A Simplified Energy Model Approach for the Determination of Long-Term Crack Width in Reinforced Concrete Elements

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ABSTRACT

The deformation of reinforced concrete elements is a main consideration during the design process due to its importance for the service life of structures. Deformation (deflections and cracking) control has an effect at both the design and construction stages. The determination of crack width based on simple code provisions, leads to conservative results, due to many affecting parameters being unknown. The exact determination of the crack width is a very complicated process with significant computational load. In the current paper, a simplified procedure is proposed for the prediction of crack width at different loading stages. The procedure is based on a previously published energy model that uses the integration of the moment-curvature relationship to take into consideration the load stage and most affecting design variables. The obtained results using the simplified model are then tested against previously published experimental data and a good agreement is shown.

Keywords-reinforced concrete elements; deformation; state diagrams; energy model; general and residual resources

I. INTRODUCTION

The serviceability limit state of concrete structures is one of the most important steps within the design. The determination of deflection and crack width using code provisions commonly results in conservative values due to many important variables affecting design being unknown [1-3]. The idealized stress-strain curves for both concrete and reinforcing steel as well as the equivalent stress block in concrete, represent some major causes of conservative estimations. Another design approach for crack width calculation in reinforced concrete structures under short-term loading was developed in [4], based on the actual strain distribution along the spacing between two adjacent cracks. Determination of the real stress-strain state of reinforced concrete elements at different stages of loading has always been a key task in reinforced concrete theory. Its implementation is largely associated with the development of general models of deformation. The fulfillment of most of validity requirements in the so-called "strength" models [5] has been impractical due to the use of an idealized equivalent rectangular stress block. In "deformation" models [6-8], the methodological unity of all calculations of reinforced concrete elements according to limit states can generally be ensured with the help of complete concrete stress-strain curves ($\sigma_c - \varepsilon_c$). However, when integrating these curves or even using simplified forms of these diagrams [7, 8], the internal static uncertainty of the cross-section of reinforced concrete elements must be calculated through numerous iterations, involving several empirical parameters and coefficients which result in a significant computational load.

The concrete deformation model [9] can be considered the closest one to the generalized model of reinforced concrete elements deformation today. It shows the reason for being impossible to build a generalized model of the deformation of reinforced concrete elements with just the help of a stress-strain diagram of concrete. The calculation of concrete deformation at any stage is not controlled by the concrete deformation diagrams, but by the moment-curvature relationship of the reinforced concrete element ($M - \phi$), which characterizes its stiffness. However, the concrete deformation model needs further development, especially in matters of assessing the actual technical condition of reinforced concrete elements and structures and calculating their capacity and deformations.

The current paper aims to predict long-term crack width based on energy modeling. The proposed procedure is largely based on the concrete deformation model [9], modified in order to simplify the calculations and lighten the computational load. The simplified model is then applied on a previously published case study.

II. MOMENT CURVATURE RELATIONSHIP

For the case of short-term loading, the relation between the moment and curvature will have the form presented in [9]:

$$M = \frac{E_c I_{cr} \cdot \phi - M_{uf} \cdot (\phi/\phi_{uf})^2}{1+[E_c I_{cr}/M_{uf} - 2/\phi_{uf}] \cdot \phi} \quad (1)$$

where E_c is the concrete's modulus of elasticity, I_{cr} is the moment of inertia of the cracked section, M_{uf} is the moment capacity at failure, and ϕ_{uf} is the limit value of the curvature at failure. For the case of long-term loading, the relation will follow the same curve of the short-term loading up to the value of service load (M_a, ϕ_a), then it will have a constant value of moment with the increase of the curvature. The increase in curvature after applying the service load will be due to the creep of concrete.

Solving (1) for the curvature, the following expression is provided in [10]:

$$\phi = \frac{\phi_{uf}}{2M_{uf}} \left[\left(1 - \frac{M}{M_{uf}}\right) \cdot E_c I_{cr} \phi_{uf} + 2M - \sqrt{\left(\left(1 - \frac{M}{M_{uf}}\right) \cdot E_c I_{cr} \phi_{uf} + 2M\right)^2 - 4M \cdot M_{uf}} \right] \quad (2)$$

For the case of short-term loading (Figure 1), three points must be defined. For a given concrete element with defined section properties, the value of moment at the three defined points in the shown moment-curvature curve can be calculated (M_a, M_u, and M_{uf}). The curvature at failure (ϕ_{uf}) can be determined using the hypothesis of concrete plane section under bending. Then the corresponding curvature at service and ultimate load (ϕ_a and ϕ_u) can be calculated using (2). These parameters are essential for use in the energy modeling approach.

III. A SIMPLIFIED APPROACH ON ENERGY MODELING

In general, it is known that in the deformation-force model, the main deformation and force parameters of the state of the element are interconnected by utilizing stiffness $EI = M/\phi$ [11-13]. The mentioned parameters can be combined using another characteristic: the energy consumed on the deformation of a reinforced concrete element per unit volume $W = M \cdot \phi$ [14]. Under such circumstances, the methodology for the calculation of the total energy of reinforced concrete elements can be used to determine the deformations directly. The energy modeling approach has been developed in [9, 10] and is summarized as follows: the total energy consumed at any stage of short-term loading can be determined by integrating (1) from null to the curvature value at the stage under consideration. As explained in [10], three integrations must be performed for the three defined stages on the moment curvature curve, at the service load, at the ultimate load and at the failure load. These three integration results are called $W_1 (W_a)$, W_u and W_{uf} respectively (Figures 2-4).

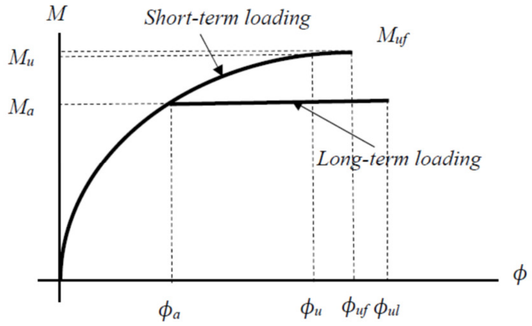

Fig. 1. Moment-curvature relationship for short-term and long-term loading.

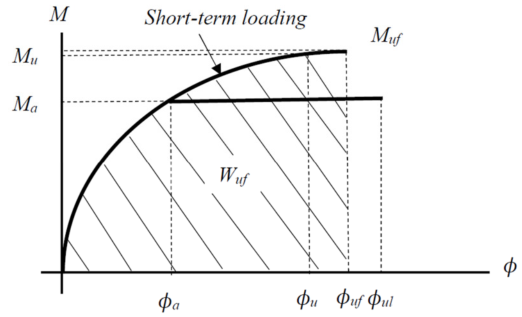

Fig. 2. Total energy consumed up to failure in case of short-term loading.

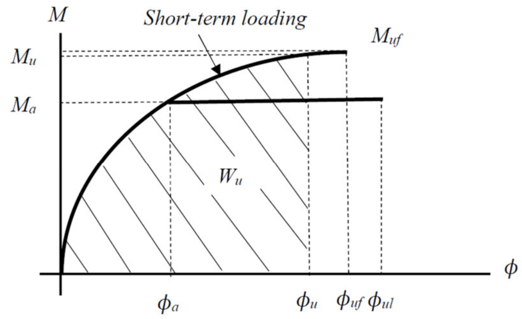

Fig. 3. Total energy consumed up to ultimate load in case of short-term loading.

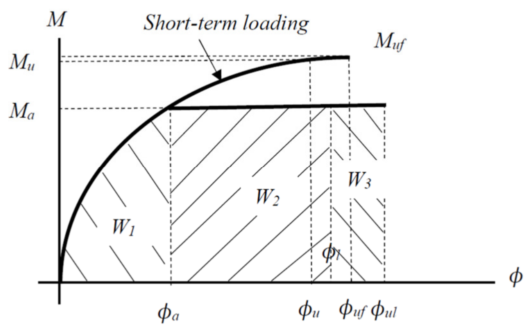

Fig. 4. Total energy consumed up to failure in case of long-term loading.

The energy consumed in both cases of the short-term and the long-term loading must be equal [9]. The following relations can be used to determine both W_2 and W_3:

$$W_u = W_1 + W_2 \tag{3}$$

$$W_{uf} = W_1 + W_2 + W_3 \tag{4}$$

The value of long-term loading curvature can be determined for both values of long-term (ϕ_l) and ultimate long-term curvature (ϕ_{ul}) using:

$$\phi_l = \frac{W_2}{M_a} + \phi_a \tag{5}$$

$$\phi_{ul} = \frac{W_{uf} - W_a}{M_a} + \phi_a \tag{6}$$

The application of the integrations presented in [10] for the determination of the consumed energy W_1, W_u, and W_{uf}, has a significant computational load. Using a rough approach for the curve line portion of long-term moment-curvature relationship from ϕ_a to ϕ_u and from ϕ_u to ϕ_{uf}, the following equations can be derived for the calculation of W_2 and W_3:

$$W_2 = \frac{M_a + M_u}{2} \cdot (\phi_u - \phi_a) \tag{7}$$

$$W_3 = \frac{M_u + M_{uf}}{2} \cdot (\phi_{uf} - \phi_u) \tag{8}$$

Solving (5) with (7) and (6) with (8), the corresponding curvatures for the long-term loading case will be:

$$\phi_l = \frac{1}{2}\left(1 + \frac{M_u}{M_a}\right)(\phi_u - \phi_a) + \phi_a \tag{9}$$

$$\phi_{ul} = \frac{1}{2}\left(1 + \frac{M_{uf}}{M_u}\right)(\phi_{uf} - \phi_u) + \phi_l \tag{10}$$

Following this scheme, once the moment and the corresponding curvature at the three defined stages of short-term loading have been defined, the values of long-term curvature ϕ_l and ϕ_{ul} can be determined directly using (9) and (10). Knowing the curvature, the following formula can be used to determine the crack width [15]:

$$\phi = \frac{\varepsilon_{c,m} + w_{cr}/S_{cr} + \varepsilon_{ctu,m}}{d} \tag{11}$$

where s_{cr} and w_{cr} are the spacing and the normal crack width, $\varepsilon_{c,m}$ and $\varepsilon_{ctu,m}$ are the mean values of compression and ultimate strain in concrete at the tension side [9], respectively, and d is the effective depth of the concrete section.

The corresponding curvature ϕ_l is related to the crack width w_l and the average strain of compressed concrete $\varepsilon_{cl,m}$, as follows:

$$w_l = (\phi_l \cdot \mathrm{d} - \varepsilon_{cl,m} - \varepsilon_{ctu,m}) \cdot S_{cr} \tag{12}$$

$$\varepsilon_{cl,m} = \varepsilon_{c,m} \cdot (1 + \varphi(t, t_0)) \tag{13}$$

where $\varphi(t, t_0)$ is the value of the creep coefficient of compressed concrete under the action of an external load with a duration of $(t - t_0)$ [7]. The value of crack width w_l can be determined from (10).

The crack width at failure under long-term loading will have the form:

$$w_{ul,cr} = (d.\phi_{ul} - \varepsilon_{clu,m} - \varepsilon_{ctu,m}) \cdot S_{cr} \tag{14}$$

IV. COMPARISON WITH EXPERIMENTAL RESULTS

The theoretical prediction of long-term crack width has been performed for the 4 beams tested in [16]. The selected beams are simply supported with spans of 3500 mm and are reinforced by two longitudinal bars of 16 mm diameter. The concrete compressive strength at 28 days was 24.8 MPa. The cross-section of the beams is shown in Figure 5 and the details are given in Table I. The maximum crack width measured from the tests under long-term loading at 14 and 400 days are also presented in Table I along with the predicted values of crack width. The detailed results of the theoretically predicted long-term crack widths were compared with the experimental ones and the result is presented in Figures 6 and 7. The theoretically predicted crack width at different ages correspond well with experimental results. From both the theoretical and experimental results, it has been observed that the crack width increases rapidly at early ages up to the first 100 days, and then slowly increases. The maximum long-term crack width has been performed at an average age of 200 days.

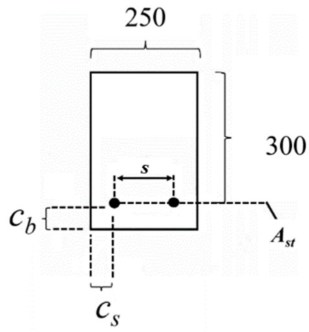

Fig. 5. Cross section of the selected beams.

TABLE I. COMPARISON OF EXPERIMENTAL AND THEORETICAL RESULTS

Beam	C_b	C_s	s	Crack width (mm)			
				Experimental		Theoretical	
				14 days	400 days	14 days	400 days
B1-a	40	40	150	0.13	0.38	0.14	0.37
B1-b	40	40	150	0.05	0.18	0.08	0.17
B2-a	25	25	180	0.10	0.36	0.14	0.37
B2-b	25	25	180	0.05	0.18	0.08	0.17

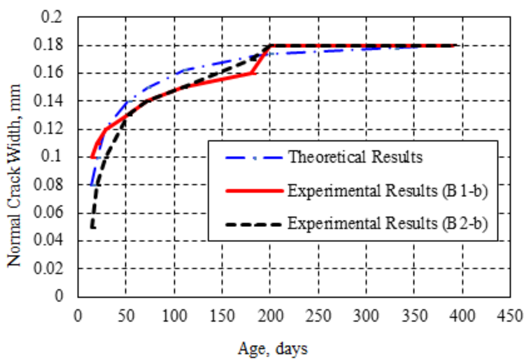

Fig. 6. Comparison of the experimental and theoretical results of the long-term crack width in reinforced concrete beam B1-a and B2-a presented in [16].

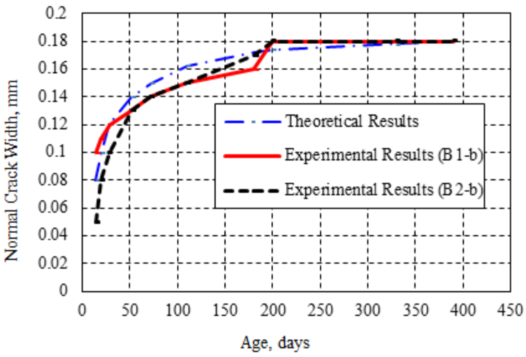

Fig. 7. Comparison of the experimental and theoretical results of the long-term crack width in reinforced concrete beams B1-b and B2-b presented in [16].

V. CONCLUSIONS

In this paper, a simplified approach is proposed for the calculation of the long-term crack width in concrete elements based on an energy model earlier model published in [10]. The simplified approach is applied on experimental data previously published in [16]. As shown, the simplified model shows good agreement with the experimental results. Future research should focus on testing the simplified model in other case studies. Further, the exact tradeoff between computational load and achieved accuracy should be investigated.

REFERENCES

[1] M. Oad, A. H. Buller, B. A. Memon, and N. A. Memon, "Impact of Long-Term Loading on Reinforced Concrete Beams Made with Partial Replacement of Coarse Aggregates with Recycled Aggregates from Old Concrete," *Engineering, Technology & Applied Science Research*, vol. 9, no. 1, pp. 3818–3821, Feb. 2019, https://doi.org/10.48084/etasr.2498.

[2] M. A. J. Hassan and A. F. Izzet, "Serviceability of Reinforced Concrete Gable Roof Beams with Openings under Static Loads," *Engineering, Technology & Applied Science Research*, vol. 9, no. 5, pp. 4813–4817, Oct. 2019, https://doi.org/10.48084/etasr.3110.

[3] M. Oad, A. H. Buller, B. A. Memon, N. A. Memon, and S. Sohu, "Long Term Impact in Reinforced Recycled Concrete Beams Under 9-Month Loading," *Engineering, Technology & Applied Science Research*, vol. 9, no. 3, pp. 4140–4143, Jun. 2019, https://doi.org/10.48084/etasr.2697.

[4] H. A. E. & M. M. Assas, "New design approach for crack width calculation in reinforced concrete structures," in *Research and Applications in Structural Engineering, Mechanics and Computation*, Boca Raton, FL, USA: CRC Press, 2013.

[5] N. I. Karpenko, T. A. Mukhamediev, and M. A. Sapozhnikov, "On the Construction of a Method for Calculating Rod Elements Based on Material Deformation Diagrams. Improvement of Methods for Calculating Statically Indeterminate Reinforced Concrete Structures," in *Improvement of Methods for Calculating Statically Indeterminate Reinforced Concrete Structures*, 1987, pp. 4–24.

[6] M. M. Attard and S. Setunge, "Stress-Strain Relationship of Confined and Unconfined Concrete," *Materials Journal*, vol. 93, no. 5, pp. 432–442, Sep. 1996, https://doi.org/10.14359/9847.

[7] *Eurocode 2: Design of concrete structures - Part 1-1 : General rules and rules for buildings*. Brussels, Belgium: CEN, 2004.

[8] V. S. Dorofeev, V. Yu, and Bardanov, *Calculation of bent elements taking into account the complete diagram of concrete deformation*. Odessa, Ukraine: OGASA, 2003.

[9] V. M. Romashko, *The deformation and power model of concrete and reinforced concrete resistance*. Rivne, Ukraine: O. Zen, 2016.

[10] O. V. Romashko-Maistruk, "Basics of energy resource modeling of reinforced concrete elements," *Collected scientific works of Ukrainian State University of Railway Transport*, no. 191, pp. 49-56, Nov. 2020, https://doi.org/10.18664/1994-7852.191.2020.217292.

[11] V. Romashko and O. Romashko, "The construction features of the deformation and force model of concrete and reinforced concrete resistance," *MATEC Web of Conferences*, vol. 116, 2017, Art. no. 02028, https://doi.org/10.1051/matecconf/201711602028.

[12] V. M. Romashko, "Деякі особливості діаграми деформування бетону. Ресурсоекономні матеріали, конструкції, будівлі та споруди," *Збірник наукових праць*, no. 14, pp. 294–301, 2006.

[13] V. M. Romashko, "Щодо визначення модуля деформацій бетону," *Вісник НУВГП*, vol. 2, no. 4, pp. 292–297, 2007.

[14] V. M. Romashko and O. V. Romashko, "Energy resource of reinforced concrete elements and structures for the deformation-force model of their deformation," *IOP Conference Series: Materials Science and Engineering*, vol. 708, no. 1, Sep. 2019, Art. no. 012068, https://doi.org/10.1088/1757-899X/708/1/012068.

[15] V. Romashko and O. Romashko, "Calculation of the crack resistance of reinforced concrete elements with allowance for the levels of normal crack formation," *MATEC Web of Conferences*, vol. 230, 2018, Art. no. 02028, https://doi.org/10.1051/matecconf/201823002028.

[16] R. I. Gilbert and S. Nejadi, *An Experimental Study of Flexural Cracking in Reinforced Concrete Members Under Sustained Loads*. Sydney, NSW, Australia: University of New South Wales, School of Civil and Environmental Engineering, 2004.