공학석사학위논문

Optimal Battery Swapping and Rebalancing in Electric Micro-mobility Sharing Systems

전기 마이크로 모빌리티 공유 시스템에서의 배터리 교체와 재배치 작업 최적화

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산업공학과

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Abstract

Optimal Battery Swapping and Rebalancing in Electric Micro-mobility Sharing Systems

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In this thesis, we consider a battery swapping and mobility inventory rebalancing problem arising in electric micro-mobility sharing systems. Vehicles are equipped with swappable batteries and they are managed by staffs' visiting each vehicle and changing depleted batteries. With the free-floating property of the system, vehicles can locate anywhere in a service area without designated stations, which increases the difficulty to visit and collect every single vehicle. In order to successfully meet user demand during the daytime, operators have to redistribute the vehicles with the right number in the right place and swap batteries with insufficient levels into fully charged ones overnight. Therefore, it is essential that operators take battery charging(swapping), staff routing, rebalancing problem all together into consideration. We aim to satisfy demand as much as possible and at the same time minimize routing and swapping costs. We formulate this problem in a mixed integer linear programming. Target inventory level for rebalancing, an important parameter used in the system, is suggested by analyzing a stochastic process that incorporates demand changes. Being a special case of vehicle routing problem with pickup and delivery, it shares the difficulty and complexity of VRP in practically large size. So as to give efficient solutions in large size problems, we develop a Cluster-first Route-second heuristic where a set partitioning problem considers inventory imbalances and approximates routing distances. We benchmark our heuristic approach on a pure MLIP formulation. The experimental result confirms that the heuristic is good at decomposing a large problem and gives efficient solutions even in practically large instances.

Keywords: Electric micro-mobility sharing, Swappable battery, Inventory rebalancing, Battery management, Staff routing, Vehicle routing problem, Clustering heuristic

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Chapter 1

Introduction

1.1 Background

Electric micro-mobility sharing has recently gained popularity thanks to its wide accessibility and convenience. It provides on-demand and electricity-based rental personal mobility services mainly with electric bikes and electric scooters. The size of vehicle (here and in what follows named **vehicle** which refers to a two-wheeled vehicle of electric micro-mobility, not a staff vehicle of Vehicle Routing Problem) is relatively small and thus it serves a short distance trip with accessibility to more specific locations on the contrary of car sharing. Micro-mobility sharing systems are categorized into a station-based system and a free-floating system. Unlike bike-sharing systems that are mostly station-based, micro-mobility sharing powered by electricity provides a free-floating rental service without stations. Users pick the nearest vehicle via phone applications and return it anywhere their trip ends. Free-floating electric micro-mobility sharing(FFEM), with no concern of end-locations, gives a rider even higher freedom of mobility and accessibility to the very specified points which cannot be reached by public transportation.

Furthermore, the use of a swappable battery has been introduced mitigating the inefficiency of charging electric vehicles at the depot. Operators run vehicles equipped with replaceable batteries. They are managed by operation staff visiting each vehicle and replacing a depleted battery with a sufficiently charged one. It means that the staff does not have to bring the vehicles back again to the charging center or the depot and charge them plugged in there for a while. They can just top up the battery level wherever a vehicle locates in.

In the free-floating electric micro-mobility sharing systems, the major goal is to respond to fluctuating demands for micro-mobility with sufficient batteries, which directly influences the service level provided to their users. When a user looks for a scooter to ride on, if there does not exist any vehicle around the user then the system immediately fails to meet demand. If there exists a vehicle but with an insufficient level of battery for a trip, it still leads to a demand loss. Namely, ride-availability is ensured in terms of both sufficiently charged batteries and the physical existence of vehicles. Especially when with competitors in the market such as other public transportation and companies, unmet demands by unavailable vehicles instantly become lost and can result in bad user experiences comparing to others. Therefore, it is very important to achieve a high degree of availability under imbalanced vehicle inventory status and demands.

To achieve this goal, two operations are required with staff routing: battery management and vehicle inventory rebalancing. A mixture of free-floating and electric-based nature makes it demanding to maintain the service level high and costly for staff to carry out the needed operations. For rebalancing, the vehicles should be collected to adjust the amount of vehicle supply to match the demands properly. Unlike station-based systems, vehicles are not parked altogether in stations with a designated location but scattered all over a service area with only a few patterns.

Even if there come some patterns of common use by users, such as high accessed areas near subway stations or central spots of a city, vehicle locations are still apart here and there within the patterns. Staffs need to visit every single vehicle point to manage it and vehicle locations are so random that it is not easy for staff to come up with a good route. Naturally, the decision becomes more complex when it comes to the large size of vehicles to manage. Furthermore, contrary to a bike-sharing, it is essential to keep track of the battery level of each vehicle at the same time as unavailability also derives from depleted batteries here. When decisions on redistributing the vehicles are made, deciding whose batteries to swap matters together.

The operational challenges in FFEM have not been well studied yet. This relatively new system is being discussed mainly in terms of social research or user behaviors[8][7]. It lacks studies for mathematical formulations with its operational characteristics taken into account or suggestions for a reasonably good working decision support tool. The problem of distributing a team of staff and reallocating the vehicle inventory is modeled as a Vehicle Routing Problem with Pickup and Delivery(VRPPD). A considerable number of studies on operational planning in electric car-sharing and bike-sharing systems are addressed upon VRPPD[18]. Both static and dynamic schemes have been dealt with. Efficient exact algorithms and heuristics are developed. When it comes to electric car-sharing, the problem of bringing the electric vehicles back to the charging center and recharging them is added. A deterministic planning for rebalancing operations and a stochastic analysis at the charging stations have been profoundly discussed [3], [2], [15]. However, little research has been done on FFEM based on electric and swappable batteries characterized by high positional variability in free-floating systems. To our best knowledge, there is only a stochastic model and analysis of the change in battery levels so far[21]. While it seems to share a huge similarity to the existing vehicle sharing systems, the mixture of the operational considerations still gives the need for a distinct remark. Especially daily operation planning of rebalancing and managing battery for meeting demand, which plays a big role in the system, needs studying.

In practice, for now, one operator usually assigns some adjacent areas to each staff and has the staff conduct operations based on intuition and experiences of them each given target inventory level, the vehicles' battery level, and locations. Another operator would just let the staff collect all the vehicles over the service areas and redistribute the whole again while in fact for some vehicles only swapping batteries without collecting would be enough. Even if staffs are well experienced, this kind of intuition-based or uniformized method causes inconsistent and inefficient operations as the staff cannot consider all the possible routes by themselves.

In this background, we study a static operational planning problem of replacing depleted batteries and rebalancing the inventory of electric vehicles in a free-floating system. We consider not only the number of vehicles but also the sufficiency of battery levels and the need for battery-swapping operations. Also, the decision is made seeking to maximize total profit. We denote the problem as a battery swapping and vehicle inventory rebalancing problem in free-floating electric micro-mobility sharing systems **(FFEM-BSR)**. As the detailed operational process can differ between operators, we set our focus on the most basic process where periodically operations are decided and conducted by moving staff. FFEM-BSR is defined as a vehicle routing problem with pickup and delivery and profitable visits.

Thus, our problem carries out selecting the most profitable visits and operations with a combination of staff routing plans. It is to determine how staff should be routed for efficient and profitable operations of rebalancing and managing batteries when sets of pickup, delivery, and battery-swap are not given.

The aim of this study lies in proposing a novel mathematical formulation by reflecting the characteristics of FFEM and developing a clustering heuristic which suits this specific environment.

Figure 1.1 depicts the situation which the operator would encounter at the decision point of a static FFEM-BSR problem.

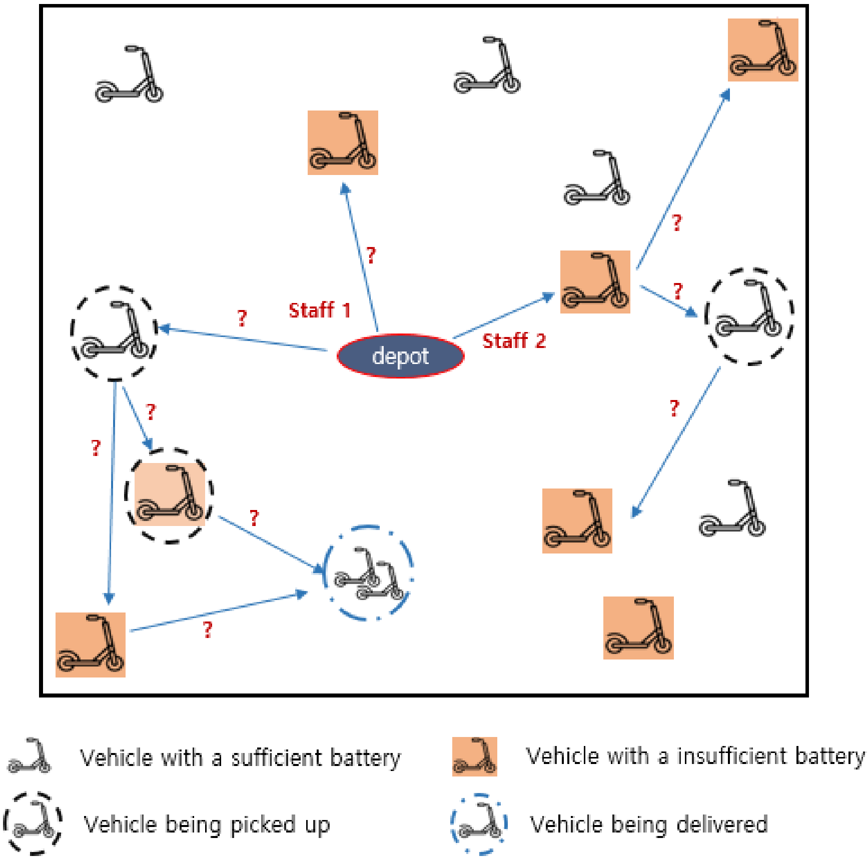

Figure 1.1: An illustration of the problem

1.2 Related literature

Our problem setting is based on mobility sharing systems where users choose to ride on vehicles located at accessible places. In this background, we introduce three streams of related researches: 1) rebalancing problems in bike-sharing systems, 2) charging and rebalancing in free-floating electric vehicle(EV) sharing, and 3) charging operations of electric mobility sharing with swappable batteries. Note that the literature on electric micro-mobility in the last stream is very rare which we want to contribute to. We report one recent study which deals with the same type of system. As mentioned already, most of the literature is based on a Vehicle Routing Problem and variants of it in deterministic and stochastic schemes. For a more comprehensive survey of the works of literature on overall mobility sharing systems, we recommend the interested readers to refer to Laporte et al.[18].

1.2.1 Rebalancing in bike sharing systems

In terms of micro-mobility that carries only one user per vehicle, rebalancing in bike-sharing systems is deeply related to our problem in that many small vehicles can be moved at once by one staff. The study on bike-sharing systems can be divided into two: station-based and free-floating. Note that our focus is on an operator-based method where a fleet of staff vehicles are often sent to move bikes from site to site. Existing bike-sharing systems are mainly station-based and rebalancing models have been constructed to address the imbalance issue. Stations are the analysis units used to perform the imbalance evaluation and the subsequent rebalancing analysis. Given a target inventory for each station, most studies' routing problem is closely related to the One-Commodity Pickup-and-Delivery VRP. Chemla et al.[6] demonstrate a static rebalancing problem with a single staff vehicle and propose a branch-and-cut algorithm for it. Approximation algorithms for the same problem are studied by Benchimol et al.[1]. Erdoğan et al.[9] develop an exact method to calculate the optimal route for the static single-vehicle rebalancing problem. Raviv et al.[24] formulate arc-, time-, sequence-indexed MIP models where the expected system-wide unmet demand and operating costs are minimized within a time limit. Raidl et al.[22] suggest a variable search metaheuristic with the Variable Neighborhood Descent method included to prevent the stations from failing to meet demand. Rainer-Harbach et al.[23] develop multiple construction heuristic approaches where the objective is minimizing the weighted sum of the deviation from target inventory, the number of loading/unloading operations, and total operating time.

Kloimüllner et al.[16], Forma et al.[13], Schuijbroek et al.[26] propose a cluster-first route-second approach. Kloimuller et al.[16] give an exact algorithm utilizing Benders composition but they assume routes alternate between pickup and delivery stations. Forma et al.[13] specifies 3-steps, where the size of a network is reduced by a clustering step and routing between clusters is considered. Schuijbroek et al.[26] combine determining service level requirements for rebalancing and finding optimal vehicle routes. Their clustering problem is to decompose the multi-vehicle problem into several single-vehicle problems and Maximum Spanning Star routing cost approximation is utilized. Those three methods all successfully find a high-quality solution in a reasonable time.

A free-floating system is considered in relatively recent studies and there exists room for more work in addition to few existing studies. This class of problem creates more challenges in rebalancing due to greater spontaneity and flexibility of user demand. Caggiani et al.[5] divides a city into several zones and develop a method to generate Spatio-temporal clusters and forecast the bike use trend in dockless bike-sharing. Pal and Zhang[20] present a mixed integer linear program for solving the multi-vehicle static rebalancing problem. They propose a hybrid nested large neighborhood search with the variable neighborhood descent algorithm which works efficiently on a large scale. Liu et al.[19] take account of the convenience level for operations and develop an enhanced version of chemical reaction optimization(CRO). Stochastic approach has been deployed in Zhai et al.[29] and Fan et al.[10]. Zhai et al.[29] focus on using the Markov stochastic process and linear programming method to optimize the fleet size and rebalancing operation.

When a whole service area is divided into a set of sub-areas and each sub-area is treated as a 'station', we can apply a station-based rebalancing model to solve the imbalance issue. Despite this plausible extension of station-based literature to free-floating systems, routing in the subareas still follows with another complex computation unsolved and other different settings need to be considered. For this reason, we apply the demand modeling and solution approach of Schuijbroek et al.[26] for the station-based to our free-floating micro-mobility systems while adjusting some portions that could not be solved by it. From the perspective of setting demand targets with stochastic processes, our research extends this approach to our problem setting but differs in that we do not have a limit on the maximum number to be returned to each region and we also try to consider an additional layer of routing within a region. Considerations on battery management are included too. Combined with the differences in the existence of stations(docks) and battery components, our problem's complexity is huge since those two additional components enlarge the size of the problem itself and decision layers.

Furthermore, Our model not only finds an optimal route but also determines which vehicle to visit for either pickup or delivery or just battery swapping. Existing VRPPDs and their heuristics usually deal with pre-determined sets of pickup node and delivery nodes, namely the amount of inventory rebalancing is given. Unlike them, our model jointly determines routing, inventory allocations, and charging.

1.2.2 Charging and rebalancing in free-floating electric vehicle(FFEV) sharing

Electric mobility sharing shares the need for vehicle relocation and additional charging operation for electric battery, which differentiates from other existing vehicle sharing. The study on electric car-sharing has gained an academic interest along with the emergence of eco-friendly and economic electric vehicles. Relocation and charging operations for electric cars usually include staff shuttle routing. Gambella et al.[14] propose a time-space-network-based formulation to relocate the vehicles in electric car-sharing systems when demand and battery level considerations. Kypriadis et al.[17] present a minimum walking car repositioning problem in free-floating electric vehicle sharing. Santos et al.[25] consider sequentially deciding on how to route a fleet of shuttles and staff for the pre-determined relocation operations. Folkestad et al.[12] consider decision-making for electric vehicle relocation to the depot and charging jointly. The study only relocates EVs with battery levels below a certain threshold and to the charging stations, not to the other demand points. Umetani et al.[27] consider the use of EVs as battery storages for stabilizing large fluctuations in the power grid through the vehicle-to-grid power system. They develop a linear programming based heuristic algorithm on a time-space network model for charge and discharge scheduling of EVs, and also present an improved two-stage heuristic algorithm to cope with uncertain demands and departure times of EVs.

The previous studies and our thesis share the topic of how to incorporate staff-based battery management considerations into mobility sharing or rebalancing of the vehicles. However, in fact, the process of shuttling staff and the constraint that moving only one car to the depot is allowed per staff generate differences in operational challenges. Ours differ that a staff shuttle and taking the vehicles to the charging depot are not considered. Vehicles are not moved one by one and a predetermined or fixed set of vehicles to be visited is not given. Thus, this thesis gives an electric-'micro-mobility'-specific model and a solution approach.

1.2.3 Charging of electric micro-mobility with swappable batteries

Only a few pieces of literature are for electric micro-mobility while electric car-sharing is mainly discussed. One deterministic planning approach is to find the optimal battery swapping stations for electric scooters in Yan et al.[28]. A solution approach based on an integer network flow problem has been proposed here. But this problem deals with electric scooters use in tourism programs and therefore all the schedules of use and battery usage can be easily expected, which does not precisely represent sharing systems.

The highly relevant literature is Pender et al.[21] which considers a system with the same features but in a stochastic approach: electric scooter sharing with swappable batteries. They modeled the use and return of vehicles, replacement of batteries by staff, and the resulting changing battery levels as a Markovian queuing model. For analysis on a large scale, it took the approach of the Empirical Process and showed the Mean Field Limit and Central Limit Theorem on the Empirical Process of Battery Life. It also proposed an algorithm to use mean and variance values to obtain the number of battery replacement staff required to keep the number of vehicles with low batteries below a certain level. However, the study only carries out the decision of the tactical level to determine the number of staff through an asymptotic solution on large scale. Our study provides an optimal charging operation strategy that works well on a small and large scale. Also, the study does not address the location of individual vehicles that must be considered for the staff visiting them. One important aspect that distinguishes our study from all previous studies is that we integrate the routing of the service staff in inventory allocation for rebalancing/charging. This, combined with that which vehicles are subject to the operations are also determined within the optimization makes the problem extremely complex

To the best of our knowledge, there is no literature on the operational level study of electric micro-mobility sharing. Thus, in our study, we set the focus on the properties and daily operations of free-floating electric micro-mobility sharing systems, which can provide a conceptual basis on the electric micro-mobility literature.

1.3 Motivation and contributions

In the presence of the operational challenges, developing a mathematically optimized and efficient decision support tool for the operators is essential to the successful implementation and expansion of the systems. Indeed, such an optimization model and operational decision could be of great use in terms of academic and practical significance for the following reasons. First, the literature on free-floating electric micro-mobility sharing systems is very few and ones on the operational level do not exist to the best of our knowledge. Although electric micro-mobility sharing has already been operated in many cities for almost five years, studies for the operational level of the problem are a lot limited in the literature. The operation process differs from that of other shared mobility services such as electric cars. There are important yet unanswered operational questions on the systems, which has motivated this work. Moreover, the problem is complex. The simplest operation would require an additional layer of determining whose battery to swap and which to move in what sequences. It would increase the complexity of the problem when it comes to practically large sizes. Thus, we aim to propose a mathematical model for the operations and figure out an efficiently well-performing and easy-to-interpret heuristic tool for FEMM.

Our main contributions can be summarized as follows.

- We propose the first mathematical modeling approach for mobility rebalancing and charging in free-floating electric micro-mobility sharing systems. We set our focus on a widely used type of operational system out of many various types and define it upon a mixed integer linear program, which considers the two operations simultaneously. Through this formulation, we give the first operational insights and analysis in FFEM where routing, inventory allocations, and charging are jointly determined.

- We incorporate the stochasticity of the demand for free-floating micro-mobility sharing systems in vehicle inventory management decisions. A dock-less property of free-floating systems makes it almost impossible to estimate the demand for each vehicle as it is. A service area is divided into regions and grouping vehicles to estimate user demand. We model the stochastic demand and utilize it in the form of deterministic value to be the goal of rebalancing.

- Our cluster-first route-second approach heuristic seeks to relieve the complexity caused by a large set of nodes in a free-floating system. An operational decision for a vehicle includes not only which staff to be assigned but also which action to be done. Since each node corresponds to each vehicle in the systems and the number of vehicles is directly linked to the complexity unlike station-based systems, a good decomposing approach alone is shown to give efficient solutions for overall staffs' operational routing.

1.4 Organization of the thesis

The remainder of this thesis is organized as follows. In Chapter 2, we develop a mathematical formulation. We define the problem on a network setting and present a stochastic demand modeling for free-floating mobility sharing systems to set target inventory for rebalancing operations. A MILP formulation is presented for operational decisions. In Chapter 3, we propose Cluster-first route-second heuristic which sequentially takes a clustering step by a set partitioning problem with a routing distance approximation of minimum spanning tree and solves a routing problem of each staff. In Chapter 4, we test how changes in size and distributional feature affect the performance of our heuristic. In the final Chapter 5, we conclude the contents and limitations of this thesis and give directions for future works.

Chapter 2

Mathematical formulations

In this chapter, we propose a mathematical formulation of FEMM-BSR. The underlying setting and basic assumptions of FEMM-BSR are suggested first to reflect the characteristics of this system. Then modeling demand as a stochastic process to use information from it and MILP formulation follows.

2.1 Basic assumptions and problem description

FEMM is composed of one service area, one depot, several rebalance points to drop-off rebalanced vehicles, and many electric micro-mobility vehicles. In a service area, electric micro vehicles are scattered and their battery levels are known to the operator which directly identifies whether they need to be battery-swapped or not. User demands occur at geographically various points and needed operations are done in advance getting ready to respond to them. Operators make a batch decision based on demand information over the observation periods. Periodically, say, every 4 hours, 8 hours, or overnight, the system determines a set of EVs that should be scheduled for battery swapping and rebalancing with the sequences. Demand information is estimated for the target observation periods ahead and we aim to serve the demand as much as the related constraints allow. After decisions are made on which vehicles to visit and which operations to do on each of them, multiple operation staffs visit every single point of vehicles that requires the operations.

We assume the systems work as follows. Total revenue is calculated based on the number of available vehicles at the end of an operation process. It means revenue is expected to be collected as much as we meet the targets assigned to regions. So long as the identified target level is not met, revenue is earned by one unit as one vehicle becomes available with sufficient batteries. If there already exist vehicles in a perfectly available condition without, then we assume to use them as they are and revenues associated with them are collected too. We set unit revenue r as an average value of revenue that can be achieved by vehicles with sufficient batteries. Namely, it is an expected revenue during the use of average battery use before unavailability.

Each of the service staff departs from the depot and routes doing battery-swap or rebalancing inventories. Which vehicles to be visited and which operations to do on each of them are not given but determined in the model. Pickup and delivery operation are for inventory rebalancing and battery swapping is for battery management. When visited to be picked up or delivered, a vehicle is to be rebalanced. For a vehicle, either of the two can happen.

All the operational cost parameters are calculated based on labor costs for the time required. Battery swapping cost, loading, and unloading cost for rebalancing occur every time the corresponding operation is done. The values are defined as the product of the time required and the staff wage per minute. Manhattan distance is used and obtained in units of 10meters or so, and the routing cost is multiplying the time required for the distance by the staff wage. The capacity and initial inventory of the depot in terms of fully charged swappable batteries are set to be large enough here so that they are not binding. For simplicity, all the vehicles in the system are assumed to circulate in the service area and not to come back to the depot. A staff is capacitated and the capacity is calculated in terms of cost, which is the times allowed multiplied by the salary.

For estimating users' ride/return demand and rebalancing operations, we shall partition the service area into S mutually exclusive nonempty and same-sized sub-regions which we call **'regions'**. We assume one depot for a whole service area and one rebalance node per region, which works as a drop-off point for each region in rebalancing operations. Without it in a free-floating system, it is not plausible to assign and move a certain amount of vehicles picked up in different regions to another region and see how many have been achieved for the goal of rebalancing target.

Given are a set of regions S in the service area and a set of service staffs K. We define a set of vehicle nodes V representing the vehicles and rebalance nodes R representing drop-off points for rebalancing. Furthermore we add the node $\{0\}$ representing the depot and define the set of total nodes $N = \{0\} \cup V \cup R$. As regions are partitioned in one service area, the set of vehicle nodes V is partitioned into S mutually exclusive nonempty subsets according to regions that each of them belongs to, which are denoted by $V(1), \ldots, V(S)$. Now the corresponding network is defined as a directed graph $G = (N, A)$ that consists of a set of nodes N and a set of arcs defined as follows: $A = (a = \{i, j\} | i, j \in N, i \neq j)$ When traversed by a vehicle each arc $a = \{i, j\}$ is associated with the distance parameter d_{ij} which is always greater than zero.

2.2 Demand Modeling and Target Inventory

A target level of how many available vehicles a region needs is required to carry out rebalancing operations. In mobility sharing, the goal of the operation is generally to satisfy a certain proportion of demands over some time. Along with the goal related to service level, we set **Target inventory level** to be the number of mobility that can meet a certain level of successful service. It should be noted that target levels in subsequent operations are achieved in terms of both vehicle inventory and battery level requirements. Usually introducing this concept of 'service level' entails a constraint where the service level should be met by any means. Whether a solution meets the constraint decides feasibility. While it is a reasonable approach when just enough vehicles to meet demands are guaranteed over the systems, this may produce total infeasibility when there might not be enough to cover high demands in the systems. Here, instead of imposing must-meet constraints with service level amounts, exploiting the service level approach only helps to construct reasonable target levels. A penalty term can be used to incur costs on violations in demand meeting. However, since we are not implementing the service level constraint but exploiting the concept, we do not define a distinct penalty term. The penalty is to be estimated and considered in a way that the same portion of revenues as unmet demands is not to be attained.

When establishing such targets, it is necessary to predict and reflect the user's demand. In this model, the targets are given as an input for a static decision. But as we seek to reflect sporadicalness and variability in demand of sharing systems, the underlying demand model for calculating the targets is viewed as a stochastic process.

The inventory of vehicles in the region fluctuates as rental and return continue. Thus, demand is analyzed from the perspective of the net demand process, which consists of the difference between rental and return. Viewing the net demand as a stochastic process, our goal is to figure out the starting number of available vehicles for each region so that service level over the observation period $[0, T]$ in the system remains above some threshold. Since there is no station and no reservation system, analysis units used to perform demand estimation are needed. Therefore, in this paper, a service area is divided into small regions where demand can be estimated. The size of the region is small enough so that assumptions of demand uniformity and similarity of demand characteristics throughout the region hold reasonably well.

We take the demand modeling approach used in Schuijbroek et al.(2017)[26], but in our own way of considering a free-floating system. As it has no limit on the number of pickup and returns of users, there is no capacity in each region and what to control is only the service level for riding.

Users' ride and return demand arrival in each region follows Poisson distribution and intervals between arrivals are exponentially distributed with rates μ_i and λ_i. Users' behavior is assumed to be stationary. The state of a region is defined as the number of vehicles in a region. Ride rates μ_i and return rates λ_i govern the evolution of each region's state upon each instance in which a vehicle is picked up and returned by users. This stationary continuous-time Markov chain is equivalent to a birth and death process. Then the vehicle inventory in a region $i \in S$ is modeled as an $M/M/1$ queuing system, with the number of customers in the queue denoting the vehicle inventory available in the region. Note that many studies on demand modeling in a frame of Markov processes verify that intervals of a ride and return demand can be estimated on exponential distribution. Additionally, Pender et al.(2020)[21] analyzed electric scooter trip data in the U.S. Their analysis presents the distributional assumption works for electric micro-mobility sharing too.

The stationary $M/M/1$ is well-studied and we can get closed-form expressions for the transient probabilities given a starting state. Taking advantage of the probabilities, we compute the target inventory level for each region.

Let $\{S_i(t) : t \geq 0\}$ denote the stochastic process on state space $\{0, 1, \dots\}$ which represents vehicle inventory available in region $i \in S$ at time $t \geq 0$. $p_{s,\sigma}(t)_i = Pr(S_i(t) = \sigma | S_i(0) = s)$ is defined as the transient probability that the inventory in region $i \in S$ ends with $\sigma \in \{0, 1, ...\}$ at time $t \geq 0$ starting with the inventory of $s \in \{0, 1, ...\}$.

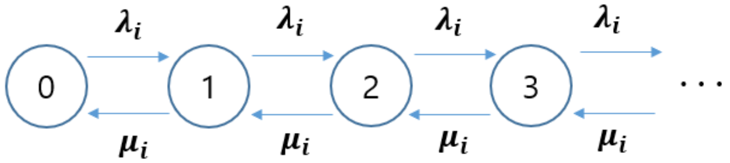

Figure 2.1: Markov chain for the inventory $S_i(t)$ in region $i \in S$ with rate λ_i to return vehicles and with rate μ_i to pick up vehicles.

The transition probabilities leads to solving Kolmogorov backward equations.

$$
\begin{aligned}
p_{s,\sigma}(t)_i' &= \mu_i p_{s,\sigma+1}(t)_i - \lambda_i p_{s,\sigma}(t)_i \\
p_{s,0}(t)_i' &= \mu_i p_{s,1}(t)_i - \lambda_i p_{s,0}(t)_i
\end{aligned}
\tag{2.1}
$$

The concept of *type* 2 service level is implemented, where the fraction of satisfied demands out of the total should be greater than a certain level. As the return is always possible here, we only consider a service level for ride demands. No backlog is assumed.

We calculate the expected ride service level for each region as follows:

$$g_i(s,0,T) = \frac{E[Satisfied\ Pickup\ Demand]}{E[Total\ Pickup\ Demand]} = 1 - \int_0^T (p_{s,0}(t)_i)dt \quad \forall i \in S$$

For an observation period $[0,T]$,

$$Target_i = \min\{s \in \{0,1,...\} : g_i(s,0,T) \geq \beta_i\}$$

, where β_i denotes the ride service level for region $i \in S$ to satisfy. Note that the resulting target inventory from the service level meeting calculations could be theoretically infeasible. Especially over the entire system, the sum of resulting target inventory can be more than the sum of vehicles. In this case, we set the upper bound for a region as round-off of a weighted amount of the whole vehicles according to the proportion of demand :

$$\left\lfloor \frac{\mu_i}{\sum_{i\in S}\mu_i} \times |V| + \frac{1}{2} \right\rfloor$$

.

Putting together the above,

$$Target_i = \min\{s \in \{0,1,..., \left\lfloor \frac{\mu_i}{\sum_{i\in S}\mu_i} \times |V| + \frac{1}{2} \right\rfloor\} : g_i(s,0,T) \geq \beta_i\}$$

With 2.2, 2.1, 2.2, now we can give inputs of target level used as parameters in our model. Note that this value is only capturing a good snapshot of the stochastic demand and our static model does not include the stochastic changes directly.

2.3 Mixed integer linear programming formulation

Given the situation that whenever you do an operation for a vehicle, you can make the system serve more users and earn more revenue, this problem is increasing revenues by operations and at the same time decreasing the corresponding operational costs. Therefore, the trade-off between revenues and costs induced by operational decisions is taken into account. Routing decisions can cost huge in the free-floating systems where vehicle locations are highly variable and widely scattered. It means doing all the needed operations can increase the total operational cost greater than the revenues by them, which is not a desirable result. Thus, we set our objective to maximize total profit instead of minimizing the cost. This frame is similar to prize-collecting in VRP with profits.

To consider cases of failing to attain the target and collect the corresponding revenues, we implicitly modeled the penalty term for unmet quantities against the targets in the objective function. With the goal of maximizing profit, the penalty is estimated in a way that restricts the number of revenues to be only the amount attained out of the $Target$. That is, the unmet amount is subtracted from the whole possible revenues that can occur by $Target$.

All the notations used in the model are given in the table 2.1.

Table 2.1: Notations of sets, Parameters, and Decision Variables

Sets	
Symbol	Definition
K	Set of service staffs $k \in K = \{1, \ldots, K\}$
S	Set of regions $s \in S = \{1, \ldots, S\}$
$V(s)$	Set of vehicles in region $s \in S$
V	Total set of vehicles in the system. $V = \{\sum V(s) : s \in S\}$
R	Set of rebalance nodes in regions. $R = \{R_1, \ldots, R_S\}$
N	Total set of nodes that can be routed in the system. $N = \{0\} \cup V \cup R$

Parameters	
Symbol	Definition
$Target_s$	target demand level for region $s \in S$
r	collected revenue by an operation
$loading$	unit loading cost per vehicle pickup
$unloading$	unit unloading cost per vehicle dropoff
w	unit battery swapping cost
d_{ij}	routing cost from node $i \in N$ to node $j \in N$
a_i	1 if battery level of vehicle $i \in V_s : s \in S$ is sufficient, 0 otherwise
a_s^+	number of vehicles with sufficient batteries in region $s \in S$
Q	flow capacity of a staff vehicle
C	maximum operating cost allowed to a staff

Decision variables	
Symbol	Definition
z_{ijk}	1 if staff k is routed from node $i \in N$ to node $j \in N$, 0 otherwise
$swap_{ik}$	1 if vehicle i' s battery is swapped by staff $k \in K$, 0 otherwise
$pick_{ik}$	1 if vehicle $i \in V$ is picked by staff $k \in K$, 0 otherwise
$drop_{jk}$	dropoff quantity at rebalance node $j \in R$ by staff $k \in K$
q_{ij}	loaded quantity of a staff traversing from node $i \in N$ to node $j \in N$
R_s	auxiliary variable to calculate $\min\{target_s, ending_s\}$ of region $s \in S$

The decision variables consist of binary variables, non-negative integer variables, and non-negative continuous auxiliary variables. Binary variables indicate decisions on routing, visiting, and doing operations. The non-negative integer variables indicate the amount of flow between nodes and vehicles within each region. The auxiliary decision variable R_s is to assign the number of fulfilled demands with which total revenue is calculated.

Before we jump into the full model, we first specify how to calculate R_s. As a first step of formulating the total revenue, we let $ending_s$ indicate the number of available vehicles with sufficient batteries in the regions after all the operations are done. It is defined as follows with $R(s)$ and $V(s)$ representing the rebalance node and vehicle nodes in region s, respectively :

$$ending_s = a_s^+ + \sum_{k \in K} \sum_{i \in V(s)} swap_{ik} + \sum_{k \in K} drop_{R(s),k} - \sum_{k \in K} \sum_{i \in V(s)} pick_{ik} \quad \forall s \in S \tag{2.2}$$

Then now, the amount for R_s is $Target_s - [Target_s – ending_s]^+$, which is same as $\min\{Target_s, ending_s\}$. We define R_s to take the minimum value of the target level and ending values like below.

$$r \times R_s = r \times \min\{Target_s, ending_s\} \quad \forall s \in S \tag{2.3}$$

This R_s term is linearized in the following and multiplied by unit revenue in the objective function.

$$\begin{aligned} R_s &\geq Target_s \quad \forall s \in S \\ R_s &\geq ending_s \quad \forall s \in S \end{aligned} \tag{2.4}$$

Finally, let P be total profit defined by the difference between total revenue and total operational costs. P is used as the objective function in our mixed integer linear program.

$$P = \sum_{s \in S} R_s \times r - \sum_{k \in K} \sum_{i \in V} loading \times pick_{ik} - \sum_{k \in K} \sum_{j \in R} unloading \times drop_{jk} - \sum_{k \in K} \sum_{i \in V} w \times swap_{ik} - \sum_{k \in K} \sum_{i,j \in N} d_{ij} \times z_{ijk} \quad (2.5)$$

Our mixed integer linear program **(FEMM-BSR)** is as follows:

$$\text{maximize} \quad P \tag{FEMM-BSR}$$

$$\text{subject to} \quad \sum_{j \in N} z_{jik} - \sum_{j \in N} z_{ijk} = 0 \qquad \forall i \in N, k \in K \tag{2.6}$$

$$\sum_{k \in K} \sum_{j \in N} z_{ijk} \leq 1 \qquad \forall i \in N \backslash \{0\} \tag{2.7}$$

$$\sum_{j \in N} z_{0jk} = 1 \qquad \forall k \in K \tag{2.8}$$

$$\sum_{j \in N} z_{ijk} = swap_{ik} + pick_{ik} \qquad \forall i \in V, k \in K \tag{2.9}$$

$$\sum_{j \in N} z_{jik} \leq drop_{ik} \qquad \forall i \in R, k \in K \tag{2.10}$$

$$\sum_{i \in B} \sum_{j \in B} z_{ijk} \leq |B| - 1 \qquad B \subseteq N \backslash \{0\}, B \neq \emptyset, \forall k \in K \tag{2.11}$$

$$0 \leq q_{ij} \leq Q \times \sum_{k \in K} z_{ijk} \qquad \forall i, j \in N \tag{2.12}$$

$$\sum_{j \in N} q_{ji} - \sum_{j \in N} q_{ij} = \begin{cases} \sum_{k \in K} drop_{ik} & \forall i \in R \\ -\sum_{k \in K} pick_{ik} & \forall i \in V \end{cases} \tag{2.13}$$

$$a_i \leq 1 - \sum_{k \in K} swap_{ik} \qquad \forall i \in V \tag{2.14}$$

$$\sum_{k \in K} swap_{ik} + \sum_{k \in K} pick_{ik} \leq 1 \qquad \forall i \in V \tag{2.15}$$

$$\sum_{j \in R} drop_{jk} = \sum_{i \in V} pick_{ik} \qquad \forall k \in K \tag{2.16}$$

$$\sum_{i \in V} loading \times pick_{ik} + \sum_{j \in R} unloading \times drop_{jk} + \sum_{i \in V} w \times swap_{ik} + \sum_{i,j \in N} d_{ij} \times z_{ijk} \leq C \qquad \forall k \in K \tag{2.17}$$

$$0 \le drop_{jk} \le Q \qquad \forall i \in R, k \in K \tag{2.18}$$

$$0 \le q_{ij} \le Q \qquad \forall i, j \in N \tag{2.19}$$

$$z_{ijk} \in \{0, 1\} \qquad \forall i, j \in N, k \in K \tag{2.20}$$

$$swap_{ik}, pick_{ik} \in \{0, 1\} \qquad \forall i \in V, k \in K \tag{2.21}$$

The objective function is total profit. The constraints consist of the routing part and the operation decision part. Constraints (2.6)-(2.12) are related to routing with the operations involved and (2.13)-(2.17) make rebalancing and battery swapping decisions. The former is similar to the basic formulation of the three-index vehicle routing problem but doesn't force that every node should be visited. Constraints (2.18)-(2.21) are domains of the variables.

Constraints (2.6) ensure that routes are connected. (2.7) indicate that each node except the depot and arc can be visited and used at most by one staff and once. Constraints (2.8) and (2.9) ensure that every staff departs from the depot and visits a vehicle node $i \in$V either for battery swapping or pickup operation. A rebalance node is to be visited for dropping off vehicles gathered from other regions by Constraints (2.10). Constraints (2.11) eliminate subtours and (2.12) represents that vehicle flow between two nodes can occur only when the corresponding arc is chosen to be used.

Constraints (2.13) update the amounts of vehicle flow on arcs following routing and rebalancing operations. Note that unlike general VRPs with pickup and delivery, our quantity flow change constraints cannot do a role as MTZ constraints and prevent subtours since visiting a node does not always lead to a change in vehicle flow here. Constraints (2.14) make sure that only insufficient batteries are swapped and (2.15) that either swapping batteries without moving or pick-up happens as mentioned in the problem description. Constraints (2.16) balance the amounts of pickups and deliveries by one staff to be equal and (2.17) indicates that each staff should get the operations done within his/her time capacity. We do not include the constraints that restrict the number of batteries to be swapped and vehicles rebalanced in each region as we can easily find this model optimally doing so by itself; if we rebalance or swap more than needed in some regions, then the cost would increase higher than other feasible solutions to compensate lacks occurring somewhere else.

Figure 2.2 illustrates a small example of solution that FEMM-BSR gives.

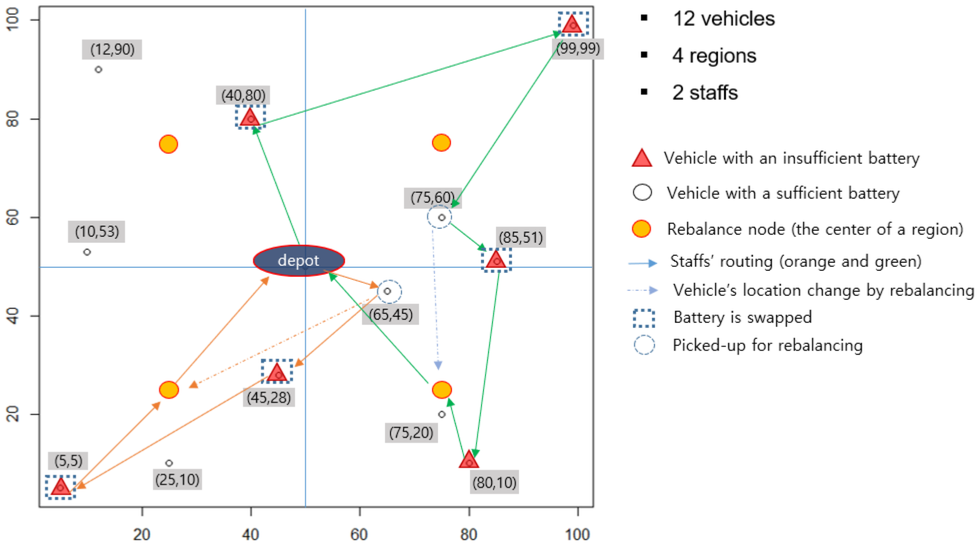

Figure 2.2: An example of a solution FEMM-BSR gives

Chapter 3

Heuristic approach

FEMM-BSR includes a vehicle routing problem and inventory allocation decisions. VRP is basically a combinatorial optimization problem in general solved by an exponential number of computations when it comes to enumerating all feasible solutions. With additional vehicle inventory decisions and logical constraints to the basic VRP, the problem requires heavy computation in a large size.

We have observed that for small instances with $|V| \leq 40$, $|S| \leq 9$ and $|K| \leq 3$, our model always succeeded to solve the problem to the optimality or reach only 0.1% difference from LP best bound. However as the size of instance grew realistically larger, we found them to get intractable. Even with $|V| \geq 350$, $|S| \geq 50$, the problem failed to give one integer solution in an hour. Thus, in this chapter we develop a heuristic decision support tool to reduce the complexity of the problem, resulting in efficient and effective solutions.

3.1 Cluster-first route-second approach

Our cluster-first route-second heuristic is composed of two steps. The first one is that regions are clustered according to considerations on geographic status as well as the inventory of vehicles in regions. Through the clustering phase, we let one staff and multiple regions be assigned to each cluster so that all the operational requirements for the regions are met within a cluster they belong to. An allocation and clustering decision process is formulated as an Assignment Problem using a routing distance approximation. In the next step, operation staff is routed through their assigned clusters with operational decisions made for an individual vehicle therein. After the first stage of partitioning regions into several portions, we get a reduced network and single-vehicle problem for each staff. That is, we bring the original FEMM-BSR again to solve this problem but with one staff and less complexity.

Although this process is built on another integer program, it only deals with the same number of nodes as that of regions in the service area. That is a much smaller size than the original formulation and the size hardly goes too large considering realistic service area in a system with one depot. Indeed, experiments confirm that it successfully solves and decomposes a large size of problems with $|V| \geq 350$, $|S| \geq 50$, in which FEMM-BSR was not able to come up with a solution in a reasonable time. This shows that even if the heuristic is with an integer program that still has complexity in increasing size, it gives a good and applicable solution approach with a clear advantage.

Clustering is implemented mainly based on matching supply and demand for vehicle inventory and this can underestimate operational costs as the visits for the battery-swap count as well. An operational decision on whether to swap an insufficient battery obviously determines the following routes and the system's status. Not to ignore it completely in the clustering phase, we consider battery swapping operations by adding a constraint on the number that batteries can be swapped and.

As [11] has shown that an AP is relatively well solved and working for this kind of problem, though it has IP structure we find it to present reasonably good performance.

3.2 Clustering problem with routing cost approximation

For clustering, not only inventory balancing but routing distance between vehicles should be taken into account as well. However, computing and utilizing all the combinations of distances requires enormously heavy computational efforts almost to impossibility. Having the structure of VRP in the first clustering stage leads to a still complex problem only with reduced size of nodes. So as to develop a reasonably fast- and well- working heuristic, a routing distance approximation is needed.

Our clustering stage is modeled as an assignment problem including routing distance approximations. We incorporated a Minimum Spanning Tree(MST) approximation in a single-commodity flow formulation into our model. We also take visiting costs into account as a part of distance approximation.

3.2.1 Minimum spanning tree approximation

A Minimum Spanning Tree(MST) is widely used as a distance approximation of TSP. MST works as a lower bound of TSP routes and accordingly it can be extended to VRP. In other literature on the cluster-first route-second approach in station-based bike-sharing systems, stations are clustered and routing ends with visiting those stations in each cluster. Their cases can integrate distance approximations with their models directly as the distance approximations represent staffs' routing approximations in the original problem as well. However, ours is for assigning several 'regions' to staffs and within the clusters staffs are required to route not only between regions but also between specific vehicles in each cluster. Routing does not end at the level of between-regions, which requires modifications to exploit the existing heuristic approaches.

In Schuijbroek et al.[26], a *Maximum Spanning Star Approximation* as an upper bound of TSP is implemented. Their objective is to minimize the makespan of routes, the maximum of all routes. However, ours rather seeks the shortest route for each. MST approximation which prevents overestimating the routing cost fits our objective.

Kloimüllner et al.[16] use an MST in their clustering problem. They assume to alternately route between pickup and delivery stations and station nodes are separated into two without the number of inventory considered. Our clustering with MST differs considerably in that the sequence or rule of a route is not assumed and we explicitly consider inventory balancing between regions. Furthermore, all the regions are not restricted to a fixed role in clusters. There could be some regions where only battery swapping operations occur or no operations are done.

Our problem clusters regions, not vehicles directly, and approximation is carried out in regional units. Although we still cannot include all the needed distance approximations between vehicles, we strive not to underestimate the routing distances too low due to only considering moving between regions. The number of vehicles that are subject to battery-swapping and rebalancing is multiplied with their costs and considered in a staff's routing distance.

3.2.2 Clustering problem

The clustering problem is newly defined on an undirected network $G = (V_c, E)$ with a set of vertices V_c and a set of edges E. Given are the number of regions S in the service area and a set of service staffs K which are equivalent to clusters. We let a set of vertices S denote corresponding regional points, which can be described as a representative vertex of a region such as the rebalance point or the centroid point of existing vehicles in a region. The region vertices are subject to routing and distance approximations. V_c includes the depot $\{0\}$ and eventually $V_c = \{0\} \cup S$. Edges are defined to connect the total vertices, $E := (e = (i, j)|i, j \in V_c, i \neq j)$

The results obtained from the second step do not correspond with a feasible solution to the original problem since the travel times between vehicle locations belonging to the same cluster are ignored. The purpose of the first step is utterly to determine which regions would be visited by each staff in the second step of the algorithm. Therefore, the objective values are not directly related to the total problem-solving. Meanwhile, to incorporate the cost structures into the clustering problem leading to a tight connection between the original problem and clustering problem, we take and use the same parameters to our problems. *loading* parameter in FEMM-BSR is used as unit vehicle pickup cost approximation. New notations used here are described additionally.

Specifically, we take s_i to approximate the number of vehicles that are subject to battery swapping. When the Target level is smaller than the vehicle inventory, only the number needed to meet the Target level out of depleted batteries can be swapped. Otherwise, at most the number of vehicles with insufficient batteries can be swapped. Those numbers give the upper bound of battery managing operations for regions and accordingly for clusters.

$$s_i = \begin{cases} \left[Target_i - a_i^+\right]^+ & \text{if } Target_i \leq A_i \\ \\ a_i^- & \text{otherwise} \end{cases}$$

A single commodity flow formulation is used for computing MST to form an efficient clustering problem. The clustering problem allows reasonably fast decomposition of the multi-vehicle problem into smaller single-vehicle problems. Note that for the single-commodity flow formulation of minimum spanning trees, the edges can be used with directions just like arcs when it comes to modeling the flow on edges.

Table 3.1 provides the notation used in this chapter.

Table 3.1: Notations of sets, parameters, and decision Variables for Clustering Problem

Sets	
Symbol	Definition
K	Set of service staffs (clusters) $k \in K = \{1, \ldots, K\}$
S	Set of region $s \in S = \{1, \ldots, S\}$
V_c	Total set of vertices. $V_c = \{0\} \cup S$

Parameters	
Symbol	Definition
r	per unit penalty for imbalance within a cluster
d_{ij}	routing cost between region $i \in S$ and $j \in S$
a_i^+	number of vehicle with sufficient batteries in region $i \in S$
a_i^-	number of vehicle with sufficient batteries in region $i \in S$
A_i	number of vehicles in region $i \in S$
s_i	number of vehicles in region $i \in S$ which needs battery swapping $= \begin{cases} \left[Target_i - a_i^+\right]^+ & \text{if } Target_i \leq A_i \\ a_i^- & \text{otherwise} \end{cases}$
SC	capacity of battery swapping per cluster

Decision Variables	
Symbol	Definition
x_{ik}	1 if vertex $i \in V_c$ is assigned to cluster(staff) $k \in K$, 0 otherwise
y_{ijk}	1 if vertices $i, j \in V_c$ are connected in MST for cluster $k \in K$, 0 otherwise
f_{ijk}	auxiliary flow between vertices $i, j \in V_c$ in MST for cluster $k \in K$
h_k	routing cost approximation by MST for cluster $k \in K$
σ_k	auxiliary surplus of vehicle inventory supply in cluster $k \in K$
ϵ_k	auxiliary surplus of vehicle inventory demand in cluster $k \in K$

In the clustering problem, the objective is to minimize total cost approximation including routing costs and deviations in regional matching for clusters. The latter is made of auxiliary variables σ_k and ϵ_k which denotes the surplus of vehicle inventory supply and demand respectively. Those are introduced not only to achieve better inventory matches by avoiding the penalty but more importantly to mitigate possible infeasibility occurring when the number of available demand and supply within a region cannot perfectly match in equal. They allow some deviations of balancing and make the inventory partitioning always feasible. If the value of σ_k is positive, then there are more vehicle inventory that can be taken to other needy regions in the cluster k than demanded. Those deviations are multiplied by expected unit revenue r, which reflects the financial penalty in the original problem. With those, the objective function results in a set of clusters with the closest distances and a feasible matching of vehicle inventory.

$$
\begin{aligned}
\text{minimize} \quad & \sum_{k \in K} \left(h_k + r \times (\epsilon_k + \sigma_k)\right) && && \text{(CPMST)} \\
\text{subject to} \quad & \sum_{k \in K} x_{ik} = 1 && \forall i \in S && (3.1) \\
& x_{0k} = 1 && \forall k \in K && (3.2) \\
& \sum_{i \in S} x_{ik} \left[Target_i - A_i\right]^+ + \sigma_k && && \\
& \qquad = \epsilon_k + \sum_{i \in S} x_{ik} \left[A_i - Target_i\right]^+ && \forall k \in K && (3.3) \\
& \sum_{i \in S} s_i x_{ik} \leq SC && \forall k \in K && (3.4) \\
& \sum_{i,j \in S} y_{ijk} = \sum_{i \in S} x_{ik} - 1 && \forall k \in K && (3.5) \\
& y_{ijk} \leq x_{ik} && \forall i,j \in S, k \in K && (3.6) \\
& y_{ijk} \leq x_{jk} && \forall i,j \in S, k \in K && (3.7) \\
& \sum_{i \in V_c:\ i<j} f_{ijk} - \sum_{i \in V_c:\ j<i} f_{jik} = x_{jk} && \forall j \in S, k \in K && (3.8) \\
& f_{ijk} \leq Q \times y_{ijk} && \forall i,j \in S, k \in K && (3.9) \\
& h_k \geq \sum_{i,j \in V_c} d_{ij} y_{ijk} + w \sum_{i \in S} s_i x_{ik} && \forall k \in K && \\
& \qquad + loading \times \left(\sum_{i \in S} x_{ik} \left[A_i - Target_i\right]^+\right) && && (3.10) \\
& x_{ik} \in \{0,1\} && \forall i \in V_c, k \in K && (3.11) \\
& y_{ijk} \in \{0,1\} && \forall i,j \in V_c, k \in K && (3.12) \\
& f_{i,j,k} \geq 0 && \forall i,j \in V_c, k \in K && (3.13) \\
& h_k, \sigma_k, \epsilon_k \geq 0 && \forall k \in K && (3.14)
\end{aligned}
$$

The objective function of (CPMST) is the sum of total operational cost approximation and imbalance penalty for clusters. The clustering problem consists of *Assignment* for rebalancing part and *Minimum Spanning Tree* polytope for routing distance approximation.

Constraints (3.1) and (3.2) ensure a set partitioning where each region must be assigned to one cluster(staff) and all clusters include the depot since all the staffs would depart from the depot. Constraints (3.3) balances demand and supply of vehicle inventory within each cluster, preventing match infeasibility here by auxiliary surplus variables This part is only considering the number of vehicles, not jointly with availability by battery sufficiency. Constraints (3.4) represent that we take account of swapping operations within a cluster besides inventory-based assignments.

Constraints (3.5)-(3.10) constitute the Minimum Spanning Tree polytope for routing cost approximation. (3.5) ensures spanning trees for clusters. (3.6) and (3.7) choose a edge for a MST when both sides of the edge is selected in cluster k. Constraints (3.8) represent that a region node must consume 1 flow for cluster k when the region is assigned to cluster k. constraints (3.9) define the flow to be 0 if the edge is not chosen by the MST.

Constraints (3.10) assigns the approximated routing costs by MST and other approximated operational costs for each cluster to the variable h_k. As our MST cannot consider all the distances between vehicles but only between regional points in approximation, we included battery swapping cost and picking up cost which requires pretty much routing. They were calculated based on the upper bound of the number of vehicles that operations are needed. Likewise in FEMM-BSR, we use the same parameter of swapping cost and loading cost for pickup to keep close ties with it. We excluded drop-off cost here since delivery operations occur only at a rebalance point of a region and it is regarded that the routing approximation for it is already done with MST. (3.11)-(3.14) are domains of variables.

3.2.3 Cluster-first Route-second heuristic

We now formally introduce our heuristic as a sequence of decomposition by the clustering problem (CPMST) and single-staff versions of **FEMM-BSR**.

Cluster-first Route-second Heuristic
1. Solve the clustering problem **CPMST** and extract the assignment of regions for each staff as $C_1, \ldots, C_k$
2. For each staff $k \in K$, solve the original problem **FEMM-BSR**with $V = C_k$, the corresponding rebalance nodes, and $K = \{1\}$ to obtain the objective value (P)
3. Sum the Ps obtained from step 2 for all $k \in K$.

Chapter 4

Computational experiments

In this chapter, we run computational experiments with our FEMM-BSR model and heuristic. We report the results comparing our heuristic **(HEUR)** with our FEMM-BSR**(ORI)** to verify effectiveness and efficiency. Our comparison is done varying the size of instances and distributional status of the vehicles in a realistic sense.

4.1 Design of experiment

Input data are generated for the experiments as follows.

- Service area and regions: the service area is created to be square and the number of regions is a square number so that the service area can be divided into equal-sized regions. The horizontal and vertical length of a region is set equal(e.g. 100m).
- The number of vehicles in each region: vehicle inventory status in each region are given randomly imbalanced.
- Vehicle location: when the number of vehicles in a region is given, the coordinate of each vehicle is generated randomly within the region.

- Battery level insufficiency: we set the proportion of vehicles with depleted batteries to be 70% in each region and insufficiency is assigned randomly therein.
- Target inventory level for each region: since we are not testing on real data, we artificially generates target inventory levels randomly within the number of total vehicles in the system.
- Location of the depot and rebalance nodes: the depot locates in the center of the service area and each rebalance node is at the central point of each region.

The imbalance in the distribution is defined in terms of the initial availability status of the regions and target levels for the operation before the operation starts. The distributional features of the initial vehicle status and target levels work as parameters reflecting the system dynamics and therefore affect the overall decisions including inventory rebalancing operations. This imbalance is where the problem begins, so it is the first piece of information you need when deciding how to relocate and recharge a fleet of vehicles. The imbalances in the system usually occur due to a random type of user demand. Sometimes, however, a specific pattern is observed where common usage behaviors prevail. It can be interpreted as information on some regions and at least for these regions and circumstances, we can come up with how our operational efforts vary.

At this point, the performance of the solution may vary depending on how the initial distribution and target distribution are combined. That is, not only the difference in performance according to the size of the system, such as the number of vehicles, but also the dynamics of the system also plays a large role in solving the problem. For example, if the current distribution is very imbalanced, the target is also imbalanced, and the difference between the current and target per region is very large, it is expected that a considerable number of vehicles be selected for a balanced status. When the target is expected to occur evenly and the existing status is equal, it can be easily considered that the operation for rebalancing is small, and it can be reduced to a problem of simply visiting and swapping. Our experiment attempts to check the performance in the situations in which the pattern of imbalance is clearly reflected.

To this end, three scenarios to test instances are generated varying the degree of imbalance in initial vehicle distribution and rebalancing target levels. A low imbalance here is assumed where the difference between regions is less than 20% of the average demand value. High imbalance assumes the difference to be more than 50% of the average demand value.

Table 4.1: Configuration of scenarios

Scenario	Initial distribution	Target distribution	Gap within a region
A	Low imbalance	Low imbalance	Small
B	High imbalance	Low imbalance	Random
C	High imbalance	High imbalance	Large

Scenario A indicates that vehicles are scattered almost evenly over the area and the expected demand for each region is not with a big difference from the status. This is expected to require a less heavy decision as the load on rebalancing is obviously expected less in a highly balanced system. Scenario B refers to where the initial vehicle distribution is highly imbalanced after very random uses and expected demand are similar between regions. In B, for some regions the gap between the two degrees can be high and for some not. The gaps are set randomly here. Scenario C indicates the situation opposite to A, where initial vehicle status is highly imbalanced and a rebalancing goal induces a large gap from the initial one. With the highly imbalanced target levels, a situation with small gaps only makes the same with Scenario A therefore we assume a large amount of change within each region.

A plausible size of instances should be tested. To narrow the exploring range of test instances, we conducted pre-experiments several times and came up with an appropriate combination of the number of vehicles, regions, and staffs from the analysis of the results in addition to the experiments of the related papers such as [20], [13]. While a larger number of vehicles or regions(stations) can be dealt with in the station-based systems, ours consider every single vehicle's condition and thus 300-350 vehicles and 100 divided-regions for one service area with one depot are practically big enough. The number of staff is determined following the size of total regions and capacity.

Table 4.2 summarizes the instances and their features.

Table 4.2: Instances

Instance	Num. of Vehicles	Num. of Regions	Num. of Staffs
1	100	9	3
2	150	16	5
3	250	25	7
4	350	36	9

Next, solutions clearly depend on how the operational parameters are set. They are calculated and given based on the hourly wage of service staff. The travel time matrix was calculated in a unit of 10 meters based on the Manhattan metric. Then routing cost is multiplying the time required for the distance by the staff wage and detour factor. The loading and unloading times were set to be one minute per vehicle. Battery swapping takes 1 min per vehicle. Staff vehicle's capacity was set to 30, which is the capacity of the light trucks used for the rebalancing operations widely in practice. r is set to be the expected revenues that can be earned by a fully charged vehicle: (average usage with a vehicle) $\times$ (average price per ride).

All the experiments were done on an Intel Core 3.10 GHz CPU, 16 GB RAM, and 16-bit Windows 10 using Python API of CPLEX 12.10.0.

4.2 Comparative Analysis

For each scenario, we report the result of FEMM-BSR and our heuristic varying the size of instances. It is also shown that as the distributional features, the scenarios, change the overall complexity and performance also gets affected.

ORI represents original FEMM-BSR and **HEUR** refers to our cluster-first route-second heuristic. The time limit for FEMM-BSR is set to an hour and for heuristic is set to 1 minute on each subproblem. The tolerance is also set to 0.001. If the optimal solution is not found within that time, the best found solution is obtained instead.

The results are reported after one hour execution of **ORI** and shorter execution of **HEUR** as mentioned above. All the numerical results are averaged over 10 runs of each instance.

Instance 1 and Instance 2 are regarded as small-sized with a small number of vehicles. They are solved to the 0.01% gap against the best bound by LP relaxations or sometimes to optimality within the time limit. However, when it comes to the large-sized instances like 3 and 4, all of them are shown to fail to attain the optimality with huge gaps against the LP best bound. Thus, we take the best found solution as **ORI**'s solution, which also can include optimality.

In table 4.3, table 4.4, table 4.5 we report the instance size, the best upper bound found, the best found solution, , the percentage gap between the best bound and the best found solution, and the one between the best found solution of ORI and HEUR.

For comparison, we obtain *LP bound* and optimality gap *Opt GAP%* which is calculated with respect to the best found solution. Optimality gap means the difference between a best found solution and a best LP bound. Therefore, we can obtain it by the following:

$$\frac{LP\ bound\ -\ Best\ found\ solution}{Best\ found\ solution} \times 100\ \%$$

HEUR-ORI% is described in the result table to present how much the heuristic solution outperforms or is close to the best found solution of **ORI**. We can say it is in a similar form to the optimality gap with respect to the heuristic. Since for most of the instances **HEUR** usually outperforms **ORI** within the time limit, the objective value of FEMM-BSR is subtracted from that of the heuristic here instead of the other way around. As our goal is to maximize total profit, the higher the objective value reaches the better the method performs.

HEUR-ORI% is calculated as follows:

$$\frac{Obj_{HEUR} - Obj_{ORI}}{Obj_{ORI}} \times 100\ \%$$

LP-H% is additionally used to estimate the performance with respect to LP bound. Since FEMM-BSR is shown to work not so good when compared to the best LP bound, we cannot promise that the heuristic which is at least better than FEMM-BSR actually gives fine solutions. If the heuristic solution is still far smaller than the best LP bound while it attains higher values than FEMM-BSR, we cannot say that the heuristic is reasonably working well. *LP-H%* is calculated as follows:

$$\frac{LP\ bound - Obj_{HEUR}}{Obj_{HEUR}} \times 100\ \%$$

The average results over the 10 instances for the scenario A, B, and C are summarized in Table 4.3, Table 4.4, Table 4.5 respectively. For simplicity, the best found solution value of **ORI** and objective solution value of **HEUR** are value in thousands.

Table 4.3: Test results for Scenario A

Instance	*ORI*			*HEUR*			
	LP bound	*Best found*	*Opt GAP%*	*Obj.*	*HEUR-ORI%*	*LP-H%*	*Time(s)*
1	2229.41	2228.34	0.048	2229.00	0.03	0.019	2.2
2	3701.31	1697.55	32.360	3700.79	32.33	0.020	6.6
3	6196.35	2536.47	67.191	6195.63	67.15	0.024	132.5
4	8660.27	2970.68	89.567	8659.40	89.51	0.026	643.7

Table 4.4: Test results for Scenario B

Instance	*ORI*			*HEUR*			
	LP bound	*Best found*	*Opt GAP%*	*Obj.*	*HEUR-ORI%*	*LP-H%*	*Time(s)*
1	2215.00	2111.23	6.061	2214.51	6.04	0.022	8.3
2	3660.40	1346.62	50.105	3639.85	173.88	0.570	93.3
3	6160.45	2220.62	78.224	6090.08	175.71	1.167	79.3
4	8601.52	2644.44	106.815	8579.49	227.36	0.257	674.2

Table 4.5: Test results for Scenario C

Instance	*ORI*			*HEUR*			
	LP bound	*Best found*	*Opt GAP%*	*Obj.*	*HEUR-ORI%*	*LP-H%*	*Time(s)*
1	2314.15	1020.32	147.920	1826.59	92.50	21.716	416.0
2	3676.85	1381.66	188.036	2923.59	146.61	20.268	282.4
3	5929.19	2681.43	197.836	5536.70	115.13	6.624	29.5
4	8276.64	2539.33	224.108	7569.95	203.08	8.526	585.0

The results of **ORI** can make some findings on a scenario aspect and an instance size aspect. First of all, when looking at the results based on the Scenario, the complexity is high in the order of A, B, and C as expected when designing the experiment. The results of the optimality gap of **ORI** show that as the degree of imbalance and the required range of work increase, there is a greater difference between the LP bound and the best found solution of FEMM-BSR. Next, for all the scenarios, an increase of the instance size from 1 to 4 results in a greater optimality gap. FEMM-BSR comes up with a good solution for the small-sized instance, instance 1, but for other bigger-sized instances it fails to give one. This means that another efficient method to solve the problem is needed in practical settings with a large number of vehicles and regions.

The results of **HEUR** suggest that our heuristic actually works very well for the problem. It always beat the best found solution of FEMM-BSR and works even better in complex scenarios(B, C) than the less(A). Following the results of **ORI**, *HEUR-ORI%* is supposed to increase in the order of the instance 1 to 4. This present that our heuristic approach is suitable for large size instances where the decomposition process works more efficiently. *LP-H%* confirms that the solution by the heuristic is not only better than the one of FEMM-BSR but also closer to the upper bound of possible values. This verifies that our heuristic approach works effectively as well. In a time-wise, **HEUR** took only 2.2 seconds on average for instance 1 of scenario A while achieving a better solution than **ORI**. Given that **ORI** took 640.9 seconds to get the solution of good quality, our heuristic is shown to be an efficient method to solve the rebalancing and battery swapping problem. The other instances in scenario A required **ORI** to take the full time limit. For all the scenarios and instances, the heuristic solutions are better than those of FEMM-BSR and within at most 21.7% gap from the LP bound of the problem. Meanwhile, the scenario C is expected to occur a heavy load of decisions since it is assumed to have a large amount to be adjusted by rebalancing and battery-swapping. The results also confirm that expectation and relatively higher values of *LP-H%* in scenario C reflect it. However, considering the degree of difficulty that the optimality gap result of **ORI** reveals, the heuristic gives a solution with a twice higher value.

An important finding to note is that in the highly imbalanced situation of the scenario C, *HEUR-ORI%* becomes higher and *LP-H%* becomes smaller for the instance 3 and 4. *HEUR-ORI%* becoming higher is general here as having been observed above but, this direction of change in *LP-H%* doesn't happen in the other scenarios. Given that usually the instance 3 or 4 induces more difficulty to get to the LP bound, the heuristic is working better in the larger-sized instances especially when the degree of rebalancing required is high.

Chapter 5

Conclusion

In this thesis, we have specified the problem of battery managing and inventory rebalancing in free-floating electric micro-mobility sharing systems. With the properties of the operation in an account, we formulate the problem mathematically based on a vehicle routing problem with pickup and delivery in a revenue-collecting way. Operational target levels for vehicle inventory rebalancing are determined by estimation in a stochastic demand process given that demand in the free-floating sharing systems is regarded as stochastic. To alleviate the computational efforts in practical use, a cluster-first route-second heuristic is proposed with a routing distance approximation. Computational results have shown that the heuristic outperforms or performs as good as the original mixed integer linear program in a relatively much faster time. Additionally, the experiments show that the complexity of the problem gets higher when the degree of imbalance, derived from the difference between the initial distribution of vehicle inventory over the area and target levels, comes high. The heuristic generally works reasonably good in large-sized instances and when especially with a high imbalance in the distributional aspect works even better than smaller ones. It is confirmed that the heuristic is good at decomposing a large problem.

This study entails the following limitations. First, no exact algorithms for the problem are not suggested and compared. A considerable number of exact algorithms for the variants of vehicle routing problems are developed. Though our aim is not at algorithmic structure but at proposing an intuitive heuristic approach to the complex problem, studying an exact algorithm for this will help us figure out how complex this problem is and propose better suggestions on the heuristic with a more precise comparison of performances. Moreover, the heuristic we propose is composed of multiple integer programs without any cuts being added to them. This hurts computational performance when it comes to complex instances with a large size of nodes even if the nodes considered become just the region units. We can improve the heuristic by devising effective cuts to add on or algorithms to apply. Another limitation is that our MST approximation does not reflect the distance traveled between all vehicles, but between regions. With the property that vehicles are scattered again in a region, jointly considering two of them is difficult. Therefore, a good approximation should be suggested to handle a lot of nodes in free-floating systems. Instead of the basic minimum spanning tree approach, a perspective of generalized minimum spanning tree which picks one point out of several points in each region can be a possible alternative. The future work will be extending this study to overcome the limitations above and an analysis of the real-world data.

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국문초록

본 연구는 교체형 배터리를 이용하는 전기 마이크로 모빌리티 공유 시스템에서의 배터리 교체 및 차량 재배치를 효율적으로 수행하는 방법을 제시하고자 한다. 수요를 성공적으로 충족시키기 위해선 모빌리티의 공급과 이용자의 수요를 맞춰주기 위한 차량 재고 차원에서의 재배치 작업과 배터리 수준을 유지시켜주는 배터리 관리 차원에서의 교체 작업이 필수적이다. 또한 충전소로 차량을 옮길 필요 없이 바로 교체할 수 있으므로 담당 직원이 산발적으로 위치한 각 모빌리티들을 순회하며 위 작업들을 진행해야 한다. 이동하며 작업하는 비용과 시간이 대부분이기 때문에 이동 순서를 함께 최적화하는 것이 비용 개선에 필수적이다. 따라서 작업 결정과 경로 결정을 동시에 고려하는 충전 및 재배치 모형을 제시한다. 이때 free-floating 모빌리티 공유시스템의 이용 수요를 효과적으로 반영하고자 수요를 stochastic process로 모델링하고 이를 이용하여 재배치 목표 수량을 구한다. 문제의 크기가 큰 경우 효율적으로 본 충전 및 재배치 모형의 좋은 해를 얻기 위한 방법으로, 해당 서비스지역의 각 구역들을 클러스터링하고 그 뒤에 스태프들의 경로와 작업을 결정하는 휴리스틱을 제안한다. 여러 스태프를 순회시키는 복잡한 형태를 클러스터링으로써 작은 크기의 문제들로 분해하여 빠르게 문제를 풀고자 한다. 이를 위해 각 클러스터에는 한 명의 스태프가 배정되고, 한 클러스터 내에서 소속된 구역들이 필요로 하는 작업들을 한 명의 스태프가 모두 진행하도록 구성한다. 최소걸침나무 근사법을 적용한 set partitioning 문제를 풀어 클러스터링을 진행한다. 계산실험 결과, 고안된 휴리스틱은 차량의 수가 많아 크기가 큰 상황에서도 빠른 시간내에 더 좋은 해를 냈다.

주요어: 전기마이크로모빌리티 공유시스템, 재배치, 배터리 교체, 다수차량경로계획

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