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Jill C. Bonner
University of Rhode Island

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Jill C. Bonner

ONE-DIMENSIONAL MODEL SYSTEMS: THEORETICAL SURVEY*

Jill C. Bonner

Physics Department, University of Rhode Island
Kingston, RI 02881

ABSTRACT

In the early 1960's, one dimensional model systems were regarded as amusing toys with the advantage of being far more easily solvable than their "real" three-dimensional counterparts. Now essentially 1-D (quasi-1-D) magnets can be "tailor-made" in the laboratory. Even more popular is the field of organic conductors like TTF·TCNQ, which are naturally quasi-1-D. Currently solitons and related solutions of non-linear, dispersive 1-D differential equations are ubiquitous in physics, including the area of 1-D magnetism. These developments are discussed in the Introduction. The rest of this paper is concerned with model Hamiltonians, model comparisons, critical singularities in 1-D (quasi-1-D) systems, accuracy of numerical techniques in comparison with exact solutions, brief accounts of dilute and disordered 1-D systems, and 1-D spin dynamics. Finally, a comment is made on a variety of interesting isomorphisms between 1-D magnets and phenomena in several other areas of physics, for example 2-D ferroelectrics, field-theoretic models, and realistic fluids. Comparison of theory and experiment has been the subject of several excellent reviews and is therefore not discussed here.

INTRODUCTION

The Ising model of cooperative magnetism was suggested by Lenz and solved in one-dimension (1-D) by Ising in 1925 [1]. The solution was essentially complete as a function of both temperature, T, and applied magnetic field, H. Later work has discussed in some detail the correlation functions [2]. In 1944, a very famous solution of the two-dimensional (2-D) Ising model by Onsager appeared [1]. The solution provided information on the thermal properties in zero-field only: A complete solution, including $H \neq 0$, is lacking to this day. Again, considerable additional work has been done on the correlation functions [3]. The 1-D Ising model is interesting as a soluble, non-trivial N-body problem and widespread and ingenious exploitation of the 2-D Ising model solution has given tremendous impetus to the developing field of critical phenomena [3,4]. Unfortunately, the Ising model has its limitations as a model for magnetic phenomena. It is a purely classical, as opposed to a quantum-mechanical, model, and is therefore inadequate to describe both static and dynamic phenomena. Good experimental examples of Ising model magnets are rare.

In the early 1960's an important sequence of solutions of a variety of 1-D magnetic insulator models appeared. Fisher was able to demonstrate a remarkably simple analytic solution for the classical (spin $S = \infty$) linear chain in zero field. The solution yielded both thermal properties and correlation functions [5]. The 1-D XY model with spin $S = 1/2$ was solved analytically, independently by Lieb, Schultz and Mattis, with emphasis on the correlation functions, and by Katsura, with emphasis on the thermal and magnetic properties [6]. The Katsura solution included also exact results for the 1-D transverse Ising model. An analytic solution for the $S = 1/2$ Heisenberg linear chain is not available even now, though important progress has been made over the past few years. Therefore, Bonner and Fisher [7] and, independently Griffiths, developed a technique of numerical extrapolation of exact results for a sequence of <u>finite</u> Heisenberg chains of increasing N to the $N = \infty$ limit. In this way an extensive set of results for all H and T and some results for the $T = 0$ correlation functions were obtained with accuracy comparable to experiment. Other important results for the $S=1/2$ Heisenberg antiferromagnetic (AF) chain were an analytic calculation by des Cloizeaux and Pearson of the

lowest excited states (spin-wave triplets) and an analytic result for the zero-point susceptibility by Griffiths [8]. The relevance of these 1-D solutions to real experimental systems was demonstrated for the first time by Griffiths [9] in conjunction with workers at the Kamerlingh Onnes Laboratorium, Leiden. Interest in 1-D magnets among experimentalists and theorists was stimulated by this result and, particularly, by an annotated collection of reprinted papers on a variety of 1-D model systems, including lattice gases, dynamical disordered crystal lattices, many-fermion gases (electron gases) as well as magnets. The collection appeared in book form, and remains today an important and influential introduction to 1-D theory [10].

In the early 1960's 1-D model systems were interesting for a variety of reasons, most of which remain in force today. First of all, an essentially complete, exact or accurate numerical, solution could be obtained in one dimension, whereas the corresponding 3-D problem was hopelessly intractable. The enormous difficulty in 3-D naturally led to the development of a variety of approximate techniques for problems in many-body cooperative phenomena. These included mean-field theory, Bethe, Bethe-Peierls-Weiss and higher-order cluster approximations, the random phase approximation and higher-order Green's function calculations, and spin-wave theory. The 1-D results provided a testing ground for such general approximate techniques, and it is remarkable how inaccurate they are for, say, the 1-D Heisenberg antiferromagnet. The failure is not only quantitative but even qualitative. To be fair, there are reasons to argue that the inadequacies of the above theories are most pronounced for low dimensionality and low spin value. Nevertheless, more caution than is often the case should perhaps be exercised in relation to the use of theories of the above type. Very recently the renormalization group (RG) approach in the position space formulation has been tested and evaluated on 1-D spin-1/2 Ising models and spin-1 Ising models (showing $T_c = 0$ tricritical phenomena) [11]. Considerably greater difficulty is experienced in applying RG theory to quantum-mechanical systems (for example, the $S = 1/2$ Heisenberg model or XY model) [12] than to classical systems (Ising model or $S = \infty$ Heisenberg model). The Q.M. models are not only of interest in their own right, especially the 2-D cases [13], but also as models of quark confinement [12]. The study of 1-D quantum systems might well aid the development of an effective RG approach.

In some magnetic crystals, 1-D magnetic interaction pathways occur naturally between metallic ions as a consequence of magnetic structure (e.g. copper tetramine sulfate, $Cu(NH_3)_4SO_4H_2O$,'CTS' [14]); special alignment of orbitals (dγ orbitals of Cu^{2+} in $KCuF_3$ [14]); or between ionic components of magnetic organic complexes (π-orbital interactions in TTF^+ $Pt S_4C_4(CF_3)_4^-$, TTF BDT(Pt) [17]). Now it is possible to grow 1-D magnets "to order" by organic spacer techniques. For example, $CoCl_2 \cdot 2H_2O$ is a "poor" 1-D magnet. If, however, the H_2O molecules are replaced by large organic pyridines to give $CoCl_2 \cdot 2NC_5H_5$, the Co^{2+} ion chains are pushed much further apart, and the 1-D character is enhanced [14]. Consequently, over the past few years there has been considerable activity in finding "good" experimental 1-D magnets, and a wide assortment is now available. This situation is especially interesting to coordination chemists. Since the theory of superexchange remains on a far from secure footing, chemists are tending to investigate empirically the variation of exchange constant with bond length and bond angle using families of 1-D magnets, relying on the complete and accurate statistical mechanical knowledge not available in higher-D systems. One important source of uncertainty in the processing of the data is thus removed.

An important area of current hot pursuit in 1-D physics is the field of quasi-1-D organic conductors, semiconductors, and even insulators [15]. Materials like the famous TTF·TCNQ are called organic metals because of their high conductivity at low temperature and its negative temperature coefficient. They are plenti-

ful and naturally anisotropic, i.e. quasi-1-D, unlike the 1-D magnets which often have to be specially "engineered" [14]. Considerable interest, and also controversy, was stimulated by a prediction of Little that organic conductors might show "high temperature" superconductivity at, say, 50 K. The discovery recently of superconductivity at 0.27 K in polysulfur nitride $(SN)_x$, unique in having no metallic components, boosted activity in this area. The technological importance of a 50 K superconductor, or even "good" organic conductors and photoconductors, is, of course, immense. Unfortunately, there is an inherent lattice instability in 1-D systems, the famous Peierls transition [16], which distorts the underlying lattice so as to induce a phase transition from "metallic" to insulating (semiconducting) behavior as the temperature is lowered. Clearly, Peierls transitions are both interesting and technologically undesirable, for they compete with the putative superconducting transition and act to suppress it. Recently, the insulating analogue of the Peierls transition has been discovered in some members of a class of quasi-1-D metallo-organic complex compounds, bisdithiolenes with TTF [17]. In this transition, an insulator-insulator transition, a uniform 1-D AF Heisenberg chain with a single exchange constant J, undergoes a dimerization at the spin-Peierls transition temperature to an alternating AF Heisenberg chain. That is, pairs of spins move respectively closer and further apart and the 1-D magnetic system has two alternating exchange constants J_1, J_2, with $J_1 > J_2$, say. According to careful numerical calculations on the alternating chain [18], a magnetic energy gap opens up between the singlet ground state and the lowest excited states (triplet excitons) for all $J_1 \neq J_2$. The existence of such a gap is crucial to spin-Peierls theory and also to the extensive field of exciton theory in physical chemistry [19]. Since the alternating linear chain Hamiltonian still possesses complete isotropic spin symmetry, doubts have been raised, based on classical (spin ∞) 1-D systems and approximate techniques, about the existence of such a gap [20], but the numerical evidence for it is strong [18].

It appears that 1-D physics is likely to exert its special fascination for some time to come. As fast as one area of interest attains a degree of maturity, e.g. 1-D magnetic insulators, a new area takes over, e.g., the field of organic conductors and 1-D metals [15]. Recently the soliton phenomenon has become very fashionable in all areas of physics, not just solid state physics. Solitons, kinks, and solitary travelling wave phenomena are special solutions of 1-D differential equations which have the characteristics of being both non-linear and dispersive. Solitons have the unusual property that after they interact, they proceed on their way with shape and velocity essentially unchanged. Solitons have turned up in particle and field theories, water waves, plasmas, SQUID theory, charge and spin density waves in 1-D conductors, and displacive ferroelectrics [21]. There is now reason to believe that solitons are characteristic excitations in 1-D continuum ferromagnets, both classical (S = ∞) and quantum [22]. Solitons may also be important in 1-D antiferromagnets, in connection with spin-Peierls and generalised Peierls transitions.

Other areas of solid state physics where exact 1-D solutions are appearing thick and fast include Fermi and Bose gas delta function models [23], the linear Hubbard model [24], a 1-D plasma which crystallises as a Wigner solid [25], and the Lai-Sutherland model which includes the Hubbard model and a dilute magnetic model as special cases [26].

While the major emphasis of this review concerns 1-D magnetic insulators, it should be noted that there has been considerable recent progress in solving 1-D electron gas models stimulated by interest in 1-D organic electrical conductors noted above. The electron gases include both continuum and lattice versions, and theories can incorporate the effects of spin and backward scattering [27]. An important equivalence between a 1-D electron gas and a 2-D Coulomb gas has been noted [28]. These solutions are informative about charge and spin density wave phenomena.

Returning to 1-D magnetic insulators, the model solutions of the early 1960's have been augmented by the following important recent developments. Blume and co-workers have extended the solution of the S = ∞ Heisenberg linear chain to include a non-zero magnetic field by numerically based transfer matrix techniques [29]. The quantum, S = 1/2 Heisenberg model has been solved exactly in a continuum (as opposed to discrete lattice) version on the basis of techniques for solving the 1-D many-fermion Tomonaga-Luttinger models [30]. A solution for the S = 1 XY continuum model has also appeared [13a], and essentially exact calculations on 1-D Ising models for S > 1/2 have been developed [31]. An interesting sequence of 1-D "n-vector" models has been solved (in zero field) by Stanley [32]. The number of spin-components is denoted by n : n = 1 is the spin 1/2 Ising model, n = 2 is the classical (spin ∞) XY model, n = 3 is the classical Heisenberg model and, finally n = ∞ gives the Berlin-Kac spherical model. Extension can be made to include biquadratic as well as bilinear spin interactions [33]. The numerical extrapolation approach of Bonner and Fisher has been applied to a variety of 1-D models where the possibility of exact solutions seems remote. Examples are the 1-D Hubbard model [34] (an extension to the alternating version would be of interest), the Heisenberg ladder model suggested to describe copper nitrate [35], and Heisenberg-type models for S $\geq$ 1/2, including single-ion anisotropy [36].

To complete the Introduction, it seems appropriate to review the present status of exact, analytic results for the spin 1/2 Ising-Heisenberg-XY linear chain. The mathematical analysis required for this 1-D model is definitely more complicated than that required to solve the 2-D Ising model. Since the 2-D Ising solution has been so fruitful in the development of the modern theories of critical phenomena, it would seem profitable to pursue the 1-D model solution in equivalent detail. Early analytic work beginning in the 1930's has been reviewed by Thompson [2]. A breakthrough by Yang and Yang about 1966 [37] resulted in a detailed solution as a function of anisotropy and magnetic field for T = 0 only. However, this was sufficient to determine the character of the interesting critical singularities. The finite temperature properties presented a greater problem. Progress has been made by Johnson and McCoy [38], and independently by Takahashi [39], on the basis of a formulation due to Gaudin [40]. An important paper by Johnson, Krinsky and McCoy deals with the elementary excitations of the general XYZ model [41]. Nevertheless, information is still most notably lacking for the cases of the Heisenberg ferro- and antiferromagnet, and the Ising-Heisenberg ferromagnet for H = 0.

MAGNETIC MODEL SOLUTIONS

The effective spin Hamiltonian on which attention is primarily focussed in this review is

$$H = -2J\sum_{i=1}^{N}\{aS_i^z S_{i+1}^z + bS_i^x S_{i+1}^x + cS_i^y S_{i+1}^y\} \qquad (1)$$

We have bilinear spin coupling, assume nearest-neighbor spin interactions only, and the effective exchange constant is J modified for the spin couplings in different directions by factors a, b and c. The Ising model is obtained by putting b = c = 0: (In general by letting any two of a, b, c equal zero). The XY model is obtained by putting a = 0: (In general by letting any one of a, b, c equal zero). If a = b = c, we have complete rotational symmetry in spin space and obtain the Heisenberg model which may, of course, be written in vector form

$$H = -2J\sum_{i=1}^{N}\vec{S}_i \cdot \vec{S}_{i+1}.$$

If J > 0, the spins may lower their energy by aligning parallel and we have a Heisenberg ferromagnet. If J < 0, the antiparallel spin alignment is favored and we have

an antiferromagnet. If a, b and c are not equal spin anisotropy prevails. If a > b ≈ c, we have uniaxial (easy axis) anisotropy. On the other hand, if a < b≈c, planar (easy plane) anisotropy results. The magnitude and type of anisotropy in the effective spin Hamiltonian results from crystal field effects, as noted, for example, by de Jongh [14]. The effects of an applied magnetic field can be represented by the addition of a Zeeman term $-g\beta \vec{H} \cdot \sum_i \vec{S}_i$ to the Hamiltonian (1). Crystal

field effects may be accounted for by a term $\Delta \Sigma_i (S_i^z)^2$. Finally, the spin value may range over $1/2 \le S \le \infty$.

The static properties commonly measured by experimentalists are the initial (zero-field) susceptibility, χ, the specific heat at constant field, C_H, and the magnetization isotherms as a function of applied field, M_T vs. H. Theoretical information, which is essentially complete for the major magnetic models Ising, Heisenberg and XY, is now available for comparison with experiment as a function both of model type and spin value. In addition, the effects of magnetic dimensionality are now reasonably well understood, qualitatively and quantitatively. These three major factors, dimensionality, spin, and model variation, will be reviewed briefly, for their value in identifying the magnetic model underlying experimental results on (preferably more than) one of the static properties.

Concerning magnetic dimensionality, the characteristics of (quasi-) 1-D magnets are relatively low, broad maxima in the magnetic specific heat, and also in the AF and perpendicular susceptibilities. These maxima do not correspond to cooperative long-range-order (LRO) transitions, since for short-range (e.g., nearest-neighbor) forces $T_c = 0$ for ideal 1-D systems, and T_c is relatively small for "good" 1-D experimental systems. The broad maxima therefore correspond to short-range-order (SRO). More specific details are available in previous articles and reviews. [2,14,42,43]. However, we note that the best method of determining magnetic dimensionality, pioneered by Shirane at Brookhaven, is quasi-elastic neutron scattering. Scattering in reciprocal space shows dramatic dimensionality effects, occuring at points for 3-D systems, lines for 2-D systems and planes for 1-D systems.

Information on spin variation is relatively recent. In general, the magnetic thermodynamic properties show a steady progressive variation in, say, the height and position of the rounded maximum as S varies from 1/2 to ∞. Results for the Ising parallel susceptibility are available [31], and for the Heisenberg susceptibility, which owing to spin-flop effects has a perpendicular character [7, 44]. (The susceptibility parallel to the field direction vanishes as T → 0 for uniaxially anisotropic AF's. Susceptibilities measured in a direction perpendicular to the easy axis show a non-zero value, χ_o, as T → 0.) Work of Blöte [36] gives information on the spin variation of C_H, and Blöte has in addition estimated the T = 0 magnetization isotherms [4 5]. The spin variation is not very striking, except in one important respect. The S = ∞ curves all show non-physical features at low temperatures. For example, the classical Heisenberg specific heat rises monotonically to a non-zero value as T → 0, instead of vanishing in accordance with the third law of thermodynamics, as do the curves for S < ∞.

Finally the model dependence of the S = 1/2 magnets will be discussed. In Fig. 1 we see the perpendicular susceptibility for the Ising model, and χ for the XY model and Heisenberg AF. All three curves show qualitatively similar features. Fig. 2 shows the zero-field specific heats. Note that the Ising and XY model specific heats are independent of the sign of J. (This is also true of the S = ∞ Heisenberg model, not shown.) The S = 1/2 Heisenberg model, however, shows pronounced differences between F and AF. Fig. 3 displays the T = 0 magnetization curves, M_T vs H, for various models, and model dependent differences become significant. For the Ising model, the curve is a step function. For the Heisenberg AF, the curve starts out linearly (with slope=χ_o) and saturates quadratically with field [7]. The critical
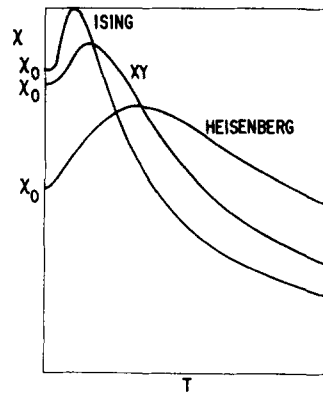

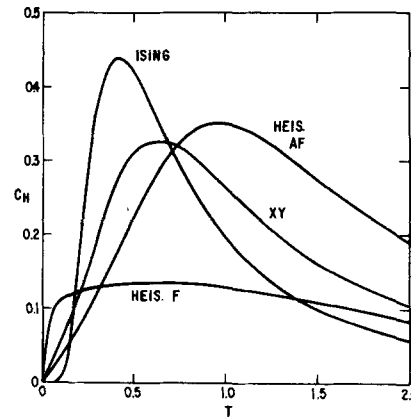Fig. 1. Perpendicular susceptibilities of various S = 1/2 1-D magnets. The χ scale has been normalized.

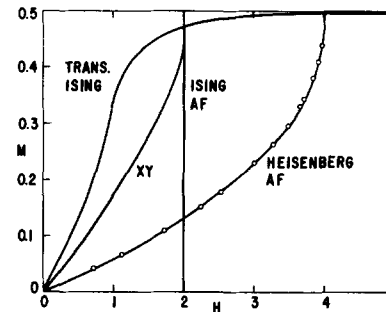Fig. 2. Zero-field specific heats of various S = 1/2 1-D magnets.

Fig. 3. T = 0 magnetization curves vs. applied magnetic field for various S = 1/2 magnets.

field is twice the Ising value. The XY curve is similar in shape to the Heisenberg model, but the critical field is the same as that for the Ising model. By contrast we show an interesting curve for the Ising chain in a transverse field. Saturation is only attained for H → ∞, but a critical field is present as an inflection point, such that the field derivative (the zero-point susceptibility) diverges logarithmically on each side. This is the same critical behavior as the zero-field specific heat of the 2-D regular Ising model, and illustrates a general result. This result strongly suggests that the critical singularities in a D-dimensional model for T=0, H ≠ 0 should be the same as those for a (D + 1)- dimensional model for H = 0, T ≠ 0 [46]. More detailed discussions of model dependences are available [7,14,43].

One point of difference between the 1-D Ising model, and both XY and Heisenberg models, is the behavior of the correlation length $\xi(T)$. This quantity is a measure of the degree of correlation (i.e. degree of

spin alignment) of spins along the chain. ξ diverges as $T \to 0$, corresponding to a $T_c = 0$ critical point. However, for the Ising model ξ diverges exponentially fast (corresponding to relatively more rapid growth of correlation), and more slowly as T^{-1} for the $S = \infty$ XY and Heisenberg chains[53]. Quasi-elastic neutron scattering has provided experimental demonstration of the behavior of $\xi(T)$ [47]. The nature of the divergence of $\xi(T)$ for the S=1/2 Heisenberg chain is not, as yet, known exactly.

CRITICAL SINGULARITIES IN 1-D MODELS

It is commonly said that 3-D systems show $T_c > 0$ phase transitions whereas 1-D systems show no phase transitions (and, by implication, are therefore less interesting). It should be noted that higher-D systems may have T_c depressed by the presence of competing interactions. In fact, exactly solvable models do exist with no phase transition (even at $T = 0$) in zero field. Transitions occur only in non-zero magnetic fields [48]. Most 1-D systems with short-range forces show interesting critical singularities situated at $T_c = 0$. However, even for 1-D models, if the magnetic interactions become sufficiently long-ranged, then $T_c > 0$. (In the limit of infinitely weak, infinite range interactions, (equivalent neighbor model), the critical behavior is mean-field-like). An important paper of Joyce [49, 50] gives analytic solutions for the long-range spherical model in both 1-D and higher D. In 1-D, 2-D and 3-D, three distinct regimes appear. For interaction range less than a special value, the critical behavior has the nearest-neighbor character. For interaction range greater than a second, larger, special value, the critical behavior is mean-field-like. For intermediate range values, the critical exponents vary continuously between short-range and long-range limits. Similar behavior has been observed in numerical studies on the 1-D long-range Ising model [51] and in renormalization group treatments of the 3-D long-range Ising model [52].

It might be noted that for $T_c = 0$ critical systems, hyperscaling relations may be formulated which are somewhat different from the usual relations, because factors of T and T^2 in the basic definitions of thermodynamic properties give rise to confluent singularities at the $T_c = 0$ critical point [53].

ACCURACY OF NUMERICAL TECHNIQUES

Since analytic treatments are not always possible, even in 1-D, it is interesting to evaluate the accuracy of numerical extrapolation methods described in the introduction. Fig. 4 is a comparison of numerical estimates of the zero-field C_H for intermediate anisotropy $b = c = a/2$, based on rings of up to 10 spins [43], with numerical solutions of coupled non-linear integral equations resulting from an analytic treatment of
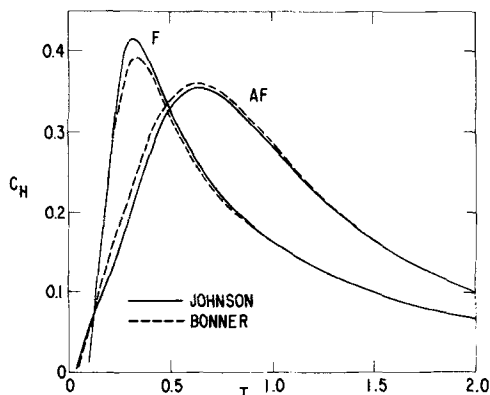

Fig. 4. Comparison of zero-field specific heats resulting from theoretical work and numerical extrapolations, for the anisotropic S = 1/2 chain.
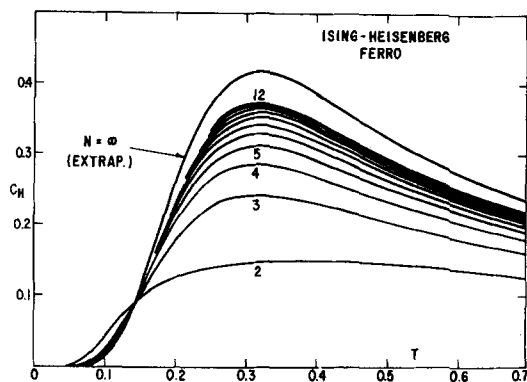

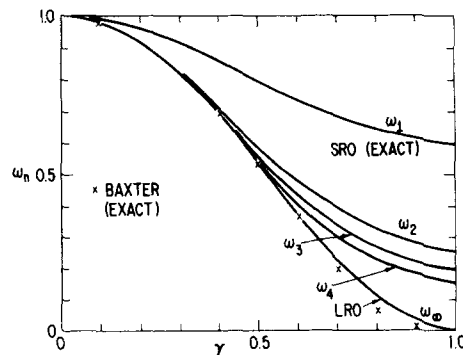Fig. 5. Numerical extrapolations of Blöte for the S=1/2 anisotropic linear ferromagnet.

Fig. 6. Comparison of numerical extrapolations with exact analytic work for the T = 0 LRO of the 1-D anisotropic antiferromagnet. Other pair correlation function ($\omega_n(\gamma) \simeq \langle S_0 S_n \rangle$) extrapolations are shown also. For b = c, the uniaxial anisotropy parameter is defined to be $\gamma = b/a$.

Johnson [38b]. Uncertainty is present in both methods, increasing as $T \to 0$. The agreement, however, is generally satisfactory, discrepancies being comparable with experimental error. More recent calculations of Blöte [54] for the anisotropic ferromagnet for free-ended chains of up to 12 spins are given in Fig. 5 and illustrate the convergence of the finite systems to the extrapolated $N \to \infty$ limit. The Blöte extrapolation agrees with more refined recent calculations of Johnson [54] down to the lowest temperatures.

Two important recent analytic results for the Heisenberg 1-D AF are not generally well-known. The AF specific heat vanishes linearly with T as $T \to 0$, with a coefficient numerically estimated to be 0.35 [7], and now exactly known to be 1/3 [39]. The long-range order parameter (LRO) for the Ising-Heisenberg AF was estimated numerically to vanish only at the Heisenberg limit. This prediction is confirmed by an exact result due to Baxter [55], and discrepancies between extrapolation and analytic result as a function of uniaxial anisotropy are reasonably small ($\sim$ 5 - 10% at most), as shown in Fig. 6.

QUASI-1-D SYSTEMS

In "real" 1-D systems, weak inter-chain interactions ultimately result in a 3-D cooperative ordering anomaly, situated at a lower temperature than the characteristic 1-D rounded maximum in a "good" example. The only available exact result illustrating this phenomenon is the well-known work of Onsager [1,14] for an Ising 2-D net with unequal "horizontal" and "vertical" exchange constants. Many approximate theories have attempted to locate T_c for a specific model and ratio of inter-chain to intra-chain coupling constants, but quantitative a-

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greement among different approaches is not good [56]. A phenomenon recently observed experimentally in a series of quasi-1-D AF's, where the phase transition initially increases in T with increasing applied field, in contradiction to the predictions of mean-field theory, can be interpreted in connection with work of Kosterlitz, Nelson and Fisher [57]. Essentially, as soon as $H > 0$, the quasi-1-D Heisenberg AF behaves as a quasi-1-D XY model. Since XY ordering temperatures are relatively higher than Heisenberg ordering temperatures, the increase is qualitatively explained.

DISORDERED AND DILUTE 1-D MAGNETS

One-dimensional topology, or, equivalently, the low 1-D coordination number, emphasize strongly the effects of impurity and disorder in 1-D relative to higher D, where interaction pathways around the impurity are available. This feature, plus the much greater ease of solvability in 1-D, have made 1-D systems a happy hunting ground for researchers in the highly popular and growing area of dilute and disordered systems. The volume of interesting articles is already too vast for any realistic attempt at review in this short paper. The subject appears ripe for review on its own: unfortunately no such review is presently available, to the author's knowledge. Brief mention might be made of a comprehensive series of papers on 1-D solvable magnets with a distribution of exchange constants [58]; approximate treatments of quasi-1-D systems with impurities [59]; exact solutions for randomly dilute Ising chains and the XY chain with a random applied field [60]; exact calculations for random and impure classical Heisenberg chains [61]; and Monte Carlo studies on Ising spin glasses [62]. Work of Richards has represented a significant contribution to a correspondingly large body of experimental work [63].

SPIN DYNAMICS OF QUASI-1-D SYSTEMS

Theoretical study of dynamical properties is considerably more difficult than studies on static properties. Much work has been done, but the subject generally is on a relatively insecure footing owing to a lack of exact solutions for testing approximate theories. Accordingly, there has been considerable activity in 1-D, which has been extensively reviewed in articles by Richards [64], Hone and Richards [64] and Steiner, Villain and Windsor [65]. Considerable attention has been paid to the classical Heisenberg chain because of its relative ease of solubility [66].

Infinite temperature correlations have been studied by numerical approximation techniques based on work of Bonner and Fisher [7] and Baker [67] for Heisenberg and XY chains [68]. A conjecture on the Gaussian time-dependence of the XY auto-correlation function [68] has recently been confirmed by exact work. A particularly complete treatment of the $T = 0$ dynamical spin correlations of the XY chain is now available [70]. At low temperatures, the spin dynamics of 1-D Heisenberg AF's is interesting. By contrast with the classical Heisenberg chain, where all spectral weight is concentrated

at the well-known double-sine (spin-wave) dispersion frequencies, numerical and approximate calculations [71,72] for the spin-1/2 Heisenberg AF indicate a "tail" of spectral weight above the des Cloizeaux and Pearson triplet spin-wave states. The situation is sketched in Fig. 7. Numerical [71] and analytical [41] studies indicate that the des Cloizeaux-Pearson states form the lower boundary of a two-parameter continuum (shown shaded) comprising degenerate singlet and triplet states. Numerical studies indicate that only the triplet states carry spectral weight, and also that non-negligible weight occurs at the top of the double continuum. If this theoretical prediction is confirmed by more refined neutron scattering experiments [73], then a further important difference between quantum ($S = 1/2$) and classical ($S = \infty$) linear chains appears. This difference in the low temperature spin dynamics is sketched in Fig. 7b.

1-D MAGNETS, 2-D FERROELECTRICS AND OTHER CORRESPONDENCES

An important relation exists between the 1-D archetypical XYZ model and a class of 2-D hydrogen-bonded ferro- and antiferroelectrics, which has proved fruitful in the theory of cooperative phenomena. The isomorphism occurs because the transfer matrix for the 2-D problem commutes with the Hamiltonian matrix for a 1-D system. Hence the correlation functions in the two problems are related, and spin anisotropy in the 1-D magnet corresponds to temperature in the 2-D ferroelectrics. Specifically, the 2-D Rys antiferroelectric model corresponds to the Ising-Heisenberg AF, the 2-D Slater KDP ferroelectric corresponds to the Ising-Heisenberg ferromagnet, and the ice problem is isomorphous to a 1-D ferromagnet with anisotropy intermediate between Heisenberg and XY. Extensive further detail is provided in reviews by Lieb and Wu [74], Nagle [50] and Nagle and Bonner [75].

To conclude this review let us survey further interesting transcriptions between 1-D magnets and other physical phenomena. A well-known correspondence between magnets and lattice gases (fluids) is utilized in exact calculations for a 1-D continuum cluster interaction fluid [76]. This transcription has also been used to transform a 1-D Ising AF with a Baker-Kac ferromagnetic tail [77] into continuum fluids [78] and lattice gases with extended repulsive hard cores [78], which show both a liquid-vapor phase line ending in a critical point and a solid-liquid phase line. In general, 1-D magnetic systems with competing long range interactions show multicritical behavior [77, 79]. The behavior of dilute magnetic alloys near the percolation threshold has been related to essentially 1-D properties of self-avoiding walks [80] and exact solutions for 1-D site percolation problems show breakdown of universality [80]. Excitations of the 1-D XYZ spin 1/2 model are equivalent to those of the field-theoretic (one space and one time dimension) Thirring model with suitable relations between coupling constants [22a].

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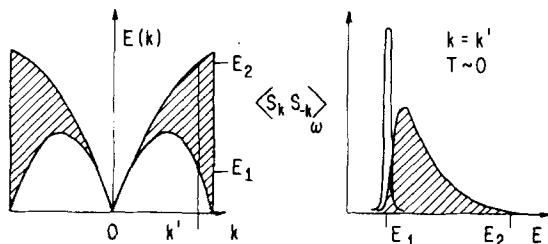4. M.E. Fisher, Rept. Prog. Phys.,30 615 (1967).

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