공학박사학위논문

Network Design and Route Planning for Integrated Logistics with Drones

드론을 활용한 통합 물류의 네트워크 설계 및 경로 계획

2021 년 2 월

서울대학교 대학원

산업공학과

김 동 욱

Network Design and Route Planning for Integrated Logistics with Drones

드론을 활용한 통합 물류의 네트워크 설계 및 경로 계획

지도교수 문 일 경

이 논문을 공학박사 학위논문으로 제출함

2020 년 10 월

서울대학교 대학원

산업공학과

김 동 욱

김동욱의 공학박사 학위논문을 인준함

2021 년 1 월

위 원 장	홍 유 석	(인)
부위원장	문 일 경	(인)
위 원	장 우 진	(인)
위 원	장 윤 석	(인)
위 원	김 병 수	(인)

Abstract

Network Design and Route Planning for Integrated Logistics with Drones

Dongwook Kim
Department of Industrial Engineering
The Graduate School
Seoul National University

Along with the new trend called the Fourth Industrial Revolution, structural changes are continuously taking place throughout society, and new driving forces, encompassing science, technology, and industry, are drawing attention. In particular, the economic and social changes brought by the rapidly emerging drone technology are the key elements underpinning the Fourth Industrial Revolution. Academia and industry already extensively conduct technical research for the commercial use of drones and achievements in the public service sector. On the other hand, operations research related to drone application is relatively insufficient. To maximize the utility value of drones, an operational plan that takes into account the physical limitations of the drone while fully revealing its strengths should be laid out. Therefore, it is necessary to propose the optimization problem from a new point of view because only limited application is possible for existing problems.

In this dissertation, we carry out research on advanced logistics system with drone operations. Specifically, a new methodology is proposed for the network design and route planning of the logistics system in association with drones. For a logistics network design, the facility location plan must be first preceded. In order to do so, the inherent uncertainty of drone operations is addressed through a stochastic approach. Based on this modelling framework, the locations of facilities and the deployment of drones stationed in each facility are determined. Subsequently, we present an integrated model that simultaneously determines the facility location of the strategic-level decision and the delivery schedules of operational-level decisions. Lastly, we propose a system in which drones work with trucks to perform delivery missions together. Swifter and cost-efficient delivery can be achieved by incorporating the complementary characteristics of two types of vehicles. In summary, the new variants of the optimization problems are proposed for stunning applications of drone technology. Practical solving techniques for the developed models are provided together.

We believe that the results obtained from this dissertation will alleviate the burden of operating drones and serve as the basis for further drone application. This research will be the one of starting points for drones to play a key role in contributing to new paradigm in logistics, not only limited to the delivery service.

Keywords: Facility location problem, Scheduling-location problem,

Vehicle routing problem, Drone, Heuristic

Student Number: 2015-30234

Contents

Abstract i

Contents vi

List of Tables vii

List of Figures viii

Chapter 1 Introduction 1

1.1 Facility location problems 3

1.2 Vehicle routing problem 6

1.3 Research motivations and contributions 9

1.4 Outline of the dissertation 12

Chapter 2 Facility Location Problem with Drones 13

2.1 Introduction 13

2.2 Problem description and mathematical model 16

2.2.1 Chance constraints 18

2.2.2 Mathematical formulation 21

2.2.3 Discussion of the FLP-D 23

2.3 Solution techniques using Benders decomposition and linear programming relaxation . . . 25
2.3.1 Master problem and slave problem . . . 26
2.3.2 Generating Benders cuts . . . 27
2.3.3 Heuristic algorithm for the FLP-D . . . 29
2.3.4 Discussion of the heuristic algorithm . . . 31
2.4 Computational experiments . . . 32
2.4.1 Description of experiments . . . 32
2.4.2 Sensitivity analysis on different parameter . . . 34
2.4.3 Comparison between deterministic approach and stochastic approach . . . 35
2.4.4 Comparison between the FLP-D and heuristic algorithm . 38
2.5 Summary . . . 39

Chapter 3 Scheduling-location Problem with Drones 42
3.1 Introduction . . . 42
3.2 Problem description and mathematical model . . . 46
3.2.1 Mathematical model . . . 47
3.2.2 Discussion of the ScheLoc-D . . . 50
3.3 Pattern-based approach for the ScheLoc-D . . . 51
3.3.1 Set-covering reformulation . . . 51
3.3.2 Generating attractive patterns (columns) . . . 55
3.3.3 Restricted master heuristic . . . 60
3.4 Computational experiments . . . 62

3.4.1 Description of experiments 62

3.4.2 Comparing the RMH to the MILP formulation 64

3.5 Summary 66

Chapter 4 Vehicle Routing Problem with Time Windows and Drones **68**

4.1 Introduction 68

4.2 Problem description and mathematical model 74

4.2.1 Mathematical formulation 77

4.2.2 Discussion of VRPTW-D 82

4.3 Solution approach for the VRPTW-D 85

4.3.1 Finding an initial VRPTW tour 86

4.3.2 Drone assignment algorithm 87

4.3.3 Route combination algorithm 88

4.3.4 Remarks for the TSH 90

4.4 Computational experiments 92

4.4.1 Description of experiments 92

4.4.2 Comparing the TSH to the mathematical model 93

4.4.3 Comparing a coordinated delivery system to truck-only delivery 96

4.4.4 Sensitivity Analysis with the drone features 101

4.5 Summary 103

Chapter 5 Conclusions **105**

Bibliography **108**

국문초록 **126**

감사의 글 **128**

List of Tables

Table 2.2 Problem classes generated from Benchmarks Library . . . 33

Table 2.3 Range of the parameter values 34

Table 2.4 Results of sensitivity analysis for different α values 35

Table 2.5 Results of sensitivity analysis for different λ values 36

Table 2.6 Results of the deterministic and stochastic approaches . . 37

Table 2.7 Comparison between IP approach and the heuristic 41

Table 3.1 Results on small-sized instances: summary 65

Table 3.2 Results on large-sized instances: summary 66

Table 4.1 Results on 10-customer Solomon instances: summary . . . 95

Table 4.2 Results on 15-customer Solomon instances: summary . . . 96

Table 4.3 Cost-efficiency analysis of 10-customer Solomon instances 97

Table 4.4 Cost-efficiency analysis of large-sized Solomon instances . 100

Table 4.5 Results of sensitivity analysis for different drone speeds . 101

Table 4.6 Results of sensitivity analysis for different delivery ranges 102

Table 4.7 Results of sensitivity analysis for different capacities of drones 103

List of Figures

Figure 3.1 Overview of the ScheLoc-D 46

Figure 4.1 Overview and the originality of the VRPTW-D 75

Figure 4.2 Difference between total travel time and completion time as an objective function 83

Figure 4.3 Comparison of the optimal solutions solved with the VRPTW and the VRPTW-D for Instance C201 98

Figure 4.4 Comparison of the optimal solutions solved with the VRPTW and the VRPTW-D for Instance R101 99

Chapter 1

Introduction

The spread of COVID-19 has led to an explosive surge in demands for e-commerce and last mile delivery. Accordingly, last mile delivery, the final stage of the delivery process, has emerged as an important business opportunity for inland transportation. With the recent popularization of e-commerce and the progress of the Fourth Industrial Revolution, the study of logistics has regained momentum. In particular, an emerging technology of unmanned aerial vehicles, also known as drones, has offered new potential for innovations in logistics. Drones are evolving beyond their military origin to become powerful business tools, and incorporating them makes delivery service faster and more efficient. To be precise, drones are becoming an essential component of the last-mile logistics of the next generation. Several pioneer logistics companies, including Amazon, JD.com, UPS, and Alibaba, are actively exploring the possibility of using delivery drones.

Drone-delivery service offers significant advantages as a form of cost-efficient and fast delivery. In addition, the use of drones is environmentally friendly. Since drones are more energy-efficient than other vehicles using diesel or gasoline, greenhouse gas emissions can be reduced. Therefore, the introduction of drones into logistics enhances cost efficiency, sustainability, and competitiveness, and also improves customer service. In particular, when drones are used in islands, mountains, and remote areas where transportation costs are high, the positive effect of drone application is further increased. Along with the applications of drones in commercial logistics field, drones can become very useful as a mode of transportation in disaster management and humanitarian logistics because they do not need preexisting paths to distribute relief goods. While conventional vehicles should follow restricted pathways, drones can move anywhere and everywhere. Therefore, if a natural disaster strikes and roads are damaged, drones can be used as an alternative to serve the destroyed region.

Network design and route planning play a key role in logistics and supply chain management. In order to successfully introduce a new concept, a drone, modeling formulations of these core activities should be redesigned. When integrating drones into the existing system, both the advantages and the limitations of operating drones have to be considered. In this dissertation, new variants of optimization problems are developed; facility location problem with drones (FLP-D), scheduling-location problem with drones (ScheLoc-D), and vehicle routing problem with time windows and drones (VRPTW-D). The FLP-D is developed to determine the locations and transport capacities of drone facilities, based on a stochastic modelling framework. The developed model is applicable to both commercial operation and emergency planning that incorporate drones into logistics while taking into account the uncertain characteristics of drone operation. The ScheLoc-D is developed to determine the optimal locations of drone facilities and delivery schedules of drones. The developed model makes two decisions simultaneously, which are generally classified into strategic (location-allocation) and operational (scheduling) level. The VRPTW-D is developed to determine the cooperative delivery route of the two types of vehicles, trucks and drones. The developed model is applicable to a delivery system in which several drones and trucks work together to provide service to customers within given time windows.

Remarkably many papers inspired this study, but there is not enough space in the margin of the dissertation to write them. Thus, the introduction only summarizes the most basic problems in the field of network design, a facility location problem (FLP), and the most classic optimization problem of routing planning, a vehicle routing problem (VRP). A brief explanation and literature review of the two problems, FLP and VRP, are provided in 1.1 and 1.2, respectively. Previous studies closely related to the problems developed in each chapter will be reviewed in the corresponding chapter. At the end of this chapter, the contributions and outline of the dissertation are presented.

1.1 Facility location problems

The FLP has been applied with great success in various areas. This field has been continually evolving, stimulated by real-world problems. In addition, the theoretical challenges of location decisions lie in various discipline, including industrial engineering. Therefore, research related to location decision is a rich and informative, and gathers a considerable amount of knowledge, both in terms of theoretical properties and practicality. The wide variety and richness of methodologies developed for the variants of the FLP can be found in many survey works, such as those by [16, 92, 56, 105, 106, 115, 73].

Hakimi [54] first introduced the modern concept of facility location problem. A new problem in determining the location of the facility to minimize the sum of the distance to the facility was presented. The author proved that the optimal solution, in which absolute median is the vertex of the graph, always exists. This property is of great significance because it means that many network location problems can be addressed in a discrete setting. This problem was extended to the multi-facility case by Hakimi [55]. The seminal works by [54, 55] became a new beginning point for explosive development of facility location problems.

Manne [81] developed the first mixed-integer linear programming (MILP) formulation for the uncapacitated facility location problem (UFLP), a fundamental problem in this field. The UFLP involves locating an undetermined number of facilities to minimize the sum of total operating cost, including those of the fixed setup costs of the facilities and the variable costs of serving demand from these facilities. The UFLP is also known as the simple facility location problem, where the predetermined facility candidate zones, location and demands of customers, and costs are considered. A more detailed investigation on the UFLP was given by [5, 40, 119, 42, 61, 29, 30, 89]. Natural extensions of the UFLP is the capacitated facility location problem (CFLP) in which the capacity is considered for the maximum demand that can be supplied from each facility. Benders decomposition (Davis and Ray, [35]), dominance rule (Akinc and Khumawala, [3]), Lagrangian relaxation (Christofides and Beasley, [23]), hybrid approach (Van Roy, [124]), and a number of solution techniques are utilized to solve the CFLP efficiently.

Facility location problems can take a variety of forms, depending on the modelling, objectives and constraints. Klincewicz and Luss [70] is one of the fist papers to introduce a multi-commodity facility location model that relaxes the single product assumption. Pirkul and Jayaraman [97] presented a new formulation to the multi-commodity facility location problem that determined the locations and number of production plants and distribution centers, and provided an efficient heuristic solution procedure. An important extension involves increasing the number of echelons incorporated in the problem formulation. Kaufman et al. [65] developed multi-echelon facility location problem that determined the locations of facilities and warehouses simultaneously, and Gao and Robinson [44] proposed an efficient dual-based Branch-and-Bound algorithm for the two-echelon case. Another stream of research focuses on relaxing the single period assumption. In general, facilities serve services over a long period of time. Parameters such as distribution costs and demands can fluctuate due to the long time that design decisions are in effect. Wesolowsky [130] extended the static single facility location problem to a model that considers changes in the location of the facility. Van Roy and Erlenkotter [125] extended the dual-based algorithm of Erlenkotter [42] to handle multiple time periods. A number of researchers also focused on the models and solution procedures for the dynamic facility location problem [130, 21, 60, 123, 43, 127].

Another important extension regards the inclusion of stochastic components in the FLP. [83]. Facility location decisions are costly, and their impact spans a long time horizon. Parameter estimates may also be inaccurate due to poor measurements or to assumptions made in the modeling process. In addition, many of the constants considered in the FLP are likely to change during this long period of time. Regarding this, Research into facility location under uncertainty is exploding. A large number of the approaches that have been proposed for the FLP under uncertainty is well reviewed [79, 116, 31]. The growing interest in uncertainty is due to the increased recognition of the uncertainty faced by most businesses due to natural disasters or epidemics such as Covid-19. These facts has resulted in many other research avenues, including the need for sustainable developments. The study of the FLP-D in Chapter 2 is one way to keep up with this trend, and will contribute to facilitate future research in this area.

1.2 Vehicle routing problem

The VRP is to find a set of vehicle routes to satisfy demands for transportation with the given vehicle fleet. The decisions to be taken concern the assignment of transportation requests to vehicles and the sequencing of them depending on the specific problem setting. Additional practical constraints can be jointly considered for routes to be feasibly executed. The wide variety and richness of methodologies developed for the variants of the VRP can be found in many survey works, such as those by [17, 18, 49, 72, 38, 122, 15]. Because of the wide range of VRP research, the topic of literature intersects with a number of other related topics that are beyond the scope of this dissertation. Therefore, We focus especially on literature for the most classical VRP and the vehicle routing problem with time windows (VPRTW).

Dantzig and Ramser [32], which first introduced the VRP in 1959 as a practical application concerning the dispatching trucks, is the starting point for VRP research. The authors proposed the first mathematical programming formulation for the VRP. A few years later, Clarke and Wright [26] proposed a constructive heuristic, so-called savings heuristic that improved on the algorithmic approach in Dantzig and Ramser [32]. Following these two seminal papers, a huge number of models and algorithms were proposed for the different variants of the VRP. The VRP is an extension of the well-known problem, Traveling salesman problem (TSP). Therefore, initial solution approaches for the VRP were derived from the successful work done for solving the TSP. Despite some great advancements, early approaches based on the TSP's solution algorithm did not satisfactorily address the VRP. In the following decades, the main families of approaches, from Branch-and-Bound schemes to highly sophisticated mathematical techniques, are developed.

It is natural to extend this problem by varying the types of vehicles, which results in the heterogeneous fleet VRP. Another popular extension, the VRPTW, involves time interval, called a time window. Time window constraints arise naturally in problems dealing with cases highly sensitive to time schedules. Two types of time window constraints, hard or soft constraints, is generally considered in the VRPTW. In case of hard time windows, a vehicle that arrives early at a customer must wait until the time window of customer opens. On the other hand, in the case of soft time window, every time window can be violated by paying a penalty cost. According to Lenstra and Rinnooy [75] the VRP belongs to the class of the NP-hard combinatorial optimization problems. Since the VRP is $\mathcal{NP}-$hard, by restriction, the VRPTW is also $\mathcal{NP}-$hard problem. In addition, finding a feasible solution to the VRPTW for a fixed number of vehicles is an $\mathcal{NP}-$complete problem.

The early works on the VRPTW [103, 71] focused on exploring feasible solutions through relatively simple heuristics. Significant advances in exact solution methods have been made for the VRPTW over following decades. Cordeau et al. [28] thoroughly reviewed the early methods that were developed before 2000. Most researchers admit that the representative of the exact algorithm for solving the VRPTW is Branch-and-Price algorithm. Branch-and-Price algorithm is a methodology dealing with Branch-and-Bound procedure in which the linear relaxations are solved by column generation. It is a leading methodology for solving time-constrained vehicle routing problems [8, 36, 80]. Desrochers, Desrosiers, and Solomon [37] first applied Branch-and-Price algorithm to the VRPTW. Branch-and-Cut algorithm is another popular method for solving combinatorial optimization problems [93]. It adds cutting planes into Branch-and-Bound procedure to tighten the linear relaxation. Bard et al. [7] were the first to propose a Branch-and-Cut algorithm for the VRPTW. Concerning the exact methods the research has concentrated on stabilization of the dual variables and smarter Branch-and-Bound techniques. On the other hand, the research for heuristic methods has focused almost solely on metaheuristics. The history of heuristics for the VRPTW is as old as the problem itself, and the field of heuristics is so flourished. Therefore, it makes no sense to provide an exhaustive compilation of them. The lack of a summary of the heuristic certainly does not mean that a heuristic is not important in VRPTW field. Currently, most routing decisions encountered in the industry are solved using heuristics because of ability to handle large instances. In addition, efforts to organize important heuristics are continuing [118, 17, 18, 122].

The latest trend in the VRP field is led by drones. Lots of papers on the routing of drones are published in recent years. The reasons are related to an additional degree of freedom that drones can provide. Logistic is one of the most promising areas to apply drone. In particular, the issue of cooperation between drones and other vehicles is rapidly emerging. Now, the question that arises naturally is how to cooperate drones with conventional vehicles. The study of the VRPTW-D in Chapter 4 is one answer to this question.

1.3 Research motivations and contributions

Drones have opened up new opportunities as useful tools to address a variety of challenges. Public research developers and several major online retailers already started investing a considerable amount of resources for the applications of drones. As drones play a growing role in business operations, questions of planning and optimization arises in both practical and academic field. Much research using drones has already been carried out to answer these new questions. However, existing literature on drones has mostly focused on topics other than the planning or optimization of drone operations (Otto et al. [91]). Previous studies on the technology and simple applicability of drones are insufficient for practical drone operations. In-depth insights into drone operations can be obtained through the development of mathematical models and management scientific analysis.

The planning model of the drone operation system has to consider the characteristics of the drone, which provides an additional degree of freedom and additional constraints, also. Therefore, drone operations pose new methodological challenges, which should be addressed. The aim of this dissertation is to propose new optimization problems arising in the operations planning of drones in logistics. As mentioned before, the most basic research problems for implementing the logistics system, the FLP, the ScheLoc, and the VRPTW are extended by integrating the operation of drones. The principal contributions of the dissertation are summarized as follows:

1. For the FLP-D,
    - A stochastic modelling framework is developed to determine the locations and transport capacities of drone facilities for effectively coping with a fluctuating situation such as disaster.
    - The chance constraint, which represents the probability that a drone returns safely to the drone facility is greater than given target level is used to deal with uncertainty in delivery range.
    - A heuristic and exact algorithm were developed based on Benders decomposition and the structure of the FLP-D, and remarkable effectiveness of Benders decomposition as a computational strategy for disaster management is validated.

2. For the ScheLoc-D,
    - A cost-minimization problem is developed to deal with the needs for simultaneously making decisions in different level.
    - Locations of facilities and delivery schedules of drones deployed at each facility are formulated for the first time.
    - The problem is reformulated to the extended formulation and solved by the restricted master heuristic which provides time-efficient high-quality solution.
    - By using the extended formulation, the concept of the parallel machine scheduling can be utilized based on the structure of the sub-problem.
3. For the VRPTW-D,
    - A new variant of the VRP, VRPTW-D, the most generalized mathematical model that takes into account both time windows and drones, is developed based on the multi-commodity network flow model.
    - While most previous studies have used the way of dealing with physical limitations of drones implicitly included in the structure of decision variables, this study formulates the problem based on two-index formulation.
    - A three-stage savings-based heuristic that utilized the nature of the routing problem is proposed to provide a time-efficient feasible solu-

tion to large-sized problems.

- The benefits of using drones in the last-mile delivery is verified through a sensitivity analysis.

1.4 Outline of the dissertation

In this dissertation, we consider three optimization problems related to the drone operation in logistics, and introduce the three problems in each chapter. More specifically, we newly developed facility location problem with drones (FLP-D), vehicle routing problem with time windows and drones (VRPTW-D), and scheduling-location problem with drones (ScheLoc-D). In Chapter 2, we provided a stochastic design framework of ways to deploy drones economically to serve a disaster-affected region with uncertain drone flight distance. The developed model is used to determine the optimal locations for the drone facilities and the capacity, which is the number of drones deployed from each facility. In Chapter 4, we proposed an advanced delivery system that provides delivery services to customers within a given time window in collaboration with drones and trucks. In detail, a drone launches from a truck on a mission, delivers itself to a customer, and returns to the truck or the depot. In Chapter 3, we proposed a new methodology for integrating operational planning decision (scheduling) with strategic planning decision (location-allocation). Specifically, new logistics system in which simple facilities equipped with drones are installed to provide direct delivery services is formulated. Finally, Chapter 5 concludes the dissertation.

Chapter 2

Facility Location Problem with Drones

2.1 Introduction

In addition to commercial use, logistics activities constitute key part of the disaster management. Various disasters do a great deal of damage and destroy tangible assets such as buildings and equipment. To minimize damage and encourage fast recovery, outstanding response-facility location models, involving the location and selection of distribution centers, warehouses, shelters, medical centers, and other facilities, are essential. An emerging approach is the use of drone facilities in a disaster area. A drone facility is an infrastructure where drones can be docked, recharged, and restocked before flying out again for another delivery. Compared to terrestrial base stations, a drone facility, such as a drone dock, can be easily established to provide on-demand coverage for demand points. By quickly installing drone facilities where necessary, logistics operations in disaster relief can be effectively carried out while supporting the existing damaged infrastructure. In this chapter, we presented a drone facility location problem that determines the locations, numbers, and transport capacities of drone facilities.

For this study, we were concerned with battery capacity, which is a key element of drone operation research. The energy consumption of drones is heavily influenced by payload, weather, and other environmental conditions. Because of the fluctuation in battery consumption, the maximum flight distance of a drone also inevitably varies. These characteristics hinder the direct usage of the existing literature, a new optimization model has to be designed. Therefore, we proposed a method to utilize drones efficiently while considering this uncertainty.

A great many methodologies have been studied to manage the uncertainty. Rosenhead et al. [107] specifically divide decision-making under uncertainty into three categories: certainty; risk; and uncertainty. In certainty situations, all parameters are deterministic and known, whereas risk and uncertainty situations both involve randomness. In risk situations, uncertain parameters are known and governed by probability distributions. In uncertainty situations, parameters are uncertain, and furthermore, no information about probabilities is known. A stochastic optimization problems is applied to the problems in risk situations to optimize the expected value of some objective functions. Problems under uncertainty are usually solved by robust optimization approaches to optimize the worst-case performance of the system. Since this chapter deals with the uncertainty inherent in supply, not demand, which is an exogenous variable, it is reasonable to assume a risky situation. This is because technologies that have not been sufficiently analyzed for performance cannot be commercialized. For classification, Rosenhead et al. [107] represents uncertainty in two terms, but in this chapter only the general term “uncertainty” was used. Previous studies dealing with the FLP considering uncertainty from the perspective of stochastic optimization are summarized as follows.

Meng and Shia [84] formulated a new set covering model based on customer-determined stochastic critical distance. Pereira and Averbakh [96] studied the robust set covering problem featuring uncertain costs. They developed three exact algorithms to find a robust deviation solution, two of which were based on Benders decomposition. Paul and MacDonald [95] developed a stochastic optimization model to determine the locations and capacities of distribution centers such that losses were minimized in the event of a disaster. Data from an earthquake-related disaster in the Northridge region of California were used to validate the model. Tayal et al. [120] formulated a new sustainable stochastic dynamic facility layout problem and optimized the material handling and rearrangement costs using various meta-heuristic techniques. Grass et al. [50] proposed an accelerated L-shaped method to solve a realistic large-scale two-stage stochastic problem. They developed a realistic large-scale case study for the hurricane-prone south-eastern coast of the United States. Research by [74, 132, 66, 82] also proposed set covering location models that accounted for uncertainty and developed efficient solution algorithms in recent years.

As can be seen from the literature review, stochastic facility location research has been actively conducted. However, few studies on integrating drones into a facility location problem have been published. Unlike that of previous studies, the developed model presented in this paper was used to consider the location of drone facilities for the efficient operation of drones, which is expected to be a new transportation mode and catalyst for innovation in disaster management and logistics. To the best of our knowledge, no attempts have been made to determine the location of drone facilities for which the stochastic flight distance of the drones was addressed. In addition, the FLP-D was formulated as a set covering model in which the allocation of drones and the probability that a drone returns to a facility were considered. These practical constraints narrow the gap between the model and practice.

The remainder of this study was organized as follows: Section 2.2 proposes the mathematical formulation of a drone facility location problem with stochastic coverage. Section 2.3 presents the development of an efficient heuristic algorithm using Benders decomposition. Numerical experiments were reported in Section 2.4, and summary of this chapter is offered in Section 2.5.

2.2 Problem description and mathematical model

This section presents a detailed description of the FLP-D. When deciding where to locate facilities, it is often considered that only demand points located within a certain distance from the nearest facility receive the service. The problems that take into account the case is called, *covering location problem*. In the covering-based approach, the actual distance is binarized, and it is said that the demand point is covered if it is in the coverage distance from the nearest facility. Covering location problems generally make the decision using given candidates with predefined positions for facility locations. Because every location of the facility is already known, the constraint on the coverage radius is easily considered by checking the binary feasibility of each facility and demand point pair. A broad literature exists on covering location problem. From seminal works [12, 55, 121, 25, 34], to recent paper [77, 27, 94, 76], numerous covering location model can be found in the literature. The FLP-D. is also formulated using covering-based approach to determine the locations of facilities.

As mentioned before, one of the critical factors that should be considered when operating drones is battery capacity, which can translate to traveling time. Payload, flying altitude, and weather circumstances, such as typhoons, have a great impact on the fuel consumption of a drone. In other words, expected flight distance of a drone is not deterministic. In this study, we examined the effects of these uncertainties on disaster management and applied the stochastic optimization.

Uncertainty in travel time was addressed by Goldberg et al. [48], who analyzed the mean and standard deviation of travel time using data from the Tucson Emergency Medical Services system. Possible traveling time leads to the maximum flight distance. So, the maximum flying duration of each drone is stochastic in this model. Because of the randomness of the maximum flight distance of drones, customer demand covered by a drone facility varies. This variability of coverage requires more difficult decision making than the previous set-covering location models Therefore, we studied an extended version of the set-covering location model by introducing the stochastic flight distance of drones.

The objective of the FLP-D is to find the optimal locations and transport capacities of facilities according to the minimization of total relevant costs. The sets of candidate locations, distance between them, and demands are assumed to be known. The unit of demand was defined as the quantity of goods that can be transported by one drone delivery. Regardless of delivery distance, a drone can transport supplies only once per period, due to the need for maintenance and battery charging. Therefore, 50 drones are required for delivery to meet 50 demands in a particular location at a specific time. Generally, the facility location problem yields a decision outcome that has long-term effects, so the parameters of the system, such as demand points, operating and distribution costs, and environmental factors, may vary over time. Also, facility additions can be made at different times. In disaster situations, the parameters are particularly subject to variability; that is, decision of the location and the timing for building a facility becomes more critical. Therefore, FLP-D decides the operational timing and location for the drone facilities and the process for allocating customers to these drone facilities. The transport capacity of the drone facility, which is the number of drones that should be deployed, is determined simultaneously.

2.2.1 Chance constraints

A *chance constraint*, also known as a *probabilistic constraint*, is necessary to manage the stochasticity of the proposed model. Chance-constrained programming usually involves a certain constraint that guarantees the probability of maintaining a state above a target. A chance constraint was originated by Charnes and Cooper [22] to deal with the tractability of temporal planning in which controllable uncertainty is considered. Miller and Wagner [85] analyzed the mathematical properties of chance constraints which considers the joint probability of a multivariate random event. Carbone [19] used chance constraints to cover situations in which the demand at each node is a random variable. The chance constraint in the FLP-D involves the probability that a drone returns safely to the drone facility should be guaranteed. The constraints can be represented as follows:

$$Prob(\tilde{d} \geq 2d_{ij}) \geq \alpha \tag{2.1}$$

where $\tilde{d}$ is a random variable that represents the distance traveled by a drone and α is the objective probability that a drone returns to a facility. In this study, the probability that a drone returns to the drone facility is considered instead of service level. If we had considered service level as a constraint, the drone delivery would be determined from the customer standpoint. So the drone may not return to the drone facility under a service level constraint. This failure to return causes disruption in the next delivery because the number of drones deployed in each drone facility is minimized. Therefore, the probability that a drone returns safely is a more important consideration than service level.

As an application of the chance constraints, the exponential effect of distance was applied. The exponential effect has been widely used in logistics because it is convenient to implement. Beckmann [11] addressed the effect when the market share of goods or services decreases exponentially as the distance between supplier and customers increases. Berman et al. [13] suggested the exponential effect as an alternative function for the coverage level. The exponential effect is partially covered when a demand node is between the lower and upper limits of the critical distance within the level of coverage; this position reflects a decreasing function of distance between a node and the closest facility. In our study, the method for dealing with the exponential effect differed from that of other studies. Our model captures the exponential effect by assuming that the flight distance of drones are exponentially distributed. This assumption leads to stochastic coverage of a facility. The probability density function and the cumulative distribution function of $\tilde{d}$, respectively, are shown as follows:

$$f(d) = \begin{cases} \lambda e^{-\lambda d}, & d \geq 0 \\ 0, & d < 0 \end{cases}$$

$$F(d) = \begin{cases} 1 - e^{\lambda d}, & d \geq 0 \\ 0, & d < 0 \end{cases}$$

Constraint (2.1) transformed to a tractable constraint using exponential distribution. Remark 2.1 demonstrates the way we dealt with chance constraints under the assumption of the flight distance of drones.

Remark 2.1 (Reformulation of chance constraints)**.** *Constraints* (2.1) *can be transformed into linear deterministic inequalities when random variable* $\tilde{d}$ *follows an exponential distribution.*

$$Prob(\tilde{d} \geq 2d_{ij}) \geq \alpha \quad \Longrightarrow 1 - F(2d_{ij}) \geq \alpha \quad \Longrightarrow e^{-2\lambda d_{ij}} \geq \alpha$$

$$\therefore \quad Prob(\tilde{d} \geq 2d_{ij}) \times X_{it} \geq \alpha \times Y_{ijt} \Longrightarrow e^{-2\lambda d_{ij}} \times X_{it} \geq \alpha \times Y_{ijt}$$

2.2.2 Mathematical formulation

In this section, all the used sets, parameters, and decision variables are presented and discussed. The developed drone facility location problem involves three given sets: customers, potential locations of facilities to serve customer demands, and time periods. Four decision variables are used to construct the developed mathematical model. The sets, parameters, and decision variables are defined to develop the mathematical model as follows:

Set

- I set of candidate locations at which facilities can be sited.
- J set of customer locations
- T set of time buckets over the planning horizon

Parameters

- s_{jt} demand of customer zone $j \in J$ in time $t \in T$
- S maximum value of demands (i.e. $\max_{j \in J, t \in T} s_{jt}$)
- d_{ij} distance between candidate location $i \in I$ and customer zone $j \in J$
- f_i opening cost of a drone facility at location $i \in I$
- o_i operating cost of a drone facility at location $i \in I$
- c_i operation and maintenance cost of a drone at location $i \in I$
- n maximum number of drones that a drone facility can operate in a unit time
- N total number of drones that can be operated in a unit time
- α objective probability that a drone returns to a facility
- λ parameter of the flight distance distribution

Decision variables

$$X_{it} = \begin{cases} 1, & \text{if a drone facility is operated at site } i \in I \text{ in time } t \in T \\ 0, & \text{otherwise} \end{cases}$$

$$Y_{ijt} = \begin{cases} 1, & \text{if customer } j \in J \text{ is covered by a drone facility at site } i \in I \text{ in time } t \\ 0, & \text{otherwise} \end{cases}$$

$$Z_{it} = \begin{cases} 1, & \text{if a drone facility at site } i \in I \text{ is opened in time } t \in T \\ 0, & \text{otherwise} \end{cases}$$

U_{it} number of drones operating at a drone facility at site $i \in I$ in time $t \in T$

Based on sets, parameters, and decision variables defined above, the mathematical model of FLP-D is developed.

$$\min \quad \sum_{i \in I} \sum_{t \in T} (o_i X_{it} + f_i Z_{it} + c_i U_{it}) \tag{2.2}$$

$$\text{s.t.} \quad e^{-2\lambda d_{ij}} X_{it} \geq \alpha Y_{ijt}, \qquad \forall i \in I, \forall j \in J, \forall t \in T \tag{2.3}$$

$$\sum_{i \in I} S \cdot Y_{ijt} \geq s_{jt}, \qquad \forall j \in J, \forall t \in T \tag{2.4}$$

$$X_{i,t+1} - X_{i,t} \leq Z_{i,t}, \qquad \forall i \in I, \forall t \in \{1, ..., |T| - 1\} \tag{2.5}$$

$$U_{it} \geq \sum_{j \in J} s_{jt} Y_{ijt}, \qquad \forall i \in I, \forall t \in T \tag{2.6}$$

$$U_{it} \leq n X_{it}, \qquad \forall i \in I, \forall t \in T \tag{2.7}$$

$$\sum_{i \in I} U_{it} \leq N, \qquad \forall t \in T \tag{2.8}$$

$$X_{it}, Z_{it} \in \mathbb{B}, \qquad \forall i \in I, \forall t \in T \tag{2.9}$$

$$Y_{ijt} \in \mathbb{B}, \qquad \forall i \in I, \forall j \in J, \forall t \in T \tag{2.10}$$

$$U_{it} \in \mathbb{Z}^{+} \qquad \forall i \in I, \forall t \in T \tag{2.11}$$

The objective function (2.2) minimizes the sum of the total relevant costs comprising the costs of opening drone facilities and the operation and maintenance of drones and drone facilities. Constraints (2.3) guarantees that the probability of drone return to a facility is higher than a target level. Constraints (2.4) guarantees that demands are covered. Constraints (2.5) links constraints of the decision for opening a facility. Constraints (2.5) represents the minimum number of drones needed to satisfy demand. Constraints (2.7) ensures that drones can be operated only in operating facilities. Constraints (2.8) indicates that the maximum number of drones can be operated in a unit time. Constraints (2.9), (2.10), and (2.11) demonstrate the binary and integer nature of the decision variables.

2.2.3 Discussion of the FLP-D

Several features of the model warrant some discussion either to indicate practical constraints for which the FLP-D differs from the related models found in the literature or to point out the flexibility that the FLP-D affords. Remarks on the FLP-D are in order.

First, it is necessary to analyze how the chance constraints affect the coverage of a drone facility. In the deterministic approach, the coverage of the drone facility is fixed, whereas in the stochastic approach, there is no clear boundary of coverage. From a practical point of view, the chance constraints do not provide a broader feasible solution region than deterministic constraints. Constraints (2.3) can be replaced by the following Constraints (2.12) under a general deterministic assumption:

$$\frac{1}{\lambda}X_{it} \geq 2d_{ij}Y_{ijt}, \qquad \forall i \in I, \forall j \in J, \forall t \in T \tag{2.12}$$

Constraints (2.12) mean that a customer's round-trip distance from the operating drone facility should be shorter than the possible flight distance that a drone can cover from the drone facility. Constraints (2.3) and (2.12) are similar in form, with the difference in the cost of the decision variables. Because λ and d_{ij} are positive values, the following inequality is obtained:

$$\alpha e^{2\lambda d_{ij}} \geq 2\lambda d_{ij} \qquad \forall \frac{1}{e} \leq \alpha$$

The objective probability that a drone returns to a facility is greater than 0.8 or 0.9 considering the practicality. Therefore, the chance constraint was seen to enforce tighter bounds on the coverage of a drone facility in the real application. Second, it may be questioned whether it is reasonable to consider the operation and maintenance costs of the drones, not the purchase cost. There was a lack of evidence that the purchase cost of the drones would vary depending on the location of the facility. Therefore, the costs associated with the drones are imposed as constants in proportion to the total amount of demand, and taking the purchase cost into account does not cause trade-off with other decision variables. Third, the number of drone facilities can be easily adjusted as needed. In a typical multi-period facility location model, operating facilities are maintained until all deliveries are made, but in this model, the facility may be abandoned. These changes are undertaken because not only the costs of opening the drone facility but also the operating costs incurred during each period were considered. If the allocated demand area were satisfied by other nearby facilities, then a facility could temporarily suspend operations. This ease of response to change by drone facilities provides flexibility for decision making and enables the FLP-D to be used for relatively quick and accurate reactions to disasters or for the relatively more efficient design of supply chains.

2.3 Solution techniques using Benders decomposition and linear programming relaxation

The very core of decision making in humanitarian logistics is the agility that can save as much time as possible. Most real-life applications of the response facility location problem are extensive and difficult to solve economically. Furthermore, the lag time due to slow decision making takes away precious moments from the golden time during which the majority of causalities are saved. In addition, multiple runs are often required in disaster management because of difficulties in precisely ascertaining future networks, demands and costs. Benders decomposition, a strategy for solving large-scale optimization problems, offers a fix to this situation. The Benders algorithm is used to solve a master problem and a slave problem iteratively. The SP solution provides information on the assignment of the MP variables in every iteration. Such information, expressed as a Benders cut, restricts assignments that are unacceptable. The Benders cut narrows the search space of the feasible region, which eventually leads to optimality. Benders cuts generated to solve one problem can be valid in a modified version of the same problem, with few revisions or additional effort [47]. Therefore, the Benders decomposition approach offers the possibility of making sequences of related runs in considerably reduced computing times. These useful characteristics enable effective responses to a changing disaster situation.

2.3.1 Master problem and slave problem

By the use of Benders decomposition, the developed integer pogramming (IP) model in Section 2.2.3 can be decomposed into the MP that solves X,Y,and Z variables and the slave problem, $SP(\bar{X}, \bar{Y}, \bar{Z})$, that solves only the U variables by fixing the X,Y,and Z variables to the MP solution and referring to them as $\bar{X}, \bar{Y}, \bar{Z}$. The MP is solved to plan the location and the opening time of a drone facility, and the SP is solved to determine the optimal number of drones that should be deployed from each drone facility. The mathematical models of the MP and SP for the FLP-D are as follows:

[Master problem, *MP*]

$$\begin{aligned} \min \quad & \sum_{i \in I} \sum_{t \in T} (o_i X_{it} + f_i Z_{it}) + \sigma \\ \text{s.t.} \quad & (2.3) - (2.5), (2.9), (2.10), \\ & \sigma \geq 0 \end{aligned}$$

[Slave problem, *SP*]

$$\min \quad \sum_{i \in I} \sum_{t \in T} c_i U_{it}$$

$$\text{s.t.} \quad (2.8), (2.11),$$

$$U_{it} \geq \sum_{j \in J} s_{jt} \bar{Y}_{ijt}, \qquad \forall i \in I, \forall t \in T \tag{2.13}$$

$$U_{it} \leq n \bar{X}_{it}, \qquad \forall i \in I, \forall t \in T \tag{2.14}$$

2.3.2 Generating Benders cuts

The generation of valid Benders cuts is the basis of Benders decomposition. Good Benders cuts guarantee the convergence of the iterations and determine how fast the algorithm converges [24]. However, it is difficult to generate valid Benders cuts for the model in this study because of the duality gap of the integer programming in the SP. One possible way to use Benders cuts is to employ the no-good cut method to exclude only current tentative assignment of MP variables that are unacceptable. Such no-good cuts result in an enumerative search and thus slow convergence. However, quick decision making is important in humanitarian logistics. Therefore, this strategy is impractical because it may lead to enumerating all extreme points in the SP. Linear programming (LP) relaxation can be an alternative to this matter. LP relaxation is a standard technique for designing approximation algorithms. It is done by assuming that decision variables in the SP are real numbers. Then, valid Benders cuts can be generated efficiently according to the strong duality property. Fractional solutions can be rounded off to obtain feasible integer solutions. This approach is usually called an LP-rounding method, and it has been successfully used as a strategy for the relaxed Benders algorithm. Shmoys et al. [114] offered exemplary research by applying LP-rounding strategy to relax the SP into an LP problem. The relaxed SP (RSP) can be formulated with decision variable V which is the relaxation of U. Dual values and variables are very useful in the Benders cut generation. The RSP can be converted to the dual problem as follows:

[Dual slave problem, *DSP*]

$$
\begin{aligned}
\max \quad & \sum_{i\in I}\sum_{t\in T}(\sum_{j\in J} s_{jt}\bar{Y}_{ijt}P_{it} - n\bar{X}_{it}Q_{it}) - \sum_{t\in T} NR_t & \\
\text{s.t.} \quad & P_{it} - Q_{it} - R_t \leq c_i, & \forall i \in I, \forall t \in T \\
& P_{it}, Q_{it} \geq 0, & \forall i \in I, \forall t \in T \\
& R_t \geq 0, & \forall t \in T
\end{aligned}
$$

where P_{it}, Q_{it} and R_t are extreme points. If the DSP is unbounded (such that the primal SP is infeasible), the extreme rays are used to add feasibility cuts to the restricted MP, RMP. If both the primal and dual SP have finite optimal solutions, but the two optimal solutions are different, then a new optimality cut is added to the RMP. Feasibility cuts and optimality cuts are defined in Remark 2.2.

Remark 2.2 (Feasibility and Optimality Cuts)**.** *Generated feasibility cut is*

$$\sum_{i\in I}\sum_{t\in T}(\sum_{j\in J} s_{jt}\overline{PR}_{it}^{(k)}Y_{ijt} - n\overline{QR}_{it}^{(k)}X_{it}) - \sum_{t\in T} N\overline{RR}_t^{(k)} \leq 0$$

and, the optimality cut is

$$\sum_{i\in I}\sum_{t\in T}(\sum_{j\in J} s_{jt}\bar{P}_{it}^{(k)}Y_{ijt} - n\bar{R}_{it}^{(k)}X_{it}) - \sum_{t\in T} N\bar{R}_t^{(k)} \leq \sigma$$

where $\overline{PR}_{it}^{(k)}$, $\overline{QR}_{it}^{(k)}$, *and* $\overline{RR}_t^{(k)}$ *are extreme rays of the feasible regions in SP for iteration* k*.*

2.3.3 Heuristic algorithm for the FLP-D

According to Benders decomposition and LP-rounding, the heuristic algorithm was developed for fast decision making. Algorithm 2.1 presents a pseudo-code of the developed heuristic approach.

Finite convergence of Algorithm 2.1 is assured for any given $\epsilon \geq 0$. Decision variables X,Y, and Z are binary, and a feasible region of MP consists of a discrete set. Therefore, Algorithm 2.1 terminates in a finite number of steps according Geoffrion [46]. The process of rounding the relaxed decision variables to the next integer for the number of drones appears in **Step 4**. Fractional solutions are infeasible because the FLP-D is originally an integer programming problem. Hence, this process is necessary and can guarantee the feasibility of the solution. Rounding off to the nearest integer, rather than the higher one, offers a possible solution for a decision maker who prefers a lower cost and is not concerned that the number of returning drones might fall below the

Algorithm 2.1: Heuristic algorithm for the FLP-D

Step 1. Initialization
Construct the initial master problem, $MP^{(0)}$.
Set the number of iteration $k = 0$ and a tolerance parameter $\epsilon \geq 0$.
Initialize $\bar{\phi}_{MP}^{(0)} = 0$, $\bar{\phi}_{SP}^{(0)} = 0$, and $UB = \infty$.

Step 2. Solving Master Problem
Solve $MP^{(k)}$. If it is feasible, obtain the optimal solution $\bar{X}^{(k)}$, $\bar{Y}^{(k)}$, $\bar{Z}^{(k)}$ and the optimal objective value $\bar{\phi}_{MP}^{(k)}$; otherwise, set $\bar{\phi}_{MP}^{(k)} = \infty$, and go to **Step 5**.

Step 3. Solving Slave Problem
Construct slave problem $SP(\bar{X}^{(k)}, \bar{Y}^{(k)}, \bar{Z}^{(k)})$.
Relax $SP(\bar{X}^{(k)}, \bar{Y}^{(k)}, \bar{Z}^{(k)})$ to the LP problem $RSP(\bar{X}^{(k)}, \bar{Y}^{(k)}, \bar{Z}^{(k)})$.
if $RSP(\bar{X}^{(k)}, \bar{Y}^{(k)}, \bar{Z}^{(k)})$ is feasible **then**
obtain the optimal solution $\bar{V}^{(k)}$ and the optimal objective value $\bar{\phi}_{SP}^{(k)}$.
end if

Step 4. Cut Generation Procedure
Return the current $\bar{\phi}_{MP}^{(k)}$, $\bar{\phi}_{SP}^{(k)}$ as the result value.
Update $UB = \sum_{i \in I} \sum_{t \in T} (o_i \bar{X}^{(k)} + f_i \bar{Z}^{(k)}) + \bar{\phi}_{SP}^{(k)}$
Generate a valid Benders cut and add it to the master problem, $MP^{(k)}$ to construct $MP^{(k+1)}$.
if $RSP(\bar{X}^{(k)}, \bar{Y}^{(k)}, \bar{Z}^{(k)})$ is feasible **then**
Add optimality cut.
else
Add feasibility cut.
end if

Step 5. Termination
if $\frac{UB - \bar{\phi}_{MP}^{(k)}}{\bar{\phi}_{MP}^{(k)}} \leq \epsilon$ **then**
Terminate the algorithm.
else
Set $k \leftarrow k + 1$ and go back to **Step 2**.
end if

if $\bar{\phi}_{MP}^{(k)} = \infty$ **then**
return Infeasible
else
Round up $\bar{V}^{(k)}$ to $\bar{U}^{(k)}$
return $\bar{X}^{(k)}, \bar{Y}^{(k)}, \bar{Z}^{(k)}, \bar{U}^{(k)}$
end if

target level. In our opinion, satisfying the constraints takes priority over costs in humanitarian logistics. The objective probability for a drone returning to a facility is directly related to a smooth restoration. Therefore, a rounding-up constraints-oriented approach was chosen for the algorithm. The performance of the heuristic was analyzed in the perspective of the solution quality in Section 2.4.

2.3.4 Discussion of the heuristic algorithm

Some notable characteristics of the algorithm are described in this section. A drone facility can presumably operate a sufficient number of drones because drones incur lower fixed and operation costs than other means of transportation. Condition (2.15) means the bound that allows for a sufficient number of drones for delivery.

$$\min(n, N) \geq \max_{t} \sum_{j \in J} s_{jt} \tag{2.15}$$

It is not difficult to verify that these assumptions make the SPs and RSPs always feasible. If Condition (2.15) is satisfied, Constraints (2.7) and (2.8) in the FLP-D become redundant. Then, Constraints (2.8) and (2.14) can be removed from the SP without changing the solution, and the SP can always find a feasible solution. Therefore, in this situation, only the optimality cut is added to MP. Under the condition that demand s_{jt} is given as an integer, SP is easily solved. Because $\bar{y}_{ijt}$ is the binary solution of Y_{ijt} satisfying the constraints in MP. $U_{it} = \sum_{j \in J} s_{jt} \bar{y}_{ijt}$ for $\forall i \in I, \forall t \in T$ is an optimal solution for a feasible

SP. For the SP with integer demand, an optimal solution to the respective LP relaxation is the same. The duality gap is zero in this case, and the SP holds strong duality. Therefore, if ϵ converges to zero, the heuristic algorithm for the FLP-D becomes exact algorithm and finds an optimal solution.

2.4 Computational experiments

Section 2.4.1 describes how the experiment was done and what data set was used. To verify the performance of the FLP-D in Section 2.2 and the developed algorithm in Section 2.1, computational experiments were conducted with Intel ® Core ™ 3.2GHz processor with 8GB RAM in the Microsoft Windows 10 operating system. The mathematical model and the heuristic algorithm for the FLP-D were solved with FICO Xpress version 7.3 (http://www.fico.com). Stochastic and deterministic plannings are compared in Section 2.4.3.

2.4.1 Description of experiments

Instances were generated according to instances on the Euclidean plane of simple location problems from the OR Benchmarks Library. The transportation costs between nodes in the benchmark sets were converted to distances between nodes. According to the FLP-D, the size of a problem was determined by the number of facility candidate locations ($|I|$), the number of customers ($|J|$), and the size of the time bucket ($|T|$). The maximum numbers of facility candidate locations and customers were limited to 100 because of the dimension of the matrix in Benchmarks Library. Table 2.2 shows 30 different problem classes ranging from small to large.

Table 2.2: Problem classes generated from Benchmarks Library

Class	Code in Library	$\|I\|$	$\|J\|$	$\|T\|$
C1	111	10	10	3
C2	211	10	20	3
C3	311	10	30	3
C4	411	10	40	3
C5	511	10	50	3
C6	611	10	60	3
C7	711	10	70	3
C8	811	10	80	3
C9	911	10	90	3
C10	1011	10	100	3
C11	1111	100	10	3
C12	1211	100	20	3
C13	1311	100	30	3
C14	1411	100	40	3
C15	1511	100	50	3
C16	1611	100	60	3
C17	1711	100	70	3
C18	1811	100	80	3
C19	1911	100	90	3
C20	2011	100	100	4
C21	2111	10	10	4
C22	2211	20	20	3
C23	2311	30	30	3
C24	2411	40	40	3
C25	2511	50	50	3
C26	2611	60	60	3
C27	2711	70	70	3
C28	2811	80	80	3
C29	2911	90	90	3
C30	3011	100	100	5

Many parameters have been generated on the basis of work by Shavarani et al. [113], who studied the San Francisco case. As noted in Shavarani et al. [113], no accurate data on the costs related to a response facility are available. Therefore, we randomly generated the opening and the operating costs of a drone facility from the uniform distribution. Operation and maintenance cost of a drone was generated by referring to Shavarani et al. [113]. Table 2.3 shows the range of the costs and demands that had integer values.

Table 2.3: Range of the parameter values

f_i	o_i	c_i	s_{jt}
$U[300,000,400,000]$	$U[30,000,40,000]$	$U[2,8]$	$U[1,100]$

Amazon reported that the drones to be used for delivery feature a travel radius of 16 km, which is equivalent to an endurance of 32 km for round trip delivery (Shavarani et al., [113]). Therefore, the parameter for the flight distance distribution of a drone was set to 0.00003125. The default setting for the objective probability that a drone returns to a facility is 0.8. For convenience in the analysis, parameters n and N were each set to 10,000 to prevent an infeasible solution due to lack of available drones. In the following three subsections, computational results and the analysis were presented.

2.4.2 Sensitivity analysis on different parameter

Sensitivity analysis can be done by changing some input parameters in the FLP-D. First, we examined the probability that a drone returns to a facility, which affects the chance constraints and is related to the success rate of delivery. Table 3 shows a trade-off between costs and target levels. Some obvious managerial insights can be confirmed through the results. The higher the target level, the more facilities need to be built to close the distance to the demand area, which results in higher costs. As can be seen, the facilities were built in more costly locations to achieve higher target levels in some cases.

Table 2.4: Results of sensitivity analysis for different α values

α	Class	Total cost($)	$\sum_{i\in I}\sum_{t\in T}\bar{Z}_{it}$	Class	Total cost($)	$\sum_{i\in I}\sum_{t\in T}\bar{Z}_{it}$
0.9	C1	2,157,495	5	C11	1,768,044	4
0.8		849,160	2		819,192	2
0.7		436,774	1		407,325	1
0.6		417,470	1		407,325	1
0.5		417,470	1		407,325	1
0.9	C2	Infeasible	-	C21	2,626,039	6
0.8		859,718	2		862,415	2
0.7		429,552	1		415,340	1
0.6		429,545	1		410,709	1
0.5		429,545	1		410,709	1
0.9	C3	Infeasible	-	C31	2,599,149	6
0.8		909,376	2		850,948	2
0.7		846,914	2		422,750	1
0.6		435,048	1		413,673	1
0.5		435,048	1		413,673	1

For the second analysis, computational experiments were performed with different distributions of flight distance that a drone can fly. The results are summarized in Table 2.5. Because the smaller λ refers to the longer distance a drone can fly, the results show that the cost decreases as λ decreases. Therefore, fewer drone facilities are necessary when drones can fly longer distances. If the distance that a drone can fly is too short, solving the FLP-D is impossible under the given conditions.

2.4.3 Comparison between deterministic approach and stochastic approach

The business performance of the FLP-D and a pure deterministic approach mentioned in section 2.3.4 were compared. Because the flight distance of a drone is not deterministic, in some circumstances the drones may not return to

Table 2.5: Results of sensitivity analysis for different λ values

λ	Class	Total cost($)	$\sum_{i\in I}\sum_{t\in T}\bar{Z}_{it}$	Class	Total cost($)	$\sum_{i\in I}\sum_{t\in T}\bar{Z}_{it}$
0.00009	C1	2,157,560	5	C11	2,280,740	5
0.00008		2,157,495	5		2,158,056	5
0.00007		2,157,495	5		2,123,846	5
0.00006		2,157,495	5		1,707,678	4
0.00005		1,707,987	4		1,282,127	3
0.00004		852,520	2		831,025	2
0.00003		849,160	2		819,192	2
0.00002		436,774	1		407,325	1
0.00001		417,470	1		407,325	1
0.00009	C2	Infeasible	-	C21	4,687,675	11
0.00008		Infeasible	-		3,422,670	8
0.00007		Infeasible	-		2,798,596	6
0.00006		Infeasible	-		2,137,032	5
0.00005		Infeasible	-		1,700,968	4
0.00004		890,065	2		1,304,549	3
0.00003		839,465	2		862,415	2
0.00002		429,952	1		415,340	1
0.00001		429,545	1		410,709	1
0.00009	C3	Infeasible	-	C31	3,566,052	8
0.00008		Infeasible	-		3,457,633	8
0.00007		Infeasible	-		3,005,912	7
0.00006		Infeasible	-		2,171,188	5
0.00005		Infeasible	-		1,748,157	4
0.00004		Infeasible	-		1,338,633	3
0.00003		909,376	2		834,187	2
0.00002		847,418	2		422,750	1
0.00001		435,048	1		413,673	1

the drone facility. When a drone does not return, the next delivery by drone is disrupted. More seriously, the delivery that the drone was assigned might have failed to transpire. Satisfying demand during a disaster and the subsequent recovery is critical because it is linked to the safety of the disaster victims. In this subsection, we verified the need for stochastic modeling in the design of the network considering the drones. The networks were designed by solving the same problem through the deterministic and stochastic model. Afterwards, we looked at how the results changed when the flight distance of a drone follows the normal distribution and the exponential distribution. The mean and variance of the two distributions were $1/\lambda$ and $1/\lambda^2$, respectively. The evaluation metrics used for comparison were as follows:

- $\#fac$: Number of facilities opened
- $Prob_N$: Probability that a drone returns to a facility when flight distance of a drone follows normal distribution
- $Prob_E$: Probability that a drone returns to a facility when flight distance of a drone follows exponential distribution
- $\#fail_N$: Number of drones that failed to return to the drone facility when flight distance of a drone follows normal distribution
- $\#fail_E$: Number of drones that failed to return to the drone facility when flight distance of a drone follows exponential distribution

Instance C1 was used in the experiment, and each evaluation metric value was averaged after 20 repeated experiments. Table 2.6 summarizes the experimental results of how each approach copes with uncertainty in terms of evaluation metrics.

Table 2.6: Results of the deterministic and stochastic approaches

Approach	λ	Total cost($)	$\#fac$	$Prob_N$	$\#fail_N$	$Prob_E$	$\#fail_E$
Deterministic	0.00003125	417,470	1	78.3%	293.7	81.5%	239.5
	0.0000625	417,470	1	74.5%	345.8	74.5%	348.5
Stochastic	0.00003125	849,160	2	81.5%	250.8	90.1%	134.2
	0.0000625	2,157,495	5	83.5%	224.3	96.5%	47.0

Simply looking at network design costs, a pure deterministic approach can lead to overly optimistic judgments about efficiency. However, looking at the evaluation metrics, we can see that the stochastic model copes with uncertainty much better than the deterministic model does. The difference became clear when the flight distance of drones follows an exponential distribution. This is a natural result because the network was designed assuming that the flight distance of drones follows an exponential distribution. If we consider the penalty for the delay in subsequent shipping, it is difficult to say which approach dominates in economic terms.

To obtain additional insight, we looked at the case of reducing the flight range of drones. After doubling the parameters, the differences between the two approaches became clearer. More facilities were established to achieve the target level as the flight range of drones was reduced. As a result, drones returned to the drone facility with a very high probability. It was shown that the distances between the drone facility and customers have a significant impact on the return probability of drones. It was also found that the network was designed to be sustainable overall because even the farthest customer assigned to each drone facility satisfied the target level.

2.4.4 Comparison between the FLP-D and heuristic algorithm

Experiments on various problem classes were performed to confirm the time efficiency of the developed heuristic algorithm. Results of the instances were summarized in Table 2.7. The gap between the best solution and the best bound was small in the FLP-D. However, experimenting with increasing the time limit, it was difficult to find the optimal solution for large size problems through the original model. Some problems could not be solved in the time limit of 3,600 seconds by the FLP-D while the heuristic algorithm found a solution. Even if both methods could not solve the problem, the heuristic algorithm yielded a better integer feasible solution within the time limit. As the size of the problem increased, the heuristic algorithm yielded better solutions than the original FLP-D.

2.5 Summary

Integrating drones into logistics is expected to be efficient and convenient. However, operation methods determined without considering the uncertain conditions of drone operations might produce a negative outcome. This chapter provided a stochastic design framework of ways to deploy drones economically to serve a disaster-affected region with uncertain drone flight distance. The developed model is used to determine the optimal locations for the drone facilities and the capacity, which is the number of drones deployed from each facility. For agile decision making, we developed a heuristic algorithm that produces a high-quality solution. The heuristic algorithm was developed according to Benders decomposition and the LP-rounding technique. The computational results showed that using a heuristic algorithm can reduce delays in decision making. This time efficiency enables effective real-time response in disaster situations. Another meaningful conclusion is the remarkable effectiveness of Benders decomposition as a computational strategy for disaster management.

The results of this chapter, which can also be found in Kim et al [67], have the following academic and practical contributions. For researchers, we believe that the approach developed is applicable to a range of disaster management operations and opens up a number of future research opportunities. For practitioners, the approach of this study can give answers to their questions about practicality. Designing constraints by considering the uncertain features of drones and making them tractable provide practitioners with a guideline for the practical use of drones. Furthermore, the FLP-D can be generalized to other facility location problems dealing with uncertainty. It can be used not only for disaster management but also for commercial purpose which helps to fully utilize this emerging technology.

Table 2.7: Comparison between IP approach and the heuristic

Problem Class	FLP-D			Alg. 2.1 ($\epsilon = 10^{-9}$)		Alg. 2.1 ($\epsilon = 10^{-2}$)	
	T.C.($)	B.B.	Time(s)	T.C.($)	Time(s)	T.C.($)	Time(s)
C1	849,160	849,160	0.59	849,160	0.36	849,160	0.35
C2	859,718	859,718	1.65	859,718	0.69	859,718	0.87
C3	909,376	909,376	1.11	909,376	0.98	909,376	0.84
C4	1,423,998	1,423,998	1199.65	1,423,998	2.06	1,423,998	2.16
C5	1,376,959	1,374,376	3, 600*	1,376,943	1.69	1,376,943	1.45
C6	1,881,041	1,874,218	3, 600*	1,880,078	2.52	1,880,078	2.25
C7	1,414,787	1,410,625	3, 600*	1,414,787	6.11	1,414,787	3.87
C8	infeasible	-	3.75	infeasible	1.48	infeasible	1.82
C9	1,324,900	1,310,827	3, 600*	1,323,078	7.14	1,323,078	6.24
C10	1,381,471	1,369,414	3, 600*	1,381,195	6.27	1,381,195	5.08
C11	819,192	819,192	321.75	819,192	21.19	819,192	16.31
C12	862,415	862,415	66.15	862,415	12.56	862,415	14.32
C13	850,948	850,948	500.48	850,948	53.18	851,620	24.78
C14	896,188	895,740	3, 600*	896,188	189.73	896,359	29.71
C15	1,259,889	1,250,508	3, 600*	1,255,679	442.15	1,255,679	324.78
C16	1,250,085	1,243,141	3, 600*	1,261,363	3, 600*	1,261,363	3, 600*
C17	955,895	949,376	3, 600*	955,895	3, 600*	955,895	3409.63
C18	1,268,390	1,261,861	3, 600*	1,268,358	3, 600*	1,268,358	3, 600*
C19	1,300,603	1,159,146	3, 600*	1,283,427	3, 600*	1,283,427	3109.42
C20	2,173,227	1,290,315	3, 600*	1,406,073	3, 600*	1,406,073	3, 600*
C21	899,799	899,799	0.95	899,799	0.44	899,799	0.56
C22	1,262,362	1,262,362	4.11	1,262,362	3.43	1,262,362	3.73
C23	1,261,634	1,261,508	3, 600*	1,261,634	12.76	1,261,634	12.03
C24	976,570	974,590	3, 600*	976,570	90.27	976,570	25.04
C25	1,281,181	1,279,701	3, 600*	1,280,919	53.68	1,287,061	51.25
C26	1,294,840	1,294,711	3, 600*	1,294,840	3063.97	1,296,371	2490.28
C27	924,775	921,781	3, 600*	924,409	153.05	924,409	120.56
C28	1,280,807	1,271,467	3, 600*	1,280,553	1328.62	1,280,553	818,52
C29	1,278,449	1,233,711	3, 600*	1,275,421	3, 600*	1,275,421	3089.27
C30	1,005,983	977,363	3, 600*	999,453	3, 600*	999,453	3, 600*

$*$: Time limit reached.
T.C : Total cost
B.B : Best bound found by commercial solver

Chapter 3

Scheduling-location Problem with Drones

3.1 Introduction

Supply chains are designed to meet customers' demands in a timely manner. Decision problems in supply chains can be categorized into three types of decision phases: strategic, tactical, or operational, based on the time planning horizon. Strategic decision planning sets a long-term decision horizon that spans annually or sometimes decades. At this strategic level, decisions are generally the first step of planning, and lay the groundwork for the supply chain process. This strategic level is associated with choosing the sites of facilities, assessing capacity requirements, and making long-term improvements and innovations. Mid-term planning decisions along the supply chain are determined at the tactical level. Operational planning decisions are short term decisions. Daily delivery routing and scheduling fall under the operational planning decision category. An operational planning decision usually comes as a consequence of strategic and tactical decisions.

Recently, much research has looked into integrating decision problems within different planning phases. A relatively new topic that deals with integrated decision-making, the scheduling-location (ScheLoc) problem, has received a lot of attention. In the ScheLoc problem, tactical decisions (selecting locations for machines) and operational decisions (scheduling of jobs) are integrated. One industry this problem applies to is mining, where minerals need to be moved to crushing machines. The best positions for crushing machines and the optimal schedule of mineral transport to those machines needs to be determined. The use of movable machines in production systems is also a sector where the ScheLoc problem can be applied (Kalsch [62]). Another application for this type of problem can be found in a container harbors, where containers need to be loaded onto ships (Kalsch and Drezner [63]). In this application, decisions on the positions for ships in the berth (location problem) and the sequence for loading containers (scheduling problem) must be determined simultaneously. Unfortunately, the amount of research considering the ScheLoc problem in this area is limited.

Hamacher and Hennes [58] first proposed an integrated model of a scheduling and location problem, in which release times for jobs were determined based on the locations of the machine to which the job was assigned. They considered a single machine within the ScheLoc problem and proposed a polynomial algorithm in which the scheduling of jobs was determined by the earliest release date (ERD) rule. Elvikis et al. [41] develeped three polynomial algorithms based on the ERD rule to solve a single machine planar ScheLoc problem. Kalsch and Drezner [63] investigated a single-machine ScheLoc problem in the plane, in order to minimize the makespan and the total completion time. They also developed a Branch-and-Bound algorithm based on the properties of models found. Rajabzadeh et al. [104] proposed the mathematical model for the parallel machine ScheLoc problem in discrete and continuous spaces, to minimize the makespan. Heßler and Deghdak [59] investigated the parallel machine ScheLoc problem, in which the candidate locations for machines were discrete. An IP model and different versions of clustering heuristics, in which jobs were split into clusters, were proposed. Liu and Liu [78] proposed a parallel machine ScheLoc problem under stochastic processing times with only partial distributional information in order to minimize the cost of operating machines and controlling the service level. The service level was measured by the probability of ensuring an on-time schedule. For the problem, a distributionally robust formulation is developed, in which the service level is restricted by a joint chance constraint.

In summary, it seems that some excellent research is leading the field of the ScheLoc problem. However, a small number of existing studies indicate that the history of research on the ScheLoc is not long and that field has not flourished. Although ScheLoc problem has been applied in some fields, researchers have not realized much of a need to integrate decision-making at different levels. In other words, on-site needs were not clearly defined, and the limitations on the application did not lead to a practical incentive to mature the field of study.

Drone use, a new technology field that requires the integration of decision phases, is on the rise. Drones appear to be a great alternative for logistics innovation, but technology still needs to be advanced to overcome realistic challenges. In particular, the limited battery capacity of drones is a major concern for drone utilization. Except for tethered drones that receive energy through a power cord, most drones use a relatively small capacity battery [88, 51]. As existing distribution centers are typically located far from city centers, relatively few customers can benefit from drone-delivery services. For this reason, leading retailers such as Amazon are working to build more distribution centers near large cities, but the cost of building distribution centers is still a big obstacle. Plans to use streetlights, gas stations, or church steeples as drone docking stations are also being studied as alternatives. A cargo truck equipped with a drone docking system may serve as a simple drone facility. In addition, different concepts of drone facilities are continually proposed to address these issues [108, 45].

Decision-making issues, due to the introduction of new technologies and concepts, are not sufficiently solved with existing models. This chapter therefore proposes a scheduling-location problem with drones (ScheLoc-D), a new methodology for integrating operational planning decisions with strategic planning decisions. Specifically, advanced logistics systems in which drone facilities are installed at optimal locations to provide direct delivery services is formulated. To the best of our knowledge, there is yet no consensus on the ScheLoc problem under the FLP scheme. As can be guessed from the order of terms called "ScheLoc," existing studies that attempted to integrate the scheduling and location problems focused more on scheduling. Research by [58, 41, 63, 104] aimed to minimize the makespan in a traditional scheduling perspective, and the economic perspective was considered to a lesser degree. On the other hand, the ScheLoc-D is relatively oriented toward the FLP-oriented setup rather than to scheduling.

The remainder of this chapter is organized as follows: A problem description and a mathematical model is provided in Section 3.2. In Section 3.3, we developed the restricted master heuristic based on the problem structure of the ScheLoc-D. The computational experiments and their results are summarized in Section 3.4. Finally, concluding remarks on this chapter are provided in Section 3.5.

3.2 Problem description and mathematical model

This section provides a detailed definition of the ScheLoc-D. The ScheLoc-D consists of determining the location of the facilities and the scheduling of delivery by drones deployed at each facility. Figure 3.1 presents an overview of the ScheLoc-D setup. As can be seen in Figure 3.1, the problem not only aims to cover all customers, but also to aims determine the order of delivery. If only one drone is on the delivery mission and two customers feature the same time window of urgency and tightness, additional facility should be established to deliver both customers.

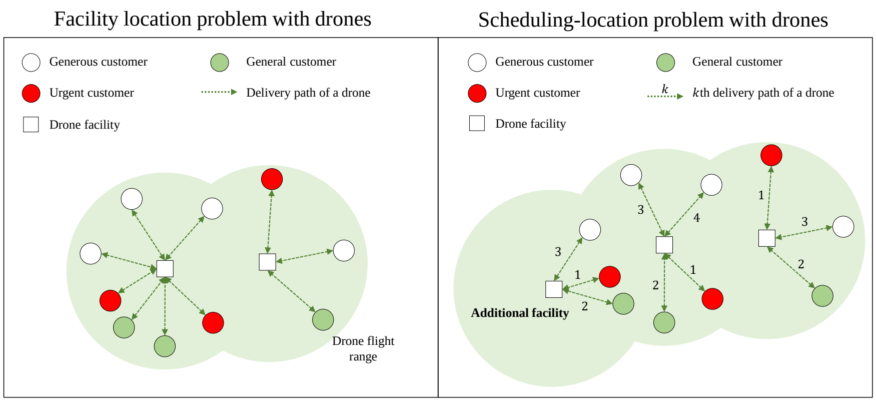

Figure 3.1: Overview of the ScheLoc-D

The objective of the ScheLoc-D is to find the optimal locations of drone facilities and feasible delivery schedules while minimizing total relevant costs. In most FLP scheme, customer allocation is determined according to the capacity of the facility, and the operational decision-making on how to do the allocation is omitted. However, the ScheLoc-D determines the coverage of a facility within which delivery scheduling is possible. Therefore, the developed model makes decisions simultaneously, which are generally classified into strategic and operational levels.

3.2.1 Mathematical model

In this section, all the used sets, parameters, and decision variables are presented and discussed. The developed scheduling-location problem involves four given sets: candidate locations of facilities, customer zones, available drones, and delivery schedules. $|J| = n$ is the number of customers to be satisfied. $|D| = m$ is the maximum number of drones available at each facility, and $|R|$ is an upper bound of the number of repeated departures of each drone. $|R|$ can be roughly set to $n - m + 1$.

Set

- I set of candidate locations at which facilities can be sited.
- J set of customer locations
- D set of available drones in each operating facility
- R set of repeated departures of each drone from the facility for delivery

Parameters

τ_{ij}	travel time between candidate site $i \in I$ and customer zone $j \in J$
$\bar{\tau}$	maximum travel time of a drone (Shipping range)
s_i	service time required to meet customer demand
$\bar{s}$	setup time to prepare for the next shipment
f_i	opening cost of a drone facility at location $i \in I$ (fixed cost)
ρ	cost factor for travel time (variable cost)
a_i	earliest time customer i can receive delivery
b_i	latest time customer i can receive delivery
MT	sufficiently large constant

Decision variables

$$x_{ij}^{dr} = \begin{cases} 1, & \text{if drone } d \in D \text{ deployed at facility } i \in I \text{ covers customer} \\ & j \in J \text{ with } r \in R\text{th shipment} \\ 0, & \text{otherwise} \end{cases}$$

$$y_i = \begin{cases} 1, & \text{if a drone facility is operated at site } i \in I \\ 0, & \text{otherwise} \end{cases}$$

T_i^{dr} number of drones operating at a drone facility at site $i \in I$ in time $t \in T$

Based on the sets, parameters, and decision variables defined above, the mathematical model of the ScheLoc-D is developed.

[Standard formulation (SF)]

$$\min \quad \sum_{i\in I} f_i y_i + \sum_{i\in I}\sum_{j\in J}\sum_{d\in D}\sum_{r\in R} 2\rho\tau_{ij} x_{ij}^{dr} \tag{3.1}$$

$$\text{s.t.} \quad \sum_{i\in I}\sum_{d\in D}\sum_{r\in R} x_{ij}^{dr} = 1, \qquad \forall j \in J \tag{3.2}$$

$$\sum_{j\in J}\sum_{d\in D}\sum_{r\in R} x_{ij}^{dr} \leq n y_i, \qquad \forall i \in I \tag{3.3}$$

$$\sum_{j\in J} x_{ij}^{dr} \leq 1, \qquad \forall i \in I, \forall d \in D, \forall r \in R \tag{3.4}$$

$$\sum_{j\in J} x_{ij}^{d,r+1} \leq \sum_{j\in J} x_{ij}^{d,r}, \qquad \forall i \in I, \forall d \in D, \forall r \in R \setminus \{|R|\} \tag{3.5}$$

$$2\tau_{ij} x_{ij}^{dr} \leq \overline{\tau}, \qquad \forall i \in I, \forall j \in J, \forall d \in D, \forall r \in R \tag{3.6}$$

$$\sum_{j\in J} \{(2\tau_{ij} + \overline{s} + s_j) x_{ij}^{dr}\} \leq T_i^{d,r+1} - T_i^{d,r}, \forall i \in I, \forall d \in D, \forall r \in R \setminus \{|R|\} \tag{3.7}$$

$$e_j x_{ij}^{dr} \leq T_i^{dr} + \tau_{ij} x_{ij}^{dr} \leq l_j x_{ij}^{dr} + MT, \quad \forall i \in I, \forall j \in j, \forall d \in D, \forall r \in R \tag{3.8}$$

$$T_i^{dr} \in \mathbb{R}^+ \qquad \forall i \in I, \forall d \in D, \forall r \in R \tag{3.9}$$

$$x_{ij}^{dr} \in \mathbb{B}, \qquad \forall i \in I, \forall j \in J, \forall d \in D, \forall r \in R \tag{3.10}$$

$$y_i \in \mathbb{B}, \qquad \forall i \in I \tag{3.11}$$

The objective function (3.1) minimizes the sum of the total relevant costs comprising the fixed costs of the facilities and the variable costs of serving demand from these facilities. Constraints (3.2) guarantees that each customer is assigned to delivery schedules. In other words, all demands should be covered.

Constraints (3.3) is linking constraints of the decision for opening a facility. Constraints (3.4) and (3.5) support defining a feasible delivery schedule for a drone. Constraints (3.6) limits the shipping range of each facility. Constraints (3.7) and (3.8) ensure the schedule feasibility with respect to time windows. MT should be large enough that the potentially optimal solution does not violate the constraints. In this case, MT can be set to $\max_{i \in I, j \in J}\{l_j + s_j + \tau_{ij} + \overline{s}\}$. Constraints (3.9), (3.10), and (3.11) demonstrate the integer and binary nature of the decision variables.

3.2.2 Discussion of the ScheLoc-D

When a sufficiently large number of drones can deliver and the maximum travel time of a drone is longer than the planning horizon, the ScheLoc-D is an equivalent problem to the FLP. The FLP on general graphs is $\mathcal{NP}-$hard, by reduction from the set-covering problem, one of Karp's 21 $\mathcal{NP}-$complete problems [64]. Therefore, the ScheLoc-D also belongs to the class of $\mathcal{NP}-$hard optimization problems. In order to highlight the originality of ScheLoc-D from the related models found in the existing literature, it is necessary to discuss several features of the model.

The ScheLoc-D covers different aspects related to facility location problems involving time-dependent decision variables. Although most existing facility-location models focus on a discrete setting as we did in Chapter 2, the ScheLoc-D focuses on a continuous time planning horizon. In the continuous time models, there are no given moments for making decisions. The best time to make a decision is only known after finding the optimal solution. Although several works by [39, 90, 101, 43] explores the features of that type of setup, continuous-time facility location problems are far less covered. Therefore, the ScheLoc-D problem can provide a fresh angle within this field of research.

Features in terms of model structure are as follows. The LP-relaxed bound for the ScheLoc-D can be very weak, due to the time window constraints. The SF of the ScheLoc-D is formulated based on four-index decision variables. Since the index contains the number of drone departures, which can be known only after finding a solution, a large number of variables should be generated by setting $|R|$ close to n to deal with the worst case scenario. Furthermore, the ScheLoc-D has symmetric solution space. Different solutions to the model can correspond to the same objective value. For example, if two drones exchange their assigned delivery schedules, two different solutions with the same objective function value are created. In this case, up to $m!$ identical delivery plans can be generated for each facility. Therefore, it is challenging to solve the SF of the ScheLoc-D with a commercial solver, and an efficient solution approach is necessary to solve large-sized instances. To deal with this computational complexity,in Section 3.3, we developed a restricted master heuristic (RMH) that can solve ScheLoc-D.

3.3 Pattern-based approach for the ScheLoc-D

3.3.1 Set-covering reformulation

A decomposition-based approach can reduce the number of variables by handling them implicitly. Minkowski's theorem proves that a polyhedron can be represented by its extreme points and extreme rays instead of by the original variables. In other words, a vector in a polyhedron can be represented as a summation of a convex combination of the extreme points and a conic combination of the extreme rays of the polyhedron. Dantzig-Wolfe decomposition reformulates the original problem by decomposing the block-diagonal structure of the constraint into smaller subproblems and the extended formulation [33]. Each subproblem consists of the structural constraints, and the resulting variables in the extended formulation implicitly satisfy those structural constraints. By definition, even when the extended formulation is LP relaxed, tighter bounds than the LP-relaxed bound of the original formulation can be provided. This is because the LP relaxation of the extended formulation is the dual of the Lagrangian subproblem. Thus, the LP-relaxed bound of the extended formulation has the same value of the Lagrangian dual bound.

In this section, Dantzig-Wolfe decomposition is applied to the ScheLoc-D. The solution of the ScheLoc-D consists of individual decisions about each facility, which construct a set-covering structure. Based on Dantzig-Wolfe decomposition, the SF can be reformulated with pattern-based decisions. Each variable in the extended formulation defines a set of customers covered by a facility. In other words, based on each facility, several feasible allocations of the set of demand points are given in advance. Ω_i is a set of feasible columns for a facility at site i. The parameters and the decision variables of the extended formulation model of the ScheLoc-D are presented as follows:

Parameters

c_{ik} cost associated to column k of facility i

$$a_{ij}^k = \begin{cases} 1, & \text{if pattern } k \text{ of facility } i \text{ covered customer } j \\ 0, & \text{otherwise} \end{cases}$$

Decision variables

$$z_{ik} = \begin{cases} 1, & \text{if pattern } k \text{ of facility } i \text{ is used} \\ 0, & \text{otherwise} \end{cases} \quad \forall i \in I, \forall k \in \Omega_i$$

The cost of each column is defined as $c_{ik} \coloneqq f_i + 2\rho\tau_{ij}a_{ij}^k$. The set covering model of the ScheLoc-D is represented in the following IP:

[Extended formulation (EF)

$$\min \sum_{i \in I} \sum_{k \in \Omega_i} c_{ik} z_{ik} \tag{3.12}$$

$$\text{s.t.} \sum_{i \in I} \sum_{k \in \Omega_i} a_{ij}^k z_{ik} \geq 1 \qquad \forall j \in J \tag{3.13}$$

$$\sum_{k \in \Omega_i} z_{ik} \leq 1 \qquad \forall i \in I \tag{3.14}$$

$$z_{ik} \in \mathbb{B} \qquad \forall i \in I, \forall k \in \Omega_i \tag{3.15}$$

The extended formulation only remains as the set-covering structure, while the scheduling-related constraints are considered implicitly in the variable. Objective function (3.12) is equivalent to Objective function (3.1) and seeks to minimize total cost. Constraints (3.13) imposes that all customers be covered. Constraints (3.14) restricts each facility to be opened at most once at a site. The binary requirements on the pattern-choice variables are expressed by (3.15).

The LP dual of the master problem can be viewed as the Lagrangian dual. Thus, the master problem can provide the Lagrangian dual, which is better than the LP-relaxed bound of the SF. By Minkowski's theorem, every solution of the compact formulation can be represented in the extended formulation. If Ω_i contains every feasible pattern for every facility, i, then the solution set of the master problem defines the convex hull of the ScheLoc-D. However, this requires an exponential number of patterns. The pattern (column)-based technique can be implemented to solve the problem. Let π_j and σ_i be dual prices associated with Constraints (3.13) and (3.14). To generate patterns, the subproblem of the ScheLoc-D is defined as follows.

[Subproblem]

$$\min \quad f_i + \sum_{j \in J} \sum_{d \in D} \sum_{r \in R} (2\rho\tau_{ij} x_j^{dr} - \pi_j) - \sigma_i \tag{3.16}$$

$$\text{s.t.} \quad \sum_{j \in J} x_j^{dr} \leq 1, \qquad \forall d \in D, \forall r \in R \tag{3.17}$$

$$\sum_{j \in J} x_j^{d,r+1} \leq \sum_{j \in J} x_j^{d,r}, \qquad \forall d \in D, \forall r \in R \setminus \{|R|\} \tag{3.18}$$

$$2\tau_{ij} x_j^{dr} \leq \overline{\tau}, \qquad \forall j \in J, \forall d \in D, \forall r \in R \tag{3.19}$$

$$\sum_{j \in J} \{(2\tau_{ij} + \overline{s} + s_j) x_j^{dr}\} \leq T_i^{d,r+1} - T_i^{d,r}, \forall d \in D, \forall r \in R \setminus \{|R|\} \tag{3.20}$$

$$a_j x_j^{dr} \leq T^{dr} + \tau_{ij} x_j^{dr} \leq b_j x_j^{dr} + MT, \quad \forall j \in j, \forall d \in D, \forall r \in R \tag{3.21}$$

$$x_j^{dr} \in \mathbb{B}, \qquad \forall j \in j, \forall d \in D, \forall r \in R \tag{3.22}$$

$$T^{dr} \in \mathbb{R}^+ \qquad \forall d \in D, \forall r \in R \tag{3.23}$$

Constraints (3.17)-(3.23) correspond to Constraints (3.4)-(3.10) of any specific facility, i, and index i is omitted in the decision variables. The objective function (3.16) minimizes the allocation reduced cost, and ensures that a negative reduced cost pattern is found when one exists.

Given the difficulty of solving the subproblem, a large part of the computing time is devoted to solving the subproblem. Therefore, maintaining a reasonable number of variables is essential to solving the problem efficiently. As is well known for various applications, heuristics can be used to generate attractive patterns (columns) with their ability to solve problems efficiently. In the following sections, a heuristic approach to solving the ScheLoc-D is proposed.

3.3.2 Generating attractive patterns (columns)

The subproblem in Section 3.3.1 is equivalent to the parallel machine scheduling problem given by considering deliverable customers as a set of jobs. Therefore, the attractive column can be generated by the solution of the following heuristic. Each facility contains $|N_i| = n_i$ jobs (transportation requests) $\mathcal{J} =< J_1, \cdots, J_{n_i} >$ and m identical machines (drones). Each job, J_j, is characterized by the quadruple (e'_j, l'_j, p_j, w_j). The interpretation is that job, J_j, is available at time a_j, *release time* (earliest time that customer j can receive delivery); it must be delivered by time b_j, *deadline* (latest time that customer j can receive delivery); its *processing time* (sum of round-trip travel time and service time at customer j) is p_j; and w_j is the *weight* (variable cost converted to negative value) associated with the job.

A feasible scheduling of job, J_j, delivered by drone, $d \in D$, at time, $e'_j \leq$

$t \leq l'_j - p_j$, is referred to as a *temporary schedule*, denoted by $J_{jd}(t)$. A temporary schedule can be expressed by an interval on the time line. Interval $J_{jd}(t) = [t, t + p_j]$ belongs to job, J_j, and several intervals can belong to a job. Temporary schedules, $J_{1d}(t_1), \cdots, J_{hd}(t_h)$, is a feasible schedule of a drone, if the corresponding intervals do not overlap, and if they belong to distinct jobs.

Based on the work by [6], a dispatching rule for drones in facility, i, Algorithm 3.1, is developed to maximize the throughput of all schedules. At each time step, t, the algorithm determines the temporary schedule that finishes first among all jobs that can be scheduled at t or later. Procedure, $Next(t, j, \mathcal{J})$, needs to be defined in order to execute Algorithm 3.1. The procedure determines temporary schedule $J_{i,j}(t'),\ t' \geq t$, such that $t' + p_{i,j}$ is the earliest among all temporary schedule in $\mathcal{J}$ that start at time t or later on machine (drone) d. If no such a job exists, the procedure returns *null*. Note that(k^d) jobs are scheduled on machine (drone) $d \in D$ without loss of generality, and tie breaks arbitrarily. A pseudo code of the algorithm is given in Algorithm 3.1.

Algorithm 3.1: Greedy algorithm for maximizing throughput

Input : $\mathcal{J} =< J_1, \cdots, J_n >$

Output : Schedules for drone delivery

Initialization

if $Next(\min_{j\in N_i}\{e'_j\}, 1, \mathcal{J}) \neq null$ **then**

$J_{j_1,1}(t_1) = Next(\min_{j\in N_i}\{e'_j\}, 1, \mathcal{J})$

$\mathcal{J} \coloneqq \mathcal{J} \setminus J_{i_1,1}(t_1)$

$t^1 = \min_{j\in N_i}\{e'_j\} + p_{i_1}$

Current time, $ct = \min_{d\in D} t^d$

Current machine (drone), $cm = \underset{d\in D}{\text{argmin}}\ t^d$

else

return empty schedule

end if

Main loop

while $Next(ct, cm, \mathcal{J}) \neq null$ **do**

for kth iteration of machine (drone) cm

$J_{j_k,cm}(ct) = Next(ct, cm, \mathcal{J})$

$\mathcal{J} \coloneqq \mathcal{J} \setminus J_{j_k,cm}(ct)$

$t^{cm} \leftarrow t^{cm} + p_{i_1}$

Current time, $ct = \min_{d\in D} t^d$

Current machine (drone), $cm = \underset{d\in D}{\text{argmin}}\ t^d$

end while

return schedules $\{J_{j_1,d}(t_1), \cdots, J_{j_{k_d},d}(t_{k_d})\} \quad \forall d \in D$

The following properties of Algorithm 3.1 are used in the analysis of the developed algorithms. Based on Proposition 1, Proposition 2 supports the performance guarantee of Algorithm 3.1.

Proposition 1 (Proposition 3.1 in Bar-noy et al. [6]). *Let S be the schedule found by Algorithm 3.1 for a job system, $\mathcal{J}$, and let F be any feasible schedule of a drone among the jobs in $\mathcal{J} \setminus S$. Then, $|F| < |S|$*

Proof. For each temporary schedule in F, there exists an interval in S that overlaps with it and terminates earlier. Otherwise, Algorithm 3.1 would have chosen this interval. The proposition follows from the feasibility of F, since at most one interval in F can overlap with the endpoint of any interval in S. □

Proposition 2. *Algorithm 3.1 generates a pattern within an approximation factor of 2.*

Proof. Let $S(m) = S^1 \cup \cdots \cup S^m$ be the output of Algorithm 3.1 and let OPT(m) $= O^1 \cup \cdots \cup O^m$ be the sets of intervals scheduled on the m machines (drones) by an optimal solution OPT. Let $R(m) = R^1 \cup \cdots \cup R^m$, where $R_d = O_d\ S(d)$ is the set of all jobs scheduled by OPT on machine (drone) d that Algorithm 3.1 did not schedule on any machine. Let $OS = OPT(k) \cap S(k)$ be the set of jobs scheduled by both Algorithm 3.1 and OPT. It follows that OPT$(k) =$ $OS \cup R$ Proposition 1 implies that $|H^d| \leq |S^(d)|$. This is true since H^d is a feasible schedule on machine (drone) d among the jobs that were not picked by Algorithm 3.1 while constructing the schedule for machine (drone) d. Since the sets R^k are mutually disjoint and the same holds for the sets S^k, $|R| \leq$ $|S(m)|$. Since $|OG| \leq |S(m)|$, we get that $|\text{OPT}(k)| \leq 2|S(k)|$ and the theorem follows. □

Algorithm 3.2 was developed to maximize the sum of weights. The algorithm is inspired by on-line call admission algorithms in [9, 4, 6]. Temporary schedules (or intervals) are checked one by one, and each temporary schedule is determined whether to be scheduled. Even if a temporary schedule is assigned, the decision may change in the future, while previously unscheduled jobs are no longer considered.

Similar to procedure $Next(t, j, \mathcal{J})$, defined in Algorithm 3.1, jobs are re-ordered based on when they finish early. Set, $\mathcal{S}$, the set of currently scheduled intervals, and $\mathcal{U}$, the set of unassigned temporary schedules, are defined. When a new temporary schedule, J_{ij}, is considered according to the sorted order, it is immediately scheduled if it does not overlap with any other interval in $\mathcal{S}$. If J_{ij} overlaps with one or more temporary schedules in $\mathcal{S}$, it is accepted only if its weight is less than β times the sum of the weights of all overlapping temporary schedules. J_{ij} is added to $\mathcal{S}$ and discards all the overlapped temporary schedules from $\mathcal{S}$. The process ends when there are no more temporary schedules to check. A pseudo code of the algorithm is given in Algorithm 3.2.

For the case of multiple machines (drones), Algorithm 3.2 is repeated machine (drone) by machine (drone), each time updating the set $\mathcal{J} = \{J_1, \cdots, J_{n_i}\} \setminus \bigcup_{d \in P} S^d$ of jobs to be scheduled, where P is the set of processed machines (drones).

Algorithm 3.2: Greedy algorithm for maximizing the sum of weights

Input : $\mathcal{J} =< J_1, \cdots, J_n >$

Output : Schedules for drone delivery

Initialization

$\mathcal{S} = \varnothing$

Let $\mathcal{U}$ be the set of unchecked temporary schedules

$\mathcal{U} = \mathcal{J} =< J_1, \cdots, J_n >$

Main loop

while $\mathcal{U} \neq \varnothing$ **do**

  Let $I \in J_d$ be the temporary schedule that finishes earliest among all instances in $\mathcal{U}$ and let w be its weight.

  Let W be the sum of the weights of all instances.

  $I_1, \cdots, I_h$ in $\mathcal{U}$ that overlap I.

  $\mathcal{U} \coloneqq \mathcal{U} \setminus I$.

  if $J_d \cap \mathcal{U} \neq \varnothing$ **then**

    Discard I.

  else if $W = 0$ **then**

    Schedule I, $\mathcal{U} \coloneqq \mathcal{U} \cup \{I\}$

  else if $\frac{w}{W} < \beta$ **then**

    Schedule I, $\mathcal{U} \coloneqq \mathcal{U} \cup \{I\} \setminus \{I_1, \cdots, I_h\}$.

  else

    Discard I.

  end if

end while

return schedules $\{J_{j_1,d}(t_1), \cdots, J_{j_{k_d},d}(t_{k_d})\} \quad \forall d \in D$

3.3.3 Restricted master heuristic

Heuristics based on exact methodologies have gained some recognition from both researchers and practitioners. RMH, one of the most widely used heuristics related to column generation, is developed to solve large-sized problems in a reasonable computing time. So, a pattern-wise decision, instead of the location-scheduling decision, is necessary to solve the master problem. A set of demand points that can be covered by one facility is defined as a decision variable in the extended formulation. Restricted master problem is then defined only by a limited subset of decision variables and is solved as a static IP.

As mentioned earlier, a large part of the computing time is usually devoted to solving the subproblem and generating columns. The restricted set of columns can either be generated heuristically. Based on the heuristically generated variables (columns) only, the ScheLoc-D can be solved with the extended formulation. In general, restricted master heuristic often finds the good primal bound of the master problem if the column-generating procedure maintains good column-wise decisions. The main drawback of the RMH is that the resulting restricted master integer problem is often infeasible. To deal with feasibility, the shortest-processing-time rule is used to generate additional columns. According to the shortest-processing-time rule, customers at the same location as the facility are always allocated. If all customer nodes are considered as candidate locations for the facility, the column generated by the rule prevents an infeasible solution. A pseudo code of the algorithm is given in Algorithm 3.3. Since the set covering problem, not the set partitioning problem, is used as the basic form of the master problem, there may be customers who are redundantly assigned to multiple facilities. In order to derive the final solution, the process of assigning these customers only to the nearest facility and recalculating costs is added.

Algorithm 3.3: Restricted master heuristic for the ScheLoc-D

Input : All sets, parameters defined in the ScheLoc-D

Output : Solution of the ScheLoc-D

Initialization

Create temporary schedules for the subproblem by adjusting parameters

$\mathcal{J} =< J_1, \cdots, J_n >$

Procedures for the subproblem

Call Algorithm 3.1

Call Algorithm 3.2

Generate columns according to shortest-processing-time rule

Procedures for the master problem

Solve the restricted master problem with generated columns.

Reallocate customers to opened facilities

return Feasible heuristic solution for the ScheLoc-D

3.4 Computational experiments

We carried out computational experiments on both small- and large-sized instances, and the computational results are provided in this section. Section 3.4.1 describes how the experiment was conducted and what data set was used. In Section 3.4.2, the performances of two solution approaches, the MILP and the RMH, are discussed.

3.4.1 Description of experiments

We utilized Solomon benchmark instances to verify the performance of the developed heuristic, the RMH. The detailed description of Solomon benchmark will be given in Chapter 4, which studies new variants of the VRPTW. Although Solomon benchmark was originally generated for studying the VRPTW, by using the information they have we can generate a data set that can be used for computational experiments in the ScheLoc-D. In the generated data set, the data on x-y coordinates, demands, time windows, and service times for customers are set the same as the values provided by Solomon benchmark. The information on the depot is fixed to the first candidate site of the facility in the generated instance. The remaining candidate sites are randomly selected from the customer's location. τ_{ij} is calculated as the following equation.

$$\tau_{ij} = \frac{\lfloor 10\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \rfloor}{10}$$

where (x_i, y_i) and (x_j, y_j) denote the coordinates for customers i and j, respectively. Parameters $\overline{s}$ and $\overline{\tau}$ are set to average value of all $s_{j\in J}$ and average value of all $\tau_{i\in I, j\in J}$, respectively. The units of all parameters used in the following experiments, including travel times and costs, follow the same scale in the benchmark instance. Facility setup costs were generated using a uniform distribution on the interval [1500,2300] based on the cost values reported in [2, 100]. The variable cost is the same as the travel time. The SF of the ScheLoc-D and RMP were solved with FICO Xpress version 8.5 (http://www.fico.com) and the heuristic procedures in the RMH were implemented in JAVA SE 8. Computational experiments were conducted with an AMD Ryzen 7 2700X eight-core 3.7GHz processor with 16GB RAM in the Microsoft Windows 10 operating system.

3.4.2 Comparing the RMH to the MILP formulation

To assess the solution quality of the RMH, we compared heuristic solution values to the optimal solution values obtained by solving the mathematical model for small-sized instances. When solving the ScheLoc-D as a MILP, we limited the computing time of the solver to 1,800 seconds. It was observed that 100-customer instances could not even be loaded to the commercial solver within that given time limit. Table 3.1 reports the class name (Class), the number of instances in each problem class (NP), the number of customers (NC), the number of candidate locations for a facility (NS), the number of instances solved optimally (Solved), the average computing time taken to find a solution with MILP in seconds (Time-M), the average objective function value of the solution found by MILP (Obj-M), the average computing time taken to find a solution with RMH in seconds (Time-H), the average objective function value of the solution found by RMH (Obj-H), and the average gap between two solutions in percentages (Δ_S). We compute the optimality gap as,

$$\Delta_S = \frac{obj.\,value\,(heuristic) - \text{OPT(IP)}}{\text{OPT(IP)}}$$

where $obj.\,value\,(heuristic)$ represents objective function value of heuristic solution and OPT(IP) is optimal solution value of an instance.

The computing times for the RMH are faster than the time needed to solve the integer programming model by a commercial solver. The RMH quickly obtained a solution, with costs lying within 3 percent of those of the optimal solutions. The solution quality of the RMH can be verified through a small gap

Table 3.1: Results on small-sized instances: summary

Class	NP	NC	NS	Solved	Obj-M	Time-M	Obj-H	Time-H	Δ_S
R1	12	10	6	5	5592.56	0.10	5656.92	0.09	1.15%
			11	12	5303.20	0.25	5303.20	0.21	0.00%
		25	13	12	6128.95	14.62	6189.82	0.21	0.99%
			26	12	5504.80	81.04	5519.65	0.20	0.27%
R2	11	10	6	8	5672.48	0.09	5674.00	0.09	0.03%
			11	11	5303.20	0.25	5303.20	0.25	0.00%
		25	13	11	5746.93	13.08	5779.66	0.20	0.57%
			26	11	5504.8	38.85	5524.60	0.20	0.36%
RC1	8	10	6	8	3460.73	0.13	3460.73	0.11	0.00%
			11	8	3341.80	0.18	3341.80	0.24	0.00%
		25	13	8	5746.93	13.08	5779.66	0.20	0.002%
			26	8	4925.60	17.60	4966.00	0.20	0.82%
RC2	8	10	6	8	3635.83	0.08	3635.83	0.17	0.00%
			11	8	3341.80	0.17	3341.80	0.20	0.00%
		25	13	8	5158.08	4.30	5270.45	0.19	2.18%
			26	8	4925.60	11.87	5067.00	0.19	2.87%
C1	9	10	6	9	3592.42	0.23	3592.42	0.19	0.00%
			11	9	3580.40	0.41	3598.80	0.25	0.51%
		25	13	9	5059.09	6.27	5059.09	0.19	0.00%
			26	9	4865.80	38.50	4865.80	0.20	0.00%
C2	8	10	6	8	4948.53	0.10	5011.17	0.08	1.27%
			11	8	3845.80	0.20	3845.80	0.22	0.00%
		25	13	7	5471.54	9.07	5489.66	0.27	0.33%
			26	8	5288.40	79.23	5288.40	0.21	0.00%

for all tested instances. Experiments were conducted on large-sized instances to clearly prove the performance of the heuristic algorithm. The descriptions in each column in Table 3.2 are similar to those in Table 3.1. We compute the average gap between two solutions in percentages as the following.

$$\Delta_L = \frac{obj.\,value\,(heuristic) \; - \; obj.\,value\,(IP)}{obj.\,value\,(IP)}$$

Table 3.2 shows that heuristic solutions are usually much better than the solutions that a commercial solver provides within given computing time limits.

Table 3.2: Results on large-sized instances: summary

Class	NP	NC	NS	Solved	Obj-M	Time-M	Obj-H	Time-H	Δ_L
R1	12	50	26	0	8846.48	1800.00	6988.65	0.23	-21.00%
			51	0	67069.26	1800.00	6581.22	0.30	-90.19%
R2	11	50	26	1	6936.31	1793.52	6960.49	0.25	0.35%
			51	0	66622.80	1800.00	6679.80	0.27	-89.97%
RC1	8	50	26	0	7508.73	1800.00	6598.55	0.23	-12.12%
			51	0	66351.53	1800.00	6420.90	0.28	-90.32%
RC2	8	50	26	4	6547.35	1623.04	6551.65	0.24	0.07%
			51	0	27260.6	1800.00	6172.20	0.25	-77.36%
C1	9	50	26	4	5364.04	1718.92	5387.24	0.21	0.43%
			51	0	16318.40	1800.00	4824.40	0.27	-70.44%
C2	8	50	26	7	6539.78	1445.36	6662.93	0.23	1.88%
			51	0	62056.00	1800.00	6450.20	0.26	-89.61%

Time efficiency of current heuristics is validated clearly in the larger instance. It was also observed that the lower bound found by the solver creating the cutting plane was very weak. A good solution was found for some instances, but could not converge, due to a weak bound. To sum up, the RMH provides a good solution for small-sized instances, and as the instance grows in size, it outperforms the performance of the commercial solver. So, we can state it is ready for integrating drones into logistics.

3.5 Summary

The advancement of drone technology is accelerating the introduction of drones into various elements of logistics. A drone-integrated system would seemingly lead to new operation problems, which would integrate different levels of decision making. In the ScheLoc-D, both the locations of drone facilities and delivery schedules of drones are considered. Because of the highly fractional solution of the LP relaxation, the standard formulation of the ScheLoc-D has a weak

LP-relaxed bound. Despite the weak LP bound, in the small-sized problems, the SF could find the optimal solution in a short time. However, in the larger problems, the SF could not be solved within the given time limit. Therefore, an extended formulation and restricted master heuristic were proposed in this chapter to improve the LP bound and utilize it. To generate attractive patterns, the subproblem is considered as a parallel machine scheduling problem, and an approximation algorithm based on a simple dispatching rule is used. The computational results showed that the restricted master heuristic outperforms the commercial solver in finding solutions for large-sized instances.

Chapter 4

Vehicle Routing Problem with Time Windows and Drones

4.1 Introduction

Drone research is emerging as a new field of study within routing problems. Compared to trucks, which are the most traditional means of transportation, drones have attractive advantages, such as their ability to avoid traffic congestion, their faster delivery time, their lower transportation costs, and their ability to be operated without costly human pilots [129]. However, although drones offer significant benefits, they also have some operational constraints due to their shortcomings. Specifically, several important technical barriers remain to be overcome before drones can be widely adopted in commercial fields. For example, the delivery capacity of the drone is technically restricted to just one or a few parcels, and their delivery range is significantly limited, as drones rely on relatively small batteries. The complementary nature of the two vehicles (trucks and drones) has been the driving force behind a novel delivery method called “coordinated logistics with a truck and a drone” [20].

In addition to the collaboration of trucks and drones, another significant trend change in delivery is worth mentioning. In the past, customers mostly wanted free delivery when they ordered goods from suppliers or retailers. Recently, more and more customers have been willing to pay a premium if they can receive goods in a faster way. Therefore, time window constraints are necessary in order to support routing decisions that take into account premium service. To pursue efficient delivery while meeting these customers' special rush needs, we develop in this dissertation a vehicle routing problem with time windows and drones (VRPTW-D), in which the service rendered at each customer starts within associated time interval. In this step, we attempted to model a delivery system in which drones are attached to trucks and dispatched to deliver a single package while the truck continues to serve other customers. After the drone delivers its payload, it should return to the truck or the depot for battery replacement and then prepare packages for the next delivery.

Related studies in this field have mostly been published recently, and significant advances has been made for different variants of the coordinated logistics with a truck and a drone. Murray and Chu [86] offers seminal work in this field of study. They developed MILP formulations for two delivery-by-drone problems, the flying sidekick TSP (FSTSP) and the parallel drone scheduling TSP. They also developed two simple heuristic algorithms. Starting with their research, studies in the field began in earnest. Agatz et al. [1] studied TSP-D with the objective of minimizing the logistics cost. In this problem, a key difference to the FSTSP is that the truck can wait for the drone in the same position from which the drone was launched. Moreover, they provided a theoretical bound on the maximum attainable gains that could be achieved by using the two different vehicles simultaneously. They constructed an integer programming model and developed a route-first, cluster-second heuristic algorithm. Bouman et al. [14] provided an exact algorithm for the TSP-D based on dynamic programming. They showed that restricting truck movement while drones are on delivery missions significantly reduced the computation time with relatively little impact on the overall solution quality. They also highlighted that their approach was able to solve large problems that the integer-programming presented by Agatz et al. [1] could not handle. Yurek and Ozmutlu [131] proposed an iterative algorithm by decomposing the TSP-D into two stages. They solved a MILP model in order to determine the drone route by fixing the truck route, and then the assignment decisions are made. They verified the proposed algorithm's efficiency by solving the problem with a setup of 12 customers in a reasonable time, whereas existing studies optimally solved problems with a maximum of 10 customers. Poikonen et al. [98] presented four heuristics to solve the TSP-D, based on the branch-and-bound scheme. Their algorithms could solve the instances of practical size in a reasonable amount of computing time. Recently, Murray and Raj [87] extended FSTSP to the version of the problem that assigned one truck and multiple heterogeneous drones to deliver parcels. They proposed a three-phase heuristic to solve the 100-customer instances.

Research on variants of the TSP-D has been actively conducted. Carlsson and Song [20] worked on another variant of the TSP-D, called the Horsefly Routing Problem. In this problem, compared to the TSP-D, a truck can collaborate with one or more drones. They used a continuous approximation technique to determine the best set of parameters that resulted in the minimum completion time of all truck-drone deliveries in the Euclidean plane. One of their key findings was that the benefit of using a drone along with truck is proportional to the square root of the relative velocity between the truck and the drone. Ha et al. [52] proposed a new variant of the TSP-D to minimize operational costs including costs incurred by transportation and waiting time. They also proposed the advanced heuristic adapted from the algorithm proposed by Murray and Chu [86] and high-performance greedy randomized adaptive search procedure. Kim and Moon [68] developed the traveling salesman problem with a drone station. A drone station is the facility in which drones are deployed and easily installed any place. They proved that their model can be divided into the models for the traveling salesman problem and the parallel machine scheduling problem under special conditions. They proved that decomposition approaches effectively deal with the complexity of their problem. Ha et al. [53] recently proposed a hybrid genetic search with dynamic population management and adaptive diversity control to solve TSP-D in which the objective is to either minimize the total operational cost or minimize the completion time for the truck and drone. Their algorithm outperformed existing solution approaches in terms of solution quality and found many new best solutions.

Wang et al. [128] introduced the VRP-D as a generalization of the TSP-D, and derived worst-case bounds for the ratios of the total delivery times with or without drones. The worst-case results depended on the number of drones per truck and the speed of the drones relative to the speed of the truck. Poikonen et al. [99] extended the results of Wang et al. [128] by relaxing the assumptions about the limited battery life, different distance metrics, and operational expenditures of deploying drones and trucks respectively. Wang and Sheu [129] proposed an arc-based integer programming model for the VRP-D, and reformulated it as a path-based model. One of the features of the model was that a backup drone could be utilized, because they assumed a service hub. Thus, there was no need to consider synchronization between the two vehicles. They developed a branch-and-price algorithm that included a pulse algorithm. Sacramento et al. [109] defined a problem similar to the FSTSP, which featured the capacitated multiple-truck case with the maximum duration and minimizing operational cost as the objective function. They proposed the adaptive large neighborhood search procedure and several problem-specific destroy-and-repair methods in order to solve large instances. They performed a detailed sensitivity analysis on several drone parameters of interest and investigated how beneficial the inclusion of the drone-delivery option was. Schermer et al. [111] proposed a matheuristic by exploiting the structure of the VRP-D. As part of the matheuristic, they introduced the drone assignment and scheduling problem that found an optimal assignment and schedule of drones to minimize the makespan. Schermer et al. [110] proposed an extension of the VRP-D in which a drone could be retrieved at some discrete points located on each arc. They also developed a hybrid algorithm based on variable neighborhood search and Tabu search, in order to solve large-scale instances. Kitjacharoenchai et al. [69] recently proposed a two-echelon vehicle routing problem with drones, which extended the FSTSP by allowing multiple trucks and drones to make deliveries while taking into account the capacities and two efficient heuristic algorithms to solve the problem.

The existing study that seems most closely related to our study is Di Puglia

Pugliese and Guerriero [102]. They introduced the vehicle drone routing problem with time windows and provided the mathematical model of the proposed problem. They used a commercial solver to reach a solution for randomly generated instances with five or ten customers and did not provide a methodology to address the high complexity of the problem. The main difference between their model and the problem proposed in this study is the freedom we offer to launch and land the drone. The VRPTW-D allows the drone to return to a different truck than the one from which it launched. Another study that considered both drones and time constraints can be found in Ham [57]. Ham [57] studied a multi-truck and multi-drone scheduling problem constrained by time windows, drop-off/pickup synchronization, and multi visits. This problem is uniquely modeled as an unrelated parallel machine scheduling problem with a sequence-dependent setup. A constraint programming (CP) approach is proposed, and CP formulations are further improved by using variable ordering heuristics.

With respect to the reviewed literature, our approach has several novel features, and the contributions of our work are manifold. First, we introduce new variants of the VRP; that is the VRPTW-D. To the best of our knowledge, the VRPTW-D is the most generalized mathematical model that takes into account both time windows and drones. In particular, unlike previous studies that considered time windows and drones in [102], it is possible for a drone to land on another truck than the truck it originally launched from. Considering a truck and a drone as a set provides limited delivery routes and does not achieve the global optimality that we are pursuing in our study. Therefore, the VRPTW-D defined in this dissertation is very flexible and allows for more efficient delivery routes. Second, achieving the optimal solution through a mixed-integer linear programming (MILP) formulation in a reasonable time is only possible on a small scale. However, instances encountered in real-world settings are usually complicated and large-scale, which means that efficient heuristic algorithm development is required in practice. To address larger instances of the VRPTW-D efficiently, we proposed a heuristic algorithm that utilizes the nature of the routing problem. Finally, we performed a sensitivity analysis of the relevant drone parameters in order to provide insights from the perspective of management. We showed that applying drones can reduce operating costs, and we highlighted the advantages of using drones in the last-mile delivery.

The remainder of this chapter is organized as follows: Problem description and the mathematical model of the VRPTW-D is proposed in Section 4.2. In Section 4.3, we develop the three-stage savings-based heuristic (TSH) and analyze how it effectively exploits the problem structure of the VRPTW-D. The computational experiments and their results are summarized in Section 4.4. Finally, Section 4.5 concludes this chapter.

4.2 Problem description and mathematical model

The VRPTW-D determines the cooperative delivery route of the two types of vehicles, trucks and drones. During delivery, a truck can launch a drone when serving a customer, and the drone performs a delivery for another customer and returns to the vehicle at a different location. A drone can perform delivery missions repeatedly after returning. Most previous studies assumed *pair constraints*, which considered a truck and a drone as a pair. A distinguishing feature of this study, with respect to recent literature, is that *pair constraints* are relaxed. In other words, the drone can land on a different truck than the truck from which it originally started delivering. This new variant of the VRP is not aimed at local cooperation in the way that drones are assigned to each truck, but is geared instead to a global cooperation for all vehicles. Figure 4.1 presents an overview of the VRPTW-D, and highlights the originality of the problem.

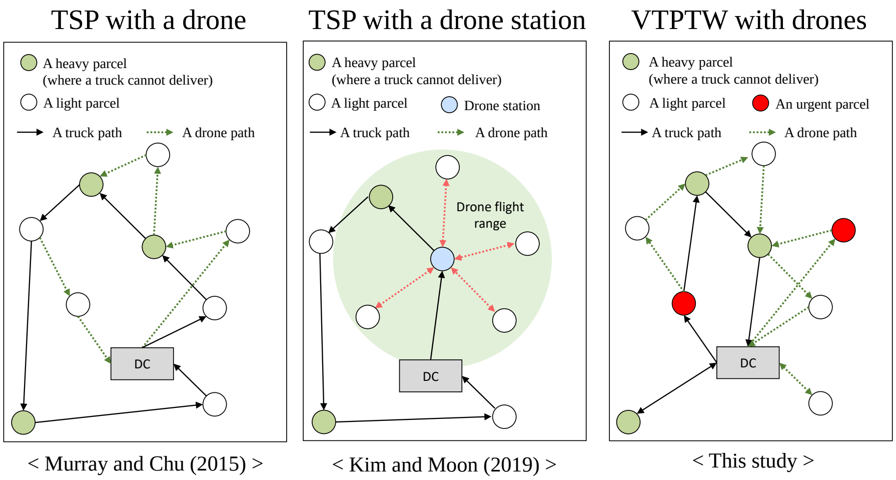

Figure 4.1: Overview and the originality of the VRPTW-D

Unlike previous studies, the VRPTW-D considers the use of multiple trucks and drones. As multiple drones taking over some delivery missions, the truck route has been simplified and the overall delivery plan has become cost-efficient. In addition, it shows that the direction the truck travels has changed due to the time window constraint. To formulate the VRPTW-D, the following assumptions are made regarding the behavior of the drones and the cooperation between truck and drones.

First, a drone can visit only one customer per flight, but the truck may visit multiple customers while the drone is in flight. This assumption is necessary because of the physical limitations of the drones. The drones still lack delivery coverage and transport capacity compared to conventional vehicles. Therefore, this assumption is a practical constraint required for actual application of drones. Indeed, most of the outstanding papers in this field also use similar assumptions [86, 1, 111]. Second, picking up parcels or replacing batteries in drones is only possible at customer locations or depots; that is, drones cannot leave any other deliveries when the truck is in motion. Third, the set-up time for preparing the drone for a new drone delivery is negligible. This is because drone-deliverable parcels are easy for drivers to handle, and because delivery preparation time can be reduced by replacing drone batteries rather than by charging them. Fourth, in order for the drone to return to the truck, the truck must first arrive at the node. This is a practical assumption to deal with safety issues and the lack of space for drones to wait. The remaining assumptions are the same as the assumptions required to define the VRPTW. In addition to the assumptions, the delivery plan should ensure the maximum duration constraints that each drone can travel and synchronization requirements between a truck and a drone.

The VRPTW-D is an extension of the VRPTW and minimizes the total costs of providing the transport services by determining optimal customer assignments for drones working in tandem with trucks. The overall routing costs involved in fixed vehicle utilization costs and variable logistics costs. As mentioned in Sacramento et al. [109], advances in technology relatively reduce the importance of fixed costs. Consequently, the most common factor used in the latter variable cost is the cost for distance and time. Therefore, the objective of the VRPTW-D is to find the shortest tour, in terms of total time, to serve all customer locations by either the truck or the drone. Moreover, the objective function is directly related to the emerging issue of sustainability and carbon emission minimization.

4.2.1 Mathematical formulation

The VRPTW-D is formulated over a complete graph $G = (N, A)$, where $N = \{0, \cdots, n+1\}$ is the node set and A is the arc set. Although only a single depot location exists, for the convenience of mathematical formulation we assign it to two unique node numbers. That is, vehicles depart from the depot at node 0 and return to the depot at node $n+1$. Thus, $C = N \setminus \{0, n+1\}$ becomes the set of customer nodes. To further facilitate the network structure of the problem, let $N_+ = \{0\} \cup C = \{0, \cdots, n\}$ represent the set of nodes from which a truck or a drone may depart, and let $N_- = C \cup \{n+1\} = \{1, \cdots, n+1\}$ represent the set of nodes to which a vehicle may return. The quantity of demand that has to be delivered to customer $i \in C$ is given by $q_i \geq 0$.

The truck, which is a ground vehicle, $g \in GV = \{1, \cdots, |GV| = m\}$, is assumed to be homogeneous, meaning that $|GV|$ vehicles have the same transport speed, capacity, $Q^{GV} \geq 0$, and identical cost efficiency. A truck moving from node i to j incurs travel time, $t_{ij} \geq 0$, and travel cost, $c_{ij} \geq 0$. Each truck can carry up to ζ drones. The drone, $d \in D = \{1, \cdots, |D|\}$ is also assumed to be homogeneous and α times faster and more cost-efficient than the truck. Thus, a drone moving from node i to j incurs travel time $\tau_{ij} = t_{ij} \; / \alpha \geq 0$ and travel cost $\rho_{ij} = c_{ij} \; / \alpha \geq 0$. The reason for this is that the drone is not affected by any traffic congestion and can use shortcuts. Hence, faster delivery could potentially be made by the drones. Cost efficiency comes from reduced labor costs from unmanned operations and the nature of using electricity. Along with the advantages of drone operation, the physical limitations of drones should also be considered. Each drone has a shipping range limited to a maximum travel time, $\overline{\tau}$, and a relatively low capacity, $Q^{GV} > Q^{D} \geq 0$. $C_{Dr} := \{n_i \in N | q_i < Q^D\}$ denotes the subset of customers who can be serviced by a drone.

The time dimension has been incorporated in the VRPTW-D in the form of customer-imposed time window constraints. Each customer, i, has a time window, $[e_i, l_i]$. Time window constraints restrict the start of service at a customer point to begin at e_i, or later thane_i and to begin earlier than l_i, or at l_i. The vehicle may arrive before the time windows open but the customer cannot be serviced until the time windows open. The vehicle is not allowed to arrive after the time window has closed. The service time for trucks and drones required by the customer, i, is s_i. To simplify notation, zero demands and zero service times are defined for the depot (i.e., $q_0 = q_{(n+1)} = s_0 = s_{(n+1)} = 0$). Furthermore, a time window is associated with them (i.e., $[e_0, l_0] = [e_{(n+1)}, l_{(n+1)}]$, where e_0 and l_0 are the earliest possible departure time from the depot and the latest possible arrival time at the depot, respectively). Given that the travel time matrix satisfies the triangle inequality, feasible solutions exist only if $e_0 \leq \min_{i \in N_-}\{l_i - \tau_{0i}\}$, and $l_0 \geq \max_{i \in N-}\{max\{e_0 + \tau_{0i}, e_i\} + s_i + \tau_{i,n+1}\}$. The following decision variables are used for modeling the VRPTW-D. A description for each of the decision variables follows:

$$X_{ij}^{g} = \begin{cases} 1, & \text{if truck } g \in GV \text{ travels arc } (i,j) \in A \\ 0, & \text{otherwise} \end{cases}$$

$$Y_{ij}^{d} = \begin{cases} 1, & \text{if drone } d \in D \text{ travels arc } (i,j) \in A\text{, independently} \\ 0, & \text{otherwise} \end{cases}$$

$$Z_{ij}^{gd} = \begin{cases} 1, & \text{if truck } g \in GV \text{ carries } d \in D \text{ and travels arc } (i,j) \in A \\ 0, & \text{otherwise} \end{cases}$$

$$o_{ij}^{g} = \begin{cases} 1, & \text{if the item in truck } g \in GV \text{ is delivered to customer } i \in C \\ 0, & \text{otherwise} \end{cases}$$

T_i^g Start of service time at node $i \in N$ when serviced by truck $g \in GV$

S_i^d Start of service time at node $i \in N$ when serviced by drone $d \in D$

Based on sets, parameters, and decision variables defined above, the mathematical model of VRPTW-D is developed. The VRPTW-D can be formulated as the following multi-commodity network flow model with time window and capacity constraints. The MILP formulation of the VRPTW-D is proposed, where Equation (4.1) is the objective function and the constraints are given by (4.2)-(4.21).

$$\min \quad \sum_{(i,j) \in A} \sum_{g \in GV} c_{ij} x_{ij}^{g} + \sum_{(i,j) \in A} \sum_{d \in D} \rho_{ij} y_{ij}^{d} \tag{4.1}$$

$$\text{s.t.} \quad \sum_{g \in GV} \sum_{i \in N_+, i \neq j} x_{ij}^g + \sum_{d \in D} \sum_{i \in N_+, i \neq j} y_{ij}^d \geq 1, \forall j \in C \tag{4.2}$$

$$\sum_{j \in N_-} x_{0j}^g = 1, \forall g \in GV \tag{4.3}$$

$$\sum_{j \in N_-} (y_{0j}^d + \sum_{g \in GV} z_{0j}^{gd}) = 1, \forall d \in D \tag{4.4}$$

$$\sum_{i \in N_+} x_{i,n+1}^g = 1, \forall i \in I, \forall g \in GV \tag{4.5}$$

$$\sum_{i \in N_+} (y_{i,n+1}^d + \sum_{g \in GV} z_{i,n+1}^{gd}) = 1, \forall d \in D \tag{4.6}$$

$$\sum_{i \in N_+, i \neq j} x_{ij}^g = \sum_{k \in N_-, j \neq k} x_{jk}^g, \forall j \in C, \forall g \in GV \tag{4.7}$$

$$\sum_{j \in N_+, i \neq j} (y_{ij}^d + \sum_{g \in GV} z_{ij}^{gd}) = \sum_{k \in N_-, k \neq j} (y_{jk}^d + \sum_{g \in GV} z_{jk}^{gd}), \forall j \in C, \forall d \in D \tag{4.8}$$

$$\sum_{d \in D} z_{ij}^{gd} \leq \zeta x_{ij}^g, \forall (i,j) \in A, \forall g \in GV \tag{4.9}$$

$$\sum_{d \in D} y_{ij}^d \leq \sum_{g \in GV} \sum_{h \in N_+} (x_{hi}^g + x_{hj}^g) + \sum_{g \in GV} \sum_{h \in N_-} (x_{ik}^g + x_{jk}^g), \forall (i,j) \in A \tag{4.10}$$

$$\sum_{i \in N_+} \tau_{ij} y_{ij}^d + \sum_{k \in N_-} \tau_{jk} y_{jk}^d \leq \overline{\tau}(1 + \sum_{i \in N_+} \sum_{g \in GV} x_{ij}^g), \forall j \in C, \forall d \in D \tag{4.11}$$

$$T_i^g + s_i + t_{ij} - T_j^g \leq (1 - x_{ij}^g) MGV_{ij}, \forall (i,j) \in A, \forall g \in GV \tag{4.12}$$

$$S_i^d + s_i + \tau_{ij} - S_j^d \leq (1 - y_{ij}^d) MD_{ij}, \forall (i,j) \in A, \forall d \in D \tag{4.13}$$

$$S_i^d + s_i + t_{ij} - T_j^g \leq (1 - z_{ij}^{gd}) MGV_{ij}, \forall (i,j) \in A, \forall g \in GV, \forall d \in D \tag{4.14}$$

$$e_i \leq T_i^g \leq l_i, \forall i \in N, \forall g \in GV \tag{4.15}$$

$$e_i \leq S_i^d \leq l_i, \forall i \in N, \forall d \in D \tag{4.16}$$

$$\sum_{i \in N_+} q_j y_{ij}^d \leq Q^D, \forall j \in C, \forall d \in D \tag{4.17}$$

$$\sum_{h \in N_+} x_{hi}^g + \sum_{d \in D} y_{ij}^d \geq 2o_j^g, \forall i \in N_+, \forall j \in C, \forall g \in GV \tag{4.18}$$

$$\sum_{j \in N_-} q_j (\sum_{i \in N_+} x_{ij}^g + o_j^g) \leq Q^{GV}, \forall g \in GV \tag{4.19}$$

$$x_{ij}^g, y_{ij}^d, z_{ij}^{gd}, o_i^g \in \mathbb{B}, \forall (i,j) \in A, \forall g \in GV, \forall d \in D \tag{4.20}$$

$$T_i^g, S_i^d \in \mathbb{R}^+, \forall i \in N, \forall g \in GV, \forall d \in D \tag{4.21}$$

Objective function (4.1) aims at minimizing the total route cost, based on total delivery time. Constraint (4.2) ensures that each customer is assigned to routes. In other words, all transportation requests must be satisfied. Constraints (4.3)–(4.6) ensure that the trucks and drones that leave the depot should return to the depot. Additionally, Constraints (4.7) and (4.8) are path-flow constraints. Constraint (4.9) limits the number of drones a truck can carry. Constraint (4.10) allows the drone to deliver to only one customer during a single flight. Constraint (4.11) guarantees the feasibility of drone flying duration. Next, Constraints (4.12)-(4.16) guarantee the schedule feasibility with respect to time windows. The values of T_i^g and S_i^d are meaningless whenever customer, i, is not visited by truck, g, and drone, d, respectively. To get better lower bounds and accelerate problem-solving, $MGV_{ij}, (i,j) \in A$ and $MD_{ij}, (i,j) \in A$ are set to $l_i + s_i + t_{ij} - e_j$ and $l_i + s_i + \tau_{ij} - e_j$, respectively. It is also possible to simply apply large constants, $MGV_{ij} = MD_{ij} = \max_{(i,j) \in A}\{l_i + s_i + t_{ij} - e_j\}$. In addition, Constraints (4.12)-(4.14) act as subtour elimination constraints.

Constraints (4.17)-(4.19) are capacity constraints related to the weight that can be carried in each type of vehicle. Specifically, Constraint (4.18) is a linking constraint on which truck the drone replenishes supplies, and Constraint (4.19) limits the transport capacity of the truck. Finally, the arc-flow variables are subject to binary requirements that can be expressed as in Constraint (4.20), and the other decision variables are subject to nonnegative integer requirements that can be expressed as in Constraints (4.21).

4.2.2 Discussion of VRPTW-D

The VRPTW-D is $\mathcal{NP}-$hard in the strong sense. This is because if the drones are not available, then the VRPTW-D is equivalent to the VRPTW which is a well-known $\mathcal{NP}-$hard combinatorial optimization problem. Several other features of the model warrant some discussion as well, in order to indicate the practicality for which the VRPTW-D differs from the related models found in existing literature.

A recently developed VRP considering the drone is particularly concerned with minimizing the delivery completion time. Many studies evaluate the performance by comparing the completion time of the truck-and-drone delivery system with the classical delivery system. In this dissertation, however, the use of completion time as a performance measure may distort the original intention. This is because the newly considered time window constraints significantly affect the complete time of delivery. If a customer who wishes to receive a delivery lately, the delivery completion time will be large regardless of other customers and previous delivery schedules. The VRPTW-D can provide a meaningless

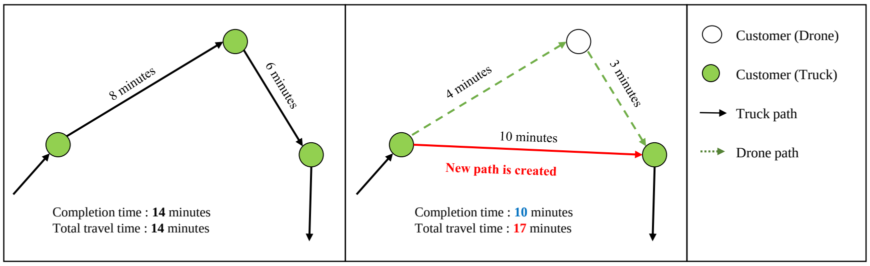

Figure 4.2: Difference between total travel time and completion time as an objective function

solution if the objective function is not set properly. Therefore, setting the minimization of total travel time or distance as the objective function in the VRPTW-D is necessary in terms of the validity of the model, as well as in terms of economic perspectives.

Figure 4.2 illustrates another rationale for setting total travel time as the objective function. It is better to use more drones in the decision of cooperative delivery with the aim of minimizing delivery completion time. Under the assumption that drones take less travel time than trucks, the drone route in an optimal solution serves to improve the objective value. This can be easily proved by applying the triangular inequality inductively. However, the situation is different for cooperative delivery in terms of minimizing total travel time or distance. This is because each time a drone is assigned to a new shipment, newly generated path of a truck adversely affects the objective function. Therefore, in order to determine the route that minimizes total travel time or distance, the delivery mission must be distributed more carefully between trucks and drones.

We assign a single depot to two unique node numbers for the convenience of mathematical formulation. This very conventional way of formulation also plays an important role in planning cooperative delivery. Since the unused truck is considered to be moving to two different depot nodes, the drone can be used for additional deliveries. In other words, the drone can deliver directly to nearby customers around the depot. A drone's direct delivery eliminates inefficient truck routes and significantly improves objective functions by performing deliveries to remote isolated customers. Another unique feature of the model is that the decision variables are defined in a different manner from previous studies. Conventionally, the physical limitations of drones are implicitly included in the structure of decision variables. In contrast to this, we utilized the general form of arc-flow decision variables. Similar approaches can also be found in Wang and Sheu [129]. These types of decision variables are easy to generalize in the model if the flight limitation and the number of drones assigned to each truck are relaxed. Additionally, the number of variables regarding the drone operations required for modeling the VRPTW-D can be reduced. Whereas existing studies required decision variables of n^3 to determine drone operations, only those of n^2 are required in this approach.

Finally, we will discuss the originality that makes this study different from other TSP-D and VRP-D studies. Research that considers cooperation between drones and trucks, has not long been in the limelight. In other words, this field of study is in an introductory stage, or growth stage, and it has been mainstream up until recently to research and analyze the tractable model using strong assumptions. The number of drones assigned to each truck was usually limited to one, or the synchronization between vehicles was not considered at all. Our study considers additional flexibility in the delivery route of drones. Drones can return to a different truck than the one from which they originally started delivering. Such a setup is not be covered by a simple extension of existing research, making the existing solution algorithms immediately inapplicable and raising the need for new solution approaches. Therefore, in Section 4.3, we developed a very simple and efficient heuristic algorithm that can solve VRPTW-D considering not only intra-route but also inter-route cooperative delivery.

4.3 Solution approach for the VRPTW-D

Given the intrinsic difficulty of the VRPTW-D, it is difficult to find an optimal solution of the VRPTW-D for practical-sized instances in a way that solves a MILP directly. Most TSP-D studies cannot find an optimal solution for a 10-customer instance, and some VRP-D studies cannot solve the problem for even six nodes. Therefore, the development of heuristics would be of interest for practical applications. For solving the VRPTW-D, we use an advanced RFCS heuristic approach (e.g., Beasley [10]), where in the first phase an initial solution is obtained by solving the VRPTW and the solution is improved using the developed heuristic algorithms. More precisely, we adapt the parallel Clarke-Wright-savings heuristic (e.g., Clarke and Wright, [26]) as an improvement procedure. Agatz et al. [1] also developed the RFCS heuristics based on local search and dynamic programming to solve the TSP-D. However, their algorithm can be applied to cases in which only one truck is utilized, so it is not suitable for solving the VRPTW-D. Moreover, existing solution approaches, including the one developed by Agatz et al. [1], do not take into account the delivery route in which the drone lands on a different truck than the truck from which it originally launched. This study newly proposes a three-stage saving-based heuristic (TSH) that adds new procedures to overcome prior limitations to solving the VRPTW-D. In what follows, the developed TSH will be described in more detail. In Section 4.3.1 we explain the procedure that is used to generate an initial solution. Sections 4.3.2 and 4.3.3 are devoted to the presentation of the drone assignment algorithm and the route combination algorithm, respectively. Potential benefits and further remarks on the heuristic are summarized in Section 4.3.4.

4.3.1 Finding an initial VRPTW tour

As the first step of a heuristic, we seek the solution of the VRPTW that does not take drones into account. Naturally, the solution of the VRPTW can be that of the VRPTW-D. Over the past 40 years, the VRPTW has been an area of research that has attracted numerous researchers. Numerous exact algorithms, which can be classified into the following three families, branch-and-price, branch-and-cut, and reduced set partitioning, were designed for the VRPTW, producing a significant improvement on the size of the instances that can be solved to optimality. Despite decades of intensive study, only relatively small instances involving around 100 customers could be solved optimally. The scale of the problems encountered in the industrial field was sometimes too large to be handled by a mathematical approach. For instance, global parcel volume surpassed 100 billion in 2019, reaching 103 billion [126]. Therefore, to generate the initial solution, the ruin and recreate (R&R) algorithm, inspired by the work of Schrimpf et al. [112], was used. The R&R algorithm is a generalization of simulated annealing, and is very similar to the large neighborhood search heuristic. According to Bräysy and Gendreau [17], the methods of Schrimpf et al. [112] are the best ones with respect to solution quality. After generating the initial solutions of the VRPTW using the R&R algorithm, improved solutions can be explored through the developed heuristics described in Sections 4.3.2 and 4.3.3.

4.3.2 Drone assignment algorithm

In this procedure, the VRPTW-D solution is constructed by distributing some of the deliveries to the drone. The basic idea of the heuristic is to find a delivery mission that would reduce total delivery time when assigned to a drone. A pseudo code of the heuristic that distributes the delivery mission to the drones is summarized in Algorithms 4.1 and 4.2. To simplify notation, we renamed the nodes included in the truck tour, $\mathfrak{R}^g$, in consecutive order (i.e., $\mathfrak{R}^g = (r_0^g, \cdots, r_i^g, \cdots, r_{|V_g|}^g)$ where $r_0^g = r_{|V_g|}^g$ refers to the depot). This algorithm performs inter-route improvements. In other words, the algorithm finds the drone's return point among all unlabeled nodes, not just within the same truck route. Based on the calculated savings, a decision is then made whether to allocate drones. In each iteration, the label of the node is set from unlabeled to truck or drone, and only nodes that are visited later than the just-labeled node are updated. Therefore, a drone assignment algorithm can be run in $O(n^2)$

time. While Algorithms 4.1 and 4.2 are running, the order in which the nodes are visited remains unchanged, and the feasibility of the solution is always guaranteed.

Algorithm 4.1: Heuristic for assigning drones - Initialization

---

Input :

$$\text{VRPTW solution}: \begin{bmatrix} (r_0^1, \cdots, r_i^1, \cdots, r_{|V_1|}^1), \\ \vdots \\ (r_0^g, \cdots, r_i^g, \cdots, r_{|V_g|}^g), \\ \vdots \\ (r_0^m, \cdots, r_i^m, \cdots, r_{|V_m|}^m) \end{bmatrix}$$

Sets of *"unlabeled"*, *"depot"*, *"truck"* and *"drone"* : $\mho, \Omega, N_T, N_D \in \varnothing$

Initialization

Initialize labels for all customer nodes as *"unlabeled"* and the depot nodes as *"depot"*.

$(r_1^1, \cdots, r_i^g, \cdots, r_{|V_g|}^g, \cdots, r_{|V_m|}^m) \in \mho$,

$(r_0^1, r_{|V_1|}^1 \cdots, r_0^g, \cdots, r_{|V_i|}^g, \cdots, r_0^m, \cdots, r_{|V_m|}^m) \in \Omega$

Calculate the number of drones available on each node, $\delta_{i \in C}$

Calculate the amount of time the truck can wait for the drone on each route to maintain the feasibility of the solution, w^g

return Input for Algorithm 4.2

4.3.3 Route combination algorithm

This section describes how to effectively adjust the number of vehicles used. This procedure is not necessary for the TSP-D, which considers only one truck. However, it is crucial in order to utilize the proper number of trucks for the VRPTW-D. The number of trucks used was initially determined by solving the

Algorithm 4.2: Heuristic for assigning drones - Main loop

Input : Output of Algorithm 4.1

Output : feasible VRPTW-D solution

while $\mho \neq \varnothing$ **do**

Select $r_i^g \in \mho$ with the fastest arrival time.

if $q_{r_i^g} \leq Q^D$ & $\delta_{r_i^g} > 0$ **then**

Find the nearest precedent *"truck"* node from r_i^g, p_i^g

Define a set of candidate nodes for drones to return,

$CN_{r_i^g} = \{\gamma | \tau_{p_i^g, r_i^g} + \tau_{r_i^g, \gamma} \leq \overline{\tau},\ \overline{T}_{p_i^g}^g + \tau_{p_i^g, r_i^g} + \tau_{r_i^g, \gamma} \leq \overline{T}_{\gamma}^g + w^g\}$

Define the following sets for the drone path that returned to node r_i^g.

$N_F(r_i^g, \gamma \in CN_{r_i^g}) = \{v_p | \sum_{d \in D} \overline{y}_{v_p, r_i^g}^d = 1,\ \tau_{v_p, r_i^g} + \tau_{r_i^g, \gamma} \leq \overline{\tau}\}$

$N_I(r_i^g, \gamma \in CN_{r_i^g}) = \{v_p | \sum_{d \in D} \overline{y}_{v_p, r_i^g}^d = 1,\ \tau_{v_p, r_i^g} + \tau_{r_i^g, \gamma} > \overline{\tau}\}$

Calculate possible savings,

$savings(r_i^g, \gamma \in CN_{r_i^g}) = c_{ps_i^g, r_i^g} + c_{r_i^g, r_{i+1}^g} - c_{ps_i^g, r_i^g} - \rho_{ps_i^g, r_i^g} - \rho_{r_i^g, \gamma} + \sum_{v \in N_F(r_i^g, \gamma)} (\rho_{v, r_i^g} - \rho_{v, \gamma}) - \sum_{v \in N_F(r_i^g, \gamma)} savings(v, r_i^g)$

where ps_i^g is the nearest precednet *"truck"* node from r_i^g) in the same route

if $max_{\gamma \in CN_{r_i^g}} savings(r_i^g, \gamma \in CN_{r_i^g}) \leq 0$ **then**

label node r_i^g as "*truck*"

else

Select the node γ with the largest savings. Node γ is chosen as return node for a drone where and label node r_i^g as "*drone*".

$\mho \leftarrow \mho \setminus \{r_i^g\}, V_D \leftarrow V_D \cup \{r_i^g\}, \overline{y}_{r_i^g, \gamma} = 1$

end if

else

$\mho \leftarrow \mho \setminus \{r_i^g\}, V_T \leftarrow V_T \cup \{r_i^g\},$

end if

Update the arrival time and the number of drones available of all node.

end while

return VRPTW-D solution, $(\mathfrak{R}, \mathfrak{T})$

classical VRPTW. If the number of trucks is kept the same as in the initial solution and then is assigned on a cooperative delivery route with the drone, trucks may be overused. An optimal or near-optimal delivery route can only be planned when a reasonable number of trucks is used. Therefore, a new procedure, a route combination algorithm, is proposed to determine the appropriate number of trucks used.

A route combination algorithm is based on an extension of the Clarke and Wright savings heuristic [26], one of the most well-known heuristics for solving the VRP. The algorithm calculates the savings achieved by combining two routes by connecting the last customer on one route to the first customer on another route. The algorithm greedily selects and combines two associated routes without violating route constraints, and terminates when two routes can no longer be combined. A route combination algorithm can be run in $O(m^2)$ time.

4.3.4 Remarks for the TSH

Solution quality and computation time are usually the two most obvious criteria used to assess the quality of an algorithm. These indicators are discussed in Section 5. Here we discuss other criteria that are also recognized as important when evaluating newly developed heuristics.

A heuristic algorithm needs to be simple in that it is easy to figure out and implement, and should not be too sensitive to the parameters. Usually, an algorithm that is controlled by few parameters is preferred. The TSH is an intuitive algorithm that applies the savings approach in Clarke and Wright [26] to drone delivery. No parameter tuning is required and no random choice is made while the algorithms in Sections 4.2 and 4.3 are running. While simplicity can lead to adverse effects on solution quality, the TSH algorithm strikes a reasonable balance between the performance criteria, as shown in the computational results in Section 4.4.

Since research in this field is still evolving, and no standardized problems have yet been defined, it becomes necessary, in order to handle various objectives and additional constraints, to develop heuristics that are sufficiently flexible. The TSH is easily able to incorporate additional constraints that may arise in practical applications. By modifying the process of calculating the savings and redefining the candidate sets, we can successfully reassign the drone's delivery mission to fit the goal of the new problem. In addition, full feasibility is guaranteed at all steps while the heuristic is running. Artificial constructions like penalty terms in the objective function are not considered at all. This property has significant implications for commercial applications.

The output of the TSH relies on the initial VRPTW solution. Using the optimal solution of the VRPTW as the initial solution generally finds a good final solution. However, a good initial solution may not lead to a near-optimal or optimal solution of the VRPTW-D. In the solution that the TSH algorithm found, the order of customers the truck visits is almost the same as the order of the initial solution. In the VRPTW solution, the time window often dictates the paths of the trucks, resulting in inefficiency. The VRPTW-D solution pursues a more efficient delivery route through the use of drones, and sometimes finds a completely different order of visits from the route found in the VRPTW solution. Therefore, the use of the optimal solution of the VRPTW is very costly in terms of computation time but has little corresponding advantage.

4.4 Computational experiments

We carried out computational experiments, on both small- and large-sized instances, and the computational results are provided in this section. Section 4.4.1 describes how the experiment was conducted and what data set was used. In Section 4.4.2, we discuss the performance of two solution approaches, the MILP introduced in Section 4.2.1 and the heuristic proposed in Section 4.3. The economic superiority of the coordinated delivery system compared to the truck-only delivery system is presented in Section 4.4.3. Afterward, we provide some sensitivity analysis on relevant drone parameters.

4.4.1 Description of experiments

We utilized Solomon benchmark instances to verify the performance of the developed heuristic, the TSH. Solomon [117] introduced VRPTW benchmark instances involving 100 customers that have since been accepted as standard benchmark problems by most researchers working on related issues. Six sets of problems are generated, and the instances differ in geographical data, the tightness and positioning of the time windows, and the several other factors that affect routing and scheduling. Problem sets include problems in which customers are located randomly (the R-problems), or in clusters (the C-problems). Problem sets also include a mixture of two geographic features (the RC-problems). Each problem consists of 100 customers, yet smaller problems can be created by considering only the first n customers. Cost and travel time will be calculated with one decimal point and truncation, a technique commonly used in this field. To check the results intuitively, the cost factor of the truck is assumed to be one. That is, the travel time and the cost incurred are expressed as the same value. The parameters c_{ij} and t_{ij} are calculated as the following equation,

$$c_{ij} = t_{ij} = \frac{\lfloor 10\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \rfloor}{10}$$

where (x_i, y_i) and (x_j, y_j) denote the coordinates for customers i and j, respectively. The units of all parameters used in following experiments, including travel times and costs, follow the same scale in the benchmark instance.

The VRPTW-D as MILP was solved with FICO Xpress version 8.5, and the TSH is implemented in JAVA SE 8. A pilot test was conducted to find the appropriate parameters and option settings to solve VRPTW-D. Computational experiments were conducted with an AMD Ryzen 7 2700X Eight-core 3.7GHz processor with 16GB RAM in the Microsoft Windows 10 operating system. All numerical results in the following sections were rounded to the second decimal place.

4.4.2 Comparing the TSH to the mathematical model

In this section, we describe the computational experiments on small-sized instances and show their numerical results. To assess the solution quality of the TSH, we compared heuristic solution values to the optimal solution values obtained by solving the mathematical model for 10-customer and 15-customer instances. The default parameter settings of the computational experiments are as follows. We assumed that the drone is twice as fast and cost-efficient as the truck (i.e., $\alpha = 2$). One truck can carry up to two drones waiting for delivery. The capacity of the drone is set to 20. For reference, the capacities of the trucks provided by the Solomon benchmark are 200, 700, or 1,000 depending on the type of problem. The maximum travel time of the drone is set to 45. The sensitivity analysis of drones set to various speeds and maximum travel times can be found in Section 4.4.4.

When solving the VRPTW-D as a MILP, we limited the computing time of the solver to 1,800 seconds. Originally, the number of trucks available was 25, provided by the Solomon benchmark. However, in order to find the optimal solution within the time limit, the number of trucks available was set to four, the maximum number of trucks used when instances are solved with the VRPTW. The comparative evaluation between the two approaches in large-sized instances is meaningless because the solver often provides an absurd solution within a limited running time. Table 4.1 reports the class name, (Class), the number of instances in each problem class, (NP), the number of instances solved optimally, (Solved), the average computing time taken to find a solution with MILP in seconds, (Time-M), the average objective function value of the solution found by MILP, (Obj-M), the average computing time taken to find a solution with TSH in seconds, (Time-H), the average objective function value of the solution found by TSH, (Obj-H), and the average gap between two solutions in percentages, (Δ_{10}). We compute the optimality gap as,

$$\Delta_{10} = \frac{obj.\,value\,(heuristic) - \text{OPT(MILP)}}{\text{OPT(MILP)}}$$

where $obj.\,value\,(heuristic)$ represents objective function value of heuristic solution and OPT(IP) is optimal solution value of an instance.

Table 4.1: Results on 10-customer Solomon instances: summary

Class	NP	Solved	Obj-M	Time-M	Obj-H	Time-H	Δ_{10}
R1	12	12	189.29	243.43	207.26	0.84	9.49%
R2	11	11	177.05	53.10	177.74	0.78	0.39%
RC1	8	8	166.98	99.38	170.91	0.97	2.35%
RC2	8	8	159.18	92.88	162.25	0.79	1.93%
C1	9	9	55.42	61.69	55.54	0.71	0.22%
C2	8	8	120.39	22.16	122.23	0.75	1.53%

The computing times for the TSH are faster than the time needed to solve the integer programming model by a commercial solver. The TSH quickly obtained a solution, with costs lying within 10 percent of those of the optimal solutions. The solution quality of the TSH can be verified through a small gap for all tested instances. Further experiments were conducted to clearly prove the performance of the heuristic algorithm. The number of trucks available was set to four in the following experiments. The descriptions in each column in Table 4.2 are similar to those in Table 4.1. We compute the average gap between two solutions in percentages as the following.

$$\Delta_{15} = \frac{obj.\,value\,(heuristic)\ -\ obj.\,value\,(MILP)}{obj.\,value\,(MILP)}$$

Table 4.2: Results on 15-customer Solomon instances: summary

Class	NP	Solved	Obj-M	Time-M	Obj-H	Time-H	Δ_{15}
R1	12	3	301.28	1,507.87	292.97	25.24	-2.76%
R2	11	1	300.26	1,680.28	254.7	11.65	-15.17%
RC1	8	3	202.47	1,469.6	205.14	26.01	1.32%
RC2	8	2	204.84	1,523.7	194.09	10.66	-5.25%
C1	9	6	136.93	697.67	139.92	10.34	2.18%
C2	8	7	162.84	729.94	157.98	10.86	-2.98%

Table 4.2 shows that heuristic solutions are usually much better than the solutions that commercial solver provides within given computing time limits. Time efficiency of current heuristics is validated clearly in the larger instance. Even though the optimal solutions for most of the 15-customer instances are unknown, it is safe to say that the TSH can provide fine solutions in a short time. To sum up, the TSH provides a good solution for small-sized instances, and as the instance grows in size, it overwhelms the performance of the commercial solver.

4.4.3 Comparing a coordinated delivery system to truck-only delivery

We conducted a comparative analysis from an economic point of view to see the efficiency of coordinated delivery of trucks and drones. We compared the solutions found by solving the VRPTW-D and the classical VRPTW. First, an experiment was conducted on 10-customer instances to compare the optimal solutions of the two problems. Table 4.3 reports the class name, (Class), the number of instances in each problem class, (NP), the objective value of VRPTW, (Truck-only), average number of trucks used in the solutions of the VRPTW, (NT_0), the objective value of VRPTW-D in situations in which only one drone can be accommodated per truck, (1 dr/tr), average number of trucks used in the solutions of the VRPTW-D considering one drone/truck, (NT_1), the gap in percentage between the objective value of VRPTW-D considering one drone/truck and the VRPTW solution (Δ^1), the objective value of VRPTW-D in situations in which each truck can accommodate up to two drones, (2 drs/tr), average number of trucks used in the solutions of the VRPTW-D considering two drones/truck, (NT_2), and the gap in percentage between the objective value of VRPTW-D considering 1-drone/truck and the VRPTW solution (Δ^2). The gaps are calculated as follows.

$$\Delta^1, \Delta^2 = \frac{\text{OPT(VRPTW) - OPT(VRPTW-D)}}{\text{OPT(VRPTW)}}$$

Table 4.3: Cost-efficiency analysis of 10-customer Solomon instances

Class	NP	Truck-only	NT_0	1 dr/tr	NT_1	Δ^1	2 drs/tr	NT_2	Δ^2
R1	12	223.23	2.75	192.89	1.17	13.59%	189.29	1	15.20%
R2	11	193.65	1.45	177.72	1	8.23%	177.05	1	8.57%
RC1	8	172.54	2	167.37	1.875	3.00%	166.98	1.875	3.22%
RC2	8	162.86	1.75	159.19	1.625	2.26%	159.18	1.625	2.26%
C1	9	57.49	1	55.42	1	3.59%	55.42	1	3.59%
C2	8	146.81	1.5	120.39	1	18.00%	120.39	1	18.00%

The objective values of the VRPTW-D were much lower than those of the VRPTW, which justifies mixed use of trucks and drones. Computational results also show the difference in the performance of the coordinated delivery system according to the geographical distribution of customers. In general, the cost efficiency of drone delivery increased when customers are distant from each other. For example, one customer is far from the remaining clustered customers in 10-customer instances of Problem C2. Therefore, significant cost saving could be achieved by delivering the drone directly to the customer, as shown in Figure 4.3.

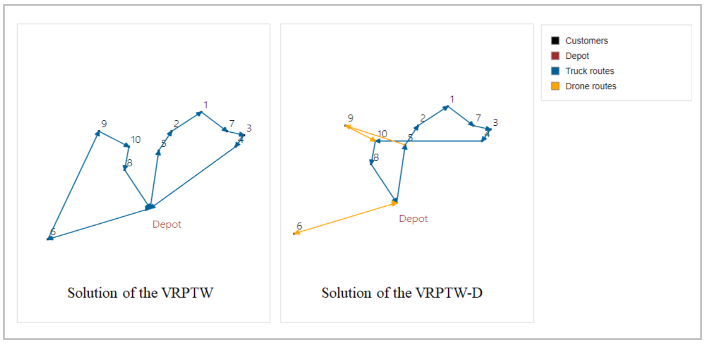

Figure 4.3: Comparison of the optimal solutions solved with the VRPTW and the VRPTW-D for Instance C201

In addition to our objective function, we could see that the number of trucks used reduced. This is a natural result of pursuing an optimal route from the perspective of minimizing total travel time. Figure 4.4 shows how the number of trucks used reduces as the use of drones increases. In the optimal solution of R101 found with the VRPTW, four trucks were required, but in the optimal solution found with the VRPTW-D considering one drone/truck, two trucks were used. Furthermore, in the optimal solution found with the VRPTW-D considering two drones/truck, only one truck was used. The fixed cost of trucks and the labor cost of men to drive the trucks are very costly compared to the fixed and operating costs of drones. Therefore, reducing the number of trucks is a very significant secondary result from an economic point of view. If the objective function was to minimize the completion time of delivery, a solution exploiting all available resources would be found.

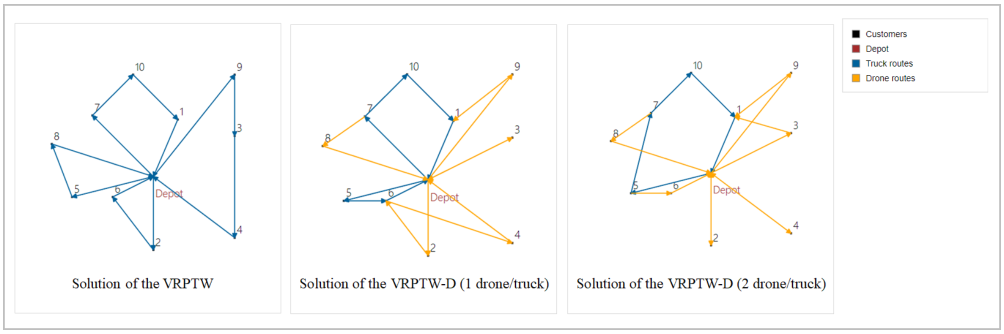

Figure 4.4: Comparison of the optimal solutions solved with the VRPTW and the VRPTW-D for Instance R101

We also conducted an economic analysis for large-sized instances. The optimal solution of the VRPTW-D for large-sized instances is unknown, so the experiment was conducted using the TSH. The TSH seems to be appropriate to analyze large-sized instances, as demonstrated in the previous section. The experimental results in Table 4.4 present the guaranteed minimum economic effect, not the ideal cost reduction. The descriptions in each column in Table 4.4 are the same as in Table 4.3. The effect of cost reduction was less than that of the previous experiment. The main reason for this was the comparison between the optimal solution for the VRPTW and the heuristic solution for the VRPTW-D. Aside from this, one potential reason for the difference may be that in large instances, the average distance between customer locations is smaller, resulting in less benefit from allocating delivery missions to drones. Also, since the number of drones was limited, they might not have been able to adequately cover the increased number of customers. Nonetheless, the economic superiority of coordinated delivery was still evident. Therefore, successful implementation of the coordinated delivery could bring about cost efficiency.

Table 4.4: Cost-efficiency analysis of large-sized Solomon instances

Class	NP	NC	Truck-only	1 dr/tr	Δ^1	2 drs/tr	Δ^2
R1	12	25	463.37	453.50	2.13%	453.26	2.18%
		50	766.13	749.91	2.12%	745.20	2.73%
		100	1173.74	1148.43	2.16%	1137.42	3.09%
R2	11	25	376.20	376.27	1.55%	376.20	1.55%
		50	615.41	614.26	0.19%	610.08	0.87%
		100	872.53	856.8	1.80%	848.09	2.80%
RC1	8	25	350.24	344.24	1.71%	343.51	1.92%
		50	730.31	716.56	1.88%	714.44	2.17%
		100	1334.48	1325.08	0.71%	1318.35	1.21%
RC2	8	25	319.28	311.44	2.45%	309.72	2.99%
		50	571.63	571.56	0.01%	568.33	0.58%
		100	1000.68	989.04	1.04%	987.49	1.32%
C1	9	25	190.59	189.68	0.48%	189.59	0.52%
		50	361.69	361.28	0.11%	360.99	0.19%
		100	826.70	825.49	0.15%	825.21	0.18%
C2	8	25	214.45	213.70	0.35%	213.14	0.61%
		50	357.50	356.08	0.40%	355.99	0.42%
		100	587.37	582.83	0.77%	582.83	0.77%

4.4.4 Sensitivity Analysis with the drone features

In this section, we carried out a sensitivity analysis when some fundamental parameters were varied. In addition to setting the features we wanted to analyze, we set the other parameters to the same values as in previous experiments. First, we examined the impact of the speed of the drone on the performance of a coordinated delivery system. We assumed that a drone is twice as fast and cost-efficient as a truck. In the following experiments, the relative drone speeds and cost efficiency metrics were changed from one to four. Table 5 shows that cost savings increase with the relative speed of the drone. A drone can deliver only to nodes that are close to the truck route when the speed of the drone is low. A drone traveling at a higher speed can cover more distance and deliver to nodes farther away. As the use of drones increases, it is possible to eliminate the inefficient routes of trucks that were dictated by time window constraints.

Table 4.5: Results of sensitivity analysis for different drone speeds

Class	NP	$\alpha = 1$	$\alpha = 2$	$\alpha = 3$	$\alpha = 4$
R1	12	1181.60	1137.42	1089.72	879.85
R2	11	875.53	848.09	819.69	804.72
RC1	8	1344.74	1318.35	1285.61	804.38
RC2	8	1008.30	987.49	961.48	949.86
C1	9	826.70	825.21	813.22	802.90
C2	8	587.38	582.83	563.90	545.35

We also investigated the impact of the delivery range of drones on system performance. To this point, the maximum travel time of drones was set to 45.

In the following experiments, we varied the maximum travel time of the drone, between 10 and 50. Table 6 shows that the cost savings of a coordinated delivery system increased with the flying duration of the drone.

Table 4.6: Results of sensitivity analysis for different delivery ranges

Class	NP	$\overline{\tau} = 10$	$\overline{\tau} = 30$	$\overline{\tau} = 50$
R1	12	1156.01	1139.36	1137.42
R2	11	848.09	848.09	848.09
RC1	8	1330.86	1318.35	1318.35
RC2	8	993.01	984.94	984.94
C1	9	825.21	825.21	825.21
C2	8	582.83	582.83	582.83

The increased flying duration of the drone increased cost savings because deliveries made by drones replaced inefficient truck routes. However, flying duration did not bring about as dramatic an effect as did increasing the speed of the drone. The reason for this is that increasing the speed of the drone in our model included the effect of expanding the delivery range of the drone. Lastly, we carried out the following experiments, changing the delivery capacity of the drone from a relatively small capacity to a capacity that could cover all customers. Table 7 shows that the cost savings of a coordinated delivery system increase with the transport capacity of the drone.

Table 4.7: Results of sensitivity analysis for different capacities of drones

Class	NP	$Q^D = 10$	$Q^D = 20$	$Q^D = 30$	$Q^D = \max_{i \in N} q_i$
R1	12	1158.45	1137.42	1132.25	1130.92
R2	11	863.48	848.09	845.20	844.91
RC1	8	1332.38	1318.35	1316.08	1315.46
RC2	8	1001.08	984.94	983.38	981.13
C1	9	826.01	825.21	824.81	824.81
C2	8	582.15	582.83	582.83	582.92

We conclude the discussion of computational results with some insights into the characteristics of a coordinated delivery system. As expected, we saw cost savings increase as the physical limitations of the drone were relaxed. In particular, the effect of drone speed on the efficiency of the coordinated delivery system was very strong. Technological advances and innovations of drones themselves must accompany such findings in order to easily prove the feasibility of drone delivery and to quickly commercialize it.

4.5 Summary

Drones have emerged as attractive options to supplement traditional delivery vehicles. Research on how to integrate drones into logistics and how to prove the effectiveness of drone delivery is constantly being conducted. In this chapter, we defined a new variant of the VRP, a vehicle routing problem with time windows and drones in which several trucks and drones worked together to provide service to customers within given time windows. A drone can move with a truck, take off from the truck to serve customers, and land on the same or a different truck from which it took off. The VRPTW-D was formulated based on a MILP, and we discussed characteristics of the newly developed problem. The VRP considering drones has been optimally solved only on a very small scale. Therefore, we presented a three-stage savings-based heuristic, a simple yet time-efficient heuristic framework for solving large-sized instances of the VRPTW-D. The three levels of this heuristic consisted of generating an initial solution, assigning a drone to a delivery, and a combining procedure to reduce the number of trucks used. Competitive results were obtained without exploiting sophisticated mathematical programming, decomposition algorithms, or meta-heuristics. Heuristic solutions are often better than those obtained by the solver and not far from the optimal solution, though obtained in a shorter time.

We presented the results and analysis of our computational experiments. Results showed that the coordinated delivery system has significant economic benefits over the truck-only system. One of the weak points in our analysis is the fact that we used a heuristic algorithm, rather than an exact algorithm. We are unaware of any techniques that find optimal solutions for large-sized instances, but we expect them to become available as interest in VRPTW-D increases. We believe that our modeling and algorithmic contributions can be a means to overcome the limits of drone-aided routing and to accelerate the commercialization of coordinated logistics with trucks and drones.

Chapter 5

Conclusions

A drone is becoming an essential component of future logistics. It can be operated as a multipurpose agent in various industry fields, owing to its low price, rapid speed, and flexible operation by the aerial operation. When integrating drones into the existing system, both the advantages and the limitations of operating drones have to be considered. Therefore, drone operations pose new methodological challenges and the need for creating new knowledge. In this dissertation, new variants of optimization problems are developed; facility location problem with drones (FLP-D), scheduling-location problem with drones (ScheLoc-D), and vehicle routing problem with time windows and drones (VRPTW-D).

In Chapter 2, we developed a stochastic facility location model considering the uncertain characteristics of drone operation. A chance constraint, also known as a probabilistic constraint, is used to manage the uncertainty. The solution approach based on Benders decomposition is developed. The proposed algorithm can be used as an exact algorithm if the termination condition is tightly set. There might be a controversy about the assumption that the flight distance of a drone follows an exponential distribution. Although similar assumptions are used in various fields, some researchers may not accept this assumption. A distributionally robust approach, instead of a stochastic optimization, might be applied to this model in the future research. This approach can alleviate complaints about the distributional assumption. We also did not consider costs and times for battery recharging. We assumed that these costs have a relatively small impact on decision making. However, further consideration of various cost factors can also yield meaningful conclusions. There may be a need to check service levels, the most representative performance indicator of supply chain management. So, a model that manages both the service level from the customer's perspective and the drone return probability from the supply perspective can be developed using conditional probability. With consideration of these points, solutions that incorporate drones into disaster management might become more practical and can provide deeper insights.

In Chapter 3, we proposed a scheduling-location problem with drones (ScheLoc-D), a new methodology for integrating operational planning decision with strategic planning decision. Based on Dantzig-Wolfe decomposition, an extended formulation is proposed to improve the LP-relaxed bound and utilize it. Further research on developing an exact algorithm is promising. Dynamic programming could be used to efficiently solve the pricing subproblem to develop a Branch-and-Price algorithm. The solution structure and the progress of column generation could also be investigated more for insightful findings.

In Chapter 4, we defined a new variant of the VRP, VRPTW-D, a vehicle routing problem with time windows and drones in which several trucks and drones worked together to provide service to customers within given time windows. We also proposed a three-stage savings-based heuristic, a simple yet time-efficient heuristic framework for solving large-sized instances of the VRPTW-D. As this is one of the first attempts to consider time window constraints in mixed use of trucks and drones for delivery, there are many promising areas for future research. For example, the VRPTW-D can be extended to account for the time cost of waiting. This is of particular interest when the waiting time causes major operational problems for the delivery system. Time-constrained routing and scheduling problems with drones can lead to more natural synchronization of the two different types of vehicles used in tandem. Another challenging topic for future research would be to develop exact solution approaches that can find optimal solutions for the VRPTW-D within reasonable computing times.

In short, we have thoroughly concerned and analyzed the drone research, and newly proposed the basis of the drone-operation system to be applied in the future. Since this dissertation provided new models related to drone operation, as mentioned above, promising follow-up studies can be performed in various ways. Also, the results of this study can be applied to various fields. Automatic mobile robot industry, which features unmanned operation, is the field where the results of this study can be best applied. Therefore, we believe that our modeling and algorithmic procedure contribute to overcoming the limits of drone-integrated systems and accelerating the commercialization of the applications of drones.

Bibliography

[1] N. Agatz, P. Bouman, and M. Schmidt, *Optimization approaches for the traveling salesman problem with drone*, Transportation Science, 52 (2018), pp. 965–981.

[2] Z. Akca, R. Berger, and T. Ralphs, *Modeling and solving location routing and scheduling problems*, in Proceedings of the eleventh INFORMS computing society meeting, 2008, pp. 309–330.

[3] U. Akinc and B. M. Khumawala, *An efficient branch and bound algorithm for the capacitated warehouse location problem*, Management Science, 23 (1977), pp. 585–594.

[4] S. Albers, N. Garg, and S. Leonardi, *Minimizing stall time in single and parallel disk systems*, Journal of the ACM (JACM), 47 (2000), pp. 969–986.

[5] M. L. Balinski, *Integer programming: methods, uses, computations*, Management science, 12 (1965), pp. 253–313.

[6] A. Bar-Noy, S. Guha, J. Naor, and B. Schieber, *Approximating the throughput of multiple machines in real-time scheduling*, SIAM Journal on Computing, 31 (2001), pp. 331–352.

[7] J. F. Bard, G. Kontoravdis, and G. Yu, *A branch-and-cut procedure for the vehicle routing problem with time windows*, Transportation Science, 36 (2002), pp. 250–269.

[8] C. Barnhart, E. L. Johnson, G. L. Nemhauser, M. W. Savelsbergh, and P. H. Vance, *Branch-and-price: Column generation for solving huge integer programs*, Operations research, 46 (1998), pp. 316–329.

[9] S. Baruah, G. Koren, D. Mao, B. Mishra, A. Raghunathan, L. Rosier, D. Shasha, and F. Wang, *On the competitiveness of on-line real-time task scheduling*, Real-Time Systems, 4 (1992), pp. 125–144.

[10] J. E. Beasley, *Route first—cluster second methods for vehicle routing*, Omega, 11 (1983), pp. 403–408.

[11] M. J. Beckmann, *Location of an economic activity*, in Lectures on Location Theory, Springer, 1999, pp. 61–69.

[12] C. Berge, *Two theorems in graph theory*, Proceedings of the National Academy of Sciences of the United States of America, 43 (1957), p. 842.

[13] O. Berman, D. Krass, and Z. Drezner, *The gradual covering decay location problem on a network*, European Journal of Operational Research, 151 (2003), pp. 474–480.

[14] P. Bouman, N. Agatz, and M. Schmidt, *Dynamic programming approaches for the traveling salesman problem with drone*, Networks, 72 (2018), pp. 528–542.

[15] K. Braekers, K. Ramaekers, and I. Van Nieuwenhuyse, *The vehicle routing problem: State of the art classification and review*, Computers & Industrial Engineering, 99 (2016), pp. 300–313.

[16] M. L. Brandeau and S. S. Chiu, *An overview of representative problems in location research*, Management science, 35 (1989), pp. 645–674.

[17] O. Bräysy and M. Gendreau, *Vehicle routing problem with time windows, part i: Route construction and local search algorithms*, Transportation science, 39 (2005), pp. 104–118.

[18] ——, *Vehicle routing problem with time windows, part ii: Metaheuristics*, Transportation science, 39 (2005), pp. 119–139.

[19] R. Carbone, *Public facilities location under stochastic demand*, INFOR: Information Systems and Operational Research, 12 (1974), pp. 261–270.

[20] J. G. Carlsson and S. Song, *Coordinated logistics with a truck and a drone*, Management Science, 64 (2018), pp. 4052–4069.

[21] S. Chand, *Decision/forecast horizon for a single facility dynamic location/relocation problem*, Operations Research Letters, 7 (1988), pp. 247–251.

[22] A. Charnes and W. W. Cooper, *Chance-constrained programming*, Management science, 6 (1959), pp. 73–79.

[23] N. Christofides and J. E. Beasley, *Extensions to a lagrangean relaxation approach for the capacitated warehouse location problem*, European Journal of Operational Research, 12 (1983), pp. 19–28.

[24] Y. Chu and Q. Xia, *Generating benders cuts for a general class of integer programming problems*, in International Conference on Integration of Artificial Intelligence (AI) and Operations Research (OR) Techniques in Constraint Programming, Springer, 2004, pp. 127–141.

[25] V. Chvatal, *A greedy heuristic for the set-covering problem*, Mathematics of operations research, 4 (1979), pp. 233–235.

[26] G. Clarke and J. W. Wright, *Scheduling of vehicles from a central depot to a number of delivery points*, Operations research, 12 (1964), pp. 568–581.

[27] J.-F. Cordeau, F. Furini, and I. Ljubić, *Benders decomposition for very large scale partial set covering and maximal covering location problems*, European Journal of Operational Research, 275 (2019), pp. 882–896.

[28] J.-F. Cordeau and Q. Groupe d'études et de recherche en analyse des décisions (Montréal, *The VRP with time windows*, Groupe d'études et de recherche en analyse des décisions Montréal, 2000.

[29] G. Cornnejols, M. Fisher, and G. Nemhauser, *Location of bank accounts of optimize float: An analytic study of exact and approximate algorithm*, Management Science, 23 (1977), pp. 789–810.

[30] G. Cornuéjols, G. Nemhauser, and L. Wolsey, *The uncapicitated facility location problem*, tech. rep., Cornell University Operations Research and Industrial Engineering, 1983.

[31] I. Correia and F. Saldanha-da Gama, *Facility location under uncertainty*, in Location science, Springer, 2019, pp. 185–213.

[32] G. B. Dantzig and J. H. Ramser, *The truck dispatching problem*, Management science, 6 (1959), pp. 80–91.

[33] G. B. Dantzig and P. Wolfe, *Decomposition principle for linear programs*, Operations research, 8 (1960), pp. 101–111.

[34] M. S. Daskin, *Network and discrete location: models, algorithms, and applications*, John Wiley & Sons, 2011.

[35] P. Davis and T. Ray, *A branch-bound algorithm for the capacitated facilities location problem*, Naval Research Logistics Quarterly, 16 (1969), pp. 331–344.

[36] G. Desaulniers, J. Desrosiers, and M. M. Solomon, *Column generation*, vol. 5, Springer Science & Business Media, 2006.

[37] M. Desrochers, J. Desrosiers, and M. Solomon, *A new optimization algorithm for the vehicle routing problem with time windows*, Operations research, 40 (1992), pp. 342–354.

[38] M. Drexl, *Synchronization in vehicle routing—a survey of vrps with multiple synchronization constraints*, Transportation Science, 46 (2012), pp. 297–316.

[39] Z. Drezner and G. Wesolowsky, *Facility location when demand is time dependent*, Naval Research Logistics (NRL), 38 (1991), pp. 763–777.

[40] M. Efroymson and T. Ray, *A branch-bound algorithm for plant location*, Operations Research, 14 (1966), pp. 361–368.

[41] D. Elvikis, H. W. Hamacher, and M. T. Kalsch, *Simultaneous scheduling and location (scheloc): the planar scheloc makespan problem*, Journal of Scheduling, 12 (2009), pp. 361–374.

[42] D. Erlenkotter, *A dual-based procedure for uncapacitated facility location*, Operations Research, 26 (1978), pp. 992–1009.

[43] R. Z. Farahani, Z. Drezner, and N. Asgari, *Single facility location and relocation problem with time dependent weights and discrete planning horizon*, Annals of Operations Research, 167 (2009), pp. 353–368.

[44] L.-L. Gao and E. P. Robinson Jr, *A dual-based optimization procedure for the two-echelon uncapacitated facility location problem*, Naval Research Logistics (NRL), 39 (1992), pp. 191–212.

[45] N. K. Gentry, R. Hsieh, and L. K. Nguyen, *Multi-use uav docking station systems and methods*, July 12 2016. US Patent 9,387,928.

[46] A. M. Geoffrion, *Generalized benders decomposition*, Journal of optimization theory and applications, 10 (1972), pp. 237–260.

[47] A. M. Geoffrion and G. W. Graves, *Multicommodity distribution system design by benders decomposition*, Management science, 20 (1974), pp. 822–844.

[48] J. Goldberg, R. Dietrich, J. M. Chen, M. G. Mitwasi, T. Valenzuela, and E. Criss, *Validating and applying a model for locating emergency medical vehicles in tuczon, az*, European Journal of Operational Research, 49 (1990), pp. 308–324.

[49] B. L. Golden, S. Raghavan, and E. A. Wasil, *The vehicle routing problem: latest advances and new challenges*, vol. 43, Springer Science & Business Media, 2008.

[50] E. Grass, K. Fischer, and A. Rams, *An accelerated l-shaped method for solving two-stage stochastic programs in disaster management*, Annals of Operations Research, 284 (2020), pp. 557–582.

[51] B. W. Gu, S. Y. Choi, Y. S. Choi, G. Cai, L. Seneviratne, and C. T. Rim, *Novel roaming and stationary tethered aerial robots for continuous mobile missions in nuclear power plants*, Nuclear Engineering and Technology, 48 (2016), pp. 982–996.

[52] Q. M. Ha, Y. Deville, Q. D. Pham, and M. H. Ha, *On the min-cost traveling salesman problem with drone*, Transportation Research Part C: Emerging Technologies, 86 (2018), pp. 597–621.

[53] Q. M. Ha, Y. Deville, Q. D. Pham, and M. H. Hà, *A hybrid genetic algorithm for the traveling salesman problem with drone*, Journal of Heuristics, 26 (2020), pp. 219–247.

[54] S. L. Hakimi, *Optimum locations of switching centers and the absolute centers and medians of a graph*, Operations research, 12 (1964), pp. 450–459.

[55] ——, *Optimum distribution of switching centers in a communication network and some related graph theoretic problems*, Operations research, 13 (1965), pp. 462–475.

[56] T. S. Hale and C. R. Moberg, *Location science research: a review*, Annals of operations research, 123 (2003), pp. 21–35.

[57] A. M. Ham, *Integrated scheduling of m-truck, m-drone, and m-depot constrained by time-window, drop-pickup, and m-visit using constraint programming*, Transportation Research Part C: Emerging Technologies, 91 (2018), pp. 1–14.

[58] H. Hamacher and H. Hennes, *Integrated scheduling and location models: single machine makespan problems*, Studies in Locational Analysis, 16 (2007), pp. 77–90.

[59] C. Hessler and K. Deghdak, *Discrete parallel machine makespan scheloc problem*, Journal of Combinatorial Optimization, 34 (2017), pp. 1159–1186.

[60] A. M. Hormozi and B. M. Khumawala, *An improved algorithm for solving a multi-period facility location problem*, IIE transactions, 28 (1996), pp. 105–114.

[61] K. Jakob and P. M. Pruzan, *The simple plant location problem: survey and synthesis*, European journal of operational research, 12 (1983), pp. 36–81.

[62] M. T. Kalsch, *Scheduling-location (ScheLoc) models, theory and algorithms*, Verlag Dr. Hut, 2009.

[63] M. T. Kalsch and Z. Drezner, *Solving scheduling and location problems in the plane simultaneously*, Computers & operations research, 37 (2010), pp. 256–264.

[64] R. M. Karp, *Reducibility among combinatorial problems*, in Complexity of computer computations, Springer, 1972, pp. 85–103.

[65] L. Kaufman, M. V. Eede, and P. Hansen, *A plant and warehouse location problem*, Journal of the Operational Research Society, 28 (1977), pp. 547–554.

[66] S. Khodaparasti, M. E. Bruni, P. Beraldi, H. Maleki, and S. Jahedi, *A multi-period location-allocation model for nursing home network planning under uncertainty*, Operations Research for Health Care, 18 (2018), pp. 4–15.

[67] D. Kim, K. Lee, and I. Moon, *Stochastic facility location model for drones considering uncertain flight distance*, Annals of Operations Research, 283 (2019), pp. 1283–1302.

[68] S. Kim and I. Moon, *Traveling salesman problem with a drone station*, IEEE Transactions on Systems, Man, and Cybernetics: Systems, 49 (2018), pp. 42–52.

[69] P. Kitjacharoenchai, B.-C. Min, and S. Lee, *Two echelon vehicle routing problem with drones in last mile delivery*, International Journal of Production Economics, 225 (2020), p. 107598.

[70] J. G. Klincewicz and H. Luss, *A dual-based algorithm for multiproduct uncapacitated facility location*, Transportation Science, 21 (1987), pp. 198–206.

[71] K. Knight and J. Hofer, *Vehicle scheduling with timed and connected calls: A case study*, Journal of the Operational Research Society, 19 (1968), pp. 299–310.

[72] G. Laporte, *Fifty years of vehicle routing*, Transportation science, 43 (2009), pp. 408–416.

[73] G. LAPORTE, S. Nickel, and F. S. d. Gama, *Location science*, SPRINGER INTERNATIONAL PU, 2016.

[74] C. Lee and J. Han, *Benders-and-price approach for electric vehicle charging station location problem under probabilistic travel range*, Transportation Research Part B: Methodological, 106 (2017), pp. 130–152.

[75] J. K. Lenstra and A. R. Kan, *Complexity of vehicle routing and scheduling problems*, Networks, 11 (1981), pp. 221–227.

[76] H. Li, S. K. Mukhopadhyay, J.-j. Wu, L. Zhou, Z. Du, et al., *Balanced maximal covering location problem and its application in bike-sharing*, International Journal of Production Economics, 223 (2020), p. 107513.

[77] X. Li, M. Ramshani, and Y. Huang, *Cooperative maximal covering models for humanitarian relief chain management*, Computers & industrial engineering, 119 (2018), pp. 301–308.

[78] M. Liu and X. Liu, *Distributionally robust parallel machine scheloc problem under service level constraints*, IFAC-PapersOnLine, 52 (2019), pp. 875–880.

[79] F. V. Louveaux, *Discrete stochastic location models*, Annals of Operations research, 6 (1986), pp. 21–34.

[80] M. E. Lübbecke and J. Desrosiers, *Selected topics in column generation*, Operations research, 53 (2005), pp. 1007–1023.

[81] A. S. Manne, *Plant location under economies-of-scale—decentralization and computation*, Management Science, 11 (1964), pp. 213–235.

[82] A. Marín, L. I. Martínez-Merino, A. M. Rodríguez-Chía, and F. Saldanha-da Gama, *Multi-period stochastic covering location problems: Modeling framework and solution approach*, European journal of operational research, 268 (2018), pp. 432–449.

[83] M. T. Melo, S. Nickel, and F. Saldanha-Da-Gama, *Facility location and supply chain management–a review*, European journal of operational research, 196 (2009), pp. 401–412.

[84] S. Meng and B. Shia, *Set covering location models with stochastic critical distances*, Journal of the Operational Research Society, 64 (2013), pp. 945–958.

[85] B. L. Miller and H. M. Wagner, *Chance constrained programming with joint constraints*, Operations Research, 13 (1965), pp. 930–945.

[86] C. C. Murray and A. G. Chu, *The flying sidekick traveling salesman problem: Optimization of drone-assisted parcel delivery*, Transportation Research Part C: Emerging Technologies, 54 (2015), pp. 86–109.

[87] C. C. Murray and R. Raj, *The multiple flying sidekicks traveling salesman problem: Parcel delivery with multiple drones*, Transportation Research Part C: Emerging Technologies, 110 (2020), pp. 368–398.

[88] F. Muttin, *Umbilical deployment modeling for tethered uav detecting oil pollution from ship*, Applied Ocean Research, 33 (2011), pp. 332–343.

[89] Y.-S. Myung, H.-g. Kim, and D.-w. Tcha, *A bi-objective uncapacitated facility location problem*, European Journal of Operational Research, 100 (1997), pp. 608–616.

[90] A. Orda and R. Rom, *Location of central nodes in time varying computer networks*, Operations research letters, 10 (1991), pp. 143–152.

[91] A. Otto, N. Agatz, J. Campbell, B. Golden, and E. Pesch, *Optimization approaches for civil applications of unmanned aerial vehicles (uavs) or aerial drones: A survey*, Networks, 72 (2018), pp. 411–458.

[92] S. H. Owen and M. S. Daskin, *Strategic facility location: A review*, European journal of operational research, 111 (1998), pp. 423–447.

[93] M. Padberg and G. Rinaldi, *Optimization of a 532-city symmetric traveling salesman problem by branch and cut*, Operations research letters, 6 (1987), pp. 1–7.

[94] Y. Park, P. Nielsen, and I. Moon, *Unmanned aerial vehicle set covering problem considering fixed-radius coverage constraint*, Computers & Operations Research, (2020), p. 104936.

[95] J. A. Paul and L. MacDonald, *Location and capacity allocations decisions to mitigate the impacts of unexpected disasters*, European Journal of Operational Research, 251 (2016), pp. 252–263.

[96] J. Pereira and I. Averbakh, *The robust set covering problem with interval data*, Annals of Operations Research, 207 (2013), pp. 217–235.

[97] H. Pirkul and V. Jayaraman, *A multi-commodity, multi-plant, capacitated facility location problem: formulation and efficient heuristic solution*, Computers & Operations Research, 25 (1998), pp. 869–878.

[98] S. Poikonen, B. Golden, and E. A. Wasil, *A branch-and-bound approach to the traveling salesman problem with a drone*, INFORMS Journal on Computing, 31 (2019), pp. 335–346.

[99] S. Poikonen, X. Wang, and B. Golden, *The vehicle routing problem with drones: Extended models and connections*, Networks, 70 (2017), pp. 34–43.

[100] S. Ponboon, A. G. Qureshi, and E. Taniguchi, *Branch-and-price algorithm for the location-routing problem with time windows*, Transportation Research Part E: Logistics and Transportation Review, 86 (2016), pp. 1–19.

[101] J. Puerto and A. M. Rodríguez-Chía, *Location of a moving service facility*, Mathematical methods of operations research, 49 (1999), pp. 373–393.

[102] L. D. P. Pugliese and F. Guerriero, *Last-mile deliveries by using drones and classical vehicles*, in International Conference on Optimization and Decision Science, Springer, 2017, pp. 557–565.

[103] H. Pullen and M. Webb, *A computer application to a transport scheduling problem*, The computer journal, 10 (1967), pp. 10–13.

[104] M. Rajabzadeh, M. Ziaee, and A. Bozorgi-Amiri, *Integrated approach in solving parallel machine scheduling and location (scheloc) problem*, International Journal of Industrial Engineering Computations, 7 (2016), pp. 573–584.

[105] C. S. ReVelle and H. A. Eiselt, *Location analysis: A synthesis and survey*, European journal of operational research, 165 (2005), pp. 1–19.

[106] C. S. Revelle, H. A. Eiselt, and M. S. Daskin, *A bibliography for some fundamental problem categories in discrete location science*, European journal of operational research, 184 (2008), pp. 817–848.

[107] J. Rosenhead, M. Elton, and S. K. Gupta, *Robustness and optimality as criteria for strategic decisions*, Journal of the Operational Research Society, 23 (1972), pp. 413–431.

[108] E. W. Saad, J. L. Vian, M. A. Vavrina, J. A. Nisbett, and D. C. Wunsch, *Vehicle base station*, Dec. 2 2014. US Patent 8,899,903.

[109] D. Sacramento, D. Pisinger, and S. Ropke, *An adaptive large neighborhood search metaheuristic for the vehicle routing problem with drones*, Transportation Research Part C: Emerging Technologies, 102 (2019), pp. 289–315.

[110] D. Schermer, M. Moeini, and O. Wendt, *A hybrid vns/tabu search algorithm for solving the vehicle routing problem with drones and en route operations*, Computers & Operations Research, 109 (2019), pp. 134–158.

[111] ——, *A matheuristic for the vehicle routing problem with drones and its variants*, Transportation Research Part C: Emerging Technologies, 106 (2019), pp. 166–204.

[112] G. Schrimpf, J. Schneider, H. Stamm-Wilbrandt, and G. Dueck, *Record breaking optimization results using the ruin and recreate principle*, Journal of Computational Physics, 159 (2000), pp. 139–171.

[113] S. M. Shavarani, M. G. Nejad, F. Rismanchian, and G. Izbirak, *Application of hierarchical facility location problem for optimization of a drone delivery system: a case study of amazon prime air in the city of san francisco*, The International Journal of Advanced Manufacturing Technology, 95 (2018), pp. 3141–3153.

[114] D. B. Shmoys, É. Tardos, and K. Aardal, *Approximation algorithms for facility location problems*, in Proceedings of the twenty-ninth annual ACM symposium on Theory of computing, 1997, pp. 265–274.

[115] H. K. Smith, G. Laporte, and P. R. Harper, *Locational analysis: highlights of growth to maturity*, Journal of the Operational Research Society, 60 (2009), pp. S140–S148.

[116] L. V. Snyder, *Facility location under uncertainty: a review*, IIE transactions, 38 (2006), pp. 547–564.

[117] M. M. Solomon, *Algorithms for the vehicle routing and scheduling problems with time window constraints*, Operations research, 35 (1987), pp. 254–265.

[118] M. M. Solomon and J. Desrosiers, *Survey paper—time window constrained routing and scheduling problems*, Transportation science, 22 (1988), pp. 1–13.

[119] K. Spielberg, *Algorithms for the simple plant-location problem with some side conditions*, Operations Research, 17 (1969), pp. 85–111.

[120] A. Tayal, A. Gunasekaran, S. P. Singh, R. Dubey, and T. Papadopoulos, *Formulating and solving sustainable stochastic dynamic facility layout problem: A key to sustainable operations*, Annals of Operations Research, 253 (2017), pp. 621–655.

[121] C. Toregas, R. Swain, C. ReVelle, and L. Bergman, *The location of emergency service facilities*, Operations research, 19 (1971), pp. 1363–1373.

[122] P. Toth and D. Vigo, *Vehicle routing: problems, methods, and applications*, SIAM, 2014.

[123] T. L. Urban, *Solution procedures for the dynamic facility layout problem*, Annals of operations research, 76 (1998), pp. 323–342.

[124] T. J. Van Roy, *A cross decomposition algorithm for capacitated facility location*, Operations Research, 34 (1986), pp. 145–163.

[125] T. J. Van Roy and D. Erlenkotter, *A dual-based procedure for dynamic facility location*, Management Science, 28 (1982), pp. 1091–1105.

[126] K. A. Verckens and C. Macmillan, *Pitney bowes parcel shipping index reports continued growth as global parcel volume exceeds 100 billion for first time ever*, Pitney Bowes, (2020).

[127] X. Wang, M. K. Lim, and Y. Ouyang, *A continuum approximation approach to the dynamic facility location problem in a growing market*, Transportation Science, 51 (2017), pp. 343–357.

[128] X. Wang, S. Poikonen, and B. Golden, *The vehicle routing problem with drones: several worst-case results*, Optimization Letters, 11 (2017), pp. 679–697.

[129] Z. Wang and J.-B. Sheu, *Vehicle routing problem with drones*, Transportation research part B: methodological, 122 (2019), pp. 350–364.

[130] G. O. Wesolowsky, *Dynamic facility location*, Management Science, 19 (1973), pp. 1241–1248.

[131] E. E. Yurek and H. C. Ozmutlu, *A decomposition-based iterative optimization algorithm for traveling salesman problem with drone*, Transportation Research Part C: Emerging Technologies, 91 (2018), pp. 249–262.

[132] B. Zhang, J. Peng, and S. Li, *Covering location problem of emergency service facilities in an uncertain environment*, Applied Mathematical Modelling, 51 (2017), pp. 429–447.

국문초록

4차 산업혁명이라 불리는 새로운 흐름에 따라, 사회 전반에서 구조적인 변화가 지속적으로 일어나고 있으며 과학기술 및 산업분야를 아우르는 신성장동력들이 주목받고 있다. 특히, 빠른 속도로 발전 중인 드론 기술이 가져오는 경제, 사회적 변화는 4차 산업혁명의 핵심요소이다. 학계 및 산업계는 이미 드론의 상업적 활용과 공공서비스 영역에서의 성과를 위한 기술적 연구를 활발히 수행 중이다. 반면에 드론 활용과 관련된 운영과학적 연구는 상대적으로 미흡하다. 드론의 활용 가치를 최대화하기 위해서는 드론이 가진 장점을 충분히 활용하면서도 드론의 물리적 한계를 고려한 운영 계획이 필요하다. 따라서 기존의 정의된 문제로는 제한적인 적용만이 가능하기 때문에 새로운 관점에서의 문제 정의가 필요하다.

본 논문에서는 드론 운용이 고려된 선진 물류 체계에 대한 연구를 수행한다. 구체적으로는, 드론을 고려한 통합 물류 체계의 네트워크 설계와 경로 계획을 위한 새로운 방법론을 제안한다. 물류 네트워크를 구성하기 위해서는 시설의 위치를 결정하는 계획이 선행적으로 수립되어야 한다. 시설의 위치를 합리적으로 결정하기 위해서 드론 운용의 내재된 불확실성들을 추계학적으로 대응한다. 이를 기반으로 시설의 위치와 드론의 배치를 동시에 결정한다. 그 다음으로, 전략적 수준의 계획인 시설위치결정과 운영적 수준의 계획인 배송 스케줄링을 동시에 의사 결정하는 통합모형을 제시한다. 마지막으로 드론이 트럭과 협력하여 배송 임무를 함께 수행하는 시스템에 대해서 연구한다. 두 운송수단의 상호보완적 특성을 활용하여 더 빠르고, 비용 효율적인 배송을 추구한다. 요약하면, 드론의 성공적인 활용을 위해 전통적인 최적화 문제의 새로운 확장 문제들을 제안하였다. 새롭게 개발된 모든

모형들의 실용적인 풀이 기법들도 함께 제시된다.

본 연구의 결과는 드론 운용의 부담을 완화시키며 드론의 활용 분야를 더욱 확대하는 기반을 조성할 것이다. 환언하면 본 연구는 드론이 물류 분야에서 단순한 배송 영역이 아닌 패러다임 자체를 변화시키는 역할을 수행하는 출발점이 될 것이다.

주요어: 위치결정문제, 일정계획-위치결정문제, 경로결정문제, 드론, 발견적기법
학번: 2015-30234

감사의 글

당찬 포부를 가지고 대학원에 입학하여 영광스런 졸업에 이르기까지 정말 오랜 시간이 흘렀습니다. 그동안 받은 관심과 사랑을 회상해보면 지나온 모든 시간이 행운이었고, 또 행복이었다고 자신있게 말할 수 있습니다. 학업의 결실인 학위 논문이 완성되기까지 수많은 사람들의 도움을 받았습니다. 감사의 마음을 모두 표현하기에는 어떠한 말로도 부족하지만, 소중한 사람들과 함께 했던 값진 시간을 추억하기 위해 짧게나마 적어보고자 합니다.

부족한 모습이 많았던 저에게 따뜻한 격려와 귀중한 가르침을 주신 문일경 교수님께 깊이 감사드립니다. 맡으신 일에 항상 주인의식과 책임감을 가지시는 모습을 바라보며 성실한 삶의 태도에 대해 많이 배울 수 있었습니다. 또한 저에게 항상 용기를 복돋아주시고 끊임없이 조언해주셔서 연구에 정진할 수 있었고 매사에 자신감을 가질 수 있었습니다. 교수님의 가르침으로 인해 저는 학문적뿐만 아니라 인성적으로도 한층 더 성장할 수 있었습니다. 졸업한 후에도 교수님께 자랑스런 제자가 될 수 있도록 온 힘을 다하겠습니다.

본 학위 논문의 심사를 맡아주신 홍유석 교수님, 장우진 교수님, 장윤석 교수님, 그리고 김병수 교수님께 감사의 인사를 드립니다. 심사위원장을 맡아주신 홍유석 교수님, 마지막 순간까지 꼼꼼하게 심사해주셔서 본 연구가 더욱 큰 의미를 갖게 된 것 같습니다. 심사 후에 조금은 안주할 뻔했던 저에게 또 다시 깨달음을 주셨고 박사 학위에 걸맞는 논문을 작성할 수 있었습니다. 장우진 교수님, 연구의 타당성 및 합리성에 대해서 조언해주셔서 연구의 완성도를 제고할 수 있었습니다. 제가 생각하지 못했던 내용들을 말씀해주셔서 연구에 많은 도움이 되었습니다. 장윤석 교수님, 실용적인 조언과 다양한 기술 지식들을 전달해주셔서 의미 있는 연구를

수행할 수 있었습니다. 교수님의 조언으로 본 연구의 본질에 대해 다시 한번 깊이 생각할 수 있었으며 올바른 방향성을 찾을 수 있었습니다. 김병수 교수님, 세부적인 내용까지 날카롭게 짚어주셨던 교수님의 세심함에 감동하였습니다. 논문에 대한 조언과 더불어 심사과정 동안 해주셨던 따뜻한 조언과 격려는 앞으로 연구자로서의 삶에 큰 힘이 될 것 같습니다. 또한 다양한 전공지식과 폭넓은 안목을 가질 수 있도록 지식과 지혜를 가르쳐주신 산업공학과 모든 교수님들께도 감사드립니다. 대학원생들이 연구에 집중할 수 있도록 교수님들께서 지원해주시고 지도해주셔서 좋은 환경에서 공부하고 연구할 수 있었습니다.

오랜 시간 함께 생활했던 공급망관리 연구실 동료들에게도 감사의 인사를 전합니다. 힘들고 어려울 때마다 서로에게 버팀목이 되어주고 연구와 미래에 대해 이야기를 나누었던 뜻깊은 시간을 잊지 못할 것 같습니다. 힘든 순간이 찾아오더라도 현명하게 극복하여 무탈하고 행복한 연구실 생활을 영위할 것이라 믿습니다. 고락을 같이한 인연이 영원하기를 바라며 앞날의 무궁한 발전과 건승을 기원합니다. 연구실 동료뿐만 아니라 재학기간 동안 학교에서 만난 소중한 인연들에게도 모두 감사의 마음을 전합니다.

석별의 아쉬움은 함께 했던 소중한 추억들로 대신 채우고 제가 받았던 도움과 배려는 절대 잊지 않겠습니다. 그리고 저 또한 누군가에게 도움을 줄 수 있는 사람이 되기 위해 더욱 정진하고 발전하겠습니다. 마지막으로 언제나 변함없이 아들을 믿고 지원해주시는 부모님께 감사의 인사를 드립니다. 그리고 항상 형을 믿고 따라주는 동생에게도 감사합니다. 더 좋은 아들이자 형이 될 수 있도록 최선을 다하겠습니다. 내 인생의 의미가 되어준 우리 가족에게 항상 좋은 일만 있기를 기원합니다. 사랑합니다.

2021년 1월

김동욱 올림