DIGITALCOMMONS @WAYNESTATE

Journal of Modern Applied Statistical Methods

Volume 15 | Issue 2 Article 27

11-1-2016

Limited Failure Censored Life Test Sampling Plan in Burr Type X Distribution

R. R. L. Kantam
Acharya Nagarjuna University, Guntur, Andhra Pradesh, India, kantam.rrl@gmail.com

M. S. Ravikumar
Konaseema Institute of Medical Sciences & Research Foundation, Andhra Pradesh, India., msrk.raama@gmail.com

Recommended Citation

Kantam, R. R. L. and Ravikumar, M. S. (2016) "Limited Failure Censored Life Test Sampling Plan in Burr Type X Distribution," *Journal of Modern Applied Statistical Methods*: Vol. 15: Iss. 2, Article 27.
DOI: 10.22237/jmasm/1478003100
Available at: http://digitalcommons.wayne.edu/jmasm/vol15/iss2/27

Limited Failure Censored Life Test Sampling Plan in Burr Type X Distribution

Cover Page Footnote

R.R.L.Kantam and M.S.Ravikumar

Journal of Modern Applied Statistical Methods
November 2016, Vol. 15, No. 2, 428-454.
doi: 10.22237/jmasm/1478003100

ISSN 1538 − 9472

Limited Failure Censored Life Test Sampling Plan in Burr Type X Distribution

R. R. L. Kantam
Acharya Nagarjuna University
Andhra Pradesh, India

M. S. Ravikumar
Konaseema Institute of Medical Sciences & Research Foundation
Andhra Pradesh, India

The Burr type X distribution is considered as a life time random variable of a product whose lots are to be decided for acceptance or otherwise on the basis of sample lifetimes drawn from the lot. The sample is divided into various groups in order to develop a group sampling plan in such a way that the life testing experiment is terminated as soon as the first failure in each group is observed. The acceptance criterion based on the theory of order statistics is proposed and is shown to be more economical than a criterion proposed in the earlier similar works.

Keywords: Single sampling, lot acceptance, group sampling plan, truncated life tests, reliability test plans, order statistics

Introduction

Acceptance sampling is concerned with inspection and decision making regarding products. Life tests are experiments carried out on sample products in order to assess the life time of an item (time to its failure or the time it stops working satisfactorily). A common practice in life test is to terminate the test at a prefixed time and record the number of failures that occurred during that time period or when a prefixed number of failures is realised. The former termination is generally called truncated life tests/time censored life test and the latter is called a failure censored life test. If the quality of a product is measured through the life time, sampling plans to determine acceptability of a product with respect to life time are called Reliability Sampling Plans.

In life test sampling plans a common constraint is the duration of total time spent on testing. Sampling plans based on time truncated life tests would address this constraint to some extent. When the life time random variable is assumed to

Dr. Kantam is an Professor of Statistics. Email him at: kantam.rrl@gmail.com. M. S. Ravikumar is a Lecturer and Biostatistician. Email at: msrk.raama@gmail.com.

follow a specific continuous probability distribution, sampling plans are developed by various researchers covering a wide spectrum of probability models.

Epstein (1954) was one of the foremost works about acceptance sampling plans based on truncated life tests with the exponential distribution as the probability model. Other researchers in this direction are as follows: Goode and Kao (1961) worked with the Weibull model which includes the exponential distribution as a particular case. Gupta and Groll (1961) and Gupta (1962) considered the gamma and log-normal distributions, respectively. More recently, the studies of Kantam, Rosaiah, and Srinivasa Rao (2001), Baklizi (2003), Baklizi and El-Masri (2004), Rosaiah and Kantam (2005), Balakrishanan, Lieva and López (2007), Aslam and Kantam (2008), Srinivasa Rao, Ghitany, and Kantam (2009), Rosaiah, Kantam, and Srinivasa Rao (2009), Srinivasa Rao and Kantam (2010), Lio, Tsai, and Wu (2009), Lio, Tsai, and Wu (2010), Lu (2011), Kantam, Sriram, and Suhasini (2012), Srinivasa Rao, Kantam, Rosaiah, and Pratapa Reddy (2012), Srinivasa Rao and Kantam (2013), Kantam and Sriram (2013), Subba Rao, Prasad, and Kantam (2013), Kantam, Sriram, and Suhasini (2013), Rosaiah, Kantam, Rama Krishnan, and Siva Kumar (2014), Subba Rao, Naga Durgamamba, and Kantam (2014) and the references therein, are related to construction of acceptance sampling plans based on truncated life tests with different probability models. In all these works, given the termination time of a life test, the construction of the sampling plan consists of determining the minimum number of sample items that are to be life-tested and the acceptance number beyond which the observed failures out of the life-tested items of the sample lead to rejection of the submitted lot, conditioned on pre specified producer’s and consumer’s risks.

However, if a failure censored life test is under consideration, one has to wait till a pre specified number of failures out of the sample items that are being tested is realised. Sometimes the life of product might be quite long possibly resulting in even a failure censored life-testing plan to be long time consuming. Johnson (1964) proposed a sampling plan in which the experimenter can decide to group the test units into several groups and then conduct the life-tests on all the groups simultaneously until the first failure in each group is realised. Based on the recorded first failure time in each group if a decision process about the acceptance/rejection of submitted lot is developed the procedure may be named as Limited Failure Censored Life Test Sampling Plan (LFCLTSP). Balasooriya (1995) developed such a sampling plan for the two parameter exponential distribution though the specific name is not given as LFCLTSP. Wu and Tsai (2000), Wu, Tsai, and Ouyang (2001), Jun, Balamurali, and Lee (2006) have proposed LFCLTSP when the underlying lifetime random variable follows

Weibull distribution, with respective distinct approaches in working out the parameters of the sampling plan. The scheme of life testing and termination process of LFCLTSP is named by some researchers as ‘Sudden death testing’ (for example Pascual & Meeker, 1998; Jun et al., 2006). ‘Limited failure censored life tests’ is the name proposed by Wu et al. (2001). Our suggested name is Limited Failure Censored Life Test Sampling Plan (LFCLTSP). Thus, the purpose of this study is to develop LFCLTSPs for one of the models of Burr (1942) – Burr type X distribution on lines of Jun et al. (2006). A new criterion is also suggested that is more economical.

Construction of LFCLTSP (Jun et al. 2006)

The purpose of proposing LFCLTSP is to reduce testing time. The total number of products to be tested, say N is divided into groups of equal size according to the number of available experimental testers. Thus there are n items in each group and a total of m groups may be considered for this grouping so that $N = m \times n$. The items in each group are tested identically and simultaneously on different testers. The first group of items is run until the first failure occurs. At this point the surviving items are suspended and removed from testing. An equal set of new items numbering n is next tested until the first failure. This process is repeated until one failure is generated from each of the m groups. In the end, m failures are observed while $(n - 1)\, m$ items are suspended. Wu et al. (2001) named this testing process as “limited failure censored life test”. The sample information so obtained can be utilized for deciding upon the acceptance of the lot from which the original sample of N is put for testing. According to the characteristics of testers a group size n is usually specified but the total number of groups m should be determined. For that a variable sampling plan is proposed by Jun et al. (2006) with the following assumptions/specifications

- The life time X follows a Weibull distribution with a known shape parameter (k).
- There is a lower specification limit (L) regarding the life time.
- p_0 is a desirable lot quality level (proportion of non conformities) at the pre specified producer’s risk α.
- $p_1\,(> p_0)$ is an undesirable lot quality level (proportion of non conformities) at the pre specified consumer’s risk β.

Sampling Plan

The cumulative distribution function (cdf) of the base line distribution (Weibull) is given by

$$F(x) = 1 - \exp\{-x^k\} \tag{1}$$

The fraction non-conforming or unreliability is expressed by

$$p = \Pr\{X < L\} = F(L) \tag{2}$$

If p is given, the corresponding L is obtained from

$$w = L^k = -\ln(1-p). \tag{3}$$

The proposed sampling plan of Jun et al. (2006) is as follows:

(i) Draw a random sample of size $N = m \times n$ and allocate n items to each of the m groups.

(ii) Observe Y_i the time to the first failure in the i^{th} group ($i = 1, 2, \ldots, m$).

(iii) Calculate the quantity $V = \sum_{i=1}^{m} Y_i^k$.

(iv) Accept the lot if $V \geq cL^k$ and reject the lot otherwise (c may be called acceptability constant - a concept similar to the acceptance number in time truncated reliability test plans).

The number of groups m and the acceptability constant c are called the parameters of the sampling plan and will be determined by the following procedure:

Since Y_i is the first order statistic in a sample of size n from Weibull distribution with shape parameter k its cdf is given by

$$\Pr(Y_i \leq y) = 1 - \exp(-ny^k), \tag{4}$$

which is the cdf of a Weibull distribution with shape parameter k and scale parameter $n^{1/k}$. Therefore the variables Y_i^k follow i.i.d exponential with scale parameter n and as such $V = \sum_{i=1}^{m} Y_i^k$ follows a gamma distribution with shape parameter m and scale parameter n. Thus the quantity $2nV$ follows a chi-square distribution with $2m$ degrees of freedom so that the probability of acceptance of the lot for a lot quality level p is given by

$$P_a(p) = \Pr\left\{V \geq cL^k \middle| p\right\} = \Pr\left\{2nV \geq 2ncL^k \middle| p\right\} = 1 - G_{2m}(2ncw), \tag{5}$$

where w is the solution of equation (3) and G_l is the cdf of a chisquare variate with l degrees of freedom. As in Fertig and Mann (1980), the probability of acceptance should be at least (1 - α) at the desirable/acceptable lot quality level p_0 where α is producer's risk. Similarly, the probability of acceptance should not be more than β at the undesirable/tolerance lot quality level p_1, where β is consumer's risk. These two remarks lead to the following two inequalities

$$1 - G_{2m}(2ncw_0) \geq 1 - \alpha \tag{6}$$

$$1 - G_{2m}(2ncw_1) \leq \beta, \tag{7}$$

If $\chi^2_{q,l}$ denotes the percentile point of tail probability q in the chi-square distribution with l degrees of freedom then, from (6), (7),

$$2ncw_0 \leq \chi^2_{1-\alpha,2m} \tag{8}$$

$$2ncw_1 \geq \chi^2_{\beta,2m} \tag{9}$$

which jointly lead to

$$\frac{w_0}{w_1} \leq \frac{\chi^2_{1-\alpha,2m}}{\chi^2_{\beta,2m}}. \tag{10}$$

Therefore, m can be obtained by the smallest integer satisfying (10). The acceptability constant c can be obtained from the equality case in either of the expressions (8), (9). It can be noticed that the number of groups m is determined independently of the group size n and also of the shape parameter k. Jun et al.

(2006) have evaluated m, nc for $\alpha = 0.05$ and $\beta = 0.1$ at selected combinations of p_0, p_1. The corresponding table is reproduced below:

Table 1. Design parameters of sampling plans ($\alpha = 0.05$, $\beta = 0.1$)

p_0	p_1	g	rk
0.001	0.002	18.7	12201.0
	0.004	5.1	2025.6
	0.005	3.9	1308.0
	0.010	2.1	408.5
	0.050	1.0	44.4
	0.100	0.8	18.6
0.005	0.010	18.6	2417.6
	0.015	7.7	757.7
	0.020	5.1	399.3
	0.025	3.9	256.5
	0.05	2.1	79.0
	0.25	0.9	7.6
0.01	0.02	18.5	1195.3
	0.04	5.0	196.0
	0.05	3.8	125.7
	0.10	2.1	37.8
	0.15	1.6	20.0
	0.3	1.1	7.0
0.05	0.1	17.4	217.7
	0.2	4.6	33.5
	0.25	3.5	20.7
	0.3	2.8	14.3
	0.5	1.8	5.1
0.1	0.2	16.1	95.9
	0.4	4.0	13.2
	0.5	3.0	7.7

For the sake of convenience in presentation, this procedure of Jun et al. (2006) is called Method-I and adopts the same for Burr type X distribution to construct LFCLTSP below.

LFCLTSP for Burr type X distributed Lifetimes: Method-I

Let the life time of a product be given by Burr type X distribution with shape parameter k so that cdf is given by

$$F(x) = \left(1 - e^{-x^2}\right)^k \tag{11}$$

Let L denote the p^{th} quantile of a Burr type X variate.

$$\text{i.e.,}\quad F\left(L\right)=p \tag{12}$$

If p is given, the corresponding L is obtained from

$$w=L=\sqrt{-\ln\left(1-p^{1/k}\right)} \tag{13}$$

Product with life time less than L is considered nonconforming. Suppose the producer and the consumer have an agreement that lots with nonconforming fraction less than or equal to p_0 are presumed to be good and have to be accepted with probability of at least 1 - α. Here α is called producer's risk. Furthermore suppose that lots with non conforming fraction greater than p_1 ($> p_0$) are not acceptable to the consumer and should be rejected with a probability of at least 1 - β. Here β is called consumer's risk.

If a random sample of N items grouped into m groups of size n each is put to test, an LFCLTSP on lines of Jun et al. (2006) can be constructed with the following decision process.

- Observe Y_i the time to the first failure in the i^{th} group (i = 1, 2,…, m).
- Calculate the quantity $V=\sum_{i=1}^{m} Y_i$.
- Accept the lot if $V \geq cL$ and reject the lot otherwise (c may be called acceptability constant - a concept similar to the acceptance number in time truncated reliability test plans).

In order to get the plan parameters m and c, the percentiles of the sampling distribution of V are needed, which is the sum of m i.i.d observations on the first order statistic in a random sample of size n modelled by Burr type X distribution with shape parameter k. In view of the mathematical structure of the Burr type X model the sampling distribution of V cannot be analytically tractable. Hence, consider the empirical sampling distribution of V for various known values of the shape parameter k and tabulated the percentiles of V for k = 1.5(0.5)3; m = 2(1)10; n = 5,10 in Tables 2 through 5.

Table 2. Percentiles of $V = \sum_{i=1}^{m} Y_i$ at k = 1.5

m	n \ p	0.99865	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005	0.00135
2	5	2.177892	2.004307	1.895265	1.754279	1.641112	1.511261	0.726597	0.628004	0.545044	0.454419	0.400317	0.308623
	10	1.575851	1.49449	1.445218	1.351926	1.27116	1.170157	0.569494	0.490655	0.429326	0.363279	0.321958	0.276647
3	5	2.831163	2.706339	2.58504	2.437557	2.313499	2.17402	1.198332	1.071406	0.970376	0.87162	0.801438	0.695547
	10	2.197636	2.064846	1.995025	1.881046	1.78807	1.684527	0.935104	0.839882	0.758188	0.671293	0.62377	0.559463
4	5	3.647301	3.449295	3.310793	3.12908	2.97969	2.80371	1.67707	1.5263	1.389551	1.26126	1.149709	1.052957
	10	2.826993	2.664092	2.570817	2.413215	2.301425	2.173745	1.315808	1.197378	1.098862	0.997235	0.92406	0.81006
5	5	4.350084	4.118747	3.969957	3.7743	3.612841	3.416829	2.170072	2.013894	1.878884	1.720819	1.618063	1.444002
	10	12.35316	10.94002	10.13745	8.904167	7.859382	6.615174	1.765038	1.607898	1.474932	1.27609	1.109361	0.734325
6	5	5.034357	4.809434	4.648151	4.403913	4.239143	4.02923	2.640285	2.47937	2.336891	2.162175	2.042361	1.836678
	10	3.875745	3.709052	3.602054	3.436095	3.28752	3.127581	2.081978	1.947834	1.82679	1.704452	1.610753	1.434844
7	5	5.753342	5.502226	5.357075	5.07924	4.8843	4.659395	3.158127	2.937147	2.796224	2.619996	2.491437	2.274716
	10	4.474	4.197021	4.080527	3.92798	3.770507	3.610251	2.466902	2.325082	2.18972	2.048043	1.969056	1.812718
8	5	6.395463	6.164418	5.99307	5.723568	5.509403	5.256372	3.66206	3.466886	3.297804	3.083538	2.943184	2.722184
	10	4.958382	4.753225	4.612685	4.430195	4.271718	4.088844	2.869967	2.709541	2.575134	2.42246	2.313287	2.101565
9	5	7.094235	6.772925	6.588319	6.343406	6.142674	5.868219	4.172239	3.952084	3.755656	3.52943	3.368455	3.118947
	10	5.431611	5.218219	5.081496	4.902999	4.740855	4.549297	3.260287	3.085957	2.930402	2.757371	2.63884	2.414776
10	5	7.802554	7.440521	7.246377	7.000981	6.743447	6.488565	4.678347	4.460186	4.265542	4.0395	3.91871	3.691152
	10	5.922311	5.721521	5.599249	5.377788	5.224957	5.021406	3.664299	3.467201	3.329653	3.15404	3.027771	2.857264

Table 3. Percentiles of $V=\sum_{i=1}^{m} Y_i$ at k = 2

m	p / n	0.99865	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005	0.00135
2	5	2.334626	2.197253	2.107378	2.00437	1.890244	1.755955	0.951999	0.85061	0.770884	0.6691	0.598262	0.491655
	10	1.837277	1.740968	1.683879	1.590693	1.515541	1.422791	0.789355	0.707058	0.634459	0.559604	0.508588	0.425463
3	5	3.22835	3.041119	2.953291	2.801527	2.669792	2.521244	1.532702	1.404577	1.304342	1.169315	1.108815	0.998243
	10	2.586898	2.426711	2.357428	2.246726	2.15384	2.038838	1.267581	1.165772	1.08047	0.985238	0.921347	0.830126
4	5	4.09782	3.860049	3.758922	3.59698	3.446504	3.272358	2.134587	1.979105	1.851842	1.715081	1.636566	1.496763
	10	3.25933	3.113185	3.03314	2.910781	2.797988	2.658206	1.760559	1.64645	1.540285	1.430059	1.356785	1.208283
5	5	4.910748	4.690049	4.535921	4.355348	4.197799	4.015986	2.767222	2.597823	2.436065	2.270212	2.149996	1.928493
	10	3.926676	3.772913	3.676726	3.525628	3.403761	3.260695	2.254587	2.118493	1.990416	1.882084	1.787045	1.662601
6	5	5.68631	5.456727	5.323453	5.119146	4.942976	4.743159	3.364193	3.188782	3.028525	2.861073	2.750664	2.51819
	10	4.67039	4.4496	4.342586	4.163724	4.019068	3.85217	2.755476	2.61692	2.470129	2.328519	2.238464	2.092608
7	5	6.450999	6.221178	6.119936	5.888118	5.703747	5.487192	3.98455	3.782213	3.615684	3.414431	3.27489	3.007482
	10	5.301514	5.121319	4.957179	4.764079	4.625306	4.454871	3.270242	3.11377	2.977862	2.836983	2.716608	2.564213
8	5	7.257875	7.028246	6.920731	6.647952	6.453962	6.211597	4.620773	4.419366	4.24421	4.02282	3.883549	3.6345
	10	5.884038	5.674714	5.559945	5.374511	5.222974	5.03431	3.783271	3.616054	3.472336	3.30749	3.207673	3.001297
9	5	8.09619	7.859907	7.663112	7.404836	7.185362	6.929384	5.234979	4.991544	4.793696	4.601039	4.480408	4.229538
	10	6.522471	6.308458	6.216781	6.005329	5.845744	5.63685	4.29928	4.11053	3.946653	3.751242	3.63625	3.409376
10	5	9.004663	8.570265	8.405551	8.140922	7.901065	7.642368	5.878805	5.634311	5.412461	5.177099	5.016886	4.7515
	10	7.219127	6.952429	6.825122	6.602637	6.428021	6.218982	4.822058	4.635115	4.473158	4.275834	4.119699	3.913186

Table 4. Percentiles of $V = \sum_{i=1}^{m} Y_i$ at k = 2.5

m	*p* / *n*	0.99865	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005	0.00135
2	5	2.468259	2.346736	2.268899	2.159342	2.055733	1.933521	1.140584	1.035646	0.945613	0.842214	0.761873	0.664364
	10	2.049563	1.941881	1.874171	1.777782	1.69742	1.610381	0.98222	0.892383	0.825022	0.747394	0.686008	0.581007
3	5	3.477266	3.304669	3.206421	3.074597	2.947724	2.798189	1.828369	1.705361	1.589362	1.444766	1.362453	1.234796
	10	2.848432	2.708178	2.635159	2.524335	2.429885	2.322525	1.53788	1.434265	1.346467	1.240148	1.178894	1.081697
4	5	4.34472	4.233665	4.130113	3.943907	3.803379	3.636949	2.516907	2.364628	2.225065	2.067376	1.964329	1.693457
	10	3.63085	3.47622	3.37126	3.252098	3.147937	3.019605	2.132412	2.012767	1.904756	1.774562	1.705587	1.610282
5	5	5.302715	5.14715	5.013002	4.815496	4.644212	4.463374	3.226286	3.05078	2.913382	2.749309	2.649468	2.465423
	10	4.419297	4.238808	4.137307	3.99215	3.860642	3.724371	2.719253	2.578875	2.445154	2.309705	2.210798	2.077999
6	5	6.310054	6.046179	5.863146	5.671537	5.510671	5.306801	3.914177	3.734768	3.566162	3.40839	3.280709	3.105496
	10	5.149388	4.954964	4.874275	4.727925	4.583576	4.420357	3.322092	3.177341	3.050563	2.894142	2.764584	2.559992
7	5	7.205234	6.949778	6.813811	6.554688	6.356546	6.131491	4.652032	4.439628	4.251104	4.045369	3.886589	3.644996
	10	5.871197	5.708455	5.592766	5.429955	5.273217	5.110513	3.931232	3.779749	3.646526	3.489141	3.375452	3.204921
8	5	7.984277	7.757269	7.597745	7.367686	7.159602	6.938196	5.367689	5.141553	4.930601	4.724564	4.605638	4.358607
	10	6.67911	6.430736	6.31225	6.143098	5.980323	5.790149	4.524136	4.356781	4.203633	3.990518	3.874942	3.713165
9	5	8.985175	8.649383	8.484864	8.252281	8.020122	7.76861	6.08215	5.845327	5.649002	5.409941	5.242056	4.959961
	10	7.448108	7.168401	7.019431	6.843368	6.664836	6.467801	5.140985	4.959848	4.799899	4.579462	4.474995	4.237564
10	5	9.800283	9.510817	9.340881	9.026834	8.817583	8.572455	6.806433	6.554773	6.34551	6.116703	5.947915	5.690806
	10	8.165481	7.897884	7.751006	7.536716	7.355737	7.151375	5.737844	5.531289	5.360084	5.169283	5.035914	4.815527

Table 5. Percentiles of $V = \sum_{i=1}^{m} Y_i$ at k = 3

m	n \ p	0.99865	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005	0.00135
2	5	2.608441	2.473904	2.411283	2.292199	2.192362	2.077583	1.297579	1.197155	1.109135	1.004346	0.939656	0.829669
	10	2.165301	2.075306	2.019814	1.919237	1.84604	1.757886	1.134309	1.045123	0.968032	0.883864	0.833268	0.742663
3	5	3.683211	3.521215	3.433714	3.297114	3.164566	3.024332	2.054279	1.929827	1.817847	1.703655	1.628594	1.463765
	10	3.070413	2.948811	2.859479	2.751485	2.647107	2.544672	1.77378	1.669841	1.579924	1.480296	1.418421	1.302424
4	5	4.664032	4.523223	4.389786	4.229283	4.108187	3.939568	2.82075	2.675108	2.546479	2.380471	2.26397	2.127763
	10	3.873799	3.754457	3.677123	3.566902	3.458809	3.32237	2.444186	2.319325	2.200637	2.067358	2.005182	1.859126
5	5	5.748489	5.546665	5.401009	5.195914	5.022025	4.837675	3.599143	3.430413	3.281878	3.119621	3.007574	2.832034
	10	4.792635	4.623547	4.509994	4.37538	4.249666	4.103716	3.108066	2.963333	2.845844	2.706644	2.627772	2.482635
6	5	6.69915	6.489924	6.349406	6.138978	5.953869	5.751055	4.376599	4.196276	4.034022	3.858248	3.724574	3.472215
	10	5.630339	5.43406	5.32593	5.184179	5.043455	4.883834	3.791391	3.629649	3.505016	3.354572	3.259626	3.033245
7	5	7.676058	7.437145	7.273444	7.065268	6.877302	6.656454	5.167565	4.962574	4.780826	4.583857	4.456209	4.271837
	10	6.379116	6.22337	6.107936	5.950745	5.80989	5.641027	4.463186	4.289654	4.148583	3.983681	3.881626	3.707637
8	5	8.597227	8.333983	8.168087	7.933648	7.749028	7.53491	5.966976	5.74813	5.543404	5.322675	5.162271	4.849312
	10	7.229972	7.069915	6.958532	6.73594	6.591474	6.399465	5.142293	4.971053	4.82937	4.649567	4.553311	4.325541
9	5	9.592283	9.262802	9.095726	8.845289	8.653274	8.412515	6.770802	6.530221	6.326654	6.088762	5.95204	5.680317
	10	7.980247	7.808114	7.680098	7.476421	7.340753	7.151641	5.819063	5.618981	5.458145	5.287944	5.141031	4.962647
10	5	10.65452	10.25902	10.07409	9.798465	9.57316	9.320885	7.579234	7.356013	7.111197	6.872162	6.685894	6.374154
	10	8.888584	8.627152	8.485921	8.290417	8.122677	7.916383	6.519429	6.308877	6.138436	5.943742	5.811467	5.586315

If $G(.)$ stands for the cdf of the random variable V, the percentiles in Tables 2 through 5 are the values of $G^{-1}(p)$. If $G_k^{-1}(q)$ stands for the q^{th} percentile of V with the shape parameter k the following inequalities are parallel to the expressions (6) through (10).

$$G_k(ncw_0) \le \alpha \tag{14}$$

$$G_k(ncw_1) \ge 1-\beta \tag{15}$$

$$ncw_0 \le G_k^{-1}(1-\alpha) \tag{16}$$

$$ncw_1 \ge G_k^{-1}(\beta) \tag{17}$$

which jointly lead to

$$\frac{w_0}{w_1} \le \frac{G_k^{-1}(1-\alpha)}{G_k^{-1}(\beta)}. \tag{18}$$

Therefore, m can be obtained by the smallest integer satisfying (18). The acceptability constant c can be obtained from the equality case in either of the expressions (16), (17). We have tabulated the values of m and c determined for the same combinations of p_0, p_1 as chosen by Jun et al. (2006) and are presented in Tables 6 through 9 for k = 1.5(0.5)3.

Table 6. Design parameters of LFCLTSP ($\alpha = 0.05$, $\beta = 0.1$, $k = 1.5$)

p_0	p_1	m	c	
			$n = 5$	$n = 10$
0.001	0.002	-----	-----	-----
	0.004	7	29.29783	23.19252
	0.005	5	20.08845	16.03866
	0.010	3	10.6872	8.377761
	0.050	2	6.264294	4.894248
	0.100	2	6.264294	4.894248
0.005	0.010	----	-----	-----
	0.015	10	25.891	20.12681
	0.020	6	14.39253	11.30701
	0.025	5	11.69048	9.333712
	0.05	2	3.64551	2.848211
	0.25	2	3.64551	2.848211
0.01	0.02	-----	-----	-----
	0.04	6	11.37282	8.934676
	0.05	5	9.237692	7.375396
	0.10	2	2.880642	2.250625
	0.15	2	2.880642	2.250625
	0.3	2	2.880642	2.250625
0.05	0.1	----	-----	-----
	0.2	5	5.273139	4.210088
	0.25	4	3.996433	3.13519
	0.3	3	2.805348	2.19913
	0.5	2	1.644353	1.284721
0.1	0.2	19	18.30972	----
	0.4	4	3.098573	2.430822
	0.5	3	2.175083	1.705062

Table 7. Design parameters of LFCLTSP (α = 0.05, β = 0.1, k = 2)

p_0	p_1	m	c: n = 5	c: n = 10
0.001	0.002	-----	-----	-----
	0.004	8	24.65361	20.1723
	0.005	6	17.78875	14.59859
	0.010	3	7.835488	6.503305
	0.050	2	4.745161	3.944351
	0.100	2	4.745161	3.944351
0.005	0.010	-----	-----	-----
	0.015	11	23.02939	-----
	0.020	7	13.96657	11.49821
	0.025	5	9.592978	7.822957
	0.05	3	5.18668	4.304845
	0.25	2	3.141047	2.610952
0.01	0.02	-----	-----	-----
	0.04	7	11.65218	9.592849
	0.05	5	8.003328	6.526616
	0.10	3	4.327196	3.59149
	0.15	2	2.620545	2.178292
	0.3	2	2.620545	2.178292
0.05	0.1	-----	-----	-----
	0.2	5	5.163768	4.21099
	0.25	4	3.933925	3.272697
	0.3	3	2.791919	2.317239
	0.5	2	1.690782	1.40544
0.1	0.2	19	18.27998	-----
	0.4	4	3.209982	2.670436
	0.5	3	2.278134	1.890807

Table 8. Design parameters of LFCLTSP (α = 0.05, β = 0.1, k = 2.5)

p_0	p_1	m	c n = 5	n = 5
0.001	0.002	-----	-----	-----
	0.004	8	22.8966	19.42811
	0.005	6	17.39037	14.80557
	0.010	3	6.680032	5.618128
	0.050	2	4.056706	3.495534
	0.100	2	4.056706	3.495534
0.005	0.010	-----	-----	-----
	0.015	11	24.39888	-----
	0.020	7	14.37326	12.17942
	0.025	5	10.44058	8.882286
	0.05	3	4.767352	4.009501
	0.25	2	2.895158	2.494665
0.01	0.02	-----	-----	-----
	0.04	7	10.68762	9.099074
	0.05	5	7.344211	6.208183
	0.10	3	4.105354	3.452738
	0.15	2	2.493134	2.148254
	0.3	2	2.493134	2.148254
0.05	0.1	-----	-----	-----
	0.2	5	6.232243	5.302059
	0.25	4	3.945877	3.358724
	0.3	3	2.845752	2.393372
	0.5	2	1.728192	1.489128
0.1	0.2	19	19.32857	-----
	0.4	4	3.318712	2.824882
	0.5	3	2.393443	2.012965

Table 9. Design parameters of LFCLTSP (α = 0.05, β = 0.1, k = 3)

p_0	p_1	m	c n = 5	n = 5
0.001	0.002	-----	-----	-----
	0.004	9	20.11819	17.31086
	0.005	7	15.28861	13.21549
	0.010	4	8.241426	7.145336
	0.050	2	3.688174	3.219797
	0.100	2	3.688174	3.219797
0.005	0.010	-----	-----	-----
	0.015	14	24.21504	-----
	0.020	8	13.2736	11.47917
	0.025	6	9.690052	8.381595
	0.05	3	4.456362	3.856001
	0.25	2	2.764474	2.413401
0.01	0.02	-----	-----	-----
	0.04	8	11.6694	10.09184
	0.05	6	8.518946	7.368625
	0.10	3	3.917781	3.389978
	0.15	2	2.430369	2.121726
	0.3	2	2.430369	2.121726
0.05	0.1	-----	-----	-----
	0.2	6	6.19041	5.354513
	0.25	4	3.94636	3.421503
	0.3	3	2.84691	2.463375
	0.5	2	1.766061	1.541781
0.1	0.2	20	19.35645	-----
	0.4	4	3.386708	2.936284
	0.5	3	2.443176	2.114032

It may be noted that m is solved as integer values only and m, c depend on the shape parameter k of the Burr type X distribution.

LFCLTSP for Burr type X distributed Lifetimes: Method-II

The statistic $V = \sum_{i=1}^{m} Y_i$ introduced for the decision process of the sampling plan seems to have been considered as the total test time to get the limited failure censored sample – $Y_1, Y_2, \ldots, Y_m$ which are m first order statistics in m independent random samples of size n each. If Z denotes the maximum of $Y_1, Y_2, \ldots, Y_m$ it may also be viewed as the total test time/experimental time as opined by Kantam and Srinivasa Rao (2004). Hence, larger realized value of Z can be considered as an indication that the products in the submitted lot have longer life prompting one to consider the lot as a good lot for acceptability. In other words "Z > cL" can be taken as a criterion of acceptance of the lot. Thus, for Method-II the following decision rule is proposed:

(i) Draw a random sample of size $N = m \times n$ and allocate n items to each of the m groups.

(ii) Observe Y_i the time to the first failure in the i^{th} group (I = 1, 2,…., m).

(iii) Identify the quantity $Z = Max\ (Y_1, Y_2, Y_3, \ldots, Y_m)$.

(iv) Accept the lot if $Z \geq cL$ and reject the lot otherwise (c may be called acceptability constant - a concept similar to the acceptance number in time truncated reliability test plans).

Using the theory of order statistics, the cdf of Z may be obtained in a closed form as long as the cdf of the base line distribution is in a closed form. Hence, the percentiles of Z can be used to get the design parameters m, c analytically. For the focal distribution, Burr type X distribution with shape parameter k, the following is the analytical procedure of calculating design parameters of LFCLTSP by Method-II.

The cdf of Burr type X with shape parameter k is

$$F(x) = \left(1 - e^{-x^2}\right)^k. \tag{19}$$

Let $X_1, X_2, X_3, \ldots, X_n$ be a random sample of size n from (19) The cdf of least of $X_1, X_2, X_3, \ldots, X_n$ is given by

$$F_{(1)}(x) = 1 - \left[1 - F(x)\right]^n. \tag{20}$$

That is,

$$F_{(1)}(x) = 1 - \left[1 - \left(1 - e^{-x^2}\right)^k\right]^n. \tag{21}$$

$Y_1, Y_2, Y_3, \ldots, Y_m$ of the limited failure censored test are now a random sample of size m from $F_{(1)}(x)$. Hence, the cdf of Z – the largest of $Y_1, Y_2, Y_3, \ldots, Y_m$ is given by

$$G_{(m)}(z) = \left[F_1(z)\right]^m \tag{22}$$

$$\text{i.e.,}\quad G_{(m)}(z) = \left[1 - \left[1 - \left(1 - e^{-z^2}\right)^k\right]^n\right]^m. \tag{23}$$

As a corollary if $k = 1$ then RHS of (23) becomes

$$G_{(m)}(z) = \left(1 - e^{-nz^2}\right)^m, \tag{24}$$

which correspondents to the cdf of Z when the base line distribution is the well known Rayleigh distribution which in turn is a special case of Weibull distribution. The design parameters m and c of LFCLTSP are obtained with the help of percentiles of $G_{(m)}(z)$ given in (23). If α and β are respectively the producer's and consumer's risks for desirable/acceptable lot quality level p_0, undesirable/lot tolerance quality level p_1 then m and c are the solutions of the following two inequalities.

$$G_m(cw_0) \le \alpha \tag{25}$$

$$G_m(cw_1) \ge 1 - \beta \tag{26}$$

where w_0 and w_1 are as defined above.

The inequalities (25), (26) respectively imply

$$cw_0 \le G_m^{-1}(1-\alpha) \tag{27}$$

$$cw_1 \ge G_m^{-1}(\beta) \tag{28}$$

which jointly lead to

$$\frac{w_0}{w_1} \le \frac{G_m^{-1}(1-\alpha)}{G_m^{-1}(\beta)} \tag{29}$$

Therefore, m can be obtained by the smallest integer satisfying (29). The acceptability constant c can be obtained from the equality case in either of the expressions (27), (28). The values of m and c were analytically determined for the same combinations of p_0, p_1 as chosen by Jun et al. (2006) and are presented in Tables 10 through 13 for $k = 1.5(0.5)3$ along with the values of the design parameters of LFCLTSP of Method-I also for the sake of comparison. The values of m obtained for Method-II can be seen to be consistently smaller than or equal to those of Method-I, thus indicating less number of items to be put to life test in Method-II and hence giving a preference to Method-II over Method-I.

Table 10. Design parameters of LFCLTSP of Methods –I and II at k = 1.5, α = 0.05 and β = 0.1

		m				*c*			
p_0	p_1	n = 5		n = 10		n = 5		n = 10	
		I	II	I	II	I	II	I	II
0.001	0.002	-----	7	-----	6	-----	6.321585	-----	4.706341
	0.004	7	3	7	3	29.29783	4.679569	23.19252	3.666006
	0.005	5	3	5	3	20.08845	4.679569	16.03866	3.666006
	0.010	3	2	3	2	10.6872	3.791998	8.377761	2.981762
	0.050	2	2	2	2	6.264294	3.791998	4.894248	2.981762
	0.100	2	2	2	2	6.264294	3.791998	4.894248	2.981762
0.005	0.010	----	7	----	6	-----	3.678851	-----	2.738859
	0.015	10	4	10	4	25.891	3.066907	20.12681	2.396384
	0.020	6	3	6	3	14.39253	2.723279	11.30701	2.133435
	0.025	5	3	5	2	11.69048	2.723279	9.333712	1.735239
	0.05	2	2	2	2	3.64551	2.206756	2.848211	1.735239
	0.25	2	2	2	2	3.64551	2.206756	2.848211	1.735239
0.01	0.02	-----	7	-----	6	-----	2.906987	-----	2.164216
	0.04	6	3	6	3	11.37282	2.151905	8.934676	1.685816
	0.05	5	3	5	2	9.237692	2.151905	7.375396	1.371166
	0.10	2	2	2	2	2.880642	1.743754	2.250625	1.371166
	0.15	2	2	2	2	2.880642	1.743754	2.250625	1.371166
	0.3	2	2	2	2	2.880642	1.743754	2.250625	1.371166
0.05	0.1	----	6	----	5	-----	1.586625	-----	1.16773
	0.2	5	3	5	3	5.273139	1.228369	4.210088	0.962312
	0.25	4	2	4	2	3.996433	0.995385	3.13519	0.782701
	0.3	3	2	3	2	2.805348	0.995385	2.19913	0.782701
	0.5	2	2	2	2	1.644353	0.995385	1.284721	0.782701
0.1	0.2	19	6	19	5	18.30972	1.230166	-----	0.905382
	0.4	4	3	4	2	3.098573	0.952397	2.430822	0.606855
	0.5	3	2	3	2	2.175083	0.771756	1.705062	0.606855

Table 11. Design parameters of LFCLTSP of Methods-I and II at k = 2, α = 0.05 and β = 0.1

		m				*c*			
p_0	p_1	n = 5		n = 10		n = 5		n = 10	
		I	II	I	II	I	II	I	II
0.001	0.002	-----	7	-----	6	-----	4.222746	-----	3.322232
	0.004	8	3	8	3	24.65361	3.307038	20.1723	2.722404
	0.005	6	3	6	2	17.78875	3.307038	14.59859	2.314123
	0.010	3	2	3	2	7.835488	2.796218	6.503305	2.314123
	0.050	2	2	2	2	4.745161	2.796218	3.944351	2.314123
	0.100	2	2	2	2	4.745161	2.796218	3.944351	2.314123
0.005	0.010	-----	7	-----	5	-----	2.795235	-----	2.101985
	0.015	11	4	11	3	23.02939	2.409381	-----	1.802088
	0.020	7	3	7	3	13.96657	2.189085	11.49821	1.802088
	0.025	5	3	5	2	9.592978	2.189085	7.822957	1.531827
	0.05	3	2	3	2	5.18668	1.850949	4.304845	1.531827
	0.25	2	2	2	2	3.141047	1.850949	2.610952	1.531827
0.01	0.02	-----	6	-----	5	-----	2.247727	-----	1.753666
	0.04	7	3	7	3	11.65218	1.826332	9.592849	1.503464
	0.05	5	2	5	2	8.003328	1.544228	6.526616	1.277988
	0.10	3	2	3	2	4.327196	1.544228	3.59149	1.277988
	0.15	2	2	2	2	2.620545	1.544228	2.178292	1.277988
	0.3	2	2	2	2	2.620545	1.544228	2.178292	1.277988
0.05	0.1	-----	6	-----	5	-----	1.450239	-----	1.13147
	0.2	5	3	5	2	5.163768	1.178354	4.21099	0.824562
	0.25	4	2	4	2	3.933925	0.99634	3.272697	0.824562
	0.3	3	2	3	2	2.791919	0.99634	2.317239	0.824562
	0.5	2	2	2	2	1.690782	0.99634	1.40544	0.824562
0.1	0.2	19	6	19	5	18.27998	1.183358	-----	0.92325
	0.4	4	2	4	2	3.209982	0.812988	2.670436	0.672821
	0.5	3	2	3	2	2.278134	0.812988	1.890807	0.672821

Table 12. Design parameters of LFCLTSP of Methods-I and II at k = 2.5, α = 0.05 and β = 0.1

		m				*c*			
p_0	p_1	n = 5		n = 10		n = 5		n = 10	
		I	II	I	II	I	II	I	II
0.001	0.002	-----	6	-----	5	-----	3.226744	-----	2.606892
	0.004	8	3	8	2	22.8966	2.698795	19.42811	1.995706
	0.005	6	2	6	2	17.39037	2.340102	14.80557	1.995706
	0.010	3	2	3	2	6.680032	2.340102	5.618128	1.995706
	0.050	2	2	2	2	4.056706	2.340102	3.495534	1.995706
	0.100	2	2	2	2	4.056706	2.340102	3.495534	1.995706
0.005	0.010	-----	6	-----	5	-----	2.302837	-----	1.860466
	0.015	11	4	11	3	24.39888	2.091057	-----	1.632736
	0.020	7	3	7	2	14.37326	1.926055	12.17942	1.42428
	0.025	5	2	5	2	10.44058	1.670065	8.882286	1.42428
	0.05	3	2	3	2	4.767352	1.670065	4.009501	1.42428
	0.25	2	2	2	2	2.895158	1.670065	2.494665	1.42428
0.01	0.02	-----	6	-----	5	-----	1.983063	-----	1.60212
	0.04	7	3	7	2	10.68762	1.658601	9.099074	1.226503
	0.05	5	2	5	2	7.344211	1.438159	6.208183	1.226503
	0.10	3	2	3	2	4.105354	1.438159	3.452738	1.226503
	0.15	2	2	2	2	2.493134	1.438159	2.148254	1.226503
	0.3	2	2	2	2	2.493134	1.438159	2.148254	1.226503
0.05	0.1	-----	6	-----	4	-----	1.374621	-----	1.05379
	0.2	5	2	5	2	6.232243	0.996904	5.302059	0.850188
	0.25	4	2	4	2	3.945877	0.996904	3.358724	0.850188
	0.3	3	2	3	2	2.845752	0.996904	2.393372	0.850188
	0.5	2	2	2	2	1.728192	0.996904	1.489128	0.850188
0.1	0.2	19	5	19	4	19.32857	1.10974	-----	0.886299
	0.4	4	2	4	2	3.318712	0.838454	2.824882	0.715058
	0.5	3	2	3	2	2.393443	0.838454	2.012965	0.715058

Table 13. Design parameters of LFCLTSP of Methods-I and II at k = 3, α = 0.05 and β = 0.1

		m				*c*			
		n = 5		n = 10		n = 5		n = 10	
p_0	p_1	I	II	I	II	I	II	I	II
0.001	0.002	-----	6	-----	5	-----	2.769706	-----	2.290677
	0.004	9	2	9	2	20.11819	2.082676	17.31086	1.811738
	0.005	7	2	7	2	15.28861	2.082676	13.21549	1.811738
	0.010	4	2	4	2	8.241426	2.082676	7.145336	1.811738
	0.050	2	2	2	2	3.688174	2.082676	3.219797	1.811738
	0.100	2	2	2	2	3.688174	2.082676	3.219797	1.811738
0.005	0.010	-----	6	-----	5	-----	2.076035	-----	1.716979
	0.015	14	3	14	3	24.21504	1.770461	-----	1.530374
	0.020	8	2	8	2	13.2736	1.561071	11.47917	1.357989
	0.025	6	2	6	2	9.690052	1.561071	8.381595	1.357989
	0.05	3	2	3	2	4.456362	1.561071	3.856001	1.357989
	0.25	2	2	2	2	2.764474	1.561071	2.413401	1.357989
0.01	0.02	-----	6	-----	5	-----	1.825133	-----	1.509471
	0.04	8	2	8	2	11.6694	1.372406	10.09184	1.193868
	0.05	6	2	6	2	8.518946	1.372406	7.368625	1.193868
	0.10	3	2	3	2	3.917781	1.372406	3.389978	1.193868
	0.15	2	2	2	2	2.430369	1.372406	2.121726	1.193868
	0.3	2	2	2	2	2.430369	1.372406	2.121726	1.193868
0.05	0.1	-----	5	-----	4	-----	1.278518	-----	1.047212
	0.2	6	2	6	2	6.19041	0.997278	5.354513	0.86754
	0.25	4	2	4	2	3.94636	0.997278	3.421503	0.86754
	0.3	3	2	3	2	2.84691	0.997278	2.463375	0.86754
	0.5	2	2	2	2	1.766061	0.997278	1.541781	0.86754
0.1	0.2	20	5	20	4	19.35645	1.097205	-----	0.898702
	0.4	4	2	4	2	3.386708	0.855849	2.936284	0.74451
	0.5	3	2	3	2	2.443176	0.855849	2.114032	0.74451

When $k = 1$ Burr type X is a Rayleigh distribution which is a Weibull distribution with shape parameter = 2. Jun et al. (2006) observed that their LFCLTSP for Weibull distribution is invariant of its shape parameter. As matter of comparison, design parameters of LFCLTSP of Method-II were computed for Burr type X at $k = 1$ also, so that these become the parameters of LFCLTSP for Weibull distribution with shape 2. These are given Table 14.

Table 14. Design parameters of LFCLTSP of Method-II at $k = 1$, $\alpha = 0.05$ and $\beta = 0.1$

p_0	p_1	m $n = 5$	$n = 5$	c $n = 5$	$n = 10$
0.001	0.002	5	4	12.6215	7.999934
	0.004	2	2	7.112941	5.029608
	0.005	2	2	7.112941	5.029608
	0.010	2	2	7.112941	5.029608
	0.050	2	2	7.112941	5.029608
	0.100	2	2	7.112941	5.029608
0.005	0.010	5	4	5.63885	3.574094
	0.015	3	2	4.281842	2.247055
	0.020	2	2	3.177816	2.247055
	0.025	2	2	3.177816	2.247055
	0.05	2	2	3.177816	2.247055
	0.25	2	2	3.177816	2.247055
0.01	0.02	5	4	3.982257	2.524089
	0.04	2	2	2.244231	1.586911
	0.05	2	2	2.244231	1.586911
	0.10	2	2	2.244231	1.586911
	0.15	2	2	2.244231	1.586911
	0.3	2	2	2.244231	1.586911
0.05	0.1	5	4	1.762744	1.117287
	0.2	2	2	0.993408	0.702445
	0.25	2	2	0.993408	0.702445
	0.3	2	2	0.993408	0.702445
	0.5	2	2	0.993408	0.702445
0.1	0.2	5	3	1.229931	0.660398
	0.4	3	2	0.933944	0.490122
	0.5	2	2	0.693137	0.490122

Comparison of Tables 1 and 14 also indicate that Method-II is preferable to Method-I in constructing LFCLTSP for Rayleigh distributed life times.

Illustration

The quality assurance in a bearing manufacturing process states that $p_0 = 0.01$, $p_1 = 0.04$, $\alpha = 0.05$, $\beta = 0.1$ the number of test positions (size of each group, n) = 10. For this information Table – 2.1 of Jun et al. (2006) suggests $m = 5$, $c = 196$. Accordingly a random sample of size $N = 50$ items are put to test in five groups with 10 items in each group. The observed first failure times in the five groups are $Y_1 = 120$, $Y_2 = 200$, $Y_3 = 185$, $Y_4 = 55$, $Y_5 = 265$. Assuming that the life times follow Weibull distribution with shape parameter 2 and a lower specification of $L = 100$ they have calculated $V = \sum_{i=1}^{5} Y_i^2 = 161875$ and the acceptability constant $cL^2 = 196000$ since $V < cL^2$ they decided the submitted lot to be rejected.

Adopting the same information to Burr type X distribution we take the shape parameter of Burr type X namely $k = 1$. Then it becomes the Rayleigh distribution which is also a Weibull with shape parameter 2. For the sake of comparison with the sampling plan of Jun et al. (2006), at the above p_0, p_1, α, β, $n = 10$, we get from Table 14 as $m = 2$, and acceptability constant $c = 1.586911$ then $cL = 158.6911$. $Z =$ the maximum of $55{,}120 = 120$. Since $Z < cL$. i.e., $120 < 158.691$, the lot is to be rejected.

From this example, the approach reached the decision of rejecting the lot by conducting limited failure censored life test for only two groups of 10 items each, whereas that of Jun et al. (2006) required the experiment to be conducted for 5 groups of 10 items each resulting in higher cost of experimentation and larger number of destructions. In that way, the Method-II is preferable to the Method-I proposed by Jun et al. (2006). Moreover, it may be recalled that V, Z are defined as

$$V = \sum_{i=1}^{m} Y_i$$

$$Z = Max\left(Y_1, Y_2, \ldots, Y_m\right).$$

If c is the acceptability constant and L is the lower specification, $Z > cL \Rightarrow V > cL$. That is acceptance by Method-II implies acceptance by Method-I, so that as far as acceptance decision is considered Method-II gives a stronger conclusion implying the same decision by Method-I.

References

Aslam, M., & Kantam, R. R. L. (2008). Economic reliability acceptance sampling based on truncated life tests in the Birnbaum-Saunders distribution. *Pakistan Journal of Statistics*, *24*(4), 269-276.

Baklizi, A. (2003). Acceptance sampling based on truncated Life tests in the Pareto distribution of the second kind. *Advances and Applications in Statistics*, *3*(1), 33-48.

Baklizi, A., & El Masri, A. E. Q. (2004). Acceptance sampling based on truncated life tests in the Birnbaum Saunders model. *Risk Analysis, 24*(6), 1453-1457. doi: 10.1111/j.0272-4332.2004.00541.x

Balakrishnan, N., Leiva, V., & López, J. (2007). Acceptance sampling plans from truncated life tests based on the generalised Birnbaum-Saunders distribution. *Communications in Statistics - Simulation and Computation*, *36*(3), 643-656. doi: 10.1080/03610910701207819

Balasooriya, U. (1995). Failure-censored reliability sampling plans for the exponential distribution. *Journal of Statistical Computation and Simulation*, *52*(4), 337-349. doi: 10.1080/00949659508811684

Burr, I. W. (1942). Cumulative frequency functions. *The Annals of Mathematical Statistics, 13*(2), 215-232. doi: 10.1214/aoms/1177731607

Epstein, B. (1954). Truncated life tests in the exponential case. *Annals of Mathematical Statistics*, *25*(3), 555-564. doi: 10.1214/aoms/1177728723

Fertig, K. W., & Mann, N. R. (1980). Life-test sampling plans for two-parameter Weibull populations. *Technometrics, 22*(2), 165-177. doi: 10.1080/00401706.1980.10486131

Goode, H. P., & Kao, J. H. K. (1961). Sampling plans based on the Weibull distribution. *Proceedings of Seventh National Symposium on Reliability and Quality Control* (pp. 24-40), Philadelphia, PA: Institute of Radio Engineers.

Gupta, S. S. (1962). Life test sampling plans for normal and lognormal distribution. *Technometrics, 4*(2), 151-175. doi: 10.1080/00401706.1962.10490002

Gupta, S. S., & Groll, P. A. (1961). Gamma distribution in acceptance sampling based on life tests. *Journal of the American Statistical Association*, *56*(296), 942-970. doi: 10.1080/01621459.1961.10482137

Johnson, L. G. (1964). *Theory and technique of variation research*. Amsterdam: Elsevier.

Jun, C.-H., Balamurali, S., & Lee, S.-H. (2006). Variables sampling plans for Weibull distributed lifetimes under sudden death testing. *IEEE Transactions Reliability*, *55*(1), 53-58. doi: 10.1109/TR.2005.863802

Kantam, R. R. L., Rosaiah, K., & Srinivasa Rao, G. (2001). Acceptance sampling based on life tests: Log-logistic model. *Journal of Applied Statistics*, *28*(1), 121-128. doi: 10.1080/02664760120011644

Kantam, R. R. L., & Srinivasa Rao, G. (2004). A note on savings in experimental time under type II censoring. *Economic Quality Control, 19*(1), 91-95. doi: 10.1515/EQC.2004.91

Kantam, R. R. L., & Sriram, B. (2013). Economic reliability test plans based on Rayleigh distribution. *Journal of Statistics*, *20*, 88-101.

Kantam, R. R. L., Sriram, B., & Suhasini, A. (2012). Reliability test plans: Exponentiated exponential distribution. *Mathematical Sciences International Research Journal, 1*(3), 1022-1032.

Kantam, R. R. L., Sriram, B., & Suhasini, A. (2013). Reliability test plans based on log-logistic distribution. *International Journal of Research in Commerce, IT & Management, 3*(9), 49-57.

Lio, Y. L., Tsai, T.-R. & Wu, S.-J. (2009). Acceptance sampling plans from truncated life tests based on the Birnbaum - Saunders distribution for percentiles. *Communications in Statistics – Simulation and Computation, 39*(1), 119-136. doi: 10.1080/03610910903350508

Lio, Y. L., Tsai, T.-R. & Wu, S.-J. (2010). Acceptance sampling plans from truncated life tests based on the Burr type XII percentiles. *Journal of the Chinese Institute of Industrial Engineers, 27*(4), 270-280. doi: 10.1080/10170661003791029

Lu, W. (2011). Acceptance sampling plans based on truncated life tests for Maxwell distribution. *Pakistan Journal of Statistics, 27*(2), 159-170.

Pascual, F. G., & Meeker, W. Q. (1998). The modified sudden death test: Planning life tests with a limited number of test positions. *Journal of Testing and Evaluation*, *26*(5), 434-443. doi: 10.1520/jte12692j

Rosaiah, K., & Kantam, R. R. L. (2005). Acceptance sampling based on the inverse Rayleigh distribution. *Economic Quality Control*, *20*(2), 277-286.

Rosaiah, K., Kantam, R. R. L., Rama Krishna, V., & Siva Kumar, D. C. U. (2014). An economic reliability test plan for type – I generalized half logistic distribution. *Journal of Chemical, Biological and Physical Sciences, 4*(2), 1486-1493.

Rosaiah, K., Kantam, R. R. L., & Srinivasa Rao, B. (2009). Reliability test plan for half logistic distribution. *Calcutta Statistical Association Bulletin, 61*(241/244), 183-196.

Srinivasa Rao, G., Ghitany, M. E., & Kantam, R. R. L. (2009). Marshall-Olkin extended Lomax distribution: An economic reliability test plan. *International Journal of Applied Mathematics*, *22*(1), 139-148.

Srinivasa Rao, G., & Kantam, R. R. L. (2010). Acceptance sampling plans from truncated life tests based on the log-logistic distributions for percentiles. *Economic Quality Control, 25*(2), 153-167. doi: 10.1515/eqc.2010.008

Srinivasa Rao, G., & Kantam, R. R. L. (2012). Two-stage estimation in log-logistic model. *Pakistan Journal of Statistics, 28*(3), 331-339.

Srinivasa Rao, B., & Kantam, R. R. L. (2013). Acceptance sampling plans for percentiles of half logistic distribution. *International Journal of Reliability, Quality and Safety Engineering, 20*(5), 1-13. doi: 10.1142/S0218539313500162

Srinivasa Rao, G., Kantam, R. R. L., Rosaiah, K., & Pratapa Reddy, J. (2012). Acceptance sampling plans for percentiles based on the inverse Rayleigh distribution. *Electronic Journal of Applied Statistical Analysis, 5*(2), 164-177. doi: 10.1285/i20705948v5n2p164

Subba Rao, R., Naga Durgamamba, A., & Kantam, R. R. L. (2014). Acceptance sampling plans: Size biased Lomax model. *Universal Journal of Applied Mathematics, 2*(4), 176-183. doi: 10.13189/ujam.2014.020403

Subba Rao, R., Prasad, G., & Kantam, R. R. L. (2013). Acceptance sampling on life tests: Exponentiated Pareto model. *Engineering Sciences International Research Journal, 1*(1), 152-158.

Wu, J.-W., & Tsai, W.-L. (2000). Failure-censored sampling plan for the Weibull distribution. *Information and Management Sciences, 11*(2), 13-25.

Wu, J.-W., Tsai, T.-R., & Ouyang, L.-Y. (2001). Limited failure-censored life test for the Weibull distribution. *IEEE Transactions on Reliability, 50*(1), 107-111.doi:10.1109/24.935024