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Research on Point-sharing Policy under Competition

경쟁 하의 포인트 공유정책 연구

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산업공학과

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Abstract

Research on Point-sharing Policy under Competition

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To encourage reward redemption and effectiveness of loyalty programs, many retailers have found ways to augment and improve conventional marketing methods. For instance, retailers from different industries in South Korea have created a point-sharing policy in which customers are able to enjoy the privileges of spending at multiple participating outlets while enjoying flexible point redemption at any participant retailer, thus increasing demand of all retailers under the umbrella of the joint point policy. This thesis develops a supply chain model consisting of one supplier and two competitive retailers under the point-sharing policy, enabling retailers to optimize their marketing strategies in order to maximize long-term profits. Meanwhile, this thesis also proposes a two-stage transfer payment contract for supply chain coordination, in order to achieve profit optimization for the whole supply chain.

Keywords: Point Sharing, Channel Coordination, Externalities, Supply Chain Coordination Contract

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Chapter 1

Introduction

With the development of the modern commercial market, promotional activities are becoming increasingly important for the retail industry. An effective promotional strategy can improve both total profits and customer loyalty, which is beneficial for long-term sustainability. However, methods for the design and implementation of promotional strategies require significant thought. There are various mechanisms for sales promotion, such as temporary price reductions, coupons, points, and combinations with other attributes of non-price-related promotions. In fact, the sales promotion mechanism also needs to consider other constraints, such as competition or market demand, among other factors.

Loyalty programs (LPs) frequently implement a point collection and redemption system in a bid to earn customer loyalty. The amount of points each customer is awarded at purchase depends on the policy of each LP and is usually based on the total purchases of a customer as well as a conversion ratio set by the retailer. Customers can then redeem any accumulated points in exchange for vouchers, which can be used at the same retailer.

In this way, retailers benefit by securing increased customer demand, while cultivating loyalty at the same time. For example, Singaporean developer CapitaLand Limited encouraged customers' loyalty of all participating outlets in their 19 shopping malls by implementing a rewards program that rewarded shoppers with STAR$® for making purchases. Redeemed Capita Vouchers could be used in any of the malls as a replacement for cash. Due to the effectiveness of such programs, more and more retailers are now employing similar schemes. In the United States, previous studies have found that there were upward of 2.65 billion LP membership schemes in 2012, an increase of 26.7 percent, since 2010.[1]

Retailers have recently found a way to increase the performance of such point reward systems. They have done this by establishing a cooperative promotional scheme, in which a number of retailers adopt the same point-sharing policy (PS). Under this policy, customers can redeem points for their purchases at participating retailers. This system is operated by third-party companies who are responsible for the management of the PS and who maintain the network of participating retailers. CJ ONE, a membership card company in South Korea, provides a points reward system for its customers that includes more than 20 different brands. These brands belong to different market sectors, including food and drink, education, and entertainment, as well as consumer shopping. Each brand provides unique goods and services within the scheme, meaning there is little or no competition between participating brands. Companies subscribing to this particular scheme range from CGV, a leading movie theater chain in South Korea, to the British company GEM, which provides English learning courses, to Tous les Jours, a leading coffee chain.

In this way, a member of CJ ONE's scheme can both collect and spend points at any participating company, regardless of the goods and services being offered. Another popular loyalty program in Singapore is Plus! Link Points, which has more than 1 million members and more than 600 participating merchants.

As we mentioned above, there is little competition between participating retailers in a point alliance , because they come from different market sectors and sell different types of products. However, with the expansion of the point alliance, more retailers join the point scheme and the product differentiation thereby decreases, which causes competition. Here, we consider only the competitive relationship among retailers selling the same products in a point alliance. We take two retailers, CJmall and Olive Young, which both participate in CJ ONE's point alliance, as case examples. CJmall is a comprehensive shopping mall that sells clothing, beauty products, and health food, among other goods. Olive Young is the first health and beauty (H&B) retailer in South Korea that provides beauty and self-care products, health care products, and health food. Both of the two retailers sell beauty products and health food, and many brands within both stores are the same. For instance, customers can buy cosmetics made by AHC (a South Korean beauty product brand) at both stores. And Olive Young is even established as a famous brand in CJmall, which means there is a big overlap between the products that each retailer sells, which leads to competition.

After examining the setup between CJmall and Olive Young, we then focus on the T-point card program in Japan, which is a typical loyalty points reward program. When customers buy things, they automatically receive points and can then use those points toward future purchases. A number of different shops and businesses support the T-point card system and the points, namely well-known stores such as Tsutaya, Family Mart, Yahoo Japan, Maruetsu (a Japanese supermarket), Doutor (a Japanese coffee shop), Excelsior Caffé, and a whole list of family restaurants, Demae-kan and Tabe-log (a Japanese restaurant guide and reservation site) among them. Compared with Korea's CJ ONE program, the number of participating retailers is much bigger in the T-point card program. Meanwhile, there are several retailers in every field that share target customers and market share. For example, the participating book stores include TSUTAYA Books, BOOksmisumi, マイナビBOOKS, and others. These are different retailers with different point systems, selling almost the same products, and each has the right to decide the point conversion ratios, respectively, which results in competition.

Many case studies show an increase in demand and customer loyalty from retailers' use of the PS policy. However, there are very few relative studies, especially from a mathematical perspective, on this phenomenon, and there is no research on the PS policy that considers competition. Therefore, based on the previous research, this thesis develops a point-sharing model under competition in which a goal is to contribute to helping retailers design optimal strategies to maximize their own profits and the profits of the whole supply chain.

Meanwhile, this thesis also proposes a contract for the coordination of a supply chain to optimize global profits.

The remainder of this thesis is organized as follows. Chapter 2 presents a literature review. Chapter 3 illustrates the formula of the proposed model and the corresponding numerical results. In Chapter 4, a two-stage contract for coordinating the supply chain is outlined. Finally, Chapter 5 presents the conclusion and potential avenues for future research.

Chapter 2

Literature Review

A topic closely related to the PS policy is the coalition loyalty program, which involves a group of companies banding together to develop a joint program. Previous studies have developed analytical models to support the planning and management of CLPs. Kim et al. [2] studied the influence of reward programs on price competition, the relationship between users, and the benefits attained from the programs. Similarly, Lola et al. [3] studied the joint benefit of redemption of customers with membership cards accumulated by maximizing value toward owners, employees, and customers. While these studies are relevant to the management of CLPs, there is a need to directly address the issue of planning the supply of rewards efficiently and effectively. Cao et al. [4] described the problem of planning LP rewards using a two-stage stochastic linear program. Researchers have also included the use of various other methods to increase sales and to target certain customers. Cao et al. [5] explored the use of bonus points, referring to cooperative promotional agreements made by hosts and partners, to give members extra points when they purchased from a specific retailer.

Given that the PS policy is a marketing tool used to improve the amount of sales and increase the demand for a retailer's goods, it is necessary to study other forms of sales promotion methods, like advertising, coupons, and group buying. Aust et al. [6] reviewed literature regarding cooperative advertising models and discovered gaps regarding demand-relevant variables. Krishnan et al. [7] explored buyback policies in cost-sharing and found that buybacks adversely affect supply chain profits. Other related research studied different scenarios wherein channel members competed in both price and sales promotion without considering which party should provide sales promotion efforts.[8] One relatively closely linked paper written by Xia et al. [9] considers a supply chain with one manufacturer and one retailer, where the manufacturer sets sales promotion itself and the retailer decides the price of the product either jointly or separately.

The third category of research examines supply chain coordination with contracts, and there is a large body of literature present in this field. Lian et al. [10] explored supply chain contracts where buyers received discounts for committing to purchases in advance. Wong et al. [11] explored how sales rebate contracts helped to achieve supply chain coordination to allow chain members to make centralized decisions for the whole system under decentralized control. In a way, this follows the setup of the PS policy, where demand is affected by the point policy redemption stimulus in the channel. There is also plentiful research related to TR (target rebate) contracts. Under this policy, suppliers pay a rebate to retailers for each unit sold beyond a particular target value. Taylor et al. [12] compared the difference between two common forms of rebates, which are linear rebates and target rebates, and found that a properly designed target rebate and returns contract achieves coordination where demand is influenced by retailers' sales efforts. Chiu et al. [13] studied the case of a supply chain with a single supplier and a single risk-averse retailer, and proposed TR contracts for achieving coordination. In addition to considering the risk sensitivity of retailers, the supply chain model was extended to include sales effort decisions of the retailer.

As for research on the point-sharing policy, which is highly related to the research of this paper, Moon et al. [14] proposed a theoretical model in which two independent retailers utilized a point-sharing policy and discussed their decision-making without considering the competitive relationship among retailers. Although it is evident that the PS policy is being quickly implemented all over the world, there is still insufficient research to fully understand the effectiveness of this policy. Few studies consider how collaborative promotional efforts in a point-sharing scheme can function best under competition, including how individual decisions are made by scheme members and the externalities that may result on the supply chain as a whole because of such schemes.

Chapter 3

Supply Chain Model under the Point-sharing policy

Demands for different products are not independent of each other but have intricate connections. For example, the best-selling quality goods sold by retailers with good reputations will always attract a large number of customers, and the increase in customer flow will drive sales of other products in stores. If a retailer sells two products that are substitute goods, the demand for one product increases, while the demand for the other will decrease. This phenomenon is referred to as demand externalities. When one product serves to promote the demand for another, it is referred to as having positive externalities. When its effect on other products is negative, it is known as having negative externalities. Demand externalities are very common in the retail sector.

Competition between retailers selling the same products is usually manifested in the behavior of their products' externalities. This is often expressed by externalities being either positive or negative. For example, when retailers are in the same region, due to the relatively fixed customer sources, the externalities between retailers are often negative, while the promotional behavior of one retailer often results in a decline of sales for the other retailer. However, when retailers are from different regions, the externalities between them are usually positive, due to geographical positions. The promotional activities of retailers in one region often have a positive impact on the sales of retailers in another region. For products early in the life cycle, the behaviors of retailers also tend to affect products' externalities positively as a result of the immaturity of the market. When one retailer offers promotional efforts to increase sales, the effect of this advertising often leads to an increase in sales for another retailer. Products in the later stages of their product life cycles often display negative externalities as a result of the fully developed market and its limited scalability.

Under the point-sharing policy, retailers can reward customers with points at a certain point-conversion ratio. This means that customers are free to choose any retailer participating in the scheme to redeem their points. It is clear that when a retailer runs at a higher point-conversion ratio, customers are more likely to consume at that retailer, thus increasing product sales. Therefore, it can be considered that the demand for a product can be a function of the point-conversion ratio. Additionally, since customers are free to choose any participating retailers at which to redeem their points, it is possible that every retailer will bear a different percentage of total consumption points generated from the point-sharing scheme. This would affect retailers' expenses and furthermore influence their managerial decision-making.

3.1 Supply Chain Model under the Point-sharing Policy

A point-sharing policy model with two competitive retailers (Retailer 1 and Retailer 2) is investigated as a case example in which retailers sell identical products provided by the same supplier. We assume that the supplier has sufficient supply capacity and the retailer's sales depend on its promotion efforts. D_i refers to the demand at Retailer i in one selling season. Here, the point conversion ratio, λ_i, represents the promotion efforts of Retailer i. Therefore, the demand function at Retailer i is

$$D_i(\lambda_1, \lambda_2) = A_i + f_i(\lambda_i) + b_{ji} f_j(\lambda_j) \quad i, j = 1,2 \qquad (3.1)$$

$-1 < b_{ji} < 1$ indicates the influencing factor of Retailer j's promotional behaviors on Retailer i. When b_{ji} is negative, the promotion efforts of the retailer have negative externalities, which means the promotion efforts of Retailer i will reduce the sales of Retailer j. When b_{ji} is zero, the promotion efforts between retailers are independent of each other. When b_{ji} is positive, the promotion efforts between retailers have positive externalities. That is, the promotion efforts of Retailer i will increase the sales volume of Retailer j; Ai is constant and represents the base sales volume of Retailer i; $f_i'(\lambda_i) > 0$ represents the positive marginal sales volume at the level of promotion efforts. That is, the harder the retailer works on sales promotion, the higher the sales volume. $b_{ji} f_j(\lambda_j)$ represents the impact of promotion efforts from Retailer j. For simplicity, $f_i(\lambda_i)$ is set as $B_i\lambda_i$. B_i is constant and $B_i > 0$. Then, the demand function of Retailer i is as follows:

$$D_i(\lambda_1, \lambda_2) = A_i + B_i\lambda_i + b_{ji}B_j\lambda_j \quad i, j = 1,2 \qquad (3.2)$$

Denote the retail price as p and the wholesale price and unit production cost as w and c, respectively. θ_i refers to the percentage of points converted by Retailer i and redeemed at Retailer i.

To further analyze this model, the following assumptions were made.

Assumption 1: Customers can only use their points when redeeming at a retailer. In other words, they do not pay any cash as partial payment.

Assumption 2: When customers redeem their points, no new points are added to their cards.

Assumption 3: Customers are willing to redeem all their points.

Assumption 4: Retailers can only change point-conversion ratios but not product price. This assumption holds true in a competitive market where buyers are price-takers. In a real-world scenario, the prices of products are fixed and the price schedule has been running for a long time. Existing research proves that it can be hard to change retail prices. Therefore, it is better to improve customer loyalty by CLPs instead of by changing retail prices.

For convenience, the supply chain consisting of the supplier and Retailer i is referred to as

Channel *i*. Then, the profit function of Retailer *i* is

$$\Pi_i(\lambda_1,\lambda_2)=(p-w)D_i(\lambda_1,\lambda_2)-\theta_i w\lambda_i D_i(\lambda_1,\lambda_2)-(1-\theta_j)w\lambda_j D_j(\lambda_1,\lambda_2)\quad i,j=1,2 \qquad (3.3)$$

On the right side of Equation (3.3), the first term refers to the gross profit without customers' redemption, the second term refers to the cost of products from customers' local redemption, and the third term refers to the cost of products bought with points issued by the other retailer.

The profit functions of Retailer 1 and Retailer 2 can be described as

$$\Pi_1(\lambda_1,\lambda_2)=(p-w)(A_1+B_1\lambda_1+b_{21}B_2\lambda_2)-\theta_1 w\lambda_1(A_1+B_1\lambda_1+b_{21}B_2\lambda_2)-(1-\theta_2)w\lambda_2(A_2+B_2\lambda_2+b_{12}B_1\lambda_1) \qquad (3.4)$$

$$\Pi_2(\lambda_1,\lambda_2)=(p-w)(A_2+B_2\lambda_2+b_{12}B_1\lambda_1)-\theta_2 w\lambda_2(A_2+B_2\lambda_2+b_{12}B_1\lambda_1)-(1-\theta_1)w\lambda_1(A_1+B_1\lambda_1+b_{21}B_2\lambda_2) \qquad (3.5)$$

The profit function of the supplier at Channel *i* is

$$\Pi_{si}(\lambda_1,\lambda_2)=(w-c)(A_i+B_i\lambda_i+b_{ji}B_j\lambda_j) \qquad (3.6)$$

The profit function of Channel *i* is

$$\Pi_{ci}(\lambda_1,\lambda_2)=(p-c)(A_i+B_i\lambda_i+b_{ji}B_j\lambda_j)-\theta_i w\lambda_i(A_i+B_i\lambda_i+b_{ji}B_j\lambda_j)-(1-\theta_j)w\lambda_j(A_j+B_j\lambda_j+b_{ij}B_i\lambda_i) \qquad (3.7)$$

3.2 Model Analysis

The performance of the centralized supply chain is analyzed first, so as to make reference to the performance of the subsequent collaboration mechanism. In a centralized supply chain, all enterprises are seamlessly integrated, and all enterprises are coordinated by a unified controlling organization, so that the entire supply chain system operates in an optimal state.

The profit function of the centralized supply chain is

$$
\begin{aligned}
\Pi_{ch}(\lambda_1,\lambda_2) &= \Pi_1(\lambda_1,\lambda_2)+\Pi_2(\lambda_1,\lambda_2)+\Pi_{s1}(\lambda_1,\lambda_2)+\Pi_{s2}(\lambda_1,\lambda_2) \\
&= (p-c)(A_1+B_1\lambda_1+b_{21}B_2\lambda_2)-w\lambda_1(A_1+B_1\lambda_1+b_{21}B_2\lambda_2)-w\lambda_2(A_2+B_2\lambda_2+b_{12}B_1\lambda_1) \\
&\quad +(p-c)(A_2+B_2\lambda_2+b_{12}B_1\lambda_1)
\end{aligned}
\tag{3.8}
$$

We can see that the total profit contributed by Retailer *i* to the supply chain consists of the basic profit from sales, the direct sales profit generated by the points policy, the change in total profit caused by the demand externalities of Retailer *i*, and the cost of promotion efforts.

We set the optimal solutions of retailers to maximize the profit of the whole supply chain channel as $\{\lambda_1^{c},\lambda_2^{c}\}$.

Theorem 1.

In a centralized supply chain, the retailers' optimal solutions, $\{\lambda_1^{c},\lambda_2^{c}\}$, can be obtained from the first-order condition of Equation (3.7) $\frac{\partial \Pi_{ch}(\lambda_1,\lambda_2)}{\partial \lambda_1}=\frac{\partial \Pi_{ch}(\lambda_1,\lambda_2)}{\partial \lambda_2}=0$, and the following conditions can be satisfied:

$$
\begin{cases}
\lambda_1^{c}=\dfrac{(p-c)(1+b_{12})B_1-w\lambda_2^{c}b_{12}B_1-w(A_1+b_{21}B_2\lambda_2^{c})}{2wB_1} & (3.9)\\[2ex]
\lambda_2^{c}=\dfrac{(p-c)(1+b_{21})B_2-w\lambda_1^{c}b_{21}B_2-w(A_2+b_{12}B_1\lambda_1^{c})}{2wB_2} & (3.10)
\end{cases}
$$

Proof.

Differentiating $\Pi_{ch}(\lambda_1,\lambda_2)$ with respect to λ_1, then we obtain

$$\frac{\partial \prod_{ch}(\lambda_1,\lambda_2)}{\partial \lambda_1} = (p-c)B_1 - \theta_1 w\lambda_1 B_1 - \theta_1 w(A_1 + B_1\lambda_1 + b_{21}B_2\lambda_2) - (1-\theta_2)w\lambda_2 b_{12}B_1$$
$$+(p-c)b_{12}B_1 - \theta_2 w\lambda_2 b_{12}B_1 - (1-\theta_1)w(A_1 + B_1\lambda_1 + b_{21}B_2\lambda_2) - (1-\theta_1)w\lambda_1 B_1 \quad (3.11)$$

$$\frac{\partial^2 \prod_{ch}(\lambda_1,\lambda_2)}{\partial {\lambda_1}^2} = -2wB_1 \quad (3.12)$$

Suppose that λ_1^c satisfies $\frac{\partial \prod_{ch}(\lambda_1,\lambda_2)}{\partial \lambda_1}=0$, $\frac{\partial^2 \prod_{ch}(\lambda_1,\lambda_2)}{\partial {\lambda_1}^2}<0$, then we get

$$\lambda_1^c = \frac{(p-c)(1+b_{12})B_1 - w\lambda_2^c b_{12}B_1 - w(A_1 + b_{21}B_2\lambda_2^c)}{2wB_1}$$

With the same approach, the optimal solution of Retailer 2, λ_2^c, is reached.

We can see each retailer has a unique optimal response to any strategy of the other retailer, and $\frac{\partial^2 \prod_{ch}(\lambda_1,\lambda_2)}{\partial {\lambda_1}^2}<0$, $\frac{\partial^2 \prod_{ch}(\lambda_1,\lambda_2)}{\partial {\lambda_2}^2}<0$, which means $\prod_{ch}(\lambda_1,\lambda_2)$ is strictly concave for strategy λ_1 and λ_2, therefore there exists a unique optimal strategy, $\{\lambda_1^c,\lambda_2^c\}$, to maximize $\prod_{ch}(\lambda_1,\lambda_2)$. According to the definition of the Nash Equilibrium (NE), the strategy $\{\lambda_1^c,\lambda_2^c\}$ is a pure-strategy NE.

Proposition 1.

λ_i^c (Optimal promotion effort level of Retailer *i*) will increase as its coefficient of externalities, b_{ji}, increases.

Proof.

$$\lambda_1^{c} = \frac{B_2\left\{B_1\left[b_{12}(b_{21}-1)-2\right]+B_2 b_{21}(1+b_{21})\right\}(c-p)+(A_2B_1b_{12}-2A_1B_2+A_2B_2b_{21})w}{\left[2B_1B_2(2-b_{12}b_{21})-B_1^2{b_{12}}^2-B_2^2{b_{21}}^2\right]w} \quad (3.13)$$

Take the first derivative of b_{12}

$$\frac{d\lambda_1^{c}}{db_{12}} = \frac{\left[B_1B_2(b_{21}-1)(c-p)+A_2B_1w\right]\left[2B_1B_2(2-b_{12}b_{21})-B_1^2{b_{12}}^2-B_2^2{b_{21}}^2\right]w+(2b_{12}B_1+2b_{21}B_1B_2)\left\{wB_2\left\{B_1\left[b_{12}(b_{21}-1)-2\right]+B_2b_{21}(1+b_{21})\right\}(c-p)+(A_2B_1b_{12}-2A_1B_2+A_2B_2b_{21})w\right\}}{\left[2B_1B_2(2-b_{12}b_{21})-B_1^2{b_{12}}^2-B_2^2{b_{21}}^2\right]^2 w^2}$$

(3.14)

We easily get

$$\begin{cases} B_1B_2(b_{21}-1)(c-p)+A_2B_1w>0 \\ 2B_1B_2(2-b_{12}b_{21})-B_1^2{b_{12}}^2-B_2^2{b_{21}}^2>0 \\ 2b_{12}B_1+2b_{21}B_1B_2>0 \\ wB_2\left\{B_1\left[b_{12}(b_{21}-1)-2\right]+B_2b_{21}(1+b_{21})\right\}(c-p)+(A_2B_1b_{12}-2A_1B_2+A_2B_2b_{21})w>0 \end{cases}$$

Subsequently, we can get

$\frac{d\lambda_1^{c}}{db_{12}}>0$, similarly, $\frac{d\lambda_2^{c}}{db_{21}}>0$.

Therefore, λ_i^{c} (optimal promotion effort level of Retailer i) will increase along with b_{ij}, its coefficient of externalities.

In the centralized supply chain, the profit function of the total profit can be written as

$$\Pi_{ch}(\lambda_1,\lambda_2)=(p-w-w\lambda_1)(A_1+B_1\lambda_1+b_{21}B_2\lambda_2)+(p-w-w\lambda_2)(A_2+B_2\lambda_2+b_{12}B_1\lambda_1) \quad (3.15)$$

When b_{ij} increases, the marginal demand increases under the promotion level, resulting in a higher marginal revenue for the supply chain. Here, the marginal revenue exceeds the marginal cost; therefore, Retailer *i* tends to set a higher λ_i^{c}. Similarly, when b_{ij} decreases, resulting in a lower marginal revenue, Retailer *i* tends to set a lower λ_i^{c}.

In the early stage of a new product being put on the market, its market recognition is low and the uncertainty of market demand is strong. Therefore, one of the burning issues for retailers is to boost the market quickly and raise market recognition. At such a time, b_{ij} is positive, meaning that the promotion efforts of one retailer will reduce the demand of the other retailer. The promotion efforts have obvious positive externalities; therefore, a relatively high point conversion ratio is set, which will be of great help to the growth of the product's market share. Conversely, when a product is in a mature stage of its life cycle, market competition will be fierce and scalability limited. Here, b_{ij} is negative, so the promotion efforts of one retailer will reduce the demand of the other retailer. Negative externalities on other retailers in a joint supply chain come up as a response. To maximize the profit of the whole supply chain, retailers will set a relatively low point conversion ratio in comparison with products that have recently entered the market.

In a decentralized supply chain, the members of the supply chain are all individuals with independent economic accounting, who make independent decisions to maximize their individual profits.

We set the optimal solutions of retailers to maximize their own profits, as well as the profit of the channel, as $\left\{\lambda_1^{\mathrm{d}}, \lambda_2^{\mathrm{d}}\right\}$ and $\left\{\lambda_{c1}^{\mathrm{d}}, \lambda_{c2}^{\mathrm{d}}\right\}$, respectively.

Theorem 2.

In a decentralized supply chain, the retailers' optimal solutions, $\left\{\lambda_1^{d}, \lambda_2^{d}\right\}$, can be obtained from the first-order condition of Equations (8) and (9), $\frac{\partial \prod_1(\lambda_1,\lambda_2)}{d\lambda_1} = \frac{\partial \prod_2(\lambda_1,\lambda_2)}{d\lambda_2} = 0$.

Therefore, the following conditions can be satisfied:

$$\begin{cases} \lambda_1^{\mathrm{d}} = \dfrac{(p-w)B_1 - (1-\theta_2)w\lambda_2^d b_{12}B_1 - \theta_1 w(A_1 + b_{21}B_2\lambda_2^d)}{2\theta_1 wB_1} & (3.16) \\ \lambda_2^{\mathrm{d}} = \dfrac{(p-w)B_2 - (1-\theta_1)w\lambda_1^d b_{21}B_2 - \theta_2 w(A_2 + b_{12}B_1\lambda_1^d)}{2\theta_2 wB_2} & (3.17) \end{cases}$$

Proof.

Differentiating $\prod_1(\lambda_1, \lambda_2)$ with respect to λ_1, then we obtain

$$\frac{\partial \prod_1(\lambda_1,\lambda_2)}{\partial \lambda_1} = (p-w)B_1 - \theta_1 w\lambda_1 B_1 - \theta_1 w(A_1 + B_1\lambda_1 + b_{21}B_2\lambda_2) - (1-\theta_2)w\lambda_2 b_{12}B_1 \quad (3.18)$$

$$\frac{\partial^2 \prod_1(\lambda_1,\lambda_2)}{\partial {\lambda_1}^2} = -2\theta_1 wB_1 \quad (3.19)$$

Suppose that λ_1^d satisfies $\dfrac{\partial \prod_1(\lambda_1,\lambda_2)}{\partial \lambda_1}$=0, and $\dfrac{\partial^2 \prod_1(\lambda_1,\lambda_2)}{\partial {\lambda_1}^2} < 0$, and the combined solution , $\{\lambda_1^{\mathrm{d}}, \lambda_2^{\mathrm{d}}\}$, is a pure-strategy NE, then we obtain:

$$\lambda_1^{\mathrm{d}} = \frac{(p-w)B_1 - (1-\theta_2)w\lambda_2^d b_{12}B_1 - \theta_1 w(A_1 + b_{21}B_2\lambda_2^d)}{2\theta_1 wB_1}$$

With the same approach, the optimal solution of Retailer 2, λ_2^{d}, is reached, and the proof of the existence of NE is similar with ***Theorem 1.***

Proposition 2.

λ_i^{d} is smaller than $\lambda_{ci}^{\mathrm{d}}$.

Proof.

With the same approach in the proof of ***Theorem 2***, $\{\lambda_{c1}^{d}, \lambda_{c2}^{d}\}$ can be obtained from the first-order condition of Equation (3.7), $\frac{\partial \prod_{c1}(\lambda_1, \lambda_2)}{d\lambda_1} = \frac{\partial \prod_{c2}(\lambda_1, \lambda_2)}{d\lambda_2} = 0$, thus, the following conditions can be satisfied.

$$\begin{cases} \lambda_{c1}^{\mathrm{d}} = \dfrac{(p-c)B_1 - (1-\theta_2)w\lambda_2^{d} b_{12}B_1 - \theta_1 w(A_1 + b_{21}B_2\lambda_2^{d})}{2\theta_1 wB_1} & (3.20) \\ \lambda_{c2}^{\mathrm{d}} = \dfrac{(p-c)B_2 - (1-\theta_1)w\lambda_1^{d} b_{21}B_2 - \theta_2 w(A_2 + b_{12}B_1\lambda_1^{d})}{2\theta_2 wB_2} & (3.21) \end{cases}$$

We can write the functions of $\lambda_{c1}^{\mathrm{d}}$ and λ_1^{d} as

$$2\theta_1 wB_1\lambda_{c1}^{\mathrm{d}} + (1-\theta_2)w\lambda_{c2}^{d} b_{12}B_1 - \theta_1 w(A_1 + b_{21}B_2\lambda_{c2}^{d}) = (p-c)B_1 \quad (3.22)$$
$$2\theta_1 wB_1\lambda_1^{\mathrm{d}} + (1-\theta_2)w\lambda_2^{d} b_{12}B_1 - \theta_1 w(A_1 + b_{21}B_2\lambda_2^{d}) = (p-w)B_1 \quad (3.23)$$

And $\lambda_{c2}^{\mathrm{d}}$, λ_2^{d} have similar expressions.

The functions of λ_1 and λ_2 are defined as

$$\begin{cases} 2\theta_1 wB_1\lambda_1 + (1-\theta_2)w\lambda_2 b_{12}B_1 - \theta_1 w(A_1 + b_{21}B_2\lambda_2) = (p-w)B_1 & (3.24) \\ 2\theta_2 wB_2\lambda_2 + (1-\theta_1)w\lambda_1 b_{21}B_2 - \theta_2 w(A_2 + b_{12}B_1\lambda_1) = (p-w)B_2 & (3.25) \end{cases}$$

If $(p-w)B_1$ grows to $(p-c)B_1$, then λ_1 remains constant, and λ_2 will get bigger, and vice versa. It is easy to understand that both λ_1 and λ_2 get bigger, which means $\lambda_{ci}^{\mathrm{d}}$ is greater than λ_i^{d}.

Retailers are closest to the consumers. This means they can learn what consumers are

Retailers are closest to consumers. This means they can learn what consumers are thinking through market feedback first, and they have more advantages in grasping market demands. There is a cost involved in promotion efforts, and retailers' promotion efforts are likely to cause an increase in sales volume, which is beneficial to the whole supply chain. This phenomenon is known as the "spillover effect." This term means that the supplier does not need to pay the cost of promotion, but shares a part of the retailers' profits, as well as the "free riding phenomenon," leading to disharmony between the supplier and the retailer. No matter how hard the retailer tries, the supplier will continue to expect the retailer to work harder. As there is a cost to be paid for the retailer, this means that the extent of a promotion does not automatically equate to profits, owing to the impact of the supplier. It is not always true, then, that the greater the promotion effort by the retailer, the more beneficial it is to the retailer. Therefore, the retailer must choose the most advantageous promotion level for its own purpose, which cannot maximize the benefits for the entire supply chain.

Therefore, it is very important to develop an appropriate contract to enable retailers to set their promotion efforts at the best level to benefit the whole supply chain, thus improving the efficiency of the supply chain. To summarize, suppliers can motivate retailers by sharing the cost of promotion or by sharing the revenue.

Proposition 3.

Under normal conditions, $\left\{\lambda_1^c, \lambda_2^c\right\}$ is different from $\left\{\lambda_1^d, \lambda_2^d\right\}$.

Proof.

When the function expressions of optimal solutions λ_1 and λ_2 are the same under centralized control and decentralized control, this means that it is easy to prove that $\left\{\lambda_1^c, \lambda_2^c\right\}$ is the same as $\left\{\lambda_1^d, \lambda_2^d\right\}$, if

$$\lambda_1 = \frac{(p-w)B_1 - (1-\theta_2)w\lambda_2 b_{12}B_1 - \theta_1 w(A_1 + b_{21}B_2\lambda_2)}{2\theta_1 w B_1} = \frac{(p-w)(1+b_{12})B_1 - w\lambda_2 b_{12}B_1 - w(A_1 + b_{21}B_2\lambda_2)}{2wB_1} \tag{3.26}$$

It is easy to prove that $\left\{\lambda_1^{\mathrm{c}}, \lambda_2^{\mathrm{c}}\right\}$ is the same with $\left\{\lambda_1^{\mathrm{d}}, \lambda_2^{\mathrm{d}}\right\}$, if

$$\begin{cases} \theta_1(1+b_{12}) = 1 \\ \theta_2(1+b_{21}) = 1 \\ \theta_1 + \theta_2 = 1 \end{cases}$$

This functional relationship shows very strong constraints on parameters, θ_1 , θ_2 , b_{12} and b_{21}, which are not decision variables. Under normal circumstances, such constraints cannot be satisfied, so the individual optimal solution is often different from the global optimal solution.

The total profit contributed to the supply chain by Retailer *i* consists of the basic sales profit, the direct sales profit generated by the point scheme, the change in total profit caused by the demand externalities of Retailer *i*, and the cost of promotion efforts. Then, the optimal, λ_i, is independent of the point redemption ratio, θ_i. Since it does not matter which retailer the points are released to, the promotion cost of the whole supply chain for point promotion is equal to $w\lambda_1(A_1 + B_1\lambda_1 + b_{21}B_2\lambda_2) + w\lambda_2(A_2 + B_2\lambda_2 + b_{12}B_1\lambda_1)$.

However, in a decentralized supply chain, retailers need to consider the proportion of point redemption, since retailers need to make decisions according to the profit brought by sales growth and the cost of promotion and may fail to make optimal solutions for the whole system. This is the prevalent problem of double marginalization in the decentralized supply chain. This means that retailers only make decisions from their own point of view, rather than considering the other retailers and the entire supply chain, which leads to inefficiency.

3.3 Numerical Experiments and Analysis

Numerical experiments are always a necessary part of scientific research, because it can be a complex task to derive conclusions directly from theories and propositions. Conducting a numerical experiment allows for deeper insights to be gained into the hidden properties of a model. Another use of numerical experiments is to verify theoretical propositions through concrete numerical examples. This chapter provides the specific numerical results for the proposed model above.

3.3.1 Initial Parameters Setup

To better show the results on retailers' optimal decisions in order for retailers to gain maximum profits from point scheme participation, the following parameters have been set up, which are presented in Table 3.1 (below), to conduct experiments. As is shown in Table 3.1, $D_1(\lambda_1,\lambda_2)=1100+1800\lambda_1-60\lambda_2$, $D_2(\lambda_1,\lambda_2)=900+1500\lambda_2-90\lambda_1$, and b_{12}=-0.05, $b_{21}=-0.04$, which means the demand externalities between the two retailers are negative.

Table 3.1: Values of initial parameters

	Retailer 1	Retailer 2
A_i	1100	900
B_i	1800	1500
b_{ij}	-0.05	-0.04
θ_i	0.5	0.4
p	10	10
w	6	6
c	4	4

3.3.2 Numerical Results and Analysis

Table 3.2: Influence of different values of b_{12} under decentralized control

b_{12}	Decentralized control						
	λ_1	λ_2	$\prod_1$	$\prod_2$	$\prod_{s1}$	$\prod_{s2}$	$\prod_{ch}$
-0.1	0.405	0.568	1567.96	2249.54	3588.57	3357.51	10763.58
-0.05	0.387	0.555	1617.61	2482.24	3526.62	3394.2	11020.67
0	0.37	0.543	1661.91	2698.46	3467.44	3427.76	11255.57
0.05	0.354	0.532	1701.14	2899.59	3410.7	3458.41	11469.83
0.1	0.339	0.521	1735.51	3086.85	3356.07	3486.32	11664.75

Table 3.3: Influence of different values of b_{12} under centralized control

b_{12}	Centralized control						
	λ_1	λ_2	$\prod_1$	$\prod_2$	$\prod_{s1}$	$\prod_{s2}$	$\prod_{ch}$
-0.1	0.157	0.193	4034.44	3458.67	2743.11	2321.13	12557.35
-0.05	0.177	0.189	4088.75	3391.35	2815.67	2334.68	12630.45
0	0.198	0.184	4143.3	3328.65	2888.96	2351.85	12712.77
0.05	0.218	0.178	4198.37	3270.25	2963.33	2372.69	12804.64
0.1	0.239	0.17	4254.24	3215.83	3039.09	2397.3	12906.46

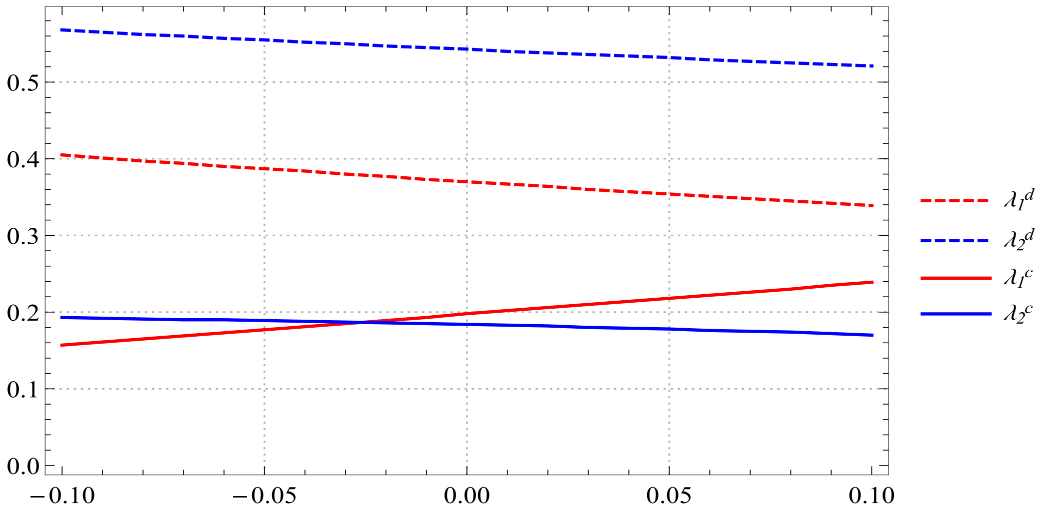

Figure 3.1: Comparison of $\{\lambda_1^d, \lambda_2^d\}$ and $\{\lambda_1^c, \lambda_2^c\}$ under different values of b_{12}

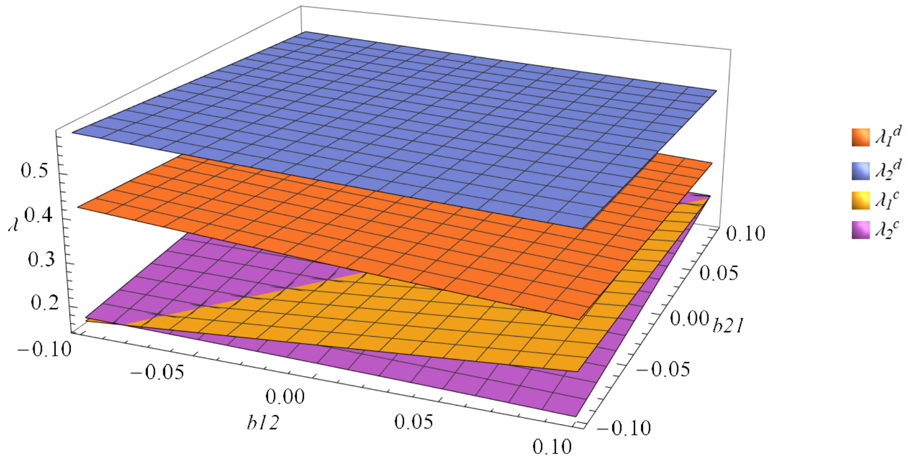

Figure 3.2: Comparison of $\{\lambda_1^d, \lambda_2^d\}$ and $\{\lambda_1^c, \lambda_2^c\}$ under different values of b_{12} and b_{21}

Table 3.2 and Table 3.3 (above) show the influence of different values of b_{12} under decentralized control and centralized controlling authorities. We can see that retailers always choose a higher point conversion ratio under decentralized control than under centralized control. Also, the total profit of the supply chain cannot be maximized under decentralized control. Figure 3.1 and Figure 3.2 (above) show that λ_1^c increases will b_{12}, λ_1^c and λ_2^c decreases will b_{12}, and the effect that changes of b_{12} have on λ_2^c is not obvious. As is discussed in ***Proposition 1***, when b_{12} increases, the marginal demand increases under the promotion level λ_1 , resulting in a higher marginal revenue of the supply chain. Therefore, Retailer 1 tends to set a higher λ_1^c. In a decentralized supply chain, an increase of b_{12} increases the promotion cost of both parties, which may be greater than the value of the increased sales revenue. This may be the reason why retailers reduce the point conversion ratio. In addition, the profit of the whole supply chain increases with b_{12}, due to the profit brought about by the increase in positive externalities by Retailer 1.

Table 3.4: Influence of different values of θ_1 under decentralized control

θ_1	Decentralized control						
	λ_1	λ_2	$\prod_1$	$\prod_2$	$\prod_{s1}$	$\prod_{s2}$	$\prod_{ch}$
0.4	0.558	0.567	2039.73	322.95	4142.51	3400	9905.19
0.55	0.325	0.55	1475.13	3096.94	3302.83	3392.69	11267.6
0.7	0.191	0.542	1204.67	4091.18	2823.58	3391.39	11510.82
0.85	0.105	0.537	1072.8	4479.51	2513.66	3392.91	11458.88
1	0.045	0.535	1017.19	4613.06	2296.79	3395.98	11323.01

Table 3.5: Influence of different values of θ_1 under centralized control

θ_1	Centralized control						
	λ_1	λ_2	$\prod_1$	$\prod_2$	$\prod_{s1}$	$\prod_{s2}$	$\prod_{ch}$
0.4	0.177	0.189	4238.53	3241.57	2815.67	2334.68	12630.45
0.55	0.177	0.189	4013.86	3466.24	2815.67	2334.68	12630.45
0.7	0.177	0.189	3789.2	3690.9	2815.67	2334.68	12630.45
0.85	0.177	0.189	3564.53	3915.57	2815.67	2334.68	12630.45
1	0.177	0.189	3339.87	4140.23	2815.67	2334.68	12630.45

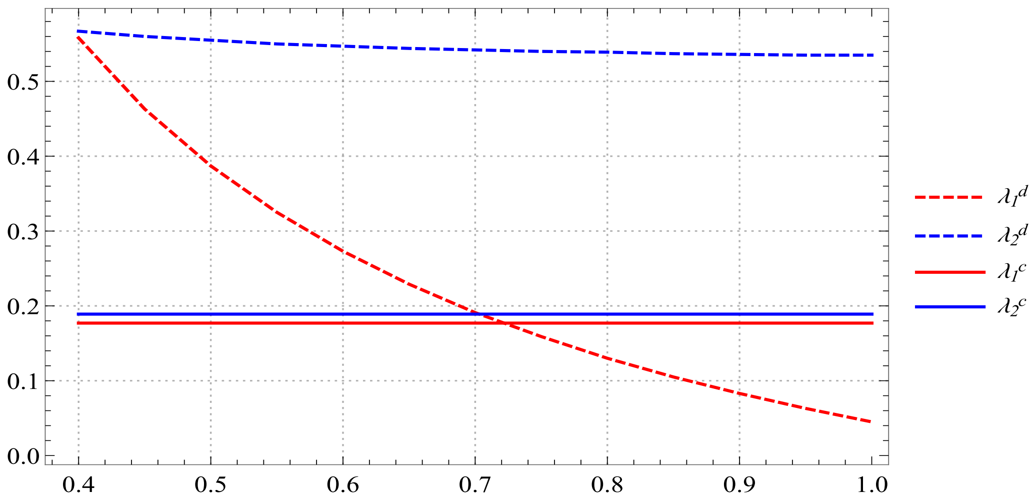

Figure 3.3: Comparison of $\{\lambda_1^d, \lambda_2^d\}$ and $\{\lambda_1^c, \lambda_2^c\}$ under different values of θ_1

Tables 3.4 and 3.5 (above) show the influence of different values of θ_1 under decentralized control and centralized control. The global optimal solution, $\{\lambda_1^c, \lambda_2^c\}$, is not affected by the change of θ_1, which is discussed in ***Proposition 3***. Obviously, λ_1^d decreases with θ_1, meaning it is easy to explain. Since the point proportion increases, the promotion cost of Retailer 1 increases at the same time. A rational retailer will choose a lower point conversion ratio to reduce the cost of promotion.

Table 3.6: Influence of different values of *w* under decentralized control

w	Decentralized control						
	λ_1	λ_2	$\prod_1$	$\prod_2$	$\prod_{s1}$	$\prod_{s2}$	$\prod_{ch}$
5	0.741	0.991	579.49	2605.79	2373.78	2319.44	7878.5
5.5	0.548	0.753	1190.35	2556.39	3061.19	2969.97	9777.9
6	0.387	0.555	1617.61	2482.24	3526.62	3394.2	11020.67
6.5	0.251	0.387	1903.64	2389.06	3821.29	3644.33	11758.33
7	0.134	0.243	2078.71	2280.92	3981.79	3757.69	12099.11

Table 3.7: Influence of different values of *w* under centralized control

w	Centralized control						
	λ_1	λ_2	$\prod_1$	$\prod_2$	$\prod_{s1}$	$\prod_{s2}$	$\prod_{ch}$
5	0.277	0.29	5670.35	4697.51	1580.35	1309.85	13258.06
5.5	0.222	0.235	4824.58	3999.17	2229.37	1848.18	12901.29
6	0.177	0.189	4088.75	3391.35	2815.67	2334.68	12630.45
6.5	0.139	0.15	3437.5	2853.16	3353.7	2781.32	12425.69
7	0.106	0.117	2852.71	2369.69	3853.83	3196.64	12272.86

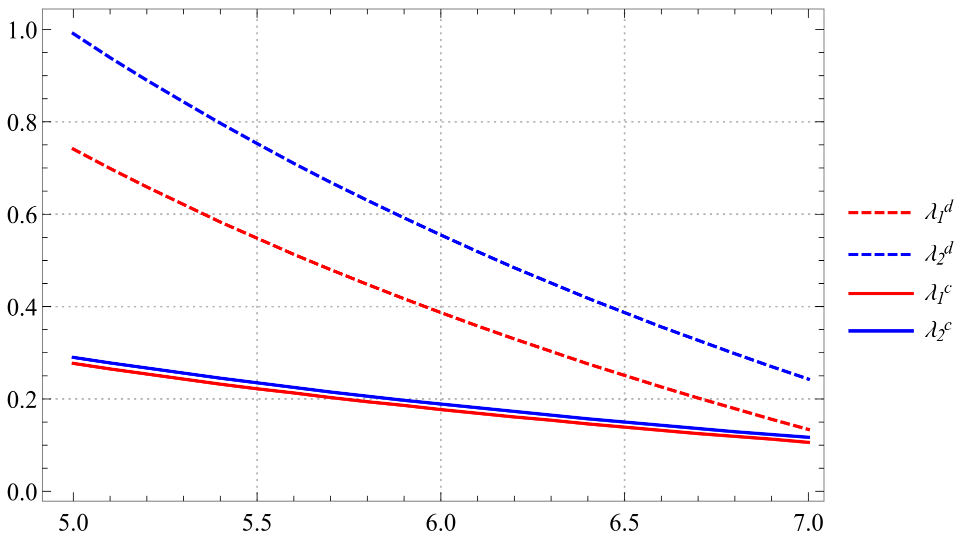

Figure 3.4: Comparison of $\{\lambda_1^d, \lambda_2^d\}$ and $\{\lambda_1^c, \lambda_2^c\}$ under different values of w

If the retail price, p, and the unit production cost, c, remain constant, Table 3.6 and Table 3.7 (above) show the influence of different values of w under decentralized control and centralized control. $\{\lambda_1^d, \lambda_2^d\}$ and $\{\lambda_1^c, \lambda_2^c\}$ both decreases with w. This is because the increase of w will reduce sales revenue for both retailers. Furthermore, the cost of points becomes greater than before, leading to a lower marginal revenue and higher marginal cost under the promotion level. A rational retailer will choose a lower promotion level regardless of the nature of the supply chain.

Table 3.8: Influence of different values of p under decentralized control

p	Decentralized control						
	λ_1	λ_2	$\prod_1$	$\prod_2$	$\prod_{s1}$	$\prod_{s2}$	$\prod_{ch}$
8	0.033	0.118	1732.01	1728.65	2305.69	2149.5	7915.85
9	0.21	0.337	1775.79	2119.06	2916.15	2771.85	9582.85
10	0.387	0.555	1617.61	2482.24	3526.62	3394.2	11020.67
11	0.564	0.773	1257.48	2818.21	4137.09	4016.54	12229.31
12	0.741	0.991	695.39	3126.94	4747.56	4638.89	13208.78

Table 3.9: Influence of different values of *p* under centralized control

p	Centralized control						
	λ_1	λ_2	$\prod_1$	$\prod_2$	$\prod_{s1}$	$\prod_{s2}$	$\prod_{ch}$
8	0.012	0.021	2131.44	1773.45	2240.62	1859.64	8005.14
9	0.095	0.105	3037.96	2523.27	2528.14	2097.16	10186.53
10	0.177	0.189	4088.75	3391.35	2815.67	2334.68	12630.45
11	0.26	0.273	5283.82	4377.67	3103.19	2572.2	15336.88
12	0.343	0.357	6623.16	5482.24	3390.71	2809.72	18305.84

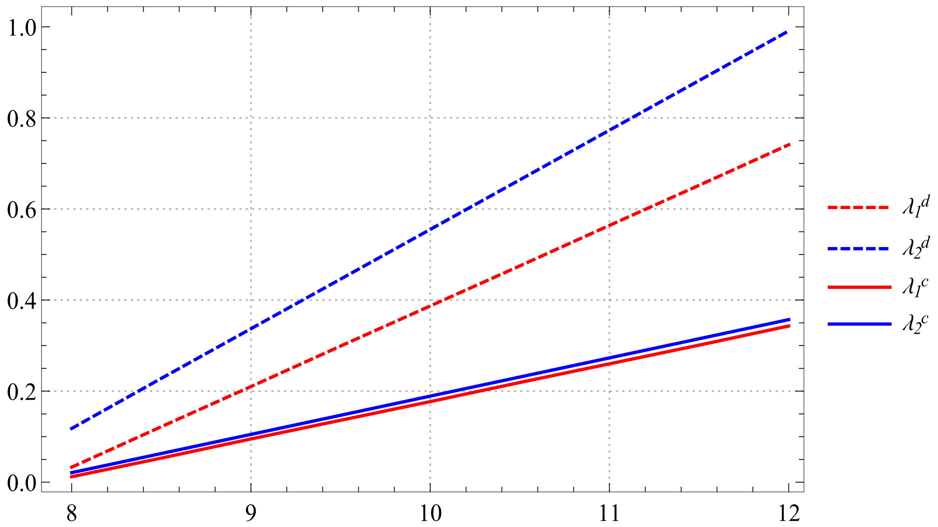

Figure 3.5: Comparison of $\{\lambda_1^d, \lambda_2^d\}$ and $\{\lambda_1^c, \lambda_2^c\}$ under different values of *p*

Table 3.8 and Table 3.9 (above) show the influence of different values of *p* under decentralized control and centralized control. $\{\lambda_1^d, \lambda_2^d\}$ and $\{\lambda_1^c, \lambda_2^c\}$ both increase with *p*. This is because the increase of *p* will increase sales revenue for both retailers, leading to higher marginal revenue under the promotion level. Retailers tend to choose a higher promotion level.

An interesting phenomenon appears when each retailer chooses a higher λ_i in the decentralized supply chain, although its individual profit is lower than the individual profit in the centralized supply chain. In this case, it is the supplier who receives greater profit because of the high demand, and the free-riding phenomenon becomes obvious. In a non-cooperative game, the retailer always makes decisions according to the other retailer's optimal decision. This proves that the Nash equilibrium in a decentralized supply chain is not Pareto optimal and it represents an appearance of the prisoner's dilemma.

Price competition is a typical prisoner's dilemma in many industries. Such a setup occurs when each company takes the others as its competitors and caters only to its own interests. In a price game, as long as the competitor is the opponent, no matter what the opponent's decision is, the company always decides that adopting a low-price strategy will be advantageous, which prompts both parties to adopt a low-price strategy. Examples of this are the competition between Coca-Cola and Pepsi, the price competition between major airlines, and so on. The model of this thesis related to sales promotion is no exception, since the membership point program is a stratagem used to maximize profits despite price competition.

Take the competition model of this thesis as an example. In the mature period of a product, two retailers in the same region compete with each other, and their promotional behaviors influence each other. If one retailer has a high point conversion ratio, its sales revenue will increase greatly. Meanwhile, its cost of promotion will increase, while some of the cost will be undertaken by the other retailer. If a lower ratio is set, the sales revenue will decrease and the cost of promotion will decrease as well.

This means there are two options for these two retailers:

- Reach an agreement with each other to set modest point conversion ratios and reduce the cost of promotion—cooperate.
- Set a high point conversion ratio to gain high sales revenue and make the other retailer bear the brunt of promotion expenditures—defect.

	Retailer 1 (Cooperate)	Retailer 1 (Defect)
Retailer 2 (Cooperate)	Low promotion cost; low promotion cost.	Low sales revenue but higher promotion cost; high sales revenue and higher promotion cost.
Retailer 2 (Defect)	High sales revenue and medium higher promotion cost; low sales revenue but medium higher promotion cost.	High sales revenue and extremely high promotion cost; high sales revenue and extremely high promotion cost.

Figure 3.6: Game between Retailer 1 and Retailer 2

If the two retailers do not trust each other and cannot cooperate with each other, and the "defect" option becomes the dominant strategy, then the two companies will be involved in the war of promotion, and the increase of promotional expenditure will damage the profits of both retailers, which is a prisoner's dilemma.

One strategy to solve the prisoner's dilemma is to construct a collaborative relationship with the other party, such as is achieved through a supply chain coordination contract, an example of a cooperation agreement mentioned in the next chapter.

Chapter 4

Coordination Mechanism of a Supply Chain

Pasternack [15] first proposed the concept of a supply chain contract in 1985. Since then, scholars have conducted significant research into the supply chain contract, which has made great progress in a number of directions. The incentive measures in the supply chain contract allow the risks brought by various uncertainties and the benefits caused by channel coordination to be shared among the members of the supply chain. All members of the supply chain are centralized to make decisions on the total profit of the channel, and the whole supply chain therefore achieves a state of Pareto optimality.

4.1 The Wholesale Price Contract

For the wholesale price contract, the supplier sets the wholesale price, w_i, in Channel *i* (supplier and retailer *i)*, and the rational Retailer *i* adopts the optimal promotion effort level, λ_i, according to w_i, with the goal of maximizing individual profit, which is also the optimal solution for the whole supply chain. In the decentralized supply chain discussed in Chapter 3.2, the wholesale price of supplier w is fixed and the same across all channels. In the wholesale price contract, however, the wholesale price of supplier w_i is a decision variable that tries to make retailers centralized in order to maximize the whole supply chain's profit. In order for the system to reach its optimal state, it must be different across different channels. If the supplier sets the wholesale price of Channel *i* as w_i, then the profit functions of the participants in the supply chain will be as follows

$$\Pi_1^{w}(\lambda_1,\lambda_2)=(p-w_1)(A_1+B_1\lambda_1+b_{21}B_2\lambda_2)-\theta_1 w_1\lambda_1(A_1+B_1\lambda_1+b_{21}B_2\lambda_2)-(1-\theta_2)w_1\lambda_2(A_2+B_2\lambda_2+b_{12}B_1\lambda_1) \quad (4.1)$$

$$\prod_2^w(\lambda_1,\lambda_2)=(p-w_2)(A_2+B_2\lambda_2+b_{12}B_1\lambda_1)-\theta_2 w_2\lambda_2(A_2+B_2\lambda_2+b_{12}B_1\lambda_1)-(1-\theta_1)w_2\lambda_1(A_1+B_1\lambda_1+b_{21}B_2\lambda_2) \quad (4.2)$$

$$\prod_{s1}^w(\lambda_1,\lambda_2)=(w_1-c)(A_1+B_1\lambda_1+b_{21}B_2\lambda_2) \quad (4.3)$$

$$\prod_{s2}^w(\lambda_1,\lambda_2)=(w_2-c)(A_2+B_2\lambda_2+b_{12}B_1\lambda_1) \quad (4.4)$$

Let $\left\{\lambda_1^{\mathrm{cw}},\lambda_2^{\mathrm{cw}}\right\}$, $\left\{\lambda_1^{\mathrm{dw}},\lambda_2^{\mathrm{dw}}\right\}$ be the individual optimal solutions and global optimal solutions under the wholesale price contract.

Proposition 4.

Under usual conditions, $\left\{\lambda_1^{\mathrm{cw}},\lambda_2^{\mathrm{cw}}\right\}$ differs from $\left\{\lambda_1^{\mathrm{dw}},\lambda_2^{\mathrm{dw}}\right\}$.

Proof.

By differentiating $\prod_i^w(\lambda_1,\lambda_2)$and $\prod_{ch}^w(\lambda_1,\lambda_2)$ in regard to λ_1 and λ_2,

$\left\{\lambda_1^{\mathrm{cw}},\lambda_2^{\mathrm{cw}}\right\}$satisfies:

$$\begin{cases}\lambda_1^{cw}=\dfrac{(p-c)(1+b_{12})B_1-(1-\theta_2)w_1\lambda_2^{cw}b_{12}B_1-\theta_1 w_1(A_1+b_{21}B_2\lambda_2^{cw})-\theta_2 w_2\lambda_2^{cw}b_{12}B_1-(1\text{-}\theta_1)w_2(A_1+b_{21}B_2\lambda_2^{cw})}{2\theta_1 w_1 B_1+2(1-\theta_1)w_2 B_1} & (4.5)\\ \lambda_2^{cw}=\dfrac{(p-c)(1+b_{21})B_2-(1-\theta_1)w_2\lambda_1^{cw}b_{21}B_2-\theta_2 w_2(A_2+b_{12}B_1\lambda_1^{cw})-\theta_1 w_1\lambda_1^{cw}b_{21}B_2-(1\text{-}\theta_2)w_1(A_2+b_{12}B_1\lambda_1^{cw})}{2\theta_2 w_2 B_2+2(1-\theta_2)w_1 B_2} & (4.6)\end{cases}$$

$$\begin{cases}\lambda_1^{\mathrm{dw}}=\dfrac{(p-w_1)B_1-(1-\theta_2)w_1\lambda_2^{dw}b_{12}B_1-\theta_1 w_1(A_1+b_{21}B_2\lambda_2^{dw})}{2\theta_1 w_1 B_1} & (4.7)\\ \lambda_2^{\mathrm{dw}}=\dfrac{(p-w_2)B_2-(1-\theta_1)w_2\lambda_1^{dw}b_{21}B_2-\theta_2 w_2(A_2+b_{12}B_1\lambda_1^{dw})}{2\theta_2 w_2 B_2} & (4.8)\end{cases}$$

It is easy to prove that $\left\{\lambda_1^{\mathrm{cw}},\lambda_2^{\mathrm{cw}}\right\}=\left\{\lambda_1^{\mathrm{dw}},\lambda_2^{\mathrm{dw}}\right\}$, when

$$
\begin{cases}
\dfrac{p-w_1}{(p-c)(1+b_{12})-(p-w_1)}=\dfrac{\theta_1 w_1}{\theta_2 w_2} \\
\dfrac{p-w_2}{(p-c)(1+b_{21})-(p-w_2)}=\dfrac{\theta_2 w_2}{\theta_1 w_1} \\
\theta_1+\theta_2=1
\end{cases}
$$

Since the parameters are θ_1 and θ_2, which means the point redemption ratios are not decision variables. We cannot artificially limit the proportion of redeeming points. Therefore, excluding particular circumstances, the conditions to form $\{\lambda_1^{\mathrm{c}},\lambda_2^{\mathrm{c}}\}=\{\lambda_1^{\mathrm{d}},\lambda_2^{\mathrm{d}}\}$ cannot be satisfied. Therefore, the individual optimal solution is often different from the global optimal solution, which results in the wholesale price contract not being able to coordinate the supply chain.

An interesting phenomenon appears here, which is that whether in the basic model or in the wholesale price contract, in order to achieve $\{\lambda_1^{\mathrm{c}},\lambda_2^{\mathrm{c}}\}=\{\lambda_1^{\mathrm{d}},\lambda_2^{\mathrm{d}}\}$, the constraint condition $\theta_1+\theta_2=1$ is needed. Why is this?

In the decentralized supply chain, the impact of the decision variable of Retailer 1, λ_1, by itself is divided into the direct sales profit generated by promotion at Retailer 1 and the cost of promotion expenditure consisting of the point cost from customers' local redemption at Retailer 1, $\theta_1 w D_1(\lambda_1,\lambda_1)$, and the point cost released by the sales growth of Retailer 2 created by the externalities of Retailer 1, $(1-\theta_2)wb_{12}B_1\lambda_1$. In the centralized supply chain, the impact of λ_1 on the whole supply chain is divided into three parts: the direct sales profit generated from promotion at Retailer 1, the sales profit from sales growth at Retailer 2 created by the externalities of Retailer 1, and the cost of promotion expenditure, which consists of point costs converted by Retailer 1, $wD_1(\lambda_1,\lambda_1)$, and the sales profit from the sales growth at Retailer 2 caused by the externalities of Retailer 1, $wb_{12}B_1\lambda_1$. If the individual optimal solution is consistently equal to the global optimal solution, then the sensitivity of the impact of λ_1 on the point cost should in theory be equal to the sensitivity of the impact on the whole supply chain, which means $\frac{\theta_1 w_1 D_1(\lambda_1,\lambda_1)_1}{w_1 D_1(\lambda_1,\lambda_1)_1} = \frac{(1\text{-}\theta_2)w_2 b_{12} B_1 \lambda}{w_2 b_{12} B_1 \lambda_1}$.

Here, we see why condition $\theta_1+\theta_2=1$ is required for maintaining supply chain coordination, and the other constraints are not discussed here.

Since the benefit of the point-sharing policy is its high flexibility in terms of point redemption, if the rules of point redemption change and customers do not have the freedom to select any retailer to redeem points, the PS policy would lose its meaning. We must consider how, even if we make rules of point redemption, the wholesale price contract will not achieve the arbitrary allocation of a supply chain's profit among members. Therefore, the wholesale price contract is not able to coordinate the supply chain.

4.2 A Two-stage Transfer Payment Contract

It is evident that the wholesale price contract cannot achieve the goal of profit maximization of the entire supply chain, which is primarily due to the "double marginalization effect" between suppliers and retailers. Instead of considering the marginal profit of the whole supply chain channel, both sides attempt to maximize their own profits. Therefore, this makes it impossible to achieve the goal of profit maximization for the whole supply chain.[16]

To tackle this, we develop a two-stage transfer payment contract based on the idea of the target rebate contract, which can make the profits of members be linear functions of the overall profit.[14] The fundamental design procedure of the contract is that Channel 1 and Channel 2 share the profit of the supply chain in a proportion, Φ, which means that Channel

1 gains Φ of the total profit and Channel 2 achieves the rest, 1-Φ, of the total profit. Meanwhile, in Channel *i*, Retailer *i* and the supplier should share the profit within the channel in the proportion, α_i. The disharmony between channels mainly comes from flexible point redemption, which causes part of the cost spillover effect. If points converted by Retailer 1 are mostly redeemed at Retailer 2, Retailer 2 has to undertake high promotion costs.

Let $S_1(\lambda_1,\lambda_2)$ be the value of points switching from Channel 1 to Channel 2, and $S_2(\lambda_1,\lambda_2)$ be the value of points switching from Channel 2 to Channel 1. Then, we get

$$S_1(\lambda_1,\lambda_2) = p\lambda_1(1-\theta_1)D_1(\lambda_1,\lambda_2) \quad (4.9)$$
$$S_2(\lambda_1,\lambda_2) = p\lambda_2(1-\theta_2)D_2(\lambda_1,\lambda_2) \quad (4.10)$$

In order to compensate for the spillover of cost, Channel 1 must provide a transfer payment, X, to Channel 2 after the end of one sales quarter.

$$\mathrm{X}(\lambda_1,\lambda_2) = \gamma\left(\frac{S_1(\lambda_1,\lambda_2)}{S_1(\lambda_1,\lambda_2)+S_2(\lambda_1,\lambda_2)} - a\right) \quad (4.11)$$

a represents the minimum value of $\frac{S_1(\lambda_1,\lambda_2)}{S_1(\lambda_1,\lambda_2)+S_2(\lambda_1,\lambda_2)}$ when Retailer 2 receives the positive transfer payment, otherwise it may receive a negative transfer payment, which means Retailer 2 should pay Retailer 1 –X as compensation. γ is related to the strength of the transfer payment, which represents the amount of the transfer payment for each unit.

In Channel *i*, the supplier does not need to pay for the cost of promotion, but shares some of the profit from the promotion efforts by Retailer *i*, which is a typical free riding phenomenon.

Therefore, to compensate for this spillover effect, and to encourage Retailer *i* to increase the point conversion ratio and thereby increase the sales volume of products, the supplier provides a transfer payment, Y_i, to Retailer *i*

$$Y_i(\lambda_1, \lambda_2) = \delta_i(\lambda_i - b_i) \qquad (4.12)$$

b_i represents the minimum value of λ_i when Retailer i receives a positive transfer payment; otherwise, it may receive a negative transfer payment. A negative transfer payment means Retailer i will pay the supplier $-Y_i$ as a fine for not reaching the target value. δ_i relates to the strength of the transfer payment, which represents the amount of the transfer payment for each unit.

The timing of the scenario is as follows:

1. Before the beginning of a sales quarter, the proportion of profit allocation, Φ, between Channel 1 and Channel 2, and the proportion, α_i, shared within the channel should be determined by the retailers and the supplier.
2. The members of the supply chain calculate a, γ, b_i and δ_i, based on Φ and α_i.
3. Retailer i decides λ_i.
4. At the end of the sales quarter, in Channel 1, the supplier pays Retailer 1 $\delta_1(\lambda_1 - b_1)$ and provides $(1-\alpha_1)\gamma(\frac{S_1(\lambda_1,\lambda_2)}{S_1(\lambda_1,\lambda_2)+S_2(\lambda_1,\lambda_2)} - a)$ as the channel transfer payment. Retailer 1 provides the remaining $\alpha_1\gamma(\frac{S_1(\lambda_1,\lambda_2)}{S_1(\lambda_1,\lambda_2)+S_2(\lambda_1,\lambda_2)} - a)$; in Channel 2, Retailer 2 receives $\alpha_2\gamma(\frac{S_1(\lambda_1,\lambda_2)}{S_1(\lambda_1,\lambda_2)+S_2(\lambda_1,\lambda_2)} - a)$ and the supplier receives the remaining transfer payment, $(1-\alpha_2)\gamma(\frac{S_1(\lambda_1,\lambda_2)}{S_1(\lambda_1,\lambda_2)+S_2(\lambda_1,\lambda_2)} - a)$. In addition, the

supplier pays Retailer 2 $\delta_2(\lambda_2 - b_2)$.

Theorem 3.

For any $\Phi \in (0,1)$, if the following conditions are satisfied

$$a = \frac{t_1}{t_1 + t_2} \qquad (4.13)$$

$$\gamma = \frac{(t_1 + t_2)(S_1(\lambda_1 + \lambda_2) + S_1(\lambda_1 + \lambda_2))}{p(1-\theta_1)(1-\theta_2)\lambda_1\lambda_2} \qquad (4.14)$$

wherein

$$t_1 = \lambda_1(1-\theta_1)(\Phi(p - c - w\theta_2\lambda_2) + (1-\Phi)(1-\theta_2)w\lambda_2) \qquad (4.15)$$

$$t_2 = \lambda_2(1-\theta_1)((1-\Phi)(p - c - w\theta_1\lambda_1) + \Phi(1-\theta_1)w\lambda_1) \qquad (4.16)$$

then

$$\Pi_{c1}(\lambda_1, \lambda_2) = \Phi \Pi_{ch}(\lambda_1, \lambda_2), \quad \Pi_{c2}(\lambda_1, \lambda_2) = (1-\Phi)\Pi_{ch}(\lambda_1, \lambda_2)$$

Proof.

Within this transfer payment contract, the profit functions of Channel 1 and Channel 2 are as follows:

$$\Pi_{c1}(\lambda_1, \lambda_2) = (p - c - \theta_1 w\lambda_1)D_1(\lambda_1, \lambda_2) - (1-\theta_2)w\lambda_2 D_2(\lambda_1, \lambda_2) - \gamma\left(\frac{S_1(\lambda_1, \lambda_2)}{S_1(\lambda_1, \lambda_2) + S_2(\lambda_1, \lambda_2)} - a\right) \qquad (4.17)$$

$$\Pi_{c1}(\lambda_1, \lambda_2) = (p - c - \theta_1 w\lambda_1)D_1(\lambda_1, \lambda_2) - (1-\theta_2)w\lambda_2 D_2(\lambda_1, \lambda_2) + \gamma\left(\frac{S_1(\lambda_1, \lambda_2)}{S_1(\lambda_1, \lambda_2) + S_2(\lambda_1, \lambda_2)} - a\right) \qquad (4.18)$$

By inserting

$$t_1 = \lambda_1(1-\theta_1)(\Phi(p - c - w\theta_2\lambda_2) + (1-\Phi)(1-\theta_2)w\lambda_2)$$

$$t_2 = \lambda_2(1-\theta_1)((1-\Phi)(p - c - w\theta_1\lambda_1) + \Phi(1-\theta_1)w\lambda_1)$$

$$S_1(\lambda_1,\lambda_2)=p\lambda_1(1-\theta_1)D_1(\lambda_1,\lambda_2)$$
$$S_2(\lambda_1,\lambda_2)=p\lambda_2(1-\theta_2)D_2(\lambda_1,\lambda_2)$$

into the profit functions, we can obtain

$$\prod_{c1}(\lambda_1,\lambda_2)=\Phi\prod_{ch}(\lambda_1,\lambda_2),\quad \prod_{c2}(\lambda_1,\lambda_2)=(1-\Phi)\prod_{ch}(\lambda_1,\lambda_2)$$

Therefore, for any $\Phi\in(0,1)$, the channels' profits can be maximized only if the overall maximum profit is achieved and if it is more profitable to choose $\{\lambda_1^c,\lambda_2^c\}$.

Theorem 4.

In Channel *i*, if the conditions in ***Theorem 3*** hold, for any $b_i\in(0,1)$, if the following conditions are satisfied:

$$b_i=\frac{[(p-w)-\alpha_i(p-c)]D_i(\lambda_1,\lambda_2)-(1-\alpha_i)(1-\theta_j)w\lambda_jD_j(\lambda_1,\lambda_2)}{(1-\alpha_i)\theta_iwD_i(\lambda_1,\lambda_2)} \qquad (4.19)$$

$$\delta_i=(1-\alpha_i)\theta_iwD_i(\lambda_1,\lambda_2) \qquad (4.20)$$

then

$$\prod_i(\lambda_1,\lambda_2)=\alpha_i\prod_{ci}(\lambda_1,\lambda_2)=\alpha_i\Phi\prod_{ch}(\lambda_1,\lambda_2),$$
$$\prod_{si}(\lambda_1,\lambda_2)=(1-\alpha_i)\prod_{ci}(\lambda_1,\lambda_2)=(1-\alpha_i)\Phi\prod_{ch}(\lambda_1,\lambda_2)$$

Proof.

In Channel *i*, the profit functions of Retailer *i* and the supplier are

$$\prod_{\mathrm{i}}(\lambda_1,\lambda_2)=(p-w-w\theta_i\lambda_i)D_i(\lambda_1,\lambda_2)-(1-\theta_j)w\lambda_jD_j(\lambda_1,\lambda_2)+\delta_i(\lambda_i-b_i)-\alpha_iX(\lambda_1,\lambda_2) \qquad (4.21)$$

$$\prod_{\mathrm{si}}(\lambda_1,\lambda_2)=(w-c)D_i(\lambda_1,\lambda_2)-\delta_i(\lambda_i-b_i)-(1-\alpha_i)X(\lambda_1,\lambda_2) \qquad (4.22)$$

By inserting b_i, δ_i into the profit functions, which is equivalent to

$$\Pi_i(\lambda_1,\lambda_2)=\alpha_i \Pi_{ci}(\lambda_1,\lambda_2), \quad \Pi_{si}(\lambda_1,\lambda_2)=(1-\alpha_i)\Pi_{ci}(\lambda_1,\lambda_2).$$

substituting b_i, δ_i into Equation (4.12), we can obtain

$$\begin{aligned} Y_i(\lambda_1,\lambda_2) &= [\alpha_i(p-c)-(p-w)]D_i(\lambda_1,\lambda_2)+(1-\alpha_i)\theta_i w\lambda_j D_i(\lambda_1,\lambda_2) \\ &\quad +(1-\alpha_i)(1-\theta_j)w\lambda_j D_j(\lambda_1,\lambda_2) \qquad (4.23) \end{aligned}$$

This demonstrates that the retailer undertakes part of the promotion cost and shares sales revenue. By maintaining other parameters, the value of the transfer payment is related only to the profit allocation scenario, Φ. This suggests that the more profit that is shared by Retailer *i* within the channel, the greater the value of the transfer payment will be.

Using this contract, the optimal promotion efforts of retailers will achieve the optimal levels of promotion efforts under the centralized system, and the total profit can be distributed arbitrarily among the supplier and the retailers. Following the implementation of the contract, the total profit of the supply chain is equal to the total profit of the centralized system.

The supply chain system can achieve the optimal equilibrium state under the contract mechanism, and the equilibrium is improved by the Pareto optimality setup. This means that in the equilibrium state, the profit of each party must be greater than their retained profits. In this thesis, we are able to define the retained profit of the retailers and the supplier as their maximum profit in the decentralized supply chain. Therefore, the profit of each party after allocation will not be less than the maximum profit under the decentralized mode before coordination.

Chapter 5

Conclusion

5.1 Research Conclusions

Competition among retailers is usually manifested in externalities between their behaviors, and this thesis examines the point-sharing policy within this environment.

In this thesis, we explored the presence of the competitive relationship between two retailers under the point-sharing policy, which has not been extensively studied in previous literature. We further developed a supply chain model consisting of one supplier and two retailers in order to investigate the feasibility and viability of the point-sharing policy. The coordination of the supply chain in which retailers' promotion efforts affect demand was also explored. Furthermore, we designed a contract to coordinate the supply chain in order to optimize the whole system, and we detailed the decision-making behaviors of the members of the supply chain in various situations, so as to improve their performance and the performance of the whole supply chain. The conclusions are as follows:

- We considered how demand relates to a supplier's promotion efforts. When the retailer's demand externalities increase, this can lead to a higher promotion level in a centralized supply chain.
- The pure point-sharing policy we examined demonstrated that it is unable to achieve supply chain coordination.
- Based on the experimental results we obtained, we saw an appearance of the prisoner's

dilemma under decentralized control. Both retailers are self-serving in selecting a high promotion level, which resulted in a lose-lose situation.

- We found that the wholesale price contract could not coordinate the supply chain, due to its inability to make the cost of promotion expenditure be shared by the supplier and the retailers. Nor could it solve the spillover effect.
- We proposed a two-stage transfer payment contract in order to coordinate the supply chain. Under this contract, the optimal promotion efforts of the retailers could reach the levels of optimal promotion efforts under the centralized system, and the total profit could be arbitrarily distributed among the supplier and the retailers. After the implementation of the contract, the total profit of the supply chain would be equal to the total profit of the centralized system.

5.2 Future Work

We can extend our approaches to apply to other problems. There are some limitations to this thesis, which need further research and improvement. The future directions for research are as follows：

- This thesis only considers the effect of promotion efforts on demand, but in the market reality, price has a larger impact on the uncertainty of demand, so we can consider the situation that the retail price is also a decision variable.
- This thesis assumes that the supplier has sufficient supply capacity, which ignores the potential effects of the supplier's inventory level. We should therefore consider the impact of the supplier's inventory, such as in cases where the supplier no longer has sufficient supply capacity and the supplier's inventory level thus affects the performance of the supply chain. At this time, in the coordination mechanism, the supplier's decision on inventory, as well as the retailer's level of promotion efforts, should be considered.
- This thesis assumes the information is complete and perfect. However, in the actual market, suppliers and retailers have their own private information for improving the

efficiency of their profits. If we consider the market environment with asymmetric information, it will be more accurate and relevant to the actual situation and have more guiding significance for the development of the supply chain.

With continuous development of the PS policy, the issues to be considered will be more comprehensive, and new problems will arise. The research on the coordination and optimization of the supply chain will be more extensive and in-depth. Furthermore, to improve the PS model of the supply chain in practice is also an important direction of further research.

Bibliography

[1] Chiu, C. H., Choi, T. M., & Li, X. (2011). Supply chain coordination with risk sensitive retailer under target sales rebate. *Automatica*, *47*(8), 1617-1625.

[2] Kim, B. D., Shi, M., & Srinivasan, K. (2001). Reward programs and tacit collusion. *Marketing Science*, *20*(2), 99-120.

[3] Lola, M. S., Kamar, N. S., & Zainuddin, N. H. Development a New Concept of Redemption Point Model towards Genuine Profit Sharing. *Sosyal Bilimler Araştırma Dergisi*, *4*(2), 164-175.

[4] Cao, Y., Nsakanda, A. L., & Diaby, M. (2012). A stochastic linear programming modelling and solution approach for planning the supply of rewards in loyalty reward programs. *International Journal of Mathematics in Operational Research, 4*(4), 400-421.

[5] Cao, Y., Nsakanda, A. L., & Diaby, M. (2015). Planning the supply of rewards with cooperative promotion considerations in coalition loyalty programmes management. *Journal of the Operational Research Society*, *66*(7), 1140-1154.

[6] Aust, G., &Buscher, U. (2014). Cooperative advertising models in supply chain management: A review. *European Journal of Operational Research*, 2*34*(1), 1-14.

[7] Krishnan, H., Kapuscinski, R., & Butz, D. A. (2004). Coordinating contracts for decentralized supply chains with retailer promotional effort. *Management Science*, *50*(1), 48-63.

[8] Tsay, A. A., & Agrawal, N. (2000). Channel dynamics under price and service competition. *Manufacturing & Service Operations Management*, *2*(4), 372-391.

[9] Xia, Y., & Gilbert, S. M. (2007). Strategic interactions between channel structure and demand enhancing services. *European Journal of Operational Research*, *181*(1), 252-265.

[10] Lian, Z., & Deshmukh, A. (2009). Analysis of supply contracts with quantity flexibility. *European Journal of Operational Research*, *196*(2), 526-533.

[11] Wong, W. K., Qi, J., & Leung, S. Y. S. (2009). Coordinating supply chains with sales rebate contracts and vendor-managed inventory. *International Journal of Production Economics*, *120*(1), 151-161.

[12] Taylor, T. A. (2002). Supply chain coordination under channel rebates with sales effort effects. *Management Science*, *48*(8), 992-1007.

[13] Dorotic, M., Verhoef, P. C., Fok, D., & Bijmolt, T. H. (2014). Reward redemption effects in a loyalty program when customers choose how much and when to redeem. *International Journal of Research in Marketing*, *31*(4), 339-355

[14] Moon, I., Xu, J., Feng, X., & Ruan, X. (2020). Cooperative sales promotion with a point-sharing policy: Advantages and limitations. *Omega*, *94*, 102038.

[15] Pasternack, B. A. (1985). Optimal pricing and return policies for perishable commodities. *Marketing Science*, *4*(2), 166-176.

[16] Corbett, C. J., DeCroix, G. A., & Ha, A. Y. (2005). Optimal shared-savings contracts in supply chains: Linear contracts and double moral hazard. *European Journal of Operational Research*, *163*(3), 653-667.

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