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Selective arc-ng pricing for vehicle routing

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Abstract

Column generation algorithms for solving vehicle routing problems often rely on a relaxed pricing subproblem where routes may be nonelementary and which is solved by a labeling algorithm. This pricing algorithm is said to be selective if it can discard nonelementary paths that may be Pareto-optimal but guarantees finding at least one (not necessarily elementary) path of negative reduced cost when there exists at least one elementary path of negative reduced cost. In this paper, we propose a selective pricing mechanism for a recently introduced variant of the *ng*-route relaxation, in which the neighborhoods are associated with arcs instead of nodes. We extend a state-of-the-art set-based dominance criterion for this problem and show, by means of an exhaustive computational campaign, that the resulting mechanism is effective at reducing the number of nondominated labels as compared to the original pricing algorithm for the same problem. As a result, typically shorter computing times are required to compute similar lower bounds when the proposed mechanism is embedded within a column generation solver.

Keywords: column generation; shortest path problem with resource constraints; selective pricing; vehicle routing

1. Introduction

The vehicle routing problem (VRP) is a classic optimization problem arising in many applications in logistics. Since the seminal work of Dantzig and Ramser (1959), many researchers and practitioners have attempted to tackle this problem as efficiently as possible. Unfortunately, solving the VRP is \mathcal{NP} -hard as the traveling salesman problem (TSP, Dantzig et al., 1954) can be reduced to a

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VRP in polynomial time. Over the years, several variants of the VRP have been proposed to model the different attributes associated with real-life problems, namely: heterogeneous fleet, multiple depots, combined pickup and delivery, split services, and vehicles with special capabilities (e.g., electric vehicles), among others. For recent compendiums and a broad view of the recent trends in vehicle routing applications, the reader is referred to Toth and Vigo (2014) and Vidal et al. (2020).

Nowadays, the dominant methodology for solving many variants of the VRP is branch-price-and-cut applied to a set-partitioning-type formulation (Costa et al., 2019), which can be described generically as follows. Let N^+ be a set of customers that need to be serviced by exactly one vehicle each. Let Ω be the set of feasible vehicle routes, where the feasibility of a route depends on the VRP variant considered. With each route $r \in \Omega$, associate a cost c_r and, for each customer $i \in N^+$, a binary parameter a_i^r that indicates whether customer i is visited in route r . Furthermore, define a binary variable ω_r , which is equal to 1 if route r is selected in the solution, or 0 otherwise. With this notation, a pure set-partitioning formulation for a VRP is as follows:

$$\min \sum_{r \in \Omega} c_r \omega_r \quad (1)$$

$$\text{s.t. } \sum_{r \in \Omega} a_i^r \omega_r = 1 \quad \forall i \in N^+ \quad (2)$$

$$\omega_r \in \{0, 1\} \quad \forall r \in \Omega. \quad (3)$$

The objective function (1) aims at minimizing the total routing cost. Constraints (2) impose a single visit to each customer $i \in N^+$. Finally, constraints (3) define the domain of the variables. For certain problem variants, additional constraints and variables may be required, yielding a set-partitioning-type model. Note also that, under certain conditions, models (1) and (2) can be replaced by a set-covering model, obtained by replacing the equality in (2) by a greater-than-or-equal relation.

Formulations like (1)–(3) have an exponential number of variables with respect to the number of customers. Handling these variables all at once may be computationally prohibitive. This drawback can be overcome using branch-and-price, that is a branch-and-bound algorithm applying column generation to solve the linear relaxations encountered throughout the search tree. Column generation is an iterative algorithm that solves at each iteration a restricted master problem (RMP) and a pricing subproblem. The RMP is defined as the linear relaxation of (1)–(3) considering only a subset of its variables. It is solved by a linear programming solver to yield a pair of primal and dual solutions. The pricing subproblem looks for negative reduced cost variables to add to the RMP. For most VRPs, it is cast as an elementary shortest path problem with resource constraints (ESPPRC) defined over an application-dependent network and solved by a labeling algorithm. When negative reduced cost columns are identified, they are added to the RMP before starting a new iteration. Otherwise, the column generation algorithm stops and the current RMP solution yields a lower bound. To strengthen the linear relaxations, cutting planes can be added dynamically to yield a branch-price-and-cut algorithm.

As mentioned above, the pricing subproblem is often an ESPPRC, which is strongly \mathcal{NP} -hard (Dror, 1994). At the opposite, the subproblem arising from completely dropping the elementary constraints, called the shortest path problem with resource constraints (SPPRC), is solvable

in pseudo-polynomial time (Desrosiers et al., 1984), but proves to be weak when the resource constraints are not sufficiently tight. To overcome this issue, several authors have focused their attention on strengthening the bounds by partially imposing path elementarity. Two main frameworks can be found in the literature to deal with the issue of partially addressing the elementarity constraints in an efficient manner: decremental state-space relaxation (DSSR, Boland et al., 2006; Righini and Salani, 2006); and cycle elimination (Desrochers et al., 1992; Irnich and Villeneuve, 2006; Baldacci et al., 2011; Bulhões et al., 2018).

In DSSR, the elementarity constraints are imposed gradually. Starting from a pure SPPRC, the path with the least reduced cost is checked for feasibility. If it is, an optimal solution has been found; otherwise, a cycle is detected and the elementarity requirements are strengthened for the subsequent iterations. Cycle elimination techniques, in turn, are closely related to the notion of route relaxations. In the seminal article of Desrochers et al. (1992), the authors presented the first successful application of column generation for VRPs. They considered the vehicle routing problem with times windows (VRPTW, Desaulniers et al., 2014) and solved by branch-and-price. The authors recognized the difficulty of the underlying pricing subproblem and described for the first time the SPPRC without cycles of length 2 (*2-cyc-SPPRC*). Unlike the ESPPRC, this problem can be solved in pseudo-polynomial time. Later, Irnich and Villeneuve (2006) studied the *k-cyc-SPPRC*, for values of *k* that range between 3 and 5, which provides bounds that are much stronger than those obtained by the *2-cyc-SPPRC*, yet, in longer computing times. Baldacci et al. (2011) introduced the *ng-route relaxation (ng-SPPRC)*, which relies on a memory mechanism to remember nodes that have been visited recently within a route. Thus, a feasible path may still contain cycles, but those cycles should be long and have a limited negative impact on the resulting bounds. Some hybrid strategies have been proposed by Martinelli et al. (2014), who successfully achieved improved results by adapting DSSR to tackle the *ng-SPPRC*, and Contardo et al. (2015), who showed that elementary paths can be obtained by imposing partial elementarity in an escalated way within a column-generation-based solver for routing problems. More recently, Bulhões et al. (2018) proposed the *arc-ng-SPPRC*, an extension of the classical *ng-SPPRC*, in which the neighborhoods are associated with arcs instead of nodes. Arc-based neighborhoods allow better management of the undesirable cycles, although, when embedded into a DSSR framework, they usually induce many more iterations compared to a traditional node-based *ng-SPPRC*. Moreover, the authors also devised a novel set-based dominance mechanism that enables to anticipate that certain label extensions may become dominated without the need of performing them.

Desaulniers et al. (2019) introduced the concept of *selective* pricing when the elementarity constraints are dropped at least partially. A pricing algorithm (associated with a column generation pricing subproblem) is said to be *selective* if it can discard nonelementary paths that may be Pareto-optimal but guarantees finding at least one path (possibly with cycles) of negative reduced cost when there exists at least one elementary path with a negative reduced cost. Such an algorithm can stop when it proves that no such elementary paths exist, even if there are still some nonelementary paths with a negative reduced cost. A selective pricing strategy can be used to derive valid dual bounds, and those bounds have the potential of being stronger than those that would be achieved using a nonselective pricing strategy. Yet, Desaulniers et al. (2019) pointed out that the gains in the bounds are only theoretical. They observed in their experiments that the differences between the bounds produced by the selective algorithm and the standard pricing algorithm are not significant. The authors illustrate the selective pricing

strategy on the *ng*-SPPRC, but the algorithm is quite demanding and requires the addition of several data structures and sub-procedures, whose overhead is not always beneficial in the long run.

In this article, we extend the set-based dominance criterion of Bulhões et al. (2018) by making it selective. Unlike the original approach of using additional data structures to keep track of the non-dominated labels (Desaulniers et al., 2019), the proposed mechanism requires minimal modifications of the labeling algorithm and is therefore easy to implement. When applied to *arc-ng*-SPPRC, the proposed selective pricing strategy proves to be faster in practice than the original (nonselective) *arc-ng*-SPPRC when embedded within a column generation algorithm for solving benchmark instances of the VRPTW.

The remainder of this article is organized as follows: Section 2 presents some of the main route relaxations proposed in the literature, with an emphasis on the *arc-ng*-SPPRC. In Section 3, we describe the selective *arc-ng*-SPPRC, provide some theoretical proofs of its correctness, and discuss how this new framework extends the traditional *arc-ng*-SPPRC. In Section 4, we report the results of our computational experiments to show the impact of considering the new framework when tackling some hard VRPTW instances. Finally, some conclusions are drawn in Section 5.

2. Route relaxations

As discussed in the previous section, the natural way of modeling pricing subproblems arising from most VRP applications is by considering an ESPPRC. However, due to the high computational complexity of this problem, route relaxations have been preferred to design efficient column-generation-based algorithms. In this section, we discuss some of the main route relaxations proposed in the literature, namely: the SPPRC, which is obtained when all elementarity requirements are relaxed; and the *ng*-SPPRC and *arc-ng*-SPPRC, which tend to avoid the formation of cycles containing nodes that might be located close to each other. To simplify the exposition, we consider that the pricing subproblem is defined over a digraph $G = (N, A)$, where $N = \{0, \dots, n, n + 1\}$ is its set of nodes and $A \subseteq N \times N$ its set of arcs. Nodes 0 (source) and $n + 1$ (sink) represent the origin and the destination of a path, respectively, whereas $N^+ = N \setminus \{0, n + 1\}$ is the set of customer nodes. The following discussion can be easily extended to other networks.

2.1. The SPPRC

The SPPRC can be formally described as follows. Consider network G and let \mathcal{R} be a set of resources, which are used to model some route feasibility rules for the problem at hand, such as vehicle capacity, time windows, or precedence, among others (Irnich and Desaulniers, 2005). The objective of the SPPRC is to find a least-cost path, not necessarily elementary, from node 0 to node $n + 1$, in such a way that the path respects the following resource constraints.

With each node $v \in N$ and resource $r \in \mathcal{R}$, define resource windows $[e_v^r, l_v^r]$, with $0 \leq e_v^r \leq l_v^r$, representing the allowed limits on the consumption of each resource r by a path ending at node v . Specifically, it is assumed that $l_0^r = 0$ and $l_{n+1}^r = H^r$, where H^r represents the maximum consumption allowed for resource r when arriving at the sink.

With each arc $(u, v) \in A$, we associate a modified routing cost $\bar{c}_{uv} \in \mathbb{R}$ that depends on the RMP dual values such that the sum of the modified costs of the arcs composing a path between nodes 0 and $n + 1$ is equal to the reduced cost of the corresponding route variable. For example, if we denote by α_i , $i \in N^+$, the dual variables associated with constraints (2), then the modified cost of arc $(u, v) \in A$ is given by

$$\bar{c}_{uv} = \begin{cases} c_{uv} - \alpha_u & \text{if } u \in N^+ \\ c_{uv} & \text{otherwise (i.e., } u = 0), \end{cases} \quad (4)$$

where c_{uv} is the routing cost along arc (u, v) . With each arc (u, v) , we also associate a vector of resource consumptions $(\gamma_{uv}^r)_{r \in \mathcal{R}} \in \mathbb{N}_+^{|\mathcal{R}|}$ (e.g., travel time and load delivered). In many classical SPPRC variants, it is assumed that, if a path ending at node v uses less than e_v^r for some resource $r \in \mathcal{R}$, it is still possible to consume the extra amount of resource, so as to respect the lower bound e_v^r . In other words, when a path is extended along an arc (u, v) , assuming the resource consumptions at node u given by ξ_u^r , $\forall r \in \mathcal{R}$, they are updated at node v with the resource extension functions $\xi_v^r \leftarrow \max\{e_v^r, \xi_u^r + \gamma_{uv}^r\}$, $\forall r \in \mathcal{R}$. The extension is deemed *resource-feasible* (i.e., it satisfies the resource constraints) if $e_v^r \leq \xi_v^r \leq l_v^r$, $\forall r \in \mathcal{R}$, otherwise it is deemed infeasible. Note that, depending on the problem at hand, the resources may vary along the paths according to more complex resource extension functions (Irnich, 2008). For the sake of simplicity and without loss of generality, we limit our discussion to the above resource extension functions.

2.2. The *ng*-SPPRC of Baldacci et al. (2011)

Currently, the state-of-the-art—and widely adopted—framework to eliminate cycles is the *ng*-route relaxation (*ng*-SPPRC) proposed by Baldacci et al. (2011). The *ng*-SPPRC differs from the *2-cyc*-SPPRC and *k-cyc*-SPPRC because the notion of length of a cycle does not depend on the number of arcs (or edges) traversed along the cycle, but rather on the notion of proximity between the nodes that belong to it. In the *ng*-SPPRC, cycles are forbidden whenever they contain customers that are close to each other. For every customer $v \in N^+$, one considers a *ng*-set (neighborhood) $\mathcal{N}_v \subseteq N^+$. Because the goal is to prevent short cycles, it is common to define the *ng*-set \mathcal{N}_v as containing the Δ geographically closest customers to v , and the vertex v itself, where $\Delta > 0$ is a predefined parameter. Let $P = \{v_1, \dots, v_h = v, \dots, v_{h'} = v, \dots, v_\ell\}$ be a path generated while solving the *ng*-SPPRC. The cycle $H = \{v_h = v, \dots, v_{h'} = v\}$ is feasible if and only if $v \notin \bigcap_{i=h+1}^{h'-1} \mathcal{N}_{v_i}$, that is, v does not belong to the *ng*-set of at least one intermediate node in the cycle. The theoretical complexity of the *ng*-SPPRC is also pseudo-polynomial if $|\mathcal{N}_v| \leq \Delta + 1$ for every $v \in N^+$. In this case, a labeling algorithm can solve the *ng*-SPPRC in pseudo-polynomial time with a multiplying constant that depends on $2^{2\Delta}$. The *ng*-SPPRC normally provides much tighter bounds than those provided by the *k-cyc*-SPPRC in comparable CPU times. In fact, experiments carried out by Poggi and Uchoa (2014) using some hard capacitated VRP instances have shown that *ng*-sets with $\Delta = 8$ yield bounds similar to those obtained by the *5-cyc*-SPPRC, but spending a time comparable to the *4-cyc*-SPPRC. For this reason, the *ng*-SPPRC has become the route relaxation of preference ever since.

2.3. The arc-ng-SPPRC of Bulhões et al. (2018)

In a recent article by Bulhões et al. (2018), the *ng*-SPPRC is extended to consider neighborhoods associated with arcs instead of nodes. We call this problem the *arc-ng*-SPPRC with arc-based neighborhoods, and denote it *arc-ng*-SPPRC. The *arc-ng*-SPPRC can be defined as follows. With every arc $a = (u, v) \in A$, we associate a *ng*-set $\mathcal{N}_a \subseteq N^+$ that corresponds to the set of customer nodes that can be remembered by arc a . The *ng*-sets \mathcal{N}_a may be defined according to any suitable criterion. Most successful implementations use some notion of distance to favor neighborhoods containing nodes that are close to each other. An example is to consider the set of nodes that are close to both extremities of the arcs, that is, $\mathcal{N}_a \subseteq \mathcal{N}_u \cap \mathcal{N}_v$, with \mathcal{N}_u and \mathcal{N}_v defined as for the *ng*-SPPRC. Let $P = (a_0, a_1, \dots, a_p)$ be a partial path, composed of arcs $a_0 = (0, v_1)$, $a_1 = (v_1, v_2)$, \dots , $a_p = (v_p, v_{p+1})$ and $\Pi(v_p)$ be the memory of this path upon arrival at node v_p , which is a subset of nodes. An extension using arc $a = (v_p, v_{p+1})$ is deemed *arc-ng-feasible* only if $v_{p+1} \notin \Pi(v_p)$. The memory upon arrival to v_{p+1} is updated according to the formula $\Pi(v_{p+1}) \leftarrow (\Pi(v_p) \cap \mathcal{N}_a) \cup \{v_{p+1}\}$. If $\mathcal{N}_a = \mathcal{N}_u \cap \mathcal{N}_v$, the *arc-ng*-SPPRC coincides with the classical *ng*-SPPRC, and, therefore, it is more general.

By considering the notions presented in Section 2.1, a path $P' = (v_1 = 0, \dots, v_{p+1} = n + 1)$ is said to be feasible only if every prefix $P'_i = (v_1, \dots, v_i)$, $i \in \{2, \dots, p + 1\}$ is both resource- and *ng*-feasible. The cost of P' is given by $c(P') = \sum_{1 \leq q \leq p} c_{v_q, v_{q+1}}$. The *arc-ng*-SPPRC consists of finding a feasible path P' of minimum total cost. Not only this mechanism allows a better management of the undesired cycles, but also yields a sharper dominance rule.

In trying to enhance the performance of their algorithm, Bulhões et al. (2018) proposed a novel set-based dominance criterion (multiple dominance) that extends the traditional dominance, which only allows pairwise comparisons to be made. This new strategy was embedded into the algorithm used to solve the *arc-ng*-SPPRC. We provide further details about this new dominance relation in Section 2.3.2.

2.3.1. Labeling algorithm

A general description of a labeling algorithm similar to the one adopted by Bulhões et al. (2018) to solve the *arc-ng*-SPPRC is as follows. A label L encodes a partial path $(v_1 = 0, \dots, v_p)$ from the source until reaching a node v_p through the following data structures: a terminal node $v(L) = v_p$; resource consumptions $\xi^r(L)$, $r \in \mathcal{R}$; a (reduced) cost $\bar{c}(L)$; a memory $\Pi(L)$; and a pointer $pred(L)$, which indicates the predecessor of label L .

An extension to a node w over an arc $a = (v_p, w)$ yields a label L' with the associated data structures computed as follows:

$$v(L') \leftarrow w \tag{5}$$

$$\xi^r(L') \leftarrow \max\{e_w^r, \xi^r(L) + \gamma_a^r\} \quad \forall r \in \mathcal{R} \tag{6}$$

$$\bar{c}(L') \leftarrow \bar{c}(L) + \bar{c}_{v_p, w} \tag{7}$$

$$\Pi(L') \leftarrow (\Pi(L) \cap \mathcal{N}_a) \cup \{w\} \tag{8}$$

$$pred(L') \leftarrow L. \tag{9}$$

Algorithm 1. Labeling algorithm

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1:   $\mathcal{V} \leftarrow \{L_0\}$ ,  $L_{best} \leftarrow nil$ ,  $\bar{c}_{min} \leftarrow +\infty$ ,
2:   $\mathcal{B}(v, \xi^{r^*}) \leftarrow \emptyset$ ,  $\forall v \in N$ ,  $\forall \xi^{r^*} \in \{0, 1, \dots, H^{r^*}\}$ 
3:  while  $\mathcal{V} \neq \emptyset$  do
4:      Let  $L$  be the first label in  $\mathcal{V}$ 
5:      Set  $\mathcal{V} \leftarrow \mathcal{V} \setminus \{L\}$ 
6:      Set  $\mathcal{B}(v(L), \xi^{r^*}(L)) \leftarrow \mathcal{B}(v(L), \xi^{r^*}(L)) \cup \{L\}$ 
7:      for all  $a \in \mathcal{A}(L)$  do
8:          Let  $L'$  be the label resulting from extending  $L$  along arc  $a$  using (5)–(9)
9:          if  $v(L') = n + 1$  then
10:             if  $\bar{c}(L') < \bar{c}_{min}$  then
11:                 Set  $L_{best} \leftarrow L'$ ,  $\bar{c}_{min} \leftarrow \bar{c}(L')$ 
12:             else if there exists no label  $L'' \in \mathcal{B}(v(L'), \xi^{r^*}(L'))$  such that  $L'' < L'$  then
13:                 Set  $\mathcal{B}(v(L'), \xi^{r^*}(L')) \leftarrow \mathcal{B}(v(L'), \xi^{r^*}(L')) \cup \{L'\}$ 
14:                 Remove from  $\mathcal{V}$  all labels  $L''$  such that  $L' < L''$ 
15: return  $L_{best}$ 

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Label L' is deemed feasible if and only if it is both resource- and *ng*-feasible. For the sake of conciseness, we denote from now on an extension of label L to a node w by $L \oplus w$.

A generic representation of a labeling algorithm is outlined in Algorithm 1. We point out that this pseudocode is only used for illustration purposes. We do not intend to provide an exact description of Bulhões et al.'s (2018) algorithm. Several implementation details, among which bi-directional search (Righini and Salani, 2006) and acceleration tricks were intentionally left out. The interested reader is referred to Bulhões et al. (2018) and Costa et al. (2019) for further details on the implementation of labeling algorithms, in particular of the *arc-ng*-SPPRC. Similarly to what is done in most of the recent implementations (e.g., Pecin et al., 2017a,b), Bulhões et al. adopt label buckets to store nondominated labels. This data structure allows a more efficient application of dominance rules. For each node $v \in N$ and possible consumption ξ^{r^*} of a given resource $r^* \in \mathcal{R}$, one defines a bucket $\mathcal{B}(v, \xi^{r^*})$ that stores all the nondominated labels associated with node v and having consumption ξ^{r^*} for resource r^* . It is initialized as empty and enlarged dynamically. The choice for the resource defining the buckets may vary according to the problem at hand (e.g., load or time). In Algorithm 1, the set \mathcal{V} keeps the unextended labels waiting in a queue. This set is kept sorted at all times in lexicographic order with regard to the consumption of resource r^* , that is, the labels with lowest consumptions of r^* are positioned before those with larger consumptions (Desrochers and Soumis, 1988). As mentioned before, the maximum consumption allowed for resource r^* is H^{r^*} . The set $\mathcal{A}(L) = \{a = (u, v) \in A : u = v(L), v \in N \setminus \Pi(L) \text{ and } L \oplus v \text{ is resource-feasible}\}$ contains the arcs that can be used to extend L . For two labels L and L' , $L < L'$ means that L dominates L' (according to the rules defined in Sections 2.3.2 and 3). Let L_0 be the label encoding an empty path of cost zero, with zero resource consumptions, empty memory, and such that $pred(L_0) = nil$. Algorithm 1 returns the label L_{best} representing a path with lowest cost. This labeling algorithm extends nondominated labels to create new labels (line 8), coupled with a dominance rule to discard nonpromising labels (lines 12 and 14). The dominance procedure prevents a combinatorial explosion and, therefore, keeps the computational burden within reasonable limits.

2.3.2. Multiple partial label dominance

Dominance in Bulhões et al. (2018) is not only performed pairwise. The authors use a sharper *multiple partial label dominance rule* similar to that introduced by Irnich and Villeneuve (2006) for the *k-cyc-SPPRC*. To formalize this idea in the context of the *arc-ng-SPPRC*, we use the notion of *weak* and *strong* dominance described in Irnich and Villeneuve (2006). Let L and L' be two feasible labels. Label L is said to *weakly dominate* L' if all the following conditions hold:

$$v(L) = v(L'), \quad (10)$$

$$\xi^r(L) \leq \xi^r(L'), \quad \forall r \in \mathcal{R} \quad (11)$$

$$\bar{c}(L) \leq \bar{c}(L'). \quad (12)$$

Conditions (10)–(12), however, do not ensure a proper dominance of L' by L . In fact, depending on the label memories $\Pi(L)$ and $\Pi(L')$, it may still be possible for L' to be extended to a node w that would not be an *ng-feasible* extension of L . This would happen for every node $w \in \Pi(L) \setminus \Pi(L')$. The additional condition

$$\Pi(L) \subseteq \Pi(L') \quad (13)$$

ensures the correctness of the pairwise dominance.

Bulhões et al. (2018) realized that condition (13) may be unnecessarily too restrictive. In fact, if a label L weakly dominates another label L' , one may restrict the extensions of L' only to the nodes $w \in \Pi(L) \setminus \Pi(L') \cup \{j \in N \setminus \Pi(L') \mid \mathcal{N}_{(v(L),j)} \cap (\Pi(L) \setminus \Pi(L')) \neq \emptyset\}$, that is, the nodes where L' can be directly extended but not L or the nodes that remember at least one such node. This brings out the following set-based dominance criterion. Let \mathcal{L} be a collection of labels $(L_i)_{i \in \mathcal{L}}$. It is said to *strongly dominate* a label $L' \notin \mathcal{L}$ if the two following conditions hold:

- i. L_i weakly dominates L' for all $i \in \mathcal{L}$.
- ii. For every *ng-feasible* extension of L' to a node w , there exists $i \in \mathcal{L}$ such that L_i can also be extended to w and $\Pi(L_i \oplus w) \subseteq \Pi(L' \oplus w)$.

Conditions (i) and (ii) combined assure that any path obtained by feasibly extending label L' would be dominated. Hence, L' can be discarded. Moreover, the authors also demonstrate that, whenever a weakly dominating label L_i is identified, one can immediately restrict feasible extensions of L' , by discarding those that would be dominated by path $L_i \oplus w$ for all $w \in N \setminus \Pi(L_i)$ such that $L_i \oplus w$ is resource-feasible. As a result, not only full dominance reduces the number of labels, but also partial dominance contributes to avoiding certain extensions. On the other hand, their computational experiments showed that it may be computationally expensive to store the set of dominated extensions for each label. Consequently, as a compromise, instead of representing this set explicitly, they rely on an implicit representation that requires to only extend the memory $\Pi(L')$ of a dominated label L' along a dominated extension, thus, avoiding the extension of the label cost and resource components.

In a framework similar to Algorithm 1, partial dominance using implicit representation of dominated extensions is applied as follows. Let $L, L' \in \mathcal{B}(u, \xi^{r^*})$ be two labels for which conditions (10)–(12) hold, but not (13). When L, L' were extended over the same arc (u, v) , resource

consumptions and the reduced cost for both labels increase by the same amount (assuming that nonrobust cuts are not employed). Consequently, (10)–(12) remain valid for the resulting labels $L \oplus v$ and $L' \oplus v$. Hence, the only condition that needs to be verified to determine if $L \oplus v$ dominates $L' \oplus v$ is (13). To apply partial dominance using implicit representation, one assigns to each bucket $\mathcal{B}(u, \xi^{r^*})$, $\forall u \in N$, $\forall \xi^{r^*} \in \{0, 1, \dots, H^{r^*}\}$, and arc $(u, v) \in A$, a list for storing the sets $\Pi(\cdot)$ associated with labels in $\mathcal{B}(u, \xi^{r^*})$ that have been extended over (u, v) . In the case of the label L' mentioned above, to avoid its complete extension over (u, v) , it suffices to check if $\Pi(L' \oplus v)$ is a subset of some set $\Pi(\cdot)$ stored in the list of memories associated with $\mathcal{B}(u, \xi^{r^*})$ and (u, v) , and which was obtained from a label L such that (10)–(12) hold. If it is the case, the extension can be discarded. Note that, as in Bulhões et al. (2018), when there is a single resource in \mathcal{R} and the labels in a bucket are kept in a nonincreasing order of their reduced cost, all stored sets $\Pi(\cdot)$ are obtained from labels L such that (10)–(12) hold. This is, however, not the case when there are multiple resources in \mathcal{R} .

When $|\mathcal{R}| > 1$, it may be necessary to adopt a more sophisticated data structure to verify (10)–(12) efficiently. For example, when $|\mathcal{R}| = 2$ as for the VRPTW, lists of sets $\Pi(\cdot)$ can be stored for each possible value of the resource $r \neq r^*$. For larger values of $|\mathcal{R}|$, one may need to consider additional structures to check when dominance relations for the resources that are not used in the definition of the bucket structure are respected. The overhead caused by the extra tests may impact negatively on the overall performance of the algorithm.

3. Selective arc-ng-SPPRC

In this section, we redefine strong dominance. For the sake of readability, we present the mechanism in two steps. First of all, we show that the pairwise comparison criterion (13) can be replaced by

$$\Pi(L) \subseteq \Pi(\text{pred}(L')) \cup \{v(L')\}. \quad (14)$$

Proposition 1 below formalizes the correctness of this criterion. Its proof relies on the following lemma.

Lemma 1. *A labeling algorithm using conditions (10)–(12), (14) to establish pairwise dominance produces, for every label E representing an elementary partial path P , at least one nondominated label L such that $v(L) = v(E)$, $\xi^r(L) \leq \xi^r(E)$, $\forall r \in \mathcal{R}$, $\bar{c}(L) \leq \bar{c}(E)$, and $\Pi(L) \subseteq V(E)$, where $V(E)$ denotes the set of customer nodes visited by P .*

Proof. The proof works by induction on the size p of $P = \{v_1 = 0, \dots, v_p\}$. If $p = 1$, then E is nondominated and $L = E$. If $p \geq 2$, let us assume that E is obtained by extending another label E' over an arc $(v_{p-1}, v_p) \in A$, with $v_{p-1} = v(E')$ and $v_p \notin V(E')$. Using the induction hypothesis, the labeling algorithm generates a nondominated label L' such that $v(L') = v(E')$, $\xi^r(L') \leq \xi^r(E')$, $\forall r \in \mathcal{R}$, $c(L') \leq c(E')$, $\Pi(L') \subseteq V(E')$. Denote by L'' the label produced by extending L' along arc (v_{p-1}, v_p) . It is easy to see that L'' is a feasible extension of L' , and that $v(L'') = v(E)$, $c(L'') \leq c(E)$ and $\xi^r(L'') \leq \xi^r(E)$, $\forall r \in \mathcal{R}$, hold. In addition, $\Pi(L'') \subseteq \Pi(L') \cup \{v_p\} \subseteq V(E') \cup \{v_p\} = V(E)$. If L'' is not dominated, then $L = L''$. Otherwise, if L'' is dominated by a nondominated label L''' , then L''' satisfies $v(L''') = v(E)$, $c(L''') \leq c(L'') \leq c(E)$, $\xi^r(L''') \leq \xi^r(L'') \leq \xi^r(E)$, $\forall r \in \mathcal{R}$, and $\Pi(L''') \subseteq \Pi(\text{pred}(L'')) \cup \{v_p\} = \Pi(L') \cup \{v_p\} \subseteq V(E') \cup \{v_p\} = V(E)$. In this case, $L = L'''$. \square

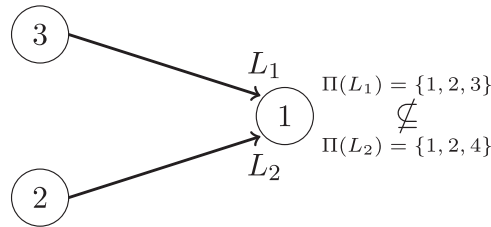


Fig. 1. Standard dominance.

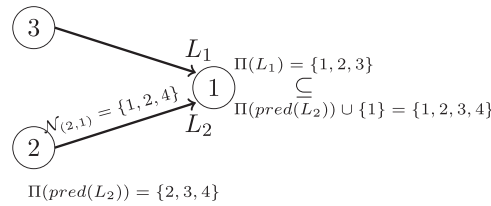


Fig. 2. Selective dominance.

From this lemma, we deduce the following result.

Proposition 1. *If there exists an elementary path P from 0 to $n + 1$ with a negative reduced cost, then a labeling algorithm based on the dominance rule (10)–(12) and (14) finds at least one (possibly nonelementary) path P' from 0 to $n + 1$ with a negative reduced cost.*

Proof. Assume that there exists such a path P which is represented by a label E . If E is not dominated, then $P' = P$. Otherwise, according to Lemma 1, there exists a nondominated label L such that $v(L) = v(E)$ and $\bar{c}(L) \leq \bar{c}(E)$. This label represents path P' . \square

To illustrate the strength of this new dominance rule, let us consider the following example. Let L_1 and L_2 be two labels such that $v(L_1) = v(L_2) = 1$, $c(L_1) \leq c(L_2)$, $\xi^r(L_1) \leq \xi^r(L_2)$, $\forall r \in R$, and let us assume that $\Pi(L_1) = \{1, 2, 3\}$ and $\Pi(L_2) = \{1, 2, 4\}$. Although L_1 weakly dominates L_2 , the latter cannot be discarded according to (10)–(13) because $\Pi(L_1) \not\subseteq \Pi(L_2)$ (Fig. 1). Now, let us assume that label L_2 was extended from label $pred(L_2)$ with $v(pred(L_2)) = 2$, over the arc $a = (2, 1)$, and such that $N_a = \{1, 2, 4\}$ and $\Pi(pred(L_2)) = \{2, 3, 4\}$. We now have that $\Pi(L_1) = \{1, 2, 3\} \subseteq \{1, 2, 3, 4\} = \Pi(pred(L_2)) \cup \{1\}$, and label L_2 can be safely discarded according to condition (14) (Fig. 2).

Observe that, in this example, it might be feasible to extend label L_2 to node 3 and that this extension might subsequently yield nondominated ng -paths between 0 and $n + 1$, possibly an optimal one. Nevertheless, when label L_2 is dominated according to conditions (10)–(12) and (14), this extension is not possible and the algorithm remains valid because the discarded ng -paths are not elementary (they visit node 3 more than once). The pricing is, thus, selective.

The second step in the presentation of our selective set-based dominance rule is stated in a corollary of the pairwise dominance condition, and extends the strong dominance criterion given by conditions (i) and (ii).

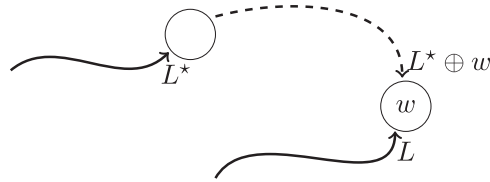


Fig. 3. Selective set-based comparison.

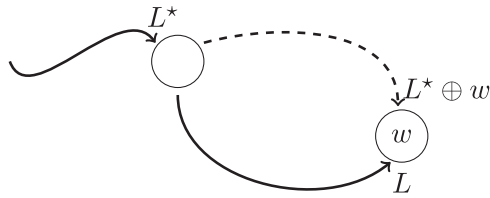


Fig. 4. Bulhões et al.'s mechanism.

Corollary 1. For a given label L^* , an extension to a node $w \in N^+ \setminus \Pi(L^*)$ can be safely omitted if there exists a label L such that all the following conditions hold:

$$v(L) = v(L^* \oplus w) = w, \tag{15}$$

$$\bar{c}(L) \leq \bar{c}(L^* \oplus w), \tag{16}$$

$$\xi^r(L) \leq \xi^r(L^* \oplus w), \quad \forall r \in R \tag{17}$$

$$\Pi(L) \subseteq \Pi(L^*) \cup \{w\}. \tag{18}$$

In fact, one can observe that (15)–(18) correspond to (10)–(12) and (14) when setting $L' = L^* \oplus w$, showing that, if L exists and L^* was extended along arc $(v(L^*), w)$, the resulting label $L' = L^* \oplus w$ would be dominated by L .

Similarly to the idea proposed by Bulhões et al. (2018), the dominance rule given by conditions (15)–(18) intends to consider some available information to identify dominated label extensions. Yet, the dominance rule described in Corollary 1 is stronger than the one proposed by Bulhões et al. (2018) because it uses the selection mechanism given by condition (18), which is stronger than condition (13) (with $L' = L^* \oplus w$). Another difference between the two algorithms is that our set-based dominance rule does not assume L and $L^* \oplus w$ to be extended from the same node (see Fig. 3), unlike that of Bulhões et al. (2018) (see Fig. 4). In these cases, label $L^* \oplus w$ can be identified as dominated by L without extending the cost and resource components of L^* in the algorithm of Bulhões et al. (2018) and without extending the memory of L^* in our selective algorithm. Therefore, if the average memory size is much larger than the number of cost and resource components, the selective dominance rule proposed in our algorithm can yield an additional speedup.

One way to reinforce dominance relations (14) and (18) is by adding to the set Π of the dominating label, the unreachable nodes, that is, the nodes to which extending the path would not be resource-feasible (equivalently, it is possible to do the same for relation (13)). Assuming that the resource consumptions $\gamma_{ij}^r, (i, j) \in A$ satisfy the triangle inequality for each resource $r \in R$, the set

of unreachable nodes of a label L is defined as $\mathcal{U}(L) = \{w \in N^+ \mid \exists r \in R \text{ such that } \xi^r(L) + \gamma_{v(L),w}^r > l_w^r\}$. Given that L and $L^* \oplus w$ cannot be feasibly extended to any node in $\mathcal{U}(L)$ and $\mathcal{U}(L^* \oplus w)$, respectively, these sets can be included in the right-hand side of their respective dominance conditions (14) and (18), which can be rewritten as

$$\Pi(L) \subseteq \Pi(\text{pred}(L')) \cup \{v(L')\} \cup \mathcal{U}(L') \quad (19)$$

$$\Pi(L) \subseteq \Pi(L^*) \cup \{w\} \cup \mathcal{U}(L^* \oplus w). \quad (20)$$

To avoid computing the unreachable set of label $L^* \oplus w$ before testing dominance condition (20), the following slightly weaker condition can be used:

$$\Pi(L) \subseteq \Pi(L^*) \cup \mathcal{U}(L^*) \cup \{w\}. \quad (21)$$

Finally, we mention that, even if Lemma 1 is valid for the case when ng-sets are associated with nodes, adopting relation (14) to devise an algorithm to solve the traditional *ng*-SPPRC does not have any impact on the solution process. This issue is further discussed in Appendix A.

4. Computational experiments

To assess the effectiveness of the proposed selective dominance rules, we performed a series of computational experiments on well-known benchmark instances of the VRPTW that can be defined as follows. Consider the network $G = (N, A)$ described in Section 2.1, where N is the set of nodes, A is the set of arcs, and N^+ is the set of customer nodes. Each customer $i \in N^+$ is associated with a demand q_i , a service time s_i , and a time window $[e_i, l_i]$ within which the service must start. Moreover, we set $q_0 = s_0 = e_0 = q_{n+1} = s_{n+1} = e_{n+1} = 0$ and $l_0 = l_{n+1} = H$, where H is the planning horizon duration. Each arc $(i, j) \in A$ is associated with a routing cost c_{ij} and a traveling time t_{ij} . An unlimited fleet of homogeneous vehicles with capacity Q is available at a single depot. The VRPTW consists of finding feasible routes visiting all customers $i \in N^+$ exactly once such that the total routing cost is minimized. A route is deemed feasible if: it is elementary; all visited customers are served within their time windows; the total demand of these customers does not exceed the vehicle capacity Q . Furthermore, the cost of a route is computed as the sum of the costs of the arcs traversed by the route.

For the VRPTW, the pricing subproblem involves a set R of two resources, namely, time and load. At a node $i \in N$, the resource windows $[e_i^r, l_i^r]$, $r \in R$, are defined by their time window $[e_i, l_i]$ and by the load window $[0, Q]$. Along an arc $(i, j) \in A$, the resource consumptions γ_{ij}^r , $r \in R$, are given by $t_{ij} + s_i$ for the time resource and by q_j for the load resource. The choice of the VRPTW as the test problem comes from the fact that, despite being simple to understand, it remains challenging. This aspect allows us to assess the performance of our selective algorithm when combined with different enhancing techniques that are typically required to solve the VRPTW efficiently. We detail some of these techniques in Section 4.1.

In this section, we provide some details on how we conducted our computational experiments and report the results obtained. The performance of the selective *arc-ng*-SPPRC pricing is compared against the default *arc-ng*-SPPRC pricing. Both of these settings are implemented over the

same column-and-cut generation framework, which is described in Section 4.1. In Section 4.2, we provide details on the experiments design. Finally, in Section 4.3, we report and discuss the computational results obtained.

4.1. Column-and-cut-generation framework

In our experiments, we consider a column-and-cut-generation framework very similar to the one developed by Pecin et al. (2017a) for the VRPTW. The algorithm includes several refinements that enhance its performance. The labeling algorithm used to solve the pricing subproblem relies on a bidirectional DSSR procedure (without completion bounds), having the time as the critical resource (Righini and Salani, 2006). The maximum size of *ng*-sets in the algorithm varies according to the instance being solved as discussed below. Furthermore, three fast heuristic labeling algorithms are always executed before any execution of the exact pricing algorithm. The first heuristic keeps only the least-cost label associated with a given time arriving at each customer node. The second and third heuristics rely on a reduced version of the network G , where around 7 and 12 (for the second and third, respectively) arcs entering and leaving each customer node are kept (see Desaulniers et al., 2008, for details on how these arcs are selected). At each column generation iteration, these three heuristic labeling algorithms are called in sequence until one of them finds negative reduced cost columns. If they all fail, the cut separation routines (see below) are invoked. When cuts are found, they are added to the RMP and column generation is re-started. Otherwise, the exact pricing algorithm is called. If it finds negative reduced cost columns, the process starts over again. Otherwise, the algorithm stops.

Because we wanted to make the implementation of our column generation algorithm as simple as possible, we decided not to consider the dynamic mechanism described by Bulhões et al. to increase and reduce the size of the *ng*-sets when needed. We judged that this procedure would make the implementation of the algorithm very cumbersome. Hence, we decided to rely on Martinelli et al.'s (2014) framework to handle the size of the *ng*-sets. In our implementation, *ng*-sets are initially empty and are updated throughout the solution process. Two strategies were considered to update the arc *ng*-sets: (i) *only-cycle*, where only *ng*-sets associated with arcs appearing in an infeasible cycle are increased; and (ii) *all-arcs*, where all the arcs induced by the customer nodes in an infeasible cycle have their *ng*-sets increased. The strategy *only-cycle* is the same considered by Bulhões et al. (2018). As pointed out by the authors, and also as we have observed in our preliminary experiments, the *only-cycle* strategy produces a higher number of DSSR iterations. However, the computational complexity of each such iteration remains lower (this is more notorious toward the end).

The baseline algorithm also includes the separation of several cuts, namely: rounded capacity cuts (RCCs, Laporte et al., 1985), limited-memory rank-1 cuts (Pecin et al., 2017a, 2017b), and elementary cuts (Pecin et al., 2017a). Except for the RCCs, all the other cuts are nonrobust, that is, their dual variables cannot be considered directly in the modified arc costs when solving the pricing subproblem. The explicit handling of these dual variables increases the difficulty of solving it. Consequently, the solution process is divided in two phases, called the *robust* and *nonrobust* phases. In the robust phase, the search for violated cuts is restricted to the RCCs. This phase ends with a call to the exact pricing algorithm that yielded no new columns and the application of

variable arc fixing (Irnich et al., 2010). The nonrobust phase then starts by searching for nonrobust violated cuts. If some are found, the solution process is re-started. This phase also ends with a final unsuccessful call to the exact pricing algorithm, ensuring that the value of the current RMP optimal solution provides a valid lower bound.

4.2. Experiments design

Our tests were performed on two datasets: (1) the 14 hardest 100-customer VRPTW instances from Solomon (1987); and (2) the 200-customer instances of Gehring and Homberger (2001) that are reported to be solved in less than five hours by Sadykov et al. (2017). These instances are called hereafter the S and the GH instances. Both datasets contain instances whose node locations are: chosen at random, clustered, and mixed (random and clustered). This is denoted in the S and GH instances, respectively, as: C, R, and RC; and C1, C2, R1, R2, RC1 and RC2. For the GH instances, specifically, instances in sets C1, R1, and RC1 have tight time windows and vehicle capacity, and typically admit optimal solutions with short vehicle routes. On the other hand, sets C2, R2, and RC2 have large time windows and loose vehicle capacity and, typically, admit optimal solutions with fewer but longer vehicle routes.

Similarly to what is done in Pecin et al. (2017a), for all S instances except R208, and all C1, RC1, and R1 GH instances, the cardinality of *ng*-sets \mathcal{N}_i , $i \in N^+$ are limited to 10. For the other instances, where allowing more cycles can be very harmful to the lower bound, we consider $|\mathcal{N}_i| = 20$. These node *ng*-sets are used to generate the arc *ng*-sets \mathcal{N}_a , $a = (u, v) \in A$, by using the formula $\mathcal{N}_a \leftarrow \mathcal{N}_u \cap \mathcal{N}_v$. Furthermore, for all instances except the C1, RC1, and R1 GH instances, we observed that the capacity constraint is not binding. Therefore, to alleviate the solution of the pricing subproblem, we did not consider this constraint, which is later enforced using RCCs whenever needed.

Given that our goal is not to solve to optimality the considered instances but rather to illustrate the impact of applying the selective *arc-ng*-SPPRC pricing, we focus only on the root node results. The algorithm described previously was coded in C++, and IBM ILOG CPLEX Optimizer 12.8 was used as the LP solver. The experiments were carried out on an Intel Xeon ES-2637 3.5GHz with 128GB RAM, running Linux Oracle Server 7.6. A time limit of 48 hours was set to solve all instances.

In our experiments, we consider three versions of the selective *arc-ng*-SPPRC pricing, which differ by the dominance rule employed. We compare these variants against the *arc-ng*-SPPRC considering a default dominance rule and the multiple partial label dominance. The latter consists of the default *arc-ng*-SPPRC with partial dominance being applied. We adopt the implicit representation of dominated extensions described in Section 2.3.2. All settings considered in our study are detailed below:

1. **Default:** The baseline algorithm that considers relations (10)–(13) as dominance rule. Relation (13) is reinforced by including the set $\mathcal{U}(L')$ of unreachable nodes in its right-hand side (which becomes $\Pi(L') \cup \mathcal{U}(L')$).
2. **PartialDom:** It is the same as **Default**, but considers the partial dominance using implicit representation of dominated extensions described in Section 2.3.2.

Table 1
Aggregated results using the only-cycle DSSR strategy before adding nonrobust cuts

Classes		T(s)					#labels				
		Default	PartialDom	SetBased	Pairwise	SetPair	Default	PartialDom	SetBased	Pairwise	SetPair
S-C	Average	17,534	12,272	8524	10,637	14,315	49,247	57,038	47,188	52,392	35,872
	Median	17,534	12,272	8524	10,637	14,315	49,247	57,038	47,188	52,392	35,872
S-R	Average	1621	1713	1232	1345	1366	69,557	72,606	55,929	58,002	56,907
	Median	649	665	533	520	542	40,523	36,038	32,088	33,020	31,437
S-RC	Average	6772	8297	4576	4335	4726	433,549	451,283	337,074	318,183	365,446
	Median	3043	3167	2363	2280	2516	589,868	583,749	452,280	419,077	441,036
GH-C1	Average	578	643	475	554	491	317,915	306,198	267,573	244,796	255,764
	Median	535	594	472	548	474	282,538	298,233	253,558	233,975	254,303
GH-C2	Average	5763	6564	4745	4509	5223	20,185	20,002	17,992	15,704	16,630
	Median	6158	7352	4811	4483	5458	14,186	14,377	13,307	11,640	13,866
GH-R1	Average	176	199	146	176	146	202,795	215,188	199,190	186,380	190,431
	Median	152	165	139	157	142	201,404	182,425	181,273	188,565	194,376
GH-R	Average	4807	4969	4370	4293	4334	15,255	16,182	12,289	12,290	12,687
	Median	4140	4379	4003	4076	4049	13,263	11,681	10,589	10,192	10,103
GH-RC	Average	206	227	167	192	167	436,372	439,793	443,949	402,631	371,387
	Median	143	153	127	140	127	273,251	270,804	260,652	237,269	221,346
GH-RC2	Average	6799	6694	5577	5688	5551	46,331	49,874	38,017	36,721	37,847
	Median	6769	6863	5441	5725	5567	44,015	42,940	35,663	33,906	35,051
# Best		0/45	0/45	18/45	13/45	14/45	3/45	1/45	9/45	21/45	16/45
# OPD		–	8/45	42/45	39/45	44/45	–	20/45	35/45	39/45	40/45
# OPP		37/45	–	44/45	44/45	44/45	24/45	–	36/45	39/45	39/45

3. SetBased: Consider relations (10)–(12) and the set-based relation (21) as dominance rule.
4. Pairwise: Consider relations (10)–(12) and the pairwise relation (19) as dominance rule.
5. SetPair: Consider relations (10)–(12), both pairwise (19) and set-based (21) relations as dominance rule.

Additionally, each pricing setting is executed twice, each time considering a different DSSR strategy, that is, only-cycle or all-arcs. Hence, in total, we test 10 distinct configurations.

For each instance, we retrieve the following information: lower bound (**lb**), number of column generation iterations performed by each algorithm (**#iters**), CPU time in seconds (**T(s)**), average number of DSSR iterations per column generation iteration performed by each algorithm (**#DSSR**), and average number of labels generated per iteration (**#labels**). This information is collected before (robust phase) and after (nonrobust phase) the addition of nonrobust cuts. Yet, because the main indicators for evaluating the efficiency of the selective algorithm are computational time and average number of labels generated per iteration, in Section 4.3, we focus our analysis on these two measures. Detailed results for all indicators are presented in Appendix B.

4.3. Computational results

In this section, we discuss the results obtained during our experiments. For the sake of conciseness, we only report summary results (see Appendix B for complete results). In Tables 1–4, we

Table 2
 Aggregated results using the all-arcs DSSR strategy before adding nonrobust cuts

Classes		T(s)					#labels				
		Default	PartialDom	SetBased	Pairwise	SetPair	Default	PartialDom	SetBased	Pairwise	SetPair
S-C	Average	8744	7194	10,703	7399	10,495	32,462	36,091	31,839	22,682	29,243
	Median	8744	7194	10,703	7399	10,495	32,462	36,091	31,839	22,682	29,243
S-R	Average	815	857	813	755	789	41,340	47,478	42,687	40,071	41,562
	Median	368	419	373	360	362	23,608	30,685	24,467	25,960	25,301
S-RC	Average	2000	2532	1490	1531	1462	173,236	183,334	133,280	157,321	128,918
	Median	842	1036	761	759	765	223,594	240,585	160,031	199,839	155,315
GH-C1	Average	354	429	326	357	327	170,597	170,982	142,030	151,625	145,292
	Median	353	433	323	356	325	131,610	125,898	111,027	121,422	114,389
GH-C2	Average	4325	3311	2966	2781	2774	13,135	13,500	12,554	11,508	12,963
	Median	4014	3639	3476	2985	3097	10,609	11,426	9659	9440	12,609
GH-R1	Average	138	149	126	140	122	131,318	133,633	126,176	123,834	127,299
	Median	131	144	120	132	116	152,863	126,463	115,984	117,794	138,251
GH-R2	Average	3523	3649	3445	3435	3398	10,362	11,054	9731	9610	9959
	Median	3298	3543	3371	3272	3416	8490	9481	8318	8243	8129
GH-RC1	Average	134	141	117	127	117	212,577	221,608	244,069	229,380	216,139
	Median	117	126	109	113	108	161,529	157,078	193,357	172,433	164,655
GH-RC2	Average	3746	3916	3542	3593	3342	25,103	25,060	25,165	22,525	22,744
	Median	3482	3645	3247	3221	3111	23,951	26,726	24,889	21,480	24,995
# Best		3/45	1/45	10/45	8/45	23/45	10/45	3/45	9/45	17/45	11/45
# OPD		–	5/45	34/45	32/45	38/45	–	18/45	25/45	30/45	29/45
# OPP		40/45	–	42/45	43/45	44/45	26/45	–	28/45	35/45	34/45

provide the average and the median for **T(s)** and **#labels** per instance class, where an instance class is preceded by S or GH to identify if they correspond to S or GH instances. In each table, we compare the performance of settings Default, PartialDom, SetBased, Pairwise, and SetPair. We analyze separately the performance of the algorithms with respect to the DSSR strategy employed (only-cycle and all-arcs). We provide distinct tables for the algorithms before and after separating nonrobust cuts. Finally, line **#Best** shows the number of times that each setting provides the best results for each indicator, and lines **#OPD** and **#OPP** display the number of times that the corresponding setting outperforms the Default and PartialDom settings, respectively. Note that, in the case where both algorithms produce equal results for a given instance, this instance is counted for both algorithms. As a consequence, the sum of the **#Best** values can exceed the number of instances.

In Tables 1 and 2, we present the summary results produced by the algorithms before the separation of nonrobust cuts. These results show a clear advantage of the selective settings over both default algorithms (Default and PartialDom) in terms of running time and the number of labels produced. This statement is supported by the values of **#OPD** and **#OPP**. By considering the values of **#Best**, among the three selective variants, settings SetBased and SetPair are the ones yielding the best gains in terms of running time for only-cycle and all-arcs, respectively. Regarding the number of labels, for both DSSR strategies, the setting Pairwise is the most effective at reducing the number of nondominated labels kept for both DSSR strategies. We point out that the best performance of the selective algorithms become more evident when only-cycle strategy is adopted. Moreover, as one can see from detailed results in Appendix B, during the robust phase of

Table 3
Aggregated results using the `only-cycle` DSSR strategy after adding nonrobust cuts

Classes		T(s)					#labels				
		Default	PartialDom	SetBased	Pairwise	SetPair	Default	PartialDom	SetBased	Pairwise	SetPair
S-C	Average	17,535	12,273	8525	10,637	14,315	49,247	57,038	47,188	52,392	35,872
	Median	17,535	12,273	8525	10,637	14,315	49,247	57,038	47,188	52,392	35,872
S-R	Average	28,394	22,927	23,543	23,739	23,217	173,658	135,068	165,085	154,845	133,561
	Median	2854	2796	2348	2572	2644	32,796	30,983	31,385	28,540	29,710
S-RC	Average	8371	10,096	6274	5904	6492	259,595	257,033	203,106	187,660	195,340
	Median	5054	5232	4157	3864	4309	358,790	346,185	256,083	246,708	246,296
GH-C1	Average	44,147	44,099	43,933	44,657	43,825	260,127	255,879	231,525	232,520	227,062
	Median	1132	1341	997	1122	1052	291,258	286,609	263,977	268,100	246,535
GH-C2	Average	8914	9450	7292	7828	8644	15,744	15,437	14,587	12,807	12,439
	Median	8523	8425	5572	6271	7544	10,098	9613	8921	8643	9316
GH-R1	Average	63,024	65,315	61,748	62,681	64,377	200,621	196,682	200,899	197,020	195,001
	Median	26,455	28,930	22,703	28,276	30,800	94,172	93,539	96,043	88,250	91,813
GH-R2	Average	9940	9521	8726	8427	8481	6091	6313	5758	5636	5475
	Median	6913	7216	6841	6254	6547	5796	5899	5016	5033	4904
GH-RC1	Average	77,254	78,590	75,859	81,790	87,095	348,714	356,581	367,944	324,192	336,289
	Median	65,555	65,340	61,909	73,183	82,086	143,555	141,917	145,699	139,747	139,481
GH-RC2	Average	26,932	23,606	22,929	26,219	18,461	14,999	16,885	12,949	11,926	12,394
	Median	15,307	12,941	12,889	18,543	15,025	13,961	14,485	12,514	12,812	12,798
# Best		3/45	2/45	24/45	7/45	9/45	1/45	5/45	9/45	17/45	13/45
# OPD		–	17/45	40/45	32/45	29/45	–	24/45	35/45	39/45	41/45
# OPP		28/45	–	39/45	37/45	36/45	21/45	–	30/45	38/45	38/45

the algorithm, the selective algorithms allow better bounds to be achieved, although the differences in the bounds compared with the standard algorithms are not significant. The selective strategies also tend to require less DSSR iterations. When comparing the performance of the selective algorithms employing different DSSR strategies, the results confirm what has already been hinted by Bulhões et al. (2018), that is, the algorithms employing `only-cycle` produce a larger number of DSSR iterations. Also, they tend to keep more nondominated labels.

Tables 3 and 4 show the results obtained once the algorithms have completed their nonrobust phase. In the presence of nonrobust cuts, the selective algorithms can become more time-consuming. Nevertheless, they are still faster than the `Default` and `PartialDom` for the majority of the instances. The better performance of the selective algorithms is confirmed when we analyze the performance of the algorithms in the pairwise comparisons of Appendix B. Regarding the performance of the settings, the behavior of the algorithms during the nonrobust phase is similar to that observed during the robust phase. `SetBased` is the fastest selective setting and `Pairwise` is the most effective in reducing the number of nondominated labels kept. The less impressive advantage of the selective algorithms in the nonrobust phase may be explained by the impact that the nonrobust cuts have on the complexity of the labeling algorithms. Even if the selective algorithms rely on less restrictive dominance rules associated with `arc-ng-SPPRC` memory sets, as nonrobust cuts are incorporated into the RMP, the complexity of the labeling algorithms increase.

Finally, we present time profiles (Dolan and More, 2002) to compare the performance of the algorithms, which are generated as follows. Let \mathcal{A} be a set of algorithms, namely, $\mathcal{A} = \{\text{Default},$

Table 4

Aggregated results using the all-arcs DSSR strategy after adding nonrobust cuts

Classes		T(s)					#labels				
		Default	PartialDom	SetBased	Pairwise	SetPair	Default	PartialDom	SetBased	Pairwise	SetPair
S-C	Average	8744	7195	10,704	7399	10,495	32,462	36,091	31,839	22,682	29,243
	Median	8744	7195	10,704	7399	10,495	32,462	36,091	31,839	22,682	29,243
S-R	Average	22,253	23,550	22,447	23,984	23,742	127,362	114,715	124,953	133,732	110,891
	Median	1947	2186	2139	1868	1979	23,670	24,935	27,022	25,738	23,459
S-RC	Average	3173	4054	2745	2496	2808	104,460	106,102	83,246	91,149	84,988
	Median	2116	2521	1930	1493	2111	143,464	146,739	97,395	116,605	98,136
GH-C1	Average	44,288	43,750	43,663	43,978	43,997	161,729	156,948	151,531	147,506	147,533
	Median	659	865	667	686	714	159,911	158,767	144,722	140,424	143,434
GH-C2	Average	6607	6037	5956	5646	5287	10,286	10,357	10,007	9173	9796
	Median	5128	4770	5002	3959	4147	7082	6671	7360	7878	7873
GH-R1	Average	60,841	62,291	60,474	60,520	62,658	172,083	164,023	169,285	164,297	163,595
	Median	25,055	23,514	26,704	22,911	27,965	80,401	80,352	83,754	79,696	82,338
GH-R2	Average	7704	7927	7626	7486	7218	5248	5278	4970	4711	4972
	Median	6074	6130	5944	5714	6143	4570	4768	4597	4442	4360
GH-RC1	Average	74,751	73,509	70,540	77,213	81,305	208,928	211,301	216,317	203,785	201,337
	Median	56,572	57,263	52,908	65,389	73,765	124,358	124,054	123,466	123,456	122,347
GH-RC2	Average	22,709	22,449	22,851	19,354	15,725	10,049	11,085	11,575	9579	10,337
	Median	12,754	12,792	9052	11,967	8319	9879	11,266	10,536	9181	9966
# Best		11/45	5/45	14/45	7/45	8/45	2/45	8/45	6/45	19/45	10/45
# OPD		–	17/45	23/45	25/45	24/45	–	19/45	21/45	32/45	32/45
# OPP		28/45	–	35/45	30/45	31/45	26/45	–	22/45	33/45	33/45

PartialDom, SetBased, Pairwise, SetPair} in our case. Let I be a set of instances, namely, the 45 instances considered in our experiments. For each algorithm $A \in \mathcal{A}$ and each instance $i \in I$, denote by T_A^i the time spent by A to solve instance i and let $T_{\text{Best}}^i = \min_{A \in \mathcal{A}} \{T_A^i\}$ be the best time achieved by an algorithm in \mathcal{A} to solve this instance. For an algorithm $A \in \mathcal{A}$, a time profile is a function $\rho_A(\tau)$ of a ratio $\tau \geq 1$ that is equal to the percentage of instances such that $\frac{T_A^i}{T_{\text{Best}}^i} \leq \tau$, that is,

$$\rho_A(\tau) = 100 \frac{\left| \left\{ i \in I \mid \frac{T_A^i}{T_{\text{Best}}^i} \leq \tau \right\} \right|}{|I|}. \quad (22)$$

In particular, $\rho_A(1)$ represents the percentage of instances for which algorithm A was the fastest. Figures 5 and 6 display the time profiles of the five algorithms for the robust and the nonrobust phases, respectively. From these profiles, one can see that the selective algorithms yield considerable speed-ups when compared to Default and PartialDom. The acceleration is more notable when comparing algorithms employing the only-cycle DSSR strategy. The time profiles confirm what has been observed in Tables 1–4, that is, SetBased seems to be the most time-efficient setting. SetPair, on the other hand, is the most time-consuming among the selective settings, to the point where it is slower than Default for 70% of the instances during the nonrobust phase when only-cycle DSSR strategy is employed. In the presence of nonrobust cuts, the overhead of using both pairwise (19) and set-based (21) dominance relations may not pay off.

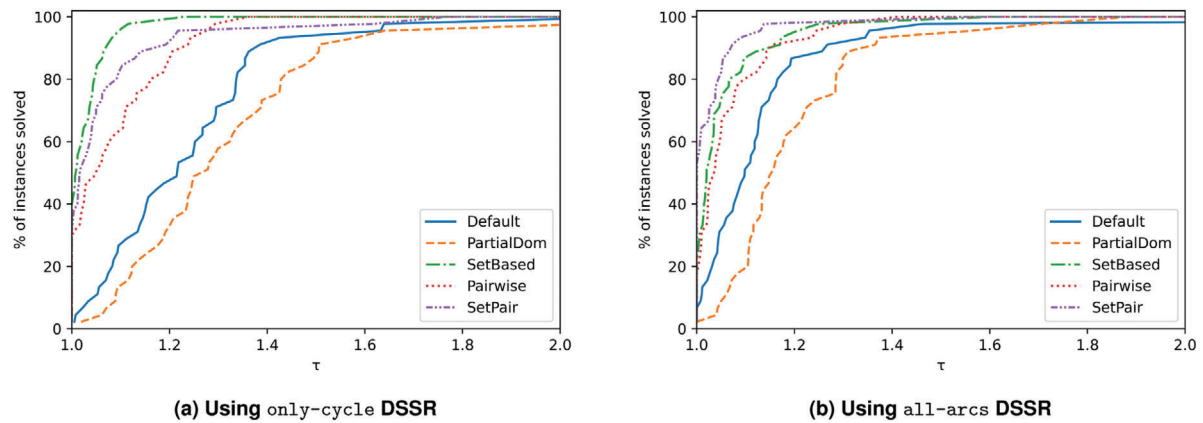


Fig. 5. Time profiles for all instances before adding nonrobust cuts.

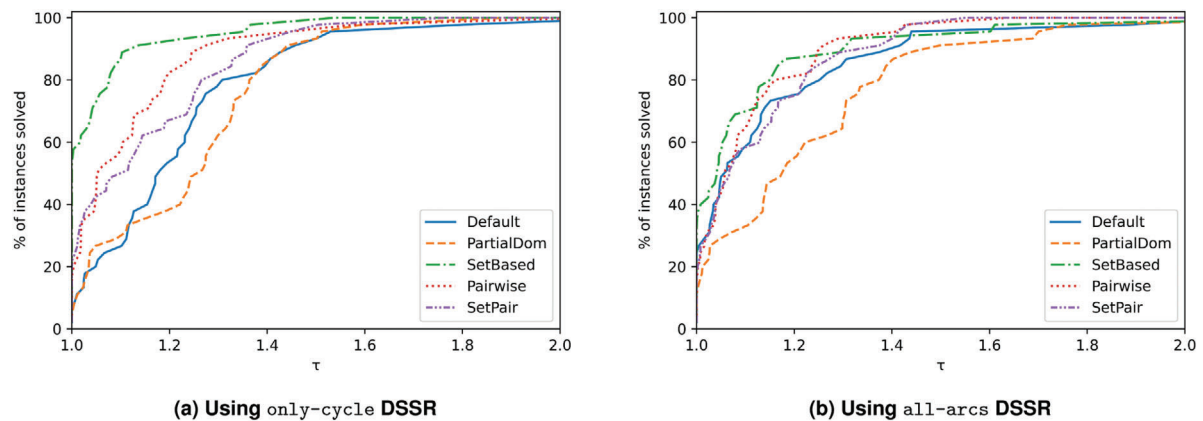


Fig. 6. Time profiles for all instances after adding nonrobust cuts.

5. Concluding remarks

In this paper, we have presented two new selective dominance rules for the *arc-ng*-SPPRC, namely, one that applies pairwise label comparison and one that extends the set-based rule of Bulhões et al. (2018). The latter rule is stronger than the one proposed by Bulhões et al. as it considers the selective mechanism expressed by condition (18), which increases the chances that a label L dominates a label $L' = L^* \oplus w$. Furthermore, an additional speedup may be observed if the average memory size is much larger than the number of cost and resource components in a label. Our computational experiments on VRPTW benchmark instances showed that, in general, the new mechanism allows a reduction in terms of the number of treated labels and, consequently, of the computational time. Note also that one of the main interests for using this mechanism is that it does not rely on complex data structures like the ones described by Bulhões et al. (2018, Section 4.2) and Desaulniers et al. (2019, Section 3.2) to work satisfactorily.

An extension of our idea would be to investigate the impact of generalizing relation (14) to consider other predecessors further down in the path. Theoretically, this generalization would strengthen even more the dominance rule. However, it seems that an elaborated data structure would be required to address the overhead incurred by the new relations.

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Appendix A: Effect of applying the newly pairwise comparison over the ng -SPPRC

In this section, we demonstrate that the selective dominance proposed in this article has no effect in reducing the number of nondominated labels when applied to a traditional ng -SPPRC, and becomes a useless overhead when compared to the traditional dominance. For a label L , let $\Pi(L)$ be the nodes currently in the memory of label L . For every node $v \in N^+$, denote by \mathcal{N}_v its neighborhood. When extending a label L to a node $v \in N^+ \setminus \Pi(L)$, the memory of the new label (that we denote L') is $\Pi(L') \leftarrow \{\Pi(L) \cap \mathcal{N}_v\} \cup \{v\}$.

Lemma 2. For every pair of labels L_1, L_2 with the same terminal node k , $\Pi(L_1) \cap [\Pi(pred(L_2)) \cup \{k\}] = \Pi(L_1) \cap \Pi(L_2)$.

Proof.

$$\begin{aligned}
 \Pi(L_1) \cap [\Pi(pred(L_2)) \cup \{k\}] &= ([\Pi(pred(L_1) \cap \mathcal{N}_k] \cup \{k\}) \cap [\Pi(pred(L_2)) \cup \{k\}]) \\
 &= (\Pi(pred(L_1)) \cap \Pi(pred(L_2)) \cap \mathcal{N}_k) \cup \{k\} \\
 &= ([\Pi(pred(L_1)) \cap \mathcal{N}_k] \cup \{k\}) \cap ([\Pi(pred(L_2)) \cap \mathcal{N}_k] \cup \{k\}) \\
 &= \Pi(L_1) \cap \Pi(L_2). \quad \square
 \end{aligned}$$

Proposition 2. For every pair of labels L_1, L_2 with the same terminal node k , $\Pi(L_1) \subseteq \Pi(L_2)$ if and only if $\Pi(L_1) \subseteq \Pi(pred(L_2)) \cup \{k\}$.

Proof. We have that

$$\begin{aligned} \Pi(L_1) \subseteq \Pi(L_2) &\Leftrightarrow \Pi(L_1) \cap \Pi(L_2) = \Pi(L_1) \\ &\Leftrightarrow \Pi(L_1) \cap [\Pi(pred(L_2)) \cup \{k\}] = \Pi(L_1) \quad (\text{from Lemma 2}) \\ &\Leftrightarrow \Pi(L_1) \subseteq [\Pi(pred(L_2)) \cup \{k\}]. \quad \square \end{aligned}$$

Appendix B: Detailed results

In this section, we report detailed results for all instances solved by settings `Default`, `SetBased`, `Pairwise`, and `SetPair`. We provide pairwise comparisons between the `Default` algorithm and each of the selective settings. Sections B.1 and B.2 present results for `S` and `GH` instances, respectively. For each instance, we report: the lower bound (**lb**) achieved, the number of column generation iterations performed by each algorithm (**#iters**), the CPU time in seconds (**T(s)**), the average number of `DSSR` iterations per column generation iteration performed by each algorithm (**#DSSR**), and the average number of labels generated per iteration (**#labels**). When comparing the performance of two algorithms, we highlight in boldface the best results for each indicator. Line **#Best** shows the total number of instances for which each algorithm was capable of achieving the best values. Note that, in the case where both algorithms produce equal results for a given instance, this instance is counted for both algorithms.

B.1. Solomon's instances

Table B1

Detailed results for comparing `Default` versus `SetBased` settings using only-cycle `DSSR` before adding nonrobust cuts

Instance	Default					SetBased				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C203	5,887.0	3423	406	1315	21,380	5,887.0	2400	375	1029	14,139
C204	5,881.0	31,645	1553	3492	77,113	5,881.0	14,649	676	2182	80,236
R202	10,164.8	284	139	547	8564	10,164.8	249	106	445	8975
R203	8,597.0	761	172	852	30,072	8,597.6	625	136	670	29,180
R204	7,202.0	2162	182	1189	100,223	7,202.0	1638	184	874	66,260
R206	8,600.1	608	116	797	34,964	8,600.1	511	102	691	32,088
R207	7,805.0	1217	159	921	55,715	7,805.6	983	125	760	54,710
R208	6,910.9	8133	285	2167	263,264	6,910.9	5872	256	1741	203,856
R209	8,371.8	346	97	609	24,750	8,371.8	304	86	535	22,607
R210	8,813.2	426	78	746	40,523	8,813.8	376	83	626	30,214
R211	7,311.0	649	127	867	67,939	7,311.2	533	128	743	55,474
RC204	7,764.9	16,472	320	3264	618,355	7,764.9	10,746	275	2895	452,280
RC207	9,415.8	802	102	1596	92,424	9,415.8	619	102	1393	68,303
RC208	7,636.4	3043	167	3166	589,868	7,636.4	2363	136	2958	490,639
#Best	10/14	0/14	4/14	0/14	2/14	14/14	14/14	11/14	14/14	12/14

Table B2

Detailed results for comparing Default versus SetBased settings using only-cycle DSSR after adding nonrobust cuts

Instance	Default					SetBased				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C203	5,887.0	3424	406	1315	21,380	5,887.0	2400	375	1029	14,139
C204	5,881.0	31,645	1553	3492	77,113	5,881.0	14,649	676	2182	80,236
R202	10,296.0	966	815	1329	3948	10,296.0	866	709	1140	3792
R203	8,708.0	1232	642	1375	11,972	8,708.0	1052	578	1175	11,568
R204	7,313.0	5512	661	1882	68,200	7,313.0	5463	684	1654	61,430
R206	8,759.0	1559	514	1324	20,537	8,759.0	1332	479	1198	18,776
R207	7,940.0	2854	606	1515	32,796	7,940.0	2348	529	1357	31,385
R208	6,998.0	229,080	653	3209	1,194,476	7,001.9	185,708	680	3094	1,129,873
R209	8,548.0	2594	603	1420	28,561	8,548.0	2171	596	1357	27,691
R210	9,005.0	4538	620	1549	37,429	9,005.0	4061	661	1433	33,007
R211	7,467.0	7215	373	1583	165,002	7,467.0	8884	374	1466	168,246
RC204	7,835.0	18,239	560	3558	381,153	7,835.0	13,049	561	3263	256,083
RC207	9,629.0	1822	461	2173	38,842	9,629.0	1617	493	1989	30,905
RC208	7,761.0	5054	396	3667	358,790	7,761.0	4157	341	3508	322,332
#Best	13/14	1/14	6/14	0/14	2/14	14/14	13/14	8/14	14/14	12/14

Table B3

Detailed results for comparing Default versus Pairwise settings using only-cycle DSSR before adding nonrobust cuts

Instance	Default					Pairwise				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C203	5,887.0	3423	406	1315	21,380	5,887.0	3060	360	1143	18,735
C204	5,881.0	31,645	1553	3492	77,113	5,881.0	18,214	787	2089	86,049
R202	10,164.8	284	139	547	8564	10,164.8	252	99	461	9175
R203	8,597.0	761	172	852	30,072	8,597.0	691	145	734	26,536
R204	7,202.0	2162	182	1189	100,223	7,202.0	1753	166	944	80,729
R206	8,600.1	608	116	797	34,964	8,600.1	486	111	647	27,500
R207	7,805.0	1217	159	921	55,715	7,805.0	943	126	726	48,319
R208	6,910.9	8133	285	2167	263,264	6,910.9	6789	272	1920	218,370
R209	8,371.8	346	97	609	24,750	8,371.8	305	74	521	24,866
R210	8,813.2	426	78	746	40,523	8,814.4	363	72	632	33,020
R211	7,311.0	649	127	867	67,939	7,311.4	520	125	729	53,503
RC204	7,764.9	16,472	320	3264	618,355	7,764.9	10,090	282	2968	419,077
RC207	9,415.8	802	102	1596	92,424	9,415.8	635	94	1379	75,578
RC208	7,636.4	3043	167	3166	589,868	7,636.4	2280	135	2937	459,893
#Best	12/14	0/14	0/14	0/14	3/14	14/14	14/14	14/14	14/14	11/14

Table B4

Detailed results for comparing Default versus Pairwise settings using only-cycle DSSR after adding nonrobust cuts

Instance	Default					Pairwise				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C203	5,887.0	3424	406	1315	21,380	5,887.0	3060	360	1143	18,735
C204	5,881.0	31,645	1553	3492	77,113	5,881.0	18,214	787	2089	86,049
R202	10,296.0	966	815	1329	3948	10,296.0	860	802	1235	3368
R203	8,708.0	1232	642	1375	11,972	8,708.0	1148	635	1269	10,019
R204	7,313.0	5512	661	1882	68,200	7,313.0	6206	657	1776	66,589
R206	8,759.0	1559	514	1324	20,537	8,759.0	1395	482	1150	16,487
R207	7,940.0	2854	606	1515	32,796	7,940.0	2477	579	1330	28,540
R208	6,998.0	229,080	653	3209	1,194,476	7,001.9	185,645	703	3152	1,046,169
R209	8,548.0	2594	603	1420	28,561	8,548.0	2572	614	1387	26,942
R210	9,005.0	4538	620	1549	37,429	9,005.0	3842	598	1401	33,483
R211	7,467.0	7215	373	1583	165,002	7,467.0	9510	388	1461	162,013
RC204	7,835.0	18,239	560	3558	381,153	7,835.0	12,150	555	3311	246,708
RC207	9,629.0	1822	461	2173	38,842	9,629.0	1698	432	1942	34,256
RC208	7,761.0	5054	396	3667	358,790	7,761.0	3864	358	3470	282,017
#Best	13/14	2/14	3/14	0/14	1/14	14/14	12/14	11/14	14/14	13/14

Table B5

Detailed results for comparing Default versus SetPair settings using only-cycle DSSR before adding nonrobust cuts

Instance	Default					SetPair				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C203	5,887.0	3423	406	1315	21,380	5,887.0	2737	440	1147	13,784
C204	5,881.0	31,645	1553	3492	77,113	5,881.0	25,892	1576	3019	57,959
R202	10,164.8	284	139	547	8564	10,164.8	246	98	480	9754
R203	8,597.0	761	172	852	30,072	8,597.0	650	155	704	24,398
R204	7,202.0	2162	182	1189	100,223	7,202.0	1587	173	934	74,028
R206	8,600.1	608	116	797	34,964	8,600.1	533	116	711	27,342
R207	7,805.0	1217	159	921	55,715	7,805.1	954	149	727	44,896
R208	6,910.9	8133	285	2167	263,264	6,910.9	7087	277	1954	219,155
R209	8,371.8	346	97	609	24,750	8,371.8	316	80	560	25,636
R210	8,813.2	426	78	746	40,523	8,813.2	382	77	650	31,437
R211	7,311.0	649	127	867	67,939	7,311.0	542	124	739	55,515
RC204	7,764.9	16,472	320	3264	618,355	7,764.9	11,012	271	2815	441,036
RC207	9,415.8	802	102	1596	92,424	9,415.8	650	97	1366	73,623
RC208	7,636.4	3043	167	3166	589,868	7,636.4	2516	119	2952	581,679
#Best	14/14	0/14	3/14	0/14	2/14	14/14	14/14	12/14	14/14	12/14

Table B6

Detailed results for comparing Default versus SetPair settings using only-cycle DSSR after adding nonrobust cuts

Instance	Default					SetPair				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C203	5,887.0	3424	406	1315	21,380	5,887.0	2738	440	1147	13,784
C204	5,881.0	31,645	1553	3492	77,113	5,881.0	25,893	1576	3019	57,959
R202	10,296.0	966	815	1329	3948	10,296.0	762	659	1118	3654
R203	8,708.0	1232	642	1375	11,972	8,708.0	1080	600	1203	10,074
R204	7,313.0	5512	661	1882	68,200	7,313.0	5255	611	1621	62,939
R206	8,759.0	1559	514	1324	20,537	8,759.0	1586	498	1203	19,174
R207	7,940.0	2854	606	1515	32,796	7,940.0	2644	594	1367	29,710
R208	6,998.0	229,080	653	3209	1,194,476	6,996.4	179,708	642	2979	855,593
R209	8,548.0	2594	603	1420	28,561	8,548.0	2324	587	1370	25,630
R210	9,005.0	4538	620	1549	37,429	9,005.0	4742	620	1404	30,323
R211	7,467.0	7215	373	1583	165,002	7,467.0	10,856	409	1477	164,952
RC204	7,835.0	18,239	560	3558	381,153	7,835.0	13,148	549	3162	246,296
RC207	9,629.0	1822	461	2173	38,842	9,629.0	2021	445	1953	35,634
RC208	7,761.0	5054	396	3667	358,790	7,761.0	4309	353	3462	304,091
#Best	14/14	4/14	4/14	0/14	0/14	13/14	10/14	11/14	14/14	14/14

Table B7

Detailed results for comparing Default versus SetBased settings using all-arcs DSSR before adding nonrobust cuts

Instance	Default					SetBased				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C203	5,887.0	1747	319	639	11,902	5,887.0	1517	324	564	10,446
C204	5,881.0	15,741	1107	1557	53,022	5,881.0	19,889	1368	1808	53,233
R202	10,164.8	216	104	366	7822	10,164.8	219	116	338	6198
R203	8,597.0	531	139	525	22,779	8,597.0	478	130	462	21,196
R204	7,201.6	1174	167	602	55,959	7,201.6	1110	139	523	57,509
R206	8,600.1	368	92	420	23,608	8,600.1	373	87	418	24,467
R207	7,805.0	708	115	476	40,871	7,805.0	653	97	453	43,833
R208	6,910.9	3441	203	792	146,423	6,910.9	3608	211	808	145,786
R209	8,371.8	243	90	362	15,426	8,371.8	245	80	364	17,652
R210	8,813.2	308	84	475	22,313	8,813.2	282	75	420	24,123
R211	7,311.0	344	112	426	36,857	7,311.0	348	97	435	43,420
RC204	7,764.9	4843	215	946	252,430	7,764.9	3413	213	829	160,031
RC207	9,415.8	316	73	580	43,685	9,415.8	295	68	521	38,995
RC208	7,636.4	842	105	867	223,594	7,636.4	761	97	830	200,815
#Best	14/14	6/14	4/14	4/14	7/14	14/14	8/14	10/14	10/14	7/14

Table B8

Detailed results for comparing Default versus SetBased settings using all-arcs DSSR after adding nonrobust cuts

Instance	Default					SetBased				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C203	5,887.0	1747	319	639	11,902	5,887.0	1517	324	564	10,446
C204	5,881.0	15,741	1107	1557	53,022	5,881.0	19,890	1368	1808	53,233
R202	10,296.0	717	677	973	3232	10,296.0	736	660	912	3494
R203	8,708.0	1140	746	1156	8708	8,708.0	830	502	864	9819
R204	7,313.0	3146	566	1048	42,611	7,313.0	3642	554	997	44,628
R206	8,759.0	1086	414	779	15,333	8,759.0	1099	453	826	14,891
R207	7,940.0	1947	529	924	23,670	7,940.0	2083	505	907	27,022
R208	7,009.7	182,430	581	1287	911,617	7,006.1	183,931	558	1246	873,352
R209	8,548.0	1638	545	903	18,992	8,548.0	2139	596	958	22,394
R210	9,005.0	4378	634	1082	29,615	9,005.0	3580	576	993	30,062
R211	7,467.0	3793	325	768	92,478	7,467.0	3985	300	759	98,913
RC204	7,835.0	6464	474	1221	143,464	7,835.0	5250	493	1127	97,395
RC207	9,629.0	940	354	917	21,801	9,629.0	1056	403	922	20,609
RC208	7,761.0	2116	326	1125	148,114	7,761.0	1930	331	1109	131,734
#Best	14/14	9/14	7/14	4/14	8/14	13/14	5/14	7/14	10/14	6/14

Table B9

Detailed results for comparing Default versus Pairwise settings using all-arcs DSSR before adding nonrobust cuts

Instance	Default					Pairwise				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C203	5,887.0	1747	319	639	11,902	5,887.0	1889	491	679	8099
C204	5,881.0	15,741	1107	1557	53,022	5,881.0	12,909	1091	1466	37,265
R202	10,164.8	216	104	366	7822	10,164.8	210	104	334	6575
R203	8,597.0	531	139	525	22,779	8,597.0	479	133	473	21,009
R204	7,201.6	1174	167	602	55,959	7,201.6	1126	151	546	52,530
R206	8,600.1	368	92	420	23,608	8,600.1	360	98	420	19,897
R207	7,805.0	708	115	476	40,871	7,805.0	660	92	457	47,529
R208	6,910.9	3441	203	792	146,423	6,910.9	3077	199	742	130,870
R209	8,371.8	243	90	362	15,426	8,371.8	248	79	365	17,136
R210	8,813.2	308	84	475	22,313	8,813.2	277	63	416	25,960
R211	7,311.0	344	112	426	36,857	7,311.0	355	110	453	39,132
RC204	7,764.9	4843	215	946	252,430	7,764.9	3547	191	865	199,839
RC207	9,415.8	316	73	580	43,685	9,415.8	288	78	513	33,072
RC208	7,636.4	842	105	867	223,594	7,636.0	759	81	820	239,051
#Best	14/14	3/14	4/14	4/14	5/14	13/14	11/14	11/14	11/14	9/14

Table B10

Detailed results for comparing Default versus Pairwise settings using all-arcs DSSR after adding nonrobust cuts

Instance	Default					Pairwise				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C203	5,887.0	1747	319	639	11,902	5,887.0	1889	491	679	8099
C204	5,881.0	15,741	1107	1557	53,022	5,881.0	12,909	1091	1466	37,265
R202	10,296.0	717	677	973	3232	10,296.0	699	622	891	3161
R203	8,708.0	1140	746	1156	8708	8,708.0	933	575	934	9083
R204	7,313.0	3146	566	1048	42,611	7,313.0	3887	548	991	49,736
R206	8,759.0	1086	414	779	15,333	8,759.0	1088	444	796	14,029
R207	7,940.0	1947	529	924	23,670	7,940.0	1868	468	874	25,738
R208	7,009.7	182,430	581	1287	911,617	7,008.4	197,603	555	1195	962,739
R209	8,548.0	1638	545	903	18,992	8,548.0	1709	565	929	19,346
R210	9,005.0	4378	634	1082	29,615	9,005.0	3718	587	1010	28,225
R211	7,467.0	3793	325	768	92,478	7,467.0	4350	329	792	91,532
RC204	7,835.0	6464	474	1221	143,464	7,835.0	4949	425	1115	116,605
RC207	9,629.0	940	354	917	21,801	9,629.0	1046	410	907	19,866
RC208	7,761.0	2116	326	1125	148,114	7,761.0	1493	267	1041	136,976
#Best	14/14	7/14	5/14	4/14	5/14	13/14	7/14	9/14	10/14	9/14

Table B11

Detailed results for comparing Default versus SetPair settings using all-arcs DSSR before adding nonrobust cuts

Instance	Default					SetPair				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C203	5,887.0	1747	319	639	11,902	5,887.0	1726	321	637	12,275
C204	5,881.0	15,741	1107	1557	53,022	5,881.0	19,263	1526	1934	46,212
R202	10,164.8	216	104	366	7822	10,164.8	208	102	332	6754
R203	8,597.0	531	139	525	22,779	8,598.4	469	159	493	18,365
R204	7,201.6	1174	167	602	55,959	7,202.0	1042	140	504	51,514
R206	8,600.1	368	92	420	23,608	8,600.1	356	97	405	20,186
R207	7,805.0	708	115	476	40,871	7,805.0	645	100	451	39,837
R208	6,910.9	3441	203	792	146,423	6,910.9	3477	175	770	153,871
R209	8,371.8	243	90	362	15,426	8,371.8	244	70	358	18,608
R210	8,813.2	308	84	475	22,313	8,813.2	295	70	448	25,301
R211	7,311.0	344	112	426	36,857	7,311.0	362	108	455	39,622
RC204	7,764.9	4843	215	946	252,430	7,764.9	3306	201	807	155,315
RC207	9,415.8	316	73	580	43,685	9,415.8	314	78	535	36,044
RC208	7,636.4	842	105	867	223,594	7,636.0	765	92	797	195,395
#Best	12/14	4/14	5/14	2/14	5/14	13/14	10/14	9/14	12/14	9/14

Table B12

Detailed results for comparing Default versus SetPair settings using all-arcs DSSR after adding nonrobust cuts

Instance	Default					SetPair				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C203	5,887.0	1747	319	639	11,902	5,887.0	1727	321	637	12,275
C204	5,881.0	15,741	1107	1557	53,022	5,881.0	19,263	1526	1934	46,212
R202	10,296.0	717	677	973	3232	10,296.0	724	618	874	3135
R203	8,708.0	1140	746	1156	8708	8,708.0	896	650	999	8447
R204	7,313.0	3146	566	1048	42,611	7,313.0	3916	544	959	45,432
R206	8,759.0	1086	414	779	15,333	8,759.0	1051	414	757	14,440
R207	7,940.0	1947	529	924	23,670	7,940.0	1979	505	885	23,459
R208	7,009.7	182,430	581	1287	911,617	7,005.3	194,490	506	1170	764,528
R209	8,548.0	1638	545	903	18,992	8,548.0	1911	520	886	19,705
R210	9,005.0	4378	634	1082	29,615	9,005.0	4340	600	1054	28,787
R211	7,467.0	3793	325	768	92,478	7,467.0	4373	320	779	90,083
RC204	7,835.0	6464	474	1221	143,464	7,835.0	5174	450	1078	98,136
RC207	9,629.0	940	354	917	21,801	9,629.0	1139	398	920	20,313
RC208	7,761.0	2116	326	1125	148,114	7,761.0	2111	297	1063	136,516
#Best	14/14	8/14	4/14	3/14	3/14	13/14	6/14	11/14	11/14	11/14

Table B13

Detailed results for comparing PartialDom versus SetBased settings using only-cycle DSSR before adding nonrobust cuts

Instance	PartialDom					SetBased				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C203	5,887.0	3427	422	1332	19,401	5,887.0	2400	375	1029	14,139
C204	5,881.0	21,117	813	2623	94,674	5,881.0	14,649	676	2182	80,236
R202	10,164.8	304	130	558	9265	10,164.8	249	106	445	8975
R203	8,597.0	836	178	881	28,547	8,597.6	625	136	670	29,180
R204	7,201.6	2203	184	1159	96,974	7,202.0	1638	184	874	66,260
R206	8,600.1	646	123	828	36,038	8,600.1	511	102	691	32,088
R207	7,805.0	1175	153	896	57,911	7,805.6	983	125	760	54,710
R208	6,910.9	8774	268	2256	294,003	6,910.9	5872	256	1741	203,856
R209	8,371.8	358	93	592	24,458	8,371.8	304	86	535	22,607
R210	8,813.2	454	91	758	33,615	8,813.8	376	83	626	30,214
R211	7,311.0	665	120	862	72,642	7,311.2	533	128	743	55,474
RC204	7,764.9	20,875	320	3198	664,362	7,764.9	10,746	275	2895	452,280
RC207	9,415.8	850	97	1580	105,739	9,415.8	619	102	1393	68,303
RC208	7,636.4	3167	156	3164	583,749	7,636.4	2363	136	2958	490,639
#Best	9/14	0/14	3/14	0/14	1/14	14/14	14/14	12/14	14/14	13/14

Table B14

Detailed results for comparing PartialDom versus SetBased settings using only-cycle DSSR after adding nonrobust cuts

Instance	PartialDom					SetBased				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C203	5,887.0	3428	422	1332	19,401	5,887.0	2400	375	1029	14,139
C204	5,881.0	21,118	813	2623	94,674	5,881.0	14,649	676	2182	80,236
R202	10,296.0	1012	860	1369	3783	10,296.0	866	709	1140	3792
R203	8,708.0	1292	640	1396	11,821	8,708.0	1052	578	1175	11,568
R204	7,313.0	5288	654	1868	63,573	7,313.0	5463	684	1654	61,430
R206	8,759.0	1648	488	1301	20,682	8,759.0	1332	479	1198	18,776
R207	7,940.0	2796	619	1533	30,983	7,940.0	2348	529	1357	31,385
R208	6,993.1	176,704	625	3186	845,641	7,001.9	185,708	680	3094	1,129,873
R209	8,548.0	2766	604	1433	27,355	8,548.0	2171	596	1357	27,691
R210	9,005.0	5233	625	1519	34,038	9,005.0	4061	661	1433	33,007
R211	7,467.0	9605	340	1519	177,736	7,467.0	8884	374	1466	168,246
RC204	7,835.0	22,986	593	3523	383,749	7,835.0	13,049	561	3263	256,083
RC207	9,629.0	2071	453	2121	41,165	9,629.0	1617	493	1989	30,905
RC208	7,761.0	5232	376	3646	346,185	7,761.0	4157	341	3508	322,332
#Best	13/14	2/14	5/14	0/14	4/14	14/14	12/14	9/14	14/14	10/14

Table B15

Detailed results for comparing PartialDom versus Pairwise settings using only-cycle DSSR before adding nonrobust cuts

Instance	PartialDom					Pairwise				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C203	5,887.0	3427	422	1332	19,401	5,887.0	3060	360	1143	18,735
C204	5,881.0	21,117	813	2623	94,674	5,881.0	18,214	787	2089	86,049
R202	10,164.8	304	130	558	9265	10,164.8	252	99	461	9175
R203	8,597.0	836	178	881	28,547	8,597.0	691	145	734	26,536
R204	7,201.6	2203	184	1159	96,974	7,202.0	1753	166	944	80,729
R206	8,600.1	646	123	828	36,038	8,600.1	486	111	647	27,500
R207	7,805.0	1175	153	896	57,911	7,805.0	943	126	726	48,319
R208	6,910.9	8774	268	2256	294,003	6,910.9	6789	272	1920	218,370
R209	8,371.8	358	93	592	24,458	8,371.8	305	74	521	24,866
R210	8,813.2	454	91	758	33,615	8,814.4	363	72	632	33,020
R211	7,311.0	665	120	862	72,642	7,311.4	520	125	729	53,503
RC204	7,764.9	20,875	320	3198	664,362	7,764.9	10,090	282	2968	419,077
RC207	9,415.8	850	97	1580	105,739	9,415.8	635	94	1379	75,578
RC208	7,636.4	3167	156	3164	583,749	7,636.4	2280	135	2937	459,893
#Best	11/14	0/14	2/14	0/14	1/14	14/14	14/14	12/14	14/14	13/14

Table B16

Detailed results for comparing PartialDom versus Pairwise settings using only-cycle DSSR after adding nonrobust cuts

Instance	PartialDom					Pairwise				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C203	5,887.0	3428	422	1332	19,401	5,887.0	3060	360	1143	18,735
C204	5,881.0	21,118	813	2623	94,674	5,881.0	18,214	787	2089	86,049
R202	10,296.0	1012	860	1369	3783	10,296.0	860	802	1235	3368
R203	8,708.0	1292	640	1396	11,821	8,708.0	1148	635	1269	10,019
R204	7,313.0	5288	654	1868	63,573	7,313.0	6206	657	1776	66,589
R206	8,759.0	1648	488	1301	20,682	8,759.0	1395	482	1150	16,487
R207	7,940.0	2796	619	1533	30,983	7,940.0	2477	579	1330	28,540
R208	6,993.1	176,704	625	3186	845,641	7,001.9	185,645	703	3152	1,046,169
R209	8,548.0	2766	604	1433	27,355	8,548.0	2572	614	1387	26,942
R210	9,005.0	5233	625	1519	34,038	9,005.0	3842	598	1401	33,483
R211	7,467.0	9605	340	1519	177,736	7,467.0	9510	388	1461	162,013
RC204	7,835.0	22,986	593	3523	383,749	7,835.0	12,150	555	3311	246,708
RC207	9,629.0	2071	453	2121	41,165	9,629.0	1698	432	1942	34,256
RC208	7,761.0	5232	376	3646	346,185	7,761.0	3864	358	3470	282,017
#Best	13/14	2/14	4/14	0/14	2/14	14/14	12/14	10/14	14/14	12/14

Table B17

Detailed results for comparing PartialDom versus SetPair settings using only-cycle DSSR before adding nonrobust cuts

Instance	PartialDom					SetPair				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C203	5,887.0	3427	422	1332	19,401	5,887.0	2737	440	1147	13,784
C204	5,881.0	21,117	813	2623	94,674	5,881.0	25,892	1576	3019	57,959
R202	10,164.8	304	130	558	9265	10,164.8	246	98	480	9754
R203	8,597.0	836	178	881	28,547	8,597.0	650	155	704	24,398
R204	7,201.6	2203	184	1159	96,974	7,202.0	1587	173	934	74,028
R206	8,600.1	646	123	828	36,038	8,600.1	533	116	711	27,342
R207	7,805.0	1175	153	896	57,911	7,805.1	954	149	727	44,896
R208	6,910.9	8774	268	2256	294,003	6,910.9	7087	277	1954	219,155
R209	8,371.8	358	93	592	24,458	8,371.8	316	80	560	25,636
R210	8,813.2	454	91	758	33,615	8,813.2	382	77	650	31,437
R211	7,311.0	665	120	862	72,642	7,311.0	542	124	739	55,515
RC204	7,764.9	20,875	320	3198	664,362	7,764.9	11,012	271	2815	441,036
RC207	9,415.8	850	97	1580	105,739	9,415.8	650	97	1366	73,623
RC208	7,636.4	3167	156	3164	583,749	7,636.4	2516	119	2952	581,679
#Best	13/14	1/14	5/14	1/14	2/14	14/14	13/14	10/14	13/14	12/14

Table B18

Detailed results for comparing PartialDom versus SetPair settings using only-cycle DSSR after adding nonrobust cuts

Instance	PartialDom					SetPair				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C203	5,887.0	3428	422	1332	19,401	5,887.0	2738	440	1147	13,784
C204	5,881.0	21,118	813	2623	94,674	5,881.0	25,893	1576	3019	57,959
R202	10,296.0	1012	860	1369	3783	10,296.0	762	659	1118	3654
R203	8,708.0	1292	640	1396	11,821	8,708.0	1080	600	1203	10,074
R204	7,313.0	5288	654	1868	63,573	7,313.0	5255	611	1621	62,939
R206	8,759.0	1648	488	1301	20,682	8,759.0	1586	498	1203	19,174
R207	7,940.0	2796	619	1533	30,983	7,940.0	2644	594	1367	29,710
R208	6,993.1	176,704	625	3186	845,641	6,996.4	179,708	642	2979	855,593
R209	8,548.0	2766	604	1433	27,355	8,548.0	2324	587	1370	25,630
R210	9,005.0	5233	625	1519	34,038	9,005.0	4742	620	1404	30,323
R211	7,467.0	9605	340	1519	177,736	7,467.0	10,856	409	1477	164,952
RC204	7,835.0	22,986	593	3523	383,749	7,835.0	13,148	549	3162	246,296
RC207	9,629.0	2071	453	2121	41,165	9,629.0	2021	445	1953	35,634
RC208	7,761.0	5232	376	3646	346,185	7,761.0	4309	353	3462	304,091
#Best	13/14	3/14	5/14	1/14	1/14	14/14	11/14	9/14	13/14	13/14

Table B19

Detailed results for comparing PartialDom versus SetBased settings using all-arcs DSSR before adding nonrobust cuts

Instance	PartialDom					SetBased				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C203	5,887.0	1970	307	686	14,256	5,887.0	1517	324	564	10,446
C204	5,881.0	12,418	757	1206	57,927	5,881.0	19,889	1368	1808	53,233
R202	10,164.8	236	115	365	6995	10,164.8	219	116	338	6198
R203	8,597.0	538	146	511	20,678	8,597.0	478	130	462	21,196
R204	7,202.0	1338	170	610	55,653	7,201.6	1110	139	523	57,509
R206	8,600.1	419	98	445	23,433	8,600.1	373	87	418	24,467
R207	7,805.0	714	103	468	45,511	7,805.0	653	97	453	43,833
R208	6,910.9	3486	164	777	171,881	6,910.9	3608	211	808	145,786
R209	8,371.8	260	66	370	20,844	8,371.8	245	80	364	17,652
R210	8,813.2	314	62	458	30,685	8,813.2	282	75	420	24,123
R211	7,311.0	407	95	472	51,620	7,311.0	348	97	435	43,420
RC204	7,764.9	6220	182	936	273,583	7,764.9	3413	213	829	160,031
RC207	9,415.8	339	85	555	35,834	9,415.8	295	68	521	38,995
RC208	7,636.4	1036	103	888	240,585	7,636.4	761	97	830	200,815
#Best	14/14	2/14	8/14	2/14	4/14	13/14	12/14	6/14	12/14	10/14

Table B20

Detailed results for comparing PartialDom versus SetBased settings using all-arcs DSSR after adding nonrobust cuts

Instance	PartialDom					SetBased				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C203	5,887.0	1971	307	686	14,256	5,887.0	1517	324	564	10,446
C204	5,881.0	12,419	757	1206	57,927	5,881.0	19,890	1368	1808	53,233
R202	10,296.0	854	713	993	3272	10,296.0	736	660	912	3494
R203	8,708.0	948	563	943	9147	8,708.0	830	502	864	9819
R204	7,313.0	4712	564	1055	50,449	7,313.0	3642	554	997	44,628
R206	8,759.0	1194	426	800	15,427	8,759.0	1099	453	826	14,891
R207	7,940.0	1854	475	893	24,935	7,940.0	2083	505	907	27,022
R208	7,003.4	187,409	459	1156	749,700	7,006.1	183,931	558	1246	873,352
R209	8,548.0	2186	533	931	20,620	8,548.0	2139	596	958	22,394
R210	9,005.0	3623	564	1031	28,315	9,005.0	3580	576	993	30,062
R211	7,467.0	9167	300	799	130,566	7,467.0	3985	300	759	98,913
RC204	7,835.0	8414	441	1211	146,739	7,835.0	5250	493	1127	97,395
RC207	9,629.0	1226	373	914	21,959	9,629.0	1056	403	922	20,609
RC208	7,761.0	2521	327	1166	149,609	7,761.0	1930	331	1109	131,734
#Best	13/14	2/14	11/14	6/14	6/14	14/14	12/14	4/14	8/14	8/14

Table B21

Detailed results for comparing PartialDom versus Pairwise settings using all-arcs DSSR before adding nonrobust cuts

Instance	PartialDom					Pairwise				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C203	5,887.0	1970	307	686	14,256	5,887.0	1889	491	679	8099
C204	5,881.0	12,418	757	1206	57,927	5,881.0	12,909	1091	1466	37,265
R202	10,164.8	236	115	365	6995	10,164.8	210	104	334	6575
R203	8,597.0	538	146	511	20,678	8,597.0	479	133	473	21,009
R204	7,202.0	1338	170	610	55,653	7,201.6	1126	151	546	52,530
R206	8,600.1	419	98	445	23,433	8,600.1	360	98	420	19,897
R207	7,805.0	714	103	468	45,511	7,805.0	660	92	457	47,529
R208	6,910.9	3486	164	777	171,881	6,910.9	3077	199	742	130,870
R209	8,371.8	260	66	370	20,844	8,371.8	248	79	365	17,136
R210	8,813.2	314	62	458	30,685	8,813.2	277	63	416	25,960
R211	7,311.0	407	95	472	51,620	7,311.0	355	110	453	39,132
RC204	7,764.9	6220	182	936	273,583	7,764.9	3547	191	865	199,839
RC207	9,415.8	339	85	555	35,834	9,415.8	288	78	513	33,072
RC208	7,636.4	1036	103	888	240,585	7,636.0	759	81	820	239,051
#Best	14/14	1/14	8/14	1/14	2/14	12/14	13/14	7/14	13/14	12/14

Table B22

Detailed results for comparing PartialDom versus Pairwise settings using all-arcs DSSR after adding nonrobust cuts

Instance	PartialDom					Pairwise				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C203	5,887.0	1971	307	686	14,256	5,887.0	1889	491	679	8099
C204	5,881.0	12,419	757	1206	57,927	5,881.0	12,909	1091	1466	37,265
R202	10,296.0	854	713	993	3272	10,296.0	699	622	891	3161
R203	8,708.0	948	563	943	9147	8,708.0	933	575	934	9083
R204	7,313.0	4712	564	1055	50,449	7,313.0	3887	548	991	49,736
R206	8,759.0	1194	426	800	15,427	8,759.0	1088	444	796	14,029
R207	7,940.0	1854	475	893	24,935	7,940.0	1868	468	874	25,738
R208	7,003.4	187,409	459	1156	749,700	7,008.4	197,603	555	1195	962,739
R209	8,548.0	2186	533	931	20,620	8,548.0	1709	565	929	19,346
R210	9,005.0	3623	564	1031	28,315	9,005.0	3718	587	1010	28,225
R211	7,467.0	9167	300	799	130,566	7,467.0	4350	329	792	91,532
RC204	7,835.0	8414	441	1211	146,739	7,835.0	4949	425	1115	116,605
RC207	9,629.0	1226	373	914	21,959	9,629.0	1046	410	907	19,866
RC208	7,761.0	2521	327	1166	149,609	7,761.0	1493	267	1041	136,976
#Best	13/14	4/14	9/14	2/14	2/14	14/14	10/14	5/14	12/14	12/14

Table B23

Detailed results for comparing PartialDom versus SetPair settings using all-arcs DSSR before adding nonrobust cuts

Instance	PartialDom					SetPair				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C203	5,887.0	1970	307	686	14,256	5,887.0	1726	321	637	12,275
C204	5,881.0	12,418	757	1206	57,927	5,881.0	19,263	1526	1934	46,212
R202	10,164.8	236	115	365	6995	10,164.8	208	102	332	6754
R203	8,597.0	538	146	511	20,678	8,598.4	469	159	493	18,365
R204	7,202.0	1338	170	610	55,653	7,202.0	1042	140	504	51,514
R206	8,600.1	419	98	445	23,433	8,600.1	356	97	405	20,186
R207	7,805.0	714	103	468	45,511	7,805.0	645	100	451	39,837
R208	6,910.9	3486	164	777	171,881	6,910.9	3477	175	770	153,871
R209	8,371.8	260	66	370	20,844	8,371.8	244	70	358	18,608
R210	8,813.2	314	62	458	30,685	8,813.2	295	70	448	25,301
R211	7,311.0	407	95	472	51,620	7,311.0	362	108	455	39,622
RC204	7,764.9	6220	182	936	273,583	7,764.9	3306	201	807	155,315
RC207	9,415.8	339	85	555	35,834	9,415.8	314	78	535	36,044
RC208	7,636.4	1036	103	888	240,585	7,636.0	765	92	797	195,395
#Best	13/14	1/14	8/14	1/14	1/14	13/14	13/14	6/14	13/14	13/14

Table B24

Detailed results for comparing PartialDom versus SetPair settings using all-arcs DSSR after adding nonrobust cuts

Instance	PartialDom					SetPair				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C203	5,887.0	1971	307	686	14,256	5,887.0	1727	321	637	12,275
C204	5,881.0	12,419	757	1206	57,927	5,881.0	19,263	1526	1934	46,212
R202	10,296.0	854	713	993	3272	10,296.0	724	618	874	3135
R203	8,708.0	948	563	943	9147	8,708.0	896	650	999	8447
R204	7,313.0	4712	564	1055	50,449	7,313.0	3916	544	959	45,432
R206	8,759.0	1194	426	800	15,427	8,759.0	1051	414	757	14,440
R207	7,940.0	1854	475	893	24,935	7,940.0	1979	505	885	23,459
R208	7,003.4	187,409	459	1156	749,700	7,005.3	194,490	506	1170	764,528
R209	8,548.0	2186	533	931	20,620	8,548.0	1911	520	886	19,705
R210	9,005.0	3623	564	1031	28,315	9,005.0	4340	600	1054	28,787
R211	7,467.0	9167	300	799	130,566	7,467.0	4373	320	779	90,083
RC204	7,835.0	8414	441	1211	146,739	7,835.0	5174	450	1078	98,136
RC207	9,629.0	1226	373	914	21,959	9,629.0	1139	398	920	20,313
RC208	7,761.0	2521	327	1166	149,609	7,761.0	2111	297	1063	136,516
#Best	13/14	4/14	9/14	5/14	2/14	14/14	10/14	5/14	9/14	12/14

B.2. Gehring and Homberger's instances

Table B25

Detailed results for comparing Default versus SetBased settings using only-cycle DSSR before adding nonrobust cuts

Instance	Default					SetBased				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C1_2_3	26,597.4	427	120	1381	174,713	26,597.4	319	124	1219	136,113
C1_2_4	26,197.7	999	200	2834	390,363	26,197.7	737	181	2582	371,003
C1_2_9	26,396.0	245	54	748	126,697	26,396.0	220	56	638	109,625
C1_210	26,197.3	644	71	2201	579,885	26,197.3	624	85	1961	453,549
C2_2_1	19,150.3	348	7	7	435	19,150.3	347	7	7	435
C2_2_2	18,395.5	6158	583	1517	9693	18,396.0	6366	708	1464	7354
C2_2_5	18,607.1	1346	141	630	5369	18,607.1	1423	148	565	4885
C2_2_6	18,406.7	4696	377	1779	14,186	18,406.7	4104	342	1528	13,307
C2_2_7	18,356.4	7349	513	2698	22,335	18,365.4	4811	331	1792	20,601
C2_2_8	18,082.4	7841	434	2819	31,722	18,082.4	5854	314	2206	33,208
C2_2_9	18,011.9	12,604	453	3088	57,553	18,011.9	10,311	488	2750	46,156
R1_2_4	30,009.9	295	55	591	299,338	30,009.9	232	46	522	311,214
R1_2_5	40,068.1	42	11	23	10,792	40,068.1	41	11	23	10,864
R1_2_6	34,930.6	100	44	181	46,927	34,930.6	97	46	180	47,956
R1_2_7	31,000.4	271	50	451	201,404	30,999.1	211	49	398	181,273
R1_2_8	29,032.3	307	34	664	582,132	29,032.3	246	31	595	559,334
R1_2_9	36,787.8	64	16	126	56,508	36,787.8	60	16	119	53,852
R1_210	32,451.1	152	25	427	222,464	32,451.1	139	23	399	229,834
R2_2_1	34,625.1	1724	66	226	1610	34,625.2	1748	69	212	1441
R2_2_2	30,006.1	5304	270	1454	13,263	30,006.1	4964	288	1244	10,589
R2_2_5	30,290.3	2838	148	689	5854	30,290.3	2755	174	658	4709
R2_2_6	26,455.6	10,031	435	2601	41,857	26,456.2	8379	397	2141	33,639
R2_2_9	28,206.0	4140	227	1220	13,690	28,206.4	4003	214	1009	11,066
RC1_2_1	34,595.9	57	18	133	44,460	34,595.9	53	17	131	46,746
RC1_2_2	31,774.7	136	37	347	124,573	31,774.7	123	38	300	110,289
RC1_2_6	32,554.4	150	19	683	421,929	32,554.4	131	19	645	411,015
RC1_2_8	30,216.1	479	35	1504	1,154,526	30,216.1	359	31	1439	1,207,744
RC2_2_1	27,891.0	2018	125	707	5248	27,891.3	1859	131	643	4500
RC2_2_2	24,588.6	11,640	582	3940	37,446	24,589.8	9569	568	3218	28,827
RC2_2_5	24,788.5	7731	519	4326	92,047	24,788.6	6100	444	3622	76,239
RC2_2_6	24,724.4	5808	414	3467	50,584	24,724.4	4782	372	2965	42,500
#Best	24/31	3/31	16/31	2/31	8/31	30/31	28/31	19/31	31/31	24/31

Table B26

Detailed results for comparing Default versus SetBased settings using only-cycle DSSR after adding nonrobust cuts

Instance	Default					SetBased				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C1_2_3	26,752.7	174,077	1903	4274	278,173	26,752.6	173,517	1884	4137	262,970
C1_2_4	26,256.0	1403	348	3605	331,293	26,256.0	1152	358	3376	288,520
C1_2_9	26,396.0	245	54	748	126,697	26,396.0	221	56	638	109,625
C1_210	26,247.0	861	196	2995	304,343	26,247.0	843	214	2629	264,985
C2_2_1	19,221.0	1007	414	414	1405	19,221.0	914	479	479	1033
C2_2_2	18,514.0	13,187	1830	2979	7847	18,514.0	11,811	1558	2420	7697
C2_2_5	18,696.0	2463	717	1269	3294	18,696.0	2292	615	1073	3539
C2_2_6	18,448.0	5770	716	2277	10,098	18,448.0	5180	737	2008	8921
C2_2_7	18,422.0	8523	875	3117	15,455	18,423.3	5572	572	2114	14,329
C2_2_8	18,137.0	9446	800	3348	22,370	18,137.0	7534	706	2764	21,520
C2_2_9	18,150.0	22,001	1049	4311	49,739	18,150.0	17,741	1010	3810	45,067
R1_2_4	30,381.8	178,343	928	2520	467,945	30,381.2	177,114	903	2504	478,440
R1_2_5	40,471.9	2034	694	706	10,679	40,471.6	1782	628	640	10,706
R1_2_6	35,486.2	26,455	1209	1706	80,659	35,485.7	22,703	1220	1711	79,309
R1_2_7	31,404.5	40,594	1061	2223	216,549	31,404.4	38,044	1036	2189	212,635
R1_2_8	29,345.9	173,240	1104	3018	506,789	29,346.2	173,559	1165	3086	502,407
R1_2_9	37,284.5	4118	778	983	27,555	37,283.6	2992	665	864	26,756
R1_210	32,812.4	16,387	1086	1963	94,172	32,812.1	16,045	1073	1951	96,043
R2_2_1	34,680.0	1851	215	386	860	34,680.0	1880	272	430	743
R2_2_2	30,082.0	6677	686	1955	6538	30,082.0	5896	599	1580	6130
R2_2_5	30,611.0	7700	1864	2583	3892	30,611.0	6898	1640	2270	4224
R2_2_6	26,750.5	26,558	2995	5367	13,371	26,750.5	22,113	2473	4426	12,679
R2_2_9	28,433.0	6913	1261	2334	5796	28,433.0	6841	1269	2166	5016
RC1_2_1	35,017.8	4855	938	1080	20,427	35,016.8	3331	770	909	19,631
RC1_2_2	32,087.2	93,147	1114	2025	141,613	32,087.7	90,927	1139	1980	140,173
RC1_2_6	32,960.2	37,964	1269	2740	145,496	32,960.2	32,890	1250	2715	151,225
RC1_2_8	30,508.3	173,052	701	4572	1,087,318	30,508.7	176,286	686	4591	1,160,747
RC2_2_1	27,974.0	2789	1239	1959	1448	27,974.0	2483	913	1549	1690
RC2_2_2	24,780.0	74,326	7382	11,203	9576	24,780.0	63,458	6212	9261	9275
RC2_2_5	24,914.0	19,218	3954	8621	30,624	24,914.0	13,672	3099	6510	25,079
RC2_2_6	24,951.0	11,396	2406	6109	18,346	24,951.0	12,105	2737	5862	15,752
#Best	27/31	4/31	11/31	5/31	8/31	24/31	27/31	20/31	26/31	23/31

Table B27

Detailed results for comparing Default versus Pairwise settings using only-cycle DSSR before adding nonrobust cuts

Instance	Default					Pairwise				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C1_2_3	26,597.4	427	120	1381	174,713	26,597.4	395	126	1284	149,825
C1_2_4	26,197.7	999	200	2834	390,363	26,197.7	889	199	2534	318,126
C1_2_9	26,396.0	245	54	748	126,697	26,396.0	230	56	629	101,539
C1_210	26,197.3	644	71	2201	579,885	26,197.3	701	87	1956	409,693
C2_2_1	19,150.3	348	7	7	435	19,150.3	346	7	7	435
C2_2_2	18,395.5	6158	583	1517	9693	18,396.0	5180	593	1187	6541
C2_2_5	18,607.1	1346	141	630	5369	18,607.1	1514	168	594	4328
C2_2_6	18,406.7	4696	377	1779	14,186	18,406.7	4340	363	1504	11,640
C2_2_7	18,356.4	7349	513	2698	22,335	18,356.4	4483	285	1846	21,371
C2_2_8	18,082.4	7841	434	2819	31,722	18,082.4	6293	380	2164	24,979
C2_2_9	18,011.9	12,604	453	3088	57,553	18,011.9	9410	421	2482	40,634
R1_2_4	30,009.9	295	55	591	299,338	30,009.9	287	49	539	292,363
R1_2_5	40,068.1	42	11	23	10,792	40,068.1	42	11	23	10,792
R1_2_6	34,930.6	100	44	181	46,927	34,930.6	106	47	172	44,706
R1_2_7	31,000.4	271	50	451	201,404	31,000.0	242	46	406	188,565
R1_2_8	29,032.3	307	34	664	582,132	29,032.3	334	36	619	506,166
R1_2_9	36,787.8	64	16	126	56,508	36,787.8	64	16	125	56,068
R1_210	32,451.1	152	25	427	222,464	32,451.1	157	25	397	206,001
R2_2_1	34,625.1	1724	66	226	1610	34,625.2	1711	63	209	1530
R2_2_2	30,006.1	5304	270	1454	13,263	30,006.1	4585	276	1223	10,192
R2_2_5	30,290.3	2838	148	689	5854	30,290.3	2621	133	590	5236
R2_2_6	26,455.6	10,031	435	2601	41,857	26,456.0	8471	384	2066	32,993
R2_2_9	28,206.0	4140	227	1220	13,690	28,206.0	4076	201	1015	11,496
RC1_2_1	34,595.9	57	18	133	44,460	34,595.9	56	18	133	44,460
RC1_2_2	31,774.7	136	37	347	124,573	31,774.7	132	41	325	103,418
RC1_2_6	32,554.4	150	19	683	421,929	32,554.4	149	20	636	371,121
RC1_2_8	30,216.1	479	35	1504	1,154,526	30,216.1	430	33	1435	1,091,526
RC2_2_1	27,891.0	2018	125	707	5248	27,891.3	1972	124	601	4258
RC2_2_2	24,588.6	11,640	582	3940	37,446	24,590.6	9328	516	3210	29,730
RC2_2_5	24,788.5	7731	519	4326	92,047	24,788.6	6530	480	3813	74,813
RC2_2_6	24,724.4	5808	414	3467	50,584	24,724.4	4921	393	2956	38,082
#Best	26/31	6/31	15/31	3/31	3/31	30/31	25/31	21/31	31/31	31/31

Table B28

Detailed results for comparing Default versus Pairwise settings using only-cycle DSSR after adding nonrobust cuts

Instance	Default					Pairwise				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C1_2_3	26,752.7	174,077	1903	4274	278,173	26,752.2	176,153	1843	4017	270,184
C1_2_4	26,256.0	1403	348	3605	331,293	26,256.0	1294	343	3336	292,342
C1_2_9	26,396.0	245	54	748	126,697	26,396.0	231	56	629	101,539
C1_210	26,247.0	861	196	2995	304,343	26,247.0	950	206	2708	266,016
C2_2_1	19,221.0	1007	414	414	1405	19,221.0	1009	414	414	1405
C2_2_2	18,514.0	13,187	1830	2979	7847	18,514.0	13,283	1667	2415	7983
C2_2_5	18,696.0	2463	717	1269	3294	18,696.0	2231	572	1034	2913
C2_2_6	18,448.0	5770	716	2277	10,098	18,448.0	5322	653	1897	8643
C2_2_7	18,422.0	8523	875	3117	15,455	18,422.0	6271	874	2625	10,538
C2_2_8	18,137.0	9446	800	3348	22,370	18,137.0	8065	832	2832	17,545
C2_2_9	18,150.0	22,001	1049	4311	49,739	18,150.0	18,612	1087	3794	40,620
R1_2_4	30,381.8	178,343	928	2520	467,945	30,381.8	174,763	903	2485	464,605
R1_2_5	40,471.9	2034	694	706	10,679	40,471.9	1981	694	706	10,679
R1_2_6	35,486.2	26,455	1209	1706	80,659	35,485.3	28,276	1317	1814	80,153
R1_2_7	31,404.5	40,594	1061	2223	216,549	31,404.4	45,569	1131	2263	206,701
R1_2_8	29,345.9	173,240	1104	3018	506,789	29,345.6	173,387	1106	3034	501,744
R1_2_9	37,284.5	4118	778	983	27,555	37,283.4	3049	659	872	27,008
R1_210	32,812.4	16,387	1086	1963	94,172	32,811.1	11,744	937	1766	88,250
R2_2_1	34,680.0	1851	215	386	860	34,680.0	1837	225	382	800
R2_2_2	30,082.0	6677	686	1955	6538	30,082.0	5421	580	1562	5808
R2_2_5	30,611.0	7700	1864	2583	3892	30,611.0	6254	1461	2089	3566
R2_2_6	26,750.5	26,558	2995	5367	13,371	26,750.5	21,719	2291	4223	12,972
R2_2_9	28,433.0	6913	1261	2334	5796	28,433.0	6903	1236	2141	5033
RC1_2_1	35,017.8	4855	938	1080	20,427	35,017.8	4825	938	1080	20,427
RC1_2_2	32,087.2	93,147	1114	2025	141,613	32,087.6	108,209	1140	1986	137,733
RC1_2_6	32,960.2	37,964	1269	2740	145,496	32,960.4	38,157	1269	2745	141,760
RC1_2_8	30,508.3	173,052	701	4572	1,087,318	30,509.0	175,969	697	4543	996,846
RC2_2_1	27,974.0	2789	1239	1959	1448	27,974.0	2735	1205	1875	1464
RC2_2_2	24,780.0	74,326	7382	11,203	9576	24,780.0	65,053	6954	10,101	8688
RC2_2_5	24,914.0	19,218	3954	8621	30,624	24,914.0	28,143	6011	10,046	20,619
RC2_2_6	24,951.0	11,396	2406	6109	18,346	24,951.0	8944	1944	5107	16,935
#Best	28/31	11/31	14/31	8/31	5/31	25/31	20/31	21/31	26/31	29/31

Table B29

Detailed results for comparing Default versus SetPair settings using only-cycle DSSR before adding nonrobust cuts

Instance	Default					SetPair				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C1_2_3	26,597.4	427	120	1381	174,713	26,597.4	360	133	1237	128,396
C1_2_4	26,197.7	999	200	2834	390,363	26,197.7	791	200	2945	380,210
C1_2_9	26,396.0	245	54	748	126,697	26,396.0	223	58	663	102,973
C1_2_10	26,197.3	644	71	2201	579,885	26,197.3	588	85	1917	411,479
C2_2_1	19,150.3	348	7	7	435	19,150.3	347	7	7	435
C2_2_2	18,395.5	6158	583	1517	9693	18,396.0	8386	724	1664	9571
C2_2_5	18,607.1	1346	141	630	5369	18,607.1	1276	133	529	4859
C2_2_6	18,406.7	4696	377	1779	14,186	18,406.7	4084	295	1455	13,866
C2_2_7	18,356.4	7349	513	2698	22,335	18,370.4	5458	393	2034	18,636
C2_2_8	18,082.4	7841	434	2819	31,722	18,082.4	5788	380	2244	25,504
C2_2_9	18,011.9	12,604	453	3088	57,553	18,011.9	11,220	469	2672	43,541
R1_2_4	30,009.9	295	55	591	299,338	30,009.9	222	42	498	315,121
R1_2_5	40,068.1	42	11	23	10,792	40,068.1	41	10	22	11,161
R1_2_6	34,930.6	100	44	181	46,927	34,930.6	95	46	171	44,276
R1_2_7	31,000.4	271	50	451	201,404	30,998.9	209	46	415	194,376
R1_2_8	29,032.3	307	34	664	582,132	29,032.3	255	34	593	508,686
R1_2_9	36,787.8	64	16	126	56,508	36,787.8	59	16	119	52,895
R1_2_10	32,451.1	152	25	427	222,464	32,451.1	142	25	398	206,501
R2_2_1	34,625.1	1724	66	226	1610	34,625.2	1717	65	211	1505
R2_2_2	30,006.1	5304	270	1454	13,263	30,006.1	4871	282	1214	10,282
R2_2_5	30,290.3	2838	148	689	5854	30,290.3	2783	173	623	4274
R2_2_6	26,455.6	10,031	435	2601	41,857	26,455.6	8248	365	2054	37,270
R2_2_9	28,206.0	4140	227	1220	13,690	28,206.2	4049	229	1012	10,103
RC1_2_1	34,595.9	57	18	133	44,460	34,595.9	54	18	133	44,324
RC1_2_2	31,774.7	136	37	347	124,573	31,774.7	119	43	315	99,413
RC1_2_6	32,554.4	150	19	683	421,929	32,554.6	136	22	651	343,279
RC1_2_8	30,216.1	479	35	1504	1,154,526	30,216.1	359	34	1390	998,532
RC2_2_1	27,891.0	2018	125	707	5248	27,891.3	1890	118	618	4553
RC2_2_2	24,588.6	11,640	582	3940	37,446	24,590.6	9181	511	3102	29,002
RC2_2_5	24,788.5	7731	519	4326	92,047	24,788.5	6277	448	3744	76,733
RC2_2_6	24,724.4	5808	414	3467	50,584	24,724.4	4856	367	2929	41,101
#Best	24/31	1/31	17/31	4/31	3/31	30/31	30/31	20/31	29/31	29/31

Table B30

Detailed results for comparing Default versus SetPair settings using only-cycle DSSR after adding nonrobust cuts

Instance	Default					SetPair				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C1_2_3	26,752.7	174,077	1903	4274	278,173	26,751.3	172,971	1841	3991	242,010
C1_2_4	26,256.0	1403	348	3605	331,293	26,256.0	1287	359	3703	312,205
C1_2_9	26,396.0	245	54	748	126,697	26,396.0	224	58	663	102,973
C1_210	26,247.0	861	196	2995	304,343	26,247.0	818	213	2686	251,061
C2_2_1	19,221.0	1007	414	414	1405	19,221.0	1086	566	566	1147
C2_2_2	18,514.0	13,187	1830	2979	7847	18,514.0	14,946	1838	2841	7905
C2_2_5	18,696.0	2463	717	1269	3294	18,696.0	2123	555	984	3258
C2_2_6	18,448.0	5770	716	2277	10,098	18,448.0	5215	635	1953	9316
C2_2_7	18,422.0	8523	875	3117	15,455	18,422.0	7570	912	2930	13,018
C2_2_8	18,137.0	9446	800	3348	22,370	18,137.0	7544	802	2879	17,826
C2_2_9	18,150.0	22,001	1049	4311	49,739	18,150.0	22,022	1389	4258	34,605
R1_2_4	30,381.8	178,343	928	2520	467,945	30,378.9	174,117	782	2329	451,245
R1_2_5	40,471.9	2034	694	706	10,679	40,471.0	2040	684	697	10,716
R1_2_6	35,486.2	26,455	1209	1706	80,659	35,485.3	30,800	1298	1802	79,940
R1_2_7	31,404.5	40,594	1061	2223	216,549	31,404.6	50,470	1015	2161	219,204
R1_2_8	29,345.9	173,240	1104	3018	506,789	29,345.5	175,173	1073	2980	485,150
R1_2_9	37,284.5	4118	778	983	27,555	37,284.1	3390	711	914	26,937
R1_210	32,812.4	16,387	1086	1963	94,172	32,811.6	14,648	1001	1870	91,813
R2_2_1	34,680.0	1851	215	386	860	34,680.0	1804	189	338	844
R2_2_2	30,082.0	6677	686	1955	6538	30,082.0	5806	592	1540	5887
R2_2_5	30,611.0	7700	1864	2583	3892	30,611.0	6547	1633	2205	3392
R2_2_6	26,750.5	26,558	2995	5367	13,371	26,750.5	21,322	2457	4378	12,350
R2_2_9	28,433.0	6913	1261	2334	5796	28,433.0	6925	1279	2178	4904
RC1_2_1	35,017.8	4855	938	1080	20,427	35,017.3	4427	831	969	20,095
RC1_2_2	32,087.2	93,147	1114	2025	141,613	32,088.0	118,036	1186	2005	135,460
RC1_2_6	32,960.2	37,964	1269	2740	145,496	32,960.6	46,136	1289	2757	143,502
RC1_2_8	30,508.3	173,052	701	4572	1,087,318	30,505.7	179,781	623	4417	1,046,100
RC2_2_1	27,974.0	2789	1239	1959	1448	27,974.0	2542	976	1631	1501
RC2_2_2	24,780.0	74,326	7382	11,203	9576	24,780.0	41,251	4738	7679	8973
RC2_2_5	24,914.0	19,218	3954	8621	30,624	24,914.0	19,812	4388	8265	22,480
RC2_2_6	24,951.0	11,396	2406	6109	18,346	24,951.0	10,238	2196	5446	16,623
#Best	29/31	12/31	13/31	4/31	4/31	22/31	19/31	18/31	27/31	27/31

Table B31

Detailed results for comparing Default versus SetBased settings using all-arcs DSSR before adding nonrobust cuts

Instance	Default					SetBased				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C1_2_3	26,597.4	249	106	478	71,318	26,597.4	215	106	452	65,229
C1_2_4	26,197.7	484	162	902	169,325	26,197.7	431	182	906	134,003
C1_2_9	26,396.0	225	55	559	93,896	26,396.0	207	57	522	88,052
C1_210	26,197.3	457	64	1189	347,848	26,197.4	452	78	1122	280,835
C2_2_1	19,150.3	358	7	7	435	19,150.3	353	7	7	435
C2_2_2	18,396.0	12,186	1176	1406	6656	18,396.0	3346	432	619	5484
C2_2_5	18,607.1	1328	153	538	4423	18,607.1	1277	137	493	4644
C2_2_6	18,406.7	2901	266	997	10,609	18,406.7	3476	328	993	9659
C2_2_7	18,356.4	4014	401	1262	12,577	18,356.4	3598	324	1090	14,090
C2_2_8	18,082.4	4019	319	1297	19,148	18,082.4	3516	273	1152	20,758
C2_2_9	18,011.9	5466	284	1315	38,099	18,011.9	5194	318	1207	32,809
R1_2_4	30,009.9	210	41	320	210,946	30,009.9	177	46	296	170,241
R1_2_5	40,068.1	42	11	23	10,792	40,068.1	42	11	23	10,864
R1_2_6	34,930.6	94	39	134	40,197	34,930.6	94	47	151	39,320
R1_2_7	31,000.8	206	42	295	152,863	31,000.5	194	52	282	115,984
R1_2_8	29,032.3	218	39	344	263,303	29,032.2	200	33	360	330,244
R1_2_9	36,787.8	62	16	113	49,861	36,787.8	58	17	110	45,953
R1_210	32,451.1	131	21	311	191,263	32,451.1	120	23	302	170,625
R2_2_1	34,625.1	1668	61	199	1518	34,625.1	1625	58	185	1499
R2_2_2	30,006.1	4211	287	927	8767	30,006.1	4085	248	825	8971
R2_2_5	30,290.3	2655	165	527	4055	30,290.3	2473	140	460	4139
R2_2_6	26,455.5	5783	288	1203	28,978	26,455.5	5669	288	1076	25,726
R2_2_9	28,206.0	3298	201	697	8490	28,206.0	3371	175	617	8318
RC1_2_1	34,595.9	56	18	133	44,460	34,595.9	54	17	131	46,746
RC1_2_2	31,774.8	122	41	263	90,794	31,774.7	117	37	266	101,657
RC1_2_6	32,554.4	112	22	442	232,263	32,554.4	101	18	424	285,058
RC1_2_8	30,216.1	246	33	580	482,789	30,216.1	196	29	572	542,817
RC2_2_1	27,891.0	1751	131	540	3854	27,891.0	1867	129	475	3534
RC2_2_2	24,588.6	6270	472	1763	20,783	24,589.4	5806	369	1524	23,857
RC2_2_5	24,788.5	3723	329	1464	48,659	24,788.5	3245	264	1305	47,348
RC2_2_6	24,724.4	3241	278	1315	27,118	24,724.4	3249	278	1277	25,921
#Best	29/31	5/31	15/31	6/31	13/31	30/31	26/31	21/31	27/31	19/31

Table B32

Detailed results for comparing Default versus SetBased settings using all-arcs DSSR after adding nonrobust cuts

Instance	Default					SetBased				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C1_2_3	26,752.8	175,608	1865	2526	233,199	26,753.2	173,111	1985	2635	228,628
C1_2_4	26,256.0	708	295	1162	146,396	26,256.0	709	350	1205	120,837
C1_2_9	26,396.0	226	55	559	93,896	26,396.0	208	57	522	88,052
C1_210	26,247.0	610	188	1593	173,425	26,247.0	625	198	1505	168,608
C2_2_1	19,221.0	1037	414	414	1405	19,221.0	935	479	479	1033
C2_2_2	18,514.0	18,036	1937	2186	7081	18,514.0	8511	1296	1505	7360
C2_2_5	18,696.0	2259	692	1125	2962	18,696.0	2031	556	947	3272
C2_2_6	18,448.0	3725	576	1346	7082	18,448.0	4766	795	1498	7195
C2_2_7	18,422.0	5128	795	1679	8776	18,422.0	9523	1994	2891	8236
C2_2_8	18,137.0	5213	676	1677	14,250	18,137.0	5002	705	1654	13,938
C2_2_9	18,150.0	10,850	855	2056	30,447	18,150.0	10,928	934	1969	29,016
R1_2_4	30,382.9	178,950	995	1620	392,300	30,382.5	174,279	932	1501	375,703
R1_2_5	40,471.9	2027	694	706	10,679	40,471.6	1730	628	640	10,706
R1_2_6	35,485.6	25,055	1295	1576	71,917	35,485.8	26,704	1306	1574	75,130
R1_2_7	31,404.6	29,284	989	1515	172,649	31,404.3	30,365	1002	1493	173,444
R1_2_8	29,346.5	174,477	1215	1904	449,750	29,346.8	174,211	1311	1988	439,039
R1_2_9	37,284.2	3658	729	912	26,890	37,283.9	3026	643	817	27,223
R1_210	32,811.1	12,433	996	1514	80,401	32,811.5	13,001	1030	1551	83,754
R2_2_1	34,680.0	1769	194	337	829	34,680.0	1811	294	430	725
R2_2_2	30,082.0	4859	508	1158	5904	30,082.0	5343	706	1294	4597
R2_2_5	30,611.0	6339	1629	2029	3579	30,611.0	6727	1709	2106	3886
R2_2_6	26,750.5	19,476	2620	3582	11,358	26,750.5	18,306	2363	3232	10,879
R2_2_9	28,433.0	6074	1251	1794	4570	28,433.0	5944	1113	1605	4762
RC1_2_1	35,017.8	4874	938	1080	20,427	35,016.8	3385	770	909	19,631
RC1_2_2	32,087.7	82,456	1092	1586	123,907	32,087.8	78,701	1132	1628	120,445
RC1_2_6	32,959.7	30,687	1196	2086	124,809	32,960.2	27,114	1193	2044	126,487
RC1_2_8	30,514.2	180,988	898	2108	566,568	30,514.7	172,960	897	2072	598,704
RC2_2_1	27,974.0	2230	873	1335	1447	27,974.0	2507	940	1382	1619
RC2_2_2	24,780.0	63,098	7402	8837	8189	24,780.0	70,794	7815	9078	9037
RC2_2_5	24,914.0	16,895	4447	5684	18,991	24,914.0	10,548	2593	3713	23,611
RC2_2_6	24,951.0	8612	2473	3561	11,569	24,951.0	7557	1932	2976	12,034
#Best	24/31	13/31	19/31	13/31	16/31	26/31	18/31	12/31	18/31	15/31

Table B33

Detailed results for comparing Default versus Pairwise settings using all-arcs DSSR before adding nonrobust cuts

Instance	Default					Pairwise				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C1_2_3	26,597.4	249	106	478	71,318	26,597.4	259	112	451	61,612
C1_2_4	26,197.7	484	162	902	169,325	26,197.7	492	179	951	157,224
C1_2_9	26,396.0	225	55	559	93,896	26,396.0	223	55	516	85,621
C1_210	26,197.3	457	64	1189	347,848	26,197.3	454	70	1156	302,044
C2_2_1	19,150.3	358	7	7	435	19,150.3	346	7	7	435
C2_2_2	18,396.0	12,186	1176	1406	6656	18,394.0	3258	416	612	5208
C2_2_5	18,607.1	1328	153	538	4423	18,607.1	1281	128	490	4733
C2_2_6	18,406.7	2901	266	997	10,609	18,406.7	2809	263	935	9440
C2_2_7	18,356.4	4014	401	1262	12,577	18,356.4	2985	265	998	13,361
C2_2_8	18,082.4	4019	319	1297	19,148	18,082.4	3533	297	1146	16,726
C2_2_9	18,011.9	5466	284	1315	38,099	18,011.8	5259	311	1234	30,655
R1_2_4	30,009.9	210	41	320	210,946	30,009.9	229	44	308	189,214
R1_2_5	40,068.1	42	11	23	10,792	40,068.1	42	11	23	10,792
R1_2_6	34,930.6	94	39	134	40,197	34,930.5	97	46	146	36,742
R1_2_7	31,000.8	206	42	295	152,863	31,000.5	199	49	271	117,794
R1_2_8	29,032.3	218	39	344	263,303	29,032.4	226	35	343	290,122
R1_2_9	36,787.8	62	16	113	49,861	36,787.8	60	16	109	47,384
R1_210	32,451.1	131	21	311	191,263	32,451.1	132	22	302	174,792
R2_2_1	34,625.1	1668	61	199	1518	34,625.1	1719	76	210	1263
R2_2_2	30,006.1	4211	287	927	8767	30,006.1	3917	242	824	8778
R2_2_5	30,290.3	2655	165	527	4055	30,290.3	2665	158	490	3761
R2_2_6	26,455.5	5783	288	1203	28,978	26,455.5	5601	280	1099	26,008
R2_2_9	28,206.0	3298	201	697	8490	28,206.0	3272	171	622	8243
RC1_2_1	34,595.9	56	18	133	44,460	34,595.9	56	18	133	44,460
RC1_2_2	31,774.8	122	41	263	90,794	31,774.7	117	39	257	90,986
RC1_2_6	32,554.4	112	22	442	232,263	32,554.4	109	19	423	253,881
RC1_2_8	30,216.1	246	33	580	482,789	30,216.1	226	28	571	528,193
RC2_2_1	27,891.0	1751	131	540	3854	27,891.3	1841	125	477	3532
RC2_2_2	24,588.6	6270	472	1763	20,783	24,588.8	6091	421	1561	19,566
RC2_2_5	24,788.5	3723	329	1464	48,659	24,788.5	3408	286	1343	43,608
RC2_2_6	24,724.4	3241	278	1315	27,118	24,724.4	3035	269	1196	23,394
#Best	29/31	10/31	14/31	6/31	10/31	27/31	21/31	22/31	28/31	24/31

Table B34

Detailed results for comparing Default versus Pairwise settings using all-arcs DSSR after adding nonrobust cuts

Instance	Default					Pairwise				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C1_2_3	26,752.8	175,608	1865	2526	233,199	26,752.8	174,314	1951	2601	223,555
C1_2_4	26,256.0	708	295	1162	146,396	26,256.0	750	334	1209	128,992
C1_2_9	26,396.0	226	55	559	93,896	26,396.0	224	55	516	85,621
C1_210	26,247.0	610	188	1593	173,425	26,247.0	623	205	1547	151,855
C2_2_1	19,221.0	1037	414	414	1405	19,221.0	1011	414	414	1405
C2_2_2	18,514.0	18,036	1937	2186	7081	18,514.0	12,010	1754	1965	7878
C2_2_5	18,696.0	2259	692	1125	2962	18,696.0	2108	602	995	2823
C2_2_6	18,448.0	3725	576	1346	7082	18,448.0	3823	630	1346	6806
C2_2_7	18,422.0	5128	795	1679	8776	18,422.0	3959	614	1390	8432
C2_2_8	18,137.0	5213	676	1677	14,250	18,137.0	4882	712	1615	11,765
C2_2_9	18,150.0	10,850	855	2056	30,447	18,150.0	11,728	1066	2166	25,098
R1_2_4	30,382.9	178,950	995	1620	392,300	30,382.1	177,048	916	1479	372,593
R1_2_5	40,471.9	2027	694	706	10,679	40,471.9	2039	694	706	10,679
R1_2_6	35,485.6	25,055	1295	1576	71,917	35,485.6	22,911	1211	1480	72,105
R1_2_7	31,404.6	29,284	989	1515	172,649	31,404.7	31,416	1033	1507	172,905
R1_2_8	29,346.5	174,477	1215	1904	449,750	29,346.8	174,130	1286	1969	415,519
R1_2_9	37,284.2	3658	729	912	26,890	37,284.0	3451	701	866	26,581
R1_210	32,811.1	12,433	996	1514	80,401	32,811.5	12,644	1009	1530	79,696
R2_2_1	34,680.0	1769	194	337	829	34,680.0	1915	353	494	678
R2_2_2	30,082.0	4859	508	1158	5904	30,082.0	4902	596	1190	4794
R2_2_5	30,611.0	6339	1629	2029	3579	30,611.0	5967	1405	1778	3636
R2_2_6	26,750.5	19,476	2620	3582	11,358	26,750.5	18,934	2493	3392	10,006
R2_2_9	28,433.0	6074	1251	1794	4570	28,433.0	5714	1089	1566	4442
RC1_2_1	35,017.8	4874	938	1080	20,427	35,017.8	4832	938	1080	20,427
RC1_2_2	32,087.7	82,456	1092	1586	123,907	32,087.6	97,015	1156	1627	124,469
RC1_2_6	32,959.7	30,687	1196	2086	124,809	32,960.2	33,762	1229	2092	122,444
RC1_2_8	30,514.2	180,988	898	2108	566,568	30,514.1	173,242	874	2085	547,801
RC2_2_1	27,974.0	2230	873	1335	1447	27,974.0	2468	1022	1442	1380
RC2_2_2	24,780.0	63,098	7402	8837	8189	24,780.0	51,013	6063	7308	7725
RC2_2_5	24,914.0	16,895	4447	5684	18,991	24,914.0	11,536	2890	3996	18,575
RC2_2_6	24,951.0	8612	2473	3561	11,569	24,951.0	12,398	3136	4117	10,638
#Best	27/31	13/31	19/31	15/31	8/31	28/31	18/31	16/31	20/31	26/31

Table B35

Detailed results for comparing Default versus SetPair settings using all-arcs DSSR before adding non-robust cuts

Instance	Default					SetPair				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C1_2_3	26,597.4	249	106	478	71,318	26,597.4	184	104	468	71,359
C1_2_4	26,197.7	484	162	902	169,325	26,197.7	435	171	816	137,328
C1_2_9	26,396.0	225	55	559	93,896	26,396.0	215	52	523	91,451
C1_210	26,197.3	457	64	1189	347,848	26,197.3	476	69	1053	281,031
C2_2_1	19,150.3	358	7	7	435	19,150.3	348	7	7	435
C2_2_2	18,396.0	12,186	1176	1406	6656	18,396.0	3495	440	695	6377
C2_2_5	18,607.1	1328	153	538	4423	18,607.1	1234	130	498	4742
C2_2_6	18,406.7	2901	266	997	10,609	18,406.7	2781	199	931	12,609
C2_2_7	18,356.4	4014	401	1262	12,577	18,356.4	3097	234	979	15,181
C2_2_8	18,082.4	4019	319	1297	19,148	18,082.4	3449	264	1138	19,031
C2_2_9	18,011.9	5466	284	1315	38,099	18,011.8	5014	289	1212	32,366
R1_2_4	30,009.9	210	41	320	210,946	30,009.9	185	44	313	188,885
R1_2_5	40,068.1	42	11	23	10,792	40,068.1	41	10	22	11,161
R1_2_6	34,930.6	94	39	134	40,197	34,930.6	93	46	159	39,948
R1_2_7	31,000.8	206	42	295	152,863	31,000.2	177	44	286	138,251
R1_2_8	29,032.3	218	39	344	263,303	29,032.3	183	35	345	275,634
R1_2_9	36,787.8	62	16	113	49,861	36,787.8	58	16	109	47,399
R1_210	32,451.1	131	21	311	191,263	32,451.1	116	19	284	189,812
R2_2_1	34,625.1	1668	61	199	1518	34,625.1	1666	70	197	1290
R2_2_2	30,006.1	4211	287	927	8767	30,006.1	3895	222	846	9565
R2_2_5	30,290.3	2655	165	527	4055	30,290.3	2536	182	514	3457
R2_2_6	26,455.5	5783	288	1203	28,978	26,455.5	5477	273	1092	27,355
R2_2_9	28,206.0	3298	201	697	8490	28,206.0	3416	176	633	8129
RC1_2_1	34,595.9	56	18	133	44,460	34,595.9	54	18	133	44,324
RC1_2_2	31,774.8	122	41	263	90,794	31,774.7	116	43	262	84,050
RC1_2_6	32,554.4	112	22	442	232,263	32,554.4	100	20	423	245,260
RC1_2_8	30,216.1	246	33	580	482,789	30,216.1	196	31	572	490,921
RC2_2_1	27,891.0	1751	131	540	3854	27,892.5	1818	131	487	3448
RC2_2_2	24,588.6	6270	472	1763	20,783	24,588.8	5330	379	1492	20,620
RC2_2_5	24,788.5	3723	329	1464	48,659	24,788.5	3324	330	1394	37,539
RC2_2_6	24,724.4	3241	278	1315	27,118	24,724.4	2898	207	1163	29,369
#Best	29/31	3/31	14/31	4/31	11/31	29/31	28/31	21/31	29/31	21/31

Table B36

Detailed results for comparing Default versus SetPair settings using all-arcs DSSR after adding nonrobust cuts

Instance	Default					SetPair				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C1_2_3	26,752.8	175,608	1865	2526	233,199	26,751.9	174,346	1821	2460	211,811
C1_2_4	26,256.0	708	295	1162	146,396	26,256.0	800	343	1153	129,352
C1_2_9	26,396.0	226	55	559	93,896	26,396.0	216	52	523	91,451
C1_210	26,247.0	610	188	1593	173,425	26,247.0	628	192	1483	157,517
C2_2_1	19,221.0	1037	414	414	1405	19,221.0	1091	566	566	1147
C2_2_2	18,514.0	18,036	1937	2186	7081	18,514.0	10,819	1366	1633	7385
C2_2_5	18,696.0	2259	692	1125	2962	18,696.0	2211	688	1104	2863
C2_2_6	18,448.0	3725	576	1346	7082	18,448.0	3617	498	1252	7873
C2_2_7	18,422.0	5128	795	1679	8776	18,422.0	4147	620	1381	8749
C2_2_8	18,137.0	5213	676	1677	14,250	18,137.0	4637	593	1523	13,588
C2_2_9	18,150.0	10,850	855	2056	30,447	18,150.0	10,488	899	1967	26,966
R1_2_4	30,382.9	178,950	995	1620	392,300	30,380.0	179,044	842	1441	366,007
R1_2_5	40,471.9	2027	694	706	10,679	40,471.0	1986	684	697	10,716
R1_2_6	35,485.6	25,055	1295	1576	71,917	35,485.3	27,965	1205	1505	74,688
R1_2_7	31,404.6	29,284	989	1515	172,649	31,404.6	33,773	968	1461	168,974
R1_2_8	29,346.5	174,477	1215	1904	449,750	29,346.1	174,610	1147	1846	416,161
R1_2_9	37,284.2	3658	729	912	26,890	37,283.2	3421	698	863	26,282
R1_210	32,811.1	12,433	996	1514	80,401	32,812.3	17,806	1146	1665	82,338
R2_2_1	34,680.0	1769	194	337	829	34,680.0	1783	218	354	756
R2_2_2	30,082.0	4859	508	1158	5904	30,082.0	4646	495	1132	5412
R2_2_5	30,611.0	6339	1629	2029	3579	30,611.0	6300	1652	2040	3690
R2_2_6	26,750.5	19,476	2620	3582	11,358	26,750.5	17,219	2235	3139	10,640
R2_2_9	28,433.0	6074	1251	1794	4570	28,433.0	6143	1236	1726	4360
RC1_2_1	35,017.8	4874	938	1080	20,427	35,017.3	4376	831	969	20,095
RC1_2_2	32,087.7	82,456	1092	1586	123,907	32,087.6	110,251	1132	1618	118,878
RC1_2_6	32,959.7	30,687	1196	2086	124,809	32,960.3	37,278	1203	2053	125,815
RC1_2_8	30,514.2	180,988	898	2108	566,568	30,512.5	173,314	830	1976	540,561
RC2_2_1	27,974.0	2230	873	1335	1447	27,974.0	2314	839	1279	1464
RC2_2_2	24,780.0	63,098	7402	8837	8189	24,780.0	43,946	5701	6928	8103
RC2_2_5	24,914.0	16,895	4447	5684	18,991	24,914.0	8939	2407	3533	19,953
RC2_2_6	24,951.0	8612	2473	3561	11,569	24,951.0	7700	2189	3232	11,829
#Best	29/31	13/31	9/31	5/31	10/31	23/31	18/31	22/31	26/31	21/31

Table B37

Detailed results for comparing PartialDom versus SetBased settings using only-cycle DSSR before adding nonrobust cuts

Instance	PartialDom					SetBased				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C1_2_3	26,597.4	394	124	1385	173,802	26,597.4	319	124	1219	136,113
C1_2_4	26,197.7	1110	196	2999	422,665	26,197.7	737	181	2582	371,003
C1_2_9	26,396.0	274	62	694	103,678	26,396.0	220	56	638	109,625
C1_210	26,197.3	795	83	2249	524,649	26,197.3	624	85	1961	453,549
C2_2_1	19,150.3	371	7	7	435	19,150.3	347	7	7	435
C2_2_2	18,394.0	11,347	773	2043	11,156	18,396.0	6366	708	1464	7354
C2_2_5	18,607.1	1542	170	656	4847	18,607.1	1423	148	565	4885
C2_2_6	18,406.7	4451	327	1673	14,377	18,406.7	4104	342	1528	13,307
C2_2_7	18,356.4	7352	480	2461	21,229	18,365.4	4811	331	1792	20,601
C2_2_8	18,082.9	7479	369	2749	35,935	18,082.4	5854	314	2206	33,208
C2_2_9	18,011.9	13,406	510	3142	52,033	18,011.9	10,311	488	2750	46,156
R1_2_4	30,009.9	328	44	576	360,513	30,009.9	232	46	522	311,214
R1_2_5	40,068.1	44	11	23	10,796	40,068.1	41	11	23	10,864
R1_2_6	34,930.1	106	43	167	45,657	34,930.6	97	46	180	47,956
R1_2_7	30,999.1	298	54	441	182,425	30,999.1	211	49	398	181,273
R1_2_8	29,032.4	388	33	648	595,520	29,032.3	246	31	595	559,334
R1_2_9	36,787.8	66	16	123	54,489	36,787.8	60	16	119	53,852
R1_210	32,451.1	165	21	414	256,912	32,451.1	139	23	399	229,834
R2_2_1	34,625.1	1743	71	234	1546	34,625.2	1748	69	212	1441
R2_2_2	30,006.1	6068	328	1575	11,681	30,006.1	4964	288	1244	10,589
R2_2_5	30,290.3	3124	197	741	4739	30,290.3	2755	174	658	4709
R2_2_6	26,455.5	9531	377	2641	48,567	26,456.2	8379	397	2141	33,639
R2_2_9	28,206.0	4379	203	1167	14,379	28,206.4	4003	214	1009	11,066
RC1_2_1	34,595.9	60	18	133	44,486	34,595.9	53	17	131	46,746
RC1_2_2	31,774.7	135	43	353	109,271	31,774.7	123	38	300	110,289
RC1_2_6	32,554.4	171	19	703	432,336	32,554.4	131	19	645	411,015
RC1_2_8	30,216.1	540	36	1576	1,173,078	30,216.1	359	31	1439	1,207,744
RC2_2_1	27,891.0	2024	123	720	5359	27,891.3	1859	131	643	4500
RC2_2_2	24,588.6	11,026	595	3696	33,713	24,589.8	9569	568	3218	28,827
RC2_2_5	24,788.5	7814	445	4284	108,258	24,788.6	6100	444	3622	76,239
RC2_2_6	24,724.4	5913	396	3411	52,168	24,724.4	4782	372	2965	42,500
#Best	23/31	1/31	13/31	3/31	8/31	30/31	30/31	23/31	30/31	24/31

Table B38

Detailed results for comparing PartialDom versus SetBased settings using only-cycle DSSR after adding nonrobust cuts

Instance	PartialDom					SetBased				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C1_2_3	26,751.8	173,438	1861	4059	264,466	26,752.6	173,517	1884	4137	262,970
C1_2_4	26,256.0	1620	347	3768	346,619	26,256.0	1152	358	3376	288,520
C1_2_9	26,396.0	275	62	694	103,678	26,396.0	221	56	638	109,625
C1_210	26,247.0	1063	203	3037	308,752	26,247.0	843	214	2629	264,985
C2_2_1	19,221.0	1116	587	587	954	19,221.0	914	479	479	1033
C2_2_2	18,514.0	16,282	1678	3034	8686	18,514.0	11,811	1558	2420	7697
C2_2_5	18,696.0	2444	675	1239	2999	18,696.0	2292	615	1073	3539
C2_2_6	18,448.0	5433	661	2097	9613	18,448.0	5180	737	2008	8921
C2_2_7	18,422.0	8425	806	2854	15,121	18,423.3	5572	572	2114	14,329
C2_2_8	18,137.0	9548	813	3447	23,318	18,137.0	7534	706	2764	21,520
C2_2_9	18,150.0	22,898	1146	4461	47,370	18,150.0	17,741	1010	3810	45,067
R1_2_4	30,379.9	178,129	844	2444	468,566	30,381.2	177,114	903	2504	478,440
R1_2_5	40,471.0	1620	573	585	10,317	40,471.6	1782	628	640	10,706
R1_2_6	35,485.2	28,930	1193	1686	81,141	35,485.7	22,703	1220	1711	79,309
R1_2_7	31,404.4	51,878	1111	2243	215,687	31,404.4	38,044	1036	2189	212,635
R1_2_8	29,344.9	175,303	1038	2954	480,037	29,346.2	173,559	1165	3086	502,407
R1_2_9	37,284.2	3711	670	875	27,485	37,283.6	2992	665	864	26,756
R1_210	32,811.9	17,630	1094	1996	93,539	32,812.1	16,045	1073	1951	96,043
R2_2_1	34,680.0	1871	214	387	864	34,680.0	1880	272	430	743
R2_2_2	30,082.0	7216	710	1981	6415	30,082.0	5896	599	1580	6130
R2_2_5	30,611.0	8223	1955	2677	4004	30,611.0	6898	1640	2270	4224
R2_2_6	26,750.5	23,238	2556	5012	14,381	26,750.5	22,113	2473	4426	12,679
R2_2_9	28,433.0	7057	1144	2220	5899	28,433.0	6841	1269	2166	5016
RC1_2_1	35,017.2	4623	868	1007	20,579	35,016.8	3331	770	909	19,631
RC1_2_2	32,087.9	94,153	1111	2012	142,075	32,087.7	90,927	1139	1980	140,173
RC1_2_6	32,960.3	36,527	1245	2714	141,759	32,960.2	32,890	1250	2715	151,225
RC1_2_8	30,508.7	179,057	702	4660	1,121,911	30,508.7	176,286	686	4591	1,160,747
RC2_2_1	27,974.0	2766	1083	1813	1617	27,974.0	2483	913	1549	1690
RC2_2_2	24,780.0	65,777	7220	10,924	9515	24,780.0	63,458	6212	9261	9275
RC2_2_5	24,914.0	14,037	2384	6699	36,953	24,914.0	13,672	3099	6510	25,079
RC2_2_6	24,951.0	11,845	2346	5953	19,456	24,951.0	12,105	2737	5862	15,752
#Best	24/31	4/31	14/31	7/31	11/31	27/31	27/31	17/31	24/31	20/31

Table B39

Detailed results for comparing PartialDom versus Pairwise settings using only-cycle DSSR before adding nonrobust cuts

Instance	PartialDom					Pairwise				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C1_2_3	26,597.4	394	124	1385	173,802	26,597.4	395	126	1284	149,825
C1_2_4	26,197.7	1110	196	2999	422,665	26,197.7	889	199	2534	318,126
C1_2_9	26,396.0	274	62	694	103,678	26,396.0	230	56	629	101,539
C1_210	26,197.3	795	83	2249	524,649	26,197.3	701	87	1956	409,693
C2_2_1	19,150.3	371	7	7	435	19,150.3	346	7	7	435
C2_2_2	18,394.0	11,347	773	2043	11,156	18,396.0	5180	593	1187	6541
C2_2_5	18,607.1	1542	170	656	4847	18,607.1	1514	168	594	4328
C2_2_6	18,406.7	4451	327	1673	14,377	18,406.7	4340	363	1504	11,640
C2_2_7	18,356.4	7352	480	2461	21,229	18,356.4	4483	285	1846	21,371
C2_2_8	18,082.9	7479	369	2749	35,935	18,082.4	6293	380	2164	24,979
C2_2_9	18,011.9	13,406	510	3142	52,033	18,011.9	9410	421	2482	40,634
R1_2_4	30,009.9	328	44	576	360,513	30,009.9	287	49	539	292,363
R1_2_5	40,068.1	44	11	23	10,796	40,068.1	42	11	23	10,792
R1_2_6	34,930.1	106	43	167	45,657	34,930.6	106	47	172	44,706
R1_2_7	30,999.1	298	54	441	182,425	31,000.0	242	46	406	188,565
R1_2_8	29,032.4	388	33	648	595,520	29,032.3	334	36	619	506,166
R1_2_9	36,787.8	66	16	123	54,489	36,787.8	64	16	125	56,068
R1_210	32,451.1	165	21	414	256,912	32,451.1	157	25	397	206,001
R2_2_1	34,625.1	1743	71	234	1546	34,625.2	1711	63	209	1530
R2_2_2	30,006.1	6068	328	1575	11,681	30,006.1	4585	276	1223	10,192
R2_2_5	30,290.3	3124	197	741	4739	30,290.3	2621	133	590	5236
R2_2_6	26,455.5	9531	377	2641	48,567	26,456.0	8471	384	2066	32,993
R2_2_9	28,206.0	4379	203	1167	14,379	28,206.0	4076	201	1015	11,496
RC1_2_1	34,595.9	60	18	133	44,486	34,595.9	56	18	133	44,460
RC1_2_2	31,774.7	135	43	353	109,271	31,774.7	132	41	325	103,418
RC1_2_6	32,554.4	171	19	703	432,336	32,554.4	149	20	636	371,121
RC1_2_8	30,216.1	540	36	1576	1,173,078	30,216.1	430	33	1435	1,091,526
RC2_2_1	27,891.0	2024	123	720	5359	27,891.3	1972	124	601	4258
RC2_2_2	24,588.6	11,026	595	3696	33,713	24,590.6	9328	516	3210	29,730
RC2_2_5	24,788.5	7814	445	4284	108,258	24,788.6	6530	480	3813	74,813
RC2_2_6	24,724.4	5913	396	3411	52,168	24,724.4	4921	393	2956	38,082
#Best	24/31	1/31	17/31	5/31	5/31	30/31	30/31	18/31	29/31	27/31

Table B40

Detailed results for comparing PartialDom versus Pairwise settings using only-cycle DSSR after adding nonrobust cuts

Instance	PartialDom					Pairwise				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C1_2_3	26,751.8	173,438	1861	4059	264,466	26,752.2	176,153	1843	4017	270,184
C1_2_4	26,256.0	1620	347	3768	346,619	26,256.0	1294	343	3336	292,342
C1_2_9	26,396.0	275	62	694	103,678	26,396.0	231	56	629	101,539
C1_210	26,247.0	1063	203	3037	308,752	26,247.0	950	206	2708	266,016
C2_2_1	19,221.0	1116	587	587	954	19,221.0	1009	414	414	1405
C2_2_2	18,514.0	16,282	1678	3034	8686	18,514.0	13,283	1667	2415	7983
C2_2_5	18,696.0	2444	675	1239	2999	18,696.0	2231	572	1034	2913
C2_2_6	18,448.0	5433	661	2097	9613	18,448.0	5322	653	1897	8643
C2_2_7	18,422.0	8425	806	2854	15,121	18,422.0	6271	874	2625	10,538
C2_2_8	18,137.0	9548	813	3447	23,318	18,137.0	8065	832	2832	17,545
C2_2_9	18,150.0	22,898	1146	4461	47,370	18,150.0	18,612	1087	3794	40,620
R1_2_4	30,379.9	178,129	844	2444	468,566	30,381.8	174,763	903	2485	464,605
R1_2_5	40,471.0	1620	573	585	10,317	40,471.9	1981	694	706	10,679
R1_2_6	35,485.2	28,930	1193	1686	81,141	35,485.3	28,276	1317	1814	80,153
R1_2_7	31,404.4	51,878	1111	2243	215,687	31,404.4	45,569	1131	2263	206,701
R1_2_8	29,344.9	175,303	1038	2954	480,037	29,345.6	173,387	1106	3034	501,744
R1_2_9	37,284.2	3711	670	875	27,485	37,283.4	3049	659	872	27,008
R1_210	32,811.9	17,630	1094	1996	93,539	32,811.1	11,744	937	1766	88,250
R2_2_1	34,680.0	1871	214	387	864	34,680.0	1837	225	382	800
R2_2_2	30,082.0	7216	710	1981	6415	30,082.0	5421	580	1562	5808
R2_2_5	30,611.0	8223	1955	2677	4004	30,611.0	6254	1461	2089	3566
R2_2_6	26,750.5	23,238	2556	5012	14,381	26,750.5	21,719	2291	4223	12,972
R2_2_9	28,433.0	7057	1144	2220	5899	28,433.0	6903	1236	2141	5033
RC1_2_1	35,017.2	4623	868	1007	20,579	35,017.8	4825	938	1080	20,427
RC1_2_2	32,087.9	94,153	1111	2012	142,075	32,087.6	108,209	1140	1986	137,733
RC1_2_6	32,960.3	36,527	1245	2714	141,759	32,960.4	38,157	1269	2745	141,760
RC1_2_8	30,508.7	179,057	702	4660	1,121,911	30,509.0	175,969	697	4543	996,846
RC2_2_1	27,974.0	2766	1083	1813	1617	27,974.0	2735	1205	1875	1464
RC2_2_2	24,780.0	65,777	7220	10,924	9515	24,780.0	65,053	6954	10,101	8688
RC2_2_5	24,914.0	14,037	2384	6699	36,953	24,914.0	28,143	6011	10,046	20,619
RC2_2_6	24,951.0	11,845	2346	5953	19,456	24,951.0	8944	1944	5107	16,935
#Best	24/31	6/31	15/31	9/31	5/31	28/31	25/31	16/31	22/31	26/31

Table B41

Detailed results for comparing PartialDom versus SetPair settings using only-cycle DSSR before adding nonrobust cuts

Instance	PartialDom					SetPair				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C1_2_3	26,597.4	394	124	1385	173,802	26,597.4	360	133	1237	128,396
C1_2_4	26,197.7	1110	196	2999	422,665	26,197.7	791	200	2945	380,210
C1_2_9	26,396.0	274	62	694	103,678	26,396.0	223	58	663	102,973
C1_210	26,197.3	795	83	2249	524,649	26,197.3	588	85	1917	411,479
C2_2_1	19,150.3	371	7	7	435	19,150.3	347	7	7	435
C2_2_2	18,394.0	11,347	773	2043	11,156	18,396.0	8386	724	1664	9571
C2_2_5	18,607.1	1542	170	656	4847	18,607.1	1276	133	529	4859
C2_2_6	18,406.7	4451	327	1673	14,377	18,406.7	4084	295	1455	13,866
C2_2_7	18,356.4	7352	480	2461	21,229	18,370.4	5458	393	2034	18,636
C2_2_8	18,082.9	7479	369	2749	35,935	18,082.4	5788	380	2244	25,504
C2_2_9	18,011.9	13,406	510	3142	52,033	18,011.9	11,220	469	2672	43,541
R1_2_4	30,009.9	328	44	576	360,513	30,009.9	222	42	498	315,121
R1_2_5	40,068.1	44	11	23	10,796	40,068.1	41	10	22	11,161
R1_2_6	34,930.1	106	43	167	45,657	34,930.6	95	46	171	44,276
R1_2_7	30,999.1	298	54	441	182,425	30,998.9	209	46	415	194,376
R1_2_8	29,032.4	388	33	648	595,520	29,032.3	255	34	593	508,686
R1_2_9	36,787.8	66	16	123	54,489	36,787.8	59	16	119	52,895
R1_210	32,451.1	165	21	414	256,912	32,451.1	142	25	398	206,501
R2_2_1	34,625.1	1743	71	234	1546	34,625.2	1717	65	211	1505
R2_2_2	30,006.1	6068	328	1575	11,681	30,006.1	4871	282	1214	10,282
R2_2_5	30,290.3	3124	197	741	4739	30,290.3	2783	173	623	4274
R2_2_6	26,455.5	9531	377	2641	48,567	26,455.6	8248	365	2054	37,270
R2_2_9	28,206.0	4379	203	1167	14,379	28,206.2	4049	229	1012	10,103
RC1_2_1	34,595.9	60	18	133	44,486	34,595.9	54	18	133	44,324
RC1_2_2	31,774.7	135	43	353	109,271	31,774.7	119	43	315	99,413
RC1_2_6	32,554.4	171	19	703	432,336	32,554.6	136	22	651	343,279
RC1_2_8	30,216.1	540	36	1576	1,173,078	30,216.1	359	34	1390	998,532
RC2_2_1	27,891.0	2024	123	720	5359	27,891.3	1890	118	618	4553
RC2_2_2	24,588.6	11,026	595	3696	33,713	24,590.6	9181	511	3102	29,002
RC2_2_5	24,788.5	7814	445	4284	108,258	24,788.5	6277	448	3744	76,733
RC2_2_6	24,724.4	5913	396	3411	52,168	24,724.4	4856	367	2929	41,101
#Best	23/31	0/31	14/31	3/31	4/31	29/31	31/31	21/31	30/31	28/31

Table B42

Detailed results for comparing PartialDom versus SetPair settings using only-cycle DSSR after adding nonrobust cuts

Instance	PartialDom					SetPair				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C1_2_3	26,751.8	173,438	1861	4059	264,466	26,751.3	172,971	1841	3991	242,010
C1_2_4	26,256.0	1620	347	3768	346,619	26,256.0	1287	359	3703	312,205
C1_2_9	26,396.0	275	62	694	103,678	26,396.0	224	58	663	102,973
C1_210	26,247.0	1063	203	3037	308,752	26,247.0	818	213	2686	251,061
C2_2_1	19,221.0	1116	587	587	954	19,221.0	1086	566	566	1147
C2_2_2	18,514.0	16,282	1678	3034	8686	18,514.0	14,946	1838	2841	7905
C2_2_5	18,696.0	2444	675	1239	2999	18,696.0	2123	555	984	3258
C2_2_6	18,448.0	5433	661	2097	9613	18,448.0	5215	635	1953	9316
C2_2_7	18,422.0	8425	806	2854	15,121	18,422.0	7570	912	2930	13,018
C2_2_8	18,137.0	9548	813	3447	23,318	18,137.0	7544	802	2879	17,826
C2_2_9	18,150.0	22,898	1146	4461	47,370	18,150.0	22,022	1389	4258	34,605
R1_2_4	30,379.9	178,129	844	2444	468,566	30,378.9	174,117	782	2329	451,245
R1_2_5	40,471.0	1620	573	585	10,317	40,471.0	2040	684	697	10,716
R1_2_6	35,485.2	28,930	1193	1686	81,141	35,485.3	30,800	1298	1802	79,940
R1_2_7	31,404.4	51,878	1111	2243	215,687	31,404.6	50,470	1015	2161	219,204
R1_2_8	29,344.9	175,303	1038	2954	480,037	29,345.5	175,173	1073	2980	485,150
R1_2_9	37,284.2	3711	670	875	27,485	37,284.1	3390	711	914	26,937
R1_210	32,811.9	17,630	1094	1996	93,539	32,811.6	14,648	1001	1870	91,813
R2_2_1	34,680.0	1871	214	387	864	34,680.0	1804	189	338	844
R2_2_2	30,082.0	7216	710	1981	6415	30,082.0	5806	592	1540	5887
R2_2_5	30,611.0	8223	1955	2677	4004	30,611.0	6547	1633	2205	3392
R2_2_6	26,750.5	23,238	2556	5012	14,381	26,750.5	21,322	2457	4378	12,350
R2_2_9	28,433.0	7057	1144	2220	5899	28,433.0	6925	1279	2178	4904
RC1_2_1	35,017.2	4623	868	1007	20,579	35,017.3	4427	831	969	20,095
RC1_2_2	32,087.9	94,153	1111	2012	142,075	32,088.0	118,036	1186	2005	135,460
RC1_2_6	32,960.3	36,527	1245	2714	141,759	32,960.6	46,136	1289	2757	143,502
RC1_2_8	30,508.7	179,057	702	4660	1,121,911	30,505.7	179,781	623	4417	1,046,100
RC2_2_1	27,974.0	2766	1083	1813	1617	27,974.0	2542	976	1631	1501
RC2_2_2	24,780.0	65,777	7220	10,924	9515	24,780.0	41,251	4738	7679	8973
RC2_2_5	24,914.0	14,037	2384	6699	36,953	24,914.0	19,812	4388	8265	22,480
RC2_2_6	24,951.0	11,845	2346	5953	19,456	24,951.0	10,238	2196	5446	16,623
#Best	26/31	6/31	13/31	7/31	6/31	26/31	25/31	18/31	24/31	25/31

Table B43

Detailed results for comparing PartialDom versus SetBased settings using all-arcs DSSR before adding nonrobust cuts

Instance	PartialDom					SetBased				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C1_2_3	26,597.4	314	112	509	74,968	26,597.4	215	106	452	65,229
C1_2_4	26,197.7	553	166	883	157,596	26,197.7	431	182	906	134,003
C1_2_9	26,396.0	229	53	540	94,199	26,396.0	207	57	522	88,052
C1_210	26,197.3	620	71	1301	357,164	26,197.4	452	78	1122	280,835
C2_2_1	19,150.3	360	7	7	435	19,150.3	353	7	7	435
C2_2_2	18,395.5	5091	568	799	5914	18,396.0	3346	432	619	5484
C2_2_5	18,607.1	1367	149	534	4416	18,607.1	1277	137	493	4644
C2_2_6	18,406.7	3103	250	1032	11,426	18,406.7	3476	328	993	9659
C2_2_7	18,356.4	3639	297	1169	15,727	18,356.4	3598	324	1090	14,090
C2_2_8	18,082.4	4018	285	1251	20,206	18,082.4	3516	273	1152	20,758
C2_2_9	18,011.9	5596	297	1342	36,377	18,011.9	5194	318	1207	32,809
R1_2_4	30,009.9	231	43	315	193,243	30,009.9	177	46	296	170,241
R1_2_5	40,068.1	44	11	23	10,796	40,068.1	42	11	23	10,864
R1_2_6	34,930.1	107	46	147	38,119	34,930.6	94	47	151	39,320
R1_2_7	30,999.5	217	45	282	126,463	31,000.5	194	52	282	115,984
R1_2_8	29,032.4	237	31	358	342,000	29,032.2	200	33	360	330,244
R1_2_9	36,787.8	65	16	113	49,843	36,787.8	58	17	110	45,953
R1_210	32,451.2	144	22	301	174,968	32,451.1	120	23	302	170,625
R2_2_1	34,625.1	1706	51	193	1779	34,625.1	1625	58	185	1499
R2_2_2	30,006.1	4414	252	921	9803	30,006.1	4085	248	825	8971
R2_2_5	30,290.3	2869	189	553	3649	30,290.3	2473	140	460	4139
R2_2_6	26,455.5	5711	267	1144	30,557	26,455.5	5669	288	1076	25,726
R2_2_9	28,206.0	3543	177	684	9481	28,206.0	3371	175	617	8318
RC1_2_1	34,595.9	59	18	133	44,486	34,595.9	54	17	131	46,746
RC1_2_2	31,774.7	124	46	271	79,659	31,774.7	117	37	266	101,657
RC1_2_6	32,554.4	128	23	462	234,497	32,554.4	101	18	424	285,058
RC1_2_8	30,216.1	252	30	577	527,790	30,216.1	196	29	572	542,817
RC2_2_1	27,891.0	1976	144	543	3499	27,891.0	1867	129	475	3534
RC2_2_2	24,588.6	6397	472	1723	20,948	24,589.4	5806	369	1524	23,857
RC2_2_5	24,788.5	3938	356	1487	43,291	24,788.5	3245	264	1305	47,348
RC2_2_6	24,724.4	3353	228	1302	32,504	24,724.4	3249	278	1277	25,921
#Best	27/31	1/31	17/31	7/31	13/31	30/31	30/31	16/31	27/31	19/31

Table B44

Detailed results for comparing PartialDom versus SetBased settings using all-arcs DSSR after adding nonrobust cuts

Instance	PartialDom					SetBased				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C1_2_3	26,752.3	173,041	2005	2678	216,061	26,753.2	173,111	1985	2635	228,628
C1_2_4	26,256.0	919	333	1206	140,079	26,256.0	709	350	1205	120,837
C1_2_9	26,396.0	230	53	540	94,199	26,396.0	208	57	522	88,052
C1_210	26,247.0	811	199	1670	177,454	26,247.0	625	198	1505	168,608
C2_2_1	19,221.0	1101	587	587	954	19,221.0	935	479	479	1033
C2_2_2	18,514.0	11,757	1681	1943	6661	18,514.0	8511	1296	1505	7360
C2_2_5	18,696.0	2044	514	932	3141	18,696.0	2031	556	947	3272
C2_2_6	18,448.0	4586	787	1617	6671	18,448.0	4766	795	1498	7195
C2_2_7	18,422.0	4770	673	1574	9874	18,422.0	9523	1994	2891	8236
C2_2_8	18,137.0	5268	644	1661	13,761	18,137.0	5002	705	1654	13,938
C2_2_9	18,150.0	12,732	944	2167	31,434	18,150.0	10,928	934	1969	29,016
R1_2_4	30,382.2	174,746	908	1519	368,626	30,382.5	174,279	932	1501	375,703
R1_2_5	40,471.0	1620	573	585	10,317	40,471.6	1730	628	640	10,706
R1_2_6	35,485.0	23,514	1155	1441	71,429	35,485.8	26,704	1306	1574	75,130
R1_2_7	31,404.6	42,064	1053	1548	174,493	31,404.3	30,365	1002	1493	173,444
R1_2_8	29,346.5	173,694	1193	1887	416,710	29,346.8	174,211	1311	1988	439,039
R1_2_9	37,283.9	4155	733	903	26,232	37,283.9	3026	643	817	27,223
R1_210	32,811.8	16,242	1092	1612	80,352	32,811.5	13,001	1030	1551	83,754
R2_2_1	34,680.0	1811	198	349	851	34,680.0	1811	294	430	725
R2_2_2	30,082.0	5212	541	1221	5727	30,082.0	5343	706	1294	4597
R2_2_5	30,611.0	6790	1603	2023	3577	30,611.0	6727	1709	2106	3886
R2_2_6	26,750.5	19,690	2610	3540	11,468	26,750.5	18,306	2363	3232	10,879
R2_2_9	28,433.0	6130	1094	1646	4768	28,433.0	5944	1113	1605	4762
RC1_2_1	35,017.2	4698	868	1007	20,579	35,016.8	3385	770	909	19,631
RC1_2_2	32,087.5	82,382	1080	1553	123,616	32,087.8	78,701	1132	1628	120,445
RC1_2_6	32,960.1	32,145	1180	2070	124,493	32,960.2	27,114	1193	2044	126,487
RC1_2_8	30,514.2	174,813	878	2067	576,518	30,514.7	172,960	897	2072	598,704
RC2_2_1	27,974.0	2610	962	1449	1429	27,974.0	2507	940	1382	1619
RC2_2_2	24,780.0	61,601	7158	8589	9289	24,780.0	70,794	7815	9078	9037
RC2_2_5	24,914.0	15,738	3752	4964	20,378	24,914.0	10,548	2593	3713	23,611
RC2_2_6	24,951.0	9846	2330	3484	13,243	24,951.0	7557	1932	2976	12,034
#Best	23/31	8/31	18/31	11/31	17/31	28/31	23/31	13/31	20/31	14/31

Table B45

Detailed results for comparing PartialDom versus Pairwise settings using all-arcs DSSR before adding nonrobust cuts

Instance	PartialDom					Pairwise				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C1_2_3	26,597.4	314	112	509	74,968	26,597.4	259	112	451	61,612
C1_2_4	26,197.7	553	166	883	157,596	26,197.7	492	179	951	157,224
C1_2_9	26,396.0	229	53	540	94,199	26,396.0	223	55	516	85,621
C1_210	26,197.3	620	71	1301	357,164	26,197.3	454	70	1156	302,044
C2_2_1	19,150.3	360	7	7	435	19,150.3	346	7	7	435
C2_2_2	18,395.5	5091	568	799	5914	18,394.0	3258	416	612	5208
C2_2_5	18,607.1	1367	149	534	4416	18,607.1	1281	128	490	4733
C2_2_6	18,406.7	3103	250	1032	11,426	18,406.7	2809	263	935	9440
C2_2_7	18,356.4	3639	297	1169	15,727	18,356.4	2985	265	998	13,361
C2_2_8	18,082.4	4018	285	1251	20,206	18,082.4	3533	297	1146	16,726
C2_2_9	18,011.9	5596	297	1342	36,377	18,011.8	5259	311	1234	30,655
R1_2_4	30,009.9	231	43	315	193,243	30,009.9	229	44	308	189,214
R1_2_5	40,068.1	44	11	23	10,796	40,068.1	42	11	23	10,792
R1_2_6	34,930.1	107	46	147	38,119	34,930.5	97	46	146	36,742
R1_2_7	30,999.5	217	45	282	126,463	31,000.5	199	49	271	117,794
R1_2_8	29,032.4	237	31	358	342,000	29,032.4	226	35	343	290,122
R1_2_9	36,787.8	65	16	113	49,843	36,787.8	60	16	109	47,384
R1_210	32,451.2	144	22	301	174,968	32,451.1	132	22	302	174,792
R2_2_1	34,625.1	1706	51	193	1779	34,625.1	1719	76	210	1263
R2_2_2	30,006.1	4414	252	921	9803	30,006.1	3917	242	824	8778
R2_2_5	30,290.3	2869	189	553	3649	30,290.3	2665	158	490	3761
R2_2_6	26,455.5	5711	267	1144	30,557	26,455.5	5601	280	1099	26,008
R2_2_9	28,206.0	3543	177	684	9481	28,206.0	3272	171	622	8243
RC1_2_1	34,595.9	59	18	133	44,486	34,595.9	56	18	133	44,460
RC1_2_2	31,774.7	124	46	271	79,659	31,774.7	117	39	257	90,986
RC1_2_6	32,554.4	128	23	462	234,497	32,554.4	109	19	423	253,881
RC1_2_8	30,216.1	252	30	577	527,790	30,216.1	226	28	571	528,193
RC2_2_1	27,891.0	1976	144	543	3499	27,891.3	1841	125	477	3532
RC2_2_2	24,588.6	6397	472	1723	20,948	24,588.8	6091	421	1561	19,566
RC2_2_5	24,788.5	3938	356	1487	43,291	24,788.5	3408	286	1343	43,608
RC2_2_6	24,724.4	3353	228	1302	32,504	24,724.4	3035	269	1196	23,394
#Best	27/31	1/31	18/31	6/31	8/31	29/31	30/31	20/31	28/31	24/31

Table B46

Detailed results for comparing PartialDom versus Pairwise settings using all-arcs DSSR after adding nonrobust cuts

Instance	PartialDom					Pairwise				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C1_2_3	26,752.3	173,041	2005	2678	216,061	26,752.8	174,314	1951	2601	223,555
C1_2_4	26,256.0	919	333	1206	140,079	26,256.0	750	334	1209	128,992
C1_2_9	26,396.0	230	53	540	94,199	26,396.0	224	55	516	85,621
C1_210	26,247.0	811	199	1670	177,454	26,247.0	623	205	1547	151,855
C2_2_1	19,221.0	1101	587	587	954	19,221.0	1011	414	414	1405
C2_2_2	18,514.0	11,757	1681	1943	6661	18,514.0	12,010	1754	1965	7878
C2_2_5	18,696.0	2044	514	932	3141	18,696.0	2108	602	995	2823
C2_2_6	18,448.0	4586	787	1617	6671	18,448.0	3823	630	1346	6806
C2_2_7	18,422.0	4770	673	1574	9874	18,422.0	3959	614	1390	8432
C2_2_8	18,137.0	5268	644	1661	13,761	18,137.0	4882	712	1615	11,765
C2_2_9	18,150.0	12,732	944	2167	31,434	18,150.0	11,728	1066	2166	25,098
R1_2_4	30,382.2	174,746	908	1519	368,626	30,382.1	177,048	916	1479	372,593
R1_2_5	40,471.0	1620	573	585	10,317	40,471.9	2039	694	706	10,679
R1_2_6	35,485.0	23,514	1155	1441	71,429	35,485.6	22,911	1211	1480	72,105
R1_2_7	31,404.6	42,064	1053	1548	174,493	31,404.7	31,416	1033	1507	172,905
R1_2_8	29,346.5	173,694	1193	1887	416,710	29,346.8	174,130	1286	1969	415,519
R1_2_9	37,283.9	4155	733	903	26,232	37,284.0	3451	701	866	26,581
R1_210	32,811.8	16,242	1092	1612	80,352	32,811.5	12,644	1009	1530	79,696
R2_2_1	34,680.0	1811	198	349	851	34,680.0	1915	353	494	678
R2_2_2	30,082.0	5212	541	1221	5727	30,082.0	4902	596	1190	4794
R2_2_5	30,611.0	6790	1603	2023	3577	30,611.0	5967	1405	1778	3636
R2_2_6	26,750.5	19,690	2610	3540	11,468	26,750.5	18,934	2493	3392	10,006
R2_2_9	28,433.0	6130	1094	1646	4768	28,433.0	5714	1089	1566	4442
RC1_2_1	35,017.2	4698	868	1007	20,579	35,017.8	4832	938	1080	20,427
RC1_2_2	32,087.5	82,382	1080	1553	123,616	32,087.6	97,015	1156	1627	124,469
RC1_2_6	32,960.1	32,145	1180	2070	124,493	32,960.2	33,762	1229	2092	122,444
RC1_2_8	30,514.2	174,813	878	2067	576,518	30,514.1	173,242	874	2085	547,801
RC2_2_1	27,974.0	2610	962	1449	1429	27,974.0	2468	1022	1442	1380
RC2_2_2	24,780.0	61,601	7158	8589	9289	24,780.0	51,013	6063	7308	7725
RC2_2_5	24,914.0	15,738	3752	4964	20,378	24,914.0	11,536	2890	3996	18,575
RC2_2_6	24,951.0	9846	2330	3484	13,243	24,951.0	12,398	3136	4117	10,638
#Best	24/31	11/31	18/31	12/31	10/31	29/31	20/31	13/31	19/31	21/31

Table B47

Detailed results for comparing PartialDom versus SetPair settings using all-arcs DSSR before adding nonrobust cuts

Instance	PartialDom					SetPair				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C1_2_3	26,597.4	314	112	509	74,968	26,597.4	184	104	468	71,359
C1_2_4	26,197.7	553	166	883	157,596	26,197.7	435	171	816	137,328
C1_2_9	26,396.0	229	53	540	94,199	26,396.0	215	52	523	91,451
C1_210	26,197.3	620	71	1301	357,164	26,197.3	476	69	1053	281,031
C2_2_1	19,150.3	360	7	7	435	19,150.3	348	7	7	435
C2_2_2	18,395.5	5091	568	799	5914	18,396.0	3495	440	695	6377
C2_2_5	18,607.1	1367	149	534	4416	18,607.1	1234	130	498	4742
C2_2_6	18,406.7	3103	250	1032	11,426	18,406.7	2781	199	931	12,609
C2_2_7	18,356.4	3639	297	1169	15,727	18,356.4	3097	234	979	15,181
C2_2_8	18,082.4	4018	285	1251	20,206	18,082.4	3449	264	1138	19,031
C2_2_9	18,011.9	5596	297	1342	36,377	18,011.8	5014	289	1212	32,366
R1_2_4	30,009.9	231	43	315	193,243	30,009.9	185	44	313	188,885
R1_2_5	40,068.1	44	11	23	10,796	40,068.1	41	10	22	11,161
R1_2_6	34,930.1	107	46	147	38,119	34,930.6	93	46	159	39,948
R1_2_7	30,999.5	217	45	282	126,463	31,000.2	177	44	286	138,251
R1_2_8	29,032.4	237	31	358	342,000	29,032.3	183	35	345	275,634
R1_2_9	36,787.8	65	16	113	49,843	36,787.8	58	16	109	47,399
R1_210	32,451.2	144	22	301	174,968	32,451.1	116	19	284	189,812
R2_2_1	34,625.1	1706	51	193	1779	34,625.1	1666	70	197	1290
R2_2_2	30,006.1	4414	252	921	9803	30,006.1	3895	222	846	9565
R2_2_5	30,290.3	2869	189	553	3649	30,290.3	2536	182	514	3457
R2_2_6	26,455.5	5711	267	1144	30,557	26,455.5	5477	273	1092	27,355
R2_2_9	28,206.0	3543	177	684	9481	28,206.0	3416	176	633	8129
RC1_2_1	34,595.9	59	18	133	44,486	34,595.9	54	18	133	44,324
RC1_2_2	31,774.7	124	46	271	79,659	31,774.7	116	43	262	84,050
RC1_2_6	32,554.4	128	23	462	234,497	32,554.4	100	20	423	245,260
RC1_2_8	30,216.1	252	30	577	527,790	30,216.1	196	31	572	490,921
RC2_2_1	27,891.0	1976	144	543	3499	27,892.5	1818	131	487	3448
RC2_2_2	24,588.6	6397	472	1723	20,948	24,588.8	5330	379	1492	20,620
RC2_2_5	24,788.5	3938	356	1487	43,291	24,788.5	3324	330	1394	37,539
RC2_2_6	24,724.4	3353	228	1302	32,504	24,724.4	2898	207	1163	29,369
#Best	26/31	0/31	10/31	5/31	10/31	29/31	31/31	25/31	28/31	22/31

Table B48

Detailed results for comparing PartialDom versus SetPair settings using all-arcs DSSR after adding nonrobust cuts

Instance	PartialDom					SetPair				
	lb	T(s)	#iters	#DSSR	#labels	lb	T(s)	#iters	#DSSR	#labels
C1_2_3	26,752.3	173,041	2005	2678	216,061	26,751.9	174,346	1821	2460	211,811
C1_2_4	26,256.0	919	333	1206	140,079	26,256.0	800	343	1153	129,352
C1_2_9	26,396.0	230	53	540	94,199	26,396.0	216	52	523	91,451
C1_210	26,247.0	811	199	1670	177,454	26,247.0	628	192	1483	157,517
C2_2_1	19,221.0	1101	587	587	954	19,221.0	1091	566	566	1147
C2_2_2	18,514.0	11,757	1681	1943	6661	18,514.0	10,819	1366	1633	7385
C2_2_5	18,696.0	2044	514	932	3141	18,696.0	2211	688	1104	2863
C2_2_6	18,448.0	4586	787	1617	6671	18,448.0	3617	498	1252	7873
C2_2_7	18,422.0	4770	673	1574	9874	18,422.0	4147	620	1381	8749
C2_2_8	18,137.0	5268	644	1661	13,761	18,137.0	4637	593	1523	13,588
C2_2_9	18,150.0	12,732	944	2167	31,434	18,150.0	10,488	899	1967	26,966
R1_2_4	30,382.2	174,746	908	1519	368,626	30,380.0	179,044	842	1441	366,007
R1_2_5	40,471.0	1620	573	585	10,317	40,471.0	1986	684	697	10,716
R1_2_6	35,485.0	23,514	1155	1441	71,429	35,485.3	27,965	1205	1505	74,688
R1_2_7	31,404.6	42,064	1053	1548	174,493	31,404.6	33,773	968	1461	168,974
R1_2_8	29,346.5	173,694	1193	1887	416,710	29,346.1	174,610	1147	1846	416,161
R1_2_9	37,283.9	4155	733	903	26,232	37,283.2	3421	698	863	26,282
R1_210	32,811.8	16,242	1092	1612	80,352	32,812.3	17,806	1146	1665	82,338
R2_2_1	34,680.0	1811	198	349	851	34,680.0	1783	218	354	756
R2_2_2	30,082.0	5212	541	1221	5727	30,082.0	4646	495	1132	5412
R2_2_5	30,611.0	6790	1603	2023	3577	30,611.0	6300	1652	2040	3690
R2_2_6	26,750.5	19,690	2610	3540	11,468	26,750.5	17,219	2235	3139	10,640
R2_2_9	28,433.0	6130	1094	1646	4768	28,433.0	6143	1236	1726	4360
RC1_2_1	35,017.2	4698	868	1007	20,579	35,017.3	4376	831	969	20,095
RC1_2_2	32,087.5	82,382	1080	1553	123,616	32,087.6	110,251	1132	1618	118,878
RC1_2_6	32,960.1	32,145	1180	2070	124,493	32,960.3	37,278	1203	2053	125,815
RC1_2_8	30,514.2	174,813	878	2067	576,518	30,512.5	173,314	830	1976	540,561
RC2_2_1	27,974.0	2610	962	1449	1429	27,974.0	2314	839	1279	1464
RC2_2_2	24,780.0	61,601	7158	8589	9289	24,780.0	43,946	5701	6928	8103
RC2_2_5	24,914.0	15,738	3752	4964	20,378	24,914.0	8939	2407	3533	19,953
RC2_2_6	24,951.0	9846	2330	3484	13,243	24,951.0	7700	2189	3232	11,829
#Best	27/31	10/31	10/31	8/31	10/31	26/31	21/31	21/31	23/31	21/31