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The Square of Opposition: Innovations in Teaching Logic

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Honors 491

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The Square of Opposition: Innovations in Teaching Logic

Introduction

Teaching logic of any type to novices can come with a variety of challenges. Many concepts, when approached linguistically, appear to be intuitive. However, the abstraction of language into logical symbols may transform what was once intuitively understood into something that appears to be foreign or not immediately relatable to a given student's modes of everyday reasoning. Specifically, new students in critical thinking courses may struggle with making inferences of categorical statements using the classical square of opposition.

There is much debate among philosophers and others over the proper approach to teaching critical thinking (Robinson, 278-85) (McPeck, 66-95). Many instructors teach only informal logic in their courses, perhaps believing that the ability to find and identify informal fallacies applies more readily to the "real world." Even in courses where formal logic is taught, however, there are those who believe that modern logic has rendered the teaching of classical categorical logic –which includes the classical square of opposition – as obsolete. By contrast, this paper takes the position that there is still value in teaching classical logic in introductory critical thinking courses.

First of all, classical categorical logic reflects the intuitions of everyday reasoning. For example, in modern logic, the inference from a true universal affirmative statement to the truth of the associated particular affirmative cannot be made due to the issue of existential import, and rightly so under the commonly accepted interpretation (The accepted interpretation of existential import results from answers to these questions: Do universal statements imply their associated particular statements? And, to what degree do our particular statements imply the existence of their subjects?). With the assumption that universal statements *never* imply the existence of their subjects, paired with the assumption that particular statements *always* imply the existence of their subjects, it logically follows that we cannot infer the truth of an existing particular based on a universal statement that does *not* imply existence. In a common, everyday context, however, reasoning from the truth of a universal to the truth of a particular is not always considered to be erroneous. In the types of informal discussions that we have regularly, the issue of existential import rarely surfaces. When someone infers that “Some particular turtle has a shell” from the statement that “All turtles have shells,” there is nothing within the typical interpretation of those statements that indicates the possibility that turtles *may not exist*. To put it more simply, basic discussions with pragmatic goals do not focus on the assertion (via statements) of the existence of their subjects, because in most cases, their existence is assumed.

Additionally, in more academic or specialized contexts, classical logic has value for students. Primarily, a given person cannot be expected to fully understand modern logic without understanding issues such as the problem of existential import, and the problem of existential import cannot be properly understood without a grasp of classical logic issues that

are being addressed by modern logic. In short, understanding where we come from, logically, tells us about where we are, and where we may be heading.

Regarding the shift toward a focus on informal reasoning in entry level critical thinking courses, an argument can be made that the type of approach stressing informal reasoning does not give students a functional understanding of the procedures used in such reasoning, thus denying the student an opportunity to practice and develop the skills necessary to make proper inferences. No doubt, the ability to recognize and identify informal fallacies is a useful one. Even so, this is only half of the battle. What remains untouched in pedagogical techniques focusing exclusively on informal reasoning is the exposure to actual logical procedures at play and the reasoning behind them. Students can then end up lacking the skills required to translate ordinary statements into their standard symbolic form or to work with such formal statements at an abstract level - a skill that contains benefits for any given student. Primarily, it is beneficial for students to be able to work with the symbolic representations of statements. Instead of parsing out inferences linguistically; standardized symbols allow students to focus solely on the logical operations required, thus avoiding any misleading ambiguities that may be present in the language.

Detractors of teaching classical logic sometimes object that making inferences from basic categorical propositions is intuitively obvious, and therefore should not be afforded the class time it takes to teach the technical aspects of such inferences. While it can be generally agreed upon that many inferences of categorical propositions are intuitively obvious, such dismissiveness oversimplifies the issue. Students may have differing levels of intuition, and, as

stated, the abstraction into unfamiliar symbols can undercut even intuitively obvious inferences. In any case, a person's intuitions can potentially be wrong, and it is always beneficial to supplement individual or *prima facie* intuitions through the cultivation and practice of technical skills.

Apologetics aside, many instructors still incorporate classical logic into their curriculum, and as long as they do, there will be the need for innovation in those teaching methods and presentation styles related to it. To that end, the remainder of this paper presents a new technique for teaching one specific model often featured in classical logic: the Square of Opposition.

The Traditional Square of Opposition

One important aspect of critical thinking is the ability to reason and draw inferences from simple categorical statements. The square of opposition, a logical diagram, was designed to visually indicate the relationships between four different types of categorical statements:

- the universal affirmative (the A statement, meaning "all [subject] are [predicate],” symbolized as SaP)
- the universal negation (the E statement, meaning "no [subject] are [predicate],” symbolized as SeP)
- the particular affirmative (the I statement, meaning "one or more [subject] are [predicate],” symbolized as SiP)
- the particular negation (the O statement, meaning "one or more [subject] are not [predicate],” symbolized as SoP)

These statements relate to each other in a variety of types of opposition. An A statement and its corresponding E statement have what is known as a *contrary* relationship, meaning that they cannot both be simultaneously true, but they can be simultaneously false. Similarly, an I statement and its corresponding O statement have what is known as a *subcontrary* relationship, meaning that they cannot be simultaneously false, but they can be simultaneously true.

Furthermore, an A statement and its corresponding O statement, and an E statement and its corresponding I statement, relate as *contradictories*. Statements that relate as contradictories cannot have identical truth values (if one contradictory is true, the other must be false. If one contradictory is false, the other must be true). Lastly, an A statement and its corresponding I statement, and an E statement and its corresponding O statement relate as *alterns*. The universal statements are referred to as *superalterns* and the particular statements are referred to as *subalterns*. The relationship between superalterns and their corresponding subalterns can be described with two sub-rules: (a) The truth of a superaltern (an A statement or an E statement) implies the truth of its corresponding subaltern (an I statement or an O statement, respectively), and (b) The falsehood of a subaltern implies the falsehood of its associated superaltern.

The idea behind making categorical inferences is this: Given one of the four basic categorical propositions and a truth value, and using the described relations as rules of inference, a given person can infer the truth value of the remaining three

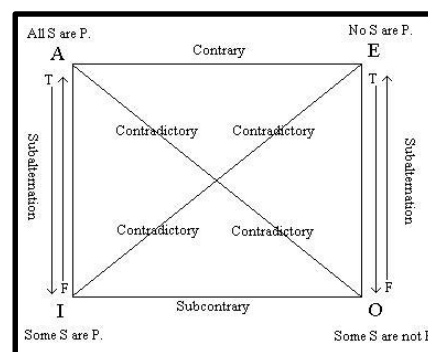


Figure 1

propositions. This process is visualized in the square of opposition (*fig.1*). While there are historical and contemporary variations in how the square of opposition is depicted (Bernhard), figure 1 represents a common presentation.

Pedagogical Stumbling Blocks to Mastering the Logic of the Square

While many of the rules of inference are linguistically intuitive, the first step in making immediate inferences is the proper translation from language into logical symbols. Once a student is dealing with symbolic representations of statements, these intuitively easy inferences can become obscured by viewing them in a simplified, symbolic state. Additionally, there is not a standardized visualization of the square of opposition, and many depictions include potentially confusing elements. For example, some depictions of the square of opposition utilize arrows to indicate the relationships between a superaltern and its related subaltern. This can become problematic. For example: when making an inference from a true superaltern, we can carry that truth value downward in the direction of the arrow. If the given superaltern is false, however, we cannot simply carry the false value downward in the same way due to the altern sub-rule (b). Yet, the inclusion of an arrow as described above - arrows imply directionality and motion, not necessarily relationships such as those focused on in immediate inferences - may mislead students into thinking that this inference is correct, because they are following the direction of the arrow.

Some diagrams attempt to account for this type of error by including additional arrows pointing upwards from a subaltern to its associated superaltern. Despite this attempt to mitigate confusion, there is not usually any visual indicator as to *when* one should follow the

downward arrow instead of the upwards arrow, because the nature of the diagram is to represent these concepts without the use of words. Therefore confusion can be paradoxically compounded by an adaptation meant to lessen that confusion.

Aside from these problems, there are certain inferences that cannot be made, which results in a truth value of *unknown*. For example, when given a superaltern as false, we cannot make the inference that its associated subaltern is either true or false, and at best can only infer that the truth value is unknown. While the existence of unknown inferences is included implicitly in the rules of inference of categorical propositions, this fact is never visually indicated in the tools and diagrams that are typically available to students.

To add to the number of potential roadblocks that a new student of logic might face, oftentimes the required textbooks used in critical thinking courses do little in terms of offering a variety of useful tools to help students of differing backgrounds and capabilities to come to a higher level of understanding. Introductory logic textbooks tend to focus on a text-based description of the processes involved, which is often less clear than a verbal lecture or real-time demonstration. They are also, quite typically, jargon heavy, which can lead to frustration if a student is already struggling. Additionally, most textbooks repeat or reuse the same or similar visual representations of the square of opposition (Kelly, 152-57) (Baronett, 173-77) (Parker, Moore, 253-54) (Salmon, 326-30) (Cohen, McMahon, Copi, 80-83). Others discard the arrows, but do little else to alter how the information is conveyed (Hurley, 211).

While such depictions can vary slightly from text to text, all lack innovative and *new* ways of visualizing the necessary processes of making immediate inferences from categorical

propositions. When the producers of a text book do decide to innovate, the resulting product may become even further abstracted from a student's grasp of essential relationships, resulting in a model that is no more intuitive than the traditional depiction.

Aside from difficulties that arise from issues related to the specific subject matter, teaching classical logic can also be hampered by more general issues of teaching that manifest themselves in any given classroom. Large classrooms in particular are comprised of a diverse collection of students, each with their own talents, strengths, weaknesses, and approaches to the material. Some students may respond positively to text based explanations, whereas others may have difficulties memorizing long or complex lists of rules and axioms. All of this can make planning a curriculum and setting standards a challenge. Instructors in any field can benefit from adding more tools to their heuristic tool belt for those students that might not respond well to standardized approaches.

Creating New Tools for Teaching Classical Logic: Dimo's Square

Creating new educational tools can be easy and effective once specific difficulty areas are discovered and identified. Further, the development of visual models and other heuristic devices, even for less common areas of difficulty, can be beneficial on a larger scale; the resulting method or model may have other intrinsic attributes that can enhance the learning outcomes of students whose difficulties stem from completely unrelated areas. An example of one alternative approach to teaching immediate inferences of categorical propositions is "Dimo's Square (*fig.2*)," a newly innovated model of the traditional square of opposition.

Dimo's Square, like the traditional square of opposition, visually indicates the process of making immediate inferences of categorical propositions. Furthermore, Dimo's Square and the traditional models are logically identical, meaning that all inferences made will yield identical results¹. The difference emerges from the manner of inference delivery.

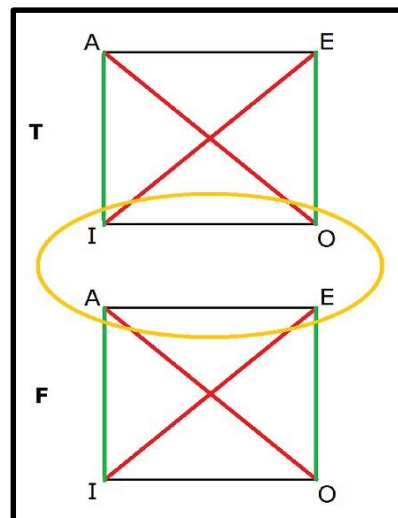


Figure 2

Dimo's Square separates the traditional square into two different squares, one for true statements, and one for false statements. Once given a case as true or false, only the square that corresponds to that truth value is used (in other words, if given a true statement, use the top square only. If given a false statement, use the bottom square only). Overlapping the two squares is a circled area that indicates a zone containing the statements which produce unknown inferences.

Dimo's Square can be operated by implementing two rules and one exception. *The rule of contradiction* states that any two statements connected by a diagonal line cannot have the same truth value. *The rule of alternans* states that, if a statement on an upper corner of either square is true, then any statement connected to it by a vertical line is also true. There is *one exception* to the rule of alternans: when given a case that is within the circled area, use the rule of contradiction *only*; all other inferences are unknown.

As an example: When given "SaP is true" as your starting case, begin at the A corner of the true square. Then, employ the *rule of contradiction* to determine that SoP is false. Using the

¹ Formal proofs are included on pages 14 to 23

rule of alternans, it can be determined that SiP is also true. Then, employing the *rule of contradiction* on the valid inference “SiP is true,” it can be concluded that SeP is false.

Another example that includes the use of *one exception*: When given “SeP is false” as your starting case, begin at the E corner of the false square. Note that the given case is within the circle, and the rule of alternans cannot be used. Then, employ the *rule of contradiction* to determine that SiP is true. Without another rule, no further inferences can be made, so the remaining statements, SaP and SoP, are unknown.

Comparative Analysis: Dimo’s Square and the Traditional Model

The primary purpose behind creating Dimo’s Square was to assist students who had difficulties identifying the statements that produced unknown inferences. Instead of using just four basic statements that are then assigned a truth value, and consequently processed through the diagram, Dimo’s Square “bundles” the truth value along with the basic categorical statements, which gives a total of eight possible base cases (a true and a false version of each of the four basic statements). This allows for the square to be separated into a true square and a false square, giving the user a dedicated corner for all eight possible cases. Stacking the two on top of each other neatly groups the corners representing the statements that result in unknown inferences. The circle around them strongly indicates the group as such.

This particular presentation of the square of oppositions has many benefits. Most notably, on a surface level, it visually indicates the statements that will produce unknown inferences. Unknown inferences, while clearly indicated in the *rules* of the traditional square,

are never *visually* indicated within the traditional diagram. Dimo's Square addresses this problem directly in *both*, through its rules and in its visualization.

Dimo's Square also gives the user a dedicated corner for each possible combination of true or false basic statements. The traditional representation has four corners that can accept either true or false variables. This may seem to be simpler, as the number of basic statements in this approach is limited to four. However, a true value in a given corner on the traditional square may call for a different directional procedure than a false value. Having dedicated starting points for each possible combination insures that no corner is doing two jobs. Each corner deals exclusively with one type of true or false statement.

Another minor but beneficial consequence of Dimo's Square is, due to its arrangement, there is no need for arrows in any way. As mentioned in the earlier discussion of problems with traditional models of the square, arrows imply a directionality of motion that can be misleading for novice logicians. By contrast, with Dimo's Square, all rules are described and defined in terms of their horizontal and diagonal connector lines, which indicates a type of relationship, *not* a direction. This greatly reduces the chances of a student making an error when dealing with the superaltern or subaltern of a given case.

Finally, and perhaps the most notable property of Dimo's Square, is that it requires fewer rules to operate. The traditional square requires the memorization of four rules (but only if you count the rule of alterns as one rule, which may be a little disingenuous due to the fact that it is comprised of two sub-rules. This may be perceived by students as an attempt to "sneak in" an additional rule without admitting that one had done so). Dimo's Square, as

described above, requires only two rules and one exception. Due to the arrangement of Dimo's square, it is not necessary to have specific rules for contraries and subcontraries. Additionally, the rule of alterns remains as a single rule, not a pair of sub-rules. And while Dimo's Square could also be criticized for "sneaking in" a rule by having an exception (which could be treated as a third rule), it is always easier to remember *not* to do something than it is to remember a thing that you need to do. Regardless, even including the exception of the rule of alterns as the third in its list of rules, Dimo's Square still operates with two fewer rules than its traditional counterpart. This attribute of Dimo's Square may be enough to distinguish itself from its predecessor in terms of its elegance. To produce the same results with fewer axioms and, even in some cases, fewer steps, is almost always more desirable than any other alternative, and surely in line with the spirit of Occam's Razor.

A possible criticism of Dimo's Square is that it trades visual simplicity for verbal or rhetorical simplicity. While it may be true that Dimo's Square can be operated with fewer rules, the visualization required for this method is more complex. There are two squares instead of one, and it requires a circle to indicate when to enact the exception. That said, the degree to which two squares and a circle are harder to memorize than a single square is possibly negligible, and Dimo's Square has the advantage of being visually distinctive as well as verbally legible.

In light of this comparative discussion, it is important to note that many students may become overwhelmed when confronted with a barrage of diagrams and visualizations. It is almost always better to begin with a generalized approach until it becomes clear that there are

specific difficulties being experienced by a student. In other words, only when there are specialized problems should there be specialized solutions. The development of Dimo's Square arose in response to one such specialized problem, hence its presentation of one such specialized solution.

Conclusion

When it comes to teaching logic, the common approaches tend not to stray far from traditional methods of representation. While having a general strategy is essential to teaching within any field, it can result in a failure to provide the necessary skills and information to those students who may not respond well to such generalized methods. While some believe that it may be pragmatically useless to focus time and effort on developing methods for outliers - that it is always more efficient to use a "best fit curve" approach in designing curricula and teaching tools - Dimo's Square shows otherwise. It is rooted in the teleology of mastery and driven by (i) a desire to convey the information in a way that can be understood, (ii) a functional understanding of what aspects of the material are confusing to students and (iii) a determination to reimagine a heuristic that conventional teaching assumes is long exhausted of its possibilities. Hopefully, this paper has demonstrated the persistence of possibilities.

PROOFS

General Rules

All given cases must: Contain one Subject and one Predicate, in that order. Be of one and only one type (either A, E, I, or O). Be given as either True or False, but not both.

Rules of Inference (Traditional square)

Contradictories: Contradictory statements are connected by diagonal lines. Contradictory statements cannot have the same truth value; if one is true, the other must be false. If one is false, the other must be true.

Alterns: Alterns are connected by vertical lines. Universal statements (A and E) are “superalterns.” Particular statements (I and O) are “subalterns.”

a) If a superaltern is true, its related subaltern is also true. If a superaltern is false, then its related subaltern is unknown.

b) If a subaltern is true, its related superaltern is unknown. If a subaltern is false, then its related superaltern is also false.

Contraries: Universal statements (A and E) that are connected by a horizontal line are “contraries.” If a given contrary statement is true, the other must be false. If a given contrary statement is false, the other is unknown.

Sub-Contraries: Particular statements (I and O) that are connected by a horizontal line are “sub-contraries.” If a given sub-contrary statement is true, the other is unknown. If a given sub-contrary is false, the other is unknown.

Rules of Inference (Dimo’s Square) Note: when given true statements, use the top square only. When given false statements, use the bottom square only.

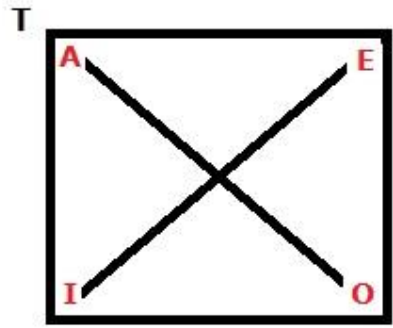
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Alterns: Alterns are connected by vertical lines. Universal statements (A and E) are “superalterns.” Particular statements (I and O) are “subalterns.” If a superaltern is true, its related subaltern is also true.

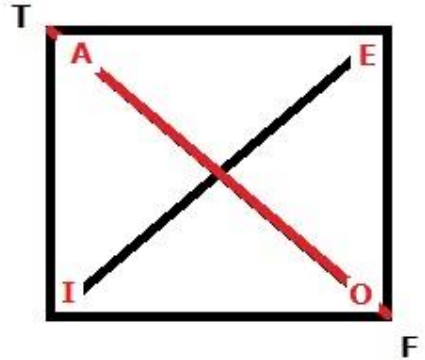
Exception: If the given case falls within the circled area, use the rule of contradiction *only*. Any other inferences that would require the use of an additional rule are unknown.

Traditional Square

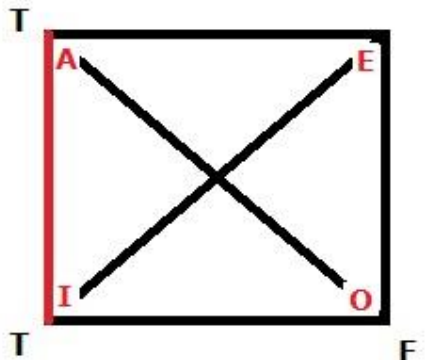
SaP is True - Given



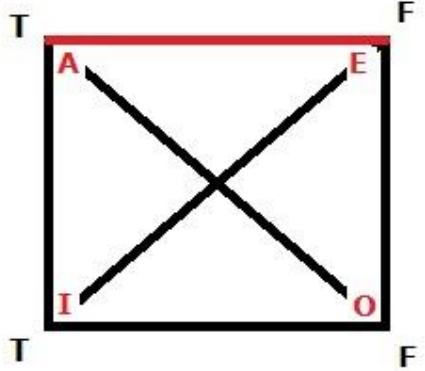
SoP is False - Rule of contradiction



SiP is True - Rule of alternans (a)

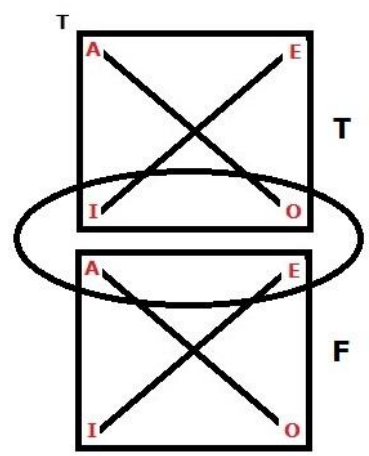


SeP is False - Rule of contraries

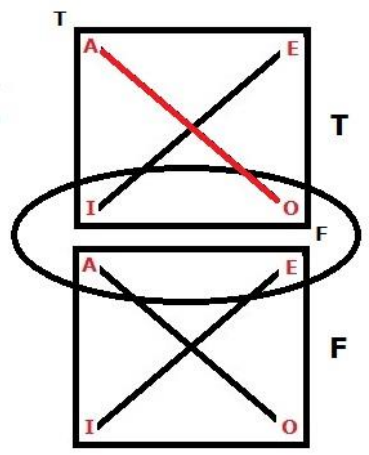


Dimo's Square

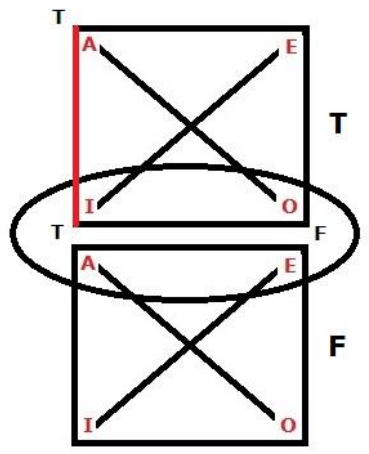
SaP is True - Given



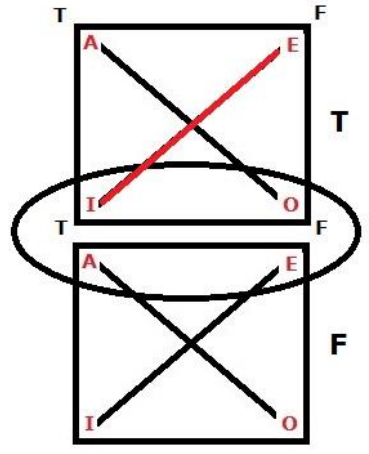
SoP is False - Rule of contradiction



SiP is True - Rule of alternans

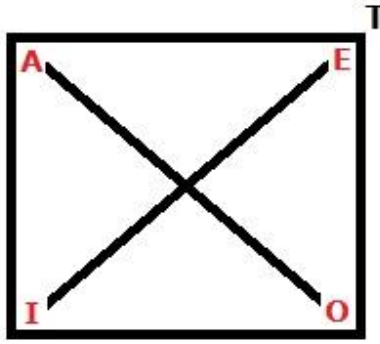


SeP is False - Rule of contradiction

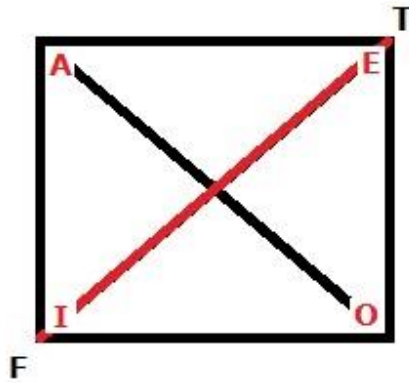


Traditional Square

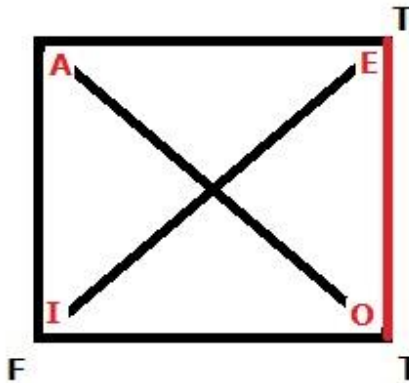
SeP is True - Given



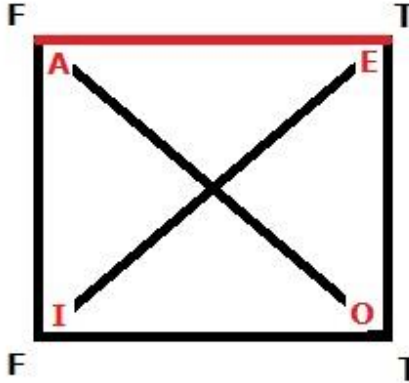
SiP is False - Rule of contradiction



SoP is True - Rule of alterns (a)

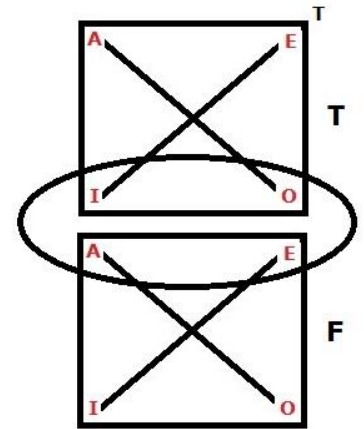


SeP is False - Rule of contraries

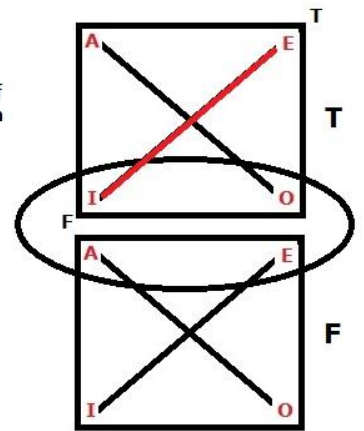


Dimo's Square

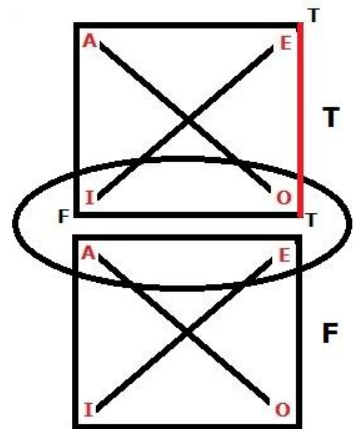
SeP is True - Given



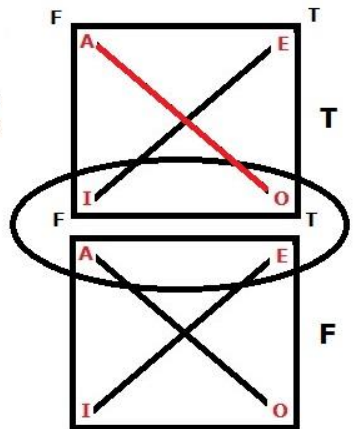
SiP is False - Rule of contradiction



SoP is True - Rule of alterns

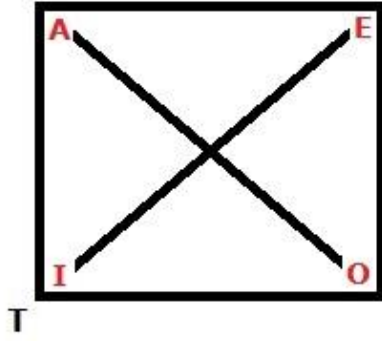


SaP is False - Rule of contradiction

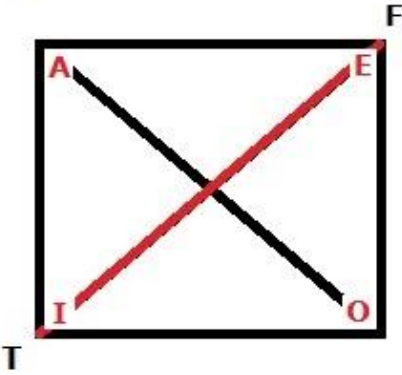


Traditional Square

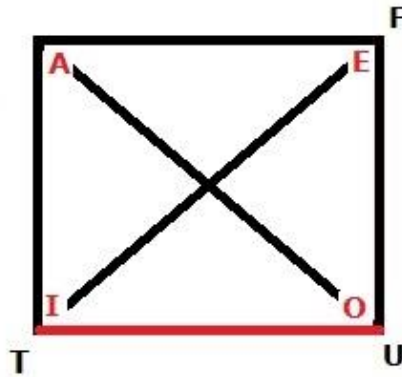
SiP is True - Given



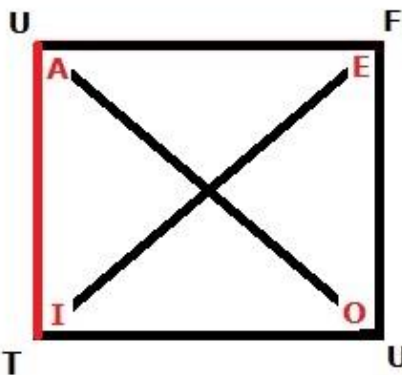
SeP is False - Rule of contradiction



SoP is Unknown - Rule of subcontraries

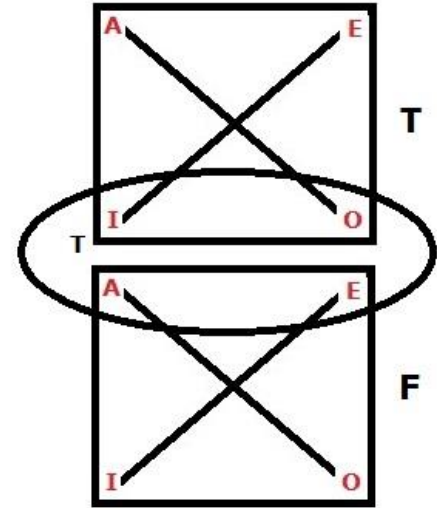


SaP is Unknown - Rule of alternans (b)

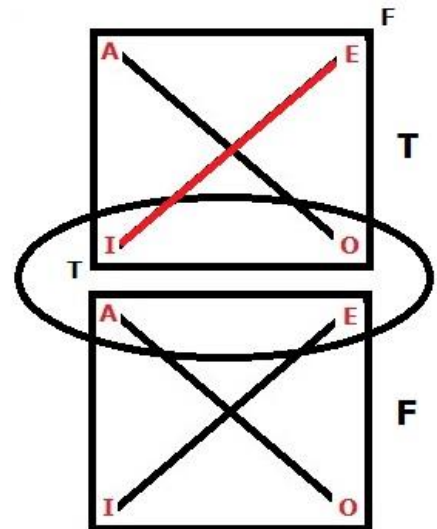


Dimo's Square

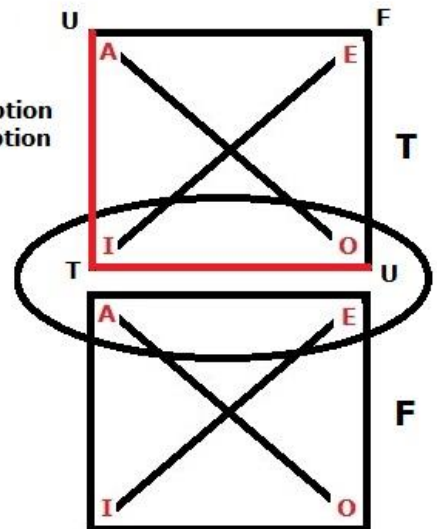
SiP is True - Given



SeP is False - Rule of contradiction

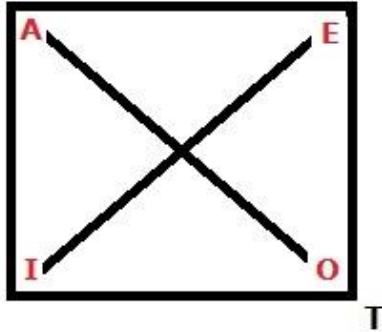


SaP is Unknown - Exception
Sop is Unknown - Exception

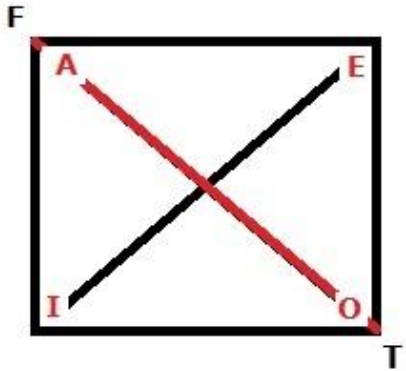


Traditional Square

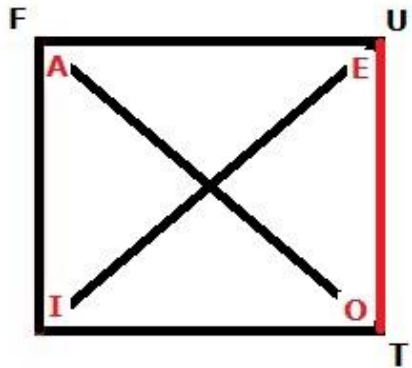
SoP is True - Given



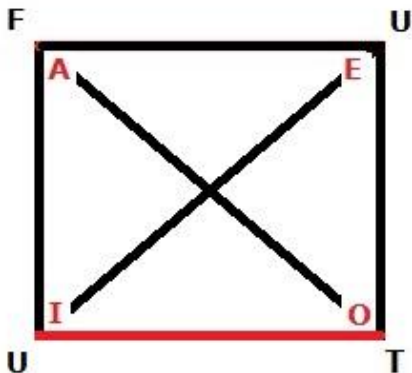
SaP is False - Rule of contradiction



SeP is Unknown - Rule of alternans (b)

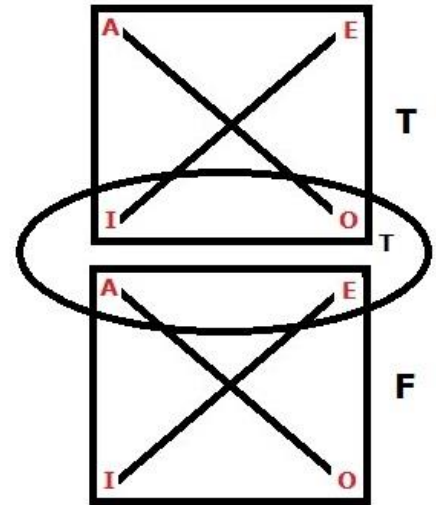


SiP is Unknown - Rule of subcontraries

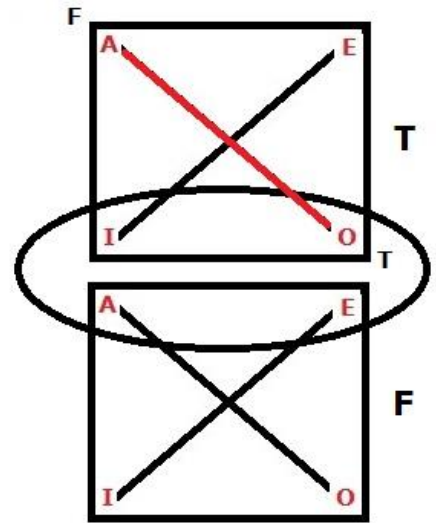


Dimo's Square

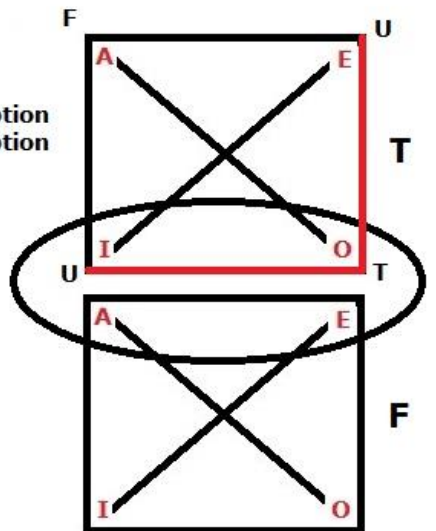
SoP is True - Given



SaP is False - Rule of contradiction

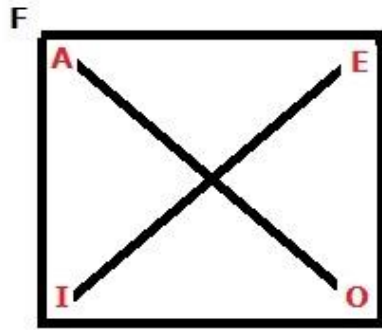


SeP is Unknown - Exception
SiP is Unknown - Exception

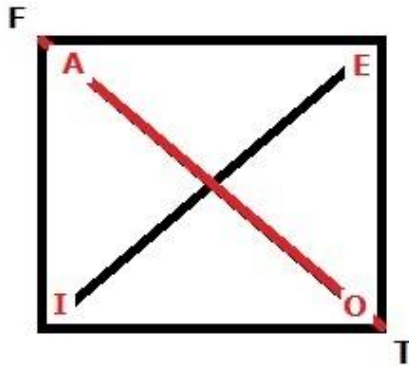


Traditional Square

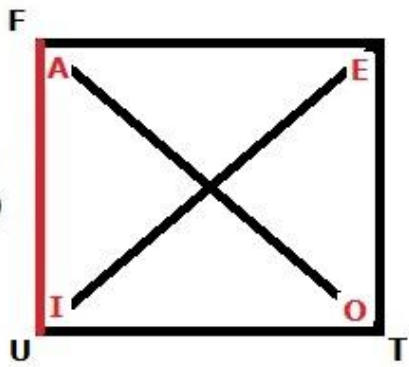
SaP is False - Given



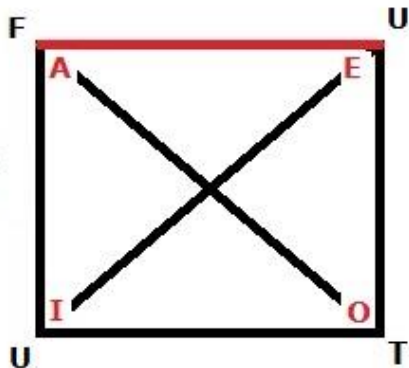
SoP is True - Rule of contradiction



SiP is Unknown - Rule of alternans (a)

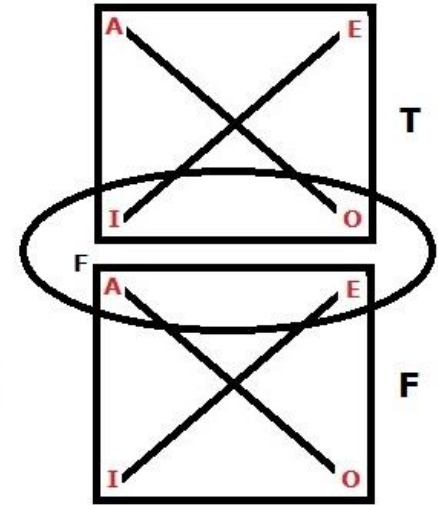


SeP is Unknown - Rule of contraries

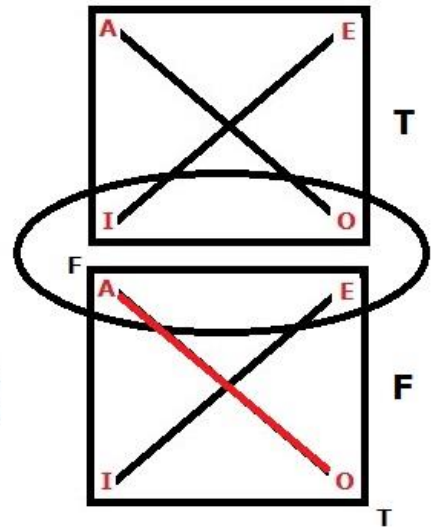


Dimo's Square

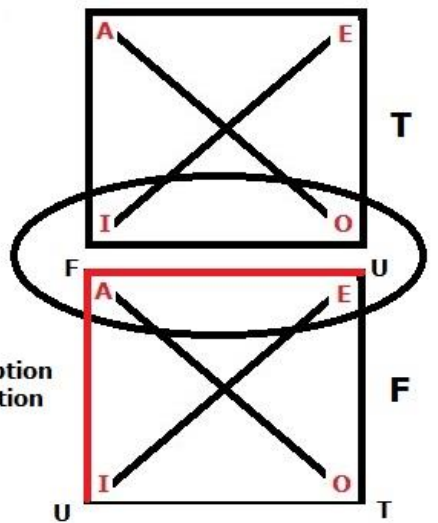
SaP is False - Given



SoP is True - Rule of contradiction

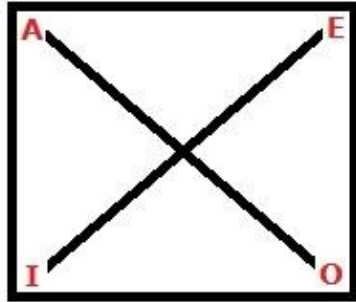


SeP is Unknown - Exception
SiP is Unknown - Exception

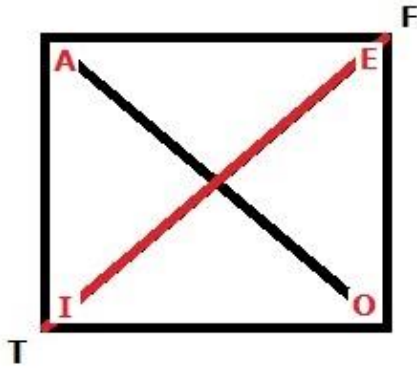


Traditional Square

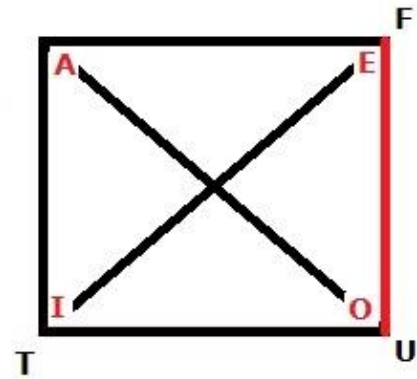
SeP is False - Given



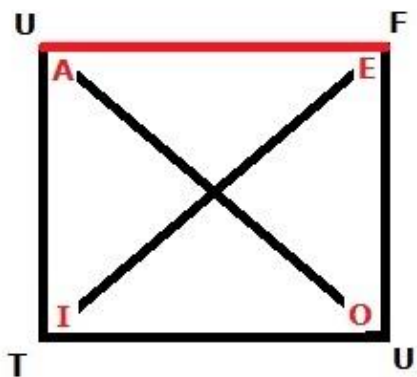
SiP is True - Rule of contradiction



SoP is Unknown - Rule of alterns (a)

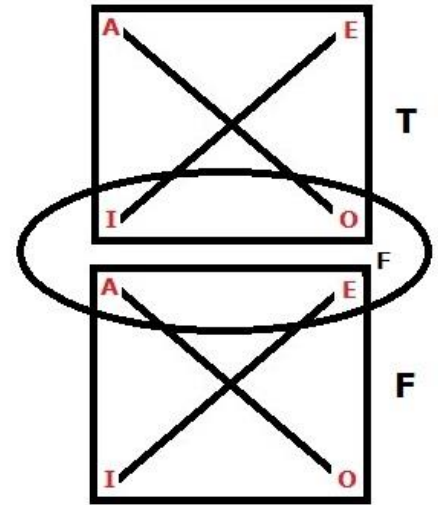


SaP is Unknown - Rule of contraries

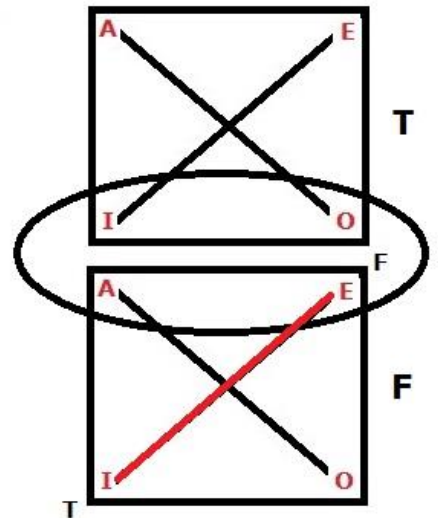


Dimo's Square

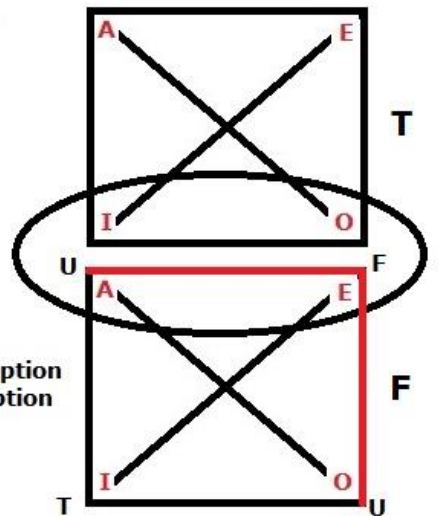
SeP is False - Given



SiP is True - Rule of contradiction

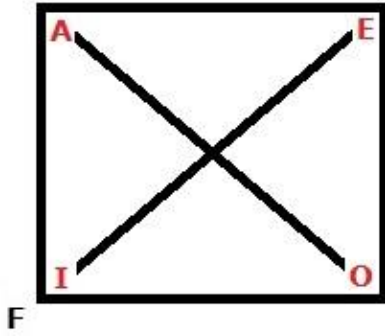


SaP is Unknown - Exception
SoP is Unknown - Exception

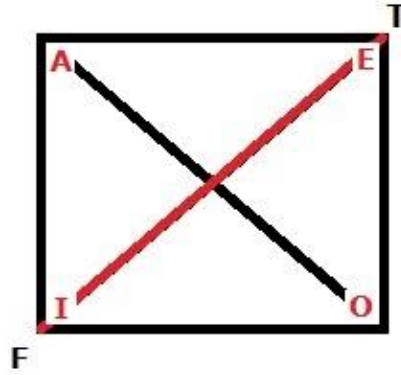


Traditional Square

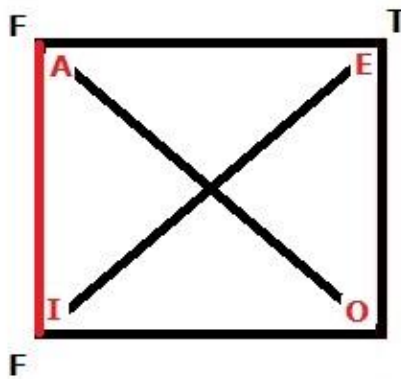
SiP is False - Given



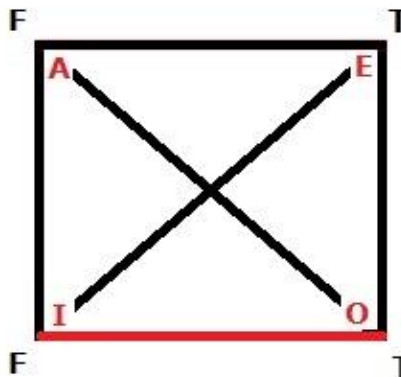
SeP is True - Rule of contradiction



SaP is False - Rule of alternans (b)

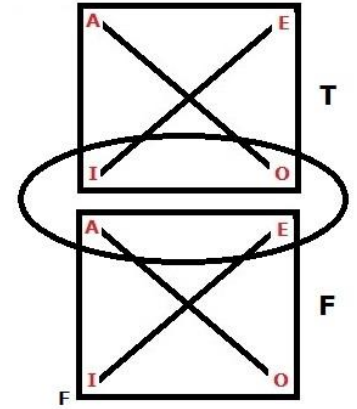


SoP is True - Rule of subcontraries

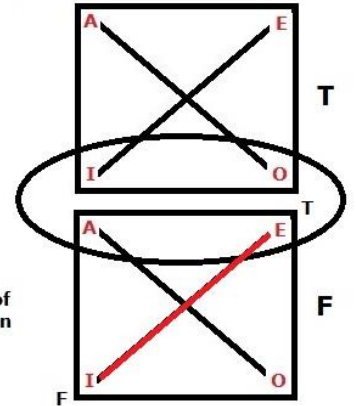


Dimo's Square

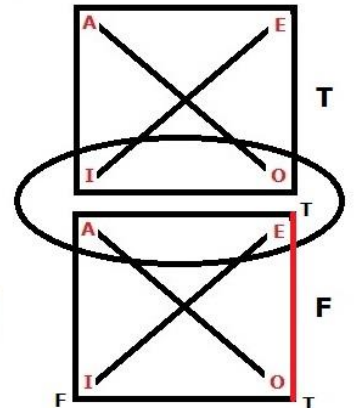
SiP is False - Given



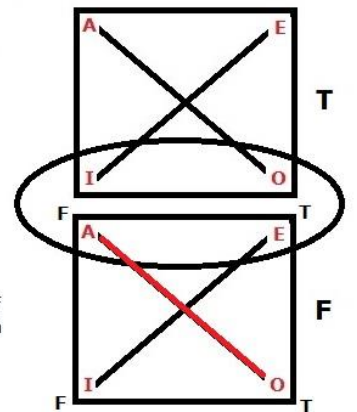
SeP is True - Rule of contradiction



SoP is True - Rule of alternans

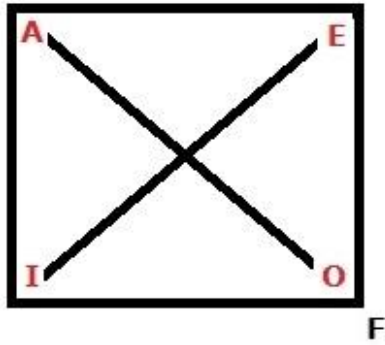


SaP is False - Rule of contradiction

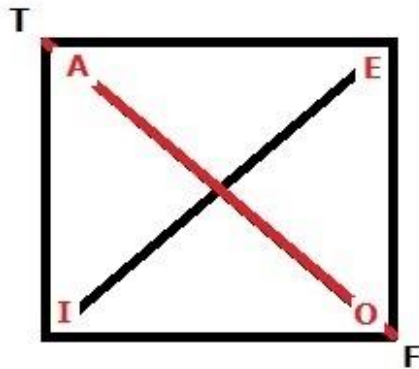


Traditional Square

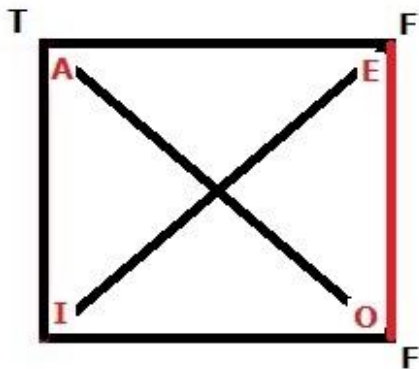
SoP is False - Given



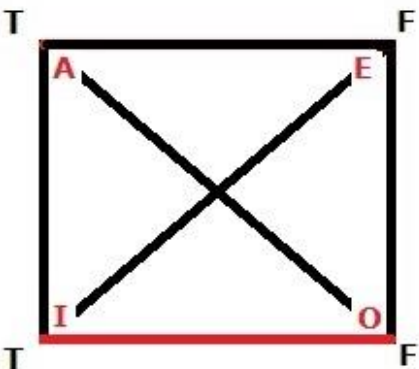
SaP is True - Rule of contradiction



SeP is False - Rule of alternans (b)

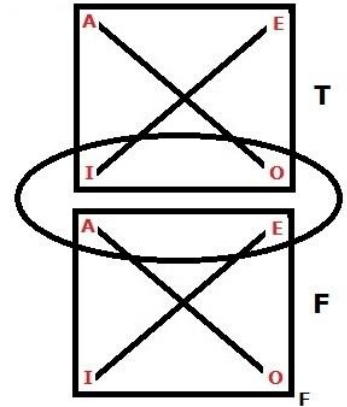


SiP is True - Rule of subcontraries

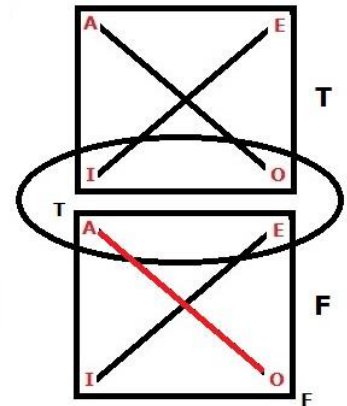


Dimo's Square

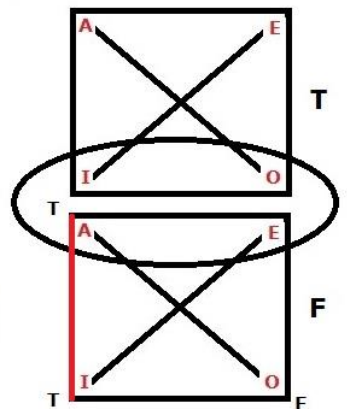
SoP is False - Given



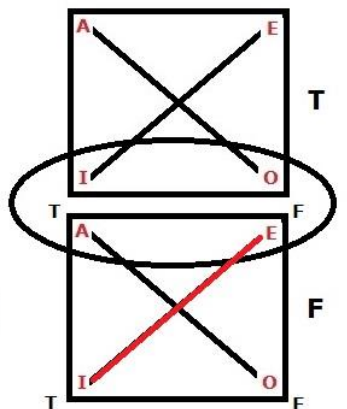
SaP is True - Rule of contradiction



SiP is True - Rule of alternans



SeP is False - Rule of contradiction



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