Competition among Liquidity Providers with Access to High-Frequency Trading Technology*

Dion Bongaerts† and Mark Van Achter‡

February 4, 2020

Abstract

We model endogenous technology adoption and competition among liquidity providers with access to High-Frequency Trading (HFT) technology. HFT technology provides speed and informational advantages. Information advantages may restore excessively toxic markets. Speed technology may reduce resource costs for liquidity provision. Both effects increase liquidity and welfare. However, informationally advantaged HFTs may impose a winner's curse on traditional market makers, who in response reduce their participation. This increases resource costs and lowers the execution likelihood for market orders, thereby reducing liquidity and welfare. This result also holds when HFT dominates traditional technology in terms of costs and informational advantages.

Keywords: Adverse Selection, Liquidity, Latency, Informed Trading, Trading Technology.

JEL: D53, G01, G10, G18.

*We would like to thank Shmuel Baruch, Jean-Edouard Colliard, Sugato Chakravarty, Hans Degryse, Jérôme Dugast, Frank de Jong, Thierry Foucault, Nicolae Gârleanu, Jasmin Gider, Terry Hendershott, Johan Hombert, Andrei Kirilenko, Pete Kyle, Katya Malinova, Albert Menkveld, Sophie Moinas, Christine Parlour, Ioanid Roşu, Mathijs van Dijk, Wolf Wagner, Jing Zhao, conference participants at the 2016 AFA Meeting (San Francisco), the 2016 Paris Dauphine Market Microstructure Conference, the 2015 Conference on the Industrial Organization of Securities Markets (Frankfurt), the 2015 CIFR Symposium Celebrating the 30 Years Since Kyle Met Glosten & Milgrom (Sydney), the 2014 FIRS Conference (Quebec), the 2014 EFA Conference (Lugano), TSE High-Frequency Trading Conference 2013 (Paris), and seminar participants at ESMA, SEC, CFTC, HEC Paris, and Erasmus University Rotterdam for helpful comments and suggestions. This paper was awarded with the 2015 De la Vega prize granted by the Federation of European Securities Exchanges. Mark Van Achter gratefully acknowledges financial support from Trustfonds Erasmus University Rotterdam.

†Rotterdam School of Management, Erasmus University, Department of Finance, Burgemeester Oudlaan 50, P.O. Box 1738, 3000 DR Rotterdam, The Netherlands. E-mail: *dbongaerts@rsm.nl*, corresponding author.

‡KU Leuven, Faculty of Economics and Business (FEB), Antwerp Carolus Campus, Korte Nieuwstraat 33, 2000 Antwerp, Belgium. E-mail: *mark.vanachter@kuleuven.be*.

1 Introduction

One of the most striking developments in financial markets of the past years is the rise of high-frequency traders (HFTs). HFTs invest heavily in trading technology that allows them to benefit from a combination of "speed" i.e., low-latency market access, and "superior information processing" generating (imperfect) signals about future order flows.

The participation of HFTs spurred an intense debate on their impact on markets. HFTs have acquired substantial market shares, thereby largely crowding out traditional liquidity providers (low frequency traders, LFTs). Large investors complain about increased "slippage"as a result of HFT presence. HFTs, however, point at tighter spreads due to their presence.

Our paper examines how competition among differentially paced and differentially informed liquidity providers affects markets, using a model with endogenous participation and technology adoption. We find that the effect of the availability of HFT technology depends on market circumstances. If adverse selection through informed liquidity demand is severe, markets can shut down. Informationally advantaged HFTs may then restore markets, thereby improving liquidity. If adverse selection is moderate or small and the informational advantage of HFTs is small, the availability of HFT technology may improve liquidity by reducing the resource costs for providing liquidity. Finally, if adverse selection is moderate and the informational advantage of HFTs is substantial, HFTs may impose a winner's curse on LFTs due to their informational advantage. This winner's curse impairs LFT profitability. In response, LFTs become more cautious in providing liquidity and reduce their participation. HFT participation by contrast increases. Liquidity reduces due to a lower likelihood of liquidity demand being served and a potential increase in the resource costs for providing liquidity. The availability of HFT technology affects welfare negatively if adverse selection is moderate and the informational advantage of HFTs is substantial, and positively otherwise.

Our model has two stages. In the participation stage, candidate liquidity providers maximize expected profits by investing in liquidity provision technology to become HFTs, LFTs, or refrain from participation. The subsequent trading stage is inspired by Cordella and Foucault (1999). It features sequential Poisson arrivals of liquidity providers posting limit orders, and a liquidity demander posting a market order. For tractability, we focus on competition at quote levels close to the mid price, which is where most action is found (e.g., Brogaard et al. (2019)). Upon arrival, liquidity providers post quotes that maximize expected profits conditional on their information sets and the standing best quote. Liquidity demand is exogenous and reflected by a market order, which executes against the standing best quote (if any). Liquidity demand is either informed or uninformed, drawn by nature with given probabilities. The liquidity provider in a transaction incurs a loss in case of informed liquidity demand, and a gain otherwise. The game ends upon market order execution or if no liquidity provider is willing to provide liquidity.

HFTs benefit from superior speed technology and superior information processing technology. Superior *speed technology* allows to monitor markets faster (e.g., due to colocation), which is reflected by a higher arrival intensity. Following Aït-Sahalia and Saglam (2017), superior *information processing technology* allows HFTs to (imperfectly) infer the nature of the incoming order flow (e.g., from recognizing informed trade clustering; see Admati and Pfleiderer, 1988), which is reflected by a common signal about the nature of the incoming market order. The adoption of LFT and HFT technology involves upfront, technology-specific participation costs.

We study liquidity and analyze two dimensions: the expected half spread for executed market orders and the likelihood of serving liquidity demand. We also analyze welfare implications. The execution of uninformed market orders increases welfare and investments in trading technology decrease welfare. Executing informed market orders is welfare neutral (zero sum transfers).

Poisson intensities and participation costs are additive across liquidity providers. As a result, we can show that when information processing technology is useless, it is the participation cost per unit of speed that determines which type of liquidity provider survives in equilibrium. Provided that order flow is not very likely to be informed, HFT technology is adopted only when its cost per unit of speed is lowest, and LFT technology is adopted otherwise. Moreover, since there is no information to condition on, liquidity is always offered. As a result, the availability of HFT technology is (weakly) beneficial for liquidity.

If liquidity demand is very likely to be informed, adverse selection losses are prohibitively severe for LFTs to participate. Signals about the nature of order flow may now allow HFTs to participate and provide liquidity only when order flow is less likely to be informed. Thereby HFTs (partially) restore markets and improve liquidity.

When information processing technology is useful, when order flow is moderately likely to be informed, and when HFT technology is not prohibitively costly, LFTs may suffer from a winner's curse. The reason is that HFTs are likely to avoid informed order flow, leaving it for LFTs. At the same time, HFTs compete with LFTs for uninformed order flow. This winner's curse makes LFTs less willing and more cautious to provide liquidity. Its severity increases as HFT presence (relative to LFT presence) increases. The increased adverse selection and increased caution when providing liquidity impair LFT profitability, thereby reducing scope for LFT participation. This reduction in LFT participation creates space for more HFT participation due to reduced competition, which in turn aggravates the winner's curse and reduces LFT profitability further. Hence, endogenous participation and technology adoption may aggravate adverse selection concerns for LFTs. As a result, LFTs may largely or completely abstain from providing liquidity. Consequently, the likelihood that liquidity demand is served is reduced due to LFTs abstaining from quoting and HFTs retracting their liquidity when they suspect informed liquidity demand.

When HFTs have higher costs per unit of speed than LFTs, multiple equilibria may arise, since adverse section losses for LFTs increase in HFT presence and decrease in LFT presence. There is then always one LFT Dominance equilibrium, and one equilibrium with large HFT presence.[1] The latter may involve coexistence when cost advantages of LFTs (vis-a-vis HFTs) are exactly offset by reduced profitability due to more cautious liquidity provision.

When HFT have slightly higher or even lower costs per unit of speed than LFT, HFT Dominance is the only equilibrium possible since informational advantages of HFTs more than offset LFT cost advantages, if any. Expected half spreads are lower than if all liquidity were provided by LFTs. Yet, the likelihood of serving liquidity demand is reduced, which may more than offset the expected half spread reduction.

The availability of HFT technology affects welfare negatively if adverse selection is moderate and the informational advantage of HFTs is substantial, due to a reduced likelihood of serving liquidity demand and potentially excessive HFT participation. The latter is associated with increased resource costs for providing liquidity. In all other situations HFT technology improves welfare by restoring markets or reducing the resource costs for providing liquidity.

Our welfare analysis indicates a scope for policy measures to curtail the negative welfare effects that the availability of HFT technology may impose on markets. We analyze three policy measures that have been explicitly or implicitly used in markets: HFT transaction taxes, mandatory liquidity provision requirements, and contingent quote subsidies. All three can prevent low welfare equilibria if HFTs have higher costs per unit of speed than LFTs. The latter two also capture welfare benefits from speed technology if HFTs have lower costs per unit

[1] There may be another equilibrium with coexistence. However, this equilibrium is trembling-hand-imperfect, and therefore ignored.

of speed than LFTs.

Our study contributes to the theoretical literature on informed trading and information production in financial markets. In this literature, traders typically use information for speculative trading by demanding liquidity. In such models (e.g., Kyle, 1985), informed liquidity demanders face price impact, which limits their demand. This attenuated demand in turn limits information acquisition and production when endogenized, like in Biais et al. (2015). Such effects are stronger when multiple informed investors compete (Nöldeke and Tröger, 2001), giving additional disincentives for information acquisition. Such competition can even increase endogenously due to learning from prices and order flows (called backrunning; see Yang and Zhu, 2019). By contrast, we model information acquisition that allows liquidity providers to avoid informed order flow. Hence, diseconomies of scale due to price impact or information-based competition do not arise. If anything, learning by LFTs from the state of the book reduces rather than increases competition due to a winners' curse.

Our paper also relates to the literature on common value auctions with differentially informed bidders. Liquidity provision in a limit order book can be viewed as a common value auction if there are no participation frictions or market power. Calcagno and Lovo (2006) is a good example. A winners' curse is a prime concern for bidders in common value auctions (see e.g., Hausch (1987)). This winners' curse would make it optimal for uninformed bidders to not participate in a one-shot auction with other informed bidders. This winner's curse is also present in our paper, but uninformed agents may still participate. The prime reason is market power due to a discrete tick size and the Poisson arrival process. In Calcagno and Lovo (2006), uninformed traders also post limit orders since they can learn from informed limit orders (similar to backrunning). Another difference is the nature of the signal. In Calcagno and Lovo (2006) informed traders suffer from learning-induced competition from uninformed traders. As a result informed traders do not fully reflect information in orders, similar to insiders in Kyle (1985). Such effects are absent in our setting for aforementioned reasons. Finally, we endogenize technology adoption and participation, which is not done in Calcagno and Lovo (2006).

Our paper also fits into the literature modeling dynamic trading in financial markets through limit order books, which includes Foucault (1999), Goettler et al. (2005),Goettler et al. (2009), Foucault et al. (2005), Parlour (1998), Li et al. (2018), Aït-Sahalia and Saglam (2017), Bernales (2014), and Roşu (2009). It also relates to the literature assessing the impact of HFT activity in financial markets. Numerous theoretical contributions emerged in recent years on this topic (see

Menkveld, 2016, for a review). However, only a limited number of those endogenize participation and technology adoption (e.g., Li et al., 2018, do this to a limited degree). We thereby take a long-run Industrial Organization perspective of the modern liquidity provision industry. In addition, we show that it is the product of speed and participation rate that matters for market outcomes (in contrast to Li et al., 2018, in which high speed is infinite).

The closest paper to ours is the one by Hoffmann (2014). His paper also models competition for liquidity provision between HFTs and LFTs, where HFTs are better able to prevent their quotes from being picked off. Yet, the economic implications differ at several important points. First, we find that higher HFT presence in equilibrium can increase expected half spreads even for the trades executed by HFTs. This is not the case in Hoffmann (2014). The reason is that in our setup, participation is always endogenous, whereas in his setup it is (in most cases) exogenous. Second, we find that HFTs can (but need not) be good for welfare, in particular when they have lower costs per unit of speed than LFTs, or when markets suffer from severe adverse selection in order flow. In Hoffmann (2014) HFTs are always bad for welfare when technology adoption is endogenized (his setting closest to ours). We allow for a tradeoff between an adverse selection-induced reduction of expected gains from trade against a better resource allocation due to HFTs having lower costs per unit of speed. This tradeoff is absent in Hoffmann (2014). Moreover, we allow HFTs to resolve no-trade, which also improves welfare. The third important difference is that in our paper LFTs become less prominent in response to HFT presence when HFT presence is high. In Hoffmann (2014) the same happens, but when HFT presence is low. The reason is that our effect is driven by a winner's curse, which is aggravated by endogenous participation. In Hoffmann (2014) the effect is driven by an exogenous link between the degree of informed trading (in his case specifically pick-off risk) and HFT presence. The fourth difference in implications is that when LFTs scale back their presence, their caution is reflected in timing in our paper (only offer liquidity when informed trading suspicions are low), whereas in Hoffmann (2014) that is not possible by assumption and their caution is reflected by less aggressive quotes. Interestingly, in these situations LFTs quote more rather than less aggressive quotes in our setting.

Our findings and model setup allow to explain and are supported by several empirical findings. A large number of studies has shown that market liquidity increases with the emergence of HFTs, especially at quote levels close to the mid price (e.g., Brogaard et al., 2014; Hasbrouck and Saar, 2012; Hendershott et al., 2011; Malinova et al., 2013). Our results on expected half spreads largely align with these empirical results. Moreover, the finding in Lyle et al. (2015) that enhanced market maker monitoring explains the majority of liquidity improvements in the 2000s is consistent with our model. HFT liquidity providers have been shown to be better informed (see Menkveld, 2013; Brogaard et al., 2014). Moreover, some recent studies show that HFTs retract liquidity in times of high information asymmetry (e.g., Anand and Venkataraman, 2016; Baldauf and Mollner, 2016; Korajczyk and Murphy, 2018). This observation is in line with the liquidity provider behavior in our model.

2 Setup

We model the participation and trading decisions of market makers with access to different trading technologies in a two-stage model. In the initial *participation stage*, potential liquidity providers decide to participate or not and decide which technology they adopt, if any. In the subsequent *trading stage*, liquidity providers compete for a market order. The game ends when the market order is executed or when all liquidity providers refuse to submit limit orders.[2] We now introduce the two stages in reverse order. For the reader's convenience, we provide a time line of the game in Fig. 1 and a notation summary in C.

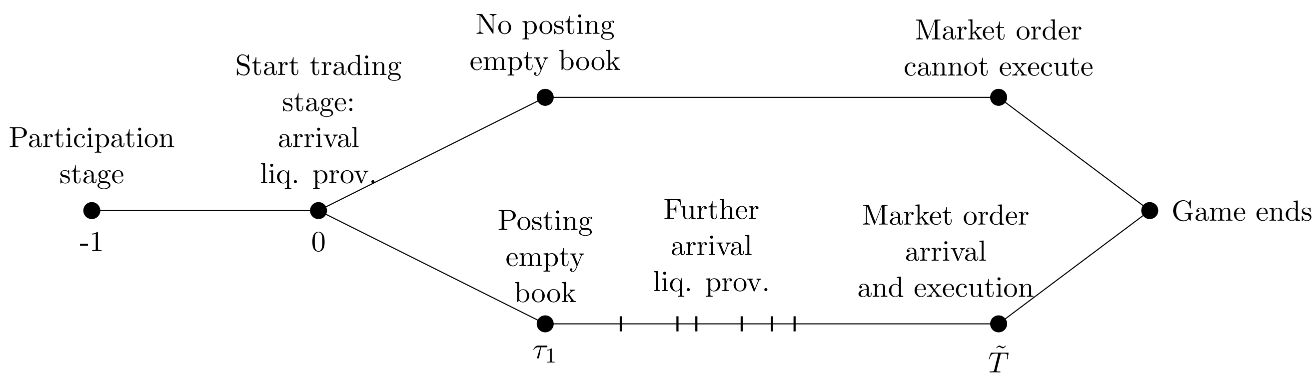


Figure 1: Time line of the game.

2.1 Trading Stage

The trading stage is inspired by Cordella and Foucault (1999). The market is characterized by a limit order book for a security with stochastic payoff $\widetilde{V}$. We only consider the ask side of the book, as the bid side is analogous. Conditional on public information, the expected value

[2] Hence, the baseline version of the model is static in nature (i.e., trading decisions are not serially correlated or path dependent based on previous trades). In Internet Appendix IA.3, we do provide a dynamic extension in which previous transactions do affect current quoting behavior through HFT learning. This way, we micro-found the signal production in the HFT information processing technology.

of the security, $E(\tilde{V})$, is given by μ. We also call this the fundamental price. The structure of the limit order book is provided in Fig. 2 below.

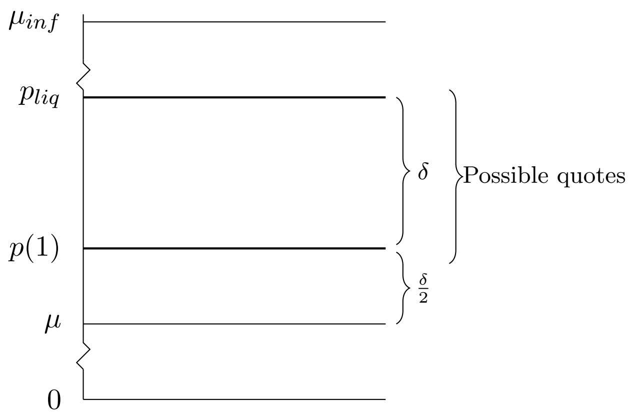


Figure 2: Price grid on which quotes can be posted (bold) on the ask side of the book.

The grid on which liquidity providers can post their quotes is discrete and characterized by the minimum tick size δ. A smaller δ implies a finer grid. As shown in Fig. 2, we assume that μ lies halfway between two ticks. Moreover, we define $p(1) = \mu + \frac{\delta}{2}$ as the "competitive price", the lowest possible price on the grid at which potentially profitable quotes can be posted (i.e., quotes that exceed μ). Furthermore, time and price priority hold.

At an exponentially distributed random time $\widetilde{T} \in [\tau_1, +\infty]$, a liquidity demander arrives, where τ_1 is the time at which the first limit order is posted. The liquidity demander is a passive player with exogenous behavior. Upon arrival, she submits a market order to buy 1 unit of the security if the best standing ask quote is lower than her reservation price. Upon the resulting transaction, the game ends. Liquidity demand is either uninformed (i.e., liquidity-induced) or informed. We model the type of liquidity demand as a randomly drawn state of nature $\zeta \in \{liq, inf\}$, where *liq* and *inf* denote the states with uninformed and informed order flow, respectively. The unconditional probabilities for states $\zeta = inf$ and $\zeta = liq$ are given by $\bar{\pi}$ and $1 - \bar{\pi}$, respectively. We assume that $E(\tilde{V}) = \mu$ when order flow is uninformed. We define order flow to be informed when the liquidity demander knows that $\tilde{V} = \mu_{inf} > \mu$. Under the assumptions made on reservation prices below, providing liquidity to informed liquidity demand yields trading losses.[3] The arrival intensity of the liquidity demand is information-specific

[3]The link between informed trading and a high value for $\tilde{V}$ is without loss of generality. The combination of an informed liquidity demander and asset value $\tilde{V} = \mu$ is irrelevant since she would be unwilling to pay a price strictly exceeding μ. An asset value $\tilde{V} = \mu_{inf}$ without informed trading need not be considered separately because μ is the expectation over all possible realizations. Some of these are high (potentially including μ_{inf}),

and given by ν_{liq} and ν_{inf} for uninformed and informed liquidity demand, respectively. In particular, we assume that informed liquidity demand is more impatient than a uninformed liquidity demand ($\nu_{inf} \neq \nu_{liq}$) for reasons outside of the model, such as perishability of private information. [4] To facilitate tractability, we assume that $\nu_{inf} = \infty$ throughout the paper. We further motivate this assumption and explain its contribution to tractability in Subsection 4.2.

The reservation price of the liquidity demander is independent of whether liquidity demand is informed or not and given by $p_{liq} \in (\mu, \mu_{inf})$.[5] In the baseline of the model, we assume that $p_{liq} = \mu + \frac{3}{2}\delta$, as also shown in Fig. 2. Due to this assumption, there are only two possible (non loss making) quote levels, p_{liq} and $p(1)$. The assumption that $p_{liq} = \mu + \frac{3}{2}\delta$ results in high tractability. We can, for example, derive the equilibrium participation rates and expected half spreads in closed form as a result of this assumption. The model with two quote levels is simple, but rich enough to analyze the strategic interactions of differentially informed traders at different speeds and with different participation costs. In Internet Appendix IA.2, we derive results for a more general model in which p_{liq} can be of arbitrary size and we get similar results. In the more general model, we can derive undercutting patterns along the lines of those in Hasbrouck (2018) and derive additional results on limit order aggressiveness. However, this simplification comes at the expense of higher complexity and the inability to solve equilibrium participation rates and expected half spreads in closed form.[6]

There are masses m and n of sophisticated (HFT) and unsophisticated (low-frequency trader, LFT) atomistic liquidity providers, respectively. These masses are determined endogenously in the participation stage, which is described later. Liquidity providers arrive to the market following Poisson processes that are characterized by the masses of HFTs and LFTs and their respective speeds. In particular, HFT and LFT arrival intensities are given by $\lambda\gamma m$ and λn, respectively, such that γ measures the *speed* advantage of HFTs relative to LFTs. This setup reflects the higher frequency with which HFts monitor the market and submit limit orders (as shown in, e.g., Baron et al., 2014; Brogaard et al., 2015; Hagströmer and Nordén, 2013;

while others are low (below μ). Hence, realizations may exceed or fall short of μ, but differences w.r.t. μ cancel out in expectation. However, if order flow depends on the realization of $\tilde{V}$, adverse selection losses can systematically materialize, which happens exactly when informed trading is paired with $\tilde{V} = \mu_{inf}$.

[4] In Internet Appendix IA.3 we provide micro-foundations for HFT information production. The assumption that $\nu_{liq} \neq \nu_{inf}$ is a necessary condition for these micro-foundations to generate meaningful signals.

[5] In the Internet Appendix IA.1.2 we show that model outcomes are identical for reservation prices that do depend on whether order flow is informed or not, as long as the reservation price for informed order flow is strictly (but arbitrarily) smaller than μ_{inf}.

[6] The generalized setting also allows for further extensions such as one in which liquidity demand becomes sensitive to the level of liquidity provision. See Internet Appendix IA.2.2.3.

Hendershott and Riordan, 2013).

Upon arrival to the market, each liquidity provider k posts a quote a to maximize her own profits given her information set ψ_k.[7] This way, LFTs act strategically despite being less sophisticated. LFTs do not have access to advanced information processing technology and their information set ψ_{LFT} consists solely of the standing best quote in the book or the absence of one (i.e., the book being empty). HFTs have access to superior information processing technology as compared to LFTs, which is captured by a different information set ψ_{HFT}. ψ_{HFT} contains a noisy (but informative) signal $s \in \{inf, liq\}$ about the state of nature, which is common to all HFTs. The signal $s = liq$ is correct with probability ϕ_1. Moreover, we assume that signals are unbiased, such that $P(s = liq) = P(\zeta = liq) = (1 - \bar{\pi})$. The unbiasedness assumption also implies that $P(s = inf) = P(\zeta = inf) = \bar{\pi}$. Moreover, it has the convenient implication that $P(s = liq|\zeta = liq) = P(\zeta = liq|s = liq) = \phi_1$ and $P(s = inf|\zeta = inf) = P(\zeta = inf|s = inf) = 1 - \frac{1-\bar{\pi}}{\bar{\pi}}(1 - \phi_1)$, due to Bayes' Rule. For notational convenience, we define

$$\phi_2 \equiv 1 - \frac{1 - \bar{\pi}}{\bar{\pi}}(1 - \phi_1) \tag{1}$$

for the rest of the paper. HFT signals are informative and therefore useful if $\phi_1 > (1 - \bar{\pi})$, which, due to Eq. (1), is equivalent to $\phi_2 > \bar{\pi}$.

Two information asymmetries arise in our model: one between the liquidity demander and liquidity providers, and one among liquidity providers. The resulting adverse selection concerns may be sufficiently severe that neither type of liquidity provider is willing to provide liquidity in an empty book. In this case, the game also ends, as shown in the upper branch of Fig. 1 (essentially, this is a market breakdown; see Milgrom and Stokey, 1982).

2.2 Participation Stage

In the initial participation stage, there is a unit mass of atomistic risk neutral agents, which simultaneously choose to invest in technology.[8] In particular, agents endogenously choose to become an LFT, an HFT, or stay out of the market. We denote the mass of agents that become HFTs and LFTs by $m \in [0, 1]$, and $n \in [0, 1-m]$, respectively.[9] In accordance with their choice, HFTs and LFTs incur total participation costs mC_{HFT} and nC_{LFT}, respectively. These costs

[7] In this setup, liquidity providers cannot demand liquidity. We show in Internet Appendix IA.1.4 that relaxing this assumption would if anything strengthen the effects we report.

[8] We consider a setting with a continuum of liquidity providers for tractability. It can be derived as the limit of a discrete case with large numbers of LFTs and HFTs. For more details, see Internet Appendix IA.4.

[9] We assume that the mass of potential liquidity providers is so large that the upper bounds do not bind.

are borne equally by all constituents in each respective group. Hence, individual HFTs and LFTs face costs per capita of C_{HFT} and C_{LFT}, respectively.These costs could be associated with IT infrastructure, accounts fees, or colocation fees.

Potential liquidity providers rationally maximize expected profits by choosing to adopt HFT, LFT, or no technology. In doing so, they account for expected trading profits and participation costs. While costs per capita are independent of m and n, expected trading profits may depend on m and n. For example, expected per capita trading profits are lower as m and n increase because expected trading surplus is shared by a larger mass of liquidity providers.

Our setting corresponds to a long-term equilibrium with free entry reflecting the a mature HFT industry in which the initial oligopoly due to unique technology access has dissolved.

In our analyses, we explore how market liquidity is affected by the availability of HFT technology. In doing so, we look at several dimensions. First, we look at expected transaction costs for trades that materialize, which we measure by the expected half spread S. Second, we look at the likelihood that liquidity demand is served.

3 Equilibrium Definition

In this section we provide a formal equilibrium definition. We work backwards, starting by deriving optimal quoting strategies R^*_{HFT}, R^*_{LFT}, for HFTs and LFTs, respectively, in the trading stage, taking m, n, and the state of the order book as given. Next, we derive expected profits as functions of m and n, given these optimal quoting strategies. The participation stage is in equilibrium if for a pair (m^*, n^*), HFTs, LFTs, and nonparticipants cannot benefit from changing their participation decisions. Hence, an equilibrium is fully characterized by a tuple $(R^*_{HFT}, R^*_{LFT}, m^*, n^*)$. For a given set of parameters, multiple equilibria may exist.

3.1 Trading Stage

We analyze liquidity provider k's order placement strategy, given standing best ask quote $\widehat{a} \in \{Q, \emptyset\}$. Given information set ψ_k, k's expected profit of posting a quote a is given by:

$$\Pi_k(a, \widehat{a}) = E\left(\Phi\left(a, \zeta\right) \cdot (a - \widetilde{V}) | \psi_k\right), \tag{2}$$

where $\Phi\left(a, \zeta\right)$ is the liquidity provider's execution probability corresponding to quote a and conditional on the nature of incoming order flow ζ, and $E(\cdot|\psi_k)$ is the liquidity provider's expectation over states of nature conditional on her information set. As price and time priority hold, a quote $a \geq \widehat{a}$ has zero execution probability and therefore zero expected profit. The same holds for a quote larger than the reservation price p_{liq}.

Each liquidity provider k optimizes her reaction function R_k to maximize expected profits:

$$R_k^*(\widehat{a}) = \arg\max_{R_k \in Q} \Pi_k(R_k, \widehat{a}), \tag{3}$$

where all liquidity providers behave according to R_{LFT}^* and R_{HFT}^*. As players are atomistic, the probability for a liquidity provider to arrive to the market repeatedly is zero.[10] Optimizing Eq. (3) yields the optimal order placement strategies R_{HFT}^* and R_{LFT}^*. The conditional execution probabilities Φ depend on optimal order placement strategies. Liquidity providers' optimal order placement strategies in turn depend on the expected execution probabilities. We define the trading stage to be in equilibrium when the liquidity provision strategies of HFTs and LFTs are characterized by a pair (R_{HFT}^*, R_{LFT}^*) such that R_{HFT}^* and R_{LFT}^* are optimal for any HFT or any LFT, respectively, given that all other HFTs and LFTs use the reaction functions R_{HFT}^* and R_{LFT}^*, respectively.

3.2 Participation Stage

Before the trading stage starts, equilibrium participation masses, m^* and n^* are determined in the participation stage. All potential liquidity providers decide simultaneously which technology to adopt, anticipating optimal reaction functions R_{HFT}^* and R_{LFT}^* and optimal participation rates (m^*, n^*) of all HFTs and LFTs, respectively. As a result, in equilibrium (potential) liquidity providers cannot (strictly) benefit from deviating from their participation decision. Optimality in participation results in zero-profit or indifference conditions (in line with empirical evidence in Baron et al., 2014). Since all HFTs and LFTs are identical within a group, they all solve the same optimization problem, and hence have identical equilibrium strategies.

HFT participation optimality implies zero profit, such that:

$$m^* = \begin{cases} 0, \text{ if } E_{\widehat{a}}\left(\Pi_{HFT}(R_{HFT}^*(\widehat{a}), \widehat{a}) | m^*, n^*\right) - C_{HFT} < 0 \ \ \forall m, \\ m^* \text{ that solves } E_{\widehat{a}}\left(\Pi_{HFT}(R_{HFT}^*(\widehat{a}), \widehat{a}) | m^*, n^*\right) - C_{HFT} = 0, \text{ otherwise}, \end{cases} \tag{4}$$

where $E_{\widehat{a}}$ is the expectation over all possible standing best quotes. Similarly, LFT participation

[10] It is possible to set up the model with a discrete number of LFTs and HFTs and allow for reentry. This model relaxation hardly affects the results substantially reduces tractability. See also Internet Appendix IA.4.

optimality implies zero profit, such that:

$$n^* = \begin{cases} 0, & \text{if } E_{\widehat{a}}\left(\Pi_{LFT}(R^*_{LFT}(\widehat{a}), \widehat{a})|m^*, n^*\right) - C_{LFT} < 0 \;\; \forall n, \\ n^* \text{ that solves } E_{\widehat{a}}\left(\Pi_{LFT}(R^*_{LFT}(\widehat{a}), \widehat{a})|m^*, n^*\right) - C_{LFT} = 0, & \text{otherwise.} \end{cases} \tag{5}$$

The participation stage is in equilibrium when zero-profit conditions (4) and (5) hold simultaneously.

4 Quote dynamics and trading costs

In this section, we characterize the equilibrium order placement strategies in the trading stage, taking the masses of liquidity providers m and n as given. We first derive equilibrium strategies for the *uninformed trading case*, in which order flow is uninformed with certainty. The uninformed case is illustrative for our model setup and an important building block for the more general case with informed trading. Next, we develop the *informed trading case* in which order flow can be informed and HFTs may receive informative signals about its nature.

4.1 Uninformed Trading Case

The uninformed trading case is obtained by setting $\bar{\pi} = 0$, such that $\zeta = liq$ with certainty. As divergences in information processing capacities do not matter in the uninformed case, we abstract from the information sets ψ_k in this Subsection for notational convenience.

Consider the arrival of liquidity provider k at time τ before the arrival time $\widetilde{T}$ of the market order. If the standing best quote $\widehat{a} = p(1)$, it is impossible to post a strictly profitable quote. Hence, queue joining at $p(1)$ with zero execution probability is (weakly) optimal. If the standing best quote $\widehat{a} = p_{liq}$, undercutting to $p(1)$ yields a strictly positive, guaranteed profit of $\frac{1}{2}\delta$. Finally, if the standing best quote exceeds the reservation price or the book is empty, an arriving liquidity provider trades off an uncertain profit of $\frac{3}{2}\delta$ by posting at p_{liq} against a certain profit of $\frac{1}{2}\delta$ by quoting $p(1)$. Posting $a = p_{liq}$ is relatively more attractive if there is a higher execution probability for that quote. We formalize this intuition by characterizing the optimal quote submission strategies in the following Proposition.

Proposition 1. *(Equilibrium Order Placement Strategies - Uninformed Trading Case). Any liquidity provider* $k \in \{HFT, LFT\}$ *optimally follows the following strategy given a standing* *best ask quote $\widehat{a}$:*

$$R_k^* = \begin{cases} p_{liq} & \textit{if } (\widehat{a} - \delta \geq p_{liq} \textit{ or } \widehat{a} = \emptyset) \textit{ and } \Phi \geq \frac{1}{3} \\ p(1) & \textit{otherwise} \end{cases}, \tag{6}$$

where

$$\Phi \equiv \frac{\nu_{liq}}{\nu_{liq} + \lambda(n + \gamma m)}. \tag{7}$$

Proof. See Appendix. □

Since the setup of the trading stage is inspired by Cordella and Foucault (1999), the result in Proposition 1 is in line with their results (for the case with only two quote levels).

We define the half spread of a trade as the difference between the transaction price and μ. In expectation it is given by

$$S = \begin{cases} \frac{1}{2}\delta + \Phi\delta, & \text{if } \Phi \geq \frac{1}{3}, \\ \frac{1}{2}\delta, & \text{otherwise.} \end{cases} \tag{8}$$

Holding everything else equal, the expected half spread decreases in the aggregate arrival intensity of liquidity providers and increases in the arrival intensity of liquidity demanders for two reasons. Lower arrival intensity of liquidity demand and higher aggregate arrival intensity of liquidity supply both increase the probability that a quote $a = p_{liq}$ is undercut and hence decrease the execution probability Φ at $a = p_{liq}$. If a market order is more likely to execute at $p(1)$, the expected half spread is lower. Moreover, with a low Φ, it is less likely that an initial quote at p_{liq} is more profitable in expectation than one at $p(1)$ and the aggressiveness of initial quotes increases. Hence, liquidity increases in HFT presence m, LFT presence n, and HFT speed γ as these all intensify competition. Proposition 1 shows how HFTs improve market liquidity absent information processing asymmetry (as shown by Brogaard et al., 2014; Hasbrouck and Saar, 2012; Hendershott et al., 2011; Malinova et al., 2013). Proposition 1 is also consistent with Jiang et al. (2014), who find that in noninformational periods, HFTs heavily compete in the U.S. Treasury market, and thereby drive spreads down.

Eq. (8) shows that if $n+\gamma m$ is sufficiently large, spreads become insensitive to participation, which is less interesting for analyzing how liquidity depends on technology availability. Moreover, in a more general (but less tractable) model with $p_{liq} = (r + \frac{1}{2})\delta + \mu$ for $r \in \mathbb{N}^+$, p_{liq} is optimally initially quoted when $\Phi \geq \frac{1}{2r+1}$. Hence, the parameter region in which $p(1)$ is initially quoted shrinks as r increases. For these reasons we assume throughout the paper that $\Phi \geq 1/3$, or equivalently, $(n+\gamma m) < \frac{2\nu_{liq}}{\lambda}$. In B we derive conditions expressed only in structural parameters for this assumption to hold. Moreover, we also show in B that if this condition holds in the uninformed case, it must also hold in the informed case.

4.2 Informed Trading Case

In this subsection, we work out the trading stage of the model in the presence of information asymmetry among liquidity providers, (i.e., $\bar{\pi} > 0$ and $\phi_2 > \bar{\pi}$). HFTs can process information better than LFTs. Therefore, HFTs can (partially) avoid informed order flow, leaving it to LFTs (if any). Hereby, HFTs reduce expected profits for LFTs and increase their own. In other words, the informational advantage of HFT can increase adverse selection experienced by LFTs.

To facilitate exposition and tractability, we assume an infinitely impatient informed liquidity demander, that is $\nu_{inf} = \infty$ (for reasons outside of the model such as perishable information).[11] The informed liquidity demander monitors the market constantly and arrives instantaneously when a quote (weakly) below her reservation price is posted. The advantage of setting $\nu_{inf} = \infty$ is that the state of nature is revealed upon posting a quote. As a result, the inference for liquidity providers that subsequently arrive is trivial: order flow is uninformed. Hence, if an initial quotes survives, the trading game reduces to the uninformed case (see Proposition 1). Therefore, potentially informed order flow only affects HFT and LFT strategies upon arrival to an empty book.

When a liquidity provider arrives to an empty book, she will only quote $a = p_{liq}$ when the expected profits from doing so outweigh the expected adverse selection losses. HFTs can condition their strategies on their signal. In some situations, however, the signal does not affect their optimal quoting strategy because of being insufficiently accurate. Not quoting is optimal for HFTs if expected profits conditional on a signal $s = liq$ being correct are small given the expected losses conditional on the signal being incorrect. Similarly, posting quotes in an empty book is always optimal for HFTs if expected profits conditional on the a signal $s = inf$ being incorrect are large given the expected losses conditional on the signal being correct. In all other scenarios, HFTs optimally post $a = p_{liq}$ in an empty book if $s = liq$ and refrain from quoting in

[11] We can allow for more patient informed liquidity demanders, at the expense of reduced tractability and increased complexity. The main results are largely unaffected. See Internet Appendix IA.1.3.

an empty book when $s = inf$. The above notions are summarized in the following Proposition.

Proposition 2. *HFTs optimally quote $a = p_{liq}$ in an empty limit order book if*

$$(p_{liq} - \mu)\hat{P}(\zeta = liq|\psi_{HFT})\Phi(\zeta = liq) \geq (\mu_{inf} - p_{liq})\hat{P}(\zeta = inf|\psi_{HFT})\Phi(\zeta = inf), \tag{9}$$

where $\hat{P}(\zeta = inf|\psi_{HFT})$ and $\hat{P}(\zeta = liq|\psi_{HFT})$ are the posterior probabilities for HFTs of having an informed or uninformed trader as the first liquidity demander to come to the market, respectively, and $\Phi(\zeta)$ is the execution probability conditional on ζ.

This inequality is always satisfied if

$$\phi_2 \leq \frac{(p_{liq} - \mu)\Phi}{(p_{liq} - \mu)\Phi + (\mu_{inf} - p_{liq})}, \tag{10}$$

and is never satisfied if

$$\phi_1 \leq \frac{\mu_{inf} - p_{liq}}{(p_{liq} - \mu)\Phi + (\mu_{inf} - p_{liq})}. \tag{11}$$

Upon arrival to a nonempty book with standing best quote $\widehat{a} \leq p_{liq}$, HFTs optimally quote $a = p(1)$.

Proof. See Appendix. □

Note that nobody posts quotes when Condition (11) holds. If it is never profitable for HFTs to post in an empty book, the same must be true for LFTs, as HFTs have superior information.

For an LFT to post in an empty book, the expected trading profit conditional on her information set also needs to be positive. The only difference compared to the HFT profitability criterion (9) is the information set ψ_{LFT} which, contrary to ψ_{HFT}, does not contain a signal. LFTs only observe whether the book is empty or not upon arrival. Subsequently they use Bayes' Rule to form rational expectations about the HFT signal s and ultimately ζ. Intuitively, when the presence of HFTs is high compared to LFTs ($\gamma m >> n$) and $s = liq$, it is very unlikely that an LFT would arrive to an empty book first. Yet, when $s = inf$ this probability equals 1 (assuming that HFTs condition their quote strategy on signal s). This high conditional probability gives rise to a winner's curse: LFTs can provide liquidity at p_{liq} almost exclusively when it is unfavorable to do so. By contrast, when the presence of HFTs compared to LFTs is low ($\gamma m << n$) the probability of an LFT arriving to an empty book when $s = liq$ is high and the winner's curse is much less of a concern for LFTs. The winner's curse is also more harmful as the adverse selection losses $(\mu_{inf} - p_{liq})$ are larger and the uninformed trading gains are lower (lower δ). We summarize these notions in the following Proposition.

Proposition 3. *LFTs optimally post quotes $a = p_{liq}$ in an empty book if*

$$(p_{liq} - \mu)\hat{P}(\zeta = liq|\psi_{LFT})\Phi(\zeta = liq) \geq (\mu_{inf} - p_{liq})\hat{P}(\zeta = inf|\psi_{LFT})\Phi(\zeta = inf) \quad (12)$$

where $\hat{P}(\zeta = inf|\psi_{LFT}) = 1 - \hat{P}(\zeta = liq|\psi_{LFT})$ is the LFTs' posterior probability for the order flow to be informed. $\hat{P}(\zeta = inf|\psi_{LFT})$ is increasing in the mass of HFTs (m), HFT speed (γ), and decreasing in the mass of LFTs (n). Given a standing best quote $\widehat{a} \leq p_{liq}$, LFTs optimally quote $a = p(1)$.

Proof. See Appendix. □

The expected half spread in the informed case is given by

$$S = \begin{cases} \frac{1}{2}\delta + \delta(\bar{\pi} + (1-\bar{\pi})\Phi), & \text{if (12) is satisfied,} \\ \frac{1}{2}\delta + \delta((1-\phi_1) + \phi_1\Phi), & \text{otherwise,} \end{cases} \quad (13)$$

where Φ is defined as in (7).

5 Profitability and Participation

In this section, we determine the equilibrium masses m^* and n^* in the participation stage, taking optimal quoting strategies in the trading stage as given. First, we express the zero-profit conditions (4) and (5) as functions of m and n. Thereafter, we derive equilibrium participation rates (m^*, n^*) and analyze all liquidity dimensions. Up to Subsection 5.2.2, we assume adverse selection in market orders to be sufficiently low to prevent market breakdowns in the absence of signals. In Subsection 5.2.3, we analyze the case in which incoming order flow is so toxic that markets would break down in the absence of additional signals for liquidity providers.

5.1 Uninformed Trading Case

To calculate the equilibrium masses, we first need to derive the expected per capita trading profits for HFTs and LFTs. Transactions materialize either at the competitive price $p(1)$ or at the reservation price p_{liq}. The latter only happens with probability Φ. Moreover, due to the Poisson arrival process, liquidity providers participate in such transactions according to relative presence in the market. We formalize these notions in the following Lemma.

Lemma 1. *The unconditional expected per capita trading profits for LFTs and HFTs are respectively given by:*

$$E_{\widehat{a}}\left(\Pi_{HFT}(R^*_{HFT}(\widehat{a})|m,n)\right) = \frac{1}{m}\frac{\gamma m}{\gamma m+n}\Pi = \frac{\gamma\Pi}{\gamma m+n}, \tag{14}$$

$$E_{\widehat{a}}\left(\Pi_{LFT}(R^*_{LFT}(\widehat{a})|m,n)\right) = \frac{1}{n}\frac{n}{\gamma m+n}\Pi = \frac{\Pi}{\gamma m+n}, \tag{15}$$

where

$$\Pi = \frac{1}{2}\delta + \Phi\delta. \tag{16}$$

Proof. See Appendix. □

The interpretation of the expressions in Lemma 1 is as follows. HFTs and LFTs share aggregate expected profits from trading Π according to their relative presence ($\frac{\gamma m}{n+\gamma m}$ and $\frac{n}{n+\gamma m}$, respectively). Moreover, each trader shares proportionally in its own expected group profits (with factors $\frac{1}{m}$, $\frac{1}{n}$, respectively). The total trading profits are derived as the probability-weighted average transaction price minus the fundamental value μ. As expected trading profits for both HFTs and LFTs are monotonically decreasing in m and n, and per capita participation costs are constant, there is always an equilibrium with strictly positive participation.

At this point, we can rewrite net expected HFT profits in Eq. (14) as

$$\begin{aligned} E_{\widehat{a}}\left(\Pi_{HFT}(R^*_{HFT}(\widehat{a})|m,n)\right) - C_{HFT} &= \frac{1}{m}\frac{\gamma m}{\gamma m+n}\Pi - C_{HFT}, \\ &= \gamma\left(\frac{1}{\gamma m+n}\Pi - \frac{C_{HFT}}{\gamma}\right). \end{aligned} \tag{17}$$

In the last expression, m only shows up in a product with γ. Moreover, up to a scalar multiplication, this expression corresponds to expected LFT profits in Eq. (5), but with C_{LFT} replaced by $\frac{C_{HFT}}{\gamma}$. Hence, participation cost per unit of speed drive technology adoption.[12]

We can now derive equilibrium HFT and LFT masses. There is a competitive market with free entry. Therefore, equilibrium prices must equal production costs of the liquidity provider type with the lowest cost per unit of speed. Liquidity is then exclusively provided by liquidity providers with lowest cost per unit of speed, as only for them liquidity provision is (weakly) profitable. Moreover, a high arrival intensity of liquidity demand, boosts participation

[12] An alternative interpretation of this result is that our original problem is equivalent to solving a related problem in which all HFTs have speed 1 and participation cost $\frac{C_{HFT}}{\gamma}$.

of liquidity providers, such that expected trading profits still equal zero. We formalize these notions in the following proposition.

Proposition 4. *In the uninformed trading case, liquidity provision in equilibrium is conducted only by HFTs when $C_{LFT} \geq \frac{C_{HFT}}{\gamma}$, and only by LFTs otherwise. This equilibrium is unique. Equilibrium participation rates are given by*

$$(m^*, n^*) = \begin{cases} \left(\frac{-(\nu_{liq}\frac{C_{HFT}}{\gamma} - \frac{\lambda\delta}{2}) + \sqrt{(\nu_{liq}\frac{C_{HFT}}{\gamma} - \frac{\lambda\delta}{2})^2 + 6\frac{C_{HFT}}{\gamma}\lambda\nu_{liq}\delta}}{2\lambda C_{HFT}}, 0 \right), & \text{if } \frac{C_{HFT}}{\gamma} \leq C_{LFT} \\ \left(0, \frac{-(\nu_{liq}C_{LFT} - \frac{\lambda\delta}{2}) + \sqrt{(\nu_{liq}C_{LFT} - \frac{\lambda\delta}{2})^2 + 6C_{LFT}\lambda\nu_{liq}\delta}}{2\lambda C_{LFT}} \right), & \text{otherwise.} \end{cases} \tag{18}$$

The expected half spread is given by

$$S = \frac{1}{2}\delta + \frac{\nu_{liq}}{\nu_{liq} + \lambda(n^* + \gamma m^*)}\delta. \tag{19}$$

The availability of HFT technology strictly reduces S iff $\frac{C_{HFT}}{\gamma} < C_{LFT}$.

Proof. See Appendix. □

For the rest of the paper, we refer to an equilibrium with $m^* > 0$, $n^* = 0$, and HFTs not using any signal as "Nonconditioning HFT Dominance."

Summarizing, only if HFTs incur lower costs per unit of speed than LFTs, they participate. They then completely take over and lead to lower expected half spreads than if HFT technology were not available. This result is also presented graphically in Panel A of Fig. 3.

5.2 Informed Trading Case

This subsection provides the main results of the paper. There are three possible scenarios in which the availability of HFT technology can affect market outcomes. First, if information processing technology for HFTs is insufficiently useful (Subsection 5.2.1), HFTs only provide liquidity (and are the only liquidity providers) when they have the lowest cost per unit of speed. Second, if HFTs have material informational advantages over LFTs and these compete with one another (Subsection 5.2.2), HFTs may impose a winner's curse on LFTs. This winner's curse may, depending on the relative presence of HFTs vs LFTs, prevent LFTs from supplying liquidity in an empty book. As a result of reduced profits, LFTs may not participate. Third, if adverse selection is sufficiently severe to make markets break down (Subsection 5.2.3), informationally advantaged HFTs may (partially) restore markets.

5.2.1 Equilibrium with no or useless Information Processing Technology

Consider the case with a strictly positive probability of informed liquidity demand ($\bar{\pi} > 0$), but HFT information processing technology that is inaccurate ($\phi_2 = \pi$). The equilibrium can be derived from the uninformed case in Subsection 5.1, if adverse selection from informed order flow is sufficiently small. We derive an upper bound on $\bar{\pi}$ for this condition to be met.

As a first step, we assume all liquidity providers to act in the trading stage as in the uninformed case (see Proposition 1). Informed liquidity demand generates unavoidable losses for HFTs and LFTs alike, as neither can use conditioning information. Moreover, trades are profitable only with probability $(1 - \bar{\pi})$. This profit base needs to cover participation costs and adverse selection losses. It turns out that expected trading profits are given by a linear transformation of those in the uninformed case. Expected trading profits are reduced by expected adverse selection losses ($-\bar{\pi}(\mu_{inf} - p_{liq})$) and by a lower profit base (a factor $(1 - \bar{\pi}) < 1$). The effect for HFTs is γ times as strong due to their speed being γ times as high.

Lemma 2. *If liquidity providers quote as in Proposition 1, expected trading profits correspond to those in the uninformed case, but with aggregate expected trading profits* $\tilde{\Pi}$ *given by*

$$\tilde{\Pi} \quad = (1 - \bar{\pi})\Pi - \bar{\pi}(\mu_{inf} - p_{liq}). \tag{20}$$

Proof. See Appendix. □

It follows from Lemma 2 that per capita expected trading profits for HFTs and LFTs are given by $\gamma \tilde{f}(n + \gamma m)$ and $\tilde{f}(n + \gamma m)$, respectively, where

$$\tilde{f}(n + \gamma m) = \frac{\tilde{\Pi}}{n + \gamma m}. \tag{21}$$

Since $\tilde{f}(\cdot)$ is hyperbolic in $n + \gamma m$, a strictly positive mass of liquidity providers must materialize in equilibrium if aggregate expected trading profits are strictly positive. Expected trading profits are strictly positive if $\bar{\pi}$ in (20) is sufficiently small. If not, neither type of liquidity provider participates.

Lemma 3. *In the absence of an informative signal, some potential liquidity providers optimally* *participate when*

$$\bar{\pi} < \frac{3\delta}{2(\mu_{inf} - \mu)} \equiv \pi^{tox}. \tag{22}$$

Proof. See Appendix. □

We assume for the remainder of this subsection that $\bar{\pi} < \pi^{tox}$. Since signals are useless, quote submission strategies follow Proposition 1, which validates the assumption that we started with. In line with the results in Subsection 5.1, we conclude that the availability of HFT technology either improves liquidity or leaves it unaffected. Since HFTs cannot condition on superior information, liquidity providers always provide liquidity. Lemma 2 and Proposition 4 then imply that HFT technology is adopted iff its cost per unit of speed is lowest. We summarize our results in the following proposition.

Proposition 5. *If $\bar{\pi} \in (0, \pi^{tox})$ and, $\phi_2 = \bar{\pi}$, the availability of HFT technology never reduces market liquidity and liquidity is always offered. HFT technology takes over completely and market liquidity improves iff HFT technology has a lower cost per unit of speed than LFT technology. Equilibrium participation rates are given by*

$$(m^*, n^*) = \begin{cases} \left(0, \frac{\lambda c - \nu_{liq} C_{LFT} + \sqrt{(\nu_{liq} C_{LFT} - \lambda c)^2 + 4\lambda \nu_{liq} C_{LFT}(c + (1-\bar{\pi})\delta)}}{2\lambda C_{LFT}}\right), & \textit{if } \frac{C_{HFT}}{\gamma} > C_{LFT}, \\ \left(\frac{\lambda c - \nu_{liq} \frac{C_{HFT}}{\gamma} + \sqrt{(\nu_{liq} \frac{C_{HFT}}{\gamma} - \lambda c)^2 + 4\lambda \nu_{liq} \frac{C_{HFT}}{\gamma}(c + (1-\bar{\pi})\delta)}}{2\lambda C_{HFT}}, 0\right), & \textit{otherwise}, \end{cases} \tag{23}$$

where

$$c = (1 - \bar{\pi})\frac{\delta}{2} - \bar{\pi}(\mu_{inf} - p_{liq}). \tag{24}$$

Proof. See Appendix. □

The results in Proposition 5 immediately extend to situations in which information processing technology is accurate ($\phi_2 > \bar{\pi}$), but irrelevant for quoting strategies. If 1.) informed trading losses are small compared to reservation prices, 2.) the execution probability is high due to a high arrival intensity of (uninformed) market orders relative to that of limit orders, or 3.) the signal is informative, but still rather inaccurate, Condition (10) holds and HFTs always quote (and hence ignore their signal). Condition (10) depends on the endogenous variables $n^* + \gamma m^*$ through Φ. Equating $\tilde{f}(\cdot)$ to participation costs (because of the zero-profit condition) and solving yields $n^* + \gamma m^*$ expressed in exogenous parameters, which can then be substituted into (10), leading to the following corollary.

Corollary 1. *The results from Proposition 5 extend to the setting with $\bar{\pi} \in (0, \pi^{tox})$ and $\phi_2 > \bar{\pi}$ when*

$$\phi_2 \leq \left(1 + \frac{\mu_{inf} - p_{liq}}{p_{liq} - \mu}\left(1 + \frac{\lambda}{\nu_{liq}}\tilde{f}^{-1}\left(\min\left(\frac{C_{HFT}}{\gamma}, C_{LFT}\right)\right)\right)\right)^{-1} \equiv \phi_2^{ul}. \tag{25}$$

Proof. See Appendix. □

5.2.2 Equilibria with useful Information Processing Technology

When $\bar{\pi} \in (0, \pi^{tox})$, $\phi_2 > \max(\bar{\pi}, \phi_2^{ul})$, the availability of superior information processing technology may expose LFTs to a winners' curse problem. The extent to which this problem arises depends on model parameters. As a result, we obtain different equilibrium types, which we analyze in this subsection.

If HFTs have higher costs per unit of speed than LFTs, multiple equilibria can materialize. The reason is that the winner's curse in Proposition 3 is particularly severe if the presence of HFTs (γm) relative to LFTs (n) is high. By contrast, with a low presence of HFTs relative to LFTs, LFTs are hardly affected by a winner's curse and have a cost advantage.

With multiple equilibria, coexistence is possible. The reason is that the LFT cost advantage (vis-a-vis HFTs) is offset by an informational disadvantage and (relatedly) reduced profitability due to not quoting in an empty book.

We start our analysis by deriving expected per capita trading profits for individual LFTs and HFTs, given their optimal quote posting strategies. To focus on adverse selection-induced effects, we assume that $\bar{\pi} \in (0, \pi^{tox})$ and $\phi_2 > \max(\phi_2^{ul}, \bar{\pi})$ throughout this section.

With the availability of information processing technology, expected per capita profits depend on whether LFTs post in an empty book (i.e., whether Condition (12) is satisfied) and are given in the following Lemma.

Lemma 4. *Unconditional expected per capita trading profits for HFTs and LFTs are given by*

$$E_{\widehat{a}}\left(\Pi_{HFT}(R^*_{HFT}(\widehat{a}))\right) = \begin{cases} g_{HFT}(m,n) \text{ , if (12) is not satisfied,} \\ h_{HFT}(m,n) \text{ , otherwise,} \end{cases} \tag{26}$$

$$E_{\widehat{a}}\left(\Pi_{LFT}(R^*_{LFT}(\widehat{a}))\right) = \begin{cases} g_{LFT}(m,n) \text{ , if (12) is not satisfied,} \\ h_{LFT}(m,n) \text{ , otherwise,} \end{cases} \tag{27}$$

respectively, where

$$
\begin{aligned}
g_{LFT}(m,n) &= (1-\bar{\pi})\phi_1(1-\Phi)\frac{\frac{1}{2}\delta}{n+\gamma m},\\
g_{HFT}(m,n) &= \gamma\left(g_{LFT}(m,n) + (1-\bar{\pi})\frac{\phi_1\Phi(p_{liq}-\mu)-(1-\phi_1)(\mu_{inf}-p_{liq})}{\gamma m}\right),\\
h_{HFT}(m,n) &= \gamma\left((1-\bar{\pi})\frac{(1-\Phi)\frac{1}{2}\delta+\phi_1\Phi(p_{liq}-\mu)-(1-\phi_1)(\mu_{inf}-p_{liq})}{n+\gamma m}\right),\\
h_{LFT}(m,n) &= \frac{1}{\gamma}h_{HFT}(m,n) + \bar{\pi}\frac{(1-\phi_2)\Phi(p_{liq}-\mu)-\phi_2(\mu_{inf}-p_{liq})}{n}.
\end{aligned}
\tag{28}
$$

If LFTs do not quote in an empty book, they only participate to undercut a quote $\hat{a} = p_{liq}$. LFTs undercut when signal $s = liq$ (with probability $(1-\bar{\pi})$), the signal is correct (with probability ϕ_1), and the initial quote is not executed (with probability $(1-\Phi)$). The expected profit of undercutting equals $\frac{1}{2}\delta$ for both LFTs and HFTs. HFTs also expect profits from quoting in an empty book (the last term in $g_{HFT}(m,n)$). HFTs quote in an empty book when $s = liq$ (with probability $(1-\bar{\pi})$). The quote yields a profit $(p_{liq}-\mu)$ when the signal is correct (with probability ϕ_1) and it is executed (with probability Φ), or a loss of $(\mu_{inf}-p_{liq})$ when the signal is incorrect (with probability $(1-\phi_1)$). Expected profits are divided equally among the mass of HFTs, m. HFTs also have a factor γ in their expected trading profit functions, reflecting their higher speed and therefore higher market presence.

Now assume that LFTs quote in an empty book. The initial quote is undercut when $\zeta = liq$ (with probability $(1-\bar{\pi})$) and it is not executed (with probability $(1-\Phi)$). Undercutting yields a profit of $\frac{1}{2}\delta$. In addition HFTs expect profits from providing liquidity in an empty book as before, but now share expected profits with LFTs (the last term in $h_{HFT}(m,n)$). LFTs have the same additional expected profits from providing liquidity in an empty book as HFTs, but also incur expected losses due to trading when HFTs suspect order flow to be toxic (last term in $h_{LFT}(m,n)$). LFTs incur additional trading losses when $s = inf$ (with probability $\bar{\pi}$). LFTs then share among themselves a loss of $(\mu_{inf}-p_{liq})$ if the HFT signal s is correct (with probability ϕ_2) and a profit of $(p_{liq}-\mu)$ if the signal is incorrect (with probability $(1-\phi_2)$) and the first quote is executed (with probability Φ).

We continue by analyzing equilibria in different ranges of participation cost parameters. We first formalize our results in Proposition 6, which is graphically illustrated by Panel B of Fig. 3. Next, we discuss the different equilibria and associated cost parameter ranges in more detail by providing intuition as well as graphical representations. A sensitivity analysis for the other model parameters is provided in Internet Appendix IA.1.1.

We start out by formally describing all possible equilibria and the cost parameter ranges in which they can materialize. To separate equilibria with $n^* = 0$, $m^* > 0$ and HFTs conditioning their quote strategy on their signal s from those without conditioning on s, we refer to the former as "Conditioning HFT Dominance equilibria."

Proposition 6. *There are unique thresholds $K_1 > K_2 > K_3 \in (0,1)$, such that when*

- $\frac{C_{HFT}}{\gamma C_{LFT}} > K_1$, *LFT Dominance is the only possible equilibrium,*
- $\frac{C_{HFT}}{\gamma C_{LFT}} \in (K_2, K_1]$, *LFT Dominance and a Coexistence equilibrium are possible,*
- $\frac{C_{HFT}}{\gamma C_{LFT}} \in [K_3, K_2]$, *LFT Dominance and Conditioning HFT Dominance are possible,*
- $\frac{C_{HFT}}{\gamma C_{LFT}} < K_3$, *Conditioning HFT Dominance is the only possible equilibrium.*

Proof. See Appendix. Closed form expressions for equilibrium participation rates for both HFTs and LFTs, and thresholds K_1, K_2, K_3 are provided in the proof. □

The result in Proposition 6 is represented graphically in Fig. 3, Panel B. We discuss and illustrate the different parameter regions and their associated equilibria below.

Consider Fig. 4 to 7. The mass of LFTs (n) and mass of HFTs (m, scaled by γ for exposition) are on the x- and y-axis, respectively. The red solid curve is the indifference curve for LFTs between posting in an empty book or not (the values for which Condition (12) binds exactly). The blue circled and green squared curves correspond to the zero-profit conditions (4) and (5) for HFTs and LFTs, respectively. These curves partition the $(n, \gamma m)$ space such that expected profits from participation are strictly positive below the curve and strictly negative above the curve. These curves can also be interpreted as indifference curves (relative to nonparticipation) for HFTs and LFTs, respectively. Equilibria at internal values of $(n, \gamma m)$ are located at points where the green and blue indifference curves intersect. Alternatively, equilibria could manifest as corner solutions with either $m = 0$ or $n = 0$. The equilibria are indicated by numbered black markers in Fig. 4 to 7.

We can use these curves to get more intuition for the effects at work. Higher aggregate participation ($n + \gamma m$) intensifies competition (competition effect). The competition effect makes LFTs and HFTs substitutes and gives downward sloping indifference curves. For the LFT indifference curve corresponding to $h_{LFT}(m, n) = C_{LFT}$ (right of the red curve), there is another effect. A higher presence of LFTs relative to HFTs weakens the winner's curse that LFTs are exposed to (winner's curse effect). When n is low, the winner's curse effect is dominant for LFTs, while the competition effect is dominant when n is high. As a result, the indifference curve in the region right of the red curve is hump-shaped. When the red curve is crossed, LFTs stop providing liquidity in an empty book, leading to a small upward jump in HFT profitability (due to lower competition) and a small downward jump in LFT profitability (due to reduced likelihood of liquidity demand being served).

We first analyze the case in which LFTs have much lower costs per unit of speed than HFTs (Fig. 4 and the solid blue range in Panel B of Fig. 3). HFTs optimally refrain from participation for cost reasons, despite their informational advantage. As a result, all liquidity is provided by LFTs. Since no signals are used in this *LFT Dominance* equilibrium, market outcomes conform to Proposition 5: liquidity is always provided and solely by the liquidity provider with the lowest cost per unit of speed.

As HFT participation costs decline, we get multiple equilibria (Fig. 5 and the green horizontally striped region of Panel B in Fig. 3). LFT Dominance is one equilibrium. Yet, if the mass of HFTs is sufficiently shocked upwards, LFTs are increasingly adversely selected and expected LFT profits decline. As a result, LFT participation is reduced compared to LFT Dominance. The adverse selection losses are now borne by fewer LFTs, such that per capita adverse selection losses for LFTs increase. Simultaneously, the drop in LFT participation reduces competition intensity and thereby provides room increased HFT participation. Increased HFT participation in turn further reduces LFTs profits from providing liquidity to uninformed order flow, reducing LFT participation further, etc. At some point LFTs optimally refrain from quoting in an empty book and only undercut (the red curve in Fig. 5 is crossed). A *Coexistence Equilibrium* then materializes (marker 2 in Fig. 5), in which the cost advantage of LFTs (vs HFTs) is exactly offset by an information disadvantage. Moreover, HFTs do not quote in an empty book when $s = inf$. Hence, with strictly positive probability no liquidity is offered.

As HFT participation costs decline further we again get multiple equilibria, but some are of different nature (Fig. 6 and the orange diagonally striped area in Panel B of Fig. 3). LFT Dominance is still an equilibrium, but, there are also two others. One is a *Conditioning HFT Dominance Equilibrium* (indicated by marker 2 in Fig. 6) in which the LFT cost advantage is more than offset by an information disadvantage. The last one is an *Instable Coexistence Equilibrium* (indicated by marker 3), in which LFTs always participate in an empty book.

Expected half spreads are high because HFTs provide some liquidity despite having higher cost per unit of speed than LFTs. The equilibrium is instable (trembling-hand-imperfect) and therefore further ignored in the analysis.

As HFT participation costs decline further, Conditioning HFT Dominance becomes the only equilibrium (Fig. 7 and the red checked area in Panel B of Fig. 3), because the informational advantage of HFTs always outweighs the LFT cost advantage, if any. As before, no liquidity is offered with strictly positive probability.

We now analyze the liquidity implications of technology availability. As we move away from LFT Dominance, expected half spreads are affected in three ways. First, LFTs do not participate in an empty book, which lowers their profitability and thereby their participation. As a result competition is impaired. Second, HFTs attain a sizeable market share. To the extend that HFTs have higher costs per unit of speed than LFTs, aggregate costs for liquidity provision go up, which reduces the total amount of liquidity provision and increases expected half spreads. A countervailing effect on expected half spreads originates from an aggregate reduction in adverse selection losses. As a result, expected half spreads can in- or decrease as we move away from LFT Dominance. Yet, with strictly positive probability liquidity demand is not served in Coexistence and Conditioning HFT Dominance equilibria. Hence, the availability of HFT technology could move liquidity measures in opposite directions, which makes it harder to make conclusive statements on liquidity. To overcome this problem, we construct a measure $\hat{S}$, which reflects a lower bound on illiquidity. It is defined as the hypothetical expected half spread if nonexecuted liquidity demand was executed at p_{liq}. More formally, it is given by

$$\hat{S} = \begin{cases} S = \frac{1}{2}\delta + \delta(\bar{\pi} + (1-\bar{\pi})\Phi), & \text{if (12) is satisfied,} \\ (1-\bar{\pi})S + \bar{\pi}(p_{liq} - \mu) = \frac{1}{2}\delta + \delta(\bar{\pi} + (1-\bar{\pi})((1-\phi_1) + \phi_1\Phi)), & \text{otherwise.} \end{cases} \tag{29}$$

It turns out that when multiple equilibria exist, aggregate liquidity always deteriorates as we move away from LFT Dominance, even if expected half spreads improve. When costs per unit of speed for LFT technology exceed those for HFT technology ($\frac{C_{HFT}}{\gamma} < C_{LFT}$), both technology costs as well as informational superiority contribute to higher aggregate participation rates. Expected half spreads then improve because of HFT technology availability. Yet, we show, using $\hat{S}$, that even in this case, liquidity as a whole can suffer from the availability of HFT technology. Hence, at an aggregate level, it is possible for the forgone profits from incorrectly classifying uninformed market orders to outweigh the combined cost savings from reduced adverse selection risk and reduced participation costs. This is important because analyzing empirically observable measures such as expected spreads may yield incorrect conclusions on the effect of HFT technology on market liquidity. We formalize the aforementioned results in the following Proposition.

Proposition 7. *When HFT technology has a higher cost per unit of speed than LFT technology, its availability can increase or reduce expected half spreads S. When multiple equilibria exist, $\hat{S}$ for LFT Dominance is lower than that of either Coexistence or Conditioning HFT Dominance. Even when HFT technology has the lowest cost per unit of speed (i.e., $\frac{C_{HFT}}{\gamma C_{LFT}} < 1$), $\hat{S}$ can be higher than if HFT technology had not been available.*

Proof. See Appendix. □

Summing up, when HFTs have high cost per unit of speed, LFTs provide all liquidity and markets are liquid. As HFT participation costs decline, multiple equilibria arise and HFTs can, but need not, crowd out LFTs. As a result, HFTs can coexist with LFTs or even dominate, in which case liquidity deteriorates. As HFT participation costs decline even further, HFTs dominate completely. As a result, liquidity can improve. However, it can also deteriorate if the cost advantage of HFTs is small.

5.2.3 Equilibria with very Informed Order Flow

We now analyze the case in which order flow is very likely to be informed ($\bar{\pi} \geq \pi^{tox}$). In the absence of a signal, Lemma 3 implies that markets break down and are infinitely illiquid. However, with a sufficiently accurate signal, informed HFTs may find it optimal to participate. The signal allows HFTs to avoid informed order flow and (partially) restore markets.

Lemma 5. *HFTs optimally participate and provide liquidity to an empty book following a signal $s = liq$ if*

$$\phi_1 \geq \frac{\mu_{inf} - \mu - \frac{3}{2}\delta}{\mu_{inf} - \mu}. \tag{30}$$

Proof. See Appendix. □

A direct implication of Lemma 5 is that the availability of HFT technology increases market liquidity by overcoming market failures if (30) is satisfied. This result is true irrespective of the participation costs, as they do not show up in Condition (30).

Proposition 8. *When $\bar{\pi} \geq \pi^{tox}$, the presence of HFT technology improves liquidity by resolving market failures if Condition (30) is satisfied. This results holds irrespective of participation cost parameters. The resulting equilibrium corresponds to either Conditioning HFT Dominance or Coexistence as in Proposition 6.*

Proof. See Appendix. □

Summing up, if Condition (30) is satisfied, the effect of HFT technology availability on liquidity in toxic markets ($\bar{\pi} \in (0, \pi^{tox})$) is opposite of that in nontoxic markets. The availability of HFT technology improves liquidity in toxic markets, even when HFTs have higher cost per unit of speed than LFTs. HFTs can even induce LFTs to participate (LFTs then only undercut).

6 Welfare and Policy Implications

In this section, we analyze welfare effects of HFT technology availability in the equilibria derived in Section 5. We use our welfare analysis to evaluate policy measures.

6.1 Welfare Analysis

To make welfare statements, we first define the sources of welfare gains and losses. Next, we measure the welfare in each equilibrium. To further our understanding of how frictions affect welfare, we compare our equilibrium outcomes to different welfare benchmarks.

In our model, gains from trade for uninformed liquidity demanders contribute positively to welfare. Spending resources on participation costs contribute negatively to welfare. Finally, informed trades are welfare neutral due to being zero-sum transfers.

We then continue by quantifying welfare in the different equilibria derived in Section 5. We quantify welfare in the equilibria derived in Section 5 in two ways, which are presented in Table 1. Aggregate expected welfare for each equilibrium type is obtained by summing over the corresponding row elements.

In panel A of Table 1, we add up all positive and negative welfare effects at the origin. When an uninformed liquidity demanders trades, the difference between reservation value and fundamental value ($p_{liq} - \mu$) is realized as a welfare gain. an uninformed liquidity demanders trades with probability $(1 - \bar{\pi})$ in LFT Dominance and Nonconditioning HFT Dominance and with probability $\phi_1(1-\bar{\pi})$ in other equilibrium types with strictly positive participation. Trades conducted by informed liquidity demanders are welfare neutral. Resources spent by LFTs and HFTs on participation costs reduce welfare by n^*C_{LFT} and m^*C_{HFT}, respectively. Total expected welfare effects are the sum of individual welfare effects. Welfare in no-participation equilibria equals zero.

Alternatively, in Panel B of Table 1 we add up welfare effects by ultimate utility recipients. Uninformed liquidity demanders capture an expected utility increase of $(p_{liq} - \mu - S)$ from an executed market order. This utility increase materializes with (unconditional) probability $(1-\bar{\pi})$ in LFT Dominance and Nonconditioning HFT Dominance equilibria and with probability $\phi_1(1-\bar{\pi})$ in all others with strictly positive participation. Liquidity providers break even in expectation, yielding them an expected utility increase of zero. Informed liquidity demanders capture an expected utility increase of $(\mu_{inf} - \mu - S)$ from an executed market order. This utility increase is realized with probability $\bar{\pi}$ in LFT Dominance and Nonconditioning HFT Dominance equilibria and with probability $(1-\bar{\pi})(1-\phi_1)$ in all others with strictly positive participation.

Table 1: Welfare in different equilibrium types

Panel A: Expected welfare gains and losses by source

Equilibrium	Uninf. Liq. Dem.	Inf. Liq. Dem.	Liq Prov.
No participation	0	$+\ 0$	$+0$
LFT Dominance	$(1-\bar{\pi})\frac{3}{2}\delta$	$+\ 0$	$-n^*C_{LFT}$
Nonconditioning HFT Dominance	$(1-\bar{\pi})\frac{3}{2}\delta$	$+\ 0$	$-m^*C_{HFT}$
Conditioning HFT Dominance	$\phi_1(1-\bar{\pi})\frac{3}{2}\delta$	$+\ 0$	$-m^*C_{HFT}$
Coexistence	$\phi_1(1-\bar{\pi})\frac{3}{2}\delta$	$+\ 0$	$-m^*C_{HFT} - n^*C_{LFT}$

Panel B: Expected welfare gains and losses by agent the welfare accrues to

Equilibrium	Uninf. Liq. Dem.	Liq. Prov.	Inf. Liq. Dem.
No participation	0	$+\ 0$	$+0$
LFT Dominance	$(1-\bar{\pi})(\frac{3}{2}\delta - S(0,n^*))$	$+\ 0$	$+\bar{\pi}(\mu_{inf} - \mu - S(0,n^*))$
Nonconditioning HFT Dominance	$(1-\bar{\pi})(\frac{3}{2}\delta - S(m^*,0))$	$+\ 0$	$+\bar{\pi}(\mu_{inf} - \mu - S(m^*,0))$
Conditioning HFT Dominance	$\phi_1(1-\bar{\pi})(\frac{3}{2}\delta - S(m^*,0))$	$+\ 0$	$+(1-\bar{\pi})(1-\phi_1)(\mu_{inf} - \mu - S(m^*,0))$
Coexistence	$\phi_1(1-\bar{\pi})(\frac{3}{2}\delta - S(m^*,n^*))$	$+\ 0$	$+(1-\bar{\pi})(1-\phi_1)(\mu_{inf} - \mu - S(m^*,n^*))$

The table presents expected welfare by equilibrium type. Panel A presents welfare components by source. Panel B presents welfare components by recipient. Aggregate expected welfare is obtained by summing over row elements.

To understand the origins of welfare losses, we construct three welfare benchmarks. For the first welfare benchmark, BM^{inf}, a social planner decides whether HFTs use their signals, while potential liquidity providers optimize their participation and other trading decisions. For the second benchmark, BM^{part}, a social planner makes participation decisions, but leaves all other decisions to the liquidity providers. For the third benchmark, BM^{comb}, a social planner decides on the use of signals and participation, while liquidity providers optimize all other decisions. For all welfare benchmarks, we require liquidity providers to break even. Hence, our welfare benchmarks correspond to a social planner's control over market entry, information access, or both, while preserving participation incentives and precluding oligopoly rents.

When $\bar{\pi} \geq \pi^{tox}$ and Condition (30) is satisfied, Conditioning HFT Dominance or Coexistence materialize as equilibria. These equilibria coincide with all welfare benchmarks, since any deviation leads to zero welfare or negative expected profits for LFTs.

Lemma 6. *When $\bar{\pi} \geq \pi^{tox}$ and Condition (30) is satisfied, all welfare benchmarks coincide with the (Conditioning HFT Dominance or Coexistence) equilibrium materializing.*

Proof. See Appendix. □

When $\bar{\pi} \in (0, \pi^{tox})$ and $\phi_2 \in [\pi, \phi_2^{ul}]$, Nonconditioning HFT Dominance is the only equilibrium if $\frac{C_{HFT}}{\gamma C_{LFT}} < 1$, and LFT Dominance is the only equilibrium otherwise. These equilibria coincide with all three welfare benchmarks in their respective scenarios. Using uninformative signals or having liquidity provided by liquidity providers with high costs per unit of speed would lower the expected profitability of the liquidity provision sector as a whole. As a result, participation would be depressed, expected half spreads increased and liquidity providers would capture more gains from trade. Since liquidity providers spend all revenues on participation costs, these effects would lower welfare.

Lemma 7. *When $\bar{\pi} \in (0, \pi^{tox})$ and $\phi_2 \in [\pi, \phi_2^{ul}]$, all welfare benchmarks coincide with Nonconditioning HFT Dominance if $\frac{C_{HFT}}{\gamma C_{LFT}} < 1$, and with LFT Dominance otherwise.*

Proof. See Appendix. □

When $\bar{\pi} \in (0, \pi^{tox})$ and $\phi_2 > \phi_2^{ul}$, BM^{inf} predicates that informative signals should not be used, since these impose a winners' curse on LFTs and may prevent gains from trade from being realized. Hence, BM^{inf} coincides with Nonconditioning HFT Dominance if $\frac{C_{HFT}}{\gamma C_{LFT}} < 1$, and LFT Dominance otherwise. BM^{part} coincides with Conditioning HFT Dominance when $\frac{C_{HFT}}{\gamma C_{LFT}} < K$ and with LFT Dominance otherwise, for some $K < 1$. The reason is that when $\frac{C_{HFT}}{\gamma C_{LFT}} > 1$, HFTs have a high cost per unit of speed, which is bad for welfare (following the intuition above). In Conditioning HFT Dominance and Coexistence equilibria, gains from trade fail to realize with strictly positive probability, which implies that $K < 1$. Finally,

$BM^{comb} = BM^{inf}$, since markets allocate liquidity provision efficiently in the absence of information asymmetry among liquidity providers.

Lemma 8. *When $\bar{\pi} \in (0, \pi^{tox})$ and $\phi_2 > \phi_2^{ul}$, BM^{inf} and BM^{part} coincide with Nonconditioning HFT Dominance when $\frac{C_{HFT}}{\gamma C_{LFT}} < 1$ and LFT Dominance otherwise. BM^{part} coincides with Conditioning HFT Dominance if $\frac{C_{HFT}}{\gamma C_{LFT}} < K$ and with LFT Dominance otherwise, where $K < 1$.*

Proof. See Appendix. □

We can now compare the equilibria in the different settings to the welfare benchmarks derived above to arrive at the main proposition of this Section:

Proposition 9. *When $\bar{\pi} \in (0, \pi^{tox})$ and $\phi_2 > \phi_2^{ul}$, the availability of HFT technology may depress welfare due to excessive information usage and HFT overparticipation. Overparticipation may even occur if HFTs have the lowest cost per unit of speed ($\frac{C_{HFT}}{\gamma C_{LFT}} < 1$). In all other cases, the availability of HFT technology improves welfare or leaves it unaffected.*

Proof. See Appendix. □

We can further analyze how welfare is impaired when $\bar{\pi} \in (0, \pi^{tox})$ and $\phi_2 > \phi_2^{ul}$. First, some gains from trade are not realized in expectation (ϕ_1 in front of $\frac{3}{2}\delta$). Second, more gains from trade are used to cover welfare reducing participation costs rather than welfare-neutral informed trading losses. Third, liquidity is provided by parties with a high cost per unit of speed. When HFTs have a higher cost per unit of speed than LFTs, all three channels are at work. Otherwise, only the first two channels lead to welfare losses while HFTs cost advantages partially counter these effects.

Panel B of Table 1 also shows how welfare is distributed. Liquidity providers are welfare-neutral since they break even. Liquidity demanders capture a larger welfare share as expected half spreads shrink. Unserved liquidity demand negatively affects the utility of both types of liquidity demanders. Yet, informed demanders suffer relatively more since they are more likely to be screened out. Hence, as participation costs for HFT technology decline, uninformed liquidity demanders capture a larger welfare share. Yet, total expected welfare may shrink, leaving the absolute effect for them ambiguous. If the availability of HFT technology prevents market failures, both types of liquidity demanders benefit, but uninformed more since HFTs must have accurate screening technology.

6.2 Policy Implications

We now use our model to analyze policy measures that have been recently proposed to curb negative welfare effects induced by HFTs. In particular, we discuss the following measures

1. imposing transaction taxes on HFTs (directly, or indirectly through exchanges),

2. imposing mandatory liquidity provision,

3. subsidizing liquidity provision when nobody is interested in doing so.

Naturally, these policies are only relevant and desirable to the extent that the availability of HFT technology impairs welfare. That is, if $\bar{\pi} \in (0, \pi^{tox})$ and $\phi_2 > \phi_2^{ul}$.

6.2.1 HFT Transaction Taxes

HFT transaction taxes have been introduced in some jurisdictions, such as France. We model HFT transaction taxes as an additional cost η for each trade conducted by an HFT. The per capita expected HFT trading profits in Lemma 4 change to:

$$\hat{g}_{HFT}(m,n) = g_{HFT}(m,n) - E(\eta), \tag{31}$$

$$\hat{h}_{HFT}(m,n) = h_{HFT}(m,n) - E(\eta), \tag{32}$$

where

$$E(\eta) = \begin{cases} \eta(1-\bar{\pi})\left(\frac{\phi_1(1-\Phi)}{n+\gamma m} + \frac{\phi_1\Phi+(1-\phi)}{\gamma m}\right), & \text{if (12) is not satisfied,} \\ \eta(1-\bar{\pi})\left(\frac{(1-\Phi)+\phi_1\Phi+(1-\phi)}{n+\gamma m}\right), & \text{otherwise.} \end{cases} \tag{33}$$

We assume the transaction tax η to be chosen such that

$$\begin{aligned} \hat{g}_{HFT}(m,n) - \gamma g_{LFT}(m,n) &< C_{HFT} - \gamma C_{LFT} \; \forall (m,n), \\ \hat{h}_{HFT}(m,n) - \gamma h_{LFT}(m,n) &< C_{HFT} - \gamma C_{LFT} \; \forall (m,n). \end{aligned} \tag{34}$$

This transaction tax impairs HFT profitability to the extent that HFTs optimally do not participate and that LFT Dominance remains as the only equilibrium. Interestingly, imposing such a tax would prevent itself from ever being collected since it would eliminate all HFTs.

Proposition 10. *The HFT transaction tax η prevents HFTs from participating. When $\frac{C_{HFT}}{\gamma C_{LFT}}$ is sufficiently high, welfare improves. The tax is never collected in equilibrium.*

Proof. See Appendix. □

As $\frac{C_{HFT}}{\gamma}$ declines, the right hand sides of Eq. (34) decline and the required transaction tax is higher. Transaction taxes allow to attain welfare benchmark BM^{part} by preventing HFT participation. Yet, transaction taxes are ineffective against inefficient information use and thereby fail to achieve BM^{comb}. Therefore, transaction taxes cannot capitalize all welfare gains if HFTs have the lowest cost per unit of speed.

6.2.2 Mandatory Liquidity Provision

Mandatory liquidity provision forces liquidity providers to quote irrespective of the signal received. As such, this measure provides regulatory control over information usage. Hence, it allows to attain BM^{inf}, which by Lemma 8 coincides with BM^{comb}. We model mandatory liquidity provision by having a regulator set $\phi_1 = 1 - \bar{\pi}$.

Proposition 11. *Equilibria with mandatory liquidity provision yield the same outcomes as equilibria without informative signals. Welfare then improves.*

Proof. See Appendix. □

Since mandatory Liquidity Provision allows to attain BM^{comb}, it is possible to capitalize on welfare benefits resulting from low costs per unit of speed of HFT technology.

The problem with mandatory liquidity provision is enforceability. With advanced trading equipment technical glitches can occur. Arguably, these are more likely in turbulent times (in which adverse selection concerns are also higher). Disentangling technical malfunctions from deliberate quote retraction may be a very difficult.

6.2.3 Contingent Quote Subsidies

Contingent quote subsidies per se do not exist (at least not at large scale). Yet, mechanisms in a similar spirit have been used in markets. Examples are Designated Market Makers (DMMs) that are compensated based on the level of their liquidity provision in liquid and illiquid markets, or better primary market allocations for treasury market dealers that support liquidity in bad times. One could also directly implement contingent quote subsidies, potentially in combination with a small transaction fee to make it cost neutral.

We model contingent quote subsidies by supplementing Condition (25) with a conditional subsidy ω for scenarios in which nobody provides liquidity otherwise. Moreover, we impose a transaction fee η that satisfies:

$$\eta = \begin{cases} 0, & \text{if } \frac{C_{HFT}}{\gamma C_{LFT}} > 1 \\ \frac{\bar{\pi}}{1-\bar{\pi}}\omega, & \text{otherwise.} \end{cases} \tag{35}$$

The subsidy ω we is chosen such that

$$\phi_2 \leq \left(1 + \frac{\mu_{inf} - p_{liq} - \omega}{p_{liq} - \mu + \omega}\left(1 + \frac{\lambda}{\nu_{liq}}\tilde{f}^{-1}\left(\min\left(\frac{C_{HFT}}{\gamma}, C_{LFT}\right)\right)\right)\right)^{-1}. \tag{36}$$

Proposition 12. *The quote subsidy system outlined above induces HFTs to always quote in an empty book. As a result, HFTs provide liquidity iff $\frac{C_{HFT}}{\gamma C_{LFT}} \leq 1$. Welfare improves as the result of HFT technology being available.*

Proof. See Appendix. □

The reason why contingent quote subsidies work is as follows. The subsidy is chosen to make sure that HFTs always post in an empty book. Consequently Corollary 1 applies, but with (25) replaced by (36). As a result, liquidity is provided by the liquidity provider with lowest costs per unit of speed, which maximizes welfare. The transaction fee is chosen to make the policy self-financing. Therefore, expected trading profits are unaffected compared to the case without contingent quote subsidies. When HFTs have higher costs per unit of speed than LFTs, HFTs do not participate (see Proposition 5) and the subsidy is never used. Therefore, a zero transaction fee is sufficient. When HFTs have lower costs per unit of speed than LFTs, HFTs provide all liquidity. The subsidy is then paid with probability $\bar{\pi}$. It is funded with a transaction fee on uninformed trades, which is collected with probability $1 - \bar{\pi}$.

Contingent quote subsidies allow to attain BM^{inf}, since they make HFTs always quote in an empty book. Because subsidies offset transaction fees in expectation, expected trading profits are given by Lemma 2. Hence, BM^{inf} coincides with BM^{comb} (see Lemma 8). As a result, one can capture welfare benefits resulting from low costs per unit of speed of HFT technology.

7 Empirical Implications

The model yields several empirical implications. Some are not unique to our model and are therefore not discussed (e.g., liquidly provision shifting from LFT- to HFT-based).

7.1 Expected Half Spreads

As $\frac{C_{HFT}}{\gamma}$ decreases the average half spread can in- or decrease, depending on a switch in possible equilibrium types. If a decrease in $\frac{C_{HFT}}{\gamma}$ does not induce such a switch, a cost decrease lowers expected half spreads in all equilibria except LFT Dominance (which is unaffected). The reason is that lower participation costs lead to more entry and therefor faster undercutting to $p(1)$. If a drop in $\frac{C_{HFT}}{\gamma}$ induces a switch, expected half spreads may go down (e.g., a switch from Coexistence to Conditioning HFT Dominance), or go up (e.g., a switch from LFT Dominance to Coexistence). The latter implication is new.

Corollary 2. *As* $\frac{C_{HFT}}{\gamma}$ *declines, expected half spreads decrease when there is no change in possible equilibrium types. If there is a change in possible equilibrium types as a result, expected half spreads may either increase or decrease.*

Proof. See Appendix. □

In line with our prediction, the literature has found effects in both directions. Generally, half spreads have been found to decrease as HFT costs decrease and HFT presence becomes more prominent (e.g., Hendershott et al., 2011). Yet, some studies document the opposite effect. Jørgensen et al. (2018) investigate the effect of imposing a fee on the Oslo stock exchange for high message to trade ratios. As these are only relevant for limit order users, and more likely to affect HFTs, we can interpret this policy as a cost increase for HFTs. The diff-in-diff estimate of this policy measure suggests that increases in $\frac{C_{HFT}}{\gamma}$ decrease spreads. [13]

7.2 The willingness of LFTs to quote at high spreads

The model predicts that in the presence of HFTs, LFTs are less willing to quote far (multiple ticks) away from the fundamental value due to increased adverse selection. This effect is more severe as HFT presence is higher.

Corollary 3. *The presence of HFTs reduces the willingness of LFTs to quote far from the fundamental value.*

Proof. See Appendix. □

This prediction is in line with recent empirical evidence by O'Hara et al. (2019). They find that HFT presence reduces the willingness of LFTs to quote far from the mid. The mechanism

[13] Other studies on similar natural experiments in Italy and Canada yield opposite results. This divergence in findings is consistent with only the Oslo fee change leading to a change in equilibrium type. Our study provides a possible explanation to reconcile these opposed findings.

they document differs slightly from ours. They argue that informed HFTs only undercut when the probability of informed order flow is low. As a result, LFTs are disproportionally likely to have their quotes executed by informed market orders.[14]

7.3 Fraction of Liquidity Demand that is Served

As $\frac{C_{HFT}}{\gamma}$ decreases and the equilibrium shifts away from LFT Dominance, the likelihood of liquidity demand being served drops. There are two reasons for this liquidity reduction. First, LFTs provide less liquidity out of fear for a winners' curse. Second, HFTs as a group are more likely to strategically stay away since their market share is larger. Together, these lead to a strictly positive probability of liquidity not being served (which is zero in LFT Dominance).

Corollary 4. *As $\frac{C_{HFT}}{\gamma}$ declines, the likelihood of liquidity demand being served decreases.*

Proof. See Appendix. □

This empirical implication is hard to empirically test for two reasons. First, the effect is only observable with changes in equilibrium type, which are rare. Second, liquidity demand that is not served is hard to observe, identify, and measure. One could look for drops in trading volume, but these are endogenous and involve demand and supply effects (e.g., as in Internet Appendix IA.2.2.3). Therefore, we leave the testing of this implication for future research.

7.4 Adverse Selection Costs for LFTs

As $\frac{C_{HFT}}{\gamma}$ decreases and the possible equilibria shift away from LFT Dominance only, adverse selection costs for LFTs are affected in two ways. As HFTs enter, adverse selection costs for LFTs increase as they provide liquidity to order flow that is disproportionately likely to be informed. This adverse selection cost increase would be the full effect if n were fixed. Yet, LFT participation is endogenous. As a result, LFTs limit liquidity provision to undercutting, or abstain altogether. As a result, and counter-intuitively, adverse selection losses for LFTs may decline as $\frac{C_{HFT}}{\gamma}$ drops.

Corollary 5. *As $\frac{C_{HFT}}{\gamma}$ declines, the fraction of adverse selection losses borne by LFTs may decline.*

Proof. See Appendix. □

[14] We are confident that our empirical prediction and their finding have the same origin: the increased adverse selection/winner's curse that HFTs impose on LFTs. The more general model we present in Internet Appendix IA.1.3, with $\nu_{inf} < \infty$, would generate the same effect resulting from strategic undercutting by HFTs as found in O'Hara et al. (2019).

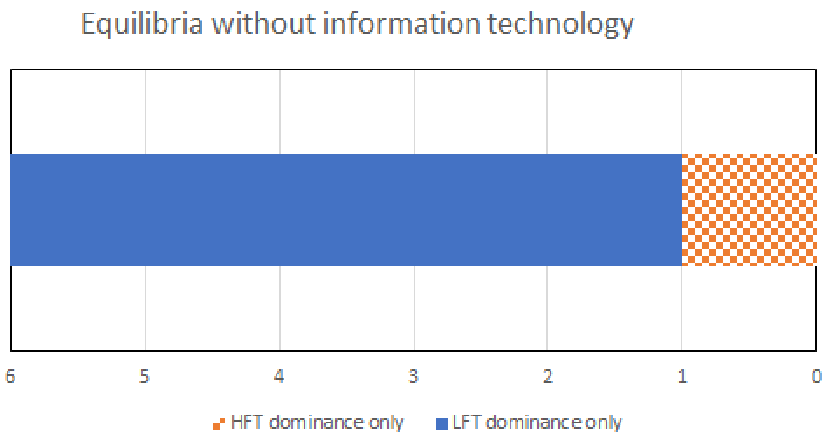
Testing this implication is hard as it is only observable when equilibrium types switch. If liquidity providers had the same motives for trading, the model would imply that adverse selection losses for HFTs exceed those for LFTs. This implication is easier to test since it does not depend on changes in equilibrium types. Brogaard et al. (2014) show that HFTs suffer adverse selection losses when providing liquidity, but that these are even larger for nonHFTs. These findings would be evidence against our implication. Yet, the results in Brogaard et al. (2014) suggest that the assumption of equal trading motives is violated. In particular, nonHFTs on average lose money on their limit orders, suggesting other trading motives (such as fundamental trading needs). Therefore, we leave the testing of this implication for future research.

8 Conclusion

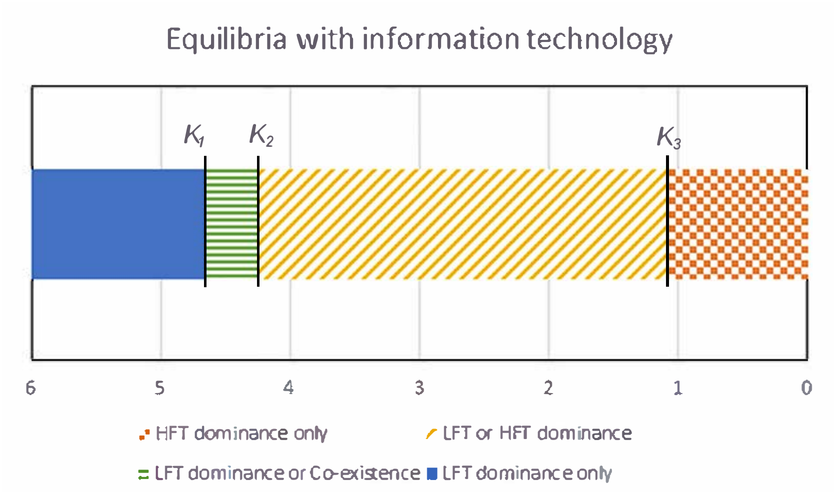
We analyze the consequences of the emergence of HFTs, complementing or replacing LFTs on financial markets in a long-term equilibrium model with endogenous participation and technology adoption. We find that with low levels of informed trading HFT speed technology improves market liquidity and welfare, which is reflected in lower transaction cost for end-users (as e.g., shown in Menkveld, 2016). However, asymmetric information problems can arise when HFTs retract in anticipation of toxic order flow (as shown in Anand and Venkataraman, 2016; Baldauf and Mollner, 2016; Korajczyk and Murphy, 2018). In such situations, only LFTs keep markets liquid. Yet, they may have been largely crowded out by HFTs (as e.g., described in Kirilenko and Lo, 2013). Interestingly, this is the situation that markets are heading to as HFT technology becomes more affordable. This situation will result in a proliferation of information processing technology of market participants at an unprecedented scale. In the transition period, markets may adopt HFT technology too early, leading to liquidity and welfare deteriorations.

Figures

Figure 3: **Equilibrium Ranges**
Panel A:

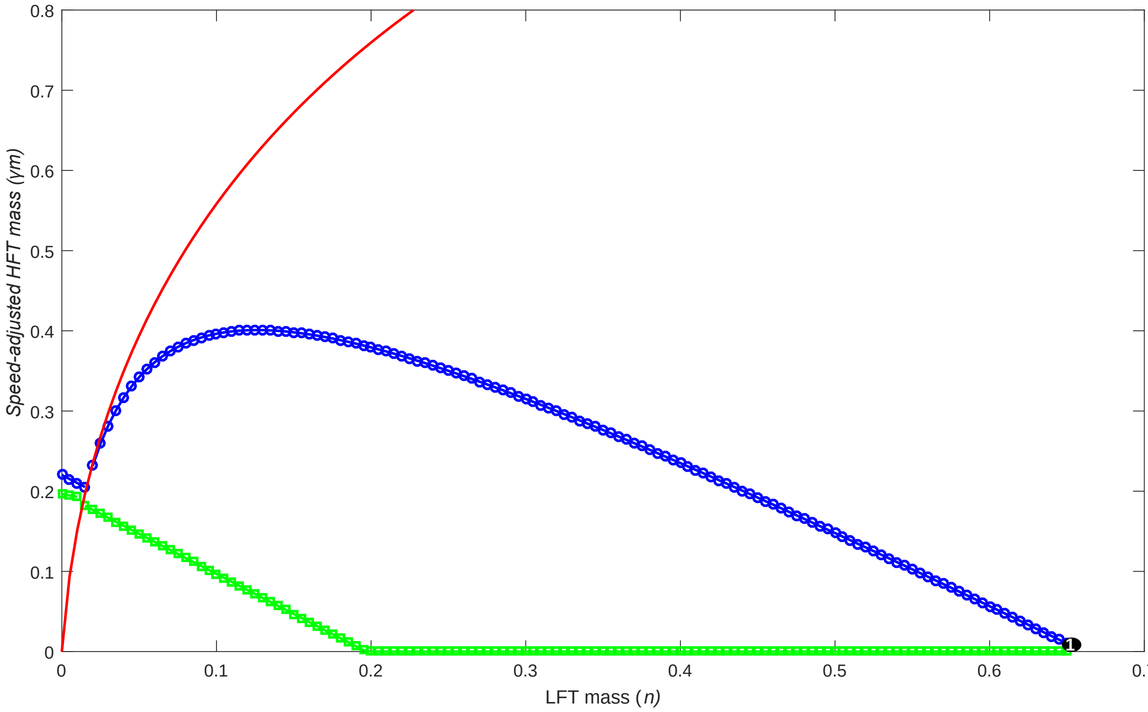


Panel B:



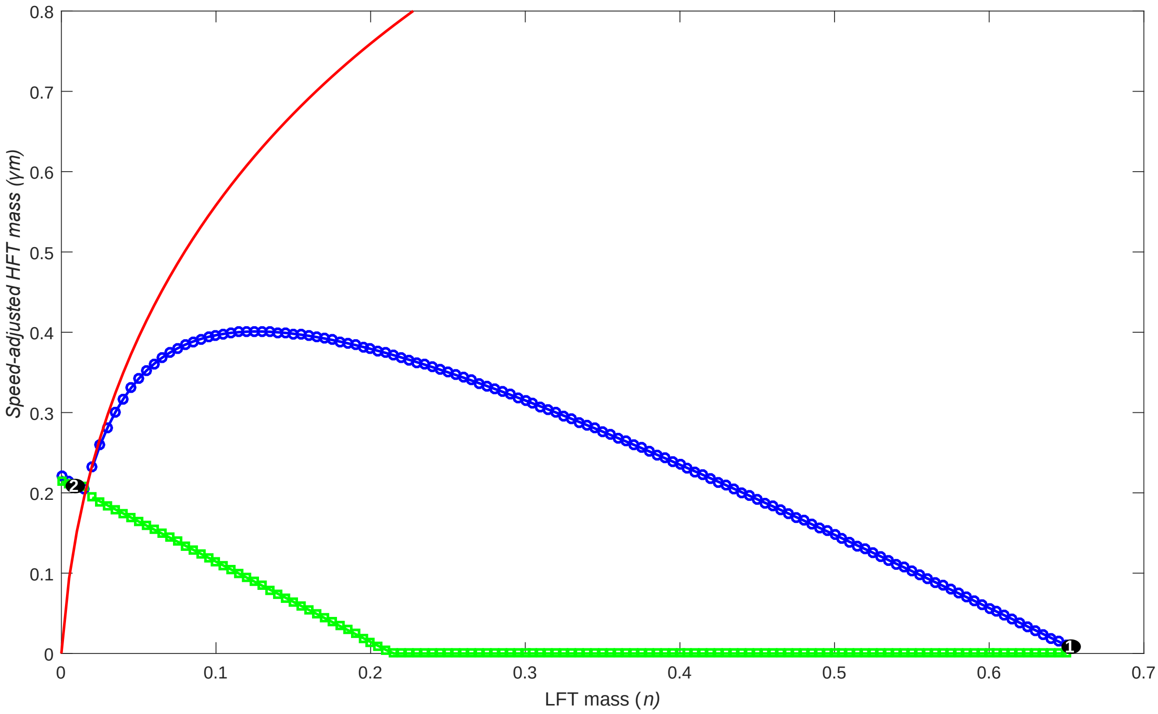
This graph displays the parameter ranges in which the different equilibrium types manifest themselves for the uninformed case (panel A) and the informed case (Panel B). The ratio of HFT speed cost to LFT speed costs $\left(\frac{C_{HFT}}{\gamma C_{LFT}}\right)$ is on the x-axis. The order of the x-axis is reversed to align with the order of exposition in Section 5.2.2.

Figure 4: **LFT Dominance Equilibrium**



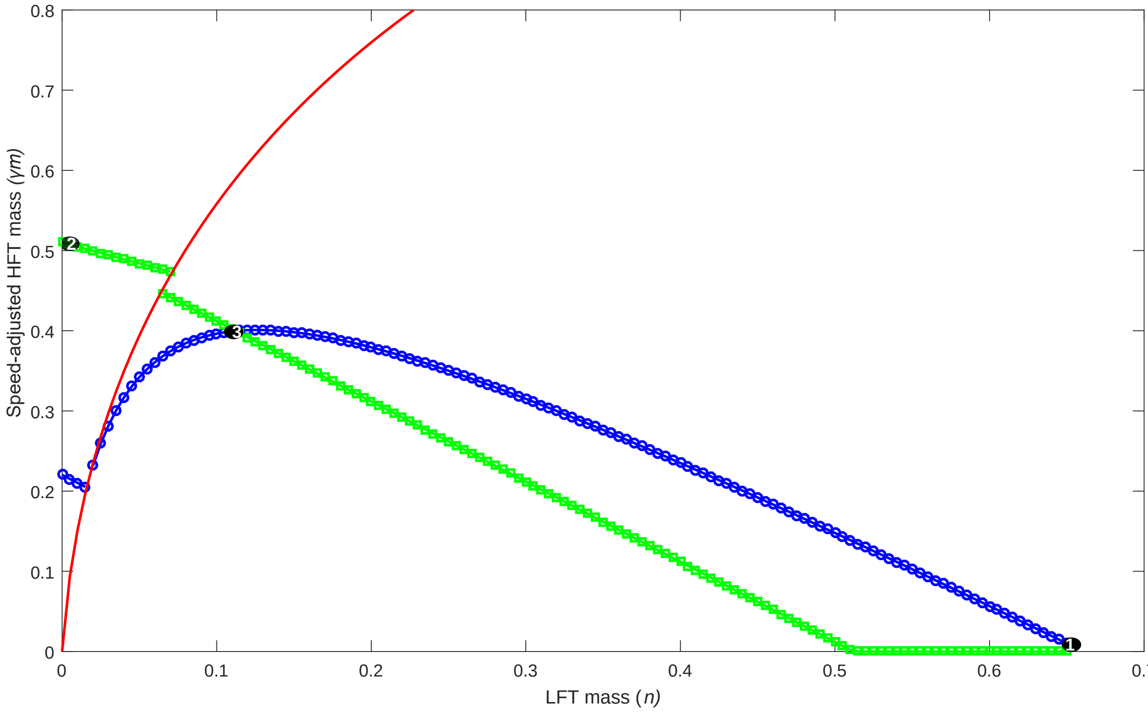
This graph displays the indifference curves for LFTs (blue circles) and HFTs (green squares) for different masses of LFTs and HFTs. The mass of LFTs (n) is displayed on the x-axis and the mass of HFTs corrected for the speed advantage ($m\gamma$) is indicated on the y-axis. The red curve is the indifference curve for LFTs to post a quote when arriving to an empty book (i.e., to the left of the red curve there is no posting, and vice versa to the right). Higher positioned indifference curves imply more competitive traders.

Figure 5: **Multiple Equilibria: LFT Dominance and coexistence**



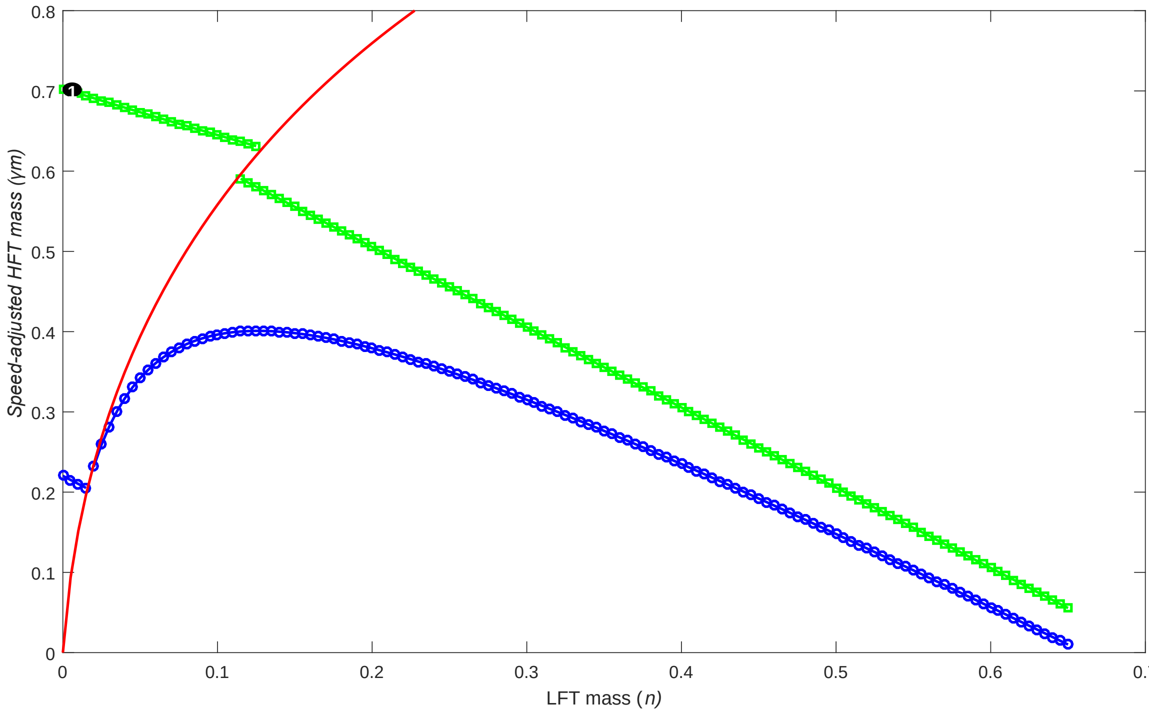
This graph displays the indifference curves for LFTs (blue circles) and HFTs (green squares) for different masses of LFTs and HFTs. The mass of LFTs (n) is displayed on the x-axis and the mass of HFTs corrected for the speed advantage ($m\gamma$) is indicated on the y-axis. The red curve is the indifference curve for LFTs to post a quote when arriving to an empty book (i.e., to the left of the red curve there is no posting, and vice versa to the right). Higher positioned indifference curves imply more competitive traders.

Figure 6: **Multiple Equilibria: LFT Dominance, Conditioning HFT Dominance, Instable coexistence**



This graph displays the indifference curves for LFTs (blue circles) and HFTs (green squares) for different masses of LFTs and HFTs. The mass of LFTs (n) is displayed on the x-axis and the mass of HFTs corrected for the speed advantage ($m\gamma$) is indicated on the y-axis. The red curve is the indifference curve for LFTs to post a quote when arriving to an empty book (i.e., to the left of the red curve there is no posting, and vice versa to the right). Higher positioned indifference curves imply more competitive traders.

Figure 7: **Conditioning HFT Dominance Equilibrium**



This graph displays the indifference curves for LFTs (blue circles) and HFTs (green squares) for different masses of LFTs and HFTs. The mass of LFTs (n) is displayed on the x-axis and the mass of HFTs corrected for the speed advantage ($m\gamma$) is indicated on the y-axis. The red curve is the indifference curve for LFTs to post a quote when arriving to an empty book (i.e., to the left of the red curve there is no posting, and vice versa to the right). Higher positioned indifference curves imply more competitive traders.

Appendices

A Proofs

Proof of Proposition 1. Due to time and price priority, $\Phi = 0$ if $a \geq \widehat{a}$. Moreover, by assumption, $\Phi = 0$ if $a > p_{liq}$. We define $\min(\widehat{a} - \delta, p_{liq}) = p_{liq}$ if $\widehat{a} = \emptyset$ to simplify notation. Assume that $\Phi > 0$ if $a \leq \min(p_{liq}, \widehat{a} - \delta)$ (proven later). Any quotes $a \leq \min(p(1) - \delta, \widehat{a} - \delta) < \mu$ are loss-making with strictly positive probability and are hence suboptimal. Any quotes $a \in [p(1), \min(\widehat{a} - \delta, p_{liq})]$ yield strictly positive profits with strictly positive execution probability and hence carry strictly positive expected profits. Hence, if $\widehat{a} \in (p(1), p_{liq}]$, undercutting to $a = p(1)$ is optimal, while posting a quote with zero execution probability (such as $a = p(1)$) is (weakly) optimal if $\widehat{a} \leq p(1)$.

In equilibrium, $a = p(1)$ is optimally not undercut and has execution probability $\Phi = 1$ if $\widehat{a} > p(1)$. A quote $a = p_{liq}$ executes if the liquidity demander arrives before the next liquidity provider arrives to undercut. The arrival rate of liquidity suppliers is given by $\lambda(\gamma m + n)$ and, because liquidity providers are atomistic, is independent of the type of liquidity provider. The arrival rate of liquidity demanders is given by ν_{liq}. Applying standard rules for the calculations with exponential distributions yields Eq. (7).

Given $\widehat{a} > p_{liq}$ or $\widehat{a} = \emptyset$, a liquidity supplier optimally either posts $a = p_{liq}$ or $a = p(1)$. The former has an execution probability Φ as derived above. The latter has guaranteed execution and a profit of $p(1) - \mu$. It follows that posting $a = p(1)$ is strictly optimal if

$$(p_{liq} - \mu)\Phi < p(1) - \mu = \frac{\delta}{2}, \Rightarrow \tag{37}$$

$$\frac{3}{2}\delta\Phi < \frac{1}{2}\delta, \Rightarrow \tag{38}$$

$$\Phi < \frac{1}{3} \tag{39}$$

while setting $a = p_{liq}$ is optimal otherwise. □

Proof of Proposition 2. Submitting a quote is only optimal when the expected profit of doing so is positive. Since $\Phi \geq \frac{1}{3}$ by assumption, if it is optimal to quote in an empty book, the optimal quote equals $a = p_{liq}$. The expected profit of doing so is positive if

$$(p_{liq} - \mu)\hat{P}(\zeta = liq|\psi_{HFT})\Phi(\zeta = liq) - (\mu_{inf} - p_{liq})\hat{P}(\zeta = inf|\psi_{HFT})\Phi(\zeta = inf) \geq 0. \tag{40}$$

Rewriting gives Expression (9). Moreover, we have that:

$$\hat{P}(\zeta = inf|\psi_{HFT}) = \begin{cases} \phi_2 \text{ if } s = inf, \\ 1 - \phi_1 \text{ if } s = liq. \end{cases} \tag{41}$$

The execution probabilities are also completely defined, because in the case of informed trading execution is guaranteed and immediate. In contrast, with uninformed trading, the game reduces to the uninformed trading game immediately after posting the first quote in an empty book. Hence, we have:

$$\Phi(\zeta = inf) = 1, \tag{42}$$

$$\Phi(\zeta = liq) = \Phi. \tag{43}$$

Substituting these expressions into Inequality (9) and rewriting implies that an HFT will always quote in an empty book if Condition (10) is satisfied, and never when (11) is satisfied.

Upon arrival to a book with standing best quote $\widehat{a} = p_{liq}$, there is no risk of informed trading, and hence Proposition 1 applies, implying that posting $a = p(1)$ is optimal. □

Proof of Proposition 3. Submitting a quote is only optimal when the expected profit of doing so is positive. Since $\Phi \geq \frac{1}{3}$ by assumption, if it is optimal to quote in an empty book, the optimal quote equals $a = p_{liq}$. The expected profit of doing so is positive if

$$(p_{liq} - \mu)\hat{P}(\zeta = liq|\psi_{LFT})\Phi(\zeta = liq) - (\mu_{inf} - p_{liq})\hat{P}(\zeta = inf|\psi_{LFT})\Phi(\zeta = inf) \geq 0. \tag{44}$$

Rewriting gives Condition (12).

To prove the second part, define the event B that a specific LFT arrives to an empty order book, let the event HS denote the situation when $s = inf$ (high suspicion by HFTs) and NS the situation that $s = liq$ (no suspicion by HFTs). Then Bayes rule implies

$$P(\zeta = inf|B) = \frac{\chi(B, \zeta = inf)}{\chi(B)}, \tag{45}$$

$$\chi(B, \zeta = inf) = \chi(B|\zeta = inf, HS)P(\zeta = inf|HS)P(HS)+ \\ \chi(B|\zeta = inf, NS)P(\zeta = inf|NS)P(NS), \quad (46)$$

$$\chi(B) = \chi(B, \zeta = inf) + \chi(B|\zeta = liq, HS)P(\zeta = liq|HS)P(HS)+ \\ \chi(B|\zeta = liq, NS)P(\zeta = liq|NS)P(NS), \quad (47)$$

where $\chi(\cdot)$ is the probability density function of a specific LFT arriving to an empty LOB. Moreover, we have that

$$\chi(B|\zeta = inf, HS) = \chi(B|\zeta = liq, HS) = \frac{1}{n}, \quad (48)$$

$$\chi(B|\zeta = inf, NS) = \chi(B|\zeta = liq, NS) = \frac{1}{n + \gamma m}, \quad (49)$$

$$P(HS) = \bar{\pi}, \qquad P(NS) = 1 - \bar{\pi}, \quad (50)$$

$$P(\zeta = inf|HS) = \phi_2, \qquad P(\zeta = liq|HS) = 1 - \phi_2 \quad (51)$$

$$P(\zeta = inf|NS) = 1 - \phi_1, \qquad P(\zeta = liq|NS) = \phi_1 \quad (52)$$

Substituting in, we get

$$\hat{P}(\zeta = inf|B) = \frac{\frac{1}{n}\phi_2\bar{\pi} + \frac{1}{n+\gamma m}(1 - \phi_1)(1 - \bar{\pi})}{\frac{1}{n}\phi_2\bar{\pi} + \frac{1}{n+\gamma m}(1 - \phi_1)(1 - \bar{\pi}) + \frac{1}{n}(1 - \phi_2)\bar{\pi} + \frac{1}{n+\gamma m}\phi_1(1 - \bar{\pi})} \quad (53)$$

$$= \frac{\frac{1}{n}\phi_2\bar{\pi} + \frac{1}{n+\gamma m}(1 - \phi_1)(1 - \bar{\pi})}{\frac{1}{n}\bar{\pi} + \frac{1}{n+\gamma m}(1 - \bar{\pi})}. \quad (54)$$

The partial derivatives (assuming $\phi_2 > \bar{\pi}$) are given by:

$$\frac{\partial \hat{P}(\zeta = inf|B)}{\partial n} = \frac{-\gamma m(1 - \bar{\pi})\bar{\pi}(\phi_1 + \phi_2 - 1)}{(n + \gamma m\bar{\pi})^2} < 0 \quad (55)$$

$$\frac{\partial \hat{P}(\zeta = inf|B)}{\partial m} = \frac{n\gamma(1 - \bar{\pi})\bar{\pi}(\phi_1 + \phi_2 - 1)}{(n + \gamma m\bar{\pi})^2} > 0. \quad (56)$$

If HFTs do not employ a differential strategy upon observing an informed trade (i.e., HFTs always or never submit a first quote), LFTs cannot learn anything about the state of the world from observing an empty book and we have that $\hat{P}(\zeta = inf|B) = \bar{\pi}$. Upon arrival to a book with standing best quote $\hat{a} = p_{liq}$, there is no risk of informed liquidity demand, and hence Proposition 1 applies, implying that posting $a = p(1)$ is optimal. □

Proof of Lemma 1. Conditional on execution at a certain price level (i.e., p_{liq} or $p(1)$), the first liquidity provider to post a quote at this level transacts. Without loss of generality, let us assume that the transaction executes at p_{liq}. Moreover, let us denote τ_{LFT} and τ_{HFT} be the arrival times of the first LFT and first HFT, respectively. Since waiting times for Poisson arrivals are exponentially distributed, we have that

$$P(\tau_{LFT} < \tau_{HFT}) = \int_0^\infty P(\tau_{LFT} < \tau_{HFT} | \tau_{LFT} = t) \lambda n e^{-\lambda n t} dt, \tag{57}$$

$$= \int_0^\infty P(t < \tau_{HFT}) \lambda n e^{-\lambda n t} dt. \tag{58}$$

We have that $P(t < \tau_{HFT}) = e^{-\lambda \gamma m t}$ (CDF of an exponential distribution). Substituting yields

$$P(\tau_{LFT} < \tau_{HFT}) = \int_0^\infty \lambda n e^{-\lambda n t} e^{-\lambda \gamma m t} dt, \tag{59}$$

$$= \lambda n \int_0^\infty e^{-\lambda (n+\gamma m) t} dt, \tag{60}$$

$$= \lambda n \left. \frac{-e^{-\lambda (n+\gamma m) t}}{\lambda (n + \gamma m)} \right|_0^\infty, \tag{61}$$

$$= \frac{\lambda n}{\lambda (n + \gamma m)} = \frac{n}{(n + \gamma m)}. \tag{62}$$

Hence, each group participates in trading profits according to relative market presence so that the profit share of HFTs equals $\frac{\gamma m}{\gamma m + n}$, while that for LFTs equals $\frac{n}{\gamma m + n}$. Moreover, since all liquidity providers within a group are homogeneous, expected profits are evenly distributed across the group members, which yields Expressions (14) and (15). The expected aggregate profits are given by $\frac{1}{2}\delta$, plus an extra tick δ multiplied with the execution probability Φ because quoting starts at p_{liq}, which yields Expression (16). □

Proof of Proposition 4. Consider Equations (14) to (17). It follows that given $n + \gamma m$, expected trading profits net of participation costs for HFTs always strictly exceed those for LFTs iff $C_{LFT} > \frac{C_{HFT}}{\gamma}$. This result must be true in particular when $m = 0$ or $n = 0$. Moreover, the partial derivatives of per capita expected trading profits in Expressions (14) to (17) with respect to n and m are all strictly negative. By our equilibrium definition, we must have for each player type in equilibrium that either participation costs equal expected trading profits or that the participation rate equals zero. As a result we have that $n = 0, m > 0$ if $C_{LFT} > \frac{C_{HFT}}{\gamma}$ and $n > 0, m = 0$ if $C_{LFT} < \frac{C_{HFT}}{\gamma}$.

Now define

$$f(n+\gamma m) = \frac{\Pi}{n+\gamma m}, \tag{63}$$

such that

$$f(n+\gamma m) = \min\left(\frac{C_{HFT}}{\gamma}, C_{LFT}\right) \tag{64}$$

is an equilibrium condition. Imposing this condition allows us to invert the function $f(\cdot)$ to obtain equilibrium participation rates of LFTs and HFTs. Substituting Expression (7) into Eq. (63) yields

$$y = f(n+\gamma m) = \left(\frac{1}{2}\delta + \frac{\delta\nu_{liq}}{\nu_{liq}+\lambda(n+\gamma m)}\right)\frac{1}{n+\gamma m} \Rightarrow \tag{65}$$

$$(n+\gamma m)y = \frac{1}{2}\delta + \frac{\delta\nu_{liq}}{\nu_{liq}+\lambda(n+\gamma m)} \Rightarrow \tag{66}$$

$$(n+\gamma m)y(\nu_{liq}+\lambda(n+\gamma m)) = \frac{1}{2}\delta(\nu_{liq}+\lambda(n+\gamma m)) + \delta\nu_{liq} \Rightarrow \tag{67}$$

$$0 = \lambda y(n+\gamma m)^2 + (\nu_{liq}y - \frac{\lambda\delta}{2})(n+\gamma m) - \frac{3}{2}\delta\nu_{liq} \Rightarrow \tag{68}$$

$$n+\gamma m = f^{-1}(y) = \frac{-(\nu_{liq}y - \frac{\lambda\delta}{2}) \pm \sqrt{(\nu_{liq}y - \frac{\lambda\delta}{2})^2 + 6\lambda y\delta\nu_{liq}}}{2\lambda y}. \tag{69}$$

n and γm need to be positive and hence their sum needs to be positive too. There is only one positive root, namely

$$n+\gamma m = f^{-1}(y) = \frac{-(\nu_{liq}y - \frac{\lambda\delta}{2}) + \sqrt{(\nu_{liq}y - \frac{\lambda\delta}{2})^2 + 6\lambda y\delta\nu_{liq}}}{2\lambda y}, \tag{70}$$

$$= \frac{\frac{1}{2}\delta}{2y} - \frac{\nu_{liq}}{2\lambda} + \sqrt{\left(\frac{\nu_{liq}}{2\lambda} - \frac{\frac{1}{2}\delta}{2y}\right)^2 + \frac{3}{2}\frac{\delta\nu_{liq}}{\lambda y}}, \tag{71}$$

$$\geq \frac{\frac{1}{2}\delta}{2y} - \frac{\nu_{liq}}{2\lambda} + \sqrt{\left(\frac{\nu_{liq}}{2\lambda} + \frac{\frac{1}{2}\delta}{2y}\right)^2} = \frac{\delta}{2y}. \tag{72}$$

Hence the inverse of $f(\cdot)$ is one-to-one and monotonic, as basic calculus predicts (since $f(\cdot)$ is continuous and monotonic on its domain). When we set $y = \min(C_{LFT}, \frac{C_{HFT}}{\gamma})$, a closed form solution for n^* or γm^* can be obtained in the uninformed case. To obtain m^*, we need to divide by γ to obtain (18) for m^*.

We have that $S = \frac{1}{2}\delta + \Phi\delta$. Substituting the expression for Φ and imposing that $(n+\gamma m) =$

$(n^* + \gamma m^*)$ yields Expression (19).

Finally, we prove that the availability of HFT technology strictly improves liquidity iff $\frac{C_{HFT}}{\gamma} < C_{LFT}$. Since n and γm only show up in the denominator of Φ and only summed together and moreover all elements of Φ are positive, Φ and thereby S are monotonically declining in n and γm. It follows that the availability of HFT technology strictly reduces S if and only if $\tilde{n} < \gamma m^*$, where $\tilde{n}$ is the n that solves $h_{LFT}(0, n) = C_{LFT}$ (the mass of LFTs that would materialize in the absence of HFT technology). $\tilde{n}$ can be derived as n^* for the hypothetical case that $C_{HFT} = \infty$. Now we assume that $\frac{C_{HFT}}{\gamma} \leq C_{LFT}$ and differentiate the expressions for γm^* and $\tilde{n}$ to get

$$\frac{\partial \gamma m^*}{\partial \frac{C_{HFT}}{\gamma}} = \frac{\partial \tilde{n}}{\partial C_{LFT}} < 0. \tag{73}$$

Now assume that $\frac{C_{HFT}}{\gamma} = C_{LFT}$. Substituting into (18) yields that $\gamma m^* = \tilde{n}$. Together with Eq. (73), it shows that $\gamma m^* > \tilde{n}$ iff $\frac{C_{HFT}}{\gamma} < C_{LFT}$. Hence, the availability HFT technology strictly improves liquidity iff $\frac{C_{HFT}}{\gamma} < C_{LFT}$. □

Proof of Lemma 2. By assumption all players act in the trading stage as in Proposition 1. With probability $(1 - \bar{\pi})$, there is no informed liquidity demand and expected per capita profits of providing liquidity to uninformed order flow for HFTs and LFTs are given by Expression (14) and (15), respectively. With probability $\bar{\pi}$, there is informed liquidity demand leading to informed trading losses of size $(\mu_{inf} - p_{liq})$. In expectation these are borne by all liquidity providers according to their relative presence due to the Poisson arrival processes. Expected trading profits for HFTs and LFTs are then given by

$$E_{\widehat{a}}\left(\Pi_{HFT}(R^*_{HFT}(\widehat{a})|m,n)\right) = (1-\bar{\pi})\frac{\gamma \Pi}{\gamma m + n} - \bar{\pi}\frac{\gamma}{n + \gamma m}(\mu_{inf} - p_{liq}) = \frac{\gamma \tilde{\Pi}}{\gamma m + n}, \tag{74}$$

$$E_{\widehat{a}}\left(\Pi_{LFT}(R^*_{LFT}(\widehat{a})|m,n)\right) = (1-\bar{\pi})\frac{\Pi}{\gamma m + n} - \bar{\pi}\frac{1}{n + \gamma m}(\mu_{inf} - p_{liq}) = \frac{\tilde{\Pi}}{\gamma m + n}, \tag{75}$$

with $\tilde{\Pi}$ given by (20). Participation costs are unchanged since these are unaffected by informed trading losses, which completes the proof. □

Proof of Lemma 3. If expected trading profits are negative, participation is never optimal since participation costs are strictly positive. Aggregate expected trading profits in the absence of informed signals are given by $\tilde{\Pi}$ due to Lemma 2. In order for participation by some liquidity provider to be profitable we need that

$$\tilde{\Pi} - nC_{LFT} - mC_{HFT} > 0 \Rightarrow \tag{76}$$

$$(1-\bar{\pi})\delta(\frac{1}{2}+\Phi) > \bar{\pi}(\mu_{inf} - \mu - \frac{3}{2}\delta) + nC_{LFT} + mC_{HFT}. \tag{77}$$

We have that

$$\lim_{n+\gamma m\to 0} \Phi = 1, \lim_{n+\gamma m\to 0} + nC_{LFT} + mC_{HFT} = 0, \tag{78}$$

such that (76) implies that

$$(1-\bar{\pi})\delta(\frac{3}{2}) > \bar{\pi}(\mu_{inf} - \mu - \frac{3}{2}\delta). \tag{79}$$

is a sufficient condition for participation. Rewriting yields (22). □

Proof of Proposition 5. Because of Lemma 2, the results of Proposition 4 directly apply, but with expected trading profits for HFTs and LFTs given by Expressions (74) and (75), respectively and n^* and m^* derived as follows. We have that

$$\tilde{f}(n+\gamma m) = \min\left(\frac{C_{HFT}}{\gamma}, C_{LFT}\right) \tag{80}$$

is an equilibrium condition. Imposing this condition allows us to invert the function $\tilde{f}(\cdot)$ to obtain equilibrium participation rates of LFTs and HFTs. Substituting Expression (7) into Eq. (21) yields

$$y = \tilde{f}(n+\gamma m) = \left(c + \frac{(1-\bar{\pi})\delta\nu_{liq}}{\nu_{liq}+\lambda(n+\gamma m)}\right)\frac{1}{n+\gamma m} \Rightarrow \tag{81}$$

$$(n+\gamma m)y = c + \frac{(1-\bar{\pi})\delta\nu_{liq}}{\nu_{liq}+\lambda(n+\gamma m)} \Rightarrow \tag{82}$$

$$(n+\gamma m)y(\nu_{liq}+\lambda(n+\gamma m)) = c(\nu_{liq}+\lambda(n+\gamma m)) + (1-\bar{\pi})\delta\nu_{liq}, \tag{83}$$

where

$$c = (1-\bar{\pi})\frac{\delta}{2} - \bar{\pi}(\mu_{inf} - p_{liq}). \tag{84}$$

Rewriting yields

$$0 = \lambda y(n + \gamma m)^2 + (\nu_{liq} y - \lambda c)(n + \gamma m) - \nu_{liq}(c + \delta(1 - \bar{\pi})) \Rightarrow \tag{85}$$

$$n + \gamma m = \tilde{f}^{-1}(y) = \frac{-(\nu_{liq} y - \lambda c) \pm \sqrt{(\nu_{liq} y - \lambda c)^2 + 4\lambda y \nu_{liq}(c + (1 - \bar{\pi})\delta)}}{2\lambda y}. \tag{86}$$

n and γm need to be positive and hence their sum needs to be positive too. There is at most one positive root, namely

$$n + \gamma m = \tilde{f}^{-1}(y) = \frac{-(\nu_{liq} y - \lambda c) + \sqrt{(\nu_{liq} y - \lambda c)^2 + 4\lambda y \nu_{liq}(c + (1 - \bar{\pi})\delta)}}{2\lambda y}. \tag{87}$$

Hence the inverse of $\tilde{f}(\cdot)$ is one-to-one and monotonic, as basic calculus predicts (since $\tilde{f}(\cdot)$ is continuous and monotonic on its domain). When we set $y = \min(C_{LFT}, \frac{C_{HFT}}{\gamma})$, a closed form solution for n^* or γm^* arises. To obtain m^*, we need to divide by γ to obtain (23) for m^*. □

Proof of Corollary 1. Let us assume that Inequality (10) is satisfied in equilibrium. In this setting, HFTs always quote in an empty book and hence we resort to the case with speed only. Due to Proposition 5, liquidity is provided exclusively by the player type with the lowest cost per unit of speed (i.e., HFTs when $\min\left(\frac{C_{HFT}}{\gamma}, C_{LFT}\right) = \frac{C_{HFT}}{\gamma}$ and LFTs otherwise). Hence, $\tilde{f}^{-1}\left(\min\left(\frac{C_{HFT}}{\gamma}, C_{LFT}\right)\right)$ yields the equilibrium participation mass n^* if LFTs have the lowest cost per unit of speed and γm^* otherwise. Inequality (25) then ensures that Inequality (10) is satisfied in equilibrium. □

Proof of Proposition 6. The proof is structured as follows. We first show existence of different equilibrium types and combinations of possible equilibria (in case of multiple equilibria). Next, we derive equilibrium participation rates for LFTs and HFTs in closed form for each of the possible equilibrium types. Finally, we derive the thresholds K_1, K_2, K_3 as the unique solutions for the equations that arise from intersections of equilibrium conditions.

As a preliminary step, we define auxiliary functions to facilitate our exposition

$$\tilde{h}_{HFT}(m, n) = \frac{h_{HFT}}{\gamma}, \tag{88}$$

$$\tilde{g}_{HFT}(m, n) = \frac{g_{HFT}}{\gamma}. \tag{89}$$

Note that in equilibrium, we need to have that $m = 0$, $g_{HFT}(m,n) = C_{HFT}$, or $h_{HFT}(m,n) = C_{HFT}$. This condition is equivalent to having $m = 0$, $\tilde{g}_{HFT}(m,n) = \frac{C_{HFT}}{\gamma}$, or $\tilde{h}_{HFT}(m,n) = \frac{C_{HFT}}{\gamma}$, since $\tilde{h}_{HFT}(m,n)$ and $\tilde{g}_{HFT}(m,n)$ are scalar multiplications of $h_{HFT}(m,n)$ and $g_{HFT}(m,n)$, respectively. We also define D_g as the domain of $g_k(m,n)$ and D_h the domain of $h_k(m,n)$.

We first prove existence of LFT Dominance only, Conditioning HFT Dominance only, and multiple equilibria regions.

We start by assuming that $C_{LFT} << \frac{C_{HFT}}{\gamma}$, such that

$$h_{LFT}(m,n) - C_{LFT} > \tilde{h}_{HFT}(m,n) - \frac{C_{LFT}}{\gamma}, \tag{90}$$

$$g_{LFT}(m,n) - C_{LFT} > \tilde{g}_{HFT}(m,n) - \frac{C_{HFT}}{\gamma}, \tag{91}$$

for all (m,n) on their respective domains. These inequalities hold in particular when $E_{\widehat{a}}(\Pi_{LFT}(R^*_{LFT}(\widehat{a}), \widehat{a}|0,n))$ C_{LFT}. Hence, an equilibrium exists with $m = 0$. When $m = 0$, Condition (12) is trivially satisfied and LFTs participate in an empty book. Hence, liquidity is always offered. Moreover, since $m = 0$, all liquidity is provided by the liquidity provider with the lowest cost per unit of speed. Since LFTs have no conditioning information, the expressions for equilibrium LFT participation rates and expected half spreads are identical to those in Proposition 5. Moreover, this equilibrium is unique since $\frac{\partial h_{LFT}(0,n) - C_{LFT}}{\partial n} < 0$ for all n and by definition, Condition (4) cannot be satisfied with strictly positive m, which completes the proof that LFT dominance exists as a unique equilibrium.

Now we show the existence of multiple equilibria. We start by assuming Condition (91) to be violated in at least one point on D_g. Since

$$0 > -\frac{\frac{\partial \tilde{g}_{HFT}}{\partial n}}{\frac{\tilde{g}_{HFT}}{\partial \gamma m}} \geq -\frac{\frac{\partial g_{LFT}}{\partial n}}{\frac{g_{LFT}}{\partial \gamma m}} \tag{92}$$

on D_g[15], the condition that $g_{LFT}(m,n) - C_{LFT} < \tilde{g}_{HFT}(m,n) - \frac{C_{HFT}}{\gamma}$ in at least one point on D_g implies that either Condition (91) is violated throughout D_g, or that there is exactly one point where $g_{LFT}(m,n) - C_{LFT} = \tilde{g}_{HFT}(m,n) - \frac{C_{HFT}}{\gamma}$. As a result, an equilibrium other LFT Dominance exists.

[15] $\frac{\frac{\partial \tilde{g}_{HFT}}{\partial n}}{\frac{\tilde{g}_{HFT}}{\partial \gamma m}}$ and $\frac{\frac{\partial g_{LFT}}{\partial n}}{\frac{g_{LFT}}{\partial \gamma m}}$ are essentially the equilibrium marginal rates of substitution for γm vs n for HFTs and LFTs, respectively. These correspond, respectively, to the slopes of the green and blue curves in Fig. 4 to 7.

To complete the proof that multiple equilibria can materialize, we must show that a combination $(C_{LFT}, \frac{C_{HFT}}{\gamma})$ exists such that Condition (91) is violated in some point(s) on D_g and that there is an n such that $h_{LFT}(0,n) - C_{LFT} = 0$ and $\tilde{h}_{HFT}(0,n) - \frac{C_{HFT}}{\gamma} < 0$. Now we pick $(C_L, \frac{C_{HFT}}{\gamma})$ such that $g_{LFT}(\tilde{m}, 0) = C_{LFT}$ and $\tilde{g}_{HFT}(\tilde{m}, 0) = \frac{C_{HFT}}{\gamma}$ for some $\tilde{m}$. This way of picking $(C_L, \frac{C_{HFT}}{\gamma})$ ensures that Condition (91) is violated in some point and that $(\tilde{m}, 0)$ is an equilibrium on D_g. It follows that

$$\frac{C_{HFT}}{\gamma} - C_{LFT} = (1-\bar{\pi})\frac{\phi_1 \Phi(p_{liq} - \mu) - (1-\phi_1)(\mu_{inf} - p_{liq})}{\gamma \tilde{m}} > 0. \tag{93}$$

Moreover, we have that

$$\frac{\frac{\partial g_{LFT}}{\partial n}}{\frac{g_{LFT}}{\partial \gamma m}} = 1, \tag{94}$$

and $h_{LFT}(0, n | n = \gamma\tilde{m}) > g_{LFT}(0, n | n = \gamma\tilde{m})$. It follows that $\tilde{n} > \gamma\tilde{m}$, where $\tilde{n}$ is defined as in the proof of Proposition 4. Now we need to show that $\tilde{h}_{HFT}(0, \tilde{n}) < \frac{C_{HFT}}{\gamma}$. We have that

$$\begin{aligned}
&(h_{LFT}(0,\tilde{n}) - C_{LFT}) - \left(\tilde{h}_{HFT}(0,\tilde{n}) - \frac{C_{HFT}}{\gamma}\right) \\
&\quad = \frac{C_{HFT}}{\gamma} - C_{LFT} + \bar{\pi}\frac{(1-\phi_2)\Phi(p_{liq} - \mu) - \phi_2(\mu_{inf} - p_{liq})}{\tilde{n}} && (95) \\
&\quad = (1-\bar{\pi})\frac{\phi_1 \Phi(p_{liq} - \mu) - (1-\phi_1)(\mu_{inf} - p_{liq})}{\gamma\tilde{m}} + \bar{\pi}\frac{(1-\phi_2)\Phi(p_{liq} - \mu) - \phi_2(\mu_{inf} - p_{liq})}{\tilde{n}} && (96) \\
&\quad \geq (1-\bar{\pi})\frac{\phi_1 \Phi(p_{liq} - \mu) - (1-\phi_1)(\mu_{inf} - p_{liq})}{\tilde{n}} + \bar{\pi}\frac{(1-\phi_2)\Phi(p_{liq} - \mu) - \phi_2(\mu_{inf} - p_{liq})}{\tilde{n}}. && (97)
\end{aligned}$$

Substituting ϕ_2 by Expression (1) and rewriting yields

$$\begin{aligned}
(h_{LFT}(0,\tilde{n}) - C_{LFT}) - \left(\tilde{h}_{HFT}(0,\tilde{n}) - \frac{C_{HFT}}{\gamma}\right) &\geq \frac{(1-\bar{\pi})\Phi(p_{liq} - \mu) - \bar{\pi}(\mu_{inf} - p_{liq})}{\tilde{n}} && (98) \\
&\geq 0, && (99)
\end{aligned}$$

where the last inequality is due to Inequality (12) being satisfied on D_h by definition. Hence, $(0, \tilde{n})$ is also an equilibrium, which completes the proof on the existence of multiple equilibria.

Now assume that Conditions (90) and (91) are violated in every point in D_h and D_g, respectively. Since $h_{LFT}(m,n) < \tilde{h}_{HFT}(m,n)$ and $g_{LFT}(m,n) < \tilde{g}_{HFT}(m,n)$, Conditions (90) and (91) are violated particularly if $C_{LFT} \geq \frac{C_{HFT}}{\gamma}$. Requiring zero profit in equilibrium implies that Conditioning HFT Dominance is the only possible equilibrium.

We now derive and participation rates in all possible equilibria in closed form below (except for the LFT Dominance ones, which have been derived before in Proposition 5). We have that

$$g_{LFT}(m,n) = (1-\bar{\pi})\phi_1(1-\Phi)\frac{\frac{1}{2}\delta}{n+\gamma m}, \tag{100}$$

$$= (1-\bar{\pi})\phi_1\left(1-\frac{\nu_{liq}}{\nu_{liq}+\lambda(n+\gamma m)}\right)\frac{\frac{1}{2}\delta}{n+\gamma m}, \tag{101}$$

$$= (1-\bar{\pi})\phi_1\frac{\lambda(n+\gamma m)}{\nu_{liq}+\lambda(n+\gamma m)}\frac{\frac{1}{2}\delta}{n+\gamma m}, \tag{102}$$

$$= (1-\bar{\pi})\phi_1\frac{\lambda\frac{1}{2}\delta}{\nu_{liq}+\lambda(n+\gamma m)}. \tag{103}$$

Equating $g_{LFT}(m,n)$ to C_{LFT} and solving for $(n+\gamma m)$ yields

$$g_{LFT}(m,n) = C_{LFT} \Rightarrow \tag{104}$$

$$(n+\gamma m) = \frac{(1-\bar{\pi})\phi_1\lambda\frac{1}{2}\delta}{\lambda C_{LFT}} - \frac{\nu_{liq}}{\lambda} = \frac{(1-\bar{\pi})\phi_1\frac{1}{2}\delta}{C_{LFT}} - \frac{\nu_{liq}}{\lambda}. \tag{105}$$

Equilibria on D_g yield an interior solution with $g_{LFT}(m,n) = C_{LFT}$ as derived above, or a corner solution with $g_{LFT}(m,n) < C_{LFT}$ such that $n = 0$. We work out both scenarios to get a full characterization of all possible equilibria.

We have that

$$\tilde{g}_{HFT}(m,n) = g_{LFT}(m,n) + \frac{1}{\gamma m}(1-\bar{\pi})\left(\phi_1\Phi\frac{3}{2}\delta - (1-\phi_1)(\mu_{inf}-\mu-\frac{3}{2}\delta)\right), \tag{106}$$

$$= g_{LFT}(m,n) + \frac{1}{\gamma m}(1-\bar{\pi})\left(\phi_1\frac{\nu_{liq}}{\nu_{liq}+\lambda(n+\gamma m)}\frac{3}{2}\delta - (1-\phi_1)(\mu_{inf}-\mu-\frac{3}{2}\delta)\right). \tag{107}$$

For any interior solution, we substitute from Eq. (104) to get

$$\tilde{g}_{HFT}(m,n|g_{LFT}(m,n) = C_{LFT}) = C_{LFT} + \frac{1}{\gamma m}(1-\bar{\pi})\left(\phi_1\frac{\nu_{liq}C_{LFT}}{\frac{1}{2}\delta\lambda(1-\bar{\pi})}\frac{3}{2}\delta - (1-\phi_1)(\mu_{inf}-\mu-\frac{3}{2}\delta)\right). \tag{108}$$

Note that this expression does not depend on n anymore. Setting $g_{HFT}(m,n) = \frac{C_{HFT}}{\gamma}$ and solving for m yields

$$m = \frac{1}{C_{HFT} - \gamma C_{LFT}}(1-\bar{\pi})\left(\phi_1 \frac{\nu_{liq} C_{LFT}}{\frac{1}{2}\delta\lambda(1-\bar{\pi})}\frac{3}{2}\delta - (1-\phi_1)(\mu_{inf} - \mu - \frac{3}{2}\delta)\right). \tag{109}$$

For a corner solution where $n = 0$, we substitute $n = 0$ to get

$$\tilde{g}_{HFT}(m, 0) = \frac{1}{\gamma m}(1-\bar{\pi})\left(\frac{\frac{1}{2}\phi_1\delta\lambda\gamma m + \frac{3}{2}\phi_1\delta\nu_{liq}}{\nu_{liq} + \lambda\gamma m} - (1-\phi_1)(\mu_{inf} - \mu - \frac{3}{2}\delta)\right). \tag{110}$$

Setting $\tilde{g}_{HFT}(m, 0) = \frac{C_{HFT}}{\gamma}$ and solving for m yields a quadratic expression of the form $em^2 + \xi m + c = 0$, which can be solved with standard quadrature rules:

$$m = \frac{-\xi \pm \sqrt{\xi^2 - 4e\sigma}}{2e}, \tag{111}$$

where

$$e = \lambda\gamma C_{HFT}, \tag{112}$$

$$\xi = \nu_{liq} C_{HFT} - (1-\bar{\pi})\left(\frac{1}{2}\phi_1\delta\lambda\gamma - (1-\phi_1)\lambda(\mu_{inf} - \mu - \frac{3}{2}\delta)\right), \tag{113}$$

$$\sigma = (1-\bar{\pi})\nu_{liq}((1-\phi_1)(\mu_{inf} - \mu) - \frac{3}{2}\delta). \tag{114}$$

This equation only has at most one positive root since $(1-\phi_1)(\mu_{inf} - \mu) - \frac{3}{2}\delta < 0$ and hence $\sigma < 0$, which gives at most one corner equilibrium.

On D_h we can have a corner equilibrium with $m = 0$. Since this equilibrium involves only LFTs, it conforms to the results from Proposition 5. There could also be an interior equilibrium where $h_{LFT} = C_{LFT}$ and $\tilde{h}_{HFT} = \frac{C_{HFT}}{\gamma}$ are jointly satisfied. Solving for $(n + \gamma m)$ on the indifference curve $\tilde{h}_{HFT} = \frac{C_{HFT}}{\gamma}$ once again involves solving for the positive root of a quadratic function (which we do not do in the interest of brevity since this equilibrium is trembling-hand-imperfect).

Now we solve for the thresholds K_1, K_2, and K_3. We solve first for K_3. LFT Dominance is not an equilibrium when

$$\tilde{h}_{HFT}(0,\tilde{n}) - \frac{C_{HFT}}{\gamma} > h_{LFT}(0,\tilde{n}) - C_{LFT} = 0, \Rightarrow \tag{115}$$

$$C_{LFT} - \frac{C_{HFT}}{\gamma} > \bar{\pi}\frac{(1-\phi_2)\Phi(p_{liq}-\mu) - \phi_2(\mu_{inf}-p_{liq})}{\tilde{n}}, \Rightarrow \tag{116}$$

$$\frac{C_{HFT}}{\gamma C_{LFT}} < K_3 = 1 - \bar{\pi}\frac{(1-\phi_2)\Phi(p_{liq}-\mu) - \phi_2(\mu_{inf}-p_{liq})}{\tilde{n}C_{LFT}}. \tag{117}$$

We now solve for K_2. When $\frac{C_{HFT}}{\gamma} = K_2$, we must have that

$$g_{LFT}(\bar{m},0) = C_{LFT}, \tag{118}$$

because $\frac{\partial g_{LFT}}{\partial m} < 0, \frac{\partial g_{LFT}}{\partial n} < 0$ on D_g. We can solve for $\bar{m}$:

$$g_{LFT}(\bar{m},0) = C_{LFT}, \Rightarrow \tag{119}$$

$$C_{LFT} = (1-\bar{\pi})\phi_1(1-\Phi)\frac{\frac{1}{2}\delta}{\gamma\bar{m}}, \Rightarrow \tag{120}$$

$$\frac{C_{LFT}}{(1-\bar{\pi})\phi_1} = \frac{\frac{1}{2}\delta}{\gamma\bar{m}}\frac{\lambda\gamma\bar{m}}{\nu_{liq}+\lambda\gamma\bar{m}} = \frac{\frac{1}{2}\delta\lambda}{\nu_{liq}+\lambda\gamma\bar{m}}, \Rightarrow \tag{121}$$

$$\gamma\bar{m} = \frac{\frac{1}{2}\delta(1-\bar{\pi})\phi_1}{C_{LFT}} - \frac{\nu_{liq}}{\lambda}, \tag{122}$$

$$\bar{m} = \frac{\frac{1}{2}\delta(1-\bar{\pi})\phi_1}{\gamma C_{LFT}} - \frac{\nu_{liq}}{\gamma\lambda}. \tag{123}$$

We can now rearrange (109) and substitute $\bar{m}$ to obtain the condition that Coexistence equilibria are not possible anymore:

$$\frac{C_{HFT}}{\gamma} < C_{LFT} + \frac{1}{\bar{m}}(1-\bar{\pi})\left(\phi_1\frac{\nu_{liq}C_{LFT}}{\frac{1}{2}\delta\lambda(1-\bar{\pi})}\frac{3}{2}\delta - \phi_1(\mu_{inf}-\mu-\frac{3}{2}\delta)\right) = K_2. \tag{124}$$

Finally, we solve for K_1. Co-existence is possible when for some m, (109) holds. Rewriting gives that Coexistence is possible when

$$K_2 \leq \frac{C_{HFT}}{\gamma} < K_1 = C_{LFT} + \frac{1}{\breve{m}}(1-\bar{\pi})\left(\phi_1\frac{\nu_{liq}C_{LFT}}{\frac{1}{2}\delta\lambda(1-\bar{\pi})}\frac{3}{2}\delta - \phi_1(\mu_{inf}-\mu-\frac{3}{2}\delta)\right), \tag{125}$$

where $\breve{m}$ is the minimum value for m such that (109) holds. Let $\breve{n}$ be the corresponding n in the Coexistence equilibrium with $m = \breve{m}$. Now we need to derive $\breve{m}$, which we cannot do directly. However, we can first derive all combinations (m,n) that correspond to Coexistence. For Coexistence, we need $g_{LFT}(m,n) = C_{LFT}$, which is particularly true for $(m,n) = (\breve{m},\breve{n})$.

We can solve this equation towards $(\breve{n} + \gamma\breve{m})$ in the same way we solved for $\tilde{m}$:

$$\breve{n} + \gamma\breve{m} = \frac{\frac{1}{2}\delta(1-\bar{\pi})\phi_1}{C_{LFT}} - \frac{\nu_{liq}}{\lambda}, \Rightarrow \tag{126}$$

$$\breve{m} = \frac{\frac{1}{2}\delta(1-\bar{\pi})\phi_1}{\gamma C_{LFT}} - \frac{\nu_{liq}}{\gamma\lambda} - \frac{\breve{n}}{\gamma}. \tag{127}$$

We finally solve for $\breve{n}$ to complete our derivation. Condition (12) binds for this Coexistence equilibrium. We first define

$$X = \breve{n} + \gamma\breve{m} = \frac{\frac{1}{2}\delta(1-\bar{\pi})\phi_1}{C_{LFT}} - \frac{\nu_{liq}}{\lambda}. \tag{128}$$

Substituting X into (54) and imposing $n = \breve{n}$, we obtain

$$\hat{P}(\zeta = inf|B) = \frac{\frac{1}{\breve{n}}\phi_2\bar{\pi} + (1-\phi_1)(1-\bar{\pi})X^{-1}}{\frac{1}{\breve{n}}\bar{\pi} + (1-\bar{\pi})X^{-1}}. \tag{129}$$

Making (12) bind, we get

$$\frac{\frac{3}{2}\delta\nu_{liq}}{\nu_{liq} + \lambda X}(1 - \hat{P}(\zeta = inf|B)) = (\mu_{inf} - p_{liq})\hat{P}(\zeta = inf|B). \tag{130}$$

We define

$$F = \frac{\frac{3}{2}\delta\nu_{liq}}{\nu_{liq} + \lambda X}, \tag{131}$$

$$E = (\mu_{inf} - p_{liq}). \tag{132}$$

Substituting and solving, we get

$$\hat{P}(\zeta = inf|B) = \frac{F}{F+E}. \tag{133}$$

Substituting (129) into (133), we get

$$\frac{\frac{1}{\breve{n}}\phi_2\bar{\pi} + (1-\phi_1)(1-\bar{\pi})X^{-1}}{\frac{1}{\breve{n}}\bar{\pi} + (1-\bar{\pi})X^{-1}} = \frac{F}{F+E}. \tag{134}$$

Rearranging, we get

$$\breve{n} = \frac{\bar{\pi}(\phi_2(F+E)-F)}{(1-\bar{\pi})X^{-1}(\phi_1(F+E)-E)}, \tag{135}$$

which allows us to get K_1 by substitution of the expressions for $\breve{n}$ and $\breve{m}$. □

Proof of Proposition 7. We first prove that the availability of HFT technology with a higher cost per unit of speed than LFT technology can both increase and decrease expected half spreads. The proof is by numerical example. We set $\nu_{liq} = 0.05$, $\lambda = 0.1$, $\mu = 8$, $\mu_{inf} = 11$, $\bar{\pi} = 0.05$, $\phi_1 = 0.98$, $\delta = 1$, $C_{LFT} = 1.1801$ and obtain the following expected half spreads for different values of $\frac{C_{HFT}}{\gamma}$.

$\frac{C_{HFT}}{\gamma}$	20	14	1.5	1.3	1.25
m	0	0.08	0.54	0.6	0.62
n	0.61	0.01	0	0	0
Φ	0.339	0.776	0.367	0.342	0.335
S	0.872	1.281	0.879	0.856	0.848
$\hat{S}$	0.872	1.292	0.910	0.888	0.881

We continue by showing that with multiple equilibria, $\hat{S}$ in Coexistence and Conditioning HFT Dominance equilibria is higher than in LFT Dominance. For LFT Dominance (12) is satisfied, so $\hat{S} = S$. It follows from (29) that it is sufficient to show that Φ for LFT Dominance is lower than for Coexistence and Conditioning HFT Dominance. Showing this is equivalent to showing that $n^* + \gamma m^*$ for LFT Dominance is higher than for Coexistence and Conditioning HFT Dominance. We have that

$$-\frac{\frac{\partial h_{LFT}}{\partial n}}{\frac{h_{LFT}}{\partial \gamma m}} > -\frac{\frac{\partial \tilde{h}_{HFT}}{\partial n}}{\frac{\tilde{h}_{HFT}}{\partial \gamma m}} = -1, \tag{136}$$

and that $\tilde{h}_{HFT}$ and h_{LFT} are continuously differentiable on D_h. Moreover, we have that

$$\tilde{h}_{HFT}(m,0) > \tilde{g}_{HFT}(m,0), \tag{137}$$

if $\tilde{h}_{HFT}$ were to exist on D_g. Now assume that multiple equilibria are possible, which implies that $h_{LFT}(0,n^*) - C_{LFT} \geq \tilde{h}_{HFT}(0,n^*) - \frac{C_{HFT}}{\gamma}$. Hence, it must be that the aggregate equilibrium competition intensity $(n^* + \gamma m^*)$ in LFT Dominance is higher than in any equilibrium on D_g.

Finally, we show that $\hat{S}$ can increase as a result of the availability of HFT technology, even when $\frac{C_{HFT}}{\gamma C_{LFT}} < 1$. The proof is by numerical example. We set $\nu_{liq} = 0.05$, $\lambda = 0.1$, $\mu = 8$, $\mu_{inf} = 11$, $\bar{\pi} = 0.05$, $\phi_1 = 0.98$, $\delta = 1$, $\frac{C_{HFT}}{\gamma} = 1.18$, $C_{LFT} = 1.1801$. Solving yields $\tilde{n} = 0.68, \gamma m^* = 0.70$. Filling in the expressions for Φ and $\hat{S}$, we get $\hat{S} = 0.9569$ for the Conditioning HFT Dominance equilibrium when evaluated at these parameter values and m^* and $\hat{S} = 0.9525$ for the hypothetical LFT Dominance equilibrium in the absence of HFT technology evaluated at these parameters and $\tilde{n}$. □

Proof of Lemma 5. Aggregate participation costs for HFTs scale linearly with m. Hence, when aggregate expected trading profits are strictly positive, it is always possible to find an m such that expected trading profits equal expected participation costs. Aggregate expected trading profits for HFTs with informed signals are given by $mg_{HFT}(m, n)$. We have that

$$mg_{HFT}(m, n) > 0 \Rightarrow \tag{138}$$

$$g_{LFT}(m, n) + (1 - \bar{\pi})\phi_1 \frac{3}{2}\delta\Phi > (1 - \bar{\pi})(1 - \phi_1)(\mu_{inf} - \mu - \frac{3}{2}\delta). \tag{139}$$

When $\phi_1 > (1 - \bar{\pi})$, the signal is informative and the condition above is tighter than (22), such that no HFT participation implies no LFT participation. Moreover, we have that

$$\lim_{n+\gamma m \to 0} \Phi = 1, \tag{140}$$

Hence $g_{LFT}(m, n)$ disappears in the limit and

$$(1 - \bar{\pi})\phi_1 \frac{3}{2}\delta > (1 - \bar{\pi})(1 - \phi_1)(\mu_{inf} - \mu - \frac{3}{2}\delta) \tag{141}$$

is a sufficient condition for participation. Rewriting yields (30). □

Proof of Proposition 8. Due to Lemma 5, there is no HFT nor LFT participation when (30) is violated. When $\bar{\pi} \geq \pi^{tox}$, but (30) is satisfied, HFTs optimally post in an empty book when $s = liq$ due to Proposition 2. Moreover, due to Proposition 3, it is never optimal for LFTs to post in an empty book. Hence, expected trading profits for HFTs and LFTs are given by $g_{HFT}(m, n)$ and $g_{LFT}(m, n)$, respectively. Requiring zero profit for HFTs and LFTs requires solving the system $g_{HFTs}(m,n) = C_{HFT}, g_{LFT}(m,n) = C_{LFT}$ or $g_{HFTs}(m,0) = C_{HFT}$. Since

$$-\frac{\frac{\partial g_{LFT}(m,n)}{\partial n}}{\frac{\partial g_{LFT}(m,n)}{\partial m}} < -\frac{\frac{\partial g_{HFT}(m,n)}{\partial n}}{\frac{\partial g_{HFT}(m,n)}{\partial m}}, \tag{142}$$

$$\frac{\partial g_{HFT}(m,n)}{\partial m}, \frac{\partial g_{LFT}(m,n)}{\partial m}, \frac{\partial g_{HFT}(m,n)}{\partial n}, \frac{\partial g_{LFT}(m,n)}{\partial n} < 0, \tag{143}$$

and

$$\lim_{n\to\infty} g_{HFT}(m,n) > 0 \forall m, \tag{144}$$

a solution always exists and is unique. Since the same equations are solved as in Proposition 6, it corresponds to Coexistence with $(m,n) = (m^*, n^*)$ as in Proposition 6 or to Conditioning HFT Dominance with $(m,n) = (m^*, 0)$ as in Proposition 6. Due to Proposition 6, transactions take place with strictly positive probability, such that liquidity is higher than in the case without HFT technology in which there was a complete absence of trading. □

Proof of Lemma 6. The zero-profit requirement implies that not using information yields zero welfare as otherwise there are no strictly positive values for m or n that allow for strictly positive expected trading profits. Proposition 8 and Panel B of Table 1 indicate that strictly positive expected trading profits and welfare are possible in Conditioning HFT Dominance or Coexistence equilibria. Hence, using information is strictly optimal from a welfare perspective. Irrespective of whether HFTs use information, LFTs optimally refrain from posting in an empty book. Given that HFTs use informative signals and LFTs refrain from posting in an empty book, expected trading profits for HFTs and LFTs are given by $g_{HFT}(m,n)$ and $g_{LFT}(m,n)$, respectively. Imposing a zero-profit requirement for HFTs and LFTs requires solving the system $g_{HFTs}(m,n) = C_{HFT}, g_{LFT}(m,n) = C_{LFT}$ or $g_{HFTs}(m,0) = C_{HFT}$. Since

$$-\frac{\frac{\partial g_{LFT}(m,n)}{\partial n}}{\frac{\partial g_{LFT}(m,n)}{\partial m}} < -\frac{\frac{\partial g_{HFT}(m,n)}{\partial n}}{\frac{\partial g_{HFT}(m,n)}{\partial m}}, \tag{145}$$

$$\frac{\partial g_{HFT}(m,n)}{\partial m}, \frac{\partial g_{LFT}(m,n)}{\partial m}, \frac{\partial g_{HFT}(m,n)}{\partial n}, \frac{\partial g_{LFT}(m,n)}{\partial n} < 0, \tag{146}$$

and

$$\lim_{n\to\infty} g_{HFT}(m,n) > 0 \forall m, \tag{147}$$

a solution always exists and is unique. Since the same equations are solved as in Proposition 6, the solution corresponds to Coexistence with $(m,n) = (m^*, n^*)$ as in Proposition 6 or to Conditioning HFT Dominance with $(m,n) = (m^*, 0)$ as in Proposition 6. Since the socially optimal use of information coincides with the equilibrium use of information and the socially optimal participation rates coincide with equilibrium participation rates, the equilibrium outcome coincides with all three welfare benchmarks. □

Proof of Lemma 7. The proof is by contradiction. Assume that informative signals are used when $\phi_2 \in [\bar{\pi}, \phi_2^{ul}]$. $\phi_2 \in [\bar{\pi}, \phi_2^{ul}]$, implies that Condition (12) is satisfied, because $P(\zeta = inf|B) < \phi_2$, where B denotes the event of arriving to an empty book (substitute (1) into (54)). Hence, LFTs optimally quote in an empty book. Moreover, since all uncertainty about ζ is resolved after the first quote has been posted, undercutting happens with intensity $\lambda(\gamma m + n)$, as before. Hence, aggregate expected trading profits for given values of m and n must equal $\tilde{\Pi}$ due to Lemma 2. Using informative signals reduces HFT profitability, which implies that LFT profitability must improve. When LFTs have the lowest cost per unit of speed, requiring zero profit leads to LFT Dominance with $n = n^*$ as in LFT Dominance of Proposition 6, because LFT profits always strictly exceed HFT profits. When HFTs have the lowest cost per unit of speed, requiring zero profit may still lead to $n > 0$. In that case, some liquidity is provided by a liquidity provider with the highest cost per unit of speed. Since aggregate trading profits $\tilde{\Pi}$ are monotonically decreasing in n and γm, it must be that $n + \gamma m < \gamma m^*$ with m^* as in NonConditioning HFT Dominance. As a result, $S(m,n) > S(m^*, 0)$. When all liquidity demand is served, expected utility of uninformed and informed liquidity demanders is given by $(1-\bar{\pi})(\frac{3}{2}\delta - S)$ and $\bar{\pi}(\mu_{inf} - \mu - S)$, respectively, while liquidity providers break even. Hence, aggregate welfare is declining in S. Hence, using informative signals cannot be socially optimal. Given the expected profits in Lemma 2, Proposition 5 shows that imposing zero-profit conditions yield a unique solution for (m^*, n^*). Hence, the equilibrium outcome must coincide with all welfare benchmarks. □

Proof of Lemma 8. We start out by proving the result for BM^{part}. Take as given that signals are used. Optimal behavior of HFTs and LFTs in the trading stage is then identical to that in Propositions 2 and 3, respectively. As a result, expected profits are given by $g_{HFT}(m, n)$ and $h_{HFT}(m, n)$ for HFTs and $g_{LFT}(m, n)$ and $h_{LFT}(m, n)$ for LFTs. Requiring zero profit for HFTs and LFTs results in fixed points corresponding to Conditioning HFT Dominance, LFT Dominance, Coexistence and Instable coexistence equilibria. Whenever Instable Coexistence is possible, LFT Dominance is also possible. In Instable coexistence some liquidity is provided by liquidity providers with the highest cost per unit of speed while liquidity demand is always served. By the same argument that is used in the proof of Lemma 7, Instable coexistence must be welfare inferior to LFT Dominance so it can never coincide with any of the welfare benchmarks.

We now compare Conditioning HFT Dominance, LFT Dominance, and Coexistence. As a first step, we substitute (29) into the expressions in Panel B of Table 1 to obtain the expected welfare in case of Conditioning HFT Dominance:

$$(1-\bar{\pi})\phi_1 \frac{3}{2}\delta - \hat{S}(m^*, 0) + (1-\bar{\pi})(1-\phi_1)(\mu_{inf} - \mu) + \bar{\pi}(p_{liq} - \mu), \tag{148}$$

and in the case of Coexistence:

$$(1-\bar{\pi})\phi_1 \frac{3}{2}\delta - \hat{S}(m^*, n^*) + (1-\bar{\pi})(1-\phi_1)(\mu_{inf} - \mu) + \bar{\pi}(p_{liq} - \mu), \tag{149}$$

expressed as a function of $\hat{S}$.

We now show that when multiple equilibria exist, LFT Dominance always strictly dominates the other equilibria in terms of aggregate welfare. We have that $\phi_1 \in (1-\bar{\pi}, 1]$, such that

$$\bar{\pi}(\mu_{inf} - \mu) > (1-\phi_1)(\mu_{inf} - \mu) + \bar{\pi}(p_{liq} - \mu). \tag{150}$$

Moreover, we have that in case of multiple equilibria, $\hat{S}$ for LFT Dominance is lower than that $\hat{S}$ for Conditioning HFT Dominance or Coexistence due to Proposition 7. It then follows immediately from expressions (148) and (149) that any other equilibrium than LFT Dominance comes at a welfare loss as long as LFT Dominance is one of the possible equilibria.

We now show that there are always values for $\frac{C_{HFT}}{\gamma} \leq C_{LFT}$ for which welfare is lower than with LFT Dominance, were it to exist (i.e., if HFTs were banned). Assume that $\frac{C_{HFT}}{\gamma} =$

C_{LFT}. Define $\tilde{n}$ as in the proof of Proposition 4. We have that $\frac{g_{HFT}(\frac{\tilde{n}}{\gamma},0)}{\gamma} > h_{LFT}(0,\tilde{n})$ due to information processing technology and LFTs shunning an empty book. As a result, $\gamma m^* > \tilde{n}$, which implies $m^* C_{HFT} > \tilde{n} C_{LFT}$. Welfare gains from uninformed trades are only a fraction $\phi_1 < 1$ of those when HFT technology were absent. It then follows from the expressions in Panel A of Table 1 that aggregate welfare is strictly lower when $\frac{C_{HFT}}{\gamma} = C_{LFT}$. Since aggregate welfare of the Conditioning HFT Dominance equilibrium is continuously declining in $\frac{C_{HFT}}{\gamma}$ there must be values for $\frac{C_{HFT}}{\gamma} < C_{LFT}$ for which welfare is also lower. Hence, BM^{part} coincides with Conditioning HFT Dominance if $\frac{C_{HFT}}{\gamma C_{LFT}} < K \leq 1$, where $K < 1$, and with LFT Dominance otherwise.

We continue by showing the results for BM^{inf} and BM^{comb}. Now take as given that signals are not used. Analogous to the proof of Lemma 7, it follows that a fixed point corresponding with Nonconditioning HFT Dominance maximizes welfare when $\frac{C_{HFT}}{\gamma C_{LFT}} < 1$ and one corresponding with LFT Dominance otherwise. These also happen to be the respective equilibria in case signals are not used. To determine BM^{comb}, we must compare the maximum welfare when information is used vs the maximum welfare when information is not used. Panel A of Table 1 shows that when m^* for Conditioning HFT Dominance exceeds m^* for Nonconditioning HFT Dominance, Nonconditioning HFT Dominance must come at higher welfare since $\phi_1 \leq 1$ and $C_{HFT} > 0$. Panel B of Table shows that when m^* for Conditioning HFT Dominance is smaller or equal to m^* for Nonconditioning HFT Dominance, Nonconditioning HFT Dominance must come at higher welfare since $\phi_1 \leq 1$, $(1-\bar{\pi})(1-\phi_1) < \bar{\pi}$, and $S(m^*,0)$ is declining in m^*. Hence, BM^{comb} coincides with Nonconditioning HFT Dominance when $\frac{C_{HFT}}{\gamma C_{LFT}} < 1$ and with LFT Dominance otherwise. Since privately optimal participation decisions conditional on not using signals coincide with socially optimal participation decisions, BM^{comb} and BM^{inf} coincide. □

Proof of Proposition 9. Lemmas 6 and 7 show that when $\bar{\pi} \geq \pi^{tox}$ and Condition (30) is satisfied or when $\bar{\pi} \in (0, \pi^{tox})$ and $\phi_2 \leq \phi_2^{ul}$, equilibrium outcomes coincide with all welfare benchmarks. When $\bar{\pi} \in (0, \pi^{tox})$ and $\phi_2 > \phi_2^{ul}$, Lemma 8 shows that the equilibria that can materialize with strictly positive HFT participation can differ from the different welfare benchmarks established. In particular, the proof of Lemma 8 shows Conditioning HFT Dominance falls short of BM^{inf}, which leads to the conclusion that welfare is forgone due to inefficient use of information. Lemma 8 also shows that when $\frac{C_{HFT}}{\gamma C_{LFT}}$ is sufficiently large, the Conditioning HFT Dominance and Coexistence equilibria with $m^* > 0$ fall short of BM^{part} for which $m^* = 0$. This result implies HFT overparticipation. □

Proof of Proposition 10. Assume that $\frac{C_{HFT}}{\gamma C_{LFT}}$ is large enough to rule out Conditioning HFT Dominance equilibria in which welfare is higher than LFT Dominance, were it to exist. Moreover, assume that η is implemented according to Conditions (34).

Consider any pair (m, n) for which $g_{LFT}(m, n) = C_{LFT}$ or $h_{LFT}(m, n) = C_{LFT}$. Due to (34), it must be that $g_{HFT}(m, n) < C_{LFT}$ or $h_{HFT}(m, n) < C_{LFT}$, which is particularly true for $g_{HFT}(m, 0)$ and $h_{HFT}(0, n)$. Moreover, Conditions (34) preclude the situation in which $\hat{g}_{HFT}(m, n) - \gamma g_{LFT}(m, n) = C_{HFT} - \gamma C_{LFT}$ or $\hat{h}_{HFT}(m, n) - \gamma h_{LFT}(m, n) = C_{HFT} - \gamma C_{LFT}$ for any (m, n). As a result, $m^* = 0$ and LFT dominance is the only possible equilibrium. Due to Proposition 9, LFT Dominance yields strictly higher welfare than any other equilibrium in these situations. Consequently, there must be a welfare improvement. Since $m^* = 0$ and taxes are only collected on trades conducted by HFTs, the tax is never collected in equilibrium. □

Proof of Proposition 11. Identity (1) and Proposition 2 imply that if $\phi_1 = 1 - \bar{\pi}$, a quote is always posted irrespective of the signal. Proposition 5 then applies. When $\frac{C_{HFT}}{\gamma C_{LFT}} > 1$, then LFT Dominance arises and Nonconditioning HFT Dominance otherwise. Proposition 9 shows that LFT Dominance yields higher welfare than Conditioning HFT Dominance when $\frac{C_{HFT}}{\gamma C_{LFT}} > 1$. Moreover, Proposition 9 shows that Nonconditioning HFT Dominance yields higher welfare than Conditioning HFT dominance, which completes the proof that welfare improves as a consequence of mandatory liquidity provision. □

Proof of Proposition 12. It is always optimal for an HFT to quote in an empty book when

$$\phi_2(\mu_{inf} - p_{liq} - \omega) \leq (p_{liq} - \mu + \omega)(1 - \phi_2)\Phi, \Rightarrow \tag{151}$$

$$\phi_2 \leq \left(1 + \frac{\mu_{inf} - p_{liq} - \omega}{p_{liq} - \mu + \omega}\left(1 + \frac{\lambda}{\nu_{liq}}(n^* + \gamma m^*)\right)\right)^{-1}. \tag{152}$$

Now assume that $n^* + \gamma m^*$ is such that (152) holds. In that case, Proposition 5 applies and $m^* > 0$ iff $\frac{C_{HFT}}{\gamma C_{LFT}} \leq 1$.

Now if $\frac{C_{HFT}}{\gamma C_{LFT}} > 1$, $m^* = 0$, such that LFT Dominance materializes and expected LFT trading profits are given by $\frac{\tilde{\Pi}}{n^*}$, such that $\tilde{f}^{-1}(C_{LFT}) = n^*$ and (36) ensures that (152) holds.

If on the other hand $\frac{C_{HFT}}{\gamma C_{LFT}} \leq 1$, then Proposition 5 dictates that $n^* = 0$. The expected HFT trading profits are given by

$$\gamma \frac{\tilde{\Pi}}{\gamma m^*} + \frac{\bar{\pi}\omega - (1-\bar{\pi})\eta}{m^*} = \gamma \frac{\tilde{\Pi}}{\gamma m^*} + 0. \tag{153}$$

As a result, we have that $\tilde{f}\left(\frac{C_{HFT}}{\gamma}\right) = \gamma m^*$ and (36) ensures that (152) holds. □

Proof of Corollary 2. If the equilibrium type is unchanged, S is strictly declining in $n+\gamma m$ since Φ is. Due to Proposition 6, we have either of three options. We can have LFT Dominance, in which case $\frac{C_{HFT}}{\gamma}$ is irrelevant, so S weakly declines when $\frac{C_{HFT}}{\gamma}$ decreases. We can have Coexistence, which still satisfies $g_{LFT}(m,n) = C_{LFT}$. Since n and m only show up as $n + \gamma m$ in $g_{LFT}(m,n)$, the marginal rate of HFT/LFT substitution equals γ, and hence liquidity is unaffected too, so S weakly declines when $\frac{C_{HFT}}{\gamma}$ decreases. We can have Conditioning HFT Dominance, in which case $n = 0$. Moreover, we have that $\frac{\partial g_{HFT}(m,n)}{\partial m} < 0$. Satisfying the condition $g_{HFT}(m,n) = C_{HFT}$ implies that m^* strictly increases as $\frac{C_{HFT}}{\gamma}$ declines (leaving γ constant). As a result, S strictly declines as C_{HFT} declines.

When equilibrium types can change as a result of a change in C_{HFT}, we can go from LFT Dominance to Coexistence. Fig. 4 and 5 show a numerical example where $n+\gamma m$ can decline and therefore S can increase as a result of a drop in $\frac{C_{HFT}}{\gamma}$ (because Coexistence becomes possible). Similarly, Fig. 4 and 7 show a numerical example where $n + \gamma m$ increases and therefore S declines as a result of a drop in $\frac{C_{HFT}}{\gamma}$. □

Proof of Corollary 3. In the model, the only quote level far from the competitive price is the reservation price p_{liq}. It follows from Proposition 3 that LFTs are less willing to post a quote there when γm is large compared to n. □

Proof of Corollary 4. Due to Proposition 6, liquidity demand is served with certainty with LFT Dominance, and only with probability $(1 - \bar{\pi}) < 1$ with Coexistence or Conditioning HFT Dominance. Due to Proposition 6, Coexistence and Conditioning HFT Dominance are only possible when $\frac{C_{HFT}}{\gamma} \leq K_1$. Hence, as $\frac{C_{HFT}}{\gamma}$ declines, the likelihood of orders being executed (weakly) declines. □

Proof of Corollary 5. Due to Proposition 3, LFTs do not quote in an empty book when (12) is violated and LFTs do not incur adverse selection losses. Due to Proposition 6, Coexistence and Conditioning HFT Dominance are only possible when $\frac{C_{HFT}}{\gamma} \leq K_1$, and only in these equilibria adverse selection losses for LFTs equal zero. With LFT Dominance these are strictly positive and independent of $\frac{C_{HFT}}{\gamma}$. Hence, as $\frac{C_{HFT}}{\gamma}$ declines the LFT adverse selection losses (weakly) decline. □

B Requirements for Nonaggressive Opening Quotes

In this section, we derive conditions for nonaggressive optimal opening quotes in an empty book (i.e., $a = p_{liq}$ being optimal rather than $a = p(1)$), and show that when this condition holds in the uninformed case, it must also hold in the informed case.

We start from the uninformed case. The expected trading profit of posting p_{liq} is given by $\frac{3}{2}\delta\Phi$. The expected trading profit of quoting $p(1)$ is given by $\frac{1}{2}\delta$. Hence posting $p(1)$ is strictly optimal when

$$\frac{3}{2}\delta\Phi < \frac{1}{2}\delta, \Rightarrow \tag{154}$$

$$\Phi < \frac{1}{3}. \tag{155}$$

Substituting for Φ, we get

$$\frac{\nu_{liq}}{\nu_{liq} + \lambda(n + \gamma m)} < \frac{1}{3}, \Rightarrow \tag{156}$$

$$n + \gamma m > \frac{2\nu_{liq}}{\lambda}. \tag{157}$$

In equilibrium $(m, n) = (m^*, n^*)$. Moreover, in equilibrium, expected trading profits should equal participation costs:

$$\frac{\Pi}{(n^* + \gamma m^*)} = \min\left(C_{LFT}, \frac{C_{HFT}}{\gamma}\right), \Rightarrow \tag{158}$$

$$(n^* + \gamma m^*) = \frac{-(\nu_{liq} y - \frac{\lambda\delta}{2}) + \sqrt{(\nu_{liq} y - \frac{\lambda\delta}{2})^2 + 6y\lambda\nu_{liq}\delta}}{2\lambda y}, \tag{159}$$

where $y = \min\left(\frac{C_{HFT}}{\gamma}, C_{LFT}\right)$ and the latter expression follows from Proposition 4. Imposing that $(m, n) = (m^*, n^*)$ on (157) and substituting in from (159) yields

$$\frac{-(\nu_{liq} y - \frac{\lambda\delta}{2}) + \sqrt{(\nu_{liq} y - \frac{\lambda\delta}{2})^2 + 6y\lambda\nu_{liq}\delta}}{2\lambda y} \leq \frac{2\nu_{liq}}{\lambda}. \tag{160}$$

Condition (160) also assures that the first quote posted in a book, if any, always coincides with p_{liq} in the informed case. The reason is that expected informed trading losses are declining in the starting quote in the book, and therefore ensuring that the starting point equals p_{liq} in the uninformed case must imply that p_{liq} is also the starting point in the informed case.

C Notation Summary

Parameters	
Symbol	*Description*
δ	tick size
μ	fundamental value conditional on public information only
p_{liq}	reservation price liquidity demanders
$p(1)$	lowest profitable price level on the grid (competitive price)
μ_{inf}	true value of the asset in the informed state
C_k	participation costs for liquidity provider of type k
λ	arrival intensity liquidity providers
γ	speed advantage of HFTs
ν_{inf}, ν_{liq}	arrival intensities for informed and uninformed liquidity demanders, respectively
ϕ_1, ϕ_2	accuracy of signals $s = liq$ and $s = inf$, respectively
$\bar{\pi}$	(unconditional) probability of $\zeta = inf$ state
States of nature	
$\tilde{V}$	asset value
ζ	liquidity demand type
s	signal about liquidity demand type
ψ_k	information set
$\widehat{a}$	standing best quote upon arrival
Indices	
k	liquidity provider type
t	time
Decision variables	
m, n	masses of HFTs and LFTs respectively
a	ask price quote to be submitted
Market outcomes	
S	expected half spread
$\hat{S}$	expected half spread corrected for nonexecuted market orders

References

Admati, A., Pfleiderer, P., 1988. A theory of intraday patterns: Volume and price variability. Review of Financial Studies 1, 3–40.

Aït-Sahalia, Y., Saglam, M., 2017. High frequency market making: Optimal quoting, working Paper.

Anand, A., Venkataraman, K., 2016. Market conditions, fragility and the economics of market making. Journal of Financial Economics 121, 327–349.

Baldauf, M., Mollner, J., 2016. Fast traders make a quick buck: The role of speed in liquidity provision, working Paper.

Baron, M., Brogaard, J., Kirilenko, A., 2014. The trading profits of high-frequency traders, working Paper.

Bernales, A., 2014. Algorithmic and high frequency trading in dynamic limit order markets, working Paper.

Biais, B., Foucault, T., Moinas, S., 2015. Equilibrium fast trading. Journal of Financial Economics 116, 292–313.

Brogaard, J., Hagströmer, B., Nordén, L., Riordan, R., 2015. Trading fast and slow: Colocation and liquidity. Review of Financial Studies 28, 3407–3443.

Brogaard, J., Hendershott, T., Riordan, R., 2014. High frequency trading and price, discovery. Review of Financial Studies 27, 2267–2306.

Brogaard, J., Hendershott, T., Riordan, R., 2019. Price discovery without trading: Evidence from limit orders. Journal of Finance 74, 1621–1658.

Calcagno, R., Lovo, S., 2006. Bid-ask price competition with asymmetric information between market-makers. The Review of Economic Studies 73, 329–355.

Cordella, T., Foucault, T., 1999. Minimum price variations, time priority, and quote dynamics. Journal of Financial Intermediation 8, 141–173.

Foucault, T., 1999. Order flow composition and trading costs in a dynamic limit order market. Journal of Financial Markets 2, 99–134.

Foucault, T., Kadan, O., Kandel, E., 2005. Limit order book as a market for liquidity. Review of Financial Studies 18, 1171–1217.

Goettler, R., Parlour, C., Rajan, U., 2005. Equilibrium in a dynamic limit order market. Journal of Finance 60, 2149–2192.

Goettler, R., Parlour, C., Rajan, U., 2009. Informed traders and limit order markets. Journal of Financial Economics 93, 67–87.

Hagströmer, B., Nordén, L., 2013. The diversity of high-frequency traders. Journal of Financial Markets 16, 741 – 770.

Hasbrouck, J., 2018. High-frequency quoting: short-term volatility in bids and offers. Journal of Financial and Quantitative Analysis 53, 613–641.

Hasbrouck, J., Saar, G., 2012. Low-latency trading. Journal of Financial Markets 16, 646 – 679.

Hausch, D. B., 1987. An asymmetric common-value auction model. The RAND Journal of Economics pp. 611–621.

Hendershott, T., Jones, C., Menkveld, A., 2011. Does algorithmic trading improve liquidity. Journal of Finance 66, 1–33.

Hendershott, T., Riordan, R., 2013. Algorithmic trading and the market for liquidity. Journal of Financial and Quantitative Analysis 48, 1001–1024.

Hoffmann, P., 2014. A dynamic limit order market with fast and slow traders. Journal of Financial Economics 113, 156 – 169.

Jiang, G., Lo, I., Valente, G., 2014. High-frequency trading around macroeconomic news announcements: Evidence from the U.S. treasury market, working Paper.

Jørgensen, K., Skjeltorp, J., Ødegaard, B. A., 2018. Throttling hyperactive robots–order-to-trade ratios at the oslo stock exchange. Journal of Financial Markets 37, 1–16.

Kirilenko, A., Lo, A., 2013. Moore's law versus Murphy's law: Algorithmic trading and its discontents. Journal of Economic Perspectives 27, 51–72.

Korajczyk, R. A., Murphy, D., 2018. High-frequency market making to large institutional trades. The Review of Financial Studies 32, 1034–1067.

Kyle, A., 1985. Continuous auctions and insider trading. Econometrica 53, 1315–1335.

Li, S., Wang, X., Ye, M., 2018. Who supplies liquidity, and when?, working Paper.

Lyle, M., Naughton, J., Weller, B., 2015. How does algorithmic trading improve market quality?, working Paper.

Malinova, K., Park, A., Riordan, R., 2013. Do retail traders suffer from high frequency traders?, working Paper.

Menkveld, A., 2013. High frequency trading and the new market makers. Journal of Financial Markets 16, 712–740.

Menkveld, A., 2016. The economics of high-frequency trading: Taking stock. Annual Review of Financial Economics 8, 1–24.

Milgrom, P., Stokey, N., 1982. Information, trade and common knowledge. Journal of Economic Theory 26, 17–27.

Nöldeke, G., Tröger, T., 2001. Existence of linear equilibria in the Kyle model with multiple informed traders. Economics Letters 72, 159 – 164.

O'Hara, M., Saar, G., Zhong, Z., 2019. Relative tick size and the trading environment. The Review of Asset Pricing Studies 9, 47–90.

Parlour, C., 1998. Price dynamics in limit order markets. Review of Financial Studies 11, 789–816.

Pedersen, L. H., 2015. Efficiently inefficient: how smart money invests and market prices are determined. Princeton University Press.

Roşu, I., 2009. A dynamic model of the limit order book. Review of Financial Studies 22, 4601–4641.

Yang, L., Zhu, H., 2019. Back-running: Seeking and hiding fundamental information in order flows. Review of Financial Studies Forthcoming.

Internet Appendix for "Competition among Liquidity Providers with Access to High-Frequency Trading Technology"

In this internet Appendix we present several robustness tests of our results. We also present a special case of the model in which the reservation price p_{liq} lies more than $\frac{3}{2}$ ticks away from μ, the best estimate of the true value based on public information. This setting offers lower tractability, but allows for richer results in terms of order aggressiveness and undercutting patterns. Moreover, we present a dynamic extension of the model with embedded micro-foundations on information production by HFTs. Finally, we discuss how our setting with continuums of HFTs and LFTs can be seen as the limit of a setting with finite numbers of HFTs and LFTs where the number of players tends to infinity.

IA.1 Robustness

In this subsection, we show that our results are robust to a wide array of settings. We first show the sensitivity of the results in Subsection 5.2.2 to various model parameters. Then, we show robustness to relaxing the assumption of state-independent reservation prices (in Subsection IA.1.2). Next, we consider patient informed liquidity demanders (in Subsection IA.1.3). We also discuss the implications of allowing LFTs to switch to market order strategies (in Subsection IA.1.4).

IA.1.1 Sensitivity to other Model Parameters

In Subsection 5.2.2, the main focus has been on how the materialization of different equilibrium types depends on participation cost parameters. In this subsection, we discuss the sensitivity of the results to changes in the other input parameters.

To start with, consider an increase in the *HFTs' signal accuracy* (i.e., ϕ_1 and linked to that ϕ_2). HFT profits in the trading stage then increase as HFTs can now avoid adverse selection costs more effectively and miss out on profitable trading opportunities less often. In graphical terms, it means that the HFT indifference curves in Fig. 4 to 7 shift up. Yet, the increased adverse selection will also make LFTs less willing to supply liquidity in an empty book, implying Condition (12) becomes more binding and the red curves shift to the right. To the right of the red curve, LFTs are susceptible to the increased adverse selection costs, which induces their indifference curve (corresponding to $h_{LFT} = C_{LFT}$) to become flatter. In terms of equilibria that materialize, LFT Dominance becomes less likely (due to the drop in LFT profitability).

In contrast, the equilibria left of the red curve (i.e., Conditioning HFT Dominance and the Coexistence equilibrium) become more likely.

The effect of an increase in the *informed trading losses* (i.e., larger μ_{inf}), and the effect of an increase in the *unconditional probability of an informed market order* (i.e., larger $\bar{\pi}$) are qualitatively similar as these result in a relative increase of the HFT indifference curve compared to the LFT one and discourage LFTs from posting quotes in an empty book. As a result, equilibria with Conditioning HFT Dominance or Coexistence become relatively more likely.

The effect of the tick size δ is ambiguous and depends on the arrival rate of uninformed liquidity demanders ν_{liq} relative to the arrival intensity of liquidity providers λ. If p_{liq} is left unaffected, a smaller tick size slows down undercutting and changes our setting to the more general setting described in Section IA.2. Moreover, it makes jumping to the competitive price in that setting less attractive, and as a result, undercutting will continue for longer. As a result, liquidity provision is more profitable if most transactions realize along the undercutting path, that is, if ν_{liq} is relatively small. As a result, expected half spreads increase. Moreover, higher profitability of liquidity provision attracts both HFTs and LFTs. Due to more intense competition, execution probabilities go down and posting in an empty book becomes relatively less attractive. Hence, in this case the probability that liquidity is always offered deteriorates. If, on the other hand, most (uninformed) transactions take place at the competitive price (ν_{liq} is large), then lowering the tick size makes liquidity provision relatively less attractive. Hence, there is lower presence of LFTs and HFTs, less intense competition, and posting in an empty book is relatively more attractive. As a result, the probability that liquidity is always offered increases.

Finally, if the *arrival intensity* (ν_{liq}) or the *reservation price* (p_{liq}) *of the uninformed liquidity demander* increase, liquidity provision becomes more profitable. As a result, the red curve shifts to the left, and both LFT and HFT indifference curves shift upward. Since the shift in expected profits for HFTs and LFTs is very similar, changing these parameters will hardly affect the outcomes that realize (all else equal).

IA.1.2 State-dependent Reservation Prices

In our setup, we assume that the reservation price for the incoming liquidity demander equals p_{liq}, irrespective of the state of nature. In this section, we show that any reservation price of the informed liquidity demanders $p_{inf} \in (p_{liq}, \mu_{inf})$ gives the same outcome as $p_{inf} = p_{liq}$. The argument runs as follows. Let us assume that $p_{inf} \in (p_{liq}, \mu_{inf})$. Now, consider a liquidity provider (irrespective of its technology) arriving to an empty book. Trivially, posting a quote strictly exceeding p_{inf} is useless as its execution probability equals zero. Posting a quote anywhere in the interval $(p_{liq}, p_{inf}]$ gives a zero execution probability for an uninformed market order and an execution probability of 1 for an informed one. However, since $p_{inf} < \mu_{inf}$, such a transaction is always loss making for the liquidity provider. Posting a quote equal to p_{liq} in the book is potentially profitable as shown in Section 4. Hence, even if reservation prices of informed liquidity demanders exceed p_{liq}, market outcomes are the same as with a uniform reservation price p_{liq}. The only condition required is that $p_{inf} < \mu_{inf}$. In a long-run equilibrium, it is reasonable to assume that such condition is satisfied when there are information production costs (see also Pedersen, 2015).

IA.1.3 Patient Informed Liquidity Demand

In itself, the restricted version of the model featuring $\nu_{inf} = \infty$ is sufficient to illustrate the main insights of the paper.

To show robustness to this assumption, we now assume that $\nu_{inf} < \infty$, LFTs can track the status of self-submitted orders, quotes are cancellable, and adverse selection concerns are sufficiently high. HFTs will still not post quotes in an empty book when $s = inf$ and LFTs will be exposed to a similar winner's curse problem as in the baseline model. As a result, LFTs may abstain from posting in an empty book altogether. Yet, there is another potential effect. An LFT can now try to infer the HFT signal from the survival of its quote as the standing best quote. If the quote survives for long, it is likely that HFTs shun the market out of adverse selection fear. In this case, the LFT optimally cancels the quote. Hence, if LFTs are present in equilibrium, liquidity may fully dry up, but also may temporarily dry up, resolving itself as time passes by and the market learns. Moreover, the increased adverse selection imposed on LFTs gives them a competitive disadvantage, as also in the baseline model. As a result, LFT participation will be lower, HFT participation will be higher and adverse selection concerns worsen even further as was also the case before. Hence, results are qualitatively similar to the base case.

IA.1.4 Dual roles in limit order markets

One of the features that crucially characterize a limit order market is that participants can trade either using limit orders or using market orders.

One could imagine that HFTs, upon getting a signal, also start to send informed market orders. This notion could be incorporated in a reduced form way by making $\bar{\pi}$ an increasing function of m and simultaneously reducing C_{HFT}. As a result, HFT participation affects adverse selection concerns of LFTs even more severely. If anything, our results regarding adverse selection are understated. The only way this assumption would affect our results is that average liquidity might deteriorate even more quickly with wider scale adoption of HFT technology, as incoming order flow becomes more toxic (as in Biais et al., 2015).

IA.2 Generalized Version of the Model

In this section, we provide a generalized version of the model. The only difference compared to the baseline model is that p_{liq} can now be more than $\frac{3}{2}$ ticks away from the fundamental price μ. In this setup, we will show that price competition in the trading stage results in undercutting patterns that are very much similar to those reported in Hasbrouck (2018). We will also show that expected profits for LFTs and HFTs can be derived in similar ways as before, but because of the added complexity, equilibrium expressions for (m^*, n^*) and expected half spreads S cannot be derived in closed form anymore. Finally, we show that in such a setting, it is relatively straightforward to make liquidity demand sensitive to liquidity supply (which is not featured in the baseline model).

IA.2.1 Quote Dynamics Trading Stage

Let us first discuss the uninformed case. As before, liquidity providers trade off posting potentially more profitable quotes away from μ with lower execution probability with certain execution at $p(1)$. When submitting a quote on the price grid in the interval $(p(1), p_{liq}]$, the execution probability is unaffected by the level of the quote posted, only by the arrival intensities of the market order and other liquidity providers. Hence, when a price is initially posted it is done at either p_{liq} or $p(1)$, and any undercutting also happens either in steps of one tick, or directly to $p(1)$. We formalize this intuition below:

Proposition IA.1. *(Equilibrium Order Placement Strategies - Uninformed Trading Case). With time and price priority enforced, any liquidity provider $k \in \{LFT, HFT\}$ follows the* *following strategy when observing an ask quote $\widehat{a}$ upon arrival:*

$$R_k^* = \begin{cases} p_{liq} & \text{if } (\widehat{a} - \delta \geq p_{liq} \text{ or } \widehat{a} = \emptyset) \text{ and } p_{liq} \geq \tilde{p}^* \\ \widehat{a} - \delta & \text{if } p_{liq} > \widehat{a} - \delta \geq \tilde{p}^* \\ p(1) & \text{if } \widehat{a} - \delta < \tilde{p}^* \text{ or } (\widehat{a} = \emptyset \text{ and } p_{liq} < \tilde{p}^*) \end{cases}, \tag{IA.1}$$

where

$$\tilde{p}^* = p(1) + \frac{1 - \Phi}{2\Phi} \cdot \delta, \qquad \Phi \equiv \frac{\nu_{liq}}{\nu_{liq} + \lambda(n + \gamma m)}. \tag{IA.2}$$

Proof. See Proofs Section. □

Proposition IA.1 states that at some point, liquidity providers switch from an undercutting-one-tick-at-a-time strategy to a strategy of undercutting to the competitive price. The intuition for this result is as follows. Consider a liquidity provider k arriving in the market at time τ, observing a standing limit order at quote $\widehat{a} > p(1)$. This liquidity provider faces the following tradeoff. If she quotes the competitive price $p(1)$, she secures execution and certainly obtains a profit of a half tick (i.e., $p(1) - \mu = \frac{\delta}{2}$). If instead she undercuts $\widehat{a}$ by only one tick (δ), she obtains a larger profit (i.e., $\widehat{a} - \delta - \mu$) in case of execution. Yet, she then runs the risk of being undercut by a subsequently arriving liquidity provider before the market order has arrived. Hence, the limit order only pays off with execution probability Φ. When $\tilde{p}^*$ is reached in the sequential undercutting process, liquidity providers switch strategies from tick-by-tick undercutting directly to $p(1)$. Fig. IA.1 shows an example of a possible undercutting path in the uninformed setting. The undercutting starts at p_{liq} when the first liquidity provider comes to the market and continues with all players undercutting each other. After $\tilde{p}^*$ is reached, all arriving liquidity providers jump to $p(1)$, which is the quote at which execution takes place when the liquidity demander arrives. Of course, if the liquidity demander were to arrive before $p(1)$ was reached, the transaction would take place at the prevailing quote at the time the liquidity demander arrived to the market to submit a market order. At the micro level, these trading patterns are in line with the fast-paced quote undercutting sequences with high-frequency liquidity provision shown in Hasbrouck (2018).

The half spread of a trade is the difference between the transaction price and μ. The expected half spread declines in the undercutting speed and the level of the switching point $\tilde{p}^*$. Both, the undercutting speed and $\tilde{p}^*$ are increasing in HFT presence m, LFT presence n, and

HFT speed γ as these all intensify competition for order flow.

In the trading stage of the informed case, essentially nothing changes. The tradeoffs and resulting conditions for HFT and LFT quote posting in an empty book are unaffected by the generalization of the reservation price. Once it is clear that there is no informed trading (because of the survival of the initial quote), the game reduces to the uninformed case starting at $p_{liq} - \delta$.

IA.2.2 Participation Stage

IA.2.2.1 Uninformed Case

To calculate the equilibrium masses, we first need to derive the expected per capita profits for HFTs and LFTs. If, conditional on m and n, the strategies R^*_{HFT} and R^*_{LFT} are played by all liquidity providers, we can distinguish two regions along the equilibrium path in Fig. IA.1. In the first region from p_{liq} down to $\tilde{p}^*$ inclusive, denoted "*UC*", both HFTs and LFTs undercut the standing best quote tick-by-tick upon arrival to the market. In the second region, denoted "*comp*", each liquidity provider that enters the market will post a quote at the competitive price $p(1)$. Next, let us define $\bar{\lambda} = (n + \gamma m)\lambda$, the overall arrival intensity of liquidity providers. Moreover, let us define Z as the number of ticks from p_{liq} to $\tilde{p}^*$ inclusive. Lemma IA.1 then presents the unconditional expected per capita profits for both liquidity provider types.

Lemma IA.1. *For an LFT and an HFT, the unconditional expected per capita profits are respectively given by:*

$$E\left(\sum_{\widehat{a}} \Pi_{HFT}(R^*_{HFT}(\widehat{a}))\right) = \frac{(1 - \frac{n}{n+\gamma m})(E(\Pi^{UC} + \Pi^{comp}))}{m}, \tag{IA.3}$$

$$E\left(\sum_{\widehat{a}} \Pi_{LFT}(R^*_{LFT}(\widehat{a}))\right) = \frac{\left(\frac{n}{n+\gamma m}\right)(E(\Pi^{UC} + \Pi^{comp}))}{n}, \tag{IA.4}$$

where

$$E(\Pi^{UC}) = \sum_{i=0}^{Z} \frac{\nu_{liq}\bar{\lambda}^i}{(\nu_{liq} + \bar{\lambda})^{i+1}}(p_{liq} - i \cdot \delta - \mu), \tag{IA.5}$$

$$E(\Pi^{comp}) = (1 - P_{UC})(p(1) - \mu), \tag{IA.6}$$

$$P_{UC} = \sum_{i=0}^{Z} \frac{\nu_{liq}\bar{\lambda}^i}{(\nu_{liq} + \bar{\lambda})^{i+1}}, \tag{IA.7}$$

Proof. See Proofs Section. ☐

The interpretation of the expressions in Lemma IA.1 is as follows. HFTs and LFTs share in the aggregate expected surplus from trading according to their relative presence in the market given by $\frac{\gamma m}{n+\gamma m}$ and $\frac{n}{n+\gamma m}$, respectively. The aggregate expected profits in the UC region are given by the probability-weighted average trading profit at each tick in this range (where probabilities sum to less than one). The aggregate expected profit in the *comp* region is given by the probability of reaching it (i.e., $(1 - P^{UC})$) times the guaranteed profit of half a tick $\frac{\delta}{2}$. With the expressions in Lemma IA.1, we can derive the equilibrium masses of HFTs and LFTs. As expected profits for both HFTs and LFTs are monotonically decreasing in m and n, and per capita costs are constant, it is always possible to find an equilibrium with a strictly positive mass of at least one type of liquidity providers.

IA.2.2.2 Informed Case

In case there are no useful signals, the results from Section IA.2.2.1 just carry over. In case signals are useful conditioning variables, the per capita expected trading revenue functions are expressed as follows.

Lemma IA.2. *The expected per capita profits for HFTs and LFTs are given by*

$$E\left(\sum_{\widehat{a}} \Pi_{HFT}(R^*_{HFT}(\widehat{a}))\right) = \begin{cases} g_{HFT}(m,n) \text{ , if (12) is not satisfied,} \\ h_{HFT}(m,n) \text{ , otherwise,} \end{cases} \tag{IA.8}$$

$$E\left(\sum_{\widehat{a}} \Pi_{LFT}(R^*_{LFT}(\widehat{a}))\right) = \begin{cases} g_{LFT}(m,n) \text{ , if (12) is not satisfied,} \\ h_{LFT}(m,n) \text{ , otherwise,} \end{cases} \tag{IA.9}$$

respectively, where

$$\begin{aligned} g_{LFT}(m,n) &= (1-\bar{\pi})\phi_1(1-\Phi)f(n+\gamma m|p_{liq}-\delta), \\ g_{HFT}(m,n) &= \gamma\left(g_{LFT}(m,n) + (1-\bar{\pi})\frac{\phi_1\Phi(p_{liq}-\mu)-(1-\phi_1)(\mu_{inf}-p_{liq})}{b}\right), \\ h_{HFT}(m,n) &= \gamma\left((1-\bar{\pi})\left((1-\Phi)f(\gamma m|p_{liq}-\delta) + \frac{\phi_1\Phi(p_{liq}-\mu)-(1-\phi_1)(\mu_{inf}-p_{liq})}{n+\gamma m}\right)\right), \\ h_{LFT}(m,n) &= \frac{1}{\gamma}h_{HFT}(m,n) + \bar{\pi}\frac{(1-\phi_2)\Phi(p_{liq}-\mu)-\phi_2(\mu_{inf}-p_{liq})}{n}, \\ f(n+\gamma m) &= \frac{(E(\Pi^{UC}+\Pi^{comp}))}{n+\gamma m}. \end{aligned} \tag{IA.10}$$

The idea here is that once a quote survives after having been posted, all uncertainty about potentially informed trading is resolved (there is none). Hence, the trading game then reduces to the uninformed trading game, but at a starting level that is one tick lower. Other than that, everything works through as in the baseline model. While quantitatively different from the baseline model, qualitatively, all results carry over.

IA.2.2.3 Heterogeneous Liquidity Demanders

In the baseline and the generalized model with arbitrary p_{liq}, the execution probability of a limit order is independent from its aggressiveness, which may be at odds with empirical patterns. Therefore, we now consider the implications of a positive correlation between quote aggressiveness and execution probability. To this end, we introduce U different groups of uninformed liquidity demanders, and assume that individual group u has relative size w_u and reservation price p_{liq}^u. Given a standing best quote $\widehat{a}$, with $p_{liq}^{u-1} < \widehat{a} \leq p_{liq}^u$, we have that the arrival intensity of an uninformed liquidity demander is given by $\tilde{\nu}_{liq} = \nu_{liq} \sum_{i=u}^{U} w_i$. That is, in group 1, we have the liquidity demanders with the lowest reservation price and in group U with the highest. Hence, as quotes become more aggressive, more uninformed liquidity demanders will consider trading.[IA.1]

Interestingly, our main results remain largely unchanged. In this extended setting, it is still impossible to separate the informed from the uninformed liquidity demanders. Moreover, as all informed trading occurs when a quote is added to an empty book, this change in setup can only affect trading strategies in two ways. First, through a different tradeoff of posting in an empty book or not. Posting is now less profitable, implying that Condition (12) is tightened. Second, through a different tradeoff between undercutting by a single tick or jumping to the competitive price. Expected undercutting profits now decrease, and thus jumping becomes relatively more attractive. As a result, both LFT and HFT expected profits shift downwards, so do equilibrium participation rates (m^*, n^*).

Most crucially, the participation outcomes obtained with uninformed liquidity demanders with heterogeneous reservation prices can also be obtained in the generalized model with arbitrary p_{liq} with adjusted parameters. In particular, the tradeoff for HFTs and LFTs posting to an empty book (red curve) can be attained by having $\nu'_{liq} = \nu_{liq} \times w_U$ in Conditions (10) to (12). Moreover, the lower expected profits for LFTs and HFTs (the blue and the green indifference curves, respectively) can be mimicked by increasing C_{LFT} and C_{HFT} by

[IA.1] The intuition is that liquidity demanders with a reservation price that is strictly lower than the best standing ask quote still arrive to the market but would not send a market order. Furthermore, we assume that the reservation price of the informed liquidity demander corresponds with the reservation price of group U (i.e., the highest reservation price in the market).

amounts that equal the lower expected profits. Hence, the model is qualitatively robust to the simplification of a uniform reservation price.

IA.3 Endogenous Information Production in a Dynamic Model

So far, the information production technology in our model has been exogenously given. To endogenize the information production process of HFTs, we extend the model to a dynamic setting, which comes at the cost of additional complexity. Let us consider an infinitely repeated version of our trading game. In every stage game l, a state of nature ζ_l is drawn according to a Markov switching process with transition matrix:

$$\begin{bmatrix} \alpha & 1-\alpha \\ 1-\beta & \beta \end{bmatrix}, \tag{IA.11}$$

where α and β denote the probabilities of continued liquidity trading and continued informed trading, respectively. In turn, $1-\alpha$ and $1-\beta$ denote the switching probabilities from liquidity to informed and from informed to liquidity trading, respectively. Unconditional steady state probabilities are then given by $\bar{\pi} = \frac{1-\alpha}{2-\beta-\alpha}$ and $1-\bar{\pi} = \frac{1-\beta}{2-\beta-\alpha}$. This setup allows to capture the clustering of informed trades.[IA.2]

Next, we assume that informed order flow is less patient than uninformed order flow (i.e., $\nu_{inf} > \nu_{liq}$) and that informationally advanced liquidity providers can perfectly observe the historical evolution of the book. In this dynamic setting, the difference in patience between informed and uninformed liquidity demanders allows for inference about trading types in previous periods by informationally advanced liquidity providers. This information is particularly useful when $\beta \neq 1-\alpha$, because information about the previous liquidity-demanding trader type will then help to better forecast the current trader type. In particular, when we assume $\nu_{inf} = \infty$ as in our main model, informationally advanced liquidity providers can perfectly infer the state of nature of the previous stage game. In that case, we obtain a perfect Bayesian equilibrium, and the signal accuracy parameters are given by:

$$\phi_1 = \alpha,\ \phi_2 = \beta. \tag{IA.12}$$

[IA.2] Informed trade clustering may for instance arise because at some times there is more private information available than at others, or because a single informed trader slices his trading volume into smaller trades and feeds them consecutively to the market (Admati and Pfleiderer, 1988).

The dynamic version of the model is essentially a repeated version of the baseline model.[IA.3] As a result, the results from earlier sections carry over. Moreover, because the signals are a result of superior technology for processing public information, signals are common across HFTs. This commonality result validates the assumption in the static baseline model of identical signals for all HFTs.

IA.3.1 Internally Consistent News Announcements

In the dynamic extension of the model, we need to make sure that price movements are consistent with informed trading. In other words, it is important that prices move in the direction of the information in the market when the state of nature switches from inf to liq. However, we want to prevent LFTs from learning from price paths to keep tractability. To this end, we assume that public information can be released between iterations. In particular, we assume that information releases always occur if ζ_l switches from informed to uninformed, such that the efficient price μ can be updated to the value μ_{inf} from last period. Moreover, we assume that information from either side of the book is impounded in prices in a similar way such that there is no price drift up or down.[IA.4] To have that information releases contain no information about ζ_l, certain conditions about the frequencies of public information releases need to be satisfied. Let us define the event A_l as a public information release (announcement) between iteration $l-1$ and l.

Assumption 1. *(Announcement uninformativeness) When the state of nature switches from inf to liq, public information is released (i.e., $P(A_l|\zeta_{l-1} = inf, \zeta_l = liq) = 1$). Moreover, information releases satisfy the following constraint*

$$\beta(1-\pi)P(A_l|\zeta_l = inf, \zeta_{l-1} = inf) + (1-\alpha)(1-\bar{\pi})(\frac{1}{\bar{\pi}} - 1)P(A_l|\zeta_l = inf, \zeta_{l-1} = liq) = (1-\beta)\bar{\pi} + \alpha(1-\bar{\pi})P(A_l|\zeta_{l-1} = liq, \zeta_l = liq) \quad \text{(IA.13)}$$

Under Assumption 1, we show below that public information releases are uninformative about the state of nature ζ_l. Note that the assumptions in this paragraph are not necessary to

[IA.3] To be complete, for the fully dynamic setting some conditions need to be satisfied for LFTs to be unable to learn, and for the learning of informationally advanced traders from order flow to be rational and internally consistent. In particular, we need to have that signals are indeed informative of future price moves, while LFTs cannot learn anything from price moves. One can achieve signal informativeness by letting prices react to public information releases and set conditions on the news release process. These conditions on public information releases and price processes are described and derived in Appendix IA.3.1.

[IA.4] For tractability reasons, we refrain from also explicitly modeling the other side of the book.

obtain our main results, but merely to show that the setup of our model is internally consistent.

To have information asymmetry that is consistent with future price movements, we have under Assumption 1 that

$$P(A_l|\zeta_{l-1} = inf, \zeta_l = liq) = 1. \tag{IA.14}$$

Moreover, we want the event A_l to be uninformative about the state of nature (to LFTs), which is the case when

$$P(\zeta_l = inf|A_l) = P(\zeta_l = inf) \ \rightarrow \tag{IA.15}$$

$$\frac{P(A_l|\zeta_l = inf)P(\zeta_l = inf)}{P(A_l)} = P(\zeta_l = inf) \ \rightarrow \tag{IA.16}$$

$$P(A_l|\zeta_l = inf) = P(A_l). \tag{IA.17}$$

The only thing left to do now is to work out this constraint in terms of public news release probabilities for each type of transition. We can work out $P(A_l|\zeta_l = inf)$ first:

$$P(A_l|\zeta_l = inf) = P(A_l|\zeta_l = inf, \zeta_{l-1} = inf)P(\zeta_{l-1} = inf|\zeta_l = inf)+ \\ P(A_l|\zeta_l = inf, \zeta_{l-1} = liq)P(\zeta_{l-1} = liq|\zeta_l = inf). \tag{IA.18}$$

Applying Bayes rule twice, we have

$$P(\zeta_{l-1} = inf|\zeta_l = inf) = \frac{P(\zeta_l = inf|\zeta_{l-1} = inf)P(\zeta_{l-1} = inf)}{P(\zeta_l = inf)} = \frac{\beta\bar{\pi}}{\bar{\pi}} = \beta, \tag{IA.19}$$

where $\bar{\pi} = \frac{1-\alpha}{2-\beta-\alpha}$, the long-term (unconditional) steady state probability of being in the informed state of nature. Similarly, we have

$$P(\zeta_{l-1} = liq|\zeta_l = inf) = \frac{(1-\alpha)(1-\bar{\pi})}{\bar{\pi}}. \tag{IA.20}$$

Substituting these expressions into Eq. (IA.18), we get

$$P(A_l|\zeta_l = inf) =$$
$$P(A_l|\zeta_l = inf, \zeta_{l-1} = inf)\beta + P(A_l|\zeta_l = inf, \zeta_{l-1} = liq)(1-\alpha)(\frac{1}{\bar{\pi}} - 1). \quad \text{(IA.21)}$$

Similarly, we can work out $P(A_l)$ as

$$\begin{aligned} P(A_l) = P(A_l|\zeta_{l-1} = inf, \zeta_l = inf)P(\zeta_{l-1} = inf, \zeta_l = inf)+ \\ P(A_l|\zeta_{l-1} = inf, \zeta_l = liq)P(\zeta_{l-1} = inf, \zeta_l = liq)+ \\ P(A_l|\zeta_{l-1} = liq, \zeta_l = inf)P(\zeta_{l-1} = liq, \zeta_l = inf)+ \\ P(A_l|\zeta_{l-1} = liq, \zeta_l = liq)P(\zeta_{l-1} = liq, \zeta_l = liq). \end{aligned} \quad \text{(IA.22)}$$

Working out basic statistical identities, we have

$$P(\zeta_{l-1} = inf, \zeta_l = inf) = P(\zeta_l = inf|\zeta_{l-1} = inf)P(\zeta_{l-1} = inf) = \beta\bar{\pi}, \quad \text{(IA.23)}$$

and similarly

$$P(\zeta_{l-1} = inf, \zeta_l = liq) = (1-\beta)\bar{\pi}, \quad \text{(IA.24)}$$
$$P(\zeta_{l-1} = liq, \zeta_l = inf) = (1-\alpha)(1-\bar{\pi}), \quad \text{(IA.25)}$$
$$P(\zeta_{l-1} = liq, \zeta_l = liq) = \alpha(1-\bar{\pi}). \quad \text{(IA.26)}$$

Substituting everything into Eq. (IA.17) and realizing that probabilities must be contained in the unit interval, any set of announcement probabilities satisfying the following set of constraints can be allowed:

$$\beta(1-\pi)P(A_l|\zeta_l = inf, \zeta_{l-1} = inf) + (1-\alpha)(1-\bar{\pi})(\frac{1}{\bar{\pi}} - 1)P(A_l|\zeta_l = inf, \zeta_{l-1} = liq) =$$
$$(1-\beta)\bar{\pi} + \alpha(1-\bar{\pi})P(A_l|\zeta_{l-1} = liq, \zeta_l = liq), \quad \text{(IA.27)}$$

and

$$P(A_l|\zeta_l = inf, \zeta_{l-1} = inf) \in [0, 1], \tag{IA.28}$$

$$P(A_l|\zeta_l = inf, \zeta_{l-1} = liq) \in [0, 1], \tag{IA.29}$$

$$P(A_l|\zeta_{l-1} = liq, \zeta_l = liq) \in [0, 1]. \tag{IA.30}$$

IA.4 Limits with a Continuum of Liquidity Providers

In our model, we have a continuum of atomistic liquidity providers that endogenously choose to participate as LFTs (a mass n), as HFTs (a mass m), or to not participate at all. Whenever $\max(m, n) > 0$, we have infinitely many liquidity providers. If we were to assume finite arrival intensity and finite participation cost per liquidity provider, the aggregate arrival intensity as well as the aggregate participation costs would become infinite. Therefore, we use infinitesimal per capita expected trading profits and participation costs, such that population aggregates are still finite. Below we show that our setting is a limiting case of a setting with a finite number of (potential) liquidity providers.

First, let us assume that we have a finite number W of potential liquidity providers. We also assume that there is a fixed amount of resources available for monitoring where each potential liquidity provider has equal access to these resources. Employing these resources comes at a cost that scales linearly in the amount of resources used (i.e., constant cost to scale). Moreover, the monitoring intensity also scales linearly with the amount of resources employed. Hence, we have for an individual potential liquidity provider i that

$$\lambda_i = \frac{\lambda}{W}, \tag{IA.31}$$

$$C_{HFT,i} = \frac{C_{HFT}}{W}, \tag{IA.32}$$

$$C_{LFT,i} = \frac{C_{LFT}}{W}, \tag{IA.33}$$

where C_{HFT}, C_{LFT} are the costs involved if all potential liquidity providers were to implement HFT or LFT technology. As a result, the maximum possible monitoring intensity equals λ if everyone adopts LFT technology and $\gamma\lambda$ if everyone adopts HFT technology.

Now let W approach infinity, which implies that

$$\lim_{W\to\infty} \lambda_i = 0, \tag{IA.34}$$

$$\lim_{W\to\infty} C_{HFT,i} = 0, \tag{IA.35}$$

$$\lim_{W\to\infty} C_{LFT,i} = 0. \tag{IA.36}$$

Yet, because the number of potential liquidity providers approaches infinity, we still have (by construction) that

$$\lim_{W\to\infty} \sum_{i=0}^{W} \lambda_i = \lambda, \tag{IA.37}$$

$$\lim_{W\to\infty} \sum_{i=0}^{W} C_{HFT,i} = C_{HFT}, \tag{IA.38}$$

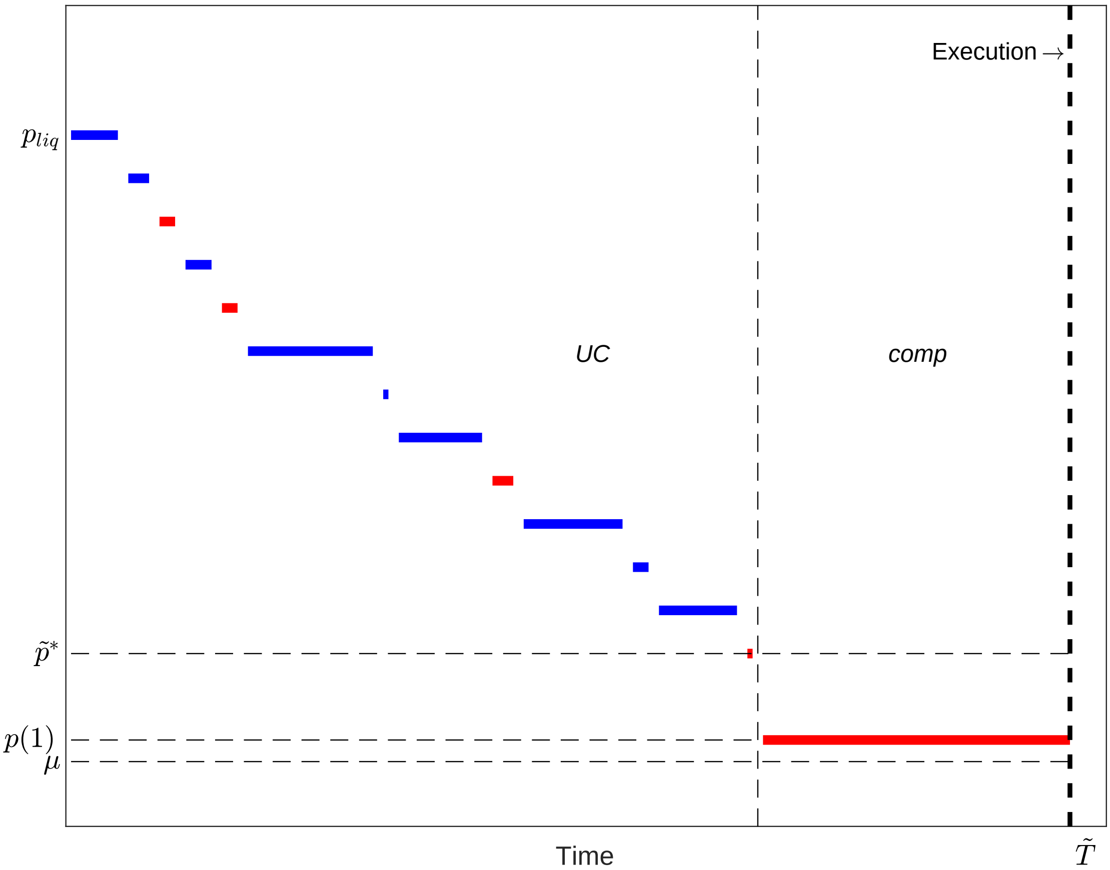
$$\lim_{W\to\infty} \sum_{i=0}^{W} C_{LFT,i} = C_{LFT}. \tag{IA.39}$$

Going from a discrete number of liquidity providers to a continuum is rather innocuous in our model as costs and intensities add linearly. With regard to aggregate arrival intensity and aggregate costs to the economy, it does not matter whether 10 out of a total of 100 potential liquidity providers adopt advanced technology or whether it is a 10% fraction of a continuous mass of potential liquidity providers.

Yet, analyzing the continuous rather than a discrete case comes with substantial advantages to tractability. First, in the trading stage, it matters because it is never optimal for a player to undercut herself and because players have heterogeneous speed. As a result, the undercutting intensity following a quote by an LFT would be $\lambda(\gamma M + N - 1)$, while following an HFT it would be $\lambda(\gamma(M-1)+N)$. As a result, the undercutting speed becomes path-dependent, making it much harder to take expectations. Second, because we have endogenous technology adoption and participation, we need to solve for zero expected profit conditions in terms of m and n. This requirement creates a need for rounding as mW or nW may not be integers. As a result, several nuisance terms may not cancel out in the discrete case while they do in the continuous one. Finally, we manage to simplify our model substantially by deriving Eq. (17), which shows that it is $\frac{C_{HFT}}{\gamma}$ rather than C_{HFT} relative to C_{LFT} that matters for technology adoption and participation decisions. In the discrete case, this derivation would only be possible if $\frac{W}{\gamma}$ is an integer, and even then the first issue of not undercutting oneself would play up.

IA.5 Figures

Figure IA.1: **Example of an Undercutting Path in the Uninformed Setting**



This figure shows an example undercutting path when there is no asymmetric information. The x-axis features time elapsed since the first quote has been posted, while the y-axis displays price ticks. Blue curves are HFT exposures while red curves are LFT exposures. *UC* stands for the undercutting region, while *comp* stands for the region in which the competitive price $p(1)$ is quoted.

IA.6 Proofs

Proof of Proposition IA.1. Due to time and price priority, $\Phi = 0$ if $a \geq \widehat{a}$. Moreover, by assumption, $\Phi = 0$ if $a > p_{liq}$. We assume for the moment that $\Phi > 0$ if $a \leq \min(p_{liq}, \widehat{a} - \delta)$, which we prove to be true later. Any quotes $a \leq \min(p(1) - \delta, \widehat{a} - \delta) < \mu$ are loss-making with strictly positive probability and are hence suboptimal. Any quotes $a \in [p(1), \min(\widehat{a} - \delta, p_{liq})]$ yield strictly positive profits with strictly positive execution probability and hence carry strictly positive expected profits. Hence, if $\widehat{a} > p(1)$, undercutting to a quote in $a \in [p(1), \min(\widehat{a} - \delta, p_{liq})]$ is optimal, while posting a quote with zero execution probability (such as $a = p(1)$) is optimal if $\widehat{a} \leq p(1)$.

In equilibrium, a quote $a = p(1)$ is optimally not undercut and hence, such a quote has execution probability $\Phi = 1$. A quote $a \in (p(1), \min(\widehat{a} - \delta, p_{liq})]$ executes whenever the liquidity demander arrives before the next liquidity provider arrives to undercut. The arrival rate of liquidity providers is given by $\lambda(\gamma m + n)$ and is independent of k, because liquidity providers are atomistic. The arrival rate of liquidity demanders is given by ν_{liq}. Applying standard rules for the calculations with exponential distributions yields Eq. (IA.2).

Given $\min(\widehat{a}, p_{liq}) > p(1)$, an arriving liquidity provider optimally either undercuts to a quote $a \in (p(1), \min(\widehat{a} - \delta, p_{liq})]$ or to $p(1)$. The former has an execution probability Φ as derived above that is independent of the exact quote in the range and hence, setting $a = \min(\widehat{a} - \delta, p_{liq})$ is optimal since this yields largest profits in case of execution. The latter has guaranteed execution and a profit of $p(1) - \mu$. It follows that undercutting to $p(1)$ is strictly optimal if

$$(\min(\widehat{a} - \delta, p_{liq}) - \mu)\Phi \leq p(1) - \mu = \frac{\delta}{2}, \Rightarrow \tag{IA.40}$$

$$\min(\widehat{a} - \delta, p_{liq}) \leq \mu + \frac{\delta}{2\Phi} = p(1) + \frac{1 - \Phi}{2\Phi}\delta, \tag{IA.41}$$

while setting $\min(\widehat{a} - \delta, p_{liq})$ is optimal otherwise. □

Proof of Lemma IA.1. We will now work out the unconditional expected profits in each of the two parts along the equilibrium path.

Let us start with region UC. To facilitate exposition, let us define the random variables d as the number of ticks away from p_{liq} on which execution takes place, and q_t the number of ticks the best standing quote is away from p_{liq}. The market-wide expected aggregate profit earned in region UC is given by

$$E(\Pi^{UC}) = \sum_{i=0}^{Z} P(d=i)(p_{liq} - i\delta - \mu).$$

The probability of execution i ticks away from p_{liq} can be derived as follows. We have that

$$P(d=i) = \int_{t=0}^{\infty} P(q_t = i) P(\tilde{T} > t) \nu_{liq} dt. \tag{IA.42}$$

The probability $P(q_t = i)$ is given by a Poisson distribution with parameter $\bar{\lambda}t$, while $P(\tilde{T} > t) = \exp(-\nu_{liq} t)$. Substituting these distribution functions into Eq. (IA.42), we get

$$P(d=i) = \int_{t=0}^{\infty} \frac{1}{i!} (\bar{\lambda} t)^i \exp(-\bar{\lambda} t) \exp(-\nu_{liq} t) \nu_{liq} dt, \tag{IA.43}$$

$$= \int_{t=0}^{\infty} \frac{\nu_{liq} \bar{\lambda}^i}{(\nu_{liq} + \bar{\lambda})^{i+1}} \left[(\nu_{liq} + \bar{\lambda})^{i+1} \frac{1}{i!} t^i \exp(-(\nu_{liq} + \bar{\lambda}) t) \right] dt. \tag{IA.44}$$

The part in square brackets can be recognized as the pdf of a Gamma distribution with parameters $(i+1, \nu_{liq} + \bar{\lambda})$, while all other terms are multiplicative, do not depend on t and can therefore be put in front of the integration. By definition, a pdf integrates to 1 over its support, such that we have

$$P(d=i) = \frac{\nu_{liq} \bar{\lambda}^i}{(\nu_{liq} + \bar{\lambda})^{i+1}}. \tag{IA.45}$$

Let us now continue with the *comp* region. Let us define the probability of execution in the UC region

$$P_{UC} = \sum_{i=0}^{Z} P(d=i). \tag{IA.46}$$

If execution takes place outside the UC region, it must take place in the *comp* region where execution is guaranteed to the first one posting a quote $p(1)$. Hence,

$$E(\Pi^{comp}) = (1 - P_{UC})(p(1) - \mu)$$

trivially follows.

Now we still need to show how expected aggregate profits accrue to LFTs and HFTs. This depends on the expected exposures of both groups. Due to the Poisson arrival assumption, expected quote life is independent of the liquidity provider's type, the expected exposure of a group depends on how often it can be expected to post an undercutting quote relative to the other group. Hence, the fraction of time that the market is exposed to LFT quotes is given by $\frac{n}{n+\gamma m}$. $\square$