공학석사학위논문

COMPUTATION OF MINIMUM BANDWIDTH OF Q-FILTER FOR ROBUST STABILITY OF DISTURBANCE OBSERVER-BASED CONTROL SYSTEMS

외란 관측기 기반 제어 시스템의 강인 안정성을 위한 Q-필터의 최소 대역폭 계산

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ABSTRACT

Computation of Minimum Bandwidth of Q-filter for Robust Stability of Disturbance Observer-based Control Systems

by

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As the disturbance observer (DOB)-based controller has been widely applied in practice, various aspects of the disturbance observer have been theoretically studied. In particular, robust stability of the linear closed-loop system with single-input single-output (SISO) Q-filter-based DOB has been rigorously analyzed, and finally, a necessary and sufficient condition for robust stability was obtained under the premise that the bandwidth of Q-filter is large. However, even the most recent study about the design of Q-filter-based DOB for robust stability does not offer a practical method for the determination of the Q-filter's bandwidth.

In this thesis, we present several lemmas regarding the determination of the bandwidth, from which a procedure is developed that can exactly compute the threshold of the bandwidth, so that robust stability (against parametric variations of the plant within a prescribed range) is lost if the bandwidth of the Q-filter becomes lower than that. The proposed procedure is implemented in a MATLAB toolbox named DO-DAT, which is now available at `https://do-dat.github.io`.

Keywords: Disturbance observer, disturbance rejection, robust stability, nominal performance recovery, uncertain polynomials
Student Number: 2019–20672

To my family and friends

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Symbols and Acronyms

$\mathbb{Z}$	set of integers
$\mathbb{N}$	set of positive integers
$\square$	end of theorems, lemmas, propositions, assumptions, remarks, and corollaries
$\mathbb{R}$	set of real numbers
$\cup$	union of sets
$\max\{X\}$	maximum of the set X
$\min\{X\}$	minimum of the set X
$\blacksquare$	end of proof
$\sum_{i=0}^{n} b_i$	$:= b_0 + b_1 + \cdots + b_n$
$\forall$	for all
$\mathbb{C}$	set of complex numbers

Acronyms

DOB	Disturbanc OBserver

Chapter 1

Introduction

The Q-filter-based disturbance observer (DOB) has been a powerful robust control scheme to reject disturbances and compensate plant uncertainties since it was first introduced by [OOM83]. The DOB has been frequently employed in the industry from the time when it was regarded as a rather heuristic method, and now several theories are available about the robust stability of the DOB-based control systems. Among others, [SJ07] and [BS08] introduced singular perturbation theory into the analysis of DOB-based control systems, and this insight yielded a necessary and sufficient condition for robust stability [SJ09]. This finding, in turn, enabled the systematic design of DOBs for robust stability against arbitrarily large parameter variations.

Based on the robust stability result, more insights about the DOB have been discovered. For instance, it was found that a high-gain observer is already embedded in the seemingly different structure of the Q-filter-based DOB and that the zero-dynamics of the plant becomes decoupled when the DOB is installed in the feedback loop [SPJ+16]. This finding provides an insightful explanation for how the DOB works as a robust controller, by which both the benefits and the limitations of DOB are clarified. It was also figured out how imprecise identification of the relative degree of an uncertain plant affects stability [JJS14] and how the classical measure of robustness, the gain/phase margin, is affected by a DOB in the loop [KPSJ16].

Based on these analyses, a few modified DOBs are also proposed to overcome the limitations of the classical DOB. For example, a way to modify the classical

DOB for robust transient performance was presented in [BS08], and a way to embed an internal model that generates external disturbances so that the modeled disturbances are rejected perfectly while the unmodeled disturbances are attenuated at the desired level was presented in [JPBS15]. On top of those theoretical developments, the DOB is replacing traditional robust control methods. Examples include flight control of drones [KCK+17], platooning of multi-vehicles [NPT+20], load-frequency control of power-grid [HSN16], robustifying the reinforcement learning based controller [KSY19], and even generating stealthy attacking signals for control systems [PLS+19].

However, most of these results are based on the premise that the bandwidth of the Q-filter is sufficiently large. For example, the necessary and sufficient condition for robust stability in [SJ09] is derived when the time constant τ of the Q-filter is less than a threshold τ^*. While the threshold τ^* is presented in [SJ09], it is just a conservative value, and in practice, the selection of τ^* should be obtained by a repeated simulation or by trial and error.

In this dissertation, we study how to choose the minimum bandwidth of Q-filter, i.e., the non-conservative value of τ^*, under which robust stability is guaranteed against parameter uncertainties within prescribed ranges. Having non-conservative τ^* is desirable because there might exist unavoidable physical constraints that limit the available bandwidth of the Q-filter. The existence of unmodeled dynamics in the model of the plant is another reason why we need to avoid unnecessarily large bandwidth of Q-filter. Moreover, succinct computation of τ^* is desired, which does not rely on an iterative method. In this dissertation, a few lemmas are presented with which exact computation of τ^* is enabled. This work will pave a road to building a computer-aided toolbox for designing DOBs that are robust against given uncertain variation of parameters.

Finally, it will be shown that the proposed procedure to find the minimum bandwidth, or the value of τ^*, of the Q-filter, can also be used for finding suitable bandwidths or the values τ even for non-minimum phase plants. No universal design methods of DOB for non-minimum phase plants are available yet. However, since we are using a numerical method whatsoever, DOB can be designed regardless of whether the plant is of minimum phase or not.

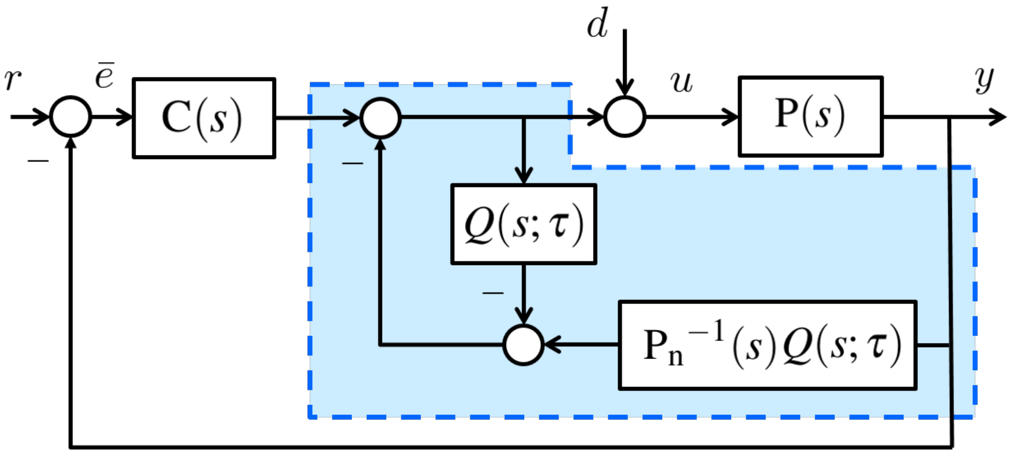

Figure 1.1: Block diagram of the closed-loop system with Q-filter-based DOB (blue dashed block).

This dissertation is organized as follows. The rest of this chapter begins with Chapter 1.1, an overview of the Q-filter-based DOB. Chapter 1.2 describes a couple of assumptions and primary results of [SJ09] as preliminary. In Chapter 2, we propose several necessary and sufficient conditions for the robust stability of the DOB-based control system as lemmas. Based on the suggested lemmas, we show a procedure to find an appropriate bandwidth of the Q-filter and give some illustrative examples that demonstrate the usefulness of the procedure in Chapter 3. Chapter 4 introduces a MATLAB toolbox named DO-DAT (Disturbance Observer - Design and Analysis Toolbox) that contains the procedure to find an appropriate bandwidth of the Q-filter. Finally, this thesis is summarized and concluded in Chapter 5. In Appendix, details of theorems which are used in the body of the thesis are provided.

1.1 Overview of Q-filter-based Disturbance Observer

The standard structure of the Q-filter-based DOB and the closed-loop system are depicted in Figure 1.1. In the figure, $\mathrm{P}(s)$ and $\mathrm{P_n}(s)$ represent a single-input single-output (SISO) real plant and its nominal model, respectively, $\mathrm{C}(s)$ is a proper (implementable) controller which is usually designed a priori for $\mathrm{P_n}(s)$, and $Q(s;\tau)$ is a stable low-pass filter called Q-filter with a parameter τ. This

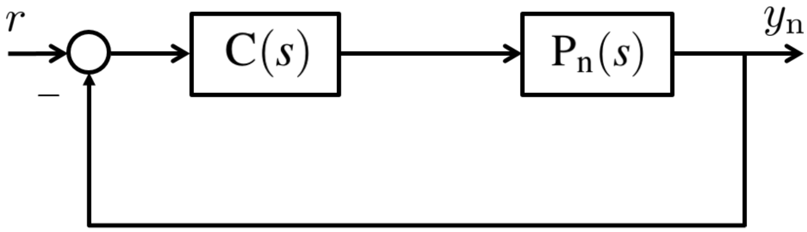

Figure 1.2: Block diagram of the nominal closed-loop system.

dissertation focuses on the design of the suitable value τ that decides the time constant or the bandwidth of the Q-filter for robust stability of the closed-loop system against a given variation of uncertain parameters. It is well-known that, if the reference r and disturbance d consist of low-frequency components and if all other parameters of Q-filter are properly set, then the Q-filter-based DOB with a large bandwidth of the Q-filter (that is, a small magnitude of τ) enables the system in Figure 1.1 to approximate the nominal closed-loop system in Figure 1.2 (see, e.g., [SPJ$^+$16]). In other words, the following approximation

$$y(j\omega) \approx \frac{\mathrm{P_n}(j\omega)\mathrm{C}(j\omega)}{1+\mathrm{P_n}(j\omega)\mathrm{C}(j\omega)} r(j\omega) = y_{\mathrm{n}}(j\omega)$$

holds with a sufficiently large bandwidth of the Q-filter, where y and y_{n} are the outputs of the DOB-based control system in Figure 1.1 and the nominal closed-loop system in Figure 1.2, respectively. This capability of approximation is one of the main features of the Q-filter-based DOB scheme, which is often called *nominal performance recovery.*

1.2 Necessary and Sufficient Condition for Robust Stability

In this dissertation, parametric uncertainty of the plant $\mathrm{P}(s)$ is assumed to satisfy the following.

Assumption 1.2.1. The real plant $\mathrm{P}(s)$ and its nominal model $\mathrm{P_n}(s)$ belong to the set of uncertain plants:

$$\mathcal{P} := \Bigg\{ \mathrm{P}(s) = \frac{\beta_{n-\nu}s^{n-\nu} + \beta_{n-\nu-1}s^{n-\nu-1} + \ldots + \beta_0}{\alpha_n s^n + \alpha_{n-1}s^{n-1} + \ldots + \alpha_0} \\ : \alpha_i \in [\underline{\alpha}_i, \overline{\alpha}_i], \beta_i \in [\underline{\beta}_i, \overline{\beta}_i] \Bigg\}, \tag{1.2.1}$$

where n and ν are positive integers such that $n \geq \nu$ and $\underline{\alpha}_i$, $\overline{\alpha}_i$, $\underline{\beta}_i$, and $\overline{\beta}_i$ are known constants such that $[\underline{\alpha}_n, \overline{\alpha}_n]$, $[\underline{\beta}_{n-\nu}, \overline{\beta}_{n-\nu}] \subset (0, \infty)$, where $(0, \infty)$ denotes the positive real line. □

In the assumption, the condition $[\underline{\alpha}_n, \overline{\alpha}_n]$, $[\underline{\beta}_{n-\nu}, \overline{\beta}_{n-\nu}] \subset (0, \infty)$ implies that all the plants in the set have the same relative degree and have the same sign of the high frequency gain (which is positive, without loss of generality). From the assumption, it is clear that the set $\mathcal{P}$ incorporates arbitrarily large but bounded uncertainties of the parameters. Note that the description of the set $\mathcal{P}$ in (1.2.1) has redundancy. This redundancy disappears by letting, for example, $\underline{\alpha}_n = \overline{\alpha}_n = 1$, but for the general purpose, we let all the parameters are independent of one another.

The stable low-pass filter $Q(s;\tau)$ is usually designed in the form

$$\begin{aligned} Q(s;\tau) &= \frac{c_k(\tau s)^k + c_{k-1}(\tau s)^{k-1} + \cdots + c_0}{(\tau s)^l + a_{l-1}(\tau s)^{l-1} + \cdots + a_1(\tau s) + a_0} \\ &=: \frac{N_Q(s;\tau)}{D_Q(s;\tau)}, \end{aligned} \tag{1.2.2}$$

where N_Q and D_Q are the numerator polynomial and the denominator polynomial of $Q(s;\tau)$, respectively, and k and l are some non-negative integers such that $k \leq l - \nu$, where ν is the relative degree of $\mathrm{P_n}(s)$. For the unity dc gain, we set $a_0 = c_0$. Note that the real positive τ determines the time constant or the bandwidth. Now we assume the following necessary condition (see [SJ09]), which is relevant to $\mathrm{P_n}(s)$, $\mathrm{C}(s)$, and $Q(s;1)$, for robust stability under large bandwidth of Q-filter.

Assumption 1.2.2. The nominal closed-loop system

$$\frac{\mathrm{P_n}(s)\mathrm{C}(s)}{1 + \mathrm{P_n}(s)\mathrm{C}(s)}$$

is internally stable, and the polynomial

$$p_f(s) := D_Q(s;1) + \left(\lim_{s\to\infty} \frac{\mathrm{P}(s)}{\mathrm{P_n}(s)} - 1\right) N_Q(s;1)$$

is Hurwitz for all $\mathrm{P}(s) \in \mathcal{P}$ in (1.2.1). □

Note that $\tau = 1$ in the assumption, and thus, the assumption is independent of the choice of τ. In fact, a systematic way to choose the parameters a_i and c_i of the Q-filter in (1.2.2) such that the second condition of Assumption 1.2.2 holds has been presented in [SJ09, Sec. 2.3].

The following theorem, taken from [SJ09], plays a crucial role to design Q-filter-based DOB for robust stability of the closed-loop system in Figure 1.1.

Theorem 1.2.1. Suppose that Assumptions 1.2.1 and 1.2.2 hold. If all the plants $\mathrm{P}(s) \in \mathcal{P}$ are of minimum phase, then there exists a constant τ^* such that, for all $0 < \tau < \tau^*$, the closed-loop system in Figure 1.1 is robustly internally stable (against the uncertainty of $\mathcal{P}$). On the contrary, if $\mathcal{P}$ contains a non-minimum phase plant such that at least one zero has positive real parts, then there is τ^* such that, for all $0 < \tau < \tau^*$, the closed-loop system is not robustly internally stable. □

While the former part of Theorem 1.2.1 guarantees the existence of the threshold τ^* (or, the minimum bandwidth of Q-filter), its proof in [SJ09] presents a conservative choice of τ^*. In fact, no method to find the exact and non-conservative value of τ^* has been reported in the literature yet. In the next chapter, some useful lemmas are introduced which can be utilized to obtain the exact value of τ^* under Assumptions 1.2.1 and 1.2.2.

Chapter 2

Lemmas on Robust Stability of Closed-loop System

In this chapter, we first observe some properties of the characteristic polynomial of the closed-loop system in Figure 1.1, and then present a couple of equivalent statements which are necessary and sufficient conditions for robust stability of the DOB-based control system.

2.1 Observations on Characteristic Polynomial

In the configuration of Figure 1.1, the following equation

$$\begin{bmatrix} \bar{e} \\ u \end{bmatrix} = \frac{1}{\gamma(s)} \begin{bmatrix} Q(\mathrm{P} - \mathrm{P_n}) + \mathrm{P_n} & (Q-1)\mathrm{P}\mathrm{P_n} \\ \mathrm{C}\mathrm{P_n} & (1-Q)\mathrm{P_n} \end{bmatrix} \begin{bmatrix} r \\ d \end{bmatrix}$$

holds, where

$$\gamma(s) = (1 + \mathrm{CP})\mathrm{P_n} + Q(\mathrm{P} - \mathrm{P_n}).$$

Now, let $\mathrm{P}(s)$, $\mathrm{P_n}(s)$, and $\mathrm{C}(s)$ be represented by the ratios of coprime polynomials such as $\mathrm{P}(s) = N(s)/D(s)$, $\mathrm{P_n}(s) = N_\mathrm{n}(s)/D_\mathrm{n}(s)$, and $\mathrm{C}(s) = N_\mathrm{C}(s)/D_\mathrm{C}(s)$. Then we can express the characteristic polynomial of the closed-loop system in Figure 1.1 in the following lemma which is the main concern throughout this dissertation.

Lemma 2.1.1. The closed-loop system in Figure 1.1 is robustly internally stable if and only if the characteristic polynomial

$$\begin{aligned}\delta(s;\tau) := \Big(D(s)D_{\mathrm{C}}(s) + N(s)N_{\mathrm{C}}(s)\Big)N_{\mathrm{n}}(s)D_Q(s;\tau) \\ + N_Q(s;\tau)D_{\mathrm{C}}(s)\Big(N(s)D_{\mathrm{n}}(s) - N_{\mathrm{n}}(s)D(s)\Big)\end{aligned} \tag{2.1.1}$$

is Hurwitz for all $\mathrm{P}(s) \in \mathcal{P}$ in (1.2.1). □

Now, we are going to define 'polytope of polynomials', 'edge polynomial', and 'exposed edge polynomial' with respect to the characteristic polynomial $\delta(s;\tau)$ in (2.1.1) to make use of the Edge theorem in [BHL88]. A set of polynomials T is called polytope of polynomials if the set T is a convex hull of a finite number of vertex polynomials. If T is a polytope of polynomials, we define an edge polynomial of T as

$$\{\lambda t_1 + (1-\lambda)t_2 : \lambda \in [0,\ 1]\}$$

for any vertices $t_1,\ t_2 \in T$. Finally, an exposed edge polynomial of T is defined as the edge polynomial which is contained in a nontrivial supporting hyperplane of the set T.

Under such terminologies, the characteristic polynomial $\delta(s;\tau)$ can be rewritten with the uncertain polynomials $D(s)$ and $N(s)$ as

$$\begin{aligned}\delta(s;\tau) &= p_D(s;\tau)\cdot D(s) + p_N(s;\tau)\cdot N(s), \\ &= p_D(s;\tau)\cdot\sum_{i=0}^{n}\alpha_i s^i + p_N(s;\tau)\cdot\sum_{j=0}^{n-\nu}\beta_j s^j,\end{aligned}$$

where

$$p_D(s;\tau) = D_{\mathrm{C}}(s)N_{\mathrm{n}}(s)D_Q(s;\tau) - N_Q(s;\tau)D_{\mathrm{C}}(s)N_{\mathrm{n}}(s)$$

and

$$p_N(s;\tau) = N_{\mathrm{C}}(s)N_{\mathrm{n}}(s)D_Q(s;\tau) + N_Q(s;\tau)D_{\mathrm{C}}(s)D_{\mathrm{n}}(s)$$

which are not uncertain. At this point, we define

$$\Omega := \{\delta(s;\tau) : \mathrm{P}(s) \in \mathcal{P}\} \tag{2.1.2}$$

as the set of all characteristic polynomials corresponding to every possible plants $\mathrm{P}(s) \in \mathcal{P}$ in (1.2.1). Then for any $\alpha_i^{(1)}$, $\alpha_i^{(2)} \in [\underline{\alpha}_i, \overline{\alpha}_i]$ and $\beta_j^{(1)}$, $\beta_j^{(2)} \in [\underline{\beta}_j, \overline{\beta}_j]$, where $i = 0, 1, \ldots, n$ and $j = 0, 1, \ldots, n - \nu$, the polynomials

$$\omega_1 := p_D(s;\tau) \cdot \sum_{i=0}^{n} \alpha_i^{(1)} s^i + p_N(s;\tau) \cdot \sum_{j=0}^{n-\nu} \beta_j^{(1)} s^j$$

and

$$\omega_2 := p_D(s;\tau) \cdot \sum_{i=0}^{n} \alpha_i^{(2)} s^i + p_N(s;\tau) \cdot \sum_{j=0}^{n-\nu} \beta_j^{(2)} s^j$$

belong to Ω. Moreover, for any $\lambda \in [0, 1]$, the convex combination of ω_1 and ω_2,

$$\begin{aligned} \lambda\omega_1 + (1-\lambda)\omega_2 = p_D(s;\tau) &\cdot \sum_{i=0}^{n} \Big(\lambda\alpha_i^{(1)} + (1-\lambda)\alpha_i^{(2)}\Big) s^i \\ + p_N(s;\tau) &\cdot \sum_{j=0}^{n-\nu} \Big(\lambda\beta_j^{(1)} + (1-\lambda)\beta_j^{(2)}\Big) s^j, \end{aligned}$$

is also in Ω because $\lambda\alpha_i^{(1)} + (1-\lambda)\alpha_i^{(2)} \in [\underline{\alpha}_i, \overline{\alpha}_i]$ and $\lambda\beta_j^{(1)} + (1-\lambda)\beta_j^{(2)} \in [\underline{\beta}_j, \overline{\beta}_j]$. Thus, the set Ω in (2.1.2) is a polytope of polynomials, namely the convex hull of a finite number of polynomials. Indeed, we have

$$m := 2^{2n-\nu+2}$$

polynomials depending on $\underline{\alpha}_i$, $\overline{\alpha}_i$, $\underline{\beta}_j$, and $\overline{\beta}_j$, where $i = 0, 1, \ldots, n$ and $j = 0, 1, \ldots, n - \nu$, as

$$\begin{aligned} \delta_1(s;\tau) &:= (\underline{\alpha}_n s^n + \underline{\alpha}_{n-1} s^{n-1} + \cdots + \underline{\alpha}_0) \cdot p_D(s;\tau) \\ &\quad + (\underline{\beta}_{n-\nu} s^{n-\nu} + \underline{\beta}_{n-\nu-1} s^{n-\nu-1} + \cdots + \underline{\beta}_0) \cdot p_N(s;\tau), \\ \delta_2(s;\tau) &:= (\overline{\alpha}_n s^n + \underline{\alpha}_{n-1} s^{n-1} + \cdots + \underline{\alpha}_0) \cdot p_D(s;\tau) \\ &\quad + (\underline{\beta}_{n-\nu} s^{n-\nu} + \underline{\beta}_{n-\nu-1} s^{n-\nu-1} + \cdots + \underline{\beta}_0) \cdot p_N(s;\tau), \\ \delta_3(s;\tau) &:= (\overline{\alpha}_n s^n + \overline{\alpha}_{n-1} s^{n-1} + \cdots + \underline{\alpha}_0) \cdot p_D(s;\tau) \\ &\quad + (\underline{\beta}_{n-\nu} s^{n-\nu} + \underline{\beta}_{n-\nu-1} s^{n-\nu-1} + \cdots + \underline{\beta}_0) \cdot p_N(s;\tau), \\ &\;\;\vdots \end{aligned}$$

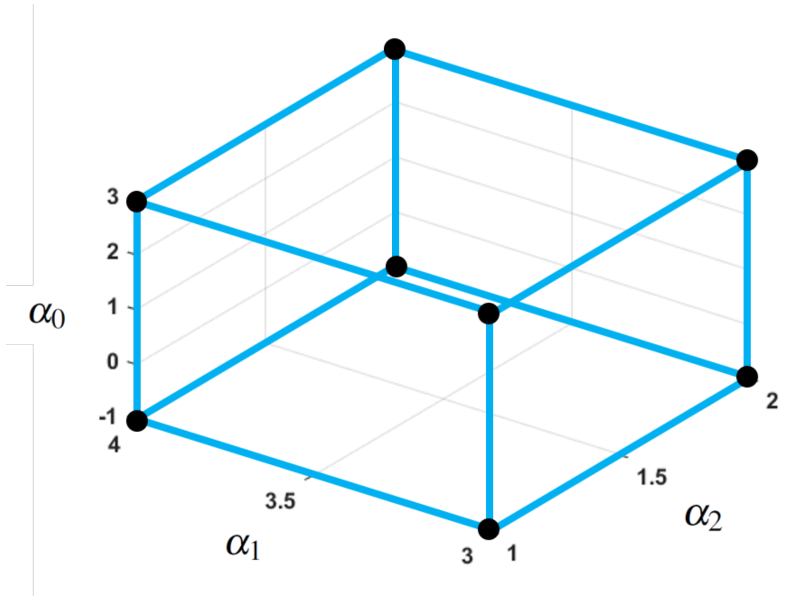

Figure 2.1: The vertex polynomials (black dots) and the exposed edge polynomials (blue line segments) of the polytope Ω in the coefficient space.

$$\delta_m(s;\tau) := (\overline{\alpha}_n s^n + \overline{\alpha}_{n-1} s^{n-1} + \cdots + \overline{\alpha}_0) \cdot p_D(s;\tau) + (\overline{\beta}_{n-\nu} s^{n-\nu} + \overline{\beta}_{n-\nu-1} s^{n-\nu-1} + \cdots + \overline{\beta}_0) \cdot p_N(s;\tau)$$

which become vertex polynomials of the polytope Ω, and the polytope Ω is the convex hull of them. Let the set of those vertex polynomials as

$$\Delta(s;\tau) := \{\delta_i(s;\tau) : i = 1,\ 2,\ \ldots,\ m\}.$$

Then an edge polynomial of the polytope Ω is

$$\{\lambda\delta_i(s;\tau) + (1-\lambda)\delta_j(s;\tau) : \lambda \in [0,\ 1]\},$$

where $\delta_i(s;\tau)$, $\delta_j(s;\tau) \in \Delta(s;\tau)$, $1 \leq i, j \leq m$ and $i \neq j$. The following example clarifies the definition of 'polytope of polynomials', 'edge polynomial', and 'exposed edge polynomial' with respect to a given uncertain polynomial.

Example 2.1.1. Consider an uncertain polynomial $p(s) = \alpha_2 s^2 + \alpha_1 s + \alpha_0$, where uncertain coefficients $\alpha_2 \in [1,\ 2]$, $\alpha_1 \in [3,\ 4]$, and $\alpha_0 \in [-1,\ 3]$. In this case, the polytope Ω can be expressed as

$$\{p(s) = \alpha_2 s^2 + \alpha_1 s + \alpha_0 : \alpha_2 \in [1,\ 2],\ \alpha_1 \in [3,\ 4],\ \alpha_0 \in [-1,\ 3]\}.$$

There are $2^3 = 8$ vertex polynomials for this example. If we represent each polynomial in Ω as a point in the coefficient space, which is three-dimensional space in this case, the vertex polynomials can be represented as black dots as in Figure 2.1. There are $\binom{8}{2} = 28$ edge polynomials, which are line segments connecting each pair of black dots in Figure 2.1. Out of 28 edge polynomials, only 12 edge polynomials are drawn in blue color in Figure 2.1, which are called exposed edge polynomials. □

2.2 Application of Edge theorem and Bialas' theorem

In this chapter, a couple of equivalent statements of Lemma 2.1.1 are presented.

Lemma 2.2.1. The closed-loop system in Figure 1.1 is robustly internally stable if and only if for each pair $\delta_i(s;\tau)$, $\delta_j(s;\tau) \in \Delta(s;\tau)$, $1 \leq i, j \leq m$, $i \neq j$, and for each $\lambda \in [0,1]$, the edge polynomial $\lambda\delta_i(s;\tau) + (1-\lambda)\delta_j(s;\tau)$ is Hurwitz. □

Proof: The proof of the lemma follows from the Edge theorem [BHL88] (reviewed in A.1 for convenience). Let us take D as the open left-half complex plane. If all the edge polynomials are Hurwitz, then all exposed edge polynomials are Hurwitz as well, and the sufficiency follows. The necessity is trivial. ■

It is worthy to note that $\delta_i(s;\tau) \in \Delta(s;\tau)$ in Lemma 2.2.1 is no longer an uncertain polynomial. However, we still need to check infinitely many polynomials in terms of $\lambda \in [0,1]$ to decide robust stability of the closed-loop system. The following lemma eliminates the λ-dependency in Lemma 2.2.1. Before stating the next lemma, let us define the Hurwitz matrix of a polynomial. For a given polynomial

$$p(s) = a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0$$

with real coefficients, the $n \times n$ matrix

$$H(p) = \begin{bmatrix} a_{n-1} & a_{n-3} & a_{n-5} & \cdots & \cdots & \cdots & 0 & 0 & 0 \\ a_n & a_{n-2} & a_{n-4} & & & & \vdots & \vdots & \vdots \\ 0 & a_{n-1} & a_{n-3} & & & & \vdots & \vdots & \vdots \\ \vdots & a_n & a_{n-2} & \ddots & & & 0 & \vdots & \vdots \\ \vdots & 0 & a_{n-1} & & \ddots & & a_0 & \vdots & \vdots \\ \vdots & \vdots & a_n & & & \ddots & a_1 & \vdots & \vdots \\ \vdots & \vdots & 0 & & & & a_2 & a_0 & \vdots \\ \vdots & \vdots & \vdots & & & & a_3 & a_1 & \vdots \\ 0 & 0 & 0 & & & & a_4 & a_2 & a_0 \end{bmatrix}$$

is called *Hurwitz matrix* of the polynomial $p(s)$. Moreover, when $a_n > 0$, the polynomial $p(s)$ is Hurwitz if and only if all the leading principal minors of the matrix $H(p)$ are positive [Kha02]. Therefore, if $p(s)$ is Hurwitz, then $|H(p)| > 0$ so that $H(p)$ is invertible.

Lemma 2.2.2. The closed-loop system in Figure 1.1 is robustly internally stable if and only if, for all $\delta_i(s;\tau) \in \Delta(s;\tau)$, the polynomial $\delta_i(s;\tau)$ is Hurwitz, and for each pair $\delta_i(s;\tau)$, $\delta_j(s;\tau) \in \Delta(s;\tau)$, no eigenvalues of the matrix $H^{-1}(\delta_i(s;\tau))H(\delta_j(s;\tau))$ are located in the negative real axis $(-\infty, 0)$ in the complex plane. □

Proof: The proof uses Bialas' theorem [Bia04] (also reviewed in A.2). In our case, the degree of each $\delta_i(s;\tau) \in \Delta(s;\tau)$ is determined only by the term

$$D(s)D_{\mathrm{C}}(s)N_{\mathrm{n}}(s)D_Q(s;\tau)$$

in (2.1.1), and thus, its leading coefficient is always nonzero. Therefore, the degrees of all vertex polynomials in the set $\Delta(s;\tau)$ are equal, and Bialas' theorem is ready to be applied to Lemma 2.2.1. ■

Lemma 2.2.2 gives a necessary and sufficient condition on the robust stability of the DOB-based control system for a given τ, without the need to check infinitely many polynomials. Now, with the help of Routh-Hurwitz stability criterion, one can compute the exact value of τ^* and the detailed procedure is proposed in the next chapter.

Chapter 3

Computation of Minimum Bandwidth of Q-filter for Robust Stability

In this chapter, we present a procedure to compute the value of τ^* in the former part of Theorem 1.2.1 based on the given lemmas and discuss the case when non-minimum phase systems belong to the set $\mathcal{P}$. Also, we provide a couple of numerical examples to describe the utility of the proposed computation procedure.

3.1 Procedure for Computing τ^*

The following procedure is for computing a range for τ, on which the closed-loop system is robustly stable against the parametric variations in Assumption 1.2.1. Once the range is computed, τ^* is obtained straightforwardly for both cases where the set of plant consists of only minimum phase systems or not.

Step 1. For each $\delta_i(s;\tau) \in \Delta(s;\tau)$, $i = 1, \ldots, m$, find the largest range $R_i \subset (0, \infty)$ such that for all $\tau \in R_i$, the polynomial $\delta_i(s;\tau)$ is Hurwitz. □

In particular, if the plant set $\mathcal{P}$ consists of minimum phase systems only, existence of the largest τ_1^i (including the case when $\tau_1^i = \infty$) such that $(0, \tau_1^i) \subset R_i$ is guaranteed by Theorem 1.2.1 for every $1 \leq i \leq m$. For the computation of R_i and τ_1^i, one can employ Routh-Hurwitz stability criterion (reviewed in A.3 for convenience).

Step 2. For each pair $\delta_i(s;\tau), \delta_j(s;\tau) \in \Delta(s;\tau)$, obtain the largest range $R_{ij} \subset (0, \infty)$ such that for all $\tau \in R_{ij}$, no eigenvalues of $H^{-1}(\delta_i(s;\tau))H(\delta_j(s;\tau))$ are in

$(-\infty, 0)$. □

Similar to the previous step, existence of the largest τ_2^{ij} such that $(0, \tau_2^{ij}) \subset R_{ij}$ is also guaranteed by Theorem 1.2.1 for every $1 \leq i,\ j \leq m$ when the plant set $\mathcal{P}$ consists of only minimum phase systems. For the computation of R_{ij} and τ_2^{ij}, one can employ Sturm's theorem, which is described in A.4.

Step 3. If the plant set $\mathcal{P}$ consists of only minimum phase systems, let

$$\tau^* = \min_{i,j}\{\tau_1^i,\ \tau_2^{ij}\} \leq \infty.$$

□

Remark 3.1.1. In fact, one can choose any $\tau \in R^*$ in order that the Q-filter-based DOB works for a given plant, where

$$R^* := \Big(\bigcap_i R_i\Big) \cap \Big(\bigcap_{i,j} R_{ij}\Big) \tag{3.1.1}$$

regardless of whether the plant is of minimum phase or not. Therefore, if the plant set $\mathcal{P}$ contains non-minimum phase systems, let τ^* be the largest $\bar{\tau} \leq \infty$ such that $(0,\ \bar{\tau}) \cap R^* = \emptyset$, where $\emptyset$ denotes the empty set. Again, the latter part of Theorem 1.2.1 guarantees the existence of such $\bar{\tau}$. Even though R^* might be the empty set so that the closed-loop system is not robustly internally stable for all $\tau > 0$ and $\bar{\tau}$ becomes ∞, one can at least demonstrate if a given real plant set that contains non-minimum phase systems is suitable to employ the Q-filter-based DOB or not. □

Remark 3.1.2. The above procedure has been implemented in MATLAB as a toolbox named DO-DAT, whose first version was introduced in [CKPS18]. Updated DO-DAT is available at `https://do-dat.github.io`. The operating principles of DO-DAT are presented in Chapter 4. □

In the following chapter, illustrative examples that show the advantages of the proposed computation procedure are given.

3.2 Examples and Simulations

In this chapter, two numerical examples are presented to describe the utility of the proposed computation procedure.

Example 3.2.1. Suppose that the components in Figure 1.1 are given as:

- $\mathrm{C}(s) = 2/(s+4)$,
- $\mathrm{P_n}(s) = 5/(s-2)$,
- $\mathrm{P}(s) = \beta_0/(s+\alpha_0)$, where $4 \le \beta_0 \le 10$, $-10 \le \alpha_0 \le 10$,
- $Q(s;\tau) = 1/(\tau s + 1)$.

It is obvious that the given $\mathcal{P}$ consists of only minimum phase systems since there is no zero-dynamics. Then, there are four vertex polynomials,

$$\begin{aligned}
\delta_1(s;\tau) &= 5\tau s^3 + (-30\tau + 4)s^2 + (-160\tau + 8)s + 8,\\
\delta_2(s;\tau) &= 5\tau s^3 + (70\tau + 4)s^2 + (240\tau + 8)s + 8,\\
\delta_3(s;\tau) &= 5\tau s^3 + (-30\tau + 10)s^2 + (-100\tau + 20)s + 20,\\
\delta_4(s;\tau) &= 5\tau s^3 + (70\tau + 10)s^2 + (300\tau + 20)s + 20.
\end{aligned}$$

As the first step, the largest $\tau_1^1 > 0$ with which $\delta_1(s;\tau)$ is Hurwitz for all $\tau \in (0, \tau_1^1)$ is computed as 0.0457 by Routh-Hurwitz stability criterion. Secondly, the largest $\tau_2^{3,4} > 0$ such that no eigenvalues of $H^{-1}(\delta_3(s;\tau))H(\delta_4(s;\tau))$ are in $(-\infty, 0)$ for all $\tau \in (0, \tau_2^{3,4})$, is obtained as 0.1667 by Sturm's theorem. Continuing the computation, we get

$$\tau^* = \min_{i,j}\{\tau_1^i,\ \tau_2^{ij}\} = 0.0457.$$

The result can be verified by `wcgain` function (that calculates the worst-case peak gain of given uncertain system) in MATLAB and it is observed that for $\tau = 0.0458$, transfer functions of r to y and d to y can have infinite gain because of the plant uncertainty. On the other hand, it is verified that gains of the same transfer functions for $\tau = 0.0456$ are bounded despite the plant uncertainty. □

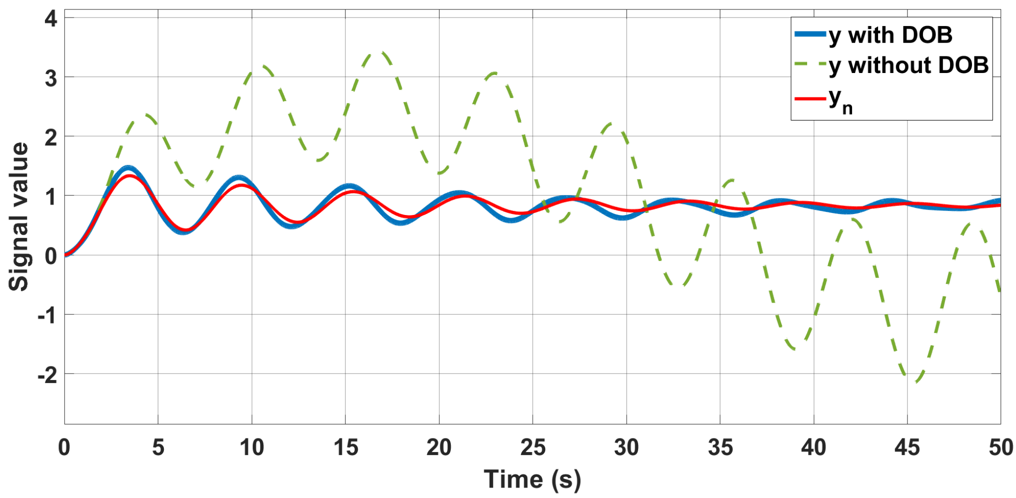

Figure 3.1: Nominal performance recovery with non-minimum phase plant $\mathrm{P}(s) = (s^2 - 0.2s + 5)/(s^3 + 3s^2 + 3s + 1)$ for $\tau = 0.21$.

Example 3.2.2. Suppose that in Figure 1.1,

- $\mathrm{C}(s) = 1/(s+1)$,
- $\mathrm{P_n}(s) = (s^2 + s + 5)/(s^3 + 3s^2 + 3s + 1)$,
- $\mathrm{P}(s) = (s^2 + \beta_1 s + 5)/(s^3 + 3s^2 + 3s + 1)$,
  where $-0.2 \leq \beta_1 \leq 2$,
- $Q(s;\tau) = 1/(\tau s + 1)$.

Although the given set $\mathcal{P}$ contains non-minimum phase systems such as

$$\mathrm{P}(s) = \frac{s^2 - 0.2s + 5}{s^3 + 3s^2 + 3s + 1},$$

the procedure provides that the closed-loop system with Q-filter-based DOB is robustly internally stable at least for

$$\tau \in (0.206, 0.627) \subset R^*,$$

where R^* is a set of τ defined in (3.1.1).

For $r(t) = 1(t)$ (Heaviside step function) and $d(t) = 2\sin(0.1t)$, Figs. 3.1 and 3.2 illustrate stability of the closed-loop system and nominal performance

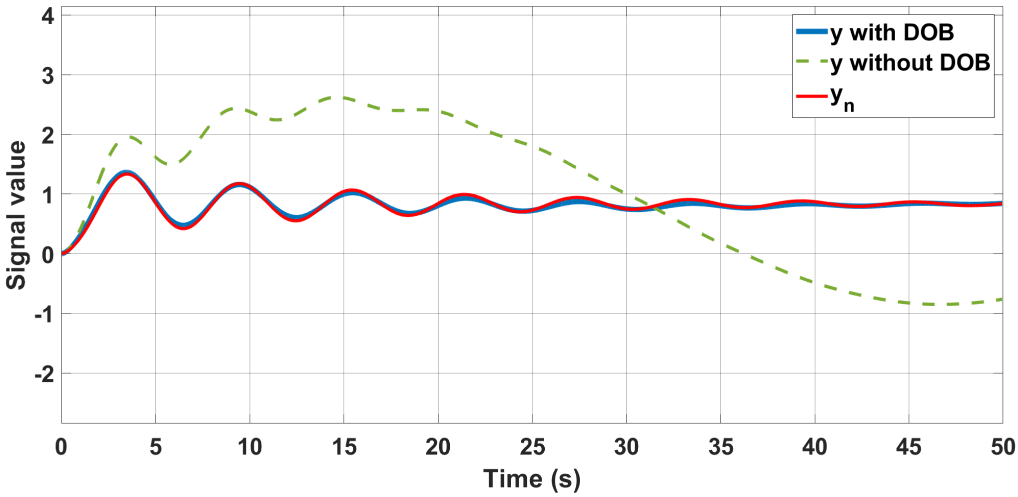

Figure 3.2: Nominal performance recovery with minimum phase plant $\mathrm{P}(s) = (s^2 + 2s + 5)/(s^3 + 3s^2 + 3s + 1)$ for $\tau = 0.21$.

recovery with two different real plant models for $\tau = 0.21$. It is observed that the closed-loop system is robustly stable even if the plant is of non-minimum phase, and the nominal performance recovery is achieved to some extent. On the other hand, if we choose τ outside the range $(0.206, 0.627)$, for example, $\tau = 0.16$, then the closed-loop system with a plant $\mathrm{P}(s) \in \mathcal{P}$ shows unstable behavior, as seen in Figure 3.3, as expected. $\square$

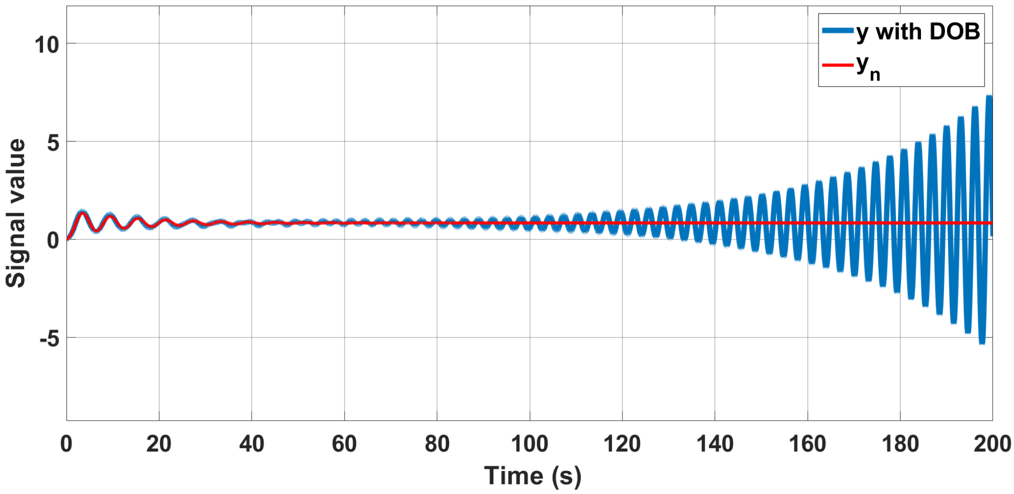

Figure 3.3: Unstable closed-loop system with $\mathrm{P}(s) = (s^2 - 0.2s + 5)/(s^3 + 3s^2 + 3s + 1)$ for $\tau = 0.16$.

Chapter 4

DO-DAT: MATLAB Toolbox for Design and Analysis of Disturbance Observer

As mentioned in Remark 3.1.2, a MATLAB toolbox DO-DAT that contains the procedure for computing a range for τ, on which the closed-loop system is robustly stable, has been developed. The flowchart of DO-DAT is given in Figure 4.1. In the figure, each block represents a function supported by DO-DAT. Users should decide which function to use depending on whether there user-defined $Q(s;1)$ and τ exist or not.

In this chapter, the operating principles of DO-DAT are presented including the manual of each supported function. For convenience, it is supposed that the components in Figure 1.1 are given by the same

- $\mathrm{C}(s) = 2/(s+4)$,
- $\mathrm{P_n}(s) = 5/(s-2)$,
- $\mathrm{P}(s) = \beta_0/(s+\alpha_0)$, where $4 \leq \beta_0 \leq 10$, $-10 \leq \alpha_0 \leq 10$,

as in Example 3.2.1 in the rest of this chapter.

4.1 Setup

For a given nominal controller, plant and a set of uncertain real plants, setup is needed before using the main functions of the toolbox.

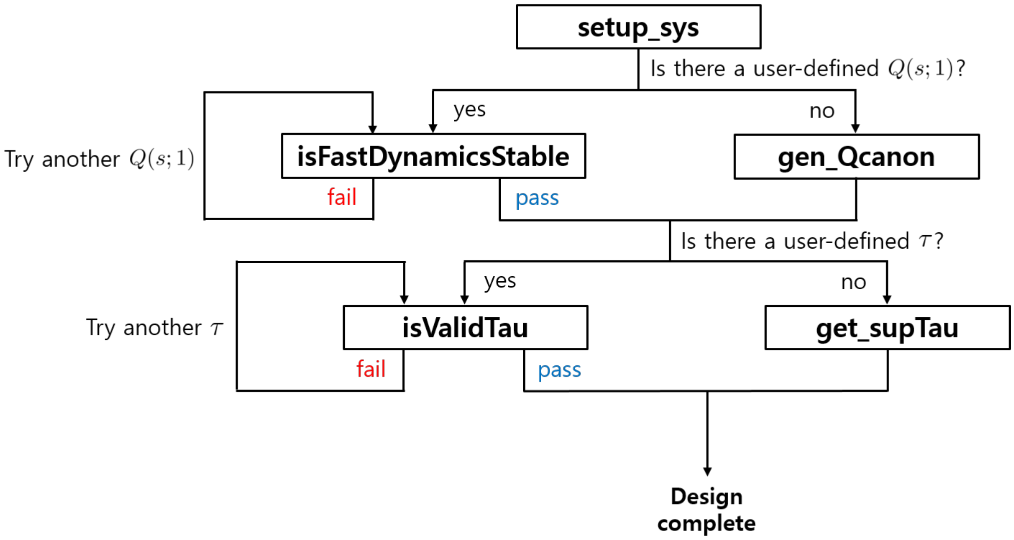

Figure 4.1: The flowchart of DO-DAT.

4.1.1 setup_sys.m

As input variables,

- N and D represent the numerator and denominator of a given uncertain plant $\mathrm{P}(s)$, respectively, and they must be entered in the form of a cell that contains both upper and lower bounds of all the coefficients as follows.

```
N = {[4, 10]};
D = {1, [-10, 10]};
```

- nominal plant P_n and controller C must be entered in the form of a transfer function model as follows.

```
P_n = tf(5, [1, -2]);
C = tf(2, [1, 4]);
```

So the function

```
sysEnv= setup_sys(N, D, P_n, C)
```

returns a structure variable sysEnv that contains the information about the system environment, such as the nominal controller $\mathrm{C}(s)$, nominal plant $\mathrm{P_n}(s)$, and

```
>> sysEnv = setup_sys(N, D, P_n, C)

sysEnv =

  struct with fields:

             P_n: [1×1 tf]
               C: [1×1 tf]
               N: [2×1 double]
               D: [2×2 double]
     nominalStab: 1
        minPhase: 1
```

Figure 4.2: The output of the function setup_sys.

the uncertain plant P(s), that will be used later in designing the DOB.

The output `sysEnv` also contains the information about the stability of the nominal closed-loop system and minimum phaseness of a given set of uncertain plants. The field `nominalStab` is 1 if the nominal closed-loop system is stable or 0 otherwise. The field `minPhase` is 1 if a given set of uncertain plants does not contain any non-minimum phase systems or 0 otherwise. Figure 4.2 shows the output of the function `setup_sys`.

4.2 Design of Coefficients of Q-filter

The coefficients a_i and c_i in (1.2.2), i.e., $Q(s;1)$, should be designed prior to determining the bandwidth of the Q-filter. If there is no user-defined $Q(s;1)$ available, the following function `gen_Qcanon` generates a transfer function model $Q(s;1)$ that can be used.

4.2.1 gen_Qcanon.m

As input variables,

- `sysEnv` is expected to be the output of the function `setup_sys`.
- desired relative degree of $Q(s;1)$ (i.e., the degree of $D_Q(s;1)$) `n` must be entered as a positive integer.

- n−1 (≥ 1) number of stable roots (that lie on the left-half plane) must be entered in the form of a row vector for the option `rhoRoots`. If the option is not used, $\rho(s)$ is set as $(s+1)^{\mathtt{n}-1}$.

As a result, the function

```
Qcanon = gen_Qcanon(sysEnv, n)
```

or

```
Qcanon = gen_Qcanon(sysEnv, n, 'rhoRoots', LHP roots)
```

returns a transfer function model $Q(s;1)$, `Qcanon`, with a constant numerator a_0 that robustly stabilizes the fast dynamics of the closed-loop system. In other words, the output of this function

$$Q(s;1) = \frac{N_Q(s;1)}{D_Q(s;1)} = \frac{a_0}{s\rho(s) + a_0}$$

guarantees that the characteristic polynomial of the fast dynamics in Assumption 1.2.2

$$\begin{aligned} p_f(s) &= D_Q(s;1) + \left(\lim_{s\to\infty} \frac{\mathrm{P}(s)}{\mathrm{P_n}(s)} - 1 \right) N_Q(s;1) \\ &= s\rho(s) + a_0 \lim_{s\to\infty} \frac{\mathrm{P}(s)}{\mathrm{P_n}(s)} \end{aligned}$$

is Hurwitz for all $\mathrm{P}(s) \in \mathcal{P}$. Figure 4.3 shows the output of the function `gen_Qcanon` for two different cases.

4.2.2 isFastDynamicsStable.m

If there is an available user-defined $Q(s;1)$ unlike the case in the previous chapter, the following function `isFastDynamicsStable` decides whether the $p_f(s)$ is Hurwitz for all $\mathrm{P}(s) \in \mathcal{P}$ under the user-defined $Q(s;1)$.

As input variables,

- `sysEnv` is expected to be the output of the function `setup_sys`.

```
>> Qcanon_1 = gen_Qcanon(sysEnv, 3)

Qcanon_1 =

          0.5
  ---------------------
  s^3 + 2 s^2 + s + 0.5

Continuous-time transfer function.

>> Qcanon_2 = gen_Qcanon(sysEnv, 3, 'rhoRoots', [-2, -4])

Qcanon_2 =

           1
  ---------------------
  s^3 + 6 s^2 + 8 s + 1

Continuous-time transfer function.
```

Figure 4.3: The output of the function gen_Qcanon.

- user-defined $Q(s;1)$ `udQcanon` must be entered in the form of a transfer function model and thus, it can be entered as the output of the function `gen_Qcanon` (Obviously, the fast dynamics $p_f(s)$ is Hurwitz for all $\mathrm{P}(s) \in \mathcal{P}$ in that case.).

Consequently, the function

```
fastDynamicsStab = isFastDynamicsStable(sysEnv, udQcanon)
```

returns a logical output `fastDynamicsStab` that equals to 1 if the fast dynamics of the closed-loop system is robustly stable, i.e., the latter part of Assumption 1.2.2 is satisfied, or 0 otherwise. Figure 4.4 shows the output of the function `isFastDynamicsStable` for three different cases.

4.3 Determination of Bandwidth of Q-filter

As the final step in designing the Q-filter in disturbance observer, the bandwidth of the Q-filter or the value of τ should be determined. The computation procedure introduced in Chapter 3 is implemented in the following functions of

```
>> fastDynamicsStab_1 = isFastDynamicsStable(sysEnv, tf(1, [1, 1]))

fastDynamicsStab_1 =

     1

>> fastDynamicsStab_2 = isFastDynamicsStable(sysEnv, Qcanon_1)

fastDynamicsStab_2 =

     1

>> fastDynamicsStab_3 = isFastDynamicsStable(sysEnv, tf([0.25, 0.25], [1, 4, 6, 4, 9, 0.25]))

fastDynamicsStab_3 =

     0
```

Figure 4.4: The output of the function isFastDynamicsStable.

DO-DAT.

4.3.1 isValidTau.m

If there is a user-defined τ, the function `isValidTau` decides whether the given τ is in R^*, where R^* is defined as in (3.1.1).

As input variables,

- `sysEnv` is expected to be the output of the function `setup_sys`.
- user-defined $Q(s;1)$ `udQcanon` that robustly stabilizes the fast dynamics of the closed-loop system ($p_f(s)$ in Assumption 1.2.2) must be entered in the form of a transfer function model.
- `tau` must be entered as a positive real number.

So the function

```
validity = isValiudTau(sysEnv, udQcanon, tau)
```

returns a logical output `validity` that equals to 1 if the closed-loop system with the DOB which is designed under given $Q(s;1)$ and τ is robustly stable or 0 otherwise. Figure 4.5 shows the output of the function `isValidTau` for two different cases.

```
>> validity_1 = isValidTau(sysEnv, Qcanon_1, 0.005)

validity_1 =

     1

>> validity_2 = isValidTau(sysEnv, Qcanon_1, 0.05)

validity_2 =

     0
```

Figure 4.5: The output of the function isValidTau.

4.3.2 get_supTau.m

If Assumptions 1.2.2 and 1.2.1 hold, the former part of Theorem 1.2.1 guarantees the existence of τ^*. Then the function `get_supTau` computes the value of τ^* using the procedure proposed in Chapter 3.

As input variables,

- `sysEnv` is expected to be the output of the function `setup_sys`.
- user-defined $Q(s;1)$ `udQcanon` that robustly stabilizes the fast dynamics of the closed-loop system ($p_f(s)$ in Assumption 1.2.2) must be entered in the form of a transfer function model.

Then the output becomes

- the (almost, in the sense of minor numerical errors) exact value of the supremum τ^* for the option `exact`. At least 2018a and Symbolic Math Toolbox are required for this option.
- an approximate value of the supremum τ^* for the option `approx`. In this case, the resolution `res` must be entered as a positive integer.

Finally, the function

```
supTau = get_supTau(sysEnv, udQcanon, 'exact')
```

or

```
supTau = get_supTau(sysEnv, udQcanon, 'approx', res)
```

```
>> supTau_exact = get_supTau(sysEnv, Qcanon_1, 'exact')

supTau_exact =

    0.0232

>> supTau_approx = get_supTau(sysEnv, Qcanon_1, 'approx', 10)

supTau_approx =

    0.0232
```

Figure 4.6: The output of the function get_supTau.

returns the supremum τ^* **supTau** such that for all $0 < \tau < \tau^*$, the closed-loop system with the DOB designed under $Q(s;1)$ is robustly stable. Figure 4.6 shows the output of the function `get_supTau` for each option.

In Figure 4.6, it is seen that $\tau^* = 0.0232$. However, it is observed that $\tau^* = 0.0457$ in Example 3.2.1. This difference is caused by which $Q(s;1)$ is used. Clearly, the range of available bandwidth of the Q-filter (or τ^*) varies depending on how the coefficients of the Q-filter or $Q(s;1)$ is designed.

Chapter 5

Conclusion

In this dissertation, several lemmas regarding necessary and sufficient conditions for robust stability of the DOB-based control system were presented, and the design of the Q-filter-based DOB including computation of the minimum bandwidth of the Q-filter was proposed. We also demonstrated that the proposed procedure can be used to find a suitable Q-filter for non-minimum phase uncertain plants by numerical computation. Finally, a MATLAB toolbox DO-DAT was introduced and the operating principles of the toolbox were presented.

APPENDIX

A.1 Edge theorem

Let $D \in \mathbb{C}$ be a simply connected domain in the complex plane $\mathbb{C}$, and let Ω be a polytope of polynomials. Then the set of the roots of Ω

$$R(\Omega) := \{s : f(s) = 0,\ f \in \Omega\} \subset \mathbb{C}$$

is contained in D if and only if the collection of the roots of all the exposed edge polynomials of Ω is contained in D.

A.2 Bialas' theorem

Let two polynomials with real coefficients

$$f_1(s) = a_n^{(1)} s^n + a_{n-1}^{(1)} s^{n-1} + \cdots + a_0^{(1)},$$
$$f_2(s) = a_n^{(2)} s^n + a_{n-1}^{(2)} s^{n-1} + \cdots + a_0^{(2)},$$

where $a_n^{(1)}, a_n^{(2)} \neq 0$, are Hurwitz. Then, the convex combination

$$\lambda f_1(s) + (1 - \lambda) f_2(s),$$

where $\lambda \in [0, 1]$, is Hurwitz if and only if no eigenvalues of $H^{-1}(f_1)H(f_2)$ are located in the negative real axis $(-\infty, 0)$, where H is the Hurwitz matrix.

A.3 Routh-Hurwitz Stability Criterion

[Nis20] Let $p(s) = a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0$ be a polynomial of degree n. The Routh-Hurwitz table of $p(s)$ can be made up as follows:

a_n	a_{n-2}	a_{n-4}	$\cdots$
a_{n-1}	a_{n-3}	a_{n-5}	$\cdots$
b_1	b_2	b_3	$\cdots$
c_1	c_2	c_3	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\ddots$

where

$$b_i = \frac{a_{n-1}a_{n-2i} - a_n a_{n-(2i+1)}}{a_{n-1}}, \quad c_i = \frac{b_1 a_{n-(2i+1)} - a_{n-1}b_{i+1}}{b_1},$$

for $i = 1, 2, \cdots$. Then, the number of sign changes in the first column of the Routh-Hurwitz table of $p(s)$ is equal to the number of roots with non-negative real part of $p(s)$.

A.4 Sturm's theorem

[Yap00] $p(s)$ be a polynomial with real coefficients and define

$$\begin{aligned} p_0(s) &:= p(s), \\ p_1(s) &:= p'(s), \\ p_{i+1}(s) &:= -\operatorname{rem}(p_{i-1}(s), p_i(s)), \qquad i = 1, 2, \ldots \end{aligned}$$

where $p'(s)$ is the derivative of $p(s)$ and $\operatorname{rem}(p_{i-1}(s), p_i(s))$ represents the remainder of the division of $p_{i-1}(s)$ by $p_i(s)$. Then, the sequence of polynomials $p_0, p_1, \ldots$ is called *Sturm sequence* of $p(s)$, which is a finite sequence. Let $\#(\zeta, p)$ be the number of sign changes in the Sturm sequence of $p(s)$ at $s = \zeta \in \mathbb{R}$. Then, the number of distinct real roots of $p(s)$ in the interval $(a, b]$ of the real axis is equal to

$$\#(a, p) - \#(b, p).$$

In order to apply Sturm's theorem for the interval $(-\infty, 0)$, one has to compute

$\#(-\infty, p)$. The sign of a polynomial $p(s)$ at $s = -\infty$ is defined as the sign of the leading coefficient, if $p(s)$ has even degree, and the opposite sign of the leading coefficient, if $p(s)$ has odd degree.

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국문초록

Computation of minimum bandwidth of Q-filter for robust stability of disturbance observer-based control systems

외란 관측기 기반 제어 시스템의 강인 안정성을 위한 Q-필터의 최소 대역폭 계산

외란 관측기가 널리 활용됨에 따라 외란 관측기가 가지는 다양한 측면이 이론적으로 연구되어 왔다. 특히, Q-필터를 기반으로 하는 단일 입출력 외란 관측기를 포함하는 선형 폐루프 시스템의 강인 안정성에 대해 많은 연구가 이루어졌으며 결국 폐 루프 시스템의 강인 안정성에 대한 필요 충분 조건이 제안된 바 있다. 그러나 제안된 필요 충분 조건은 충분히 큰 Q-필터의 대역폭을 전제하는데, 폐 루프 시스템의 강인 안정성을 보장하기 위해 필요한 Q-필터의 최소 대역폭을 정확히 알아내는 방법은 제시된 바 없다.

본 학위논문에서는 몇 가지 보조정리와 함께 Q-필터 기반 외란 관측기가 포함된 폐 루프 시스템의 강인 안정성을 보장하는 Q-필터의 최소 대역폭을 정확히 계산할 수 있는 방법론을 제안한다. 본 학위논문에서 제안하는 방법론은 DO-DAT이라는 이름의 MATLAB 툴박스로 구현되어 있으며 관련 내용은 `https://do-dat.github.io`에서 확인할 수 있다.

주요어: 외란 관측기, 외란 제거, 강인 안정성, 공칭 성능 회복, 불확실한 다항식

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