# Decision Making Under Uncertainty

# -An investigation from economic and psychological perspective

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# Decision Making Under Uncertainty -An investigation from economic and psychological perspective

Beslissen bij onzekerheid

– Een onderzoek vanuit een economisch en een psychologisch perspectief

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# Contents

# Chapter 1 | Introduction

In ancient times, when a hungry man saw a beast or animal, he needed to decide whether he should fight against it to get the food. The potential gain is a bunch of meat and the potential loss is his life. Based on his past experience, the man starts to estimate the chance of each result and makes the final decision. Life or death, he has to make a choice under uncertainty. Today, when a person, say a man, enters a stock market, he needs to decide which stock to buy. The estimated chances of potential gains or losses are based on his understanding of the market. Like his ancestor in the forest, he needs to make a decision where each result has a chance to happen. During the history of mankind, decision making under uncertainty has been an eternal topic, from surviving to living a better life.

The first formal attempt to study this problem dates back into the 17$^{th}$ century when some great mathematicians Blaise Pascal, Pierre de Fermat, and Johan de Wit developed a theory of rational decision making known as Expected Value Theory (EVT). According to EVT, an act is valued according to the sum of its probability-weighted outcomes and a rational person should choose the act with the highest expected value in an uncertain environment. However, this theory was challenged by Bernoulli in the 18$^{th}$ century, who argued that utility increments should decrease with the wealth of the person, violating the constant utility increments implied by expected value. Bernoulli further developed Expected Utility Theory (EUT), where the probability-weighted average utility rather than probability-weighted outcome is used to evaluate an act. Kenyes (1921) and Knight (1921) first made a clear distinction between risk (known probability) and ambiguity (unknown probability), which shaped the development of decision theory afterwards. In 1961, Ellsburg proposed a classical paradox to illustrate the difference of decision making between risk and ambiguity. People tend to bet on events with known probability (risk) than on events with unknown probability (ambiguity). The study of decision making under ambiguity further developed afterwards. Among them are maximin expected utility (Gilboa and Schmeidler 1989), the α-maxmin model (Ghiradato, Maccheroni &

Marinacci 2004), Choquet expected utility (Schmeidler 1989) and the source method (Abdellaoui et al 2011).

There has so far been no perfect model to explain or predict the behavior of human beings. One of the difficulties is that the cognitive process during decision under uncertainty is still a blackbox to be explored. Such a process is affected by emotions, cognitive abilities, bounded rationalities, environmental constraints and other psycholotical factors. Therefore, this thesis investigates decision under uncertainty from both economic and psychological perspectives.

Chapter 2 discusses multiplier preferences, a model that describes behavior under ambiguity, where people have subjective beliefs but do not know exact probabilities. The subjective expected utility model (Savage 1954) prescribes that ambiguity should not affect a person's preferences (only subjective beliefs matter), but numerous studies (most notably Ellsberg 1961) have found that people do behave differently when ambiguity is involved. Multiplier preferences are widely used in macroeconomic models, but they have not been applied in microeconomic settings, largely because they do not permit behavior in which people are ambiguity seeking. We give a preference foundation for an extension of this model, such that it allows for ambiguity seeking as well as ambiguity aversion. We then present a simple method to measure extended multiplier preferences, which is easy to apply and measures multiplier preferences at the individual subject level. Our method is illustrated in two large representative samples from the Dutch and the US population involving over 5,000 subjects in total. Most subjects were moderately ambiguity averse, but between 23% (Dutch sample) and 36% (US sample) were ambiguity seeking, which illustrates the desirability of our extension of multiplier preferences.

Chapter 3 examines decisions under ambiguity when those decisions are taken under time pressure (TP). In behavioral economnics, it has long been understood that human memory and mental capacities are limited and often lead to suboptimal decisions. This suboptimality is especially pronounced when decisions have to be made under TP. Therefore, TP has received special attention throught the history of decision theory. Many papers have investigated the effect of TP on decision under risk, but this effect has not been studied under ambiguity. In a lab experiment with real incentives, subjects are asked to choose between risky and ambiguous bets. The ambiguous bets are based on the movement of the Amsterdam Stock Market Index (AEX), and therefore the probabilities associated with the outcomes were unknown. In the risky bets, however, outcomes are presented with their respective objective probabilities. In the treatment group, subjects have to state their preferences within a time limit. For the analysis of ambiguity, a new simplified method is introduced which relies on a tool called matching probabilities to measure ambiguity behavior. Ambiguity behavior can be dissected into two components: ambiguity aversion and ambiguity-generated insensitivity (a-insensitivity). Ambiguity aversion is an affective component that refers to how much less people like ambiguity as compared to risk, whereas a-insensitivity is a cognitive component indicating how much less people understand ambiguity compared to risk. The results of this experiment demonstrate that making choices under time pressure does not change how people react to ambiguity affectively; the ambiguity aversion factor remains unaffected. However, the a-insensitivity component is negatively impacted by time pressure. Therefore, the results agree with past research showing that cognitive faculties are compromised under time pressure.

Chapter 4 discusses how people update their beliefs under ambiguity using three different approaches. One is the traditional Bayesian updating, where only ambiguity neutral behavior is accommodated. The other two approaches are non-Bayesian, introduced by *Gilboa and Schmeidler* (1993) (*GS*) as well as *Dempster* (1967) and *Shafer* (1976) (*DS*) respectively, where both ambiguity averse and ambiguity seeking behavior is accommodated. Under the framework of decision theory, this paper compares Bayesian and non-Bayesian updating and their numerical implications. Ambiguity attitudes affect not only static decisions, but also the way in which new information is incorporated. For an ambiguity averse (seeking) decision maker, GS updating leads to more pessimistic (optimistic) behavior than DS updating, and favorable or unfavorable information has bigger (smaller) impact on GS updating than on DS updating.

Finally, Chapter 5 deals with the problem of intertemporal decisions, i.e. how people evaluate future outcomes. It introduces a new method to measure the temporal discounting of money. Unlike preceding methods, this method requires neither knowledge nor measurement of utility. It is easier to implement, clearer to subjects, and requires fewer measurements than existing methods. Because the method directly measures discounting, and utility plays no role, it is called the direct method (DM). The basic idea of the DM is as follows. Assume that a decision maker is indifferent between: (a) an extra payment of $10 per week during weeks 1-30; and (b) the same extra payment during weeks 31-65. Then the total discount weight of weeks 1-30 is equal to that of weeks 31-65. We can derive the entire discount function from such equalities. Knowledge of utility is not required because it drops from the equations. Even though this method is elementary, it has not been known before. In an experiment, we compare it with a traditional, utility based, method (UM) and find that the DM needs fewer questions than the UM but gives similar results.

# Chapter 2 | Measuring Ambiguity Attitude: (Extended) Multiplier Preferences for the American and the Dutch Population

*Joint work with Aurélien Baillon, Han Bleichrodt and Rogier Potter van Loon*

## 2.1 Introduction

While both the theoretical and the empirical literature on ambiguity are rich[1], there is only limited interaction between the two. A reason is that most ambiguity models use concepts that are hard if not impossible to observe empirically. Empirical measurements of ambiguity have therefore resorted to pragmatic measures that lack a foundation in theory. The purpose of this paper is to bridge this gap between theory and empirics. We use Hansen and Sargent's (2001) multiplier preferences model, which captures ambiguity aversion by a single parameter, to derive a theoretically-founded measure of ambiguity aversion. We extend the multiplier preference model to capture all kinds of ambiguity attitudes, we present a method to measure the ambiguity aversion parameter, and we apply this method in two large representative surveys.

Multiplier preferences are widely used in macroeconomics and finance to permit that decision makers' beliefs about economic phenomena are non-unique. In the multiplier preferences model, decision makers rank payoff profiles $f$ according to the criterion:

$$V(f) = \min_{p} \quad \int u(f)\, dp + \frac{1}{\sigma} R(p||q), \tag{2.1}$$

where $u$ is a utility function, $q$ is a subjective probability distribution on the states of the world, $\sigma$ is a behavioral parameter, and $R(p||q)$ is the relative entropy of any probability distribution $p$ with respect to $q$. The intuition underlying Eq. (2.1) is that

[1] See Trautman and van de Kuilen (2015) for a recent survey of the empirical literature and Machina and Siniscalchi (2014) for a survey of the theoretical literature.

the decision maker has some best guess $q$ of the probability distribution, but he does not have full confidence in his guess and also considers other probability distributions $p$. The plausibility of these other distributions decreases with their distance from $q$, as measured by the relative entropy $R$. The parameter $\frac{1}{\sigma}$ captures the degree to which the decision maker takes alternative probability distributions into account. The lower is $\sigma$, the more the decision maker trusts that $q$ is the correct distribution. In the limit, if $\sigma$ goes to zero, Eq. (2.1) becomes subjective expected utility.

The lack of trust decision makers have in their beliefs may result from ambiguity (Hansen and Sargent 2001). In empirical studies, most subjects are not neutral towards ambiguity, as assumed by expected utility, but are ambiguity averse. Multiplier preferences capture ambiguity aversion while remaining analytically convenient and easy to incorporate in economic models of aggregate behavior. However, they do not accommodate ambiguity seeking, which limits their applicability at the micro level where a wide range of ambiguity attitudes is typically observed and a substantial proportion of respondents is ambiguity seeking.

This paper extends multiplier preferences to accommodate both ambiguity aversion and ambiguity seeking. We give a preference foundation of this extended model that complements Strzalecki (2011) and that makes multiplier preferences suitable for microeconomic applications.

We then present a simple method to measure extended multiplier preferences. Our method is easy to apply and measures multiplier preferences at the individual subject level. Hence, we obtain an axiomatically founded measure of ambiguity aversion that can easily be used in empirical research and that captures the heterogeneity in individual ambiguity attitudes.

We illustrate our method in two large representative samples of the Dutch and the US population involving over 5,000 subjects in total and provide the first micro estimates of (extended) multiplier preferences. Most subjects were moderately ambiguity averse, but between 23% (Dutch sample) and 36% (US sample) were ambiguity seeking. In both samples, we observed that education and income were uncorrelated with ambiguity aversion but negatively correlated with the deviation from ambiguity neutrality.

## 2.2 Extended Multiplier Preferences

We use the Anscombe-Aumann setting. Let $S$ be the *state space,* i.e. the set of all possible *states of the world* $s$. $S$ can be finite or infinite. One state $s$ will occur but the decision maker does not know which one. $\Sigma$ denotes a sigma-algebra on $S$. Its elements are called *events* and are typically denoted $E$. The set of all countably additive probability measures on $(S, \Sigma)$ is denoted by $\Delta(S)$ and is endowed with the weak* topology. A probability measure $p \in \Delta(S)$ is *absolutely continuous* with respect to $q \in \Delta(S)$ if for all $E \in \Sigma$, $q(E) = 0$ implies $p(E) = 0$. Let $\Delta(q)$ denote the set of all countably additive probability measures that are absolutely continuous with respect to $q$. For any $p, q \in \Delta(S)$, the *relative entropy* of $p$ with respect to $q$ is given by $R(p||q) = \int_S \log\left(\frac{dp}{dq}\right) dp$ if $p \in \Delta(q)$ and $R(p||q) = \infty$ otherwise.

We denote the *outcome set* by $Z$. $\Delta(Z)$ is the set of all simple lotteries on $Z$. Elements of $\Delta(Z)$ are denoted $x$ or $y$. The decision maker chooses between *acts*, finite-valued mappings from $S$ to $\Delta(Z)$, which are $\Sigma$-measurable. Acts are usually denoted $f$ or $g$. For event $E$, $f_E g$ denotes the act that gives $f(s)$ if $s \in E$ and $g(s)$ if $s \in E^c$ with $E^c$ the complement of $E$. The set of all acts is $\mathcal{F}$. Acts have two stages: the first stage corresponds to the uncertainty modeled by $S$ and the second stage to the risks modeled by $\Delta(Z)$. The *mixture act* $\alpha f + (1-\alpha)g$ for $\alpha \in [0,1]$ is the act that assigns the lottery $\alpha f(s) + (1-\alpha)g(s)$ to state $s$ for all $s \in S$. The decision maker's preferences over acts in $\mathcal{F}$ are denoted by $\succcurlyeq$ (with $\sim$, $\succ$, $\preccurlyeq$, and $\prec$ defined as usual). A functional $V$ *represents* $\succcurlyeq$ if $V: \mathcal{F} \to \mathbb{R}$ is such that $f \succcurlyeq g \Leftrightarrow V(f) \geq V(g)$.

**Definition 2.1:** We call $\succcurlyeq$ *extended multiplier preferences* if $\succcurlyeq$ can be represented by

$$V(f) = \begin{cases} \min_{p \in \Delta(S)} \int_S u(f(s))dp(s) + \frac{1}{\sigma} R(p||q) & if\ \sigma > 0 \\ \int_S u(f(s))dp(s) & if\ \sigma = 0 \\ \max_{p \in \Delta(S)} \int_S u(f(s))dp(s) + \frac{1}{\sigma} R(p||q) & if\ \sigma < 0 \end{cases}$$

where $u$ is a nonconstant expected utility functional, $q \in \Delta(S)$, and $\sigma \in \mathbb{R}$. We call these preferences robust if $\sigma \geq 0$ and opportunity seeking if $\sigma \leq 0$.

A decision maker whose preferences are opportunity seeking chooses the probabilities that will maximize his expected utility minus a cost, which depends on the distance between these probabilities and his best guess. A decision maker with robust preferences tries to find options that are maximally insensitive to remaining uncertainties. By contrast, an opportunity seeking decision maker is looking for possibilities to improve his expected utility and he values options for which the remaining uncertainties can lead to high expected utilities.

An alternative interpretation of extended multiplier preferences approach comes from a comparison with $\int u(f)\, dp + \theta[R(p||q) - \eta]$, the Lagrange function deduced from minimizing (in the robust approach) or maximizing (in the opportunity seeking approach) $\int u(f)\, dp$ such that the relative entropy does not exceed a threshold ($R(p||q) < \eta$). This comparison shows that the multiplier parameter $\theta = \frac{1}{\sigma}$ is the Lagrange multiplier of the optimization problem and can be interpreted as the shadow price of relaxing the constraint imposed on the relative entropy (Hansen and Sargent, 2001).

There is a third interpretation of the multiplier parameter as an index of ambiguity aversion. Lemma 2.1 in the Appendix shows that extended multiplier preferences are ordinally equivalent to Neilson's (2010) second-order expected utility[2] (SOEU)

[2] SOEU shows that ambiguity attitudes can be modeled by relaxing the assumption of reduction of compound lotteries between the objective stage (the lottery $f(s)$) and the subjective stage (the subjective probability q(s)). Segal (1987) first made this point using rank-dependent utility in both stages. Dillenberger and Segal (2015) showed that Segal's model also accommodates examples of ambiguity behavior proposed by Machina (2009, 2014) that most other ambiguity models cannot accomodate.

$$V(f) = \int_S \varphi_\sigma\left(u\big(f(s)\big)\right) dq(s)$$

when $\varphi_\sigma$ is exponential:

$$\varphi_\sigma(t) = \begin{cases} -e^{-\sigma t} & if\ \sigma > 0 \\ t & if\ \sigma = 0 \\ e^{-\sigma t} & if\ \sigma < 0 \end{cases}$$

and $u$, $q$, and $\sigma$ as in Definition 1. Axiomatizations of SOEU were given by Grant, Polak, and Strzalecki (2009), Nau (2006), and Neilson (2010). We know from Pratt (1964) that under expected utility the exponential utility function is equivalent to constant absolute risk aversion. This implies that adding an amount $c$ to all outcomes of the lotteries under comparison does not change the preferences between these lotteries. For the exponential function, the Arrow-Pratt index of risk attitude $-\frac{u''}{u'}$ is constant and equal to the exponential parameter. Under SOEU, we can give a similar interpretation to the exponential $\varphi_\sigma$ function in terms of utility: adding the same (expected) utility to each state of the acts under comparison does not change the preferences between these acts. Grant and Polak (2013) coin the term constant absolute uncertainty aversion to describe this property. The index $-\frac{\varphi''}{\varphi'} = \sigma$ is then an Arrow-Pratt index of ambiguity attitude. We used $\sigma$ instead of $\theta$, because $\sigma$ is a monotonic and continuous measure and, therefore, more convenient for statistical analysis.

## 2.3 Axiomatization

Strzalecki (2011) axiomatized extended multiplier preferences for $\sigma \geq 0$, i.e. for decision makers with robust preferences. We will characterize extended multiplier preferences, i.e. including the case of opportunity seeking ($\sigma \leq 0$). We do so by dropping uncertainty aversion (his A.5) from Strzalecki's set of axioms and by replacing results in his proof that depend on this axiom by other results that do not depend on it.

We impose the following conditions on $\succcurlyeq$:

1. *Weak order:* $\succcurlyeq$ is complete and transitive.

2. *Weak certainty independence:* for all $f, g \in \mathcal{F}$, for all $x, y \in \Delta(Z)$, and for all $\alpha \in (0,1)$, $\alpha f + (1-\alpha)x \succcurlyeq \alpha g + (1-\alpha)x \Rightarrow \alpha f + (1-\alpha)\mathrm{y} \succcurlyeq \alpha g + (1-\alpha)y$.

3. *Continuity:* for all $f, g, h \in, \mathcal{F}$, the sets $\{\alpha \in [0,1]: \alpha f + (1-\alpha)\mathrm{g} \succcurlyeq h\}$ and $\{\alpha \in [0,1]: \alpha f + (1-\alpha)\mathrm{g} \preccurlyeq h\}$ are closed.

4. *Monotonicity:* for all $f, g \in \mathcal{F}$ if $f(s) \succcurlyeq g(s)$ for all $s \in S$ then $f \succcurlyeq g$.

5. *Nondegeneracy*: there exist acts $f, g \in \mathcal{F}$ such that $f \succ g$.

6. *Weak monotone continuity:* for all $f, g \in \mathcal{F}$, for all $x \in \Delta(Z)$, and for all $\{E_n\}_{n\geq 1} \in \Sigma$ with $E_1 \supseteq E_2$ …. and $\cap_{n\geq 1} E_n = \emptyset$, $f \succ g$ implies that there exists an $n_0$ such that $x_{E_{n_0}} f \succ g$.

7. *Sure thing principle:* for all $E \in \Sigma$ and for all $f, g, h, h' \in \mathcal{F}$, $f_E h \succcurlyeq g_E h \;\Rightarrow f_E h' \succcurlyeq g_E h'$.

An event is *essential* if there exist $f, g, h \in \mathcal{F}$ such that $f_E h \succ g_E h$.

**Theorem 2.1:** If $S$ has at least three disjoint essential events[3] then the following two statements are equivalent:

1. $\succcurlyeq$ is a continuous, nondegenerate weak order that satisfies weak certainty independence, monotonicity, weak monotone continuity and the sure thing principle.

2. $\succcurlyeq$ has an extended multiplier representation.

[3] If only one event is essential then the Theorem also holds but the uniqueness properties are different. If exactly two disjoint events are essential then the sure thing principle should be strengthened to the hexagon condition (Wakker 1989).

**Observation 2.1:** Two triples $(\sigma, u, q)$ and $(\sigma', u', q')$ represent the same extended multiplier preference if and only if $q$ and $q'$ are identical and there exist $\alpha > 0$ and $\beta \in \mathbb{R}$ such that $u^{'} = \alpha u + \beta$ and $\sigma^{'} = \sigma/\alpha$.

All proofs are in Appendix.

We can distinguish the robust and the opportunity seeking approaches using Schmeidler's (1989) condition of ambiguity aversion and its counterpart of ambiguity seeking.

**Definition 2.2:** *Ambiguity aversion* (*seeking*) holds if for all acts $f, g$ in $\mathcal{F}$and for all $\alpha$ in (0,1), $f \sim g \Rightarrow \alpha f + (1-\alpha)g \succcurlyeq (\preccurlyeq) f$.

**Theorem 2.2:** Under extended multiplier preferences, ambiguity aversion is equivalent to robust preferences and ambiguity seeking is equivalent to opportunity seeking preferences.

According to Theorem 2.2, the sign of $\sigma$ determines whether an agent is ambiguity averse or ambiguity seeking. But for σ to be a proper index of ambiguity aversion, it should also satisfy the property that a higher value represents more ambiguity aversion. Consider two decision makers i ∈ {1,2} represented by preferences $\succcurlyeq_{\mathrm{i}}$. We use the definition of "more ambiguity averse" proposed by Ghirardato and Marinacci (2002).

**Definition 2.3**: $\succcurlyeq_2$ is more ambiguity averse than $\succcurlyeq_1$ if for all acts f in $\mathcal{F}$ and lotteries x in Δ(Z), $\mathrm{x} \succcurlyeq_1 \mathrm{f} \Rightarrow \mathrm{x} \succcurlyeq_2 \mathrm{f}$.

This definition adapts the definition of "more risk averse" introduced by Yaari (1969) to ambiguity. It implies that the ambiguity attitudes of two decision makers can only be compared if they share the same beliefs (here, the same q). Moreover, as shown by

Ghirardato and Marinacci (2002, Proposition 11), the decision makers need to have the same risk attitudes, which corresponds to their utility functions being cardinally equivalent: $u_1 \approx u_2$ if there exist $\alpha > 0$ and $\beta \in \mathbb{R}$ such that $u_1 = \alpha u_2 + \beta$.

**Theorem 2.3**: Given two extended multiplier preferences $\succcurlyeq_1$ and $\succcurlyeq_2$ represented by $(\sigma_1, u_1, q_1)$ and $(\sigma_2, u_2, q_2)$, the following two statements are equivalent:

1. $\succcurlyeq_2$ is more ambiguity averse than $\succcurlyeq_1$.
2. $u_1 \approx u_2$, $q_1 = q_2$, and $\sigma_1 \leq \sigma_2$ (if we scale utility such that $u_1 = u_2$).

Theorem 2.3 shows that $\sigma$ is a proper measure of ambiguity aversion.

## 2.4 Measuring Extended Multiplier Preferences

### *2.4.1 Method*

Strzalecki (2011, Example 3) explained how the multiplier parameter $\sigma$ could be measured under the assumption that utility $u$ is a power function. We describe an alternative method that makes no assumptions about utility and requires fewer questions. Because extended multiplier preferences are ordinally equivalent to SOEU with $\varphi_\sigma$ exponential, we will display our results using SOEU for ease of understanding. Suppose that a ball will be drawn from an urn with an unknown number of yellow and purple balls. Let $S = \{Y, P\}$ where $Y$ stands for "the ball is yellow" and $P$ for "the ball is purple". The decision maker can win either \$15 or nothing, depending on the color of the ball. Hence, $Z = \{0,15\}$. The act $f_Y$ pays \$15 if the ball is yellow and nothing otherwise and the act $f_P$ pays \$15 if the ball is purple and nothing otherwise. Each lottery from $\Delta(Z)$ can be written as $15_r0$, where $r$ is the probability to get 15. We scale utility so that $u(0) = 0$ and $u(15) = 15$. Then $u(15_r0) = r * 15 + (1 - r) * 0 = 15r$.

Assume that $f_Y \sim f_P \sim 15_r0$ for some probability $r$. We call this probability $r$ a *matching probability* of the acts $f_Y$ and $f_P$. Under SOEU, we obtain from $f_Y \sim f_P$ that $q(Y) =$

$q(P) = ½$ . The second indifference, $f_P \sim 15_r 0$, then implies $\varphi_\sigma(15r) = ½\ \varphi_\sigma(15) + ½\ \varphi_\sigma(0)$. We prove in the Appendix that this equation has a unique solution $\sigma$ for each value of $r \in (0,1)$. If $r = ½$ , then $\sigma = 0$ and the decision maker is indifferent between an objective and a subjective probability of ½ . If $r < ½$ then $\sigma > 0$and the decision maker prefers an objective probability of ½ to a subjective probability of ½.. This corresponds to ambiguity aversion. Similarly, $r > ½$ implies ambiguity seeking ($\sigma < 0$). If $r \to 0$, preferences are extremely robust (ambiguity averse) and $\sigma \to +\infty$. If $r \to 1$, preferences are extremely opportunity seeking and $\sigma \to -\infty$.

### 2.4.2 *Calibration*

Observation 1 shows that the sign of the multiplier parameter does not depend on the scaling of the utility function, but its magnitude does. In the empirical study reported in Section 2.4.3, we scale utility such that the utility of initial wealth $W$is 0 and that of $W + 15$ is 15. For any utility function $v$, the multiplier parameter $\sigma_v$ can be computed from the $\sigma$ that we report below using $\sigma_v = \frac{15\sigma}{v(W+15)-v(W)}$. Because $\sigma$ depends only on the scaling of utility and not on utility curvature, it does not depend on a subject's risk aversion. Hence, it can be used for correlation analysis if the same scaling is used for all subjects.

### 2.4.3 *Empirical Illustration*

Two surveys have been held in which subjects answered questions of the form described in section 2.4.1. Dimmock, Kouwenberg and Wakker (2015) ran a survey among 1,900 participants of the Dutch Longitudinal Internet Study for the Social Sciences (LISS). Dimmock, Kouwenberg, Mitchell and Peijnenburg (2013) ran a similar survey among 3,300 participants of the American Life Panel (ALP)4. We illustrate our method by showing the σ values obtain from the responses in these two datasets.

[4] Both papers analyzed a subset of their respondents, excluding subjects who took too much or too little time in answering., For example. Dimmock et al. (2015) excluded more than half of their subjects as these were not incentivized. In our analyses, we chose to include all subjects as any exclusion criterion is to some extent arbitrary. The results we present were unaffected if we used the same inclusion criteria as Dimmock et al. (2013, 2015).

In both surveys, subjects had to choose between two urns: a known urn K and an ambiguous urn A. Urn K contained 100 yellow (orange) and purple balls in known proportions. Urn A contained 100 yellow and purple balls in unknown proportions. By default, purple was the winning color, but subjects could change the winning color to yellow. Only 1% of all subjects did so, which indicates that most subjects were not suspicious and had no preference between the two winning colors. This implies that $f_Y \sim f_P$.

The survey measured the matching probability $r$ for which subjects were indifferent between urn A and urn K with $r * 100$ balls of their winning color. Subjects made a series of choices between urn A and urn K, where urn A remained the same while the proportion of winning balls in urn K changed depending on previous choices.

At the end of the experiment, one randomly selected choice was played for real. A ball was drawn from the urn that the subject preferred in that choice. The subject received €15 euro (dollar) if the ball had his winning color and nothing otherwise. Advantages and disadvantages of the approached followed are discussed in Dimmock et al. (2013, 2015).

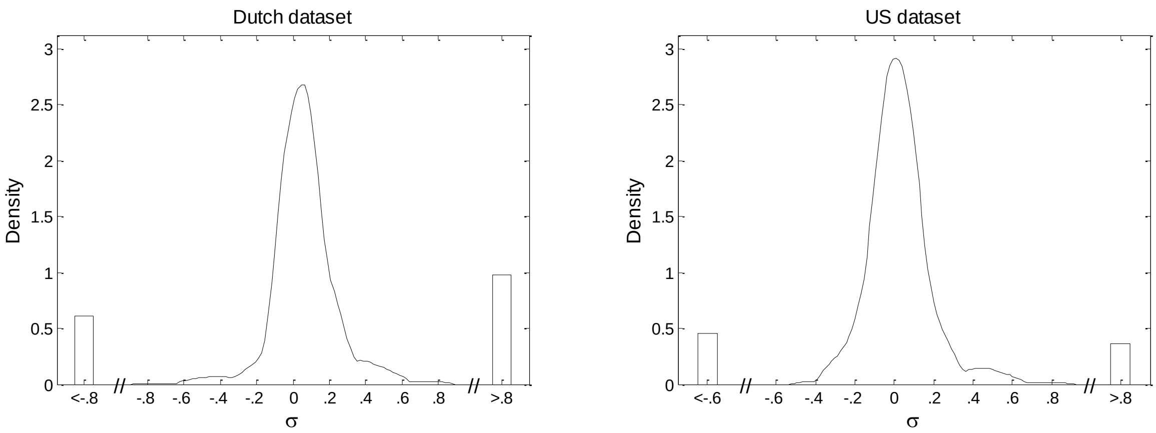


**Figure 2.1: Kernel density estimates of respondents' $\sigma$ values. The Epanechnikov function was used, with a kernel width of 0.07. The boxes at the upper and lower end indicate the proportion of subjects with $\sigma$ values greater than .8 and less than -.8/-.6.**

Figure 2.1 shows the estimated distribution of σ in the two datasets using a kernel density estimate. In the Dutch (US) dataset, the median value of σ was equal to 0.05

(0.02)), which corresponds with a matching probability of 40.6% (47.0%). Both distributions are centered slightly to the right of zero and concentrated in the ambiguity averse domain. Still, 22.5% (35.9%) of subjects were found to be ambiguity seeking. The box at the far left of the distribution show that 6.2% (4.5%) of the subjects gave matching probabilities close to 1, which corresponds with a value of σ that is below -0.8 (-0.6)[5]. Similarly, the boxes on the far right indicate that 9.6% (3.6%) gave matching probabilities close to zero, which corresponds with a σ value greater than 0.8.

| | Dutch dataset | | US dataset | |
|---|---|---|---|---|
| | $\sigma$ | $\|\sigma\|$ | $\sigma$ | $\|\sigma\|$ |
| Gender (female = 1) | 0.012 | −0.002 | −0.053*** | 0.011 |
| Age | −0.070*** | 0.142*** | −0.032* | 0.001 |
| High income | −0.003 | −0.047** | 0.020 | −0.060*** |
| High education | 0.003 | −0.064*** | 0.031* | −0.064*** |
| N | 1,821 | 1,821 | 3,217 | 3,217 |

**Table 2.1: Correlations between demographic variables and ambiguity aversion ($\sigma$) and deviation from ambiguity neutrality ($|\sigma|$). *significant at 10% level, **5%, ***1%.**

As a further illustration, Table 2.1 shows correlations between σ and demographic variables in the first and third column, and correlations between |σ| and demographic variables in the second and fourth column. Correlations with |σ| are also analyzed because some effects may be correlated with the deviation from ambiguity neutrality, rather than with the degree of ambiguity aversion. Such a deviation implies a violation of either probabilistic sophistication or dynamic consistency, two conditions that are generally considered normative. Ambiguity neutrality is therefore often

[5] These thresholds are not of the same absolute value due to an asymmetry in the question design of Dimmock et al. (2013).

perceived as the rational model of choice under uncertainty (e.g., Wakker 2010, p. 326).

In the Dutch sample, the only variable that is correlated with σ is age, with older respondents being more ambiguity seeking. The second column shows that age is positively correlated with |σ|, which suggests that they have more extreme ambiguity attitudes. Income and education are negatively correlated with the deviation from ambiguity neutrality, which seems consistent with the finding that people with higher cognitive abilities deviate less from models of rational choice (Frederick 2005, Dohmen et al. 2010).

In the US sample, women are more ambiguity seeking, as are older and less educated people (marginally significant). Although there is no correlation between age and |σ| as in the Dutch dataset, the correlation coefficients for income and education are remarkably similar to their Dutch counterparts. All correlations are negative, indicating that those with higher income and education are closer to ambiguity neutrality.

## 2.5 Concluding Remarks

Multiplier preferences, proposed by Hansen and Sargent (2001), are a popular model in macroeconomics and finance. In its original form, multiplier preferences only capture ambiguity aversion, which make them less suitable for applications at the micro level where substantial ambiguity seeking has also been observed. This chapter extends multiplier preferences to include ambiguity seeking and it gives a preference foundation for these extended multiplier preferences. We also show how extended multiplier preferences can be measured and thereby obtain an axiomatically-founded measure of ambiguity aversion that can easily be applied in empirical studies and that captures the substantial heterogeneity in ambiguity attitudes that typically exists in micro data. As an illustration, we applied our method to two large scale representative surveys, one from the Netherlands and one from the US. In both samples a substantial fraction of the respondents was ambiguity seeking, which illustrates the desirability of our extension of multiplier preferences.

## Appendix 2: *Proofs*

**Lemma 2.1:** Preferences $\succcurlyeq$ are extended multiplier preferences if and only if there exists $\sigma \in \mathbb{R}$ such that $\succcurlyeq$ can be represented by SOEU with $q \in \Delta(S)$ and $\varphi = \varphi_\sigma$.

*Proof:*

The equivalence between robust preferences and $\varphi(t) = -e^{-\sigma t}$ has been shown by Strzalecki (2011). It is based on Proposition 1.4.2 of Dupuis and Ellis (1997) stating that for all countably additive probability measures $q \in \Delta(S)$ and for all $\Sigma$-measurable functions $v$:

$\min_{p\in\Delta(S)} \int_S \; v(s)dp(s) + \frac{1}{\lambda} R(p||q) = \varphi_\lambda^{-1}\left(\int_S \; \varphi_\lambda\big(v(s)\big)dq(s)\right).$

For $\sigma < 0$, we apply this formula to $v = -u \circ f$ and $\lambda = -\sigma$ and we obtain:

$$\begin{aligned}
\max_{p\in\Delta(S)} \int_S \; u\big(f(s)\big)dp(s) + \frac{1}{\sigma}R(p||q) \quad &= \quad -\left[\min_{p\in\Delta(S)} \int_S \; v(s)dp(s) + \frac{1}{\lambda}R(p||q)\right] \\
&= \quad -\varphi_\lambda^{-1}\left(\int_S \; \varphi_\lambda\big(v(s)\big)dq(s)\right) \\
&= \quad \varphi_\sigma^{-1}\left(\int_S \; \varphi_\sigma\left(u\big(f(s)\big)\right)dq(s)\right).
\end{aligned}$$

The last equality follows from $\varphi_\sigma^{-1}(t) = -\frac{\ln(t)}{\sigma} = \frac{\ln(t)}{\lambda} = -\varphi_\lambda^{-1}(-t)$ and

$\varphi_\lambda\big(v(s)\big) = -e^{-\lambda v(s)} = -e^{-\sigma u(f(s))} = -\varphi_\sigma\left(u\big(f(s)\big)\right).$

Hence, both robust and opportunity seeking preferences are equivalent to SOEU with an exponential $\varphi$ function. □

*Proof of Theorem 2.1:*

(ii) $\Rightarrow$ (i). Because (ii) is a normalized niveloid that represents $\succcurlyeq$ and u is nonconstant and affine, Lemma 28 in Maccheroni, Marinacci and Rustichini (2006) implies that $\succcurlyeq$ is a constinuous, nondegenerate weak order that satisfies weak certainty independence and monotonicity. Because q is countably additive, $\succcurlyeq$ satisfies uniform continuity by Theorem 5.4 in Krantz et al. (1971). Finally, by Proposition 1.4.2 in Dupuis and Ellis (1997), (ii) is equivalent to a second order expected utility representation. Consequently, the sure thing principle must hold.

We show that (i) $\Rightarrow$ (ii) by closely following Strzalecki's proof without imposing uncertainty aversion. First we introduce some new notation. Let $B_0(\Sigma)$ denote the set of all real-valued $\Sigma$-measurable simple functions[6] and let $B_0(\Sigma, \mathrm{K})$ denote the set of functions in $B_0(\Sigma)$ that take values in a convex set $K \subseteq \mathbb{R}$. Let $\Phi_3$denote the set of finite partitions of $S$ that contain at least three essential events. For all $G \in \Phi_3$, let $\mathcal{A}(G)$ be the algebra generated by $G$ and let $\mathcal{F}_G$ denote the set of acts in $\mathcal{F}$ that are measurable with respect to $\mathcal{A}(G)$.

By Lemmas 25 and 28 of Maccheroni et al. , there exist a real-valued nonconstant affine function $u$ on $\Delta(Z)$ and a normalized real-valued functional $I: B_0(\Sigma, \mathcal{U}) \to \mathbb{R}$ where $\mathcal{U}$ is the range of $u\big(\Delta(Z)\big)$ and such that for all acts $f, g \in \mathcal{F}$, $f \succcurlyeq g$ iff $I(u \circ f) \geq I(u \circ g)$ and $I(\alpha\psi + (1-\alpha)k) = I(\alpha\psi) + (1-\alpha)k$ for all $\psi \in B_0(\Sigma, \mathcal{U})$, $k \in \mathcal{U}$ and $\alpha \in (0,1)$.

Theorem 1 in Grant, Polak, and Strzalecki (2009) ensures that for finite $S \succcurlyeq$ can be represented by $f \longmapsto \sum_{s \in S} v_s\big(u\big(f(s)\big)\big)$ with $u$ nonconstant and affine and with range $\mathcal{U}$ and $v_s$ continuous, nondecreasing, and with at least three $v_s$ nonconstant. Weak certainty independence then ensures that indifference curves in the utility space are parallel and have common supporting hyperplanes at the set of constant vectors in $\mathcal{U}^S$. By the proof of Theorem 3 in Grant et al. it follows that for all $G \in \Phi_3$the restriction of $\succcurlyeq$ to $\mathcal{F}_G$ can be represented by $f \longmapsto \sum_{s \in S} p_G(s)\varphi_G\big(u_G(f_s)\big)$ with $u_G$ nonconstant and affine, $\varphi_G$ continuous and strictly increasing, and measure $p_G: \mathcal{A}(\mathrm{G}) \to [0,1]$ such that at least three events in G are nonzero. In applying Theorem 3, we replace uncertainty aversion and their Axiom A.7 by weak certainty independence. Uncertainty aversion is used in the application of Theorem 3 in Debreu

[6] A function is simple if it takes no more than countably many distinct values.

and Koopmans (1982) to derive differentiability of the functions $v_s$. However, as noted by Grant et al. and Maccheroni et al. ( p.1475, 1491), weak certainty independence implies Lipschitz continuity and hence differentiability. By Theorem 4 in Strzalecki (2011), the proof of which does not use uncertainty aversion, $\succcurlyeq$ can be represented by second order expected utility $f \longmapsto \int_S \varphi\big(u(f_s)\big)dq(s)$ with $q \in \Delta(Z)$ and $\varphi$ continuous and strictly increasing. $q$ is countably additive by uniform continuity (Villegas 1964, Theorem 1). Moreover, if $(u, \varphi, q)$ and $(u', \varphi', q')$ both represent $\succcurlyeq$ then there exist $\alpha, A > 0, \beta, B \in \mathbb{R}$ such that $q' = q, u' = \alpha u + \beta, \varphi'(\alpha r + \beta) = A\varphi(r) + B$ for all $r$ in $\mathcal{U}$.

$I$ represents $\succcurlyeq$ and is translation invariant, i.e. for all $f, g \in \mathcal{F}$ and $k$ such that $f(s) + k, g(s) + k \in \mathcal{U}$ for all $s \in S, I(u \circ f) \geq I(u \circ f)$ iff $I(u \circ f + k) = I(u \circ f) + k \geq I(u \circ f) + k = I(u \circ f + k)$. It then follows that for all acts $f, g \in \mathcal{F}$ and $k$ such that $f(s) + k, g(s) + k \in \mathcal{U}$ for all $s \in S$, $\int_S \varphi(u(f(s)))dq(s) \geq \int_S \varphi(u(g(s)))dq(s)$ iff $\int_S \varphi(u(f(s) + k)dq(s) \geq \int_S \varphi(u(g(s) + k)dq(s)$.

Hence, $(u, \varphi, q)$ and $(u, \varphi_k, q)$ defined by $\varphi_k(l) = \varphi(l + k)\ \forall l, l + k \in \mathcal{U}$ are both SOEU representations of $\succcurlyeq$. Consequently, $\varphi(l + k) = A(k)\varphi(l) + B(k)$. Because $\varphi$ is nonconstant, if $\mathcal{U}$ is unbounded, it follows from Corollary 1 in Aczél (1966, Section 3.1.3) that $\varphi$ equals $\varphi_\sigma$. If $\mathcal{U}$ is bounded then because $\varphi$ is nonconstant Theorem 4 in Aczél (2005) implies $\varphi = \varphi_\sigma$ on the interior of $\mathcal{U}$. Because $\varphi$ is continuous, the extension to all of $\mathcal{U}$ follows.

By Proposition 1.4.2 in Dupuis and Ellis (1997) and Lemma 2.1, we then obtain the extended multiplier representation. □

*Proof of Observation 2.1:*

The proof of Theorem 2.1 already showed that the probability measure $q$ is unique and that the utility function $u$ is unique up to positive affine transformations. We also know that for $A > 0$ and $B \in \mathbb{R}$, $\varphi' = A\varphi + B$. Because $e^{-\sigma' u'} = e^{-\sigma'(\alpha u + \beta)} = e^{-\beta} e^{-\alpha\sigma' u}$, it follows from the uniqueness properties of $\varphi$ that $\sigma' = \frac{1}{\alpha}\sigma$. □

*Proof of Theorem 2.2:*

Ambiguity aversion states that preferences are convex. Hence it is equivalent to a concave representation. Since $u$ is linear with respect to mixture of lotteries, ambiguity aversion is equivalent to the SOEU with $\varphi$ concave, which means $\sigma \geq 0$. The opposite reasoning applies to ambiguity seeking. □

*Proof of Theorem 2.3:*

(2) ⇒ (1) is trivial. Assume (1). It implies $u_1 \approx u_2$ (Ghirardato and Marinacci, 2002, Proposition 11). We scale utility such that $u_1 = u_2$. Recode lotteries into expected utilities. Using the second-order expected utility formulation of extended variational preferences and the results of Yaari (1969), we immediately obtain $q_1 = q_2$ and $\varphi_2$ more concave than $\varphi_1$, which implies $\sigma_1 \leq \sigma_2$. □

*Proof that there is a unique solution $\sigma$ for each value of r.*

$f_Y \sim f_P$ and $f_P \sim 15_r0$ jointly imply $\varphi_\sigma(15r) = ½\varphi_\sigma(15) + ½\varphi_\sigma(0)$, which is equivalent to $15r = ½(15) + ½(0)$ if $\sigma = 0$ and to
$\exp(-15\sigma r) = ½\exp(-15\sigma) + ½\exp(0)$ otherwise. Hence,

$r = ½$ if $\sigma = 0$

$r = -\frac{\ln(½\exp(-15\sigma)+½)}{15\sigma}$ if $\sigma \neq 0$

The proof that $r$ is continuous and decreasing as a function of $\sigma$ is elementary. By the intermediate value theorem, there is a unique solution $\sigma$ for each $r \in (0,1)$. □

# Chapter 3 | Ambiguity Attitude under Time Pressure[7]

*Joint work with Aurélien Baillon, Asli Selim, & Peter P. Wakker*

## 3.1 Introduction

This chapter examines decisions under ambiguity when those decisions are taken under time pressure (TP). We aim to contribute both to the literature on ambiguity and to that on TP. Decision researchers have long recognized that real life decisions have to be performed under circumstances that are far from ideal, and that environmental constraints have significant impact on the quality and the outcome of decisions. Yet, for the sake of simplification, most economic studies have disregarded environmental constraints. Time pressure is one such important environmental constraint. For some professions, such as stockbrokers, emergency room doctors and fire fighters to name a few, TP is ubiquitous. Therefore, taking TP into account improves the relevance of decision theory to real life decisions.

In behavioral economics, it has long been understood that human memory and mental capacities are limited and often lead to suboptimal decisions. This suboptimality is signaled by the many violations of elementary rationality principles that have been widely documented for human decisions (Kahneman, 2011). Consequently, the classical models of decision making, which describe the rational homo economicus, fail to predict many of our decisions, and many alternative models have been developed to capture our suboptimal decisions. The deviations from rationality are especially pronounced when decisions have to be made under TP. Therefore, TP has received special attention throughout the history of decision theory—as yet mostly in the psychological literature—because, in addition to its direct practical relevance, it provides a good context for studying and learning about heuristics and suboptimal decisions.

[7] Chen Li made helpful comments

TP enhances the use of simple noncompensatory decision heuristics, which efficiently lead to good but not optimal decisions (satisficing; Simon, 1982). Although simplified heuristics actually improve our decisions under some specific circumstances (Baumeister, Masicampo, & Vohs 2011; Wilson & Schooler, 1991), in most cases they lead to bad decisions. TP usually aggravates violations of rationality principles. This general finding has been investigated and confirmed in many decision contexts; see the general TP survey by Ariely & Zakay (2001).[8]

In addition to the aforementioned contexts, many papers have investigated the effects of TP on decision making under risk (uncertainty with known probabilities), confirming the above general findings.[9] We will investigate the effects of TP on decisions under ambiguity, where no probabilities are known for the uncertain events faced. Closest to our study is Young et al. (2012), in which the authors use prospect theory to study the effects of TP as we will do, except that they focus on risk instead of ambiguity.

Since Keynes (1921) and Knight (1921) it has been understood that ambiguity is more prevailing and is more important than risk. However, as ambiguity is more difficult to analyze, the development of this field is relatively new (Gilboa 1987; Gilboa & Schmeidler 1989; Schmeidler 1989; Tversky & Kahneman 1992; see Etner, Jeleva, & Tallon (2012) and Trautmann & van de Kuilen (2013) for recent surveys). For this reason, TP has not yet been investigated for ambiguity.

In our analysis of TP, we will consider two components of ambiguity. The first component reflects the well-known aversion to ambiguity. Ellsberg (1961) showed that people are mostly ambiguity averse, i.e., they dislike it more than risk. Aversion to ambiguity is empirically prevailing, but recent empirical studies have shown that there can be systematic ambiguity seeking in many situations (Binmore, Stewart, & Voorhoeve 2012; Charness, Karni, & Levin 2013; Ivanov 2011; reviewed by Trautmann & van de Kuilen 2013).

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[8] Further references include Ordonez & Benson (1997) for consumer decisions, Reutskaja, Nagel, & Camerer (2011) for search dynamics, and several experimental studies in games (Dror, Busemeyer, & Basola 1999; Kocher & Stutter 2006; Sutter, Kocher, & Strauss 2003; Tinghög et al. 2013).

[9] See the references in Ariely & Zakay (2001), and Chandler & Pronin (2012) Kocher, Pahlke, & Trautmann (2013), Maule, Hockey, & Bdzola (2000), and Payne, Bettman, & Luce (1996).

The second component of ambiguity that we will consider is called ambiguity-generated insensitivity, or a-insensitivity for short. It is a cognitive component, reflecting to what extent people do not understand uncertainty and are insensitive to changes in uncertainty-information. A-insensitivity is a relative component that indicates how much less people understand uncertainty than they understand risk, just as how ambiguity aversion shows how much less people like ambiguity than they like risk. Empirical studies have commonly confirmed the importance of a-insensitivity for understanding empirical data on ambiguity (Baillon, Cabantous, & Wakker 2012; Trautmann & van de Kuilen 2013; Wakker 2010 §10.4.2). One contribution of our paper is a simplified method for measuring the two aforementioned components. Our method differs from the method of Abdellaoui et al. (2011), and its simplification by Dimmock, Kouwenberg, & Wakker (2013), by not requiring any estimation of subjective probabilities. It formalizes tests used by Baillon & Bleichrodt (2013). We introduce this method and provide global source-dependent indexes of ambiguity attitudes that are not tied to particular events, in §3.2. Thus, our indexes are not ad hoc but have decision-theoretic foundations.

The new ambiguity theories are especially relevant to finance. In our experiment, subjects have to make decisions about the performance of the AEX (Amsterdam stock exchange) index. As for most uncertain events, neither we nor the subjects know the probabilities of future stock movements, the economy being in a state it was never in before. In the central treatment in our experiment, subjects have to make their decisions under TP. We compare this treatment with several control treatments and investigate the effects of ambiguity and learning on decision making.

For our understanding of TP it is important to know how TP is associated with ambiguity attitudes, given the ubiquity of ambiguity in all our decisions. Our main finding will be that TP affects the cognitive component of a-insensitivity, but not the aversion component. When subjects must decide fast, they understand and process the uncertain information to a lesser extent, but they do not feel more or less aversion. Therefore, our results indicate that future studies on TP may want to focus on cognitive components of ambiguity rather than ambiguity aversion, and that the drawbacks of TP can primarily be mitigated by increasing knowledge and understanding, rather than by neutralizing unreasonable dislike.

Our findings are, obviously, also relevant to the field of ambiguity. In their general survey of TP, which, of course, could not yet incorporate ambiguity, Ariely & Zakay (2001) noted that TP provides an easy way to generate stress, which enhances the use of suboptimal cognitive processes, cognitive biases, and errors. Hence, TP provides an easy tool to investigate the role of biases in ambiguous decisions. Our results confirm Ariely & Zakay's (2001) findings for ambiguity and underscore the importance of the cognitive ambiguity-sensitivity component by showing that it is the primary component associated with bounded rationality.

## 3.2 Measuring Ambiguity Attitudes: Theory

For each subject, in each of three parts, we consider a triple of exhaustive and mutually exclusive uncertain events E1, E2, and E3, always referring to changes in the AEX index. For instance, in Part 2, we have $E_1 = (-\infty,-0.2)$, $E_2 = [-0.2,0.2]$, and $E_3 = (0.2,+\infty)$, where intervals describe percentage increases in the AEX index. $E_{ij}$ denotes the union $E_i \cup E_j$, where $i \neq j$ is implicit. For each event E ($E = E_i$ or $E = E_j$) we define its *matching probability* m ($m = m_i$ or $m = m_{ij}$) by:

Receiving €20 under event E is equivalent to receiving €20 with probability m. (3.1)

In each case it is understood that the complementary payoff is nil.

The more ambiguity averse a person is the lower the matching probabilities will be. Hence, the following index of ambiguity aversion is plausible:

**Definition 3.1**. The *ambiguity aversion index* is defined as

$$b = 1 - \overline{m_c} - \overline{m_s}, \tag{3.2}$$

where $\overline{m_s} = (m_1 + m_2 + m_3)/3$ denotes the average single-element-event matching probability, and $\overline{m_c} = (m_{12} + m_{23} + m_{13})/3$ denotes the average two-element-event matching probability. Under expected utility (Savage 1954), $\overline{m_s} = 1/3$ and $\overline{m_c} = 2/3$. Hence, then b = 0, reflecting ambiguity neutrality. This index has been suggested before, always under the assumption of expected utility under risk (Dow & Werlang 1992; Schmeidler 1989 pp. 572, 574; explained in our Appendix). Under Schmeidler's

(1989) model, his ambiguity aversion (called uncertainty aversion there) implies b > 0, and his ambiguity neutrality implies b = 0, and his ambiguity seeking implies b < 0 (Observation A.2). Observation A.1 in the appendix will show that index b is a pragmatic version of Abdellaoui et al.'s (2011) ambiguity aversion index. This index is based on a decision-theoretic justification that does not require a commitment to expected utility under risk (or ambiguity).

Insensitivity refers to a regression-to-the-mean type of effect, where ambiguity weights move towards fifty-fifty. It leads to relatively large values of $\overline{m_s}$ and relatively low values of $\overline{m_c}$. It is plausible to have an index that is increasing in the former, but decreasing in the latter. This is reflected by the following definition:

**Definition 3.2**. The *ambiguity-generated insensitivity* ((a-)insensitivity) index is defined as

$$a = 1 + 3\overline{m_s} - 3\overline{m_c}, \qquad (3.3)$$

Under Savage's expected utility, a = 0. Observation A.1 will show that this index, again, is a pragmatic version of Abdellaoui et al.'s (2011) a-insensitivity index, with a decision-theoretic justification that does not require expected utility under risk.

## 3.3 Experiment: Method

*Participants*

N = 104 subjects participated (56 male, median age 20). They were all students of a Dutch university and were recruited from a subject pool.

*Procedure*

Computers were separated by wooden panels to minimize interaction between subjects. A brief set of instructions were read aloud, and tickets with ID numbers were handed out. Subjects typed in their ID numbers to start the experiment. The subjects were randomly allocated to treatment groups through their ID numbers.

Talking was not allowed during the experiment. Instructions were given with detailed information about the payment process, user interface, and the type of questions subject would face. The subjects could ask questions to the experimenters at any time. In each session, all subjects started the experiment at the same time.

*Stimuli: Within- and between-subject treatments*

**TABLE 3.1: Organization of the experiment**

| Within subject / Between subject | *Part 1 (Training)* | *Part 2* | *Part 3* |
|---|---|---|---|
| **Time pressure group** | No time pressure | **Time pressure** | No time pressure |
| **Control group** | No time pressure | No time pressure | No time pressure |

The experiment consisted of three parts, each containing eight questions. Table 3.1 shows the design of the experiment. The first part served for training purposes. All subjects faced the same set of decision tasks, the only difference being that, in the second part of the experiment, the subjects in the time pressure (TP) treatment had to make their choices under time pressure.

In the TP treatment, we took two measures to make sure that TP would not have any effects in Part 1 and 3. First, we imposed a two-minute break after Parts 1 and 2, to avoid spill-over of stress from Part 2 to Part 3. Second, we did not tell the subjects they will be put under TP prior to Part 2, so as to avoid stress generated by such an announcement in Part 1 (Ordonez & Benson 1997). There were 42 subjects in the control treatment and 62 in the TP treatment. The TP sample had more subjects because it is more interesting, and we expected more variance in it.

*Stimuli: Choice lists*

In each question, subjects were asked to choose between two options:

OPTION 1: You win €20 if the AEX (Amsterdam stock exchange) index increases/decreases by more/less than XX% between the beginning and the end of the experiment, and nothing otherwise.

OPTION 2: You win €20 with p% probability and nothing otherwise.

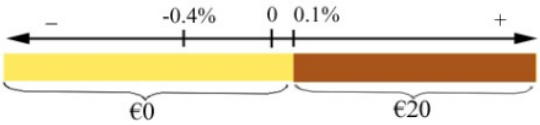
Thus, in each question, the uncertainty that subjects faced for the first option concerned the variation of the AEX index between the beginning and the end of the experiment. Further details are given later.

We used these choice questions in a choice list so as to infer the value of p that gives indifference (Figure 3.1). To be more precise, subjects were asked to state which one of these two options they preferred for different values of p, ascending from 0 to 100. The midpoint between the two values of p where they switched preference was taken as the indifference point, and this indifference probability was treated as the matching probability of the AEX event.

To help subjects answer the questions quickly, which was crucial under TP, the experimental webpage allowed them to state their preferences with a single click. For example, if they clicked on Option 2 when the probability of winning was 50%, then for all p > 50%, the option boxes for Option 2 were automatically filled out and for all p < 50% the option boxes for Option 1 were automatically filled out. This procedure also precluded violations of stochastic dominance by preventing multiple preference switches. After clicking on their choices, subjects clicked on a "Submit" button to move to the next question. The response times were also tracked.

**Figure 3.1: Screenshot of the experiment software for single event $E_3$ in Part 1**

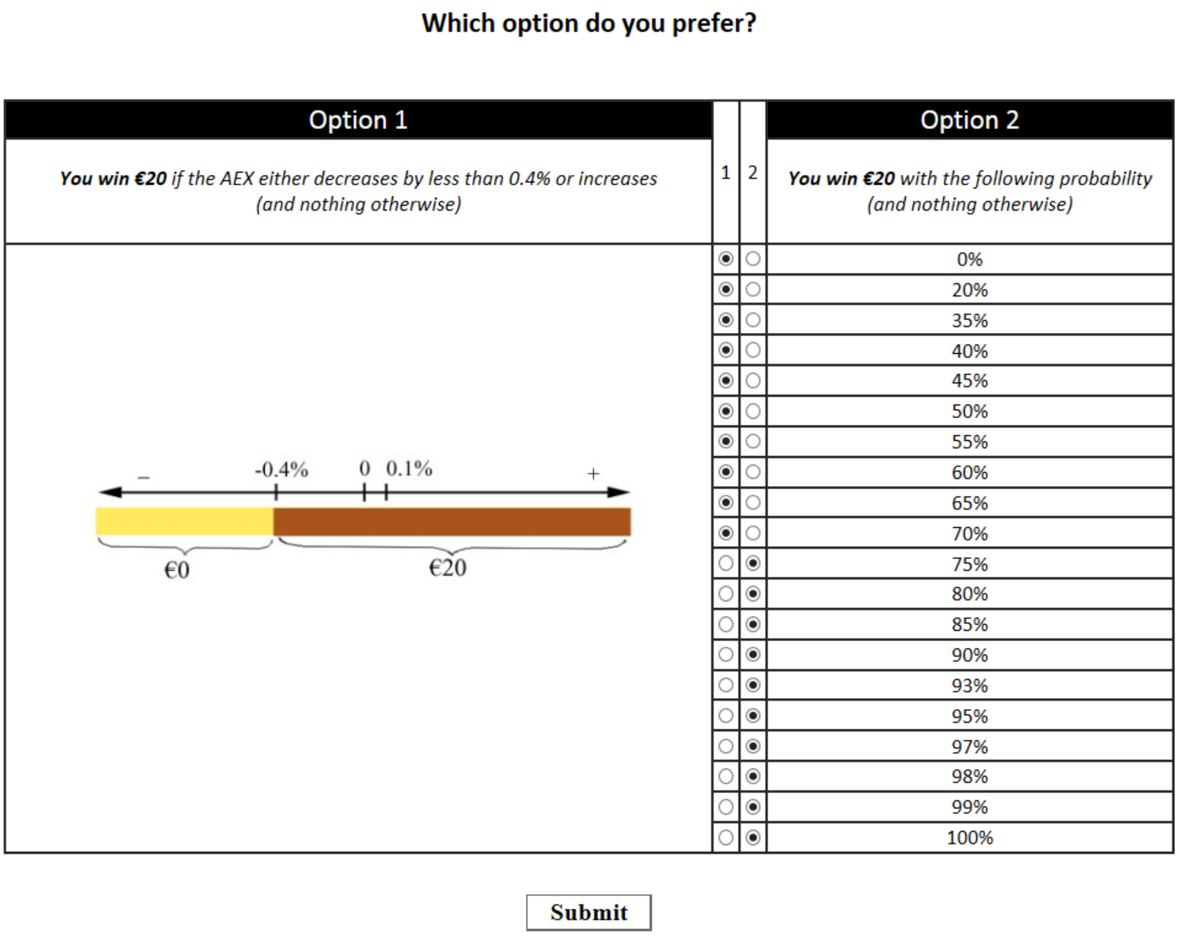
**Which option do you prefer?**

| Option 1 | 1 | 2 | Option 2 |
|---|---|---|---|
| ***You win €20*** *if the AEX increases by strictly more than 0.1% (and nothing otherwise)* | | | ***You win €20*** *with the following probability (and nothing otherwise)* |
|  | ◉ | ○ | 0% |
| | ◉ | ○ | 1% |
| | ◉ | ○ | 2% |
| | ◉ | ○ | 5% |
| | ◉ | ○ | 10% |
| | ◉ | ○ | 15% |
| | ◉ | ○ | 20% |
| | ◉ | ○ | 25% |
| | ◉ | ○ | 30% |
| | ◉ | ○ | 35% |
| | ○ | ◉ | 40% |
| | ○ | ◉ | 45% |
| | ○ | ◉ | 50% |
| | ○ | ◉ | 55% |
| | ○ | ◉ | 60% |
| | ○ | ◉ | 65% |
| | ○ | ◉ | 70% |
| | ○ | ◉ | 75% |
| | ○ | ◉ | 85% |
| | ○ | ◉ | 100% |



**Submit**

**Figure 3.2: Screenshot of the experiment software for composite event $E_{23}$ in Part 1**




*Stimuli: Time pressure*

TP was imposed by setting a 25-second time limit by which subjects had to submit their choices. A timer was displayed showing the time left to answer. If subjects failed to submit their choices before the time limit expired, their choices would be registered but not be paid. This happened only 5 out of the 496 times (62 subjects × 8 choices). In a pilot, the average response time without TP was 36 seconds, and another session of the pilot showed that, under a 30-second time limit, subjects did not experience much TP. Therefore, we decided to set the time limit to 25 seconds.

*Stimuli: Uncertain events*

In each part we consider a triple of mutually exclusive and exhaustive events, called *single events*, and unions of pairs of such events, called *composite events.* See Table 3.2.

**Table 3.2: Single AEX-change events for different parts**

| | Event E1 | Event E2 | Event E3 |
|---|---|---|---|
| Part 1 | $(-\infty,-0.4)$ | $[-0.4,0.1]$ | $(0.1,\infty)$ |
| Part 2 | $(-\infty,-0.2)$ | $[-0.2,0.2]$ | $(0.2,\infty)$ |
| Part 3 | $(-\infty,-0.1)$ | $[-0.1,0.3]$ | $(0.3,\infty)$ |

For each part, we considered all six nontrivial events generated by unions of the three single events, of which two were repeated to test consistency. This results in eight questions per part. The order in which the subjects faced the corresponding prospects was randomized for each subject within each part. Table 3.3 lists all events.

**Table 3.3: List of events on which the AEX prospects were based**

| Part | Prospect number | Event | Event description |
|---|---|---|---|
| 1 (Training) | 1 | $E_1$ | the AEX decreases by strictly more than 0.4% |
| | 2 | $E_1$ | the AEX decreases by strictly more than 0.4% |
| | 3 | $E_2$ | the AEX either decreases by less than 0.4% or increases by less than 0.1% |
| | 4 | $E_3$ | the AEX increases by strictly more than 0.1% |
| | 5 | $E_{12}$ | the AEX either increases by less than 0.1% or decreases |
| | 6 | $E_{23}$ | the AEX either decreases by less than 0.4% or increases |
| | 7 | $E_{23}$ | the AEX either decreases by less than 0.4% or increases |
| | 8 | $E_{13}$ | the AEX either decreases by strictly more than 0.4% or increases by strictly more than 0.1% |
| 2 | 1 | $F_1$ | the AEX decreases by strictly more than 0.2% |
| | 2 | $F_2$ | the AEX either decreases by less than 0.2% or increases by less than 0.2% |
| | 3 | $F_2$ | the AEX either decreases by less than 0.2% or increases by less than 0.2% |
| | 4 | $F_3$ | the AEX increases by strictly more than 0.2% |
| | 5 | $F_{12}$ | the AEX either increases by less than 0.2% or decreases |
| | 6 | $F_{12}$ | the AEX either increases by less than 0.2% or decreases |
| | 7 | $F_{23}$ | the AEX either decreases by less than 0.2% or increases |
| | 8 | $F_{13}$ | the AEX either decreases by strictly more than 0.2% or increases by strictly more than 0.2% |
| 3 | 1 | $G_1$ | the AEX decreases by strictly more than 0.1% |
| | 2 | $G_2$ | the AEX either decreases by less than 0.1% or increases by less than 0.3% |
| | 3 | $G_3$ | the AEX increases by strictly more than 0.3% |
| | 4 | $G_3$ | the AEX increases by strictly more than 0.3% |
| | 5 | $G_{12}$ | the AEX either increases by less than 0.3% or decreases |
| | 6 | $G_{23}$ | the AEX either decreases by less than 0.1% or increases |
| | 7 | $G_{13}$ | the AEX either decreases by strictly more than 0.1% or increases by strictly more than 0.3% |
| | 8 | $G_{13}$ | the AEX either decreases by strictly more than 0.1% or increases by strictly more than 0.3% |

*Stimuli: Avoiding middle bias*

A problem that using choice lists brings with it is the middle bias: subjects mostly choose the options, in our case the preference switch, that are located in the middle of the range provided (Erev & Ert 2013; Poulton 1989). TP can be expected to augment this bias. Had we used a common equally-spaced choice list with, say, 5% incremental steps, then the middle bias would have moved matching probabilities in the direction of 50% (both for the single and composite events). This bias would have enhanced the main phenomenon found in this paper, a-insensitivity, and render our findings less convincing. To get around this problem, we designed choice lists that are not equally spaced. In our design, the middle bias enhances matching probabilities 1/3 for single events and probabilities 2/3 for composite events. Thus, the middle bias enhances additivity of the matching probabilities, decreases a-insensitivity and moves our a-insensitivity index towards 0. It makes findings of nonadditivity and a-insensitivity all the more convincing.

*Stimuli: Further questions*

At the end of the experiment, subjects were asked to report their age, gender, and nationality, and to assess their knowledge of the AEX index from 1 ("I don't know this index at all") to 5 ("I know this index very well"). The median self-assessed knowledge was 2 and the maximum was 4, suggesting that most subjects were unfamiliar with the events, which would further enhance the experience of ambiguity.

*Incentives*

Subjects received a show-up fee of €5. For each subject, one preference (i.e., one row of one choice list) was randomly selected to be played for real at the end of the experiment. If subjects preferred the bet on the stock market index, then the outcome was paid according to the change in the stock market index during the duration of the experiment. Bets on the given probabilities were settled using dice. In the instructions of the experiment, subjects were presented with two examples to familiarize them with the payment scheme. If the time deadline for a TP question had not been met, the worst outcome (no payoff) resulted. Therefore, it was in the subjects' interest to submit their choices on time.

*Analysis*

We will first analyze response time, to verify that subjects answered faster in the TP treatment. We will then turn to matching probabilities. As the first part served to familiarize subjects with the tasks, we will only analyze the matching probabilities of Part 2 and 3. For some events we elicited the matching probabilities twice to test for consistency, since TP can be expected to decrease consistency. In the rest of the analysis, we only use the first matching probability elicited for each event. By monotonicity, the matching probability of a composite event should not be below the matching probability of either one of its two constituents. Thus, we can test monotonicity six times in each part. We will run non-parametric analysis to study whether time pressure had an impact on monotonicity violations.

We computed ambiguity aversion and a-insensitivity indexes as explained in Section 3.2. A-insensitivity predicts that matching probabilities of single events, $m_1$, $m_2$, and $m_3$, are higher than they are under ambiguity neutrality (when their average is 1/3) and a-insensitivity is increasing in these values. The matching probabilities of composite events, $m_{12}$, $m_{13}$, $m_{23}$, are lower than they are under ambiguity neutrality (when their average is 2/3), and it is decreasing in these values. Ambiguity aversion predicts that all matching probabilities are relatively low, and our index is decreasing in all of these. We will use an ANOVA and two-sided t-tests to study the impact of time pressure. As robustness checks, we will then run an ANCOVA (adding control variables). Moreover, 5 subjects in the TP treatment did not submit one of their matching probabilities on time. We did not exclude these subjects from our analyses, although excluding them would not affect any of our conclusions.

## 3.4 Experiment: Results

### *3.4.1 Response time*

The average response time in the training part is more than 25 seconds, but it strongly decreases in both the control and the TP treatment in the second part. An ANOVA for repeated measures (with treatment as a between-subject factor, part and event as within subject factors, and with the interaction of treatment with the two within-subject factors) shows that only the part in which an answer was made and the interaction of part with treatment had a significant impact on the response time (part: $\mathbf{F_2}$=118.73 p<0.01; part*treatment: $\mathbf{F_2}$=6.24, p<0.01). Subsequent pairwise comparisons using t-tests with the Bonferroni correction for multiple testing show: (a) For the control treatment, the response time is highest for Part 1 ( $\mathbf{t_{101}}$ = 6.00 and 7.22 against Parts 2 and 3 respectively, p < 0.01 both times), but it does not differ between Parts 2 and 3 ($\mathbf{t_{101}}$ = 2.12, p = 0.11); (b) for the TP treatment, the response time is highest in Part 1 ($\mathbf{t_{101}}$ = 12.28 and 10.00 against Parts 2 and 3, respectively, p < 0.01in both cases) and is lowest in Part 2 ($\mathbf{t_{101}}$ = −2.57, p = 0.04 against Part 3). The response time of the TP treatment differs from the response time of the control treatment only in Part 2, where it is shorter as would be expected ($\mathbf{t_{101}}$ = 3.21, p < 0.01).

**Table 3.4: Response time**

| | | Part 1 | Part 2 | Part 3 |
|---|---|---|---|---|
| TP treatment | Average | 28.14 | 12.72 | 15.04 |
| | Median | 21.00 | 12.00 | 12.00 |
| | STD | 21.98 | 5.27 | 11.92 |
| | Percentage ≥ 25s | 40 | 0 | 14 |
| Control treatment | Average | 25.79 | 16.63 | 14.30 |
| | Median | 20.00 | 13.00 | 11.00 |
| | STD | 19.24 | 13.47 | 11.58 |
| | Percentage ≥ 25s | 35 | 15 | 10 |

### *3.4.2 Consistency and monotonicity*

An ANOVA for repeated measures (with treatment as a between-subject factor, event and repetition as within subject factors, and with the interaction of treatment with the two within-subject factors) shows that neither repetition (whether the matching probability was elicited the first time or the second time) nor its interaction with treatment is significant. If we still carry out pairwise comparisons with the Bonferroni correction, we find one difference, in one of the two tests in Part 2 for the TP treatment: the second matching probability m13 is higher than the first one (mean difference = 0.03; $t_{102} = -2.22$; $p = 0.03$). The other differences are not significant.

A similar pattern is found within the monotonicity tests. Out of 6 monotonicity checks, the average number of violations is 0.60 in Part 2 for the TP treatment, while it is only 0.31 in Part 3 for the same treatment and 0.34 and 0.16 in Parts 2 and 3, respectively, for the control treatment. The difference is marginally significant in the between-subject test (Mann-Whitney U test comparing the TP treatment and the control treatment in Part 2; $Z = -1.79$, $p = 0.07$[10]) and significant in the within-subject test (Wilcoxon signed-ranks test comparing Part 2 and Part 3 for the TP treatment; $Z = -2.31$, $p = 0.03$).

[10] The difference is significant under one-sided testing, which is justified here because the direction of difference was predicted.

### *3.4.3 Ambiguity attitudes*

Surprisingly, we do not find ambiguity aversion, and cannot reject the null of ambiguity neutrality, in any treatments or parts. There is even a trend towards ambiguity seeking, reaching marginal significance in Part 2 of the TP treatment and Part 3 of the control treatment. By Observation A.2, our result also falsifies the ambiguity aversion predicted by Dow & Werlang (1992), Schmeidler (1989), and others. Table 3.5 displays corresponding statistics. An ANOVA for repeated measures (with treatment as a between-subject factor, and part as a within-subject factor, and with their interaction) shows that only the interaction of part with treatment had an impact on the ambiguity aversion index b ($\mathbf{F_1}$ = 6.13, p = 0.02). Subsequent pairwise comparisons show that index b is lower in Part 3 than in Part 2 ($\mathbf{t_{102}}$ = 2.10, p = 0.04) for the control treatment. Thus, subjects of the control treatment became slightly more ambiguity seeking in Part 3. Adding age, gender, nationality (Dutch / non-Dutch) and self-assessed knowledge of the AEX index as control variables do not change any of the results. As a by-product, age turns out to be significant in the ANCOVA, ($\mathbf{F_1}$ = 4.97, p = 0.03) with older subjects having higher b indexes.

**Table 3.5: ambiguity aversion indices b**

| | | Part 2 | Part 3 |
|---|---|---|---|
| TP treatment | Average | −0.08 | −0.06 |
| | Median | −0.07 | −0.06 |
| | STD | 0.24 | 0.23 |
| | *t-statistics* | −2.65 | −1.91 |
| | *df* | 61 | 61 |
| | *p* | 0.01 | 0.06 |
| Control treatment | Average | −0.07 | −0.11 |
| | Median | −0.08 | −0.10 |
| | STD | 0.21 | 0.24 |
| | *t-statistics* | −2.01 | −3.01 |
| | *df* | 41 | 41 |

| | | | |
|---|---|---|---|
| | *p* | 0.051 | < 0.01 |

We find a-insensitivity (a > 0) in all questions (see Table 3.6). The insensitivity index is between 0.15 and 0.17 for Parts 2 and 3 of the control treatment, and also for Part 3 of the TP treatment. An ANOVA for repeated measures (with treatment as a between-subject factor, and part as a within-subject factor, and with their interaction) shows that the part in which the a-insensitivity index was measured and the interaction of part with treatment had an impact on indexes a (part: $F_1 = 5.89$, $p = 0.02$; part*treatment: $F_1 = 9.17$, $p < 0.01$). Subsequent pairwise comparisons confirm that there is much more a-insensitivity for the TP questions (Part 2 of TP treatment), with a = 0.35. It is different both in a between-subject comparison (Part 2 of control treatment versus TP treatment; $t_{102} = -2.29$, $p = 0.02$) and in a within-subject comparison (Part 2 versus Part 3 of TP treatment; $t_{102} = -4.33$, $p < 0.01$). The direction and the significance of our results on a-insensitivity are robust to the addition of control variables (age, gender, nationality, and knowledge of the AEX index).

**Table 3.6: a-insensitivity indexes a**

| | | Part 2 | Part 3 |
|---|---|---|---|
| TP treatment | Average | 0.35 | 0.17 |
| | Median | 0.35 | 0.11 |
| | STD | 0.44 | 0.45 |
| | *t-statistics* | 6.21 | 2.96 |
| | *df* | 61 | 61 |
| | *p* | < 0.01 | < 0.01 |
| Control treatment | Average | 0.15 | 0.17 |
| | Median | 0.07 | 0.21 |
| | STD | 0.43 | 0.41 |
| | *t-statistics* | 2.21 | 2.67 |
| | *df* | 41 | 41 |

| | | | |
|---|---|---|---|
| | $p$ | 0.03 | 0.01 |

### 3.4.4 *Summary and discussion of experiment*

The results on response times, consistency, and monotonicity all confirm Ariely & Zakay's (2001) observation that TP aggravates biases and irrationalities: As regards response times, subjects need more time in the training questions, and use less time in the TP questions. Consistency is violated only in the TP questions, and violations of monotonicity are observed most frequently in the TP questions. The enhanced irrationalities and biases increase a-insensitivity (index a) in the TP questions, but do not affect ambiguity aversion (index b).

All results on TP questions are confirmed by both between-subject and within-subject analyses. The absence of ambiguity aversion is not surprising in view of the recent studies that also found similar deviations, especially since we used natural events rather than Ellsberg urns, which increases ambiguity aversion due to contrast effects (Fox & Tversky 1995). Finally, the reduction of ambiguity aversion in Part 3 of the control treatment suggests that learning increases familiarity, in agreement with the familiarity bias (Chew, Ebstein, & Zhong 2012; Fox & Levav 2000; Kilka & Weber 2001).

Similar to our results, Young et al. (2012) also found that TP increases insensitivity in their context of risk, but only over the loss domain. For gains, they found no significant change in insensitivity. The effects of TP on risk aversion are not clear and can go either direction (Young et al. 2012; Kocher, Pahlke, & Trautmann 2013). Kocher et al. (2013) find increased insensitivity towards outcomes under TP.

## 3.5 Conclusion

We have investigated the effects of TP on ambiguity attitudes. TP does not affect ambiguity aversion, but leads to an increase in insensitivity (treating events too much as "fifty-fifty"), as confirmed by both within and between subject analyses. Our findings provide clear evidence that the cognitive insensitivity component of ambiguity is important, and that it is more closely associated with cognitive limitations and bounded rationality than the ambiguity aversion component is. Given the ubiquity of TP and ambiguity, our findings contribute to the understanding of human decisions.

## Appendix 3: Estimating Ambiguity Attitudes

Our technique with triples of disjoint events shows a new way to conveniently measure ambiguity attitudes using the matching-probability function m. Under most nonexpected utility theories, m captures the ambiguity attitude (Wakker 2010 Example 11.2.2; Dimmock, Kouwenberg, & Wakker 2013 Theorem 5.1). The lower m is, the more ambiguity aversion there is, and the more inverse-S shaped m is, the more insensitivity there is. We next consider Abdellaoui et al.'s (2011) indexes, applied separately to triples $E_i$, $E_j$, and $E_{ij}$. For their calculations on general data sets, Abdellaoui et al. use best approximations

$$m = \tau + \sigma P(E), \quad (A.1)$$

say by minimizing quadratic distances, as in regular regressions. Here $\sigma > 0$ and $\tau$ are constants, and P is an additive probability measure. P need not be a subjective probability reflecting any subjective state of belief, but it is purely choice-based, justified by Chew & sagi's (2008) exchangeability conditions.

In our data set, we can directly obtain an estimation of $\tau$ from $E_1$, $E_2$, and $E_{12}$ (without estimating the P(E)s and quadratic fitting), from the equation, implied by Eq. A.1:

$$m1 + m2 - m12 = \tau . \quad (A.2)$$

Proof: With Eq. A.1 as a deterministic model, the left-hand side equals $\tau + \sigma P(E_1) + \tau + \sigma P(E_2) - (\tau + \sigma P(E_{12})) = \tau$. 〈

We similarly obtain a second estimation of τ from $E_1$ and $E_3$, and a third one from $E_2$ and $E_3$. Our final estimation of τ is the average of these three estimations, with $\overline{m_s}$ and $\overline{m_c}$ defined in §3.2:

$$\tau = 2\overline{m_s} - \overline{m_c}\,. \tag{A.3}$$

With τ available, we obtain an estimation of σ from $E_1$ and $E_{23}$ by

$$m_1 + m_{23} = 2\tau + \sigma\,. \tag{A.4}$$

PROOF: In a deterministic model, the left-hand side equals $\tau + \sigma P(E_1) + \tau + \sigma P(E_{23}) = 2\tau + \sigma$. 〈

We similarly obtain a second estimation from $E_2$ and $E_{13}$, and from $E_3$ and $E_{12}$. Our final estimation is, again, the average of these three estimations:

$$\sigma = 3\overline{m_c} - 3\overline{m_s}\,. \tag{A.5}$$

The ambiguity aversion index of Abdellaoui et al. (2011, Eq. 9), shown below to be equivalent to our b, is defined as

$$b' = 1 - 2\tau - \sigma. \tag{A.6}$$

It is a linear transform of the area below the fitting line $\tau + \sigma p$. Similarly,

$$a' = 1 - \sigma \tag{A.7}$$

is the index of a-insensitivity (Abdellaoui et al. Eq. 8). It is an anti-index of the steepness of the fitting line $\tau + \sigma p$, reflecting how sensitive the decision maker is to changes in (likelihoods of) events.

We next express a´ and b´ directly in terms of the observed matching probabilities, showing that they are identical to our indices a and b.

OBSERVATION A.1. $a' = 1 + 3\overline{m_s} - 3\overline{m_c} = a$; $b' = 1 - \overline{m_c} - \overline{m_s} = b$.

PROOF. $a' = 1 - \sigma = 1 - \overline{m_s} - \overline{m_c} + 2\tau = 1 - \overline{m_s} - \overline{m_c} + 2\overline{m_s} + 2\overline{m_s} - 2\overline{m_c} = 1 + 3\overline{m_s} - 3\overline{m_c} =$ a.

$b' = 1 - 2\tau - \sigma = 1 - 2(\overline{m_s} + \overline{m_s} - \overline{m_c}) - 3\overline{m_c} + 3\overline{m_s} = 1 - \overline{m_c} - \overline{m_s} = b.$ 〈

Thus we immediately obtain plausible estimations of Abdellaoui et al.'s (2011) indices. Unlike Abdellaoui et al. (2011) and Dimmock, Kouwenberg, & Wakker (2015), we need not specify the subjective probabilities P, because they cancel anyhow, and we need not carry out data fitting. Therefore, the indexes can be used in applications, as explained in §3.2, without knowing the mathematics and parameters of the underlying theories in detail.

We briefly note that Schmeidler's (1989) model, under ambiguity aversion, predicts $b > 0$, in agreement with our interpretation of ambiguity aversion. It assumes expected utility for risk and, hence for each event E the nonadditive measure W(E) is equal to the matching probability m(E). Convexity of W, which in his model is equivalent to ambiguity aversion (or uncertainty aversion as he called it) implies that $W(E_1) + W(E_{23}) < 1$.

The general index of ambiguity aversion (with $E^c$ the complement to E)

$$1 - W(E) - W(E^c) \tag{A.8}$$

was suggested by Schmeidler (1989 pp. 572, 574) in a particular example, and was suggested in general by Dow & Werlang (1992) and others (§3.2), always in models assuming expected utility for risk. Our index b is this index, averaged over $\{E_1, E_{23}\}$, $\{E_2, E_{13}\}$, and $\{E_3, E_{12}\}$.

OBSERVATION A.2. Our ambiguity aversion index b results from Eq. A.8. Under Schmeidler (1989), ambiguity aversion implies $b > 0$, ambiguity neutrality implies $b = 0$, and ambiguity seeking implies $b < 0$. 〈

Many authors have used parts of Observation A.2, invariably assuming expected utility for risk, which implies that matching probabilities equate the nonadditive decision weighting functions. Interpreting a value 0 in Eq. A.8 as ambiguity neutrality of the decision maker, or nonambiguity of the event E, occurred in Ghirardato &

Marinacci (2002 Proposition 22), Ivanov (2011), and others. Interpreting a positive value as ambiguity aversion of the decision maker, or as ambiguity of the event (usually in approaches that focused on ambiguity aversion), occurred in Klibanoff, Marinacci, & Mukerji (2005 Definition 7) and many others.

# Chapter 4 | Belief Updating under Ambiguity[11]

## 4.1 Introduction

Belief updating is at the heart of decision theory and statistical theory, and has been applied to economic theory and artificial intelligence. The traditional Bayesian updating approach assumes ambiguity neutral behavior, and provides a simple method to calculate posterior probabilities based on prior beliefs and received information. Its decision principles leads to expected utility (EU; de Finetti 1937, Savage 1954). However many empirical findings have demonstrated violations of the EU model. One of the biggest empirical problems for Bayesian updating is that it can only accommodate ambiguity neutral behavior due to the additivity of subjective probability. Neither ambiguity averse nor ambiguity seeking behavior can be accommodated. Much literature (Gilboa & Marinacci, 2013; Trautmann & Van de Kuilen 2015) has shown that a large proportion of people are ambiguity averse, with also considerable ambiguity seeking, showing the need to develop non-Bayesian approaches. Dempster (1967) and Shafer (1976) provided a famous non-Bayesian updating rule. They used non-additive measure of belief and weighting functions to represent perceived likelihood. Another rule was provided by Gilboa and Schmeidler (1993)

The three aforementioned belief updating rules are the most popular ones in decision theory, where both Bayesian and non-Bayesian updating rules have their pros and cons. The Bayesian approach avoids the problem of arbitrage and is often regarded as a benchmark for rationality. The non-Bayesian approaches require that likelihoods of events are represented by weights rather than by subjective probabilities. These features enable non-Bayesian approaches to allow heterogeneity in ambiguity attitudes. According to some authors, non-Bayesian approaches can be rational (Gilboa and Marinacci 2013, Schmeidler 1989).

[11] Professor Peter P. Wakker made helpful comments and suggestions

This chapter studies belief updating under ambiguity. Using the framework of decision theory, we compare Bayesian with non-Bayesian updating in its model specification and the numerical implications. This chapter is organized as follows. Section 4.2 describes Gilboa's (1987) and Schmeidler's (1989) rank dependent utility for ambiguity, which agrees with cumulative prospect theory for gains under ambiguity (Tversky & Kahneman, 1992), and which is the framework we use to study belief updating. Section 4.3 presents different models for belief updating. A numerical analysis of the three updating approaches is in Section 4.4. Section 4.5 discusses the results and their implications. Section 4.6 concludes.

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## 4.2 Rank Depdendent Utility

### *4.2.1 Definition*

Rank-dependent-utility (RDU; Gilboa 1987; Schmeidler 1989) is a model for decision under ambiguity and it is one of the most popular models to capture deviations from the classical expected utility model (Savage 1954). It can accommodate both ambiguity averse and ambiguity-seeking behavior. The model was first introduced by Quiggin (1982) for decision under risk (known probabilities). Gilboa (1987) and Schmeidler (1989) introduced it more generally for decision under uncertainty (unknown probabilities). This paper focuses on ambiguity and, hence, follows Schmeidler's (1989) assumption of expected utility for risk. Thus, deviations from expected utility are generated by ambiguity. This is the prevailing assumption in studies of ambiguity in the modern literature, leading to the Anscombe-Aumann (1963) model for ambiguity (Gilboa & Marinacci 2013).

The basic idea of RDU is that decision makers use nonadditive weighting functions rather than subjective probabilities. They may therefore overweight or underweight some events relative to subjective probabilities. This paper uses RDU to analyze choices between prospects. In particular, RDU is used to calculate the *certainty equivalent* (CE) of a prospect, i.e. a guaranteed return that is indifferent to the prospect for a decision maker.

We first introduce the general RDU model. Let S be a *state space* with its elements *states (of nature)* and *events* as its subsets. One state $\mathbf{s} \in \mathbf{S}$ is true, and the other states are not true. An event is *true* if it contains the true state. Consider a prospect

$$(\mathbf{E_1}:\mathbf{x_1}, \ldots, \mathbf{E_n}:\mathbf{x_n}) \qquad (\mathbf{4.1})$$

yielding outcome $\mathbf{x_j}$ when event $\mathbf{E_j}$ happens, $\mathbf{j = 1, \dots n}$. The events $\mathbf{E_1, \dots E_n}$ partition the state space $\mathbf{S}$. In this paper, *outcomes* are real numbers designating money. The prospect designates a state-contingent outcome, i.e., the outcome depends on which state of nature is true. However, a decision maker does not know for sure which state is the true one and cannot influence it.

We next turn to defining the RDU functional. For this purpose, we assume that the prospect in Eq. (1) is rank-ordered: $\mathbf{x_1 \geq \cdots \geq x_n}$. This can always be arranged by renumbering the events. For event $\mathbf{E_j}$, the *rank* is defined as $\mathbf{E_1 \cup \dots \cup E_{j-1}}$, which is the union of the events of receiving an outcome ranked better than $\mathbf{x_j}$. The prospect in Eq. (1) is evaluated by the following formula:

$$\sum_{j=1}^{n} \pi_j \, U(x_j). \qquad (4.2)$$

$\mathbf{U}$ is the *utility function* and the $\boldsymbol{\pi_j}$**s** are *decision weights*. For simplicity, this paper assumes linear utility, i.e., $\mathbf{U(x) = x}$, so as to focus on the novelty of ambiguity. The assumption of linear utility can alternatively be justified by the Anscombe-Aumann (1963) approach where outcomes are lotteries. For adherence to conventions in finance and, again, for simplicity, this paper focus on monetary outcomes. Decision weights are nonnegative and sum to 1, and are defined later in terms of a weighting function. Under linear utility, the CE of a prospect is also the price the decision maker is willing to pay to receive the prospect, which is useful for applications in finance.

By definition, a *weighting function* $\mathbf{W}$ maps events to [0,1] and satisfies the following three conditions:

$$(1)\ \mathbf{W(\phi) = 0},\ (2)\ \mathbf{W(S) = 1},\ (3)\ \textbf{If } \mathbf{E \supseteq F}, \textbf{ then } \mathbf{W(E) \geq W(F)}.$$

The *decision weight* $\boldsymbol{\pi_j}$ in Eq. (4.2) is defined as follows:

$$\pi_j = W(E_1 \cup \ldots \cup E_j) - W(E_1 \cup \ldots \cup E_{j-1}). \qquad (4.3)$$

An implication is that the decision weight $\pi_j$ of receiving outcome $x_j$ depends both on the event $E_j$ and on the rank. Such rank dependence can be used to model pessimism (overweighting bad outcomes), for instance, enhancing ambiguity aversion (Schmeidler 1989), which is the topic of the chapter.

### 4.2.2 *Special case of Rank Dependent Utility*

For many years it was generally thought that, because of Ellsberg's (1961) paradox, subjective probabilities could not be used in any manner to analyze ambiguity. Chew & Sagi (2008) demonstrated, surprisingly, that probabilities can still be used to analyze ambiguity, by generalizing an implicit assumption made up to that point: Chew and Sagi assumed that probabilities can be weighted differently for different kinds of events. Under ambiguity aversion, probabilities of ambiguous events will be weighted more pessimistically than objective probabilities. Their approach implies that Machina & Schmeidler's (1992) probabilistic sophistication does not hold globally, but it may still hold locally when restricted to subcollections of events, related to some suited sources of uncertainty. This will be the case considered in this paper.

Chew & Sagi's discovery was the basis of the source method of Abdellaoui et al. (2011), and will be used in this paper. We describe it briefly, referring the reader to Abdellaoui et al. (2011) for full details. For our model, Chew & Sagi's assumption amounts to the existence of a function **w** and a probability measure P such that

$$W(.) = w(P(.)) \qquad (4.4)$$

where P satisfies additivity (Wakker 2010 Exercise 10.3.2). The function $w(.)$, carrying subjective probabilities to decision weights, is called the *source function*.

Intuitively, w reflects the deviation from Bayesian beliefs. We call the model in Eq. (4.4) *CS-RDU*.

For simplicity, we use one of the most common forms of the w function, the power function, i.e., $w(p) = p^r$, to capture ambiguity aversion or seeking. When $r = 1$, the source function reduces to linear function and RDU reduces to SEU, i.e., ambiguity neutrality. When $r > 1$, the source function becomes convex, exhibiting a general underweighting of probabilities implying ambiguity aversion. When $r < 1$, the source function becomes concave, exhibiting a general overweighting of probabilities and ambiguity seeking. This paper will consider three different cases of r, specifically, $r = 0.8$, 1, and 1.2.

The source method is a restriction of Schmeidler's (1989) RDU. It has been found that RDU is too general; i.e., there exist too many nonadditive weighting functions W whenever the state space is not very small. The source method has been shown to be a sufficiently tractable specification (Kothiyal, Spinu, & Wakker 2014).

### 4.2.3 *Two probability measures*

We consider three exhaustive and mutually exclusive events A, B and C. Say A designates a stock price going up by more than 0.4%, C designates the stock price going down by more than 0.6%, and B designates the event between these two. We assume that we can consider events A, B and C each day again, and in this respect they are timeless. To distinguish them from events formalized later that will be time specific, we call A, B and C *generic events*.

This chapter assumes that a decision maker observes events at the end of the day and then updates her beliefs. Therefore, in day 0 (assumed as today), the decision maker does not observe any new events before the end of that day. Her belief then, during that day before its end, is based only on her prior beliefs. At the start of day 1, she has observed the event at the end of day 0, and her beliefs have been updated using the new information. Similarly, the decision maker observes new information at the end of day 1, and her beliefs in the last day that we consider, day 2, are updated based on the information she now has. For simplicity, we restrict the discussion of belief updating to two observations of new information. Three different approaches to update belief are discussed below.

We use symbols $\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\gamma}$ to denote general elements of $\{\mathbf{A}, \mathbf{B}, \mathbf{C}\}$. Further, $\boldsymbol{\alpha_0}$ denotes the generic event $\boldsymbol{\alpha}$ realized in day 0, $\boldsymbol{\beta_1}$ the generic event $\boldsymbol{\beta}$ realized in day 1, $\boldsymbol{\gamma_2}$ the generic event $\boldsymbol{\gamma}$ realized in day 2, and so on. In each day, one of the generic events is realized, so that a state (of nature) $\mathbf{s} \in \mathbf{S}$ can be expressed by $(\boldsymbol{\alpha_0}, \boldsymbol{\beta_1}, \boldsymbol{\gamma_2})$, for instance. The state space $\mathbf{S}$ consists of all 27 such triples (3 generic events per day, 3 days in total, i.e., $\mathbf{3^3 = 27}$ states), which are displayed in Tables 4.1 and 4.2 below. An example of an event is:

$$\beta_1 = \{(A_0, \beta_1, A_2), (A_0, \beta_1, B_2), (A_0, \beta_1, C_2), (B_0, \beta_1, A_2),$$

$$(B_0, \beta_1, B_2), (B_0, \beta_1, C_2), (C_0, \beta_1, A_2), (C_0, \beta_1, B_2), (C_0, \beta_1, C_2)\}$$

For *P* as in Eq. (4.4), $P(\alpha_0)$ denotes the probability of the realization of generic event $\alpha$ in day 0. Similarly, $P(\alpha_0, \beta_1)$ is the probability of the realization of generic event α in day 0 and β in day 1, and $P(\alpha_0, \beta_1, \gamma_2)$ means the probability of the realization of generic event α in day 0, β in day 1, and γ in day 2.

This paper considers two probability measures and the probabilities in day 0 are $P(A_0) = P(B_0) = P(C_0) = \frac{1}{3} \cong 0.33$ and $P(A_0) = 0.6, P(B_0) = 0.23, P(C_0) = 0.17$, respectively. Their extension to more general events, specifying what happens on day 1 and 2, is given in the following tables. This extension is based on an assumed Dirichlet process. This process is based on the beta family of probabilities and their multinomial extension (Wilks 1962), and is a widely used process model. Further explanations are in Appendix 4.1.

**Table 4.1: States of nature and their probabilities**

when $P(A_0) = P(B_0) = P(C_0) = \frac{1}{3} \cong 0.333$

| | | |
|---|---|---|
| $P(A_0, A_1, A_2) = 0.123$ | $P(B_0, A_1, A_2) = 0.031$ | $P(C_0, A_1, A_2) = 0.031$ |
| $P(A_0, A_1, B_2) = 0.031$ | $P(B_0, A_1, B_2) = 0.031$ | $P(C_0, A_1, B_2) = 0.012$ |
| $P(A_0, A_1, C_2) = 0.031$ | $P(B_0, A_1, C_2) = 0.012$ | $P(C_0, A_1, C_2) = 0.031$ |
| $P(A_0, B_1, A_2) = 0.031$ | $P(B_0, B_1, A_2) = 0.031$ | $P(C_0, B_1, A_2) = 0.012$ |
| $P(A_0, B_1, B_2) = 0.031$ | $P(B_0, B_1, B_2) = 0.123$ | $P(C_0, B_1, B_2) = 0.031$ |
| $P(A_0, B_1, C_2) = 0.012$ | $P(B_0, B_1, C_2) = 0.031$ | $P(C_0, B_1, C_2) = 0.031$ |
| $P(A_0, C_1, A_2) = 0.031$ | $P(B_0, C_1, A_2) = 0.012$ | $P(C_0, C_1, A_2) = 0.031$ |
| $P(A_0, C_1, B_2) = 0.012$ | $P(B_0, C_1, B_2) = 0.031$ | $P(C_0, C_1, B_2) = 0.031$ |
| $P(A_0, C_1, C_2) = 0.031$ | $P(B_0, C_1, C_2) = 0.031$ | $P(C_0, C_1, C_2) = 0.123$ |

**Table 4.2: States of nature and their probabilities**

when $P(A_0) = 0.6, P(B_0) = 0.23, P(C_0) = 0.17$

| | | |
|---|---|---|
| $P(A_0, A_1, A_2) = 0.352$ | $P(B_0, A_1, A_2) = 0.050$ | $P(C_0, A_1, A_2) = 0.037$ |
| $P(A_0, A_1, B_2) = 0.050$ | $P(B_0, A_1, B_2) = 0.034$ | $P(C_0, A_1, B_2) = 0.008$ |
| $P(A_0, A_1, C_2) = 0.037$ | $P(B_0, A_1, C_2) = 0.008$ | $P(C_0, A_1, C_2) = 0.023$ |
| $P(A_0, B_1, A_2) = 0.050$ | $P(B_0, B_1, A_2) = 0.034$ | $P(C_0, B_1, A_2) = 0.008$ |
| $P(A_0, B_1, B_2) = 0.034$ | $P(B_0, B_1, B_2) = 0.069$ | $P(C_0, B_1, B_2) = 0.010$ |
| $P(A_0, B_1, C_2) = 0.008$ | $P(B_0, B_1, C_2) = 0.010$ | $P(C_0, B_1, C_2) = 0.009$ |
| $P(A_0, C_1, A_2) = 0.037$ | $P(B_0, C_1, A_2) = 0.008$ | $P(C_0, C_1, A_2) = 0.023$ |
| $P(A_0, C_1, B_2) = 0.008$ | $P(B_0, C_1, B_2) = 0.010$ | $P(C_0, C_1, B_2) = 0.009$ |
| $P(A_0, C_1, C_2) = 0.023$ | $P(B_0, C_1, C_2) = 0.009$ | $P(C_0, C_1, C_2) = 0.044$ |

The probability of event can be calculated from the 27 states of nature, for example: $P(A_0, B_1) = P(A_0, B_1, A_2) + P(A_0, B_1, B_2) + P(A_0, B_1, C_2)$. In the Dirichlet process and, hence, also in Tables 1 and 2, the probabilities of two states of nature are the same if the total number of each generic event occurring is the same between the two states. For example, $P(A_0, C_1, C_2) = 0.023 = P(C_0, C_1, A_2)$ as both state $(A_0, C_1, C_2)$ and state $(C_0, C_1, A_2)$ contain one time the generic event A and two times the generic event C. Therefore, the probabilities of two events are the same if the total number of each generic event occurring is the same between two events. For example, $P(A_0, C_1) = P(C_0, A_1)$. Appendix 4.1 discusses this point in further detail.

## 4.3 Models for belief updating

This section discusses three different approaches to update a decision maker's beliefs based on observed information. A decision maker is assumed to update her preference in agreement with prior preference (Machina, 1989; Epstein and Le Breton, 1993). For each updating method, we first discuss a general conditional weighting function $W_c$, and then turn to the special case of CS-RDU considered in this paper. All conditional weighting functions that we will consider are all of the following general form:

$$W_c(E|G) = \frac{\pi(E \cap G)}{\pi(G)}. \tag{4.5}$$

Here we have $E, G \subset S$, the $\pi$'s are decision weights, and we have $W_c(.) \in [0,1]$. This general form was defined by Sarin and Wakker (1998). The particular decision weight $\pi$ differs for the different methods, and will be explained for each case. The differences can be interpreted as different assumptions about the ranks of the events.

### *4.3.1 Gilboa and Schmeidler's updating approach*

Gilboa (1989a, 1989b) proposed the following rule to update belief

$$W_g(E|G) = \frac{W(E \cap G)}{W(G)} \tag{4.6}$$

where $E, G \subset S$, W is weighting function and $W_g(.|G)$ denotes the updated weighting function conditional on event G. This updating rule was shown to be plausible and was axiomatized by Gilboa and Schmeidler (1993). It is referred to as *Gilboa and Schmeidler (GS)* updating. The underlying assumption in this rule is that the decision maker assumes that the event G, of which she has been informed, corresponds with the "best of all possible outcomes." The corresponding rank-ordering of events is $E \cap G \succcurlyeq G \backslash E \succcurlyeq G^c$. By applying this rank order in Eq. (4.5), we obtain

$$W_c(E|G) = \frac{\pi(E \cap G)}{\pi(G)} = \frac{W(E \cap G)}{W(G)} = W_g(E|G).$$

This shows that the general conditional weighting function coincides with GS updating if a specific ranking of events is assumed.

If we assume CS-RDU with the source function a power function, i.e. $W(E) = w(P(E)) = P(E)^r$, then G&S updating can be expressed by

$$W_g(E|G) = \frac{W(E \cap G)}{W(G)} = \frac{P(E \cap G)^r}{P(G)^r} = \frac{(P(E|G)P(G))^r}{P(G)^r} = P(E|G)^r. \quad (4.7)$$

The factor $P(G)^r$ appears both in the numerator and the denominator, and cancels. The updated weighting probability is the weighted updated probability. That is, as it so happens, weighting and updating are commutative. This is a feature typical of GS updating that does not hold in general, and that will for instance not hold for DS updating defined later. The conditional weighting function becomes a power function and its concavity depends on the power $r$. For GS updating, an increase in $r$ results directly in a decrease in $W_g(E|G)$. A small change in $r$ has considerable impact on $W_g(E|G)$, which the numerical analysis reported later will confirm.

As appears from Eq. (4.7), the conditional weight is fully determined by the probabilities of events. As discussed at the end of section 4.2.3, probabilities of two events are the same if the total number of each generic event occurring is the same for the two events, e.g., $P(A_0, C_1) = P(C_0, A_1)$. Hence, if $E$ concerns only day $j$, and $G$ or $F$ concerns only the preceding days, $W_g(E|G) = W_g(E|F)$ if the total number of each generic event occurring in $G$ and $F$ is the same. In particular, the order of the observed generic events does not affect conditional weight. This is explained in detail in Appendix 4.2.

Here are two examples to illustrate how the conditional weighting function and the CE of a prospect are calculated. The first example concerns day 0, with no updating involved. The second example concerns day 2 and does involve updating.

EXAMPLE 4.1: We use the change of the AEX (Amsterdam Stock Exchange) index as the source of uncertainty. Let, for some day, A be the generic event that the AEX index increases by more than 0.4% that day. B is the generic event that the AEX index changes between 0.4% and -0.6%, and C is the generic event that the AEX index decreases by more than 0.6%, all on the day considered. Events A, B, and C are mutually exclusive and exhaustive: they partition the universal event.

In each day, if a prospect yields outcome 130 for event A, 110 for event B, and 60 for event C, then A is ranked better than B, which is ranked better than C. If this prospect concerns day 0, then it can be denoted $(\mathrm{A_0}: 130, \mathrm{B_0}: 110, \mathrm{C_0}: 60)$. We assume the prior beliefs in day 0 to be as in Table 2: $P(A_0) = 0.6, P(B_0) = 0.23, P(C_0) = 0.17$. The power $r$ in the weighting function is assumed to be 1.2 and utility is linear, i.e. $U(x) = x$ for simplicity. At the start of day 0, no new information is available, and the weights of the three events and their unions are:

$$W_g(A_0) = \mathrm{P}(A_0)^{\mathrm{r}} = 0.6^{1.2} = 0.54.$$

$$W_g(A_0 \cup B_0) = \mathrm{P}(A_0 \cup B_0)^{\mathrm{r}} = (0.6 + 0.23)^{1.2} = 0.80.$$

Therefore, the CE of the prospect $(A_0: 130, \mathrm{B_0}: 110, \mathrm{C_0}: 60)$ can be calculated according to its RDU value. Following Eq. (4.3), we have:

$$RDU(\mathrm{A_0}: 130, B_0: 110, C_0: 60) = \sum_{j=1}^{n} \pi_j \, U(x_j)$$

$$= W_g(A_0)U(130) + \left[W_g(A_0 \cup B_0) - W_g(A_0)\right]U(110) + \left[1 - W_g(A_0 \cup B_0)\right]U(60)$$

$$= 0.54 \cdot 130 + \left[0.8 - 0.54\right] \cdot 110 + \left[1 - 0.8\right] \cdot 60$$

$$= 110.8$$

The CE of the prospect is 110.8. In other words, a decision maker is willing to pay 110.8 for this prospect. □

Next we discuss the case of day 2 when the information about days 0 and 1 has been obtained.

EXAMPLE 4.2: Assume that generic event B is realized in day 0 and generic event C in day 1. We use $(A_2: 130, \mathrm{B_2}: 110, \mathrm{C_2}: 60)$ to denote the prospect considered in day 2. As in Example 1, we assume the probabilities of Table 2, $r = 1.2$, and linear utility. We apply Eq. (4.7) to find the conditional weighting function[12]:

[12] We assume updating in one stroke in day 2. One could also update in two steps, where the first one updates the weighting function on day 1 given the information received then, and then the second one updates this on day 2 given the extra information received then. For Bayesian updating and for GS updating as considered here, this alternative always gives the same result as

$W_g(A_2|B_0, C_1) = \mathrm{P}(A_2|B_0, C_1)^{\mathrm{r}} = [\frac{P(B_0,C_1,A_2)}{P(B_0,C_1)}]^{1.2} = 0.24.$

$W_g(A_2 \cup B_2|B_0, C_1) = [\frac{P(B_0,C_1,B_2)+P(B_0,C_1,A_2)}{P(B_0,C_1)}]^{1.2} = 0.61.$

Therefore, if the generic event B is realized in day 0 and C in day 1, then the CE of the prospect $(A_2{:}\,130, \mathrm{B}_2{:}\,110, \mathrm{C}_2{:}\,60)$ can be calculated according to its RDU value. Following Eq. (4.3), we have:

$$RDU(\mathrm{A}_2{:}\,130, B_2{:}\,110, C_2{:}\,60) = \sum_{j=1}^{n} \pi_j\, U(x_j)$$

$$= W_g(A_2|B_0, C_1)U(130) + [W_g(A_2 \cup B_2|B_0, C_1) - W_g(A_2|B_0, C_1)]U(110) + [1 - W_g(A_2 \cup B_2|B_0, C_1)]U(60)$$

$$= 0.24 \cdot 130 + [0.61 - 0.24] \cdot 110 + [1 - 0.61] \cdot 60$$

$$= 95.3$$

The CE of prospect $(A_2{:}\,130, \mathrm{B}_2{:}\,110, \mathrm{C}_2{:}\,60)$ is 95.3. In other words, a decision maker is willing to pay 95.3 for this prospect. It is natural that the CE has decreased relative to day 0, as the information received was unfavorable for the prospect considered. □

### **4.3.2** *Dempster and Shafer's updating approach*

Dempster (1967) and Shafer (1976) proposed another rule to update belief:

$$W_d(E|G) = \frac{W\big((E \cap G) \cup G^c\big) - W(G^c)}{1 - W(G^c)} \tag{4.8}$$

where $E, G \subset S$, $W$ is the weighting function and $W_d(.\,|G)$ denotes the updated weighting function conditional on $G$. This rule is referred to as *Dempster-Shafer (DS)* updating. The assumption underlying this rule is that the decision maker assumes that the event $G$, of which she has been informed, corresponds with the

updating in one stroke. In general, however, such as for DS updating considered later, the two approaches can be different. For simplicity, we focus on updating in one stroke.

"worst of all possible outcomes." The corresponding rank-ordering of events is $G^c \succcurlyeq E \cap G \succcurlyeq G \backslash E$. By applying this rank order in Eq. (4.5), we obtain

$$W_c(E|G) = \frac{\pi(E \cap G)}{\pi(G)} = \frac{W((E \cap G) \cup G^c) - W(G^c)}{1 - W(G^c)} = W_d(E|G).$$

This shows that the general conditional weighting function coincides with DS updating when a specific ranking of events is assumed.

Similar to GS updating, if we assume CS-RDU and the source function as a power function, i.e., $W(E) = w(P(E)) = P(E)^r$, then the DS approach can be expressed by

$$W_d(E|G) = \frac{W((E \cap G) \cup G^c) - W(G^c)}{1 - W(G^c)} = \frac{(P(E \cap G) + P(G^c))^r - P(G^c)^r}{1 - P(G^c)^r}. \quad (4.9)$$

Unlike GS updating, the final result cannot be reduced to a power function of a conditional probability. The factor $P(G^c)$ appears both in the numerator and the denominator and does not cancel. An increase in $r$ results in a decrease in $P(G^c)$, so that the denominator increases. However, the change in the numerator is not clear as both $(P(E \cap G) + P(G^c))^r$ and $P(G^c)^r$ decline. Therefore, how the power $r$ affects the conditional weighting function $W_d(E|G)$ is not directly clear. We will discuss this point more in the numerical analysis. As in GS updating, the conditional weight is calculated from probabilities of events. Therefore, if $E$ concerns only day $j$, and $G$ or $F$ concerns only the preceding days, $W_d(E|G) = W_d(E|F)$ if the total number of each generic event occurring in $G$ and $F$ is the same. In particular, again, the order of the observed generic events does not affect conditional weight.

We again give an example to illustrate how the conditional weights and CE of a prospect are calculated. In day 0 when there is no updating, the weighting function is the same as in the GS approach. Hence the calculations here are identical to those in GS, and we refer to those. We next consider an example concerning day 2 that does involve updating. Here the DS approach differs from the GS approach.

EXAMPLE 4.3: Using the same assumptions about *P, U, r* as in Example 4.2, this example shows how to calculate the CE of the prospect $(A_2\colon 130, B_2\colon 110, C_2\colon 60)$. The conditional weighting functions are as follow:

$$W_d(A_2|B_0, C_1) = \frac{\left(P(B_0, C_1, A_2) + P((B_0, C_1)^c)\right)^r - P((B_0, C_1)^c)^r}{1 - P((B_0, C_1)^c)^r} = 0.30.$$

$$W_d(A_2 \cup B_2|B_0, C_1) = \frac{\left(P(B_0, C_1, A_2) + P(B_0, C_1, B_2) + P((B_0, C_1)^c)\right)^r - P((B_0, C_1)^c)^r}{1 - P((B_0, C_1)^c)^r} = 0.66.$$

Therefore, if the generic event B is realized in day 0 and C in day 1, then the CE of the prospect $(A_2\colon 130, B_2\colon 110, C_2\colon 60)$ can be calculated according to its RDU value. Following Eq. (4.3), we have:

$$RDU(A_2\colon 130, B_2\colon 110, C_2\colon 60) = \sum_{j=1}^{n} \pi_j\, U(x_j)$$

$$= W_d(A_2|B_0, C_1)U(130) + [W_d(A_2 \cup B_2|B_0, C_1) - W_d(A_2|B_0, C_1)]U(110) + [1 - W_d(A_2 \cup B_2|B_0, C_1)]U(60)$$

$$= 0.30 \cdot 130 + [0.66 - 0.30] \cdot 110 + [1 - 0.66] \cdot 60$$

$$= 99.2.$$

The CE of the prospect $(A_2\colon 130, B_2\colon 110, C_2\colon 60)$ is 99.2. In other words, a decision maker is willing to pay 99.2 for this prospect. Compared with GS updating, CE of the same prospect is higher in the DS approach. The unfavorable information received in days 0 and 1 has less impact under DS updating than under GS updating. This point will be confirmed in the numerical analysis given later. □

### 4.3.3 *Special case when* $r = 1$

In Section 4.3.1 and 4.3.2, we contrasted GS and DS updating under ambiguity aversion ($r = 1.2$). In the numerical analysis reported later, we consider both the ambiguity averse case ($r = 1.2$) and the ambiguity seeking case ($r = 0.8$). This section considers the case of ambiguity neutrality, $r = 1$. Then RDU becomes expected utility, which can only accommodate ambiguity neutral behavior. Then the GS and the DS updating rules coincide and are reduced to classical Bayesian updating: $W(E|G) = P(E|G)$. We denote the conditional weighting function of Bayesian updating as follows:

$$W_b(E|G) = P(E|G). \tag{4.10}$$

EXAMPLE 4.4: Using the same assumptions about $P, U$ as in Example 1, but taking $r = 1$, this example shows how to calculate the CE of the prospect $(A_0: 130, \mathrm{B}_0: 110, \mathrm{C}_0: 60)$. The unconditional weighting functions are as follows:

$W_b(A_0) = P(A_0) = 0.60.$

$W_b(A_0 \cup B_0) = P(A_0 \cup B_0) = 0.83.$

Following Eq. (4.3), we have:

$$RDU(\mathrm{A}_0: 130, B_0: 110, C_0: 60) = \sum_{j=1}^{n} \pi_j \, U(x_j)$$

$$= W_b(A_0)U(130) + \ [W_b(A_0 \cup B_0) - W_b(A_0)]U(110) + [1 - W_b(A_0 \cup B_0)]U(60)$$

$$= 0.60 \cdot 130 + \ [0.83 - 0.60] \cdot 110 + [1 - 0.83] \cdot 60$$

$$= 113.5$$

The CE of the prospect $(A_0: 130, \mathrm{B}_0: 110, \mathrm{C}_0: 60)$ is 113.5. Compared with GS updating, the CE of the same prospect is higher in the Bayesian approach, indicating a higher valuation of the prospect by the decision maker. This difference occurs because the decision maker here is ambiguity neutral rather than ambiguity averse. □

EXAMPLE 4.5: Using the same assumptions about $P, U, r$ as in Example 4.4 and assuming as in Example 4.2 that the generic event B is realized in day 0 and C in day 1, this example shows how to calculate the CE of the prospect $(A_2: 130, \mathrm{B}_2: 110, \mathrm{C}_2: 60)$. The conditional weighting functions are as follow:

$W_b(A_2|B_0, C_1) = P(A_2|B_0, C_1) = 0.30.$

$W_b(A_2 \cup B_2|B_0, C_1) = P(A_2 \cup B_2|B_0, C_1) = 0.67.$

Therefore, if the generic event B is realized in day 0 and C in day 1, then the CE of the prospect $(A_2: 130, \mathrm{B}_2: 110, \mathrm{C}_2: 60)$ can be calculated according to its RDU value (is expected value). Following Eq. (4.3), we have:

$$RDU(\mathrm{A}_2: 130, B_2: 110, C_2: 60) = \sum_{j=1}^{n} \pi_j \, U(x_j)$$

$$= W_b(A_2|B_0, C_1)U(130) + [W_b(A_2 \cup B_2|B_0, C_1) - W_b(A_2|B_0, C_1)]U(110) + [1 - W_b(A_2 \cup B_2|B_0, C_1)]U(60)$$

$$= 0.30 \cdot 130 + [0.67 - 0.30] \cdot 110 + [1 - 0.67] \cdot 60$$

$$= 99.3$$

Compared with other updatings, the CE under Bayesian updating (99.3) is very close to the one in DS updating (99.2), and higher than the one under GS updating (95.3). □

## 4.4 Numerical analysis

This section uses numerical methods to compare the three updating rules. We consider the same source of uncertainty (change of AEX index) and the definition of generic events A, B and C as in Examples 4.1-4.5. We compare $W(A)$, $W(A \cup B)$ and the CE of the prospect under the three updating rules. The reason that we only discuss two weights ($W(A)$, $W(A \cup B)$) is that rank A is better than rank B, which is better than rank C as the outcome is the highest for generic event A and the lowest for C. In order to calculate the CE of the prospect, we then only need these two weights to calculate all relevant decision weights (See Examples 4.1-4.5).

We consider the outcome (130, 110, 60) corresponding to generic events A, B, and C. Hence the prospect is $(A_j: 130, B_j: 110, C_j: 60)$, where $j = 0,1$, or 2. Besides Bayesian updating where the power $r$ of the source function $w$ is 1, this section discusses two cases with different power in the source function, $r = 0.8$ and $r = 1.2$. The former case concerns concave weighting and ambiguity seeking, and the latter case assumes convex weighting and ambiguity aversion. Two probability measures have been provided in section 4.2.3, where the first measure (Table 4.1) assumes the prior probability in day 0 to be $P(A_0) = P(B_0) = P(C_0) = \frac{1}{3} \cong 0.333$ and the second measure (Table 4.2) assumes $P(A_0) = 0.6, P(B_0) = 0.23, P(C_0) = 0.17$. The former one assumes a symmetric distribution of the three generic events before new observations, where the latter one assumes the highest chance for $A$ and the lowest chance for $B$. In the main context, we only discuss the second case. The first case is in Appendix 4.3

### 4.4.1 *Ambiguity averse source function* $(\boldsymbol{r = 1.2})$

We first discuss the case when $r = 1.2$ and the source function is convex. The table below (Table 4.3) summarizes the weights of $A$ and $A \cup B$ as well as CEs of the prospect $(A_j: 130, B_j: 110, C_j: 60)$, where $j = 0,1$, or 2, when the prior probability is $P(A_0) = 0.6, P(B_0) = 0.23, P(C_0) = 0.17$. The first column describes which updating rule is used. The weights and CEs are illustrated in column 2 for day 0, in column 3 to 5 for day 1, and in column 6 to 11 for day 2.

**Table 4.3: Weights and CEs between day 0, day 1 and day 2 when $P(A_0) = 0.6, P(B_0) = 0.23, P(C_0) = 0.17$ and $r = 1.2$**

| | Day 0 | Day 1 $(A_0)$ | Day 1 $(B_0)$ | Day 1 $(C_0)$ | Day 2 $(A_0, A_1)$ | Day 2 $(A_0, B_1)$ | Day 2 $(A_0, C_1)$ | Day 2 $(B_0, B_1)$ | Day 2 $(B_0, C_1)$ | Day 2 $(C_0, C_1)$ |
|---|---|---|---|---|---|---|---|---|---|---|
| GS | $W(A_0) = 0.542$ | $W(A_1 \mid A_0) = 0.689$ | $W(A_1 \mid B_0) = 0.333$ | $W(A_1 \mid C_0) = 0.333$ | $W(A_2 \mid A_0, A_1) = 0.765$ | $W(A_2 \mid A_0, B_1) = 0.488$ | $W(A_2 \mid A_0, C_1) = 0.488$ | $W(A_2 \mid B_0, B_1) = 0.236$ | $W(A_2 \mid B_0, C_1) = 0.236$ | $W(A_2 \mid C_0, C_1) = 0.236$ |
| DS | $W(A_0) = 0.542$ | $W(A_1 \mid A_0) = 0.717$ | $W(A_1 \mid B_0) = 0.394$ | $W(A_1 \mid C_0) = 0.396$ | $W(A_2 \mid A_0, A_1) = 0.791$ | $W(A_2 \mid A_0, B_1) = 0.547$ | $W(A_2 \mid A_0, C_1) = 0.548$ | $W(A_2 \mid B_0, B_1) = 0.297$ | $W(A_2 \mid B_0, C_1) = 0.299$ | $W(A_2 \mid C_0, C_1) = 0.298$ |
| Bayesian | $W(A_0) = 0.600$ | $W(A_1 \mid A_0) = 0.733$ | $W(A_1 \mid B_0) = 0.400$ | $W(A_1 \mid C_0) = 0.400$ | $W(A_2 \mid A_0, A_1) = 0.800$ | $W(A_2 \mid A_0, B_1) = 0.550$ | $W(A_2 \mid A_0, C_1) = 0.550$ | $W(A_2 \mid B_0, B_1) = 0.300$ | $W(A_2 \mid B_0, C_1) = 0.300$ | $W(A_2 \mid C_0, C_1) = 0.300$ |
| GS | $W(A_0 \cup B_0) = 0.800$ | $W(A_1 \cup B_1 \mid A_0) = 0.866$ | $W(A_1 \cup B_1 \mid B_0) = 0.866$ | $W(A_1 \cup B_1 \mid C_0) = 0.492$ | $W(A_2 \cup B_2 \mid A_0, A_1) = 0.899$ | $W(A_2 \cup B_2 \mid A_0, B_1) = 0.899$ | $W(A_2 \cup B_2 \mid A_0, C_1) = 0.613$ | $W(A_2 \cup B_2 \mid B_0, B_1) = 0.899$ | $W(A_2 \cup B_2 \mid B_0, C_1) = 0.613$ | $W(A_2 \cup B_2 \mid C_0, C_1) = 0.348$ |
| DS | $W(A_0 \cup B_0) = 0.800$ | $W(A_1 \cup B_1 \mid A_0) = 0.879$ | $W(A_1 \cup B_1 \mid B_0) = 0.884$ | $W(A_1 \cup B_1 \mid C_0) = 0.549$ | $W(A_2 \cup B_2 \mid A_0, A_1) = 0.911$ | $W(A_2 \cup B_2 \mid A_0, B_1) = 0.9143$ | $W(A_2 \cup B_2 \mid A_0, C_1) = 0.663$ | $W(A_2 \cup B_2 \mid B_0, B_1) = 0.9141$ | $W(A_2 \cup B_2 \mid B_0, C_1) = 0.664$ | $W(A_2 \cup B_2 \mid C_0, C_1) = 0.413$ |
| Bayesian | $W(A_0 \cup B_0) = 0.830$ | $W(A_1 \cup B_1 \mid A_0) = 0.887$ | $W(A_1 \cup B_1 \mid B_0) = 0.887$ | $W(A_1 \cup B_1 \mid C_0) = 0.553$ | $W(A_2 \cup B_2 \mid A_0, A_1) = 0.915$ | $W(A_2 \cup B_2 \mid A_0, B_1) = 0.915$ | $W(A_2 \cup B_2 \mid A_0, C_1) = 0.665$ | $W(A_2 \cup B_2 \mid B_0, B_1) = 0.915$ | $W(A_2 \cup B_2 \mid B_0, C_1) = 0.665$ | $W(A_2 \cup B_2 \mid C_0, C_1) = 0.415$ |
| GS | $CE = 110.816$ | $CE = 117.064$ | $CE = 109.940$ | $CE = 91.239$ | $CE = 120.246$ | $CE = 114.705$ | $CE = 100.405$ | $CE = 109.660$ | $CE = 95.361$ | $CE = 82.119$ |
| DS | $CE = 110.816$ | $CE = 118.264$ | $CE = 112.080$ | $CE = 95.348$ | $CE = 121.366$ | $CE = 116.665$ | $CE = 104.137$ | $CE = 111.654$ | $CE = 99.210$ | $CE = 86.621$ |
| Bayesian | $CE = 113.500$ | $CE = 119.000$ | $CE = 112.333$ | $CE = 95.667$ | $CE = 121.750$ | $CE = 116.750$ | $CE = 104.250$ | $CE = 111.750$ | $CE = 99.250$ | $CE = 86.750$ |

In day 0, no information is obtained, and therefore the weights and CEs are the same between GS and DS updating, because no updating took place. Whereas Bayesian updating with $r = 1$ involves ambiguity neutrality, for the other two approaches with $r = 1.2$, the source function demonstrates ambiguity averse behavior. Then generic events with higher outcome are underweighted, so that the weights of $A$ and $A \cup B$ as well as CEs then are smaller than under Bayesian updating.

In day 1, one observation is gained and belief is updated. There are three possibilities $(A_0, B_0, C_0)$ of the observation in day 0, and we choose one to discuss because the other two give similar findings. Assume that the generic event B is realized in day 0 (See column 4). Then, the weights and CEs are higher under DS updating than under GS updating, and both are lower than under Bayesian updating. This reflects that GS updating induces more ambiguity aversion than DS updating. The difference between Bayesian and DS updating is much smaller than that between Bayesian and GS updating. For $W(A_1|B_0)$, the difference is 0.006 (=0.400-0.394) between Bayesian and DS and 0.067 (=0.400-0.333) between Bayesian and GS. The latter difference is more than 10 times the former.

Similarly, for $W(A_1 \cup B_1|B_0)$, the difference is 0.003 (=0.887-0.884) between Bayesian and DS and 0.021 (=0.887-0.866) between Bayesian and GS. Now the latter difference is around 7 times the former. This explains why the CE difference between Bayesian and DS updating (0.253 =112.333-112.080) is much smaller than that between Bayesian and GS updating (2.14 =112.080-109.940), where the latter one is around 8 times the former. As Bayesian updating can be viewed as GS or DS updating when $r = 1$, such differences in weights and CEs between Bayesian and the other two approaches are the results of the change of power $r$ from 1 to 1.2. As discussed in section 4.3.1, GS updating is more sensitive to the change of $r$ and, hence, an increase in $r$ (from 1 to 1.2) leads to a significant decrease in the decision weights of favorable events and CEs. As is discussed in section 4.3.2, the impact of $r$ on DS updating is not clear from the model specification. This is confirmed by numerical results: DS updating is less sensitive to the change of $r$ and, hence, the change of $r$ has only a small effect on decision weights and CEs. These results also apply to the cases where other observations are obtained in day 0.

When we compare different cases in day 1, concerning different observations in day 0, we find that unfavorable information affects CEs of prospects differently between GS and DS updating. Compared with the case when favorable information is received, a decision maker's CE of the prospect decreases more under GS updating than under DS updating when unfavorable information is received. For example, the difference in CE of the prospect $(A_1: 130, B_1: 110, C_1: 60)$between the condition of $A$ in day 0 and the condition of $C$ in day 0 is 25.807 (117.064-91.239) under GS updating and 22.916 (118.264-95.348) under DS updating. This implies that unfavorable information leads to a bigger increase in ambiguity averse behavior under GS updating than under DS updating.

We also find that the weights can be the same after observing different information under GS and Bayesian updating. For example, under Bayesian updating, $W(A_1|B_0) = W(A_1|C_0) = 0.400$ and $W(A_1 \cup B_1|A_0) = W(A_1 \cup B_1|B_0) = 0.887$. In the first equation, neither of the two observations ($B_0$ or $C_0$) confirms the event ($A_1$) to be weighted. In the second equation, both of the two observations ($A_0$ or $B_0$) confirm the event ($A_1 \cup B_1$) to be weighted. We find that the conditional weight is the same as long as the number of observations that confirm the event to be weighted is equivalent. There is an underlying reason. In Bayesian updating, the conditional weight is equal to the conditional probability, which is calculated according to Carnap's rule as explained in Appendix 4.1. Disjoint causality is satisfied under Carnap's rule, which means that the conditional probability of an event, say $A_2$, is only affected by the number of observations that gave that event $A$, and it is immaterial how often the remaining observations gave $B$ or $C$. Therefore, given observations, the conditional weight of a specific event depends only on the number of observations that confirm the event to be weighted. In GS updating, the conditional weighting function is the source function $w$ applied to the conditional probability and, hence, disjoint causality also holds for weights. However, this property does not hold for DS updating. For example, in day 1, $\mathrm{W}(\mathrm{A}_1|\mathrm{B}_0) = 0.394$ and $\mathrm{W}(\mathrm{A}_1|\mathrm{C}_0) = 0.396$.

In day 2, two observations are obtained and belief is updated. As discussed in section 4.3, the order of the observed generic events does not affect conditional weight, so that CEs are also not affected. Hence there are 6 possibilities $\{(A_0, A_1), (A_0, B_1), (A_0, C_1), (B_0, B_1), (B_0, C_1), (C_0, C_1)\}$ of the observation in day 0 and day 1. All the findings about updating in day 1 are confirmed for day 2: the weights and CEs are higher in DS updating than in GS updating, and both of these are lower than in Bayesian updating. The difference between Bayesian and DS is much smaller than that between Bayesian and GS.

Unfavorable information makes a decision maker more ambiguity averse or pessimistic under GS updating than under DS updating. Disjoint causality for weights hold for Bayesian and GS updating, but not for DS updating.

When comparing different days, we find that, apart from day 0, CEs are always higher in DS than in GS updating. Therefore, when new information is received, no matter whether good or bad, a decision maker always exhibits more ambiguity aversion under GS updating than under DS updating.

### 4.4.2 *Ambiguity seeking source function ($\boldsymbol{r = 0.8}$)*

This section discusses the case when $r = 0.8$ and the source function is concave. The table below summarizes the weights of $A$ and $A \cup B$ as well as CEs of the prospect $(A_j: 130, B_j: 110, C_j: 60)$, where $j = 0,1$ or 2. The prior probability is $P(A_0) = 0.6, P(B_0) = 0.23, P(C_0) = 0.17$ and $r = 0.8$. The structure is the same as in Table 4.3.

**Table 4.4: Weights and CEs among day 0, day 1 and day 2 when $P(A_0) = 0.6, P(B_0) = 0.23, P(C_0) = 0.17$ and $r = 0.8$**

| | Day 0 | Day 1 $(A_0)$ | Day 1 $(B_0)$ | Day 1 $(C_0)$ | Day 2 $(A_0, A_1)$ | Day 2 $(A_0, B_1)$ | Day 2 $(A_0, C_1)$ | Day 2 $(B_0, B_1)$ | Day 2 $(B_0, C_1)$ | Day 2 $(C_0, C_1)$ |
|---|---|---|---|---|---|---|---|---|---|---|
| GS | $W(A_0) = 0.665$ | $W(A_1\|A_0) = 0.780$ | $W(A_1\|B_0) = 0.480$ | $W(A_1\|C_0) = 0.480$ | $W(A_2\|A_0, A_1) = 0.837$ | $W(A_2\|A_0, B_1) = 0.620$ | $W(A_2\|A_0, C_1) = 0.620$ | $W(A_2\|B_0, B_1) = 0.382$ | $W(A_2\|B_0, C_1) = 0.382$ | $W(A_2\|C_0, C_1) = 0.382$ |
| DS | $W(A_0) = 0.665$ | $W(A_1\|A_0) = 0.749$ | $W(A_1\|B_0) = 0.406$ | $W(A_1\|C_0) = 0.404$ | $W(A_2\|A_0, A_1) = 0.809$ | $W(A_2\|A_0, B_1) = 0.552$ | $W(A_2\|A_0, C_1) = 0.552$ | $W(A_2\|B_0, B_1) = 0.303$ | $W(A_2\|B_0, C_1) = 0.301$ | $W(A_2\|C_0, C_1) = 0.302$ |
| Bayesian | $W(A_0) = 0.600$ | $W(A_1\|A_0) = 0.733$ | $W(A_1\|B_0) = 0.400$ | $W(A_1\|C_0) = 0.400$ | $W(A_2\|A_0, A_1) = 0.800$ | $W(A_2\|A_0, B_1) = 0.550$ | $W(A_2\|A_0, C_1) = 0.550$ | $W(A_2\|B_0, B_1) = 0.300$ | $W(A_2\|B_0, C_1) = 0.300$ | $W(A_2\|C_0, C_1) = 0.300$ |
| GS | $W(A_0 \cup B_0) = 0.862$ | $W(A_1 \cup B_1\|A_0) = 0.908$ | $W(A_1 \cup B_1\|B_0) = 0.908$ | $W(A_1 \cup B_1\|C_0) = 0.623$ | $W(A_2 \cup B_2\|A_0, A_1) = 0.931$ | $W(A_2 \cup B_2\|A_0, B_1) = 0.931$ | $W(A_2 \cup B_2\|A_0, C_1) = 0.722$ | $W(A_2 \cup B_2\|B_0, B_1) = 0.931$ | $W(A_2 \cup B_2\|B_0, C_1) = 0.722$ | $W(A_2 \cup B_2\|C_0, C_1) = 0.495$ |
| DS | $W(A_0 \cup B_0) = 0.862$ | $W(A_1 \cup B_1\|A_0) = 0.895$ | $W(A_1 \cup B_1\|B_0) = 0.889$ | $W(A_1 \cup B_1\|C_0) = 0.558$ | $W(A_2 \cup B_2\|A_0, A_1) = 0.919$ | $W(A_2 \cup B_2\|A_0, B_1) = 0.916$ | $W(A_2 \cup B_2\|A_0, C_1) = 0.667$ | $W(A_2 \cup B_2\|B_0, B_1) = 0.9159$ | $W(A_2 \cup B_2\|B_0, C_1) = 0.666$ | $W(A_2 \cup B_2\|C_0, C_1) = 0.417$ |
| Bayesian | $W(A_0 \cup B_0) = 0.830$ | $W(A_1 \cup B_1\|A_0) = 0.887$ | $W(A_1 \cup B_1\|B_0) = 0.887$ | $W(A_1 \cup B_1\|C_0) = 0.553$ | $W(A_2 \cup B_2\|A_0, A_1) = 0.915$ | $W(A_2 \cup B_2\|A_0, B_1) = 0.915$ | $W(A_2 \cup B_2\|A_0, C_1) = 0.665$ | $W(A_2 \cup B_2\|B_0, B_1) = 0.915$ | $W(A_2 \cup B_2\|B_0, C_1) = 0.665$ | $W(A_2 \cup B_2\|C_0, C_1) = 0.415$ |
| GS | $CE = 116.367$ | $CE = 121.018$ | $CE = 115.022$ | $CE = 100.752$ | $CE = 123.300$ | $CE = 118.967$ | $CE = 108.474$ | $CE = 114.204$ | $CE = 103.710$ | $CE = 92.374$ |
| DS | $CE = 116.367$ | $CE = 119.716$ | $CE = 112.586$ | $CE = 95.986$ | $CE = 122.126$ | $CE = 116.835$ | $CE = 104.363$ | $CE = 111.846$ | $CE = 99.290$ | $CE = 86.879$ |
| Bayesian | $CE = 113.500$ | $CE = 119.000$ | $CE = 112.333$ | $CE = 95.667$ | $CE = 121.750$ | $CE = 116.750$ | $CE = 104.250$ | $CE = 111.750$ | $CE = 99.250$ | $CE = 86.750$ |

Many findings under ambiguity aversion are naturally reflected under ambiguity seeking. Now the weights and CEs in Bayesian updating are always smaller than under GS and DS updating. Apart from day 0, where no updating is involved, the weights and CEs in GS updating are larger than under DS updating. This reflects that GS updating now is in general more ambiguity seeking than DS updating.

Compared with the case when favorable information is received, a decision maker's CE of the prospect decreases less under GS updating than under DS updating when unfavorable information is received. For example, the difference in CE of the prospect $(A_1: 130, B_1: 110, C_1: 60)$between the condition of $A$ in day 0 and the condition of $C$ in day 0 is 20.266 (121.018-100.752) under GS updating and 23.73 (119.716-95.986) under DS updating. This implies that unfavorable information leads to a smaller increase of ambiguity averse behavior under GS updating than under DS updating.

For Table 4.4, as for Table 4.3, disjoint causality of weights hold for GS and Bayesian updating, but not for DS updating. The deviation in weights and CEs between DS and Bayesian updating is always smaller than that between GS and Bayesian updating.

Among the three updating approaches, GS updating is affected most by the change of power $r$ from 1.2 to 0.8. Both the weights and CEs in GS updating are the lowest when the source function is convex ($r = 1.2$) and the highest when the source function is concave ($r = 0.8$). This confirms the inference in section 4.3.1 that a small change in r has a considerable impact on weights. The change in DS updating is less significant when $r$ changes. Both the weights and CEs are slightly lower (higher) than Bayesian updating when $r = 1.2$ (0.8). Thus, GS updating accentuates deviations from ambiguity neutrality relative to DS updating.

In the main text, we discussed three updating rules using the non-uniform probability measure of $P(A_0) = 0.6, P(B_0) = 0.23, P(C_0) = 0.17$. We show in appendix 4.2 that the same results are found when another, uniform, probability measure $P(A_0) = P(B_0) = P(C_0) = \frac{1}{3} \cong 0.333$ is implemented.

## 4.5 Discussion

The numerical analysis describes how CEs and weights of the prospect changes when receiving new information under the three updating rules. Our findings can be applied in other domains, especially finance. If we take the market as the decision maker and prospects concern state-contingent assets. The CE of a prospect then is its market price, which depends on the market's belief. If the market is ambiguity averse, then the price of the prospect is lower under GS updating than under DS updating. For an ambiguity seeking market, the price of the prospect is higher under GS updating than under DS updating. Prices under DS updating are relatively stable no matter what the market's ambiguity attitude is, and they are always close to Bayesian updating. If we assume Bayesian updating to be a benchmark for rationality, then GS updating leads to more deviations from rationality than DS updating, and it is more sensitive to the ambiguity attitude of the market. A small change in ambiguity attitude of the market leads to a large change in price under GS updating no matter what its ambiguity attitude is.

Although favorable (unfavorable) information has positive (negative) effect on price in all three approaches, there are differences between these effects. For an ambiguity averse market, favorable (unfavorable) information leads to the biggest rise (decrease) in price under GS updating. For an ambiguity seeking market, favorable (unfavorable) information leads to the smallest rise (decrease) in price under GS updating. Therefore, the price change to information is very sensitive to the ambiguity attitude of the market under GS updating.

## 4.6 Conclusion

This chapter discusses belief updating and its implications in financial pricing under different approaches. The Bayesian updating rule only allows for ambiguity neutral behavior while the GS and DS approaches are more flexible and can accommodate both ambiguity averse and ambiguity seeking behavior. The Bayesian approach is the special case of GS and DS approaches when ambiguity neutrality holds. A numerical analysis is provided to compare the three updating rules. Ambiguity attitudes affect not only static decisions, but also the way in which new information is incorporated. For an ambiguity averse (seeking) decision maker, GS updating leads to a lower (higher) CE of the prospect than DS updating, and favorable or unfavorable information has bigger (smaller) impact under GS updating than under DS updating.

## Appendix 4.1

The probabilities in Table 4.1 and 4.2 are taken from *conjugate* parametric families, i.e., its members, after updating, turn into other members of that same parametric family.  In particular, the probabilities belong to the Dirichlet family, which is a multinomial extension of the beta family, a widely used conjugate family.

The calculation of the prior probability is as follows

$$P(\alpha_0, \beta_1, \gamma_2) = P(\gamma_2|\alpha_0, \beta_1) \cdot P(\beta_1|\alpha_0) \cdot P(\alpha_0)$$

where $\alpha, \beta, \gamma \in \{A, B, C\}$, and the subscript refers to the day. The conditional probabilities are calculated as follows:

$$P(\gamma_2|\alpha_0, \beta_1) = \frac{\lambda P(\gamma_0) + n_2}{\lambda + 2} \text{ and } P(\beta_1|\alpha_0) = \frac{\lambda P(\beta_0) + n_1}{\lambda + 1}$$

where

$\lambda$ is the parameter that represents the weight on one's prior belief.

$n_1$ is the number of appearances of generic event $\alpha$ in the past 1 day (day 0).

$n_2$ is the number of appearances of generic event $\beta$ in the past 2 days (day 0 and day 1).

An intuitive understanding of the conditional probability is that the decision maker predicts future events based on her prior belief and the observation of new events. $\lambda$ represents the level of trust the decision maker gives to her prior belief. A higher value of $\lambda$ indicates more trust and less attention for new observations. In the tables, I assume $\lambda = 2$.

The methods of calculation are special cases of Carnap's induction (1952, 1980). Assume that $p_0 > 0$ is the prior probability of observing a certain event E, $\lambda$ is a positive constant, N is the total number of observation and n is the number of observations giving event E. Then the conditional probability of event E after N observations, I denoted $p_N$, is a convex combination of the prior probability $p_0$ and the observed relative frequency $\frac{n}{N}$ with weights proportional to $\lambda$ and $N$. We have,

$$p_N = \frac{\lambda p_0 + N\frac{n}{N}}{\lambda + N}.$$

In Carnap's approach, the order of appearance of generic events does not affect the conditional probability. This condition is called *exchangeability* (Wakker 2002). For instance, if $N = 5$ and $n = 1$, the subjective probability is not affected by the day on which the decision maker observed the generic event.

Given $\lambda$ and $N$, the conditional probability is only affected by *n,* the total number of observations giving event E, and not by which disjoint events happened in the other *N-n* cases. This condition is called *disjoint causality* (Wakker 2002). Exchangeability and disjoint causality explain why the probabilities of two states of nature (in Table 1 and Table 2) are the same if the total number of each generic event is the same. This condition and some other plausible conditions imply Carnap's method (Carnap, 1952, 1980, Wakker, 2002).

## Appendix 4.2

We know from Eq. (4.7) that $W_g(E|G) = \frac{P(E \cap G)^r}{P(G)^r}$ and $W_g(E|F) = \frac{P(E \cap F)^r}{P(F)^r}$. If the total number of each generic event occurring in $G$ and $F$ is the same, we have $P(G)^r = P(F)^r$ as the probabilities of two events are the same if the total number of each generic event occurring is the same between two events. In addition, if $E$ concerns only day $j$, and $G$ or $F$ concerns only the preceding days, then $P(E \cap G)^r = P(E \cap$

$F)^r$ for the same reason. Therefore, $W_g(E|G) = \frac{P(E \cap G)^r}{P(G)^r} = \frac{P(E \cap F)^r}{P(F)^r} = W_g(E|F)$ if these two conditions are satisfied.

## Appendix 4.3

The two tables below summarize the weights of $A$ and $A \cup B$ as well as CEs of the prospect $(A_j: 130, B_j: 110, C_j: 60)$, where $j = 0,1,2$, when the prior probability is $P(A_0) = P(B_0) = P(C_0) = \frac{1}{3} \cong 0.333$. In Table 4.5, $r = 1.2$ and in Table 4.6, $r = 0.8$

As appears from the tables, we have the same findings as in the main text.

**Table 4.5: Weights and CEs among day 0, day 1 and day 2 when $P(A_0) = P(B_0) = P(C_0) = \frac{1}{3} \cong 0.333$ and $r = 1.2$**

| | Day 0 | Day 1 $(A_0)$ | Day 1 $(B_0)$ | Day 1 $(C_0)$ | Day 2 $(A_0, A_1)$ | Day 2 $(A_0, B_1)$ | Day 2 $(A_0, C_1)$ | Day 2 $(B_0, B_1)$ | Day 2 $(B_0, C_1)$ | Day 2 $(C_0, C_1)$ |
|---|---|---|---|---|---|---|---|---|---|---|
| GS | $W(A_0) = 0.268$ | $W(A_1\|A_0) = 0.494$ | $W(A_1\|B_0) = 0.164$ | $W(A_1\|C_0) = 0.164$ | $W(A_2\|A_0, A_1) = 0.615$ | $W(A_2\|A_0, B_1) = 0.350$ | $W(A_2\|A_0, C_1) = 0.350$ | $W(A_2\|B_0, B_1) = 0.116$ | $W(A_2\|B_0, C_1) = 0.116$ | $W(A_2\|C_0, C_1) = 0.116$ |
| DS | $W(A_0) = 0.268$ | $W(A_1\|A_0) = 0.546$ | $W(A_1\|B_0) = 0.215$ | $W(A_1\|C_0) = 0.215$ | $W(A_2\|A_0, A_1) = 0.662$ | $W(A_2\|A_0, B_1) = 0.415$ | $W(A_2\|A_0, C_1) = 0.415$ | $W(A_2\|B_0, B_1) = 0.164$ | $W(A_2\|B_0, C_1) = 0.166$ | $W(A_2\|C_0, C_1) = 0.164$ |
| Bayesian | $W(A_0) = 0.333$ | $W(A_1\|A_0) = 0.556$ | $W(A_1\|B_0) = 0.222$ | $W(A_1\|C_0) = 0.222$ | $W(A_2\|A_0, A_1) = 0.667$ | $W(A_2\|A_0, B_1) = 0.417$ | $W(A_2\|A_0, C_1) = 0.417$ | $W(A_2\|B_0, B_1) = 0.167$ | $W(A_2\|B_0, C_1) = 0.167$ | $W(A_2\|C_0, C_1) = 0.167$ |
| GS | $W(A_0 \cup B_0) = 0.615$ | $W(A_1 \cup B_1\|A_0) = 0.740$ | $W(A_1 \cup B_1\|B_0) = 0.740$ | $W(A_1 \cup B_1\|C_0) = 0.378$ | $W(A_2 \cup B_2\|A_0, A_1) = 0.803$ | $W(A_2 \cup B_2\|A_0, B_1) = 0.803$ | $W(A_2 \cup B_2\|A_0, C_1) = 0.524$ | $W(A_2 \cup B_2\|B_0, B_1) = 0.803$ | $W(A_2 \cup B_2\|B_0, C_1) = 0.524$ | $W(A_2 \cup B_2\|C_0, C_1) = 0.268$ |
| DS | $W(A_0 \cup B_0) = 0.615$ | $W(A_1 \cup B_1\|A_0) = 0.771$ | $W(A_1 \cup B_1\|B_0) = 0.771$ | $W(A_1 \cup B_1\|C_0) = 0.434$ | $W(A_2 \cup B_2\|A_0, A_1) = 0.831$ | $W(A_2 \cup B_2\|A_0, B_1) = 0.832$ | $W(A_2 \cup B_2\|A_0, C_1) = 0.581$ | $W(A_2 \cup B_2\|B_0, B_1) = 0.831$ | $W(A_2 \cup B_2\|B_0, C_1) = 0.581$ | $W(A_2 \cup B_2\|C_0, C_1) = 0.329$ |
| Bayesian | $W(A_0 \cup B_0) = 0.667$ | $W(A_1 \cup B_1\|A_0) = 0.778$ | $W(A_1 \cup B_1\|B_0) = 0.778$ | $W(A_1 \cup B_1\|C_0) = 0.444$ | $W(A_2 \cup B_2\|A_0, A_1) = 0.833$ | $W(A_2 \cup B_2\|A_0, B_1) = 0.833$ | $W(A_2 \cup B_2\|A_0, C_1) = 0.583$ | $W(A_2 \cup B_2\|B_0, B_1) = 0.833$ | $W(A_2 \cup B_2\|B_0, C_1) = 0.583$ | $W(A_2 \cup B_2\|C_0, C_1) = 0.333$ |
| GS | $CE = 96.089$ | $CE = 106.861$ | $CE = 100.272$ | $CE = 82.185$ | $CE = 112.469$ | $CE = 107.170$ | $CE = 93.181$ | $CE = 102.504$ | $CE = 88.515$ | $CE = 75.708$ |
| DS | $CE = 96.089$ | $CE = 109.465$ | $CE = 102.852$ | $CE = 86.023$ | $CE = 114.770$ | $CE = 109.910$ | $CE = 97.370$ | $CE = 104.803$ | $CE = 92.385$ | $CE = 79.713$ |
| Bayesian | $CE = 100.000$ | $CE = 110.000$ | $CE = 103.333$ | $CE = 86.667$ | $CE = 115.000$ | $CE = 110.000$ | $CE = 97.500$ | $CE = 105.000$ | $CE = 92.500$ | $CE = 80.000$ |

| Table 4.6: Weights and CEs among day 0, day 1 and day 2 when $P(A_0) = P(B_0) = P(C_0) = \frac{1}{3} \cong 0.333$ and $r = 0.8$ | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Day 0 | Day 1 ($A_0$) | Day 1 ($B_0$) | Day 1 ($C_0$) | Day 2 ($A_0, A_1$) | Day 2 ($A_0, B_1$) | Day 2 ($A_0, C_1$) | Day 2 ($B_0, B_1$) | Day 2 ($B_0, C_1$) | Day 2 ($C_0, C_1$) |
| GS | $W(A_0) = 0.415$ | $W(A_1\|A_0) = 0.625$ | $W(A_1\|B_0) = 0.300$ | $W(A_1\|C_0) = 0.300$ | $W(A_2\|A_0, A_1) = 0.723$ | $W(A_2\|A_0, B_1) = 0.496$ | $W(A_2\|A_0, C_1) = 0.496$ | $W(A_2\|B_0, B_1) = 0.238$ | $W(A_2\|B_0, C_1) = 0.238$ | $W(A_2\|C_0, C_1) = 0.238$ |
| DS | $W(A_0) = 0.415$ | $W(A_1\|A_0) = 0.565$ | $W(A_1\|B_0) = 0.230$ | $W(A_1\|C_0) = 0.230$ | $W(A_2\|A_0, A_1) = 0.671$ | $W(A_2\|A_0, B_1) = 0.419$ | $W(A_2\|A_0, C_1) = 0.419$ | $W(A_2\|B_0, B_1) = 0.170$ | $W(A_2\|B_0, C_1) = 0.168$ | $W(A_2\|C_0, C_1) = 0.170$ |
| Bayesian | $W(A_0) = 0.333$ | $W(A_1\|A_0) = 0.556$ | $W(A_1\|B_0) = 0.222$ | $W(A_1\|C_0) = 0.222$ | $W(A_2\|A_0, A_1) = 0.667$ | $W(A_2\|A_0, B_1) = 0.417$ | $W(A_2\|A_0, C_1) = 0.417$ | $W(A_2\|B_0, B_1) = 0.167$ | $W(A_2\|B_0, C_1) = 0.167$ | $W(A_2\|C_0, C_1) = 0.167$ |
| GS | $W(A_0 \cup B_0) = 0.723$ | $W(A_1 \cup B_1\|A_0) = 0.818$ | $W(A_1 \cup B_1\|B_0) = 0.818$ | $W(A_1 \cup B_1\|C_0) = 0.623$ | $W(A_2 \cup B_2\|A_0, A_1) = 0.864$ | $W(A_2 \cup B_2\|A_0, B_1) = 0.864$ | $W(A_2 \cup B_2\|A_0, C_1) = 0.650$ | $W(A_2 \cup B_2\|B_0, B_1) = 0.864$ | $W(A_2 \cup B_2\|B_0, C_1) = 0.650$ | $W(A_2 \cup B_2\|C_0, C_1) = 0.415$ |
| DS | $W(A_0 \cup B_0) = 0.723$ | $W(A_1 \cup B_1\|A_0) = 0.784$ | $W(A_1 \cup B_1\|B_0) = 0.784$ | $W(A_1 \cup B_1\|C_0) = 0.454$ | $W(A_2 \cup B_2\|A_0, A_1) = 0.836$ | $W(A_2 \cup B_2\|A_0, B_1) = 0.834$ | $W(A_2 \cup B_2\|A_0, C_1) = 0.585$ | $W(A_2 \cup B_2\|B_0, B_1) = 0.836$ | $W(A_2 \cup B_2\|B_0, C_1) = 0.585$ | $W(A_2 \cup B_2\|C_0, C_1) = 0.338$ |
| Bayesian | $W(A_0 \cup B_0) = 0.667$ | $W(A_1 \cup B_1\|A_0) = 0.778$ | $W(A_1 \cup B_1\|B_0) = 0.778$ | $W(A_1 \cup B_1\|C_0) = 0.444$ | $W(A_2 \cup B_2\|A_0, A_1) = 0.833$ | $W(A_2 \cup B_2\|A_0, B_1) = 0.833$ | $W(A_2 \cup B_2\|A_0, C_1) = 0.583$ | $W(A_2 \cup B_2\|B_0, B_1) = 0.833$ | $W(A_2 \cup B_2\|B_0, C_1) = 0.583$ | $W(A_2 \cup B_2\|C_0, C_1) = 0.333$ |
| GS | $CE = 104.454$ | $CE = 113.391$ | $CE = 106.898$ | $CE = 92.139$ | $CE = 117.674$ | $CE = 113.142$ | $CE = 102.415$ | $CE = 107.984$ | $CE = 97.256$ | $CE = 85.532$ |
| DS | $CE = 104.454$ | $CE = 110.531$ | $CE = 103.813$ | $CE = 87.315$ | $CE = 115.228$ | $CE = 110.090$ | $CE = 97.631$ | $CE = 105.197$ | $CE = 92.615$ | $CE = 80.289$ |
| Bayesian | $CE = 100.000$ | $CE = 110.000$ | $CE = 103.333$ | $CE = 86.667$ | $CE = 115.000$ | $CE = 110.000$ | $CE = 97.500$ | $CE = 105.000$ | $CE = 92.500$ | $CE = 80.000$ |

# Chapter 5 | Measuring Discounting without Measuring Utility

*Joint work with Arthur E. Attema, Han Bleichrodt, Yu Gao and Peter P. Wakker*

## 5.1 Introduction and Background

Discounted utility is the most widely used model to analyze intertemporal decisions. It evaluates future outcomes by their utility weighted by a discount factor. Measuring discount factors is difficult because they interact with utility. Most measurements simply assume that utility is linear,[13] which is unsatisfactory for many economic applications. Frederick, Loewenstein, & O'Donoghue (2002 p. 382), therefore, suggested to measure utility using risky choices while assuming expected utility, as in Chapman (1996), and then to use these utilities to measure discount factors. In the health domain, this method had been used before for flow (continuous) variables (Stiggelbout et al. 1994). In economics, Andersen et al. (2008) and Takeuchi (2011) used this method for discrete outcomes.

The aforementioned method has two limitations. First, expected utility is often violated (Starmer 2000), which distorts utility measurements. Second, the transfer of risky cardinal utility to riskless intertemporal choice is controversial (Camerer 1995 p. 619; Luce & Raiffa 1957 p. 32 Fallacy 3; Moscati 2013). Abdellaoui, Bleichrodt, & L'Haridon (2013) and Andreoni & Sprenger (2012b) provided empirical evidence against such a transfer. When introducing discounted utility, Samuelson (1937 last paragraph) immediately warned that cardinal intertemporal utility may differ from other kinds of cardinal utility. To avoid these two difficulties, some studies elicited both utility and discounting from intertemporal choices (Abdellaoui, Attema, & Bleichrodt 2010; Abdellaoui, Bleichrodt, & l'Haridon 2013; Andreoni & Sprenger

[13] See Frederick, Loewenstein, & O'Donoghue (2002 p. 381), Sutter et al. (2013), Tanaka, Camerer, & Nguyen (2010), and Warner & Pleeters (2001).

2012a, b; Epper & Fehr-Duda 2015). Such elicitations are complex and susceptible to collinearities between utility and discounting.

This chapter presents a tractable method to measure discounting that requires no knowledge of utility. We adapt a recently introduced method for flow variables in health (Attema, Bleichrodt, & Wakker 2012) to discrete monetary outcomes in economics. Flow variables, such as quality of life, are continuous in time and are consumed per time unit. Whereas theoretical economic studies sometimes take money as a flow variable, experimental studies of discounting invariably take it as discrete, received at discrete time points, and so will we do. Because our method directly measures discounting, and utility plays no role, we call it the direct method (DM).

The basic idea of the DM is as follows. Assume that a decision maker is indifferent between: (a) an extra payment of $10 per week during weeks 1-30; and (b) that extra payment during weeks 31-65. Then the total discount weight of weeks 1-30 is equal to that of weeks 31-65. From such equalities we can derive the entire discount function. Knowledge of utility is not required because it drops from the equations. Even though this method is elementary, it has not been known before.

The DM is easy to implement and subjects can easily understand it. In an experiment, we compare it with a traditional, utility based, method (UM). For the UM we use the implementation by Epper, Fehr-Duda, & Bruhin (2011; EFB henceforth), which is based on prospect theory, currently the most accurate descriptive theory of risky choice. We show that the DM needs fewer questions than the UM but gives similar results.

## 5.2 Theory

We assume a preference relation $\succcurlyeq$ over discrete *outcome streams* $(x_1,\ldots,x_T)$, yielding *outcome* (money amount) $x_j$ at time $t_j$, for each $j \leq T$. For ease of presentation, we consider the stimuli used in our experiment, where $T = 52$ and the unit of time is one week. Thus $(x_1,\ldots,x_{52})$ yields $x_j$ at the end of week j, for each j. *Discounted utility* holds if preferences maximize the *discounted utility* of outcome stream x:

$$\sum_{j=1}^{52} d_j U(x_j) \tag{5.2.1}$$

Here, U is the subjective *utility function*, which is strictly increasing and satisfies U(0) = 0, and $0 < d_j$ is the subjective *discount factor* of week j. For $E \subset \{1,\ldots,52\}$, $\alpha_E\beta$ denotes the outcome stream that gives outcome $\alpha$ at all time points in E and outcome $\beta$ at all other time points. C(E) denotes the cumulative sum $\sum_{j\in E} d_j$ and reflects the total time weight of E. C(k) denotes C({1,…,k}). C is called the *cumulative (discount) weight*. The proof of the following result clarifies why we need not know utility: it drops from the equations.

**OBSERVATION 5.2.1**. Assume discounted utility, and $\alpha > \beta$. Then:

$$\alpha_A\beta \succ \alpha_B\beta \Leftrightarrow \sum_{j\in A} d_j > \sum_{j\in B} d_j \ \ (\text{i.e.}, C(A) > C(B)); \tag{5.2.2}$$

$$\alpha_A\beta \sim \alpha_B\beta \Leftrightarrow \sum_{j\in A} d_j = \sum_{j\in B} d_j \ \ (\text{i.e.}, C(A) = C(B)); \tag{5.2.3}$$

$$\alpha_A\beta \prec \alpha_B\beta \Leftrightarrow \sum_{j\in A} d_j < \sum_{j\in B} d_j \ \ (\text{i.e.}, C(A) > C(B)); \tag{5.2.4}$$

PROOF. The preference and two inequalities in Eq. (5.2.2) are each equivalent to

$C(A)\big(U(\alpha)-U(\beta)\big) + C(\{1,\ldots,52\})U(\beta) > C(B)\big(U(\alpha)-U(\beta)\big) + C(\{1,\ldots,52\})U(\beta)$. The other results follow from similar derivations. □

Using Observation 5.2.1, we can derive equalities of sums of $d_j$s, which, in turn, define the function C on {1,…,52} and all the $d_j$s. This procedure does not need any knowledge of utility and is therefore called the *direct method* (*DM*).

In the mathematical analysis, we also consider a continuous extension of C, defined on all of (0,52], and also called the *cumulative (discount) weight*. At the timepoints 1, …, 52 it agrees with C defined above. In the continuous extension, any payoff $x_j$ is a salary received during week j. Receiving a salary of $x_j$ per week during week j amounts to receiving $x_j$ at time j. We equate j with (j–1,j] here. Salary can also be received during part of a week. In the continuous extension, C(t)U(α) is the subjective value of receiving α during period (0,t], where C(t) = C(0,t] and t may be a noninteger, $0 \leq t \leq T$. Then C(t,52] = C(0,52] – C(0,t] also for nonintegers t. In all the empirical estimations reported later we extend C from integers to nonintegers using linear interpolations. Given the small time interval of a week, a piecewise linear approximation is satisfactory. The following remark shows that the $d_j$s serve as discretized approximations of the derivative of C.

**REMARK 5.2**. d(j) = C(j) – C(j–1) is the average of the derivative C´ over the interval (j–1,j]. Thus, $d_j$ is approximately C´(t) at t = j. □

## 5.3 Measuring Discounting Using the Direct Method

We now explain how C can be measured up to any degree of precision using the DM. Of course, C(0) = 0. Normalization of C can be chosen freely because it does not affect preference. We choose C(52) = $d_1 + \cdots + d_{52} = 1$. We write $c_p = C^{-1}(p)$. Then $c_0 = 0$ and $c_1 = 52$. We take any α > 0 and measure $c_{½}$ such that $\alpha_{(0,c_{½}]}0 \sim \alpha_{(c_{½},52]}0$. By Observation 5.1, $C\big((0,c_{½}]\big) = C\big((c_{½},52]\big) = ½$. Once we know $c_{½}$ we can measure $c_{¼}$ and $c_{¾}$ by eliciting indifferences $\alpha_{(0,c_{¼}]}0 \sim \alpha_{(c_{¼},c_{½}]}0$ and $\alpha_{(c_{½},c_{¾}]}0 \sim \alpha_{(c_{¾},52]}0$. It follows that $C(c_{¼}) = ¼$ and $C(c_{¾}) = ¾$. In general, we measure subjective midpoints s of time intervals (q,t] by eliciting indifferences $\alpha_{(q,s]}\beta \sim \alpha_{(s,t]}\beta$ (α > β). By doing this repeatedly, we can measure the cumulative function C to any desired degree of precision. We can, then, derive the discount factors from C.

The DM assumes discounted utility. Its most critical property is *separability*: a preference $(x_1,\ldots,x_{52}) \succcurlyeq (y_1,\ldots,y_{52})$ with a common outcome $x_i = y_i = c$ is not affected if this common outcome is replaced by another common outcome $x_i = y_i = c´$. By repeated application, preference is independent of any number of common outcomes.

The next proposition shows that the DM permits a simple test of separability, which we implemented in our experiment. The proposition holds for any outcome $\alpha > 0$ and, more generally, for any pair of outcomes $\alpha > \beta$ with $\beta$ instead of 0. The proof is in the appendix.

**PROPOSITION 5.3.1**. Under weak ordering and separability, we must have:

(i) $\alpha_{[c_{1/4}, c_{1/2}]}0 \sim \alpha_{[c_{1/2}, c_{3/4}]}0$;

(ii) $\alpha_{[0, c_{1/4}]}0 \sim \alpha_{[c_{3/4}, 52]}0$. □

## 5.4 The Traditional Utility-based Method (UM)

Our experiment compared the DM with a traditional *utility(–based) method* (*UM*), replicating the implementation by EFB. We first measured prospect theory's utility function from elicited certainty equivalents of 20 risky options (see Table 5.5.1). Next, we measured the money amount $\lambda$ such that

$$90_3 0 \sim \lambda_j 0, \tag{5.4.1}$$

where $\lambda_j 0$ stands for receiving $\lambda$ at time (week) j and 0 at all other times. Unlike the DM, the UM only involves one-time payments. We chose $90_3 0$ (and avoided time 0) to have stimuli similar to those of the DM. Using the measured utility function U and Eq. 5.2.1 (discounted utility), we derive from Eq. 5.4.1:

$$\frac{d_j^u}{d_3^u} = \frac{U(90)}{U(\lambda)}. \tag{5.4.2}$$

Here $d_j^u$ is the discrete *utility based discount factor* of week j. Normalization can be chosen freely, and we usually normalized $d_3^u = 1$.

## 5.5 Experiment

*Subjects:*

We recruited 104 students (61% male; median age 21) from Erasmus University Rotterdam online, mostly economics or finance bachelors. The experiment was run at the EconLab of Erasmus School of Economics. The data were collected in five sessions. Seven subjects gave erratic answers[14] and their data were excluded from the analyses.

*Incentives:*

Each subject was paid a €5 participation fee right after the experiment. In addition, we randomly selected (by bingo machine) one subject in each session and then one of his choice to be played out for real. The selections were made in public. We transferred the amount won to the subject's bank account at the dates specified in the outcome streams. In the DM, subjects made choices between streams of money. Consequently, if one of the DM questions was played out for real, we made bank transfers during several weeks. The five subjects who played for real earned €290 on average. Over the whole group, the average payment per subject was €18.70.

*Procedure:*

The experiment was computerized. Subjects sat in cubicles to avoid interactions. They could ask questions at any time during the experiment. The experiment took 45 minutes on average.

The first part of the experiment consisted of the DM questions, and the second and third part consisted of the UM questions. Subjects could only start each part when they had correctly answered two comprehension questions. Training questions familiarized subjects with the stimuli. Within the DM and the UM parts, the order of the questions was counterbalanced.

*Stimuli: Part 1*

[14] Debriefings revealed that at least two of these subjects ignored all future payoffs because they had no bank account.

Part 1 consisted of five questions to measure discounting using the DM and two questions to test separability. To measure discounting, we elicited $c_{½}$, $c_{¼}$, $c_{¾}$, $c_{⅛}$, and $c_{⅞}$ from the following indifferences:

$$\alpha_{(0,c_{½}]}0 \sim \alpha_{(c_{½},52]}0,\ \alpha_{(0,c_{¼}]}0 \sim \alpha_{(c_{¼},c_{½}]}0,\ \alpha_{(c_{½},c_{¾}]}0 \sim \alpha_{(c_{¾},52]}0,\ \alpha_{(0,c_{⅛}]}0 \sim \alpha_{(c_{⅛},c_{¼}]}0,$$
$$\text{and } \alpha_{(c_{¾},c_{⅞}]}0 \sim \alpha_{(c_{⅞},52]}0. \qquad (5.5.1)$$

To test separability, we measured the indifferences in Proposition 5.3.1.

Each question was presented as a choice list in which subjects chose between two options, A and B, in each row. Figure 5.5.1 displays a screen that subjects faced. In the first choice (first row), B dominates A. Moving down the list, A becomes more attractive and in the final choice A dominates B. The computer enforced monotonicity: After a choice A [B], the computer automatically selected A [B] for all rows below [above], A [B] being more attractive there. Thus, there was a unique switch from B to A between two values. We took the indifference value as the midpoint between those two values. In Figure 5.5.1, which measures $c_{⅛}$ for $c_{¼} = 13$, the subject switched between 5 and 6 weeks and the indifference value therefore was 5.5.

We only used integer-week periods as stimuli to keep the choices simple. Hence, we could not always use the indifference values in subsequent questions and we had to make rounding assumptions.We rounded values below 26 weeks upwards (e.g., 5.5 to 6 weeks), and values above downwards (e.g., 35.5 to 35 weeks) in subsequent choices. The appendix and web appendix give details of our rounding and analyses. With one exception, mentioned later, our conclusions remained the same under different rounding rules.

**FIGURE 5.5.1. Choice list for the DM elicitation**

**Which option do you prefer?**

**Gain €20 per week**

| Option A | A | B | Option B |
|---|---|---|---|
| in week **1** [1] | ○ | ◉ | starting week **1** and ending (after) week **13** [13] |
| in week **1** [1] | ○ | ◉ | starting week **2** and ending (after) week **13** [12] |
| starting week **1** and ending (after) week **2** [2] | ○ | ◉ | starting week **3** and ending (after) week **13** [11] |
| starting week **1** and ending (after) week **3** [3] | ○ | ◉ | starting week **4** and ending (after) week **13** [10] |
| starting week **1** and ending (after) week **4** [4] | ○ | ◉ | starting week **5** and ending (after) week **13** [9] |
| starting week **1** and ending (after) week **5** [5] | ○ | ◉ | starting week **6** and ending (after) week **13** [8] |
| starting week **1** and ending (after) week **6** [6] | ◉ | ○ | starting week **7** and ending (after) week **13** [7] |
| starting week **1** and ending (after) week **7** [7] | ◉ | ○ | starting week **8** and ending (after) week **13** [6] |
| starting week **1** and ending (after) week **8** [8] | ◉ | ○ | starting week **9** and ending (after) week **13** [5] |
| starting week **1** and ending (after) week **9** [9] | ◉ | ○ | starting week **10** and ending (after) week **13** [4] |
| starting week **1** and ending (after) week **10** [10] | ◉ | ○ | starting week **11** and ending (after) week **13** [3] |
| starting week **1** and ending (after) week **11** [11] | ◉ | ○ | starting week **12** and ending (after) week **13** [2] |
| starting week **1** and ending (after) week **12** [12] | ◉ | ○ | in week **13** [1] |
| starting week **1** and ending (after) week **13** [13] | ◉ | ○ | in week **13** [1] |

submit my choices

After each choice list, we asked a control question (explained in Web Appendix WA3).

*Stimuli: Part 2*

Part 2 consisted of seven questions of the type $A = 90_30 \sim \lambda_j0 = B$ with weeks j = 4, 12, 20, 28, 36, 44, and 52. Following EFB, we kept the early outcome in Option A constant and varied the gain λ in Option B (Figure 5.5.2). As in part 1, the computer enforced monotonicity. EFB only used the time points 1 day, 2 months + 1 day, and 4 months + 1 day. We changed those to obtain more detailed measurements and to facilitate comparison with our DM measurements.

*Stimuli: Part 3*

We elicited the certainty equivalents (CE) of twenty risky prospects, shown in Table 5.5.1, to measure prospect theory's utility function. The CE choice lists appeared in random order. They consisted of choices between sure amounts (option B) and risky prospects (option A) yielding $x_1$ with probability p and $x_2 < x_1$ otherwise. We used a choice list in which the sure amount that B offered decreased from $x_1$ in the first row to $x_2$ in the final row. We used the prospects in EFB with all amounts multiplied by 10 and we used Euros instead of Swiss Francs. Figure 5.5.3 gives an example of one of the choice lists.

**FIGURE 5.5.2. Choice list for the UM elicitation**

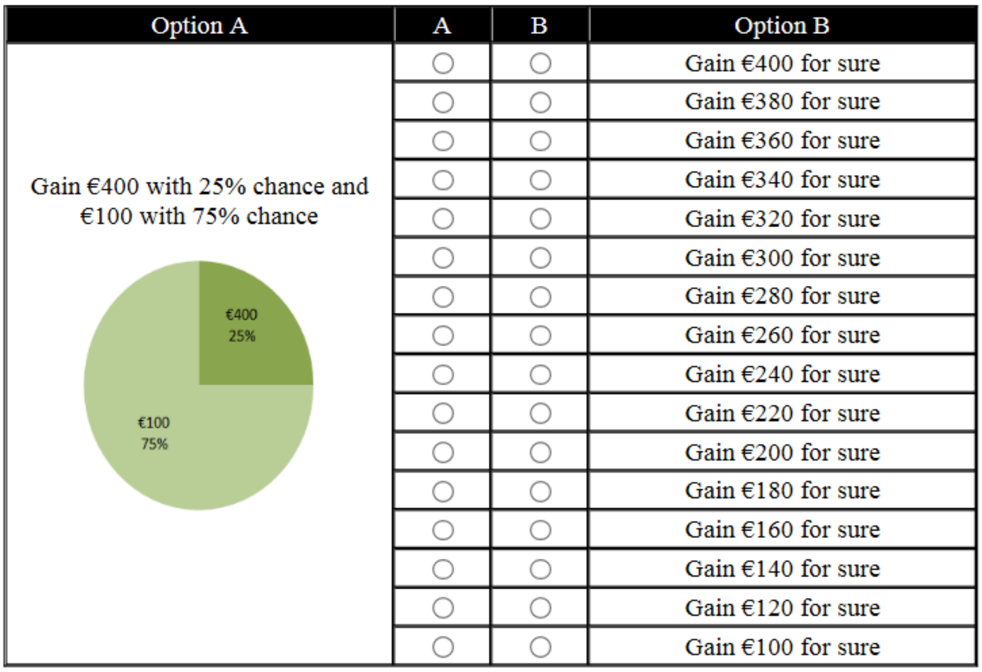
**Which option do you prefer?**

| Option A | A | B | Option B |
|---|---|---|---|
| Gain €90 after 3 weeks | ○ | ○ | Gain €220 after 36 weeks |
| | ○ | ○ | Gain €210 after 36 weeks |
| | ○ | ○ | Gain €200 after 36 weeks |
| | ○ | ○ | Gain €190 after 36 weeks |
| | ○ | ○ | Gain €180 after 36 weeks |
| | ○ | ○ | Gain €170 after 36 weeks |
| | ○ | ○ | Gain €160 after 36 weeks |
| | ○ | ○ | Gain €150 after 36 weeks |
| | ○ | ○ | Gain €140 after 36 weeks |
| | ○ | ○ | Gain €130 after 36 weeks |
| | ○ | ○ | Gain €120 after 36 weeks |
| | ○ | ○ | Gain €110 after 36 weeks |
| | ○ | ○ | Gain €100 after 36 weeks |
| | ○ | ○ | Gain €90 after 36 weeks |
| | ○ | ○ | Gain €80 after 36 weeks |

**TABLE 5.5.1. Risky prospects**

| p | $x_1$ | $x_2$ | p | $x_1$ | $x_2$ |
|---|---|---|---|---|---|
| **0.10** | 200 | 100 | **0.25** | 500 | 200 |
| **0.50** | 200 | 100 | **0.50** | 500 | 200 |
| **0.90** | 200 | 100 | **0.75** | 500 | 200 |
| **0.05** | 400 | 100 | **0.95** | 500 | 200 |
| **0.25** | 400 | 100 | **0.05** | 1500 | 500 |
| **0.50** | 400 | 100 | **0.50** | 100 | 0 |
| **0.75** | 400 | 100 | **0.50** | 200 | 0 |
| **0.95** | 400 | 100 | **0.05** | 400 | 0 |
| **0.05** | 500 | 200 | **0.95** | 500 | 0 |
| **0.10** | 1500 | 0 | **0.25** | 400 | 0 |

**FIGURE 5.5.3. Choice list of prospects for the CE elicitation**



| Option A | A | B | Option B |
|---|---|---|---|
| Gain €400 with 25% chance and €100 with 75% chance | ○ | ○ | Gain €400 for sure |
| | ○ | ○ | Gain €380 for sure |
| | ○ | ○ | Gain €360 for sure |
| | ○ | ○ | Gain €340 for sure |
| | ○ | ○ | Gain €320 for sure |
| | ○ | ○ | Gain €300 for sure |
| | ○ | ○ | Gain €280 for sure |
| | ○ | ○ | Gain €260 for sure |
| | ○ | ○ | Gain €240 for sure |
| | ○ | ○ | Gain €220 for sure |
| | ○ | ○ | Gain €200 for sure |
| | ○ | ○ | Gain €180 for sure |
| | ○ | ○ | Gain €160 for sure |
| | ○ | ○ | Gain €140 for sure |
| | ○ | ○ | Gain €120 for sure |
| | ○ | ○ | Gain €100 for sure |

## 5.6 Results

Because normality of distributions was always rejected, we used Wilcoxon signed rank tests throughout

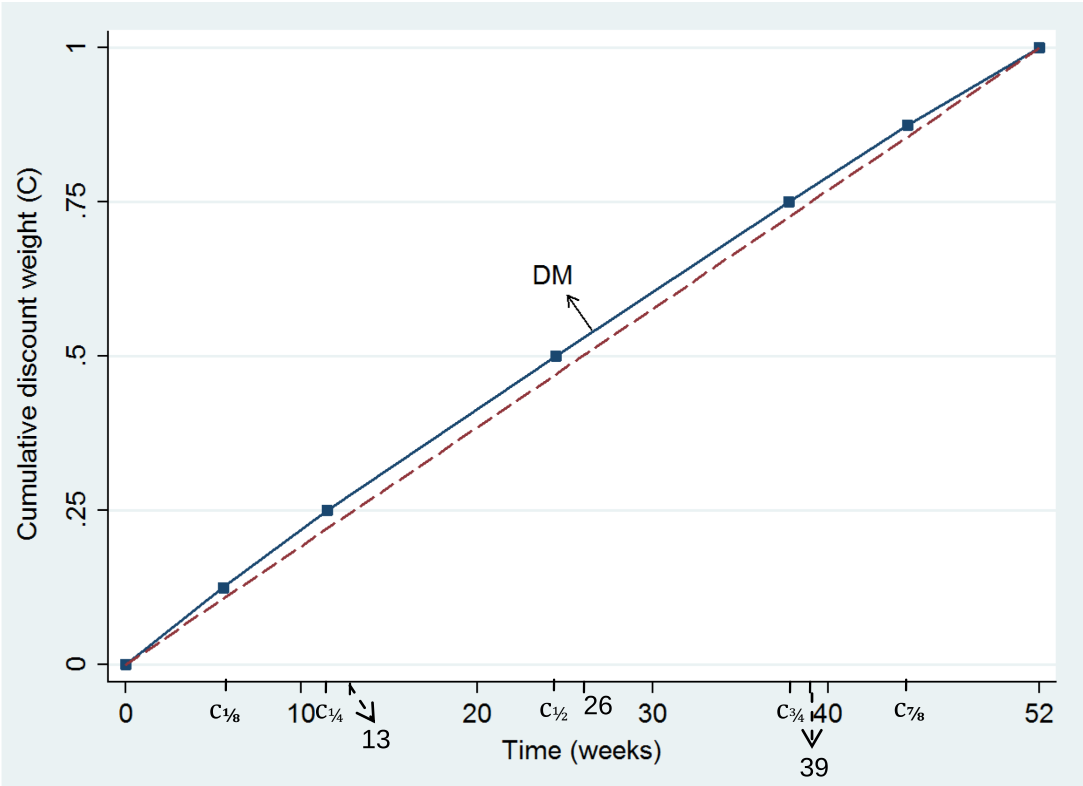
### 5.6.1 Results for the DM

In all tests reported below $p \leq 0.001$ except when noted. The DM elicits subjective midpoints of time intervals (q,t], denoted s(q,t]. Table 5.6.1 shows that s(q,t] was always closer to q than to t, which is consistent with impatience. Figure 5.6.1 shows that the cumulative C function was concave, also indicating impatience. We can derive the discount factors $d_j = C(j) - C(j{-}1)$ from C. They are in Figure 5.6.2, where they will be compared with the UM discount factors.

TABLE 5.6.1. Descriptive statistics of the direct method

| variable | mean | sd | min | median | max | N |
|---|---|---|---|---|---|---|
| $c_{1/8}$ | 5.55 | 1.25 | 2.13 | 6.13 | 9.13 | 97 |
| $c_{1/4}$ | 11.47 | 1.91 | 4.25 | 12.25 | 15.25 | 97 |
| $c_{1/2}$ | 24.47 | 2.72 | 14.50 | 25.50 | 29.50 | 97 |
| $c_{3/4}$ | 37.77 | 2.22 | 27.75 | 38.75 | 42.75 | 97 |
| $c_{7/8}$ | 44.5 | 1.45 | 39.88 | 44.88 | 47.88 | 97 |

FIGURE 5.6.1. C function of mean data




Statistical tests confirmed the above observations. In all tests, we could reject the one-sided null of no or negative impatience ($s(q,t] \geq \frac{q+t}{2}$) in favor of the alternative hypothesis of impatience ($s(q,t] < \frac{q+t}{2}$) [15].

Decreasing impatience, found in many studies, implies that $s(0,c_{1/2}] - c_{1/4} > (c_{1/2},1] - c_{3/4}$ and $s(0,c_{1/4}] - c_{1/8} > s(c_{3/4},1] - c_{7/8}$. The evidence on decreasing impatience was mixed and depended on the rounding assumption used (see the web appendix). Under one rounding assumption, we found decreasing impatience in the comparison between $(0,c_{1/2}]$ and $(c_{1/2},52]$ and increasing impatience in the comparison between $(0,c_{1/4}]$ and $(c_{3/4},52]$. Under another rounding assumption,[16] the null of constant impatience could not be rejected. For all other tests in this paper, the rounding assumptions were immaterial.

[15] $c_{1/2} < 26$, $c_{1/4} < c_{1/2}/2$, $c_{1/2} < (c_{1/4} + c_{3/4})/2$ (marginally significant), $c_{3/4} < (c_{1/2} + 52)/2$, $c_{1/8} < c_{1/4}/2$, and $c_{7/8} < (52 + c_{3/4})/2$.

[16] A large middle group (n=37) gave answers as close as possible to constant discounting. The first rounding takes them as slightly impatient. It can also be argued that the null of constant discounting should be accepted for them (our second rounding).

To test separability condition (i) in Proposition 5.3.1, we directly measured the subjective midpoint $s_{½}$ of $(c_{¼}, c_{¾}]$. That is, $\alpha_{(c_{¼},s_{½}]}0 \sim \alpha_{(s_{½},c_{¾}]}0$. By condition (i), $s_{½}$ should equal $c_{½}$. To test separability condition (ii) in Proposition 5.3.1, we directly measured the value $s_{¾}$ such that $\alpha_{(0, c_{¼}]}0 \sim \alpha_{(s_{¾}52]}0$. By Condition (ii), $s_{¾}$ should equal $c_{¾}$.

Separability was rejected in the first test ($p < 0.01$ two-sided), but not in the second. Even in the first test, we found few violations of separability at the individual level. For 54 out of 97 subjects separability was satisfied exactly. Moreover, for 80 subjects $s_{½}$ and $c_{½}$ differed by at most 1. For 13 subjects they differed by 2 or 3, and for 3 subjects by 4 or more (1 subject missing). In the second test, separability could not hold exactly due to rounding, but 55 subjects had the minimal difference of 0.5, and for 76 subjects the difference was 1.5 or less. For 17 subjects it was 2.5 or 3.5, and for 3 subjects it was 4.5 or more (1 missing).

### 5.6.2 Results for the UM

Table 5.6.2 summarizes the descriptive statistics of the discount factors $d_j^u$ under the normalization $d_3^u = 1$, the discount factor of the shortest delay in the UM. Comparing pairs of consecutive discount factors confirmed impatience (always $p < 0.001$). We could derive the *cumulative function* $C^u(j) = \sum_{i=1}^{j} d_i^u$ from the discount factors. We here normalized $C^u(52) = 1$.

**TABLE 5.6.2. Descriptive statistics of the utility-based method (UM)**

| variable | mean | sd | min | median | Max | N[17] |
|---|---|---|---|---|---|---|
| $\mathbf{d_4^u}$ | 0.93 | 0.07 | 0.67 | 0.96 | 1 | 96 |
| $\mathbf{d_{12}^u}$ | 0.87 | 0.12 | 0.44 | 0.92 | 1 | 96 |
| $\mathbf{d_{20}^u}$ | 0.85 | 0.14 | 0.33 | 0.89 | 1 | 96 |
| $\mathbf{d_{28}^u}$ | 0.79 | 0.17 | 0.33 | 0.83 | 1 | 96 |
| $\mathbf{d_{36}^u}$ | 0.77 | 0.18 | 0.33 | 0.81 | 1 | 96 |

[17] We excluded one subject because of his extreme power (−118.7; overall average is 0.53) for utility.

| | | | | | | |
|---|---|---|---|---|---|---|
| $\mathbf{d^u_{44}}$ | 0.75 | 0.20 | 0.26 | 0.78 | 1 | 96 |
| $\mathbf{d^u_{52}}$ | 0.73 | 0.21 | 0.26 | 0.76 | 1 | 96 |

### 5.6.3 *Comparing discounting and impatience under the DM and the UM*

Figure 5.6.2 shows the discount factors of the DM and the UM. For easy comparison, we normalized both to 1 at week 3 here. Both discount factors were decreasing, confirming impatience. The DM discount factors slightly exceeded the UM discount factors, but not significantly (tests provided later). According to both methods, the annual discount rate was 35%, assuming continuous compounding $e^{-rt}$ with t in years.

Figure 5.6.3 shows the cumulative functions of the DM and the UM, using linear interpolation to obtain general values $d_j^u$ from the seven measured $d_j^u$'s. We measure impatience (= concavity) for each cumulative function by the difference beween the area under this function and that under the diagonal ($t \longmapsto t/52$). For convex functions this index is negative. It is 0 if the decision maker does not discount.

The average value of the impatience index was 1.14 for the DM and 1.62 for the UM. Both indices exceded 0 ($p < 0.001$), in agreement with impatience. The index for the UM exceeded that for the DM ($p = 0.02$), suggesting more impatience under the UM.

**FIGURE 5.6.2. Comparing discount factors of the DM and the UM**

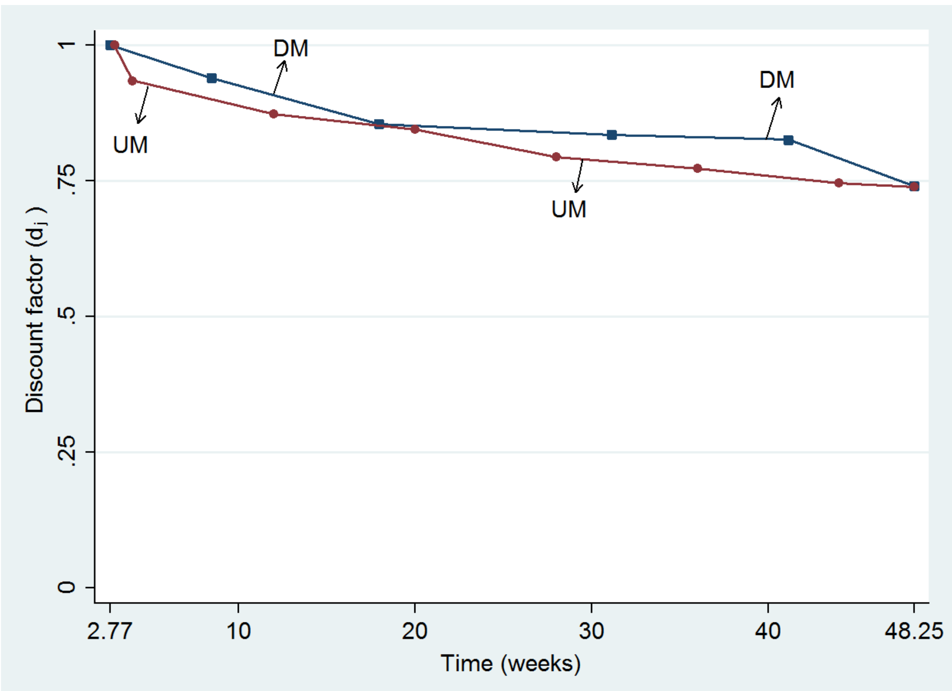
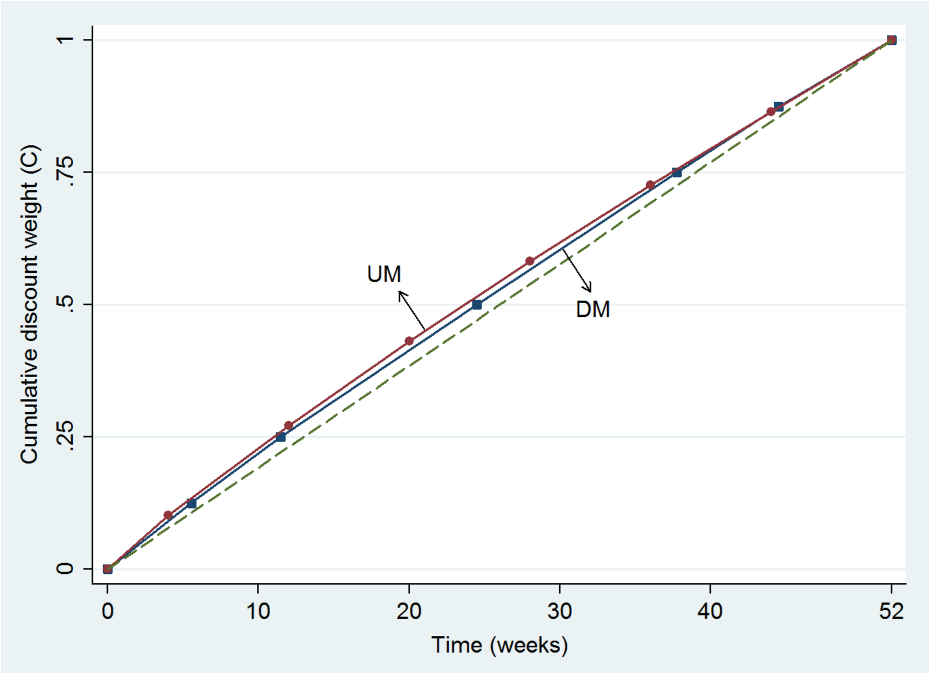


FIGURE 5.6.3. Comparing cumulative discount weights of the DM and the UM




We also estimated the DM and UM curves in Figure 5.6.3 by a power function. The median powers were 0.96 for the DM and 0.93 for the UM. Both powers were below 1 (both $p < 0.001$), again confirming impatience, but they did not differ significantly from each other. The three measures of impatience (discount factors for week 52, area-differences, and estimated power coefficients) correlated strongly ($\geq 0.90$), for both the DM and the UM. For consistency between the two methods at the individual level, we tested correlations of the three measures of impatience between the DM and UM. They were all around 0.25 ($p < 0.001$). Hence, even though the different measures led to consistent conclusions within methods, differences remained.

### 5.6.4 *Parametric estimations*

The discount factors trace out the discount function D(t) without making parametric assumptions (see Figure 5.6.2). This section reports parametric fittings. We estimated the discount function of each subject by maximum likelihood using the following three parametric families.

(1) *Constant discounting* (Samuelson 1937), with one parameter $r \geq 0$:

$D(t) = e^{-rt}$ with $r \geq 0$.

(2) *Hyperbolic discounting* (Loewenstein and Prelec 1992), with two parameters $\alpha \geq 0$ and $\beta > 0$:

For $\alpha > 0$: $D(t) = (1 + \alpha t)^{-\beta/\alpha}$;

for $\alpha = 0$: $D(t) = e^{-\beta t}$.

The $\alpha$–parameter determines how much the discount function departs from constant discounting. The limiting case $\alpha = 0$ reflects constant discounting. The $\beta$ parameter determines impatience.

(3) *Unit-invariance discounting*[18], with two parameters $r > 0$ and d:

For $d > 1$, $D(t) = \exp(rt^{1-d})$ (only if $t = 0$ is not considered);
for $d = 1$, $D(t) = t^{-r}$ (only if $t = 0$ is not considered);
for $d < 1$, $D(t) = \exp(-rt^{1-d})$.

The r-parameter determines impatience, and the d-parameter departure from constant discounting, interpreted as sensitivity to time by Ebert & Prelec (2007). The common empirical finding is $d \leq 1$, reflecting insensitivity; $d > 1$ reflects oversensitivity.

Hyperbolic discounting can only account for decreasing impatience. However, empirical studies have observed that a substantial proportion of subjects are not decreasingly, but increasingly impatient (references in §5.6.5). Unit-invariance discounting can account for both decreasing and increasing impatience. We can use the entire unit invariance family because our domain does not contain $t = 0$ (explained in the discussion section). The exclusion of $t = 0$ also implies that the popular quasi-hyperbolic family coincides with constant discounting for our stimuli.

[18] Read (2001 Eq. 16) first suggested this family. Bleichrodt, Rohde, & Wakker (2009) call it constant relative decreasing impatience, and Ebert & Prelec (2007) call it constant sensitivity. Bleichrodt et al. (2013) proposed the term unit invariance.

TABLE 5.6.3. Results of parametric fittings for the DM and the UM

| | | **exponential** | | **hyperbolic** | | **unit invariance** | |
|---|---|---|---|---|---|---|---|
| | | Mean(r) | Median(r) | Mean(α,β) | Median(α, β) | Mean(d,r) | Median(d,r) |
| Par. | **UM** | 0.009 | 0.006 | 1.88, 0.25 | 1.21, 0.06 | 0.58, 2.19 | 0.94, 0.69 |
| | **DM** | 0.005 | 0.002 | 1.66, 0.14 | 1.30, 0.05 | 0.80, 1.86 | 0.89, 0.25 |
| AIC | **UM** | –3.03 | –3.26 | –1.55 | –1.68 | –1.63 | –1.70 |
| | **DM** | –3.56 | –3.68 | –1.66 | –1.80 | –1.65 | –1.78 |

Table 5.6.3 shows the estimated parameters. The exponential discounting parameters differed between the UM and the DM (always $p < 0.001$), reflecting more discounting for the UM. The parameters of hyperbolic discounting and unit invariance discounting did not differ significantly ($p > 0.2$).

The final two rows of Table 5.6.3 show the goodness of fit of the three discount families by the Akaike information criterion (AIC). More negative values indicate better fit. The best fit was obtained by the DM method with exponential discounting. Compared with the UM, the DM fitted better for exponential discounting ($p < 0.001$). The DM also seemed to fit better for unit invariance and hyperbolic discounting, but the differences were not significant. Of the three parametric families, exponential discounting fitted best for both the DM and the UM (both $p < 0.001$). For the UM, hyperbolic discounting gave the worst fit ($p < 0.001$). For the DM we found no significant difference between unit invariance and hyperbolic discounting ($p = 0.22$). In the absence of the immediacy effect, exponential discounting performed well, which also supports quasi-hyperbolic discounting. The web appendix gives further details.

We, finally, investigated the relation between impatience (concavity of C and $C^u$) and risk attitudes, controlling for demographic variables (age, gender, and foreign versus domestic—Dutch). Impatience under the UM was negatively related with concavity of utility, which is not surprising because the UM measurements were based on utility. Under the DM, impatience was not related with utility, suggesting that these are independent components. Impatience under the UM was also negatively related with risk aversion in the form of pessimism of probability weighting, whereas impatience under the DM again was not. Age was positively related with UM impatience. Other relations were not significant.  Details are in the web appendix.

### 5.6.5 Discussion of the results

The DM and the UM led to similar conclusions. Under both methods, subjects were impatient. However, we found less discounting with the DM. Given the high estimated annual discount rate (35%), we consider this to be a desirable feature of the DM.

Even though theoretical studies commonly assume universal decreasing impatience , many empirical studies have found considerable increasing impatience at the individual level.[19] We found prevailing decreasing impatience in the UM, but mixed evidence in the DM. Statistical tests only showed weak evidence for decreasing impatience and Figure 5.6.2 suggests that impatience was not always decreasing. Increasing impatience implies that people become more reluctant to wait as time passes by. Substantial increasing impatience also explains the poor performance of the hyperbolic discount functions, which only allow for universal decreasing impatience, and cannot fit the data of increasingly impatient subjects.

Our measurement of the DM included two tests of separability. One test suggested violations of separability, but we could not reject separability in the other test and most subjects behaved in agreement with it. No decision model fits data perfectly, and we still use such decision models in the absence of better models that are sufficiently tractable. Violations of separability may, for example, be due to sequencing effects and habit formation (Dolan & Kahneman 2008 p. 228; Loewenstein & Prelec 1991 p. 350). The DM permits easy tests of separability that help to assess its restrictiveness. Because separability is used in virtually all applications of discount measurements, such tests are desirable.

Besides separability, our analysis also assumes independence of discounting from the outcome used. This condition is sometimes called separability of money and time, and its violation the magnitude effect (Loewenstein & Prelec 1992). If magnitude effects

[19] For a review see Attema et al. (2010 p. 2026).  Recent studies include Andreoni & Sprenger (2012), Burger, Charness, & Lynham (2011), and Takeuchi (2011).

exist, then our measurements are only valid for outcomes close to those used in the measurements.

Attema et al. (2010) measured discounting up to a power without the need to know utility, but needed separate measurements to identify the power. Unlike the DM but like the UM, their measurements did not need time separability but they could not test it either. In a mathematical sense, our method is similar to the measurement of subjective probability based on equal likelihood assessments, where utility also drops from the equations and separability (now over events) is assumed (Baillon 2008).

For the UM, we used the method of EFB. Our estimates of risk attitudes were close to theirs except for the curvature of utility, which we discuss in the Appendix. We could not directly compare our findings on discounting with those of EFB, because they used fewer and different time points. The negative relation between concave utility and impatience that we found for the UM is not surprising because utility plays a central role in the UM. Concave utility increases the ratio in Eq. 5.4.2 and thus decreases impatience. The negative relation between impatience and probability weighting suggests that this component of risk attitude also affects the UM measurements. Our findings suggests that there is collinearity between utility/risk attitude and discounting in the UM but not in the DM.

Several authors conjectured that discounting may be due to the inherent uncertainty about the future. Halevy (2008) gave a theoretical foundation (that also assumes time separability), and the correlations between risk and time attitudes of Epper et al. (2011) support this conjecture. However, our finding that, when measured without risk involved, time attitudes were not related to risk attitudes suggests that time attitude entails a component separate from risk attitude. It is interesting to investigate whether other riskless methods to measure discounting, such as Attema et al.'s (2010), are related to risk attitudes.

Our implementation of the DM is adaptive, with answers to questions influencing the stimuli in later questions. Theoretically, this may offer scope for manipulation: responding untruthfully to some questions may improve later stimuli. However, according to Bardsley et al.'s (2010 p. 265) classification, this possibility is only theoretical and is no cause for concern in our experiment. First, it was virtually impossible for subjects to realize that questions were adaptive because of roundings used. Second, we expect that even readers who, like us, know the adaptive nature and even the stimuli of our experiment beforehand, will not see a way in which the loss of wrongly answering one question could be compensated by advantages in follow-up questions. For our subjects this would be impossible. Web appendix WE gives details. For these two reasons, manipulation is only a theoretical concern for our experiment.

The DM can be implemented nonadaptively. For example, we can select a number of options $\alpha_{A^j}\beta$ and timepoints $s^j$ beforehand, and measure the timepoints $t^j$ such that $\alpha_{A^j}\beta \sim \alpha_{(s^j,t^j]}\beta$ for all j. Observation 5.2.1 still gives equalities $C(A^j) = C(s^j, t^j]$. We can use these in parametric fittings of C or in tests of properties such as decreasing impatience, still without requiring knowledge about U. A drawback of this nonadaptive procedure is that we then cannot readily draw a connected C-curve as in Figures 5.6.1 and 5.6.3, where we needed no parametric assumption (other than linear interpolation).

The DM always fitted better than the UM, and exponential discounting always fitted best, with unit invariance second best. Exponential discounting could perform well because we did not include the present t = 0 in our stimuli, where most violations are found due to the immediacy effect (Attema 2012). Although this effect is important and deserves further study, we decided to focus our first implementation of the DM on a better understood empirical domain, which we could compare directly with EFB. In this regard, we follow many other studies in the literature that use front-end delays.

The DM can readily investigate the immediacy effect and discrete outcomes at t = 0. The latter are then interpreted as salaries paid at the beginning, instead of at the end of periods (weeks in our case). Given that the relations that we made with flow variables only served as intermediate tool in our mathematical analysis, and played no role in the stimuli or results, we can use the interpretation mentioned. Quasi-hyperbolic discounting then implies a high weight for the first week of salary (now mathematically representing the present rather than the timepoint one week ahead), and moderate weights for the other weeks.

In experimental measurements of discounting of money, subjects may use market discounting of income rather than their subjective discounting of consumption. Cubitt & Read (2007) discuss this problem in detail, suggesting that it is often not very serious, and Reuben, Sapienza, & Zingales (2010) confirm so empirically. We conjecture that a stream of small extra payments on future occasions is more likely to be perceived as extra consumptions than as one big sum received at some future timepoint, but leave this as a topic for future research. At any rate, the DM gives a new tool to study and handle this issue.

Subjective midpoints, used by the DM to measure discounting, have a long tradition in psychophysics (bisection; Stevens 1936) and mathematics (quasi-arithmetic mean; Aczél 1966). Condition (i) in Proposition 5.3.1, a necessary condition of a quasi-arithmetic mean, is a special case of autodistributivity (Aczél (1966 Eq. 6.4.2.3, for t the midpoint of x and y).

## 5.7 General discussion

To our best knowledge, all experimental measurements of money discounting have used discrete outcomes. Real-life decisions often involve flow outcomes that are repeated per time unit. Examples are salary payments, pension saving plans, and mortgage debt repayments. In such contexts, the DM is more natural than discrete methods such as the UM. For discrete outcomes, the DM can be an alternative to the UM if the payments are sufficiently frequent and the periods are sufficiently fine, as in our experiment. For single-outcome decisions or decisions in which outcomes occur infrequently, the DM is less useful.

In the DM, subjects only make tradeoffs between periods. In the UM, subjects make tradeoffs both between outcomes and between periods, which is more complex. Hence, the DM is easier for subjects. Our experiment gave indirect support: We found a positive correlation between utility curvature and discounting for the UM, but not for the DM, showing that outcome tradeoffs impact time tradeoffs in the UM but not in the DM.

The DM is also easier to use for researchers, because of the elementary nature of Observation 5.2.1. In the DM, we only used seven questions. In the UM, we also used seven questions[20] to elicit discounting, but we needed additional questions to elicit utility. The DM took much less time.

The DM can be analyzed using parametric econometric fittings (§5.6.4), as can all existing methods, but, unlike most methods, the DM can also be analyzed in a parameter-free way (§5.6.1). This reveals the correct discount function without a commitment to a parametric family of discount functions. The DM can also be used for interactive prescriptive measurements in consultancy applications (Keeney & Raiffa 1976).

## 5.8 Conclusion

This paper has introduced a new method to measure the discounting of money, the direct method. This method is simpler than existing methods because it does not need information about utility. Consequently, the experimental tasks are easier for subjects, researchers have to ask fewer questions, and the measurements are not distorted by biases in utility. An experiment confirms the implementability and validity of the direct method.

### Appendix 5.1 Proofs

PROOF OF PROPOSITION 5.3.1. This proof is elementary in not using technical assumptions such as continuity. The proof only uses the two outcomes used in the preferences, being $\alpha$ and 0. For deriving the first indifference, we denote outcome streams as quadruples $(x_1,x_2,x_3,x_4)$, with $x_1$ received in $(0,c_{¼}]$, $x_2$ in $(c_{¼},c_{½}]$, $x_3$ in $(c_{½},c_{¾}]$, and $x_4$ in $(c_{¾},52]$. We only use the following parts of Eq. 5.5.1: $(\alpha,\alpha,0,0) \sim (0,0,\alpha,\alpha)$ (1), $(\alpha,0,0,0) \sim 0,\alpha,0,0)$ (2), and $(0,0,\alpha,0) \sim (0,0,0,\alpha)$ (3). Assume, for

[20] We used the same numbers of questions to make the methods comparable. In fact, two DM questions tested separability. The DM derived the discount functions from only five measurements.

contradiction, that (i) is violated, say $(0,\alpha,0,0) \succ (0,0,\alpha,0)$ (4). This, (2), (3), and transitivity, imply $(\alpha,0,0,0) \succ (0,0,0,\alpha)$ (5). By separability, (4) implies $(\alpha,\alpha,0,0) \succ (\alpha,0,\alpha,0)$, and (5) implies $(\alpha,0,\alpha,0) \succ (0,0,\alpha,\alpha)$. By transitivity, $(\alpha,\alpha,0,0) \succ (0,0,\alpha,\alpha)$, contradicting (1). Reversing all preferences shows that $(0,\alpha,0,0) \prec (0,0,\alpha,0)$ implies the contradictory $(\alpha,\alpha,0,0) \prec (0,0,\alpha,\alpha)$. The indifference in (i) has been proved. The second indifference follows from the first, (2), (3), and transitivity. □

## Appendix 5.2 Details of the DM method

Preferences $\alpha_{\{1,\ldots,j\}}0 \prec \alpha_{\{j+1,\ldots,52\}}0$ and $\alpha_{\{1,\ldots,j+1\}}0 \succ \alpha_{\{j+2,\ldots,52\}}0$ reveal that $c_{½}$ is in the interval (j, j+1). We then estimate $c_{½} = j+½$. For the DM, we used the following roundings to derive the discount factors from the C function (Figure 5.6.2). For each of the six periods considered (bounded by t = 0, the five $c_p$ values that we measured, and t = 52), we divided the increase of C over this period by the length of the period to obtain the average week-weight d over this period. This d value we assigned to the midpoint of the period. Between these midpoints we used linear interpolation. We normalized (setting d = 1) at the smallest positive time point considered, being $c_{1/8}/2$. Its average (2.75) was approximately 3, leading to about the same normalization as with the UM. Thus we obtained a d-function over the interval (3, 48.25], with 48.25 the average midpoint of the last interval ($c_{⅞}$,52].

Because we only presented integer-week periods to subjects, and estimates of $c_p$ usually were nonintegers, we could not present exact $c_p$ values to our subjects in our adaptive experiment. For example, to find the subjective midpoint $c_{¼}$ of (0, $c_{½}$], we rounded $c_{½}$ and took the smallest larger integer, denoted j+1 here, and then found the subjective midpoint x of (0,j+1]. To derive $c_{¼}$ from this midpoint x we corrected for the roundings. Because we had used j+1 instead of $c_{½}$, which on average is an overestimation of $c_{½}$ by ½, and half of it will propagate into x, we subtracted ¼ from x to get $c_{¼}$. In all other estimations of values $c_p$ we similarly used roundings and corrections. Complete details of the roundings and corrections for all $c_p$ are in the web appendix.

## Appendix 5.3 Details of the UM method

Following EFB, we adopted power utility (Wakker 2008) and Prelec's (1998) two-parameter probability weighting:

If $\eta > 0$, then $u(x) = x^{\eta}$;

If $\eta = 0$, then $u(x) = \ln(x)$;

If $\eta < 0$, then $u(x) = -x^{\eta}$;

$w(p) = e^{-\beta(-\ln(p))^{\alpha}}$.

For convenience, x is generic for outcomes in this appendix. The average value of the utility parameter $\eta$ was 0.47 (in EFB, $\eta = 0.87$), which reflects concavity.[21] The average insensitivity index $\alpha$ was 0.55 (in EFB, $\alpha = 0.51$), indicating departure from linear probability weighting. The average estimates for the pessimism index $\beta$ was 0.94 (in EFB, $\beta = 0.97$).

In Eq. 5.4.2 we should have $U(0) = 0$, which agrees with prospect theory's scaling. Following EFB, the estimation of utility is carried out after shifting all outcomes by one unit of money, so as to avoid mathematical complications of logarithmic or negative-power utility at $x = 0$.

We followed EFB in using choice lists to elicit $\lambda$ in Eq. 5.4.1 (details in the web appendix). If the largest value in a choice list was still too small to lead to preference, we assumed preference to switch in the first higher value to follow. We thus use censored data. It gives a smaller bias than dropping these subjects, the most impatient ones, as done by EFB, and it keeps more subjects for other measurements. The DM measurements need no censoring of data because the indifference points are always between extremes of the choice lists.

[21] The average value of the parameters in our analysis is based on 96 subjects (including subjects who have missing values). EFB removed all subjects with missing values.

# Chapter 6 | Conclusions

This thesis examines decision making under uncertainty using both empirical and theoretical analyses.

Chapter 2 analyzes a model with multiplier preferences. This is a popular model in macroeconomics and finance that was introduced by Hansen and Sargent (2001). This model allows for a deviation from expected utility (the standard model) due to a different treatment of 'ambiguous' events. People make a guess of the probabilities of these events occurring, but do not know the exact probabilities.

In its original form, the model with multiplier preferences can only capture ambiguity aversion, where people prefer known probabilities to unknown ones. This is not a problem on a macroeconomic level, but on a micro level, a substantial proportion of people is often ambiguity seeking: they prefer unknown probabilities to known ones. We give a preference foundation for an extension of multiplier preferences, such that it can be used to explain ambiguity-seeking behavior.

We also show how extended multiplier preferences can be measured and thereby obtain a measure of ambiguity aversion that can easily be applied in empirical studies. A first application of this method on two large scale representative surveys (Netherlands & US) shows that a substantial fraction (around one third) of the population was indeed ambiguity seeking.

Chapter 3 presents a study on the effects of time pressure on decision making under ambiguity. In a lab experiment with real incentives, subjects are asked to choose between risky and ambiguous bets. The ambiguous bets are based on the movement of the Amsterdam Stock Market Index (AEX), and therefore the probabilities associated with the outcomes are unknown. In the risky bets, however, outcomes are presented with their respective objective probabilities. In the treatment group, subjects have to state their preferences within a time limit. For the analysis of ambiguity, a new simplified method is introduced which relies on a tool called matching probability function to measure ambiguity behavior. Ambiguity behavior can be dissected into two components: ambiguity aversion and ambiguity-generated insensitivity (a-insensitivity). Ambiguity aversion is an affective component that refers to how much less people like ambiguity as compared to risk, whereas a-insensitivity is a cognitive component indicating how much less people understand ambiguity compared to risk. The results of this experiment demonstrate that time pressure does not change people's reactions to ambiguity affectively; the ambiguity aversion component remains unaffected. However, the a-insensitivity component is negatively impacted by time pressure. Therefore, the results agree with past research showing that cognitive faculties are compromised under time pressure.

Chapter 4 discusses how people update their beliefs under ambiguity from three approaches. One is the traditional Bayesian updating, where only ambiguity neutral behavior is accommodated. The other two approaches are non-Bayesian, introduced by *Gilboa and Schmeidler* (1993) (*GS*) as well as *Dempster* (1967) and *Shafer* (1976) (*DS*) respectively, where both ambiguity averse and ambiguity seeking behavior are accommodated. Under the framework of decision theory, this paper compares Bayesian and non-Bayesian updating in its model specification and numerical implications. Ambiguity attitudes affect not only static decisions, but also the way in which new information is incorporated. For an ambiguity averse (seeking) decision maker, GS updating leads to more pessimistic (optimistic) behavior than DS updating, and favorable or unfavorable information has a bigger (smaller) impact on GS updating than on DS updating.

Finally, Chapter 5 introduces a new method to measure the temporal discounting of money. Unlike preceding methods, this method requires neither knowledge nor measurement of utility. It is easier to implement, clearer to subjects, and requires fewer measurements than existing methods. Because the method directly measures discounting, and utility plays no role, it is called the direct method (DM). The basic idea of the DM is as follows. Assume that a decision maker is indifferent between: (a) an extra payment of $10 per week during weeks 1-30; and (b) the same extra payment during weeks 31-65. Then the total discount weight of weeks 1-30 is equal to that of weeks 31-65. We can derive the entire discount function from such equalities. Knowledge of utility is not required because it drops from the equations. Even though this method is elementary, it has not been known before. In an experiment, we compare it with a traditional, utility based, method (UM) and find that the DM needs fewer questions than the UM but gives similar results.

# Samenvatting

Dit proefschrift onderzoekt beslissingen bij onzekerheid, zowel gebruik makend van theoretische als van empirische analyses. Hoofdstuk 2 onderzoekt een model met zogenaamde multiplier preferenties. Dit is een populair model in macro-economie en finance dat is geïntroduceerd door Hansen and Sargent (2001). Dit model laat afwijkingen van verwacht nut (het standaard model) toe door een verschillende behandeling van ambigue gebeurtenissen. Mensen doen een eerste schatting van de waarschijnlijkheid dat deze gebeurtenissen optreden, maar kennen niet de precieze kansen. In zijn oorspronkelijke vorm kan het model van multiplier preferenties alleen maar afkeer van ambiguïteit beschrijven, waar mensen bekende kansen prefereren boven onbekende kansen. Dit is geen probleem op macro-economisch niveau, maar op het micro niveau is een aanzienlijk deel van de mensen ambiguïteit zoekend: zij prefereren onbekende boven bekende kansen. We geven een preferentie-fundering aan het multiplier model zodanig dat het ook gebruikt kan worden om ambiguïteit zoekendheid te verklaren.

We laten ook zien hoe het resulterende uitgebreide multiplier preferentie model kan worden gemeten en verkrijgen daarbij een maat van ambiguïteits afkeer die gemakkelijk kan worden toegepast in empirische studies. Een eerste toepassing van deze methode op twee grootschalige representieve enquetes (Nederland en de Verenigde Staten) toont dat een aanzienlijk deel (ongeveer 1/3) van de populatie inderdaad ambiguïteit zoekend is.

Hoofdstuk 3 presenteert een onderzoek naar de effecten van tijdsdruk op beslissen bij ambiguïteit. In een laboratorium experiment met real incentives wordt subjecten gevraagd te kiezen tussen riskante en ambigue prospects. De ambigue prospects zijn gebaseerd op de bewegingen van de Amsterdam Stock Exchange (AEX), en daarom zijn de kansen op de relevante uitkomsten hierbij onbekend. In de riskante prospect worden de uitkomsten echter gepresenteerd tezamen met hun objectieve kansen van optreden. In de behandel groep moeten subjecten hun preferenties kenbaar maken binnen een tijdslimiet. Voor de analyze van ambiguïteit wordt een nieuwe vereenvoudigde manier ingevoerd die gebruik maakt van matching kansen. Gedrag onder ambiguïteit kan ontleed worden in twee componenten: afkeer en ambiguïteit-gegenereerde ongevoeligheid (a-ongevoeligheid). Afkeer is een affectieve component die beschrijft in welke mate mensen ambiguïteit minder prefereren dan risico, terwijl a-ongevoeligheid een cognitieve component is die aangeeft hoeveel mensen ambiguïteit minder goed begrijpen dan risico.De resultaten van dit experiment tonen dat tijdsdruk geen invloed heeft op de reacties van subjecten op ambiguïteit in affectieve zin; de ambiguïteits component wordt niet beïnvloed door tijdsdruk. A-ongevoeligheid wordt echter negatief beïnvloed door tijdsdruk. Daarom stemmen de resultaten overeen met eerdere onderzoekingen die ook toonden dat cognitieve factoren gecompromiteerd worden door tijdsdruk.

Hoodstuk 4 onderzoekt hoe mensen hun geloof onder ambiguïteit updaten onder drie verschillende benaderingen. Een is de traditionele Bayesiaanse, waar alleen nautraal gedrag tov ambiguïteit mogelijk is. De andere twee benaderingen zijn niet-Bayesiaans, en zijn geïntroduceerd door Gilboa en Schmeidler (1993) (GS) en door Dempster (1967) en Shafer (1976), waarbij zowel ambiguïteit-afkerig en ambiguïteit-zoekend gedrag mogelijk is. In het raamwerk van beslissings-theorie vergelijkt dit paper Bayesiaanse en niet-Bayesiaanse updating betreffende numerieke implicaties. Houdingen tov ambiguïteit hebben niet alleen invloed op statische beslissingen, maar ook op de manier waarop nieuwe informatie wordt verwerkt. Voor een ambiguïteits-afkerige (zoekende) beslisser leidt GS updaten tot meer pessimistisch (optimistisch) gedrag dan DS updaten, en gunstige of ongunstige informatie heeft een grotere (kleinere) invloed op GS updaten dan op DS updaten.

Hoofdstuk 5, tenslotte, introduceert een nieuwe methode om het disconteren van geld te meten. In tegenstelling tot eerdere methoden vereist deze methode noch kennis noch een meting van utiliteit. Hij is makkelijker te implementeren, duidelijker voor subjecten, en vergt minder metingen dan bestaande methoden. Omdat de methode disconteren direct meet, en utiliteit geen rol speelt, heet hij de directe methode (DM). Het grondidee van de DM is als volgt. Veronderstel dat een beslisser indifferent is tussen: (a) een extra betaling van$10 per week gedurende weken 1-30; en (b) dezelfde betaling gedurende weken 31-65. Dan is het totale gedisconteerde gewicht van weken 1-30 gelijk aan dat van weken31-65. We kunnen de gehele disconterings-functie van zulke gelijkheden afleiden. Kennis van nut is niet nodig omdat het uit de vergelijkingen weg valt. Zelfs hoewel deze methode elementair is, is hij niet eerder bekend geweest. In een experiment vergelijken we hem met een traditionele, utiliteits-gebaseerde, methode (UM) en vinden dat de DM minder vragen nodig heeft dan de UM maar gelijke resultaten geeft.

# Bibliography


Abdellaoui, Mohammed, Arthur E. Attema, & Han Bleichrodt (2010) "Intertemporal Tradeoffs for Gains and Losses: An Experimental Measurement of Discounted Utility," *Economic Journal* 120, 845–866.

Abdellaoui, Mohammed, Aurélien Baillon, Laetitia Placido, and Peter P. Wakker. (2011), "The Rich Domain of Uncertainty: Source Functions and Their Experimental Implementation," *American Economic Review*, 101(2): 695-723

Abdellaoui, Mohammed, Han Bleichrodt, & Olivier L'Haridon (2013) "Sign-Dependence in Intertemporal Choice," *Journal of Risk and Uncertainty* 47, 225–253.

Aczél, János (1966) "*Lectures on Functional Equations and Their Applications.*" Academic Press, New York.

Aczél, János. (2005) "Utility of Extension of Functional Equations—When Possible" *Journal of Mathematical Psychology*, 49(6): 445-449.

Andersen, Steffen, Glenn W. Harrison, Morten I. Lau, & E. Elisabet Rutstrom (2008) "Eliciting Risk and Time Preferences," *Econometrica* 76, 583–618.

Andreoni, James & Charles Sprenger (2012a) "Estimating Time Preference from Convex Budgets," *American Economic Review* 3333–3356.

Andreoni, James & Charles Sprenger (2012b) "Risk Preferences are not Time Preferences," *American Economic Review* 3357–3376.

Anscombe F.J. and Aumann R.J., (1963), "A Definition of Subjective Probability," *The Annals of Mathematical Statistics*, Vol. 34, No.1

Ariely, Dan & Dan Zakay (2001) "A Timely Account of the Role of Duration in Decision Making," *Acta Psychologica* 108, 187–207.

Attema, Arthur E. (2012) "Developments in Time Preference and Their Implications for Medical Decision Making," *Journal of the Operational Research Society* 63, 1388–1399.

Attema, Arthur E., Han Bleichrodt, Kirsten I.M. Rohde, & Peter P. Wakker (2010) "Time-Tradeoff Sequences for Analyzing Discounting and Time Inconsistency," *Management Science* 56, 2015–2030.

Attema, Arthur E., Han Bleichrodt, & Peter P. Wakker (2012) "A Direct Method for Measuring Discounting and QALYs more Easily and Reliably," *Medical Decision Making* 32, 583–593.

Baillon, Aurélien & Han Bleichrodt (2015) "Testing Ambiguity Models through the Measurement of Probabilities for Gains and Losses," *American Economic Journal: Microeconomics*, Vol 7:2, pp.77-100.

Baillon, Aurélien, Laure Cabantous, & Peter P. Wakker (2012) "Aggregating Imprecise or Conflicting Beliefs: An Experimental Investigation Using Modern Ambiguity Theories," *Journal of Risk and Uncertainty* 44, 115–147.

Bardsley, Nick, Robin P., Cubitt, Graham Loomes, Peter Moffat, Chris Starmer, & Robert Sugden (2010) "*Experimental Economics; Rethinking the Rules*." Princeton University Press, Princeton, NJ.

Baumeister, Roy F., E. J. Masicampo, & Kathleen D. Vohs (2011) "Do Conscious Thoughts Cause Behavior?," *Annual Review of Psychology* 62, 331–361.

Binmore, Ken, Lisa Stewart, & Alex Voorhoeve (2012) "How Much Ambiguity Aversion? Finding Indifferences between Ellsberg's Risky and Ambiguous Bets," *Journal of Risk and Uncertainty* 45, 215–238.

Bleichrodt, Han, Amit Kothiyal, Drazen Prelec, & Peter P. Wakker (2013) "Compound Invariance Implies Prospect Theory for Simple Prospects," *Journal of Mathematical Psychology* 57, 68–77.

Bleichrodt, Han, Kirsten I.M. Rohde, & Peter P. Wakker (2009) "Non-Hyperbolic Time Inconsistency," *Games and Economic Behavior* 66, 27–38.

Burger, Nicholas, Gary Charness, & John Lynham (2011) "Field and Online Experiments on Self-Control," *Journal of Economic Behavior and Organization* 77, 393–404.

Camerer, Colin F. (1995) "Individual Decision Making." *In* John H. Kagel & Alvin E. Roth (eds.) *Handbook of Experimental Economics*, 587–703, Princeton University Press, Princeton, NJ.

Carnap, Rudolf (1952), "The Continuum of Inductive Methods," *University Press, Chicage*.

Carnap, Rudolf (1980), "A Basic System of Inductive Logic, Part II," In Richard C. Jeffrey (Ed.), *Studies in Inductive Logic and Probability*, Vol. II, 7-155, University of California Press, Berkeley.

Chapman, Gretchen B. (1996) "Expectations and Preferences for Sequences of Health and Money," *Organizational Behavior and Human Decision Processes* 67, 59–75.

Charness, Gary, Edi Karni, & Dan Levin (2013) "Ambiguity Attitudes and Social Interactions: An Experimental Investigation," *Journal of Risk and Uncertainty* 46, 1–25.

Chew, Soo Hong and Sagi J (2008), "Small worlds: Modeling Attitudes toward Sources of Uncertainty," *Journal of Economic Theory* Vol 139, Issue 1, 1-24.

Chew, Soo Hong, Richard P. Ebstein, & Songfa Zhong (2012) "Ambiguity Aversion and Familiarity Bias: Evidence from Behavioral and Gene Association Studies," *Journal of Risk and Uncertainty* 44, 1–18.

Croson, Rachel and Uri Gneezy. 2009. "Gender Differences in Preferences" *Journal of Economic Literature* 47(2), 448–474.

Cubitt, Robin P. & Daniel Read (2007) "Can Intertemporal Choice Experiments Elicit Time Preferences for Consumption?," *Experimental Economics* 10, 369–389.

Debreu, Gerard and Tjalling C. Koopmans. 1982. "Additively Decomposed Quasiconvex Functions" *Mathematical Programming*, 24(1): 1-38.

Dempster, A.P. (1967), "Upper and Lower Probabilities Induced by a Multivalued Mapping," *Annals of Operations Research* 52, 21-42.

De Finetti, Bruno (1937), "La Prevision: Ses Lois Logiques, ses Source Subjectives," Annales de l'Institut Henri Poincare 7, 1-68. Translated into English by Henry E. Kyburg Jr. "Foresight: Its Logical Laws, its Subjective Sources." in Henry E. Kyburg Jr. & Howard E. Smokler (1964, Eds.), *Studies in Subjective Probabilities,* 53-118, Wiley, New York; 2nd edition 1980, Krieger, New York.

Dillenberger, David, & Segal, Uzi. 2015. "Recursive Ambiguity and Machina's Examples" *International Economic Review* 56(1), 55-61.

Dimmock, Stephen G., Roy Kouwenberg, and Peter P. Wakker. 2015. "Ambiguity Attitudes in a Large Representative Sample" *Management Science,* forthcoming.

Dimmock, Stephen G., Roy Kouwenberg, Olivia S. Mitchell, and Kim Peijnenburg. 2013. "Ambiguity Aversion and Household Portfolio Choice: Empirical Evidence" *NBER Working Paper* No. 18743.

Dohmen, Thomas, Armin Falk, David Huffman, and Uwe Sunde. 2010. "Are Risk Aversion and Impatience Related to Cognitive Ability?" *American Economic Review*, 100(3): 1238-1260.

Dolan, Paul & Daniel Kahneman (2008) “Interpretations of Utility and Their Implications for the Valuation of Health,” *Economic Journal* 118, 215–234.

Dow, James & Sérgio R.C. Werlang (1992) “Uncertainty Aversion, Risk Aversion and the Optimal Choice of Portfolio,” *Econometrica* 60, 197–204.

Dror, Itiel E., Beth Basola, & Jeromy R. Busemeyer (1999) “Decision Making under Time Pressure: An Independent Test of Sequential Sampling Models,” *Memory & Cognition* 27, 713–725.

Dupuis, Paul and Richard S. Ellis. 2011. *A Weak Convergence Approach to the Theory of Large Deviations*. New York, NY: John Wiley & Sons.

Ebert, Jane E.J. & Drazen Prelec (2007) “The Fragility of Time: Time-Insensitivity and Valuation of the Near and Far Future,” *Management Science* 53, 1423–1438.

Ellsberg, Daniel. (1961), “Risk, Ambiguity and the Savage Axioms,” *Quartely Journal of Economics* 75, 643-669.

Epper, Thomas & Helga Fehr-Duda (2015) “Risk Preferences Are not Time Preferences: Balancing on a Budget Line: Comment (#12),” *American Economic Review* 105, 2261–2271.

Epper, Thomas, Helga Fehr-Duda, & Adrian Bruhin (2011) “Viewing the Future through a Warped Lens: Why uncertainty Generates Hyperbolic Discounting,” *Journal of Risk and Uncertainty* 43, 163–203.

Epstein, L.G. and Le Breton, M. (1993) “Dynamically Consistent Beliefs Must be Bayesian,” *Journal of Economic Theory* 61, 1-22.

Erev, Ido & Eyal Ert (2013), “On the Descriptive Value of Loss Aversion in Decisions under Risk,” *Judgment and Decision Making* 8, 214–235.

Etner, Johanna, Meglena Jeleva, & Jean-Marc Tallon (2012) “Decision Theory under Ambiguity”, *Journal of Economic Surveys* 26, 234–270.

Frederick, Shane, George F. Loewenstein, & Ted O’Donoghue (2002) “Time Discounting and Time Preference: A Critical Review,” *Journal of Economic Literature* 40, 351–401.

Fox, Craig R. & Jonathan Levav (2000) “Familiarity Bias and Belief Reversal in Relative Likelihood Judgment,” *Organizational Behavior and Human Decision Processes* 82, 268–292.

Fox, Craig R. & Amos Tversky (1995) "Ambiguity Aversion and Comparative Ignorance," *Quarterly Journal of Economics* 110, 585–603.

Frederick, Shane. 2005. "Cognitive Reflection and Decision Making" *Journal of Economic Perspectives*, 19(4): 25-42.

Ghirardato, Paolo & Massimo Marinacci (2002) "Ambiguity Made Precise: A Comparative Foundation," *Journal of Economic Theory* 102, 251–289.

Gilboa, I. (1987), "Expected Utility with Purely Subjective Non-Additive Probabilities", *Journal of Mathematical Economics*, 16, 65-88.

Gilboa, I. (1989a), "Duality in Non-additive Expected Utility Theory," *Annals of Operations Research* 19, 405-414.

Gilboa, I. (1989b), "Additivations of NonAdditive Measures," *Mathematics of Operations Research* 14, 1-17.

Gilboa, I. and M. Marinacci, (2013), "Ambiguity and the Bayesian Paradigm", *Advances in Economics and Econometrics: Theory and Applications*, Tenth World Congress of the Econometric Society. D. Acemoglu, M. Arellano, and E. Dekel (Eds.). New York: Cambridge University Press.

Gilboa, I. and Schmeidler, D (1993) "Updating Ambiguous Beliefs," *Journal of Economic Theory* 59, 33-49.

Gilboa, Itzhak & David Schmeidler (1989) "Maxmin Expected Utility with a Non-Unique Prior," *Journal of Mathematical Economics* 18, 141–153.

Grant, Simon and Ben Polak. 2013. "Mean-Dispersion Preferences and Constant Absolute Uncertainty Aversion" *Journal of Economic Theory*, 148(4): 1361-1398.

Grant, Simon, Ben Polak, and Tomasz Strzalecki. 2009. "Second-Order Expected Utility" *mimeo*.

Hansen, Lars P. and Thomas J. Sargent. 2001. "Robust Control and Model Uncertainty" *American Economic Review*, 91(2): 60-66.

Ibanez Marcela, Simon Czermak, & Matthias Sutter (2009) "Searching for a Better Deal. On the Influence of Group Decision Making, Time Pressure and Gender in a Search Experiment," *Journal of Economic Psychology* 30, 1–10.

Ivanov, Asen (2011) "Attitudes to Ambiguity in One-Shot Normal-Form Games: An Experimental Study," *Games and Economic Behavior* 71, 366–394.

Kahneman, Daniel (2011) "*Thinking: Fast and Slow*." Penguin Books, London.

Keeney, Ralph L. & Howard Raiffa (1976) "*Decisions with Multiple Objectives.*" Wiley, New York (2nd edn. 1993, Cambridge University Press, Cambridge).

Keynes, John Maynard (1921) "*A Treatise on Probability.*" McMillan, London. 2nd edn. 1948.

Kilka, Michael & Martin Weber (2001) "What Determines the Shape of the Probability Weighting Function under Uncertainty," *Management Science* 47, 1712–1726.

Klibanoff, Peter, Massimo Marinacci, & Sujoy Mukerji (2005) "A Smooth Model of Decision Making under Ambiguity," *Econometrica* 73, 1849-1892.

Knight, Frank H. (1921) "*Risk, Uncertainty, and Profit.*" Houghton Mifflin, New York.

Kocher, Martin G. &Matthias Sutter (2006) "Time Is Money—Time Pressure, Incentives, and the Quality of Decision-Making," *Journal of Economic Behavior & Organization* 61, 375–392.

Kocher, Martin G., Julius Pahlke, & Stefan T. Trautmann (2013) "Tempus Fugit: Time Pressure in Risky Decisions," *Management Science*, forthcoming.

Kothiyal, Amit, Spinu, Vitalie, & Wakker, Peter P, (2014) "An Experimental Test of Prospect Theory for Predicting Choice Under Ambiguity," *Journal of Risk and Uncertainty* 48, 1-17.

Krantz, David H., R. D. Luce, Patrick Suppes, and Amos Tversky. 1971. *Foundations of Measurement: Vol. 1: Additive and Polynomial Representations*. New York, NY: Academic Press.

Loewenstein, George F. & Drazen Prelec (1991) "Negative Time Preference," *American Economic Review, Papers and Proceedings* 81, 347–352.

Loewenstein, George F. & Drazen Prelec (1992) "Anomalies in Intertemporal Choice: Evidence and an Interpretation," *Quarterly Journal of Economics* 107, 573–597.

Luce, R. Duncan & Howard Raiffa (1957) "*Games and Decisions.*" Wiley, New York.

Maccheroni, Fabio, Massimo Marinacci, and Aldo Rustichini. 2006. "Ambiguity Aversion, Robustness, and the Variational Representation of Preferences" *Econometrica*, 74(6): 1447-1498.

Machina, Mark J. (1989), "Dynamic Consistency and Non-Expected Utility Models of Choice under Uncertainty," *Journal of Economic Literature* 27, 1622-1688.

Machina, Mark J., and David Schmeidler. (1992) "A More Robust Definition of Subjective Probability," *Econometrica*, 60(4): 745-80.

Machina, Mark J. (2009) “Risk, Ambiguity, and the Rank-Dependence Axioms,” *American Economic Review* 99, 385–392.

Machina, Mark J. (2014) "Ambiguity aversion with three or more outcomes." *American Economic Review* 104(12), 3814-40.

Machina, Mark J. et Siniscalchi, Marciano (2014) “Ambiguity and Ambiguity Aversion” *Handbook of the Economics of Uncertainty*, 1:729-807.

Maule A. John, G. Robert J. Hockey , & Larissa Bdzola (2000) “Effects of Time-Pressure on Decision Making under Uncertainty: Changes in Affective State and Information Processing Strategy,” *Acta Psychologica* 104, 283–301.

Moscati, Ivan (2013) “How Cardinal Utility Entered Economic Analysis, 1909-1944,” *European Journal of the History of Economic Thought*, forthcoming.

Nau, Robert F. (2006) "Uncertainty aversion with second-order utilities and probabilities" *Management Science* 52(1): 136-145.

Neilson, William S. 2010. "A Simplified Axiomatic Approach to Ambiguity Aversion" *Journal of Risk and Uncertainty*, 41(2): 113-124.

Ordonez, Lisa & Lehman Benson III (1997) “Decisions under Time Pressure: How Time Constraint Affects Risky Decision Making,” *Organizational Behavior and Human Decision Processes* 71, 121–140.

Payne, John W., James R. Bettman, & Mary-Frances Luce (1996) “When Time Is Money: Decision Behavior under Opportunity-Cost Time Pressure,” *Organizational Behavior and Human Decision Processes* 66, 131–152.

Poulton, E. Christopher (1989) “*Bias in Quantifying Judgments.”* Erlbaum, Hillsdale NJ.

Pratt, John W. 1964. "Risk Aversion in the Small and in the Large" *Econometrica*, 32(1-2): 122-136.

Prelec, Drazen (1998) “The Probability Weighting Function,” *Econometrica* 66, 497–527.

Quiggin, John. (1981) “Risk Perception and Risk Aversion among Australian Farmers,” *Australian Journal of Agricultural Economics* 25, 160-169.

Quiggin, John. (1982), “A Theory of Anticipated Utility,” *Journal of Economic Behavior and Organization* 3, P323-343.

Read, Daniel (2001) “Is Time-Discounting Hyperbolic or Subadditive?,” *Journal of Risk and Uncertainty* 23, 5–32.

Reuben, Ernesto, Paolo Sapienza, & Luigi Zingales (2010), "Time Discounting for Primary and Monetary Rewards," *Economic Letters* 106, 125–127.

Reutskaja Elena, Rosemarie Nagel, Colin F. Camerer, & Antonio Rangel (2011) "Search Dynamics in Consumer Choice under Time Pressure: An Eye-Tracking Study," *American Economic Review* 101, 900–926.

Samuelson, Paul A. (1937) "A Note on Measurement of Utility," *Review of Economic Studies* 4 (Issue 2, February 1937), 155–161.

Sarin, Rakesh and Wakker Peter. P. (1998), "Revealed Likelihood and Knightian Uncertainty," *Journal of Risk and Uncertainty* 16, 223-250.

Savage, Leonard J. (1954), "*The Foundations of Statistics*," Wiley, New York. (Second Edition 1972, Dover, New York).

Schmeidler, David, (1989). "Subjective Probability and Expected Utility without Additivity," *Econometrica* 57, 571-587.

Segal, Uzi. 1987. "The Ellsberg Paradox and Risk Aversion: An Anticipated Utility Approach" *International Economic Review*, 28: 175–202.

Shafer, G. (1976), "A Mathematical Theory of Evidence. Princeton," NJ: Princeton University Press.

Simon, Herbert A. (1982) "*Models of Bounded Rationality*, Vols 1 and 2." The MIT Press, London.

Starmer, Chris (2000) "Developments in Non-Expected Utility Theory: The Hunt for a Descriptive Theory of Choice under Risk," *Journal of Economic Literature* 38, 332–382.

Stevens, Stanley S. (1936) "A Scale for the Measurement of a Psychological Magnitude: Loudness," *Psychological Review* 43, 329–353.

Stiggelbout, Anne M., Gwendoline M. Kiebert, Job Kievit, Jan-Willem H. Leer, Gerrit Stoter, & Hanneke C.J.M. de Haes (1994) "Utility Assesement in Cancer Patients: Adjustment of Time Tradeoff Scores for the Utility of Life Years and Comparison with Standard Gamble Scores," *Medical Decision Making* 14, 82–90.

Strzalecki, Tomasz. 2011. "Axiomatic Foundations of Multiplier Preferences" *Econometrica*, 79(1): 47-73.

Sutter Matthias, Martin G. Kocher, Sabine Strauss (2003) "Bargaining under Time Pressure in an Experimental Ultimatum Game," *Economic Letters* 81, 341–347.

Sutter, Matthias, Martin G. Kocher, Daniela Glätzle-Rüetzler, & Stefan T. Trautmann (2013) "Impatience and Uncertainty: Experimental Decisions Predict Adolescents' Field Behavior," *American Economic Review* 103, 510–531.

Takeuchi, Kan (2011) "Non-Parametric Test of Time Consistency: Present Bias and Future Bias," *Games and Economic Behavior* 71, 456–478.

Tanaka, Tomomi, Colin F. Camerer, & Quang Nguyen (2010) "Risk and Time Preferences: Linking Experimental and Household Survey Data from Vietnam," *American Economic Review* 100, 557–571.

Tinghög, Gustav, David Andersson, Caroline Bonn, Harald Böttiger, Camilla Josephson, Gustaf Lundgren, Daniel Västfjäll, Michael Kirchler, Magnus Johannesson (2013) "Intuition and Cooperation Reconsidered," *Nature* 498 (06 June 2013), pp. E1–E2.

Trautmann, Stefan T. and Gijs van de Kuilen, (2015), "Ambiguity Attitudes." In Gideon Keren & George Wu (eds.), *The Willey Blackwell Handbook of Judgment and Decision Making*, 89-116, Blackwell, Oxford, UK.

Tversky, Amos & Daniel Kahneman (1992) "Advances in Prospect Theory: Cumulative Representation of Uncertainty," *Journal of Risk and Uncertainty* 5, 297–323.

Villegas, Carlos. 1964. "On Qualitative Probability σ-Algebras" *Annals of Mathematical Statistics*, 35(4): 1787-1796.

Wakker, Peter. 1989. *Additive Representations of Preferences, A New Foundation of Decision Analysis.* Kluwer Academic Publishers, Dordrecht.

Wakker, Peter P, (2002), "Decision-Principles to Justify Carnap's Updating Method and to Suggest Corrections of Probability Judgments," In Adnam Darwiche & Nir Friedman (Eds.) *Uncertainty in Artificial Intelligence*, Proceedings of the Eighteenth Conference, 544-551, Morgan Kaufmann, San Francisco, CA.

Wakker, Peter P. (2008) "Explaining the Characteristics of the Power (CRRA) Utility Family," *Health Economics* 17, 1329–1344.

Wakker, Peter P, (2010), "Prospect Theory for Risk and Ambiguity," Cambridge: Cambridge University Press.

Warner, John T. & Saul Pleeters (2001) "The Personal Discount Rate: Evidence from Military Downsizing Programs," *American Economic Review* 91, 33–53.

Wilks, Samuel S. (1962), "Mathematical Statistics." Wiley, New York.

Wilson, Timothy D. & Jonathan W. Schooler (1991) "Thinking too Much: Introspection Can Reduce the Quality of Preferences and Decisions," *Journal of Personality and Social Psychology* 60, 181–192.

Yaari, Menahem E. (1969) "Some Remarks on Measures of Risk Aversion and on Their Uses," *Journal of Economic Theory* 1, 315–329.

Young, Diana L., Adam S. Goodie, Daniel B. Hall, & Eric Wu (2012) "Decision Making under Time Pressure, Modeled in a Prospect Theory Framework," *Organizational Behavior and Human Decision Processes* 118, 179–188.

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