ALBERTO GIUDICI

Cooperation, Reliability, and Matching in Inland Freight Transport

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Cooperation, Reliability, and Matching in Inland Freight Transport

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Contents

1 Introduction

Container transport allowed for global trade and trade has lifted the welfare and fostered the development of nations worldwide (World Bank Group 2018). The exchange of goods required the movements of those along a chain of parties and companies. The invention of the container in the 1970s reduced transport insurance costs, increased operational efficiency of transport, and allowed for globalization (Notteboom & Rodrigue 2008). Considering the journey of a container, this thesis focuses on inland container transport, the first and end part of the trip, which connects production areas to ports and ports to warehouses and last-mile distribution (Wilmsmeier et al. 2011, Notteboom & Rodrigue 2005). In the inland, different modes of transport are used, either truck, train, or inland waterway vessels (barges), each of which has different characteristics. While intercontinental container transport allowed for global trade, the inland transport sector plays a critical role in the competitive performance of regions and countries alike (Rietveld & Nijkamp 1992). Though, differently from the ocean transport market which is consolidated around a few dozen companies, the inland transport market is harder to manage due to its highly fragmented and competitive nature with thousands of players in the Netherlands only (Fransoo & Lee 2013, Morder Intelligence 2021).

In recent years, supply chains and transport systems got under pressure to deliver an ever-increasingly efficient system. First, globalization itself, the development of e-commerce, and the need to maintain economic welfare required cheaper and cheaper transport solutions (The Economist 2021). Second, the rising climate crisis led to public concerns about the sustainability of transport (IPPC 2014), which motivated initiatives to shift towards more sustainable transport solutions. Finally, the COVID-19 pandemic with its effect on consumer demands and companies' workforce, and the Suez canal blockage led to unprecedented low records for the reliability of transport, which unveiled the fragility of current transport systems (Berger 2022). These problems motivated policymakers, industry, and academia alike to search for solutions to improve the performance of transport from several different perspectives (Ballot et al. 2020, Tavasszy et al. 2010). This thesis addresses this challenge of improving transport performance by focusing on the utilization of transport capacity given immutable transport demand, technology, and infrastructure. A particular focus is on the plan of transport execution and its effect on the utilization of transport capacity. Aspects related to infrastructural expansions[1], new technologies[2], or demand management problems[3] are not considered here.

From the transport capacity utilization perspective, this thesis relates the above problems calling for improved performance of inland container transport to three main issues of the inland container transport sector: first, the heterogeneity and the large number of small companies offering transport services, which hinder system performance by partitioning the overall transport problem into smaller ones; second, the uncertainty of transport operations and limited operational visibility in the transport chain, which lower transport reliability and increase costs; third, the traditional sourcing of transport supply with high transaction costs, which leads to sub-optimal allocation of transport demand to suppliers given the limited number of alternatives that can be explored.

A natural solution to the first issue is cooperation between transport companies. Indeed, pooling transport capacity and demand hedges against the partitioning of the overall transport problem. Whether cooperation itself is a valid solution depends on other factors, which are the focus of Chapter 2 and are illustrated in Section 1.1 below.

The second issue, i.e., uncertainty of transport operations, is tackled nowadays by sourcing information and updating transport plans manually as delays or disruptions occur. The problem with such an approach is that it is hard to manage the reliability of transport before its execution and prepare alternatives beforehand as these are costly. While reliability has become a major concern for transport companies, the capability to make transport plans that are cost-efficient while guaranteeing a certain level of reliability is missing in practice. Similarly, the understanding of the benefits of such an alternative planning approach is limited in the literature as well. This problem is focal in Chapter 3 and it is explored further in Section 1.2 of this introduction.

To address the third issue, standardization of the sourcing of transport supply has been promoted by both digital freight forwarders and digital transport marketplaces. The digitization of the information exchange process allowed for a larger than usual pool of shippers to be exposed to a larger than usual pool of carriers. This turned into a larger set of opportunities to match demand with a suitable transport operator. Unfortunately, the ever-increasing pool sizes led to increasing frictions, not noticeable at first, in finding a good party on the other side of the market and led to stationing growth for digital platforms. A solution to this problem is the focus of Chapter 4 and is introduced in Section 1.3.

[1] such as the problem of where to locate terminals (Tran et al. 2017, Wang & Qi 2020).
[2] such as truck-platooning or autonomous trucks (Nasri et al. 2018, Bhoopalam et al. 2018).
[3] such as pricing of transport to nudge demand (Riessen 2018).

The last work is motivated by and was the result of a cooperation with a company operating a digital transport marketplace for bulk barge transport.

As a final remark, the three chapters addressing each issue can be read in any order without affecting the understanding. Indeed, the arguments and theory developed in each chapter are independent of those of the others.

1.1 Sensitivity analysis of inland transport cooperation

In transportation, the larger the capacity pool a company owns the higher the potential for efficient operations. Indeed, more opportunities for optimization can be found given demand, because of a larger set of alternatives and a larger operating area. As the inland transport market is made of a myriad of small-medium-sized transport companies, the efficiency of the overall transport operation is limited by this fragmentation of the supply side.

A natural way to address this limitation is to allow transport companies to pool transport demand and capacity to achieve economies of scope, by vertical cooperation, and scale, by horizontal cooperation. Vertical cooperation is achieved in practice by transferring containers from the transport means of a company to those of another in a relay fashion. Horizontal cooperation, instead, happens when companies share (part of) their transport capacity and demands and act as a single one having larger assets and demand (Cruijssen et al. 2007b).

Frisk et al. (2010) and Houghtalen et al. (2011) showed that the gains achieved by cooperation should suffice to motivate companies to engage in this endeavor. Despite the potential gains, Basso et al. (2019) showed that real-world cases of horizontal cooperation in ground transport are rare, and disagreements regarding the division of benefits may prevent the formation of cooperation. With the main problem being related to the fair share of the gains of cooperation. Though savings can be large, cooperation tends to be unstable as self-interested parties compete to get the most benefits. In particular, design issues in the early phase of the collaboration are critical for real cooperation, such as the case of transport operators in the region of the port of Rotterdam (Ypsilantis & Zuidwijk 2019), or forestry transportation in Sweden (Frisk et al. 2010).

A range of approaches to sharing the gains of cooperation have been proposed by applying the theory of cooperative games (Cf. Nash 1953) to transport problems (Theys et al. 2008, Cruijssen et al. 2007a). Instead of proposing a novel approach ourselves, a novel sensitivity analysis of the stability of cooperation is developed. Thus, an explanatory approach is followed with the aim of understanding under which conditions it is reasonable to expect that a cooperation is stable. This approach is based on the recent theoretical results at the interface between cooperative and non-cooperative game theory. In particular, the proof by Pérez-Castrillo & Wettstein (2001) shows that a well-known approach to sharing the gain of a collaboration, the Shapley value (Shapley 1953), is the result of the natural bargaining between companies involved in a cooperation.

With the methodology developed in Chapter 2, a first step is made in understanding whether a cooperation is stable or not, and to what extent, depending on the number of cooperating companies, the heterogeneity of their operations, and the type of cooperation (either vertical or horizontal). Moreover, a measure of rational stability of cooperation is introduced as a tool to explore instability and further the understanding of this problem. Overall, with this study, the hope is to progress the understanding of cooperation in inland transport and contribute to their success.

1.2 Booking a reliable flow in a stochastic network

The inland journey of a container is subject to several uncertainties. Being moved like the baton (the stick) in a relay race from one company to the next one, this journey is affected by delays due to congestion at terminals, on the road, or the railway, technical failures, and weather. All affect the timing of the operations required to either move or transfer it from one company to the other. In this situation where multiple and different stakeholders are responsible for such a journey, visibility on the actual status of the journey is limited as it needs to be sourced from multiple companies at the right time. As a result, during the inland journey of a container is rare to find a single party that is aware of all the steps (Konrad 2022). In the past, the negative effect of uncertainty and limited operational visibility were not of main concern as cost-efficiency considerations prevailed. Nowadays, the COVID-19 pandemic and Suez canal blockage prompt the world's attention to the unreliability of container transport systems, and made reliable transport operations a requirement for carriers (Hapag-Lloyd 2021, Vernimmen et al. 2007, Berger 2022).

With the need to improve reliability, uncertainties are currently being addressed by companies either, when delays arise, by manually adjusting the disrupted transport plan, or by introducing buffer times to hedge for possible future delays. Sector-wide stakeholders as policymakers or port authorities addressed, instead, the limited operational visibility by fostering digitization in the sector. Despite containers being monitored within the scope of each company's operations, this information is not shared outside of the same company with the overall visibility not improving. This leads to a complex situation that is hard to solve. Indeed, limited operational visibility reduces the value of planning ahead, as the quality of information affects the quality of planning, and reactive or static planning practices prevent parties from sharing operational information, as providing additional information would lead to either drastic changes or earlier reactions which might hurt the party sharing operational information.

A possibility for breaking this vicious cycle is to find an alternative planning approach that takes advantage of improved operational visibility while reducing costs and improving the reliability of transport. Among the many alternatives one might consider, the investigation of Chapter 2 aligns with initiatives such as Synchromodality, a joint Dutch industry-academia born concept,[4] and the Physical Internet, an international logistics-wide endeavor for a deep concept-shift in logistics,[5] which pointed at the value of adaptation in face of uncertainties as an opportunity to improve planning performance. If the academic literature is surveyed, models for adaptive planning received only limited attention in transport (Perera et al. 2017, Spivey & Powell 2004) and considered only one of the two dimensions of either time (and reliability) or cost, or both but in specific contexts. From a general point of view, the trade-off between capacity reservation and the possibility to adapt during the execution needs to be captured. Indeed, as capacity needs to be reserved prior to the execution, the possibility to adapt to uncertainties is limited by the capacity reservations made prior to the execution of transport. Moreover, if adapting requires preparing costly alternatives ahead of time, this means that adapting might cost more than not adapting, and the gains in terms of planning reliability might be outrun by increased costs.

Chapter 2 addresses this knowledge gap by proposing an abstract model for adaptive decisions which captures the main decision trade-off between costs and reliability. This model builds upon the stochastic shortest path problem (Opasanon & Miller-Hooks 2006, Wang et al. 2016, Miller-Hooks & Mahmassani 2003, 2000, Chen et al. 2016) and recent developments in the public transport literature on adaptive passenger routing to the case of freight transport (Rambha et al. 2016, Keyhani et al. 2017). The model proposed takes the perspective of a planner that has visibility of a transport network and can, upon payment, reserve capacity on scheduled services to organize transport for multiple containers.

This focal problem is addressed by developing a mathematical model which includes both static and adaptive decisions. Using Graph Theory -to represent transport networks- and Markov Decision Processes -to represent adaptive decisions-, it is shown that a single optimization model can be built to capture the two types of decisions at once (Ahuja et al. 1993, Puterman 2005). The model is then tested through simulations on a large set of realistic instances. Crafting realistic transport networks is a difficult task as many parameters need to be tuned carefully. By sourcing empirical data on freight

[4] Synchromodality (Tavasszy et al. 2017, Giusti et al. 2019)
[5] The Physical Internet (Sternberg & Norrman 2017, Montreuil et al. 2012, Montreuil 2011).

transport networks from the literature on Complex Network Analysis (Lin & Ban 2013, Boccaletti et al. 2006), a generator of random instances which follows the realistic parameter distributions is built to obtain simulation results of realistic value. Where the empirical literature does not support the experimental design, a space-filling design[6] approach is followed to hedge against manual choices and improve the quality of the experiment design (Joseph 2016, Santner et al. 2018). Finally, the source code of the model and the instances generated for the testing phase are made available publicly in the hope to further the understanding of adaptive planning.

1.3 Online learning for two-sided sequential matching markets

Traditionally, transport capacity is sourced by shippers via phone calls or emails to suppliers (either carrier or freight forwarder) which often implies a limited number of parties can be contacted. Similarly, suppliers have little overview of the larger transport market they operate in. As a result, matching in the current transport market is a time-consuming and, to some extent, inefficient process based on traditional business relations (Brancaccio et al. 2021). This situation hinders the efficient utilization of capacity as supply and demand of transport match with a limited overview of the range of possibilities.

Recently, digitization improved matching in the transport market by standardizing communication processes. Digital transport marketplaces made a step further in this direction by standardizing the whole transactions on the market end-to-end. On those digital platforms, the two sides of the market are exposed to a larger-than-traditional set of alternatives and different dynamics (Miller et al. 2020). Similar to dating platforms, such as Tinder or Bumble, digital platforms in transportation grew with their customer base being positively affected by a larger network of alternatives (Ríos et al. 2020). As the user base increased so did the number of alternatives and searching for the right partner became more difficult. In many cases, this resulted in plateauing growth or changes in the company's strategy (Cullen & Farronato 2021).

Li & Netessine (2020) shows that the design of the marketplace plays a fundamental role in supporting matching. Indeed, the information and the set of alternatives presented impact users' decisions and the overall matching rate. The question becomes then how to choose what to show to which user to improve the overall marketplace performance. Answering this question requires knowing what are the preferences of the users in terms of transported material and operating area, for instance. To extract this knowledge from data, one needs to realize that both the availability of observations on a given

[6] A *space-filling* design approach is one where the parameters used in a simulation are generated in such a way to cover well the region of interest in the space of parameters (cf. ibid.)

user and the set of alternatives evaluated by that same user impacts the estimated preference. This problem of *learning* has been formalized in the so-called *multi-armed bandit* framework (Lattimore & Szepesvári 2020) which models the sequential decisions of an agent that needs to choose between a set of alternatives of unknown reward. After making a choice, the agent collects an observation on the outcome of that choice which can be used to drive future decisions. A key contribution of this framework is the realization that, at each decision, there is a trade-off between exploring new alternatives for which no or little observations have been collected and exploiting, i.e., opting for, those alternatives for which abundant information has been collected. If this concept is translated to the marketplace problem, it is understood that the set of alternatives each user is presented affects what one can learn about the preference of that user. Thus, the owner of the marketplace has the opportunity to leverage the alternatives they present to users to learn user preferences and cater interesting alternatives to improve the overall matching rate.

Chapter 4 focuses on this problem of improving the matching rate on a marketplace for inland waterway transport by devising a model for learning while catering assortments of alternatives to the two sides of the market. The main challenge, in this case, is to understand to what extent can such a novel approach improve the matching rate. By building upon the two-sided assortment model of Ashlagi et al. (2019b), and the application of multi-armed bandits to the one-sided assortment of Agrawal et al. (2019), a model of incremental assortment decisions is developed. A parameterized class of heuristics is proposed as a solution approach that addresses the realistic case of the company involved in this research. The algorithm developed is based on the principle of *optimism in face of uncertainty*, which suggests favoring unexplored over explored alternatives (Lattimore & Szepesvári 2020). In a realistic simulation of a heterogeneous marketplace, three different policies with different degrees of optimism are evaluated against the theoretically best decision. Such an evaluation approach estimates the value of this model against the best possible outcome and provides insight into whether this approach can be valuable in practice.

1.4 Overview of research objectives and methodology

As a final overview of this thesis, the research objectives and methodologies developed in the remainder of this work are summarized. The following research objectives are addressed:

1. To understand under which conditions cooperation between transport operators is stable depending on topology and saturation of inland container transport (cf. Chapter 2).

2. To understand whether adaptive planning can reduce transport costs and improve transport reliability by devising a model for transport planning that handles fixed capacity booking decisions and adaptive routing decisions (cf. Chapter 3).

3. To understand whether the matching rate in growing digital transport marketplaces can be improved by limiting what either side of the market can evaluate while learning carrier and shipper preferences (cf. Chapter 4).

Each objective has been considered using a specific methodology and led to a specific methodological contribution:

1. The first objective is investigated using a blend of the theory of cooperative games and the theory of parametric optimization. The first provides a framework for investigating the stability of cooperation while the second allows the development of a novel approach for sensitivity analysis of the stability of the cooperation itself.

2. The second objective is studied from the lens of dynamic optimization as it provides a framework for modeling adaptive and sequential decisions. By building upon models for adaptive routing on stochastic networks, a novel model capturing a-priori capacity booking decisions and adaptive routing for a flow of containers is devised. This model is applied to hundreds of realistic instances generated from empirical observations on the structure of real freight transport networks.

3. The third objective is addressed by combining models for two-sided matching markets and assortment optimization with multi-armed bandit models. While the first provides a theoretical framework to describe the interactions between users on two sides of the market, assortment optimization models focus on the problem of the company operating the marketplace, and the multi-armed bandit framework captures the process of learning users' preferences and balancing between exploitation of current information and exploration of alternative actions. The combination of those frameworks leads to a novel model of the operations of a digital marketplace.

The last chapter of the thesis, Chapter 5, concludes this thesis and provides an outlook on future research.

Research Statement

This Ph.D. thesis has been written during the author's work at the Erasmus University Rotterdam. The author is responsible for formulating the research questions, building the analytical models, analyzing the results, and writing all the chapters of this thesis. While carrying out the research, the author received valuable and constructive feedback from the doctoral advisors and other doctoral committee members, which subsequently increased the quality of the research.

2 An analysis of the stability of hinterland container transport cooperation

2.1 Introduction

Cooperation in transportation has the potential to reduce total costs, at the risk of exposing companies to the failure of the cooperation itself. Cost reductions have a direct positive impact on profits. Failure, in contrast, threatens the companies' market position and generates additional costs and losses (Park & Ungson 2003). Therefore, prior to engaging in a cooperation, managers need to be able to evaluate the conditions under which the cooperation will endure. While achieving cost savings motivates the formation of a cooperation, the division of these savings among participating companies may lead to failure. As highlighted by Basso et al. (2019), real-world cases of horizontal cooperation in ground transport are rare and disagreements regarding the division of benefits may prevent the formation of a cooperation. Design issues in the early phase of the collaboration are critical for real cooperations, such as the case of transport operators in the region of the port of Rotterdam (Ypsilantis & Zuidwijk 2019), or forestry transportation in Sweden (Frisk et al. 2010). Thus far, only limited guidelines exist to show how a transport setting affects the stability of a collaboration. This leads to the following question: Given the size of the cooperation as well as the transport network, costs, capacities, and orders, is it possible to predict whether a cooperation will be stable or not?

We focus on hinterland container transport, although we believe that our results are applicable to other transport domains as well.

The hinterland of a sea-port is the inland region of locations that can be served by transportation services from (import) or to (export) the sea terminals. (Notteboom & Rodrigue 2007). On its way from the terminals at the port to a warehouse (or vice versa), a loaded container is moved through a sequence of transshipment and transport operations that might involve road, rail, and inland waterway transport. Terminal operators – both at the port and inland – and transport operators as well as other stakeholders are involved in the organization of container transport in the hinterland.

In this industry, very low margins and a strong pressure on the performance of transport chains drive companies to cooperate. Hinterland costs account for 40% to 80% of the overall door-to-door container transport costs (Notteboom 2004). Moreover, hinterland accessibility has become a mayor success factor for ports (Langen 2004). Shippers, the cargo owners, increasingly require reliability of their hinterland transport chains (Port of Rotterdam 2018). Point-to-point connections between port and inland terminals, called corridors, are seen as an opportunity to alleviate the downsides of visiting congested port areas, and arise naturally in the development of ports (Notteboom & Rodrigue 2005). To make hinterland regions accessible, transport corridors between ports and inland terminals have to be formed by cooperating transport operators (Wilmsmeier et al. 2011). A successful example in the Netherlands is that of Brabant Intermodal, where barge operators cooperated to consolidate their visits to port terminals (Veenstra et al. 2012).

We study transport cooperation in which orders and transport capacity are shared. Companies strive to minimize the costs of their joint transport plans. Cost savings will then be divided following an agreed-upon mechanism, where the individual interests of each company ideally are advanced. Unfortunately, even if an agreement is reached, the cooperation is still exposed to the risk of failure. Indeed, self-interested negotiations do not take into account the interests of groups of companies, which might have incentives, as a subcoalition, to drop out (Park & Ungson 2003). The economic literature has established that the Shapley value (Shapley 1953) provides a reasonable prediction for the cost allocation when players engage in a noncooperative bargaining process (Gul 1989, Pérez-Castrillo & Wettstein 2001). For this reason, we focus on the Shapley value as the cost allocation concept in this paper. Thus, the question is under what conditions the Shapley value helps establish a stable coalition.

2.1.1 Illustrating example

As an illustrating example, consider three operators P_1, P_2, and P_3 that offer transport between a port and its hinterland. Their networks partially overlap, which allows the formation of a corridor that serves demand from the port s to a common inland terminal t as shown in Fig. 2.1. The unit transport cost faced by each operator for transport from origin to destination are given in Table 2.1. We assume that each operator chooses to share 15 units of orders and 30 units of capacity out of their total orders and capacity. The orders correspond to demand for transport from the port s to the terminal t. Capacity is mapped to each operator's arc in the graph G_T in Fig. 2.1. Either individually or as a cooperation, all orders are transported at minimum cost from node s to node t in G_T. Individually, operator P_i would face a cost equal to $15c_i$ by using only her arc, which leads to a minimum total cost of 1125 without

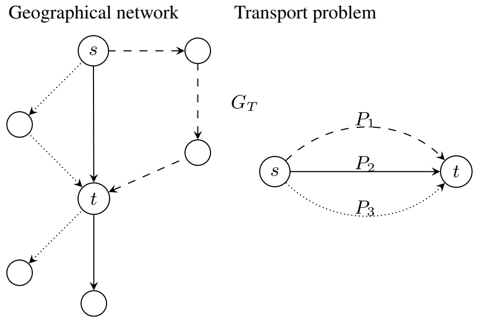


Figure 2.1: Example of a collaborative transport network, where s is the port and t an inland location up to which the transport networks of the operators overlap.

	P_1	P_2	P_3
c_i	30	25	20
Φ_i	387.5	350	237.5

Table 2.1: Parameter setting for the example. Per unit cost c_i and Shapley value cost allocation Φ_i.

cooperation. When the three operators form a cooperation, the pooled orders are transported using pooled capacities and a minimum total cost of 975 can be achieved, which corresponds to about 10% savings. We can test whether the operators are able to form a stable cooperation if the cost allocation is based on a bargaining outcome. Assuming that the bargaining process follows Gul (1989), the Shapley value cost allocation Φ_i to each operator is reported in Table 2.1. The value Φ_i represents the share of the total cost of 975 that operator P_i would agree to pay after the bargaining phase. Despite reaching such an agreement, it can be observed that operators P_1 and P_3 have an incentive to drop out of the cooperation: without operator P_2, they would face a cost of 600 for transporting their orders using only their arcs P_1 and P_3, which is lower than their allocated cost share of $\Phi_1 + \Phi_3 = 625$ in the three-way collaboration. Therefore, the cooperation would fail. Interestingly, if only a few orders more (18 rather than 15 units) were shared by each operator, or less capacity were pooled (25 rather than 30 units), no breakaway subcoalition would form and the cooperation would not suffer from failure (cf. Appendix 2.8.1). Had the operators been guided by these insights ahead of bargaining, a more durable cooperation could have been designed.

2.1.2 Our contribution

In the literature, it has been observed that sharing demand and capacity in a collaborative transport network leads to cost reductions, but may or may not result in a stable cooperation (cf. Frisk et al. 2010, Houghtalen et al. 2011). Most existing findings, however, are based on purely numerical observations.

There is a lack of theoretical results and mathematical characterization for the structure of this problem. In this paper, we attempt to fill this gap.

To this end, we propose a novel methodology that combines cooperative game theory with parametric optimization to characterize the stability of cooperation in transport networks. Specifically, we first provide a mathematical characterization of the stability of the Shapley value as a function of a cost parameter, and then derive a closed-form characterization of whether the bargaining outcome (i.e., the Shapley value allocation) is stable for a special case with identical players. The obtained closed-form solutions are then extended in order to formally analyze the same setting on a richer network structure. For general cases, we develop a parametric optimization-based algorithm to efficiently evaluate the stability of cooperation. Furthermore, in the event that the Shapley value allocation is not stable, we introduce a measure of instability of cooperation, which is inspired by the concept of the ε-Core in Shapley & Shubik (1966) and is similar to ε-stability in Karsten et al. (2015). We measure instability by computing the maximum gap between the Shapley value allocation to a coalition and the cost generated by this coalition: the larger the gap, the more willing that coalition is to drop out of the cooperation. Under mild conditions, we further prove that this measure is bounded whenever no sub-coalition achieves higher cost savings than does the group of companies taken as a whole. To the best of our knowledge, we are the first to perform a sensitivity analysis of cooperative game solution concepts in transport networks that permits analytical results in conjunction with a general computational approach that exploits the structure of the game. Our results provide a systematic understanding about how demand, capacity, and operating costs impact the stability of cooperation.

The remainder of this paper is organized as follows: Section 2.2 reviews the literature on collaborative transport, and Section 2.3 provides an overview of basic concepts of cooperative game theory used in the paper. The mathematical model is formulated in Section 2.4. Our analytical results are presented in Section 2.5, and the numerical study is reported in Section 2.6. Section 2.7 concludes the paper.

2.2 Literature review

Our work contributes to the literature on collaborative transport, which has been gaining traction over the past years. In this section, we review the studies most relevant to our work. For a broader overview, we refer interested readers to the comprehensive survey in Guajardo & Rönnqvist (2016) and to an overview of practical challenges in collaborative transport in Basso et al. (2019). The main problem considered in this stream of research is to find a suitable mechanism (or solution concept) to allocate the total costs or profits generated between collaborating companies. In this paper, we take a different angle by asking whether or not a cooperation would be stable if the cost shares were determined by a reasonable bargaining outcome. As mentioned earlier, we adopt the Shapley value as the bargaining outcome as per the theoretical foundation by Gul (1989).

Collaborative operational planning problems have been considered by Houghtalen et al. (2011) and Frisk et al. (2010), who base their models on network flow problems (Ahuja et al. 1993). Houghtalen et al. (2011) develop a capacity exchange pricing mechanism to drive self-interested behavior in a cooperative setting towards social optimum, and they observe numerically that overcapacity undermines the stability of cooperation. Frisk et al. (2010) perform a real-world case study on cooperative truck transport in the forestry industry in Sweden. The Shapley value is found to be difficult to accept by practitioners, despite being able to capture synergies between companies more effectively than other solution concepts. Along this line, Cruijssen et al. (2007a) explain that simple rules to divide the gains in horizontal cooperation cannot precisely capture the contribution of each individual player. The network flow problem considered in our paper is similar to those studied in Houghtalen et al. (2011) and Frisk et al. (2010); nevertheless, we complement their work by systematically analyzing the impact of overcapacity on the stability of cooperation given that the cost allocation is determined by a bargaining outcome. Among other results, we theoretically validate the numerical finding of Houghtalen et al. (2011) under certain conditions.

Other papers have considered tactical decision problems in cooperative settings, focusing on a very different scale than in our work. Lozano et al. (2013) study horizontal cooperation between shippers that jointly determine weekly transit frequencies of transport connections to satisfy pooled demand. Agarwal & Ergun (2010) and Zheng et al. (2015) address a network design problem in a partially decentralized setting related to maritime container transport, where service frequency and cargo flow have to be defined.

In broader contexts, the Shapley value has been applied by Engevall et al. (2004) and Özener et al. (2013) to estimate cost-to-serve customers in oil distribution networks and vendor-managed inventory settings, respectively. In Cruijssen et al. (2010), the Shapley value is used to decide the order in which a logistic service provider should approach new customers to provide increasing benefits to already confirmed ones.

Methodologically, our work is related to literature on parametric optimization. In particular, our algorithm is based on work from Eisner & Severance (1976), which solves a parametric linear problem without requiring an implementation of the simplex method for linear programs. The parametric setting of our model is further related to the work of Carstensen (1983), which constructively show that exponentially many points might be required to describe the solution of a linear parametric network flow problem. We contribute to this stream of literature by identifying an application of parametric optimization in cooperative game theory.

2.3 Basic concepts

In this section, we recall basic concepts of cooperative game theory that are used throughout the paper.

In contrast to its non-cooperative counterpart, cooperative game theory studies the bargaining process in a cooperative setting under complete information where contracts might be enforced (Nash 1953). Cooperative games can be represented as transferable utility games (TU games) in characteristic form. Given a set $N = \{1, 2, \ldots, n\}$ of companies (or *players*), the cost structure of the cooperation is described by a cost vector $c = (c_S)_{S \subseteq N} \in \mathbb{R}^{2^n}$, where the component c_S represents the cost generated by coalition $S \subseteq N$ (Serrano 2004). Games are characterized in terms of their properties. A game is called *subadditive* if $c_S + c_T \geq c_{S \cup T}$ for all $S, T \subseteq N$ with $S \cap T = \emptyset$, and *convex* if $c_{S \cup T} + c_{S \cap T} \leq c_S + c_T$ for all $S, T \subseteq N$. Subadditivity implies that joining forces does not increase costs. Convexity, on the other hand, represents an advantageous situation where the incentive to join the cooperation grows with the cooperation size (Shapley 1971).

An *allocation vector* $(x_i)_{i \in N} \in \mathbb{R}^n$ describes the cost x_i allocated to each player $i \in N$, corresponding to the agreed outcome of the bargaining.

A *solution concept* Ψ represents a cost sharing mechanism and maps TU games to allocation vectors, i.e., $\Psi : TU^n \to \mathbb{R}^n$ with $\Psi(c) = (x_i)_{i \in N}$, where TU^n is the set of n-person TU games and $c = (c_S)_{S \subseteq N} \in TU^n$. Certain properties are sought: *efficiency* requires the total generated cost c_N to be completely allocated to the players ($\sum_{i \in N} x_i = c_N$), while *individual rationality* mandates that individual players do no worse under cooperation ($x_i \leq c_i$ for each $i \in N$). We focus on the Shapley value $\Phi \in \mathbb{R}^n$ (Shapley 1953), with the component Φ_i for a player $i \in N$ defined as follows:

$$\Phi_i = \frac{1}{n} \sum_{S \subseteq N \setminus \{i\}} \binom{n-1}{|S|}^{-1} \left(c_{S \cup \{i\}} - c_S \right). \tag{2.1}$$

The allocation Φ_i is a weighted average of the marginal cost of player i to any coalitions she can join. Results from the so-called Nash-program proved that the Shapley value is the outcome of a non-cooperative game that models bargaining (Serrano 2004). Thus, (2.1) provides an explicit form for the bargained cost division. Pérez-Castrillo & Wettstein (2001) show that the Shapley value coincides with the perfect subgame equilibrium outcomes of a non-cooperative game. Engevall et al. (2004) show that, whenever the game needs to be constructed, the computation time for the Shapley value is negligible.

The concept of the *core* describes the set of allocation vectors that do not give any subcoalition an incentive to leave the cooperation (Gillies 1959). Formally, the core is the set $\mathfrak{C} \subseteq \mathbb{R}^n$ defined as

$$\mathfrak{C} := \{(x_i)_{i \in N} \in \mathbb{R}^n : \sum_{i \in N} x_i = c_N, \sum_{i \in S} x_i \leq c_S \; \forall S \subseteq N\}. \tag{2.2}$$

The inequalities in (2.2) capture the so-called *coalitional rationality* of an allocation vector. It requires a given allocation $(x_i)_{i \in N}$ not to assign a total cost $\sum_{i \in S} x_i$ to a coalition S that is higher than the cost c_S that coalition would generate by itself. If such a case occurs, the players in S would form a subcoalition and abandon the larger game. Shapley (1971) shows that, if a game is convex, the Shapley value is within the core.

2.4 Model definition

The current approach in the literature to test the properties of a collaboration is to sample the parameter space of a transport setting, generate one game for each sample, and then test the desired properties. We instead propose an evaluation of the stability of the Shapley value that uses parametric solutions of the problem and results in a parametric sensitivity analysis.

We start by defining the transport problem and the resulting cooperative game in a classical sense, later expanding them to the parametric case.

Let $N = \{1, \dots, n\}$ be the set of companies that jointly operate the transport network given by the directed graph $G = (V, R)$, with node set V and arc set R. The set of potentially shared services need not constitute the whole network operated by each company. All transport demand originates at the source node $s \in V$, for example a sea port, and must be transported to a single inland destination node $t \in V$, where $s \neq t$ (uniqueness of the destination is without loss of generality, see Appendix 2.8.3 [Online]). The single-source problem describes a scenario where demand originates either from a single terminal or from multiple, tightly-grouped terminals. It is assumed that all transport demand can be transported on time with pooled services, which allows us to exclude the time element from explicit consideration and represents a realistic assumption when considering time-insensitive cargo. For each company $i \in N$, let $R^i \subseteq R$ and $k^i \in \mathbb{N}_{\geq 0}$ be the set of services owned and the amount of demand shared by i, respectively. We assume that no arc is owned by multiple companies, i.e., $R^i \cap R^j = \varnothing$

for $i \neq j$, and that all arcs are assigned to companies, i.e., $\cup_{i \in N} R^i = R$. Transport of containers in the hinterland of a port is done either by truck, train, or barge, which are distinguished here in terms of capacity and unit transport cost. For each service $r \in R$, let $u_r \in \mathbb{N}_{\geq 0}$ be its shared capacity and $c_r \in \mathbb{N}_{\geq 0}$ the unit transport cost on service r. We assume that companies share only part of their total vehicle or barge capacity per service. Capacities of transport means vary from a few TEUs (Twenty-feet Equivalent Units, a standard in container size) for trucks to hundreds for barges and trains, which justifies discreteness of flow[1].

Given a group of operators $S \subseteq N$, let $R^S := \cup_{i \in S} R^i$ and $k^S := \sum_{r \in S} k^i$ be the set of services controlled and the amount of demand pooled by S, respectively. We concentrate only on total transport costs, requiring each coalition S to find a feasible flow allocation $(f_r)_{r \in R^S}$ transporting k^S units of flow from s to t on the graph $G^S := (V, R^S)$ such that $f_r \leq u_r$ for all $r \in R^S$, that minimizes the total transport cost $\sum_{r \in R^S} c_r f_r$. If we denote by $\delta^+(v)$ and $\delta^-(v)$ the set of outgoing and incoming arcs of node $v \in V$, respectively, we can define the following integer programming formulation P^S for this problem:

$$
\begin{aligned}
c_S := \min \quad & \sum_{r \in R^S} c_r f_r & & \text{(2.3a)} \\
\text{s.t.} \quad & f_r \leq u_r & \forall r \in R^S \quad & \text{(2.3b)} \\
& \sum_{r \in \delta^-(v)} f_r - \sum_{r \in \delta^+(v)} f_r = 0 & \forall v \in V \setminus \{s, t\} \quad & \text{(2.3c)} \\
& \sum_{r \in \delta^+(s)} f_r = k^S & & \text{(2.3d)} \\
& \sum_{r \in \delta^-(t)} f_r = k^S & & \text{(2.3e)} \\
& f_r \in \mathbb{N}_{\geq 0} & \forall r \in R^S \quad & \text{(2.3f)}
\end{aligned}
$$

Here, (2.3b) ensures that transportation orders per service do not exceed the available capacity, (2.3c) ensures that incoming and outgoing flow at each node $v \neq s, t$ are equal, (2.3d) and (2.3e) require that flow demand at source and sink nodes is met, and (2.3f) forces integrality of the flow $(f_r)_{r \in R^S}$. It is well-known that constraint (2.3f) can be relaxed whenever capacities and demands are integer. An integer optimal solution can then be found by solving the LP relaxation of P^S whenever a feasible solution exists (cf. Ahuja et al. (1993)). Assuming that problem P^S is feasible for each $S \subseteq N$, the cooperative game $c = (c_S)_{S \subseteq N}$ is obtained by solving the problems $\{P^S\}_{S \subseteq N}$.

[1]By assuming that capacity and demand are integral, we might consider continuous flows. This is in contrast with Agarwal & Ergun (2010), who assume continuity of flow variables due to homogeneity and bigger transport capacity.

In order to obtain insight into the dependence of the stability of the cooperation on costs, we perturb one of the players' arc costs by an additive parametric term $\lambda \in \Lambda \subseteq \mathbb{R}_+$. Let $i_p \in N$ be the company owning service $r_\lambda \in R^{i_p}$ having parametric unit transport cost $c_{r_\lambda}(\lambda) = c_{r_\lambda} + \lambda$, where c_{r_λ} is the original unit transport cost on arc r_λ. We assume that Λ is such that $c_{r_\lambda}(\lambda) \geq 0$ for all $\lambda \in \Lambda$. The choice of company $i_p \in N$ is arbitrary. The introduction of the parameter λ requires an update to the objective function (2.3a) in problem P^S, which now takes the form $\sum_{r \in R^S} c_r f_r + \lambda f_{r_\lambda}$. Clearly, this change affects only the cost c_S of the coalitions $S \subseteq N$ for which $i_p \in S$ and leads to the parametric version $P^S(\lambda)$ of problem P^S.

Thus, rather than the optimal value c_S, the *cost curve* $c_S(\lambda)$ will be computed as a function of λ. Given $\lambda \in \Lambda$, the cost $c_S(\lambda)$ is the optimal objective value of $P^S(\lambda)$. In our case, the cost curve is a piecewise linear, non-decreasing, concave function (Gal 1994). A parameter value λ at which the slope of the cost curve $c_S(\lambda)$ changes is called a *breakpoint*. We denote the set of breakpoints of $c_S(\lambda)$ by $\mathfrak{B}_S$. The number of breakpoints is a natural measure of problem complexity, as the set of optimal solutions changes exactly at the breakpoints. As shown by Carstensen (1983), the number of breakpoints can be exponential in the instance size. Our parametrization also changes the definition of the cooperative game. Indeed, unlike a classical cooperative game in characteristic function form, we let the cost functions $c_S(\lambda)$ be the cost curves for the parametric problems $P^S(\lambda)$. We denote the cost functions by $c(\lambda) := (c_S(\lambda))_{S \subseteq N}$ and the parametric minimum cost flow cooperative game by $(N, c(\lambda))$. However, in order to simplify notation, we usually identify the game $(N, c(\lambda))$ with $c(\lambda)$ when the set of players is clear from context.

Solution concepts themselves are now parametrized. For company $i \in N$, the Shapley value allocation Φ_i changes from (2.1) to the following expression:

$$\Phi_i(\lambda) = \frac{1}{n} \sum_{S \subseteq N \setminus \{i\}} \binom{n-1}{|S|}^{-1} \left(c_{S \cup \{i\}}(\lambda) - c_S(\lambda) \right). \tag{2.4}$$

The core $\mathfrak{C}(\lambda) \subseteq \mathbb{R}^n$ of the game $c(\lambda)$ is now defined as

$$\mathfrak{C}(\lambda) = \{x \in \mathbb{R}^n : \sum_{i \in N} x_i = c_N(\lambda), \sum_{i \in S} x_i \leq c_S(\lambda) \quad \forall S \subseteq N\}. \tag{2.5}$$

Overall, we set up a parametric description of the transport problem and apply this to the overlaying cooperative game and solution concepts. Given an interval of interest $\Lambda = [\underline{\lambda}, \overline{\lambda}]$ with $\underline{\lambda} < \overline{\lambda}$, our next step is to tackle the problem of describing the set of values $\lambda \in \Lambda$ for which $\Phi(\lambda) \in \mathfrak{C}(\lambda)$. It is clear that this will depend on the problem instance, so capacity, demands, and costs will appear in the description of this set of values, which will describe the transport settings leading to stability of the cooperation.

Two basic properties need to be tested: subadditivity of the game and non-emptiness of the core.

Proposition 1. *The parametric minimum cost flow game $c(\lambda)$ is subadditive and has a non-empty core for all $\lambda \in \Lambda$.*

Proof. The proof is given in Appendix 2.8.4 [Online]. □

Subadditivity implies that formation of the grand-coalition is optimal for the cooperation and that the Shapley value is individually rational, while non-emptiness of the core implies that testing the membership $\Phi(\lambda) \in \mathfrak{C}(\lambda)$ is a non-trivial problem for all $\lambda \in \Lambda$. Moreover, the Shapley value is not guaranteed to belong to the core in general as shown by the example provided in Section 2.1.

2.5 Results

In this section, we lay out the mathematical properties supporting our proposed sensitivity analysis (Section 2.5.1). These properties are exploited to characterize stability of cooperation in a stylized corridor setting in Section 2.5.2, which is extended to a more involved network structure in Section 2.6.3. We define the ε-distance to measure instability of cooperation in Section 2.5.3.

2.5.1 Sensitivity analysis

For any given value $\lambda \in \Lambda$, we have $\Phi(\lambda) \in \mathfrak{C}(\lambda)$ if and only if

$$\sum_{i \in N} \Phi_i(\lambda) = c_N(\lambda) \tag{2.6}$$

and

$$\Phi_S(\lambda) \leq c_S(\lambda) \qquad \forall S \subseteq N, \tag{2.7}$$

where $\Phi_S(\lambda) := \sum_{i \in S} \Phi_i(\lambda)$ is the total marginal cost of the players in coalition S. While (2.6) is always satisfied as the Shapley value is an efficient solution concept (Shapley 1953), (2.7) is not guaranteed to hold. For a given coalition S, both sides of the inequality in (2.7) are piecewise linear functions of λ. Indeed, the Shapley value is obtained as a linear combination of piecewise linear functions. Let $\mathfrak{B}$ be the set of all breakpoints of the cost functions, i.e., $\mathfrak{B} := \cup_{S \subseteq N} \mathfrak{B}_S$. Each set $\mathfrak{B}_S$ is finite since $|\mathfrak{B}_S| \le u_{r_\lambda}$ for all $S \subseteq N$, and non-empty as we add the points $\underline{\lambda}$ and $\overline{\lambda}$. We further assume that the breakpoints in $\mathfrak{B}$ are sorted in increasing order, so $\mathfrak{B} = \{\lambda_0 = \underline{\lambda}, \lambda_1, \lambda_2, \ldots, \lambda_l, \lambda_{l+1} = \overline{\lambda}\}$ with $\lambda_i < \lambda_{i+1}$, $i = 1, \ldots, l$. It follows that, for $i \in \{0, \ldots, l\}$ and $\lambda \in [\lambda_i, \lambda_{i+1}]$, the functions on both sides of (2.7) are linear. Linearity implies that for each $S \subseteq N$, the inequality $\sum_{i \in S} \Phi_i(\lambda) \le c_S(\lambda)$ is either valid for all $\lambda \in [\lambda_i, \lambda_{i+1}]$, there exists no $\lambda \in [\lambda_i, \lambda_{i+1}]$ for which it holds, or there exists $\lambda_i^S \in [\lambda_i, \lambda_{i+1}]$ for which the inequality is valid on exactly one of the subintervals $[\lambda_i, \lambda_i^S]$ and $[\lambda_i^S, \lambda_{i+1}]$ (see Fig. 2.2).

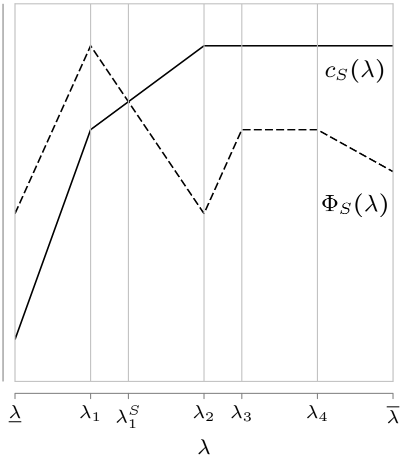


Figure 2.2: Evaluating the inequality $\Phi_S(\lambda) \le c_S(\lambda)$ to analyze the stability of the Shapley value .

It follows that the Shapley value transitions from inside the core to outside, or vice-versa, at most once between breakpoints, and that we can find those points by checking intersections between lines. Furthermore, the observations above result in the following point-wise stability tests of the cooperation: instability at one sample value extends to a neighborhood that can be computed explicitly.

Proposition 2. *If there exists $\hat{\lambda} \in \Lambda$ and $\hat{S} \subset N$ such that $\Phi_{\hat{S}}(\hat{\lambda}) > c_{\hat{S}}(\hat{\lambda})$, then*

$$\forall \varepsilon \in \left(0, \frac{\Phi_{\hat{S}}(\hat{\lambda}) - c_{\hat{S}}(\hat{\lambda})}{2K}\right): \quad \Phi(\lambda) \notin \mathfrak{C}(\lambda) \quad \forall \lambda \in (\hat{\lambda} - \varepsilon, \hat{\lambda} + \varepsilon), \tag{2.8}$$

where $K \ge 0$ is a Lipschitz constant of the functions $\{c_S(\lambda)\}_{S \subseteq N}$ and $\{\Phi_S(\lambda)\}_{S \subseteq N}$, i.e, $\forall \lambda', \lambda'' \in \Lambda$, $|c_S(\lambda') - c_S(\lambda'')| \le K \cdot |\lambda' - \lambda''|$ and $|\Phi_S(\lambda') - \Phi_S(\lambda'')| \le K \cdot |\lambda' - \lambda''|$ for all $S \subseteq N$.[2]

Proof. The proof is given in Appendix 2.8.5.

[2] In this case, the constant K can be chosen as the highest slope of all functions.

□

2.5.2 Corridor with identical players

We start by analyzing stability of the collaboration in the case of cooperation on a corridor with identical companies. The closed-form solutions we obtain here align with insights for complex problems in the literature (Agarwal & Ergun 2010, Houghtalen et al. 2011).

In a corridor, each company $i \in \{1, \dots, n\}$ offers a transport connection r_i between the port s to a single inland terminal t (see Fig. 2.3). For company i, we denote the amount of pooled demand by k^i. For service r_i, we denote its capacity by u_i and its unit cost by c_i. The amount of demand k^i, the capacity u_i, and the per unit cost c_i of arc r_i are assumed to be independent of i, i.e., $k^i = k$, $u_i = u$, and $c_i = c_0$ for all $i \in \{1, \dots, n\} \setminus \{i_p\}$. Company i_p, however, is assigned a parametric arc cost $c_{i_p} = c_0 + \lambda$.

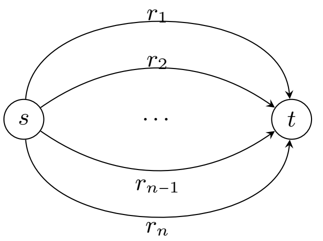


Figure 2.3: Transport network for corridor cooperation.

To characterize stability of the cooperation, we need to solve the set of inequalities (2.7) for the problem just defined. We provide a constructive proof where the cost game and the Shapley value $\Phi(\lambda)$ are explicitly computed and the inequalities $\Phi_S(\lambda) \leq c_S(\lambda)$ are solved for all $S \subseteq N$.We note that the cases $\frac{k}{u} = 0$ or $\frac{k}{u} = 1$ are trivial, meaning that the Shapley value is in the core for those cases.

Theorem 1. *In the case of cooperation on a corridor with identical players, whether the Shapley value is in the core or not depends on the value of the ratio $\frac{k}{u}$ compared to the size $n = |N| \geq 2$ of the grand* *coalition as follows:*

$$\Phi(0) \in \mathfrak{C}(0) \qquad \textit{for all} \qquad 0 \le \frac{k}{u} \le 1, \tag{2.9a}$$

$$\Phi(\lambda) \in \mathfrak{C}(\lambda) \quad \forall \lambda > 0 \quad \textit{if and only if} \quad \frac{(2n-1)(n-2)}{2n^2-3n} \le \frac{k}{u} \le 1 \tag{2.9b}$$

$$\Phi(\lambda) \in \mathfrak{C}(\lambda) \quad \forall = -c_0 \le \lambda < 0 \quad \textit{if and only if} \quad 0 \le \frac{k}{u} \le \frac{2n-2}{2n^2-3n}. \tag{2.9c}$$

Proof. Proof. We provide an intuitive explanation of the proof, which is given in Appendix 2.8.6. Here, we focus on the case $\lambda > 0$, the other cases are treated similarly.

The proof is based on a decomposition of the minimum cost flow game $c(\lambda)$ as a linear combination of two simpler, non-parametric games c^0 and c^+, and on linearity of the Shapley value on the vector space of N-person games (Shapley 1953). This decomposition means that $c_S(\lambda) = c_S^0 + \lambda c_S^+$ for all $S \subseteq N$, so linearity of the Shapley value implies that $\Phi(c(\lambda)) = \Phi(c^0 + \lambda c^+) = \Phi(c^0) + \lambda \Phi(c^+)$. This implies that each inequality $\sum_{i \in S} \Phi_i(\lambda) \le c_S(\lambda)$ for $S \subseteq N$ can be rewritten as follows:

$$\begin{aligned}
& \sum_{i \in S} \Phi_i(\lambda) \le c_S(\lambda) \\
\Leftrightarrow \quad & \sum_{i \in S} \left(\Phi_i(c^0) + \lambda \Phi_i(c^+)\right) \le c_S^0 + \lambda c_S^+ \\
\Leftrightarrow \quad & \sum_{i \in S} \Phi_i(c^0) + \lambda \sum_{i \in S} \Phi_i(c^+) \le c_S^0 + \lambda c_S^+ \\
\Leftrightarrow \quad & \lambda \sum_{i \in S} \Phi_i(c^+) \le \lambda c_S^+ && \left(\text{as we prove that } \sum_{i \in S} \Phi_i(c^0) = c_S^0\right) \\
\Leftrightarrow \quad & \sum_{i \in S} \Phi_i(c^+) \le c_S^+ && \left(\text{as } \lambda > 0\right)
\end{aligned}$$

This greatly simplifies the problem at hand by removing the dependency on λ and allowing for a direct calculation of the solutions of the last inequality. □

□

In the following, we denote the terms $\frac{(2n-1)(n-2)}{2n^2-3n}$ in (2.9b) and $\frac{2n-2}{2n^2-3n}$ in (2.9c) by $f^+(n)$ and $f^-(n)$, respectively. We note that these expressions are asymptotic, for large values of n, to the simpler expressions $\frac{n-1}{n}$ and $\frac{1}{n}$, respectively. Moreover, as can be seen in Fig. 2.4, these values and the respective asymptotes are close for $n \ge 3$ as well. Indeed, the shaded areas are obtained by the actual bounds and the dotted lines represent the simpler expressions.

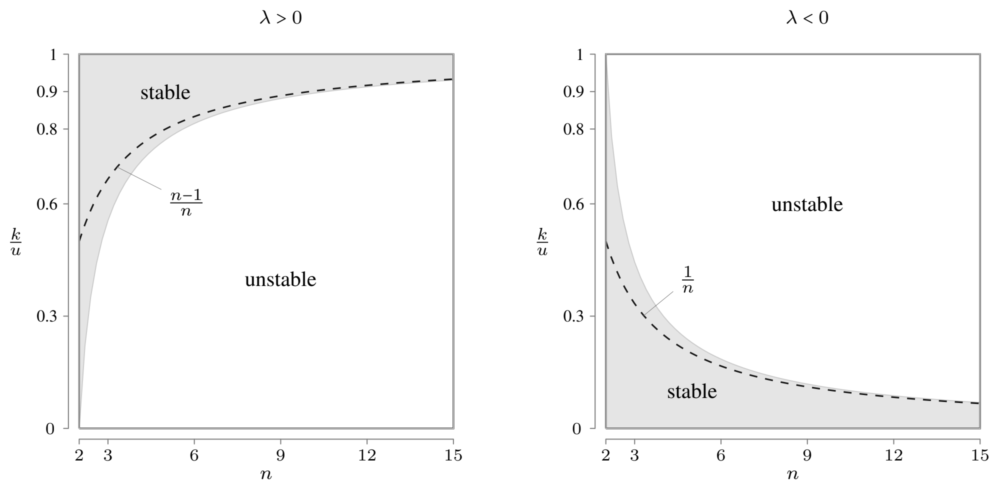


Figure 2.4: Theorem 1. Stability of cooperation on a corridor with identical players depends on the sign of λ and the demand-over-capacity ratio k/u.

In this case, stability is insensitive to the absolute value of the cost parameter λ, but depends on its sign, the network saturation, and the number of players as detailed in conditions (2.9). Mathematically, this independence of the absolute value of λ is explained by linearity of the Shapley value, as can be seen from the proof provided above. Dependency on the sign of λ follows, instead, from the impact a sign change has on the flow allocation between players. Indeed, for $\lambda < 0$, the arc of player i_p becomes the cheapest and will be used first, while it is the most expensive arc (and, thus, will be used last) for $\lambda > 0$. This dependence of the stability on the cost parameter λ holds only for this case, as shown by the numerical experiments of Sec. 2.6.

Our result shows explicitly that overcapacity hinders the stability of a cooperation when $\lambda > 0$, i.e., when a single player's cost exceeds that of the others: for low values of the demand-over-capacity ratio $\frac{k}{u}$, the cooperation is unstable. Moreover, our result shows that the overcapacity threshold is a function of the number of companies only. The threshold value $\frac{k}{u} = \frac{(2n-1)(n-2)}{2n^2-3n} = f^+(n)$ can be interpreted by looking at its asymptote $\frac{n-1}{n}$, which corresponds to the amount of orders leading to a saturation of $n-1$ companies' transport capacity. As $f^+(n) < \frac{n-1}{n}$, cooperation is achieved just before reaching saturation of $n-1$ companies. For $\lambda > 0$, we face the situation where company i_p has a transport cost that is higher than that of any other company. For $\frac{k}{u} \geq \frac{n-1}{n}$, all companies with the lowest cost have their capacity fully utilized. Despite company i_p's shared capacity being used last, the cost reduction this company achieves is spread among the other companies, reducing their total cost.

A symmetric observation holds when $\lambda < 0$, i.e., when a single player operates transport at a lower unit cost than that of all others. Indeed, for $\lambda < 0$, the cooperation is stable in the over-capacitated regime of low values of the demand-over-capacity ratio $\frac{k}{u}$. The threshold $f^-(n)$ is just greater than $\frac{1}{n}$, which corresponds to the amount of orders that can be transported by a single player, i_p in this case. This means that the cooperation is stable when all the orders of the cooperation can be executed by a single company.

In summary, by considering stylized corridors, we obtain closed-form solutions that characterize the transport setting leading to stability. While being in line with Agarwal & Ergun (2010) and Houghtalen et al. (2011), who show how overcapacity is related to instability, we extend their work by providing an exact threshold that quantifies the overcapacity level leading to instability.

In the same setting of Theorem 1, we further support practitioners in deciding an acceptable number of partners to seek to obtain a stable cooperation:

Corollary 1. *Given demand k and capacity $u > 0$ such that $k \leq u$, the maximum size of a stable cooperation on a corridor with identical players when $\lambda > 0$ is $\bar{n}^+ \coloneqq \left\lfloor \frac{5u-3k-\sqrt{9u^2-14uk+9k^2}}{4(u-k)} \right\rfloor$. In case $\lambda < 0$, the maximum size of a stable cooperation on a corridor with identical players is $\bar{n}^- \coloneqq \left\lfloor \frac{2u+3k+\sqrt{9k^2-4ku+4u^2}}{4k} \right\rfloor$.*

Proof. By solving the inequality $\frac{k}{u} \geq \frac{(2n-1)(n-2)}{2n^2-3n}$ in (2.9b) for n, one obtains $n \leq \frac{5u-3k-\sqrt{9u^2-14uk+9k^2}}{4(u-k)}$. Rounding down is necessary as n is integer. Similarly, by solving $\frac{k}{u} \leq \frac{2n-2}{2n^2-3n}$ for n in (2.9c), one obtains the result after rounding down.□ □

2.5.3 The ε-distance

Stability of the Shapley value is determined by testing the inequalities $\sum_{i \in S} \Phi_i(\lambda) \leq c_S(\lambda)$. If one is invalid, subcoalitions will form. Should a coalition $S \subseteq N$ be allocated a total share $\sum_{i \in S} \Phi_i(\lambda)$ greater than the cost $c_S(\lambda)$ it generates, it may drop out of the grand-coalition N to take advantage of the lower cost $c_S(\lambda)$ instead of $\sum_{i \in S} \Phi_i(\lambda)$. Clearly, the magnitude of the gap $\sum_{i \in S} \Phi_i(\lambda) - c_S(\lambda)$ is ignored from this perspective. To address this shortcoming, we define a measure of instability based on the concept of the ε-Core (Shapley & Shubik 1966). The ε-Core is the set of efficient pay-off allocations where coalitional rationality is relaxed by a given threshold that can be interpreted as a cost for dropping out of the grand-coalition, or an incentive to stay (Shapley & Shubik 1966). For the parametric game (λ) and $\varepsilon \in \mathbb{R}$, the ε-Core $\mathfrak{C}_\varepsilon(\lambda)$ is defined as follows:

$$\mathfrak{C}_\varepsilon(\lambda) \coloneqq \left\{ x \in \mathbb{R}^n : \sum_{i \in N} x_i = c_N(\lambda) \text{ and } \sum_{i \in S} x_i \leq c_S(\lambda) + \varepsilon \quad \forall \varnothing \neq S \subsetneq N \right\}. \tag{2.10}$$

We measure the instability of the Shapley value as its maximum deviation from coalitional rationality, and define the following distance.

Definition 1. *Given a parametric cooperative game $c(\lambda)$, the ε-*distance *$\varepsilon_\Phi(\lambda)$ of the Shapley value $\Phi(\lambda)$ is given by*

$$\varepsilon_\Phi(\lambda) \coloneqq \max_{S \subseteq N} \left\{ \sum_{i \in S} \Phi_i(\lambda) - c_S(\lambda) \right\}. \tag{2.11}$$

Efficiency of the Shapley value implies that $\varepsilon_\Phi(\lambda) \geq 0$, and, if $\Phi(\lambda) \in \mathfrak{C}(\lambda)$ then $\varepsilon_\Phi(\lambda) = 0$. If $\Phi(\lambda) \notin \mathfrak{C}(\lambda)$, it follows that $\varepsilon_\Phi(\lambda) > 0$ is the smallest ε-value for which the Shapley value belongs to the ε-Core. Moreover, if $\Phi(\lambda) \notin \mathfrak{C}(\lambda)$, then $\Phi(\lambda) \in \mathfrak{C}_{\varepsilon_\Phi(\lambda)}$, showing that stability has been violated by an amount $\varepsilon_\Phi(\lambda)$.

Our definition of ε-distance is comparable to that of $\underline{\epsilon}$-*stability* introduced by Karsten et al. (2015). In their case, for a vector $\underline{\epsilon} \in \mathbb{R}^n$, an allocation x for a game (N, c) is $\underline{\epsilon}$-stable if $\sum_{i \in S} x_i \leq c_S + \sum_{i \in S} \epsilon_i$ for all $\varnothing \neq S \subseteq N$. Note that if x is $\underline{\epsilon}$-stable, then $x \in \mathfrak{C}_{\sum_{i \in N} \epsilon_i}$ and the ε-distance ε_x for the allocation x would satisfy $\varepsilon_x \leq \sum_{i \in N} \epsilon_i$.

If we define the synergy $\sigma_S(\lambda)$ for coalition $S \subseteq N$ as the cost reduction generated by the cooperation between companies in S, i.e.,

$$\sigma_S(\lambda) \coloneqq \sum_{i \in S} c_i(\lambda) - c_S(\lambda), \tag{2.12}$$

it follows that the ε-distance and the synergy are related by the following result.

Theorem 2. *Given a subadditive parametric cost game $c(\lambda)$ and an individually rational and efficient solution concept $\Psi(\lambda)$, the following holds:*

$$\sigma_S(\lambda) \leq \sigma_N(\lambda) \;\; \forall S \subseteq N \quad \Rightarrow \quad \Psi(\lambda) \in \mathfrak{C}_{\sigma_N}(\lambda) \quad \forall \lambda \in \Lambda, \tag{2.13}$$

where $\mathfrak{C}_{\sigma_N}(\lambda)$ is the ε-Core for $\varepsilon = \sigma_N(\lambda)$.

Proof. Proof. The proof is given in Appendix 2.8.7 . □

This general result shows that if the synergy $\sigma_S(\lambda)$ of each coalition $S \subseteq N$ is lower than the grand coalition's synergy $\sigma_N(\lambda)$, then the gain from dropping out for any coalition is at most $\sigma_N(\lambda)$. Given a fixed value of the parameter λ, high values of synergy $\sigma_S(\lambda)$ stand for high reductions of transport costs generated by the cooperation. Indeed, the total cost without cooperation $\sum_{i \in S} c_i(\lambda)$ is lowered by the cost $c_S(\lambda)$ generated under cooperation. Subadditivity of the game ensures that $\sigma_S(\lambda) \geq 0$, while additivity would lead to $\sigma_S(\lambda) = 0$ for all $S \subseteq N$.

Note that $\sigma_S(\lambda)$ is defined independently of any solution concept. A similar notion of synergy of a coalition S is given in Lozano et al. (2013), who define it as: $\text{Synergy}(S) \coloneqq \frac{\sum_{i \in S} c_i - c_S}{c_S}$. In contrast, we do not rescale by the total cost generated by the coalition.

Corollary 2. *Given a parametric minimum cost flow game $c(\lambda)$, it follows that*

$$\Phi(\lambda) \in \mathfrak{C}_{\sigma_N(\lambda)} \quad \forall \lambda \in \Lambda. \tag{2.14}$$

Proof. Proof. The proof is given in Appendix 2.8.8.

□

Interpreting the value of the ε-distance might be difficult since incentives are reported in absolute terms. To overcome this problem, we define the relative ε-distance $\bar{\varepsilon}_\Phi(\lambda)$ as the ε-distance relative to the total cost generated by the subcoalition:

$$\bar{\varepsilon}_\Phi(\lambda) \coloneqq \max_{S \subseteq N} \left\{ \frac{\sum_{i \in S} \Phi_i(\lambda) - c_S(\lambda)}{c_S(\lambda)} \right\}. \tag{2.15}$$

Efficiency of the Shapley value implies that $\bar{\varepsilon}_\Phi(\lambda) \geq 0$. Moreover, $\bar{\varepsilon}_\Phi(\lambda) > 0$ if and only if $\Phi(\lambda) \notin \mathfrak{C}(\lambda)$. In case $\varepsilon_\Phi(\lambda) > 0$, each coalition S maximizing (2.15) is unstable, i.e., $\sum_{i \in S} \Phi_i(\lambda) - c_S(\lambda) > 0$. Such a measure quantifies the magnitude of the incentive to form subcoalitions relative to the total cost generated by each subcoalition itself. Clearly, from the value $\bar{\varepsilon}_\Phi(\lambda)$ it cannot be concluded if $\Phi(\lambda) \in \mathfrak{C}_{\bar{\varepsilon}_\Phi}(\lambda)$.

The computation of the relative ε-distance $\bar{\varepsilon}_\Phi(\lambda)$ requires deriving the solution concept, i.e., the Shapley value $\Phi(\lambda)$. In case this operation is complex or expensive, we provide the following upper bound that is defined by the coalitional costs $c_S(\lambda)$ only.

Corollary 3. *Given a parametric minimum cost flow game $c(\lambda)$, it follows that*

$$\bar{\varepsilon}_\Phi(\lambda) \leq \frac{\sigma_N(\lambda)}{\min\limits_{S \subsetneq N : |S| \geq 2} c_S(\lambda)} \quad \forall \lambda \in \Lambda. \tag{2.16}$$

Proof. Proof. From Corollary 2, we obtain that $\sum_{i\in S}\Phi_i(\lambda) - c_S(\lambda) \le \sigma_N(\lambda)$ for all coalitions $S \subseteq N$. Division by the smallest cost of a coalition that can violate coalitional irrationality yields the claimed upper bound on (2.15).□ □

The previous result shows that the maximum relative deviation from stability is bounded by the cooperation synergy relative to the cost generated by the cheapest coalition.

2.6 Generalizations

Obtaining a closed-form expression (in terms of the parameters of the game $c(\lambda)$) for the solution set of $\Phi(\lambda) \in \mathfrak{C}(\lambda)$ is a challenging task. This holds especially since the costs $c_S(\lambda)$ ($S \subseteq N$) are optimal solutions to an optimization problem. Therefore, as soon as we generalize the transport setting, we opt for a numerical approach that exploits the Eisner-Severance method (Eisner & Severance 1976) for the construction of the cost curves (see Appendix 2.8.2 for a detailed description). Once the parametric game $c(\lambda)$ has been constructed, the Shapley value is obtained numerically by working directly with the piecewise linear cost curves $\{c_S(\lambda)\}_{S\subseteq N}$. Thanks to the observation from Section 2.5.1, solving $\Phi(\lambda) \in \mathfrak{C}(\lambda)$ for $\lambda \in \Lambda$ translates into the problem of solving linear inequalities.

In what follows, we conduct several tests in which we drop several assumptions made in the case treated in Theorem 1. In Section 2.6.1, we consider the case where players' costs are allowed to take two different values. This is extended in Section 2.6.2, where two different demand and capacity levels are considered as well. Finally, the network structure is generalized in Section 2.6.3, where both a mathematical and a numerical analysis are carried out.

2.6.1 Corridor with high and low costs

We again consider collaboration on a corridor as in Sect. 2.5.2, and test whether our insights obtained from Theorem 1 still hold when unit costs are no longer identical. As opposed to Theorem 1, we assume different costs: $n_L < n$ companies have low cost c_L, company i_p has a cost of $c_{i_p} = c_L + \lambda$, for $\lambda \in \Lambda$, and the remaining $n_H := n - n_L - 1$ companies have cost $c_H > c_L$.

We repeatedly generate the parametric game for increasing demand levels from $k = k_0$ to $k = u$ for each value $n_L = 0, 1, \ldots, n-1$, keeping all other parameters fixed.[3] For each instance, the optimal

[3]Parameters are set as follows: $k_0 = 0$, $u = 30$; $c_L = 20$, $c_H = 30$; company $i_p = 1$ has parametric cost $c_1 = c_L + \lambda$, $\lambda \in \Lambda = [-20, 40]$; for $i \in \{2, \ldots, n_L + 1\}$, $c_i = c_L$, while for $i \in \{n_L + 2, \ldots, n\}$, $c_i = c_H$ ($c_L < c_H$).

objective value $c_S(\lambda)$ for the parametric problem $P^S(\lambda)$ is computed for each coalition $S \subseteq N$ by using Algorithm 1 described in Appendix 2.8.2. We find the intervals in Λ for which $\Phi(\lambda) \in \mathfrak{C}(\lambda)$. Parameter regions of stability are shown shaded in Figure 2.5 for $n_L = 0, 1, 2, 3, 4$ and $n = 5$. The case $n_L = 4$, which coincides with the situation studied in Theorem 1, has been inserted for comparison.

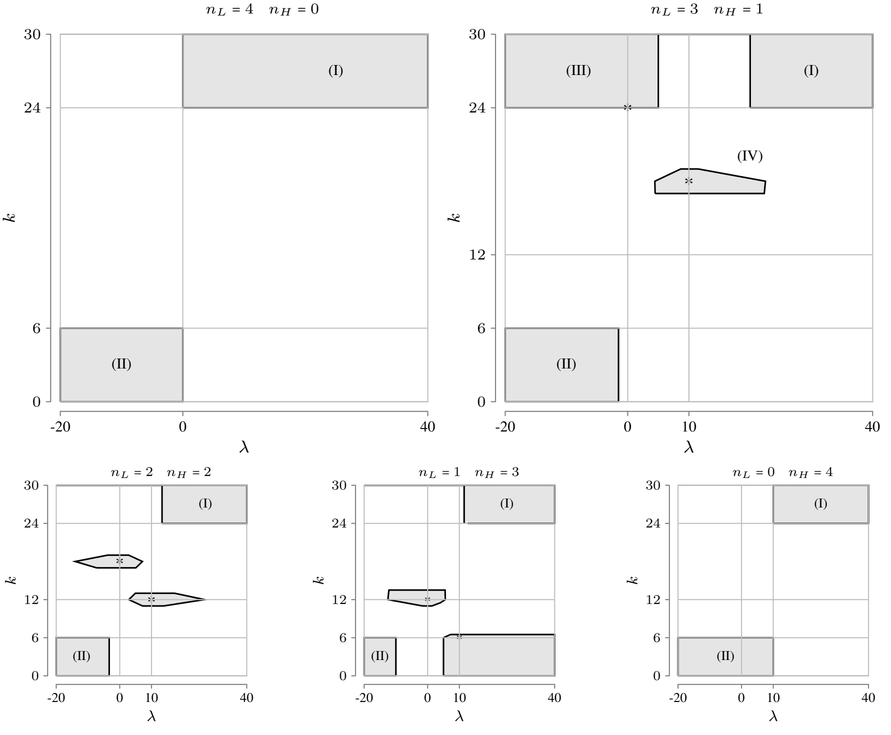


Figure 2.5: Regions of stability (shaded) of the 5-players cooperation. Note that the cases $n_L = 4, n_H = 0$ and $n_L = 0, n_H = 4$ coincide with the situation studied in Theorem 1. "$*$" indicates the case where a complete transfer of order is possible between companies with low and high cost, as explained further in the text.

We find that stability is sensitive to changes in λ as soon as players' costs are heterogeneous. Regions (I) and (II) are inherited from the scenario with identical costs studied in Theorem 1. Indeed, region (I) appears for values of $\frac{k}{u}$ higher than the threshold $\frac{(2n-1)(n-2)}{2n^2-3n}$ and positive values of λ, while region (II) is located at values of $\frac{k}{u}$ lower than the threshold $\frac{2n-2}{2n^2-3n}$ and negative values of λ. Unlike for Theorem 1, the extension of those regions now also depends on the absolute value of λ and not only on its sign. Notably, the stability regions inherited from the identical cost scenario appear consistently throughout the experiments, even when the size n of the cooperation increases.

	Case $\lambda > 0$				Case $\lambda < 0$			
		Mean relative error [%]				Mean relative error [%]		
n	$f^+(n)$	$u = 30$	$u = 40$	$u = 60$	$f^-(n)$	$u = 30$	$u = 40$	$u = 60$
4	0.70	3.17	3.17	3.17	0.30	7.41	7.41	7.41
5	0.77	3.70	3.70	3.70	0.23	12.50	12.50	12.50
6	0.81	2.27	2.27	2.27	0.19	10.00	10.00	10.00
7	0.84	2.67	2.67	2.67	0.16	14.44	14.44	14.44
8	0.87	1.80	1.80	1.80	0.13	11.56	11.56	11.56

Table 2.2: Relative deviation of the numerically obtained thresholds $\delta^+(n, n_L)$ and $\delta^-(n, n_L)$ from the theoretical ones $f^+(n) = \frac{(2n-1)(n-2)}{2n^2-3n}$ and $f^-(n) = \frac{2n-2}{2n^2-3n}$, respectively. The reported values are the average over the cases $n_L \in \{1, \dots, n-1\}$ of the absolute relative error $|\delta^\pm(n, n_L) - f^\pm(n)|/f^\pm(n)$.

From Figure 2.5, it can be seen that region (I) is formed beginning at $k^+ = 24$. Let $\delta^+(n, n_L) := \frac{k^+}{u}$ be the numerical threshold obtained, where k^+ is the lowest demand value k contained in region (I). Similarly, we denote the value below which region (II) is formed by k^- and the corresponding numerical threshold by $\delta^-(n, n_L) := \frac{k^-}{u}$. In the case of Figure 2.5, we have $k^- = 6$. Table 2.2 reports average values for the absolute relative deviation of the numerically obtained thresholds $\delta^+(n, n_L)$ and $\delta^-(n, n_L)$ from the theoretical ones obtained in Theorem 1. Given a test value of the capacity u, the values $\delta^+(n, n_L)$ have been computed for each $n = 4, \dots, 8$ and $n_L = 1, \dots, n-1$, and the absolute relative error $|\delta^+(n, n_L) - f^+(n)|/f^+(n)$ has been computed. The same procedure has been performed for $\delta^-(n, n_L)$. Table 2.2 reports the averages of values obtained in both cases. The low values obtained show that the analytical results we obtained are good indicators for the position of regions of stability when generalizing the parameter setting. Of the two thresholds, it can be observed that the mean relative error for the threshold $f^+(n)$ is lower than that for $f^-(n)$.

Non-identical costs introduce new regions of stability in the over-capacitated regime. For the case of $n_L = 1$, the regions (III) and (IV), which were not present in Theorem 1, appear. Regions (III) and (IV) form as neighborhoods of parameter values at which low cost companies are executing all orders of the high cost companies by saturating their transport capacity. The cooperation is stable when a complete transfer of orders is possible, i.e, when there is a division of roles between companies that either only share capacity or only share orders. Following this argument, the parameter values are $k = \frac{n_L+1}{n}$ and $\lambda = c_L$, or $k = \frac{n_L}{n}$ and $\lambda = c_H$ and are highlighted by "$*$" in Fig. 2.5.

We further explore how instability is sensitive to λ and how it depends on $\frac{k}{u}$ by computing the relative ε-distance $\bar{\varepsilon}_\Phi(\lambda)$. The contour plots in Fig. 2.6 show levels of 1%, 5%, and 20% for the case of $n_L = 1$ and $n = 5$. Regions of 1%-instability appear in the saturated top regions of the plots. This shows that if one relaxes the notion of stability, high values of the ratio $\frac{k}{u}$ are still related to low instability of the cooperation, but also that new regions of limited instability appear in the over-capacitated regime as well.

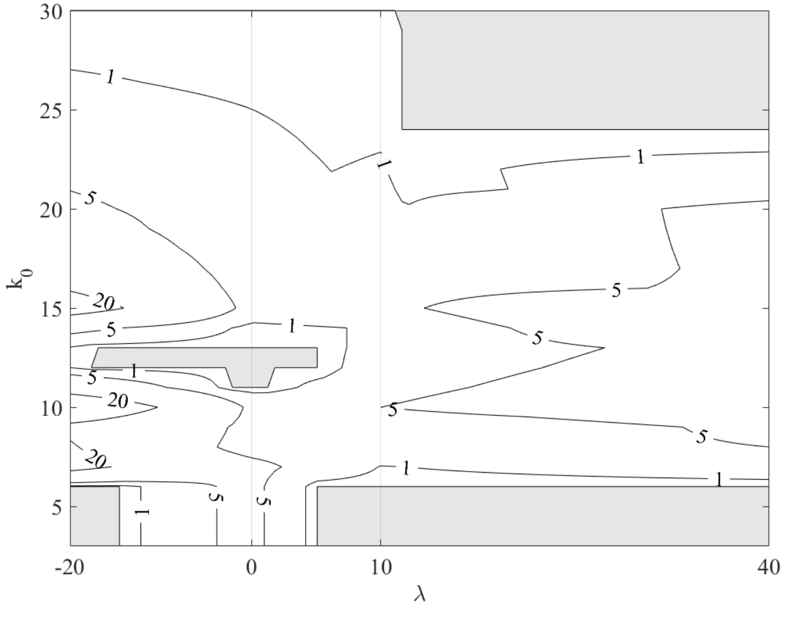


Figure 2.6: The relative ε-distance $\bar{\varepsilon}_\Phi(\lambda)$ for a 5-players cooperation with $n_L = 1$ (values in percent). Cf. Fig. 2.5.

From a sensitivity analysis point of view, $\bar{\varepsilon}_\Phi(\lambda)$ is less sensitive to λ in the saturated region than in the over-capacitated one. A similar effect can be observed for the upper bound: the higher the saturation, the lower its sensitivity to variations of λ.

It can be concluded that the capacity-over-demand ratio has a major impact in regulating the sensibility of stability on a single player's transport cost. It is still possible to obtain stable cooperation in the over-capacitated regime, which will be more sensitive to λ than in the saturated regime.

This result sheds light on corridor formation, as this structure can be stable even in the case of an over-capacitated network. This could potentially benefit areas of the hinterland where container flows are small and cooperation is not seen as an option because economies of scale cannot be achieved. Our results show that, even in the absence of this cost reduction, cooperation can nonetheless be stable and beneficial.

2.6.2 Corridor cases with varying demand and capacity

In the previous section, the network saturation $\frac{k}{u}$ could be computed from the demand k and capacity u. This is not the case when demand and capacity are not the same for all players. We observe that the total demand $K \coloneqq \sum_{i \in N} k_i$ over total capacity $U \coloneqq \sum_{i \in N} u_i$ ratio $\frac{K}{U}$ allows for a comparison with the previous case.

	Case $\lambda > 0$				Case $\lambda < 0$			
n	$f^+(n)$	Δ_k	Mean relative error [%] $k_L > k_H$	Mean relative error [%] $k_H > k_L$	$f^-(n)$	Δ_k	Mean relative error [%] $k_L > k_H$	Mean relative error [%] $k_H > k_L$
5	0.77	5	3.70	13.58	0.23	5	12.50	2.08
		10	3.70	3.70		10	2.08	12.50
6	0.81	5	1.93	3.98	0.19	5	19.00	13.00
		10	2.12	−−		10	23.00	16.00
7	0.84	5	2.10	2.10	0.16	5	9.10	12.92
		10	1.08	3.23		10	15.28	9.86
8	0.87	5	2.64	1.91	0.13	5	9.76	19.52
		10	2.07	2.07		10	15.48	16.43

Table 2.3: $(k_L \neq k_H)$: Relative deviation of the numerically obtained thresholds $\delta^+(n, n_L)$ and $\delta^-(n, n_L)$ from the theoretical ones $f^+(n) = \frac{(2n-1)(n-2)}{2n^2-3n}$ and $f^-(n) = \frac{2n-2}{2n^2-3n}$, respectively. The reported values are the average over the cases $n_L \in \{1, \dots, n-1\}$ of the absolute relative error $|\delta^\pm(n, n_L) - f^\pm(n)|/f^\pm(n)$.

Let k_L and k_H be the demand for players with low or high cost, respectively, and let $\Delta_k \coloneqq |k_H - k_L|$ be the demand gap between the players. We assume: $n_L < n$ companies with low cost c_L have demand k_L, company i_p with cost of $c_{i_p} = c_L + \lambda$, for $\lambda \in \Lambda$ has demand k_L, and the remaining $n_H \coloneqq n - n_L - 1$ companies with high cost c_H have demand k_H.

Then, we test the stability of cooperation for increasing values of $k = k_0, \dots, u - \Delta_k$ in two cases where either $k_L = k + \Delta_k$ $(k_H = k)$, or $k_L = k$ $(k_H = k + \Delta_k)$. Note that it is not necessary that $k_L < k_H$. We generate plots as in Figure 2.5 and compute the numerical thresholds $\delta^+(n, n_L)$ and $\delta^-(n, n_L)$ using the lowest total demand K^+ at which the saturated region of stability exists, and the highest total demand K^- below which the region of stability for negative values of λ exists, respectively. In Table 2.3, we report the mean relative error $|\delta^\pm(n, n_L) - f^\pm(n)|/f^\pm(n)$ over $n_L = 1, \dots, n-2$ for two sample values of Δ_k. We observe that the numerical threshold $\delta^+(n, n_L)$ is close to the value $f^+(n)$ obtained from Theorem 1, but a higher gap is present between $\delta^-(n, n_L)$ and $f^-(n)$ for $\lambda < 0$. The demand gap Δ_k can explain this difference as low values of saturation of the network cannot be reached. Indeed, even if one company is only transporting $k = 1$ order, others would have $k = 1 + \Delta_k$ orders, so the network is still far from the target values of low saturation.

Similar results are obtained if we assume that players' capacities are different. Again we test for the case where either $u_L = u + \Delta_u$ and $u_H = u$, the capacity for players with low or high costs, respectively, or $u_L = u$ and $u_H = u + \Delta_u$. The player i_p with parametric cost is counted as a player with low cost. Table 2.4 reports values of the mean relative error that again shows results close to the theoretical threshold obtained from Theorem 1 for the case of $\lambda > 0$ only.

	Case $\lambda > 0$				Case $\lambda < 0$			
			Mean relative error [%]				Mean relative error [%]	
n	$f^+(n)$	Δ_u	$u_L > u_H$	$u_H > u_L$	$f^-(n)$	Δ_u	$u_L > u_H$	$u_H > u_L$
5	0.77	5	1.34	5.60	0.23	5	21.81	3.33
		10	2.78	4.34		10	32.43	9.88
6	0.81	5	1.63	6.44	0.19	5	22.13	3.07
		10	0.06	6.41		10	36.96	12.39
7	0.84	5	1.28	3.83	0.16	5	29.94	5.31
		10	1.53	4.34		10	26.53	8.68
8	0.87	5	1.76	3.19	0.13	5	20.68	15.89
		10	2.42	3.02		10	40.27	24.59

Table 2.4: $(u_L \neq u_H)$: Relative deviation of the numerically obtained thresholds $\delta^+(n, n_L)$ and $\delta^-(n, n_L)$ from the theoretical ones $f^+(n) = \frac{(2n-1)(n-2)}{2n^2-3n}$ and $f^-(n) = \frac{2n-2}{2n^2-3n}$, respectively. The reported values are the average over the cases $n_L \in \{1, \dots, n-1\}$ of the absolute relative error $|\delta^\pm(n, n_L) - f^\pm(n)|/f^\pm(n)$.

Overall, these numerical tests show that the total demand to capacity ratio, compared to the threshold $f^+(n)$, is a main discriminant for the stability of a cooperation. We can also conclude that the region of stability for low values of the demand-over-capacity ratio for $\lambda < 0$ is more susceptible to variations in the amount of orders and of capacity in the network.

2.6.3 Vertical cooperation opportunity

In this section, we consider the case where the companies also have the opportunity of collaborating in a vertical transport setting. While being simple in its formulation, this case generalizes the network of the cases considered previously. The analysis performed here shows that our theoretical results can be extended to a slightly richer network and our algorithmic approach is not bound to specific networks.

Consider the case where each company in a cooperation executes a segment of a joint transport route and, at the same time, is able to execute direct transport from origin to destination. The sequence of segments constitutes a path that can be used only when all companies cooperate. One example of such a configuration can be found in the case of intermodal container transport where each company operates a transport service in a vertically-integrated sequence of transport operations, but also has the opportunity of self-arranging direct transport.

Formally, we consider the network given in Figure 2.7. Each company $i \in N = \{1, \dots, n\}$ executes transport on arc r_i, representing the direct transport option, and arc $\bar{r}_i$, representing the segment in the vertical cooperation path joining origin s and destination t. All companies but company i_p have identical unit transport costs $c_i = c$ and $c_{\bar{r}_i} = \bar{c}$. Company i_p has a parametric arc cost either on arc r_{i_p}, i.e.,$c_{\bar{r}_{i_p}} = \bar{c} + \lambda$, or arc $\bar{r}_{i_p}$, i.e, $c_{r_{i_p}} = c + \lambda$, that is used to inspect the sensitivity of the stability of the Shapley value. Each of these two cases will be treated separately later. The direct transport capacity $u_{r_i} = u$ on arc r_i and the vertical cooperation transport capacity $\bar{u}_{\bar{r}_i} = \bar{u}$ on arc $\bar{r}_i$ are identical for all companies $i \in N$, as well as the amount of orders $k^i = k$ to be transported from origin node s to destination node t. We let $c_S(\lambda)$ denote total cost of the minimum cost flow generated by coalition $S \subseteq N$ when the set of arcs $R^S = \cup_{i \in S}\{r_i, \bar{r}_i\}$ is used and the amount of orders $k^S = \sum_{i \in S} k^i$ is pooled. We then obtain a parametric minimum cost flow game as in Section 2.4.

We denote the resulting parametric minimum cost flow game on the graph given in Figure 2.7 by $c^v(\lambda)$ and, for simplicity, refer to it as the *vertical cooperation game*. In Section 2.6.3.1, we discuss the theoretical properties of the cooperation, while Section 2.6.3.2 presents the numerical experiments that complement the theoretical findings.

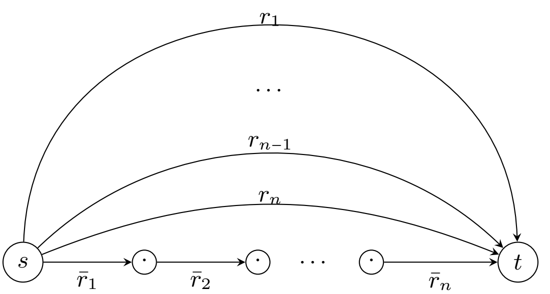


Figure 2.7: Transport network for corridor cooperation with opportunity of vertical cooperation. Arc r_i represents a direct connection, while path $(\bar{r}_1, \dots, \bar{r}_n)$ represents the joint vertical service.

2.6.3.1 Theoretical Analysis

We denote the Shapley value and the core of the vertical cooperation game $c^v(\lambda)$ by $\Phi^v(\lambda)$ and $\mathfrak{C}^v(\lambda)$, respectively.

We first consider the case where $c_{\bar{r}_{i_p}} = \bar{c} + \lambda$ and assume that $\lambda \geq -c$ in order to ensure that $c_{\bar{r}_{i_p}} \geq 0$. Here, we obtain the following theorem:

Theorem 3. *Consider the vertical cooperation game $c^v(\lambda)$, where $c_{\bar{r}_{i_p}} = \bar{c} + \lambda$. For all values of direct unit transport cost c and capacity u, vertical unit transport cost $\bar{c}$ and capacity $\bar{u}$, amount of orders k and number of players n, we have*

$$\Phi^v(\lambda) \in \mathfrak{C}^v(\lambda) \qquad \textit{for all } \lambda \in [-c, +\infty). \tag{2.17}$$

In other words, when the parametric cost is on one of the arcs in the vertical cooperation path, the Shapley value $\Phi^v(\lambda)$ is stable for all values of λ.

Proof. Proof. The proof is given in Appendix 2.8.9. □

Theorem 3 shows that, given identical costs for direct transport, adding a vertical cooperation opportunity can only benefit the stability of the Shapley value. Moreover, it becomes clear that this case does not require any further computational investigation as stability holds for every parameter setting.

We now consider the case where the parameter λ is on the direct arc r_{i_p} of player i_p, i.e., $c_{r_{i_p}} = c + \lambda$. We assume that $\lambda \geq -c$ to ensure non-negativity of the unit transport cost, and that the total unit cost $n\bar{c}$ for vertical cooperation transport is at most that of direct transport, i.e., $n\bar{c} \leq c$. Otherwise the vertical cooperation game would reduce to the horizontal cooperation case studied in Theorem 1. Indeed, if $n\bar{c} > c$, then the vertical cooperation path is never used.

Given the parameter being on arc r_{i_p}, if we consider only cooperation on the direct transport arcs $\{r_i : i \in N\}$, we obtain the case of cooperation on a corridor with identical players treated in Theorem 1. We denote the corresponding cooperative game by $c^h(\lambda)$ and show the following relation between $c^v(\lambda)$ and $c^h(\lambda)$:

Theorem 4. *In the vertical cooperation game $c^v(\lambda)$ where $c_{r_{i_p}} = c + \lambda$, we have that, for all values of direct unit transport cost c and capacity u, vertical unit transport cost $\bar{c}$ such that $n\bar{c} \leq c$ and capacity $\bar{u}$, amount of orders k and number of players n, the following holds: For each value of*

$\lambda \in [-\bar{c}, +\infty)$, stability of the Shapley value $\Phi^h(\lambda)$ in the horizontal cooperation game $c^h(\lambda)$ implies stability of the Shapley value $\Phi^v(\lambda)$ in the vertical cooperation game $c^v(\lambda)$. More formally:

$$\text{For each } \lambda \in [-\bar{c}, +\infty): \qquad \Phi^h(\lambda) \in \mathfrak{C}^h(\lambda) \quad \Rightarrow \quad \Phi^v(\lambda) \in \mathfrak{C}^v(\lambda). \tag{2.18}$$

The converse does, in general, not hold true.

Proof. Proof. We provide an intuitive explanation of the proof, which is given in Appendix 2.8.9 [Online]. Here, we focus on the case $\lambda > 0$.

Like the proof of Theorem 1, this proof is based on a decomposition of the game $c^v(\lambda)$ as a linear combination of the horizontal cooperation game $c^h(\lambda)$ and a new game $\bar{c}(\lambda)$ that is obtained as the algebraic difference of $c^v(\lambda)$ and $c^h(\lambda)$. Since, in this case, the arc r_{i_p} with parametric cost is a direct transport arc, we obtain that the horizontal cooperation game $c^h(\lambda)$ can be decomposed as in the proof of Theorem 1, i.e., $c^h(\lambda) = c^0 + \lambda c^+$.

We denote the Shapley values for the games c^+ and c^0 by $\Phi^+ = \Phi(c^+)$ and $\Phi^0 = \Phi(c^0)$, respectively. Combining the decomposition of $c^h(\lambda)$ with that of $c^v(\lambda)$, we obtain that $c^v(\lambda) = c^0 + \lambda c^+ + \bar{c}(\lambda)$ and the Shapley value $\Phi^v(\lambda)$ can be obtained by using linearity since it can be computed explicitly for each of the games c^0, c^+, and $\bar{c}(\lambda)$.

Testing coalitional rationality for $S \subsetneq N$ means testing whether $\sum_{i \in S} \Phi_i^v(\lambda) \leq c_S^v(\lambda)$, which can be rewritten as follows:

$$\begin{aligned} & \sum_{i \in S} \Phi_i^v(\lambda) \leq c_S^v(\lambda) \\ \Leftrightarrow \quad & \sum_{i \in S} \Phi_i^0 + \lambda \sum_{i \in S} \Phi_i^+ + |S| \frac{\bar{c}_N(\lambda)}{n} \leq c_S^0 + \lambda c_S^+ \\ \Leftrightarrow \quad & \lambda \sum_{i \in S} \Phi_i^+ + |S| \frac{\bar{c}_N(\lambda)}{n} \leq \lambda c_S^+ \end{aligned}$$

This greatly simplifies the problem and provides a simple way to complete the proof. Indeed, it now suffices to study the term $|S|\frac{\bar{c}_N(\lambda)}{n}$ because a closed-form solution to the set of inequalities $\sum_{i \in S} \Phi_i^+ \leq c_S^+$ $(S \subseteq N)$ has already been obtained in the proof of Theorem 1. □ □

From Theorem 4, we can conclude that, independently of the unit cost $n\bar{c}$ and capacity $\bar{u}$ on the vertical cooperation path, the vertical cooperation opportunity is never disadvantageous for stability. However, it is not clear whether and – if so – by how much the region of stability is enlarged. We test this in the numerical experiments of the following section.

2.6.3.2 Numerical study

To test whether the region of stability of the vertical cooperation game $c^v(\lambda)$ extends beyond that of the horizontal cooperation game $c^h(\lambda)$, we consider the following numerical experiments.

We assume $n = 4$, direct unit transport cost $c = 100$ and vertical unit transport cost $\bar{c} \in \{25, 24, 23, 20\}$, direct transport capacity $u = 60$ and vertical transport capacity $\bar{u} = 120$. Knowing from Theorem 1 that the stability of the Shapley value in the game $c^h(\lambda)$ depends on the ratio $\frac{k}{u}$, we test stability for a varying amount k of orders between $k = 0$ and $k = u = 60$. The parameter λ varies in the interval $\Lambda = [-100, 100]$. Note that we set vertical transport capacity equal to the direct transport capacity of two players, and generate a plot for each value of $\bar{c}$.

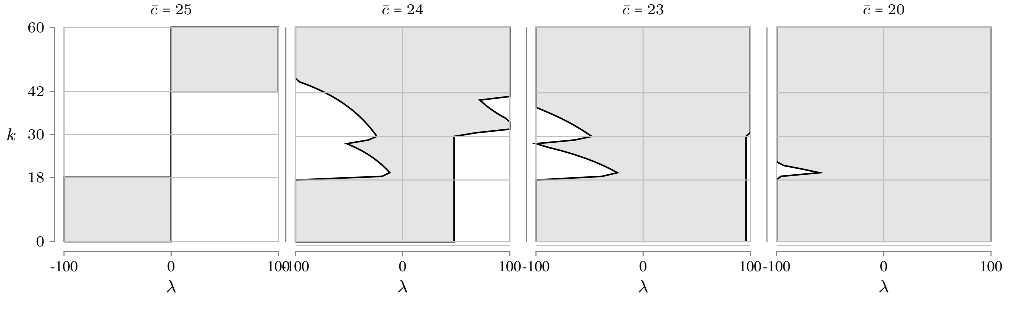


Figure 2.8: Regions of stability for the vertical cooperation game. All subfigures share the same vertical axis shown on the left.

Our results are shown in Figure 2.8, in which the shaded areas correspond to the regions of stability of the vertical cooperation game. We observe that, in case $n\bar{c} = 100 = c$, we obtain the result described in Theorem 1 for $n = 4$ players. As soon as vertical cooperation becomes beneficial for $\bar{c} = 24$, meaning that $n\bar{c} = 96 < 100 = c$, the region of stability expands for all values of the demand-over-capacity ratio $\frac{k}{u}$. This behaviour is consistent across the remaining two cases of $\bar{c} = 23$ and $\bar{c} = 20$.

From the experiments, we can conclude that, as soon as vertical transport is advantageous (i.e., its unit transport cost is lower than that of direct transport), the stability of the vertical cooperation game becomes less dependent on the demand-over-capacity ratio than in the horizontal cooperation on a corridor case (compare Figure 2.8 with Figures 2.5 and 2.6). This decreased dependency on the amount of spare capacity depends on the value of the unit cost of vertical transport. This result adds to the current understanding that horizontal cooperation in transportation is stable only for coalitions of small size (Basso et al. 2019, Agarwal & Ergun 2010). Indeed, we observe that, when a service requiring the joint effort of all companies is advantageous for all, then the cooperation gains in stability.

2.7 Conclusion

Cooperation in the hinterland container transport sector can improve the performance of hinterland connections. However, while reducing costs and improving the competitive position of ports, cooperation exposes members to the risk of its failure. For this reason, we study the relation between transport setting and stability of cooperation from a cost sharing perspective. We propose a sensitivity analysis method to test the stability of bargained cost shares. This approach combines results from linear parametric optimization with key concepts from cooperative game theory. By using methods from linear parametric optimization, we generate parametric cooperative games for more complex instances. Our approach computes parameter intervals leading to stability, thus extending the sampling-based analysis available in the literature on collaborative transport. Furthermore, we introduce a measure of instability that quantifies the deviation from stability based on the ε-Core (Shapley & Shubik 1966). Overall, we prove that the demand-over-capacity ratio – when compared to a function of the size of the cooperation – is the main discriminant for stability of horizontal cooperation in transportation for network flow-like cost structures. Moreover, we show that, even for over-capacitated networks, a stable, or limited unstable, cooperation is possible. Given the complexity of the formal analysis, our analytical results are limited to the case of identical companies cooperating on two different networks. In our numerical experiments, instead, we study the effect that heterogeneity of companies has on our theoretical results.

There are several directions for future research. First, we assume a simple transport model. Including a time dimension in the model could lead to a parametric analysis of the dependency of cooperation stability on time-related parameters, such as speed and frequency of connections. Second, extending and generalizing the type of networks studied could further reveal the role played by the network structure itself. Third, other concepts from cooperative game theory can be parametrized using the proposed definition of parametric cooperative games. Finally, a stochastic optimization model could be considered to improve the representation of the actual decision making process, at the cost of finding a suitable representation of the bargaining process.

2.8 Appendix

2.8.1 Introductory example analysis

If we denote the amount of orders and capacity of each company by k and u, respectively, then the problem to be solved for each coalition $S \subseteq N \coloneqq \{P_1, P_2, P_3\}$ is a minimum cost flow problem on the graph in Figure 2.1. Node s is the source and node t the sink (see the model definition given in Section 2.4). Table 2.5 shows the cost c_S generated by each subset S of companies as well as the Shapley value allocation to each player P_i ($i = 1, 2, 3$) and each coalition. The Shapley value has been computed using expression (2.1) in Section 2.3. Note that, for coalition $S \subseteq N$, $\Phi_S \coloneqq \sum_{P_i \in S} \Phi_{P_i}$. Table 2.5 presents three cases: the case $k = 15$ is the one described in Section 2.1, the cases $k = 18$ and $u = 25$ refer to the mentioned situations where demand is increased to 18 units or capacity is lowered to 25 units.

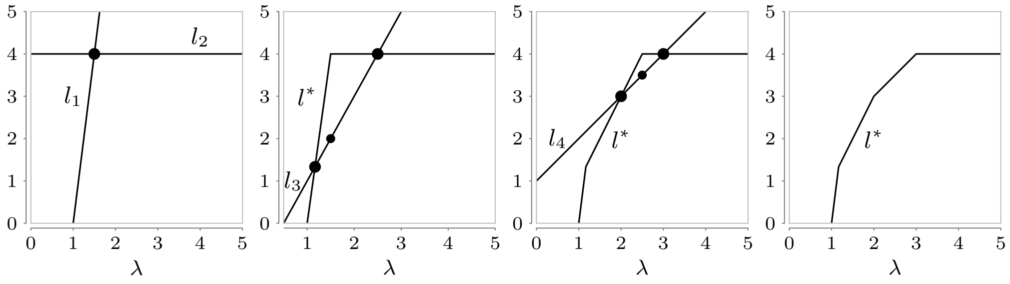
The bold value in case $k = 15$ highlights instability of coalition $\{P_1, P_3\}$ due to the cost generated by this coalition being lower then the Shapley value allocation. In all other cases, the Shapley value allocation Φ_S is at most as large as the cost c_S, proving stability of the Shapley value and of the cooperation.

Case		$\{P_1\}$	$\{P_2\}$	$\{P_3\}$	$\{P_1, P_2\}$	$\{P_1, P_3\}$	$\{P_2, P_3\}$	$\{P_1, P_2, P_3\}$
$k = 15$	c_S	450	375	300	750	600	600	975
	Φ_S	387.5	350	237.5	737.5	**625**	587.5	975
$k = 18$	c_S	540	450	360	930	780	750	1200
	Φ_S	480	420	300	900	780	720	1200
$u = 25$	c_S	450	375	300	775	650	625	1000
	Φ_S	400	350	250	750	650	600	1000

Table 2.5: Game definition and Shapley value for the introductory example.

2.8.2 Algorithmic approach

The parametric minimum cost flow game $c(\lambda)$ can be efficiently constructed by computing the cost curves $c_S(\lambda)$ for each coalition $S \subseteq N$ following the Eisner-Severance method (Eisner & Severance 1976), which we recall here. We first explain the method with the help of Figure 2.9, then provide a formal definition in Algorithm 1. We use the same notation as in Section 2.4.



(a) **(b)** **(c)** **(d)**

Figure 2.9: Sequential construction of the parametric cost curve $c_S(\lambda)$ with the Eisner-Severance method.

The construction of the cost curve $c_S(\lambda)$ over an interval $\Lambda \coloneqq [\underline{\lambda}, \overline{\lambda}]$ is performed by updating a piecewise linear curve $l^*(\lambda)$ until it converges to $c_S(\lambda)$. During the execution of the algorithm, optimal solutions to the minimum cost flow problem at query values of $\lambda \in \Lambda$ will be computed. For an optimal solution f^* found at a query value λ', let $l_{\lambda'}(\lambda) \coloneqq \sum_{r \in R^S \setminus \{r_\lambda\}} c_r f_r^* + \lambda f_{r_\lambda}^*$ be the parametric objective value for the optimal solution f^*. Note that $l_{\lambda'}(\lambda') = c_S(\lambda')$, i.e., $l_{\lambda'}$ is optimal at λ', and $l_{\lambda'}(\lambda) \geq c_S(\lambda)$ for $\lambda \neq \lambda'$, as f^* is not granted to be an optimal solution to the minimum cost flow problem for $\lambda \neq \lambda'$.

Algorithm 1 Construction of cost curve $c_S(\lambda)$ for $\lambda \in \Lambda$.

$A \leftarrow l_{\underline{\lambda}} \cap l_{\overline{\lambda}}$
if $l_{\underline{\lambda}} = l_{\overline{\lambda}}$ **then**
    $l^*(\lambda) \leftarrow l_{\underline{\lambda}}$
else
    $l^* \leftarrow l_{\underline{\lambda}} \wedge l_{\overline{\lambda}}$
    while $A \neq \varnothing$ **do**
        $\lambda' \leftarrow$ pop an element from A
        if $|l^* \cap l_{\lambda'}| \leq 2$ **then**
            $A \leftarrow A \cup (l^* \cap l_{\lambda'})$
        $l^* \leftarrow l^* \wedge l_{\lambda'}$
$l^*(\lambda)$ is the optimal cost curve $c_S(\lambda)$ for $\lambda \in \Lambda$.

Assuming that $c_S(\lambda)$ is not linear (i.e., it has at least one breakpoint) implies that $l^*(\lambda)$ is piecewise linear and that $l^*(\lambda)$ has at least one breakpoint since the start of the algorithm. The starting point, indeed, is the computation of the lines $l_1(\lambda) \coloneqq l_{\underline{\lambda}}(\lambda)$ and $l_2(\lambda) \coloneqq l_{\overline{\lambda}}(\lambda)$ obtained from the optimal solutions at query values given by the extremes of Λ. The curve $l^*(\lambda)$ is defined at first by the pointwise minimum of the lines $l_1(\lambda)$ and $l_2(\lambda)$: $l^*(\lambda) = \min\{l_1(\lambda), l_2(\lambda)\}$ (see Figure 2.9a). During each update, a new line $l_i(\lambda) := l_{\lambda'}(\lambda)$ is computed for a query value of λ' corresponding to one of the breakpoints of $l^*(\lambda)$. This operations can have two outcomes: either the point $(\lambda', l_{\lambda'}(\lambda'))$ lies on the curve $l^*(\lambda)$ and this breakpoint does not need to be tested further, or $(\lambda', l_{\lambda'}(\lambda'))$ lies below $l^*(\lambda')$. In the second case, the optimal solution found by solving the minimum cost flow problem generates the line $l_{\lambda'}(\lambda)$ which intersects $l^*(\lambda)$ (see l_3 in Figure 2.9b). In this case, $l^*(\lambda)$ can be updated to the point-wise minimum between $l^*(\lambda)$ and $l_{\lambda'}(\lambda)$, leading to new breakpoints to test in the following updates (see Figure 2.9c). The method terminates when no further breakpoints need to be tested.

From a computational perspective, these updates performed within the algorithm require only $2p_S - 1$ queries to a solver for the minimum cost flow problem, where p_S is the number of breakpoints of $c_S(\lambda)$ in the interval Λ (Jenkins 1990).

Intuitively, the result is correct because the performed operations are equivalent to updating an upper and a lower bound until the two converge to the cost curve itself. The upper bound is obtained from optimal solutions, while the lower bound is a piecewise linear and concave approximation of the cost curve. A formal proof is provided in Eisner & Severance (1976).

We now provide a formal description in Algorithm 1, where we use the following notation: given two lines $l_1(\lambda)$ and $l_2(\lambda)$ in the plane, we denote by $l_1 \wedge l_2$ the piecewise linear function obtained by the point-wise minimum of the two lines, and by $l_1 \cap l_2$ the set of the intersection points between the two lines. For ease of notation, we write $l_{\lambda'}$ for the line $l_{\lambda'}(\lambda)$ obtained from an optimal solution at $\lambda = \lambda'$.

Note that the condition at Line 2 treats the case where the cost curve $c_S(\lambda)$ has no breakpoints, meaning that it is a linear function. In this case, the fact that the two lines $l_{\underline{\lambda}}$ and $l_{\overline{\lambda}}$ coincide implies that each of them is optimal for the other extreme point as well. Lines 5–10 formalize the explanation given above. The set A contains the query points to evaluate.

2.8.3 Reduction from the multiple destination case

In this section, we show how our model can accommodate multiple destinations for different orders. We then observe that the same reasoning can be applied to multiple sources, but cannot be extended to the combination of the two, i.e., the case of multiple sources and destinations.

Given a coalition $S \subseteq N$, let $t_1^S, \dots, t_d^S$ be the destinations and $k_{t_1}^S, \dots, k_{t_d}^S$ be the total demands for each destination. We introduce a super sink t^* and arcs $r_j = (t_j, t^*)$ for all $j = 1, \dots, d$, with per unit cost $c_{r_j} = 0$ and capacity $u_{r_j} = k_{t_j}^S$. Finally, we set total demand at node t^* equal to $\sum_{j=1}^{d} k_{t_j}^S$.

This transformation of the graph leads to an equivalent formulation of the minimum cost flow problem for coalition S, where the single destination t^* is used.

A similar reduction can be applied to the case of multiple sources. The case of multiple sources and destinations, however, cannot be modeled with our formulation, as it requires the definition of a multicommodity flow problem in order to distinguish the path followed by each commodity, which might not be conserved otherwise (Ahuja et al. 1993).

2.8.4 Proof of Proposition 1

Proposition 1. *The parametric minimum cost flow game $c(\lambda)$ is subadditive and has a non-empty core for all $\lambda \in \Lambda$.*

Proof. Proof. In what follows, we assume that $\lambda \in \Lambda$ is fixed and consider the parametric minimum cost flow game $c(\lambda)$ for this given value. To simplify notation, we write c_S for $c_S(\lambda)$ for all $S \subseteq N$, and prove that the core of the minimum cost flow game (N, c) is non-empty.

We follow Owen (1975) to prove this claim by showing that the minimum cost flow game without integrality constraints is a balanced cost sharing game. Balancedness of the game is equivalent to non-emptiness of the core. Integral optimal solutions are found by solving the linear relaxation P_L^S of problem P^S obtained by substituting the flow integrality constraint $f_r^S \in \mathbb{N}_{\geq 0}$ with $f_r^S \in \mathbb{R}_{\geq 0}$. This allows for our proof while guaranteeing the same result for our case as integral optimal solutions can be found by solving the relaxed problem. We assume that problem P^S if feasible for all coalitions $S \subseteq N$, so P_L^S is also feasible.

To define balancedness of a game, the concept of a balanced map is required. A map $\gamma : 2^N \setminus \{\varnothing\} \to [0, +\infty)$ is called *balanced* for N if

$$\sum_{S \in 2^N \setminus \{\varnothing\}} \gamma(S) \vec{e}^S = \vec{e}^N \tag{2.19}$$

where, for each $\varnothing \neq S \subseteq N$, the vector $\vec{e}^S \in \mathbb{R}^{|N|}$ is such that

$$e_i^S := \begin{cases} 1 & \text{if } i \in S, \\ 0 & \text{otherwise,} \end{cases} \qquad \forall i \in N.$$

A cost sharing game (N, c) is balanced if, for every balanced map γ for N, we have $\sum_{S \in 2^N \setminus \{\varnothing\}} \gamma(S) c_S \geq c_N$.

In what follows, we prove that the minimum cost flow game is balanced. Let γ be a balanced map for N. Then we have

$$\begin{aligned}\sum_{S\in 2^N\setminus\{\varnothing\}}\gamma(S)k^S &= \sum_{S\in 2^N\setminus\{\varnothing\}}\sum_{i\in S}\gamma(S)k_i\\ &= \sum_{i\in N}\left[\sum_{S\in 2^N\setminus\{\varnothing\}:i\in S}\gamma(S)\right]k_i\\ &= \sum_{i\in N}k_i\\ &= k^N\end{aligned}$$

Now, we have $c_S = \sum_{r\in R^S} c_r f_r^S$ by definition of the cost game, where the optimal flow allocation $\{f_r^S\}_{r\in R^S}$ satisfies constraints (2.3b) (2.3c), (2.3d),(2.3e). Then,

$$\begin{aligned}\sum_{S\in 2^N\setminus\{\varnothing\}}\gamma(S)c_S &= \sum_{S\in 2^N\setminus\{\varnothing\}}\sum_{r\in R^S}\gamma(S)c_r f_r^S\\ &= \sum_{r\in R^N}c_r\sum_{S\in 2^N\setminus\{\varnothing\}:r\in R^S}\gamma(S)f_r^S\\ &= \sum_{r\in R^N}c_r\hat{f}_r\end{aligned}\tag{2.20}$$

where $\hat{f}_r := \sum_{S\in 2^N\setminus\{\varnothing\}:r\in R^S}\gamma(S)f_r^S$ for all arcs $r\in R^N$. It follows that the vector $\{\hat{f}\}_{r\in R^N}$ is feasible for the relaxed problem P_L^N of the grand-coalition. Indeed, for all $v\in V^N\setminus\{s,t\}$

$$\begin{aligned}\sum_{r\in\delta_N^-(v)}\hat{f}_r &= \sum_{r\in\delta_N^-(v)}\sum_{S\in 2^N\setminus\{\varnothing\}:r\in R^S}\gamma(S)f_r^S\\ &= \sum_{S\in 2^N\setminus\{\varnothing\}}\gamma(S)\sum_{r\in\delta_S^-(v)}f_r^S\\ &= \sum_{S\in 2^N\setminus\{\varnothing\}}\gamma(S)\sum_{r\in\delta_S^+(v)}f_r^S\\ &= \sum_{r\in\delta_N^+(v)}\sum_{S\in 2^N\setminus\{\varnothing\}:r\in R^S}\gamma(S)f_r^S\\ &= \sum_{r\in\delta_N^+(v)}\hat{f}_r.\end{aligned}$$

Similarly, we have

$$\begin{aligned}\sum_{r\in\delta_N^+(s)}\hat{f}_r &= \sum_{r\in\delta_N^+(s)}\ \sum_{S\in 2^N\setminus\{\varnothing\}:r\in R^S}\gamma(S)f_r^S\\ &= \sum_{S\in 2^N\setminus\{\varnothing\}}\gamma(S)\sum_{r\in\delta_S^+(s)}f_r^S\\ &= \sum_{S\in 2^N\setminus\{\varnothing\}}\gamma(S)k^S\\ &= k^N.\end{aligned}$$

The same holds for constraint (2.3e).

Finally, $\hat{f}_r \geq 0$ for all $r \in R^N$ because of the non-negativity of the coefficients $\gamma(S)$.

As $\{\hat{f}\}_{r\in R^N}$ is feasible for P_L^N, it follows that

$$c_N \leq \sum_{r\in R^N} c_r\hat{f}_r. \tag{2.21}$$

Then, we see from (2.20) and (2.21) that $\sum_{S\in 2^N\setminus\{\varnothing\}} c_S \geq c_N$, so (N,c) is balanced. □ □

As the proof is independent of λ, this result holds for all $\lambda \in \Lambda$.

2.8.5 Proof of Proposition 2

Proposition 2. *If there exists $\hat{\lambda}\in\Lambda$ and $\hat{S}\subset N$ such that $\Phi_{\hat{S}}(\hat{\lambda}) > c_{\hat{S}}(\hat{\lambda})$, then*

$$\forall\varepsilon\in\left(0,\frac{\Phi_{\hat{S}}(\hat{\lambda})-c_{\hat{S}}(\hat{\lambda})}{2K}\right):\quad \Phi(\lambda)\notin\mathfrak{C}(\lambda)\quad\forall\lambda\in(\hat{\lambda}-\varepsilon,\hat{\lambda}+\varepsilon), \tag{2.22}$$

where $K\geq 0$ is a Lipschitz constant of the functions $\{c_S(\lambda)\}_{S\subseteq N}$ and $\{\Phi_S(\lambda)\}_{S\subseteq N}$, i.e, $\forall\lambda',\lambda''\in\Lambda$, $|c_S(\lambda')-c_S(\lambda'')|\leq K\cdot|\lambda'-\lambda''|$ and $|\Phi_S(\lambda')-\Phi_S(\lambda'')|\leq K\cdot|\lambda'-\lambda''|$ for all $S\subseteq N$.[4]

Proof. Proof. From the hypothesis, it follows that $\Phi(\hat{\lambda})\notin\mathfrak{C}(\hat{\lambda})$. Because of piecewise linearity of the cost curves $c_S(\lambda)$ and the marginal cost functions $\Phi_S(\lambda)$ ($S\subseteq N$), these functions satisfy the Lipschitz property, i.e., there exists a constant $K\geq 0$ such that $|c_S(\lambda')-c_S(\lambda'')|\leq K\cdot|\lambda'-\lambda'|$ for all $\lambda',\lambda''\in\Lambda$ and for all $S\subseteq N$, and similarly for the marginal cost functions $\Phi_S(\lambda)$ (because we deal with a finite number of functions, we can use the same Lipschitz constant for all functions by

[4]In this case, the constant K can be chosen as the highest slope of all functions.

taking the maximum of all of their separate Lipschitz constants). In particular, we obtain that $\Phi_{\hat{S}}(\lambda)$ is bounded from below by $-K \cdot |\hat{\lambda} - \lambda| + \Phi_{\hat{S}}(\hat{\lambda})$ for all $\lambda \in \Lambda$, while $c_{\hat{S}}(\lambda)$ is bounded from above by $K \cdot |\hat{\lambda} - \lambda| + c_{\hat{S}}(\hat{\lambda})$ for all $\lambda \in \Lambda$. These bounds are obtained directly from the Lipschitz property, as shown here for the case of $c_{\hat{S}}(\lambda)$:

$$\begin{aligned}
&|c_{\hat{S}}(\hat{\lambda}) - c_{\hat{S}}(\lambda)| \le K \cdot |\hat{\lambda} - \lambda| \\
\Leftrightarrow\ &|c_{\hat{S}}(\lambda) - c_{\hat{S}}(\hat{\lambda})| \le K \cdot |\hat{\lambda} - \lambda| \\
\Leftrightarrow\ &+K \cdot |\hat{\lambda} - \lambda| \ge c_{\hat{S}}(\lambda) - c_{\hat{S}}(\hat{\lambda}) \ge -K \cdot |\hat{\lambda} - \lambda| \\
\Rightarrow\ &c_{\hat{S}}(\lambda) - c_{\hat{S}}(\hat{\lambda}) \le K \cdot |\hat{\lambda} - \lambda| \\
\Leftrightarrow\ &c_{\hat{S}}(\lambda) \le K \cdot |\hat{\lambda} - \lambda| + c_{\hat{S}}(\hat{\lambda})
\end{aligned}$$

Because of these two bounds for $\Phi_{\hat{S}}(\lambda)$ and $c_{\hat{S}}(\lambda)$, the difference $\Phi_{\hat{S}}(\lambda) - c_{\hat{S}}(\lambda)$ satisfies $\Phi_{\hat{S}}(\lambda) - c_{\hat{S}}(\lambda) \ge -2K|\hat{\lambda} - \lambda| + \Phi_{\hat{S}}(\hat{\lambda}) - c_{\hat{S}}(\hat{\lambda})$. Hence, for any $\varepsilon \in \left(0, \frac{\Phi_{\hat{S}}(\hat{\lambda}) - c_{\hat{S}}(\hat{\lambda})}{2K}\right)$ and any $\lambda \in (\hat{\lambda} - \varepsilon, \hat{\lambda} + \varepsilon)$, we obtain $\Phi_{\hat{S}}(\lambda) - c_{\hat{S}}(\lambda) > 0$, which implies that $\Phi(\lambda) \notin \mathfrak{C}(\lambda)$ and concludes the proof. □

□

2.8.6 Proof of Theorem 1

Theorem 1. *In the case of cooperation on a corridor with identical players, whether the Shapley value is in the core or not depends on the value of the ratio* $\frac{k}{u}$ *compared to the size* $n = |N| \ge 2$ *of the grand coalition as follows:*

$$\Phi(0) \in \mathfrak{C}(0) \qquad \textit{for all} \qquad 0 \le \frac{k}{u} \le 1, \tag{2.23a}$$

$$\Phi(\lambda) \in \mathfrak{C}(\lambda) \quad \forall \lambda > 0 \qquad \textit{if and only if} \qquad \frac{(2n-1)(n-2)}{2n^2 - 3n} \le \frac{k}{u} \le 1 \tag{2.23b}$$

$$\Phi(\lambda) \in \mathfrak{C}(\lambda) \quad \forall -c_0 \le \lambda < 0 \qquad \textit{if and only if} \qquad 0 \le \frac{k}{u} \le \frac{2n-2}{2n^2 - 3n}. \tag{2.23c}$$

Proof. Proof. Testing stability of the Shapley value means computing the combinations of the parameters n, c_0, λ, u, k for which all the inequalities $\sum_{i \in S} \Phi_i(\lambda) \le c_S(\lambda)$ (for $S \subseteq N$) are satisfied. To tackle this problem, we will first decompose the minimum cost flow game into two the sum of two games and exploit linearity of the Shapley value to simplify the computations. At that point, we will distinguish three cases based on the value of λ: $\lambda = 0$ is treated first as instrumental to solve the following two cases of $\lambda > 0$ and $\lambda < 0$. For each of the three cases, we will need to discuss several subcases that are required to make the expression of the Shapley value and the cost function explicit. More precisely, we will compute the cost function of each game, the marginal costs, and the Shapley value. From their expressions, the conditions defining the subcases will become clear. In each subcase, we then make the expressions required to solve the system of inequalities $\{\sum_{i \in S} \Phi_i(\lambda) \leq c_S(\lambda) : S \subseteq N\}$ explicit for the parameters mentioned above. At this stage, the inequalities can be solved exactly, which leads to the conditions in (2.23).

Before delving into the proof, we recall linearity of the Shapley value over the set of cooperative games in characteristic function form (Shapley 1953). Given two games $c^1 = (c^1_S)_{S \subseteq N} \in \mathbb{R}^{2n}$ and $c^2 = (c^2_S)_{S \subseteq N} \in \mathbb{R}^{2n}$ on the same set of players N (where $|N| = n$), the sum $c^1 + c^2$ of the two games is the game defined by $(c^1 + c^2)_S = c^1_S + c^2_S$ for each coalition $S \subseteq N$. Similarly, for a given scalar $\alpha \in \mathbb{R}$, the game αc^1 is defined by $(\alpha c^1)_S = \alpha c_S$ for each coalition $S \subseteq N$. Linearity of the Shapley value implies that $\Phi(c^1 + \alpha c^2) = \Phi(c^1) + \alpha \Phi(c^2)$ (Shapley 1953). As a final remark, we write c for c_0 in the remainder of the proof to simplify the notation.

Case 0: Special cases. We start by dealing with two special cases.

First, we note that the Shapley value is stable in the cases where $k = 0$ or $k = u$, independently of the value of λ or the size n of the grand-coalition. Indeed, if $k = 0$, no orders are transported and all costs are zero, which results in a Shapley value allocation $\Phi(\lambda) \equiv 0$ that is stable (see Inequalities 2.7). For $k = u$, instead, we observe that the cost of each coalition $S \subseteq N$ satisfies $c_S(\lambda) = \sum_{i \in S} c_i(\lambda)$ because all arcs are saturated. Thus, individual rationality of the Shapley value (i.e., $\Phi_i(\lambda) \leq c_i(\lambda)$ for all players $i \in N$) implies coalitional rationality and, therefore, stability of the Shapley value itself $(\sum_{i \in S} \Phi_i(\lambda) \leq \sum_{i \in S} c_i(\lambda) = c_S(\lambda))$.

Second, in case of a two-players cooperation (i.e., $n = 2$) the Shapley value is stable independently of the values of all other parameters. Indeed, coalitional rationality is implied by individual rationality and efficiency of the Shapley value as only the cases of $|S| = 1$ and $S = N$ remain from the inequalities in (2.7).

Having taken care of Case 0, we may assume for the rest of the proof that k and u are such that $0 < k < u$, and that $n \geq 3$.

Case 1 1: $\boldsymbol{\lambda = 0}$. For $\lambda = 0$, all players have identical costs and the cost of each coalition $S \subseteq N$ equals $c_S(0) = |S|kc$. We define the game c^0 having cost $c^0_S := |S|kc$ for each coalition $S \subseteq N$. We denote the Shapley value allocation computed for the game c^0 by $\Phi_i(c^0)$. Note that $\Phi_i(0) = \Phi_i(c^0)$. Since all players are symmetric in this game (and, thus, $\Phi_i(0) = \Phi_j(0)$ for all i, j) and the Shapley value is an efficient solution concept (i.e., $\sum_{i\in N}\Phi_i = c_N^0$), the Shapley value allocation $\Phi_i(0)$ to each player $i \in N$ is $\Phi_i(c^0) = kc$.

In this case, it can be observed that $\sum_{i\in S}\Phi_i(c^0) = |S|kc = c_S^0$ for each coalition $S \subseteq N$. Thus, for all values of k and u such that $0 < \frac{k}{u} < 1$, we have $\Phi(c^0) \in \mathfrak{C}(c^0)$ and coalitional rationality is satisfied in this case.

Case 2: $\lambda > 0$. For a given coalition $S \subseteq N$, the cost function $c_S(\lambda)$ is given by:

$$c_S(\lambda) = \begin{cases} |S|kc & i_p \notin S & \text{(2.24a)} \\ |S|kc & i_p \in S \text{ and } |S| \geq \frac{u}{u-k} & \text{(2.24b)} \\ (|S|-1)uc + [u - |S|(u-k)](c+\lambda) & i_p \in S \text{ and } |S| < \frac{u}{u-k} & \text{(2.24c)} \end{cases}$$

Cases (2.24a) and (2.24b) follow from the fact that $|S|k$ orders can be executed without sending flow on arc r_{i_p}. Note that the inequality $|S| \geq \frac{u}{u-k}$ is equivalent to $|S|k \leq (|S|-1)u$, meaning that the total amount of orders is less than the total capacity of all players in S different from i_p. The expression in (2.24c), instead, follows from the fact that arc r_{i_p} has to be used in spite of its higher cost resulting from the positive value of λ (note that the condition $|S| < \frac{u}{u-k}$ is equivalent to $|S|k > (|S|-1)u$), thus $(|S|-1)$ arcs are used fully at a unit cost of c, and the remaining $|S|k - (|S|-1)u$ orders will use arc r_{i_p} at a unit cost $c+\lambda$.

Now, we decompose the minimum cost flow game $c(\lambda)$ into the sum $c(\lambda) = c^0 + \lambda c^+$ of the two games c^0 and c^+, where c^0 has been defined in Case 1, and c^+ is defined as follows:

$$c_S^+ := \begin{cases} 0 & i_p \notin S\,, & \text{(2.25a)} \\ 0 & i_p \in S \text{ and } |S| \geq \frac{u}{u-k}\,, & \text{(2.25b)} \\ u - |S|(u-k) & i_p \in S \text{ and } |S| < \frac{u}{u-k}\,. & \text{(2.25c)} \end{cases}$$

By comparing each condition in (2.24) and (2.25), it can be checked that $c_S(\lambda) = c_S^0 + \lambda c_S^+$ for all $S \subseteq N$. Intuitively, the cost game c^+ counts only the amount of orders transported on arc r_{i_p}, so that λc^+ is the additional cost generated by transporting on arc r_{i_p}.

From this decomposition and linearity of the Shapley value, it follows that $\Phi_i(\lambda) = \Phi_i(c^0) + \lambda\Phi_i(c^+)$ for all players $i \in N$, and that, for each coalition $S \subseteq N$, the inequality $\sum_{i\in S}\Phi_i(\lambda) \leq c_S(\lambda)$ can be written as:

$$\sum_{i\in S}\Phi_i(\lambda)\leq c(\lambda) \tag{2.26a}$$
$$\Leftrightarrow \quad \sum_{i\in S}\left(\Phi_i(c^0)+\lambda\Phi_i(c^+)\right)\leq c_S^0+\lambda c_S^+ \tag{2.26b}$$
$$\Leftrightarrow \quad \sum_{i\in S}\Phi_i(c^0)+\lambda\sum_{i\in S}\Phi_i(c^+)\leq c_S^0+\lambda c_S^+ \tag{2.26c}$$
$$\Leftrightarrow \quad \lambda\sum_{i\in S}\Phi_i(c^+)\leq \lambda c_S^+ \qquad \left(\text{as } \sum_{i\in S}\Phi_i(c^0)=c_S^0\right) \tag{2.26d}$$
$$\Leftrightarrow \quad \sum_{i\in S}\Phi_i(c^+)\leq c_S^+ \qquad \left(\text{as } \lambda>0\right) \tag{2.26e}$$

Therefore, the Shapley value is stable in the game $c(\lambda)$ for $\lambda > 0$ if and only if the Shapley value is stable in the game c^+. Moreover, note that, in (2.26), we have obtained both independence of λ and a much simpler game to analyze.

In order to compute the expression of the Shapley value, we compute the marginal cost $c^+_{S\cup\{i\}} - c^+_S$ for each player $i \in N$ and each coalition $S \subseteq N \setminus \{i\}$. We distinguish two cases, namely, whether $i \neq i_p$ or $i = i_p$.

For $i \neq i_p$, we have:

$$c^+_{S\cup\{i\}} - c^+_S = \begin{cases} 0 & i_p \notin S & (2.27\text{a}) \\ k-u & i_p \in S \text{ and } |S| < \frac{k}{u-k} & (2.27\text{b}) \\ |S|(u-k)-u & i_p \in S \text{ and } \frac{k}{u-k} \leq |S| < \frac{u}{u-k} & (2.27\text{c}) \\ 0 & i_p \in S \text{ and } \frac{u}{u-k} \leq |S| & (2.27\text{d}) \end{cases}$$

For $i = i_p$, we know that $i_p \notin S$, and the marginal cost equals

$$c^+_{S\cup\{i_p\}} - c^+_S = \begin{cases} k & |S| = 0 & (2.28\text{a}) \\ k-|S|(u-k) & \text{and } 0 < |S| < \frac{k}{u-k} & (2.28\text{b}) \\ 0 & \frac{k}{u-k} \leq |S| & (2.28\text{c}) \end{cases}$$

With those expressions for the marginal costs, we compute the Shapley value allocation to each player. We first treat the case of players $i \neq i_p$ and then the case of player $i = i_p$. We denote the Shapley value allocation to a player $i \neq i_p$ by $\Phi_{-i_p}(c^+)$, which can be computed as follows:

$$\Phi_{-i_p}(c^+) = \frac{1}{n} \sum_{S \subseteq N \setminus \{i\}} \binom{n-1}{|S|}^{-1} (c^+_{S \cup \{i\}} - c^+_S) \tag{2.29a}$$

$$= \frac{1}{n} \left(\sum_{\substack{S \subseteq N \setminus \{i\}: \\ i_p \in S, \\ \frac{k}{u-k} \leq |S| < \frac{u}{u-k}}} \binom{n-1}{|S|}^{-1} (c^+_{S \cup \{i\}} - c^+_S) + \sum_{\substack{S \subseteq N \setminus \{i\}: \\ i_p \in S, \\ |S| < \frac{k}{u-k}}} \binom{n-1}{|S|}^{-1} (c^+_{S \cup \{i\}} - c^+_S) \right) \tag{2.29b}$$

$$= \frac{1}{n} \left(\sum_{\substack{l=1,\dots,n-1: \\ \frac{k}{u-k} \leq l < \frac{u}{u-k}}} \sum_{\substack{S \subseteq N \setminus \{i\}: \\ i_p \in S, \\ |S| = l}} \binom{n-1}{l}^{-1} \big(l(u-k) - u\big) + \right.$$

$$\left. + \sum_{\substack{l=1,\dots,n-1: \\ l < \frac{k}{u-k}}} \sum_{\substack{S \subseteq N \setminus \{i\}: \\ i_p \in S, \\ |S| = l}} \binom{n-1}{l}^{-1} (k-u) \right) \tag{2.29c}$$

$$= \frac{1}{n} \left(\sum_{\substack{l=1,\dots,n-1: \\ \frac{k}{u-k} \leq l < \frac{u}{u-k}}} \binom{n-1}{l}^{-1} (l(u-k) - u) \binom{n-2}{l-1} + \right.$$

$$\left. + \sum_{\substack{l=1,\dots,n-1: \\ l < \frac{k}{u-k}}} \binom{n-1}{l}^{-1} (k-u) \binom{n-2}{l-1} \right) \tag{2.29d}$$

$$= \frac{1}{n} \left(\sum_{\substack{l=1,\dots,n-1: \\ \frac{k}{u-k} \leq l < \frac{u}{u-k}}} \frac{l}{n-1} (l(u-k) - u) + \sum_{\substack{l=1,\dots,n-1: \\ l < \frac{k}{u-k}}} \frac{l}{n-1} (k-u) \right) \tag{2.29e}$$

We now compute the Shapley value allocation to player $i = i_p$:

$$\Phi_{i_p}(c^+) = \frac{1}{n} \sum_{S \subseteq N \setminus \{i\}} \binom{n-1}{|S|}^{-1} (c^+_{S \cup \{i\}} - c^+_S) \tag{2.30a}$$

$$= \frac{1}{n} \left(\sum_{\substack{S \subseteq N \setminus \{i_p\}: \\ |S| < \frac{k}{u-k}}} \binom{n-1}{|S|}^{-1} (c^+_{S \cup \{i_p\}} - c^+_S) \right) \tag{2.30b}$$

$$= \frac{1}{n}\left(\sum_{\substack{l=0,\dots,n-1:\\ l<\frac{k}{u-k}}} \sum_{\substack{S\subseteq N\setminus\{i_p\}:\\ |S|=l}} \binom{n-1}{l}^{-1} \big(k-l(u-k)\big)\right) \tag{2.30c}$$

$$= \frac{1}{n}\left(\sum_{\substack{l=0,\dots,n-1:\\ l<\frac{k}{u-k}}} \binom{n-1}{l}^{-1} (k-l(u-k))\binom{n-1}{l}\right) \tag{2.30d}$$

$$= \frac{1}{n}\left(\sum_{\substack{l=0,\dots,n-1:\\ l<\frac{k}{u-k}}} (k-l(u-k))\right) \tag{2.30e}$$

$$= \frac{1}{n}\left(k+\sum_{\substack{l=1,\dots,n-1:\\ l<\frac{k}{u-k}}} (k-l(u-k))\right) \tag{2.30f}$$

It is possible to rewrite the expressions in (2.29e) and (2.30f) further by discussing the index sets of the summations as, depending on the value of $\frac{k}{u}$, they can be empty. We distinguish three cases that are presented in Table 2.6 and discuss them now.

In case $0 < \frac{k}{u} \le \frac{1}{2}$, we have $\frac{k}{u-k} \le 1$ and $\frac{u}{u-k} \le 2$, and the index set $\{l = 1,\dots,n-1 : \frac{k}{u-k} \le l < \frac{u}{u-k}\}$ reduces to the single value $l = 1$, while the index set $\{l = 1,\dots,n-1 : l < \frac{k}{u-k}\}$ is empty and the summations with this index set are equal to zero.

In case $\frac{1}{2} < \frac{k}{u} \le \frac{n-1}{n}$, we have $\frac{k}{u-k} \le n-1$ and the index set $\{l = 1,\dots,n-1 : \frac{k}{u-k} \le l < \frac{u}{u-k}\}$ reduces to the singleton $\{l = \lceil \frac{k}{u-k} \rceil\}$, as $\lceil \frac{k}{u-k} \rceil$ is the only integer number in $\left[\frac{k}{u-k}, \frac{u}{u-k}\right)$. The index set $\{l = 1,\dots,n-1 : l < \frac{k}{u-k}\}$ equals $\{l = 1,\dots,\lceil \frac{k}{u-k} \rceil - 1\}$. Indeed, $\lceil \frac{k}{u-k} \rceil - 1$ is the largest integer number lower than $\frac{k}{u-k}$ (note that $\lceil \frac{k}{u-k} \rceil - 1 = \lfloor \frac{k}{u-k} \rfloor$, we opt for this expression for later convenience).

In case $\frac{n-1}{n} < \frac{k}{u} \le 1$, we have $\frac{k}{u-k} > n-1$ and the index set $\{l = 1,\dots,n-1 : \frac{k}{u-k} \le l < \frac{u}{u-k}\}$ is empty, while the index set $\{l = 1,\dots,n-1 : l < \frac{k}{u-k}\}$ equals $\{l = 1,\dots,n-1\}$.

For each subcase that we just defined, we now solve the following set of inequalities: $\sum_{i\in S} \Phi_i(c^+) \le c^+_S$ for all $S \subseteq N$ such that $2 \le |S| \le n-1$. Note that, because the Shapley value is an individually rational and efficient solution concept, the inequalities for $|S| = 1$ and $|S| = n$ (i.e., $S = N$) are already satisfied for any value of the parameters. Moreover, we first consider the inequalities for coalitions S such that $i_p \notin S$ first, and afterwards the ones for which $i_p \in S$.

Case	$\Phi_{-i_p}(c^+)$	$\Phi_{i_p}(c^+)$
$\mathbf{0 < \frac{k}{u} \leq \frac{1}{2}}$	$\frac{1}{n}\big(\sum\limits_{l=1}\frac{l}{n-1}(l(u-k)-u)+0\big)$	$\frac{1}{n}(k+0)$
$\mathbf{\frac{1}{2} < \frac{k}{u} \leq \frac{n-1}{n}}$	$\frac{1}{n}\big(\sum\limits_{l=\lceil\frac{k}{u-k}\rceil}\frac{l}{n-1}(l(u-k)-u)+\sum\limits_{l=1}^{\lceil\frac{k}{u-k}\rceil-1}\frac{l}{n-1}(k-u)\big)$	$\frac{1}{n}\big(k+\sum\limits_{l=1}^{\lceil\frac{k}{u-k}\rceil-1}(k-l(u-k))\big)$
$\mathbf{\frac{n-1}{n} < \frac{k}{u} \leq 1}$	$\frac{1}{n}\big(0+\sum\limits_{l=1}^{n-1}\frac{l}{n-1}(k-u)\big)$	$\frac{1}{n}\big(k+\sum\limits_{l=1}^{n-1}(k-l(u-k))\big)$

Table 2.6: Summary of the expressions of the Shapley value for the case $\lambda > 0$. Note that the expressions have not been fully simplified to highlight the effect of the various cases on the index set.

Case 2.1: $\mathbf{0 < \frac{k}{u} \leq \frac{1}{2}}$. We first simplify the expressions for $\Phi_{-i_p}(c^+)$ and $\Phi_{i_p}(c^+)$ from Table 2.6:

$$\Phi_{-i_p}(c^+) = \frac{1}{n}\Big(\sum\nolimits_{l=1}\frac{l}{n-1}(l(u-k)-u)\Big) \tag{2.31a}$$

$$= -\frac{k}{n(n-1)} \tag{2.31b}$$

$$\Phi_{i_p}(c^+) = \frac{k}{n} \tag{2.31c}$$

We now test coalitional rationality, first for coalitions $S \subsetneq N$ such that $i_p \notin S$. In this case, the cost of coalition S equals $c_S^+ = 0$ and each inequality $\sum_{i \in S}\Phi_i(c^+) \leq 0$ is equivalent to $\Phi_{-i_p}(c^+) \leq 0$. As $k > 0$ and $n \geq 2$, $\Phi_{-i_p}(c^+) < 0$ and coalitional rationality is satisfied for these coalitions.

We now consider coalitions $S \subsetneq N$ such that $i_p \in S$. In this case, $c_S^+ = 0$ and the inequality to solve is the following:

$$\sum_{i \in S \setminus \{i_p\}} \Phi_i(c^+) + \Phi_{i_p}(c^+) \leq 0 \tag{2.32a}$$

$$\Leftrightarrow \quad -\frac{(|S|-1)k}{n(n-1)} + \frac{k}{n} \leq 0 \tag{2.32b}$$

$$\Leftrightarrow \quad -(|S|-1) + n - 1 \leq 0 \tag{2.32c}$$

$$\Leftrightarrow \quad n - |S| \leq 0 \tag{2.32d}$$

Since $S \subsetneq N$, (2.32d) cannot hold, which means that coalitional rationality is not satisfied for these coalitions S. Thus, we proved that $\Phi(c^+) \notin \mathfrak{C}(c^+)$ for $0 < \frac{k}{u} \leq \frac{1}{2}$ as coalitional rationality failed for some coalitions. Note that the previous contradiction holds for all coalition sizes we need to test (i.e., $2 \leq |S| \leq n-1$).

Case 2.2: $\frac{1}{2} < \frac{k}{u} \leq \frac{n-1}{n}$. We follow the same procedure as in the previous case: we start by computing the expressions for $\Phi_{-i_p}(c^+)$ and $\Phi_{i_p}(c^+)$ from Table 2.6:

$$\Phi_{-i_p}(c^+) = \frac{1}{n}\Big(\sum_{l=\lceil \frac{k}{u-k} \rceil} \frac{l}{n-1}(l(u-k)-u) + \sum_{l=1}^{\lceil \frac{k}{u-k} \rceil - 1} \frac{l}{n-1}(k-u)\Big) \tag{2.33a}$$

$$= \frac{1}{n}\left(\frac{\lceil \frac{k}{u-k} \rceil}{n-1}(\left\lceil \frac{k}{u-k} \right\rceil (u-k)-u) + \frac{(k-u)}{n-1}\sum_{l=1}^{\lceil \frac{k}{u-k} \rceil - 1} l\right) \tag{2.33b}$$

$$= \frac{1}{n(n-1)}\left(\left\lceil \frac{k}{u-k} \right\rceil \left(\left\lceil \frac{k}{u-k} \right\rceil (u-k)-u\right) + (k-u)\frac{\lceil \frac{k}{u-k} \rceil (\lceil \frac{k}{u-k} \rceil - 1)}{2}\right) \tag{2.33c}$$

$$= \frac{1}{2n(n-1)}\left\lceil \frac{k}{u-k} \right\rceil \left(\left\lceil \frac{k}{u-k} \right\rceil (u-k)-u-k\right) \tag{2.33d}$$

$$\Phi_{i_p}(c^+) = \frac{1}{n}\Big(k + \sum_{l=1}^{\lceil \frac{k}{u-k} \rceil - 1}(k - l(u-k))\Big) \tag{2.33e}$$

$$= \frac{1}{n}\left(k + (k-u)\frac{\lceil \frac{k}{u-k} \rceil (\lceil \frac{k}{u-k} \rceil - 1)}{2} + k\left(\left\lceil \frac{k}{u-k} \right\rceil - 1\right)\right) \tag{2.33f}$$

$$= \frac{1}{n}\left\lceil \frac{k}{u-k} \right\rceil \left(\frac{(k-u)}{2}\left(\left\lceil \frac{k}{u-k} \right\rceil - 1\right) + k\right) \tag{2.33g}$$

$$= \frac{1}{2n}\left\lceil \frac{k}{u-k} \right\rceil \left((k-u)\left\lceil \frac{k}{u-k} \right\rceil + k + u\right) \tag{2.33h}$$

We now test coalitional rationality for coalitions S with $i_p \notin S$. We know that $c_S^+ = 0$ for these coalitions in this case. Thus, $\sum_{i \in S} \Phi_i(c^+) \leq c_S^+$ reduces to $\Phi_{-i_p}(c^+) \leq 0$, which can be rewritten as follows:

$$\Phi_{-i_p}(c^+) \leq 0 \tag{2.34a}$$

$$\Leftrightarrow \quad \frac{1}{2n(n-1)}\left\lceil \frac{k}{u-k} \right\rceil \left(\left\lceil \frac{k}{u-k} \right\rceil (u-k)-u-k\right) \leq 0 \tag{2.34b}$$

$$\Leftrightarrow \quad \left\lceil \frac{k}{u-k} \right\rceil \leq \frac{u+k}{u-k} \tag{2.34c}$$

$$\Leftrightarrow \quad \left\lceil \frac{k}{u-k} \right\rceil - \frac{k}{u-k} \leq \frac{u}{u-k} \tag{2.34d}$$

As for all $0 < k < u$, $\lceil \frac{k}{u-k} \rceil - \frac{k}{u-k} < 1$ by definition of the ceiling operation and $\frac{u}{u-k} \geq 1$ as $u - k < u$, we have that (2.34d) is always satisfied for coalitions S such that $i_p \notin S$.

We now test coalitional rationality for coalitions S such that $i_p \in S$. From $\frac{1}{2} < \frac{k}{u} \leq \frac{n-1}{n}$, we obtain $2 < \frac{u}{u-k} \leq n$ and, therefore, we need to distinguish two further cases depending on the size $|S|$ of coalition S compared to $\frac{u}{u-k}$ as in the definition of c_S^+:

$$c_S^+ = \begin{cases} 0 & i_p \in S \text{ and } |S| \geq \frac{u}{u-k}\ , \quad (2.35a) \\ u - |S|(u-k) & i_p \in S \text{ and } |S| < \frac{u}{u-k}\ . \quad (2.35b) \end{cases}$$

Therefore, we start with the case where $|S| \geq \frac{u}{u-k}$. It is important to note that, since $|S| \leq n-1$, for values of $\frac{u}{u-k} > n-1$ this case will not occur as no coalition has such a size and we are left with the case that will be discussed later. The condition $\frac{u}{u-k} > n-1$ is equivalent to $\frac{k}{u} > \frac{n-2}{n-1}$. Note that $\frac{n-2}{n-1} < \frac{n-1}{n}$ for all $n > 2$.

$$(|S|-1)\Phi_{-i_p}(c^+) + \Phi_{i_p}(c^+) \leq 0 \tag{2.36a}$$

$$\Leftrightarrow \quad \frac{|S|-1}{2n(n-1)}\left\lceil\frac{k}{u-k}\right\rceil\left(\left\lceil\frac{k}{u-k}\right\rceil(u-k) - u - k\right) + \frac{1}{2n}\left\lceil\frac{k}{u-k}\right\rceil\left((k-u)\left\lceil\frac{k}{u-k}\right\rceil + k + u\right) \leq 0 \tag{2.36b}$$

$$\Leftrightarrow \quad \frac{n-|S|}{2(n-1)}\left\lceil\frac{k}{u-k}\right\rceil\left(k\left\lceil\frac{k}{u-k}\right\rceil + k - u\left\lceil\frac{k}{u-k}\right\rceil + u\right) \leq 0 \tag{2.36c}$$

$$\Leftrightarrow \quad \left\lceil\frac{k}{u-k}\right\rceil(k-u) + u + k \leq 0 \tag{2.36d}$$

$$\Leftrightarrow \quad u + k \leq (u-k)\left\lceil\frac{k}{u-k}\right\rceil \tag{2.36e}$$

$$\Leftrightarrow \quad \frac{u}{u-k} \leq \left\lceil\frac{k}{u-k}\right\rceil - \frac{k}{u-k} \tag{2.36f}$$

As $1 \leq \frac{u}{u-k}$ and $\left\lceil\frac{k}{u-k}\right\rceil - \frac{k}{u-k} < 1$ by definition of the ceiling operation, (2.36f) does not hold, which means that coalitional rationality is not satisfied in this case. Thus, for $\frac{1}{2} < \frac{k}{u} \leq \frac{n-2}{n-1}$, we have $\Phi(c^+) \notin \mathfrak{C}(c^+)$.

We can now test coalitional rationality for the case $|S| < \frac{u}{u-k}$ when $\frac{n-2}{n-1} < \frac{k}{u} \leq \frac{n-1}{n}$, as for the case $\frac{1}{2} < \frac{k}{u} \leq \frac{n-2}{n-1}$ coalitional rationality was not achieved. We note that $\frac{u}{u-k} > n-1$ is equivalent to $\frac{k}{u-k} > n-2$, and that $\frac{k}{u} \leq \frac{n-1}{n}$ is equivalent to $\frac{u}{u-k} \leq n$, from which it follows that $n-2 < \frac{k}{u-k} \leq n-1$

and $\left\lceil \frac{k}{u-k} \right\rceil = n-1$. We can then solve the following

$$(|S|-1)\Phi_{-i_p}(c^+) + \Phi_{i_p}(c^+) \le u - |S|(u-k) \tag{2.37a}$$

$$\Leftrightarrow \quad \frac{|S|-1}{2n(n-1)} \left\lceil \frac{k}{u-k} \right\rceil \left(\left\lceil \frac{k}{u-k} \right\rceil (u-k) - u - k \right) + \frac{1}{2n} \left\lceil \frac{k}{u-k} \right\rceil \left((k-u) \left\lceil \frac{k}{u-k} \right\rceil + k + u \right) \le u - |S|(u-k) \tag{2.37b}$$

$$\Leftrightarrow \quad \frac{1}{2n}(kn^2 - kn|S| - 2k|S| - n^2u + n|S|u + 2nu - 2u) \le 0 \tag{2.37c}$$

$$\Leftrightarrow \quad kn^2 + |S|((3n-2)u - 3kn) - n^2u \le 0 \tag{2.37d}$$

At this point, we note that this expression is linear in $|S|$ which takes values in $\{2, 3, \dots, n-1\}$. Thus, depending on the sign of the coefficient $(3n-2)u - 3kn$, it will be sufficient to test this expression for $|S| = 2$ or $|S| = n-1$ as the maximum of the expression on the left must be achieved at one of these two extremes. We test for positivity of the coefficient:

$$(3n-2)u - 3kn > 0 \tag{2.38a}$$

$$\Leftrightarrow \quad \frac{k}{u} > \frac{3n-2}{3n} \tag{2.38b}$$

Note that $\frac{3n-2}{3n} > \frac{n-1}{n}$ and, therefore, the coefficient $(3n-2)u - 3kn$ is positive for all values of k and u such that $\frac{n-2}{n-1} < \frac{k}{u} \le \frac{n-1}{n}$. We can then replace $|S| = n-1$ in (2.37d) and obtain:

$$kn(3-2n) + u(2 - 5n + 2n^2) \le 0 \tag{2.39a}$$

$$\Leftrightarrow \quad (2 - 5n + 2n^2) \le \frac{k}{u}(2n^2 - 3n) \tag{2.39b}$$

$$\Leftrightarrow \quad \frac{2n^2 - 5n + 2}{2n^2 - 3n} \le \frac{k}{u} \tag{2.39c}$$

$$\Leftrightarrow \quad \frac{(2n-1)(n-2)}{2n^2 - 3n} \le \frac{k}{u} \tag{2.39d}$$

Because we have $\frac{(2n-1)(n-2)}{2n^2-3n} > \frac{n-2}{n-1}$ and $\frac{(2n-1)(n-2)}{2n^2-3n} < \frac{n-1}{n}$ for all $n > 2$, we can conclude that, for values of k and u such that $\frac{(2n-1)(n-2)}{2n^2-3n} \le \frac{k}{u} \le \frac{n-1}{n}$, the Shapley value is stable, i.e., $\Phi(c^+) \in \mathfrak{C}(c^+)$.

Case 2.3: $\frac{n-1}{n} < \frac{k}{u} \le 1$. Again, we follow the same procedure as above: we start by computing the expressions for $\Phi_{-i_p}(c^+)$ and $\Phi_{i_p}(c^+)$ from Table 2.6:

$$\Phi_{-i_p}(c^+) = \frac{1}{n}\Big(0 + \sum\nolimits_{l=1}^{n-1} \frac{l}{n-1}(k-u)\Big) \tag{2.40a}$$
$$= \frac{(k-u)}{n(n-1)}\frac{n(n-1)}{2} \tag{2.40b}$$
$$= \frac{k-u}{2} \tag{2.40c}$$
$$\Phi_{i_p}(c^+) = \frac{1}{n}\Big(k + \sum\nolimits_{l=1}^{n-1}(k - l(u-k))\Big) \tag{2.40d}$$
$$= \frac{1}{n}\Big(k + (n-1)k - (u-k)\frac{n(n-1)}{2}\Big) \tag{2.40e}$$
$$= k + (k-u)\frac{n-1}{2} \tag{2.40f}$$

We now check coalitional rationality starting with coalitions S such that $i_p \notin S$. In this case, $c_S^+ = 0$, and the inequality to test is:

$$|S|\Phi_{-i_p}(c^+) \leq 0 \tag{2.41a}$$
$$\Leftrightarrow \quad \Phi_{-i_p}(c^+) \leq 0 \tag{2.41b}$$
$$\Leftrightarrow \quad \frac{k-u}{2} \leq 0 \tag{2.41c}$$

Since $k < u$, (2.41c) holds true, which means that coalitional rationality is satisfied for these coalitions.

We now test coalitional rationality for coalitions S such that $i_p \in S$. In this case, we have $c_S^+ = u - |S|(u-k)$ and

$$(|S|-1)\Phi_{-i_p}(c^+) + \Phi_{i_p}(c^+) \leq c_S^+ \tag{2.42a}$$
$$\Leftrightarrow \quad (|S|-1)\frac{k-u}{2} + k + (k-u)\frac{n-1}{2} \leq u - |S|(u-k) \tag{2.42b}$$
$$\Leftrightarrow \quad (k-u)(n-2) \leq 0 \tag{2.42c}$$

Since $k < u$, (2.42c) holds true, which means that we have shown that $\Phi(c^+) \in \mathfrak{C}(c^+)$ for $\frac{n-1}{n} < \frac{k}{u} \leq 1$.

A summary of the findings in the previous three cases is presented in Table 2.7, which can be combined into

$$\Phi(c^+) \in \mathfrak{C}(c^+) \text{ if and only if } \frac{(2n-1)(n-2)}{2n^2-3n} \leq \frac{k}{u} \leq 1. \tag{2.43}$$

We can now move to consider the third and last case.

Case	Result
$\mathbf{0 < \frac{k}{u} \leq \frac{1}{2}}$	$\Phi(c^+) \notin \mathfrak{C}(c^+)$
$\mathbf{\frac{1}{2} < \frac{k}{u} \leq \frac{n-1}{n}}$	$\Phi(c^+) \in \mathfrak{C}(c^+)$ if and only if $\frac{(2n-1)(n-2)}{2n^2-3n} \leq \frac{k}{u} \leq \frac{n-1}{n}$
$\mathbf{\frac{n-1}{n} < \frac{k}{u} \leq 1}$	$\Phi(c^+) \in \mathfrak{C}(c^+)$

Table 2.7: Summary of the findings in the three cases for $\lambda > 0$.

Case 3: $\boldsymbol{\lambda < 0}$. Our discussion of this case follows the same structure as the one of Case 2.

For a given coalition $S \subseteq N$ and $\lambda \in [-c, 0]$, the cost $c(\lambda)$ of coalition S is given as follows:

$$c_S(\lambda) = \begin{cases} |S|kc & i_p \notin S & (2.44a) \\ |S|k(c+\lambda) & i_p \in S \text{ and } |S| \leq \frac{u}{k} & (2.44b) \\ u(c+\lambda) + c(|S|k - u) & i_p \in S \text{ and } |S| > \frac{u}{k} & (2.44c) \end{cases}$$

The conditions $|S| \leq \frac{u}{k}$ and $|S| > \frac{u}{k}$ in (2.44b) and (2.44c) can be rewritten as $u \geq |S|k$ and $u < |S|k$, respectively. They represent the two cases where all the $|S|k$ orders can either be transported on arc r_{i_p} or not. Indeed, as $\lambda < 0$, arc r_{i_p} is used first in any optimal solution of the minimum cost flow problem for coalition S.

We rewrite $c(\lambda)$ as $c(\lambda) = c^0 + \lambda c^-$, where

$$c_S^- = \begin{cases} 0 & i_p \notin S\,, & (2.45a) \\ |S|k & i_p \in S \text{ and } |S| \leq \frac{u}{k}\,, & (2.45b) \\ u & i_p \in S \text{ and } |S| > \frac{u}{k}\,. & (2.45c) \end{cases}$$

We now observe that, as in Case 2, this decomposition simplifies checking the stability of the Shapley value. Indeed, using linearity of the Shapley value, we can write $\Phi_i(\lambda) = \Phi_i(c^0) + \lambda\Phi_i(c^-)$ for each player $i \in N$ and, for each coalition $S \subseteq N$, we can rewrite the inequality $\sum_{i \in S} \Phi_i(\lambda) \leq c_S(\lambda)$ as follows:

$$\sum_{i \in S} \Phi_i(\lambda) \leq c(\lambda) \tag{2.46a}$$

$$\Leftrightarrow \quad \sum_{i \in S} \left(\Phi_i(c^0) + \lambda \Phi_i(c^-) \right) \leq c_S^0 + \lambda c_S^- \tag{2.46b}$$

$$\Leftrightarrow \quad \sum_{i \in S} \Phi_i(c^0) + \lambda \sum_{i \in S} \Phi_i(c^-) \leq c_S^0 + \lambda c_S^- \tag{2.46c}$$

$$\Leftrightarrow \quad \lambda \sum_{i \in S} \Phi_i(c^-) \leq \lambda c_S^- \qquad \left(\text{as } \sum_{i \in S} \Phi_i(c^0) = c_S^0\right) \tag{2.46d}$$

$$\Leftrightarrow \quad \sum_{i \in S} \Phi_i(c^-) \geq c_S^- \qquad (\text{as } \lambda < 0) \tag{2.46e}$$

Therefore, the Shapley value is stable in the game $c(\lambda)$ for $\lambda < 0$ if and only if $\sum_{i \in S} \Phi_i(c^-) \geq c_S^-$ for all coalitions $S \subseteq N$. As in Case 2, note that, in (2.46), we have obtained both independence of λ and a much simpler game to analyze.

We now compute the marginal cost for each player $i \neq i_p$:

$$c_{S \cup \{i\}}^- - c_S^- = \begin{cases} 0 & i_p \notin S & (2.47\text{a}) \\ k & i_p \in S \text{ and } |S| + 1 \leq \frac{u}{k} & (2.47\text{b}) \\ u - |S|k & i_p \in S \text{ and } \frac{u}{k} - 1 < |S| \leq \frac{u}{k} & (2.47\text{c}) \\ 0 & i_p \in S \text{ and } |S| > \frac{u}{k} & (2.47\text{d}) \end{cases}$$

For $i = i_p$, we have $i_p \notin S$, and the marginal cost equals

$$c_{S \cup \{i_p\}}^- - c_S^- = \begin{cases} (|S| + 1)k & |S| + 1 \leq \frac{u}{k} & (2.48\text{a}) \\ u & \text{and } |S| + 1 > \frac{u}{k}\ . & (2.48\text{b}) \end{cases}$$

We now compute the expressions for $\Phi_{-i_p}(c^-)$ and $\Phi_{i_p}(c^-)$.

For $i \neq i_p$:

$$\Phi_i(c^-) = \frac{1}{n} \sum_{S \subseteq N \setminus \{i\}} \binom{n-1}{|S|}^{-1} (c^-_{S \cup \{i\}} - c^-_S) \tag{2.49a}$$

$$= \frac{1}{n} \left(\sum_{\substack{S \subseteq N \setminus \{i\}: \\ i_p \in S, \\ |S| \leq \frac{u}{k} - 1}} \binom{n-1}{|S|}^{-1} (c^-_{S \cup \{i\}} - c^-_S) + \sum_{\substack{S \subseteq N \setminus \{i\}: \\ i_p \in S, \\ \frac{u}{k} - 1 < |S| \leq \frac{u}{k}}} \binom{n-1}{|S|}^{-1} (c^-_{S \cup \{i\}} - c^-_S) \right) \tag{2.49b}$$

$$= \frac{1}{n} \left(\sum_{\substack{l = 1, \ldots, n-1: \\ l \leq \frac{u}{k} - 1}} \sum_{\substack{S \subseteq N \setminus \{i\}: \\ i_p \in S, \\ |S| = l}} \binom{n-1}{l}^{-1} k + \sum_{\substack{l = 1, \ldots, n-1: \\ \frac{u}{k} - 1 < l \leq \frac{u}{k}}} \sum_{\substack{S \subseteq N \setminus \{i\}: \\ i_p \in S, \\ |S| = l}} \binom{n-1}{l}^{-1} (u - lk) \right) \tag{2.49c}$$

$$= \frac{1}{n} \left(\sum_{\substack{l = 1, \ldots, n-1: \\ l \leq \frac{u}{k} - 1}} \binom{n-1}{l}^{-1} \binom{n-2}{l-1} k + \sum_{\substack{l = 1, \ldots, n-1: \\ \frac{u}{k} - 1 < l \leq \frac{u}{k}}} \binom{n-1}{l}^{-1} \binom{n-2}{l-1} (u - lk) \right) \tag{2.49d}$$

$$= \frac{1}{n} \left(\sum_{\substack{l = 1, \ldots, n-1: \\ l \leq \frac{u}{k} - 1}} k \frac{l}{n-1} + \sum_{\substack{l = 1, \ldots, n-1: \\ \frac{u}{k} - 1 < l \leq \frac{u}{k}}} (u - lk) \frac{l}{n-1} \right) \tag{2.49e}$$

For $i = i_p$:

$$\Phi_{i_p}(c^-) = \frac{1}{n} \sum_{S \subseteq N \setminus \{i_p\}} \binom{n-1}{|S|}^{-1} (c^-_{S \cup \{i_p\}} - c^-_S) \tag{2.50a}$$

$$= \frac{1}{n} \left(\sum_{\substack{S \subseteq N \setminus \{i_p\}: \\ |S| \leq \frac{u}{k} - 1}} \binom{n-1}{|S|}^{-1} (c^-_{S \cup \{i_p\}} - c^-_S) + \sum_{\substack{S \subseteq N \setminus \{i_p\}: \\ |S| > \frac{u}{k} - 1}} \binom{n-1}{|S|}^{-1} (c^-_{S \cup \{i_p\}} - c^-_S) \right) \tag{2.50b}$$

$$= \frac{1}{n} \left(\sum_{\substack{l = 0, 1, \ldots, n-1: \\ l \leq \frac{u}{k} - 1}} \sum_{\substack{S \subseteq N \setminus \{i_p\}: \\ |S| = l}} \binom{n-1}{l}^{-1} (l+1)k + \right.$$

$$\left. + \sum_{\substack{l = 0, 1, \ldots, n-1: \\ l > \frac{u}{k} - 1}} \sum_{\substack{S \subseteq N \setminus \{i_p\}: \\ |S| = l}} \binom{n-1}{l}^{-1} u \right) \tag{2.50c}$$

$$= \frac{1}{n} \left(\sum_{\substack{l = 0, 1, \ldots, n-1: \\ l \leq \frac{u}{k} - 1}} \binom{n-1}{l}^{-1} \binom{n-1}{l} (l+1)k + \sum_{\substack{l = 0, 1, \ldots, n-1: \\ l > \frac{u}{k} - 1}} \binom{n-1}{l}^{-1} \binom{n-1}{l} u \right) \tag{2.50d}$$

$$= \frac{1}{n}\left(\sum_{\substack{l=0,1,\ldots,n-1:\\ l\le \frac{u}{k}-1}} (l+1)k + \sum_{\substack{l=0,1,\ldots,n-1:\\ l> \frac{u}{k}-1}} u\right) \tag{2.50e}$$

It is possible to rewrite the expressions in (2.50e) and in (2.49e) further by analysing the index sets of the two summations as, depending on the value of $\frac{k}{u}$, they can be empty or not. We distinguish four cases that are presented in Table 2.8 and discuss them now.

In case $0 < \frac{k}{u} \le \frac{1}{n}$, we have $\frac{u}{k} - 1 \ge n - 1$, and the index set $\{l = 1, \ldots, n-1 : l \le \frac{u}{k} - 1\}$ equals $\{l = 1, \ldots, n-1\}$, while the index set $\{l = 1, \ldots, n-1 : \frac{u}{k} - 1 < l \le \frac{u}{k}\}$ is empty and the summation with this index set is zero. Moreover, the index set $\{l = 0, 1, \ldots, n-1 : l \le \frac{u}{k} - 1\}$ equals $\{l = 0, 1, \ldots, n-1\}$, while the index set $\{l = 1, \ldots, n-1 : l > \frac{u}{k} - 1\}$ is empty.

In case $\frac{1}{n} < \frac{k}{u} \le \frac{1}{n-1}$, we have $n-2 \le \frac{u}{k} - 1 < n-1$, and the index set $\{l = 1, \ldots, n-1 : l \le \frac{u}{k} - 1\}$ equals $\{l = 1, \ldots, \lfloor \frac{u}{k} - 1 \rfloor\}$, while the index set $\{l = 1, \ldots, n-1 : \frac{u}{k} - 1 < l \le \frac{u}{k}\}$ equals $\{l = n-1\}$. Moreover, the index set $\{l = 0, 1, \ldots, n-1 : l \le \frac{u}{k} - 1\}$ equals $\{l = 0, 1, \ldots, \lfloor \frac{u}{k} - 1 \rfloor\}$, while the index set $\{l = 1, \ldots, n-1 : l > \frac{u}{k} - 1\}$ reduces to the singleton $\{l = \lceil \frac{u}{k} - 1 \rceil\}$.

In case $\frac{1}{n-1} < \frac{k}{u} \le \frac{1}{2}$, we have $1 \le \frac{u}{k} - 1 < n-2$, and the index set $\{l = 1, \ldots, n-1 : l \le \frac{u}{k} - 1\}$ equals $\{l = 1, \ldots, \lfloor \frac{u}{k} - 1 \rfloor\}$ as in the previous case, while the index set $\{l = 1, \ldots, n-1 : \frac{u}{k} - 1 < l \le \frac{u}{k}\}$ equals $\{l = \lfloor \frac{u}{k} \rfloor\}$. Moreover, the index set $\{l = 0, 1, \ldots, n-1 : l \le \frac{u}{k} - 1\}$ equals $\{l = 0, 1, \ldots, \lfloor \frac{u}{k} - 1 \rfloor\}$ as in the previous case, while the index set $\{l = 1, \ldots, n-1 : l > \frac{u}{k} - 1\}$ equals $\{l = \lfloor \frac{u}{k} \rfloor, \lfloor \frac{u}{k} \rfloor + 1, \ldots, n-1\}$.

In case $\frac{1}{2} < \frac{k}{u} \le 1$, we have $0 \le \frac{u}{k} - 1 < 1$, and the index set $\{l = 1, \ldots, n-1 : l \le \frac{u}{k} - 1\}$ is empty, while the index set $\{l = 1, \ldots, n-1 : \frac{u}{k} - 1 < l \le \frac{u}{k}\}$ equals $\{l = \lfloor \frac{u}{k} \rfloor\}$. Moreover, the index set $\{l = 0, 1, \ldots, n-1 : l \le \frac{u}{k} - 1\}$ reduces to the singleton $\{l = \lfloor \frac{u}{k} \rfloor\}$, while the index set $\{l = 1, \ldots, n-1 : l > \frac{u}{k} - 1\}$ equals $\{l = 1, \ldots, n-1\}$.

Case 3.1: $0 < \frac{k}{u} \le \frac{1}{n}$. We start by computing the expressions for $\Phi_{-i_p}(c^-)$ and $\Phi_{i_p}(c^-)$ from Table 2.8:

Case	$\Phi_{-i_p}(c^-)$	$\Phi_{i_p}(c^-)$
$\mathbf{0 < \frac{k}{u} \le \frac{1}{n}}$	$\frac{1}{n}\sum_{l=1}^{n-1} k\frac{l}{n-1}$	$\frac{1}{n}\sum_{l=0}^{n-1}(l+1)k$
$\mathbf{\frac{1}{n} < \frac{k}{u} \le \frac{1}{n-1}}$	$\frac{1}{n}\Big(\sum_{l=1}^{\lfloor \frac{u}{k}-1\rfloor} k\frac{l}{n-1} + \sum_{l=n-1}(u-lk)\frac{l}{n-1}\Big)$	$\frac{1}{n}\Big(\sum_{l=0}^{\lfloor \frac{u}{k}-1\rfloor}(l+1)k + \sum_{l=\lceil \frac{u}{k}-1\rceil} u\Big)$
$\mathbf{\frac{1}{n-1} < \frac{k}{u} \le \frac{1}{2}}$	$\frac{1}{n}\Big(\sum_{l=1}^{\lfloor \frac{u}{k}-1\rfloor} k\frac{l}{n-1} + \sum_{l=\lfloor \frac{u}{k}\rfloor}(u-lk)\frac{l}{n-1}\Big)$	$\frac{1}{n}\Big(\sum_{l=0}^{\lfloor \frac{u}{k}-1\rfloor}(l+1)k + \sum_{l=\lfloor \frac{u}{k}\rfloor}^{n-1} u\Big)$
$\mathbf{\frac{1}{2} < \frac{k}{u} \le 1}$	$\frac{1}{n}\Big(0 + \sum_{l=\lfloor \frac{u}{k}\rfloor}(u-lk)\frac{l}{n-1}\Big)$	$\frac{1}{n}\Big(\sum_{l=0}(l+1)k + \sum_{l=1}^{n-1} u\Big)$

Table 2.8: Summary of the expressions of the Shapley value for the case $\lambda < 0$. Note that the expressions have not been fully simplified to highlight the effect of the various cases on the index set.

$$\Phi_{-i_p}(c^-) = \frac{1}{n}\sum_{l=1}^{n-1} k\frac{l}{n-1} \tag{2.51a}$$

$$= \frac{k}{n(n-1)}\frac{n(n-1)}{2} \tag{2.51b}$$

$$= \frac{k}{2} \tag{2.51c}$$

$$\Phi_{i_p}(c^-) = \frac{1}{n}\sum_{l=0}^{n-1}(l+1)k \tag{2.51d}$$

$$= \frac{k}{n}\sum_{l=1}^{n} l \tag{2.51e}$$

$$= \frac{k}{n}\frac{n(n+1)}{2} \tag{2.51f}$$

$$= \frac{k(n+1)}{2} \tag{2.51g}$$

We now test coalitional rationality of the Shapley value by using the equivalent expression (2.46e) for all coalitions $S \subseteq N$ starting with coalitions S such that $i_p \notin S$. In this case, $c_S^- = 0$, and the inequality to test is the following:

$$|S|\Phi_{-i_p}(c^-) \ge 0 \tag{2.52a}$$

$$\Leftrightarrow \quad \Phi_{-i_p}(c^-) \ge 0 \tag{2.52b}$$

$$\Leftrightarrow \quad \frac{k}{2} \ge 0 \tag{2.52c}$$

Since $k > 0$, (2.52c) holds true for all coalitions S such that $i_p \notin S$. We now test coalitional rationality for coalitions S such that $i_p \in S$. In this case, $c_S^- = |S|k$, and the inequality to test is

$$
\begin{aligned}
(|S|-1)\Phi_{-i_p}(c^-) + \Phi_{i_p}(c^-) &\geq |S|k && \text{(2.53a)}\\
\Leftrightarrow \quad (|S|-1)\frac{k}{2} + \frac{k}{2}(n+1) &\geq |S|k && \text{(2.53b)}\\
\Leftrightarrow \quad kn &\geq |S|k && \text{(2.53c)}\\
\Leftrightarrow \quad n &\geq |S| && \text{(2.53d)}
\end{aligned}
$$

Since $S \subseteq N$, (2.53d) holds true and, overall, we obtain that the Shapley value $\Phi(c(\lambda))$ is stable in this case.

Case 3.2: $\frac{1}{n} < \frac{k}{u} \leq \frac{1}{n-1}$. We start by computing the expressions for $\Phi_{-i_p}(c^-)$ and $\Phi_{i_p}(c^-)$ from Table 2.8. Note that, in this case, $\lfloor \frac{u}{k} - 1 \rfloor = n-2$. Indeed, the condition defining this case is equivalent to $n-1 \leq \frac{u}{k} < n$ and, if $\frac{u}{k} \in \mathbb{N}$, then $\frac{u}{k} = n-1$. If $\frac{u}{k} \notin \mathbb{N}$ instead, then $\lceil \frac{u}{k} - 1 \rceil = \lceil \frac{u}{k} \rceil - 1 = n-2$. This makes the limit of the first summations in the expressions of $\Phi_{-i_p}(c^-)$ and $\Phi_{i_p}(c^-)$ explicit.

$$
\begin{aligned}
\Phi_{-i_p}(c^-) &= \frac{1}{n}\left(\sum_{l=1}^{\lfloor \frac{u}{k}-1 \rfloor} k\frac{l}{n-1} + \sum_{l=n-1}(u-lk)\frac{l}{n-1}\right) && \text{(2.54a)}\\
&= \frac{k}{n(n-1)}\sum_{l=1}^{n-2} l + \frac{1}{n}(u-(n-1)k)\frac{n-1}{n-1} && \text{(2.54b)}\\
&= \frac{k}{n(n-1)}\frac{(n-1)(n-2)}{2} + \frac{u-(n-1)k}{n} && \text{(2.54c)}\\
&= \frac{-nk+2u}{2n} && \text{(2.54d)}\\
\Phi_{i_p}(c^-) &= \frac{1}{n}\left(\sum_{l=0}^{\lfloor \frac{u}{k}-1 \rfloor}(l+1)k + \sum_{l=\lceil \frac{u}{k}-1 \rceil} u\right) && \text{(2.54e)}\\
&= \frac{k}{n}\sum_{l=1}^{n-1} l + \frac{u}{n} && \text{(2.54f)}\\
&= \frac{k}{n}\frac{n(n-1)}{2} + \frac{u}{n} && \text{(2.54g)}\\
&= \frac{n(n-1)k+2u}{2n} && \text{(2.54h)}
\end{aligned}
$$

Similarly to the previous case, we test coalitional rationality starting with coalitions S such that $i_p \notin S$. In this case, $c_S^- = 0$, and the inequality to test is

$$|S|\Phi_{-i_p}(c^-) \geq 0 \tag{2.55a}$$

$$\Leftrightarrow \quad \Phi_{-i_p}(c^-) \geq 0 \tag{2.55b}$$

$$\Leftrightarrow \quad \frac{2u - nk}{2n} \geq 0 \tag{2.55c}$$

$$\Leftrightarrow \quad 2u \geq nk \tag{2.55d}$$

$$\Leftrightarrow \quad \frac{k}{u} \leq \frac{2}{n} \tag{2.55e}$$

Since the case-defining condition $\frac{k}{u} \leq \frac{1}{n-1}$ implies that $\frac{k}{u} \leq \frac{2}{n}$ (as $\frac{1}{n-1} \leq \frac{2}{n}$ for $n \geq 2$), (2.55e) holds true.

We now consider coalitions S such that $i_p \in S$. In this case, $c_S^- = |S|k$, and the inequality to test is

$$(|S|-1)\Phi_{-i_p}(c^-) + \Phi_{i_p}(c^-) \geq |S|k \tag{2.56a}$$

$$\Leftrightarrow \quad (|S|-1)\frac{2u - nk}{2n} + \frac{n(n-1)k + 2u}{2n} \geq |S|k \tag{2.56b}$$

$$\Leftrightarrow \quad n^2k - 3n|S|k + 2|S|u \geq 0 \tag{2.56c}$$

$$\Leftrightarrow \quad |S|(2u - 3nk) \geq -n^2k \tag{2.56d}$$

as $(2u - 3nk) < 0$ and the inequality should hold for all $|S| = 1, 2, \ldots, n-1$, it suffices to consider $|S| = n - 1$:

$$\Leftrightarrow \quad (n-1)(2u - 3nk) \geq -n^2k \tag{2.56e}$$

$$\Leftrightarrow \quad 2\frac{u}{k}(n-1) \geq 2n^2 - 3n \tag{2.56f}$$

$$\Leftrightarrow \quad \frac{k}{u} \leq \frac{2n-2}{2n^2 - 3n} \tag{2.56g}$$

Since $\frac{1}{n} < \frac{2n-2}{2n^2-3n} \leq \frac{1}{n-1}$ for all $n \geq 2$, (2.56g) defines a new condition for stability in this case.

Combining the results just obtained yields $\Phi(c(\lambda)) \in \mathfrak{C}(\lambda)$ for $\frac{1}{n} < \frac{k}{u} \leq \frac{2n-2}{2n^2-3n}$.

Case 3.3: $\frac{1}{n-1} < \frac{k}{u} \leq \frac{1}{2}$. We start by computing the expressions for $\Phi_{-i_p}(c^-)$ and $\Phi_{i_p}(c^-)$ from Table 2.8:

$$\Phi_{-i_p}(c^-) = \frac{1}{n}\left(\sum_{l=1}^{\lfloor \frac{u}{k}-1\rfloor} k\frac{l}{n-1} + \sum_{l=\lfloor \frac{u}{k}\rfloor}(u-lk)\frac{l}{n-1}\right) \tag{2.57a}$$

$$= \frac{k}{n(n-1)}\frac{\lfloor \frac{u}{k}\rfloor\lfloor \frac{u}{k}-1\rfloor}{2} + \frac{(u-\lfloor \frac{u}{k}\rfloor k)\lfloor \frac{u}{k}\rfloor}{n(n-1)} \tag{2.57b}$$

$$= \frac{k}{2n(n-1)}\left[\left\lfloor \frac{u}{k}\right\rfloor\left(\left\lfloor \frac{u}{k}\right\rfloor - 1\right) + 2\left\lfloor \frac{u}{k}\right\rfloor\left(\frac{u}{k} - \left\lfloor \frac{u}{k}\right\rfloor\right)\right] \tag{2.57c}$$

$$\Phi_{i_p}(c^-) = \frac{1}{n}\left(\sum_{l=0}^{\lfloor \frac{u}{k}-1\rfloor}(l+1)k + \sum_{l=\lfloor \frac{u}{k}\rfloor}^{n-1} u\right) \tag{2.57d}$$

$$= \frac{k}{n}\sum_{l=2}^{\lfloor \frac{u}{k}\rfloor} l + \frac{u}{n}\left(n - \left\lfloor \frac{u}{k}\right\rfloor\right) \tag{2.57e}$$

$$= \frac{k}{2n}\left\lfloor \frac{u}{k}\right\rfloor\left(\left\lfloor \frac{u}{k}\right\rfloor - 1\right) + u - \frac{u}{n}\left\lfloor \frac{u}{k}\right\rfloor \tag{2.57f}$$

We now check whether coalitional rationality holds for all $S \subseteq N$ starting with coalitions S such that $i_p \notin S$. In this case,$c^-_S = 0$, and the inequality to test is

$$|S|\Phi_{-i_p}(c^-) \geq 0 \tag{2.58a}$$

$$\Leftrightarrow \quad \Phi_{-i_p}(c^-) \geq 0 \tag{2.58b}$$

$$\Leftrightarrow \quad \frac{k}{2n(n-1)}\left[\left\lfloor \frac{u}{k}\right\rfloor\left(\left\lfloor \frac{u}{k}\right\rfloor - 1\right) + 2\left\lfloor \frac{u}{k}\right\rfloor\left(\frac{u}{k} - \left\lfloor \frac{u}{k}\right\rfloor\right)\right] \geq 0 \tag{2.58c}$$

$$\Leftrightarrow \quad \left(\left\lfloor \frac{u}{k}\right\rfloor - 1\right) + 2\left(\frac{u}{k} - \left\lfloor \frac{u}{k}\right\rfloor\right) \geq 0 \tag{2.58d}$$

Since $\frac{u}{k} \geq 2$ in this case and $\frac{u}{k} - \left\lfloor \frac{u}{k}\right\rfloor \geq 1$ by definition of the floor operation, (2.58d) holds true.

We now consider coalitional rationality for coalitions S such that $i_p \in S$. In this case, the value of $c^- S$ depends on whether $|S| \leq \frac{u}{k}$ or not. We treat only the case $|S| > \frac{u}{k}$ as it will suffice to prove that the Shapley value is not stable in this case. For $|S| > \frac{u}{k}$, we have $c^-_S = u$ and the inequality to test is

$$(|S|-1)\Phi_{-i_p}(c^-) + \Phi_{i_p}(c^-) \geq u \tag{2.59a}$$

$$\Leftrightarrow \quad (|S|-1)\frac{k}{2n(n-1)}\left[\left\lfloor\frac{u}{k}\right\rfloor\left(\left\lfloor\frac{u}{k}\right\rfloor-1\right)+2\left\lfloor\frac{u}{k}\right\rfloor\left(\frac{u}{k}-\left\lfloor\frac{u}{k}\right\rfloor\right)\right]+ \tag{2.59b}$$

$$\frac{k}{2n}\left\lfloor\frac{u}{k}\right\rfloor\left(\left\lfloor\frac{u}{k}\right\rfloor-1\right)+u-\frac{u}{n}\left\lfloor\frac{u}{k}\right\rfloor \geq u \tag{2.59c}$$

$$\Leftrightarrow \quad \frac{|S|-1}{n-1}\left[\left\lfloor\frac{u}{k}\right\rfloor\left(\left\lfloor\frac{u}{k}\right\rfloor-1\right)+2\left\lfloor\frac{u}{k}\right\rfloor\left(\frac{u}{k}-\left\lfloor\frac{u}{k}\right\rfloor\right)\right]+\left\lfloor\frac{u}{k}\right\rfloor\left(\left\lfloor\frac{u}{k}\right\rfloor-2\frac{u}{k}\right)+\left\lfloor\frac{u}{k}\right\rfloor \geq 0 \tag{2.59d}$$

we test the case $|S| = n-1$ and obtain that

$$\Leftrightarrow \quad \left\lfloor\frac{u}{k}\right\rfloor - 2\frac{u}{k} + 1 \geq 0 \tag{2.59e}$$

$$\Leftrightarrow \quad \left\lfloor\frac{u}{k}\right\rfloor - \frac{u}{k} + 1 \geq \frac{u}{k} \tag{2.59f}$$

Since $\lfloor\frac{u}{k}\rfloor - \frac{u}{k} + 1 \leq 1$ and $\frac{u}{k} \geq 2$, (2.59f) does not hold, and we obtain that the Shapley value is not stable in this case.

Case 3.4: $\frac{1}{2} < \frac{k}{u} \leq 1$. We start by computing the expressions for $\Phi_{-i_p}(c^-)$ and $\Phi_{i_p}(c^-)$ from Table 2.8:

$$\Phi_{-i_p}(c^-) = \frac{1}{n}\sum\nolimits_{l=\lfloor\frac{u}{k}\rfloor}(u-lk)\frac{l}{n-1} \tag{2.60a}$$

$$= \frac{u-k}{n(n-1)} \tag{2.60b}$$

$$\Phi_{i_p}(c^-) = \frac{1}{n}\left(\sum\nolimits_{l=0}(l+1)k + \sum\nolimits_{l=1}^{n-1}u\right) \tag{2.60c}$$

$$= \frac{k}{n} + \frac{u}{n}(n-1) \tag{2.60d}$$

$$= \frac{k+(n-1)u}{n} \tag{2.60e}$$

We now test coalitional rationality of the Shapley value for all coalitions $S \subseteq N$ starting with coalitions S such that $i_p \notin S$. In this case, $c_S^- = 0$, and the inequality to test is

$$
\begin{aligned}
& |S|\Phi_{-i_p}(c^-) \geq 0 && (2.61a)\\
\Leftrightarrow \quad & \Phi_{-i_p}(c^-) \geq 0 && (2.61b)\\
\Leftrightarrow \quad & \frac{u-k}{n(n-1)} \geq 0 && (2.61c)
\end{aligned}
$$

Since $k < u$, (2.61c) holds true.

We now consider coalitions S such that $i_p \in S$. Moreover, we consider the case where $|S| > 1$ as, for $|S| = 1$, we would be testing individual rationality of the Shapley value, which is always satisfied. Under these assumptions, we have $c_S^- = u$ and the inequality to test is

$$
\begin{aligned}
& (|S|-1)\Phi_{-i_p}(c^-) + \Phi_{i_p}(c^-) \geq u && (2.62a)\\
\Leftrightarrow \quad & \frac{(|S|-1)(u-k)}{n(n-1)} + \frac{k+(n-1)u}{n} \geq u && (2.62b)\\
\Leftrightarrow \quad & (u-k)(|S|-n) \geq 0 && (2.62c)
\end{aligned}
$$

Since $S \subsetneq N$, (2.62c) does not hold, meaning that the Shapley value is not stable in this case.

Case	Result
$\mathbf{0 < \frac{k}{u} \leq \frac{1}{n}}$	$\Phi(c^-) \in \mathfrak{C}(c^-)$
$\mathbf{\frac{1}{n} < \frac{k}{u} \leq \frac{1}{n-1}}$	$\Phi(c^-) \in \mathfrak{C}(c^-)$ if and only if $\frac{1}{n} < \frac{k}{u} \leq \frac{2n-2}{2n^2-3n}$
$\mathbf{\frac{1}{n-1} < \frac{k}{u} \leq \frac{1}{2}}$	$\Phi(c^-) \notin \mathfrak{C}(c^-)$
$\mathbf{\frac{1}{2} < \frac{k}{u} \leq 1}$	$\Phi(c^-) \notin \mathfrak{C}(c^-)$

Table 2.9: Summary of the findings in the four cases for $\lambda < 0$.

A summary of the findings in the previous four cases is given in Table 2.9, which can be put together as

$$
\Phi(c^-) \in \mathfrak{C}(c^-) \text{ if and only if } 0 < \frac{k}{u} \leq \frac{2n-2}{2n^2-3n}. \tag{2.63}
$$

Summary. As a final remark, we note that the numerical expressions we found under the assumption that $n \geq 3$ extend to the case of a two-players cooperation, despite the fact that the subcases in Tables 2.7 and 2.8 would not lead to the correct case. We observe that the summarizing conditions $\Phi(\lambda) \in \mathfrak{C}(\lambda)$ if and only if $\frac{(2n-1)(n-2)}{2n^2-3n} \le \frac{k}{u} \le 1$ for $\lambda > 0$, and $\Phi(\lambda) \in \mathfrak{C}(\lambda)$ if and only if $0 \le \frac{k}{u} \le \frac{2n-2}{2n^2-3n}$ for $\lambda < 0$ are valid for $n = 2$ as well. Indeed, the two terms $\frac{(2n-1)(n-2)}{2n^2-3n}$ and $\frac{2n-2}{2n^2-3n}$ equal 0 and 1, respectively, for $n = 2$, thus covering the stable case of a two-players cooperation.

Thanks to the previous observation and by combining the results of Cases 2 and 3, the proof is complete.□

□

2.8.7 Proof of Theorem 2

To prove the theorem, we need the following Lemma.

Lemma 1. *Given a subadditive TU cost game (N, c) and a solution concept Ψ that is individually rational and efficient, let the individual rationality gap δ_i for company $i \in N$ be $\delta_i := c_i - \Psi_i$. Then, for any coalition $S \subsetneq N$, coalitional rationality implies bounded synergy for this coalition. Moreover, synergy greater than that of the grand-coalition leads to instability of the solution concept. In other words:*

$$\sum_{i \in S} \Psi_i \le c_S \quad \Rightarrow \quad \sigma_S \le \sigma_N \tag{2.64a}$$

$$\sigma_S > \sigma_N \quad \Rightarrow \quad \sum_{i \in S} \Psi_i > c_S \Rightarrow \mathfrak{C} = \varnothing \tag{2.64b}$$

Moreover,

$$\sum_{i \in S} \Psi_i = c_S + \varepsilon \quad \Leftrightarrow \quad \varepsilon = \sigma_S - \sigma_N + \sum_{i \in N \setminus S} \delta_i \tag{2.65}$$

Proof. Proof. We have:

$$\begin{aligned} & \sum_{i \in S} \Psi_i > c_S \\ \Leftrightarrow \quad & c_N - \sum_{i \in N \setminus S} \Psi_i + \sum_{i \in N} c_i - \sum_{i \in N} c_i > c_S \\ \Leftrightarrow \quad & \sigma_S > \sigma_N - \sum_{i \in N \setminus S} \delta_i \end{aligned}$$

Thus,

$$\sum_{i \in S} \Psi_i > c_S \quad \Leftrightarrow \quad \sigma_S > \sigma_N - \sum_{i \in N \setminus S} \delta_i \tag{2.66}$$

which implies (2.64a) and (2.64b). Equivalence (2.66) shows that instability of a coalition implies a synergy level higher than the difference of the synergy level of the grand-coalition and the individual rationality gap of the other players.

Equivalence (2.65) is obtained by repeating the same steps, but starting with $\sum_{i \in S} \Psi_i = c_S + \varepsilon$. □ □

Theorem 2. *Given a subadditive cost game (N, c) and an individually rational and efficient solution concept Ψ, the following holds:*

$$\sigma_S \leq \sigma_N \ \ \forall S \subseteq N \quad \Rightarrow \quad \Psi \in \mathfrak{C}_{\sigma_N}, \tag{2.67}$$

where $\mathfrak{C}_{\sigma_N}$ is the ε-Core for $\varepsilon = \sigma_N$.

Proof. Proof. Pick $S \subseteq N$ and assume that $\sigma_S \leq \sigma_N$. Then, either $\sigma_S \leq \sigma_N - \sum_{i \in N \setminus S} \delta_i$ or $\sigma_N - \sum_{i \in N \setminus S} \delta_i < \sigma_S \leq \sigma_N$. In the former case, we have $\sum_{i \in S} \Psi_i \leq c_S$; in the latter case, $\sum_{i \in S} \Psi_i > c_S$ and $\sum_{i \in S} \Psi_i = c_S + \varepsilon_S$, where $\varepsilon_S := \sigma_S - \sigma_N + \sum_{i \in N \setminus S} \delta_i$ (from (2.65)). From the condition $\sigma_S \leq \sigma_N$, we obtain that $\varepsilon_S \leq \varepsilon_0 := \sum_{i \in N} \delta_i$ for all coalitions $S \subseteq N$. If $\Psi \notin \mathfrak{C}$, then it follows that $\Psi \in \mathfrak{C}_\varepsilon$, where $\varepsilon = \max_{S \subseteq N} \{\sum_{i \in S} \Psi_i - c_S\} = \max_{S \subseteq N} \{\varepsilon_S\} \leq \varepsilon_0$. Thus, $\Psi \in \mathfrak{C}_{\varepsilon_0}$. If $\Psi \in \mathfrak{C}$, then $\Psi \in \mathfrak{C}_{\varepsilon_0}$ as $\mathfrak{C} \subseteq \mathfrak{C}_{\varepsilon_0}$.

Now, $\varepsilon_0 = \sum_{i \in N} (c_i - \Psi_i) = \sum_{i \in N} c_i - c_N = \sigma_N$ concludes the proof of (2.67). □ □

2.8.8 Proof of Corollary 2

Corollary 2. *Given a parametric minimum cost flow game $c(\lambda)$, it follows that*

$$\Phi(\lambda) \in \mathfrak{C}_{\sigma_N(\lambda)} \quad \forall \lambda \in \Lambda. \tag{2.68}$$

Proof. Proof.

Consider a fixed value $\lambda \in \Lambda$. Non-emptiness of the core of the game $c(\lambda)$ implies that $\sigma_S(\lambda) \leq \sigma_N(\lambda)$ for all $S \subseteq N$. Indeed, non-emptiness of the core implies that there exists $x = (x_i)_{i \in N} \in \mathfrak{C}(\lambda)$ such that $\sum_{i \in N} x_i = c_N(\lambda)$ and $\sum_{i \in S} x_i \leq c_S(\lambda)$ for all coalitions $S \subseteq N$. From the inequalities $\sum_{i \in S} x_i \leq c_S(\lambda)$, we obtain

$$\sum_{i \in S} x_i \leq c_S(\lambda) \tag{2.69a}$$

$$\Leftrightarrow \quad c_N(\lambda) - \sum_{i \in N \setminus S} x_i + \sum_{i \in N} c_i(\lambda) - \sum_{i \in N} c_i(\lambda) \leq c_S(\lambda) \tag{2.69b}$$

$$\Leftrightarrow \quad \sigma_S(\lambda) \leq \sigma_N(\lambda) - \sum_{i \in N \setminus S} (c_i(\lambda) - x_i) \tag{2.69c}$$

$$\Rightarrow \sigma_S(\lambda) \leq \sigma_N(\lambda) \tag{2.69d}$$

where, in (2.69c), we note that $c_i(\lambda) - x_i \geq 0$ because of coalitional rationality. Thus, we have $\sigma_S(\lambda) \leq \sigma_N(\lambda)$ for all $S \subseteq N$. Therefore, since the Shapley value $\Phi(\lambda)$ is an individually rational and efficient solution concept, the claim follows from Theorem 2 and the fact that our argument holds true for each value $\lambda \in \Lambda$. □

□

2.8.9 Proofs of Theorems 3 and 4

In this section, we provide the proofs of Theorems 3 and 4, which extend the results of Theorem 1 to a slightly richer network structure.

Independently of the position of the parameter λ on the arcs of the network, we can observe that the cost function $c^v(\lambda)$ of the vertical cooperation game $c^v(\lambda)$ can be decomposed into the sum of two games $c^v(\lambda) = c^h(\lambda) + \bar{c}(\lambda)$, where the horizontal cooperation game $c^h(\lambda)$ is defined by the minimum cost flow game from Section 2.4, where only the set of arcs $\{\bar{r}_i : i \in N\}$ is considered, and the cost function of the game $\bar{c}(\lambda)$ is the difference $\bar{c}(\lambda) \coloneqq c^v(\lambda) - c^h(\lambda)$. Note that, from the fact that the vertical cooperation path can be used only by the grand coalition, it follows that $\bar{c}_S(\lambda) \coloneqq 0$ for all $S \subsetneq N$, while only $\bar{c}_N(\lambda)$ can have non-zero value depending on λ and the parameters of the game.

Theorem 3. *Consider the vertical cooperation game $c^v(\lambda)$, where $c_{\bar{r}_{i_p}} = \bar{c} + \lambda$. For all values of direct unit transport cost c and capacity u, vertical unit transport cost $\bar{c}$ and capacity $\bar{u}$, amount of orders k and number of players n, we have*

$$\Phi^v(\lambda) \in \mathfrak{C}^v(\lambda) \qquad \textit{for all } \lambda \in [-c, +\infty). \tag{2.70}$$

In other words, when the parametric cost is on one of the arcs in the vertical cooperation path, the Shapley value $\Phi^v(\lambda)$ is stable for all values of λ.

Proof. Proof. We first note that the game $c^v(\lambda)$ is symmetric for every value of λ. Indeed, for any two players $i, j \in N$, we have $c^v_{S\cup\{i\}}(\lambda) = c^v_{S\cup\{j\}}(\lambda)$ for all coalitions $S \subseteq N \setminus \{i,j\}$. This is because $S \cup \{i\} \subsetneq N$ and $S \cup \{j\} \subsetneq N$, so the vertical cooperation game reduces to the horizontal cooperation game where all players and their networks (being a single arc) are identical (see Figure 2.7). Therefore, we have that $c^v_{S\cup\{i\}}(\lambda) = c^h_{S\cup\{i\}}(\lambda) = c^h_{S\cup\{j\}}(\lambda) = c^v_{S\cup\{j\}}(\lambda)$. Note that this holds true even when $i = i_p$ as the parametric cost is on arc $\bar{r}_{i_p}$, which is not considered in the game $c^h(\lambda)$. It follows that $\Phi_i(\lambda) = \Phi_j(\lambda)$ for all i, j. Using efficiency of the Shapley value, this implies that $\Phi^v_i(\lambda) = \frac{1}{n} c^v_N(\lambda)$ for all players $i \in N$.

We now distinguish two cases depending on whether the unit transport cost $\lambda + n\bar{c}$ of the vertical cooperation path is greater than the unit cost c for direct transport or not. In the first case, the vertical cooperation path is not used by the cooperation, while in the second case, it might appear in the cost $c^v_N(\lambda)$ of the grand coalition.

Case 1: $\boldsymbol{\lambda + n\bar{c} > c}$. In this case, the game $c^v(\lambda)$ reduces to the game $c^h(0)$ as the vertical cooperation path is not used, and the parameter λ does not enter the cost expressions. Therefore, $c^h(\lambda) = c^h(0)$ and, thus, the Shapley value is stable by Theorem 1, i.e., $\Phi^v(\lambda) \in \mathfrak{C}^v(\lambda)$.

Case 2: $\boldsymbol{\lambda + n\bar{c} \leq c}$. In this case, the vertical cooperation path can be used by the grand coalition since its unit transport cost $\lambda + n\bar{c}$ is at most as large as the unit cost of direct transport. Testing coalitional rationality of the Shapley value requires us to test whether $\sum_{i\in S} \Phi^v{}_i(\lambda) \leq c^v_S(\lambda)$ for all coalitions $S \subseteq N$. For any $S \subseteq N$, we can reformulate the corresponding inequality as follows:

$$\sum_{i\in S} \Phi^v{}_i(\lambda) \leq c^v_S(\lambda) \tag{2.71a}$$

$$\Leftrightarrow \quad |S| \frac{c^v_N(\lambda)}{n} \leq |S| kc \tag{2.71b}$$

$$\Leftrightarrow \quad c^v_N(\lambda) \leq knc \tag{2.71c}$$

Inequality (2.71c) holds since $\lambda + n\bar{c} \leq c$ implies that $c^v_N(\lambda) = \bar{u}(\lambda + n\bar{c}) + (kn - \bar{u})c \leq knc$. Here, the equality follows since the grand coalition can transport $\bar{u}$ orders on the vertical cooperation path at unit cost $\lambda + n\bar{c}$. Therefore, coalitional rationality is satisfied for all coalitions $S \subseteq N$ and we obtain $\Phi^v(\lambda) \in \mathfrak{C}^v(\lambda)$.□ □

We now provide the proof of Theorem 4.

Theorem 4. *In the vertical cooperation game $c^v(\lambda)$ where $c_{r_{i_p}} = c + \lambda$, we have that for all values of direct unit transport cost c and capacity u, vertical unit transport cost $\bar{c}$ such that $n\bar{c} \leq c$ and capacity $\bar{u}$, amount of orders k and number of players n, the following holds: For each value of $\lambda \in [-\bar{c}, +\infty)$, stability of the Shapley value $\Phi^h(\lambda)$ in the horizontal cooperation game $c^h(\lambda)$ implies stability of the Shapley value $\Phi^v(\lambda)$ in the vertical cooperation game $c^v(\lambda)$. More formally:*

$$\text{For each } \lambda \in [-\bar{c}, +\infty): \qquad \Phi^h(\lambda) \in \mathfrak{C}^h(\lambda) \quad \Rightarrow \quad \Phi^v(\lambda) \in \mathfrak{C}^v(\lambda). \tag{2.72}$$

The converse does, in general, not hold true.

Proof. Proof. Note that this proof is based on observations found in the proof of Theorem 1 provided in Appendix 2.8.6.

We first note that the game $\bar{c}(\lambda)$ is symmetric for every value of λ since only $\bar{c}_N(\lambda)$ can be non-zero. Therefore, for any two of players $i, j \in N$, we have $\bar{\Phi}_i(\lambda) = \bar{\Phi}_j(\lambda)$. Using efficiency of the Shapley value, this implies that $\bar{\Phi}_i(\lambda) = \frac{1}{n}\bar{c}_N(\lambda)$ for all players $i \in N$. Moreover, we note that $\bar{c}_N(\lambda) \leq 0$ for each λ. This follows since $\bar{c}_N(\lambda) = c^v_N(\lambda) - c^h_N(\lambda)$ by definition and since the graph considered in the minimum cost flow problem of the grand coalition N in $c^h(\lambda)$ is a subgraph of the graph in the minimum cost flow problem of the grand coalition in $c^v(\lambda)$, which implies that the minimum cost $c^h_N(\lambda)$ achievable by the grand coalition in $c^h(\lambda)$ is at least as large as the minimum cost $c^v_N(\lambda)$ achievable by the grand coalition in $c^v(\lambda)$.

Because the arc r_{i_p} with parametric cost is a direct transport arc, we obtain that the horizontal cooperation game $c^h(\lambda)$ can be decomposed as in the proof of Theorem 1: $c^h(\lambda) = c^0 + \lambda c^{\pm}$, where $c^{\pm} = c^+$ or $c^{\pm} = c^-$ depending on the sign of λ. The games c^0, c^+, and c^- have been defined in the proof of Theorem 1 provided in Appendix 2.8.6 (see Equations (2.25) and (2.45) for c^+ and c^-, respectively). Intuitively, the two games c^+ and c^- count only the amount of orders transported on arc r_{i_p}, so that $\lambda c^{\pm}$ is the cost difference resulting from transport on arc r_{i_p}. The dependence on the sign of λ follows from the fact that, for $\lambda < 0$, player i_p's arc r_{i_p} is used first in each coalition, while, for $\lambda > 0$, arc r_{i_p} is used last.

We let $\Phi^{\pm}$ and Φ^0 be the Shapley value for the games $c^{\pm}$, and c^0, respectively.

Combining the decomposition of $c^h(\lambda)$ with that of $c^v(\lambda)$, we obtain that $c^v(\lambda) = c^0 + \lambda c^{\pm} + \bar{c}(\lambda)$ and the Shapley value $\Phi^v(\lambda)$ can be computed by using linearity, as its expression is known explicitly for each of the games c^0, $c^{\pm}$ and $\bar{c}(\lambda)$.

Before entering the proof of coalitional rationality, we recall that $\bar{c}_S(\lambda) = 0$ for all $S \subsetneq N$.

Testing coalitional rationality for $S \subsetneq N$ means testing whether $\sum_{i \in S} \Phi_i^v(\lambda) \leq c_S^v(\lambda)$, which can be rewritten as follows:

$$\sum_{i \in S} \Phi_i^v(\lambda) \leq c_S^v(\lambda) \tag{2.73a}$$

$$\Leftrightarrow \quad \sum_{i \in S} \Phi_i^0 + \lambda \sum_{i \in S} \Phi_i^{\pm} + |S| \frac{\bar{c}_N(\lambda)}{n} \leq c_S^0 + \lambda c_S^{\pm} \tag{2.73b}$$

$$\Leftrightarrow \quad \lambda \sum_{i \in S} \Phi_i^{\pm} + |S| \frac{\bar{c}_N(\lambda)}{n} \leq \lambda c_S^{\pm} \tag{2.73c}$$

To further make the expression (2.73c) explicit, we distinguish two cases depending on the sign of λ.

Case 1: $\boldsymbol{\lambda = 0}$. In this case, (2.73c) reduces to

$$\frac{|S|}{n} \bar{c}_N(0) \leq 0, \tag{2.74}$$

which holds since $\bar{c}_N(0) \leq 0$. Thus, $\Phi^v(0) \in \mathfrak{C}^v(0)$.

Case 2: $\boldsymbol{\lambda > 0}$. In this case, we can rewrite (2.73c) as follows, knowing that $c^{\pm} = c^{+}$:

$$\sum_{i \in S} \Phi_i^{+} + \frac{1}{\lambda} \frac{|S|}{n} \bar{c}_N(\lambda) \leq c_S^{+} \tag{2.75}$$

Since $\bar{c}_N(\lambda) \leq 0$ and $\lambda > 0$, we have $\frac{1}{\lambda} \frac{|S|}{n} \bar{c}_N(\lambda) \leq 0$. Consequently, we obtain that $\sum_{i \in S} \Phi_i^{+} \leq c_S^{+}$ for all $S \subset N$, which is equivalent to $\Phi^h(\lambda) \in \mathfrak{C}^h(\lambda)$ as seen in (2.26) in the proof of Theorem 1, implies (2.75), i.e., that $\Phi^v(\lambda) \in \mathfrak{C}^v(\lambda)$.

Case 3: $\boldsymbol{-c \leq \lambda < 0}$. In this case, we rewrite (2.73c) as follows, knowing that $c^{\pm} = c^{-}$:

$$\sum_{i \in S} \Phi_i^{-} + \frac{1}{\lambda} \frac{|S|}{n} \bar{c}_N(\lambda) \geq c_S^{-} \tag{2.76}$$

Since $\bar{c}_N(\lambda) \leq 0$ and $\lambda < 0$, we have $\frac{1}{\lambda} \frac{|S|}{n} \bar{c}_N(\lambda) \geq 0$. Thus, if $\sum_{i \in S} \Phi_i^{-} \geq c_S^{-}$ for all $S \subseteq N$, then (2.76) holds and $\Phi^v(\lambda) \in \mathfrak{C}^v(\lambda)$. As $\sum_{i \in S} \Phi_i^{-} \geq c_S^{-}$ for all $S \subseteq N$ is equivalent to $\Phi^h(\lambda) \in \mathfrak{C}^h(\lambda)$ because of (2.46) in the proof of Theorem 1, we obtain that $\Phi^h(\lambda) \in \mathfrak{C}^h(\lambda)$ implies $\Phi^v(\lambda) \in \mathfrak{C}^v(\lambda)$ as claimed.

By combining the two cases, we have shown that $\Phi^h(\lambda) \in \mathfrak{C}^h(\lambda)$ implies $\Phi^v(\lambda) \in \mathfrak{C}^v(\lambda)$.

To show that the opposite implication does not hold true in general, we refer to the numerical example in Section 2.6.3.2, which shows that, for certain combinations of the parameters, we have $\Phi^h(\lambda) \in \mathfrak{C}^h(\lambda)$ but $\Phi^v(\lambda) \notin \mathfrak{C}^v(\lambda)$.□ □

5 Conclusions and Future Outlook

This thesis studied how the utilization of inland containerized and bulk freight transport capacity can improve upon current practice from the planning and execution perspective. This focal problem is analyzed from three main points of view related to the state of the industry at the time of pursuing this research.

First, to counter the effect of a fragmented transport market with a large share of small companies, cooperation among carriers that pool transport demand and capacity is analyzed. Thanks to a novel tool that allows for a sensitivity analysis of cooperative games, it is possible to analyse whether a cooperation is stable depending on a given transport setting. Indeed, as the bargaining power of a participant in the cooperation depends on their contribution to the cooperative transport setup, the cooperation is stable or not depending on the partition of the gains among the participants themselves. Such a division of the gains might be successful, as all agree, or not, because of unfairness in the partition, for instance. The effect that the transport setting has on this partition and its success is analyzed in Chapter 2.

Second, an adaptive planning approach suitable for freight transport operations is formulated and studied to address the negative impact of uncertainty of container transport operations on transport reliability. In particular, the purpose of this study is estimating the performance improvement, in terms of costs and reliability, resulting from the adoption of such a complex approach. A challenging aspect of freight operations is that, to be able to adjust a transport plan during the execution, the planner needs to pay a fee to reserve that additional capacity required to adapt the plan.

Third, digital transport marketplaces are considered as an opportunity to improve upon traditional matchmaking in the inland waterway transport market. The raising search friction and heterogeneity due to enlarging customer base are addressed with a dynamic model for two-sided assortment optimization. The model proposed tackles the challenge of learning the behavior of the users while improving the rate of matches in the marketplace.

The remainder of this chapter is organized as follows: Section 5.1 concludes the research in Chapters 2, 3 and 4; Section 5.2 provides an outlook on future research.

5.1 Conclusions

The main conclusions from each chapter are reported in the following.

Chapter 2 shows that cooperation, either horizontal or vertical, between container transport operators, can be stable under certain conditions through a novel tool to inspect this problem. Indeed, by taking advantage of both parametric optimization and cooperative game theory, an analysis of the sensitivity of cooperation stability is made possible.

Taking advantage of this approach, a theoretical study on a stylized setting shows that the main discriminant for the stability of horizontal cooperation is the demand-over-capacity ratio compared with a function of the size of the cooperation. This abstract result is in line with the business understanding of the conditions for cooperation to be most stable. In particular, it is proven that, for problems with network flow-like cost structure, cooperation is stable or limited unstable even in over-capacitated settings, where one would expect competition to prevail over cooperation.

Moreover, a novel measure of instability is introduced to further the understanding of the stability of a cooperation, which has been traditionally studied from a binary stance, i.e., either a cooperation is stable or not. Taking advantage of this measure, a numerical analysis of a stylized cooperation setting finds only a small relative instability across the whole range of parameters, provided similar transport costs between the companies. This finding shows that small and similar companies can reap the benefits of stable medium-sized cooperation against a small individual loss for some. Whether this loss is acceptable for a small company or not depends on other factors that go beyond the scope of this study. While previous analysis would have concluded not-viable cooperation for those parameters, by measuring instability new opportunities for cooperation might be considered.

Overall, it can be concluded that conflicts during the division of the gains of a cooperation should not be expected to be the main reason for cooperation to fail as the measure of instability is rather insensitive to changing configurations. As those theoretical and numerical results are in line with other empirical findings on real cases of failing cooperation in transport, it can be concluded that the operational challenge of coordinating the cooperative effort must be greater than the benefits of improved transport performance.

From the stability of cooperation considered in Chapter 2, this thesis focuses then in Chapter 3 on adaptive routing decisions as a way to hedge against uncertainties in transport operations at the planning level. The findings in Chapter 3 suggest that adaptive decisions can reduce the marginal cost of reliability when compared to non-adaptive planning approaches. The trade-off between adaptive decisions and capacity booking is tackled at once thanks to a novel model combining static and dynamic decisions. The complexity of this problem lies in the fact that the set of alternative adaptive routing options is defined by the capacity booking decisions. After showing that this problem is hard to solve (NP-complete), a solution approach is found in a MIP formulation which can be input to off-the-shelf solvers. Having a solution approach for this problem, two sets of numerical experiments show that (i) the medium-sized realistic instances can be solved by a consumer-grade computer, and suggest (ii) a constant marginal cost of reliability. The latter result is of relevance for practitioners as it highlights the potential value of adaptive decisions: the reliability of a transport plan can be increased at a cost linear to the reliability requirement. While further investigations should check whether this result holds in specific transport operations, the stylized model constructed gives first evidence of the potential of adaptive decisions.

Overall, the research in Chapter 3 showed that a model to combine adaptive routing decisions for container freight transport is possible under current business practice. By formalizing the complex interaction between a-priori and adaptive decisions in a scheduled network, this model opens a range of opportunities for container transport planning in the hope of improving the capability of practitioners to react to changes

Chapter 4 considers the problem faced by a growing digital transport marketplace, like the one of the industry partner involved in this research, that aims at improving the matching rate in a heterogeneous and dynamic freight transport market. First, it is found that the problem faced by the platform operating the marketplace is more similar to the problem faced by dating platforms than that faced by ride-hailing ones: indeed, because the platform is not in control of transport capacity, they cannot assign demand to supply like a taxi platforms assigns riders to taxis. Instead, the platform can facilitate matching by deciding what each side evaluates, a problem related to the so-called assortment optimization problem (faced by physical stores, for instance, that can decide how to display items to facilitate sales). As a second conclusion, Chapter 4 shows that the novel approach achieves an expected regret rate of 0.27 meaning that it is expected that only 0.27 of matchable transport requests cannot be matched by the model at each epoch. Moreover, this performance level is reached quickly within the first 20 transport requests. This result shows a promising approach that is ready to be tested further in field experiments. Because three different policies were tested with a different attitude towards exploring new alternatives or exploiting successful ones, numerical results suggest further that exploratory approaches should be preferred over exploitative ones because of the possible positive effect on customer adoption. Indeed, in the first transport requests, the more exploratory method achieves the lowest regret meaning they can cater the best combination of alternatives among the three policies. In the long run, though, this early gain is overridden by the learned preferences.

Overall, Chapter 4 shows that digital transport marketplaces can tackle a larger customer base by learning user preferences while limiting the number of alternatives each user effectively evaluates. This shows that digital platforms can expand further thus allowing for a better-than-traditional matching between transport demand and supply in the inland transport market.

5.2 Future outlook

This dissertation investigates how Operations Research could benefit transport planning by tackling different problems with a range of methodologies. This work opens several avenues for further investigation.

Extending the sensitivity analysis of cooperative games. The method developed in Chapter 2 to perform a sensitivity analysis of cooperative games considers only changes for a single parameter in the objective value of the optimization problem. Changes in parameters in the set of constraints have been evaluated numerically. A natural extension of the method proposed would extend the analysis to parameter changes in a constraint.

Analysing cooperation on general networks. The analysis of the stability of cooperation in inland transport networks in Chapter 2 considers only horizontal and vertical types of cooperation. While those are two well-understood and studied types of arrangement, the structure of the transport networks of multiple companies might lead to other types of power relations. A second natural extension of the research presented in Chapter 2 could consider general transport network structures to study the stability of cooperation. This would allow for far greater precision in representing specific cooperative settings and providing insights into their stability.

Extending the adaptive flow model to multi-commodity flows. The model developed and studied in Chapter 3 considered a single type of container as well as a single origin-destination pair. It would then be interesting to explore the effect of the interaction of multiple shipments having different origins and destinations. Moreover, as different types of containers exist and might not be transported on the same set of transport means, adding this real feature to the model would be a natural extension. Those extensions would benefit the understanding of the value of adaptive planning in an even more realistic setting than the one considered in this thesis.

An online extension of adaptive decision making. Transport planning requires online decision making: a stream of requests is received while plans need to be adjusted. In many cases, the planner has to decide on whether to accept or reject an order within a few minutes without having the possibility to pool requests and decide on a batch. While the model in Chapter 3 acts on pooled transport demand and supply to book capacity and devise an adaptive routing policy, extending the model to include online decisions would be very valuable both to support real operations and to study the cost and value of online decision making.

Modelling customer long-term value in assortment decisions. Mathematical models for digital marketplaces like the one considered in Chapter 4 often consider only the matching rate as their objective. As a result, the characterization of platform agents is limited to features that can characterize their matching behavior. Indeed, the reaction and acceptance behavior as well as the characteristics of transport become focal. The result of this focus is that the platform is left to address the long-term value of their users by means other than matching technology. It follows that the main measure of perceived quality of service from the users, i.e., their individual matching performance, is neglected by the matching algorithm. A model that includes estimations on the long-term value of a customer in the matching decision would be valuable for the practitioner. Moreover, such an approach would open a wealth of interesting academic problems.

Finally, the many exchanges with practitioners that were formally or informally involved in this dissertation lead to the following research question the author is particularly interested in. This question follows up naturally from the gradual approach to real problems of this thesis. While the presentation of the thesis focused on well-defined and scoped academic contributions, the research in the three Chapters progresses from a more abstract to a more realistic representation of the focal real problem. Indeed, from a theoretical study on cooperation via an adaptive model for planning, the work in Chapter 4 was the result of a cooperation with an industry partner facing a real problem. Therefore, the following question can be seen as a natural outlook on the research of this thesis.

How can one take advantage of methods from Operations Research to improve, if not maximize, the success rate of the application of Operations Research and Operations Management models and frameworks to solve real-life problems?

Bibliography

Agarwal, R. & Ergun, . (2010). Network Design and Allocation Mechanisms for Carrier Alliances in Liner Shipping. *Operations Research*, 58(6), 1726–1742.

Agatz, N., Erera, A., Savelsbergh, M., & Wang, X. (2012). Optimization for dynamic ride-sharing: A review. *European Journal of Operational Research*, 223(2), 295–303.

Agrawal, S., Avadhanula, V., Goyal, V., & Zeevi, A. (2019). MNL-Bandit: A Dynamic Learning Approach to Assortment Selection. *Operations Research*, 67(5), 1453–1485.

Ahuja, R. K., Magnanti, T. L., & Orlin, J. B. (1993). *Network Flows: Theory, Algorithms, and Applications*. Pearson.

Aouad, A., Farias, V., & Levi, R. (2020). Assortment Optimization Under Consider-Then-Choose Choice Models. *Management Science*, (pp. 1–19).

Aouad, A. & Saban, D. (2020). Online Assortment Optimization for Two-sided Matching Platforms. *SSRN Electronic Journal*.

Ashlagi, I., Burq, M., Jaillet, P., & Manshadi, V. (2019a). On matching and thickness in heterogeneous dynamic markets. *Operations Research*, 67(4), 927–949.

Ashlagi, I., Krishnaswamy, A. K., Makhijani, R., Saban, D., & Shiragur, K. (2019b). Assortment planning for two-sided sequential matching markets. *arXiv*.

Ballot, E., Barbarino, S., van Bree, B., Liesa, F., Rod Franklin, J., Nettsträter, A., Paganelli, P., & Tavasszy, L. A. (2020). *The Physical Internet*. Technical report.

Barabási, A.-L. & Albert, R. (1999). Emergence of Scaling in Random Networks. *Science*, 286(5439), 509–512.

Basso, F., D'Amours, S., Rönnqvist, M., & Weintraub, A. (2019). A Survey on Obstacles and Difficulties of Practical Implementation of Horizontal Collaboration in Logistics. *International Transactions in Operational Research*, 26(3), 775–793.

Bastani, H., Harsha, P., Perakis, G., & Singhvi, D. (2018). Sequential Learning of Product Recommendations With Customer Disengagement. *SSRN Electronic Journal*.

Berge, C. (1973). *Graphs and hypergraphs*. Amsterdam: Elsevier.

Berger, P. (2022). The U.S. Import Surge Is Skipping the Train. *The Wall Street Journal*.

Bertsimas, D., Jaillet, P., & Martin, S. (2019). Online Vehicle Routing: The Edge of Optimization in Large-Scale Applications. *Operations Research*, 67(1), 143–162.

Bertsimas, D. & Kallus, N. (2020). From predictive to prescriptive analytics. *Management Science*, 66(3), 1025–1044.

Bertsimas, D. & Tsitsiklis, J. (1993). Simulated Annealing. *Statistical Science*, 8(1), 10–15.

Bhattacharya, A. & Kharoufeh, J. P. (2017). Linear Programming Formulation for Non-stationary, Finite-horizon Markov Decision Process Models. *Operations Research Letters*, 45(6), 570–574.

Bhoopalam, A. K., Agatz, N., & Zuidwijk, R. (2018). Planning of truck platoons: A literature review and directions for future research. *Transportation Research Part B: Methodological*, 107, 212–228.

Birge, J. R. & Louveaux, F. (2011). Introduction to Stochastic Programming. In *Springer Series in Operations Research and Financial Engineering* (pp. 1–469). Springer New York, NY.

Boccaletti, S., Latora, V., Moreno, Y., Chavez, M., & Hwang, D. U. (2006). Complex networks: Structure and dynamics. *Physics Reports*, 424(4-5), 175–308.

Botev, Z. I., L'Ecuyer, P., Rubino, G., Simard, R., & Tuffin, B. (2013). Static Network Reliability Estimation via Generalized Splitting. *INFORMS Journal on Computing*, 25(1), 56–71.

Brancaccio, G., Kalouptsidi, M., Papageorgiou, T., & Rosaia, N. (2021). Search Frictions and Efficiency in Decentralized Transportation Markets. *SSRN Electronic Journal*.

Brathwaite, T., Vij, A., & Walker, J. L. (2017). Machine Learning Meets Microeconomics: The Case of Decision Trees and Discrete Choice. *arXiv*.

Breiman, L. (2001). Random Forests. *Machine Learning 2001 45:1*, 45(1), 5–32.

Butcher, M. (2020). Digital freight forwarder Forto raises another $50M in round led by Inven Capital. *techcrunch*.

Caplice, C. (2007). Electronic markets for truckload transportation. *Production and Operations Management*, 16(4), 423–436.

Caro, F. & Gallien, J. (2007). Dynamic Assortment with Demand Learning for Seasonal Consumer Goods. *Management Science*, 53(2), 276–292.

Carstensen, P. (1983). *The complexity of some problems in parametric linear and combinatorial programming*. PhD thesis, University of Michigan.

Chen, A. & Ji, Z. (2005). Path finding under uncertainty. In *Journal of Advanced Transportation*, volume 39 (pp. 19–37).

Chen, B. Y., Lam, W. H., Sumalee, A., Li, Q., Shao, H., & Fang, Z. (2013). Finding Reliable Shortest Paths in Road Networks Under Uncertainty. *Networks and Spatial Economics*, 13(2), 123–148.

Chen, B. Y., Li, Q., & Lam, W. H. (2016). Finding the k reliable shortest paths under travel time uncertainty. *Transportation Research Part B: Methodological*, 94, 189–203.

Chen, L. & Miller-Hooks, E. (2012). Resilience: An Indicator of Recovery Capability in Intermodal Freight Transport. *Transportation Science*, 46(1), 109–123.

Crainic, T. G., Hewitt, M., Toulouse, M., & Vu, D. M. (2016). Service network design with resource constraints. *Transportation Science*, 50(4), 1380–1393.

Crainic, T. G. & Kim, K. H. (2007). Chapter 8 Intermodal Transportation. *Handbooks in Operations Research and Management Science*, 14, 467–537.

Crichton, D. (2021). Father and son duo take on global logistics with Optimal Dynamics sequential decision AI platform | TechCrunch.

Cruijssen, F., Borm, P., Fleuren, H., & Hamers, H. (2010). Supplier-initiated outsourcing: A methodology to exploit synergy in transportation. *European Journal of Operational Research*, 207(2), 763–774.

Cruijssen, F., Cools, M., & Dullaert, W. (2007a). Horizontal cooperation in logistics: Opportunities and impediments. *Transportation Research Part E: Logistics and Transportation Review*, 43(2), 129–142.

Cruijssen, F., Dullaert, W., & Fleuren, H. (2007b). Horizontal Cooperation in Transport and Logistics: A Literature Review. *Transportation Journal*, 46(3), 22–39.

Cullen, Z. & Farronato, C. (2021). Outsourcing Tasks Online: Matching Supply and Demand on Peer-to-Peer Internet Platforms. *Management Science*, 67(7), 3985–4003.

Dempsey, H. (2022). Is there an end in sight to supply chain disruption? *Financial Times*.

Eisner, M. J. & Severance, D. G. (1976). Mathematical Techniques for Efficient Record Segmentation in Large Shared Databases. *Journal of the ACM*, 23(4), 619–635.

Engevall, S., Göthe-Lundgren, M., & Värbrand, P. (2004). The Heterogeneous Vehicle-Routing Game. *Transportation Science*, 38(1), 71–85.

ETP-ALICE (2016). *Corridors, Hubs and Synchromodality*. Technical report.

EuYu (2014). Degree sequence of connected graphs. Mathematics Stack Exchange.

Ford, L. R. & Fulkerson, D. R. (1958). Constructing Maximal Dynamic Flows from Static Flows. *Operations Research*, 6(3), 419–433.

Fradkin, A. (2017). Search, Matching, and the Role of Digital Marketplace Design in Enabling Trade: Evidence from Airbnb. *SSRN Electronic Journal*.

Frank, H. (1969). Shortest Paths in Probabilistic Graphs. *Operations Research*, 17(4), 583–599.

Fransoo, J. C. & Lee, C. Y. (2013). The critical role of ocean container transport in global supply chain performance. *Production and Operations Management*, 22(2), 253–268.

Frisk, M., Göthe-Lundgren, M., Jörnsten, K., & Rönnqvist, M. (2010). Cost allocation in collaborative forest transportation. *European Journal of Operational Research*, 205(2), 448–458.

Gal, T. (1994). *Postoptimal Analyses, Parametric Programming, and Related Topics*. De Gruyter.

Gillies, D. B. (1959). Solutions to general non-zero-sum games. *Contributions to the Theory of Games*, 4(40), 47–85.

Giusti, R., Manerba, D., Bruno, G., & Tadei, R. (2019). Synchromodal logistics: An overview of critical success factors, enabling technologies, and open research issues. *Transportation Research Part E: Logistics and Transportation Review*, 129, 92–110.

Greene, W. H. (2018). *Econometric Analysis*. Pearson, 8 edition.

Guajardo, M. & Rönnqvist, M. (2016). A review on cost allocation methods in collaborative transportation. *International Transactions in Operational Research*, 23(3), 371–392.

Gul, F. (1989). Bargaining Foundations of Shapley Value. *Econometrica*, 75(1), 81–95.

Hall, R. W. (1986). The Fastest Path through a Network with Random Time-Dependent Travel Times. *Transportation Science*, 20(3), 182–188.

Hapag-Lloyd (2021). Schedule Reliability: Hapag-Lloyd to provide full transparency on vessel arrivals - Hapag-Lloyd.

Hastie, T., Tibshirani, R., & Friedman, J. (2009). *The Elements of Statistical Learning*. Springer Series in Statistics. New York, NY: Springer New York.

Houghtalen, L., Ergun, ., & Sokol, J. (2011). Designing Mechanisms for the Management of Carrier Alliances. *Transportation Science*, 45(4), 465–482.

Hu, M. & Zhou, Y. (2015). Dynamic Matching in a Two-Sided Market. *SSRN Electronic Journal*.

Huang, H. & Gao, S. (2018). Trajectory-adaptive routing in dynamic networks with dependent random link travel times. *Transportation Science*, 52(1), 102–117.

IPPC (2014). *IPCC, 2014: Climate Change 2014: Mitigation of Climate Change. Contribution of Working Group III to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change*. Technical report, IPCC, , Cambridge, United Kingdom and New York, NY, USA.

Jaillet, P., Qi, J., & Sim, M. (2016). Routing Optimization Under Uncertainty. *Operations Research*, 64(1), 186–200.

Jenkins, L. (1990). Parametric Methods in Integer Linear Programming. *Annals of Operations Research*, 27, 77–96.

Johari, R., Kamble, V., & Kanoria, Y. (2021). Matching While Learning. *Operations Research*, 69(2), 655–681.

Johnson, T. (2020). Reliability on the spot. *Maersk News*.

Joseph, V. R. (2016). Space-filling designs for computer experiments: A review. In *Quality Engineering*, volume 28 (pp. 28–35).: Taylor and Francis Inc.

Joseph, V. R., Gul, E., & Ba, S. (2019). Designing computer experiments with multiple types of factors: The MaxPro approach. *Journal of Quality Technology*, 52(4), 343–354.

Kanoria, Y. & Saban, D. (2017). Facilitating the Search for Partners on Matching Platforms: Restricting Agents' Actions. *SSRN Electronic Journal*.

Karsten, F., Slikker, M., & van Houtum, G.-J. (2015). Resource Pooling and Cost Allocation Among Independent Service Providers. *Operations Research*, 63(2), 476–488.

Keyhani, M. H., Schnee, M., & Weihe, K. (2017). Arrive in time by train with high probability. *Transportation Science*, 51(4), 1122–1137.

Knolwer, G. (2021). Container shipping: Europe stuck with port bottlenecks until liner reliability improves: Rotterdam.

Konrad, A. (2022). Flexport Is Silicon Valleys Solution To The Supply Chain MessWhy Do Insiders Hope It Sinks? *Forbes*.

Kurant, M. & Thiran, P. (2006). Layered complex networks. *Physical Review Letters*, 96(13), 1–4.

Langen, P. W. D. (2004). *The Performance of Seaport Clusters; A Framework to Analyze Cluster Performance and an Application to the Seaport Clusters of Durban, Rotterdam and the Lower Mississippi*. Erasmus Research Institute of Management (ERIM), Erasmus University Rotterdam.

Lattimore, T. & Szepesvári, C. (2020). *Bandit Algorithms*. Cambridge University Press.

L'Ecuyer, P., Maillé, P., Stier-Moses, N. E., & Tuffin, B. (2017). Revenue-maximizing rankings for online platforms with quality-sensitive consumers. *Operations Research*, 65(2), 408–423.

Li, J. & Netessine, S. (2020). Higher market thickness reduces matching rate in online platforms: Evidence from a quasiexperiment. *Management Science*, 66(1), 271–289.

Li, W. & Cai, X. (2007). Empirical analysis of a scale-free railway network in China. *Physica A: Statistical Mechanics and its Applications*, 382(2), 693–703.

Lijesen, M. G. (2014). Optimal Traveler Responses to Stochastic Delays in Public Transport. *Transportation Science*, 48(January 2015), 256–264.

Lin, J. & Ban, Y. (2013). Complex Network Topology of Transportation Systems. *Transport Reviews*, 33(6), 658–685.

Lin, Y.-K. (2007). On a multicommodity stochastic-flow network with unreliable nodes subject to budget constraint. *European Journal of Operational Research*, 176(1), 347–360.

Lium, A. G., Crainic, T. G., & Wallace, S. W. (2009). A study of demand stochasticity in service network design. *Transportation Science*, 43(2), 144–157.

Lozano, S., Moreno, P., Adenso-Diaz, B., & Algaba, E. (2013). Cooperative game theory approach to allocating benefits of horizontal cooperation. *European Journal of Operational Research*, 229(2), 444–452.

Maersk (2020). *Logistics Digital Revolution*. Technical Report August, A.P. Moller Maersk.

Miller, J., Nie, Y. M., & Liu, X. (2020). Hyperpath Truck Routing in an Online Freight Exchange Platform. *Transportation Science*, 54(6), 1676–1696.

Miller-Hooks, E. & Mahmassani, H. (2003). Path comparisons for a priori and time-adaptive decisions in stochastic, time-varying networks. *European Journal of Operational Research*, 146(1), 67–82.

Miller-Hooks, E. D. & Mahmassani, H. S. (2000). Least Expected Time Paths in Stochastic, Time-Varying Transportation Networks. *Transportation Science*, 34(2), 198–215.

Montreuil, B. (2011). Toward a Physical Internet: meeting the global logistics sustainability grand challenge. *Logistics Research*, 3(2-3), 71–87.

Montreuil, B., Meller, R. D., & Ballot, E. (2012). Physical Internet Foundations. *IFAC Proceedings Volumes*, 45(6), 26–30.

Morder Intelligence (2021). *Netherlands Road Freight Transport Market*. Technical report.

Mulder, J., van Jaarsveld, W., & Dekker, R. (2019). Simultaneous Optimization of Speed and Buffer Times with an Application to Liner Shipping. *Transportation Science*, 53(2), 365–382.

Nash, J. (1953). Two-Person Cooperative Games. *Econometrica*, 21(1), 128.

Nasri, M. I., Bektaş, T., & Laporte, G. (2018). Route and speed optimization for autonomous trucks. *Computers and Operations Research*, 100, 89–101.

Nie, Y. M. & Wu, X. (2009). Shortest path problem considering on-time arrival probability. *Transportation Research Part B: Methodological*, 43(6), 597–613.

Notteboom, T. (2004). A carrier's perpective on container network configuration at sea and on land. *Journal of International Logistics and Trade*, 1(2), 65–87.

Notteboom, T. & Rodrigue, J. P. (2005). Port regionalization: Towards a new phase in port development. *Maritime Policy and Management*, 32(3), 297–313.

Notteboom, T. & Rodrigue, J.-P. (2007). Re-assessing port-hinterland relationships in the context of global commodity chains. *Inserting Port-Cities in Global Supply Chains. London: Ashgate*.

Notteboom, T. & Rodrigue, J.-P. (2008). Containerisation, Box Logistics and Global Supply Chains: The Integration of Ports and Liner Shipping Networks. *Maritime Economics & Logistics*, 10(1-2), 152–174.

Öner, K. B., Scheller-Wolf, A., & van Houtum, G.-J. (2013). Redundancy Optimization for Critical Components in High-Availability Technical Systems. *Operations Research*, 61(1), 244–264.

Opasanon, S. & Miller-Hooks, E. (2006). Multicriteria adaptive paths in stochastic, time-varying networks. *European Journal of Operational Research*, 173(1), 72–91.

Owen, G. (1975). On the core of linear production games. *Mathematical Programming*, 9(1), 358–370.

Özener, O. ., Ergun, ., & Savelsbergh, M. (2013). Allocating Cost of Service to Customers in Inventory Routing. *Operations Research*, 61(1), 112–125.

Özkan, E. & Ward, A. R. (2020). Dynamic Matching for Real-Time Ride Sharing. *Stochastic Systems*, 10(1), 29–70.

Park, S. H. & Ungson, G. R. (2003). Interfirm rivalry and managerial complexity: a conceptual framework of alliance failure. *Organization Science*, 12(1), 37–53.

Perera, S., Bell, M. G., & Bliemer, M. C. (2017). Network science approach to modelling the topology and robustness of supply chain networks: a review and perspective.

Pérez-Castrillo, D. & Wettstein, D. (2001). Bidding for the surplus: A non-cooperative approach to the Shapley value. *Journal of Economic Theory*, 100(2), 274–294.

Pérez Rivera, A. E. (2018). *Anticipatory Freight Scheduling in Synchromodal Transport*. PhD thesis, University of Twente, Enschede, The Netherlands.

Port of Rotterdam (2018). Interim verview of inland container shipping sector consultations.

Powell, W. B. (2011). *Approximate Dynamic Programming*. Wiley Series in Probability and Statistics. Hoboken, NJ, USA: John Wiley & Sons, Inc.

Powell, W. B. (2019). A unified framework for stochastic optimization. *European Journal of Operational Research*, 275(3), 795–821.

Prakash, A. A. & Srinivasan, K. K. (2016). Pruning algorithms to determine reliable paths on networks with random and correlated link travel times. *Transportation Science*, 52(1), 80–101.

Puterman, M. (2005). *Markov Decision Processes Discrete Stochastic Dynamic Programming*.

Qin, Z., Tang, X., Jiao, Y., Zhang, F., Xu, Z., Zhu, H., & Ye, J. (2020). Ride-hailing order dispatching at DiDi via reinforcement learning. *Interfaces*, 50(5), 272–286.

Rambha, T., Boyles, S. D., & Waller, S. T. (2016). Adaptive Transit Routing in Stochastic Time-Dependent Networks. *Transportation Science*, 50(3), 1043–1059.

Reis, V. (2015). Should we keep on renaming a +35-year-old baby? *Journal of Transport Geography*, 46, 173–179.

Riedl, J., Chan, T., Schöndorfer, S., Schröder, F., & Sønderby, M. (2018). *The digital imprerative in freight forwarding*. Technical report, Boston Consulting Group.

Riessen, B. v. (2018). *Optimal transportation plans and portfolios for synchromodal container networks*. PhD thesis, Erasmus University Rotterdam, Rotterdam.

Rietveld, P. & Nijkamp, P. (1992). Transport and regional development.

Ríos, I., Saban, D., & Zheng, F. (2020). Improving Match Rates in Dating Markets Through Assortment Optimization. *SSRN Electronic Journal*.

Robbins, H. (1952). Some Aspects of the Sequential Design of Experiments. *Bulletin of the American Mathematical Society*, 58(5), 527–535.

Ruß, M., Gust, G., & Neumann, D. (2021). The Constrained Reliable Shortest Path Problem in Stochastic Time-Dependent Networks. *Operations Research*, 69(3), 709–726.

Santner, T. J., Williams, B. J., & Notz, W. I. (2018). *The Design and Analysis of Computer Experiments*. Springer Series in Statistics. New York, NY: Springer New York, 2 edition.

Sauré, D. & Zeevi, A. (2013). Optimal Dynamic Assortment Planning with Demand Learning. *Manufacturing & Service Operations Management*, 15(3), 387–404.

Schuster, R., Nath, G., Rodriguez, P., OBrien, C., Aylor, B., Sidopoulos, B., Weise, D., & Datta, B. (2021). *Real-World Supply Chain Resilience*. Technical report.

Serrano, R. (2004). Fifty Years of the Nash Program, 1953-2003. *SSRN Electronic Journal*, XXIX(2), 219–258.

Shapley, L. S. (1953). A value for n-person games. *Contributions to the Theory of Games*, 2(28), 307–317.

Shapley, L. S. (1971). Cores of convex games. *International Journal of Game Theory*, 1(1), 11–26.

Shapley, L. S. & Shubik, M. (1966). Quasi-Cores in a Monetary Economy With Nonconvex Preferences. *Econometrica*, 34(4), 805–827.

Spivey, M. Z. & Powell, W. B. (2004). The Dynamic Assignment Problem. *Transportation Science*, 38(4), 399–419.

Sternberg, H. & Norrman, A. (2017). The Physical Internet review, analysis and future research agenda. *International Journal of Physical Distribution & Logistics Management*, 47(8), 736–762.

Sutton, R. & Barto, A. (2018). *Reinforcement learning: An introduction*. A Bradford Book.

Tavasszy, L., Behdani, B., & Konings, R. (2017). Intermodality and Synchromodality. In *Ports and Networks* (pp. 251–266). Abingdon, Oxon ; New York, NY : Routledge, 2018.: Routledge.

Tavasszy, L. a., Janssen, R., Lugt, L. V. d., & Hagdom, L. (2010). *Verkenning synchromodaal transportsysteem*. Technical report.

The Economist (2021). A perfect storm for container shipping. *The Economist*.

Theys, C., Dullaert, W., & Notteboom, T. (2008). Analyzing Cooperative Networks in Intermodal Transportation: a Game-Theoretic Approach. (January), 1–37.

Thompson, W. R. (1933). On the Likelihood that One Unknown Probability Exceeds Another in View of the Evidence of Two Samples. *Biometrika*, 25(3/4), 285.

Toy, J., Gesing, B., Ward, J., Noronha, J., & Bodenbenner, P. (2022). *Trend Radar, 5 th Edition*. Technical report, DHL Customer Solutions & Innovation.

Tran, T. H., OHanley, J. R., & Scaparra, M. P. (2017). Reliable Hub Network Design: Formulation and Solution Techniques. *Transportation Science*, 51(1), 358–375.

van Laarhoven, P. J. M. & Aarts, E. H. L. (1987). Simulated annealing. In *Simulated Annealing: Theory and Applications* (pp. 7–15). Dordrecht: Springer Netherlands.

Veenstra, A., Zuidwijk, R., & Van Asperen, E. (2012). The extended gate concept for container terminals: Expanding the notion of dry ports. *Maritime Economics and Logistics*, 14(1), 14–32.

Vernimmen, B., Dullaert, W., & Engelen, S. (2007). Schedule Unreliability in Liner Shipping: Origins and Consequences for the Hinterland Supply Chain. *Maritime Economics & Logistics*, 9(3), 193–213.

Viger, F. & Latapy, M. (2005). Efficient and Simple Generation of Random Simple Connected Graphs with Prescribed Degree Sequence. In *Lecture Notes in Computer Science*, volume 3595 (pp. 440–449). Springer Verlag.

Wang, L., Yang, L., & Gao, Z. (2016). The constrained shortest path problem with stochastic correlated link travel times. *European Journal of Operational Research*, 255(1), 43–57.

Wang, Z. & Qi, M. (2020). Robust Service Network Design Under Demand Uncertainty. *Transportation Science*, 54(3), 676–689.

Waters, W. (2021). Container shipping reliability drops further. *Lloyd's Loading List*.

Watts, D. J. & Strogatz, S. H. (1998). Collective dynamics of small-world networks. *Nature*, 393(6684), 440–442.

Wiegmans, B. & Konings, R. (2016). *Inland Waterway Transport*. Routledge, 1st edition.

Williams, D. (1991). *Probability with Martingales*. Cambridge University Press.

Wilmsmeier, G., Monios, J., & Lambert, B. (2011). The directional development of intermodal freight corridors in relation to inland terminals. *Journal of Transport Geography*, 19(6), 1379–1386.

World Bank Group (2018). *Stronger Open Trade Policies Enable Economic Growth for All*. Technical report.

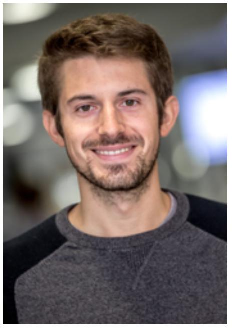
Yang, L. & Zhou, X. (2017). Optimizing on-time arrival probability and percentile travel time for elementary path finding in time-dependent transportation networks: Linear mixed integer programming reformulations. *Transportation Research Part B: Methodological*, 96, 68–91.

Ypsilantis, P. & Zuidwijk, R. (2019). Collaborative fleet deployment and routing for sustainable transport. *Sustainability*, 11(20), 5666.

Zhang, Y., Max Shen, Z. J., & Song, S. (2017). Lagrangian relaxation for the reliable shortest path problem with correlated link travel times. *Transportation Research Part B: Methodological*, 104, 501–521.

Zheng, J., Gao, Z., Yang, D., & Sun, Z. (2015). Network Design and Capacity Exchange for Liner Alliances with Fixed and Variable Container Demands. *Transportation Science*, 49(4), 886–899.

About the author



Alberto Giudici was born in Monza, Italy, on June 12, 1991. He studied mathematics in Italy (Universita' di Milano-Bicocca) and Germany (TU Kaiserslautern), following his passion for knowledge and abstract modeling. In 2016, he joined a PhD-track at RSM to discover, if not test, whether the mathematics he learned could support and improve logistical processes. This path saw him involved first in two European projects, SELIS and PLANET, and later in collaboration with a startup operating a digital marketplace. Coming towards the end of his Ph.D., he decided to attempt the road less traveled by: combining industry and academia on an equal share. Since November 2021, he has worked at Uturn as an Operations Research Analyst and at RSM, involved in the 4TEURN project.

Alberto's research interests include the theoretical and data-driven analysis and optimization of transport systems and the digital marketplace. His work has been published in Transportation Science and presented at several international conferences, including the INFORMS Annual Meeting, European Conference on Operational Research, and Triennial Symposium on Transportation Analysis. He has also served as an ad-hoc reviewer of various journals, such as the European Journal of Operations Research and the Journal of Heuristics.

Portfolio

Publications

Publications in Journals:

Giudici, A., Lu, T., Thielen, C., & Zuidwijk, R. (2021). An Analysis of the Stability of Hinterland Container Transport Cooperation. *Transportation Science, 55(5)*, 1170-1186.

Giudici, A., Halffmann, P., Ruzika, S., & Thielen, C. (2017). Approximation schemes for the parametric knapsack problem. *Information Processing Letters, 120*, 11-15.

Working Papers:

Giudici, A., Lu, T., Thielen, C., & Zuidwijk, R. (2022). Booking a reliable flow in a stochastic network: the minimum cost adaptive flow problem.

Giudici, A., van Dalen, J., Lu, T., & Zuidwijk, R. (2022). Online learning for two-sided sequential matching markets with temporal effects.

PhD Courses

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Stochastic Models & Optimisation	Facility Logistics Management
Stochastic Programming	Convex Analysis for Optimization
Freight Transport Management	Cooperative Games
Transport Logistics Modelling	Publishing Strategy
Scientific Integrity	English

Teaching

Lecturer:

Management Science 3 2019, 2020, 2021

Tutorial Lecturer:

Economics of Digitization 2020

Operations Research Methods 2019, 2020

Conferences Attendance and Invited Sessions

LOGMS 2017, Bergen, Norway	08-2017
TRAIL PhD Conference 2017, Utrecht, The Netherlands	11-2017
OML Netherlands 2018, Soesterberg, The Netherlands	04-2018
TRA, Vienna, Austria	04-2018
IWOBIP 2018, Lille, France	06-2018
EURO 2019, Valencia, Spain	07-2018
INFORMS 2018, Phoenix, Arizona, USA	11-2018
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Container transport allowed for global trade, and trade has lifted the welfare and fostered the development of nations worldwide. The exchange of goods required the movements of those along a chain of parties and companies. While intercontinental transport allowed for global trade, the inland transport sector plays a critical role in regions' and countries' competitive performance.

This thesis focuses on the inland container and bulk transport, the first and end part of the intercontinental journey, which connects production areas to ports and warehouses and last-mile distribution. Within this context, the challenge tackled in this research is improving transport performance by focusing on transport capacity utilization given immutable transport demand, technology, and infrastructure. A particular focus is on the plan of transport execution and its effect on transport capacity utilization.

From such a problem perspective, three main solutions are studied: first, cooperation among transport operators as a way for companies to improve their capacity utilization by demand pooling and supply sharing; second, advanced transport planning models based on adaptive decisions that are cost-efficient while guaranteeing a certain level of reliability as a way to improve capacity utilization when transport times are uncertain; third, matching algorithms for digital transport marketplaces as a way to enhance the utilization of capacity by effectively match demand and supply.

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