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Research article

Linear barycentric rational interpolation method for solving Kuramoto-Sivashinsky equation

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Abstract: The Kuramoto-Sivashinsky (KS) equation being solved by the linear barycentric rational interpolation method (LBRIM) is presented. Three kinds of linearization schemes, direct linearization, partial linearization and Newton linearization, are presented to get the linear equation of the Kuramoto-Sivashinsky equation. Matrix equations of the discrete Kuramoto-Sivashinsky equation are also given. The convergence rate of LBRIM for solving the KS equation is also proved. At last, two examples are given to prove the theoretical analysis.

Keywords: barycentric rational interpolation; collocation method; Kuramoto-Sivashinsky equation; non-linear PDE
Mathematics Subject Classification: 65D32, 65D30, 65R20

1. Introduction

Lots of physical phenomena can be expressed by non-linear partial differential equations (PDE), including, inter alia, dissipative and dispersive PDE. In this paper, we consider the Kuramoto-Sivashinsky (KS) equation

$$\frac{\partial\phi}{\partial t}+\gamma\frac{\partial^4\phi}{\partial s^4}+\frac{\partial^2\phi}{\partial s^2}+\phi\frac{\partial\phi}{\partial s}=\varphi(s,t)\ 0\le s\le 1,\ 0\le t\le T,\ \gamma>0, \tag{1.1}$$

$$\phi(0,t)=0,\ \phi(1,t)=0,\ \phi_{ss}(0,t)=0,\phi_{ss}(1,t)=0,\ 0<t<T, \tag{1.2}$$

$$\phi(s,0)=\varphi(s),\ 0\le s\le 1, \tag{1.3}$$

where $\gamma\in R$ is the constant.

The KS equation plays an important role in physics such as in diffusion, convection and so on. Lots of attention has been paid by researchers in recent years. An H_1-Galerkin mixed finite element method for the KS equation was proposed in [1], lattice Boltzmann models for the Kuramoto-Sivashinsky equation were studied in [2], Backward difference formulae (BDF) methods for the KS equation were investigate in [3]. Stability regions and results for the Korteweg-de Vries-Burgers and Kuramoto-Sivashinsky equations were given in [4, 5], respectively. In [6], an improvised quintic B-spline extrapolated collocation technique was used to solve the KS equation, and the stability of the technique was analyzed using the von Neumann scheme, which was found to be unconditionally stable. In [7], a septic Hermite collocation method (SHCM) was proposed to simulate the KS equation, and the nonlinear terms of the KS equation were linearized using the quasi-linearization process. In [8], a semidiscrete approach was presented to solve the variable-order (VO) time fractional 2D KS equation, and the differentiation operational matrices and the collocation technique were used to get a linear system of algebraic equations. In [9] the discrete Legendre polynomials (LPs) and the collocation scheme for nonlinear space-time fractional KdV-Burgers-Kuramoto equation were presented.

In order to avoid the Runge's phenomenon, barycentric interpolation [10–12] was developed. In recent years, linear rational interpolation (LRI) was proposed by Floater [13–15], and error of linear rational interpolation was also proved. The barycentric interpolation collocation method (BICM) has been developed by Wang et al. [22–25], and the algorithm of BICM has been used for linear/non-linear problems [21]. Volterra integro-differential equation (VIDE) [16, 20], heat equation (HE) [17], biharmonic equation (BE) [18], the Kolmogorov-Petrovskii-Piskunov (KPP) equation [19], fractional differential equations [20], fractional reaction-diffusion equation [28], semi-infinite domain problems [27] and biharmonic equation [26], plane elastic problems [29] have been studied by the linear barycentric interpolation collocation method (LBICM), and their convergence rates also have been proved.

In order to solve the KS equation efficiently, the LBRIM is presented. Because the nonlinear part of the KS equation cannot be solved directly, three kinds of linearization methods, including direct linearization, partial linearization and Newton linearization, are presented. Then, the nonlinear part of the KS equation is translated into the linear part, three kinds of iterative schemes are presented, and matrix equation of the linearization schemes are constructed. The convergence rate of the LBRCM for the KS equation is also given. At last, two numerical examples are presented to validate the theoretical analysis.

2. Linearization for KS equation

In the following, the KS equation is changed into the linear equation by the linearization scheme, including direct linearization, partial linearization and Newton linearization.

2.1. Direct linearization

For the Kuramoto-Sivashinskyr equation with the initial value of nonlinear term $\phi\frac{\partial\phi}{\partial s}$ is changed to $\phi_0\frac{\partial\phi_0}{\partial s}$,

$$\frac{\partial\phi}{\partial t}+\gamma\frac{\partial^4\phi}{\partial s^4}+\frac{\partial^2\phi}{\partial s^2}+\phi_0\frac{\partial\phi_0}{\partial s}=\varphi(s,t), \tag{2.1}$$

and then we get the linear scheme as

$$\frac{\partial \phi_n}{\partial t} + \gamma \frac{\partial^4 \phi_n}{\partial s^4} + \frac{\partial^2 \phi_n}{\partial s^2} = -\phi_{n-1} \frac{\partial \phi_{n-1}}{\partial s} + \varphi(s,t), a \le s \le b, 0 \le t \le T. \tag{2.2}$$

2.2. Partial linearization

By the partial linearization, nonlinear term $\phi \frac{\partial \phi}{\partial s}$ is changed to $\phi_0 \frac{\partial \phi}{\partial s}$,

$$\frac{\partial \phi}{\partial t} + \gamma \frac{\partial^4 \phi}{\partial s^4} + \frac{\partial^2 \phi}{\partial s^2} + \phi_0 \frac{\partial \phi}{\partial s} = \varphi(s,t), \tag{2.3}$$

and then we have

$$\frac{\partial \phi_n}{\partial t} + \gamma \frac{\partial^4 \phi_n}{\partial s^4} + \frac{\partial^2 \phi_n}{\partial s^2} + \phi_{n-1} \frac{\partial \phi_n}{\partial s} = \varphi(s,t), a \le s \le b, 0 \le t \le T. \tag{2.4}$$

2.3. Newton linearization

For the initial value $\phi \frac{\partial \phi}{\partial s} = \phi_0 \frac{\partial \phi_0}{\partial s} + (\frac{\partial \phi_0}{\partial s} + \phi_0 \frac{\partial \phi_0}{\partial s})(\phi - \phi_0)$, we have

$$\frac{\partial \phi}{\partial t} + \gamma \frac{\partial^4 \phi}{\partial s^4} + \frac{\partial^2 \phi}{\partial s^2} + \phi \frac{\partial \phi_0}{\partial s} + \phi_0 \frac{\partial \phi_0}{\partial s} \phi = \varphi(s,t) + \phi_0 \frac{\partial \phi_0}{\partial s} \phi_0, \tag{2.5}$$

and then we have

$$\frac{\partial \phi_n}{\partial t} + \gamma \frac{\partial^4 \phi_n}{\partial s^4} + \frac{\partial^2 \phi_n}{\partial s^2} + \phi_n \frac{\partial \phi_{n-1}}{\partial s} + \phi_{n-1} \frac{\partial \phi_{n-1}}{\partial s} \phi_n = \varphi(s,t) + \phi_{n-1} \frac{\partial \phi_{n-1}}{\partial s} \phi_{n-1}, \tag{2.6}$$

where $n = 1, 2, \cdots$.

3. Differentiation matrices of KS equation

Interval $[a, b]$ is divided into $a = s_0 < s_1 < s_2 < \cdots < s_{m-1} < s_m = b$, for uniform partition with $h_s = \frac{b-a}{m}$ and nonuniform partition to be the second kind of Chebychev point. Time $[0, T]$ is divided into $0 = t_0 < t_1 < t_2 < \cdots < t_{n-1} < t_n = T$ and $h_t = \frac{T}{n}$ for uniform partition. Then, we take $\phi_{nm}(s,t)$ to approximate $\phi(s,t)$ as

$$\phi_{nm}(s,t) = \sum_{i=0}^{m} \sum_{j=0}^{n} r_i(s) r_j(t) \phi_{ij} \tag{3.1}$$

where $\phi_{ij} = \phi(s_i, t_j)$,

$$r_i(s) = \frac{\dfrac{w_i}{s - s_i}}{\displaystyle\sum_{j=0}^{m} \frac{w_j}{s - s_j}}, \quad r_j(t) = \frac{\dfrac{w_j}{t - t_j}}{\displaystyle\sum_{i=0}^{n} \frac{w_i}{t - t_i}} \tag{3.2}$$

is the barycentric interpolation basis [26], and

$$w_i = \sum_{k\in J_i}(-1)^k \prod_{j=k,j\neq i}^{k+d_s} \frac{1}{s_i - s_j}, \quad w_j = \sum_{k\in J_j}(-1)^k \prod_{i=k,k\neq j}^{k+d_t} \frac{1}{t_j - t_i} \tag{3.3}$$

where $J_i = \{k \in I, i - d_s \leq k \leq i\}, I = \{0, 1, \cdots, m - d_s\}$. See [26]. We get the barycentric rational interpolation.

For the case

$$w_i = \frac{1}{\prod_{i\neq k}(s_i - s_k)}, w_j = \frac{1}{\prod_{j\neq k}\left(t_j - t_k\right)}, \tag{3.4}$$

we get the barycentric Lagrange interpolation.

So,

$$r_j'(s_i) = \frac{w_j/w_i}{s_i - s_j}, \quad j \neq i, \quad r_i'(s_i) = -\sum_{j\neq i} r_j'(s_i), \tag{3.5}$$

$$r_j^{(k)}(s_i) = k\left(r_i^{(k-1)}(s_i)\, r_i'\left(s_j\right) - \frac{r_i^{(k-1)}\left(s_j\right)}{s_i - s_j}\right), \quad j \neq i, \tag{3.6}$$

$$r_i^{(k)}(s_i) = -\sum_{j\neq i} r_j^{(k)}(s_i). \tag{3.7}$$

Then, we have

$$\mathrm{D}_{ij}^{(0,1)} = r_i'\left(t_j\right), \tag{3.8}$$

$$\mathrm{D}_{ij}^{(1,0)} = r_i'\left(s_j\right), \tag{3.9}$$

$$\mathrm{D}_{ij}^{(k,0)} = r_i^{(k)}\left(s_j\right), k = 2, 3, \cdots. \tag{3.10}$$

3.1. Matrix equation of direct linearization

Combining (3.1) and (2.2), we have

$$\left[\mathcal{I}_m \otimes \mathcal{D}^{(0,1)} + \mathcal{D}^{(2,0)} \otimes \mathcal{I}_n + \gamma\mathcal{D}^{(4,0)} \otimes \mathcal{I}_n\right]\phi_n = \Psi - diag(\phi_{n-1})\mathcal{D}^{(1,0)} \otimes \mathcal{I}_n \cdot \phi_{n-1}, \tag{3.11}$$

and then we have

$$\mathcal{L}\phi_n = \Psi_{n-1} \tag{3.12}$$

where

$$\mathcal{L} = \mathcal{I}_m \otimes \mathcal{D}^{(0,1)} + \mathcal{D}^{(2,0)} \otimes \mathcal{I}_n + \gamma\mathcal{D}^{(4,0)} \otimes \mathcal{I}_n,$$

$$\Psi_{n-1} = \Psi - diag(\phi_{n-1})\mathcal{D}^{(1,0)} \otimes \mathcal{I}_n \cdot \phi_{n-1}$$

and $\otimes$ is the Kronecher product [17].

3.2. Matrix equation of partial linearization

Combining (3.1) and (2.4), we have

$$\left[\mathcal{I}_m \otimes \mathcal{D}^{(0,1)} + \mathcal{D}^{(2,0)} \otimes \mathcal{I}_n + \gamma\mathcal{D}^{(4,0)} \otimes \mathcal{I}_n + diag(\phi_{n-1})\mathcal{D}^{(1,0)} \otimes \mathcal{I}_n\right]\phi_n = \Psi, \tag{3.13}$$

$n = 1, 2, \cdots$, and then we have

$$\mathcal{L}\phi = \Psi \tag{3.14}$$

where $\mathcal{L} = \mathcal{I}_m \otimes \mathcal{D}^{(0,1)} + \mathcal{D}^{(2,0)} \otimes \mathcal{I}_n + \gamma\mathcal{D}^{(4,0)} \otimes \mathcal{I}_n + diag(\phi_{n-1})\mathcal{D}^{(1,0)} \otimes \mathcal{I}_n$.

3.3. Matrix equation of Newton linearization

Combining (3.1) and (2.6), we have

$$\begin{aligned}&\left[\mathcal{I}_m \otimes \mathcal{D}^{(0,1)} + \mathcal{D}^{(2,0)} \otimes \mathcal{I}_n + \gamma\mathcal{D}^{(4,0)} \otimes \mathcal{I}_n + diag(\phi_{n-1})\mathcal{D}^{(1,0)} \otimes \mathcal{I}_n\right]\phi_n \\ &= \Psi + [diag(\phi_n) - diag(\phi_{n-1})]\mathcal{D}^{(1,0)} \otimes \mathcal{I}_n \cdot \phi_{n-1},\end{aligned} \tag{3.15}$$

and then we get

$$\mathcal{L}\phi = \Psi_{n-1} \tag{3.16}$$

where

$$\mathcal{L} = \mathcal{I}_m \otimes \mathcal{D}^{(0,1)} + \mathcal{D}^{(2,0)} \otimes \mathcal{I}_n + \gamma\mathcal{D}^{(4,0)} \otimes \mathcal{I}_n + diag(\phi_{n-1})\mathcal{D}^{(1,0)} \otimes \mathcal{I}_n,$$

and

$$\Psi_{n-1} = \Psi + [diag(\phi_n) - diag(\phi_{n-1})]\mathcal{D}^{(1,0)} \otimes \mathcal{I}_n \cdot \phi_{n-1}.$$

4. Convergence rate of KS equation

In this part, an error estimate of the KS equation is given with $r_n(s) = \sum_{i=0}^{n} r_i(s)\phi_i$ to replace $\phi(s)$, where $r_i(s)$ is defined as (3.2), and $\phi_i = \phi(s_i)$. We also define

$$e(s) := \phi(s) - r_n(s) = (s - s_i)\cdots(s - s_{i+d})\phi\left[s_i, s_{i+1}, \ldots, s_{i+d}, s\right]. \tag{4.1}$$

Then, we have the following.

Lemma 1. *For $e(s)$ defined by (4.1) and $\phi(s) \in C^{d+2}[a, b]$, there is*

$$\left|e^{(k)}(s)\right| \leq Ch^{d-k+1}, k = 0, 1, \cdots. \tag{4.2}$$

For KS equation, rational interpolation function of $\phi(s, t)$ is defined as $r_{mn}(s, t)$

$$r_{mn}(s,t) = \frac{\sum_{i=0}^{m+d_s}\sum_{j=0}^{n+d_t} \frac{w_{i,j}}{(s - s_i)\left(t - t_j\right)}\phi_{i,j}}{\sum_{i=0}^{m+d_s}\sum_{j=0}^{n+d_t} \frac{w_{i,j}}{(s - s_i)\left(t - t_j\right)}} \tag{4.3}$$

where

$$w_{i,j} = (-1)^{i-d_s+j-d_t} \sum_{k_1 \in J_i} \prod_{h_1=k_1, h_1 \neq j}^{k_1+d_s} \frac{1}{\left|s_i - s_{h_1}\right|} \sum_{k_2 \in J_i} \prod_{h_2=k_2, h_2 \neq j}^{k_2+d_t} \frac{1}{\left|t_j - t_{h_2}\right|}. \tag{4.4}$$

We define $e(s,t)$ to be the error of $\phi(s,t)$ as

$$\begin{aligned} e(s,t) : &= \phi(s,t) - r_{mn}(s,t) \\ &= (s - s_i) \cdots (s - s_{i+d_s}) \phi\left[s_i, s_{i+1}, \ldots, s_{i+d_1}, s, t\right] \\ &+ \left(t - t_j\right) \cdots \left(t - t_{j+d_t}\right) \phi\left[s, t_j, t_{j+1}, \ldots, t_{j+d_2}, t\right]. \end{aligned} \tag{4.5}$$

With similar analysis of Lemma 1, we have the following

Theorem 1. *For $e(s,t)$ defined as (4.5) and $\phi(s,t) \in C^{d_s+2}[a,b] \times C^{d_t+2}[0,T]$, we have*

$$\left|e^{(k_1,k_2)}(s,t)\right| \leq C(h_s^{d_s-k_1+1} + h_t^{d_t-k_2+1}), k_1, k_2 = 0, 1, \cdots. \tag{4.6}$$

We take the direct linearization of the KS equation as an example prove the convergence rate. Let $\phi(s_m, t_n)$ be the approximate function of $\phi(s,t)$ and L be a bounded operator. There holds

$$L\phi(s_m, t_n) = \varphi(s_m, t_n), \tag{4.7}$$

and

$$\lim_{m,n\to\infty} \phi(s_m, t_n) = \phi(s,t). \tag{4.8}$$

Then, we get the following

Theorem 2. *For $\phi(s_m, t_n) : L\phi(s_m, t_n) = \varphi(s,t)$ and L defined as (4.7), there*

$$|\phi(s,t) - \phi(s_m, t_n)| \leq C(h^{d_s-3} + \tau^{d_t}).$$

Proof. As

$$\begin{aligned} &L\phi(s,t) - L\phi(s_m, t_n) \\ &= \frac{\partial\phi}{\partial t} + \gamma\frac{\partial^4\phi}{\partial s^4} + \frac{\partial^2\phi}{\partial s^2} - \phi_0\frac{\partial\phi_0}{\partial s} - \varphi(s,t) \\ &-\left[\frac{\partial\phi(s_m,t_n)}{\partial t} + \gamma\frac{\partial^4\phi(s_m,t_n)}{\partial s^4} + \frac{\partial^2\phi(s_m,t_n)}{\partial s^2} + \phi_0(s_m,t_n)\frac{\partial\phi_0(s_m,t_n)}{\partial s} - \varphi(s,t)\right] \\ &= \frac{\partial\phi}{\partial t} - \frac{\partial\phi}{\partial t}(s_m,t_n) + \gamma\left[\frac{\partial^4\phi}{\partial s^4} - \frac{\partial^4\phi}{\partial s^4}(s_m,t_n)\right] \\ &+\frac{\partial^2\phi}{\partial s^2} - \frac{\partial^2\phi}{\partial s^2}(s_m,t_n) + \left[\phi_0\frac{\partial\phi_0}{\partial s} - \phi_0(s_m,t_n)\frac{\partial\phi_0}{\partial s}(s_m,t_n)\right] \\ &:= E_1(s,t) + E_2(s,t) + E_3(s,t) + E_4(s,t). \end{aligned} \tag{4.9}$$

Here,

$$E_1(s,t) = \frac{\partial \phi}{\partial t} - \frac{\partial \phi}{\partial t}(s_m, t_n),$$

$$E_2(s,t) = \gamma \left[\frac{\partial^4 \phi}{\partial s^4} - \frac{\partial^4 \phi}{\partial s^4}(s_m, t_n) \right],$$

$$E_3(s,t) = \frac{\partial^2 \phi}{\partial s^2} - \frac{\partial^2 \phi}{\partial s^2}(s_m, t_n),$$

$$E_4(s,t) = \phi_0 \frac{\partial \phi_0}{\partial s} - \phi_0(s_m, t_n) \frac{\partial \phi_0}{\partial s}(s_m, t_n).$$

With $E_2(s,t)$, we have

$$\begin{aligned} E_2(s,t) \quad &= \gamma \left[\frac{\partial^4 \phi}{\partial s^4} - \frac{\partial^4 \phi}{\partial s^4}(s_m, t_n) \right] \\ &= \gamma \left[\frac{\partial^4 \phi}{\partial s^4} - \frac{\partial^4 \phi}{\partial s^4}(s_m, t) + \frac{\partial^4 \phi}{\partial s^4}(s_m, t) - \frac{\partial^4 \phi}{\partial s^4}(s_m, t_n) \right] \\ &= \frac{\sum_{i=0}^{m-d_s} (-1)^i \frac{\partial^4 \phi}{\partial s^4}[s_i, s_{i+1}, \ldots, s_{i+d_1}, s_m, t]}{\sum_{i=0}^{m-d_s} \lambda_i(s)} \\ &+ \frac{\sum_{j=0}^{n-d_t} (-1)^j \frac{\partial^4 \phi}{\partial s^4}[t_j, t_{j+1}, \ldots, t_{j+d_2}, s_m, t_n]}{\sum_{j=0}^{n-d_t} \lambda_j(t)} \\ &= \frac{\partial^4 e}{\partial s^4}(s_m, t) + \frac{\partial^4 e}{\partial s^4}(s_m, t_n). \end{aligned}$$

For $E_2(s,t)$ we get

$$|E_2(s,t)| \leq \left| \frac{\partial^4 e}{\partial s^4}(s_m, x) + \frac{\partial^4 e}{\partial s^4}(s_m, t_n) \right| \leq C(h^{d_s-3} + \tau^{d_t+1}). \tag{4.10}$$

Then, we have

$$|E_1(s,t)| \leq \left| \frac{\partial e}{\partial t}(s_m, t) + \frac{\partial e}{\partial t}(s_m, t_n) \right| \leq C(h^{d_s+1} + \tau^{d_t}). \tag{4.11}$$

Similarly, for $E_3(s,t)$ we have

$$E_3(s,t) = \frac{\partial^2 \phi}{\partial s^2}(s,t) - \frac{\partial^2 \phi}{\partial s^2}(s_m, t_n) = \frac{\partial^2 e}{\partial s^2}(s, t_n) + \frac{\partial^2 e}{\partial s^2}(s_m, t_n), \tag{4.12}$$

and

$$|E_3(s,t)| \leq \left| \frac{\partial^2 e}{\partial s^2}(s, t_n) + \frac{\partial^2 e}{\partial s^2}(s_m, t_n) \right| \leq C(h^{d_s-1} + \tau^{d_t+1}). \tag{4.13}$$

For $E_4(s,t)$ we get

$$\begin{aligned} |E_4(s,t)| &= \left|\phi_0 \tfrac{\partial \phi}{\partial s} - \phi_0(s_m,t_n)\tfrac{\partial \phi}{\partial s}(s_m,t_n)\right| \\ &\leq \left|\tfrac{\partial e}{\partial t}(s_m,t) + \tfrac{\partial e}{\partial t}(s_m,t_n)\right| \leq C(h^{d_s+1} + \tau^{d_t}). \end{aligned} \quad (4.14)$$

Combining (4.9) and (4.11)–(4.14) together, the proof of Theorem 2 is completed. □

5. Numerical examples

All the examples are carried on a computer with Intel(R) Core(TM) i5-8265U CPU @ 1.60 GHz 1.80 GHz operating system, 16 G radon access running memory and a 512 G solid state disk memory. All simulation experiments were realized by the software Matlab (Version: R2016a). In this part, two examples are presented to test the theorem.

Example 1. *Consider the KS equation*

$$\frac{\partial \phi}{\partial t} + \gamma \frac{\partial^4 \phi}{\partial s^4} + \frac{\partial^2 \phi}{\partial s^2} + \phi \frac{\partial \phi}{\partial s} = \varphi(s,t)$$

with the condition is

$$\phi(0,t) = 0, \phi(1,t) = 0,$$

and

$$\phi(s,0) = \sin(2\pi s).$$

$$\phi_{ss}(0,t) = 0, \phi_{ss}(1,t) = 0,$$

and

$$\varphi(s,t) = e^{-t}\sin(2\pi s)[2\pi e^{-t}\cos(2\pi s) - 1 + 16\pi^4 - 4\pi^2].$$

The solution of the KS equation is

$$\phi(s,t) = e^{-t}\sin(2\pi s).$$

In Figures 1–3, errors of unform partition with direct linearization, partial linearization, Newton linearization for the KS equation are presented. In Figures 4–6, errors of non-uniform partition with direct linearization, partial linearization, Newton linearization for the KS equation are presented.

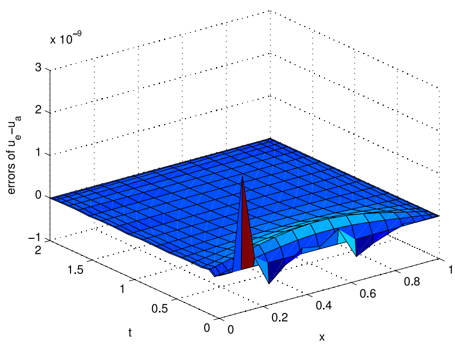

Figure 1. Errors of nonuniform partition by direct linearization with $m = n = 19$.

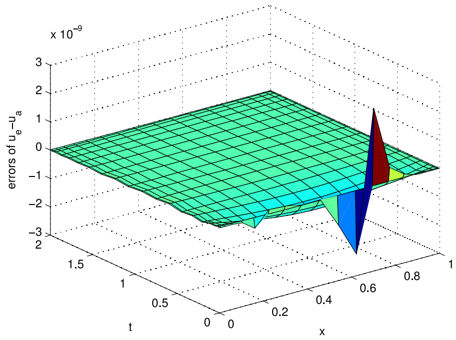

Figure 2. Errors of nonuniform partition by partial linearization with $m = n = 19$.

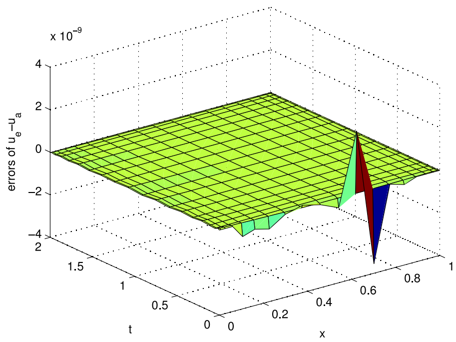

Figure 3. Errors of nonuniform partition by Newton linearization with $m = n = 19$.

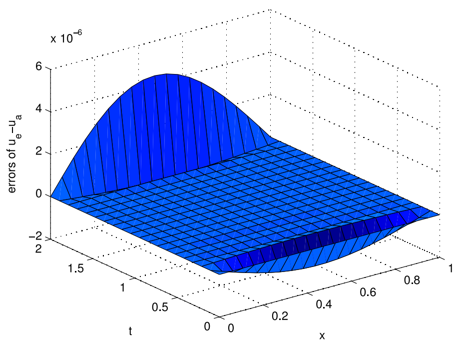

Figure 4. Errors of uniform partition by direct linearization with $m = n = 19$.

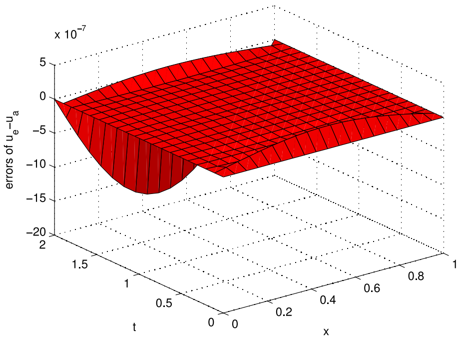

Figure 5. Errors of uniform partition by partial linearization with $m = n = 19$.

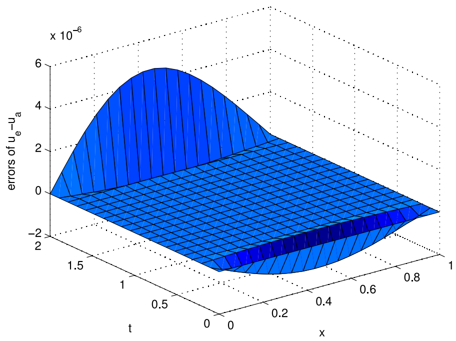

Figure 6. Errors of uniform partition by Newton linearization with $m = n = 19$.

In Tables 1 and 2, errors of LBCM and LBRCM for the KS equation with boundary condition dealt with by the method of substitution and method of addition are given. From Table 1, we know that the accuracy of LBCM is higher than LBRCM, and from Table 2 the accuracy of the method of additional is higher than the method of substitution.

Table 1. Errors of LBCM for KS equation with $m = n = 17$.

	Method of substitution		Method of additional	
Linearization	Uniform partition	Nonuniform partition	Uniform partition	Nonuniform partition
direct	1.3278e-07	5.6616e-10	1.7050e-08	4.6293e-10
partial	5.5563e-07	2.6381e-09	1.1492e-07	5.0974e-10
Newton	6.6705e-07	4.8875e-10	8.8609e-08	2.5867e-11

Table 2. Errors of LBRCM for KS equation with $m = n = 17, d_s = d_t = 12$.

	Method of substitution		Method of additional	
Linearization	Uniform partition	Nonuniform partition	Uniform partition	Nonuniform partition
direct	4.4575e-06	3.2280e-08	4.1010e-08	2.2749e-09
partial	4.4573e-06	3.2245e-08	5.4191e-07	1.5951e-07
Newton	4.4560e-06	3.2215e-08	1.2972e-06	3.5137e-07

In Table 3, we choose the Newton linearization to solve the KS equation, and the error of LBRCM for uniform and nonuniform partitions are presented with $t = 0.3, 0.9, 2, 4, 8, 16$.

The errors of LBRCM of uniform and Chebyshev partitions are presented with $(m, n, d_s, d_t) = (8, 8, 7, 7), (16, 16, 15, 15)$. From the table, comparing $(m, n) = (8, 8)$ with $(m, n) = (16, 16)$, the accuracy was higher when the number became bigger.

Table 3. Errors of Newton linearization for t.

	Uniform partition		Nonuniform partition	
t	$(8,8)d_s = d_t = 7$	$(16,16)d_s = d_t = 15$	$(8,8)d_s = d_t = 7$	$(16,16)d_s = d_t = 15$
0.3	1.5449e-01	1.3163e-06	6.2692e-02	2.4769e-08
0.9	1.4211e-01	1.1737e-06	6.1721e-02	2.3846e-08
2	1.2162e-01	1.0785e-06	5.8680e-02	2.3685e-08
4	9.1544e-02	9.4383e-07	5.3241e-02	2.3353e-08
8	5.1798e-02	7.2283e-07	4.3721e-02	2.2440e-08
16	1.6540e-02	4.1712e-07	2.9435e-02	1.9220e-08

In the following table, we take Newton linearization to present numerical results. From Tables 4 and 5, with errors of Newton linearization for uniform partition $d_t = 6; t = 1$ are given and convergence rate is $O(h^{d_s})$. From Table 5, with space variable $s, d_s = 6$, and there is superconvergence rate $O(h^{d_s - 1})$ at $t = 1$.

Table 4. Errors of Newton linearization for uniform partition $d_t = 6$.

m, n	$d_s = 2$	h^α	$d_s = 3$	h^α	$d_s = 4$	h^α
8,8	4.1317e-01		3.2652e-03		3.3180e-01	
16,16	1.8608e-01	1.1508	3.1257e-02	-	3.3919e-02	3.2902
32,32	9.5437e-02	0.9633	1.0198e-02	1.6159	3.3873e-03	3.3239
64,64	4.7221e-02	1.0151	2.6490e-03	1.9448	3.5472e-04	3.2554

Table 5. Errors of Newton linearization for uniform partition $d_s = 6$.

m, n	$d_t = 2$	τ^α	$d_t = 3$	τ^α	$d_t = 4$	τ^α
8,8	1.3997e-01		1.4004e-01		1.4008e-01	
16,16	5.4923e-03	4.6716	5.4957e-03	4.6714	5.4973e-03	4.6714
32,32	1.2850e-04	5.4176	1.2883e-04	5.4148	1.2891e-04	5.4143
64,64	2.9976e-06	5.4218	3.0728e-06	5.3898	3.0798e-06	5.3874

For Tables 6 and 7, the errors of Chebyshev partition for Newton linearization with s and t are presented. For $d_t = 6$, the convergence rate is $O(h^{d_s})$ in Table 6, while in Table 7, there are also superconvergence phenomena.

Table 6. Errors of Newton linearization for Chebyshev partition $d_t = 6$.

m, n	$d_s = 2$	h^α	$d_s = 3$	h^α	$d_s = 4$	h^α
8,8	5.4754e-01		2.9399e-02		8.5922e-02	
16,16	1.0318e-01	2.4078	4.6815e-03	2.6507	1.2658e-03	6.0849
32,32	9.6912e-02	0.0904	8.0675e-04	2.5368	1.9577e-05	6.0148
64,64	4.8014e-01	-	1.7672e-03	-	2.2716e-05	-

Table 7. Errors of Newton linearization for Chebyshev partition $d_s = 6$.

m, n	$d_t = 2$	τ^α	$d_t = 3$	τ^α	$d_t = 4$	τ^α
8,8	6.1344e-02		6.1386e-02		6.1415e-02	
16,16	8.1492e-05	9.5561	8.1163e-05	9.5629	8.0977e-05	9.5669
32,32	1.4204e-07	9.1642	1.4183e-07	9.1606	1.5487e-07	9.0303
64,64	6.3190e-06	-	3.8960e-06	-	1.4861e-06	-

Example 2. *Consider the KS equation*

$$\frac{\partial \phi}{\partial t} + \gamma \frac{\partial^4 \phi}{\partial s^4} + \frac{\partial^2 \phi}{\partial s^2} + \phi \frac{\partial \phi}{\partial s} = 0,$$

with the analytic solution

$$\phi(s,t) = c + \frac{15\sqrt{11}}{19\sqrt{19}} \left[-3\tanh \frac{\sqrt{11}}{2\sqrt{19}}(s - ct + s_0) + \tanh^3 \frac{\sqrt{11}}{2\sqrt{19}}(s - ct + s_0) \right],$$

and boundary condition

$$\phi(-10,t) = c + \frac{15\sqrt{11}}{19\sqrt{19}} \left[-3\tanh \frac{\sqrt{11}}{2\sqrt{19}}(-10 - ct + s_0) + \tanh^3 \frac{\sqrt{11}}{2\sqrt{19}}(-10 - ct + s_0) \right],$$

$$\phi(10,t) = c + \frac{15\sqrt{11}}{19\sqrt{19}} \left[-3\tanh \frac{\sqrt{11}}{2\sqrt{19}}(10 - ct + s_0) + \tanh^3 \frac{\sqrt{11}}{2\sqrt{19}}(10 - ct + s_0) \right],$$

and initial condition

$$\phi(s,0) = c + \frac{15\sqrt{11}}{19\sqrt{19}} \left[-3\tanh \frac{\sqrt{11}}{2\sqrt{19}}(s + s_0) + \tanh^3 \frac{\sqrt{11}}{2\sqrt{19}}(s + s_0) \right],$$

with $c = 2, x_0 = 10$.

In Figures 7–9, errors of direct linearization, partial linearization, Newton linearization with $m = n = 19$ KS equation are presented, respectively.

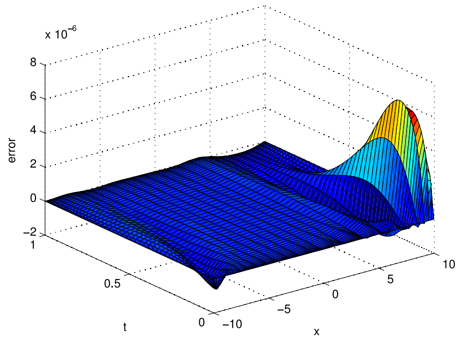

Figure 7. Errors of direct linearization with $m = n = 19$.

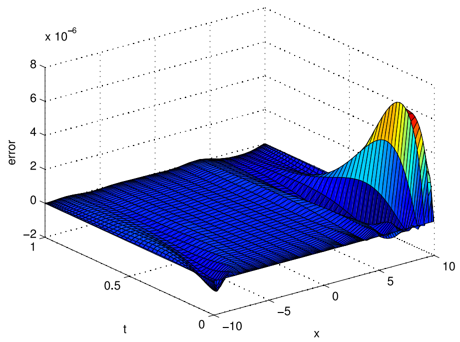

Figure 8. Errors of partial linearization with $m = n = 19$.

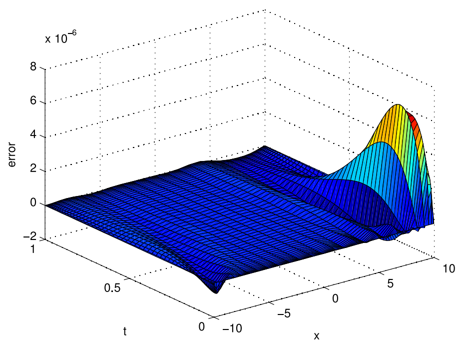

Figure 9. Errors of Newton linearization with $m = n = 19$.

In the following table, direct linearization is chosen to present numerical results. From Tables 8 and 9, errors of direct linearization for uniform partition $d_t = 7$ with different d_s are given and the convergence rate is $O(h^{d_s} - 1)$. From Table 9, with space variable $s, d_s = 7$, and there are also superconvergence phenomena.

Table 8. Errors of direct linearization for uniform partition for $d_t = 7$.

m, n	$d_s = 2$	h^{α}	$d_s = 3$	h^{α}	$d_s = 4$	h^{α}
8,8	1.3587e+00		8.9361e-01		6.3703e-01	
16,16	2.1617e-01	2.6520	2.7467e-01	1.7019	2.5682e-01	1.3106
32,32	6.7743e-02	1.6740	6.8822e-02	1.9967	4.7078e-02	2.4476
64,64	2.5175e-02	1.4281	1.3216e-02	2.3806	4.3739e-03	3.4281

Table 9. Errors of direct linearization for uniform partition for $d_s = 7$.

m, n	$d_t = 2$	τ^{α}	$d_t = 3$	τ^{α}	$d_t = 4$	τ^{α}
8,8	3.6253e-01		3.6380e-01		3.6446e-01	
16,16	1.8147e-01	0.9984	1.8124e-01	1.0052	1.8121e-01	1.0081
32,32	6.4076e-02	1.5019	6.4158e-02	1.4982	6.4141e-02	1.4983
64,64	8.9037e-04	6.1692	8.9840e-04	6.1581	8.9863e-04	6.1574

For Tables 10 and 11, the errors of Chebyshev partition for direct linearization with s and t are presented. For $d_t = 7$, the convergence rate is $O(h^{d_s})$ in Table 10, while in Table 11, there are also superconvergence phenomena.

Table 10. Errors of direct linearization for Chebyshev partition for $d_t = 7$.

m, n	$d_s = 2$	h^{α}	$d_s = 3$	h^{α}	$d_s = 4$	h^{α}
8,8	6.5990e-01		4.0742e-01		3.6175e-01	
16,16	1.1154e-01	2.5646	1.7539e-01	1.2160	2.1752e-01	0.7338
32,32	4.3052e-02	1.3735	8.6654e-03	4.3391	1.2511e-03	7.4418
64,64	3.9204e-02	0.1351	2.3776e-03	1.8658	3.5682e-04	1.8099

Table 11. Errors of direct linearization for Chebyshev partition for $d_s = 7$.

m, n	$d_t = 2$	τ^{α}	$d_t = 3$	τ^{α}	$d_t = 4$	τ^{α}
8,8	4.3760e-01		4.3745e-01		4.3739e-01	
16,16	1.1801e-01	1.8908	1.1801e-01	1.8902	1.1801e-01	1.8900
32,32	9.9842e-04	6.8850	9.9854e-04	6.8849	9.9801e-04	6.8857
64,64	2.5749e-06	8.5990	2.5052e-06	8.6388	4.8401e-06	7.6879

6. Conclusions

In this paper, LBRCM is used to solve the (1+1) dimensional SK equation. Three kinds of linearization methods are taken to translate the nonlinear part into a linear part. Matrix equations of the discrete SK equation are obtained from corresponding linearization schemes. The convergence rate of LBRCM is also presented. In the future work, LBRCM can be developed for the (2+1) dimensional SK equation and other partial differential equations classes, including Kolmogorov-Petrovskii-Piskunov (KPP) equation and, fractional reaction-diffusion equation and so on.

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Conflict of interest

The author declares no conflict of interest.

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