Essays on Networks: Theory and Applications

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Essays on Networks: Theory and Applications

Essays over netwerken: theorie en toepassingen

Thesis

to obtain the degree of Doctor from the
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Prof.dr. S.W.J. Lamberts
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by

Ana Maria Babus
born in Iasi, Romania

ERASMUS UNIVERSITEIT ROTTERDAM

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[1]

[1] The blank page gives us the chance to wonder (Gaston Bachelard).

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Amsterdam

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Chapter 1

Introduction

Networks have proven to be a useful representation of various systems. Social and economic interactions, biological and ecological systems, the internet can be understood better if modelled as networks. Intuitively, a network describes a collection of nodes and the links between them. The notion of nodes is fairly general, they may be individuals or firms or countries. A link between two nodes represents a direct relation between them; for instance, a link could be a friendship tie between people, a research and development agreement between firms, or, in the context of countries, a link may be a mutual defense pact.

The study of networks spans across disciplines. A central field in sociology and studied in depth by mathematicians over the past fifty years, networks have recently received extensive attention in statistical physics, computer science, business strategy and organization theory. While economists have occasionally showed interest in networks, a literature has emerged only in the last decade.

This thesis brings two contributions. On the one hand, it aims to enhance the role of network theories in solving economic issues, by proposing a network representation of financial systems. On the other hand, it aims to enrich the set of methods used to analyze networks, by introducing economic microfoundations to a preferential attachment model of network formation.

It has often been emphasized the contribution economists bring to the field of networks. However, it is less clear what contribution networks bring to economics. To have a relevant impact on the discipline, networks ought to be used as tools to tackle economic questions. Applications of network theories to finance, industrial organization, labor economics or marketing, for instance, will mark the role networks play in economics. Chapter 2 and 3 of this thesis take a step in this direction and propose two applications to financial systems.

The study of networks has generated both empirical studies of various networked systems and the development of new techniques and models for their analysis and interpretation. Economists, sociologists, computer scientists and physics theorists, each have developed their own set of instruments for this purpose. I will explain in more detail the developments in economics, since I will be using these tools, especially in chapter 2.

The economic models of networks consider that social and economic phenomena should ultimately be explained in terms of decisions made by individuals through a rational deliberation of costs and benefits. A natural metric determines how an agent benefits from another agent depending on their relative position in the network. Agents choose with whom they interact by weighing the costs and benefits from being connected. Externalities across players are network dependent. That individuals are supposed to form or sever relationships, depending on the benefits they bring, can be modeled through a game of network formation.

Various equilibrium concepts have been advanced in the past few years to analyze the formation of bilateral connections in settings where agents are fully aware of the shape of the network they belong to and of the benefits they derive from it.[1] The difficulty arises from the fact that bilateral connections require consent to be formed from both sides involved (the vast majority of interactions falls within this category). Hence, a non-cooperative concept, such as Nash equilibrium, is not very useful to solve a network formation game.[2] A simpler notion looks directly at stable networks and

[1] Jackson (2005) provides an extensive survey of the literature on network formation, while Bloch and Jackson (2007) provide a comprehensive summary of stability and equilibrium definitions.

[2] Myerson (1991) attempted to model a noncooperative linking game in which agents independently announce which links they would like to see formed and then standard game-theoretic equilibrium concepts can be used to make predictions about which networks will form. There are at least two drawbacks with this approach. The first is that there is a multiplicity of equilibria with different types of network. There is also always an "empty network" equilibrium. It is always a Nash equilibrium for

has been proposed by Jackson and Wolinski (1996). According to them, a network is pairwise stable i) if a link between two individuals is absent from the network then it cannot be that both individuals would benefit from forming the link and ii) if a link between two individuals is present in a network then it cannot be that either individual would strictly benefit from deleting that link. There are many alternatives to this notion that have been proposed in the literature, mainly varying how many relationships agents can manage at the same time. For example, Gilles and Sarangi (2006) consider that individuals can change multiple relationships at the same time rather than just one at a time, while Goyal and Vega-Redondo (2007) go one step further by allowing pairs of individuals to coordinate on how they change relationships.[3] Bloch and Jackson (2007) propose another linking game where players can offer or demand transfers along with the links they suggest, which allows players to subsidize the formation of particular links. For the cases when consent is not needed, so that agents can unilaterally form new relationships, Bala and Goyal (2000) return to the Nash equilibrium concept.[4].

Our understanding of financial systems can benefit greatly from the recent development of network theories. The ongoing turmoil in financial markets has shown that financial systems are deeply intertwined. While the events unfolded, it became clear that the consequences of such an interconnected system are hard to predict. What initially was seen as difficulties in the US subprime mortgage market, rapidly escalated and spilled over to debt markets all over the world. As markets plunged, investors became more risk-averse. Banks became less willing to lend money as freely. Interbank lending rates started to rise and soon the market for short-term lending dried-up. The credit crunch ultimately triggered a bank run at the British mortgage lender Northern Rock - something not seen in the UK for over 140 years and in Western Europe for the last 15 years. The theory of networks applied to financial systems may contribute to-

each agent to say that he or she does not want to form any links, anticipating that the others will do the same.

[3] This is a special case of the general setting that allows groups to coordinate their changes in relationships - see Dutta and Mutuswami (1997) and Jackson and van den Nouweland (2005).

[4] The papers mentioned here are not exhaustive. The literature goes beyond a static equilibrium approach to study dynamic processes where the network gradually evolves over time (Jackson and Watts, 2002; Dutta et al. 2005; Page et al., 2005; Mauleon and Vannetelbosch, 2004).

wards the regulation of interbank and security markets, and aid in limiting the spread of future financial crises

Connections in the financial world are varied. The dependencies between financial institutions stem from both the asset and the liability side of their balance sheet. A network representation of financial systems may capture the intricate structure of linkages, whether they are created through mutual exposures between banks acquired on the interbank market, by holding similar portfolio exposures or by sharing the same mass of depositors. In this thesis, I argue that the theory of networks provides a conceptual framework within which the various patterns of connections can be described and analyzed in a meaningful way.

In the third chapter of this thesis (*The Formation of Financial Networks*), I develop a model of network formation in the banking system. In this chapter I propose a setting where the rationale behind the linkages in the banking system is given by the danger of contagion. It is widely acknowledged that banks and other financial institutions are linked in a variety of ways. The incentives for linking are driven by the benefits these links bring. The same connections that, for instance, facilitate the transfer of liquidity between banks, may expose the banking system to the risk of contagion. That is, idiosyncratic shocks, which initially affect only a few institutions, may propagate through the entire system.

I construct a model where banks form links with each other in order to reduce the risk of contagion. A network is formed endogenously between banks and serves as an insurance mechanism. In short, I consider a framework where negatively correlated liquidity shocks affect the banking system. Banks can perfectly insure against individual risk by exchanging interbank deposits. Risk-sharing, however, involves a trade-off: transfers create links between banks, exposing the system to the risk of contagion. The model predicts a connectivity threshold above which contagion does not occur, and banks form links to rich this threshold. Thus, in an equilibrium network, the probability of contagion is virtually zero.

The fourth chapter of the thesis (*Contagion Risk in Financial Networks*) also concentrates on financial networks. This chapter explores the effect of incomplete information in banking systems, when financial institutions are connected under different network structures. In particular, I study how the trade-off between the benefits and the costs of being linked changes depending on the network structure. I show that incomplete networks give rise to incomplete information. In this situation, the transfers between banks that perfectly ensures against liquidity shocks increase, at the same time, the contagion risk. The problem is solved when the network is complete, as the liquidity can be redistributed in the system, such that the risk of contagion is minimal.

To better function as a tool to solve economic problems, the theory of networks needs to further develop techniques, methods and measures of networks specifically tailored to tackle economic issues. The development of game theory over the past decades allowed the economists to formalize new techniques for analyzing networks. Nevertheless, most of the measures of networks used by economists are borrowed from sociology and theoretical physics. Clustering coefficients, that measure the tendency of linked nodes to have common neighbors, betweenness centrality of a node, that expresses how essential a node is for connecting other nodes, the degree distribution, as the number of links per node, the maximum and the average distance between nodes in a network, are all meaningful descriptive statistics of networks. However, it is very difficult to replicate these measures in a network formation game solved with the equilibria concepts surveyed above. The gap between network techniques used by economists and the current set of measures of networks is, actually, not surprising. These measures have been initially introduced to address specific questions in sociology and statistical physics. To bridge the gap, both new methods that take into account strategic behavior of agents and new measures of networks need to be formulated.

The second chapter of this thesis (*Limited Connections*) takes a step in this direction. Network formation models advanced by economists consider that networks are a consequence of strategic linking behavior of rational agents. The modelling of networks assumed in the other non-economic literatures starts from different premises: the network formation is driven mainly by a stochastic process. While the first approach leads to networks that stylize features like unequal connections and short distances, it is the second that captures richer characteristics of the real-world networks. This chapter aims to replicate the predictive power of the second approach in a model where (i) networks expand continuously by the addition of new nodes, (ii) new nodes form links with already existent nodes in order to maximize their payoffs, and (iii) linking requires consent of both agents involved. First, we provide a simple example that explains the emergence of power-law networks when agents entering the network employ logistic choice to form links. Further, we analyze various specifications for the benefits that nodes gain from connections, both in a error-free and in a error-prone setting. The key elements that lead to networks with high centrality and many levels of hierarchy are (i) intermediary benefits to those connecting otherwise disconnected parts of the network, and (ii) linking costs that increase with individual degree.

Part I

Theory

Chapter 2

Limited Connections

2.1 Introduction

Empirical research done in the field of networks has revealed a set of common properties that describe many real-world networks. For example, the world-wide-web, the film actors network, metabolic networks that determine the physiological and biochemical properties of a cell, all seem to exhibit similar features. In particular, real-world networks can be characterized through a series of variables as follows.

1. Short average distance. That is, the shortest path length between pairs of nodes in a social network tends to be very low. The maximum distance between any pair of nodes in a network is also small.

2. High clustering. Clustering coefficients measure the tendency of linked nodes to have common neighbors. The level of clustering in a social network is very high when compared to networks where links are formed at random.

3. Unequal connections. Social networks tend to exhibit high inequality, both in the number of links nodes have as well as in the payoffs nodes get from being connected. For instance, it has often been shown that the degree distribution of nodes in a network follows a power-law distribution. That is, the probability that a node in a network interacts with k other nodes decays as a power law. This

suggests that very few nodes have a large number of links, while there exists a large number of nodes with very few links.

4. Hierarchy. A rather disregarded feature of many real networks is their hierarchical topology. As Girvan and Newman (2007) observe, networks often have a fractal-like structure in which vertices cluster together into groups that then join to form groups of groups, and so forth, from the lowest levels of organization up to the level of the entire network.

5. Positive assortativity. The degrees of linked nodes tend to be positively correlated: well connected nodes tend to have well connected neighbors, while less connected nodes are more likely to be linked to other less connected nodes.[1]

Theoretical work has been considering these features when formulating models of network formation. Two major approaches have been outlined, drawing primarily from two sources: the economics literature and the random graph literature (and the subsequent statistical physics literature). While the first approach considers that networks form depending on how agents respond to different incentives, the second one aims to reproduce the empirical findings through various probabilistic rules of linking. Although each of these approaches has strengths and limitations, the perspectives they assume remain largely incompatible.

The economic modeling of networks considers that social and economic phenomena should ultimately be explained in terms of choices made by rational agents. This approach to network formation focuses on using game-theoretic tools. Agents choose with whom they interact by weighing the costs and benefits from being connected. The network arises as a consequence of individual linking decisions. Various stability concepts are employed to identify which networks emerge in equilibrium and their characteristics (e.g. Bala and Goyal, 2000; Jackson and Wolinsky, 1996; Goyal and Vega-Redondo, 2007, Bloch and Jackson, 2007, for a comprehensive survey, see Jackson, 2006). The strength of this approach is that it provides microfoundations to the network formation process. However, this research predicts simple network architectures, like stars

[1] These properties have empirically documented in various studies. For a comprehensive survey of this work, see Newman (2003).

and circles. Despite being highly stylized, these models have managed to explain some features of real world networks, like short average distances and unequal connections

The modelling of networks assumed in the other non-economic literatures starts from different premises. Since the empirical findings listed above appear to be universal for complex biological, social, and engineered systems, most of the research has focused on explaining how they emerged. This approach is rather mechanical, as the network is formed mainly through a stochastic process. For instance, Watts and Strogatz (1998) develop a model by starting with a symmetric network and randomly rewiring some links. In Price (1976) and Barabasi and Albert (1999) nodes form links through preferential attachment: new nodes link to existing nodes with probabilities proportional to the existing nodes' degree. Recently, a few models have been advanced to incorporate the hierarchical feature of networks (Ravasz and Barabasi, 2003; Clauset et al., 2006). Models where network formation is driven by a stochastic process lie in the statistical physics literature, overlapping with literature in sociology and computer science. Although they perform better in approximating characteristics of real-world networks, these models lack an economic reasoning. Moreover, one-sided link formation is commonly assumed, that is links are formed without requiring consent. While this assumption may be justified for describing the formation of networks like the world wide web or citation networks, consent is essential when formalizing social interactions.

This chapter explores the synergies that can result from bridging these two lines of research. In particular, we formulate a model of network formation where (i) networks expand by the sequential addition of new nodes, (ii) each new node forms one link with an already existent node in order to maximize its payoff, and (iii) linking requires consent of both agents involved. More specific, a link between two agents is established only when each earns benefits that exceed the costs of the new link.

Our model retains features from both literatures. On the one hand we capture the idea of rational deliberation with our assumption that agents entering the network optimize their initial links. On the other hand we depart from an equilibrium approach, and consider that networks form through a growth process.

The assumption that the population of nodes grows over time and links are formed sequentially, has recently been adopted also by the economics literature on network formation. For instance, Jackson and Rogers (2007) employ a network growth model to explain various features of real-world networks.[2] In their model, nodes form links uniformly at random, as well as, by searching locally through the current structure of the network. They successfully fit their model to data about several real-world networks by varying the ratio of connections formed at random to those formed through search.

In our setting, as in many network growth models, links cannot be removed after they have been formed. Connections in social networks, such as friendship links, are persistent over time. In fact, many real-world networks have been analyzed empirically under the assumption of persistent links. One instance is the network of scientific co-authorship in which a link between two scientists is assumed to exist when they published a jointly written paper.[3] In consequence, many of the above listed features of real-world networks have been derived by considering that links persist over time.

Myopic optimization, our second assumption, is another typical assumption in the literature on network formation, and can be justified with the complexity of predicting the network dynamics. When forming links, individuals might find it substantially easier to neglect the links that themselves or other individuals will establish in the future. A correct prediction of the future dynamics of networks would require, among else, a perfect understanding of other individuals' reasoning. Indeed, many equilibrium concepts assume a certain degree of myopia, when they allow players to deviate if they have an immediate gain from deviations, without considering the consequent changes in the network[4].

The assumption of network growth combined with that of optimizing behavior of agents can capture some striking features of real-world networks. We demonstrate in Section 2 that a power-law network emerges when agents entering the network employ

[2] In previous work, Jackson and Rogers (2005) show that small-world features emerge in a simple economic model of network formation, where agents benefit from indirect relationships.

[3] Goyal et al. (2006) provide a study of networks of co-authors in economics.

[4] Bloch and Jackson (2005) provide a comprehensive summary of stability and equilibrium definitions. Some interesting investigations of network formation with forward-looking individuals can be found in Dutta et al. (2005) or Mauleon and Vannetelbosch (2004).

logistic choice to form links. We then proceed to analyze the network growth process under four different payoff functions, both in a error-free and in a error-prone (discrete choice) setting. The network architectures we find range from symmetric trees, where distances are increasing to infinity, to networks where the path length between any pair of nodes stays very short, like stars and hierarchical stars (see fig. 3). We identify two key elements that lead to networks with high centrality and many levels of hierarchy (i) intermediary benefits to those connecting otherwise disconnected parts of the network, and (ii) linking costs that increase with individual degree. While, our findings remain relatively stylized, they provide indications on the economic forces driving some of the above features of real-world networks, in a preferential attachment framework.

The chapter is organized as follows. In the following section we show that error-prone myopic linking can lead to the power-law degree distribution. In Section 3 we describe our network growth model in detail. We investigate the resulting network formation under deterministic and probabilistic linking in Sections 4 and 5. Section 6 contains a discussion of our results and possible extensions, and Section 7 concludes.

2.2 The Preferential Attachment Model Explained

Inequality in the number of links nodes have seems to be a very robust finding of many real world networks. Moreover, several networks, such as collaborations between move actors, co-authorship networks, or the World Wide Web are thought to exhibit a degree distribution that follows a power-law. A power-law distribution implies that major hubs are closely followed by smaller ones, and these ones are followed by other nodes with an even smaller degree and so on. The network topology implied by such a heavy-tailed distribution has several interesting properties. First it allows for fault tolerant behavior: since failures occur at random and the vast majority of nodes are those with small degree, the likelihood that a hub is affected is almost negligible(Albert et al, 2000). Second, and perhaps carrying more economic implications, diffusion of behavior or transmission of information takes place at faster pace in power-law networks (Lopez-Pintado, 2007, Pastor-Satorras and Vespignani, 2001). These properties motivated an entire body of research to seek models that predict a skewed degree distribution.

In a highly influential paper, Barabasi and Albert (1999) explain the emergence of power-law networks as a consequence of two generic mechanisms: (i) networks expand continuously by the addition of new nodes, and (ii) new nodes attach preferentially to old nodes that are well connected. The growing character of the networks is modelled through the following process: In a discrete-time setting, a new node is added to the network every period with m links that connect the new node to m different nodes already present in the system. Preferential attachment is modelled by considering that the probability Π a new node will be connected to an existent node i depends on the connectivity η_i of that node, so that $\Pi(\eta_i) = \frac{\eta_i}{\Sigma_j \eta_j}$.

Although it has a high predictive power, the preferential attachment model has hardly been adopted by economists as a network formation model. The main reason for this standpoint is that the economic modeling of networks, like much of the economic modelling in general, considers that social and economic phenomena should ultimately be explained in terms of the choices made by rational agents. Agents choose with whom they interact by weighing the costs and benefits from being connected. Thus, while the growth component of the preferential attachment model may be regarded as another dynamic process through which the network is formed (Jackson and Rogers, 2007), the probability of attachment remains an unsatisfactory explanation of how agents form links.

In this section we illustrate with a simple example how networks with a power-law distribution can emerge through a growth mechanism, while nodes choose with whom to link by considering costs and benefits from being connected.

When we model the benefits players gain from the network, we follow a standard approach in the literature on the economics of networks. It is commonly assumed that connections grant access to important information concerning, for instance, job vacancies or ideas for research. Players benefit from the information they access through their neighbors and through the other nodes they are indirectly connected to (Jackson and Wolinsky, 1996; Bala and Goyal, 2000). In our setting, players can access the information of others, as long as they are at most two links away in the network. Moreover, the value of information decreases logarithmically, depending on the number of links between agents. For simplicity, we normalize the value of information to 1. Formally, the marginal payoff a node i gains from linking with node j is given by

$$\pi_i(g+ij) - \pi_i(g) = 1 + \ln[\max(1, \eta_j)] - c \tag{2.1}$$

where $\pi_i(g)$ is the payoff of i in the current network g, $g + ij$ is the network from g by adding the link ij, and η_j is the number of neighbors j has in network g.

In other words, a player i gains 1 from a direct neighbor, logarithmically less from a neighbor's neighbors, while it has to pay a cost c for the connection. For convenience, we assume that $c = 1$ in the remaining of the section.

The network is formed over time, sequentially, through the addition of new nodes to the population. Time is a discrete set of dates $t \in \{1, 2, ...\}$. At each date t, a new node is born. A node born at date t is denoted by its birthdate t. The set of nodes existent at date t is denoted by N_t. Let g_t be the network that is in place at time t. Upon birth, node t can form at most one link with an existing node in N_{t-1}. Links are formed as follows.

We first consider a deterministic setting. A new node, t, first proposes a link to the existing node in N_{t-1} that would give it the highest payoff. In principle, the node which receives the proposal will either accept or reject the link depending on whether the marginal benefit from being linked to the new node t exceeds, or at least equals the cost of linking. However, the marginal payoff of any old node from linking to a new node is 0, according to eq. (2.1). This implies that an old node always accepts a link with a new node. Since a new node prefers to link to the old node that has the highest degree, the network grows to be a star after 3 periods.

Next, we introduce a more realistic decision process and consider that new nodes make errors when choosing with whom to link. Depending on the error structure considered, different network architectures may emerge. Here we focus on a randomized discrete choice framework, along the lines of Manski and McFadden (1981). In an environment where agents must make a single selection from a set of discrete options, their deviations from optimal decisions are negatively correlated with associated costs. In other words, agents are more likely to make better choices than worse choices, but do not necessarily choose the best option. The agents estimate the relative benefits of each choice and the probability individual i selects option k from a set of m alternatives is given by

$$\Pr(x_i = k) = \frac{e^{\rho U_{i,k}}}{\sum_{l=1}^{m} e^{\rho U_{i,l}}}$$

where x_i is the choice i makes, $U_{i,k}$ indicates the utility value of option k for agent i, and ρ is the intensity of choice parameter, such that when $\rho = 0$ choices are random and when $\rho \to \infty$ choices are perfectly rational. Discrete choice models are described in detail in Section 4.

In our setting, a new node t needs to make a choice with whom to link among the previously born $(t-1)$ nodes. Each of the $(t-1)$ options gives it a utility $U_{t,k}$ that equals the marginal payoff t gains from linking to the node k: $U_{t,k} = \ln[\max(1, \eta_k)]$. Hence, for a particular case when $\rho = 1$, the probability node t links to node k in the network g_{t-1} is given by

$$\Pr(x_t = k) = \frac{\eta_k}{\sum_{i=1}^{t-1} \eta_i}$$

This implies that the probability a new node t will be connected to an existent node k depends on the connectivity η_k of that node. This probability captures exactly the preferential attachment feature of the Barabasi and Albert (1999) that yields, in the limit, a power-law degree distribution.

A degree distribution that follows a power law arises only when $\rho = 1$. When the intensity of choice, ρ, takes values above 1, Krapivsky et al. (2000) have shown that a single node connects to nearly all other nodes. For $\rho < 1$, the number of nodes with k links, varies as a stretched exponential.

We started by considering a network formation process based on growth, where nodes are added sequentially to the network and form one link with a previously born node. Although nodes form links in order to maximize their payoff, they do not always succeed in choosing the best option. Instead, the probability a new node links to an old node depends on benefit the old node yields relative to the others. Ultimately, this implies that the probability a new node will be connected to an existent node depends on the connectivity of the respective old node. Barabasi and Albert (1999) have shown that, in the limit, such a probability of attachment in a network growth setting generates networks that have a power-law degree distribution.

2.3 The Model

In this section we proceed to analyze other specifications for the benefits agents acquire through the network. We consider the same network growth model that was described above. A new node first proposes a link to the existing node that would give it the highest payoff. We first discuss a deterministic setting, and then we introduce errors in how new nodes make choices. The node which receives the proposal will either accept or reject the link depending on whether the marginal benefit from being linked to the new node t exceeds, or at least equals the cost of linking. If the link is accepted, period t ends and a new network g_t is defined. If the link is rejected, the new node t proposes a link to the node in N_{t-1} that gives it the second highest payoff. The process is iterated until the link is accepted or until all the nodes in N_{t-1} have been exhausted.

Below we describe the payoff structure of this network growth model. When constructing the payoffs, we aim to model two types of benefits network embeddedness may bring. On the one hand, as assumed in the previous section, connections act as conduits for information. Information flows over links and players' benefit from the information that reaches them. It is commonly assumed that while these benefits are non-rival, there exists a decay in the value of information from indirect connections. On the other hand, in various instances players gain significant benefits from bridging gaps in the network. The idea that some players may extract an additional benefit from intermediating connections between other players is behind the concept of "structural holes" developed by Burt (1994). In this setting benefits are rival and they are shared among different players. We study the network growth process for the two types of benefits separately. Thus, in one case, players' payoff is determined only by access benefits. In the second case, players' payoff includes intermediation benefits, in addition to access benefits.

Links bring benefits against a cost. Again, we distinguish two cases. We first consider that the marginal cost of establishing a link between players, c, is constant, as commonly assumed in the network formation literature. However, there are settings where assuming that the cost per link is increasing in the number of links is more appealing. This captures the idea that individuals are subject to capacity constraints. Thus, the second case we consider takes the marginal cost of linking to be increasing in the number of links a node has.

A critical feature of our model is that, by construction, the resulting network can only be minimally connected - a tree - or empty[5]. In other words, in any non-empty network there exists a unique path between any pair of nodes. A path of length l between i and j is a sequence of distinct agents $(i, j_1, ..., j_{l-1}, j)$ such that ij_1,j_1j_2,..., $j_{l-1}j \in g$.

These considerations lead us to study the dynamic of this network growth model under four payoff functions:

The first payoff function is based on *access benefits and constant marginal cost of linking.* Players get benefits from accessing other nodes in the network, and these benefits decrease depending on the distance, as in Jackson and Wolinsky (1996). However, instead of the usual geometric decay, we assume that the benefits decrease proportional to the distance between players. At the same time, every player pays a cost c for each link it is involved in. Explicitly, in our setting the payoff function takes the form:

$$\pi_i^a(g_t) = \sum_{j \in N_t} \frac{1}{d(i,j;g_t)+1} - \eta_i(g_t)c \tag{2.2}$$

where $\eta_i(g_t)$ is the number of links i has in network g_t, and $d(i,j;g_t)$ represents the number of links on the shortest path that connects nodes i and j.

The second payoff function includes *intermediation benefits* in addition to *access*

[5] A new-born node is allowed to form at most one link.

benefits, while keeping *constant the marginal cost of linking.* The intermediation benefits are generated when the connection between two players is mediated by other players as in Goyal and Vega-Redondo (2004). The primary assumption is that any two players that are connected create a surplus of 1. Situations when players are traders and involved in exchanging goods are collected by this assumption. If the players are directly connected, that is when they know each other, each of them gets a half of the surplus. If the players are indirectly connected, the allocation of the surplus depends on the competition between intermediaries. To determine the intermediation rents, the authors introduce the idea of "essential players". Basically, a player i is considered to be essential for players j and k if i lies on every path that connects j and k in the network. The allocation of the surplus depends on the number of intermediaries between the two players in the following way. Non-essential players between j and k get a zero share of the surplus, while the essential players and j and k divide the surplus equally. Formally, the payoff function can be expressed as follows:

$$\pi_i^e(g_t) = \sum_{j \in N_t} \frac{1}{d(i,j;g_t)+1} + \sum_{j,k \in N_t} \frac{I_{\{i \in P(j,k)\}}}{d(j,k;g_t)+1} - \eta_i(g_t)c \qquad (2.3)$$

where $I_{\{i \in P(j,k)\}}$ takes value 1 or 0 depending on whether i lies on the path between j and k, $P(j,k)$, or not. Each link requires both players involved to pay a cost c.

The third payoff function considers again only *access benefits*, similarly to the first one. However, the *marginal cost of linking is increasing* in the number of links a node has. This conveys the idea that players become more selective with time when forming links. The cost of linking should be interpreted as a fixed cost paid initially when the link has been established. There is no further cost attached to maintaining the link. Moreover, we consider that the marginal cost increases at a constant rate c, such that the payoff function takes the following form.

$$\pi_i^{a+}(g_t) = \sum_{j \in N_t} \frac{1}{d(i,j;g_t)+1} - \sum_{l=1}^{\eta_i(g_t)} lc \qquad (2.4)$$

The fourth payoff function analyzed incorporates *access benefits, intermediation* *benefits and increasing marginal cost.* The access and the intermediation benefits are constructed in the same way as for the second payoff function, while the cost is marginally increasing in the number of links.

$$\pi_i^{e+}(g_t) = \sum_{j \in N_t} \frac{1}{d(i,j;g_t)+1} + \sum_{j,k \in N_t} \frac{I_{\{i \in P(j,k)\}}}{d(j,k;g_t)+1} - \sum_{l=1}^{\eta_i(g_t)} lc \qquad (2.5)$$

2.4 Deterministic Analysis

In this section we identify the networks that are formed following the growth process and characterize their properties. We analyze each of the four payoff functions separately, aiming to explain the differing results through differences in the benefit and cost structures.

The network formation process is best explained in terms of marginal payoffs. Recall that the network is formed through a growth mechanism: every period in time a new node is added to the existent population of nodes. Thus, we need to distinguish between the marginal payoff a new node obtains when it attaches to the network and the marginal payoff an old receives when it accepts a new link. The marginal payoff a new node gains determines where in the existent network it wants to attach. This is the first step in the preferential attachment process. The second step consists in the decision of older nodes to accept or reject links. The marginal payoff an old node gains determines whether it accepts or rejects a new link. It is the decision of the older node that mainly shapes the network. The differences in the network architectures that emerge can be explained by the different incentives old nodes have when accepting a new link in each of the four cases proposed.

2.4.1 Access benefits and constant marginal cost of linking

The case that considers that benefits are derived from accessing the information of the other players in the network, while paying a cost c per link is straightforward. On the one hand, the marginal payoff an old node i gains when accepting a new link stems only from the information it can access from the new-born node t.

$$\pi_i^a(g_{t-1} + it) - \pi_i^a(g_{t-1}) = \frac{1}{2} - c \tag{2.6}$$

On the other hand, a new node is able to access the information of everyone else in the network through the link it forms. Thus, the marginal payoff that a new node t gains when it links to an old node i is given by

$$\pi_t^a(g_{t-1} + it) - \pi_t^a(g_{t-1}) = \frac{1}{2} + \sum_{j \in N_{t-1}} \frac{1}{d(i,j) + 2} - c \tag{2.7}$$

The marginal payoff of a new node is maximized when distances are short. This creates an agglomeration pressure that drives the formation of a star network. Moreover, provided that the marginal cost of establishing a link, c, is small enough ($c \leq 1/2$), an old node always affords a new link. Based on these considerations we can formulate the next result.

Proposition 2.1 *For any $t > 1$, let g_t be the network at time t. Consider that the payoff of a node i in network g_t is $\pi_i^a(g_t)$ given by (2.2). If $c < \frac{1}{2}$ then g_t is a star. If $c > \frac{1}{2}$ then g_t is the empty network.*

Proof. See Appendix. ■

A similar case has been studied by Jackson and Wolinsky (1996). In the "connection model" they propose, both players have to agree to form a link and the value of the information decays geometrically with the distance between players.[6] The authors model the network formation using a static equilibrium approach. They advance the notion of pairwise stability in order to characterize the networks in equilibrium. Although a full characterization of the set of equilibria turns out to be difficult, stars are also found to be stable.

[6] Note that the geometric specification for modeling decay of information δ^x, behaves similarly as $\frac{1}{x+2}$, for small x and when $\delta \in (0,1)$.

2.4.2 Access benefits, intermediation benefits and constant marginal cost of linking

We study now the case where network embeddedness creates opportunity for some players to intermediate connections among other players. The payoffs players receive have been adjusted to include benefits derived from intermediation in addition to access benefits. However, since a non-empty network can only be a tree, only nodes that are not terminal nodes receive intermediation benefits. This implies that an old node i receives the following marginal payoff when accepting a new link.

$$\pi_i^e(g_{t-1} + it) - \pi_i^e(g_{t-1}) = \frac{1}{2} + \sum_{j \in N_{t-1}} \frac{1}{d(i,j)+2} - c \tag{2.8}$$

A new node when is born cannot serve as intermediary between other nodes in the network. Thus, it only benefits from the surplus created by being connected with other nodes. However, this surplus will be shared with other players that are essential for a connection to take place. In particular, a new-born node t receives the following marginal payoff from linking to an old node i

$$\pi_t^e(g_{t-1} + it) - \pi_t^e(g_{t-1}) = \frac{1}{2} + \sum_{j \in N_{t-1}} \frac{1}{d(i,j)+2} - c \tag{2.9}$$

The agglomeration pressure, that led previously to a star, continues to be the driving force of the network formation process. This becomes clear if we notice that same incentives act on a new-born node. In fact, a new-born node t receives the same marginal payoff from linking to an old node i as in (2.7) in all the four cases considered in our analysis.

Proposition 2.2 *For any $t > 1$, let g_t be the network at time t. Consider that the payoff of a node i in network g_t is $\pi_i^e(g_t)$ given by (2.3). If $c \leq \frac{1}{2}$ then g_t is a star. If $c > \frac{1}{2}$ then g_t is the empty network.*

Proof. See Appendix. ■

The same intuition developed for explaining Proposition 2.1 holds. A new node has the incentive to shorten distances, while an old node will always accept a new link as long as the cost of linking c is small enough. Figure 2.1 illustrates networks described by Proposition 2.1 and 2.2.

Goyal and Vega-Redondo (2007) analyze the network formation problem using game theoretical tools. The authors use a notion of stability that allows pairs of players to form and delete links simultaneously, and manage to isolate a star as the unique bilateral equilibrium. However, when the less demanding concept of bilateral-proofness[7] is employed, any cycle containing all players can be supported in equilibrium.

2.4.3 Access benefits and increasing marginal cost of linking

This case considers again situations when network connections facilitate access to information, as discussed in section 2.4.1. The marginal cost of adding a new link a node supports is now increasing linearly in the number of links it already has. However, this does not change the marginal payoff a new node t receives when it attaches to a node i in the existing network.

$$\pi_t^{a+}(g_{t-1} + it) - \pi_t^{a+}(g_{t-1}) = \frac{1}{2} + \sum_{j \in N_{t-1}} \frac{1}{d(i,j) + 2} - c \tag{2.10}$$

The marginal payoff of an old node needs to be adjusted to account for the new cost specification. In particular, any new link accepted costs c more than the previous one. Thus, for a node i that has a degree $\eta_i - 1$, the marginal payoff from accepting the η_ith link becomes

$$\pi_i^{a+}(g_{t-1} + it) - \pi_i^{a+}(g_{t-1}) = \frac{1}{2} - \eta_i c \tag{2.11}$$

Equation (2.11) indicates that an old node can afford only a limited number of links. In particular, an old node accepts links as long as the marginal payoff stays positive. The maximum degree of a node becomes bounded from above: $\eta_i \leq \frac{1}{2c}$. The incentives to shorten distances that drive the linking behavior of the new nodes are limited now by the capacity constraints old nodes have.

[7] A bilateral-proof equilibrium is a two-person coaltion prof equilibrium, that allows for profitable deviations from the agreed upon deviation. This only enlarges the set of equilibria, since only a subset of bilateral deviations qualify as valid.

Proposition 2.3 *For any $t > 1$, let g_t be the network at time t. Consider that the payoff of a node i in network g_t is $\pi_i^{a+}(g_t)$ given by (2.4). Then, the followings hold:*

1. *If $c \leq \frac{1}{2}$, the network g_t is a regular tree. That is, for any node i in g_t, $\eta_i = 1$ or $\eta_i = \left[\frac{1}{2c}\right]$.*[8]

2. *If $c > \frac{1}{2}$, the network g_t is empty.*

Proof. See appendix. ■

This result follows as well from an inductive argument. For the first few periods, the growth process shapes the network as a star. However, nodes are limited in the number of links they can accept. Once the linking capacity of the center is reached, new nodes start attaching to the neighbors of the center. The neighbors are bounded as well in the number of links they can receive. Next, new nodes prefer to form links with players at a distance of 2 from the initial center. As the growth process continues, the network develops as a layered structure centered around the first node. Moreover, the network is symmetric, as every node has the same number of links, except for terminal nodes. Figure 2.2 illustrates the network that is formed when the cost $c = \frac{1}{6}$.

As before, the network formation process is governed by the tendency towards a star. However, this force is dominated by capacity constraints old node have to receive new link. It is, thus, the incentives of old nodes that shape the different networks architectures across the four cases.

2.4.4 Access benefits, intermediation benefits and increasing marginal cost of linking

Introducing intermediation benefits and increasing marginal cost of linking at the same time adds complexity to the network growth process. The incentives new nodes have when forming links are the same as in the previous three cases, since the marginal payoff is calculated in the same way.

[8] We denote by $[x]$ the closest integer to x, with $[x] \leq x$.

$$\pi_t^{e+}(g_{t-1}+it)-\pi_t^{e+}(g_{t-1})=\frac{1}{2}+\sum_{j\in N_{t-1}}\frac{1}{d(i,j)+2}-c \tag{2.12}$$

The incentives of old nodes to accept link are more intricate. The increasing marginal cost does not allow nodes to accumulate links indefinitely. However, the marginal benefit from linking to a newborn node is no longer constant, as considered in section 2.4.3. The intermediation component in the benefit structure allows it to increase with the addition of new nodes to the population and depending on the evolution of the network structure over time. The marginal benefit a node i with degree $(\eta_i - 1)$ from accepting the η_ith link is given by:

$$\pi_i^{e+}(g_{t-1}+it)-\pi_i^{e+}(g_{t-1})=\frac{1}{2}+\sum_{j\in N_{t-1}}\frac{1}{d(i,j)+2}-\eta_i c \tag{2.13}$$

Proposition 2.4 *For any $t>1$, let g_t be the network at time t. The payoff of a node i in network g_t is $\pi_i^{e+}(g_t)$ given by (2.5). Suppose that for the first $\bar{t}$ periods, forming links is costless. Then, the followings hold:*

1. *If $c \leq \frac{3\bar{t}+4}{24}$ then there exists a period τ and a subnetwork h_τ of the network g_τ such that any node t born afterwards $(t \geq \tau)$ forms a link with a node in h_τ. Moreover, h_τ is a star.*

2. *If $c > \frac{3\bar{t}+4}{24}$, then g_t is empty.*

Proof. See appendix. ■

The network architecture is again a result of the incentives that drive old nodes to accept links. The increase in the marginal benefit of an old node over time is dampened by the increase in the marginal cost of linking. The interaction between these two forces creates complicated dynamics. As links become marginally more expensive, there will not be one single node that can afford to accept all the links proposed to it. However, the effects of the agglomeration forces are not entirely driven out as it happens in the third case we considered. For instance, an old node that cannot afford to form a link in period t, might accept to link a few periods later, once the network has grown sufficiently. In particular, to afford a new link, any old node needs its marginal benefit to increase by an amount "c" since it received the last link.

There are two network properties that follow from Proposition 2.4. First, the maximum distance in any network generated with the payoff functions given by eq. (2.12) and (2.13) is at most 4. Thus, irrespective of the size of the network, the distance between any pair of nodes is at most 4. Despite that the population of nodes can grow at infinitum, there is always a path of finite length between any two nodes. This property is a direct consequence of the network being dominated by a connected group of nodes. Since there exists a subnetwork h_τ of g_t that receives all the links after a certain period τ, the maximum distance in any network g_t is given by the maximum distance in h_τ extended by 2 to account for nodes that attach at the extremities of h_τ. This concludes the argument, as h_τ is a star and the diameter of a star is 2.

The second property concerns the global organization of the network. As in the case that looks only at access benefits and increasing cost of linking, the network is shaped in layers centered around the first node. Moreover, the network is characterized by a well-defined hierarchical structure. That is, the network divides naturally into groups and these groups divide themselves into subgroups until we reach the level of individual vertices. In addition, any two nodes in the network are connected by a hierarchical path: a path between nodes is called hierarchical if it consists of an "up path", where one is allowed to step from node i to node j only if their degrees η_i, η_j satisfy $\eta_i \leq \eta_j$, followed by a "down path", where only steps of lower or equal degree are allowed. Either the up or down path is allowed to have zero length.[9] Intuitively, the formation of the hierarchical network is driven initially by the preference of new nodes to form links with the first-born node. The marginal benefit of the first node increases faster with the addition of new nodes to the network than the marginal benefit of any other node.[10] Thus, it will accumulate links more frequently than other nodes. Consequently, the preference of new-born nodes to attach to the first node is reinforced over time.

The dynamics of the growth process are not straightforward. Simulations of the network formation are, therefore, able to provide further insights. We simulate the

[9] This definition of hierarchical path was proposed by Trusina et al. (2004).

[10] This happens since the first node is at the shortest distance from everyone else in the network.

network growth model as described above for an arbitrarily large number of periods. In order to allow the network formation process to start, we assume that links are costless for the first few periods, depending on the value of the parameter c. In addition, we assume that a new node attaches to the older node, when it must choose between two old nodes that give the same marginal benefit.

By mean of simulations, we can easily plot the networks. Figures 2.3 (a,b,c) illustrate what networks are formed after 1000 periods for various values of the cost parameter c. There are two immediate observations that we see from the plots. First, the networks are organized in a hierarchical structure, with one node that is a star among other nodes that are stars in turn. Second, the numbers of stars in the network is increasing with the parameter c. In fact, it the number of nodes in the subnetwork h_τ that increases with c.

Further, Table 2.1 shows a number of bar plots illustrating the degree distribution of nodes for various values of the cost parameter c. The distributions were truncated for the number of nodes that have degree 1 in order to have a better picture of the tail. Simulations were run for 1000, 2000 and 3000 periods to show that once the subnetwork h_τ is formed, the number of nodes belonging to h_τ stays constant over time. The plots also indicate that there is a certain periodicity that determines how often old nodes receive new links. Moreover, the node with the highest degree receives links most frequently over intervals of 1000 periods.

2.5 Probabilistic linking choice

In this section, we introduce a more realistic decision process and consider that new nodes make errors when choosing to whom to link. We use a discrete choice framework, along the lines of Manski and McFadden (1981), to model how the new nodes form links. The basic intuition is that individuals are more likely to select better choices than worse choices, but do not always manage to choose the best option.

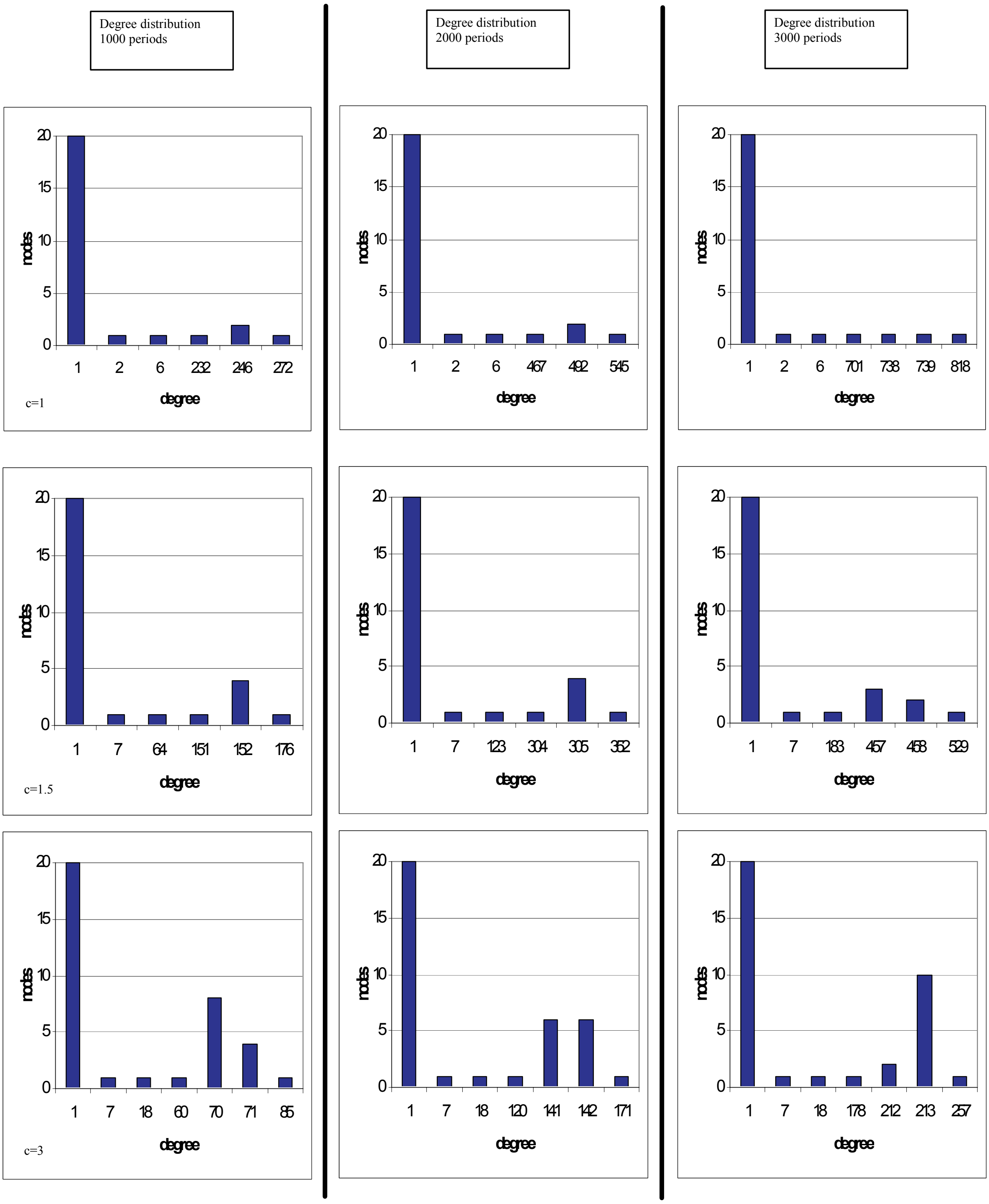


Table 2.1: The evolution of the degree distribution over time
- access benefits, intermediation benefits, increasing marginal cost of linking -

Probabilistic choice models, or random utility models, have long been used to incorporate stochastic elements in to the analysis of individual decisions. There is a vast number of papers that use logit and probit models for empirical estimations. Recently, however, discrete choice models have been adopted in other areas as well. For instance, the quantal response equilibrium, introduced by McKelvey and Palfrey (1995), is the analogous way to model games with noisy players. Brock and Hommes (1997) use as well a discrete choice model to advance the concept of adaptive rational equilibrium.[11]

These models usually apply to study situations in which agents choose between a finite number of discrete options. Consider a decision maker, labeled i, that faces a choice among K alternatives. From each alternative k, the decision maker i is assumed to obtain a certain level of utility $V_{i,k}$. Agents are utility maximizers, although they do not necessarily choose the option that gives them the highest observed utility. This can generally be attributed to either unmodelled idiosyncratic components of the agents' utility function, or to a random component in individual preferences (McFadden, 1974). In particular, the utility that a decision maker, i, obtains from alternative k can be decomposed into: $V_{i.k} = U_{i,k} + \varepsilon_{i,k}$, $\forall k$, where $V_{i,k}$ is the true utility, $U_{i,k}$ is the observed utility and $\varepsilon_{i,k}$ is the idiosyncratic component. Agents estimate the relative benefit of each choice, and choose alternative k if and only if $V_{i,k} > V_{i,j}$ $\forall j \neq k$. This implies that a decision maker i chooses alternative k with probability $\Pr(x_i = k) = \Pr(\varepsilon_{i,j} - \varepsilon_{i,k} < U_{i,k} - U_{i,j}\ \forall j \neq k)$. When the random term $\varepsilon_{i,k}$ is independently, identically Gumbel distributed[12], with a variance $\sigma^2(\pi^2/6)$, the probability of an agent i choosing option k becomes

$$\Pr(x_i = k) = \frac{e^{\rho U_{i,k}}}{\sum_j e^{\rho U_{i,j}}}$$

where ρ is an inverse function of σ. The greater the parameter ρ, the more likely agents maximize the true utility. However, when the variance σ increases, the choices agents make are nearly random.

We applying this model to our setting. A new node t needs to make a choice with

[11] The A.R.E. is a frequently emplyed tool for modeling the evolutionary process of a dynamic population.

[12] The density function of a Gumbel distribution is $f(\varepsilon_{i,k}) = e^{-\varepsilon_{i,k}} e^{-e^{-\varepsilon_{i,k}}}$, and the cumulative distribution is $F(\varepsilon_{i,k}) = e^{-e^{-\varepsilon_{i,k}}}$. The variance of this distribution is $\pi^2/6$.

whom to link among the previously born $(t-1)$ nodes. Each of the $(t-1)$ options gives it a utility $U_{t,k}$ that equals the marginal payoff t gains from linking to the node k: $U_{t,k} = \pi_t(g_{t-1} + tk) - \pi_t(g_{t-1})$, where π is one given by either of the functions 2.7, 2.9, 2.10 or 2.12. However, since the marginal payoff of a new node is the same, for all the four functions studied, we have that:

$$U_{t,k} = \frac{1}{2} + \sum_{j \in N_{t-1}} \frac{1}{d(k,j)+2} - c$$

Hence, the probability that a new node t proposes a link to an old node k is the same for all four cases considered. This implies that it is, again, the decision of the older node that mainly shapes the network.

We simulate the network growth process only for the fourth case, when nodes gain intermediation benefits in addition to access benefits, while the marginal cost of linking is increasing. Introducing errors in the decisions new nodes take does not affect the shape of the network considerably in comparison with the deterministic case. Fig. 2.4 (a) and (b) show networks that results from simulating the process for $\rho = 1$. As expected, the maximum distance in the network increases and is on average close to 8 for $c = 1$ and $\rho = 1$, to decrease when the cost c increases. However, the agglomeration pressure still dominates the network formation process. Since the payoffs from linking to the center are considerably higher than payoffs gained by linking with a node at the periphery, new nodes are less likely to make a mistake and link to a terminal node. The outcome is, again, a hierarchical star. The following table shows the degree distribution for networks that have been formed after 2000 periods when new nodes make mistakes with an intensity of choice parameter $\rho = 1$ and $\rho = 0.5$. The values are averages taken after simulations were run for 20 times. The benchmark case of choice under full rationality is represented by the first column, when $\rho = \infty$.

The results can also be interpreted in the light of the findings of Krapivsky et al. (2000). They show that when probability of attachment is non-linear in the degree, a single node connects to nearly all other nodes. In our case, the probability of attachment grows exponentially as a function of the degree.

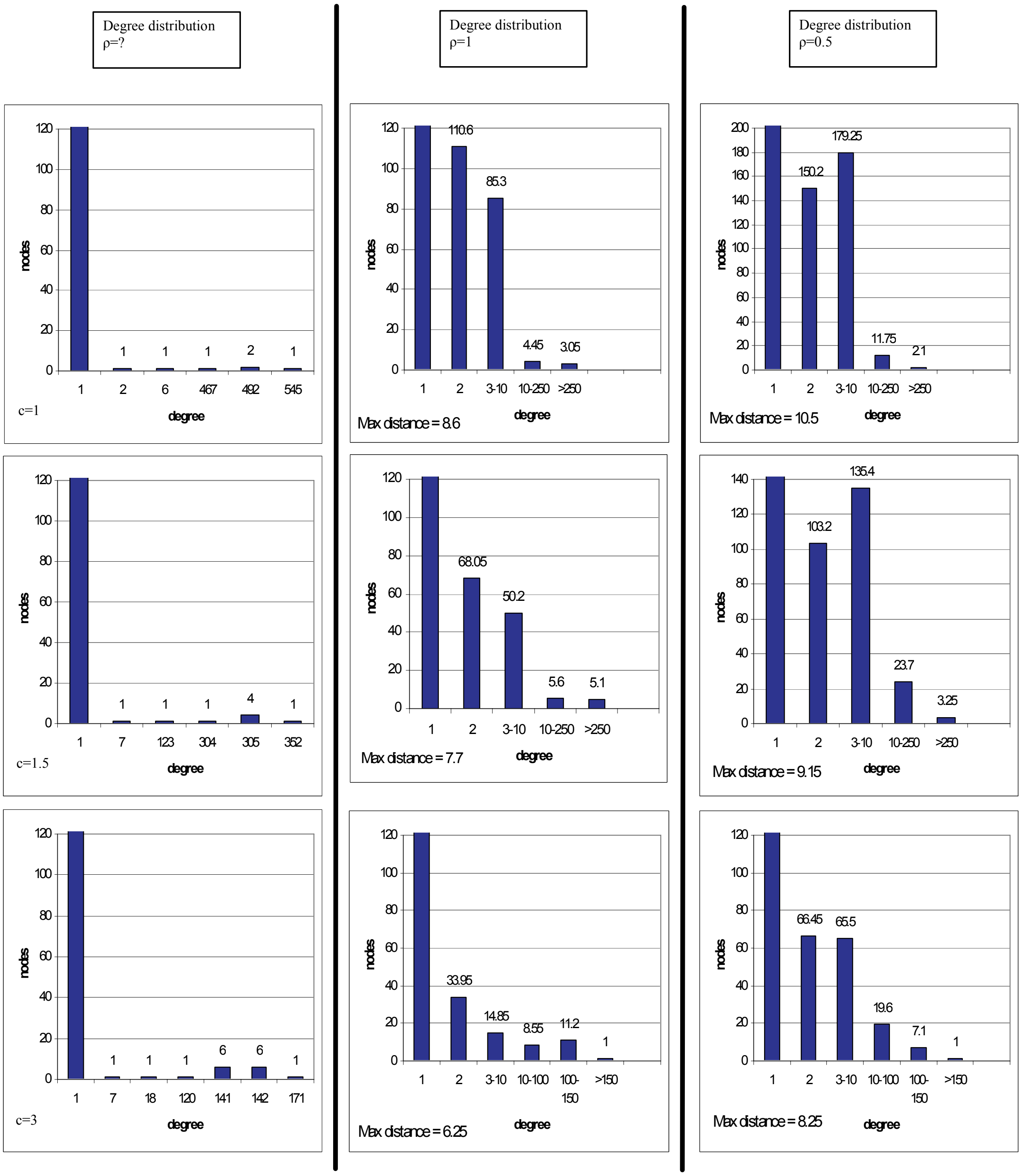


Table 2.2: Degree distribution in networks formed in an error-prone setting - access benefits, intermediation benefits and increasing marginal cost of linking; 2000 periods -

2.6 Discussions

In this section we discuss the results and possible extensions. In summary, we model a network formation process based on preferential attachment, where nodes form links to maximize their payoffs and links require consent to be established. Newborn nodes will get higher payoffs from existing nodes that are highly linked for all the payoff functions employed in our analysis. The temporal aspect of the model will generally favor older nodes to have a higher degree. This explains the agglomeration pressure that generates a star in the first two cases. The fact that links require consent might lead to deviations from a star, as is evident in the last two cases. An old node accepts a new link provided that the marginal benefit from a newborn node is higher than the marginal cost of linking. Since in the first two cases, discussed in Sections 2.4.1 and 2.4.2 the marginal cost of linking is independent of the number of links, there will be only one node that accumulates all the links ad infinitum. In the third case discussed in Section 2.4.3, links are marginally more expensive. However, the marginal benefit from linking to a new born node is fixed and independent of the population size or of the network structure. Thus, nodes can accept to form links up to the point where the marginal cost of linking at most equals the marginal benefit from linking to a newborn node. In the fourth case, discussed in Section 2.4.4, the marginal benefit from linking to a new-born node increases over time, with the addition of nodes to the network. Hence, old nodes continue to receive new links over time, with a periodicity that depends on the cost parameter c.

We construct a payoff function that generates networks with a power-law degree distribution, when agents make errors in their linking choices. For more standard payoff functions we find networks that are still fairly stylized, especially when compared to the models proposed by the statistical physics literature. However, the fourth case suggests that allowing players to gain benefits from intermediation and introducing increasing marginal cost of linking may yield interesting network architectures. Although the degree distributions are jagged, the model can explain four out the five empirical regularities identified in the beginning: short distances, unequal connections, hierarchy and positive assortativity.[13] The results indicate that further analysis of the case based both on access and intermediation benefits, with increasing marginal cost of linking is likely to bring more insights.

We take a step in this direction by proposing two extensions based on alterations of the cost function. First, we simulate the network growth process when the marginal cost of linking increases linearly only after a certain number $\bar{\eta}$ of links have been accumulated. In particular, we consider that for any node i in the network, the first $\bar{\eta}$ links cost the same amount c. Thereafter, any additional link costs c more than the previous. The marginal payoff of an old node becomes:

$$\pi_i(g_{t-1}+it)-\pi_i(g_{t-1})=\frac{1}{2}+\sum_{j\in N_{t-1}}\frac{1}{d(i,j)+2}-(\eta_i-\bar{\eta})c$$

The networks that follow from simulations exhibit the same features as before. They are organized hierarchically as stars centered around another star. After a certain period, a group of nodes shaped as a star seems to receive all the new links ad infinitum and the size of the group stays constant over time. Hence, the maximum distance in a network is bounded from above. Surprisingly however, when we simulate the network growth for $\bar{\eta}=1$, the maximum distance is 6, larger than in the networks found in section 3.4 for the same values of the cost parameter c. The maximum distance decreases with $\bar{\eta}$, such that for $\bar{\eta}=4$ it reduces to 4. Figure 2.5 illustrates the network formed for $\bar{\eta}=1$ and $c=3$.

The second extension we briefly discuss takes the marginal cost of linking to be quadratically increasing in the number of links. Thus, the marginal payoff of an old node is given by:

$$\pi_i(g_{t-1}+it)-\pi_i(g_{t-1})=\frac{1}{2}+\sum_{j\in N_{t-1}}\frac{1}{d(i,j)+2}-\eta_i^2c$$

Introducing a convex marginal cost affects the dynamics of the network growth in one particular way. Namely the size of the group that receives new links is no longer

[13] Since nodes can form only one link when they are born, the resulting network is a always a tree. Thus, by construction, the model cannot account for clustering.

constant, but increases over time. Old nodes are constantly incorporated in the group. However, the size of the group increases relatively slowly, such that the maximum distance remains very short. Table 2.3 shows several bar-plots illustrating the evolution of the group on which most mass of the degree distribution is concentrated.

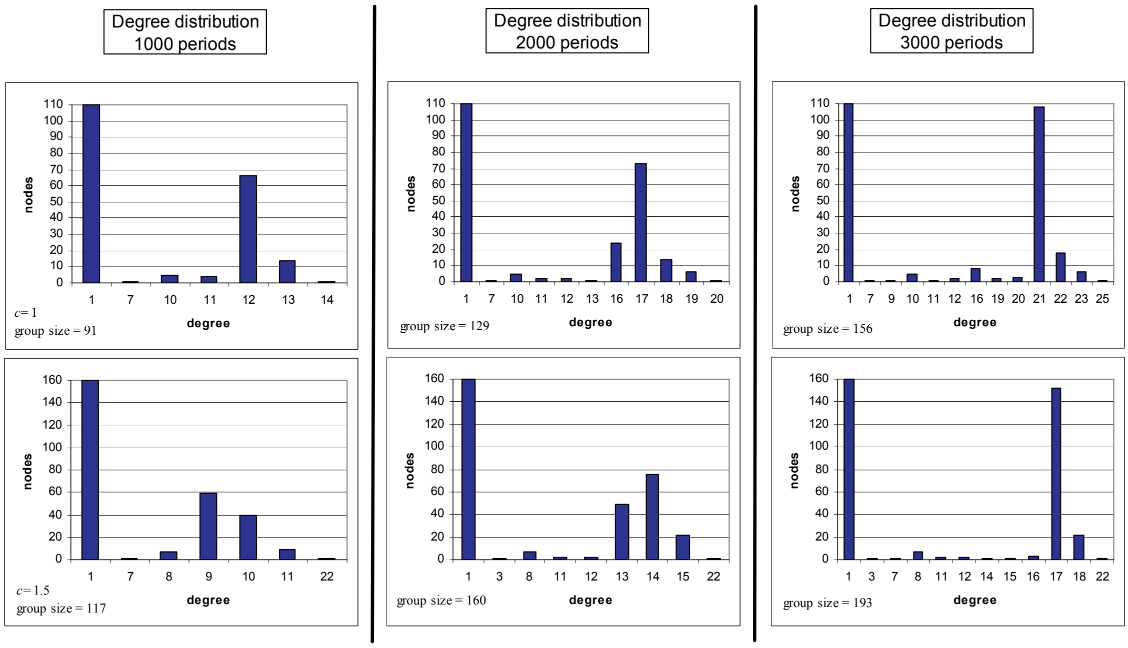


Table 2.3: The evolution of degree distribution over time
- access benefits, intermediation benefits and quadratic marginal cost of linking -

2.7 Conclusions

The preferential attachment of Barabasi and Albert (1999) enables nodes to receive new links with a probability proportional to their degree. Why new-born nodes prefer to attach to highly linked nodes can be easily explained through incentives and optimal behavior of agents. In fact, this aspect is captured by all game-theoretical models of network formation that yield stars in equilibrium. Why the Barabasi and Albert (1999) network does not ultimately converge to a star turned out to be more difficult to interpret using economic reasoning. In this chapter we introduce payoff optimization in a network growth model. We move away from the star towards a hierarchical structure by taking the marginal cost of linking to be increasing in the number of links.

At the same time, we keep distances short by allowing agents to gain benefits from intermediating connections between other nodes in the network.

2.A Appendix

Proposition 2.1 *For any $t > 1$, let g_t be the network at time t. Consider that the payoff of a node i in network g_t is $\pi_i^a(g_t)$ given by (2.2). If $c \leq \frac{1}{2}$ then g_t is a star. If $c > \frac{1}{2}$ then g_t is the empty network.*

Proof. The case $c > \frac{1}{2}$ is trivial. An old node will never accept to form a link, since the cost of the link is larger than the benefit it brings. Thus, no links will be formed since the first period. As nodes are added to the population, the only network that emerges is the empty network.

When $c \leq \frac{1}{2}$, the network settles as a star after the first few periods. We follow the network formation for the first few periods and then we show the result holds for any t using an inductive proof.

Period 1: node 1 is born.

Period 2: node 2 is born. Since $\pi_i(g_1 + 12) \geq \pi_i(g_1)$, $\forall i \in \{1, 2\}$, the link between 1 and 2 will be formed.

Period 3: node 3 is born. $\pi_3(g_2 + 31) = \pi_3(g_2 + 32) \geq \pi_3(g_2)$. Although 3 is indifferent from linking with 1 or 2, we assume, for simplicity that 3 proposes a link to 1. As $c \leq \frac{1}{2}$, the marginal payoff of an old node is always positive. The link between 1 and 3 is formed.

Period 4: node 4 is born. $\pi_4(g_3 + 41) > \pi_4(g_3 + 4i) \geq \pi_4(g_3)$, $\forall i \in N_3 - \{1\}$. Thus, 4 proposes a link to 1 and the link 14 is formed and g_4 is a star.

$\vdots$

Period m: node m is born. Suppose that $\pi_m(g_{m-1} + m1) > \pi_m(g_{m-1} + mi) \geq \pi_m(g_{m-1})$, $\forall i \in N_{m-1} - \{1\}$. The link $m1$ is formed and g_m is a star.

Period $m+1$: node $(m+1)$ is born. We show that $\pi_{m+1}(g_m + (m+1)1) > \pi_{m+1}(g_m + (m+1)i) \geq \pi_{m+1}(g_m)$, $\forall i \in N_m - \{1\}$. Since g_m is a star, $\pi_{m+1}(g_m + (m+1)1) = \frac{1}{2} + \frac{m-1}{3} - c$ and $\pi_{m+1}(g_m + (m+1)i) = \frac{1}{2} + \frac{1}{3} + \frac{m-2}{4} - c$, $\forall i \in N_m - \{1\}$. The inequality follows immediately. The link $(m+1)1$ is formed and $g_{(m+1)}$ is a star.

This step concludes the proof. ■

Proposition 2.2 *For any $t > 1$, let g_t be the network at time t. Consider that the payoff of a node i in network g_t is $\pi_i^e(g_t)$ given by (2.3). If $c \leq \frac{1}{2}$ then g_t is a star. If $c > \frac{1}{2}$ then g_t is the empty network.*

Proof. The proof is the same as the one of the previous result. It is only the marginal payoff of an old node that is different in the case of access benefits, intermediation benefits and constant cost of linking. However, for the proof to hold we simply need the marginal payoff of an old node to be positive. This holds for $c \leq \frac{1}{2}$. ■

Corollary *Let the payoff of a node i in network g_t be $\pi_i^e(g_t)$ given by (2.3). Suppose that for the first $\bar{t}$ periods, forming links is costless. If $c \leq \frac{2\bar{t}+1}{6}$ then g_t is a star. If $c > \frac{2\bar{t}+1}{6}$ then g_t is the empty network.*

Proof. Since $c = 0$ for the first $\bar{t}$ periods, Proposition **??** implies that $g_{\bar{t}}$ is a star, centered around the first node. At time $(\bar{t}+1)$, the maximum marginal benefit an old node gains from accepting a new link is $\frac{1}{2} + \frac{\bar{t}-1}{3}$. The center of $g_{\bar{t}}$ receives the most, while each of the spokes gets $\frac{1}{2} + \frac{1}{3} + \frac{\bar{t}-2}{4}$. Thus, when the cost $c \geq \frac{1}{2} + \frac{\bar{t}-1}{3}$ neither the center nor the spokes will afford to form new links. The network is virtually empty, excepting the first $\bar{t}$ nodes that are connected in a star. When $c < \frac{1}{2} + \frac{\bar{t}-1}{3}$, the same inductive argument applies and the network converges to a star. ■

Proposition 2.3 *For any $t > 1$, let g_t be the network at time t. Consider that the payoff of a node i in network g_t is $\pi_i^{a+}(g_t)$ given by (2.4). Then, the followings hold:*

1. *If $c \leq \frac{1}{2}$, the network g_t is a regular tree. That is, for any node i in g_t, $\eta_i = 1$ or $\eta_i = \left[\frac{1}{2c}\right]$.*[14]

2. *If $c > \frac{1}{2}$, the network g_t is empty.*

Proof. If $c > \frac{1}{2}$, an old node never accepts to form a link, hence the network is empty.

For $c \leq \frac{1}{2}$, the proof is based on a recursive argument. An old node will accept links as long as its marginal payoff is positive. The maximum number of links an old can accept is $\left[\frac{1}{2c}\right]$. As nodes are born they will form links with the first node, until

[14]We denote by $[x]$ the closest integer to x, with $[x] \leq x$.

it reaches its maximum capacity. At date $\left[\frac{1}{2c}\right]+2$, the new node forms a link with a neighbor i of 1. For the next $\left[\frac{1}{2c}\right]-1$ periods, i receives new links. Once i reaches its capacity, new nodes propose links to other neighbors of 1. The formation process relies on two principles that apply recursively:

- between two nodes i and j at the same distance from 1, a new node prefers to attach to the one with the highest degree;
- between two nodes i and j with the same degree, a new node prefers to attach to the one closer to 1.

■

Proposition 2.4 *For any $t>1$, let g_t be the network at time t. The payoff of a node i in network g_t is $\pi_i^{e+}(g_t)$ given by (2.5). Suppose that for the first $\bar{t}$ periods, forming links is costless. Then, the followings hold:*

1. *If $c \leq \frac{3\bar{t}+4}{24}$ then there exists a period τ and a subnetwork h_τ of the network g_τ such that any node t born afterwards $(t \geq \tau)$ forms a link with a node in h_τ. Moreover, h_τ is a star.*
2. *If $c > \frac{3\bar{t}+4}{24}$, then g_t is empty.*

Proof. Since links are costless for the first $\bar{t}$ periods, $g_{\bar{t}}$ is a star centered around the first node. The condition $c \leq \frac{3\bar{t}+4}{24}$ ensures that at time $(\bar{t}+1)$ there exists at least one node that can afford to receive a new link. The condition is equivalent to $\frac{1}{2}+\frac{1}{3}+\frac{\bar{t}-2}{4}-2c \geq 0$, which implies that the payoff of a spoke in the network $g_{\bar{t}}$ from receiving a new link is positive.

At the same time, the condition $c \leq \frac{3\bar{t}+4}{24}$ is sufficient to guarantee that the network can grow ad infinitum. Re-written for $\bar{t}$, the inequality becomes: $\bar{t} \geq 8c-\frac{4}{3} \geq 5(c-\frac{1}{3})$. Thus, the number of spokes in the initial network is sufficient to sustain the growth of the network. Any new link received by a spoke increases the marginal benefit of the others by $\frac{1}{5}$. Any node can afford a new link if its marginal benefit increased by $(c-\frac{1}{3})$ since it received the last link. Hence, a spoke needs $5(c-\frac{1}{3})$ periods to afford a new link since the last one it received. Suppose that each period a different spoke receives a new link. Then the number of spokes that sustains the growth of the network is the smallest integer larger than $5(c-\frac{1}{3})$.

We sketch the steps of the proof. We first define a partition over the set of nodes. Then we define a preferential attachment ranking for the old nodes. Finally, we show how the subnetwork h_τ forms.

Consider the following partition spans entirely the set of nodes.. The first born node is denoted by 1.

$P_1(t) = \{1\}$

$P_2(t) = \{i \in N_t \text{ s.t. } d(i,1) = 1 \text{ and } \eta_i \geq 2\}$

$P_3(t) = \{i \in N_t \text{ s.t. } d(i,1) = 1 \text{ and } \eta_i = 1\}$

$P_4(t) = \{i \in N_t \text{ s.t. } d(i,1) = 2\}$

We show that new nodes prefer to form links with nodes in P_1, P_2 and P_3 in this order. Thus, the most beneficial node is the first born, followed by its neighbors ranked according to their degree. The ranking over partitions is independent of the time t.

A node i is preferred to node j if the marginal benefit a new node gains from linking to i is higher than from linking to j. To show that 1 is preferred to any node $i \in P_2$ at date t we need to show that:

$$\frac{1}{2} + \frac{\eta_1(t)}{3} + \frac{\Sigma(t)}{4} > \frac{1}{2} + \frac{\eta_i(t)}{3} + \frac{\eta_1(t)-1}{4} + \frac{\Sigma(t)-(\eta_i(t)-1)}{5} \tag{2.14}$$

where $\Sigma(t) = \sum_{j \in N_t} (\eta_j(t) - 1)$. Rewriting and dropping the index t we have

$$\frac{\eta_1}{3} - \frac{\eta_1 - 1}{4} + \frac{\Sigma - (\eta_i - 1)}{4} > \frac{\eta_i}{3} - \frac{\eta_i - 1}{4} + \frac{\Sigma - (\eta_i - 1)}{5} \tag{2.15}$$

Showing that $\eta_1 \geq \eta_i$ for any t ensures the inequality above holds.

The links become costly as of period $\bar{t}+1$. We exclude the trivial cases when c is too small and the network converges to a star (for $c \leq \frac{1}{3}$) or to an interlinked star (for $c \in (\frac{1}{3}, \frac{7}{12}]$). Hence, for c sufficiently large, the center of $g_{\bar{t}}$ cannot afford new links and the new nodes will attach to the spokes.

Suppose there exists a period δ and a node $i \in P_2$ such that the degree of i becomes $\eta_i(\delta) + 1$ and $\eta_i(\delta) = \eta_1(\delta)$. This implies:

$$\frac{1}{2} + \frac{\eta_i(\delta)}{3} + \frac{\eta_1(\delta) - 1}{4} + \frac{\Sigma(\delta) - (\eta_i(\delta) - 1)}{5} \geq (\eta_i(\delta) + 1)c \tag{2.16}$$

Working out the inequality for $\Sigma(\delta)$ we obtain:

$$\Sigma(\delta) \geq (\eta_1 + 1)(5c - \frac{23}{12}) - \frac{1}{3} \geq 4(\eta_1 + 1)(c - \frac{1}{3}) - \frac{2}{3} \tag{2.17}$$

This implies that

$$\frac{1}{2} + \frac{\eta_1(\delta)}{3} + \frac{\Sigma(\delta)}{4} \geq (\eta_1(\delta) + 1)c \tag{2.18}$$

In other words, at time δ or even earlier, node 1 could afford to receive a new link. A new node prefers to form a link with 1 rather than i, when $\eta_1 = \eta_i$ (see eq. 2.15). Thus, if at time δ, node i receives the $(\eta_i + 1)$st link, that implies that a new node skipped the opportunity of linking such that it gains the maximum benefit.

We showed that P_1 is preferred to P_2 at any period in the network formation process. Moreover, P_2 is preferred to P_3, since new nodes gain more by linking to nodes with higher degree, when everything else is equal. Nodes in P_3 are preferred to nodes in P_4, since they have the same degree, but they are at a shorter distance from everyone else in the network. In fact, the number of nodes in $P_2 \cup P_3$ is sufficiently large, such that nodes in P_4 never receive links.

The subnetwork h_τ forms as a response to the agglomeration pressure from the new nodes. New nodes attach to the network forming links with older nodes according to their rank and their availability. We showed that the ranking of partitions stays the same over time. Any new node first proposes a link to the node in P_1, followed by nodes in P_2 depending on their degree, and then by nodes in P_3. The link is formed with the first available old node. This stable ranking keeps the number of nodes that receive links at the minimum required for the network to grow. If each period a different node receives a new link, then a subnetwork, including node 1 and a subset of $(5c - 5/4)$ of its most connected neighbors, sustains the network growth. The growth process is cyclical over time and old nodes receive new links with a periodicity that depends on their rank. ■

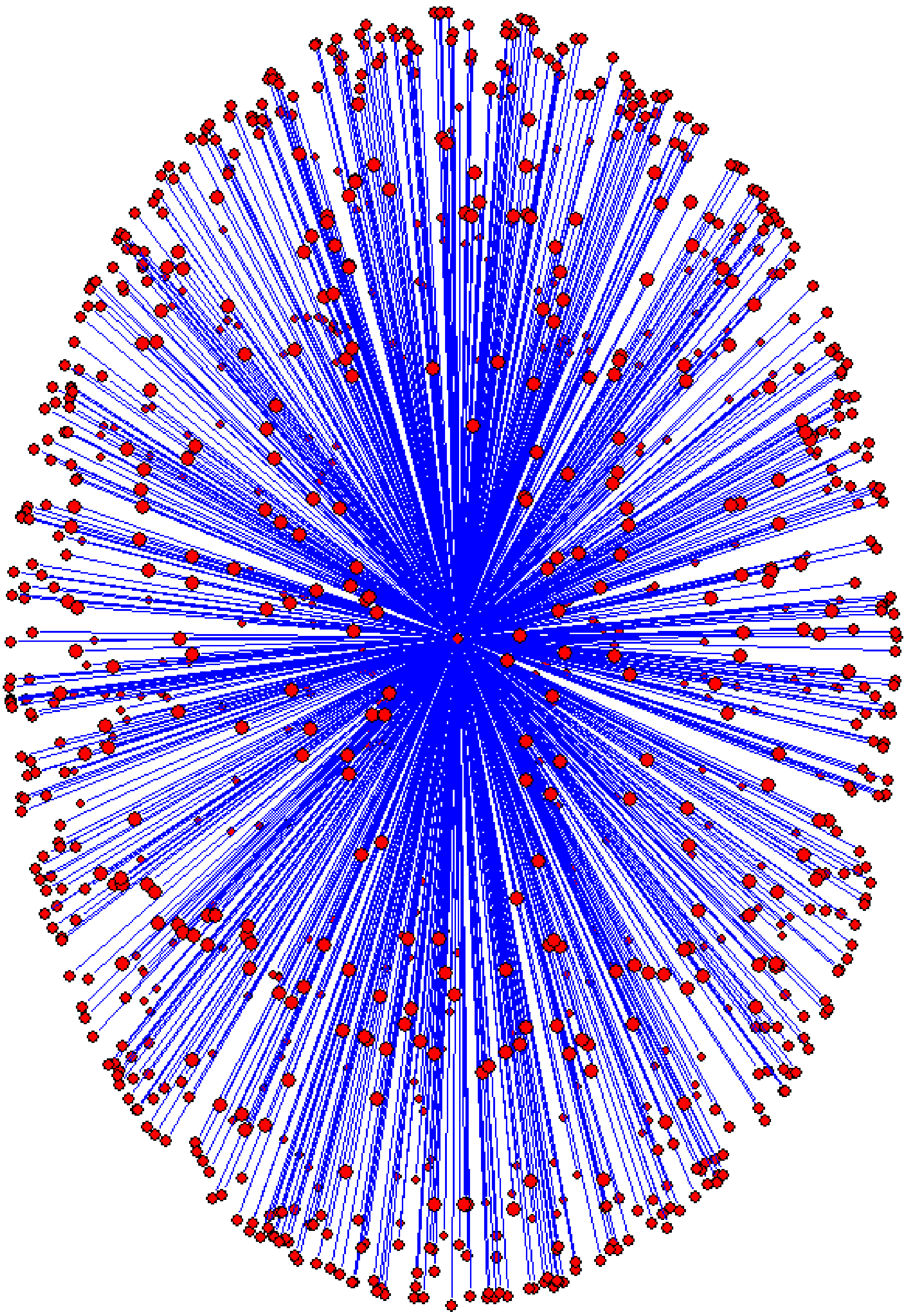


Figure 2.1: Access benefits, (intermediation benefits) and constant marginal cost of linking

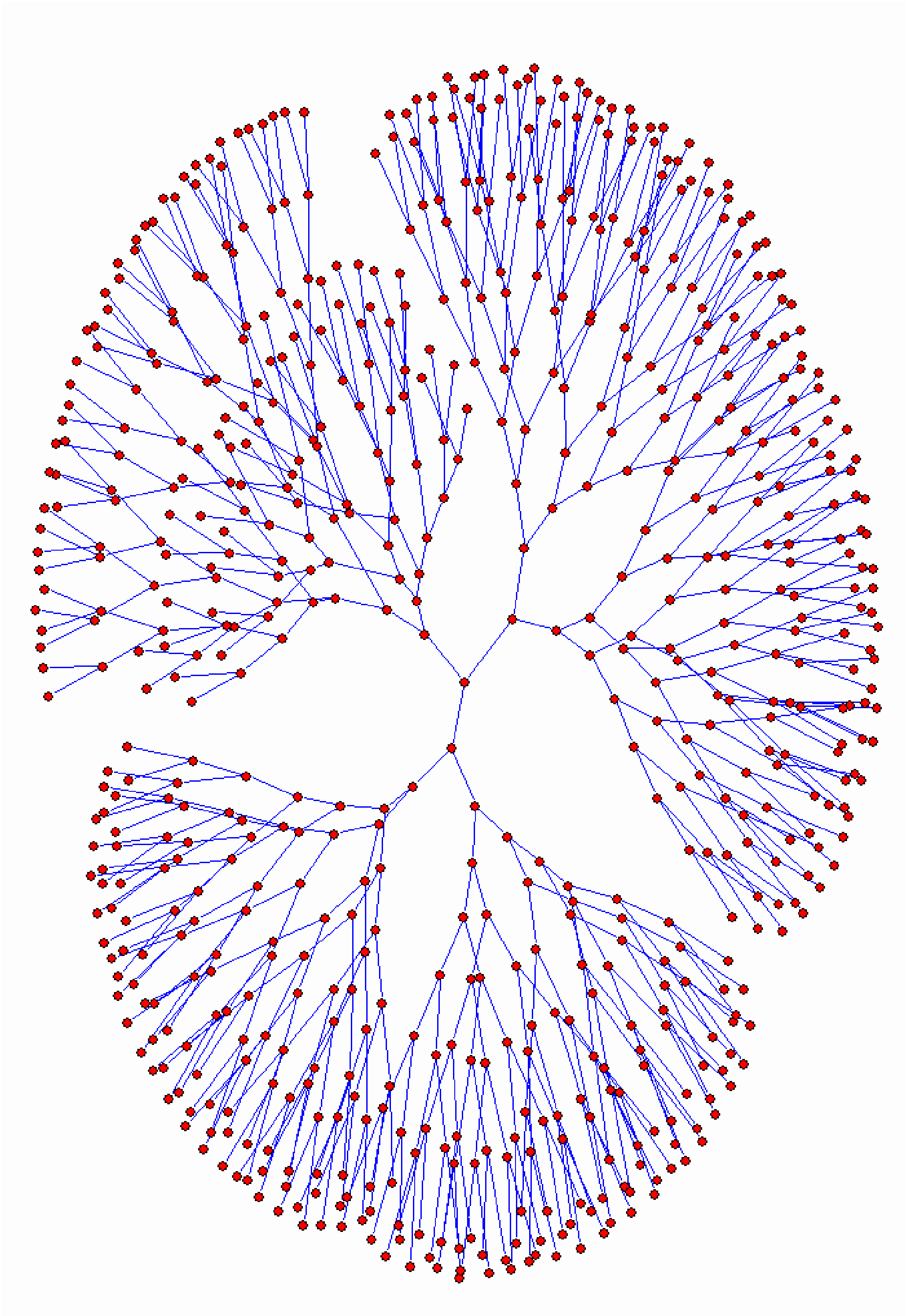


Figure 2.2: Access benefits and increasing marginal cost of linking

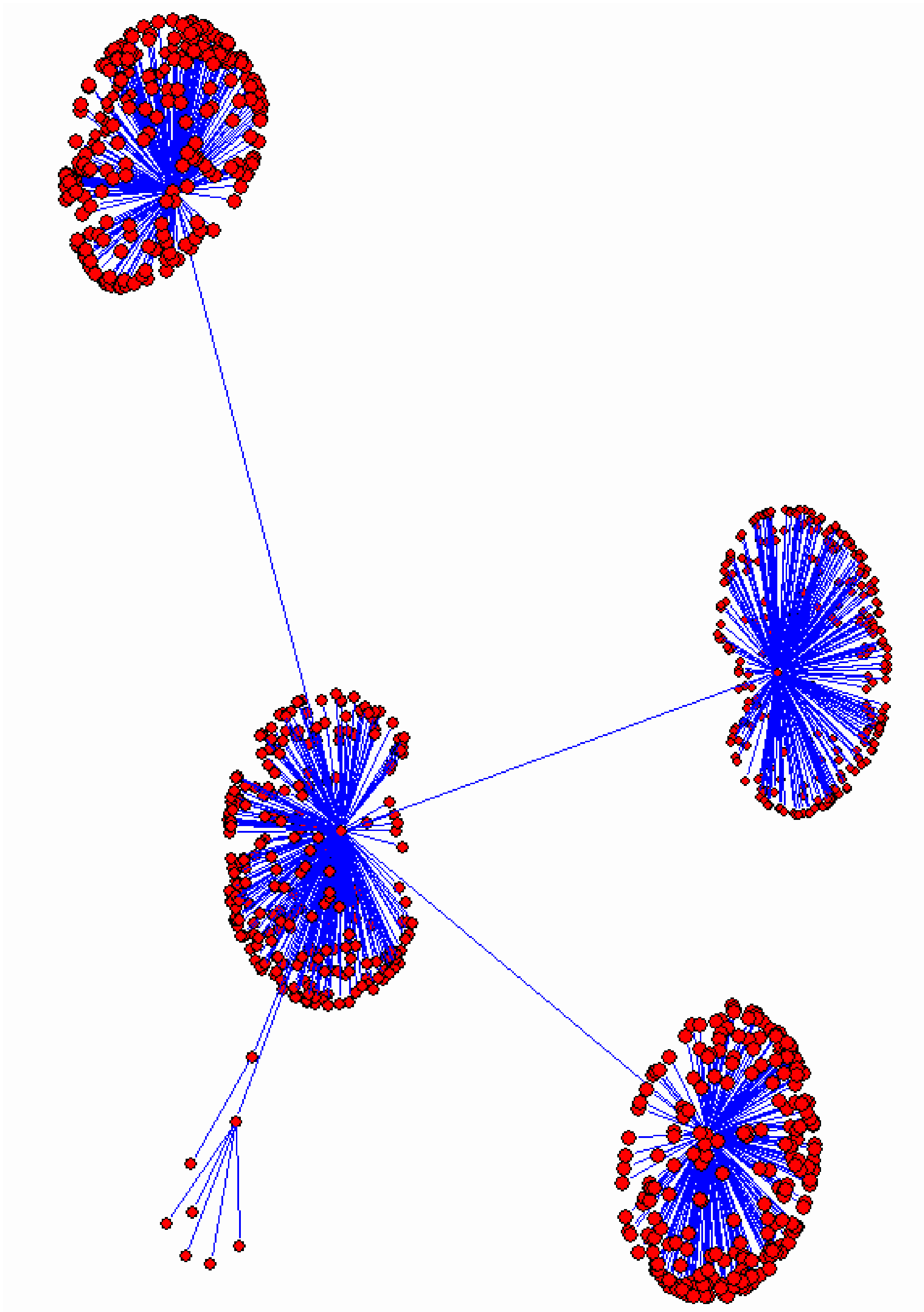


Figure 2.3 (a): Access benefits, intermediation benefits and increasing marginal cost of linking $c = 1$

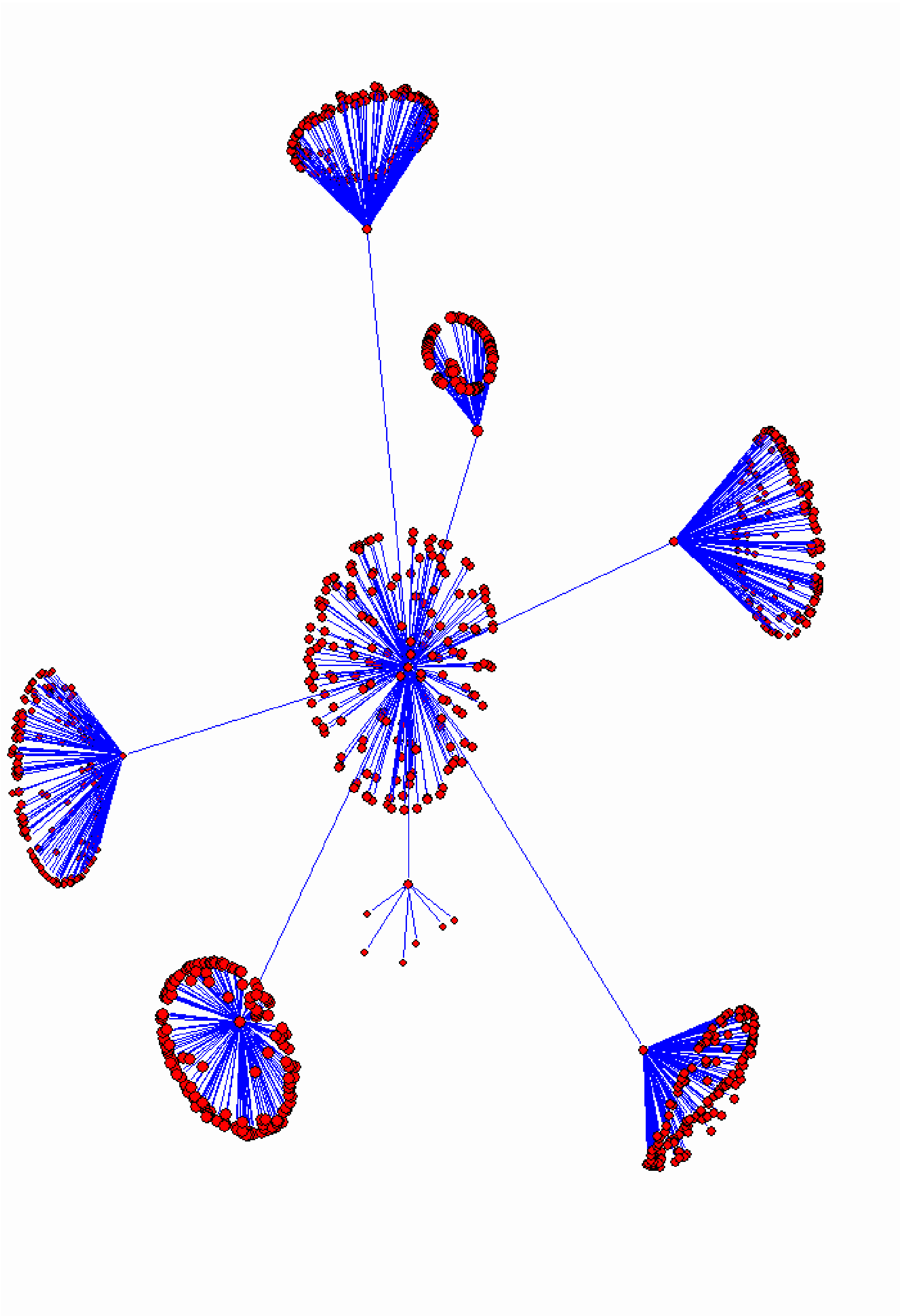


Figure 2.3 (b): Access benefits, intermediation benefits and increasing marginal cost of linking $c = 1.5$

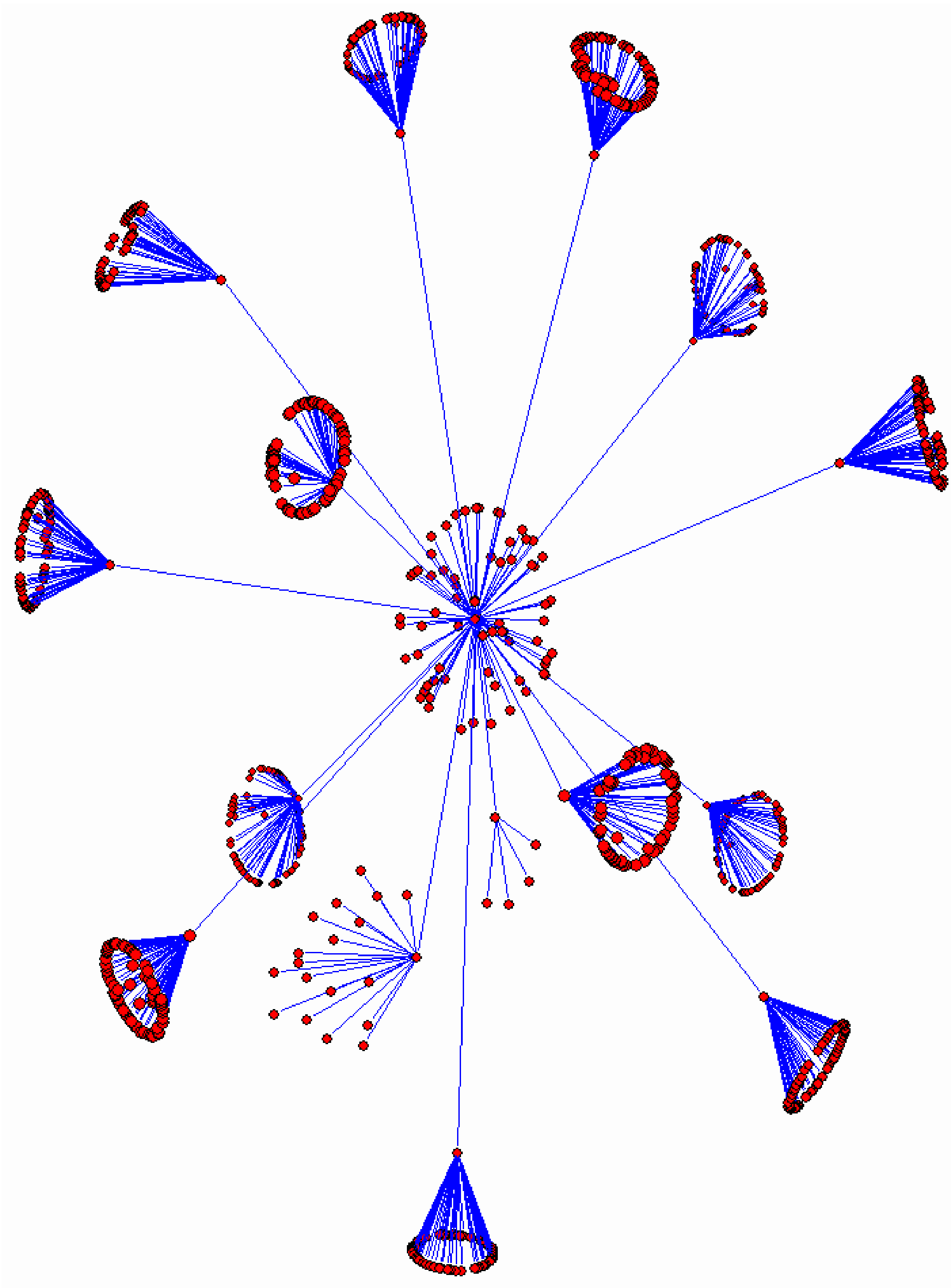


Figure 2.3 (c): Access benefits, intermediation benefits and increasing marginal cost of linking $c = 3$

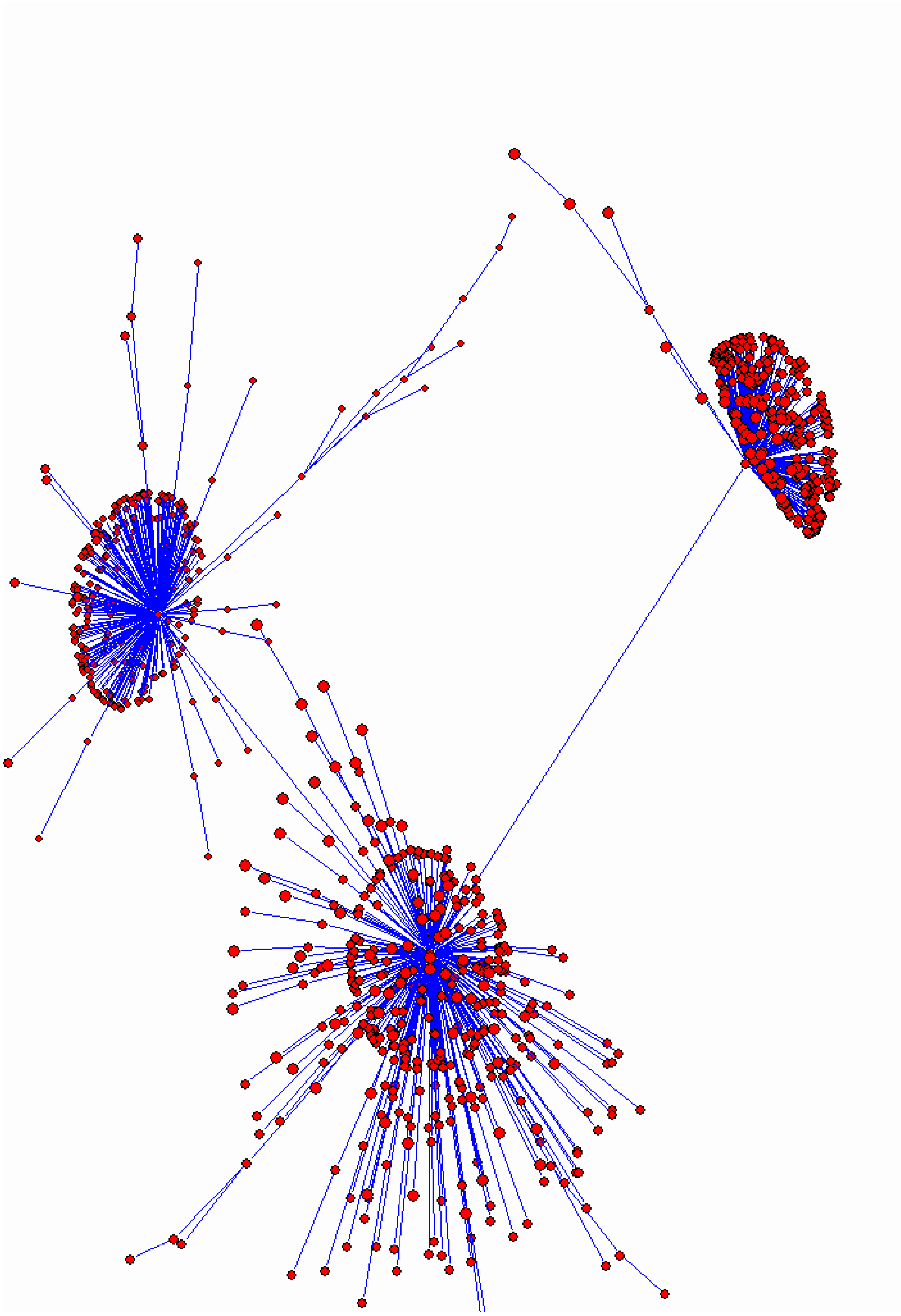


Figure 2.4 (a): Access benefits, intermediation benefits and increasing marginal cost of linking in an error-prone setting $c = 1$ and $\rho = 1$

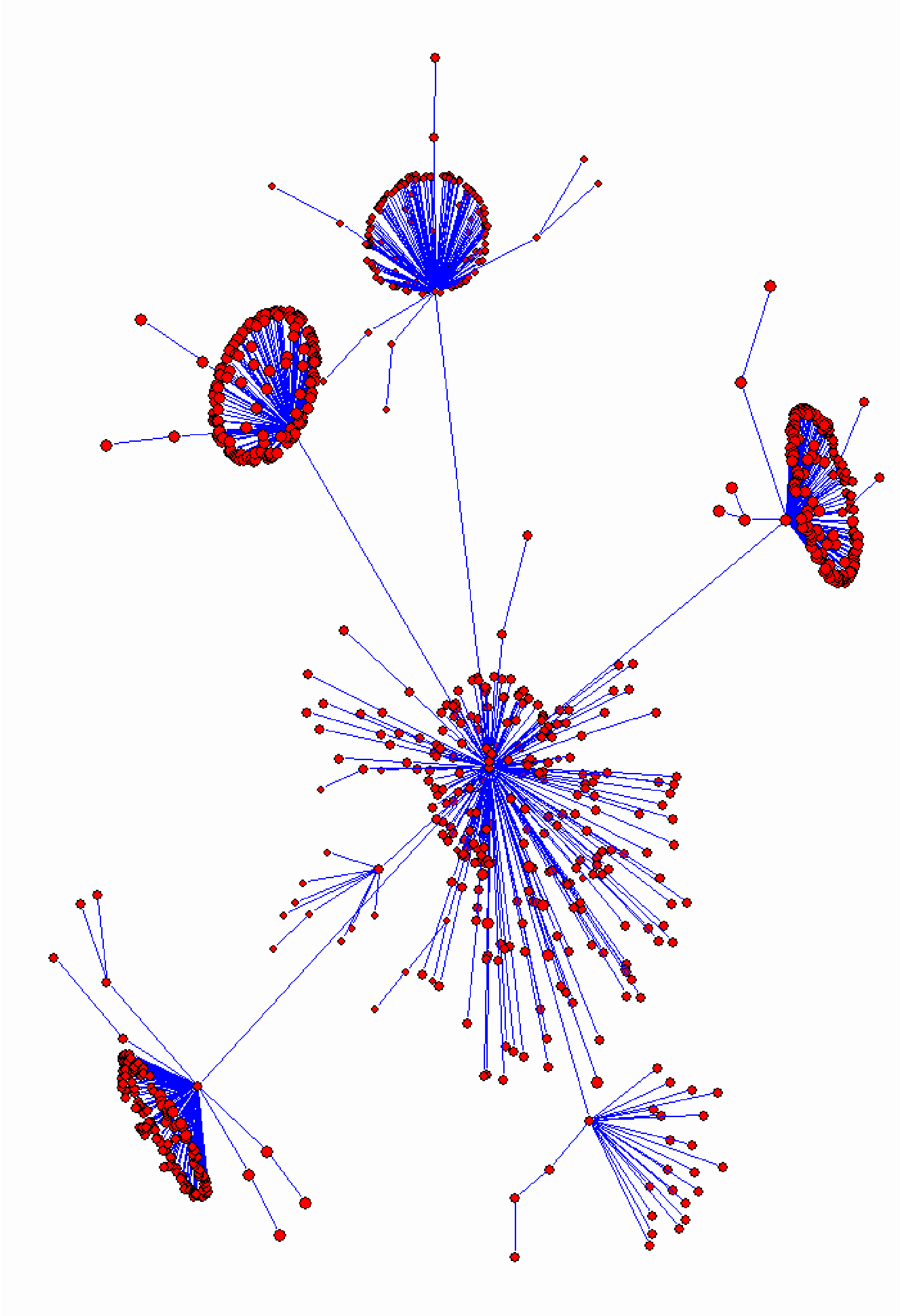


Figure 2.4 (b): Access benefits, intermediation benefits and increasing marginal cost of linking in an error-prone setting $c = 1.5$ and $\rho = 1$

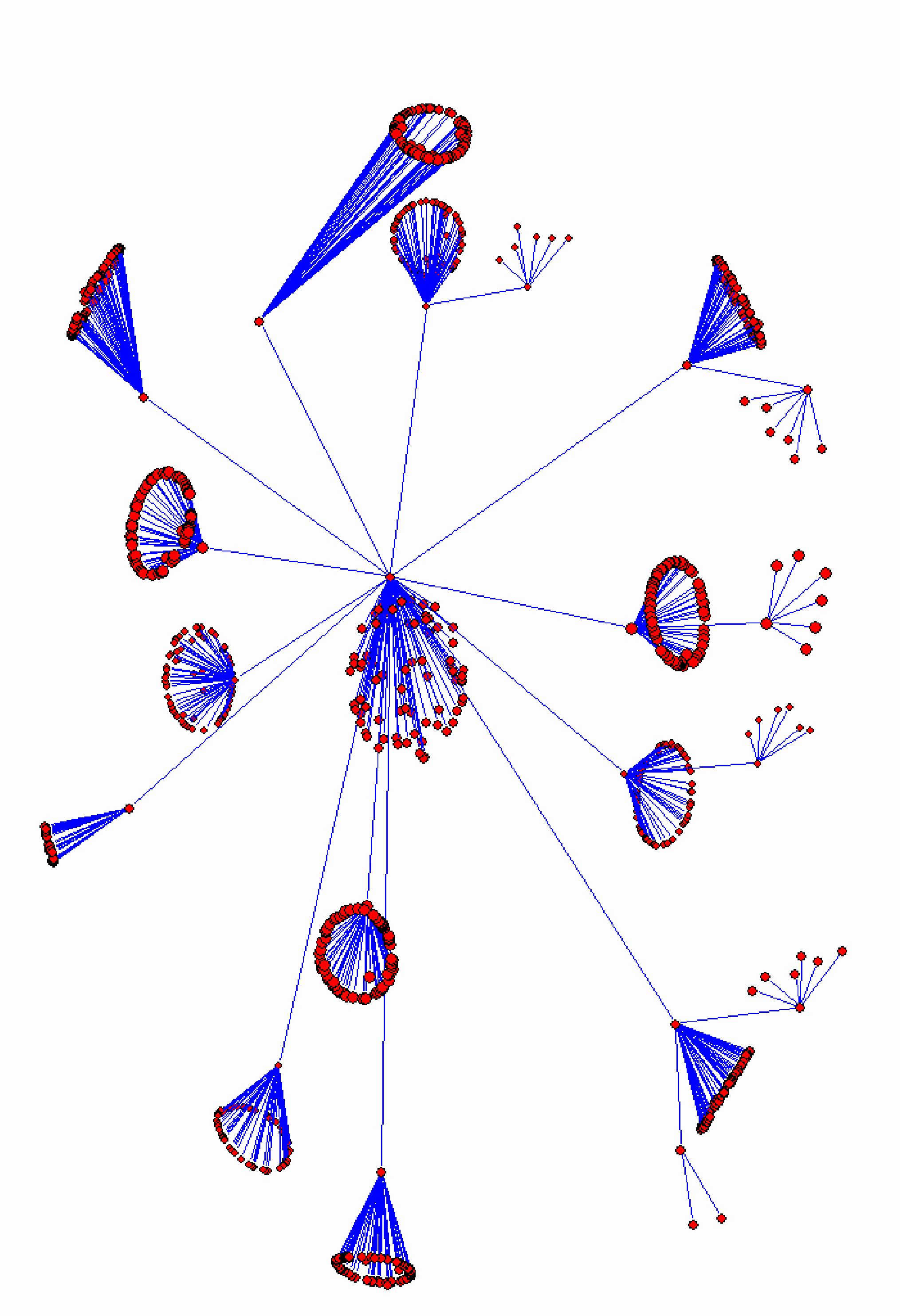


Figure 2.5: Access benefits, intermediation benefits and marginal cost of linking increases with delay

Part II

Applications: Financial Networks

Motivation

Connections in the financial world are varied. Linkages between financial institutions stem from both the asset and the liability side of their balance sheet. Frequently, banks solve their liquidity imbalances by transferring funds from the ones that have a cash surplus to those with a cash deficit. The supply and demand for liquidity connect the financial institutions into a network. Connections in the financial world extend, however, beyond the ones created by market forces. For instance, in the corporate loan market, banks often prefer syndicating loans with other banks over being the sole lender. In the primary equity and bond markets, banks tend to co-underwrite securities offerings with banks with which they have long-standing relationships.

Broadly understood as a collection of nodes and links between nodes, networks can be a useful representation of financial systems. By providing means to model the specifics of economic interactions, network analysis can better explain certain economic phenomena. In this thesis we argue that the use of network theories may enrich our understanding of financial systems, as well. Given the intertwined nature of financial systems, network theories may provide tools to better address several themes. One theme to be explored is the issue of systemic risk. That a small shock in one institution can destabilize the entire financial system is of major concern to both regulators and academics. In this context, two questions arise: How resilient financial networks are to contagion, and how financial institutions form connections when exposed to the risk of contagion. Another theme that may benefit from the developments of networks theories investigates the collapse of the interbank market of the type we have observed in the second half of 2007. Likewise, meaningful insights can be gained from using a network approach to explain how some financial institutions exploit their position as intermediaries between other institutions. Financial institutions that bridge otherwise disconnected parts of the network might gain significant payoff advantages. Financial networks will, thus, be shaped by incentives that drive institutions to acquire the intermediation gains.

Thus far, the literature has primarily adopted a network approach when exploring the issue of systemic risk. Only very recently a new line of research became concerned with the effects of social networks on investment decisions and corporate governance. We survey below the main theoretical contributions, as well as the empirical findings.

The theoretical literature takes two approaches. The first approach looks for contagious effects via direct linkages. Pioneering work in the field that generated a stream of subsequent articles is Allen and Gale (2000). The paper studies how the banking system responds to contagion when banks are connected under different network structures. The authors show that incomplete networks are more prone to contagion than complete structures. Specifically, they take the case of an incomplete network where the failure of a bank may trigger the failure of the entire banking system. They prove that, for the same set of parameters, if banks are connected in a complete structure, then the system is resilient to contagious effects.

Freixas et al. (2000) considers the case of banks that face liquidity needs as consumers are uncertain about where they are to consume. In their model the connections between banks are realized through interbank credit lines that enable these institutions to hedge regional liquidity shocks. The authors analyze different market structures and find that a system of credit lines, while it reduces the cost of holding liquidity, exposes the banking sector to gridlocks, even when all banks are solvent. Dasgupta (2004) also discusses how linkages between banks represented by crossholding of deposits can be a source of contagious breakdowns. Fragility arises when depositors, that receive a private signal about banks' fundamentals, may wish to withdraw their deposits if they believe that enough other depositors will do the same. A unique equilibrium is isolated and it depends on the value of the fundamentals. Eisenberg and Noe (2001) take a more technical approach when investigating systemic risk in a network of financial institutions. First the authors show the existence of a clearing payment vector that defines the level of connections between banks. Next, they develop an algorithm that allows them to evaluate the effects that small shocks have on the system. Similarly, Minguez-Afonso and Shin (2007) use lattice-theoretic methods to study liquidity and systemic risk in high-value payment systems, such as for the settlement of accounts receivable and payable among industrial firms, and interbank payment systems. Leitner (2005) constructs a model that shows how agents may be willing to bail out other agents, in order to prevent the collapse of the whole network. He is interested in the design of optimal financial networks that minimize the trade-off between risk sharing and the potential for collapse. Vivier-Lirimont (2004) addresses the issue of optimal networks from a different perspective: He is interested in those network architectures where transfers between banks improve depositors' utility. He finds that only very dense networks, where banks are very few links away from one another, are compatible with the Paretto optimal allocation.

The second approach focuses on indirect balance-sheet linkages. Lagunoff and Schreft (2001) construct a model where agents are linked in the sense that the return on an agent's portfolio depends on the portfolio allocations of other agents. Similarly, de Vries (2005) shows that there is dependency between banks' portfolios, given the fat tail property of the underlying assets return distributions, and this caries the potential for systemic breakdown. Cifuentes et al. (2005) present a model where financial institutions are connected via portfolio holdings. The network is complete as everyone holds the same asset. Although the authors incorporate in their model direct linkages through mutual credit exposures as well, contagion is mainly driven by changes in asset prices. These papers all share the same finding: Financial systems are inherently fragile. Fragility not only arises exogenously, from financial institutions' exposure to macro risk factors, as it is the case in de Vries (2005). It also arises endogenously, through forced sales of assets by some banks that depress the market price inducing further distress to other institutions, as in Cifuentes et al. (2004).

Besides the theoretical investigations, there has been a substantial interest in looking for empirical evidence of contagious failures of financial institutions resulting from the mutual claims they have on one another. Most of these papers use balance sheet information to estimate bilateral credit relationships for different banking systems. Subsequently, the stability of the interbank market is tested by simulating the breakdown of a single bank. Upper and Worms (2004) analyze the German banking system. Sheldon and Maurer (1998) consider the Swiss system. Cocco et al. (2005) present empirical evidence for lending relationships existent on the Portuguese interbank market. Furfine (2003) studies the interlinkages between the US banks, while Wells (2004) looks at the UK interbank market. Boss et al. (2004) provide an empirical analysis of the network structure of the Austrian interbank market and discuss its stability when a node is eliminated. In the same manner, Degryse and Nguyen (2007) evaluate the risk that a chain reaction of bank failures would occur in the Belgian interbank market. These papers find that the banking systems demonstrate a high resilience, even to large shocks. Simulations of the worst case scenarios show that banks representing less than five percent of total balance sheet assets would be affected by contagion on the Belgian interbank market, while for the German system the failure of a single bank could lead to the breakdown of up to 15% of the banking sector in terms of assets. Departing from the simulation approach, Iyer and Peydro-Alcalde (2007) test for financial contagion using data about interbank exposures at the time of the failure of a large Indian bank. They find that banks with higher interbank exposure to the failed bank experience higher deposit withdrawals, and that the impact of exposure on deposit withdrawals is higher for banks with weaker fundamentals.

In addition to this literature, a recent trend of empirical research advances a new important set of questions: the effects of social networks on investment decisions. on investment decisions and corporate governance. Cohen et al. (2007) use social networks to identify the transfer of information in security markets. Connections between mutual fund managers and corporate board members via shared education institutions proxy the social network. They find that portfolio managers place larger bets on firms they are connected to through their network, and perform significantly better on these holdings relative to their non-connected holdings. Hochberg et al. (2007) look at venture capital forms that are connected through a network of syndicated portfolio company investments. They find that better-networked VC firms experience significantly better fund performance, as measured by the proportion of investments that are successfully exited through an IPO or a sale to another company. Nguyen-Dang (2007) is concerned with the impact of social ties between CEOs and directors within a board of directors on the effectiveness of board monitoring. The paper investigates whether CEOs are less accountable for poor performance depending on their position in the social network. To map the social network, the author uses data on educational background of CEOs from the largest French quoted corporations. Social ties are also formed through interlocking directorships. He finds that when some of the board members and the CEO belong to the same social circles, the CEO is provided with a double protection. She is less likely to be punished for poor performance and more likely to find a new and good job after a forced departure. In a later work, Kramarz and Thesmar (2007) run a similar analysis and find evidence to support that social networks may strongly affect board composition and may be detrimental to corporate governance.

A comprehensive survey of the existent literature on financial networks, that also suggests several directions for future research is given by Allen and Babus (2008). There would appear to be many ways network analysis can be used to gain a better understanding of financial systems. Despite this, the literature on financial networks is still at an early stage. In this thesis, we contribute to the research on financial networks, and propose two applications of network theories to financial systems. The next two chapters of this thesis join the literature that focuses on the issue of systemic risk. The next chapter studies the endogenous formation of networks in the banking system, while the last chapter looks at network effects on contagion risk.

Chapter 3

The formation of Financial Networks

3.1 Introduction

The modern financial world exhibits a high degree of interdependence. Banks and other financial institutions are linked in various ways. For instance, banks are directly connected through mutual exposures acquired on the interbank market. Likewise, holding similar portfolios creates indirect linkages between financial institutions.

In this chapter we propose a different setting: The rationale behind the linkages in the banking system is given by the threat of contagion. We develop a model where banks form links with each other in order to reduce the risk of contagion. A network is formed endogenously between banks and serves as an insurance mechanism.

Despite their obvious benefit, linkages between banks come at the cost that small shocks, which initially affect only a few institutions, may propagate through the entire system. Two questions can be explored. First, what network structures are resilient to contagion? Second, and perhaps more importantly, how financial institutions form connections?

This chapter addresses the second question. We investigate how banks form linkages with each other, where a link represents a transfer of funds between two institutions. In particular, we study a network formation process that is mainly driven by the risk of contagion. Identifying what networks emerge in equilibrium and how these structures respond to contagion is of particular importance for emerging market economies. Lacking the sound regulatory frameworks that characterize the developed financial systems, the banking systems of developing economies have to rely on the ways banks take decisions. The network formation process can be interpreted as decentralizing an insurance scheme that a central planner would adopt.

We apply tools from the theory of networks to address this question. Situations, such as the one we study, where agents form or severe connections depending on the benefits they bring are modeled through a game of network formation. A rapidly growing literature on network formation games has developed in the past few years, introducing various approaches to model network formation and proposing several equilibrium concepts (Bala and Goyal, 2000, Bloch and Jackson, 2007, Jackson and Wolinsky, 1996, Dutta et. al, 2005). Despite the recent developments, the applications to financial networks are rather limited.

The paper that is closest related to ours is by Allen and Gale (2000). Allen and Gale (2000) study how the banking system responds to contagion when banks are connected under different network structures. In a setting where consumers have Diamond and Dybvig (1983) type of liquidity preferences, banks perfectly insure against liquidity shocks by exchanging interbank deposits. The connections created by swapping deposits expose the system to contagion. The authors show that incomplete networks are more prone to contagion than complete structures. Specifically, they take the case of an incomplete network where the failure of a bank may trigger the failure of the entire banking system. However, if banks are connected in a complete structure, then the system is resilient to contagious effects.

The main innovation in our work is to endogenize the network formation of banks. Although, we use the same framework to motivate interactions on the interbank market, we no longer consider that the network of banks is given. We allow the endogenous formation of links and analyze what are the implications for the stability of the banking system. At the base of the link formation process lies the same intuition developed in Allen and Gale (2000): better connected networks are more resilient to contagion. In fact, in our model there is a connectivity threshold above which contagion does not occur. Thus, in order to insure against the risk of contagion, banks form links to reach this threshold. However, an implicit cost associated to being involved in a link prevents banks from forming connections more than required by the connectivity threshold. We show that banks manage to form networks that are usually resilient to the propagation of shocks.

This chapter is organized as follows. Section 2 presents an illustrative example of a network formation process in a 4 - bank framework. The details of the model are presented in Section 3. Section 4 introduces the payoffs banks have from forming links, and models the network formation game. Section 6 analyses the efficiency of banks' link formation decisions. Section 7 concludes.

3.2 Example

We explain with a simple example the incentives banks have when forming links. Consider an economy that consists of two regions, and in each region there are two banks. The banking system is subject to liquidity shocks, perfectly symmetric and negatively correlated across regions. Thus, when each bank in one region has a liquidity surplus $+z$, each bank in the other region has a liquidity shortage $-z$. Since liquidity shocks are negatively correlated, banks are able to insure against illiquidity risk by transferring funds across regions. For any pattern of links that connects banks in the two regions, there exists at least one set of transfers that perfectly insures banks against liquidity shocks. Fig. 3.1 $(a), (b)$ shows two such instances, where banks 1 and 2 are in the same region, while banks 3 and 4 are in the other region.

Transfers, however, create links between banks, exposing the system to the risk of contagion. Explicitly, when a bank fails it induces a loss to all the neighboring banks. If this loss is above a threshold, the banks that incur such a loss go bankrupt as well. This way, the initial shock propagates to other banks via links.

Limiting contagion is the objective of both a social planner concerned with preventing systemic risk, as well as, of banks interested in reducing spillovers from a failed

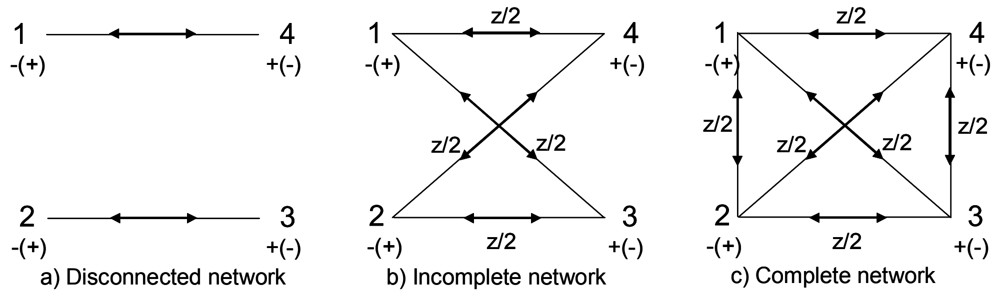


Figure 3.1: The banking system under different networks

institution. The risk of contagion depends on the size of the loss a failed bank induces to its neighbors. The loss has two important properties.[1] First, it is increasing in the size of deposits transferred between banks. The smaller the amount transferred between two banks, the lower is the loss one bank faces if the other one fails. Second, the loss is decreasing in the total number of links a bank has. Thus, increasing the connectivity level in the banking system reduces its propensity to contagion.

We illustrate below how these two properties affect the banks' link formation decisions. Figure 3.1 shows different patterns of connections between banks and the associated transfers schemes. Assume that the set of parameters is such that contagion does not occur in the complete network in fig. 3.1 (c). We study how a social planner would design a network in order to prevent contagion, in parallel to the decentralized link formation process.

Social planner approach: A social planner that is able to organize transfers between banks faces a trivial problem. Designing a fully connected banking system, as in fig. 3.1 (c), ensures that contagion does not occur. Moreover, in a partially connected network, as shown in fig. 3.1 (b), the size of deposits that need to be transferred in order fully insure banks against liquidity shocks can be reduced by half compared to the network shown in fig. 3.1 (c). Lower amounts of transfers may bring the loss produced when a bank fails below the contagion threshold. Thus, contagion need not occur in the incomplete network 3.1 (b). If this is the case, a social planner that aims to prevent systemic risk is indifferent between the two networks: 3.1 (b) and 3.1 (c).

[1] We show why the two properties hold in Section 3.3.

Banks decentralized approach. Banks seek to reduce contagious spillovers from a failed bank, while fully insuring against liquidity shocks. Links across regions provide insurance against liquidity shocks, by allowing transfer of funds from banks with a cash surplus to banks with a cash deficit. Links within a region serve solely to limit contagion, by increasing the connectivity level in the banking system. Moreover, there is an implicit cost associated to being involved in a link, that takes the form of the loss one bank would face if the other one fails. Banks will weigh the costs and benefits when deciding how to form links.

Consider a banking system connected as in fig. 3.2.1 (b) and assume that contagion can occur. For example, if bank 1 fails, banks 3 and 4 go bankrupt as well. Furthermore, the failure of bank 1 indirectly triggers the failure of 2, although there is no link between the two. The loss that a bank spreads decreases with the number of links it has. Provided the loss is lowered below the contagion threshold, both banks 1 and 2 will agree to form a link. The same incentives induce bank 3 and 4 to link. The complete network is, thus, formed.

In contrast, when the system in fig. 3.2.1 (b) is resilient to contagion, there is no benefit for banks 1 and 2 to form a link. Suppose the link exists; then bank 2 incurs a loss, for instance, when 1 fails. Links within the same region serve only to reduce spillovers from a failed bank. Since contagion does not occur in the first place, the link is detrimental for both banks.

3.3 The Model

3.3.1 Consumers and Liquidity Preferences

The economy is divided into $2n$ sectors, each populated by a continuum of risk averse consumers. There are three time periods $t = 0, 1, 2$. Each agent has an endowment equal to one unit of consumption good at date $t = 0$. Agents are uncertain about their liquidity preferences: they are either early consumers, who value consumption only at date 1, or they are late consumers, who value consumption only at date 2. In the aggregate there is no uncertainty about the liquidity demand in period 1: on average, the fraction of early consumers is q. Each sector, however, experiences random fluctuations in the need for liquidity of early consumers. With probability $(1-\pi)/2$, in each sector there is either a high proportion p_H of agents that need to consume at date 1 or a low proportion p_L of agents that value consumption in period 1. Only with a small probability π, the fraction of early consumers is the same across sectors, $q = \frac{p_H + p_L}{2}$.

All the uncertainty is resolved at date 1, when the state of the world is realized and commonly known. At date 2, the fraction of late consumers in each region will be $(1-p)$ where the value of p is known at date 1 as either p_H, p_L or q.

3.3.2 Banks, Asset Investments and Idiosyncratic Shocks

In each sector i there is a competitive representative bank. Agents deposit their endowment in the regional bank. In exchange, they receive a deposit contract that guarantees them an amount of consumption depending on the date they choose to withdraw their deposits. In particular, the deposit contract specifies that if they withdraw at date 1, they receive $C_1 > 1$, and if they withdraw at date 2, they receive $C_2 > C_1$.

Banks have two investment opportunities: a liquid asset with a return of 1 after one period, or an illiquid asset that pays a return of $r < 1$ after one period, or $R > 1$ after two periods. Let x and y be the per capita amounts invested in the liquid and illiquid asset, respectively. Naturally, banks use the liquid asset to pay depositors that need to withdraw in the first period and reserve the illiquid asset to pay the late consumers. For convenience, we assume that the investment in the liquid asset, x, equals qC_1, while the investment in the illiquid asset, y, will cover $(1-q)C_2/R$.[2] Thus, at date 1 each bank has with probability $(1-\pi)/2$ either a liquidity shortage of $zC_1 = (p_H - q)C_1$ or a liquidity surplus of $zC_1 = (q - p_L)C_1$.

The uncertainty in the liquidity preferences of consumers creates liquidity shocks in the banking system. We assume that liquidity shocks are distributed across banks such that there are two regions, A and B, in the banking system. In each region there

[2] Allen and Gale (2000) show in their paper that both the deposit contract and the investement in liquid and illiquid asset are such that they maximize the expected utility of consumers.

is a equal number of banks: $A = \{1, 2, ..., n\}$ and $B = \{n+1, n+2, ..., 2n\}$. Moreover, liquidity shocks are perfectly negatively correlated across regions. Hence, when banks in region A receive a positive shock, for instance, banks in region B receive a negative shock.

In addition, banks are subject to idiosyncratic shocks. In particular, one bank will fail in period 1, when there are no liquidity imbalances. This implies that each bank has an individual probability of $\pi/2n$ of going bankrupt in period 1.

Table 1 summarizes the states of the world for the banking system and the associated probabilities.

		Region A				Region B			
Probability	State/Bank	1	2	...	n	$n+1$	$n+2$	...	$2n$
$(1-\pi)/2$	S_1	$+z$	$+z$	$+z$	$+z$	$-z$	$-z$	$-z$	$-z$
$(1-\pi)/2$	S_2	$-z$	$-z$	$-z$	$-z$	$+z$	$+z$	$+z$	$+z$
$\pi/2n$	$\bar{S}_1$	$fail$	0	0	0	0	0	0	0
$\pi/2n$	$\bar{S}_2$	0	$fail$	0	0	0	0	0	0
$\pi/2n$	...	...	...	...	...	...	...	...	...
$\pi/2n$	$\bar{S}_{2n}$	0	0	0	0	0	0	0	$fail$

Table 3.1: Distribution of Shocks in the Banking System

3.3.3 Balance Sheet Linkages

The regional liquidity shocks create opportunities for risk-sharing. While there is no aggregate liquidity shock, banks in either region have with probability $(1-\pi)/2$ either a liquidity shortage or a liquidity surplus of zC_1. As shocks are negatively correlated across regions, individual risk can be completely hedged by exchanging interbank deposits between banks in different regions, at date 0. We assume each bank receives the same return as the consumers for the amounts transferred as deposits: C_1, if they withdraw after one period, and C_2 if they withdraw after two periods. These interactions create balance sheet linkages between banks in the two regions.

Thus, banks' portfolios consist of three assets: the liquid asset, the illiquid asset and the interbank deposits. Each of these three assets can be liquidated in either of the last two periods. However, we assume that the costliest in terms of early liquidation is the illiquid asset, followed by interbank deposits. This implies the following ordering of returns:

$$1 < \frac{C_2}{C_1} < \frac{R}{r} \tag{3.1}$$

Let a_{ij} denote the amount exchanged as deposits between banks i and j at date 0. We consider that deposit contracts are bilateral, hence we have $a_{ij} = a_{ji}$. Let N_i be the set of banks that i is linked to and let N_i^{cross} be a subset of N_i representing the banks that i is linked to in the other region. Then, the total amount of deposits i exchanges with its neighbors should balance out its liquidity shortage or excess. Since the insurance against liquidity shocks is provided only through links with banks in a different region, a_{ij} should satisfy the *feasibility constraint*:

$$\sum_{j \in N_i^{cross}} a_{ij} \geq z \tag{3.2}$$

3.3.4 Losses Given Default

The non insurable idiosyncratic shocks bear the risk of contagion. That is, the shock that initially affects only one institution can propagate through the entire system. In our setting, shocks spread sequentially: an exogenous failure of a bank in period 1 might spill over first to any neighbor bank and next, via links, to all the other banks. The contagion mechanism is detailed in the next section. As a measure to evaluate contagion risk we use *loss given default* (henceforth LGD). LGD expresses the excess of nominal liabilities over the value of the assets of the failed bank. In our setting, LGD will be given by the loss of value a bank incurs on its deposits when one of its neighbor banks is liquidated.

To calculate LGD, we need to determine the value of the assets of the failed bank. When a bank i fails, its portfolio of assets is liquidated at the current value and distributed equally among creditors. The three assets in banks' portfolio yield different returns upon liquidation in period 1. First, the amount of x per capita banks invested in the liquid asset pays a return of 1. Second, the amount y per capita banks hold in the illiquid asset, pays a return of $r < 1$ if liquidated early. And lastly, the interbank deposits, summing up to $\sum_{k \in N_i} a_{ik}$, yield a return of C_1 per unit of deposit. On the liability side, a bank has to pay its depositors, payment normalized to 1, and at the same time to repay its interbank creditors that also add up to $\sum_{k \in N_i} a_{ik}$. This yields a new return per unit of good deposited in a bank i equal to $\bar{C}_i = \frac{x+ry+\sum_{k \in N_i} a_{ik} C_1}{1+\sum_{k \in N_i} a_{ik}} < C_1$.[3] The LGD of bank j given that bank i has failed is expressed as[4]:

$$LGD_{ji} = a_{ji}(C_1 - \bar{C}_i) = a_{ji} \frac{C_1 - x - ry}{1 + \sum_{k \in N_i} a_{ik}} \tag{3.3}$$

LGD has two interesting properties that have important implications on the way banks form links.

- *First*, the LGD_{ji} is increasing in the amount of deposits a_{ji}that is exchanged between banks. This gives banks an incentive to exchange the minimum amount of deposits.

- *Second*, the LGD_{ji} is decreasing in $(\sum_{k \in N_i - \{j\}} a_{ik})$. This implies that the more connected one bank is, the smaller the loss it induces to its neighbors in case it fails. In other words, the better connected banks are, the smaller the contagious effects will be.

3.3.5 Contagion Threshold

In this section we describe the contagion mechanism. When a bank fails, its neighbors incur a loss on the value of deposits exchanged with the failed bank. This implies that, in period 1, affected banks hold the value of the liquid asset, qC_1, less the size of LGD. Hence, to meet its obligation, qC_1, towards early consumers, a bank must liquidate an amount of the illiquid asset that equals the LGD value. Liquidating the illiquid asset

[3] Eq. (3.1) ensures that the inequality holds.

[4] In principle $LGD_{ji} \neq LGD_{ij}$ since it may be that $\sum_{k \in N_i} a_{ik} \neq \sum_{k \in N_k} a_{jk}$

prematurely, however, involves a penalty rate $r < 1$ and has negative consequences for the late consumers. In fact, if too much of the illiquid asset is liquidated early, the consumption of late consumers may be reduced to a level below C_1. In this case, the late consumers gain more by imitating the early consumers and withdrawing their investment from the bank at date 1. This induces a run on the bank and, subsequently, triggers its failure.

The maximum amount of illiquid asset that can be liquidated without causing a run is $b(q)$ and it depends on the fraction of late consumers, $(1-q)$, and the return rates for early and late liquidation of the illiquid asset, r and R. Equation (3.4) captures the exact effect of these variables.

$$b(q) \equiv r\left[y - \frac{(1-q)C_1}{R}\right] \tag{3.4}$$

The maximum amount of illiquid asset that can be liquidated without causing a run on the bank can be interpreted as a *contagion threshold*. Any bank that incurs a LGD higher than $b(q)$ will, inevitably, fail. A value of LGD below the threshold $b(q)$ will not trigger the failure of a bank. However, it will be costly for the late consumers, given that their consumption is now reduced to $\tilde{C}_2 < C_2$.[5] Thus, there is an implicit cost associated with being involved in a link. Links are potentially conduits of LGD, which is detrimental for banks that incur it, even when below the contagion threshold $b(q)$.

3.4 The Network Formation Game

3.4.1 Concepts and Notations

Let $N = \{1, 2, ..., 2n\}$ denote the set of banks. A network g on the set N is a collection of g_{ij} pairs, with the interpretation that i and j are linked. Thus, if i and j are linked in the network g, then $g_{ij} \in g$.

[5] The consumption of late consumers at least equals the consumption of the early consumers: $\tilde{C}_2 \geq C_1$.

The set of *neighbors* of bank i in the network g is $N_i(g) = \{j \in N \mid g_{ij} \in g\}$. Let $\eta_i(g) = |N_i(g)|$, where $|\cdot|$ represents the cardinality of a finite set. The number of neighbors, $\eta_i(g)$, bank i has in the network g is called the *degree* of bank i. In addition, let $N_i^{inner}(g) = \{j \in N \,|g_{ij} \in g \text{ and } i,j \in A \text{ or } i,j \in B\}$ and $\eta_i^{inner}(g) = |N_i^{inner}(g)|$. The number of neighbor banks in the same region is defined as the *inner degree* of bank i. A related notation is used for the set of banks in a different region, $N_i^{cross}(g) = N_i(g) \setminus N_i^{inner}(g)$, and the number of neighbor banks in a different region is $\eta_i^{cross}(g) = |N_i(g) \setminus N_i^{inner}(g)|$.

We use the notation $g + g_{ij}$ to denote the new graph obtained from g by linking i and j, if $g_{ij} \notin g$. Similarly, we consider that $g - g_{ij}$ represents the graph obtained from g by deleting an existing link between i and j, when $g_{ij} \in g$

A path of length k between i and j is a sequence of distinct agents $(i, j_1, ..., j_{k-1}, j)$ such that g_{ij_1}, $g_{j_1j_2}$,..., $g_{j_{k-1}j} \in g$. A network g is connected if there exists a path between any two nodes i and j from N. A network g is complete if for any node $i \in N$, $\eta_i(g) = n - 1$. A network g is regular of degree k if for any node $i \in N$, $\eta_i(g) = k$.

3.4.2 Strategies, Objectives and Welfare

The interaction between banks on the interbank market can be modeled as a network formation game. The network is formed as a result of banks' actions, who decide how to form links. Since deposits are exchanged on bilateral basis (i.e. bank i agrees to pass its deposits to bank j if and only if bank j will pass its deposits to bank i in turn), the network is undirected and the formation of a link requires the consent of both parties involved. However, the severance of a link can be done unilaterally.

The strategy of bank i can be described as a linking vector $s_i = (s_{i1}, s_{i2}, ..., s_{i2n})$ such that $s_{ij} \in \{0, 1\}$ for each $j \in N \setminus\{i\}$ and $s_{ii} = 0$, where $s_{ij} = 1$ means that i intends to form a link with bank j. A link between i and j is formed if and only if $s_{ij} = s_{ji} = 1$.[6] Links between banks in different regions provide insurance against liquidity shocks. Links formed between banks in the same region serve solely to limit

[6]This condition capture that the formation of a link between two banks requires the consent of both participants.

contagion. In this chapter we explicitly model the network formation between banks in the same region for a given pattern of interactions between banks across regions. In particular we assume that between banks in the different regions there exists a complete bipartite graph. Thus, for any bank $i \in \{1, 2, ..., n\}$ the linking vector can be written as $s_i = (s_{i1}, s_{i2}, ..., s_{in}, 1, 1, ..., 1)$, while for any bank $j \in \{n+1, ..., 2n\}$ the linking vector is represented by $s_j = (1, 1, ..., 1, s_{jn+1}, ..., s_{j2n})$. In essence, we study a network formation process driven by the risk of contagion.

Since liquidity shocks are negatively correlated across regions, they are perfectly insurable. Banks in one region can hedge their potential liquidity excess or shortage by exchanging deposits with banks in the other region. Any linking pattern between banks in different regions is sufficient for insurance purposes, as long as it allows banks to exchange deposits which satisfy the feasibility constrain (3.2). However, idiosyncratic shocks can spread through links and expose the banking system to contagion risk. The risk of contagion, measured in terms of LGD, is reduced on the one hand when the size of deposits exchanged per link is small, and, on the other hand, when the number of links a bank has is high. If we only look at symmetric solutions, the amount of deposits exchanged per link is minimal when each bank in one region is linked to all the other banks in the other region. This is the rationale behind the assumption that banks of a different type form a complete bipartite graph. Next, we investigate how banks endogenously reduce contagion risk, by forming links with other banks in the same region.

Lemma 3.1 *Consider the states of the world S_1 and S_2, when liquidity shocks are negatively correlated across the two regions A and B. The minimization problem for LGD associated to each link, under the feasibility constrain (3.2), has a symmetric solution when each $i \in A$ is linked to each $i' \in B$, and $a_{ii'} = \frac{z}{n}$.*

Proof. The proof is provided in the Appendix. ■

For simplicity, we normalize the amount of deposits exchanged between banks i and j in the same region to $a_{ij} = \frac{z}{n}$.

In a network where they can fully insure against liquidity shocks, banks need only to prevent losses through contagion. Thus, banks' primary objective is to minimize the probability of failure through contagion. Secondarily, banks seek to avoid losses, even when LGD is below the contagion threshold. These two forces drive the link formation process in different directions: The first motivates banks to form densely connected networks, while the second limits the number of links that banks have.

Similarly, a social planner objective is to minimize the number of banks that fail through contagion. The main concern of a social planner is the systemic risk. In our framework, the systemic risk is given by the number of banks that fail, following a shock in a single institution. Thus, banks' and social planner's incentives to form networks are at least partially aligned.

3.4.3 Payoffs' Properties

We briefly give the intuition behind the payoffs that banks gain from linking to other banks. The *contagion threshold* we introduced in Section 3.3 is useful to describe the trade-offs of the linking process. When a failed bank induces a value of LGD higher than the contagion threshold, $b(q)$, then all the the neighboring banks will fail in turn. By assumption, each bank has at least n neighbors. Thus, the failure of a bank triggers the failure of at least n other banks. This chain of failure will affect remaining banks, causing, in the end, the failure of the entire system. However, if the LGD caused by a failed bank is below the threshold $b(q)$, only the neighboring banks experience a loss. This reduces the consumption of the late consumers to a level $\tilde{C}_2 < C_2$, as explained in Section 3.3.5. Any other bank is able to pay C_1 to the early consumers and C_2 to the late consumers.

Whether LGD is above or below the contagion threshold, $b(q)$, depends on how well connected the respective bank is. The size of deposits exchanged between banks in different regions is z/n, as shown by Proposition 3.1. The size of deposits exchanged between banks in different regions is also, by assumption, z/n. This implies that the LGD induced by a failed bank depends on how many neighbors the respective bank has.

The contagion threshold, $b(q)$, is identical for all banks and is independent of the number of links a bank has. Thus we can identify a number $t \in \mathbb{N}$ that brackets $b(q)$

as follows:

$$\frac{z}{n}\frac{C_1 - x - ry}{1 + (n+t)\frac{z}{n}} \leq b(q) < \frac{z}{n}\frac{C_1 - x - ry}{1 + (n+t-1)\frac{z}{n}} \tag{3.5}$$

The left hand side of the inequality is exactly the LGD induced by a bank with $(n+t)$ neighbors, and the right hand side is the LGD induced by a bank with $(n+t-1)$ neighbors. Inequality 3.5 relates the contagious effects of a bank failure to the number of links banks have. For a given value of the contagion threshold $b(q)$, the failure of a bank with $(n+t-1)$ links or less triggers the failure of the entire system, through the mechanism described above. The failure of a bank with at least $(n+t)$ links, however, affects only neighboring banks, which incur a loss.

We discuss in detail the implications of a bank failure for the case $t \in \{1, 2, ..., n-1\}$. Consider the failure of a bank j in a network g. Bank j is assumed to be linked to all the banks in a different region, which implies $\eta_j^{cross} = n$. We distinguish the following cases:

1. $\eta_j^{inner}(g) < t$. In this case, for any $i \in N_j(g)$ we have $LGD_{ij} > b(q)$. Consequently, any bank $k \in N$ will also fail.[7]

2. $\eta_j^{inner}(g) \geq t$. In this case, for any $i \in N_j(g)$ we have $LGD_{ij} \leq b(q)$. Thus, any bank $i \in N_j(g)$ will pay the early consumers C_1. The late consumers, however, will have their consumption reduced to $\tilde{C}_2 < C_2$. Any other non-neighboring bank $k \in N \setminus N_j(g)$ will not be affected in any way and will be able to pay its consumers C_1 at date 1 and C_2 at date 2.

3.4.4 Payoffs

We formally introduce the payoff that a bank i gains from network g. Banks seek to minimize the probability of failure through contagion. This creates incentives to form links in order to bring the LGD below the contagion threshold, $b(q)$. However, there is an implicit cost of linking: Losses are transmitted through links, even when $LGD \leq b(q)$.

[7] With this discussion, we are able to conclude that the existence of a single bank with insufficient links may trigger the failure of the entire system.

Let $b(q)$ be the contagion threshold and t an integer that satisfies 3.5. Formally, we can express the payoff of a bank $i \in N$ as a function u, that is increasing in the number of nodes with an inner degree higher than t and decreasing in the number of neighbors with an inner degree higher than t:

$$u_i(g) = f(|T|, |N_i(g) \cap T|) \tag{3.6}$$

where $T = \{j \in N \left| \eta_j^{inner}(g) \geq t \right.\}$ and $|\cdot|$ represents the cardinality of a set.

Moreover, for any node i and any inner degree η_i^{inner}, the payoff u_i has the following *properties*:

1. $u_i(g + g_{ij}) = u_i(g)$ and $u_i(g - g_{ij}) = u_i(g)$, $\forall j \in N$ s.t. $\eta_j^{inner}(g) < t - 1$

   The explanation for this indifference relies on the fact that the failure of a node with an inner degree below t will trigger the failure of the entire system. The failure of j leads to the failure of i, regardless of i creating a link or severing an existent link with j.

2. $u_i(g + g_{ij}) > u_i(g)$, $\forall j \in N$ s.t. $\eta_j^{inner}(g) = t - 1$

   If $\eta_j^{inner}(g) = t - 1$ and i creates a link with j, then the inner degree of j becomes $\eta_j^{inner}(g) = t$. Thus, i trades a situation when the failure of j induces its own failure, for a situation when the failure of j results in merely a lower utility for i's late consumers.

3. $u_i(g + g_{ij}) < u_i(g)$, $\forall j \in N$ s.t. $\eta_j^{inner}(g) \geq t$

   When j has already an inner degree sufficiently high, its failure will have only effects for the neighbor banks. Linking with j does not bring i any benefits, but it comes at the cost represented by the loss i might incur if j fails.

4. $u_i(g - g_{ij}) > u_i(g)$, $\forall j \in N$ s.t. $\eta_j^{inner}(g) \geq t + 1$

   Severing an existent link with j, will leave j with an inner degree still sufficiently high. It will, however, spare i from experiencing a loss in case j fails.

3.4.5 Stable Networks

We are now ready to approach the main goal of this paper. We can characterize the networks that arise in a banking system where financial players' incentives to form links are driven by the risk of contagion. The regional liquidity shocks in the banking system are perfectly insurable through risk-sharing agreements. Banks in one region can hedge their potential liquidity excess or shortage by exchanging deposits with banks in the other region. We assume that transfers take place when each bank in a region is linked to each bank in the other region, such that the amount of deposits exchanged per link is minimal. However, an idiosyncratic shock that hits one bank can propagate through the entire system. Banks can limit contagion by creating links with other banks in the same region. We follow the network formation process between banks in the same region.

To identify stable networks, we use the following concept introduced by Jackson and Wolinsky (1996).

Criterion 3.1 *Let* $g_{ij} = \min(s_{ij}, s_{ji})$ *and consider that* $g_{ij} \in g$ *when* $g_{ij} = 1$. *A network* g *is pairwise stable if*

1. *for all* $g_{ij} \in g$, $u_i(g) \geq u_i(g - g_{ij})$ *and* $u_j(g) \geq u_j(g - g_{ij})$ *and*

2. *for all* $g_{ij} \notin g$, *if* $u_i(g) < u_i(g + g_{ij})$ *then* $u_j(g) > u_j(g + g_{ij})$.

where $u_i(g)$ *is the payoff of bank* i *in the network* g.

The first condition of the stability criterion states that a network is stable if there is no bank that wishes to severe a link in which it is involved. The second condition requires that in a stable network there are no two unconnected banks that would both benefit by forming a link. In other words, a network is stable if there are no banks that wish to deviate either unilaterally (by severing existent links), or bilaterally (by adding a link between two banks).

The first result provides a necessary condition for a stable network to exist.[8]

[8] All results in this section hold under the assumption that the crossing degree of any bank $i \in N$, in the network g, is $\eta_i^{cross}(g) = n$.

Proposition 3.1 *Let $b(q)$ be the contagion threshold and t an integer that satisfies 3.5. If a network g is pairwise stable, then any bank $i \in N$ has an inner degree $\eta_i^{inner}(g) \leq t$.*

Proof. The proof follows immediately from the payoff properties 3 and 4 described in the previous section. Suppose that there exists a bank i such that $\eta_i^{inner}(g) > t$. Then, any neighbor $j \in N_i(g)$ has an incentive to severe the link that connects it with i. Thus, g is no longer stable. ■

This result provides only a partial characterization of stable networks. In fact, under payoffs that adhere to properties $1-4$, there are many pairwise stable networks. In fact, any network where each node i has an inner degree $\eta_i^{inner}(g) \leq t-2$ is pairwise stable. This multiplicity of equilibria is mainly driven by the indifference in forming or severing links as expressed by property 1. In what follows we alter property 1 in order to restrict the set of stable networks. Namely, we consider that banks have a weak preference to forming links with other banks. We keep the other properties as described in Section 4.4.

Formally, if $b(q)$ is the contagion threshold and t is an integer that satisfies the inequality 3.5, then for any node i and any inner degree η_i^{inner}, the payoff u_i has the following *properties*:

1'. $u_i(g + g_{ij}) = u_i(g) + \varepsilon$ and $u_i(g - g_{ij}) = u_i(g) - \varepsilon$, $\forall j \in N$ s.t. $\eta_j^{inner} < t - 1$;

2. $u_i(g + g_{ij}) > u_i(g)$, $\forall j \in N$ s.t. $\eta_j^{inner} = t - 1$;

3. $u_i(g + g_{ij}) < u_i(g)$, $\forall j \in N$ s.t. $\eta_j^{inner} \geq t$;

4. $u_i(g - g_{ij}) > u_i(g)$, $\forall j \in N$ s.t. $\eta_j^{inner} \geq t + 1$.

Under this new set of properties we can comprehensively characterize the set of stable networks and make a prediction about the stability of the banking system. Identifying stable networks architectures is not a goal in itself, but rather the goal is to evaluate their resilience to contagion. Contagion stems from banks that have insufficient links. Recall that bank i has insufficient links when $\eta_i^{inner} < t$. Hence, we aim to characterize stable networks depending on the number of banks that have at least t neighbors in the same region ($\eta_i^{inner} = t$). The following two results provide necessary conditions for the existence of stable networks.

Proposition 3.2 *Let g be a pairwise stable network and $T = \{i \in N \mid \eta_i^{inner}(g) = t\}$. Then $|T| \geq 2(n - t)$.*

Proof. The proof is provided in the Appendix. ■

Proposition 3.2 has strong implications for the stability of the banking system, especially when t is small. If t is small, proposition 3.2 shows that in equilibrium most of the banks have sufficient links to prevent a shock in one of the institutions spreading through contagion. For t large, however, the predictions are weaker. We thus need a refinement for large values of t. The following proposition provides such a refinement.

Proposition 3.3 *Let g be a pairwise stable network and $T = \{i \in N \mid \eta_i^{inner}(g) = t\}$. If $t \geq n/2$, then $|T| \geq n$.*

Proof. The proof is provided in the Appendix. ■

Proposition 3.3 states that in a stable network, at least half the banks will have a sufficiently large number of links such that the losses they may generate are small enough.

The contagion threshold, $b(q)$, depends positively on the early liquidation return of the illiquid asset, r. Since t is the smallest integer such that $b(q) < \frac{z}{n}\frac{C_1 - x - ry}{1+(n+t-1)\frac{z}{n}}$, a low value of the contagion threshold requires a large t. Hence, when the return rate for early liquidation of the illiquid asset, r, is low, banks need a large number of connections with banks in the same region, for the LGD to be below the contagion threshold. Similarly, a high opportunity cost of liquidating early the illiquid asset, $\frac{R}{r}$, decreases the contagion threshold and increases the number of links a bank requires not to be a source of contagion.

The following two results relate these findings to implications for the stability of the system.

Corollary 3.1 *Let g be a pairwise stable network and $T = \{i \in N \mid \eta_i^{inner}(g) = t\}$. If $t < n/2$, then the probability that the failure of a bank will spread through contagion is at most $t\pi/n$.*

Proof. The proof follows simply from Proposition 3.2. ■

Corollary 3.2 *Let g be a pairwise stable network and $T = \{i \in N \mid \eta_i^{inner}(g) = t\}$. If $t \geq n/2$, then the probability that the failure of a bank will spread through contagion is at most $\pi/2$.*

Proof. The proof follows simply from Proposition 3.3. ■

The first result implicitly insures that for high levels of the contagion threshold the probability of contagion is significantly low. The intuition for this result relies on the fact that the higher the contagion threshold is, the lower is the number of links that banks need in order to prevent contagion. Proposition 3.2 indicates that a lower connectivity of the banking system is easier to obtain. A low level of the contagion threshold, however, requires a high connectivity in the banking system. Hence, the probability of contagion is higher.

3.5 Efficient networks

For a given contagion threshold, $b(q)$, and an integer t that satisfies 3.5, the failure of a bank with at least $(n + t)$ links does not propagate through contagion, although it affects neighboring banks that incur a loss. Thus, there is a large number of efficient networks a social planner can design in order to prevent contagion in the banking system. The set of efficient networks is characterized in the following proposition.

Proposition 3.4 *Let g be a network such that $\eta_i^{cross}(g) = n$ and $\eta_i^{inner}(g) \geq t$, $\forall$ $i \in N$. Then g is efficient.*

Proof. Since $\eta_i^{inner} \geq t$, $\forall$ $i \in N$ then it follows immediately that $\eta_i \geq n + t$, $\forall$ $i \in N$. Thus, for any pair ij it must be that $LGD_{ij} \leq \frac{z}{n}\frac{C_1 - x - ry}{1+(n+t)\frac{z}{n}}$. As the limit loss $b(q)$ satisfies inequality 3.5, then $LGD_{ij} \leq b(q)$ for any pair of banks ij. Hence, in the network g the failure of a bank will not trigger the failure of other banks in the system. ■

The conflict between efficient outcomes and individual incentives is a classical theme in economics. In this model, however, the incentives are partially aligned. Indeed, the set of stable networks, described by proposition 3.2 and 3.3, includes an efficient network. It is easy to check that a network g such that $\eta_i^{inner}(g) = t, \forall\ i \in N$ is pairwise stable and, by proposition 3.4, is also efficient.[9]

The set of pairwise stable networks, nevertheless, incorporates many non-efficient networks, especially for large values of t. We show that we can restrict the set of equilibria to the efficient one when we use a refinement of the pairwise stability concept, that allows for deviations in which a pair of players each can delete one or more links *and/or* add a link in a coordinated manner. For this purpose, we introduce the notion of *bilateral equilibrium*.

Criterion 3.2 *Let $g_{ij} = \min(s_{ij}, s_{ji})$ and consider that $g_{ij} \in g$ when $g_{ij} = 1$. A network g^* is a bilateral equilibrium if:*

1. *There is a Nash equilibrium strategy profile s^* which yields g^*.*
2. *For any pair of players $i, j \in N$, and every strategy pair (s_i, s_j),*

$$u_i(g(s_i, s_j, s^*_{-i-j})) > u_i(g(s^*_i, s^*_j, s^*_{-i-j})) \Rightarrow u_j(g(s_i, s_j, s^*_{-i-j})) < u_j(g(s^*_i, s^*_j, s^*_{-i-j}))$$

A network g is supported in a 'bilateral equilibrium' if no player or pair of players can deviate and benefit from the deviation (at least one of them strictly). This equilibrium concept allows a pair of players to deviate by forming a link, if the link did not exist before, and, at the same time, by severing other links they are involved in. The terminology of 'bilateral equilibrium' was introduced in Goyal and Vega-Redondo (2007). Note that any bilateral equilibrium network is also pairwise stable.

The new equilibrium concept allows us to relax the assumption that the pattern of interactions between banks in different regions is fixed. Thus far we have considered a network formation game between banks in the same region, under the assumption that each bank in one region is connected with all the other banks in the other region. We can now study the network formation between banks in the same region, as well as, between banks in different regions. However, we look only at cases when the

[9] The existence of such an equilibrium efficient network is conditional on the existance of a t-regular network. Lovasz (1979) discusses in detail conditiond for the existence of a t-regular network with n nodes.

deposits exchanged between banks in different regions satisfy the feasibility constraint 3.2. Moreover, we consider only symmetric solutions, such that the amount a bank i transfers on a link is the same for all links: $a_{ij} = \frac{z}{\eta_i^{cross}}$, $\forall j \in N_i(g)$.

Proposition 3.5 *Let g be a bilateral equilibrium network. Then the probability that the failure of a bank will spread through contagion is at most $\pi/2n$.*

Proof. The proof is provided in the Appendix. ■

The basic intuition for this result relies on the idea that a bank can move from any network to a network where it has exactly one link with another bank in the other region. The concept of pairwise stability allows only one deviation at a time, either as severing a link, or as creating a new link. For a network to be a bilateral equilibrium, however, it needs to be robust to multiple deviations at the same time. Since multiple deviations are allowed, a bank can severe all its links in a network and agree to form a link with one other bank in the other region. This deviation is beneficial in any network where there are at least two banks as possible sources of contagion. In contrast, in a network where a bank has exactly one link with another bank in the other region, there is only one source of contagion, while perfect insurance against liquidity shocks is achieved as well. These deviations are possible for any pattern of connections between banks across regions. Thus the concept of bilateral equilibrium allows us to solve the network formation game between banks in different regions, as well as between banks in the same region.

The bilateral equilibrium concept rules out all the inefficient equilibria, except for one. For instance, a network such that for any j, $\eta_j^{cross} = n$, where a single node i that has an insufficient number of links, $\eta_i^{inner} < t$ and $\eta_{j \in N-\{i\}}^{inner} = t$ can be sustained in equilibrium. In this network, although the probability of contagion is very low, $\pi/2n$, the failure of bank i triggers the failure of the entire system. Thus, with probability $\pi/2n$ there will be $(2n-1)$ banks that will fail due to contagious effects.

In this paper we have discussed a particular set of equilibrium networks. Namely, we have assumed that $\eta_i^{cross} = n$, $\forall\ i \in N$ and we have modeled the link formation process that takes place between banks of the same type. In other words, we have studied the set of equilibrium networks for which each bank is linked to all the banks of the other type. It is important to note, however, that the set of pairwise stable networks is, by no means, restricted the set of networks for which $\eta_i^{cross} = n$, $\forall\ i \in N$. Due to the limitations of the pairwise stability concept, networks where there exist nodes such that the crossing degree is smaller than n can be sustained in equilibrium. The full characterization of the set of pairwise stable networks is possible. It is not, however, of much interest since there is no efficient equilibrium that can emerge when there exist nodes such that the crossing degree is smaller than n, for the given set of parameters.

The bilateral equilibrium concept solves this problem and proposition 3.5 holds without any prior assumption about the crossing degree of banks.

3.6 Conclusions

The problem of contagion within the banking system is a hot issue. Our main contribution to the existing literature is that we endogenized the degree of interdependence that exists between banks. In particular, we developed a model of network formation for the banking system. We investigate how banks form links with each other, when the banking system is exposed to contagion risk. The question we address is wether banks form networks that are resilient to the propagation of small idiosyncratic shocks.

The message this paper transmits is rather optimistic. Banks respond to contagion risk by forming links. The stable network architectures that emerge are very likely to support systemic stability. For instance, when the probability of a shock is π, then the probability that it will spread by contagion is at most $\pi/2n$. For large values of the limit loss, the probability of contagion is virtually 0, if n is sufficiently large.

3.A Appendix

In what it follows we will prove some results advanced in the main text.

Lemma 3.1 *Consider the states of the world $S_1$and S_2, when liquidity shocks are negatively correlated across the two regions A and B. The minimization problem for LGD associated to each link, under the feasibility constrain (3.2), has a symmetric solution when each $i \in A$ is linked to each $i' \in B$, and $a_{ii'} = \frac{z}{n}$.*

Proof. The optimization problem is:

$$\forall\ i \in N, i' \in N_i^{cross},\ \ \min_{a_{ii'}} LGD_{ii'}, \tag{3.7}$$

$$\text{s.t.} \sum_{i' \in N_i^{cross}} a_{ii'} = z \tag{3.8}$$

First we show that $LGD_{ii'}$ is decreasing in $a_{ii'}$. For this it is useful to express LGD as

$$LGD_{ii'} = a_{ii'} \frac{C_1 - x - ry}{1 + a_{ii'} + \sum_{\substack{k \in N_i(g) \\ k \neq i'}} a_{ik}} \tag{3.9}$$

The derivative of $LGD_{ii'}$ with respect to $a_{ii'}$ is given by

$$\frac{\partial LGD_{ii'}}{\partial a_{ii'}} = \frac{(C_1 - x - ry)(1 + \sum_{\substack{k \in N_i(g) \\ k \neq i'}} a_{ik})}{(1 + a_{ii'} + \sum_{\substack{k \in N_i(g) \\ k \neq i'}} a_{ik})^2} > 0 \tag{3.10}$$

A positive sign for the derivative implies that $LGD_{ii'}$ is increasing in $a_{ii'}$.

The only restriction in minimizing LGD_{ij} is the feasibility constraint (3.2). According to the feasibility constraint, any bank i needs to insure that the amount of deposits exchanged with banks of a different type sums up to z.

We impose that the solution is symmetric. That is $a_{ii'} = \frac{z}{\gamma}$. Since there are n banks of a different type and the $LGD_{ii'}$ is increasing in the amount of deposits a_{ij}, the solution to the minimization problem dictates that each bank creates links to all the other banks of a different type. Subsequently, the amount exchanged on each link is $a_{ij} = \frac{z}{n}$. ■

Proposition 3.2 *Let g be a pairwise stable network and $T = \{i \in N \,|\eta_i^{inner}(g) = t\}$. Then $|T| \geq 2(n-t)$.*

Proof. Consider the two regions in the banking system $A = \{1, 2, ..., n\}$ and $B = \{n+1, n=2, ..., 2n\}$. Let $T(A) = \{i \in A \,|\eta_i^{inner}(g) = t\}$ and $T(B) = \{i \in B \,|\eta_i^{inner}(g) = t\}$. Clearly we have $|T| = |T(A)| + |T(B)|$. In order to prove that $|T| \geq 2(n-t)$, we show that $|T(A)| \geq n-t$ and $|T(B)| \geq n-t$. Since the cases are symmetric, we prove only that $|T(A)| \geq n-t$. For this we assume the contrary in order to arrive to a contradiction.

Suppose that $|T(A)| < n-t$. This implies that the set $T(A)$ has at most $n-t-1$ elements. Further, this implies that $|A - T(A)| \geq n - (n-t-1)$. In other words, the set of banks with an inner degree $\eta_i^{inner}(g) < t$ has at least $t+1$ elements. By property 1' and 2 above we know that in a stable network all the banks such that $\eta_i^{inner} \leq t-1$ must be directly linked with each other. Since the set of banks with this property is at least $t+1$, it must be that each bank in $A - T(A)$ has an inner degree $\eta_i^{inner} \geq t$. We thus arrived to a contradiction. ■

Proposition 3.3 *Let g be a pairwise stable network and $T = \{i \in N \,|\eta_i^{inner}(g) = t\}$. If $t \geq n/2$, then $|T| \geq n$.*

Proof. The proof follows similar steps as the proof for the previous result. Adopting the same notations, we prove only that $|T(A)| \geq n/2$.

Let $|T(A)| = \tau$. If $\tau \geq t$, the proof is complete.

Consider the case when $\tau < t$. By property 1' and 2 above we know that in a stable network all the banks in the set $A - T(A)$ must be directly linked with each other. This implies that the total number of links[10] between banks of the same type with an inner degree $\eta_i^{inner} < t$ must be $(n-\tau)(n-\tau-1)$. In addition, since $\tau < t$, it must be that each bank in $T(A)$ has some links with banks in $A - T(A)$. Assuming that all banks in $T(A)$ are directly linked with each other, there must be at least $\tau(t-\tau+1)$ links with banks in $A - T(A)$.

Since all the banks in $A - T(A)$ have an inner degree $\eta_i^{inner} < t$, the total amount

[10] Links here are counted twice for each node. However, we maintain the same double counting for the rest of the proof, such that in the end it cancels out.

of links these banks have should not exceed $(n-\tau)t$. Thus, the following inequality must hold:

$$(n-\tau)(n-\tau-1)+\tau(t-\tau+1)<(n-\tau)t$$

This inequality can be rewritten as

$$(t-\tau+1)(2\tau-n)<(n-\tau)(2\tau-n)$$

Since $t-\tau+1<n-\tau$, it must be that $2\tau-n>0 \Leftrightarrow \tau>n/2$. This concludes the proof. ■

Proposition 3.4 *Let g be a bilateral equilibrium network. Then the probability that the failure of a bank will spread through contagion is at most $\pi/2n$.*

Proof. We show that in a bilateral equilibrium networks there exists at most one node i such that $\eta_i^{inner}<t$.

Suppose that in an equilibrium network there exist at least two nodes i and j such that $\eta_i^{inner}<t$, $\eta_j^{inner}<t$. In a network that there are at least two nodes with an insufficient number of links, there are two sources of contagious failure. Thus the probability a bank associates to failing by contagion is at least $2\pi/2n$.

Let $\tilde{g}$ be such a network. Then there exist a pair ij of nodes of a different type (i.e. $i\in A$ and $j\in B$) such that it pays off to severe the links they are involved in and form the link $\tilde{g}_{ij}$, if $\tilde{g}_{ij}\notin\tilde{g}$. Formally, let $\tilde{s}_i$ and $\tilde{s}_j$ be the strategy profile bank i and bank j follow, respectively, in network $\tilde{g}$.And let $s_i^*=(\underbrace{0,0,...,0}_{1,...,j-1},1,\underbrace{0,0,...,0}_{j+1,...,n})$ and $s_j^*=(\underbrace{0,0,...,0}_{1,...,i-1},1,\underbrace{0,0,...,0}_{i+1,...,n})$. Then

$$u_i(\tilde{g}(s_i^*,s_j^*,\tilde{s}_{-i-j}))>u_i(g(\tilde{s}_i,\tilde{s}_j,\tilde{s}_{-i-j})) \text{ and } u_j(\tilde{g}(s_i^*,s_j^*,\tilde{s}_{-i-j}))>u_j(\tilde{g}(\tilde{s}_i,\tilde{s}_j,\tilde{s}_{-i-j}))$$

In the new network, the only link i and j have is $\tilde{g}_{ij}$ and thus they are exposed to contagion stemming from only one source. If $\tilde{g}_{ij}$ is the only link banks i and j have, this link will bear the entire amount of deposits necessary to provide insurance against liquidity shocks z. Thus, if one of the banks fails, the other one fails by necessity, since the loss it incurs is above the limit loss threshold. However, the probability that one of the two banks will fail is $\pi/2n$ and smaller than in the network $\tilde{g}$. Hence, $\tilde{g}$ cannot be an equilibrium.

Since, in a bilateral equilibrium there exists at most one node i such that $\eta_i^{inner} < t$, it follows that the probability that the failure of a bank will spread through contagion is at most $\pi/2n$. ■

Chapter 4

Contagion Risk in Financial Networks

A notable feature of the modern financial world is its high degree of interdependence. The mutual exposures that financial institutions adopt towards each other connect the banking system in a network. Despite the obvious benefits, such linkages come at a cost. That is, shocks, which initially affect only a few institutions, can propagate through the entire system. Since these linkages carry the risk of contagion, an interesting question is whether the degree of interdependence in the banking system sustains systemic stability. This paper addresses this issue. In particular, we investigate how the network structure affects the cross institutional holdings and investigate the implications for contagion risk.

We study a setting where the banking system is exposed to both liquidity and idiosyncratic shocks. The connections between banks facilitate the transfer of liquidity from the ones that have a cash surplus to those with a cash deficit. Banks can insure against the liquidity shocks by exchanging deposits through links in the network. The same connections, however, make the banking system prone to contagion. The risk of contagion is increasing in the size of interbank deposits. We first investigate the size of interbank deposits transferred between banks that provides full insurance against liquidity shocks, while keeping the network structure fixed. Then we asses how the banking network responds to contagion.

We show that incomplete network structures create uncertainty about the distribution of liquidity shocks. As a result, the level of interbank deposits that insures against liquidity shocks increases, at the same time, the contagion risk. In other words, to achieve perfect insurance against liquidity shocks, banks need to accept a higher risk of contagion. The problem is solved when the network is complete, as the level of deposits that perfectly re-distribute the liquidity in the banking system, also minimizes the risk of contagion.

The paper that is closest related to our work is by Allen and Gale (2000). In particular, our paper uses the same framework as Allen and Gale (2000) to motivate interactions on the interbank market. However, Allen and Gale (2000) study the banking system when there exist correlations between the shocks in the liquidity demand that affect different regions. We extend their analyses and look at the banking system without building in any correlations between liquidity shocks. Thus, we introduce uncertainty about which regions have negatively correlated shocks. In addition, we incorporate one very important feature of real world banking systems. That is, relations between banks, in general, and deposit contracts, in particular are private information. Our setting captures this aspect and allows a link that exists between two banks not to be observed by the other banks in the system. Thus, we analyze network effects on systemic risk if these two sources of uncertainty are present.

Allen and Gale (2000) find that the losses caused by contagion in an incomplete network are larger than in a complete network. We reinforce their result by showing that incomplete networks have an additional effect. When the network is incomplete, the allocation of interbank deposits that provides insurance against liquidity shocks is unlikely at the level that minimizes contagion risk. This is no longer the case when the network becomes complete. In a complete network the degree of interdependence between banks is such that contagion risk is minimal.

The model is based on a framework introduced by Diamond and Dybvig (1983). There are three periods $t = 0, 1, 2$ and a large number of identical consumers, each endowed with one unit of a consumption good. Ex-ante, consumers are uncertain about their liquidity preferences. They might be early consumers, who value consumption at date 1, or late consumers, who value consumption at date 2. The consumers find it optimal to deposit their endowment in banks that invest on their behalf. In return, consumers are offered a fixed amount of consumption at each subsequent date, depending when they choose to withdraw. Banks can invest in two assets: there is a a liquid asset that pays a return of 1 after one period and there is an illiquid asset that pays a return of $r < 1$ after one period or $R > 1$ after two periods. In addition, liquidity shocks hit the economy randomly in the following way. Although there is no uncertainty about the average fraction of early consumers, the liquidity demand is unevenly distributed among banks in the first period. Thus, each bank experiences either a high or a low fraction of early consumers. To ensure against these regional liquidity shocks, banks exchange deposits on the interbank market in period 0.

Deposits exchanged this way constitute the links that connect the banks in a network. This view of the banking system as a network is useful in analyzing the effects that the failure of a bank may produce. If such an event occurs, the risk of contagion is evaluated in terms of the loss in value for the deposits exchanged at date 0. It becomes apparent that contagion risk depends on the size of these deposits. When the probability of a bank failure is small, the size of the interbank deposits needs to meet two criteria. First, interbank deposits should be large enough to insure perfectly against any distribution of liquidity shocks realized at date 1. And second, interbank deposits should not be larger than needed for insurance against liquidity shocks. In other words, they need to minimize the risk of contagion by diversification.

The paper is organized as follows. Section 2 introduces the main assumptions about consumers and banks and describes the interbank market as a network. We discuss the linkages between banks and how contagion may arise in section 3. In Section 4 we show how banks set the interbank deposits and investigate if they are at the level that minimizes contagion risk for different degrees of network connectedness. Section 5 considers possible extensions and ends with some concluding remarks.

4.1 The Model

4.1.1 Consumers and Liquidity Shocks

We assume that the economy is divided into 6 regions, each populated by a continuum of risk averse consumers (the reason for 6 will become clear in due course). There are three time periods $t = 0, 1, 2$. Each agent has an endowment equal to one unit of consumption good at date $t = 0$. Agents are uncertain about their liquidity preferences: they are either early consumers, who value consumption only at date 1, or they are late consumers, who value consumption only at date 2. In the aggregate there is no uncertainty about the liquidity demand in period 1. Each region, however, experiences different liquidity shocks, caused by random fluctuations in the fraction of early consumers. In other words, each region will face either a high proportion p_H of agents that need to consume at date 1 or a low proportion p_L of agents that value consumption in period 1. There are $\binom{6}{3}$ equally likely states of nature that distribute the high liquidity shocks to exactly three regions and the low liquidity shocks to the other three. Note that this set of states of the world does not build in any correlations between the liquidity shocks that affect any two regions.

To sum up, it is known with certainty that on average the fraction of early consumers in the economy is $q = (p_H + p_L)/2$. Nevertheless, the liquidity demand is not uniformly distributed among regions. All the uncertainty is resolved at date 1, when the state of the world is realized and commonly known. At date 2, the fraction of late consumers in each region will be $(1 - p)$ where the value of p is known at date 1 as either p_H or p_L.

4.1.2 Banks, Demand Deposits and Asset Investments

We consider that in each region i there is a competitive representative bank. Agents deposit their endowment in the regional bank. In exchange, they receive a deposit contract that guarantees them an amount of consumption depending on the date they choose to withdraw their deposits. In particular, the deposit contract specifies that if they withdraw at date 1, they receive $C_1 > 1$, and if they withdraw at date 2, they receive $C_2 > C_1$.

There are two possibilities to invest. First, banks can invest in a liquid asset with a return of 1 after one period. They can also choose an illiquid asset that pays a return of $r < 1$ after one period, or $R > 1$ after two periods. Let x and y be the per capita amounts invested in the liquid and illiquid asset, respectively. Banks will use the liquid asset to pay depositors that need to withdraw in the first period and will reserve the illiquid asset to pay the late consumers. Since the average level of liquidity demand at date 1 is qC_1, we assume that the investment in the liquid asset, x, will equal this amount, while the investment in the illiquid asset, y, will cover $(1-q)C_2/R$.[1]. This macro allocation will be relaxed later.

Banks are subject to idiosyncratic shocks that are not insurable. That means that, with a small probability π, the failure of a bank will occur in either period 1 or 2. This event, although anticipated, will have only a secondary effect on banks' actions for reasons that will become clear in section 4.

4.1.3 Interbank Market

Uncertainty in their depositors' preferences motivates banks to interact in order to ensure against the liquidity shocks that affect the economy. These interactions create balance sheet linkages between banks, as described below.

At date 1 each bank has with probability half either a liquidity shortage of $(p_H - q)C_1$ or a liquidity surplus of $(q - p_L)C_1$. We denote by z the deviation from the mean of the fraction of early consumers, which in turn makes the liquidity surplus or shortage of a banks equal to zC_1.[2] As in the aggregate, the liquidity demand matches the liquidity supply, all the regional imbalances can be solved by the transfer of funds from banks with a cash surplus to banks with a cash deficit. Anticipating this outcome, banks will agree to hedge the regional liquidity shocks by exchanging deposits at date 0. This way, a contract is made between two banks that gives the right to both parts to withdraw their deposit, fully or only in part, at any of the subsequent dates. For the amounts

[1] This allocation maximizes the expected utility of consumers, see Allen and Gale (2000).

[2] Since $q = \frac{p_H + p_L}{2}$, than it must be that $(p_H - q)C_1 = (q - p_L)C_1$.

exchanged as deposits, each bank receives the same return as consumers: C_1, if they withdraw after one period, and C_2 if they withdraw after two periods.

Banks' portfolios consist now of three assets: the liquid asset, the illiquid asset and the interbank deposits. Each of these three assets can be liquidated in any of the last 2 periods. However, the costliest in terms of early liquidation is the illiquid asset. This implies the following ordering of returns:

$$1 < \frac{C_2}{C_1} < \frac{R}{r} \tag{4.1}$$

An important feature of the model is that the swap of deposits occurs ex-ante, before the state of the world is realized. Note, however, that this prevents lenders to gain any monopoly power. For instance, in an ex-post market for deposits, lenders might take advantage of their position as liquidity providers to extract money from banks with a shortage of liquidity. To avoid this unfavorable situation, banks prefer to close firm contracts that set the price of liquidity ex-ante.

An interbank market, as introduced above may be very well described as a network. The network can be characterized by the pattern of interactions between banks, as well as by the amount of interbank deposits that represent the links. In this paper, we investigate how the size of interbank deposits, depends on the network structure. In particular, we are interested in the effects complete and incomplete networks have on banks' decisions when setting the level of interbank deposits. In order to illustrate the effects of incomplete structures, we restrict our analysis to regular networks (we introduce definitions below). Thus, each bank in the network is a node and each node is connected to exactly $n < 6$ other nodes. This means that each bank may, but need not, exchange deposits with other n banks. Note that we do not model explicitly how these connections are formed. Since the contracts are bilateral, and thus the amounts exchanged between any two banks are the same, the network is *undirected.* Next, we introduce a some important definitions.

A *network* g is, formally, a collection of ij pairs, with the interpretation that nodes i and j are linked. A network is *regular of degree* n (or n*-regular*) if any node in the network is directly connected with other n nodes. The *complete network* is the graph

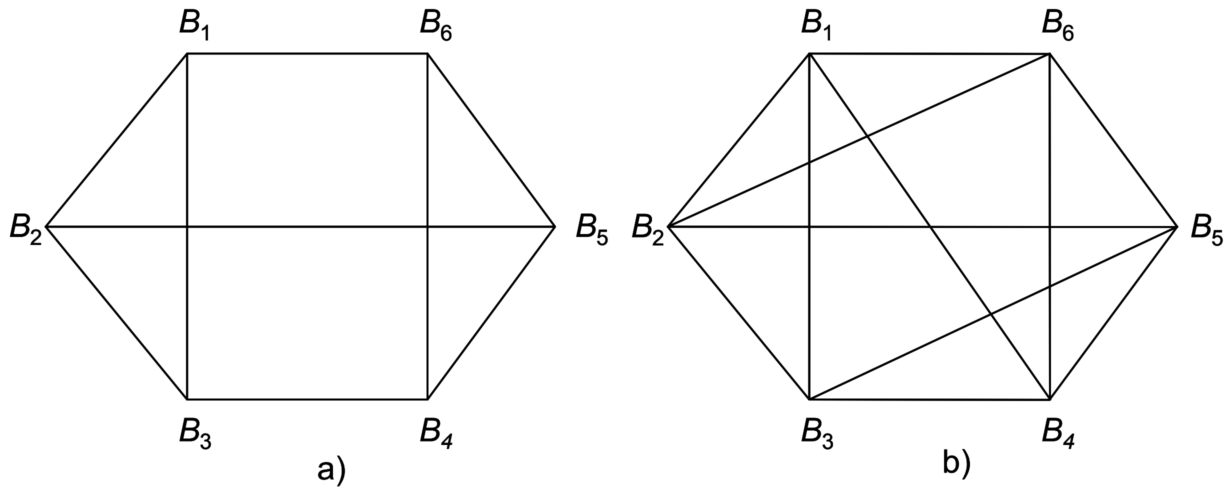


Figure 4.1: n-regular networks: a) $n = 3$; b) $n = 4$

in which all nodes are linked to one another. Any two nodes connected by a link are called *neighbors*.

We now discuss the incomplete information structure. We incorporate in our framework an important feature of real world banking systems. Namely, banks have incomplete information over the network structure. Although it is common knowledge that the network is n-regular, banks do not know the entire network architecture. Thus, they do not observe the linkages in the network, beyond their own connections. For instance, B_1 in figure 4.1 knows his set of neighbors: B_2, B_3 and B_6. Nevertheless, it cannot observe how they are connected neither between themselves, nor to the other banks in the system.

For the purposes of our analyses we consider different values of n. However, since modern banking systems are highly connected, we reasonably assume that $n \geq 3$.[3] In other words, each bank is connected to at least half of the other banks in the system. At the same time the markets are not always complete structures. In a possible interpretation, in a single country interbank market all the banks are connected to all the other banks. The connections outside the home country are nevertheless rather scarce.

[3]The cases $n = 1$ and $n = 2$ will be briefly discussed later in the paper.

4.2 Contagion Risk

4.2.1 Balance Sheet Linkages

The main goal of our paper is to study which degree of interdependence arises between banks and the implications for the fragility of the banking system. The interdependence stems from two sources. First, there is a system-wide dependence that is reflected in the size of z, the liquidity shortage or surplus of any bank. The larger z, the higher is the degree of interdependence. Second, there is pairwise dependence that is given by the size of deposits exchanged between any two banks. Since we assume z to be fixed, for the moment, we focus on explaining pairwise dependence and its potential contagious consequences.

An *allocation rule for deposits* is a mapping from the set of links to the real numbers $a : g \to \mathbb{R}$ that specifies the amount exchanged as deposits between banks i and j at date 0. For simplicity we use the following notation $a(ij) = a_{ij}$. Since deposit contracts are bilateral, we have $a_{ij} = a_{ji}$.

An allocation rule is *feasible* if in period 1 deposits can be withdrawn such that there will be no bank with a liquidity surplus nor a liquidity shortage. Formally, let d_{ij} represent the amount transferred from i to j in period 1, for any pair ij, and N_i be the set of neighbors of bank i, for any i. Than, an allocation rule is feasible if, for any bank i and for any neighbor j of i, there exist d_{ij} and d_{ji} such that $\left|\sum_{j\in N_i} d_{ji} - \sum_{j\in N_i} d_{ij}\right| C_1 = zC_1$ and $0 \leq d_{ij}, d_{ji} \leq a_{ij}$.[4]

Lemma 4.1 *For a n-regular network with $n \geq 3$ there always exists a feasible allocation rule.*

Proof. This holds true as in a n-regular network, when $n \geq 3$, there is always a path between every pair of nodes. A path is a sequence of consecutive links in a network. Moreover, it can be shown that the length of this path it is at most 2. A general proof follows in the appendix. ■

[4]Note that $d_{ij} \neq d_{ji}$ in period 1. That is because when the state of the world is realized in period 1, liquidity will flow from banks that have in excess to banks that have a deficit. Hence, the network becomes directed in period 1.

The proof of Lemma 4.1 shows in fact that there exists a feasible allocation for any connected network. A regular network with a degree larger than half the number of nodes is a particular case of connected network.

Corollary 4.1 *A feasible allocation ensures that there will be no bank with a liquidity surplus nor a liquidity shortage in period* 2 *as well.*

In period 2 each bank will have a fraction of $(1-p)$ late consumers where p has been realized for each region in period 1. Thus, the transfer of deposits between any banks i and j will simply be reversed.

4.2.2 Losses Given Default

In order to evaluate contagion risk we need to introduce a measure that quantifies it. For this purpose, we apply the same procedure as the empirical literature on contagion: We consider the event of a bank failure and analyze its implications for the banking system. In our model, the failure of a bank will occur in either period 1 or 2 with a small probability π. The risk of contagion is than evaluated in terms of *loss given default* (henceforth LGD). LGD expresses the excess of nominal liabilities over the value of the assets of the failed bank. In our setting, LGD will be given by the loss of value a bank incurs on its deposits when one of its neighbor banks is liquidated.

This measure focuses only on the loss associated to a direct link between two banks. It ignores any aspects related to the indirect effects the failure of a bank might have on the system. For instance, it does not capture the problems that arise when a bank that is a liquidity supplier fails.

Another aspect worth mentioning is that the failure of a bank might have contagious effects only if this event is realized in period 1. Once each bank reaches period 2, straightforward calculations show that the value of its assets is sufficiently large to cover all its liabilities. Hence, there is no loss in value for deposits, and LGD will be 0.

To calculate LGD we need to determine the value of the assets of the failed bank. If a bank fails, its portfolio of assets is liquidated at the current value and distributed equally among creditors. Now, recall that a bank portfolio consists of three assets.

First, banks hold an amount of x per capita invested in a liquid asset that pays a return of 1. Second, banks have invested an amount y per capita in an illiquid asset that pays a return of $r < 1$ if liquidated in the first period. And lastly, there are interbank deposits summing up to $\sum_{k \in N_i} a_{ik}$ that pay a return of C_1 per unit of deposit. On the liability side, a bank will have to pay its depositors, normalized to 1, and at the same time to repay its interbank creditors that add up to $\sum_{k \in N_i} a_{ik}$. This yields a new return per unit of good deposited in a bank i equal to $\bar{C}_i = \frac{x+ry+\sum_{k \in N_i} a_{ik} C_1}{1+\sum_{k \in N_i} a_{ik}} < C_1$.[5] The LGD of bank j given that bank i has failed is easy now to express as[6]:

$$LGD_{ji} = a_{ji}(C_1 - \bar{C}_i) = a_{ji} \frac{C_1 - x - ry}{1 + \sum_{k \in N_i} a_{ik}} \tag{4.2}$$

4.3 Deposits Allocation and their Optimality

4.3.1 Network Structures and Uncertainty

To understand how the allocation of deposits should be set in period 0, we need to characterize the sources of uncertainty that dominate the environment in the banking system.

In an incomplete network, there are two sources of uncertainty. On the one hand, there is no prior information about the distribution of liquidity shocks. That is, any of the $\binom{6}{3}$ states of the world that allows a high liquidity demand in any 3 regions and a low liquidity demand in the remaining 3 is equally likely. This further implies that there is no ex-ante correlation between the fractions of early consumers in any two regions. The lack of correlations between liquidity shocks is converted, for any bank i, into uncertainty. First, there is uncertainty about how many neighbors from N_i will be affected by a different liquidity shock than i at date 1. And second, there is uncertainty about who these neighbors are. Note that the first type of uncertainty depends on the network degree of completeness n and disappears when the network is complete. That is because the condition $n \geq 3$ guarantees that each bank has at least $n - 2$ neighbors

[5] Eq. (4.1) ensures that the inequality holds.

[6] In principle $LGD_{ji} \neq LGD_{ij}$ since it may be that $\sum_{k \in N_i} a_{ik} \neq \sum_{k \in N_k} a_{jk}$

that will face a different liquidity demand in period 1.

Example 4.1 *Suppose that the network degree is* $n = 3$*. Than a bank might have, as seen from period* 0*, one, two or three neighbors that may experience a different fraction of early consumers than itself in period* 1*.*

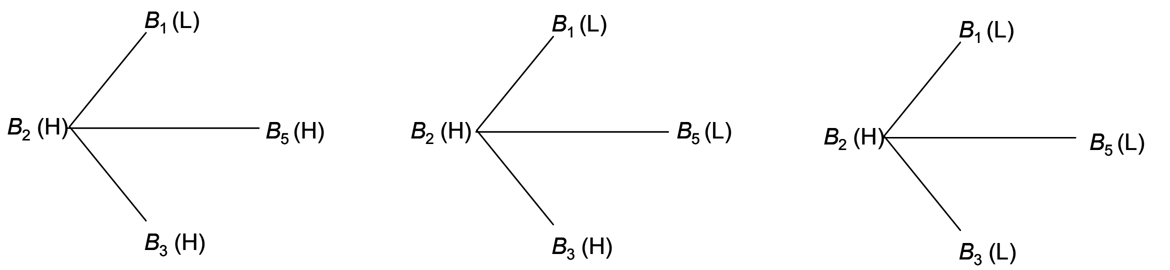


Figure 4.2 (a): Uncertainty about the number of neighbours of a different type

Moreover, any of the banks in the neighbors set of a bank i*, is equally likely to experience a different liquidity shock than* i*.*

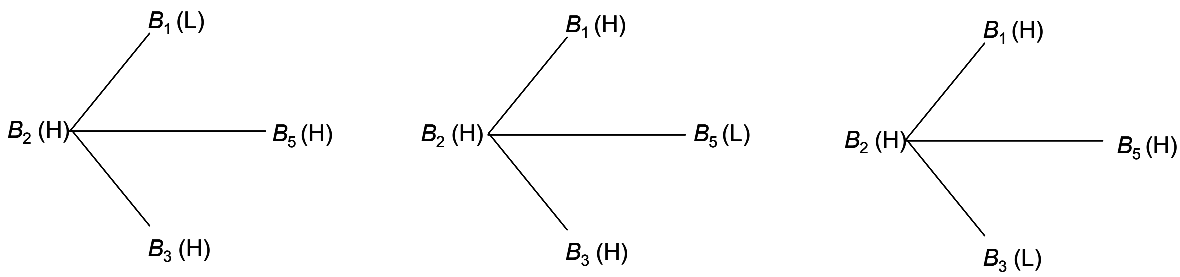


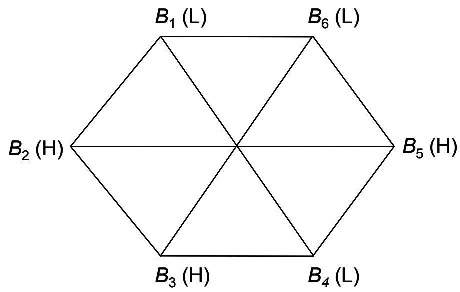
Figure 4.2 (b): Uncertainty about which neighbours are of a different type

On the other hand, any link that connects two banks is private information for the respective institutions. Even though it is common knowledge that each bank i has n links, which nodes are at the end of these links is only known by i.[7] This sort of incomplete information generates uncertainty about the minimum number of links that will connect banks of a different type. Banks are said to be of a different type if they will experience different liquidity shocks in period 1. In particular, a bank is of type

[7]This motivates our choice of 6 banks. In the 4 - bank setting proposed by Allen and Gale (2000) if n is common knowledge, each bank can make inferences and accurately guess the network structure.

H if it will face a high liquidity demand and a bank is of type L if it will face a low liquidity demand.

Example 4.2 *Suppose that $n = 3$ and the network g is represented as.*



For this structure, in period 1, there will be at most two banks each having exactly one neighbor that experiences a different fraction of early consumers, regardless of the states of the world realized. Hence, for any state of the world realized there will be at least 5 links that connect the H nodes and the L nodes. From the perspective of any bank i, however, it seems possible that each bank has exactly one neighbor of a different type, and thus the minimum number of links connecting nodes of a different type is 3.

In the case of a complete network, banks' environment simplifies considerably since most of the uncertainty is resolved. When the network is complete each bank will have with certainty 3 neighbors of a different type than itself. Moreover, every node is linked to every other node and thus there will be exactly 9 links connecting the H nodes and the L nodes, for any state of the world that is realized. The only uncertainty that banks have to consider concerns which of their neighbors will be of a different type.

4.3.2 Deposit allocations

Liquidity imbalances that occur in period 1 can be solved by the transfer of funds from banks of type L to banks of type H. When the probability of a bank failure is small, the allocation rule for deposits needs to meet two criteria. First, interbank deposits should be large enough to insure perfectly against any distribution of liquidity shocks realized at date 1. That is, after the transfer of funds takes place, each bank's cash holdings will exactly match the liquidity demand. And second, given that the liquidity shocks are hedged, the risk of a bank failure needs to be considered. Thus, the level of interbank deposits needs to be low enough to minimize the risk of contagion.

In order to meet the first criterion, the interbank system is considered to be at date 1 in the state when each bank has exactly $n-2$ nodes of a different type. Note that uncertainty about the state of the world allows *one bank* to have exactly $n-2$ neighbors of a different type, while uncertainty about the network structure allows *all the banks*, to have each exactly $n-2$ neighbors of different type. Thus, the allocation of deposits that ensure the transfer of liquidity from L nodes to H nodes, for any state of the world is the allocation that permits the transfer when each bank has exactly $n-2$ neighbors of a different type. To satisfy the second criterion, banks need to divide z, the amount they will borrow (lend), among the $n-2$ neighbors of a different type. Moreover, each bank may have any of their neighbors of a different type than itself.

To summarize, the allocation of deposits should minimize the *loss given default* associated with each link, for the *worst case scenario*.[8] We consider the *worst case scenario* to be the state of the world for which each bank has exactly $n-2$ nodes of a different type. Since for any pair ij, LGD_{ij} is decreasing in a_{ij}, the minimization problem yields an ex-ante optimal allocation of deposits exchanged at date 0 between any two banks of $\frac{z}{n-2}$.

Proposition 4.1 *Let g be a n-regular network of banks, with $n \geq 3$. The allocation rule for deposits that sets $a_{ij} = \frac{z}{n-2}$, for any pair of banks $ij \in g$, is feasible.*

Proof. The proof is provided in the appendix. ■

4.3.3 Optimality

In this section we examine whether the minimal feasible allocation rule for deposits is optimal for the risk of contagion. Moreover, we discuss when the ex-ante optimal allocation is also ex-post optimal. In other words, we are interested in how the network

[8]These loss averse actions are entirely consistent with the usual behavior of banks. The use of VaR measure in practice is a sufficient evidence to support the assumption of loss aversion.

structure affects the trade-off between perfect insurance against liquidity shocks and contagion risk.

Given that banks choose an allocation rule for deposits that sets $a_{ij} = \frac{z}{n-2}$, the loss of any bank i given the default of any neighbor j of i is given by

$$LGD^*_{ij} = \left(\frac{z}{n-2}\right)\frac{C_1 - x - ry}{1 + (nz)/(n-2)} = z\frac{C_1 - x - ry}{n - 2 + nz}$$

The following proposition relates the optimality of LGD^* to the degree of network completeness.

Proposition 4.2 *Let g be an incomplete n-regular network (i.e. $n = 3, 4$) and consider any realization of the liquidity shocks that allows at least one bank to have at least $(n-1)$ neighbors of a different type. Than there exists a feasible allocation of deposits a_{ij} such that $a_{ij}\frac{C_1-x-ry}{1+\sum_{k\in N_j} a_{jk}} < LGD^*_{ij}$, for any pair $ij \in g$.*

Proof. The proof is provided in the appendix. ■

Proposition 4.2 tells us that the allocation of deposits is sub-optimal ex-post, for any realization of the state of the world that is *not* the worst case scenario. In other words, when the network is incomplete, the degree of interdependence between banks does not insure that the corresponding losses are minimal.

Corollary 4.2 *For $n = 3$, the allocations of deposits $a_{ij} = \frac{3z}{5}$, for any pair $ij \in g$, satisfies proposition 4.2. For $n = 4$, the allocations of deposits that satisfies proposition 4.2 is $a_{ij} = \frac{3z}{8}$.*

Proposition 4.2 discusses the case for $n = 3, 4$ and the next corollary treats the case of complete networks. We briefly explain what happens for $n = 1, 2$. A network degree larger than 3 insures that the network is connected. For $n < 3$, however, the network structure could be characterized by "islands"[9]. Moreover, the liquidity demand and the liquidity supply in the separate islands might be mismatched. This would create uncertainty about the aggregate fraction of early consumers as well. Anticipating this outcome, it may be optimal for banks not to exchange deposits in the first place.

[9] For $n = 2$ the network could be structured in two 2-regular components. For $n = 1$ there is no connected network structure.

Corollary 4.3 *Let $\tilde{g}$ be the complete network. Than, there is no feasible allocation of deposits a_{ij} such that $a_{ij}\frac{C_1-x-ry}{1+\sum_{k\in N_j} a_{jk}} < LGD^*_{ij}$, for all pairs $ij \in \tilde{g}$.*

Proof. The proof is provided in the appendix. ■

To clarify, there is no allocation of deposits that reduces the loss of one bank without increasing the loss of another bank. The intuition behind corollary 4.3 relies on the fact that in a complete network the worst case scenario is realized for any distribution of the liquidity shocks.

This result is particularly important since it states that the complete network is the only network where the ex-ante optimal degree of interdependence is also ex-post optimal.

4.3.4 Varying asset portfolio

We have discussed above what implications the interbank linkages have for contagion risk, under the assumption that banks' portfolio is fixed. We have considered that the amount invested in the liquid asset, x, will be qC_1, while the amount invested in the illiquid asset, y, will cover $(1-q)C_2/R$. In other words, up to now, we have constrained banks to create linkages on the interbank market in order to insure against the liquidity shocks that will hit the economy in period 1. Moreover, by fixing the cash holdings of banks at date 1, we have imposed the dependency of each bank on the banking system to z.

Our assumption was reasonable. In fact, Allen and Gale (2000) show that the distribution (x, y) of the initial wealth in the liquid and illiquid asset is such that the expected utility of consumers is maximized. Any deviation from this distribution generates welfare losses for consumers. Nevertheless, it may be the case that, anticipating the failure of a bank and the consequent contagious losses, banks might decide on a different portfolio distribution. For instance, the higher the probability of a bank failure, the more banks prefer to hold cash reserves larger than qC_1. A larger investment in the liquid asset reduces the amount banks need to borrow from the interbank market. Thus, banks might favor a lower degree of dependency, even though it means that they need to forgo for this consumers' welfare.

Indeed, let the new portfolio distribution to be $(\bar{x}, \bar{y})$, where $\bar{x} > x$ and $\bar{y} < y$, such that $\bar{x} + \bar{y} = 1$. This further implies that $\bar{x} = \bar{q}C_1$, with $\bar{q} \in (q, p_H]$, and the amount banks need to transfer on the interbank market will be $\bar{z}C_1 = (p_H - \bar{q})C_1$. Note that $\bar{z} > 0$ provided that $\bar{q} < p_H$. Hence, as long as banks hold a positive amount of interbank deposits, the degree of interdependence in an incomplete network will be sub-optimal.

4.4 Concluding Remarks

The problem of contagion within the banking system is a hot issue. This chapter studies whether the degree of interdependence that exists between banks supports systemic stability. In particular, we investigate how the network structure affects the trade-off between perfect insurance against liquidity shocks and contagion risk. We know from Allen and Gale (2000) that in an incomplete network the losses caused by contagion are larger than in a complete network. In addition, we show that an incomplete network generates uncertainty and this makes the ex-ante optimal allocation of deposits to be sub-optimal ex-post. It is, indeed, usually the case that in an incomplete information setting the ex-ante optimal outcomes are not also ex-post optimal. The point our paper raises is that it is exactly in an incomplete network where this setting of incomplete information is created. We show that in a complete network the uncertainty is resolved, and we conclude that a complete network favors an optimal degree of interdependence.

To end, we discuss the robustness of our results and draw a parallel to the empirical research on contagion. Our model extends naturally to more than 6 regions. Recall that what drives the results is that the allocation of deposits ensures the banking system perfectly against liquidity shocks, for any realization of the states of the world. More precisely, the allocation rule for deposits is such that losses due to contagion are minimal in the worst case scenario. When the network is incomplete, the allocation of deposits turns out to be sub-optimal for any realization of the state of the world that is not the worst case scenario. In a complete network, however, the allocation of deposits always minimizes contagion risk since any state of the world implies the worst case scenario. This feature of complete networks versus incomplete network is independent of the actual number of nodes (regions).

The message this paper transmits is rather optimistic. When the network is complete, banks have the right incentives to choose the degree of interdependence for which the contagion risk is minimal. In short, in a complete network the contagion risk is very low. This result can be interpreted in the light of the empirical research on contagion, which consistently finds that the banking system demonstrates a high resilience to shocks. Recall that we use the same tool as the empirical papers to assess contagion risk. At the same time, the analyses in these papers are usually limited to a single country interbank market, where the network is likely to be complete. Our model can thus account as an explanation to support the empirical evidence.

4.A Appendix

In order to prove Proposition 4.1 and 4.2, respectively, we need to introduce further notations.

Let Ω be the set of all possible state of the worlds[10] and denote with ω an element of this set. Let $\mathbf{H}^{\omega}$ denote the set of banks of type H and $\mathbf{L}^{\omega}$ the set of banks of type L in the state of the world ω.

Let s_i^{cr} denote the number of neighbors of bank i that are of a different type than i and s_i^{in} denote the number of neighbors of bank i that are of the same type as i. For the remainder of the paper, we call s_i^{cr} the crossing degree of bank i and s_i^{in} the inner degree of bank i. If the network degree is n, than for every bank i we have $s_i^{cr} + s_i^{in} = n$. Moreover, since $s_i^{cr} \geq n-2$, the following condition holds $n-3 \leq s_i^{in} \leq 2$.

This notation is useful to understand that any state of the world can be expressed in terms of inner and crossing degree. We distinguish the following cases, independent of the network structure.

Case 1 $n = 3$.

For $n = 3$, any state of the world ω will be converted to one of the following 4 situations[11]*:*

1. *For any bank $i \in \mathbf{H}^{\omega}$, $s_i^{in} = 2$.*

2. *There exists exactly one bank $i \in \mathbf{H}^{\omega}$ such that $s_i^{in} = 2$ and for any bank $j \in \mathbf{H}^{\omega} - \{i\}$ we have $s_j^{in} = 1$.*

3. *There exists exactly one bank $i \in \mathbf{H}^{\omega}$ such that $s_i^{in} = 0$ and for any bank $j \in \mathbf{H}^{\omega} - \{i\}$ we have $s_j^{in} = 1$.*

4. *For any bank $i \in \mathbf{H}^{\omega}$, $s_i^{in} = 0$.*

   Any other possibility is excluded. For instance, consider a situation that allows two banks i and j to have a inner degree $s_i^{in} = s_j^{in} = 2$. Suppose that the link ij

[10] We established that $\mathbf{card}(\Omega) = \binom{6}{3}$, where $\mathbf{card}(\cdot)$ represents the cardinality of a set.

[11] We discuss only the case of banks of type H. Due to symmetry, the case of banks of type L is analogous.

is created, than each bank needs one more link with a bank of the same type. This implies that the third bank k must have $s_k^{in} = 2$, which falls under situation 1.

Case 2 $n = 4$.

For $n = 4$, any state of the world ω will be converted to one of the following 2 situations:

1. *For any bank $i \in \mathbf{H}^\omega$, $s_i^{in} = 2$.*

2. *There exists exactly one bank $i \in \mathbf{H}^\omega$ such that $s_i^{in} = 2$ and for any bank $j \in \mathbf{H}^\omega - \{i\}$ we have $s_j^{in} = 1$.*

   A similar reasoning as above applies to exclude any other situation.

Case 3 $n = 5$.

When the network is complete, any state of the world ω will be converted to the following situation. For any bank $i \in \mathbf{H}^\omega$, $s_i^{in} = 2$.

It is easy to check that any other situation violates the regularity of the network.

Lemma 4.2 *Let ω be the realized state of the world. Than for any bank $i \in \mathbf{H}^\omega$ with a inner degree s_i^{in} and a crossing degree s_i^{cr} there exists a bank $k \in \mathbf{L}^\omega$ such that $s_k^{in} = s_i^{in}$ and $s_k^{cr} = s_i^{cr}$.*

Proof. The proof is based on the fact that $\sum_{i \in \mathbf{H}} s_i^{cr} = \sum_{j \in \mathbf{L}} s_j^{cr}$. Consequently, $\sum_{i \in \mathbf{H}} s_i^{in} = \sum_{j \in \mathbf{L}} s_j^{in}$. This implies that when the banks in $\mathbf{H}^\omega$ are in one of the situation described above, than it is necessary that the banks in $\mathbf{L}^\omega$ are in exactly the same situation. ■

We shall now continue with the proof of proposition 1 and 2, respectively.

Proposition 4.1 *Let g be a n-regular network of banks, with $n \geq 3$. The allocation rule for deposits that sets $a_{ij} = \frac{z}{n-2}$, for any pair of banks $ij \in g$, is feasible.*

Proof. In order to prove that a_{ij} is feasible we need to show that for any bank i and for any neighbor j of i there exist d_{ij} and d_{ji} such that $\left|\sum_{j \in N_i} d_{ji} - \sum_{j \in N_i} d_{ij}\right| C_1 = zC_1$ and $0 \leq d_{ij}, d_{ji} \leq a_{ij}$.

The proof is constructive. Let ω be the state of the world. Consider the network $\bar{g} = g - (\{ij\}_{i,j\in\mathbf{H}} \cup \{ij\}_{i.j\in\mathbf{L}})$, where ij represents the link between banks i and j. In other words, $\bar{g}$ is the network formed from the initial network by deletion of links between banks of the same type. Thus, $\bar{g}$ is the set of links that exist between banks of a different type. The total number of links in the network $\bar{g}$ is $\sum_{i\in\mathbf{H}} s_i^{cr} = \sum_{j\in\mathbf{L}} s_j^{cr}$ which is larger than $3(n-2)$. In the network $\bar{g}$ we further delete links such that each bank has exactly $(n-2)$ neighbors. Let $\hat{g}$ be the new network where each has bank has exactly $(n-2)$ links. The reader may check that there exists a network $\hat{g}$ for any $n \geq 3$.

For any link $ij \in \hat{g}$, we set $d_{ij} = \frac{z}{n-2}$ if $i \in \mathbf{L}$ and $j \in \mathbf{H}$ and $d_{ij} = 0$ otherwise. Similarly, for any link $ij \in g$ and $ij \notin \hat{g}$, we set $d_{ij} = 0$. These transfers clearly satisfy $\left|\sum_{j\in N_i} d_{ji} - \sum_{j\in N_i} d_{ij}\right| C_1 = zC_1$, q.e.d. ■

Proposition 4.2 *Let g be an incomplete n-regular network (i.e. $n = 3, 4$) and consider any realization of the liquidity shocks that allows at least one bank to have minimum $(n-1)$ neighbors of a different type. Than there exists a feasible allocation of deposits a_{ij} such that $a_{ij}\frac{C_1 - x - ry}{1+\sum_{k\in N_j} a_{jk}} < LGD^*_{ij}$, for any pair $ij \in g$.*

Proof. We treat the two cases $n = 3$ and $n = 4$ separately.

For $n = 3$, we consider the following allocation of deposits: $a_{ij} = \frac{3z}{5}$ for all pairs ij. Clearly, this allocation satisfies $a_{ij}\frac{C_1 - x - ry}{1+\sum_{k\in N_j} a_{jk}} < LGD^*_{ij}$. We just need to show that $a_{ij} = \frac{3z}{5}$ is feasible for all the states of the world that allow at least one bank to have minimum $(n-1)$ neighbors of a different type. In order for at least one bank to have minimum 2 neighbors of a different type, the banking system needs to be in one of the situations $2-4$ corresponding to case 1. Moreover, lemma 4.2 ensures that there are at least 2 banks, one of type H and one of type H, each having minimum 2 neighbors of a different type.

If the system is in situation 2, we construct the transfer of deposits in the following way. Let $k \in \mathbf{L}$ be the bank such that $s_k^{cr} = 1$, and let $l \in \mathbf{H}$ be the bank such that $s_l^{cr} = 1$. Consider the transfer of deposits $d_{ij} = \frac{3z}{5}$ if $i \in \mathbf{L}$ and $j \in \mathbf{H}$. Set $d_{ki} = \frac{z}{5}$ for any $i \in \mathbf{L} - \{k\}$ and $d_{jl} = \frac{z}{5}$ for any $j \in \mathbf{H} - \{l\}$. For all the other links set $d = 0$. These transfers satisfy $\left|\sum_{j\in N_i} d_{ji} - \sum_{j\in N_i} d_{ij}\right| C_1 = zC_1$ and $0 \leq d_{ij} \leq a_{ij}$ for any pair $ij \in g$.

If the system is in situation 3 and 4, in a similar manner as above, we construct the networks $\hat{g}_3$ and $\hat{g}_4$, respectively. $\hat{g}_3$ is the network where for each bank i, $s_i^{cr} = 2$, while $\hat{g}_4$ is the network where for each bank i, $s_i^{cr} = 3$. In situation 3, for any link $ij \in \hat{g}_3$ we set the transfers to be $d_{ij} = \frac{z}{2}$ if $i \in \mathbf{L}$ and $j \in \mathbf{H}$ and $d_{ij} = 0$ otherwise. Similarly, for any link $ij \in g$ and $ij \notin \hat{g}_3$, we set $d_{ij} = 0$. These transfers satisfy $\left|\sum_{j \in N_i} d_{ji} - \sum_{j \in N_i} d_{ij}\right| C_1 = zC_1$ and $0 \leq d_{ij} \leq a_{ij}$ for any pair $ij \in g$. In situation 4, for any link $ij \in \hat{g}_4$ we set the transfers to be $d_{ij} = \frac{z}{3}$ if $i \in \mathbf{L}$ and $j \in \mathbf{H}$ and $d_{ij} = 0$ otherwise. Similarly, for any link $ij \in g$ and $ij \notin \hat{g}_4$, we set $d_{ij} = 0$. These transfers satisfy $\left|\sum_{j \in N_i} d_{ji} - \sum_{j \in N_i} d_{ij}\right| C_1 = zC_1$ and $0 \leq d_{ij} \leq a_{ij}$ for any pair $ij \in g$.

For $n = 4$, we consider the following allocation of deposits: $a_{ij} = \frac{3z}{8}$ for all pairs ij. Clearly, this allocation satisfies $a_{ij} \frac{C_1 - x - ry}{1 + \sum_{k \in N_j} a_{jk}} < LGD^*_{ij}$. We just need to show that $a_{ij} = \frac{3z}{8}$ is feasible for all the states of the world that allow at least one bank to have minimum $(n-1)$ neighbors of a different type. In order for at least one bank to have minimum 3 neighbors of a different type, the banking system needs to be in the situation 2 corresponding to case 2. When the system is in situation 2, we construct the transfer of deposits in the following way. Let $k \in \mathbf{L}$ be the bank such that $s_k^{cr} = 2$, and let $l \in \mathbf{H}$ be the bank such that $s_l^{cr} = 2$. Consider the transfer of deposits $d_{ij} = \frac{3z}{8}$ if $i \in \mathbf{L}$ and $j \in \mathbf{H}$. Set $d_{ki} = \frac{z}{8}$ for any $i \in \mathbf{L} - \{k\}$ and $d_{jl} = \frac{z}{8}$ for any $j \in \mathbf{H} - \{l\}$. For all the other links set $d = 0$. These transfers satisfy $\left|\sum_{j \in N_i} d_{ji} - \sum_{j \in N_i} d_{ij}\right| C_1 = zC_1$ and $0 \leq d_{ij} \leq a_{ij}$ for any pair $ij \in g$. q.e.d. ■

Corollary 4.3 *Let $\tilde{g}$ be the complete network. Than, there is no feasible allocation of deposits a_{ij} such that $a_{ij} \frac{C_1 - x - ry}{1 + \sum_{k \in N_j} a_{jk}} < LGD^*_{ij}$, for all pairs $ij \in \tilde{g}$.*

Proof. We assume there is a feasible allocation of deposits a_{ij} such that $a_{ij} \frac{C_1 - x - ry}{1 + \sum_{k \in N_j} a_{jk}} < LGD^*_{ij}$, for all pairs $ij \in \tilde{g}$. Since $\tilde{g}$ is the complete network $\sum_{k \in N_j} a_{jk} = \sum_{\substack{k=1 \\ k \neq j}}^{6} a_{jk} = S_j$.

Since the inequality $a_{ij} \frac{C_1 - x - ry}{1 + \sum_{k \in N_j} a_{jk}} < LGD^*_{ij}$ holds, than we must have $\frac{a_{ij}}{1 + \sum_{k \in N_j} a_{jk}} < \frac{z}{3+5z}$ or $(3+5z)a_{ij} < z + zS_j$, for any $i, j \in \{1, 2, ..., 6\}$. Keeping j fixed and aggregating these inequalities after i , we obtain: $(3 + 5z) \sum_{\substack{i=1 \\ i \neq j}}^{6} a_{ij} < 5z + 5zS_j$. This yields further

$S_j < \frac{5z}{3}$, $\forall j \in \{1, 2, ..., 6\}$. In order for an allocation of deposits a_{ij} to be feasible a necessary condition is that it exists a pair $kl \in \tilde{g}$ such that $a_{kl} > \frac{z}{3}$. Since $S_k < \frac{5z}{3}$, we must have $\frac{a_{kl}}{1+S_k} > \frac{z}{3}\frac{1}{1+5z/3}$ or $a_{kl}\frac{C_1-x-ry}{1+S_k} > LGD^*_{kl}$ which contradicts our initial assumption. q.e.d. ■

In the end we give a general proof for the connectedness property of n-regular networks that we employ in the proof of Lemma 4.1 .

Lemma 4.3 *Let* $M = \{1, 2, ..., m\}$ *be a set of nodes connected in a* n*-regular network* g*. If* $n \geq m/2$*, than the network* g *is connected and the maximum path length between any two nodes is* 2*.*

Proof. Consider the node $i \in M$ and let $N(i) = \{i_1, i_2, ...i_n\}$ be the set of nodes directly connected with i. Than $\mathbf{card}(M - N(i)) \leq m/2$. Since any node $j \in M - N(i)$ has degree $n \geq m/2$ than for $\forall j \in M - N(i)$, $\exists i_l \in N(i)$ such that j and i_l are directly connected. This further implies that j and i are connected through a path of length 2.

■

Bibliography

Albert, R., H. Jeong, and A. Barabasi, 2000, Attack and Error Tolerance of Complex Networks, *Nature* 406, 378–382.

Allen, F., and A. Babus, 2008, Networks in Finance, Working Paper 08-07, Wharton Financial Institutions Center, University of Pennsylvania.

Allen, F., and D. Gale, 2000, Financial Contagion, *Journal of Political Economy* 108, 1–33.

Bala, V., and S. Goyal, 2000, A Non-Cooperative Model of Network Formation, *Econometrica* 68, 1181–1230.

Barabasi, A., and R. Albert, 1999, Emergence of Scaling in Random Networks, *Science* 286, 509–512.

Bloch, F., and M. Jackson, 2006, Equilibrium Definitions in Network Formation Games, *International Journal of Game Theory* 34, 305–318.

——— , 2007, The Formation of Networks with Transfers Among Players, *Journal of Economic Theory* 133, 83–110.

Boss, M., H. Elsinger, S. Thurner, and M. Summer, 2004, Network Topology of the Interbank Market, *Quantitative Finance* 4, 1–8.

Burt, R., 1994, *Structural Holes* (New York: Academic Press).

Cifuentes, R., G. Ferrucci, and H. S. Shin, 2005, Liquidity Risk and Contagion, *Journal of European Economic Association* 3.

Clauset, A., C. Moore, and M. Newman, 2006, Structural Inference of Hierarchies in Networks, arXiv:physics/0610051v1.

Cocco, J., F. Gomes, and N. Martins, 2005, Lending Relationships in the Interbank Market, working paper, London Business School.

Cohen, L., A. Frazzini, and C. Malloy, 2007, The Small World of Investing: Board Connections and Mutual Fund Returns, NBER Working Paper No. 13121.

Dasgupta, A., 2004, Financial Contagion Through Capital Connections: A Model of the Origin and Spread of Bank Panics, *Journal of European Economic Association* 2, 1049–1084.

de Vries, C., 2005, The Simple Economics of Bank Fragility, *Journal of Banking and Finance* 29, 803–825.

Degryse, H., and G. Nguyen, 2007, Interbank Exposures: An Empirical Examination of Systemic Risk in the Belgian Banking System, *International Journal of Central Banking* forthcoming.

Diamond, D., and P. Dybvig, 1983, Bank Runs, Deposit Insurance and Liquidity, *Journal of Political Economy* 91, 401–419.

Dutta, B., S. Ghosal, and D. Ray, 2005, Farsighted Network Formation, *Journal of Economic Theory* 122, 143–164.

Dutta, B., and S. Mutuswami, 1997, Stable Networks, *Journal of Economic Theory* 76, 322–344.

Eisenberg, L., and T. Noe, 2001, Systemic Risk in Financial Systems, *Management Science* 47, 236–249.

Freixas, X., B. Parigi, and J. C. Rochet, 2000, Systemic Risk, Interbank Relations and Liquidity Provision by the Central Bank, *Journal of Money, Credit and Banking* 32, 611–638.

Furfine, C., 2003, Interbank Exposures: Quantifying the Risk of Contagion, *Journal of Money, Credit and Banking* 35, 111–128.

Gilles, R. P., and S. Sarangi, 2005, Stable Networks and Convex Payoffs, working paper Virginia Tech.

Girvan, M., and M. Newman, 2002, Community Structure in Social and Biological Networks, *Procl. Natl. Acad. Sci. USA* 99, 7821–7826.

Goyal, S., M. Van der Leij, and J.L. Moraga-Gonzales, 2006, Economics: An Emerging Small World, *Journal of Political Economy* 114, 403–412.

Goyal, S., and F. Vega-Rodondo, 2007, Structural Holes in Social Networks, *Journal of Economic Theory* forthcoming.

Hochberg, Y., A. Ljungqvist, and Y. Lu, 2007, Whom You Know Matters: Venture Capital Networks and Investment Performance, *Journal of Finance* 62, 251–301.

Iyer, R., and J. L. Peydro-Alcalde, 2007, The Achilles' Heel of Interbank Markets: Financial Contagion Due to Interbank Linkages, working paper ECB.

Jackson, M., 2004, A Survey of Models of Network Formation: Stability and Efficiency, Chapter 1 in "Group Formation in Economics; Networks, Clubs and Coalitions", ed. G. Demange and M. Wooders, Cambridge University Press, Cambridge U.K.

——— , 2006, *Network Formation* (MacMillan Press) in 'The New Palgrave Dictionary of Economics and the Law', forthcoming.

——— , and A. Van Den Nouweland, 2005, Strongly Stable Networks, *Games and Economic Behavior* 51, 420–444.

Jackson, M., and B. Rogers, 2005, The Economics of Small Worlds, *Journal of European Economic Association* 3, 617–627.

——— , 2007, Meeting Strangers and Friends of Friends: How Random are Social Networks?, *American Economic Review* forthcoming.

Jackson, M., and A. Watts, 2002, The Evolution of Social and Economic Networks, *Journal of Economic Theory* 106.

Jackson, M., and A. Wolinsky, 1996, A Strategic Model of Social and Economic Networks, *Journal of Economic Theory* 71, 44–74.

Kramarz, F., and D. Thesmar, 2007, Social Networks in the Boardroom, CEPR discussion paper 5496.

Krapivsky, P.L., S. Redner, and F. Leyvraz, 2000, Connectivity of Growing Random Networks, *Physical Review Letters* 85, 4629–4632.

Lagunoff, R., and L. Schreft, 2001, A Model of Financial Fragility, *Journal of Economic Theory* 99, 220–264.

Leitner, Y., 2005, Financial Networks: Contagion, Commitment, and Private Sector Bailouts, *Journal of Finance* 60, 2925–2953.

Lopez-Pintado, D., 2007, Diffusion in Complex Social Networks, *Games and Economic Behavior* forthcoming.

Lovasz, L., 1979, *Combinatorial Problems and Exercises* (Amsterdam: Noord-Holland).

Manski, C., and D. McFadden, 1981, *Structural Analysis of Discrete Data with Econometric Applications* (MIT Press: Cambridge).

Mauleon, A., and V. J. Vannetelbosch, 2004, Farsightedness and Cautiousness in Coalition Formation, *Theory and Decision* 56, 291–324.

McFadden, D., 1974, Conditional Logit Analysis of Qualitative Choice Behavior, in P. Zarembka (ed.), Frontiers of Econometrics, Academic Press.

McKelvey, R., and T. Palfrey, 1995, Quantal Response Equilibria for Normal Form Games, *Games and Economic Behavior* 10, 6–38.

Minguez-Afonso, G., and H. Shin, 2007, Systemic Risk and Liquidity in Payment Systems, working paper London School of Economics.

Myerson, R., 1991, *Game Theory: Analysis of Conflict* (HarvardUniv ersity Press: Cambridge, MA).

Newman, M. E. J., 2003, The Structure and Function of Complex Networks, *SIAM Review* 45, 167–256.

Nguyen-Dang, B., 2007, Does the Rolodex Matter? Corporate Elite's Small World and the Effectiveness of Boards of Directors, working paper Chinese University of Hong Kong.

Page, F., M. Wooders, and S. Kamat, 2005, Networks and Farsighted Stability, *Journal of Economic Theory* 120, 257–269.

Pastor-Satorras, R., and A. Vespignani, 2001, Epidemic Spreading in Scale-Free Networks, *Phys. Rev. Lett.* 86, 3200–3203.

Price, D.J.S., 1976, A General Theory of Bibliometric and Other Cumulative Advantage Processes, *J. Amer. Soc. Inform. Sci.* 27, 292–306.

Ravasz, E., and A. Barabasi, 2003, Hierarchical Organization in Complex Networks, *Physical Review E* 67, 1–6.

Sheldon, G., and M. Maurer, 1998, Interbank Lending and Systemic Risk: An Empirical Analysis for Switzerland, *Swiss Journal of Economics and Statistics* pp. 685–704.

Trusina, A., S. Maslov, P. Minnhagen, and K. Sneppen, 2004, Hierarchy Measures in Complex Networks, *Physical Review Letters* 92.

Upper, C., and A. Worms, 2004, Estimating Bilateral Exposures in the German Interbank Market: Is There a Danger of Contagion?, *European Economic Review* 48, 827–849.

Vivier-Lirimont, S., 2004, Interbanking Networks: Towards a Small Financial World?, Cahiers de la Maison Des Sciences Economiques, Université Paris Panthéon-Sorbonne.

Watts, D., and S. Strogatz, 1998, Collective Dynamics of "Small World" Networks, *Nature* 393, 440–442.

Wells, S., 2004, U.K. Interbank Exposures: Systemic Risk Implications, *The Journal of Monetary Economics* 2, 66–77.

Nederlandse samenvatting (Summary in Dutch)

Netwerken hebben zich bewezen als een nuttige weergave van verscheidene systemen. Sociale en economische interacties, biologische en ecologische systemen én het internet kunnen door modellering als netwerk beter begrepen worden. Een netwerk beschrijft een verzameling knooppunten en connecties tussen deze knooppunten. Een vriendschap tussen individuen, een onderzoeksovereenkomst tussen bedrijven en een wederzijds verdedigingspact tussen landen zijn voorbeelden van connecties.

Dit proefschrift levert twee belangrijke bijdragen. Aan de ene kant beoogt het de toepasbaarheid van netwerktheoriën op economische vraagstukken te vergroten door een netwerkweergave van financiële systemen. Aan de andere kant beoogt het de beschikbare methodes om netwerken te analyseren te verrijken door een netwerkvormingsmodel met geprefereerde connecties van een microeconomische basis te voorzien.

De bijdragen van economen aan netwerktheoriën zijn al vaak benadrukt. De bijdrage van netwerktheorie aan de economische wetenschap is daarentegen minder duidelijk. Om een impact op het vakgebied te hebben dienen netwerken vaker gebruikt te worden om economische vraagstukken op te lossen. Toepassingen in bijvoorbeeld finance, industriele organisatie, arbeidseconomie of marketing zullen de positie van netwerken in de economische wetenschap versterken. Hoofdstuk twee en drie van dit proefschrift zetten en stap in deze richting door middel van een tweetal toepassingen op financiële systemen.

De financiële wereld bevat een breed scala van connecties. De wederzijdse afhankelijkheid van financiële instellingen komen voort uit zowel de activa als de passiva op hun balans. Een netwerkweergave van financiële systemen kan inzicht verschaffen in de bestaande connecties, of deze nu zijn onstaan op de interbancaire markt, door het aanhouden van gelijkaardige portfolio's of door het delen van dezelfde groep rekeninghouders. In dit proefschrift betoog ik dat netwerktheorie een kader verschaft waarin bovengenoemde connecties op een zinvolle wijze beschreven en geanalyseerd kunnen worden.

In het derde hoofdstuk van dit proefschrift ('*The Formation of Financial Networks*') ontwikkel ik een model van netwerkvorming in het bankwezen. In dit hoofdstuk stel ik een opzet voor waarin besmettingsgevaar de reden voor het vormen van connecties tussen banken is. Het is breed onderkend dat banken en andere financiële instellingen op verschillende wijzen met elkaar verbonden zijn. De prikkels voor het vormen van een bepaalde connecties bestaan uit de voordelen die deze connecties opleveren. De connecties die het verhandelen van liquiditeit tussen banken vergemakkelijken, kunnen het bankwezen ook blootstellen aan besmetting. Hiermee bedoel ik dat idiosyncratische schokken, waaraan een beperkt aantal instellingen is blootgesteld, zich door het gehele systeem bankwezen kunnen verspreiden.

Ik zet een model op waarin banken connecties met elkaar vormen om het besmettingsgevaar te verminderen. Een netwerk tussen banken komt endogeen tot stand en dient als een verzekeringsmechanisme. In het kort onderzoek ik een kader waarin negatief gecorreleerde liquiditeitsschokken het bankwezen beinvloeden. Banken kunnen zich perfect tegen hun individueel verzekeren door het onderling verhandelen van interbancaire deposito's. Het delen van risico brengt echter een afweging met zich mee: transfers creeren connecties tussen banken die het systeem blootstellen aan besmettingsrisico. Het model voorspelt een 'connectiviteitsgrens' (connectivity threshold) waarboven besmetting niet zal optreden, en banken zullen trachten dit niveau te bereiken. Daarom is de kans op besmetting in een volgroeid (equilibrium) netwerk bijna nul.

Teneinde economische vraagstukken beter te kunnen benaderen, dient de netwerktheorie meer specifieke technieken, methodes en maatstaven voor dit doel te ontwikkelen. De vooruitgang in de speltheorie over de laatste decennia heeft economen de mogelijkheid gegeven ook de technieken om netwerken te onderzoeken te formaliseren. Desalniettemin zijn de meeste maatstaven die economen voor netwerken gebruiken afkomstig uit de sociologie en theoretische fysica. 'Clustering coefficients', die de kans meten dat verbonden knooppunten dezelfde buren hebben, 'betweeness centrality' van een knooppunt, wat aangeeft hoe cruciaal een knooppunt is voor het verbinden van andere knooppunten, het aantal verbindingen per knooppunt en de gemiddelde afstand tussen twee knooppunten in een netwerk zijn alle zinvolle maatstaven voor netwerken. Helaas is het erg moeilijk om deze maatstaven te reproduceren in een kader met netwerkvorming tot het bereiken van een evenwicht, zoals hierboven beschreven. De kloof tussen de huidige economische technieken en de maatstaven binnen de netwerktheorie zijn niet verrassend, omdat deze maatstaven voor de sociologie en statische fysica bedacht zijn. Om deze kloof te overbruggen dienen binnen de netwerktheorie zowel nieuwe methodes gebaseerd op strategisch gedrag als nieuwe maatstaven geformuleerd te worden.

Het tweede hoofdstuk van dit proefschrift ('*Limited Connections*') zet een stap in deze richting. Economische modellen met netwerkvorming propageren netwerken als het gevolg van strategisch verbindingsgedrag van rationele entiteiten. De modellering in andere vakgebieden heeft een ander uitgangspunt: netwerkvorming is voornamelijk een stochastisch proces. Hoewel het eerste uitgangspunt leidt tot gestyleerde uitkomsten als ongelijke connecties en korte afstanden, omvat het tweede uitgangspunt de zeer diverse eigenschappen van bestaande netwerken beter. Dit hoofdstuk tracht om de voorspellende kracht van het tweede uitgangspunt te dupliceren in een model waarin (i) netwerken continu uitbreiden door de toevoeging van nieuwe knooppunten, (ii) nieuwe knooppunten opbrengstmaximaliserende connecties vormen met al bestaande knooppunten en (iii) het tot stand komen van een connectie alleen kan met goedkeuring van beide knooppunten. Ten eerste geven wij een simpel voorbeeld dat het ontstaan van 'power-law' netwerken als toetreders tot het netwerk 'logistic choice' bewerkstelligen bij het vormen van connecties. Daarnaast analyseren we verschillende specificaties voor het nut dat knooppunten afleiden uit connecties, in zowel een omgeving zonder als met onzekerheid (en dus vergissingen). De cruciale elementen die leiden tot netwerken met hoge 'centrality' en andere vormen van hierarchie zijn (i) het bestaan van bemiddelingsopbrengsten voor diegenen die anders onverbonden delen van het netwerken verbinden en (ii) verbindingskosten die stijgen in het aantal verbindingen dat een knooppunt heeft.

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