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Scale Effects of Non-aerated Flow Properties over Gabion Stepped Spillways

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Abstract. One of the oldest types of human-made hydraulic structures is known as a stepped spillway, and the main use of such structures is to control and monitor the flow of water over embankment dams. However, a common problem encountered by designers of these structures is how to measure the impact of scaling on the structures' performance. This paper thus investigates the effect of scale on the Reynolds and Froude similitudes of such structures using two-dimensional numerical models developed based on Reynolds-averaged Navier-Stokes equations. Dam break conditions were selected to simulate the experiments of this study; therefore, all the essential requirements were applied to achieve model stability. The impact of scaling ratio of 1:2 in gabion stepped spillways on the location of the inception point, the velocity distribution, turbulence intensity, and pressure distribution were explored. The results were then discussed and compared with previous research which conducted over normal stepped spillways. The current study is among the first to provide guidance for the flow properties over gabion stepped spillways which could be impact significantly by the scale effects. This comparative analysis emphasised that full-scale prototype extrapolation conditions may not be feasible based on the Reynolds and Froude similitudes. The current results could be appropriate to be adopted for additional types of water flow over porous steps. Nevertheless, more comprehensive experimental tests for the properties of the water flow at the prototype scale are required.

INTRODUCTION

The stepped channel design for water management was known as early as 3,500 years ago, when it was used by Greek and Minoan engineers [1]. The step size directly impacts the energy dissipation rate at the channel face. Introducing new construction materials for the stepped channels and spillways, such as using Roller Compacted Concrete (RCC) to improve gabion strength, has highlighted the benefits of such systems. More generally, the outlet structure of a spillway is designed to ensure that discharges do not excessively erode the downstream channel bed [1].

Different flows can be obtained depending on the discharge rate, commonly being divided into nappe, transition, and skimming flow regimes [2, 3, 4, and 5]. The most common flow type over stepped spillways is the skimming flow, distinguished by high loss and the transfer of momentum from the main stream to recirculation zones. Due to the nature of this flow, two zones are generated based on air presence, and these are thus referred to as aerated and non-aerated zones, as shown in figure 1 [6].

Identifying the most favourable design of stepped spillways can substantially reduce stilling basin size or even remove the need for such basins, thus significantly decreasing construction costs. This and other benefits have been emphasised in discussions of the importance of stepped spillways as discussed in the literature. It is critical to develop accurate estimations for the main design parameters of stepped spillways, such as the inception point location, velocity distribution, and pressure distribution of the flow, due to the significant contribution of such factors to the design process. Relevant research may be performed numerically or experimentally; however, such work is expensive and time-consuming when conducted experimentally.

Modern high-performance computing now plays a more significant role in providing efficient computational approaches for problem-solving, as common practical problems such as the scale effect can now be examined numerically at reasonable time and financial cost. In the current work, therefore, the properties of skimming flows over gabion stepped spillways were studied numerically. Many studies have already shown wide-ranging use of scaled-down spillways in laboratory experiments. Heller [7] delivered a general literature review of suitable approaches in hydraulic engineering to be used to minimise scale effects.

This paper aims to develop preliminary guidance for assessing the properties of water flow where scale effects are anticipated for undistorted Reynolds and Froude similitudes. This study should thus provide more explicit guidance about those flow properties that may be impacted due to scale effects. As there is limited research data for gabion stepped spillways, the results were compared against a normal stepped spillway case [8]. A common possible scale effect, at a scaling ratio of 2:1, was thus investigated, using comparative analysis to assess the scaling impact on the non-aerated flow properties of the gabion stepped spillways. The data used in the current work included the inception point location, pressure distribution, velocity distribution, and turbulence intensity. The results might thus also be valid for other free-surface water flow types, such as breaking waves over gabion structures and hydraulic jumps.

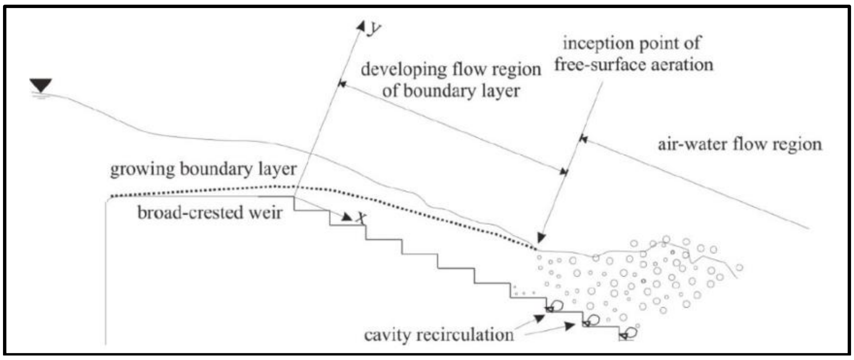


FIGURE 1. The development of inception points and boundary layer in a skimming flow regime (Felder, 2013) [9]

NUMERICAL MODELLING

Hydrodynamics studies have been conducted experimentally using water flumes for many years; however, numerical modelling has more recently become a truly viable alternative due to advances in computer technology. Ohyama and Nadaoka [10] were the first to introduce numerical wave tank (NWT) terminology, with the idea of developing a virtual tank and a set of governing equations to be resolved using computational methods to create a more precise numerical model to illustrate flow phenomena. Flow characteristics such as velocity, free surface, pressure, and other derived variables, including vorticity, strain, stress, and turbulence intensity, could then be directly collected from the numerical results.

NEWFLUME can simulate various different flow regimes, such as turbulent flow with highly shifting free surface, breaking waves, dam-break flows, tidal bores, and hydraulic jumps. NEWFLUME solves the governing equations (Reynolds-averaged Navier-Stokes) in two dimensions; those particular equations are implemented because of their ability to simulate many applications: for example, fluid-structure interactions can be conducted by modelling permeable and impermeable structures [11]. This was also the main reason for selecting this model in the current work.

This model has been validated against many different problems, including cases with seawall breaking waves with a porous armour layer, submarine pipeline flow forces, dam breaks, jets (plane and submerged), hydraulic jumps, and water exiting from a circular cylinder [12]. Moreover, the numerical model in NEWFLUME was validated previously by Reeve et al. [13] and Zuhaira et al. [14] for different cases of gabion stepped spillways using a variety of experimental data. The results demonstrated very good agreement with the experiments, providing reassurance that the code can replicate the appropriate flow conditions. The technical description of the model, following that of Lin and Xu [12], is reproduced here for the sake of completeness.

The velocity and the pressure fields for the total flow can be separated into mean velocity $\langle u_i \rangle$ and pressure $\langle p \rangle$ alongside turbulent velocity, $\grave{u}_i$ and pressure, $\grave{p}$.

$$u_i = \langle u_i \rangle + \grave{u}_i \qquad (1)$$

$$p = \langle p \rangle + \grave{p} \qquad (2)$$

where i = 1, 2 for a two-dimensional flow. The Reynolds equations for incompressible fluids are used as the governing equations of the mean motion of turbulent flows:

$$\frac{\partial\langle u_i\rangle}{\partial x_i} = 0 \quad (3)$$

$$\frac{\partial\langle u_i\rangle}{\partial t} + \langle u_j\rangle\frac{\partial\langle u_i\rangle}{\partial x_j} = -\frac{1}{\rho}\frac{\partial\langle p\rangle}{\partial x_i} + g_i + \frac{1}{\rho}\frac{\partial}{\partial x_j}\left(\mu\,\frac{\partial\langle u_i\rangle}{\partial x_j} - \rho\,\langle\,\grave{u}_i\grave{u}_j\rangle\right) \quad (4)$$

where

$\langle u_i\rangle$ = the mean velocity in the i directions measured in m.s^{-1} units

$\langle p\rangle$ = mean pressure measured in KN.m^{-2} units

ρ = fluid density measured in Kg.m^{-3} units

g_i = gravitational acceleration in the i direction measured in m.s^{-2} units

μ = molecular viscosity measured in m^2.s^{-1} units

$\langle\,\grave{u}_i\grave{u}_j\rangle$ = Reynolds stress measured in KN.m^{-2} units

The mean viscous stress is given as

$$\langle\tau_{ij}\rangle = \mu\frac{\partial\langle u_i\rangle}{\partial x_j} + \mu\frac{\partial\langle u_j\rangle}{\partial x_i} \quad (5)$$

Reynolds stress is calculated using the nonlinear eddy viscosity model, which adopts the mean velocity, turbulence kinematic energy (k) and dissipation rate of turbulence (ε) [12].

$$\frac{\partial k}{\partial t} + \langle u_j\rangle\frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\frac{v_t}{\sigma_k} + v\right)\frac{\partial k}{\partial x_j}\right] - \langle\,\grave{u}_i\grave{u}_j\rangle\frac{\partial\langle u_i\rangle}{\partial x_j} - \varepsilon\frac{\partial\varepsilon}{\partial t} + \langle u_j\rangle\frac{\partial\varepsilon}{\partial x_j} \quad (6)$$

$$\frac{\partial\varepsilon}{\partial t} + \langle u_j\rangle\frac{\partial\varepsilon}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\frac{v_t}{\sigma_\varepsilon} + v\right)\frac{\partial\varepsilon}{\partial x_j}\right] - c_{1\varepsilon}\frac{\varepsilon}{k}\langle\,\grave{u}_i\grave{u}_j\rangle\frac{\partial\langle u_i\rangle}{\partial x_j} - c_{2\varepsilon}\frac{\varepsilon^2}{k} \quad (7)$$

where

v and $v_t = \frac{c_d * k^2}{\varepsilon}$ for the kinematic and eddy viscosity, respectively (m^2.s^{-1}) and

$c_d = \frac{2}{3}\left(\frac{1}{7.4+s_{max}}\right)$ where $s_{max} = \frac{k}{\varepsilon}$ max $[|\frac{\partial\langle u_i\rangle}{\partial x_i}|$ (indices not summed)].

The recommended values for the empirical coefficients are thus $\sigma_k = 1.0, \sigma_\varepsilon = 1.3, c_{1\varepsilon} = 1.44$ and $c_{2\varepsilon} = 1.92$ [11].

The porous media mean flow is governed by

$$\frac{\partial\bar{u}_i}{\partial x_i} = 0 \quad (8)$$

$$\frac{1+C_A}{n}\frac{\partial\bar{u}_i}{\partial t} + \frac{\bar{u}_j}{n^2}\frac{\partial\bar{u}_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial\bar{p}}{\partial x_i} + g_i + \frac{v}{n}\frac{\partial^2\bar{u}_i}{\partial x_j\partial x_i} - ga_p\bar{u}_i - gb_p\sqrt{\bar{u}_k\bar{u}_k}\,\bar{u}_i \quad (9)$$

where

$\bar{u}_i$ is the i-th component of the mean velocity,

n is the porosity of the porous medium,

C_A, a_p, and b_p are the porous medium coefficients, and

The subscript k denotes the velocities' summation in two directions.

The coefficients in the previous equations are thus

$C_A = \gamma_p\frac{1-n}{n}$

$a_p = \alpha\frac{(1-n)^2}{n^3}\frac{v}{gD_{50}^2}$

$b_p = \beta\left(1 + \frac{7.5}{KC}\right)\frac{1-n}{n^3}\frac{1}{gD_{50}}$

where $\gamma_p = 0.34, \alpha = 200, \beta = 1.1, KC = \frac{\sqrt{\bar{u}_k\bar{u}_k}\,T}{nD_{50}}$

Implementing the model requires the initial conditions across the computational domain for pressure distribution and mean flow. These initial conditions can be obtained from the initial stationary flow alongside the hydrostatic pressure and zero mean velocity at the start of the test. Based on analytical solutions and laboratory measurements, mean velocities can also be determined, along with the initial displacements of the flow free surface. The mean flow is assumed to be similar to that of still water, with no waves or current motions in the initial conditions.

For free surface tracking, the NEWFLUME model uses a method referred to as the volume of fluid method (VOF). Hirt and Nichols [15] initially developed this, while Kothe et al. [16] modified it later. There are two main benefits to using VOF: the first is that it needs minimum storage for its computations, while the second is that the VOF avoids issues with intersecting surfaces due to its ability to follow the regions rather than boundaries.

The default set-up uses a fixed rectangular mesh system of m by n cells to discretise the computational domain. All scalar quantities, such as pressure, eddy viscosity, and turbulence intensity, are specified at the cell centres, while the vector quantities, such as velocities are given at the cell nodes. This model can develop uniform and non-uniform meshes: a uniform mesh is usually applied when accurate results are required, while in terms of computational cost, non-uniform meshes can reduce running time. As the accuracy of calculation was paramount in the current research, uniform grids were used.

GABION STEPPED SPILLWAYS CONFIGURATION

It is challenging to simultaneously satisfy Froude and Reynolds's similarity in a scaled case in practical design, and an actual resemblance cannot be attained. Fitting Froude or Reynolds similitude, thorough potential scale effect testing, is needed to guide the flow properties scalability.

In this work, a systematic examination of the properties of the water flow over the non-aerated zone was performed as a way to examine undistorted Froude, and Reynolds similitude scaling criteria over gabion stepped spillways. Two different geometrically scaled models were tested numerically using the NEWFLUME model. The scaling ratio was set to 1:2, with two different steps heights of 0.05 m (about 20 steps) and 0.1 m (about 10 steps). The resulting stepped configurations were then each fitted with a channel slope of θ = 26.6° in the same test section. Identical initial water depth was then applied at the upstream end over a broad-crested weir to create a dam break condition. This initial water depth was selected to achieve a range of flow rates, being set to 1.7 m. The gravel used to fill the gabions had a D_{50} of 0.017 m and a porosity of 0.325. The same experimental conditions were replicated in both cases, with the mesh size set to 0.012 m and 0.005 m in the x- and y-directions, respectively. The flow rate was determined over the critical section of the broad crested weir, and to achieve numerical stability in the NEWFLUME model, the initial time step was set to be 0.001 s. The total time for the simulation was 15 s. All boundaries in the numerical flume were set as closed, excluding the right boundary, which was fixed to be opened in order to allow the water stream to flow outside of the numerical flume.

SCALE EFFECT IN NON-AERATED FREE SURFACE FLOW

The flow conditions of the experiments used in the current work are summarised in Tables 1 and 2. These included the dimensionless discharge, dc/h, the discharge per unit width, q_w, the Reynolds number, the calculated step edge, and the inception point location (L_i) for the skimming flow regime (SK). Scale effects comparison was performed at identical distances upstream the point of the air entrainment. The results obtained from these analyses, shown in Tables 1 and 2, were then compared with the flow properties for Reynolds and Froude similitudes.

TABLE 1. Comparison of water flow properties vs. experimental flow using Froude similitude

Step height (h), 0.1m				Step height (h), 0.05m			
dc/h [-]	q_w [m²/s]	Re [-]	Inception point	dc/h [-]	q_w [m²/s]	Re [-]	Inception point
2.505	0.394	$3.92*10^5$	-	2.500	0.1372	$1.37*10^5$	End of step 9 (L_i=1.002 m)
2.254	0.3345	$3.33*10^5$	-	2.248	0.1191	$1.19*10^5$	Beginning of step 9 (L_i=0.894 m)
2.009	0.2802	$2.79*10^5$	-	2.002	0.0983	$9.79*10^4$	End of step 8 (L_i=0.881 m)
1.753	0.2304	$2.29*10^5$	End of Step 10 (L_i=2.004 m)	1.750	0.0823	$8.20*10^4$	End of step 5 (L_i=0.559 m)
1.502	0.1836	$1.83*10^5$	Middle of Step 10 (L_i=1.923 m)	1.500	0.0641	$6.38*10^4$	End of step 3 (L_i=0.331 m)

TABLE 2. Comparison of water flow properties vs. experimental flow using Reynolds similitude

Step height (h), 0.1m				Step height (h), 0.05m			
dc/h [-]	q_w [m²/s]	Re [-]	Inception point	dc/h [-]	q_w [m²/s]	Re [-]	Inception point
2.505	0.394	$3.92*10^5$	-	4.960	0.3939	$3.92*10^5$	-
2.254	0.3345	$3.33*10^5$	-	4.464	0.3355	$3.34*10^5$	-
2.009	0.2802	$2.79*10^5$	-	4.018	0.2802	$2.79*10^5$	-
1.753	0.2304	$2.29*10^5$	End of Step 10 (L_i=2.004 m)	3.506	0.2304	$2.29*10^5$	-
1.502	0.1836	$1.83*10^5$	Middle of Step 10 (L_i=1.923 m)	3.004	0.1836	$1.83*10^5$	End of step 20 (L_i=2.115 m)

Froude Similitude

As mentioned before, the properties of water flow were used to investigate scaled geometry effects on the skimming flows over gabion stepped spillways. Dimensionless terms were used to present the data for several consecutive step edges (S) as a function of the dimensionless distance perpendicular to the direction of the main flow, y/dc. Although the water flow properties indicated some differences among scaled gabion stepped spillway models, the critical parameter of velocity distribution demonstrated relatively good agreement (Fig. 2). As illustrated in figure 2, the scale effect had only minor effects at high flow rates as compared to lower discharges.

These results are in line with Felder and Chanson [17], who stated that, for skimming flow regimes over normal stepped spillways, the velocity profiles could be scaled using Froude similitude. Moreover, the results reinforce previous observations by Felder and Chanson [18], Boes [19], and Chanson and Gonzalez [20] with regard to skimming flow behaviours over normal stepped spillways.

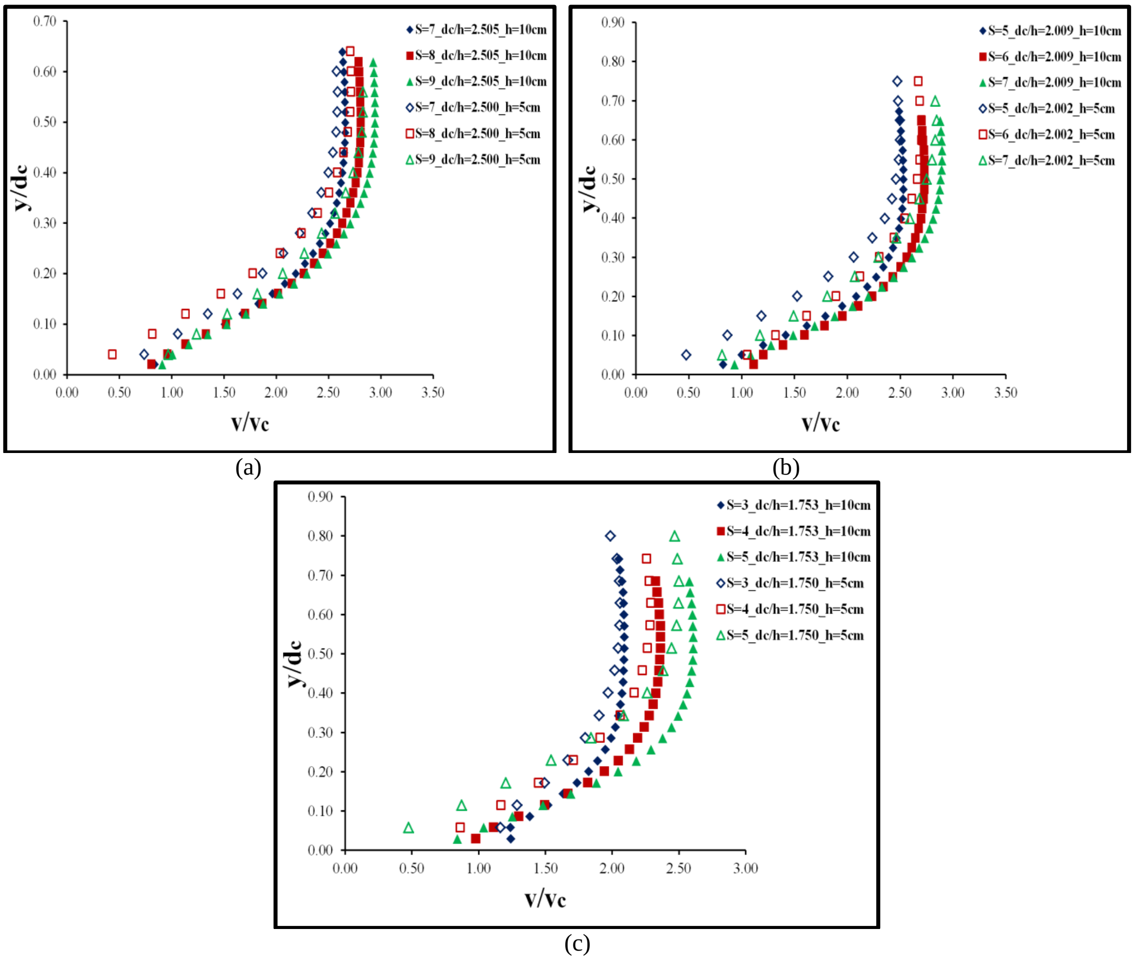
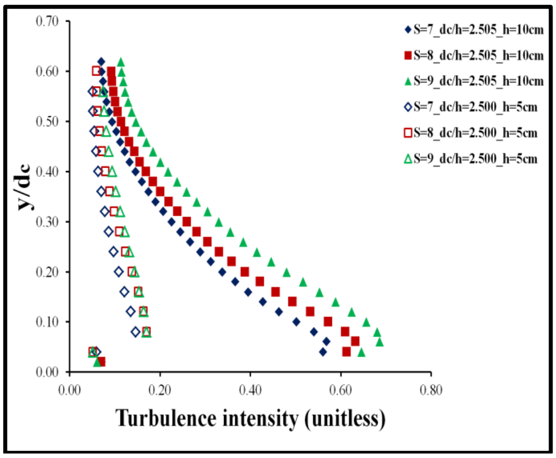


FIGURE 2. Water flow velocity distribution for Froude similitude

Considerable differences were noted among turbulence intensity distributions (Fig. 3), especially near the surface of the porous media. The turbulence intensities were consistently more significant for the higher steps (Fig. 3), and the Froude values did not accurately scale the turbulence intensity; this aligns with the observations of Felder and Chanson [17] in their stepped spillway experiments.

Figure 4 illustrates a comparison of pressure distributions for scaled gabion stepped spillways. The results obtained from the analysis indicate significant disagreements in the pressure values inside the porous media and over the first third of the distance from the gabion box. However, these differences gradually reduce towards the water-free surface, becoming negligible at the water free surface. In addition, the results showed a higher-pressure value for the highest steps for all data as compared to lower steps; thus, the scaling of the pressure distribution to the scaled prototype is very crude when based on Froude similitude.



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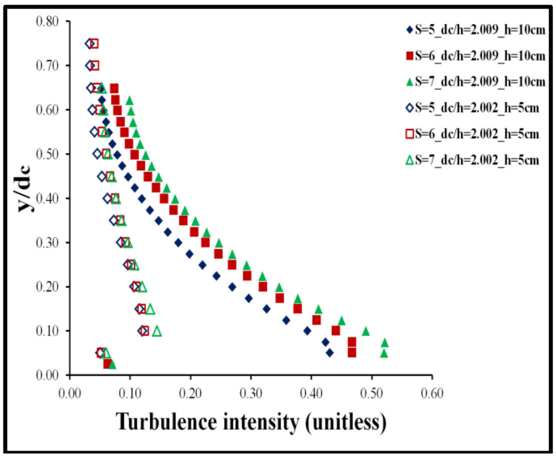

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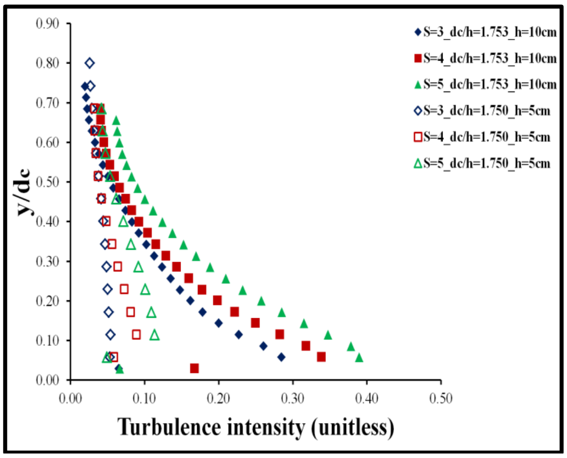

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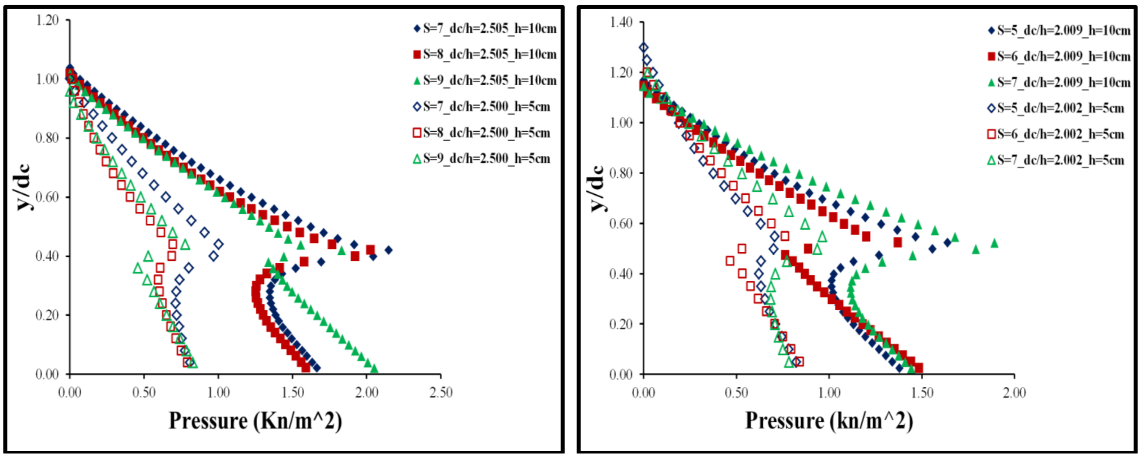
FIGURE 3. Turbulence intensity distribution for Froude similitude



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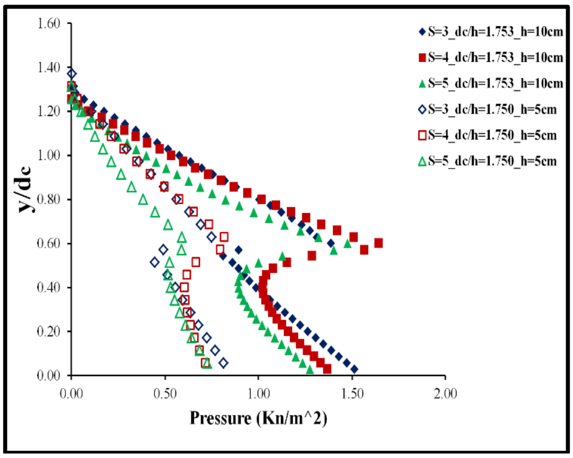

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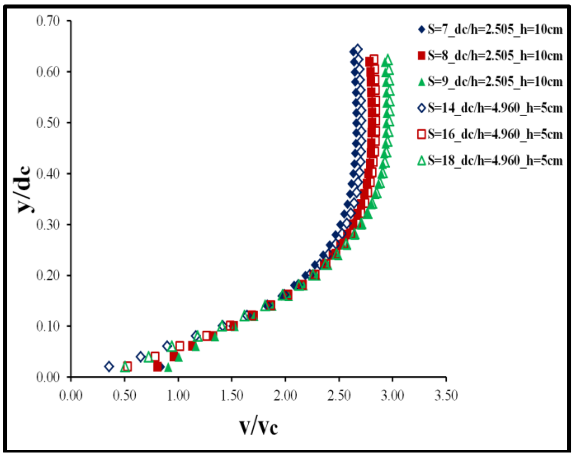
FIGURE 4. Pressure distribution for Froude similitude

Reynolds Similitude

Using Reynold similitude to capture the effect of scaling models is essential; thus, comparative analyses were performed for a range of skimming flow conditions (Table 2) at the edges of the step, with identical distances downstream of the weir crest. In order to explore the scale effects based on Reynolds similitude for the main design parameters of the flow, such as the distribution of the velocity, turbulence intensity, and pressure distribution, the comparative analysis was done as a function of y/dc. This is illustrated in figures 5, 6, and 7.

As shown in figure 5, the flow velocity distribution over gabion stepped spillways in the non-aerated zone demonstrates fairly close alignment to the scaled models; this aligns with Felder and Chanson [17], and also verifies that the flow velocity is scalable based on Reynolds similitude. Furthermore, minimal differences were observed when the flow rates were low, though values of the flow velocities were slightly larger, by 5%, for the smaller step heights as compared with larger step heights.

However, the other properties demonstrated that the most water flow characteristics were not induced in the scaled prototype, based upon the Reynolds criterion. The turbulence levels, as shown in figure 6, demonstrated more significant impact when the step height increased: that impact was estimated to be between 0 to 35% for all data sets. These differences did get smaller gradually as the model moved towards being water-free, and might reach zero for low flow rates.



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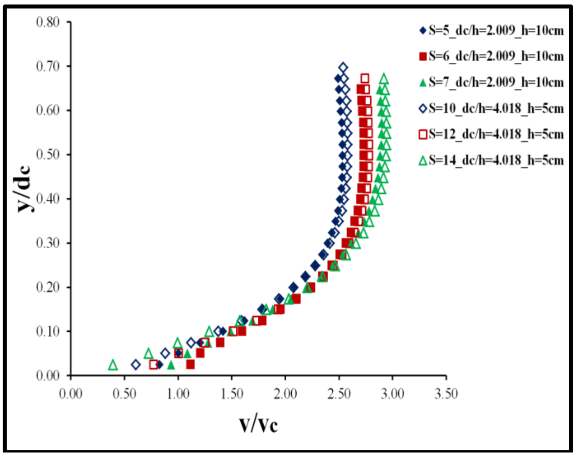

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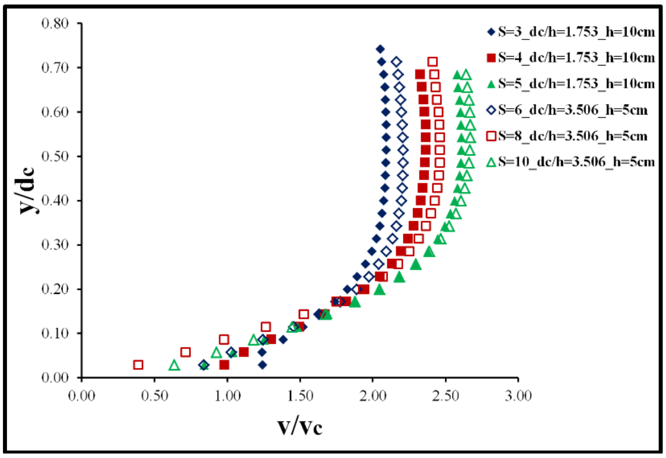

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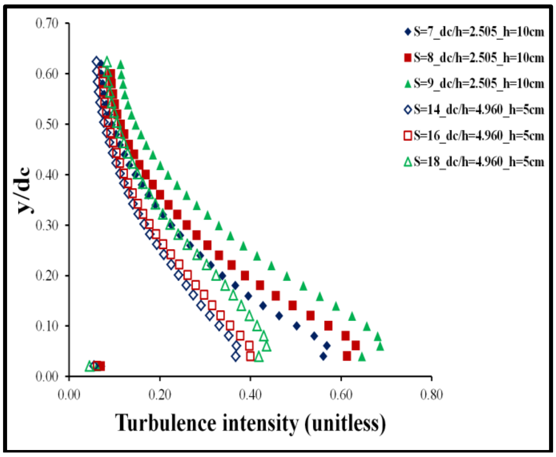
FIGURE 5. Water flow velocity distribution for Reynolds similitude



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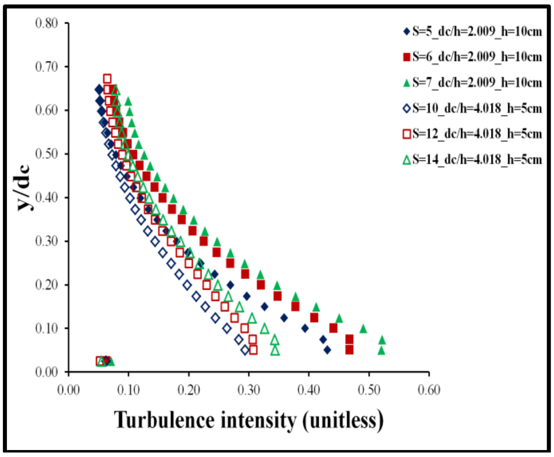

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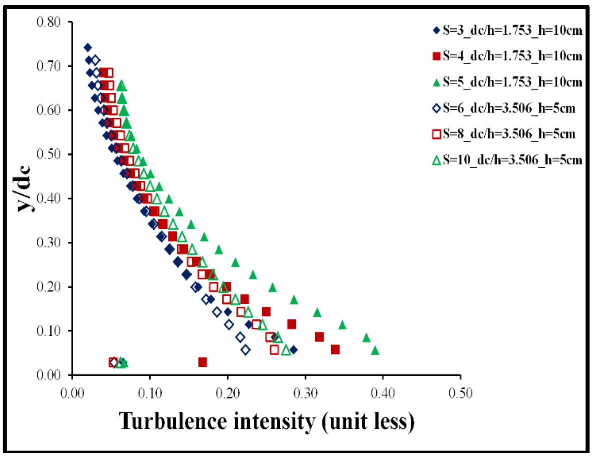

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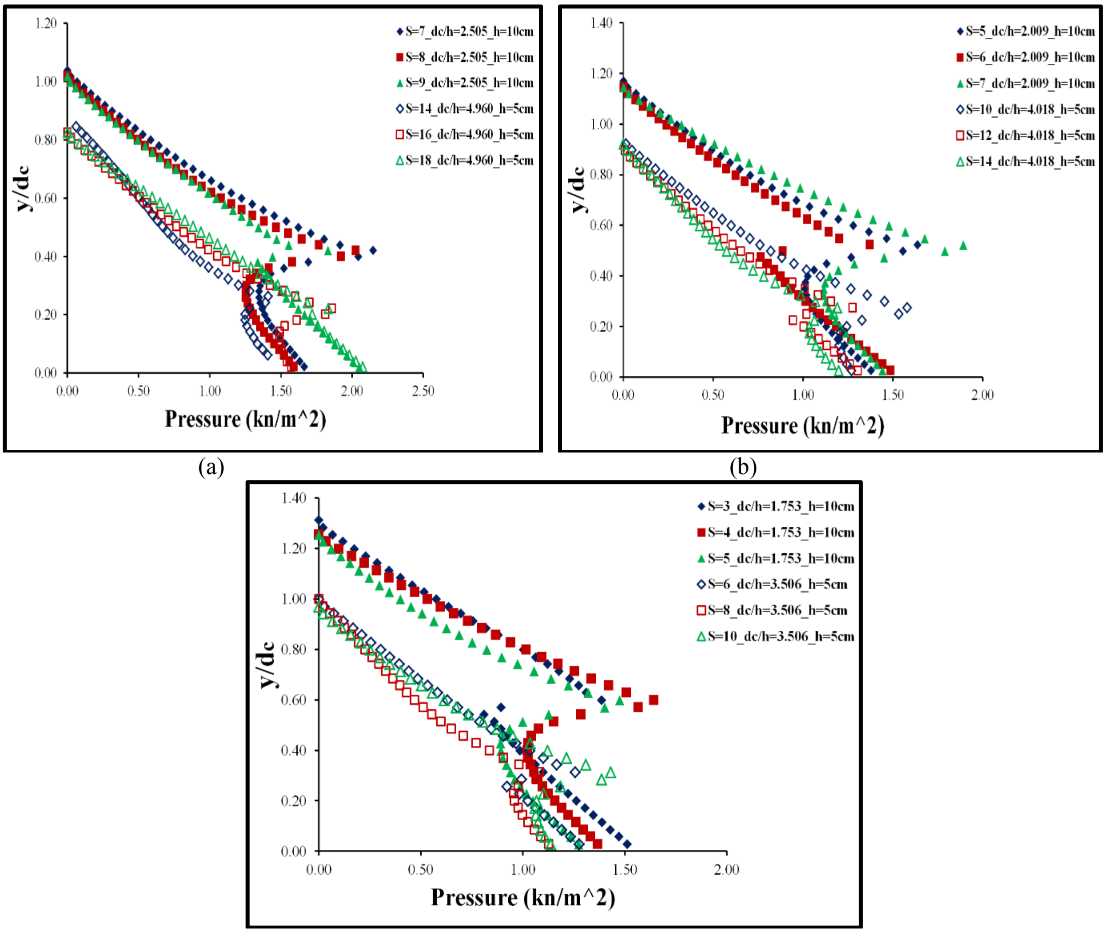
FIGURE 6. Turbulence intensity distribution for Reynolds similitude

Substantial scale effects were noted for the pressure distributions over the non-aerated zones of gabion stepped spillways, highlighting that the pressure over the non-aerated zone could not be precisely scaled using Reynolds similitude (Fig. 7). The results revealed that the differences near the impervious edge varied from 0 to 17%, yet this was nevertheless better than that seen using Froude similitude, which was around 60%. However, in the latter case, the pressure values were almost identical near the water-free surface, as mentioned earlier, while this was not achieved when the Reynolds similitude was applied, when differences varied from 15 to 21%.

Scaling the pressure distribution over the non-aerated zones in gabion stepped spillways to prototype scale is thus impossible using either Reynolds or Froude similitude.



(a) (b)

(c)

FIGURE 7. Pressure distribution for Reynolds similitude

CONCLUSION

In this paper, scaling effects for gabion stepped spillways were numerically investigated using two different models. The data were validated using experimental data obtained from the literature. Given the acknowledged limitations of this work, three main points can be concluded:

- The critical parameter of velocity distribution demonstrated relatively good agreement with Froude similitude; however, some differences were observed, especially at low flow rates. The flow velocity profile offered very good agreement throughout, based upon Reynolds similitude.
- Turbulence intensities were not scalable for either Froude or Reynolds similitude.
- Scaling of pressure distribution over the non-aerated zone in a gabion stepped spillway to prototype scale is impossible using either Reynolds or Froude similitude.

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