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Model-based forecast adjustment: With an illustration to inflation

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Abstract

This paper introduces the idea of adjusting forecasts from a linear time series model where the adjustment relies on the assumption that this linear model is an approximation of a nonlinear time series model. This way of creating forecasts could be convenient when inference for a nonlinear model is impossible, complicated or unreliable in small samples. The size of the forecast adjustment can be based on the estimation results for the linear model and on other data properties such as the first few moments or autocorrelations.

An illustration is given for a first-order diagonal bilinear time series model, which in certain properties can be approximated by a linear ARMA(1, 1) model. For this case, the forecast adjustment is easy to derive, which is convenient as the particular bilinear model is indeed cumbersome to analyze in practice.

An application to a range of inflation series for low-income countries shows that such adjustment can lead to some improved forecasts, although the gain is small for this particular bilinear time series model.

KEYWORDS

adjustment of forecasts, ARMA(1, 1), first-order diagonal bilinear time series model, inflation, method of moments

1 | INTRODUCTION

Forecasts from econometric time series models are frequently adjusted by experts who have domain knowledge (see Franses, 2014, and the many studies cited therein). There are various reasons why such econometric model-based forecasts are adjusted. The observation at the forecast origin may be an outlier, or an explanatory variable may suffer from measurement error. It can be believed that parameters will change in the future, or one may know that there will be a structural shift in the forecast sample.

There are various methods of expert adjustment of forecasts. One may simply add or subtract a number from a given quote, or one may change an estimated parameter to another value; one may multiply the quote by some number, change the observation of an explanatory variable to another value, or replace the observation at the forecast origin with another observation.

In this paper I put forward yet another reason to adjust a model-based forecast. The model-based forecast is believed to be based on an incorrectly specified model, while it is assumed known what the correct specification should be, but where the data do not allow that potentially correct model to be analyzed. In fact, here the idea is to generate a forecast from a linear time series model, and to adjust this forecast based on the assumption that a specific nonlinear time series model would be a more appropriate specification. There are many nonlinear time series models around (see De Gooijer, 2017, for a recent extensive survey) but, typically, proper parameter estimation for these models requires quite a number of observations, potentially with a high frequency. Also, for some nonlinear time series models (like the one to be discussed below), asymptotic theory is missing and there can be problems with the likelihood function.

The present paper specifically addresses one-step-ahead forecasts for annually or monthly observed inflation rates for low-income countries. For many countries in Africa, typically, data are not collected at a higher frequency than yearly, and the available samples typically cover five or six decades at the very maximum. Inflation rates in low-income countries once in a while can show periods of hyperinflation, whereas at other times inflation rates can be moderate. Inflation rate data thus seem a good candidate for nonlinear time series models, due to the potential presence of temporary shifts in the data. One may view such patterns as reflecting recurring structural shifts (see, e.g., Arize, Malindretos, & Nam, 2005; Castle, Doornik, Hendry, & Nymoen, 2014). Alternatively, one may see such longer periods with higher or lower inflation as a reflection of long memory, perhaps to be modeled by a fractionally integrated time series model (see, e.g., Bos, Franses, & Ooms, 2002). Here, I will take the angle of a nonlinear time series model.

In this paper I will focus on a specific nonlinear time series model, which is the so-called first-order diagonal bilinear time series model (introduced in Granger & Andersen, 1978). An approximate linear time series model turns out to be an autoregressive moving average model of order (1, 1), in short an ARMA(1, 1). Inference for this diagonal bilinear model is notoriously difficult, and also for many other bilinear models the asymptotic properties of the parameter estimators are unknown. For point-forecasting purposes, the latter properties may be viewed as less relevant, as long as one gets the proper estimates of the parameters.

The outline of this paper is as follows. In Section 2 the focus is on the ARMA(1, 1) model and how it relates to a first-order diagonal bilinear time series model. First, the linear model can be viewed as a proper linear approximation of this bilinear model. Second, given the expressions for the expected values of the levels and the squared levels of the data, the parameters in the bilinear model could be identified and a method-of-moments estimator could be used, although it will be shown that then the data should have rather peculiar properties. In Section 3 I illustrate the potential merits of such model-based forecast adjustment for data on annual inflation for 41 countries on the African continent and for 11 sector-specific monthly inflation series for Suriname (a country that recently experienced a period of very high inflation). For all series the ARMA(1, 1) model is fitted. Looking at the quality of the one-step-ahead forecasts, it can be learned that for eight of the 41 countries the adjusted forecasts lead to improvement (sometimes more than 30%), although the sign test indicates statistically significant improvement for only one country. For the remaining 33 countries, the adjusted forecasts are less accurate. For the 11 sector-specific inflation rates, there is moderate forecast improvement for three series, but none is statistically significant. Section 4 concludes with further ideas on the proposed indirect method to create forecasts.

2 | THE MAIN IDEA

In this section I outline the main idea of model-based forecast adjustment. First, I discuss a linear model and a specific nonlinear model. Next, the expression for the adjusted forecast will be presented.

2.1 | Linear and bilinear models

Consider a time series y_t and suppose that a reasonable model for this time series is an ARMA(1, 1) model: that is,

$$y_t = \tau + \alpha y_{t-1} + u_t + \theta u_{t-1},$$

with $|\alpha| < 1$ and $|\theta| < 1$.The first-order autocorrelation of the ARMA(1, 1) model is

$$\rho_1 = \frac{(1+\alpha\theta)(\alpha+\theta)}{1+2\alpha\theta+\theta^2}$$

(see Franses, van Dijk, & Opschoor, 2014, pp. 52–53). The next autocorrelations obey the scheme

$$\rho_k = \alpha\rho_{k-1},$$

for $k = 2, 3, \ldots$ (see Franses et al., 2014, p. 53). Simple algebra gives

$$\rho_1 - \alpha = \frac{\theta(1-\alpha^2)}{1+2\alpha\theta+\theta^2}.$$

This shows for $\alpha > 0$ that $\rho_1 > \alpha$ when $\theta > 0$ and that $\rho_1 < \alpha$ when $\theta < 0$. Hence, with a positive value of θ, there is more persistence in the process.

The one-step-ahead forecast from origin T for an ARMA(1, 1) model is based on

$$y_{T+1|T} = \tau + \alpha y_T + \theta u_T, \tag{1}$$

where in practice, of course, the parameters are replaced by estimated values. The forecast error is $u_{T+1} = y_{T+1} - y_{T+1\mid T}$. Thus too low a forecast means a positive forecast error, and when $\theta < 0$ there is a tendency to revert to the mean. In the case of inflation, this is perhaps an unwanted effect as typically inflation can peak for a few periods in a row. That is, high initial inflation levels can spur a period with again high inflation. One may therefore want to look at alternative forecasts.

To make a link with a bilinear model, one may now want to replace θ with a function of y_T to mitigate any mean-reverting effect; that is, one may want to replace the ARMA(1, 1) forecast by

$$y_{T+1|T} = \tau + \alpha y_T + \beta y_T u_T; \tag{2}$$

that is, θ in Equation 1 is replaced by βy_T in Equation 2. A closer look at this forecast function reveals that it corresponds to a so-called first-order diagonal bilinear time series model; that is,

$$y_t = \alpha y_{t-1} + \beta y_{t-1}\varepsilon_{t-1} + \varepsilon_t, \tag{3}$$

where the notation for α is kept the same, for reasons to become clear below. There is no need to include an intercept, as we will also see below. Naturally, the u_t in the ARMA(1, 1) model is not the same as the ε_t in Equation 3. This first-order diagonal bilinear model was introduced in Granger and Andersen (1978, p. 56 et seq.).

This model has acquired quite some attention in the literature. Basrak, David, and Mikosch (1999) examined the sample autocorrelation function of Equation 3. Bibi and Oyet (2004) extended the model to allow for time-varying coefficients. Brunner and Hess (1995) discussed the potential problems with the likelihood function. Charemza, Lifshits, and Makarova (2005) studied Equation 3 in the case of $\alpha = 1$. Guegan and Pham (1989) discussed estimation of the parameters using the least squares method. A method-of-moments estimator for this diagonal model was considered by Kim, Billard, and Basawa (1990). Pham and Tran (1981) discussed various other properties of this first-order bilinear time series model. Sesay and Subba Rao (1988) looked into estimation methods using higher order moments, and Subba Rao (1981) provided a general theory of bilinear models. Among the few studies where bilinear models in general are considered for forecasting are Poskitt and Tremayne (1986) and Weiss (1986), where it was found for a few cases that bilinear models could slightly improve on linear models. Finally, Turkman and Turkman (1997) derived the properties of the extremal observations corresponding to bilinear time series models.

That a bilinear time series model can be associated with extremal observations can also be seen from the following. For the bilinear model in Equation 3, Granger and Andersen (1978, p. 56) derived that

$$\mu = E(y_t) = \frac{\beta\sigma_\varepsilon^2}{1-\alpha},$$

$$\omega = E\left(y_t^2\right) = \frac{\sigma_\varepsilon^2\left(1 + 2\beta^2\sigma_\varepsilon^2 + 4\alpha\beta\mu\right)}{1 - \alpha^2 - \beta^2\sigma_\varepsilon^2}.$$

They also derived that the autocorrelation function of the first-order diagonal bilinear time series model in Equation 3 was the same as that of an ARMA(1, 1) model such as

$$y_t = \tau + \alpha y_{t-1} + u_t + \theta u_{t-1},$$

with exactly the same α (see Granger & Andersen, 1978, p. 56).

One could now think that with an expression for α and the expressions for the first and second moments, one could design separate estimators for β and σ_ε^2. However, in the Appendix it is shown that this method-of-moments-type method is quite unlikely to be successful for empirical data. In short, the reason is that $\omega = E\left(y_t^2\right)$ should be more than (about) eight times as large as $\mu^2 = (E(y_t))^2$, or ω should be very small relative to μ. For the inflation data in Africa, to be analyzed later, this occurs only for Chad and the Democratic Republic of Congo. For the data on Suriname this does not happen at all.[1] This shows that, as such, the first-order diagonal bilinear time series model may not be successfully analyzed in practice, and this may also explain the relatively small number of empirical applications of this model. But perhaps indirectly achieved forecasts based on a linear model may be useful.

2.2 | Model-based forecast adjustment

Conveniently, to create the model-adjusted forecast

$$y_{T+1|T} = \tau + \alpha y_T + \beta y_T u_T,$$

there appears to be no need to estimate β and σ_ε^2 separately. This can be seen as follows. When the first-order diagonal bilinear time series model is the data-generating process, and we fit an ARMA(1, 1) model to these data, then the estimator for σ_u^2 for the ARMA model is not an estimator for σ_ε^2. Hence the model-adjusted forecast should correct for the difference between the two, and one should properly scale the added term, like

[1]Note that a useful by-product of the exercise in this paper is that a simple diagnostic method can be implemented which can be used to see if it would be worthwhile to try to estimate the parameters in a bilinear model in the first place. This diagnostic method is based on the difference between the expected values of the levels and the squares.

$$y_{T+1|T} = \tau + \alpha y_T + \beta y_T \left(\frac{\sigma_\varepsilon^2}{\sigma_u^2}\right) u_T. \quad (4)$$

Given an estimator for the variance σ_u^2 for the ARMA model, and given the observable value y_T, we thus need to know $\beta\sigma_\varepsilon^2$. This last term can simply be found from the first moment (see earlier): that is,

$$\mu(1-\alpha) = \beta\sigma_\varepsilon^2.$$

All in all, we now have quite a simple way of finding a forecast that associates with a first-order diagonal bilinear time series model, without having to estimate its model parameters.

3 | EMPIRICAL APPLICATION

This section deals with a comparison of the forecasts from an ARMA(1, 1) model and from a model-based forecast adjustment, where it is presumed that the first-order diagonal model in Equation 3 could have generated the data. Fitting the model to the data is unlikely to work, and therefore I opted for the forecast adjustment approach.

3.1 | Forty-one countries in Africa

The first set of data concerns annual inflation rates for 41 African countries, ranging from 1960 to 2015. The data source is Franses and Janssens (2018). Table 1 presents the estimates of α, μ, $\beta\sigma_\varepsilon^2$, and $\left(\beta\sigma_\varepsilon^2\right)/\sigma_u^2$, where this latter term will be used in Equation 4 to create the adjusted forecast. The estimates of that term have a maximum value of 0.402 (Botswana) and a minimum of 2.14E − 05 (Democratic Republic of Congo).

Table 2 presents the results on measures of forecast accuracy, where here it is chosen to use the median absolute forecast error (MedAFE).[2] The MedAFE is defined as the median value of the absolute values of the prediction errors, where these errors are defined as $y_{T+1} - \hat{y}_{T+1|T}$. The forecasts are all one-step-ahead forecasts, within the sample, where the estimates are obtained for the full sample.[3] Much more refined forecast evaluation methods can be considered, but it is believed that the overall qualitative outcome will be about the same. In italics are those cases where the model-based adjusted forecasts give a

[2]As the data can have outliers, the median seems an obvious choice. And, as inflation is measured as a percentage, the absolute errors seem the natural errors to evaluate.

[3]The sample sizes are not large, and cutting the data in estimation and holdout samples seriously reduces the quality of the estimated parameters. Moreover, the prediction equation requires the estimated residuals from the ARMA model.

TABLE 1 Estimation results for annual inflation in Africa

Country	α	μ	$\beta\sigma_\varepsilon^2$	$\frac{\beta\sigma_\varepsilon^2}{\sigma_u^2}$
Algeria	0.620	8.957	3.404	0.151
Angola	0.123	339.36	297.62	9.12E − 04
Benin	0.924	7.343	0.558	0.016
Botswana	0.839	9.754	1.570	0.402
Burkina Faso	0.647	4.577	1.616	0.033
Burundi	−0.293	9.892	12.790	0.227
Cape Verde	0.816	4.504	0.829	0.078
Central African Republic	−0.615	4.132	6.673	0.170
Chad	−0.092	4.789	5.230	0.058
Republic of Congo	0.021	10.614	10.391	0.048
DR of Congo	0.644	642.67	228.79	2.14E − 05
Egypt	0.862	9.264	1.278	0.068
Equatorial Guinea	0.126	3.596	3.143	0.119
Ethiopia	−0.259	8.614	10.845	0.108
Gabon	0.167	4.995	4.161	0.084
Gambia	0.666	8.041	2.686	0.046
Guinea Bissau	0.977	31.213	0.718	0.005
Ivory Coast	−0.081	5.586	6.038	0.196
Kenya	0.373	10.271	6.440	0.138
Libya	0.731	5.303	1.427	0.038
Madagascar	0.367	11.725	7.422	0.119
Malawi	0.732	26.179	7.016	0.021
Mali	−0.212	3.180	3.854	0.271
Mauritius	0.450	7.407	4.074	0.111
Morocco	0.893	4.454	0.477	0.058
Mozambique	0.811	18.741	3.542	0.042
Niger	0.233	4.502	3.498	0.061
Nigeria	0.337	15.948	10.574	0.075
Rwanda	0.173	7.734	6.396	0.205
Senegal	0.533	5.104	2.384	0.050
Seychelles	0.075	6.959	6.437	0.185
Sierra Leone	0.892	23.770	2.567	0.004
Somalia	0.689	23.171	7.206	0.008
South Africa	0.827	8.195	1.418	0.361
Sudan	0.886	28.486	3.247	0.008
Swaziland	0.843	9.554	1.500	0.077
Tanzania	0.860	16.145	2.260	0.067
Togo	0.387	5.380	3.298	0.067
Tunisia	0.863	5.521	0.756	0.041
Uganda	0.712	30.964	8.918	0.011
Zambia	0.598	36.616	14.720	0.023

TABLE 2 Median absolute forecast error (MedAFE), based on in-sample one-step-ahead forecasts, Africa

Country	ARMA	Model-based adjusted
Algeria	2.728	2.852
Angola	126.00	143.99
Benin	*2.746*	*2.061*
Botswana	1.125	3.144
Burkina Faso	3.460	3.569
Burundi	4.126	6.689
Cape Verde	1.711	2.330
Central African Republic	3.781	5.824
Chad	5.013	6.710
Republic of Congo	6.408	7.655
DR of Congo	*498.37*	*235.61**
Egypt	2.502	2.843
Equatorial Guinea	1.786	1.900
Ethiopia	4.747	8.738
Gabon	2.260	2.713
Gambia	1.829	2.261
Guinea Bissau	2.073	2.389
Ivory Coast	2.297	2.652
Kenya	4.096	5.172
Libya	*3.072*	*2.928*
Madagascar	3.990	4.548
Malawi	*7.011*	*5.315*
Mali	0.841	1.207
Mauritius	2.398	2.713
Morocco	1.179	1.814
Mozambique	2.255	3.554
Niger	4.560	5.196
Nigeria	5.020	6.807
Rwanda	3.626	4.240
Senegal	*2.947*	*2.361*
Seychelles	2.677	3.042
Sierra Leone	*46.169*	*31.360*
Somalia	*18.197*	*6.804*
South Africa	1.163	2.725
Sudan	44.366	50.824
Swaziland	2.557	3.539
Tanzania	3.043	3.476
Togo	2.901	3.423
Tunisia	*1.312*	*1.033*

(Continues)

TABLE 2 (Continued)

Country	ARMA	Model-based adjusted
Uganda	30.390	42.712
Zambia	41.664	48.906

Note. Cases with forecast improvement are in italics. Only for the DR of Congo is the improvement statistically significant at the 5% level (*).

lower MAFE than those from the linear ARMA model. Clearly, there are eight cases with improvement, where, even though the improvement is not statistically significant, the differences can be substantial (look at Sierra Leone and Somalia, for example). On the other hand, for some of the 33 other cases, the adjusted forecasts can be less good. Upon using a sign test, only for the Democratic Republic of Congo is there a statistically significant result.

3.2 | Eleven categories in Suriname

Figure 1 presents the monthly inflation rates for the South American country Suriname. The sector-specific inflation rates concern the percentage differences between prices in a current month and that same month the year before. The prices data range from January 2013 to December 2017 and are obtained from Statistics Suriname (http://www.statistics-suriname.org/), and hence the inflation rate data start in January 2014. Clearly, there were months with exceptionally high inflation levels.

Table 3 presents similar estimation results for Suriname as given in Table 1 for Africa. The estimates for $(\beta\sigma_\varepsilon^2)/\sigma_u^2$ range from 0.007967 (category Housing,

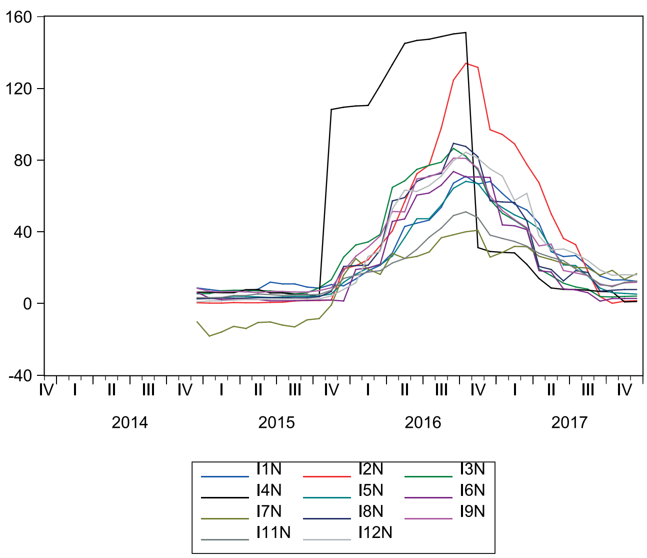

FIGURE 1 Monthly inflation in Suriname, January 2014 to December 2017 [Colour figure can be viewed at wileyonlinelibrary.com]

TABLE 3 Estimation results for monthly inflation in Suriname

Category	α	μ	$\beta\sigma_\varepsilon^2$	$\frac{\beta\sigma_\varepsilon^2}{\sigma_u^2}$
Food, Non-alcohol	0.939	27.82	1.697	0.077
Alcohol, Tobacco	0.942	38.15	2.213	0.022
Clothing, Footwear	0.947	29.75	1.577	0.030
Housing, Utilities	0.882	49.95	5.894	0.008
Household Furnishing	0.956	24.07	1.059	0.069
Health Care	0.942	23.03	1.336	0.018
Transportation	0.918	13.53	1.109	0.035
Communication	0.924	28.52	2.168	0.027
Recreation, Education	0.952	30.03	1.441	0.034
Food away from home	0.943	19.57	1.115	0.132
Other	0.943	30.90	1.761	0.033

TABLE 4 Median absolute forecast error (MedAFE), based on in-sample one-step-ahead forecasts, Suriname

Category	ARMA	Model-based adjusted
Food, Non-alcohol	2.179	2.254
Alcohol, Tobacco	3.866	4.879
Clothing, Footwear	*2.147*	*1.749*
Housing, Utilities	*5.951*	*5.126*
Household Furnishing	1.715	2.206
Health Care	1.567	2.316
Transportation	2.866	4.153
Communication	2.178	6.873
Recreation, Education	*2.639*	*2.469*
Food away from home	1.504	2.853
Other	1.767	2.599

Note. Cases with forecast improvement are in italics.

Utilities) to 0.132 (category Food away from home). Table 4 presents the MedAFE results, for the one-step-ahead in-sample forecasts. We see that for three categories there can be some slight forecast improvement, although a sign test indicates no statistically significant differences.

4 | CONCLUSION

The application of the new and simple "model-based forecast adjustment" method in this paper to one particular type of bilinear time series model did not lead to much forecast success overall, although there were a few exceptions. Indeed, the simple ARMA model seems to outperform this particular bilinear model.

This study can best be seen as an attempt to readdress attention to a model that, because of estimation problems and other issues, is rarely considered in practice. This may also hold for various other models that have features which make their empirical application cumbersome. For that matter, in this paper I therefore proposed an alternative approach, which does rely on an assumption of a nonlinear data-generating process, but which does not require its parameter estimation and asymptotic inference. This approach simply estimates a linear time series model, and then modifies the forecasts using properties of the data that associate with the nonlinear data-generating process. For 11 of the 41 + 11 = 52 cases in total, it was found that some forecast improvement is possible. It is hoped, therefore, that this new and indirect approach can bring life to nonlinear model classes that have interesting properties, but which are difficult to analyze in practice.

Further work on this approach could consider various other nonlinear models. For example, consider the bilinear model

$$y_t = \beta y_{t-2}\varepsilon_{t-1} + \varepsilon_t,$$

which is the focus of Ling, Peng, and Zhu (2015). The expected value of y_t is zero, and also the autocorrelations are zero. This means that the linear model would simply be $y_t = u_t$, where $\sigma_u^2 = \sigma_y^2 \neq \sigma_\varepsilon^2$. An adjusted forecast for $T + 1$ would then be

$$\mathrm{y}_{\mathrm{T+1|T}} = \beta \mathrm{y}_{\mathrm{T-1}} \mathrm{u}_{\mathrm{T}} \frac{\sigma_\varepsilon^2}{\sigma_\mathrm{u}^2}.$$

Grahn (1995) shows that

$$E(y_t y_{t-1}) = \beta\sigma_\varepsilon^2 y_{t-2},$$

and hence also for this model we can obtain an estimate of $\beta\sigma_\varepsilon^2$. This makes it possible to create the rather simple model-based-adjusted forecast equal to

$$\mathrm{y}_{\mathrm{T+1|T}} = \beta \mathrm{y}_{\mathrm{T-1}} \mathrm{u}_{\mathrm{T}} \frac{\sigma_\varepsilon^2}{\sigma_\mathrm{y}^2} = \beta\sigma_\varepsilon^2 \frac{\mathrm{y}_{\mathrm{T-1}}\mathrm{y}_\mathrm{T}}{\sigma_\mathrm{y}^2}$$

This bilinear model has different features than the diagonal model considered for the inflation series. For example, the autocorrelations are zero, which is not the case for the inflation data. To see if this model can perhaps be useful to financial returns data, where typically the best forecast is that the return is 0, and where autocorrelations are zero, I consider the daily returns (y_t) on the Dow Jones index, January 1, 1990 to and including December 31,

2012. Based on the data and the auxiliary regression to retrieve $\beta\sigma_\varepsilon^2$, the forecasting scheme becomes

$$y_{T+1|T} = 0.088219\frac{y_{T-1}y_T}{(1.096087)^2}.$$

This model predicts the sign of the returns correctly in 44.7% of cases. The zero mean model never predicts a sign; it always predicts 0. A prediction equal to the average returns would mean always a positive forecast, and this can hardly be believed to be a sensible forecast. This illustration suggests that a bilinear model may be useful for asset returns. Further research is needed to see whether more such models exist, for which our simple methodology can be useful.

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APPENDIX A

To show that the first-order diagonal bilinear model is difficult to handle in practice, consider the case where $\alpha = 0$ (to save notation); that is, consider

$$y_t = \beta y_{t-1}\varepsilon_{t-1} + \varepsilon_t.$$

The first and second moments are

$$\mu = E(y_t) = \beta\sigma_\varepsilon^2$$

and

$$\omega = E\left(y_t^2\right) = \frac{\sigma_\varepsilon^2\left(1 + 2\beta^2\sigma_\varepsilon^2\right)}{1 - \beta^2\sigma_\varepsilon^2}$$

(see Granger & Andersen, 1978, p. 56). This last equation can be written as

$$\left(1 - \beta^2\sigma_\varepsilon^2\right)\omega = \sigma_\varepsilon^2\left(1 + 2\beta^2\sigma_\varepsilon^2\right).$$

Replacing σ_ε^2 by μ/β (based on the first moment) and rearranging gives a second-order equation for β:

$$-\mu\omega\beta^2 + \left(\omega - 2\mu^2\right)\beta - \mu = 0.$$

To solve for β, the determinant is

$$D_\beta = \omega^2 - 8\mu^2\omega + 4\mu^4.$$

To see when D_β is positive, solve $D_\beta = 0$ for ω, to get the determinant

$$D_\omega = 48\mu^4.$$

The solutions for ω are $\left(4 + 2\sqrt{3}\right)\mu^2$ and $\left(4 - 2\sqrt{3}\right)\mu^2$. Therefore, to find estimates based on a method-of-moments estimator for σ_ε^2 and β, it should hold that

$$\omega > \left(4 + 2\sqrt{3}\right)\mu^2$$

or that

$$\omega < \left(4 - 2\sqrt{3}\right)\mu^2.$$

Both conditions are very rare for empirical data. For the African countries the first condition occurs twice, and for the Suriname data neither one of the conditions occurs.