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Research article

Intuitionistic fuzzy-based TOPSIS method for multi-criterion optimization problem: a novel compromise methodology

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Abstract: The decision-making process is characterized by some doubt or hesitation due to the existence of uncertainty among some objectives or criteria. In this sense, it is quite difficult for decision maker(s) to reach the precise/exact solutions for these objectives. In this study, a novel approach based on integrating the technique for order preference by similarity to ideal solution (TOPSIS) with the intuitionistic fuzzy set (IFS), named TOPSIS-IFS, for solving a multi-criterion optimization problem (MCOP) is proposed. In this context, the TOPSIS-IFS operates with two phases to reach the best compromise solution (BCS). First, the TOPSIS approach aims to characterize the conflicting natures among objectives by reducing these objectives into only two objectives. Second, IFS is incorporated to obtain the solution model under the concept of indeterminacy degree by defining two membership functions for each objective (i.e., satisfaction degree, dissatisfaction degree). The IFS can provide an effective framework that reflects the reality contained in any decision-making process. The proposed TOPSIS-IFS approach is validated by carrying out an illustrative example. The obtained solution by the approach is superior to those existing in the literature. Also, the TOPSIS-IFS approach has been investigated through solving the multi-objective transportation problem (MOTP) as a practical problem. Furthermore, impacts of IFS parameters are analyzed based on Taguchi method to demonstrate their effects on the BCS. Finally, this integration depicts a new philosophy in the mathematical programming field due to its interesting principles.

Keywords: multi-criterion optimization; intuitionistic fuzzy; TOPSIS; compromise solution; Taguchi method
Mathematics Subject Classification: 03E72, 08A72, 54A40

1. Introduction

Nowadays, decision-making process is becoming a complicated task, and the frequent decisions present a headache to the decision maker (DM), when an accurate decision of a system is required. Moreover, some factors (data) of a system may be characterized by existence of uncertainties. Thus, adequate models are required to deal with these uncertainties, vagueness and impreciseness. IFS is an effective model that was proposed by Attanassov [1] to handle impreciseness and uncertainties in data. The IFS represents an extension of the fuzzy set (FS) [2] which is characterized by two membership functions, a membership degree, and a non-membership degree. Angelov [3] developed the concepts of IFS in the optimization area, where his approach is equipped by maximizing membership degree (satisfaction) and minimizing the non-membership degree (dissatisfaction), and then the crisp model is computed.

Realistically, optimization problems are characterized not only by uncertainty aspects but also by the presence of a multiple criteria (objectives). In this sense, no solution can be obtained that optimizes all the objectives simultaneously. Different approaches to deal with the multiple criteria have been developed [4–6]. Among them is the TOPSIS approach that was proposed by Hwang and Yoon [7] to acquire the BCS. TOPSIS represents one of the most widely utilized methodologies for practical MCOP due to its simplicity, sound mathematical foundation, and ease of applicability [8]. TOPSIS has inspired scores of novel techniques and analysis based on it [9]. TOPSIS aggregates the alternatives based on two reference points, ideal solution and the negative-ideal solutions, which is more beneficial for the decision-making situations than the other methods. It is established based on constituting two objective functions from K number of objectives (criteria). The first step of TOPSIS is to minimize the distance from the positive ideal solution (PIS), while the second one is to maximize distance from the negative ideal solution (NIS). The final step is to obtain the solution by the max-min operation that was introduced by Bellman and Zadeh in which the maximization degree of membership function plays the major role [2]. TOPSIS has widely flourished in many areas, including manufacturing decision making [10,11], product selection [12], critical mission planning [13], economic-emission load dispatch problem [14], multi-objective non-linear program [15], and bilevel optimization [16].

Contrary to FS, IFS has been proven to be highly beneficial when dealing with vagueness and uncertainty. Through this approach, the optimization problem is reformulated by considering the degree of rejection of the constraint and the objective value, which are non-admissible. Many optimization techniques based on IFS have been developed [17–19]. The IFS based on goal programming was proposed to solve a vector optimization problem [20]. Chakrabortty et al. presented a method based on IFS technique to solve the multi-objective inventory model [21], while Garg and Rani [22] proposed a combined technique using particle swarm optimization (PSO) and IFS theory. Apart from several works employed describe the IFS theory has been proposed and given in [23–26], where most of them have studied the single and multi-objective fuzzy linear programming. However, today, much real-world decision-making is characterized by different natures including nonlinear, multi-dimensional, many-objective, conflicting nature. Therefore, the fuzzy models are considered as more realistic than the deterministic ones.

TOPSIS is one of the most widely used approaches in the decision-making process due to its simplicity, sound mathematical foundation, and ease of applicability. In this method, it aggregates the different information of the alternatives in terms of two references points. Although, several approaches appear in the literature to address the decision-making problems in terms of MCOP using TOPSIS method, they have certain limitations. For instance, the TOPSIS approaches based on the conventional FS has been shown to be satisfactory in comparisons with other multi-objective methods, but the applicability of this approach is weak due to its on reliance on the satisfaction membership degree. In a real-world problem, an analyst always pays an equal attention to the consideration of the non-satisfactory degree too during the analysis, such as in, the situations that involve the respecting and consensus of the experts' opinions which have a significant effect in the decision-making process. On the other hand, in terms of IFSs environment, the conflicting nature among the objectives is resolved with the consideration of the acceptance membership (satisfaction) and rejection membership (dissatisfaction). In such TOPSIS approaches, the two references points, the goals related to the positive and negative membership grades are considered as zero. This means that the decision maker cannot increase/decrease their satisfaction or dissatisfaction degrees towards the upper or lower bounds of the reference level. To make a decision smoother and better, there is an always a need to consider a level of satisfaction and dissatisfaction of the decision-maker in the analysis so that by increasing or decreasing their levels, the degree of the attainability can be achieved. Therefore, this perspective has motivated us to integrate the IFS with TOPSIS approach to make the decision more reasonable and reliable while treating the vagueness and uncertainty of the MCOP. Moreover, the integration of the IFS can provide a reasonable judgment while avoiding the subjectivity issue as it relies on the degree of satisfaction (acceptance) and rejection of objectives.

Motivated by these natures, this paper presents an integrated TOPSIS approach based on IFS aspect, denoted as TOPSIS-IFS, to solve MCOP. In this context, the TOPSIS approach aims to provide a compromise solution of the MCOP with conflicting natures through the underling concepts of intuitionistic fuzzy terminology. In this regards, two features are acquired by this algorithm. TOPSIS reduces the K-dimensional objectives into two-dimensional objectives with the aim to mitigate the multiple conflicts among many objectives, while IFS can provide realistic representation of objectives by defining two membership functions for each objective (i.e., satisfaction degree and dissatisfaction degree) with the aim to realize the practical consideration of the MCOP. An illustrative example is conducted to validate the proposed TOPSIS-IFS approach and the MOTP model has been taken for demonstration. The proposed TOPSIS-IFS approach gives a superior and competitive results with the existing in the literature. Finally, this integrating depicts a new philosophy in the mathematical programming field due to its interesting principles.

In a nutshell, the main contributions of this article are as follows.

1) This result broadens the literature by developing a new approach named as TOPSIS-IFS for MCOP. In this approach, TOPSIS is employed to convert any set of objectives into two objectives only.
2) As the reliance on membership function only may not serve for realistic issues, IFS is developed based on the degree of attainability and non-attainability for such situations.
3) A closeness strategy is presented to assess the obtained solutions against the other compromise alternatives and counterparts.
4) Impacts of IFS parameters on the BCS are analyzed using the Taguchi method.
5) The effectiveness of the TOPSIS-IFS approach is affirmed through the validation on numerical illustrations.

The reminder sections of this paper are structured as follows. The preliminaries of the problem formulation and related definitions are presented in Section 2. Section 3 introduces the proposed methodology with its details. The proposed methodology is investigated by an illustrative example in Section 4. Finally, the conclusion and the future research are outlined in Section 5.

2. Preliminaries

This section is devoted to introducing the formulation of the multi-criterion optimization problem (MCOP) and some related definitions.

2.1. The MCOP formulation

As its name implies, MCOP deals with multiple conflicting criteria, and there is no single best outcome that optimizes all criteria at the same time. In this sense, the DM are interested in reaching the ‘‘most compromise or preferred’’ solution rather than the optimal one. MCOP occurs in a variety of real-world circumstances, such as multi-objective design of synchronous motor [27], power system application [28], and capacitor placement problem [29]. For instance, when a family wishes to purchase a car, they must take into account factors such as cost, safety, comfort, maintenance, fuel consumption and so forth. These requirements or criteria typically conflict with one another. Finding the safest, most comfortable car at the best price is a difficult task. There are other MCOP issues that are more challenging than buying a car. Making decisions is becoming increasingly difficult due to the world's rising complexity and uncertainty. In order to express the MCOP, we can utilize the problem definition. Without the loss of generality, the following equations formulate the MCOP denoted by $\boldsymbol{P_1}$ as a maximization problem [30]:

$$\begin{gathered} P_1: \\ Max\ F(x) = (f_1(x), f_2(x), \ldots, f_K(x)) \\ Subjectto: \\ x \in \Psi = \{x \in R^n : g_l(x) \leq 0, l = 1,2,\ldots,N\} \end{gathered} \tag{1}$$

where $\boldsymbol{x} = (\boldsymbol{x_1}, \boldsymbol{x_2}, \ldots, \boldsymbol{x_n})$ defines the decision variable vector with $\boldsymbol{n}$ dimensions, $\boldsymbol{F}(\boldsymbol{x}) = (\boldsymbol{f_1}(\boldsymbol{x}), \boldsymbol{f_2}(\boldsymbol{x}), \ldots, \boldsymbol{f_K}(\boldsymbol{x}))$ is the objective vector with $\boldsymbol{K}$ objectives, and $\boldsymbol{g_l}(\boldsymbol{x})$ is the $\boldsymbol{l}$th constraint, whereas $\boldsymbol{N}$ is the total number of constraints.

2.2. The distance measures for objectives space

This section provides the definitions of distance regarding multiple objectives under some reference points [31]. Consider that the vector of an objective function, $\boldsymbol{F}(\boldsymbol{x}) = (\boldsymbol{f_1}(\boldsymbol{x}), \boldsymbol{f_2}(\boldsymbol{x}), \ldots, \boldsymbol{f_K}(\boldsymbol{x}))$, is associated with two reference points, namely, ideal point or positive ideal solution (PIS), $\boldsymbol{F}^* = (\boldsymbol{f_1^*}, \boldsymbol{f_2^*}, \ldots, \boldsymbol{f_K^*})$ and anti-ideal point or negative ideal solution (NIS), $\boldsymbol{F}^- = (\boldsymbol{f_1^-}, \boldsymbol{f_2^-}, \ldots, \boldsymbol{f_K^-})$. These reference points can be computed as follows.

$$f_k^* = \begin{cases} \max\limits_{x \in \Psi} f_k(x), & \text{Maximization problem} \\ \min\limits_{x \in \Psi} f_k(x), & \text{Minimization problem} \end{cases}, \qquad k = 1,2,\ldots,K \tag{2}$$

$$f_k^- = \begin{cases} \min\limits_{x \in \Psi} f_k(x), & \text{Maximization problem} \\ \max\limits_{x \in \Psi} f_k(x), & \text{Minimization problem} \end{cases}, \qquad k = 1,2,\ldots,K \tag{3}$$

Also, the most prominent of closeness is the $\boldsymbol{L_P}$-metric, $\boldsymbol{L_P}$-metric denotes the distance, $\boldsymbol{d_p}$, among any vectors of objective functions ($\boldsymbol{F}(\boldsymbol{x})$ and $\boldsymbol{F}^*$) as follows.

$$d_p = \left\{ \sum_{k=1}^{K} w_k^p \left[f_k^* - f_k(x)\right]^p \right\}^{\frac{1}{p}}, p = 1,2,\ldots,\infty \tag{4}$$

where $\boldsymbol{w_k}$ defines the relative importance (weight) of the $\boldsymbol{k}$th objective, where this weight expresses the DM's relative preference of the objective. Due to the units among the objectives not being unified, a scaling form is applied to obtain the dimensionless form and then the obtained value lies in the interval [0, 1]. The dimensionless metric form is formulated as follows:

$$d_p = \left\{ \sum_{k=1}^{K} w_k^p \left[\frac{f_k^* - f_k(x)}{f_k^* - f_k^-}\right]^p \right\}^{\frac{1}{p}}, p = 1,2,\ldots,\infty \tag{5}$$

In this sense, the compromise solution of MCOP is attained by converting the original model of Eq (1) to an auxiliary model through the underling concepts of distance metric as follows [31].

$$\underset{x \in \Psi}{Min}\ d_p = \left\{ \sum_{k=1}^{K} w_k^p \left[\frac{f_k^* - f_k(x)}{f_k^* - f_k^-}\right]^p \right\}^{\frac{1}{p}}, p = 1,2,\ldots,\infty \tag{6}$$

where, Eq (6) minimizes the sum of divisions of objective functions from their respective reference points (positive ideal solutions) through some weights. The parameter $\boldsymbol{p}$ reflects the way of attaining the compromise solution, where it is responsible for balancing scenario among the group utility and maximal individual regret. In this regard, increasing of $\boldsymbol{p}$ results in decreasing in the group utility (distance $\boldsymbol{d_P}$), i.e., $\boldsymbol{d_1} \geq \boldsymbol{d_2} \geq \ldots \geq \boldsymbol{d_p}$, and the largest deviation has a greater emphasis in forming the total. Where $\boldsymbol{p} = \boldsymbol{1}$ reflects the equal importance (weights) regarding all deviations, while $\boldsymbol{p} = \boldsymbol{2}$ implies that the importance of deviation has an effect according to its amount, with the largest having the largest weight [31]. Finally, $\boldsymbol{p} = \infty$ implies that the largest deviation is the prominent one, where the $\boldsymbol{L_\infty} -$ metric is defined as follows.

$$d_\infty = \max_k \{w_k[f_k^* - f_k(x)]\} \quad \text{or} \quad d_\infty = \max_k \left\{w_k \left[\frac{f_j^* - f_j(x)}{f_k^* - f_k^-}\right]\right\} \tag{7}$$

2.3. The conception of FS and IFS

This section provides some concepts of FS and IFS. Assume that $\boldsymbol{X}$ denotes a classical set of elements (objects). The conception of IFS can be considered as an alternate method to define a FS in situations where information at hand is insufficient for describing an imprecise concept by means of a classical FS. Thus, it is anticipated that IFS can be employed to simulate the human decision-making task and any duties requiring human knowledge and expertise that are inherently imprecise or not totally reliable. In this sense, IFS theory can be employed in which the degree of membership of an object is measured within an interval pattern rather than the point valued as in FS. For this, hesitation degree among the membership functions has been respected in terms of the acceptance degree (membership function) and rejection degree (non-membership function) so that the sum of them is less than one. The FS and IFS are defined as follows.

Definition 1 [32]**.** The FS, $\widetilde{\boldsymbol{A}}$, is denoted as an ordered pair: $\widetilde{\boldsymbol{A}} = \{(\boldsymbol{x}, \boldsymbol{\mu}_{\widetilde{A}}(\boldsymbol{x})) | \boldsymbol{x} \in \boldsymbol{X}\}$ and $\boldsymbol{\mu}_{\widetilde{A}}(\boldsymbol{x}): \boldsymbol{X} \to [\boldsymbol{0}, \boldsymbol{1}]$ is defined as the membership function.

Definition 2 [1]**.** The set of ordered triplet $\widetilde{\boldsymbol{A}}^{\boldsymbol{I}} = \{(\boldsymbol{x}, \boldsymbol{\mu}_{\widetilde{A}^I}(\boldsymbol{x}), \boldsymbol{\nu}_{\widetilde{A}^I}(\boldsymbol{x})) | \boldsymbol{x} \in \boldsymbol{X}\}$ is defined as IFS, whereas $\boldsymbol{\mu}_{\widetilde{A}^I}(\boldsymbol{x})$, and $\boldsymbol{\nu}_{\widetilde{A}^I}(\boldsymbol{x})$ denote the degree of membership and the degree of non-membership, respectively. Each membership is a function from $\boldsymbol{X}$ to $[\boldsymbol{0}, \boldsymbol{1}]$, i.e., $\boldsymbol{\mu}_{\widetilde{A}^I}(\boldsymbol{x}), \boldsymbol{\nu}_{\widetilde{A}^I}(\boldsymbol{x}): \boldsymbol{X} \to [\boldsymbol{0}, \boldsymbol{1}]$ such that $\boldsymbol{0} \le \boldsymbol{\mu}_{\widetilde{A}^I}(\boldsymbol{x}), \boldsymbol{\nu}_{\widetilde{A}^I}(\boldsymbol{x}) \le \boldsymbol{1}$ for all $\boldsymbol{x} \in \boldsymbol{X}$. Also, $\boldsymbol{1} - \boldsymbol{\mu}_{\widetilde{A}^I}(\boldsymbol{x}) - \boldsymbol{\nu}_{\widetilde{A}^I}(\boldsymbol{x})$ denotes the hesitation degree or indeterminacy of $\boldsymbol{x}$ being in $\widetilde{\boldsymbol{A}}^{\boldsymbol{I}} \in \boldsymbol{X}$.

Definition 3 [33]**.** The $\boldsymbol{\alpha}$ -cut or $\boldsymbol{\alpha}$ -level set of a fuzzy set $\widetilde{\boldsymbol{A}}$ is defined as the crisp set $\boldsymbol{A}_{\boldsymbol{\alpha}}$with the elements of the universal set $\boldsymbol{X}$ such that the degree of membership is at least $\boldsymbol{\alpha}$, i.e.,

$$A_\alpha = \{x \in X | \mu_{\widetilde{A}}(x) \ge \alpha\}, 0 \le \alpha \le 1 \tag{8}$$

Definition 4 [33]**.** The $\boldsymbol{\alpha}$-cut and $\boldsymbol{\beta}$-cut of IFS are denoted by $\boldsymbol{A}_{\boldsymbol{\alpha}}^{\boldsymbol{I}}$ and $\boldsymbol{A}_{\boldsymbol{\beta}}^{\boldsymbol{I}}$ respectively and are defined as follows.

$$A_\alpha^I = \{x \in X | \mu_{\widetilde{A}^I}(x) \ge \alpha\}, and\ A_\beta^I = \{x \in X | \nu_{\widetilde{A}^I}(x) \le \beta\}\ \forall \alpha, \beta \ge 0 \tag{9}$$

3. The proposed TOPSIS-IFS for MCOP

In this section, the proposed TOPSIS-IFS approach is applied to solve the MCOP. The first phase starts by formulating the distances measures of TOPSIS in which the PIS, $\boldsymbol{F}^* = (\boldsymbol{f}_{\boldsymbol{1}}^*, \boldsymbol{f}_{\boldsymbol{2}}^*, \ldots, \boldsymbol{f}_{\boldsymbol{K}}^*)$ and NIS are employed to obtain $\boldsymbol{d}_{\boldsymbol{p}}^{\boldsymbol{PIS}}$and $\boldsymbol{d}_{\boldsymbol{p}}^{\boldsymbol{NIS}}$.

$$f^* = \left\{\min_{x \in \Psi} (\text{or}\ max)\ f_j(x) \big(\text{or}\ f_i(x)\big) \forall j (\text{and}\ i)\right\} \tag{10}$$

$$f^- = \left\{\max_{x \in \Psi} (\text{or}\ min)\ f_j(x) \big(\text{or}\ f_i(x)\big) \forall j (\text{and}\ i)\right\} \tag{11}$$

where $j \in J$ and $i \in I$, whereas J and I denote the set of benefit objectives of maximization and the set cost objectives of minimization, respectively. In this sense, the TOPSIS method converts any set of objectives into two objectives only: The first objective comprises the normalized shortest distance from the PIS denoted by $d_p^{PIS}(x)$, and the second one comprises the normalized farthest from the NIS denoted $d_p^{NIS}(x)$. In this context, each of distance metric aims to aggregate all objectives in normalized form using the weighted sum, where the first term implies the benefit objective for maximization, while second term implies the cost objective for minimization. Theses distances can be expressed as follows.

$$\boldsymbol{d_p^{PIS}(x)} = \left\{\sum_{j\in J} \boldsymbol{w_j^p}\left[\frac{\boldsymbol{f_j^* - f_j(x)}}{\boldsymbol{f_j^* - f_j^-}}\right]^{\boldsymbol{p}} + \sum_{i\in I} \boldsymbol{w_i^p}\left[\frac{\boldsymbol{f_i(x) - f_i^*}}{\boldsymbol{f_i^- - f_i^*}}\right]^{\boldsymbol{p}}\right\}^{\boldsymbol{1/p}} \tag{12}$$

$$\boldsymbol{d_p^{NIS}(x)} = \left\{\sum_{j\in J} \boldsymbol{w_j^p}\left[\frac{\boldsymbol{f_j(x) - f_j^-}}{\boldsymbol{f_j^* - f_j^-}}\right]^{\boldsymbol{p}} + \sum_{i\in I} \boldsymbol{w_i^p}\left[\frac{\boldsymbol{f_i^- - f_i(x)}}{\boldsymbol{f_i^- - f_i^*}}\right]^{\boldsymbol{p}}\right\}^{\boldsymbol{1/p}} \tag{13}$$

After formulating the distances in scalarization form using the weighted sum methods, TOPSIS was developed with the aim is to minimize the distance from the PIS, and maximize distance from the NIS, simultaneously. Thus, the MCOP is reformulated according to TOPSIS model denoted by $\boldsymbol{P_2}$ as the following form.

$$\begin{gathered} P_2: \\ Min\ d_p^{PIS}(x) \\ Max\ d_p^{NIS}(x) \\ Subject\ to: x \in \Psi \end{gathered} \tag{14}$$

The second phase of the proposed TOPSIS-IFS approach employs the IFS conception. In this sense, each objective of the TOPSIS model is optimized individually to obtain its aspiration level which then is used to formulate the satisfaction and dissatisfaction degrees of the IFS model. This can be expressed mathematically as follows.

$$\left(d_p^{PIS}\right)^* = \min_{x\in\Psi} d_p^{PIS}(x) \text{ and the solution is } x^P \tag{15}$$

$$\left(d_p^{NIS}\right)^* = \max_{x\in\Psi} d_p^{NIS}(x) \text{ and the solution is } x^N \tag{16}$$

$$\left(d_p^{PIS}\right)^- = d_p^{PIS}(x^N) \text{ and } \left(d_p^{NIS}\right)^- = d_p^{NIS}(x^P) \tag{17}$$

These distances $\left(\boldsymbol{d_p^{PIS}}\right)^-$ and $\left(\boldsymbol{d_p^{NIS}}\right)^-$ are computed as

$$\left(\boldsymbol{d_p^{PIS}}\right)^- = \max_{x\in\Psi} \boldsymbol{d_p^{PIS}(x)} \text{ and } \left(\boldsymbol{d_p^{NIS}}\right)^- = \min_{x\in\Psi} \boldsymbol{d_p^{NIS}(x)}$$

Let $\left(\boldsymbol{d_p}\right)^* = \left(\left(\boldsymbol{d_p^{PIS}}\right)^*, \left(\boldsymbol{d_p^{NIS}}\right)^*\right)$ and $\left(\boldsymbol{d_p}\right)^- = \left(\left(\boldsymbol{d_p^{PIS}}\right)^-, \left(\boldsymbol{d_p^{NIS}}\right)^-\right)$.

After formulating the TOPSIS model, the conflicting nature among the $\boldsymbol{d_p^{PIS}(x)}$ and $\boldsymbol{d_p^{NIS}(x)}$ objectives is resolved with the help of the IFS technique where the acceptance membership (satisfaction) and rejection membership (dissatisfaction) are considered for each objective. To model the membership and non-membership functions for the $\boldsymbol{d_p^{PIS}(x)}$, the individual best optimal value of each objective function with its aspiration level should be set first. For example, Eq (15) finds the solitary minimum of the $\boldsymbol{d_p^{PIS}}(\boldsymbol{x})$ denoted by $\left(\boldsymbol{d_p^{PIS}}\right)^*$, where the corresponding objective value $\boldsymbol{d_p^{NIS}}(\boldsymbol{x^P})$ denoted by $\left(\boldsymbol{d_p^{PIS}}\right)^-$ is considered as the aspiration level. Therefore, the upper and lower levels of acceptability of the membership function are $\left(\boldsymbol{d_p^{PIS}}\right)^*$ and $\left(\boldsymbol{d_p^{PIS}}\right)^-$respectively. In this sense, the membership function is expressed linearly between upper and lower levels of acceptability with assigning a specific grade for each value within those levels of acceptability. A 1 grade is adopted for values less than or equal to $\left(\boldsymbol{d_p^{PIS}}\right)^*$, and a 0 grade is adopted for values greater than or equal to $\left(\boldsymbol{d_p^{PIS}}\right)^-$. Similarly, the non-membership function of the IFS is formulated using the lower and upper levels of acceptability of the non-membership function ($\left(\boldsymbol{d_p^{PIS}}\right)^*$ and $\left(\boldsymbol{d_p^{PIS}}\right)^- + \boldsymbol{\alpha_1}$), where $\boldsymbol{\alpha_1}$ defines the degree of hesitancy or neutrality for the $\boldsymbol{d_p^{PIS}}(\boldsymbol{x})$. Meanwhile, the DM is unwilling to accept values greater than $\left(\boldsymbol{d_p^{PIS}}\right)^-$ without completely rejecting the values from $\left(\boldsymbol{d_p^{PIS}}\right)^-$ to $\left(\boldsymbol{d_p^{PIS}}\right)^- + \boldsymbol{\alpha_1}$. Similarly, the membership and non-membership functions for the $\boldsymbol{d_p^{NIS}}(\boldsymbol{x})$ can be modeled. Therefore, the membership function $\boldsymbol{\mu_1}(\boldsymbol{x}) \equiv \boldsymbol{\mu}_{\boldsymbol{d_p^{PIS}}}(\boldsymbol{x})$ and non-membership function $\boldsymbol{\nu_1}(\boldsymbol{x}) \equiv \boldsymbol{\nu}_{\boldsymbol{d_p^{PIS}}}(\boldsymbol{x})$ for the $\boldsymbol{d_p^{PIS}}(\boldsymbol{x})$ can be depicted as in Figure 1 and expressed by Eqs (18) and (19). Also, the membership function $\boldsymbol{\mu_2}(\boldsymbol{x}) \equiv \boldsymbol{\mu}_{\boldsymbol{d_p^{NIS}}}(\boldsymbol{x})$, and non-membership function $\boldsymbol{\nu_2}(\boldsymbol{x}) \equiv \boldsymbol{\nu}_{\boldsymbol{d_p^{NIS}}}(\boldsymbol{x})$ for the $\boldsymbol{d_p^{NIS}}(\boldsymbol{x})$ can be depicted as in Figure 2 and Eqs (20) and (21).

$$\boldsymbol{\mu_1}(\boldsymbol{x}) = \begin{cases} \mathbf{1} & \boldsymbol{if} \quad \boldsymbol{d_p^{PIS}}(\boldsymbol{x}) \le \left(\boldsymbol{d_p^{PIS}}\right)^* \\ \mathbf{1} - \frac{d_p^{PIS}(x) - \left(d_p^{PIS}\right)^*}{\left(d_p^{PIS}\right)^- - \left(d_p^{PIS}\right)^*} & \boldsymbol{if} \quad \left(\boldsymbol{d_p^{PIS}}\right)^* \le \boldsymbol{d_p^{PIS}}(\boldsymbol{x}) \le \left(\boldsymbol{d_p^{PIS}}\right)^- \\ \mathbf{0} & \boldsymbol{if} \quad \left(\boldsymbol{d_p^{PIS}}\right)^- \le \boldsymbol{d_p^{PIS}}(\boldsymbol{x}) \end{cases} \tag{18}$$

$$\boldsymbol{\nu_1}(\boldsymbol{x}) = \begin{cases} \mathbf{0} & \boldsymbol{if} \quad \boldsymbol{d_p^{PIS}}(\boldsymbol{x}) \le \left(\boldsymbol{d_p^{PIS}}\right)^* \\ \mathbf{1} - \frac{\left(\left(d_p^{PIS}\right)^- + \alpha_1\right) - d_p^{PIS}(x)}{\left(\left(d_p^{PIS}\right)^- + \alpha_1\right) - \left(d_p^{PIS}\right)^*} & \boldsymbol{if} \quad \left(\boldsymbol{d_p^{PIS}}\right)^* \le \boldsymbol{d_p^{PIS}}(\boldsymbol{x}) \le \left(\boldsymbol{d_p^{PIS}}\right)^- + \boldsymbol{\alpha_1} \\ \mathbf{1} & \boldsymbol{if} \quad \boldsymbol{d_p^{PIS}}(\boldsymbol{x}) \ge \left(\boldsymbol{d_p^{PIS}}\right)^- + \boldsymbol{\alpha_1} \end{cases} \tag{19}$$

$$\boldsymbol{\mu_2}(\boldsymbol{x}) = \begin{cases} \mathbf{1} & \boldsymbol{if} \quad \boldsymbol{d_p^{NIS}}(\boldsymbol{x}) \ge \left(\boldsymbol{d_p^{NIS}}\right)^* \\ \mathbf{1} - \frac{\left(d_p^{NIS}\right)^* - d_p^{NIS}(x)}{\left(d_p^{NIS}\right)^* - \left(d_p^{NIS}\right)^-} & \boldsymbol{if} \quad \left(\boldsymbol{d_p^{NIS}}\right)^- \le \boldsymbol{d_p^{NIS}}(\boldsymbol{x}) \le \left(\boldsymbol{d_p^{NIS}}\right)^* \\ \mathbf{0} & \boldsymbol{if} \quad \boldsymbol{d_p^{NIS}}(\boldsymbol{x}) < \left(\boldsymbol{d_p^{NIS}}\right)^- \end{cases} \tag{20}$$

$$\boldsymbol{\nu_2}(\boldsymbol{x}) = \begin{cases} \mathbf{1} & \boldsymbol{if} \quad \boldsymbol{d_p^{NIS}}(\boldsymbol{x}) \le \left(\boldsymbol{d_p^{NIS}}\right)^- - \boldsymbol{\alpha_2} \\ \mathbf{1} - \frac{d_p^{NIS}(x) - \left(\left(d_p^{NIS}\right)^- - \alpha_2\right)}{\left(d_p^{NIS}\right)^* - \left(\left(d_p^{NIS}\right)^- - \alpha_2\right)} & \boldsymbol{if} \quad \left(\boldsymbol{d_p^{NIS}}\right)^- - \boldsymbol{\alpha_2} \le \boldsymbol{d_p^{NIS}}(\boldsymbol{x}) \le \left(\boldsymbol{d_p^{NIS}}\right)^* \\ \mathbf{0} & \boldsymbol{if} \quad \boldsymbol{d_p^{NIS}}(\boldsymbol{x}) \ge \left(\boldsymbol{d_p^{NIS}}\right)^* \end{cases} \tag{21}$$

The proposed shape of the membership and non-membership functions for $\boldsymbol{d_p^{PIS}}(\boldsymbol{x})$ has been provided in Figure 1. Based on this shape, it is noted that in the interval $\left[\left(\boldsymbol{d_p^{PIS}}\right)^-, \left(\boldsymbol{d_p^{PIS}}\right)^- + \boldsymbol{\alpha_1}\right]$, the membership degree regarding the goal is zero, while the other is not, which means that the decision maker is not concerned to take more than $\boldsymbol{d_p^{PIS}}(\boldsymbol{x})$ but at this moment not strictly rejecting the values from $\left(\boldsymbol{d_p^{PIS}}\right)^-$to $\left(\boldsymbol{d_p^{PIS}}\right)^- + \boldsymbol{\alpha_1}$. Also, for Figure 2 which represents the maximization of $\boldsymbol{d_p^{NIS}}(\boldsymbol{x})$, the satisfaction degree of the decision maker increases with an aim to approach its respective upper bound $\left(\boldsymbol{d_p^{NIS}}\right)^*$.

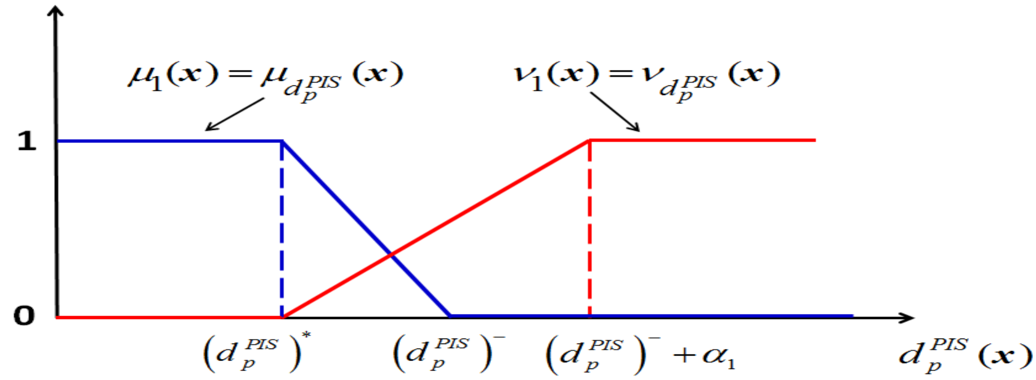


Figure 1. Membership and non-membership functions for $\boldsymbol{d_p^{PIS}}(\boldsymbol{x})$.

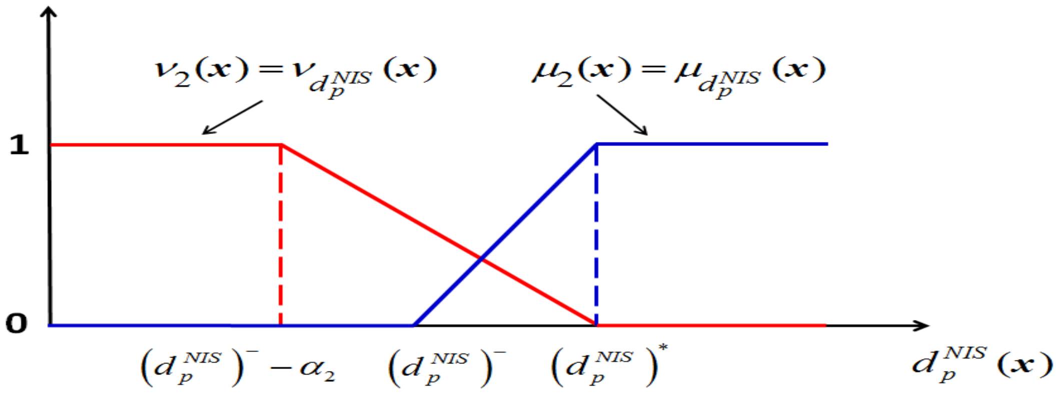


Figure 2. Membership and non-membership functions for $\boldsymbol{d_p^{NIS}}(\boldsymbol{x})$.

Now, the main goal is to increase satisfaction level ($\boldsymbol{\Delta}$) and decrease dissatisfaction level ($\boldsymbol{\Delta'}$) of the decision maker, while increasing the degree of attainability and decreasing the degree of non-attainability are the main goals of the IFS model. This can be described as follows:

Let $\boldsymbol{\Delta} = \boldsymbol{min}\{\boldsymbol{\mu_1}(\boldsymbol{x}), \boldsymbol{\mu_2}(\boldsymbol{x})\}$ and $\boldsymbol{\Delta'} = \boldsymbol{min}\{\boldsymbol{\nu_1}(\boldsymbol{x}), \boldsymbol{\nu_2}(\boldsymbol{x})\}$. Then the IFS model of problem $\boldsymbol{P_2}$ can be formulated as in problem ($\boldsymbol{P_3}$):

$$\begin{gathered} P_3: \\ \{Max\ \Delta, Min\ \Delta'\} \\ Subject\ to: \begin{cases} x \in \Psi \\ \mu_1(x) \geq \Delta, \mu_2(x) \geq \Delta \\ \nu_1(x) \leq \Delta', \nu_2(x) \leq \Delta' \\ \Delta, \Delta' \geq 0, \Delta + \Delta' \leq 1 \end{cases} \end{gathered} \tag{22}$$

This model can be rewritten as a deterministic single objective model as follows.

$$\begin{gathered} P_4: \\ Max(\Delta - \Delta') \end{gathered} \tag{23}$$

$$Subject\ to: \begin{cases} x \in \Psi \\ \left(d_p^{PIS}\right)^- - d_p^{PIS}(x) \geq \Delta\left(\left(d_p^{PIS}\right)^- - \left(d_p^{PIS}\right)^*\right) \\ d_p^{PIS}(x) - \left(d_p^{PIS}\right)^* \leq \Delta'\left(\left(\left(d_p^{PIS}\right)^- + \alpha_1\right) - \left(d_p^{PIS}\right)^*\right) \\ d_p^{NIS}(x) - \left(d_p^{NIS}\right)^- \geq \Delta\left(\left(d_p^{NIS}\right)^* - \left(d_p^{NIS}\right)^-\right) \\ \left(d_p^{NIS}\right)^* - d_p^{NIS}(x) \leq \Delta'\left(\left(d_p^{NIS}\right)^* - \left(\left(d_p^{NIS}\right)^- - \alpha_2\right)\right) \\ \Delta, \Delta' \geq 0, \Delta + \Delta' \leq 1 \end{cases}$$

Thus, the overall solution procedures of the proposed TOPSIS-IFS methodology can be stated as follows.

Step 1: Formulate the MCOP.
Step 2: Obtain the individual minimum and maximum for each objective of the candidate problem.
Step 3: Construct the equivalent model based on the distance's functions of TOPSIS, $\boldsymbol{d_p^{PIS}(x)}$ and $\boldsymbol{d_p^{NIS}(x)}$.
Step 4: Get the maximum and minimum corresponding to objectives $\boldsymbol{d_p^{PIS}(x)}$ and $\boldsymbol{d_p^{NIS}(x)}$ under the given constraints.
Step 5: Obtain the PIS payoff table of problem $\boldsymbol{P_2}$ to obtain $\left(\boldsymbol{d_p}\right)^* = \left(\left(\boldsymbol{d_p^{PIS}}\right)^*, \left(\boldsymbol{d_p^{NIS}}\right)^*\right)$ and also form the NIS payoff table to obtain $\left(\boldsymbol{d_p}\right)^- = \left(\left(\boldsymbol{d_p^{PIS}}\right)^-, \left(\boldsymbol{d_p^{NIS}}\right)^-\right)$.
Step 6: Identify the membership and non-membership functions for each objective function of the TOPSIS phase.
Step 7: Ask the DM to select $\boldsymbol{p}$, $\{\boldsymbol{p = 1, 2, \ldots, \infty}\}$.
Step 8: Formulate the IFS model as in (23).
Step 9: Solve the IFS model to get the BCS.
Step 10: Get the satisfaction level ($\boldsymbol{\Delta}$) and dissatisfaction level ($\boldsymbol{\Delta'}$).
Step 11: If the obtained solution is satisfactory for the DM, stop. Else go to Step 1 through changing the hesitancy degrees ($\boldsymbol{\alpha_1}$and $\boldsymbol{\alpha_2}$).
Step 12: Repeat the steps until the DM is convinced of the found solutions.

4. Numerical illustration

This section provides the validation of the proposed TOPSIS-IFS approach through a numerical example taken from the literature [34]. This example was solved based on classical FS approach using the satisfaction degree only, while the proposed TOPSIS-IFS approach has considered the IFS based on the two membership functions for each objective (i.e., satisfaction degree, dissatisfaction degree). The obtained results affirmed that the proposed model is superior to the classical one. Also, a multi-objective transportation problem (MOTP) has been conducted [35]. The obtained solution is compared with the result obtained from the literature.

$$\begin{gathered} Max\ f_1(x) = x_1^2 + x_2^2 + x_3^2 \\ Max\ f_2(x) = (x_1 - 1)^2 + x_2^2 + (x_3 - 2)^2 \\ Max\ f_3(x) = 2x_1 + x_2^2 + x_3 \\ \text{Subject to} \\ x = (x_1, x_2, x_3) \in \Psi = \{x_1 - 3x_2 + 4x_3 \leq 6, \\ 2x_1^2 + 2x_2 + x_3 \leq 10, \\ 0 \leq x_1 \leq 3, 0 \leq x_2 \leq 4, 0 \leq x_3 \leq 2\} \end{gathered} \tag{24}$$

The first step is to get the minimum and maximum for each objective function individually as recorded in Table 1.

Table 1. PIS payoff matrix of (24).

	f_1	f_2	f_3	x_1	x_2	x_3
$Maxf_1$	11.1111	16.1111	11.1111	0	3.3333	0
$Maxf_2$	11.1111	16.1111	11.1111	0	3.3333	0
$Minf_3$	1.6687E-7	5	0.2600E-6	0.4657E-7	0.4085E-3	0

Table 2. NIS payoff matrix of (24).

	f_1	f_2	f_3	x_1	x_2	x_3
$Minf_1$	0.1034E-6	4.9989	5.5164E-4	0.1826E-3	0.1880E-3	0.1864E-3
$Minf_2$	3.2677	0.3461	3.4270	0.8846	0.3461	1.538
$Maxf_3$	10.9337	15.4791	11.3372	0.2273	3.2988	0

From Tables 1 and 2, the PIS and NIS solutions can be obtained respectively as, $\boldsymbol{f}^* = (\mathbf{11.1111}, \mathbf{16.1111}, \mathbf{0.2600}\boldsymbol{E}-\mathbf{6})$ and $\boldsymbol{f}^- = (\mathbf{0.1034}\boldsymbol{E}-\mathbf{6}, \mathbf{0.3461}, \mathbf{11.3372})$. Then, the second step of the solution procedures is to employ the distance formulations of TOPSIS approach as follows.

$$\boldsymbol{F_1} = \boldsymbol{d_p^{PIS}} = \left(\begin{array}{c} \boldsymbol{w_1^2}\left(\dfrac{11.1111 - \left(\boldsymbol{x_1^2} + \boldsymbol{x_2^2} + \boldsymbol{x_3^2}\right)}{11.1111\text{-}0.1034\text{E-}6}\right)^{\mathbf{2}} + \\ +\boldsymbol{w_2^2}\left(\dfrac{16.1111 - ((\boldsymbol{x_1} - \mathbf{1})^{\mathbf{2}} + \boldsymbol{x_2^2} + (\boldsymbol{x_3} - \mathbf{2})^{\mathbf{2}})}{16.1111\text{-}0.3461}\right)^{\mathbf{2}} \\ +\boldsymbol{w_3^2}\left(\dfrac{(\mathbf{2}\boldsymbol{x_1} + \boldsymbol{x_2^2} + \boldsymbol{x_3}) - \mathbf{0.2600}\text{E-}6}{11.3372\text{-}\mathbf{0.2600}\text{E-}6}\right)^{\mathbf{2}} \end{array} \right)^{\frac{1}{2}}$$

$$\boldsymbol{F_2} = \boldsymbol{d_p^{NIS}} = \left(\begin{array}{c} \boldsymbol{w_1^2}\left(\dfrac{\left(\boldsymbol{x_1^2} + \boldsymbol{x_2^2} + \boldsymbol{x_3^2}\right) - 0.1034\text{E-}6}{11.1111\text{-}0.1034\text{E-}6}\right)^{\mathbf{2}} \\ +\boldsymbol{w_2^2}\left(\dfrac{((\boldsymbol{x_1} - \mathbf{1})^{\mathbf{2}} + \boldsymbol{x_2^2} + (\boldsymbol{x_3} - \mathbf{2})^{\mathbf{2}})\text{-}0.3461}{16.1111\text{-}0.3461}\right)^{\mathbf{2}} \\ +\boldsymbol{w_3^2}\left(\dfrac{11.3372\text{-}(\mathbf{2}\boldsymbol{x_1} + \boldsymbol{x_2^2} + \boldsymbol{x_3})}{11.3372\text{-}\mathbf{0.2600}\text{E-}6}\right)^{\mathbf{2}} \end{array} \right)^{\frac{1}{2}} \qquad (25)$$

Subject to

$$\boldsymbol{x} = (\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}) \in \boldsymbol{\Psi} = \{\boldsymbol{x_1} - \mathbf{3}\boldsymbol{x_2} + \mathbf{4}\boldsymbol{x_3} \le \mathbf{6}, \mathbf{2}\boldsymbol{x_1^2} + \mathbf{2}\boldsymbol{x_2} + \boldsymbol{x_3} \le \mathbf{10}, \mathbf{0} \le \boldsymbol{x_1} \le \mathbf{3}, \mathbf{0} \le \boldsymbol{x_2} \le \mathbf{4}, \mathbf{0} \le \boldsymbol{x_3} \le \mathbf{2}\}$$

For problem (25), assume that $\boldsymbol{w_1} = \boldsymbol{w_2} = \boldsymbol{w_3} = \frac{\mathbf{1}}{\mathbf{3}}$, where the model (25) of the $\boldsymbol{d_p^{PIS}}$ and $\boldsymbol{d_p^{NIS}}$ is solved, and the solution is given in Table 3.

Table 3. The payoff matrix of (25).

	F_1	F_1	x_1	x_2	x_3
$Min\ F_1$	0.4415	0.5939	0	2.6017	0
$Max\ F_2$	0.5657	0.8166	0	3.3333	0

Based on Table 3, the PIS and NIS solutions of problem (25) can be computed respectively as, $\boldsymbol{F}^* = (\mathbf{0.4415}, \mathbf{0.8166})$ and $\boldsymbol{F}^- = (\mathbf{0.5657}, \mathbf{0.5939})$. Therefore, the membership functions (i.e., $\boldsymbol{\mu}_1$and $\boldsymbol{\mu}_2$) and non- membership functions (i.e., $\boldsymbol{\nu}_1$ and $\boldsymbol{\nu}_2$) are formulated as mentioned in Eqs (18)-(21). Afterwards, the IFS model can be formed as in (23). Through solving model (23), the BCS can be obtained with satisfaction level $\boldsymbol{\Delta} = 0.6154$ and dissatisfaction level $\boldsymbol{\Delta}' = 0.2614$, $\boldsymbol{x}_1 = \mathbf{0}$, $\boldsymbol{x}_2 = 2.6667$, $\boldsymbol{x}_3 = \mathbf{2}$.

4.1. Validation on MOTP model

This subsection emphasis the validation of the proposed TOPSIS-IFS approach for MOTP associated with some descriptions. Consider a MOTP with four sources $\boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{a}_3, \boldsymbol{a}_4$, and five destinations $\boldsymbol{b}_1, \boldsymbol{b}_2, \boldsymbol{b}_3, \boldsymbol{b}_4, \boldsymbol{b}_5$. Suppose the goods are to be delivered from the $\boldsymbol{i}^{th}$ ($\boldsymbol{i} = \mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}$) source to the $\boldsymbol{j}^{th}$ ($\boldsymbol{j} = \mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}, \mathbf{5}$) destination. The costs of transportation along with supply and demand are given in Table 4. Therefore, without loss of generality, the MOTP defined as transportation cost, the transportation time and loss during the transportation can be expressed as a multi-objective optimization problem as follows [35].

Table 4. Descriptions of MOTP.

Supply	Demand	Penalty
$a_1 = 5$, $a_2 = 4$, $a_3 = 2$, $a_4 = 9$.	$b_1 = 4$, $b_2 = 4$, $b_3 = 6$, $b_4 = 2$, $b_5 = 4$.	$C_1 = \begin{bmatrix} 9 & 12 & 9 & 6 & 9 \\ 7 & 3 & 7 & 7 & 5 \\ 6 & 5 & 9 & 11 & 3 \\ 6 & 8 & 11 & 2 & 2 \end{bmatrix}, C_2 = \begin{bmatrix} 2 & 9 & 8 & 1 & 4 \\ 1 & 9 & 9 & 5 & 2 \\ 8 & 1 & 8 & 4 & 5 \\ 2 & 8 & 6 & 9 & 8 \end{bmatrix}, C_3 = \begin{bmatrix} 2 & 4 & 6 & 3 & 6 \\ 4 & 8 & 4 & 9 & 2 \\ 5 & 3 & 5 & 3 & 6 \\ 6 & 9 & 6 & 3 & 1 \end{bmatrix}$

$$\boldsymbol{Min\ f_1} = \sum_{\boldsymbol{i=1}}^{\boldsymbol{4}} \sum_{\boldsymbol{j=1}}^{\boldsymbol{5}} \boldsymbol{C_{ij}^1 x_{ij}}$$

$$\boldsymbol{Min\ f_2} = \sum_{\boldsymbol{i=1}}^{\boldsymbol{4}} \sum_{\boldsymbol{j=1}}^{\boldsymbol{5}} \boldsymbol{C_{ij}^2 x_{ij}}$$

$$\boldsymbol{Min\ f_3} = \sum_{\boldsymbol{i=1}}^{\boldsymbol{4}} \sum_{\boldsymbol{j=1}}^{\boldsymbol{5}} \boldsymbol{C_{ij}^3 x_{ij}}$$

$\boldsymbol{Subject\ to}$:

$$\sum_{\boldsymbol{j=1}}^{\boldsymbol{5}} \boldsymbol{x_{ij}} \le \boldsymbol{a_i}, \boldsymbol{i} = \mathbf{1}, \mathbf{2}, \ldots, \mathbf{4}$$

$$\sum_{i=1}^{4} \boldsymbol{x_{ij}} \leq \boldsymbol{b_j}, \boldsymbol{j} = \mathbf{1}, \mathbf{2}, \ldots, \mathbf{5}$$

$$\boldsymbol{x_{ij}} \geq \mathbf{0} \text{ and integer } \forall\, \boldsymbol{i}, \boldsymbol{j}. \tag{26}$$

where $\boldsymbol{C_{ij}^k}$ defines the $\boldsymbol{k^{th}}$penalty criterion $\boldsymbol{k}$ ($\boldsymbol{k} = \mathbf{1}, \mathbf{2}, \mathbf{3}$), that can be transportation cost, delivery time etc. related to transporting the item from the $\boldsymbol{i^{th}}$ source to the $\boldsymbol{j^{th}}$ destination, $\boldsymbol{x_{ij}}$ defines the quantity of an item that needs to be transported from the $\boldsymbol{i^{th}}$ source to the $\boldsymbol{j^{th}}$ destination, $\boldsymbol{b_j}$ and $\boldsymbol{a_i}$ refer to available demand and supply at $\boldsymbol{j^{th}}$ destination and $\boldsymbol{i^{th}}$ source respectively. It is assumed that $\boldsymbol{C_{ij}^k} \geq \mathbf{0}, \boldsymbol{a_i} \geq \mathbf{0}, \boldsymbol{b_j} \geq \mathbf{0}\ \forall\, \boldsymbol{k}, \boldsymbol{i}, \boldsymbol{j}$ and $\sum_{i=1}^{4} \boldsymbol{a_i} = \sum_{j=1}^{5} \boldsymbol{b_j}$.

The optimization process starts by finding the individual solution of each objective function is determined, where the bounds for each objective are given as follows: $\mathbf{102} \leq \boldsymbol{f_1} \leq \mathbf{157}, \mathbf{72} \leq \boldsymbol{f_2} \leq \mathbf{157}$ and $\mathbf{64} \leq \boldsymbol{f_3} \leq \mathbf{136}$. Afterwards the distance formulations based on the TOPSIS approach are formulated as in Eq (14), and then the reference points of the PIS and NIS are obtained as, $\boldsymbol{F}^* = (\mathbf{0.3321}, \mathbf{0.7146})$ and $\boldsymbol{F}^- = (\mathbf{0.4140}, \mathbf{0.6789})$. Therefore, the membership functions (i.e., $\boldsymbol{\mu_1}$ and $\boldsymbol{\mu_2}$) and non-membership functions (i.e., $\boldsymbol{\nu_1}$ and $\boldsymbol{\nu_2}$) are formulated as mentioned in Eqs (18) –(21). Afterwards, the IFS model can be formed as in (23). By solving model (23), the BCS can be obtained through obtaining the satisfaction level $\boldsymbol{\Delta}$ = 0.9277 and dissatisfaction level $\boldsymbol{\Delta'}$ = 0.7096E-1,$\boldsymbol{x_{11}} = \mathbf{3}, \boldsymbol{x_{12}} = \mathbf{0}, \boldsymbol{x_{13}} = \mathbf{0}, \boldsymbol{x_{14}} = \mathbf{2}, \boldsymbol{x_{15}} = \mathbf{0}, \boldsymbol{x_{21}} = \mathbf{0}, \boldsymbol{x_{22}} = \mathbf{2}, \boldsymbol{x_{23}} = \mathbf{2}, \boldsymbol{x_{24}} = \mathbf{0}, \boldsymbol{x_{25}} = \mathbf{0}, \boldsymbol{x_{31}} = \mathbf{0}, \boldsymbol{x_{32}} = \mathbf{2}, \boldsymbol{x_{33}} = \mathbf{0}, \boldsymbol{x_{34}} = \mathbf{0}, \boldsymbol{x_{35}} = \mathbf{0}, \boldsymbol{x_{41}} = \mathbf{1}, \boldsymbol{x_{42}} = \mathbf{0}, \boldsymbol{x_{43}} = \mathbf{4}, \boldsymbol{x_{44}} = \mathbf{0}, \boldsymbol{x_{45}} = \mathbf{4}$.

4.2. Performance evaluation

This subsection is adopted to validate the performance in the presence of the set of outcomes. Meanwhile, as TOPSIS-IFS approach acquires relative weights for scalarization of the objective functions, a variety of outcomes can be obtained not only by assigning different weights but also with employing different hesitancy degrees of the IFS model (α_1 and α_2). In this respect, to make the decision more reliable, the closeness strategy [35] is introduced as a powerful assistance tool to sample the BCS which is close to ideal or optimal solution. Moreover, this strategy can be employed for the comparisons among different outcomes. The formulation of closeness strategy is stated as follows.

$$\phi_1(w, K) = 1 - \sum_{i=1}^{K} w_i\, d_i \tag{27}$$

$$\phi_2(w, K) = \left[\sum_{i=1}^{K} w_i^2 (1 - d_i)^2\right]^{\frac{1}{2}} \tag{28}$$

$$\phi_\infty(w, K) = max_i\{w_i(1 - d_i)\} \tag{29}$$

where $\boldsymbol{d_i}$ takes the following form for minimization problem,

$$d_i = \begin{cases} \dfrac{F_i^*}{CS_i} & \text{minimization problem} \\ \dfrac{CS_i}{F_i^*} & \text{maximization problem} \end{cases} \quad (30)$$

where $\boldsymbol{F_i^*}$ and $\boldsymbol{CS_i}$ are the ideal value and the compromise value of $\boldsymbol{F_i}$, respectively. The comparison results of the closeness strategy among the proposed TOPSIS-IFS approach and the classical FS approach are given in Table 5. Based on the obtained results, it is noted that the proposed TOPSIS-IFS approach is more preferred than the classical FS for MCOP and MOTP test instances. In this context, for the MCOP test instance, the proposed TOPSIS-IFS approach is compared with the FS approach [23]. As shown from Table 5, the obtained values of $\boldsymbol{f_1}$ and $\boldsymbol{f_2}$ by the proposed approach are closer to the ideal solution than the FS approach [34] with closeness values of 0.4988, 0.6446, and 0.3333 for $\boldsymbol{\phi_1}$, $\boldsymbol{\phi_2}$, and $\boldsymbol{\phi_3}$, respectively. For the MOTP test instance, the proposed TOPSIS-IFS approach is compared with FS approach [35]. As shown from Table 5, the obtained values of $\boldsymbol{f_1}, \boldsymbol{f_2}$ and $\boldsymbol{f_3}$ by the proposed approach are closer to the ideal solution than the FS approach [24] with closeness values of 0.22081, 0.2297, and 0.1025 for $\boldsymbol{\phi_1}$, $\boldsymbol{\phi_2}$, and $\boldsymbol{\phi_3}$, respectively.

Table 5. Closeness values for fuzzy approach and the proposed approach.

	MCOP test instance			MOTP test instance		
	Ideal solution	FS approach [34]	The proposed approach	Ideal solution	FS approach [35]	The proposed approach
f_1	11.1111	1.3740	11.1111	102	133	127
f_2	16.1111	1.6850	8.1111	72	112	104
f_3	0.2600E-6	1.1722	9.1111	64	86	76
d_1	-	0.12366	1	-	0.7669	0.8031
d_2	-	0.104586	0.503448	-	0.6429	0.6923
d_3	-	2.22E-07	2.85E-08	-	0.7442	0.8421
ϕ_1	-	0.9239	0.4988	-	0.2820	0.22081
ϕ_2	-	0.9255	0.6446	-	0.2871	0.2297
ϕ_∞	-	0.3333	0.3333	-	0.1190	0.1025

4.3. Parametric study

This subsection provides a parametric study for the parameters of IFS model with the aim to demonstrate their effects on the satisfaction and dissatisfaction levels. The hesitancy degrees of satisfaction and dissatisfaction opinions play a vital role in the decision-making process, and their subjective values affect the outcome of the MCOP. Therefore, quantifying the hesitancy degrees using the experimental design technique is convenient and reliable for the optimization process. In this regard, the Taguchi approach [36,37] is employed, as it represents one of the frequently employed experimental design methodologies [38,39] which allow optimization with minimal number of experiments along with robust design solutions. Moreover, the Taguchi experimental design improves quality and reduces the cost. The prominent advantage of the Taguchi approach over the others is the simultaneous optimization of numerous parameters, which allows for the extraction of more quantitative information from fewer experimental trials.

Due to its advantages, the Taguchi approach is adapted to study the impacts of IFS model' parameters with the aim to determine their effects on compromise solutions [40]. The model contains the hesitancy degrees (α_1 and α_2). In this sense, the Taguchi approach is started with setting the levels for the IFS model parameters, where each model parameter is considered with four levels as in Table 6. Then, the effective set of combinations induced by the Taguchi-based a special design of orthogonal arrays is listed in Table 7. For each combination of the parameters α_1 and α_2, the values of satisfaction and dissatisfaction levels are obtained along with the obtained decision variables. In this context, Taguchi classifies the values of satisfaction and dissatisfaction into three categories: the "larger is better", the "smaller is better", and "the nominal is best" types. As satisfaction level the "larger is better" type is selected, while for dissatisfaction the "smaller is better" is chosen. These categories aim to attain the optimal combination of the model parameters that acquire the smallest variance in performance. Accordingly, the statistical analysis based on the signal-to-noise ratio (S/N ratio) provides an effective way to find significant model parameters with their associated levels through evaluating the minimum variance. For more details on the Taguchi selection and use of orthogonal arrays, readers are referred to Ref. [41]. By this selection, Taguchi approach can provide the S/N ratio that reflects the values of the parameters on the satisfaction and dissatisfaction levels as in Figure 3. In this regard, the analysis based on Taguchi experiment using larger is better type for the satisfaction level is performed, where the obtained values of hesitancy degrees ($\alpha_1$and α_2) are illustrated in Figure 3(a). Based on the depicted analysis, the values of $\alpha_1 = 0.35$ and $\alpha_2 = 0.002$ have significant effects on the satisfaction level. Moreover, the analysis based on Taguchi experiment using smaller is better type for the dissatisfaction level is carried out, where the obtained values of hesitancy degrees are presented in Figure 3(b). Figure 3(b) demonstrates that the values of $\alpha_1 = 0.35$ and $\alpha_2 = 0.1$ influence the performance on the dissatisfaction level.

Table 6. Parameters with their levels.

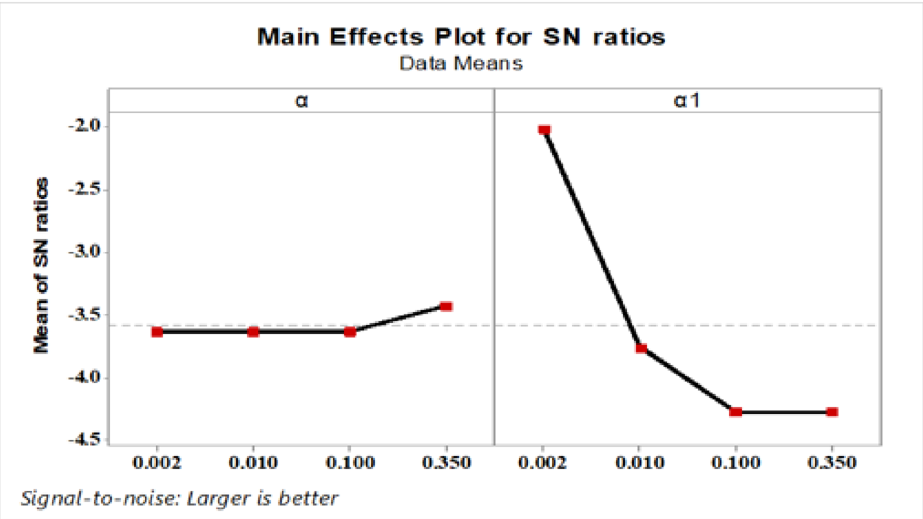
Parameter	Levels			
	Level 1	Level 2	Level 3	Level 4
α_1	0.002	0.01	0.1	0.35
α_2	0.002	0.01	0.1	0.35

Table 7. Parameters with their levels.

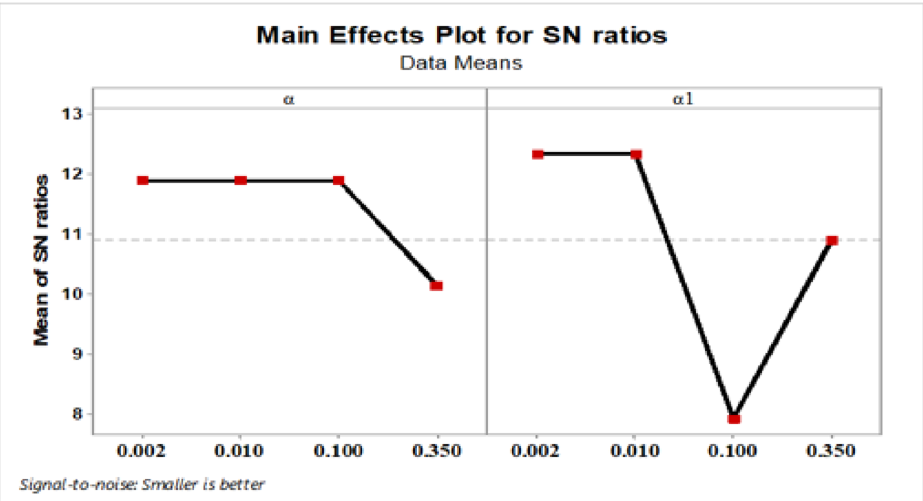
Parameter		Satisfaction level	Dissatisfaction level	Obtained solution		
α_1	α_2	Δ	Δ'	x_1	x_2	x_3
0.002	0.002	0.8051	0	0	3.1171	0
0.002	0.01	0.6154	0	0	2.6667	2
0.002	0.1	0.6154	0	0	2.6667	2
0.002	0.35	0.6154	0.2541	0	2.6667	2
0.01	0.002	0.8051	0	0	3.1171	0
0.01	0.01	0.6154	0	0	2.6667	2
0.01	0.1	0.6154	0	0	2.6667	2
0.01	0.35	0.6154	0.2541	0	2.6667	2
0.1	0.002	0.8051	0	0	3.1171	0

Continued on next page

Parameter		Satisfaction level	Dissatisfaction level	Obtained solution		
α_1	α_2	Δ	Δ'	x_1	x_2	x_3
0.1	0.01	0.6154	0	0	2.6667	2
0.1	0.1	0.6154	0	0	2.6667	2
0.1	0.35	0.6154	0.2541	0	2.6667	2
0.35	0.002	0.7587	0.2412	0	3.0597	0
0.35	0.01	0.7587	0.2412	0	3.0597	0
0.35	0.1	0.5984	0.4015	0	2.6409	2
0.35	0.35	0.5984	0.4015	0	2.6409	2



(a)



(b)

Figure 3. The SN ratio plot for the parameters values (a) Δ (b) Δ'in Example 1.

4.4. Characteristic comparison

Aside from the above comparison, we also compare the critical characteristic of the proposed approach with the various approaches present in the literature. The characteristics comparison table is shown in Table 8. In this table, the symbol ✓ indicates that the method satisfies the particular feature, but the, symbol × means that the corresponding process fails to satisfy. For instance, the present approach is suitable to describe a wider range of the information in terms of IFS features, while some approaches [12,13,19,34,35] have less range of information. Thus, their approaches have some limitations in access. On the other hand, in the proposed approach, we consider the attribute characteristic of the decision maker towards the assessment of the final best alternative, while the existing approaches [12,13,34,35] do not consider it during the evaluation. Hence, such existing approaches does not provide the more compromise solution as per the need of the expert. Aside from these features, the proposed approach has defined the membership and non-membership grades of the PIS and NIS, which helps to satisfy the region of the grades more closely, while existing approaches fails to satisfy this feature. This means that, in the existing approaches, the decision maker cannot increase/decrease their satisfaction or dissatisfaction degrees towards the upper or lower bounds of the reference level. To make a decision smoother and better, there is an always a need to consider a level of satisfaction and dissatisfaction of the decision-maker into the analysis so that by increasing or decreasing their levels, the degree of the attainability can be achieved. Finally, the ideal points in most of the existing approaches are considered as an ideal and independent of the aspiration level; however, we have considered it properly in the proposed approach.

Table 8. Characteristic comparison of the proposed approach.

	Express a wider range of information?	Consider multiple experts?	Consider the attitude character of the decision-maker?	Consider the satisfaction levels of the decision-maker?	Compute the ideal values using aspiration level?	Define the membership functions for PIS and NIS?	Consider the multi-Criterion problems?
Singh et al. [35]	×	×	×	×	×	×	×
Abo-Sinna and Amer [34]	×	✓	×	×	✓	✓	✓
Razmi et al. [19]	✓	✓	✓	✓	×	×	✓
Tavana and Hatamu-Marbini [13]	×	×	×	×	×	×	✓
Akgul et al. [12]	×	×	✓	×	×	×	✓
Proposed approach	✓	✓	✓	✓	✓	✓	✓

5. Conclusions

In this paper, an attempt has been made to provide a new concept in the optimization filed through integrating the TOPSIS approach with the IFS model, named TOPSIS-IFS. However, FS approaches were employed for solving optimization tasks; they do not always judge the real-world decision situations as they rely only the on satisfaction-based membership function. In such circumstances, IFS overcomes this dilemma by introducing membership and non-membership functions which play a vital role in modeling the practical situations. The proposed TOPSIS-IFS approach acquires two features, namely, TOPSIS phase that aims to reduce the K-dimensional objectives into two-dimensional objectives, and IFS phase that aims to provide realistic representation of objectives by defining two membership functions for each objective function (i.e., satisfaction degree, dissatisfaction degree). TOPSIS-IFS is validated using an illustrative example, and it is realized on the MOTP to affirm its practicability. The obtained solution by the TOPSIS-IFS affirmed its superiority to those existing in the literature. Finally, this integrating depicts a new philosophy in the mathematical programming field due to its interesting principles and its neutrality aspect. Also, this approach can assist the operators and designer to obtain more reasonable and reliable decisions by altering the hesitancy degrees ($\boldsymbol{\alpha_1}$ and $\boldsymbol{\alpha_2}$) of membership and non-membership functions.

The stated approach has a wide ability to capture the information related to the decision-making problem, but it is simultaneously true that the considered IFS environment encounters some problems during the rating. For instance, when an expert submits their evaluation for which the sum of their membership degress exceed 1, then such an approach is not applicable. Although the proposed TOPSIS-IFS method provides a broader model to address the decision-making process which is accompanied by the uncertainty aspect through considering the satisfaction and dissatisfaction degrees of the information, the utilization of the proposed approach on high-dimensional problems still deserves further exploration. To resolve this problem, we intend to create more adaptable mathematical frameworks in the future, which should allow us to record a noticeable greater range of evaluation. Additionally, we can generalize our approach, and it will allow us to expand the application of the approach to deal with practical cases such as the Economic-emission load dispatch problem, and multi-objective wind farm layout optimization. Also, some other modification will be suggested, and exploring the decision-making process under the rough set theory environment is a worthwhile direction. Finally, we can extend the approach to the different types of optimization problems such as fractional programming and multi-level programming problems and the diverse application by using some extended efficient optimization algorithms such as support vector machine [42], many-objective optimization [43,44], scheduling problems [45].

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Conflict of interest

The authors declare no conflict of interest regarding the publication of this article.

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