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NUMERICAL SOLUTION OF RADIATIVE BOUNDARY LAYER FLOW IN POROUS MEDIUM DUE TO EXPONENTIALLY SHRINKING PERMEABLE SHEET UNDER FUZZY ENVIRONMENT[†]

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In this paper we are considering a fluid flows problem that contains two equation of motions and more than two parameters in the governing equation of motion. Which is namely *Radiative Boundary Layer Flow in Porous Medium due to Exponentially Shrinking Permeable Sheet*. The parameters are $K = \frac{ck_0}{L\vartheta}$, $Pr = \frac{\mu c_p}{k_\infty}$, $N = \frac{4\sigma_1 T_\infty{}^3}{3k_1 k_\infty}$ and ε denote the permeability parameter, Prandtl number, and radiation parameter and is the thermal conductivity variation parameter respectively. The governing differential equation can be obtained by using similarity variable technique and then the governing equation of motion can be Fuzzified by the help of Zadeh extension theorem. The $\alpha - cut$ technique is used for the validation of the uncertainty of the equation of the motion. The effect of the K, Pr, N and ε are discussed with the fuzzified governing equation of motion under fuzzy environment. It is observed none of the parameters are directly involved in the occurrence of the uncertainty of the solutions. The uncertainty occurs in the problem is due to the assumption and the numerical computation. Finally, the solution is being carried out under fuzzy environment. It is found that the increasing values of permeability parameter, the values of both the numbers Skin friction coefficient as well as Nusselt number are increases.
Keywords: *Shrinking sheet; Fuzzified; computer codes;* $\alpha - cut$
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1. INTRODUCTION

Flow and heat Transfer in boundary layer flow of viscous fluid due to deforming surface is pivotal in many industrial processes cutting across different realms. Specially radiative thermal regime in porous medium has drawn much attention recently due to large application in gasification of oil shale waste heat storage in aquifer and many more.

Vast application of radioactive thermal in porous medium we need to study this class of problems in different ways. Due to involvement of nonlinear differential equation, there is no direct process available to solve exactly. Here we consider such a mechanical problem for our discussion in which the governing equations of motion can have two non-linear differential equations of motion (One for velocity profile and another one is for temperature profile) and four parameters in the governing equation of motion and one parameter in the boundary conditions. The specific problem is *Radiative Boundary Layer Flow in Porous Medium due to Exponentially Shrinking permeable Sheet.*

A few relevant research has been presented in recent years (2010 to cont.). Radiative flow of Jeffery fluid with variable thermal conductivity in a porous medium was discussed by Elbashbeshy and Emam (2011), Hayat et al. (2012) about the effects of radiation and heat transfer over an unsteady stretching surface embedded in a porous medium. Paresh Vyas and Nupur Srivastava studied (2016) about the flow past and exponentially shrinking placed at the bottom of fluid saturated porous medium taking variable thermal conductivity and radiation using fourth order Runge-kutta scheme together with shooting method.

Here we introduce a new approach of solving of the said problem using fuzzy set theory. In this chapter our objective is to find is there any kind of uncertainty involved in the specific problem i.e. Radiative Boundary Layer Flow in Porous Medium due to Exponentially Shrinking permeable Sheet using fuzzy environment. For the graphical interpretation we developed computer codes for the said problem and represent the parameter's effect on the uncertainty involved in the flow of motion. On the basic concept of fuzzy differential equations Chakraverty et al., (2016) proposed some numerical methods for fuzzy fractional differential equations. Hazarika and Bora (2017, 2018) studied about the fuzzification of some numerical problems. J. Bora et al (2020) discussed some fluids problems using fuzzy set theory.

2. FORMULATION OF THE PROBLEM

2.1. Derivation of The Basic Equation

Let us consider the steady 2D boundary layer flow of optical thick viscous Newtonian fluid and associated heat transfer over a permeable sheet placed at bottom of the fluid saturated porous medium having permeability of specific form. A Cartesian coordinate system is chosen where the *x*-axis is taken along the sheet and *y*-axis is normal to it. The flow is caused by the sheet shrinking in an exponential fashion. A suction is applied normal to sheet to contain the

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vorticity. The fluid considered here is without phase change, optically dense, absorbing-emitting radiation but a nonscattering medium. The thermal conductivity of the fluid is assumed to vary linearly with temperature. The radiation flux in the energy equation is presumed to follow Rosseland approximation. The boundary layer equations for the considered setup are

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\vartheta\left(\frac{\partial^2 u}{\partial y^2}\right)-\vartheta\frac{u}{k} \tag{2}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)-\frac{\partial q_r}{\partial y} \tag{3}$$

With the boundary condition

At $y=0,\quad u=U_w(x)=-ce^{\frac{x}{l}},\quad v=V_w(x)=-v_0e^{\frac{x}{2l}},\quad T=T_w(x)=T_\infty+T_0e^{\frac{x}{2l}}$

and at

$$y\to\infty,\ u\to 0,\ T\to T_\infty \tag{4}$$

where u, v are the velocity components along x and y directions, respectively, k is the permeability, c_p is the specific heat at constant pressure, υ is the kinematic viscosity, ρ is the density, and T, μ, and κ are the temperature, viscosity and thermal conductivity of the fluid, respectively. Further, L is the characteristic length, T_w is the variable temperature at the sheet, T_0 is the constant reference temperature, and T_∞ is the constant free stream temperature. U_w and V_w are the shrinking velocity of the sheet and mass transfer velocity, respectively, where $c > 0$ is the shrinking constant and v_0 is a constant (where $v_0 < 0$ corresponds to mass suction).
Let us introduce the stream function $\psi(x,y)$ as

$$u=\frac{\partial\psi}{\partial y}=-\frac{\partial\psi}{\partial x} \tag{5}$$

Thus equation (5.1) is identically satisfied and the similarity transformation can be written as

$$\psi=\sqrt{2\upsilon Lc}f(\eta)e^{\frac{x}{2l}}\ \eta=y\sqrt{\frac{c}{2\upsilon L}}e^{\frac{x}{2l}},\text{ and }\theta(x)=\frac{T-T_\infty}{T_w-T_\infty} \tag{6}$$

On using (5.5) and (6.5) we obtain the expression for velocity component in non-dimensional form as

$$u=cf'(\eta)\,e^{\frac{x}{2l}}\text{ and }v=-\sqrt{\frac{\upsilon c}{2L}}\big(\eta f'(\eta)+f(\eta)\big)e^{\frac{x}{2l}} \tag{7}$$

In order to obtain the similarity solutions, it is assumed that the permeability k of the porous medium takes the following form

$$k(x)=2k_0e^{-\frac{x}{L}} \tag{8}$$

Where k_0 is the reference permeability.
As in our setup the thermal conductivity of the fluid is assumed to vary with temperature in a linear function as

$$k=k_\infty\,(1+\in\theta) \tag{9}$$

Where $\in$ is the thermal conductivity variation parameter. In general, $\in > 0$ for fluids such as water and air, while $\in<0$ for fluids such as lubrication oils. The radiative heat flux in the energy equation is presumed to follow Rosseland approximation and is given by

$$q_r=-\frac{4\sigma_1}{3k_1}\frac{\partial T^4}{\partial y} \tag{10}$$

Where σ_1 is the Stephan-Boltzmann constant and k_1 is the mean absorption constant. It is further assumed that the temperature difference within the fluid is sufficiently small sothat T^4 may be expressed as a linear function of temperature T. This is done by expanding T^4 in a Taylor series about T_∞ and omitting higher-order terms to yield

$$T^4\cong 4T_\infty{}^3T-3T_\infty{}^4$$

Thus, the equation of momentum (5.2) and energy (5.3) reduces to the following non dimensional form

$$f'''+ff''-2f'^2-\frac{f'}{K}=0 \tag{11}$$

$$(1+\tfrac{4N}{3})\,\theta''+\epsilon\theta\theta''+\epsilon\theta'^2+\Pr(f\theta'-\theta f')=0 \tag{12}$$

With the boundary conditions

$$\eta \to 0: f'(\eta) = -1, f(\eta) = \frac{-v_0}{\sqrt{vc/2L}} = S, \theta(\eta) = 1, \eta \to \infty: f'(\eta) \to 0, \theta(\eta) \to 0 \tag{13}$$

Where

$$K = \frac{ck_0}{L\vartheta}, \quad Pr = \frac{\mu c_p}{k_\infty}, \; N = \frac{4\sigma_1 {T_\infty}^3}{3k_1 k_\infty}$$

Denote the permeability parameter, Prandtl number, and radiation parameter respectively.

2.2. Conversion of The Basic Equation into Fuzzified Form

Now we Applying Zadeh fuzzy Extension theorem in (5.11-5.12) and (5.13-5.14)

$$\widehat{f'''} + \hat{f}\widehat{f''} - \hat{2}\widehat{f'^2} - \frac{\widehat{f'}}{K} = 0 \tag{15}$$

$$(\hat{1} + \widehat{\tfrac{4N}{3}})\,\widehat{\theta''} + \hat{\epsilon}\hat{\theta}\widehat{\theta''} + \hat{\epsilon}\widehat{\theta'^2} + \widehat{\Pr}\,\left(\hat{f}\widehat{\theta'} - \hat{\theta}\widehat{f'}\right) = \hat{0} \tag{16}$$

And the boundary condition became as (Fuzzy Environment)

$$\hat{\eta} \to \hat{0} : \hat{f}'(\eta) = \widehat{-1}, \hat{f}(\eta) = \frac{\widehat{-v_0}}{\sqrt{vc/2L}} = \hat{S}, \hat{\theta}(\eta) = \hat{1}, \tag{17}$$

$$\hat{\eta} \to \hat{\infty}: \widehat{f'}(\eta) \to \hat{0}, \hat{\theta}(\eta) \to \hat{0} \tag{18}$$

Considering the Fuzzified (5.15) equations as triangular fuzzy number then the Fuzzified equation became the following:

$$[\,\underline{f'''}\,, f'''\,, \overline{f'''}] + [\,\underline{f}, f\,, \overline{f}]\,[\underline{f''}, f'', \overline{f''}] - 2\left[\underline{f'^2}, f'^2, \overline{f'^2}\right] - \frac{[\,\underline{f'}, f', \overline{f'}]}{[\underline{K},\; K, \overline{K}]} = [\underline{0}, 0, \overline{0}]$$

Using fuzzy arithmetic we have,

$$\Rightarrow\; [\,\underline{f'''}, f''', \overline{f'''}] + [\min T,\; T_0, \max T\,] - \left[2\underline{f'^2}, 2f'^2, 2\overline{f'^2}\right] - [\min \boldsymbol{S},\; \boldsymbol{S}_0, \max \boldsymbol{S}] = [\underline{0}, 0, \overline{0}]$$

$$\Rightarrow [\,\underline{f'''} + \min T, f''' + T_0, \overline{f'''} + \max T\,] - \left[2\underline{f'^2} + \min \boldsymbol{S}, 2f'^2 + \boldsymbol{S}_0, 2\overline{f'^2} + \max \boldsymbol{S}\right] = \quad [\underline{0}, 0, \overline{0}]$$

$$\Rightarrow [\,\underline{f'''} + \min T - (2\overline{f'^2} + \max \boldsymbol{S}), f''' + T_0 - (2f'^2 + \boldsymbol{S}_0), \overline{f'''} + \max T - (2\underline{f'^2} + \;\min \boldsymbol{S})\,] = \;[\underline{0}, 0, \overline{0}]$$

Thus, we have,

$$\underline{f'''} + \min T - \left(2\overline{f'^2} + \max \boldsymbol{S}\right) = \underline{0} \tag{19}$$

$$f''' + T_0 - \left(2f'^2 + S_0\right) = 0. \tag{20}$$

$$\overline{f'''} + \max T - \left(2\underline{f'^2} + \min \boldsymbol{S}\right) = \overline{0} \tag{21}$$

$$\text{Where } \boldsymbol{S} = \frac{\underline{f'}}{\underline{\underline{K}}}, \frac{\overline{f'}}{\underline{K}}, \frac{\underline{f'}}{\overline{\overline{K}}}, \frac{\overline{f'}}{\overline{K}} \text{ and } \boldsymbol{S}_0 = \frac{f'}{K}$$

$$T = \underline{f}\underline{f''}, \underline{f}\overline{f''}, \overline{f}\underline{f''}, \overline{f}\overline{f''}, \text{ and } T_0 = ff''$$

Similarly considering the Fuzzified (5.16) equations as triangular fuzzy number then the Fuzzified equation became the following:

$$\left[\underline{1 + \tfrac{4N}{3}}, 1 + \tfrac{4N}{3}, \overline{1 + \tfrac{4N}{3}}\right] [\,\underline{\theta''}, \theta'', \overline{\theta''}] + [\underline{\epsilon}, \epsilon, \overline{\epsilon}][\underline{\theta}, \theta, \overline{\theta}][\underline{\theta''}, \theta'', \overline{\theta''}] + [\underline{\epsilon}, \epsilon, \overline{\epsilon}]\; [\,\theta'^2, \theta'^2, \theta'^2] + [\,\underline{Pr}, Pr, \overline{Pr}]$$
$$\{\left[\,\underline{f}, f, \overline{f}\right]\left[\underline{\theta'}, \theta', \overline{\theta'}\right] - \left[\,\underline{f'}, f', \overline{f'}\right]\,[\underline{\theta}, \theta, \overline{\theta}]\} = [\underline{0}, 0, \overline{0}]$$

$$\Rightarrow [\min X, \widetilde{X}, \max X] + [\underline{\epsilon}, \epsilon, \overline{\epsilon}]\; [\min Y, \widetilde{Y}\,, \max Y] + [\min Z, \widetilde{Z}, max\; Z] + [\,\underline{Pr}, Pr, \overline{Pr}]\; \{[\min A, \tilde{A}, \max A] -$$
$$[\min B, \tilde{B}, \max B]\;\} = [\underline{0}, 0, \overline{0}]$$

$$\Rightarrow\; [\min X, \widetilde{X}, \max X] + \;[\min Y_0, \widetilde{Y_0}\,, \max Y_0] + \;[\min Z, \widetilde{Z}, max\; Z] + \left[\,\underline{Pr}, Pr, \overline{Pr}\right]\; [\min A - \max B, \tilde{A} -$$
$$\widetilde{B},\; \max A - \min B] = \;[\underline{0}, 0, \overline{0}]$$

$$\Rightarrow [\min X + \min Y_0 + \min Z, \tilde{X} + \widetilde{Y_0} + \tilde{Z}, \max X + \max Y_0 + \max Z] + [\min A_0 B_0, \widetilde{A_0 B_0}, \max A_0 B_0] = [\underline{0}, 0, \overline{0}]$$

$$\Rightarrow [\min X + \min Y_0 + \min Z + \min A_0B_0\,, \tilde{X} + \tilde{Y_0} + \tilde{Z} + \widetilde{A_0B_0},\ \max X + \max Y_0 + \max Z + \max A_0B_0] = [\underline{0}, 0, \overline{0}] \quad (22)$$

Where

$$X = \underline{1 + \tfrac{4N}{3}}\ \underline{\theta''}, \underline{1 + \tfrac{4N}{3}}\ \overline{\theta''}\,, \overline{1 + \tfrac{4N}{3}}\ \underline{\theta''}, \overline{1 + \tfrac{4N}{3}}\ \overline{\theta''} \quad \text{and } \tilde{X} = 1 + \tfrac{4N}{3}\ \theta''$$

$$Y = \underline{\theta}\ \underline{\theta''}, \underline{\theta}\ \overline{\theta'}', \overline{\theta}\underline{\theta''}, \overline{\theta}\ \overline{\theta''}, \text{ and } \tilde{Y} = \theta\theta''$$

$$Z = \underline{\epsilon}\ \underline{\theta'^2}, \underline{\epsilon}\ \overline{\theta'^2}, \overline{\epsilon}\ \underline{\theta'^2}, \overline{\epsilon}\ \overline{\theta}'^2 \text{ and } \tilde{Z} = \epsilon\theta'^2$$

$$Y_0 = \underline{\in} \min Y, \overline{\in} \min Y, \underline{\in} \max Y, \overline{\in} \max Y, \text{ and } \widetilde{Y_0} = \in \tilde{Y}$$

$$A = \underline{f}\,\underline{\theta'}, \underline{f}\,\overline{\theta}', \overline{f}\underline{\theta'}, \overline{f}\,\overline{\theta}', \text{ and } \tilde{A} = f\theta'$$

$$B = \underline{f'}\ \underline{\theta}, \underline{f'}\overline{\theta}, \overline{f'}\ \underline{\theta}, \overline{f'}\ \overline{\theta}, \text{ and } B = f'\theta$$

$A_0B_0 = \underline{\Pr}\ (\min A - \max B)\ , \underline{\Pr}\ (\max A - \min B)\ , \ \overline{\Pr}\ (\min A - \max B)\ , \ \overline{\Pr}\ (\max A - \min B)\ , \widetilde{A_0B_0} = Pr\ (\tilde{A} - \tilde{B})$,

Now we can re-write the equation (5.22) as follows

$$\min X + \min Y_0 + \min Z + \min A_0B_0 = \underline{0} \quad (23)$$

$$\tilde{X} + \tilde{Y_0} + \tilde{Z} + \widetilde{A_0B_0} = 0, \quad (24)$$

$$\max X + \max Y_0 + \max Z + \max A_0B_0 = \overline{0}, \quad (25)$$

Similarly if we convert the boundary condition into Fuzzified form then a new system of equation will arise as follows

$$\underline{f'''} + \min T - \left(2\overline{f'^2} + \max \boldsymbol{S}\right) = \underline{0}$$

$$\min X + \min Y_0 + \min Z + \min A_0B_0 = \underline{0}$$

With the boundary conditions

$$\underline{\eta} \to \underline{0} : \underline{f'(\eta)} = \underline{-1}, \underline{f(\eta)} = \underline{\frac{-v_0}{\sqrt{\upsilon c/2L}}} = \underline{S},\ \underline{\theta}(\eta) = \underline{1}, \underline{\eta} \to \underline{\infty}: \underline{f'(\eta)} \to \underline{0}, \underline{\theta}(\eta) \to \underline{0} \quad (26)$$

$$f''' + T_0 - \left(2f'^2 + S_0\right) = 0$$

$$\tilde{X} + \tilde{Y_0} + \tilde{Z} + \widetilde{A_0B_0} = 0,$$

With the boundary conditions

$$\eta \to 0 : f'(\eta) = -1, f(\eta) = \frac{-v_0}{\sqrt{\upsilon c/2L}} = S, \theta(\eta) = 1, \eta \to \infty: f'(\eta) \to 0, \theta(\eta) \to 0 \quad (27)$$

$$\overline{f'''} + \max T - \left(2\underline{f'^2} + \min \boldsymbol{S}\right) = \overline{0} \qquad \max X + \max Y_0 + \max Z + \max A_0B_0 = \overline{0}$$

With the boundary conditions

$$\overline{\eta} \to \overline{0} : \overline{f'(\eta)} = \overline{-1}, \overline{f(\eta)} = \overline{\frac{-v_0}{\sqrt{\upsilon c/2L}}} = \overline{S}, \overline{\theta(\eta)} = \overline{1},$$

$$\overline{\eta} \to \overline{\infty}: \overline{f'(\eta)} \to \overline{0}, \overline{\theta(\eta)} \to \overline{0} \quad (28)$$

3. Definition of Skin Friction C_f and Nusselt Number Nu_x

The physical quantities of principal interest are the skin friction coefficient C_f and the local Nusselt number Nu_x, which are defined as

$$Re_x{}^{1/2}C_f = f''(0) \text{ and } \quad Re_x{}^{-1/2}Nu_x = -\theta'(0)$$

Where $Re_x = \frac{u_x(x)x}{v}$ is the local Reynolds number.

4. Result and Discussion

The system of equations (26-28), the fuzzified equations of motion with fuzzified boundary conditions are solved numerically by using finite difference scheme. The discretized fuzzified equations are solved using an iterative method based on Gauss Seidel iterative method by developing suitable codes in python.

The numerical computations carried out for different sets of values of the parameters entering into the problem have been depicted through graphs and tables. Result is obtained for different values for the parameter $s = 1, K = .25,\ \in = .1,$ $Pr = 0.7$ and for different $\alpha - cut$ of the fuzzified system of equations (26-28)

In each of the following graphs the blue curve is the solution for the right values of the of the fuzzified velocity profile, green curve is the solution for the mid values of the fuzzified velocity profile which is same as the crisp velocity profile and blue curve is the solution for the right values of the of the Fuzzified velocity profile.

The Figure (1-3) exhibits the Fuzzified temperature profile for $\alpha - cut\ with\ \alpha = 0.3,0.6,0.9$, and $K = 0.25, n = 0.5\ , Pr = 0.7\ , \in = 0.1, s = 1$. It is observed from the graph that there is a deflection on the curve in the right solution of the temperature profile as compare to the left solution of the temperature profile from the mid value solution (i.e. crisp solution). Which is the indication of the uncertainty involved in the temperature profile.

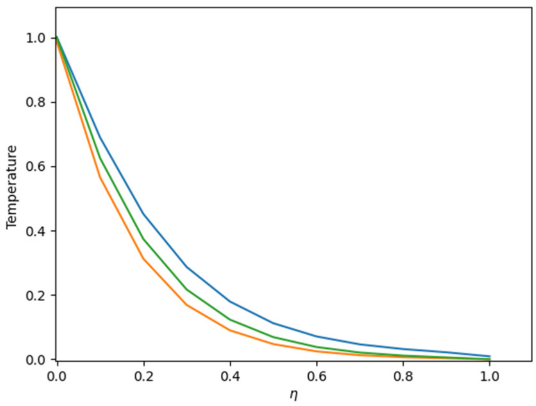

Figure 1. Fuzzified Temperature profile for $\alpha - cut, with\ \alpha = 0.3, s = 1, K = .25, \in = .1, Pr = 0.7$

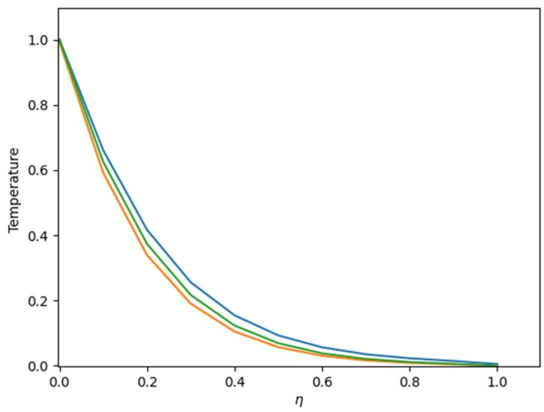

Figure 2. Fuzzified Temperature profile for $\alpha - cut\ with\ \alpha$=0.6, $s = 1, K = .25, \in = .1, Pr = 0.7$

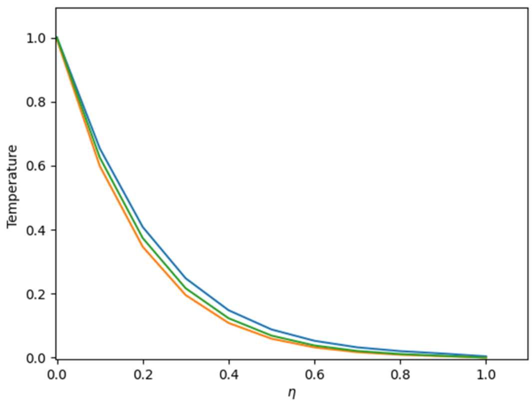

Figure 3. Fuzzified Temperature profile for $\alpha - cut\ with\ \alpha = 0.9, s = 1, K = .25, \in = .1, Pr = 0.7$

Figure (4-7) are the fuzzified temperature profiles for $\alpha - cut\ with\ \alpha = 0.5$ and different values of the parameters $s = 1\ and\ 2, K = 0.25\ and\ 0.5, Pr = 0.7\ and\ 1.1\ , \varepsilon = 0.1\ and\ 0.3.$

It is observed from the Figures that there is no significant deflection of right solution as compare to left solution from the mid value solution (Crisp solution). Which is due to the changes of these parameter are not the cause of the uncertainty involved in the solution of the temperature profile.

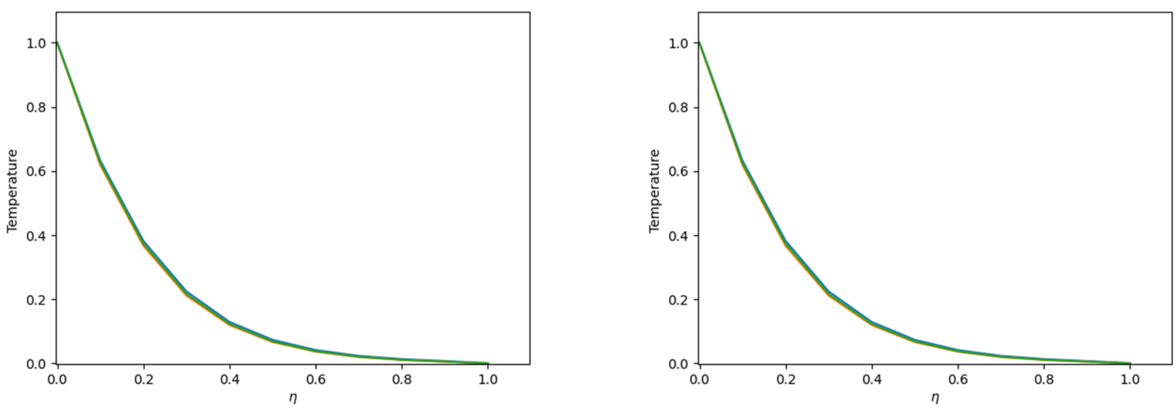

Figure 4. Fuzzified temperature profile for $\alpha - cut\ with\ \alpha$ =0.5, $\boldsymbol{s = 1\ and\ 2}, K = .25, \in = .1, Pr = 0.7$

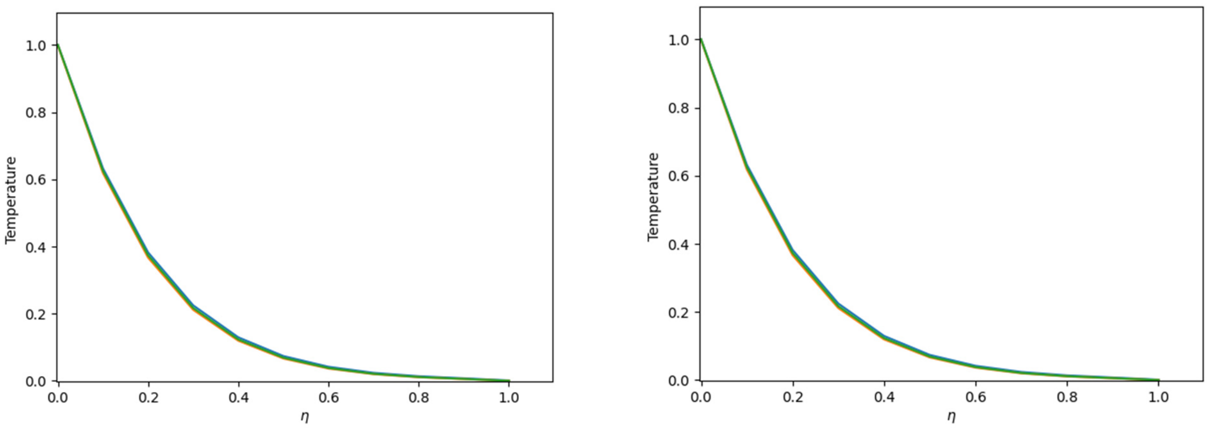

Figure 5. Fuzzified temperature profile for $\alpha - cut\ with\ \alpha$ =0.5, $s = 1, \boldsymbol{K} =. \mathbf{25}\ \boldsymbol{and}\ \mathbf{0.5}, \in = .1, Pr = 0.7$

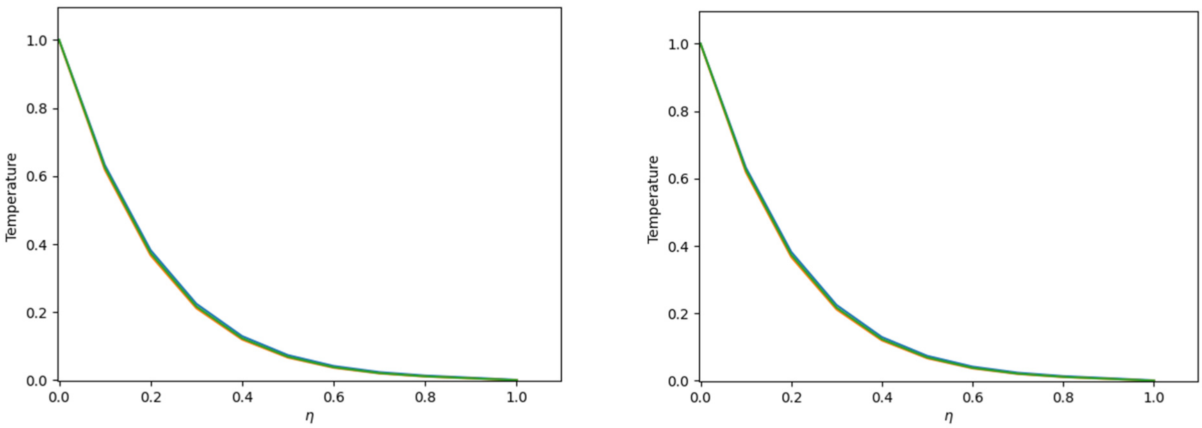

Figure 6. Fuzzified temperature profile for $\alpha - cut\ with\ \alpha$ =0.5, $s = 1, K = .25, \in = \mathbf{0.1}\ \boldsymbol{and}\ \mathbf{0.3}, Pr = 0.7$

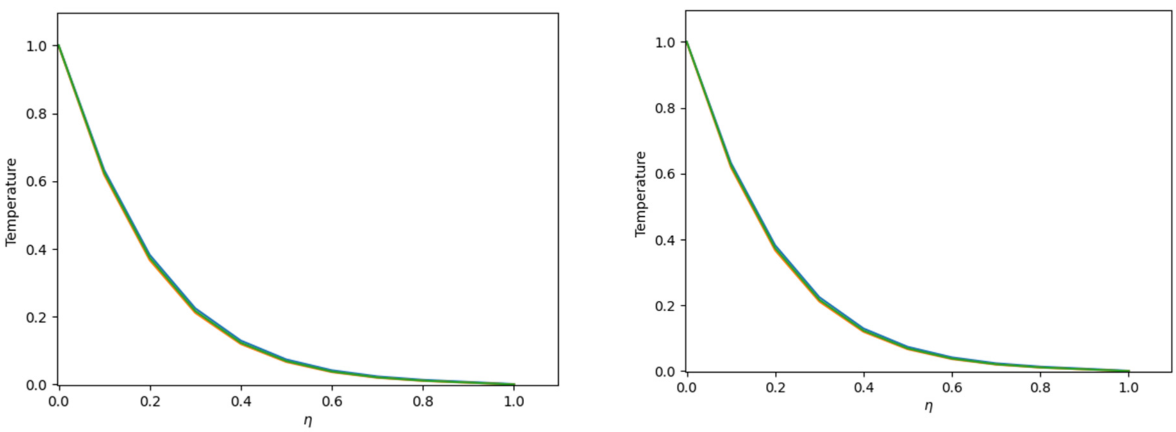

Figure 7. Fuzzified temperature profile for $\alpha - cut\ with\ \alpha$ =0.5, $s = 1, K = .25,\ \in = .1, \boldsymbol{Pr} = \mathbf{0.7}\boldsymbol{and}\ \mathbf{1.1}$

Figure (7-9) represent the crisp velocity profile for different values of $s, \in\ and\ Pr$. It is observed in Figure 7 that with the increasing values of suction parameter s the velocity decrease. Whereas velocity decreases with increase of Pr in Figure 8. It is found that the pattern of the flows is almost similar in the temperature profile for the changes of the parameter. Also, we see that $\theta(n)$ decay with the increase of Pr. Whereas $\theta(n)$ increases with increasing value of $\in$ in Figure 9.

As the parameter changes are not affect in the uncertainty of the solution of the temperature profile so we are discussed the effect of the parameter in Crisp Solution i.e., $\alpha - cut = 1$.

Figure (10) is the Fuzzified velocity profile for $\alpha - cut\ \ with\ \alpha = 0.5$, and $K = 0.25,\ n = 0.5\ ,\ Pr = 0.7, \in = 0.1$, $s = 1$. It is observed from the graph that there is a deflection on the curve in the right solution of the velocity profile (Green curve) as compare to the left solution of the velocity profile (Light yellow curve) from the mid value solution i.e. crisp solution (Violet curve). Which is the indication of the uncertainty involved in the solution of the velocity profile.

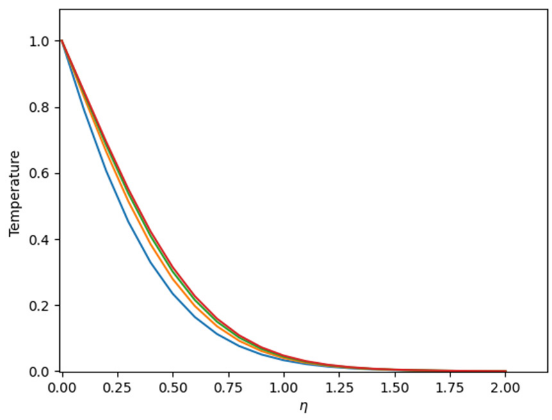

Figure 7. Crips Temperature profile for
$\in= 0.1\ (blue), 0.5\ (Orange), 0.9\ (Green), 1.2\ (Brown)$
$and\ \ s = 1, K = .25, Pr = 0.7$

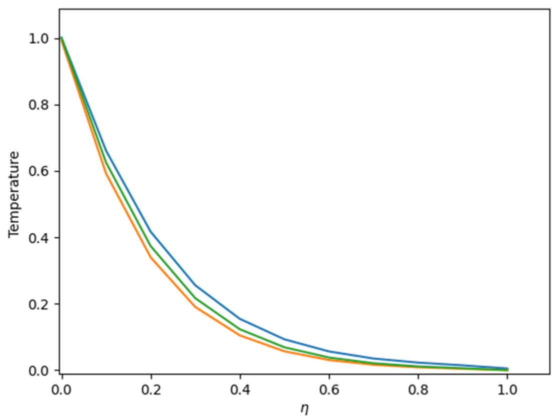

Figure 8. Crips Temperature profile for
$\boldsymbol{s} = 1(Blue), 3\ (Green), 5\ (Orange)$
$and\ \ K = .25, \in= .1, Pr = 0.7$

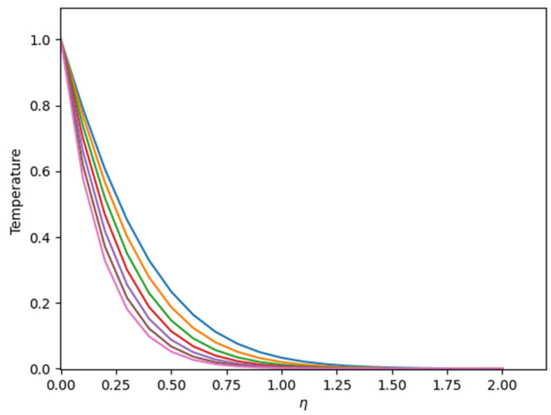

Figure 9. Crips Temperature profile for
$\boldsymbol{Pr} = 21.7\ (Light\ Pink), 17(\ dark\ brown),$
$13.7(Violet), 10.7(brown), .7(Green)$,$4.7(Orange)$,$0.7(Blue)$
and $s = 1,\ \ K = .25,\ \ \in= .1$

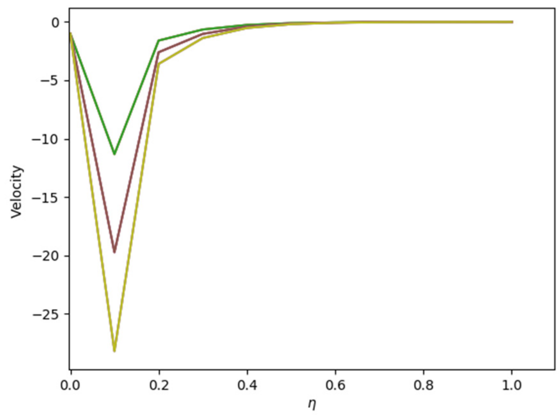

Figure 10. Fuzzified Velocity profile for
$\alpha - cut\ with\ \ \alpha$ =0.5, $s = 1, K = .25, \in= .1, Pr = 0.7$

Fig (11-14) are the fuzzified velocity profiles for $\alpha - cut\ with\ \alpha = 0.5$ and different values of the parameters $s = 1\ and\ 2,\ K = 0.25\ and\ 0.5, Pr = 0.7\ and\ 1\ , \varepsilon = 0.1\ and\ 0.5.$ It is observed from the Figures that there are no significant deflections of right solution as compare to left solution from the mid value solution (Crisp solution). This is due to the changes of these parameter are not the cause of the uncertainty involved in the solution of the velocity profile.

As the parameter changes are not affect in the uncertainty of the solution of the velocity profile so we are discussed the effect of the parameter in Crisp Solution i.e. $\alpha - cut = 1$.

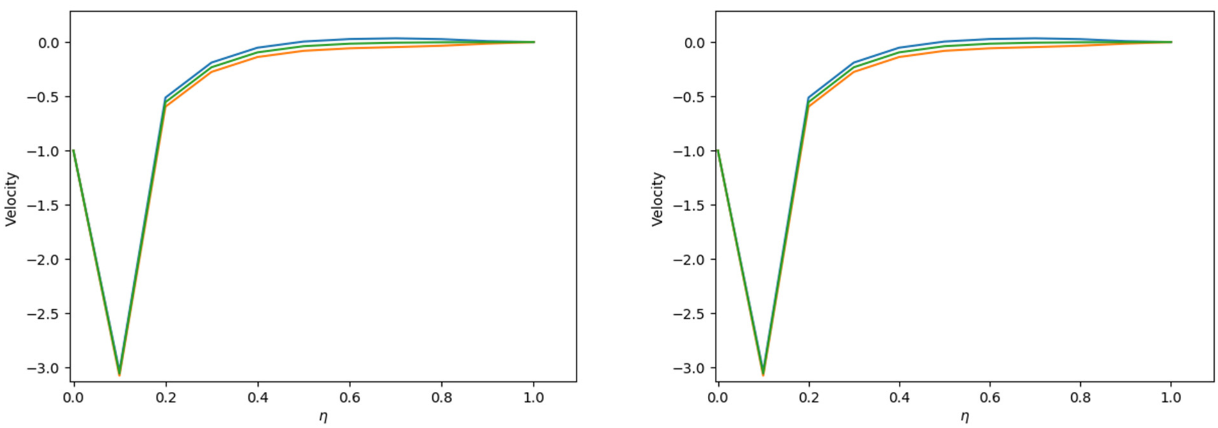

Figure 11. Fuzzified Velocity profile for $\alpha - cut\ with\ \alpha$ =0.5, $s = \mathbf{1\ and\ 2}, K = .25, \in= .1, Pr = 0.7$

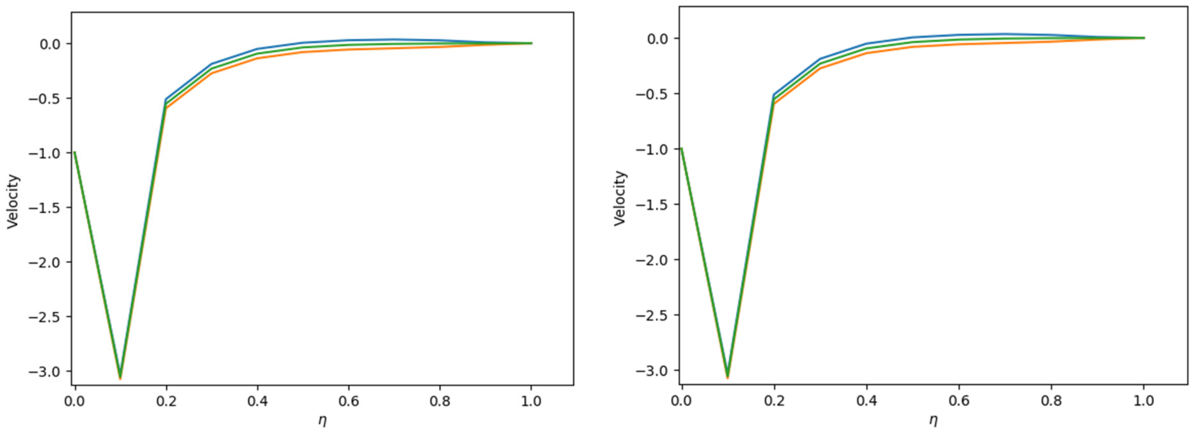

Figure 12. Fuzzified Velocity profile for $\alpha - cut\ with\ \alpha$ =0.5, $s = 1, K = .25\ and\ 0.5, \in = .1, Pr = 0.7$

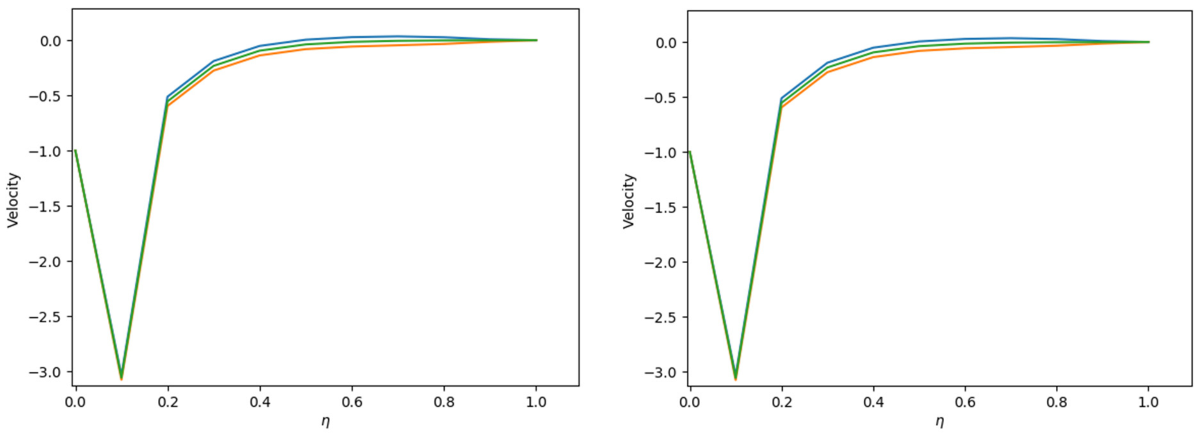

Figure 13. Fuzzified Velocity profile for $\alpha - cut\ with\ \alpha$ =0.5, $s = 1,\ K = 0.25, \in = 0.1\ \ and\ \ 0.5,\ Pr = 0.7$

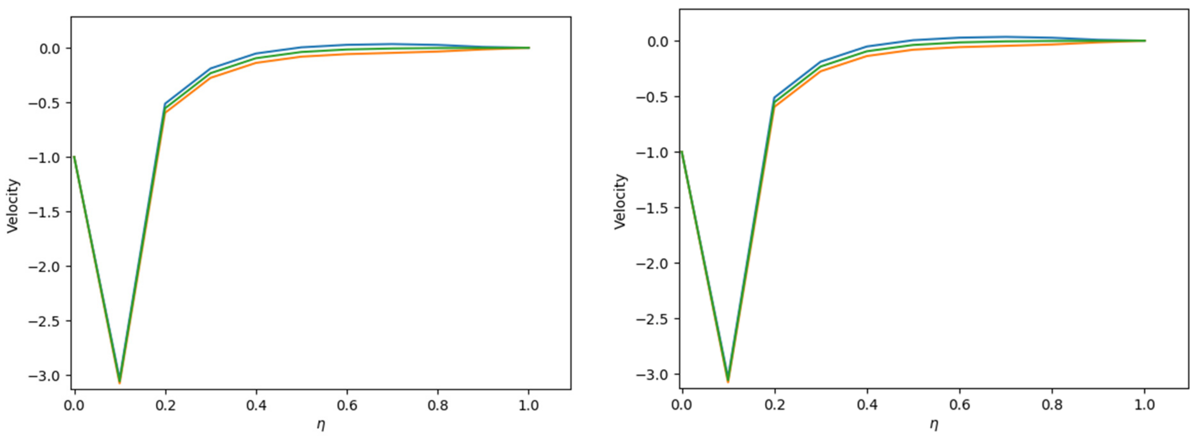

Figure 14. Fuzzified Velocity profile for $\alpha - cut\ with\ \ \alpha = 0.5,\ s = 1,\ K = 0.25,\ \in = .1,\ Pr = 0.7\ and\ 1$

Figure (15) represent the crisp velocity profile for the different values of the parameter $\in$ and fix value of the parameter $= 0.25,\ \ s = 1\ and\ \ Pr = 0.7$. It is observed that with the increasing value of $\in$ the velocity profile decreases.

Again Figure (16) represents the crisp velocity profile for the different values of the parameter s and fix value of the parameter $K = 0.25,\ \in = 0.1\ and\ \ Pr = 0.7$ respectively. It is observed that with the increasing value of s the velocity profile also increases.

It is observed from the graphs of the crisp velocity profile in Fig. (15-16) that the solution shows the occurrence of reverse flow. The occurrence of the sharp point in the back flow this is due to the numerical difficulties as the numbers of subdivision are less in number. If we increase the number of divisions to as large extend time complicity arise in the fuzzified solution but the curve would be smooth.

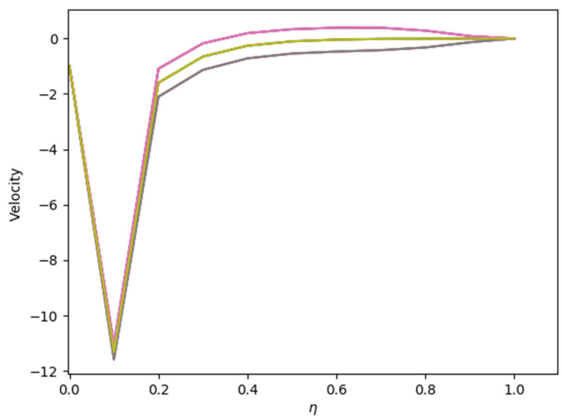

Figure 15. Crips Velocity profile for $\epsilon = 0.1(Violet), 0.5(yellow), 1(pink)\ and\ s = 1, K = .25,$ $Pr = 0.7$

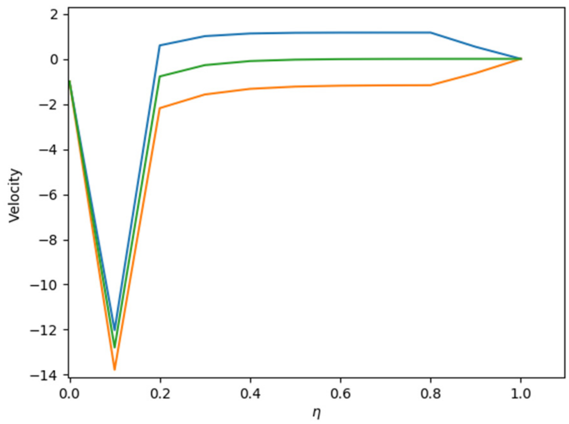

Figure 16. Crips Velocity profile for $s = 9(Blue), 5(Green), 1(Orange)\ and\ K = .25, \epsilon = .1,$ $Pr = 0.7$

5. Comparison of Skin Friction Coefficient C_f and Local Nusselt Number Nu_x

The two important parameters in fluid flow problem are the skin friction coefficient C_f and local nusselt number Nu_x we have computed these two parameters for different values of the Permiability parameter which are given in the following Table.

Table

Permiability Parameter	C_f		Nu_x	
	Crips	**Fuzzified**	**Crisp**	**Fuzzified**
10	0.264853	0.243849	-2.13187057	-2.00200567
20	0.265152	0.244158	-2.13187043	-2.0020055
30	0.265251	0.244261	-2.13187038	-2.00200544
40	0.265301	0.244313	-2.13187036	-2.00200541

It is observed from the table that with the increasing values of permeability parameter the values of the Skin friction coefficient increases. Similarly with the increasing values of permeability parameter the values of Nusselt number also increases. The results are well agreed with those of crisp values. The effect of fuzzification is also observed from the above Table.

6. Conclusion

In this chapter, the Radiative boundary layer flow in Porous medium due to exponentially shrinking steady MHD stagnation point flow due to shrinking permeable sheet has been theoretically considered under fuzzy environment. The effect of suction parameter, velocity ration parameter, Prandlt number on the flow and heat transfer have been studied under fuzzy environment. The numerical results have been obtained by developing computer codes on PYTHON. Thus, we conclude the followings from the above discussion:

(1) The involvement of uncertainty in the equation of motion of this problem.
(2) None of the parameters are directly involved in the occurrence of the uncertainty of the solutions. The uncertainty occurs in the problem is due to the assumption and the numerical computation.
(3) The crisp solution of velocity profile as well as temperature profile and the fuzzified velocity profile as well as temperature profile are in good agreements. The flow pattern for both the case velocity profile as well as Temperature profile are almost similar for different values of parameters.
(4) With the increasing values of permeability parameter, the values of both the numbers Skin friction coefficient as well as Nusselt number are increases.
(5) The effect of fuzzification is observed in the values of the physical quantities of the Skin friction coefficient C_f and local Nusselt number Nu_x.

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ЧИСЛОВЕ РІШЕННЯ ТЕЧІЇ РАДІАЦІЙНОГО ПРИКОРДОННОГО ШАРУ В ПОРИСТОМУ СЕРЕДОВИЩІ ЧЕРЕЗ ЕКСПОНЕНЦІАЛЬНО ЗТИСНУТИЙ ПРОНИКНИЙ ШАР В НЕЧІТКИХ УМОВАХ

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У цій статті розглянуто задачу про течії рідини, яка містить два рівняння руху та більше двох параметрів у визначальному рівнянні руху. Це саме радіаційний потік прикордонного шару в пористому середовищі через проникний лист, що експоненційно стискається. Параметри рівняння $K = \frac{ck_0}{L\vartheta}, Pr = \frac{\mu c_p}{k_\infty}, N = \frac{4\sigma_1 {T_\infty}^3}{3k_1 k_\infty}$, ε означають відповідно параметр проникності, число Прандтля, параметр випромінювання та параметр варіації теплопровідності. Основне диференціальне рівняння може бути отримане з використанням методу змінних подібності, а потім основне рівняння руху може бути fuzzified за допомогою теореми розширення Заде. Метод α-зрізу використовується для перевірки невизначеності рівняння руху. Обговорюється вплив K, Pr, N та ε з нечітким керуючим рівнянням руху в нечіткому середовищі. Знайдено, що жоден із параметрів не бере безпосередньої участі у виникненні невизначеності рішень. Невизначеність виникає через припущення та чисельний розрахунок. Нарешті, рішення виконано у нечіткому середовищі. Встановлено, що зі збільшенням значення параметра проникності зростають значення обох чисел: коефіцієнта поверхневого тертя, а також числа Нуссельта.

Ключові слова: *термозбіжний лист; нечіткість; комп'ютерні коди; α-зріз*