in any default state, they are able to consume out of their deposit investments. Second, the banking sector is subject to costly default. The profit-maximizing behaviour of the banking sector that fails to consider negative externalities associated with costly failure gives rise. In equilibrium, regulator's decision on optimal IOER considers the following channels: first, an optimal IOER trades off welfare gains of lower deadweight losses associated with costly bank failure, because in any default state, bank's higher revenues due to IOER increases ex-post liquidation proceeds. Second, regulators decision on optimal IOER equates marginal benefits of lower costly defaults against marginal benefits of higher transfers to households. Thirds, IOER interacts with banks asset allocation decision through its impact on solvency channel that is priced in bank's net worth. Specifically, an optimal IOER maximizes welfare gains associated with credit flow to the real economy against marginal cost of heightened default risk due to extended lending. Decision to lower IOER when the rate is above zero bound is negatively related to optimal capital regulation because lower net interest incomes increases bank's ability to meet debt liabilities and therefore increases probabilistic cost of default. Regulator's decision on optimal IOER incentivises banking sector to extend lending with an expansionary impact while tighter capital requirement provides welfare gains by reducing (expected) bank failure cost.

## 3.6 Calibration

In the aftermath of the financial crisis, the central bank in many European economies, as well as the Federal reserves, substantially increases the size of their balance sheets. The first driver of this increased size was due to the purchase of long-maturity Treasury and mortgage-backed securities. This effect was backed on the liabilities side by a steady increase in the quantities of notes in circulation and more importantly, by reserve balances deposited by the financial institutions. This dramatic balance sheet expansion prompted the regulators to examine the impact of overnight interest rate on the credit supply and financial stability. In theory, when the quantity of reserves outstanding is sufficiently large, the

| **Structural Parameterization** | | |
|---|---|---|
| Description | Notation | Value |
| Household Subjective Discount Factor | $\beta$ | 0.99 |
| Household Coefficient of Relative Risk-aversion | $\gamma$ | 1.00 |
| Household Elasticity of Intertemporal Substitution | $\psi$ | 1.00 |
| Bankruptcy Cost Parameter (proportional cost: $1-\chi$) | $\chi$ | 70.00% |
| Intermediation CostParameter (proportional cost: $1-\kappa$) | $\kappa$ | 98.50% |
| Lending Decreasing Return to Scale | $\alpha$ | 0.95 |
| Aggregate Shock (Lending) Mean Parameter | $\mu_L$ | 0.085 |
| Aggregate Shock (Lending) S.D. Parameter | $\sigma_L$ | 11.75% |
| Aggregate Shock (Lending) Expectation | $e^{\mu_L+\frac{1}{2}\sigma_L^2}$ | 0.0963 |
| Aggregate Shock (Lending) Variance | $e^{2\mu_L+\sigma_L^2}(e^{\sigma_L^2}-1)$ | 0.017 |

**Table 3.2:** Calibration Parameterization

IOER becomes a key driver of the several interest rates across the economy through financial arbitrage. During the initial episodes of the post-2008 crisis, the monetary and financial regulatory authorities anticipated that recovery will eventually revert the economic state to the pre-crisis era and a rise in the interest rates mechanically divert the funds away from reverse balances to the neighbourhood of $50b. Nevertheless, the prolonged environment of low productivity and high uncertainty indicated that very low and possibly negative IOER forms an integral part of the policy tools to determine the market clearing conditions associated with excess reserves which ultimately determines the credit supply.

The calibration exercise in this section examines the implications of marginal changes in IOER policy on the banking institutions portfolio holding, its liabilities structure and ulti-

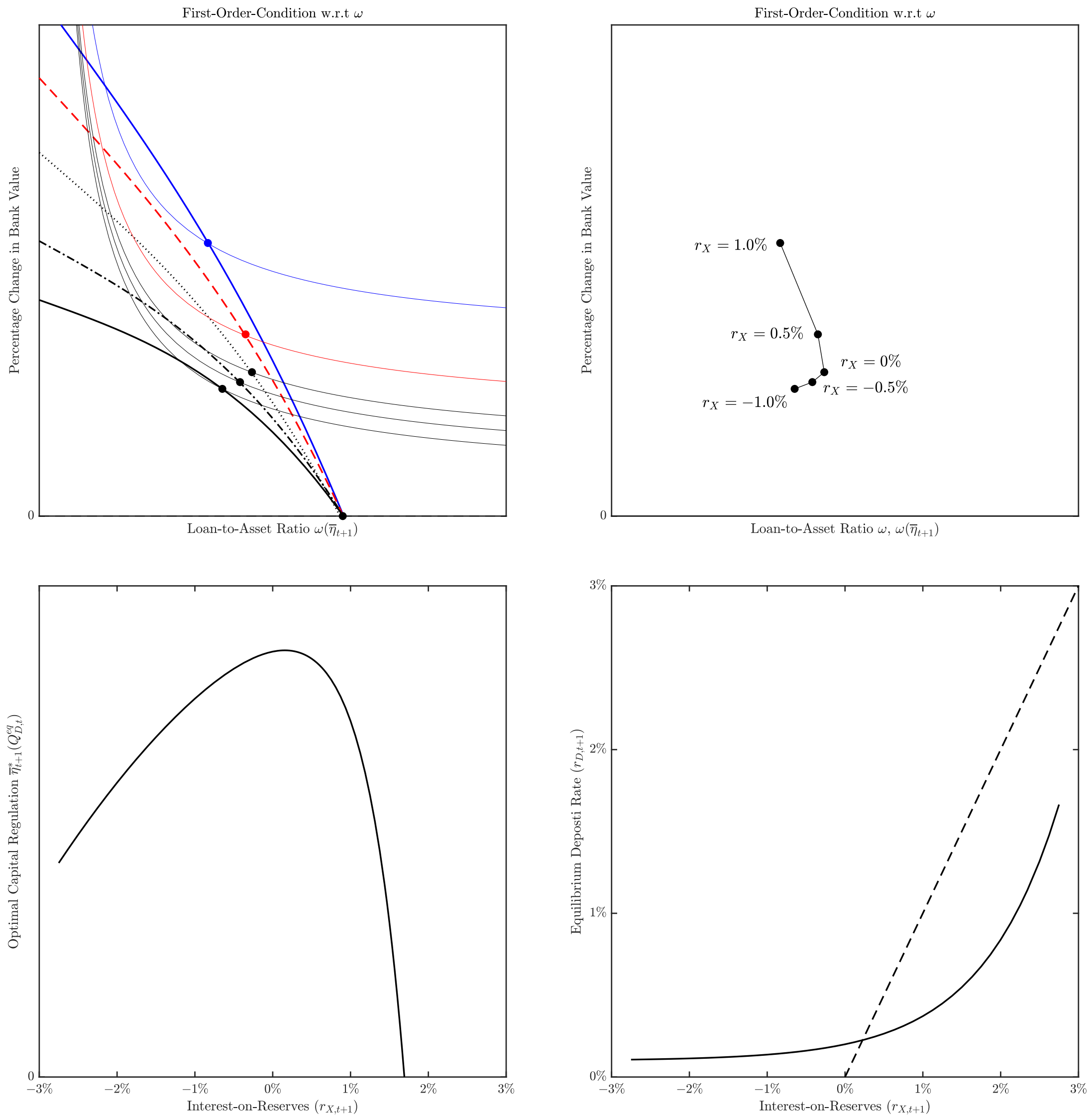



**Figure 3.7:** The figure illustrates that for a given interest-on-reserves rate, optimal capital regulation falls when bank interest expenses fall faster than the reduction in interest incomes from reserves. Conversely, the relationship between optimal capital regulation and interest-on-reserves reverses when bank's default risk increases as a results of loss of interest income from reserves and nonresponsive changes in deposit rate.The figure illustrates bank's ability to extend loans when capital regulation requires the bank to hold additional equity per unit of loan. The solid line shows percentage change in bank's value function given a unit change in allocation to loan, the dotted line shows percentage change in bank's solvency when its portfolio holding of loan increases, and dashed line shows percentage change in solvency when bank complies with capital regulation.

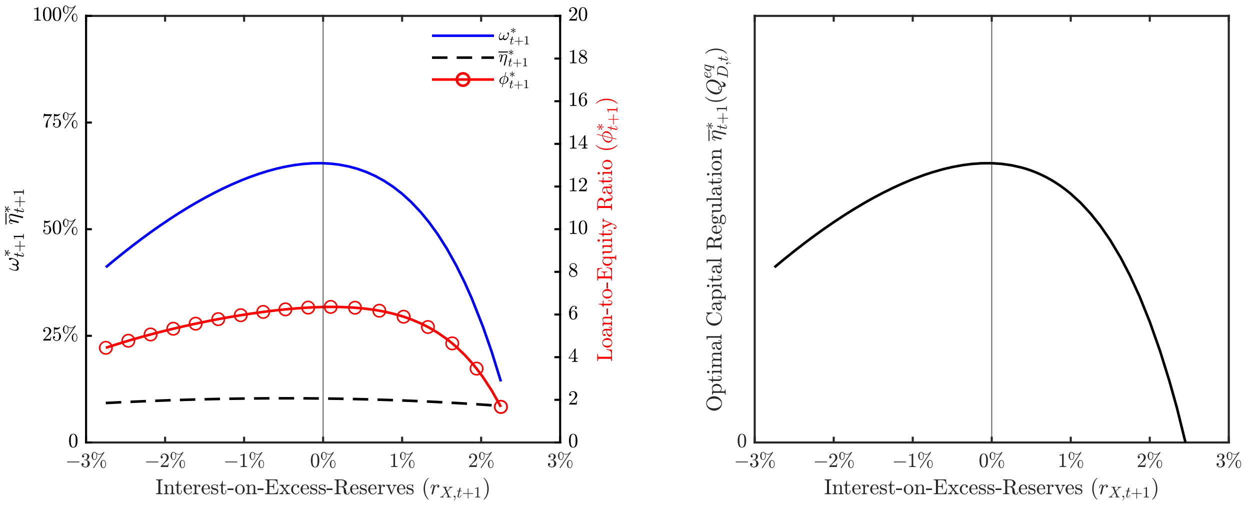



**Figure 3.8:** The figure illustrates bank's portfolio rebalancing when RW-capital regulation requires the bank to hold higher equity per loans. The solid dashed line described bank's laissez-faire loan-to-equity schedule and the solid line describes regulated loan-to-assets schedule which is always toward the outer right side of unregulated schedule.

mately the welfare. Households' preferences to hold a higher share of the wealth in bank deposits increases, as they become more risk averse. Given an exogenous risk-weighted capital regulation, the market clearing condition for the deposit market implies that households are prepared to accept a lower compensation per each unit of deposits leading to a lower cost of debt for the banking institutions that raises funds from the deposit market. The lower cost of debt lowers banks default risk because its end of period interest expenses fall, and subsequently the new equilibrium arises where bank's capital structure shifts to include higher debt to equity ratio until the marginal gains from lower cost of debt equates the marginal increase in the default probability due to higher debt. The equilibrium prediction implies that when the households have a preference for an early resolution of uncertainty with $\gamma > 1$ and $\gamma\psi > 1$, the consumption falls, and the supply of saving to the bank increases leading to an expansion of bank balance sheet size. On the margin, banks allocation of additional funds to credit and excess reserves depends on the IOER policy. Given an exogenous risk-weighted capital constraint, a percentage decrease in the IOER leads to the following effects: first, as IOER falls within the positive territory, bank's interest incomes on excess reserves falls leading to a subsequent proportional fall in the equilibrium deposits rate. Because the bank invests only a fraction deposits in reserves, banks overall interest expenses falls, its solvency improves and ultimately the optimal response by the bank leads to a substitution effect with further loans replacing the reserves.

However, when the IOER falls further down to very low or possibly negative value, the response of the equilibrium deposit rate to IOER flattens such that the bank's overall interest expenses begins to increase. At this thresholds, further reduction of the IOER leads to exacerbates bank solvency and contractionary implications for the real sector. The policy-maker's perspective to determine an optimal IOER considers the welfare gains of marginal changes in IOER over the default and solvency regions. Specifically, when the likelihood of bank defaults increases, lower IOER increases the credit risk of the bank on its asset side and prompts the capital regulation to tighten. Conversely, as the IOER increases, the bank is incentivised for substitute loans with reserves and its asset sides risk falls, which prompts the capital regulation to loosen. This impact indicates that a welfare-maximizing IOER and risk-weighted capital regulation are negatively correlated.

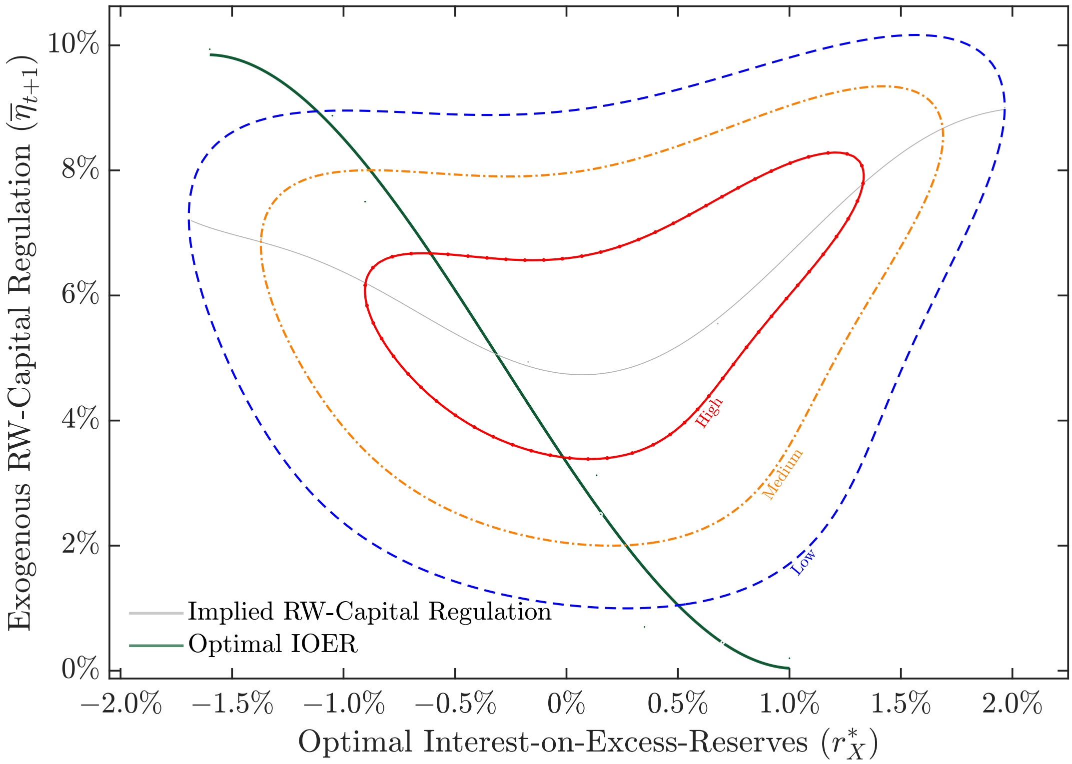



**Figure 3.9:** The figure illustrates the social welfare function level curves. The social welfare increases towards the inner contour curves depicted by dashed (blue), dashed-dotted (orange) and solid (red) contours. The optimal IOER policy rate is negatively correlated with an exogenous risk-weighted capital requirement. The welfare-maximizing optimal financial regulation including the IOER and capital regulation deliver social gains to the society at the intersection of the two optimal schedules.

The household's preference to substitution consumption over time has important implications for the conduct of IOER policy. Household's optimal consumption-saving decision become less responsive to changes in the return on wealth, that is ultimately driven by an IOER policy, when their elasticity of intertemporal substitution approaches one. In this case, a one percentage point fall in IOER policy leads and expansion of the real sector, when the rates are above effective lower bound. In particular, the supply of saving remains intact, however, as the overall interest expenses of the banking institutions falls, the bank substitutes excess reserves with loans until the marginal negative impact of lending wears off the marginal gains in solvency provided by the lower interest expenses. Conversely, when the household's preferences is characterizes by an elasticity of intertemporal substitution below unit, marginal increases in the return on wealth leads to higher consumption-wealth ration because the income effect dominates the substitution effect. As a result, a one percentage point fall in the IOER leads to opposite impacts, where the size of the real sector falls and the overreliance on excess reserves increases.

**Optimal Interest-on-Excess Reserves (%)**

| Panel A: Low Loss Given Default ($1-\chi = 10\%, \kappa = 99\%$) | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\omega_{t+1}$ | Exogenous RW-Capital Requirement ($\overline{\eta}_{t+1}$) (share of total assets) | | | | | | | | | | |
| | 1.00% | 2.00% | 3.00% | 4.00% | 5.00% | 6.00% | 7.00% | 8.00% | 9.00% | 10.00% | 15.00% |
| 10% | −0.14 | −0.16 | −0.18 | −0.20 | −0.22 | −0.24 | −0.26 | −0.28 | −0.31 | | |
| 25% | −0.12 | −0.14 | −0.16 | −0.18 | −0.20 | −0.22 | −0.25 | −0.27 | −0.30 | −0.32 | −0.40 |
| 50% | −0.11 | −0.13 | −0.15 | −0.16 | −0.18 | −0.21 | −0.23 | −0.25 | −0.28 | −0.30 | −0.38 |
| 75% | −0.09 | −0.11 | −0.13 | −0.15 | −0.17 | −0.19 | −0.21 | −0.24 | −0.26 | −0.28 | −0.37 |
| 90% | −0.07 | −0.09 | −0.11 | −0.13 | −0.15 | −0.17 | −0.20 | −0.22 | −0.24 | −0.27 | −0.35 |
| 100% | −0.06 | −0.08 | −0.09 | −0.11 | −0.13 | −0.16 | −0.18 | −0.20 | −0.22 | −0.25 | −0.33 |
| Panel B: Medium Loss Given Default ($1-\chi = 20\%, \kappa = 99\%$) | | | | | | | | | | | |
| $\omega_{t+1}$ | Exogenous RW-Capital Requirement ($\overline{\eta}_{t+1}$) (share of total assets) | | | | | | | | | | |
| | 1.00% | 2.00% | 3.00% | 4.00% | 5.00% | 6.00% | 7.00% | 8.00% | 9.00% | 10.00% | 15.00% |
| 10% | −0.11 | −0.12 | −0.14 | −0.16 | −0.18 | −0.20 | −0.23 | −0.25 | −0.28 | | |
| 25% | −0.09 | −0.11 | −0.13 | −0.15 | −0.16 | −0.19 | −0.21 | −0.23 | −0.26 | −0.28 | −0.37 |
| 50% | −0.07 | −0.09 | −0.11 | −0.13 | −0.15 | −0.17 | −0.19 | −0.22 | −0.24 | −0.26 | −0.35 |
| 75% | −0.06 | −0.08 | −0.10 | −0.11 | −0.13 | −0.16 | −0.18 | −0.20 | −0.22 | −0.25 | −0.33 |
| 90% | −0.04 | −0.06 | −0.08 | −0.10 | −0.12 | −0.14 | −0.16 | −0.18 | −0.21 | −0.23 | −0.31 |
| 100% | −0.02 | −0.04 | −0.06 | −0.08 | −0.10 | −0.12 | −0.14 | −0.17 | −0.19 | −0.21 | −0.29 |
| Panel C: High Loss Given Default ($1-\chi = 30\%, \kappa = 99\%$) | | | | | | | | | | | |
| $\omega_{t+1}$ | Exogenous RW-Capital Requirement ($\overline{\eta}_{t+1}$) (share of total assets) | | | | | | | | | | |
| | 1.00% | 2.00% | 3.00% | 4.00% | 5.00% | 6.00% | 7.00% | 8.00% | 9.00% | 10.00% | 15.00% |
| 10% | −0.05 | −0.07 | −0.09 | −0.10 | −0.12 | −0.14 | −0.17 | −0.19 | −0.21 | | |
| 25% | −0.03 | −0.05 | −0.07 | −0.09 | −0.11 | −0.13 | −0.15 | −0.17 | −0.20 | −0.22 | −0.30 |
| 50% | −0.02 | −0.04 | −0.05 | −0.07 | −0.09 | −0.11 | −0.13 | −0.16 | −0.18 | −0.20 | −0.28 |
| 75% | 0.00 | −0.02 | −0.04 | −0.06 | −0.08 | −0.10 | −0.12 | −0.14 | −0.16 | −0.19 | −0.26 |
| 90% | 0.01 | 0.00 | −0.02 | −0.04 | −0.06 | −0.08 | −0.10 | −0.12 | −0.15 | −0.17 | −0.25 |
| 100% | 0.03 | 0.01 | −0.01 | −0.02 | −0.04 | −0.06 | −0.09 | −0.11 | −0.13 | −0.15 | −0.23 |

**Table 3.3:** The table illustrates the optimal Interest-on-Excess-Reserves set by the policymaker on the reserves deposit facility accepting funds from the banking institutions. Panel (A) shows the optimal IOER in percentage points and given an exogenous RW-capital requirement where the rate decreases when the bank is mandated to hold higher capital-to-assets ratio and increases as the bank allocates further share of its assets to the loans market. As the deadweight loss associated with bank default increases, the optimal IOER increases to mitigate the default likelihood of bank failure through the increases interest expenses.

## 3.7 Conclusion

Over the past decade, oversized excess reserves consistently comprised nearly half of the total assets of central banks in charge of 40% of world economy and policymakers used IOER as a lever (inter alia) to address banks' overreliance on excess reserves. The transmission mechanism between IOER policy rate and capital requirement regulation is an important consideration with welfare implications because conflicting policies may effectively lead to under-regulation of the banking sector and therefore re-exposure to default risk, or over-regulation that disrupts credit flow to the real economy.

First, this paper provides a foundation to understand this interaction and show that policymaker's decision to lower IOER provides social benefits only when this policy rate is above zero. In general equilibrium, falling IOER is followed by an almost proportional fall in the equilibrium deposit rate when IOER is above zero but as this rate becomes very low or possibly negative, the equilibrium deposit rate remains positive and nonresponsive to further changes in IOER. Because the banking sector has only a fraction of deposits invested in reserves, a proportional decrease in equilibrium deposit rate in response to falling IOER leads to a faster drop in interest expenses on deposits than loss of interest incomes from reserves. The banking sector extends lending to the real economy as a result of lower default risk when IOER falls and subsequently, the optimal capital regulation tightens to adjust for the added risk to banks' assets.

However, when IOER becomes very low, or possibly negative, the equilibrium deposit rate exhibits an increasingly flatter response to further changes in IOER because deposit investors require a marginally positive compensation for time preference to forego consumption. When equilibrium deposit rate is increasingly nonresponsive to any further reduction in IOER, loss of interest incomes from reserves exceeds lowered interest expenses on deposits. Bank's optimally responds to increased default risk due to higher net interest expenses by lowering lending in order to maintain its shareholder value and subsequently optimal capital regulation loosen. The analysis in Section (3.4) shows that lower IOER dissuades the banking sector from over-relying on idle excess reserves which has an expansionary effect of real output only when lower rates lead to lower default risk, otherwise lowering IOER generates counterproductive results by worsening this overreliance problem and becomes contractionary.

Second, the analysis in Section (3.5) shows that for any given IOER rate, optimal capital regulation constantly addresses distortions associated with costly bank failure by requiring the banking sector to hold higher equity per unit of loan. Particularly, as IOER falls within positive territory, optimal capital regulation responds negatively, and positively when IOER becomes very low or below zero. The social value provided by the capital regulation, however, is able to address one distortion at a time at the expense of disruptions of credit flow to the real sector. An optimal IOER policy, when considered in conjunction with the optimal capital regulation, is able to provide further social value by maximizing gains from boosting the real economy while addressing costly bank failure distortions.

An optimal joint financial regulation that considers this non-monotonic relationship between two levers provides support for an integration between the monetary authority in charge of reserves management and the financial regulatory body in charge of capital regulation. The analysis in Section (3.5) also sheds light on the interconnectedness of IOER to government guarantees that protect deposits held in the banking sector. The results show that a positive IOER when combined with oversized excess reserves leads to large interest expenses and strains government resources that are intended to compensate depositors in any default state, whereas low or below zero IOER can relax government funds, raised from the households or the banking sector, and provide social benefits by increasing the size of the financial sector and ultimately credit flow to the real economy.

Finally, this paper shows a motivation for the monetary and financial regulatory policymakers to act jointly to provide further welfare gains to the society. Nonetheless, future work on joint financial regulation of banking system confronted with aggregate uncertainty needs to consider the welfare implications of deposit insurance funding regimes, an aspect that remains an open question in this paper. Under-funded deposit insurance system provides partial insurance to depositors leading to higher equilibrium cost of debt for the banking system because rational investors price potential bank defaults as well as sovereign defaults. Although sovereign default is unlikely to occur when government guarantees are met in nominal terms, nominal implications become an important consideration that extends the scope of this study to models beyond the real economy. Alternatively, a government guarantee is met by borrowing against the future which raises fiscal implications, or by borrowing from foreign with potentially a downward pressure on exchange rates and international finance considerations.

## 3.8 Appendix

### A. Price Functional Equation

Given strictly increasing and concave preferences in argument $C_t$ and intertemporal budget constraint constraint. The first order conditions for the Lagrangian problem of the household gives:

$$\begin{aligned}
\lambda_t C_t^{-\frac{1}{\psi}} &= \delta(\mu_t(U_{t+1})_{t+1})^{-\frac{1}{\psi}} \left[\mathbb{E}_t\left[U(W_{t+1})^{1-\gamma}\right]\right]^{\frac{1}{1-\gamma}-1} \mathbb{E}_t\left[U(W_{t+1})^{-\gamma} U'(W_{t+1}) R_{W,t+1}\right] \\
&= \delta(\mu_t(U_{t+1}))^{-\frac{1}{\psi}} \left[\mathbb{E}_t\left[U(W_{t+1})^{1-\gamma}\right]\right]^{\frac{1}{1-\gamma}-1} \mathbb{E}_t\left[U(W_{t+1})^{-\gamma} \lambda_{t+1} C_{t+1}^{-\frac{1}{\psi}} R_{W,t+1}\right] \quad (3.8.1)
\end{aligned}$$

the recursive structure is required to have closed-form solution. The SDF is $(M_{t+1})$ is:

$$M_{t+1} = \delta \frac{\lambda_{t+1}}{\lambda_t} \frac{U(W_{t+1})^{\frac{1}{\psi}-\gamma}}{(\mu_t(U_{t+1}))^{\frac{1}{\psi}-\gamma}} \frac{C_{t+1}^{-\frac{1}{\psi}}}{C_t^{-\frac{1}{\psi}}} \quad (3.8.2)$$

rewrite the SDF in logarithms:

$$\begin{aligned}
m_{t+1} &= \log(\delta) + \log(\frac{\lambda_{t+1}}{\lambda_t}) - \frac{1}{\psi}\Delta C_{t+1} + (\frac{1}{\psi} - \gamma)\log(\frac{U_{t+1}}{\mu_t(U_{t+1})}) \\
&= \theta\log(\delta_t) + \theta\log\left(\frac{\lambda_{t+1}}{\lambda_t}\right) - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1) r_{W,t+1} \quad (3.8.3)
\end{aligned}$$

conditional expected return to equity and to risk-free deposits are:

$$r_{f,t+1} = \frac{\mu}{\psi} - \log(\delta) - \log\left(\frac{\lambda_{t+1}}{\lambda_t}\right) + \left[\frac{1-\theta}{\theta}(1-\gamma)^2 - \gamma^2)\right]\frac{\sigma_c^2}{2} + \left[(1-\theta)(\kappa_{c,1} A_{c,1})^2\right]\frac{\sigma_\lambda^2}{2}$$

Conjecture the following affine functional equation:

$$P_t(\theta_{t+1}, R_{D,t+1}; \gamma) = \vartheta(P_{t+1} + div_{t+1}) = \frac{\vartheta}{1-\vartheta}\mathbb{E}_t[div_{t+1}] \quad (3.8.4)$$

where the first order condition with respect to intertemporal decision satisfies:

$$\begin{aligned} 1 &= R_{D,t+1}\mathbb{E}_t\left[\frac{1}{((1-\theta)R_{D,t+1}+\theta R_{E,t+1})^{\gamma}}\right] \\ &= R_{D,t+1}\mathbb{E}_t\left[\frac{1}{\left((1-\theta)R_{D,t+1}+\theta\dfrac{P_{t+1}+div_{t+1}}{P_t}\right)^{\gamma}}\right] \end{aligned}$$

the linear functional equation maximizes the household's lifetime utility when:

$$\vartheta = \left\{1-\frac{\frac{1}{1-\Phi(\delta)}\cdot\frac{1}{R_{D,t+1}}-\dfrac{\Phi(\delta)}{1-\Phi(\delta)}\frac{1}{\left[(1-\theta_{t+1})R_{D,t+1}\right]^{\gamma}}-\gamma(1-\theta_{t+1})R_{D,t+1}-\frac{\gamma^2}{2}\theta_{t+1}^2}{\gamma\theta_{t+1}}\right\}^{-1}$$

## B. Deposit Insurance Premium

The welfare gains associated with the size of taxes that provide resources for deposit insurance service is evaluated against its social cost given by:

$$0 = \mathbb{E}_t\left\{M_{t,t+1}\left(\underbrace{-\left[(1-\theta_{t+1})R_{D,t+1}+\theta_{t+1}R_{E,t+1}\right]}_{\text{loss of credit to output market}} \quad \underbrace{+\frac{d\mathrm{Tr}_{t+1}}{d\tau_{t+1}}}_{\text{gain from lower bankruptcy cost}}\right)\right\}$$

The trade-off is given by the real economic implications of nonproductive fund that cover depositors in an unlikely widespread failure against social costs of runs on the bank debt due to panic-based runs:

$$\begin{aligned} 0 = & \int_0^{\delta}\left\{M_{t,t+1}\left(-\left[(1-\theta_{t+1})R_{D,t+1}\right]+\frac{d\mathrm{Tr}_{t+1}}{d\tau_{t+1}}\right)\right\}dF + \\ & \int_{\delta}^{\infty}\left\{M_{t,t+1}\left(-\left[(1-\theta_{t+1})R_{D,t+1}+\theta_{t+1}R_{E,t+1}\right]+1\right)\right\}dF \end{aligned}$$

## C. Present Value of Equity Return

Monotonicity of the following present value problem yields ($\mathbb{E}_t M_{t,t+1}R_{E,t+1} = \int_{\Delta} M_{t,t+1}R_{E,t+1}dF$):

$$\arg\max\ \mathbb{E}_t M_{t,t+1}R_{E,t+1} = \arg\max\ \log\mathbb{E}_t M_{t,t+1}R_{E,t+1} \tag{3.8.5}$$

then,

$$\log \mathbb{E}_t M_{t,t+1} R_{E,t+1} \;=\; \mathbb{E}_t \log(M_{t,t+1} R_{E,t+1}) + \frac{1}{2}\mathbb{V}_t \log(M_{t,t+1} R_{E,t+1}) \tag{3.8.6}$$

the first term on the RHS of equation (3.8.6) is:

$$\begin{aligned}\mathbb{E}_t \log(M_{t,t+1} R_{E,t+1}) &= \mathbb{E}_t \log R_{E,t+1} + \mathbb{E}_t \log M_{t,t+1} \\ &= \mathbb{E}_t \log R_{E,t+1} - \mathbb{E}_t \log R_{h,t+1} - \frac{(1-\gamma)^2}{2}\mathbb{V}_t \log R_{h,t+1}\end{aligned} \tag{3.8.7}$$

the second term (without 1/2) on the RHS of equation (3.8.6) is:

$$\begin{aligned}\mathbb{V}_t \log(M_{t,t+1} R_{E,t+1}) &= \mathbb{V}_t \log R_{E,t+1} + \mathbb{V}_t \log M_{t,t+1} + 2\mathrm{Cov}_t(\log M_{t,t+1}, \log R_{E,t+1}) \\ &= \mathbb{V}_t \log R_{E,t+1} + \gamma^2 \mathbb{V}_t \log R_{h,t+1} + 2\mathrm{Cov}_t(\log M_{t,t+1}, \log R_{E,t+1}) \\ &= (1-\gamma\pi_{t+1})^2 \mathbb{V}_t \log R_{E,t+1}\end{aligned} \tag{3.8.8}$$

Re-writing equation (3.8.6) using (3.8.7) and (3.8.8):

$$\begin{aligned}&= \log \mathbb{E}_t M_{t,t+1} R_{E,t+1} \\ &= \mathbb{E}_t \log R_{E,t+1} - \mathbb{E}_t \log R_{h,t+1} - \frac{(1-\gamma)^2}{2}\mathbb{V}_t \log R_{h,t+1} + \frac{(1-\gamma\pi_{t+1})^2}{2}\mathbb{V}_t \log R_{E,t+1} \\ &= \mathbb{E}_t \log R_{E,t+1} - \mathbb{E}_t \log R_{h,t+1} - \frac{1}{2}\left[(1-\gamma)^2 \pi_{t+1}^2 - (1-\gamma\pi_{t+1})^2\right] \mathbb{V}_t \log R_{E,t+1} \\ &= (1-\pi_{t+1})(\mathbb{E}_t r_{E,t+1} - r_{D,t+1}) - \frac{1}{2}(1-\pi_{t+1})\left[(2\gamma-1)\pi_{t+1} - 1\right] \mathbb{V}_t \log R_{E,t+1}\end{aligned} \tag{3.8.9}$$

## D. Optimal Capital Regulation

The social welfare function is evaluated by household's utility function given regulator's resources,

$$g \;=\; \begin{cases} (1-\tau).\left[\frac{1-\overline{\eta}}{Q_{D,t}} + (1-\kappa).\overline{\eta}.R_{E,t+1}\right] + \left[\tau - (1-\tau).(1-\omega_{t+1}).r_x\right] & \text{in solvency} \\ (1-\tau).\frac{1-\overline{\eta}}{Q_{D,t}} + \left[\tau - (1-\tau).\left[\left(\frac{1-\overline{\eta}}{Q_{D,t}} - \chi.R_{p,t+1}\right) - (1-\omega_{t+1}).r_x\right]\right] & \text{in default} \end{cases}$$

The first derivative of w.r.t. capital regulation choice $\overline{\eta}_{t+1}$ is

$$0 \;=\; \frac{\chi + \Phi(\lambda_{t+1}).(1-\chi)}{1-\chi} - \frac{1-\overline{\eta}_{t+1}(1-\kappa)}{1-\kappa}\left(-\frac{1}{\sigma}\frac{\partial\Phi(\lambda_{t+1})}{\partial\tau_{t+1}}\frac{\partial\lambda_{t+1}}{\partial z_{b,t+1}}\right)\frac{\partial z_{b,t+1}}{\partial\overline{\eta}_{t+1}}$$

Approximating the term $-\frac{1}{\sigma}\frac{\partial\Phi(\lambda_{t+1})}{\partial\tau_{t+1}}\frac{\partial\lambda_{t+1}}{\partial z_{b,t+1}}$ with the following exponential affine function, $e^{a_0+a_1 z_b}$ where $a_0$ and $a_1$ are functions of $\mu$ and $\sigma$. Table (**??**) shows the optimal capital regulation given the IOER within $[-0.75\%, 1.75\%]$ when the bankruptcy cost is low (Panel A), medium (Panel B) and high (Panel C), respectively, and equity intermediation cost is 1%. Table (**??**), replicates the same exercise when the intermediation cost associated with the equity market fall to 0.5%.

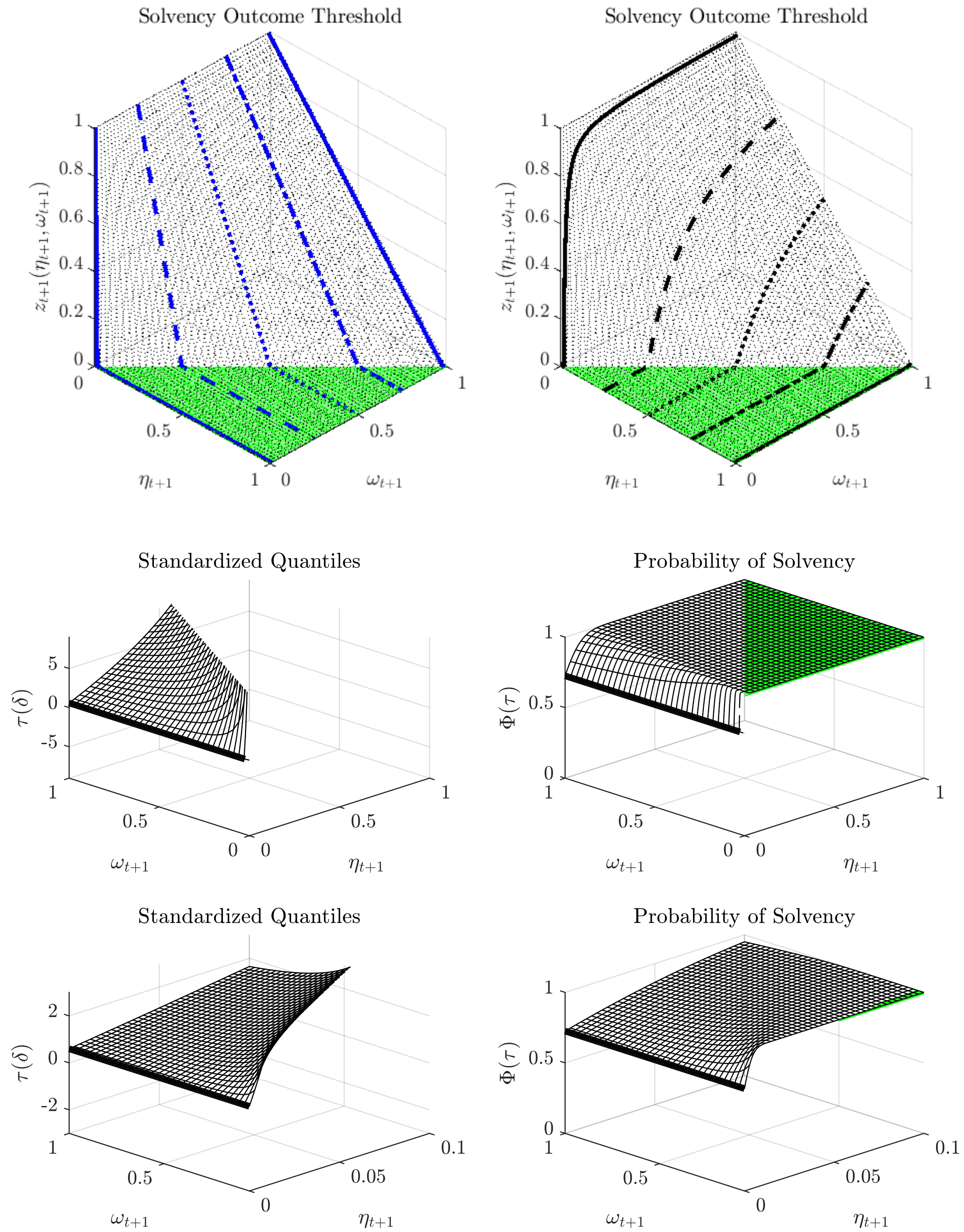



**Figure 3.10:** Figures on the top row depict the value of ex-post loan rate that makes the bank break even between its revenues and liabilities over the portfolio and capital structure space. The green section shows the region where the bank is highly capitalized and holds a zero or very small position in the loan market therefore there is no such loan rate that can force the bank into bankruptcy (zero loan rate). Each line on the surface along the capital structure dimension (fixing a portfolio weight) is decreasing in equity, for any portfolio level. Conversely, the top-right figure shows increasing lines on the surface along the position in the risky asset. The figure on middle-left shows the standardized quantile of default threshold over capital structure and portfolio decisions space. As the bank increases its capital to asset ratio, the quantile that breaks even its revenues and costs increases. Conversely, as the bank increases its position in the risky asset, the standardized quantiles fall. This surface is not defined over the region that corresponds to the fully covered section as there is no such quantile (infinity) that can default the bank. The figure on middle-right shows the probabilities associated with the quantiles. Over fully covered region, the probability of success is one, and as the bank increases its position in the risky asset or reduces its capital, the probability of success falls. Figures on bottom-left and bottom-right show the same figures in the middle row but focused on the under-capitalized and over-invested in loans scenarios.

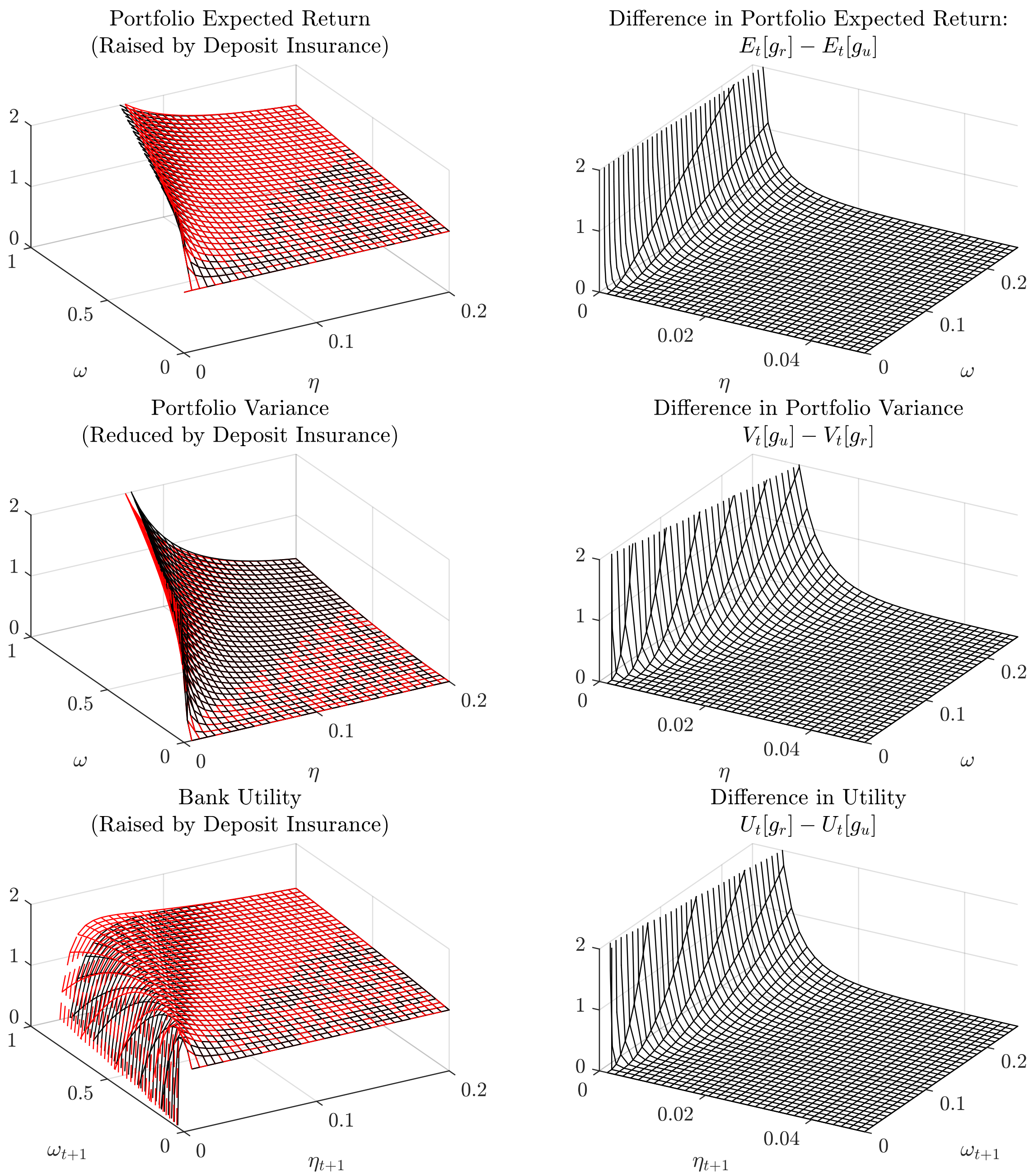



**Figure 3.11:** The top panel-left figure illustrates the expected portfolio return of the bank with and without deposit insurance. The surface value (weakly) increases for any capital structure or portfolio decisions in the presence of deposit insurance. The top-right figure shows the difference expected portfolio return with deposit insurance and without where the entire surface value is always (weakly) positive. The middle-left figure illustrates the portfolio variance of the bank with and without deposit insurance. The surface value (weakly) decreases for any capital structure or portfolio decisions in the presence of deposit insurance. The middle-right figure shows the difference in portfolio variance without and with deposit insurance, where the entire surface value is always (weakly) positive. The bottom-left figure illustrates the expected utility of the bank with and without deposit insurance. The surface value (weakly) increases for any capital structure or portfolio decisions in the presence of deposit insurance. The bottom-right figure shows the difference expected utility with deposit insurance and without where the entire surface value is always (weakly) positive.

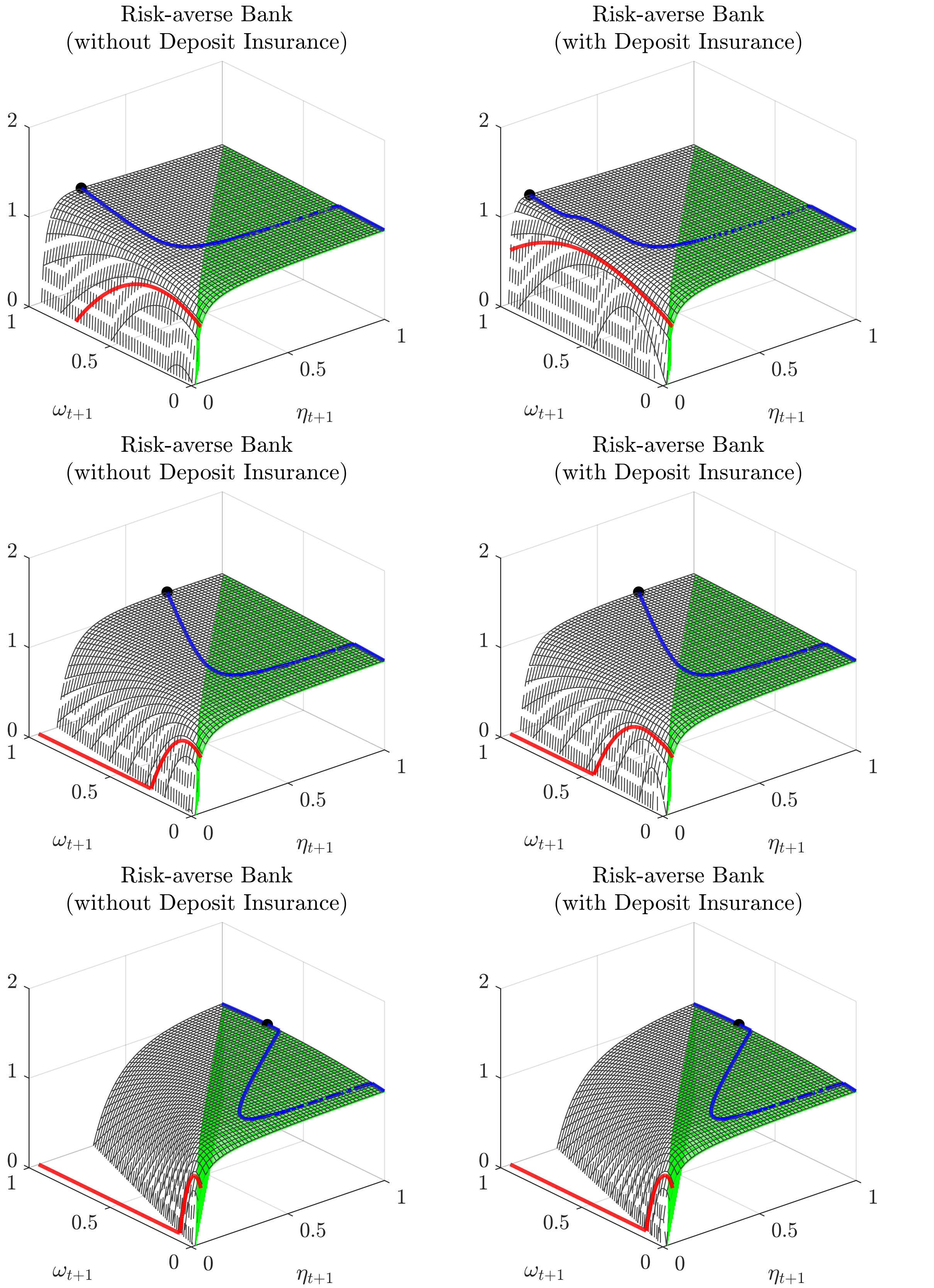



**Figure 3.12:** The figure illustrates the utility of a risk-averse bank. The top row uses $\sigma_L = 0.05$, the second row used $\sigma_L = 0.10$ and last row uses $\sigma_L = 0.15$, where increasing the standard deviation of risky loan pressures the bank to pick a higher level of equity to balance out the increased variance.

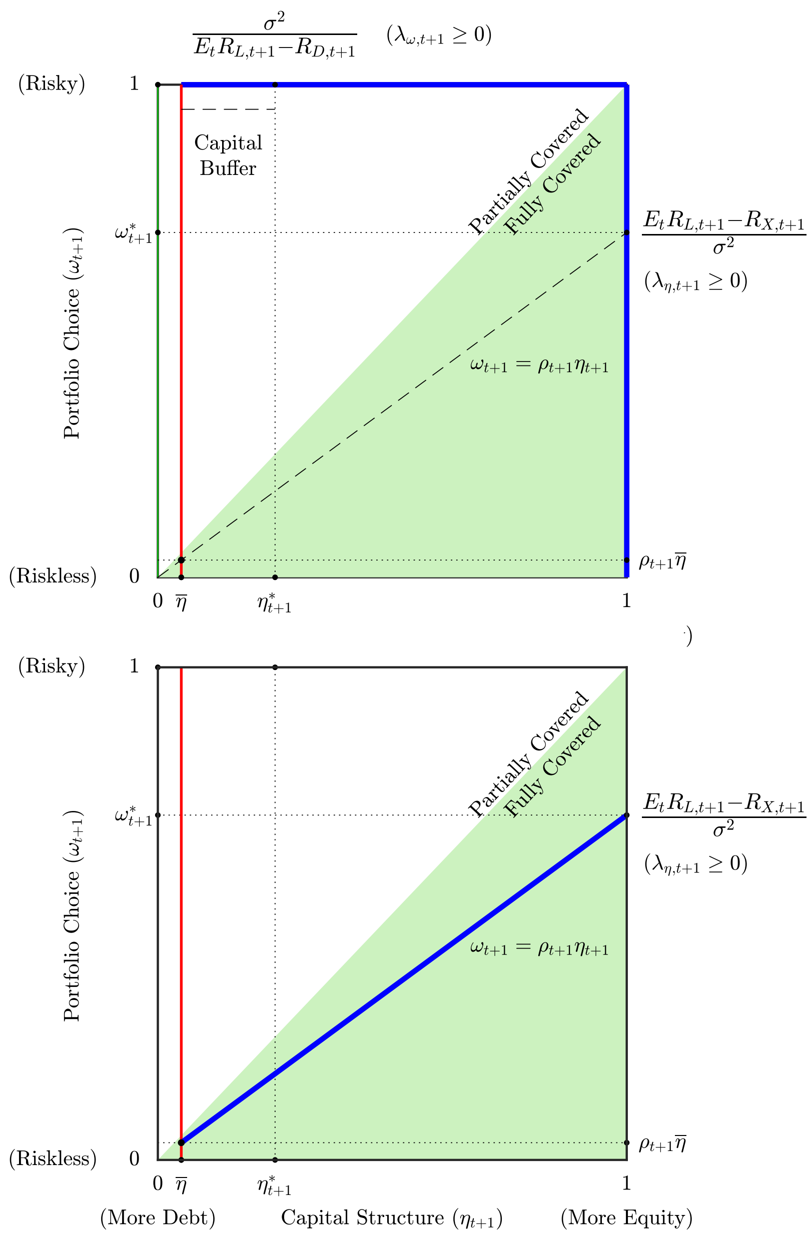



**Figure 3.13:** The top figure shows the solution to the bank's problem when the riskfree gap is positive. The solution set is then defined on the top and right border of the box, where either the capital structure is all equity or that capital structure has interior solution but the bank invests all of its assets in the risky loan. The bottom figure shows the bank's solution set in equilibrium. The reason to get an interior solution for both capital structure and portfolio choice decisions is that in the equilibrium, the risk free gap closes and the excess reserves interest become equal to deposit rates.

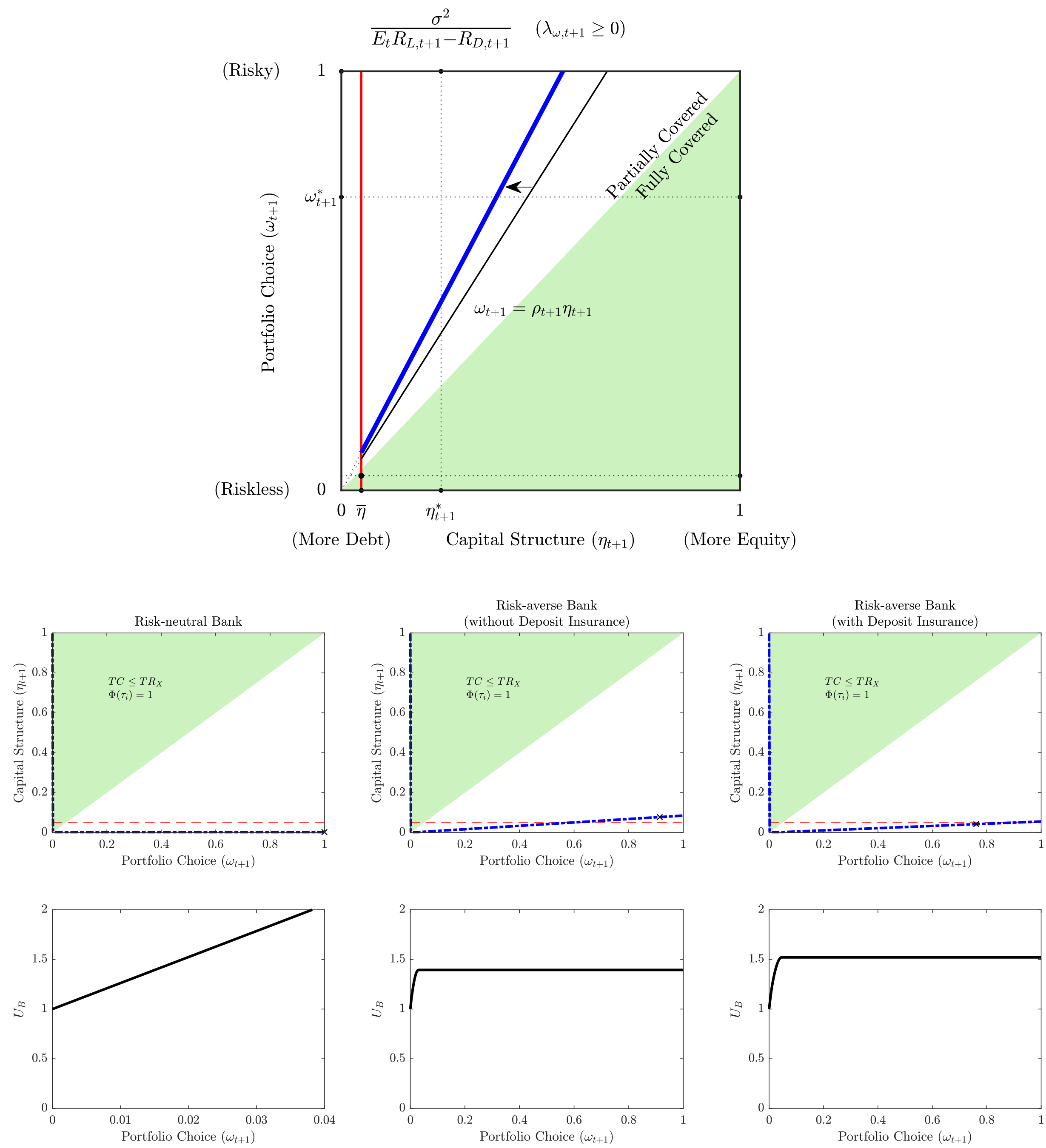
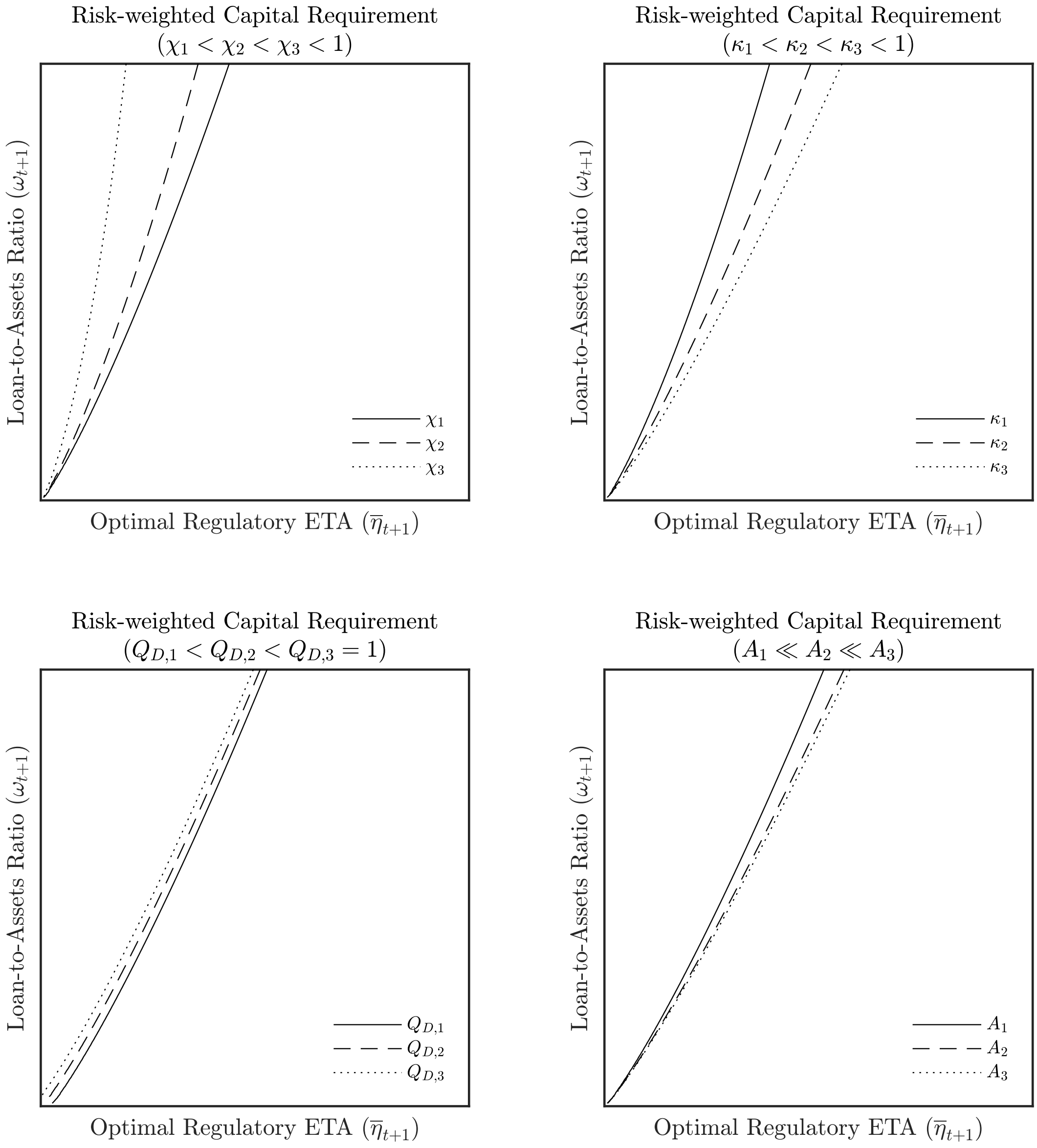



**Figure 3.14:** The top figure shows a comparative statics when the riskiness of loan investment decreases. The result is that the solution set shifts to the left as the wishes to hold more level of risky loan given the same amount of equity.

Top-left figure shows equilibrium expected loan rate as a function of excess reserves interest rate. As the regulator increases the interest rate on excess reserves, the loan rate increases but the spread (shown in top-middle) gradually narrows. The top-right figure shows expected credit spread (partial equilibrium) as a function of capital structure. When the capital structure is taken as given, then more equity (higher $\eta$) indicates lower credit spread. This is because a bank with more equity is able to withstand a more risky position and supply more loans, thereby decreasing the loan rate. The figure in middle-left shows, the equilibrium capital structure (solid line) and equilibrium portfolio position in risky loan (dashed line). Because the bank has more risky investment than its equity, it is exposed to defaults. As the interest rate on excess reserves increases, both $\eta$ and $\omega$ decrease because the bank prefers to invest more resources in the excess reserve and decrease its position in loans. Figure in middle-center shows bank's portfolio position (partial equilibrium) when capital structure decision is given. The bank increases its position in risky asset, as it increases its equity, but general equilibrium prices adjust and credit spread narrows thereby investment in risky asset becomes less attractive. The bottom-left figure shows the equilibrium break-even ex-post loan rate as a function of excess reserves interest rate. As the interest rate increases, the default threshold that sets the break even threshold decreases because the bank is encouraged to take a larger position in risk-free asset and is less likely to go bankrupt. The figure in bottom-middle shows the same threshold in standard normal distribution scale, where higher vertical values imply realization of a tail even which makes the bank break even between total revenues and total costs.



Risk-weighted Capital Requirement
$(\chi_1 < \chi_2 < \chi_3 < 1)$
Loan-to-Assets Ratio $(\omega_{t+1})$
Optimal Regulatory ETA $(\overline{\eta}_{t+1})$
$\chi_1$
$\chi_2$
$\chi_3$
Risk-weighted Capital Requirement
$(\kappa_1 < \kappa_2 < \kappa_3 < 1)$
Loan-to-Assets Ratio $(\omega_{t+1})$
Optimal Regulatory ETA $(\overline{\eta}_{t+1})$
$\kappa_1$
$\kappa_2$
$\kappa_3$
Risk-weighted Capital Requirement
$(Q_{D,1} < Q_{D,2} < Q_{D,3} = 1)$
Loan-to-Assets Ratio $(\omega_{t+1})$
Optimal Regulatory ETA $(\overline{\eta}_{t+1})$
$Q_{D,1}$
$Q_{D,2}$
$Q_{D,3}$
Risk-weighted Capital Requirement
$(A_1 \ll A_2 \ll A_3)$
Loan-to-Assets Ratio $(\omega_{t+1})$
Optimal Regulatory ETA $(\overline{\eta}_{t+1})$
$A_1$
$A_2$
$A_3$


**Figure 3.15**

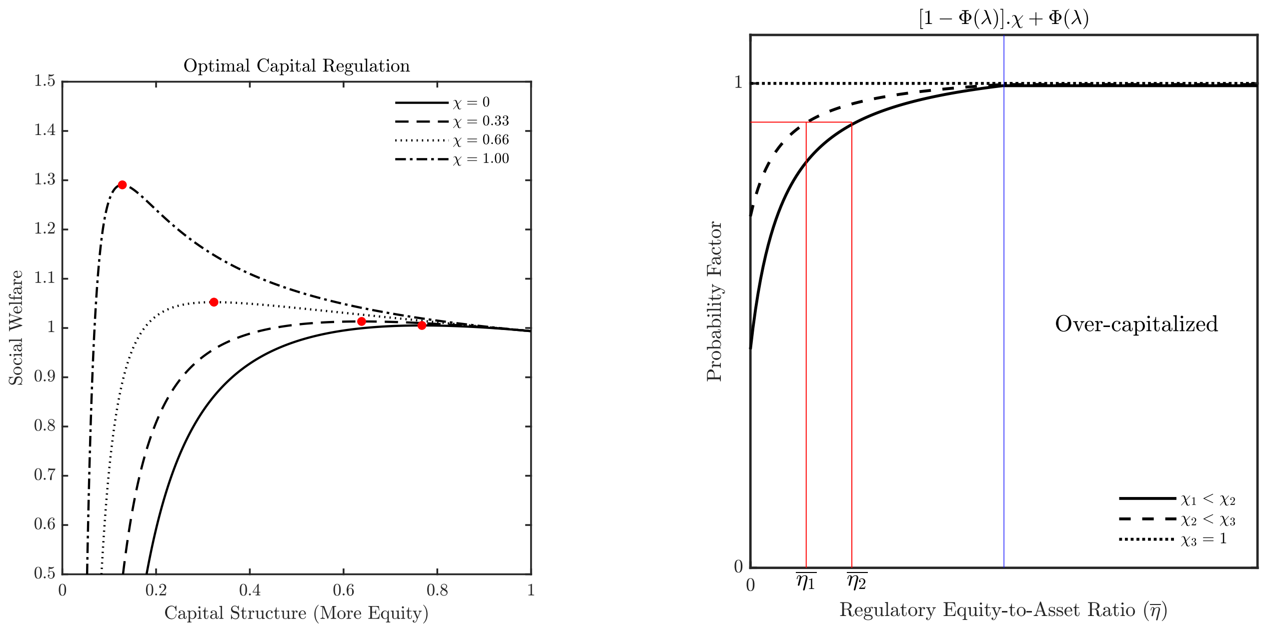



**Figure 3.16:** The figure on the left illustrates the social welfare function over capital structure. Each line shows evaluated social welfare value given a different bankruptcy cost parameter where higher bankruptcy cost (lower $\chi$) lowers the value of social welfare given any capital structure level. Each dot specifies maximum of each welfare function where higher values of $\chi$ are associated with lower optimal capital regulation decision. The figure on the right illustrates the optimal capital structure required by the regulator as a function of seizable asset fraction. Higher seizable fraction implies lower bankruptcy cost which is associated with regulator's lower optimal capital requirement value. Each line shows the relationship given a different value of regulator's risk aversion parameter where a more risk averse regulator levies higher capital requirement for any level of bankruptcy cost.

# Notation

| | |
|---|---|
| **(i) Parameters** | |
| $\beta \in (0,1)$ | HH subjective discount factor |
| $\gamma > 1$ | HH coefficient of RRA |
| $\psi > 0$ | HH EIS |
| $\mu > 0$ | commercial loan portfolio return |
| $\sigma > 0$ | commercial loan portfolio S.D. |
| $\alpha \in (0,1)$ | decreasing return to scale parameter |
| $\chi \in (0,1)$ | default cost |
| $\kappa \in (0,1)$ | underwriting cost |
| **(ii) Variables** | |
| $U(.)$ | HH Utility |
| $S$ | HH total saving |
| $\theta$ | HH portfolio choice |
| $D$ | deposit holding |
| $E$ | equity holding |
| $M$ | SDF |
| $X$ | excess reserves |
| $L$ | loans |
| $A$ | bank total balance sheet size |
| $\eta = E/A$ | bank equity to asset |
| $\omega = L/A$ | bank loan to asset |
| $Div$ | bank dividend |
| $\delta$ | default threshold |
| $\Phi(\delta)$ | bank probability of solvency |
| $z$ | aggregate shock |
| $\overline{\eta}$‘ | capital requirement |
| $\tau$ | taxation |
| **(iii) Prices** | |
| $P_E$ | price of equity |
| $Q_D$ | price of deposits |
| $R_D = 1 + r_D$ | return on deposit |
| $Q_X$ | price of reserves |
| $R_X = 1 + r_X$ | gross IOER |

# Bibliography