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Estimation of Population Mean on Recent Occasion under Non-Response in *h*-Occasion Successive Sampling

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In this article, an attempt has been made to study on general estimation procedures of population mean on recent occasion when non-response occurs in *h*-occasion successive sampling. Suggested estimators have advantageously influenced the estimation procedures in the presence of non-response. Detailed properties of the suggested estimation procedures have been examined and compared with the estimation process of the same circumstances but in the absence of non-response. Empirical studies have been carried out to demonstrate the performances of the estimates and suitable recommendations have been made.

Keywords: Non-response, successive sampling, study variable, variance

Introduction

Successive sampling was developed for estimation of population parameters on recent point of time (occasion), when the population parameters changes over successive points of time (occasion). It is a sampling method to provide reliable and fruitful estimates of population parameters over different desire points of time (occasion). Jessen (1942) initiated a technique with the help of past information to provide the effective estimates on current occasion in two-occasion successive sampling. Later, this technique was extended by Yates (1949), Patterson (1950), Tikkiwal (1951), Eckler (1955), Rao and Graham (1964), Gupta (1979), Binder and Hidiroglou (1988), Kish (1998), McLaren and Steel (2000), Singh, Kennedy and Wu (2001), Steel and McLaren (2002) among others. Sen (1971, 1973) applied this theory in designing the estimators of population mean using information on two or more auxiliary variables which was readily available on

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previous occasion in two-occasion successive sampling. Singh, Singh and Shukla (1991), Singh and Singh (2001) made an efficient use of auxiliary variable on current occasion and subsequently Singh (2003) uses this methodology for h-occasion successive sampling in estimation of current population mean.

In many situations, information on an auxiliary variable may be readily available on the first as well as on the second occasion. Utilizing the auxiliary information on both occasions, Feng and Zou (1997), Biradar and Singh (2001), Singh (2005), Singh and Karna (2009), Singh and Prasad (2010), Singh, Prasad, and Karna (2011), Singh, Majhi, Maurya, and Sarma (2015) and Singh and Sharma (2014, 2015) have proposed several estimators of population mean on current (second) occasion in two-occasion successive sampling.

Non-response is a common problem almost encountered in all sample surveys and successive sampling is more prone to this problem because of its repetitive nature. For example, in agriculture yield surveys, it might be possible that crop on certain plots are destroyed due to some natural calamities or disease so that yield on these plots are impossible to be measured. Hansen and Hurwitz (1946) suggested a method of sub sampling of non-respondents to address the problems of non-response in mail surveys. Later on Cochran (1977) and Okafor and Lee (2000) extended this technique for the case when besides the information on character under study, information is also available on one auxiliary character. More recently, Choudhary, Bathla, and Sud (2004), Singh and Priyanka (2007), and Singh and Kumar (2008) used the Hansen and Hurwitz (1946) technique for the estimation of population mean on current occasion in context of sampling on two occasions.

Motivated with the above arguments and using Hansen and Hurwitz (1946) method, the aim of the present work is to suggest the estimation procedure for population mean at h^{th} (recent) occasion when the non-response occurs on h^{th} occasion, $(h\text{-}1)^{\text{th}}$ (previous) occasion and simultaneously on both h^{th} and $(h\text{-}1)^{\text{th}}$ occasions in h-occasion successive (rotation) sampling. The properties of the proposed estimation procedure have been examined and compared with the similar estimation but under complete response. Empirical studies are carried out and suitable recommendations have been made.

Notations

Let $U = (U_1, U_2, \text{- - -}, U_N)$ be the finite population of N units, which has been sampled over h occasions. The character under studies are denoted by y_h and $y_{h\text{-}1}$ on the h^{th} and $(h\text{-}1)^{\text{th}}$ occasions respectively. Assume that the non-response occur on h^{th} occasion, $(h\text{-}1)^{th}$ occasion and simultaneously on both h^{th} and $(h\text{-}1)^{th}$ occasions, so that the population can be divided into two classes, those who will respond at the first attempt and those who will not. Let the sizes of these two classes be N_h and N^*_h on the h^{th} occasion and the corresponding sizes on $(h\text{-}1)^{th}$ occasion be $N_{h\text{-}1}$ and $N^*_{h\text{-}1}$. Let a simple random sample (without replacement) of size n be selected on the h^{th} occasion which consist of $n'_h = n\lambda_h$ units common to the units observed on the $(h\text{-}1)^{th}$ occasion and $n''_h = n\mu_h$ units drawn afresh on the h^{th} occasion i.e. $n = n'_h + n''_h$. Here λ_h and μ_h $(\lambda_h + \mu_h = 1)$ are the fractions of matched and unmatched samples, respectively, on the h^{th} occasion. The values of λ_h and μ_h should be chosen optimally. Assume that in the unmatched portion of the sample on the h^{th} occasion n''_h units respond and n''_{h2} units do not respond. Let n''_{h2s} denote the size of sub sample drawn from the non-response class in the unmatched portion of the sample on the h^{th} occasion and their response collected by direct contact or interview. Similarly, n'_{h1} units respond and n'_{h2} units do not respond in the sample of matched units and let n'_{h2s} denote the size of sub sample drawn from the non-response class in the matched portion of the sample on the h^{th} occasion and their response collected by direct contact or interview. Following are the list of notations, which are considered for their further use:

$\bar{Y}_h$: The population mean of the study variable y_h on the h^{th} occasion.

$\bar{y}'_h$: The sample mean of the study variable based on n'_h units common to the units observed on the $(h\text{-}1)^{th}$ occasion.

$\bar{y}''_h$: The sample mean of the study variable based on n''_h units drawn afresh on the h^{th} occasion.

$\rho_{h,h-1}$: The correlation between the study variables y_h and $y_{h\text{-}1}$.

S^2_{hy} : The population variance of the variable y_h on the h^{th} occasion.

$W^*_{h-1} = \frac{N^*_{h-1}}{N}$: The proportion of non-responding units in the population on the $(h\text{-}1)^{th}$ occasion.

$W^*_h = \frac{N^*_h}{N}$: The proportion of non-responding units in the population on the h^{th} occasion.

$f\left(\frac{n}{N}\right)$: The sampling fraction.

$f_1 = \frac{n'_{h2}}{n'_{h2s}}$.

$f_2 = \frac{n''_{h2}}{n''_{h2s}}$.

Formulation of Estimator

For estimating the population mean $\bar{Y}$ on the h^{th} occasion, a sample mean and a regression type estimator are suggested. First is the Hansen and Hurwitz (1946) type estimator, say ω''_h, which is based on n''_h sample units drawn afresh on h^{th} occasion such that out of these n''_h units, n''_{h1} units respond and remaining $n''_{h2}\left(= n''_h - n''_{h1}\right)$ units do not respond. Hence, ω''_h is defined as

$$\omega''_h = \bar{y}''^{*}_h \tag{1}$$

where

$$\bar{y}''^{*}_h = \frac{n''_{h1}\bar{y}''_1 + n''_{h2}\bar{y}''_{h2s}}{n''_h}$$

The second estimator is based on the sample of size n'_h, which is common to the units observed on the $(h\text{-}1)^{\text{th}}$ occasion. Because non-response is occurred on the previous occasion, therefore, again Hansen and Hurwitz (1946) type estimator are considered. The second estimator, ω'_h, for estimating the population mean on h^{th} occasion is a regression type estimator, and is defined as

$$\omega'_h = \bar{y}'^{*}_h + \beta_{h,h-1}\left(\omega_{h-1} - \bar{y}'^{*}_{h-1}\right) \tag{2}$$

where

$$\bar{y}_h^{\prime *} = \frac{n'_{h1}\bar{y}'_{h1} + n'_{h2}\bar{y}'_{h2}}{n'_h} \quad , \quad \bar{y}_{h-1}^{\prime *} = \frac{n'_{h1}\bar{y}'_{(h-1)1} + n'_{h2}\bar{y}'_{(h-1)2}}{n'_h}$$

and $\beta_{h,h-1}$ is population regression coefficient between the study variable y_h and $y_{h\text{-}1}$.

The resulting estimator ω_h is a convex linear combination of the estimators ω_h'' and ω_h'. The estimator ω_h is defined as

$$\omega_h = \varphi_h \omega_h'' + (1\text{-}\varphi_h)\omega_h' \tag{3}$$

where $\varphi_h\,(0 \le \varphi_h \le 1)$ is the unknown constant to be determined under certain criterion.

Remark 1: For estimating the mean on h^{th} occasion the estimator ω_h'' is suitable, which implies that more belief on ω_h'' could be shown by choosing φ_h as 1 (or close to 1), while for estimating the change from one occasion to the next, the estimator ω_h' could be more useful so φ_h might be chosen as 0 (or close to 0). For asserting both the problems simultaneously, the suitable (optimum) choice of φ_h is required.

Remark 2: (i) Assume that the correlation between variables observed on two occasions, more than one occasion apart is zero. (ii) For practical application the population regression coefficient will be estimated by their respective sample estimates.

Properties of the Estimator ω_h

Because ω_h'' and ω_h' are sample mean and difference type estimators respectively, they are unbiased for population mean $\bar{Y}_h$. Therefore, the resulting estimator ω_h defined in equation (3) is also an unbiased estimator of $\bar{Y}_h$. The variance of the estimator ω_h is shown in following theorem.

Theorem 1: Variance of the estimator ω_h to the first order of approximations is obtained as

$$V\left(\omega_h\right)=\varphi_h^2 V\left(\omega_h''\right)+\left(1-\varphi_h\right)^2 V\left(\omega_h'\right)+2\varphi_h\left(1-\varphi_h\right)C\left(\omega_h'',\omega_h'\right) \tag{4}$$

where

$$V\left(\omega_h''\right)=\left[\frac{1}{n_h''}\left\{1+W_h^*\left(f_2-1\right)\right\}-\frac{1}{N}\right]S_{hy}^2 \tag{5}$$

$$V\left(\omega_h'\right)=\left[\frac{1}{n_h'}\left\{1+W_{h-1}^*\left(f_1-1\right)\right\}\left(1-\rho_{h,h-1}^2\right)+\frac{\varphi_{h-1}}{n_{h-1}''}\rho_{h,h-1}^2-\frac{1}{N}\right]S_{hy}^2 \tag{6}$$

and

$$C\left(\omega_h'',\omega_h'\right)=-\frac{1}{N}S_{hy}^2 \tag{7}$$

Remark 3: Following Hansen and Hurwitz (1946) technique, some variances which are used in Theorem 1, are evaluated as given below:

$$\begin{aligned} V\left(\bar{y}_h''^*\right) &= V\left[E\left(\bar{y}_h''^*\middle| n_{h1}'',n_{h2}''\right)\right]+E\left[V\left(\bar{y}_h''^*\middle| n_{h1}'',n_{h2}''\right)\right] \\ &= V\left(\bar{y}_h''\right)+E\left[\frac{n_{h2}''}{n_h''^2}\left(f_2-1\right)S_{hy}^2\left(n_{h2}''\right)\right] \\ &= \left(\frac{1}{n_h''}-\frac{1}{N}\right)S_{hy}^2+\frac{\left(f_2-1\right)}{n''}S_{hy}^2\left(N_h^*\right)\frac{N_h^*}{N} \end{aligned}$$

where $S_{hy}^2\left(N_h^*\right)$ is the population variance of non response class on h^{th} occasion. Further we assume that $S_{hy}^2\left(N_h^*\right)=S_{hy}^2$, and hence

$$V\left(\bar{y}_h''^*\right)=\left[\left(\frac{1}{n_h''}-\frac{1}{N}\right)+\frac{W_h^*\left(f_2-1\right)}{n_h''}\right]S_{hy}^2 \tag{8}$$

Similarly

$$V\left(\bar{y}_h'^*\right)=\left[\left(\frac{1}{n_h'}-\frac{1}{N}\right)+\frac{W_{h-1}^*\left(f_1-1\right)}{n_h'}\right]S_{hy}^2 \tag{9}$$

$$V\left(\bar{y}_{h-1}'^*\right)=\left[\left(\frac{1}{n_h'}-\frac{1}{N}\right)+\frac{W_{h-1}^*\left(f_1-1\right)}{n_h'}\right]S_{(h-1)y}^2 \tag{10}$$

where

$$f_1=\frac{n_{h2}'}{n_{h2s}'}\ ;\ f_2=\frac{n_{h2}''}{n_{h2s}''}$$

Minimum Variance of the Estimator ω_h

Substituting the values of variances and covariance from equations (5), (6) and (7) in equation (4) we have the expression of the exact variance of the proposed estimator ω_h. Now, minimize the variance of ω_h, which is shown in equation (4). Define a function *f* (*x*, *y*), where the variables *x* and *y* are interpreted as φ_h and μ_h respectively, which represents the expression of the variance of ω_h given in equation (4). Thus, variance of ω_h is reduce to following equation

$$f\left(x,y\right)=\frac{S}{n}\left[\frac{x^2}{y}\Delta_2+\left(1-x\right)^2\left(\frac{\Delta_1}{1-y}+\gamma\right)-f\right] \tag{11}$$

where

$$S=S_{hy}^2,\quad \alpha=1-\rho_{h,h-1}^2,\quad \gamma=t_{h-1}\rho_{h,h-1}^2,\quad \Delta_1=\alpha+W_{h-1}^*\left(f_1-1\right)\alpha,$$

$$\Delta_2=1+W_h^*\left(f_2-1\right),\ t_{h-1}=\frac{\varphi_{h-1}}{\mu_{h-1}},\quad \mu_h=1-\lambda_h,\ \text{ and }\ f=\frac{n}{N}.$$

To find the minimum variance, we differentiate the equation (11) with respect to *x* and *y* respectively and then equate to zero,

$$\frac{x}{y}\Delta_2=\frac{1-x}{1-y}\left[\Delta_1+\gamma\left(1-y\right)\right] \tag{12}$$

and

$$\frac{x}{y}\sqrt{\Delta_2}=\frac{1-x}{1-y}\sqrt{\Delta_1} \tag{13}$$

From equations (12) and (13),

$$y=1-\sqrt{\Delta_1}\left(\sqrt{\Delta_2}-\sqrt{\Delta_1}\right)\gamma^{-1} \tag{14}$$

Again, from equations (13) and (14), if

$$\frac{y}{x}=1+\left(\sqrt{\Delta_2}-\sqrt{\Delta_1}\right)^2\gamma^{-1} \tag{15}$$

then

$$t_h=\left[1+r_h t_h^{-1}\right]^{-1}=\frac{x}{y} \tag{16}$$

where

$$r_h=\left(\sqrt{\Delta_2}-\sqrt{\Delta_1}\right)^2\left(1-\alpha\right)^{-1} \tag{17}$$

Because the values of α depend on the values of correlation. Therefore, $\alpha \geq 0$ and consequently r_h is real. After iteration,

$$t_h=\left[1-\sum_{j=1}^{h}\prod_{k=j}^{h}r_k\right]^{-1} \tag{18}$$

Hence, minimum variance of ω_h is obtained from equations (11) and (12) which is as follows

$$V\left(\omega_h\right)_{\text{opt}}=f\left(x,y\right)_{\text{opt}}=\frac{S}{n}\left[t_h\Delta_2-f\right] \tag{19}$$

Special Cases

Case 1: When non-response occurs only on (*h*-1)th *(previous) occasion.*

For the case when non-response occurs only on (h-1)th occasion, the estimator for population mean $\bar{Y}_h$ on recent occasion may be structured as

$$\omega_h^* = \varphi_h^* \tau_h'' + \left(1 - \varphi_h^*\right) \omega_h' \tag{20}$$

where $\tau_h'' = \bar{y}_h''$ and $\omega_h' = \bar{y}_h'^* + \beta_{h,h-1}\left(\omega_{h-1}^* - \bar{y}_{h-1}'^*\right)$. φ_h^* is unknown constant to be determined so as to minimize the variance of the estimator ω_h^*.

Properties of the estimator ω_h^*

Because τ_h'' and ω_h' are sample mean and difference type estimators respectively, they are unbiased for population mean $\bar{Y}_h$. Therefore, the resulting estimator ω_h^* is defined in equation (20) is also unbiased estimator of $\bar{Y}_h$.

Theorem 2: variance of the estimator ω_h^* is obtained as

$$V\left(\omega_h^*\right) = \varphi_h^{*2} V\left(\tau_h''\right) + \left(1 - \varphi_h^*\right)^2 V\left(\omega_h'\right) + 2\varphi_h^*\left(1 - \varphi_h^*\right) C\left(\tau_h'', \omega_h'\right) \tag{21}$$

where

$$V\left(\tau_h''\right) = \left(\frac{1}{n_h''} - \frac{1}{N}\right) S_{hy}^2 \tag{22}$$

$$V\left(\omega_h'\right) = \left[\frac{1}{n_h'}\left\{1 + W_{h-1}^*\left(f_1 - 1\right)\right\}\left(1 - \rho_{h,h-1}^2\right) + \frac{\varphi_{h-1}^*}{n_{h-1}''}\rho_{h,h-1}^2 - \frac{1}{N}\right] S_{hy}^2 \tag{23}$$

and

$$C\left(\tau''_h, \omega'_h\right) = -\frac{1}{N} S_{hy}^2 \tag{24}$$

Minimum Variance of the estimator ω_h^*

Similarly, represent the expression of the variance of ω_h^* in equation (21) as

$$f^*\left(x^*, y^*\right) = \frac{S}{n}\left[\frac{x^{*2}}{y^*} + \left(1 - x^*\right)^2 \left(\frac{\Delta_1}{1 - y^*} + \gamma\right) - f\right] \tag{25}$$

To find the minimum variance,

$$\frac{x^*}{y^*} = \frac{1 - x^*}{1 - y^*}\left[\Delta_1 + \gamma\left(1 - y^*\right)\right] \tag{26}$$

$$\frac{x^*}{y^*} = \frac{1 - x^*}{1 - y^*}\sqrt{\Delta_1} \tag{27}$$

From equations (26) and (27),

$$y^* = 1 - \sqrt{\Delta_1}\left(1 - \sqrt{\Delta_1}\right)\gamma^{-1} \tag{28}$$

Further,

$$\frac{y^*}{x^*} = 1 + \left(1 - \sqrt{\Delta_1}\right)^2 \gamma^{-1} \tag{29}$$

$$t_h^* = \left[1 + r_h^* t_{h-1}^{*-1}\right]^{-1} = \frac{x^*}{y^*} \tag{30}$$

where

$$r_h^* = \left(1 - \sqrt{\Delta_1}\right)^2 \left(1 - \alpha\right)^{-1} \tag{31}$$

$$t_h^* = \left[1 + \sum_{j=1}^{h} \prod_{k=j}^{h} r_k^*\right]^{-1} \tag{32}$$

From (25) and (26) minimum variance of ω_h^* is expressed as

$$V\left(\omega_h^*\right)_{\text{opt}} = f^*\left(x^*, y^*\right)_{\text{opt}} = \frac{S}{n}\left[t_h^* - f\right] \tag{33}$$

Case 2: When non-response occurs only on hth (recent) occasion

The estimator for the population mean $\bar{Y}_h$ on recent occasion for this case may be given as

$$\omega_h^{**} = \varphi_h^{**}\omega_h'' + \left(1 - \varphi_h^{**}\right)\tau_h' \tag{34}$$

where ω_h'' is defined in equation (1) and $\tau_h' = \bar{y}_h' + \beta_{h,h-1}\left(\omega_{h-1}^{**} - \bar{y}_{h-1}'\right)$ and φ_h^{**} is unknown constant to be determined so as to minimize the variance of the estimator ω_h^{**}.

Properties of the estimators ω_h^{**}

Because ω_h'' and τ_h' are sample mean and difference type estimators respectively, they are unbiased for population mean $\bar{Y}_h$. Therefore, the resulting estimator ω_h^{**} defined in equation (34) is also unbiased estimator of $\bar{Y}_h$.

Theorem 3: Variance of estimators ω_h^{**} is obtained as

$$V\left(\omega_h^{**}\right) = \varphi_h^{**2}V\left(\omega_h''\right) + \left(1 - \varphi_h^{**}\right)^2 V\left(\tau_h'\right) + 2\varphi_h^{**}\left(1 - \varphi_h^{**}\right)C\left(\omega_h'', \tau_h'\right) \tag{35}$$

where $V\left(\omega_h''\right)$ is shown in equation (5),

$$V\left(\tau_h'\right) = \left[\frac{1}{n_h'}\left(1 - \rho_{h,h-1}^2\right) + \frac{\varphi_{h-1}^{**}}{n_{h-1}''}\rho_{h,h-1}^2 - \frac{1}{N}\right]S_{hy}^2 \tag{36}$$

and

$$C\left(\Delta_h'', \tau_h'\right) = -\frac{1}{N} S_{hy}^2 \tag{37}$$

Minimum Variance of the estimator ω_h^{**}

The expression of the variance of ω_h^{**} shown in equation (35) is reduced to the following form

$$f^{**}\left(x^{**}, y^{**}\right) = \frac{S}{n}\left[\frac{x^{**2}}{y^{**}}\Delta_2 + \left(1 - x^{**}\right)^2\left(\frac{\alpha}{1 - y^{**}} + \gamma\right) - f\right] \tag{38}$$

To find the minimum variance,

$$\frac{x^{**}}{y^{**}} = \frac{1 - x^{**}}{1 - y^{**}}\left[\alpha + \gamma\left(1 - y^{**}\right)\right] \tag{39}$$

$$\frac{x^{**}}{y^{**}}\sqrt{\Delta_2} = \frac{1 - x^{**}}{1 - y^{**}}\sqrt{\alpha} \tag{40}$$

From equations (39) and (40)

$$y^{**} = 1 - \sqrt{\alpha}\left(\sqrt{\Delta_2} - \sqrt{\alpha}\right)\gamma^{-1} \tag{41}$$

then

$$\frac{y^{**}}{x^{**}} = 1 + \left(\sqrt{\omega_2} - \sqrt{\beta}\right)^2 \gamma^{-1} \tag{42}$$

$$t_h^{**} = \left[1 + r_h^{**} t_{h-1}^{**-1}\right]^{-1} = \frac{x^{**}}{y^{**}} \tag{43}$$

where

$$r_h^{**} = \left(\sqrt{\Delta_2} - \sqrt{\alpha}\right)^2 \left(1-\alpha\right)^{-1} \tag{44}$$

$$t_h^{**} = \left[1 + \sum_{j=1}^{h} \prod_{k=j}^{h} r_k^{**}\right]^{-1} \tag{45}$$

Thus, from (38) and (39) minimum variance of Δ_h^{**} is obtained as

$$V\left(\omega_h^{**}\right)_{\text{opt}} = f^{**}\left(x^{**}, y^{**}\right)_{\text{opt}} = \frac{S}{n}\left[t_h^{**}\Delta_2 - f\right] \tag{46}$$

Efficiency Comparison

To examine the loss in precision of the estimators ω_h, ω_h^{*} and ω_h^{**} due to non-response, the percent relative loss in precision of estimator ω_h, ω_h^{*} and ω_h^{**} with respect to the estimator τ_h, have been computed for different choices of $\rho_{h,h-1}$. The estimator τ_h is defined under the similar circumstances as the estimator ω_h but in the absence of non-response. Hence the estimator τ_h is given as

$$\tau_h = \varphi_h \tau_h'' + \left(1-\varphi_h\right)\tau_h' \tag{47}$$

where $\tau_h'' = \bar{y}_h'',\quad \tau_h' = \bar{y}_h' + \beta_{h,h-1}\left(\tau_{h-1} - \bar{y}_{h-1}'\right)$ and ψ_h is unknown constant to be determined by the minimization of the variance of τ_h.

Following Sukhatme, Sukhatme, Sukhatme, and Asok (1984) the optimum variance of τ_h is given by

$$V\left(\tau_h\right)_{\text{opt}} = \frac{S}{n}\left[\hat{t}_h - f\right] \tag{48}$$

where $\hat{t}_h = \left[1 + \sum_{j=1}^{h} \prod_{k=j}^{h} \hat{r}_k\right]^{-1}$ and $\hat{r}_k = \left(1-\sqrt{\alpha}\right)\left(1+\sqrt{\alpha}\right)^{-1}$.

Remark 4: To compare the performance of the estimators ω_h, ω_h^* and ω_h^{**} with respect to τ_h, the assumptions $W_h^* = W_{h-1}^*$ (say W^*) are introduced. The percent relative losses in precision of the estimators ω_h, ω_h^* and ω_h^{**} with respect to τ_h under their respective optimality conditions are given by

$$L = \frac{V(\omega_h)_{\text{opt}} - V(\tau_h)_{\text{opt}}}{V(\omega_h)_{\text{opt}}} \times 100 \qquad L^* = \frac{V(\omega_h^*)_{\text{opt}} - V(\tau_h)_{\text{opt}}}{V(\omega_h^*)_{\text{opt}}} \times 100$$

and

$$L^{**} = \frac{V(\omega_h^{**})_{\text{opt}} - V(\tau_h)_{\text{opt}}}{V(\omega_h^{**})_{\text{opt}}} \times 100$$

The expressions of $\mu_{h(\text{opt})}$, $\mu_{h(\text{opt})}^*$, $\mu_{h(\text{opt})}^{**}$ and the percent relative losses are given in terms of the population correlation coefficients. Therefore, they have been computed for different choices of correlation $\rho_{h,h-1}$. Percent relative losses in precision of the estimators ω_h, ω_h^* and ω_h^{**} have been computed for different choices of f, f_1, f_2, W_h^*, W_{h-1}^* and $\rho_{h,h-1}$.

Presented in Tables 1 - 3 are the optimum values of $\mu_{h(\text{opt})}$, $\mu_{h(\text{opt})}^*$, $\mu_{h(\text{opt})}^{**}$ and the percent relative losses with respect to τ_h.

Table 1. Percent relative loss L in precision of ω_h with respect to τ_h for f = 0.1.

Occasions (h)			$\rho_{h,h-1}$→	0.5		0.7		0.9	
	f_1↓	W^*↓	f_2↓	$\mu_{h(\text{opt})}$	L	$\mu_{h(\text{opt})}$	L	$\mu_{h(\text{opt})}$	L
2	1.0	0.2	1.5	0.3668	4.2873	0.5122	4.8124	0.6702	5.1467
			2.0	0.2053	6.5394	0.4443	8.2798	0.6451	9.2212
		0.4	1.5	0.2053	6.5394	0.4443	8.2798	0.6451	9.2212
			2.0	*	**	0.3164	12.5942	0.5978	15.1928
	2.0	0.2	1.5	0.6201	13.0095	0.5745	12.0443	0.6632	9.6643
			2.0	0.4431	17.0978	0.5001	15.8719	0.6357	13.6380
		0.4	1.5	0.7100	22.6145	0.5681	21.4502	0.6309	17.4019
			2.0	0.3503	28.9318	0.4167	26.8537	0.5750	23.0665
3	1.0	0.2	1.5	0.2822	3.1253	0.4008	2.8605	0.5172	1.5974
			2.0	0.0379	3.2392	0.2794	3.8429	0.4545	1.9800
		0.4	1.5	0.0379	3.2392	0.2794	3.8429	0.4545	1.9800
			2.0	*	**	0.0094	1.4728	0.3205	0.0642
	2.0	0.2	1.5	0.6048	13.3281	0.5128	12.6047	0.5275	9.3567
			2.0	0.3951	16.9222	0.4000	15.0408	0.4640	10.2194
		0.4	1.5	0.7042	22.9963	0.5128	22.2920	0.4778	16.7377
			2.0	0.2849	28.6079	0.2806	25.3154	0.3413	16.6839
4	1.0	0.2	1.5	0.2709	3.1253	0.3753	2.8605	0.4462	1.5974
			2.0	0.0027	3.2392	0.2305	3.8429	0.3521	1.9800
		0.4	1.5	0.0027	3.2392	0.2305	3.8429	0.3521	1.9800
			2.0	*	**	*	**	0.1294	0.0642
	2.0	0.2	1.5	0.6042	13.3281	0.5039	12.6047	0.4728	9.3567
			2.0	0.3910	16.9222	0.3800	15.0408	0.3830	10.2194
		0.4	1.5	0.7041	22.9963	0.5057	22.2920	0.4142	16.7377
			2.0	0.2784	28.6079	0.2488	25.3154	0.2127	16.6839

Note: "" indicate $\mu_{h(\text{opt})}$ does not exist.

Table 2. Percent relative loss L^* in precision of ω_h^* with respect to τ_h for $f = 0.1$.

Occasions (h)		$\rho_{h,h-1}\rightarrow$	0.5		0.6		0.8	
	$f_1\downarrow$	$W^*\downarrow$	$\mu^*_{h(\text{opt})}$	L^*	$\mu^*_{h(\text{opt})}$	L^*	$\mu^*_{h(\text{opt})}$	L^*
2	1.5	0.2	0.6668	3.9706	0.6163	3.6895	0.6936	2.4170
		0.4	0.8053	6.3585	0.6524	6.7430	0.6920	4.6254
	2.0	0.2	0.8053	6.3585	0.6524	6.7430	0.6920	4.6254
		0.4	*	**	0.7327	11.2723	0.6917	8.5195
3	1.5	0.2	0.6556	4.3688	0.5670	4.6766	0.5822	3.8818
		0.4	0.8032	6.8491	0.6188	8.3301	0.5882	7.2990
	2.0	0.2	0.8032	6.8491	0.6188	8.3301	0.5882	7.2990
		0.4	*	**	0.7196	13.3894	0.6024	13.0285
4	1.5	0.2	0.6552	4.4016	0.5607	4.8963	0.5416	4.6755
		0.4	0.8032	6.8849	0.6156	8.6488	0.5532	8.6882
	2.0	0.2	0.8032	6.8849	0.6156	8.6488	0.5532	8.6882
		0.4	*	**	0.7189	13.7590	0.5766	15.2022

Note "" indicate $\mu^*_{h(\text{opt})}$ does not exist.

Table 3. Percent relative loss L^{**} in precision of ω_h^{**} with respect to τ_h for $f = 0.1$.

Occasions (h)		$\rho_{h,h-1}\rightarrow$	0.5		0.6		0.7	
	$f_2\downarrow$	$W^*\downarrow$	$\mu^{**}_{h(\text{opt})}$	L^{**}	$\mu^{**}_{h(\text{opt})}$	L^{**}	$\mu^{**}_{h(\text{opt})}$	L^{**}
2	1.5	0.2	0.3668	4.2873	0.5122	4.8124	0.6702	5.1467
		0.4	0.2053	6.5394	0.4443	8.2798	0.6451	9.2212
	2.0	0.2	0.2053	6.5394	0.4443	8.2798	0.6451	9.2212
		0.4	*	**	0.3164	12.5942	0.5978	15.1928
3	1.5	0.2	0.2822	3.1253	0.4008	2.8605	0.5172	1.5974
		0.4	0.0379	3.2392	0.2794	3.8429	0.4545	1.9800
	2.0	0.2	0.0379	3.2392	0.2794	3.8429	0.4545	1.9800
		0.4	*	**	0.0094	1.4728	0.3205	0.0642
4	1.5	0.2	0.2709	2.9384	0.3753	2.2653	0.4462	-0.6762
		0.4	0.0027	2.4725	0.2305	2.2186	0.3521	-3.1638
	2.0	0.2	0.0027	2.4725	0.2305	2.2186	0.3521	-3.1638
		0.4	*	**	*	**	0.1294	-13.0652

Note: "" indicate $\mu^{**}_{h(\text{opt})}$ does not exist.

Results

Behavior of Estimator ω_h,

From Table 1,

(a) For the fixed values of h, f_1, f_2 and W^*, the value of $\mu_{h(\text{opt})}$ are mostly increased while the values of L are almost decreased when the values of $\rho_{h,h-1}$ are increased.

(b) For the fixed values of h, f_1, W^* and $\rho_{h,h-1}$, the values of $\mu_{h(\text{opt})}$ decrease while L increases with the increasing value of f_2. This trend shows the larger fresh sample is required to be replaced on the recent occasion.

(c) For the fixed values of h, f_2, W^*, and $\rho_{h,h-1}$, the values of $\mu_{h(\text{opt})}$ and L are increasing with the increasing values of f_1.

(d) For the fixed values of h, f_1, f_2 and $\rho_{h,h-1}$, the values of $\mu_{h(\text{opt})}$ almost decrease while L increases with the increasing value of W^*. This behavior shows that the higher the non-response rate, the larger fresh sample is required to be replaced on the recent occasion.

(e) For the fixed values of h, f_1, $W^*$and $\rho_{h,h-1}$, the values of $\mu_{h(\text{opt})}$ and L are almost decreasing with the increasing values of number of occasions (h). This phenomenon suggests that smaller fresh sample is required on the recent occasion which leads in reducing the cost of the survey.

Behavior of Estimator ω_h^*

From Table 2,

(a) For the fixed values of h, f_1, and W^*, no patterns are visible in the values of $\mu^*_{h(\text{opt})}$ and L^* with the increasing value of $\rho_{h,h-1}$.

(b) For the fixed values of h, W^*, and $\rho_{h,h-1}$, the values of $\mu^*_{h(\text{opt})}$ and L^* are increasing with the increasing values of f_1.

(c) For the fixed values of h, f_1, and $\rho_{h,h-1}$, the values of $\mu^*_{h(\text{opt})}$ and L^* increase with the increasing values of W^*.

(d) For the fixed values of f_1, W^* and $\rho_{h,h-1}$, the values of $\mu^*_{h(\text{opt})}$ are decreasing while the values of L^* are increasing with the increasing values of number of occasions (h). This event suggests that smaller fresh sample is required on the recent occasion so that cost of the survey is reduced.

Behavior of Estimator ω_h^{**}

From Table 3,

(a) For the fixed values of h, f_2, and W^* the values of $\mu^{**}_{h(\text{opt})}$ and L^{**} are almost increased when the value of $\rho_{h,h-1}$ is increased.

(b) For the fixed values of h, $\rho_{h,h-1}$, and W^* the values of $\mu^{**}_{h(\text{opt})}$ decrease while L^{**} increases with the increasing values of f_2. This phenomenon indicates that if a highly correlated auxiliary variable is available it pays in terms of reducing the cost of the survey and smaller fresh sample is required at the recent occasion.

(c) For the fixed values of h, f_2, and $\rho_{h,h-1}$, the values of $\mu^{**}_{h(\text{opt})}$ decreases while the values of L^{**} does not follow any certain pattern with the increasing value of W^*.

(d) For the fixed values of f_2, W^* and $\rho_{h,h-1}$, the values of $\mu^{**}_{h(\text{opt})}$ and L^{**} are decreasing with the increasing values of number of occasions (h). This behavior suggests that lower the non-response is useful and smaller fresh sample is required at the recent occasion which leads in the minimizing the survey cost.

Conclusion

On the basis of preceding interpretations, it may be concluded that the proposed estimation procedure is more useful and fruitful in the estimate of population mean when non-response occur on h^{th} occasion, $(h\text{-}1)^{\text{th}}$ occasion and simultaneously on both h^{th} and $(h\text{-}1)^{\text{th}}$ occasions in the h-occasion successive sampling. It is also visible from the empirical studies that the percent relative loss in precision is not so high. Hence, the proposed estimators ω_h, ω_h^*, and ω_h^{**} are performing well in terms of precision even in the presence of non-responses. Thus they are reliable and may be recommended to the survey statisticians and practitioners for its practical applications.

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