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A social image theory of information acquisition, opinion formation, and voting

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ABSTRACT

Recent empirical research on voter turnout has revealed a variety of regularities. Citizens who expect to be asked about their turnout decisions after the elections are more likely to vote. Parents whose children enter the electorate are more likely to vote when their children live home than when they left home. Citizens without social networks acquire less information about politics. We develop a model that can explain these and other empirical findings. In our model, citizens receive disutility from being perceived not to have voted. This motivates a citizen to vote. Moreover, a citizen feels worse being perceived not to have voted when he is thought to have a strong opinion as this raises expectations about his voting behavior among peers. When a citizen anticipates that he will likely vote, the latter concern motivates him to acquire information, to participate in political discussions, and to vote. However, when a citizen anticipates that he will likely abstain from voting, he shies away from politics to lower his peers' expectations.

1. Introduction

At least since Downs (1957), the observation that in large-scale elections many citizens vote has been a notorious puzzle. In the last decade, evidence has been presented that casts light on this puzzle. Gerber and Rogers (2009) and DellaVigna et al. (2017) show that citizens, who expect to be asked about their turnout decisions after an election, are much more likely to vote.[2] They view their results as supportive of the existence of social image concerns. Your friends, peers, or colleagues expect you to vote. Not meeting these expectations arouses feelings of shame.[3] The observation that social image concerns are important for turnout decisions suggests that voting is a social activity rather than a private one. Recent empirical evidence supports this view. Bond et al. (2012) show that political mobilization messages sent to 61 million Facebook users affect the voting behavior of the users' close friends. A placebo-controlled experiment by Nickerson (2008) shows that in households with two registered voters, a Get Out the Vote Message to one member of the household passed with 60% to the other member. Cantoni and Pons (2019) find that citizens who move to states with a higher share of Republicans (Democrats) are more likely to register as Republican (Democrat).[4] Dahlgaard

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[2] See also Rogers et al. (2016).

[3] These results are also in line with widespread overreporting of voting in surveys (Harbaugh, 1996; Belli et al., 1999). From 25 to 50% of the non-voters are assessed to lie when asked about their past turnout decisions.

[4] Likewise, Perez-Truglia (2018) finds that citizens who move to areas with a higher share of Democrats contribute more to Barack Obama's presidential campaign.

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(2018) finds that parents whose children enter the electorate are more likely to vote when they live at home than when they left home.[5]

The empirical evidence that a citizen's environment is important for his vote decision challenges the assumption underlying most theoretical studies on voter turnout that citizens act atomistically. Moreover, the recent empirical literature suggests that the concept of elections is broader than simply aggregating preferences. It includes debating politics with others, opinion formation, enforcing social norms, and so on.[6] Understanding individual voting acts requires understanding how citizens are affected by others.

Following Harbaugh (1996) and Ali and Lin (2013), we take social image concerns as a point of departure for developing an integrated theory of voter turnout.[7] Unlike these studies, we do not only focus on turnout decisions but also on citizens' decisions to acquire and share information. In our model, these decisions affect the strength of citizens' opinions and citizens' sensitivities to social concerns. We develop a model in which each citizen takes four actions. First, a citizen can acquire information about the main issue at stake in an upcoming election. This information may increase the probability that he has a correct private opinion about this issue. Second, he decides what information to convey to the other persons in his social environment. We refer to these persons as a citizen's peers. We model this by allowing citizens to send cheap-talk messages about their private opinions to each other. Third, based on the exchanged cheap-talk messages, each citizen forms an opinion about the issue at stake. Finally, each citizen decides whether to vote and if he votes, what to vote for.

We assume two social image concerns. First, each citizen derives utility from being perceived by his peers as having voted. This concern may reflect feelings of pride from being perceived to have voted.[8] The evidence of over-reporting of voting in surveys and the recent studies that find that people are more likely to vote when they expect to be asked about their voting behavior after an election show the relevance of this concern. Second, each citizen feels worse being perceived not to have voted when he is thought to have a strong opinion. The idea behind this assumption is that John's peers especially expect John to vote if they believe that he has a strong opinion for one of the alternatives. Not meeting this expectation yields disutility of shame. John's peers' expectations are lower when John is ambivalent towards the alternatives. Then, not voting yields less shame.

A driving force in our model is that when peers share information, consensus among peers strengthens opinions, while dissonance weakens opinions. The reason for this result is that similar private signals reinforce each other while conflicting signals cancel out (see Swank and Visser, 2019). As a citizen's strength of opinion affects his disutility from being perceived not to have voted, his decision whether or not to participate in political discussions, and if so, what to say influence his voting behavior. A citizen can influence his disutility from being perceived not to have voted directly by voting, and indirectly, by creating low expectations about his voting behavior among his peers. One key result is that on the basis of communication among citizens two equilibria can be distinguished. In a separating equilibrium, citizens share information and acquire information if it is not too expensive. Turnout is relatively high. In the pooling equilibrium, citizens never acquire information and, by the nature of the equilibrium, do not participate in political discussions with peers. Turnout is relatively low. In a pooling equilibrium, citizens do not suffer much from being perceived as not to have voted, because their opinions are relatively weak. Their peers did not expect them to vote. In the separating equilibrium, expectations are high, but often citizens meet them.

The point of departure for any theoretical discussion about turnout is the *Calculus of Voting* model (Downs, 1957; Riker and Ordeshook, 1968).[9] In this model, a citizen votes if the benefit of voting exceeds the cost. The benefit includes the probability that a citizen affects the election outcome. The benefit also includes a "reward" a citizen receives from doing his duty or expressing his opinions. The main insight from this model is that in any large election, the probability that a citizen affects the election outcome is close to zero. As a result, a citizen casts his ballot if the benefit of doing his duty exceeds the cost of voting (see also Palfrey and Rosenthal, 1985). In two respects, our theory is closely related to the *Calculus of Voting* model. First, in our model citizens are concerned with outcomes, but these concerns do not affect individuals' turnout decisions, because the probabilities of being pivotal are close to zero. Second, performing one's civic duty may create social image concerns.

In some studies, performing one's civic duty is interpreted as complying with a social norm of voting (see, for example, Gerber and Rogers, 2009). Harsanyi (1980), Coate and Conlin (2004), and Feddersen and Sandroni (2006) model citizens as rule-utilitarians. Citizens follow a turnout rule that maximizes aggregate utility or group utility. This rule stipulates who should vote under which conditions. Though the turnout model of Feddersen and Sandroni delivers comparative static results that are consistent with a variety of empirical results, it cannot explain why in surveys citizens lie about their turnout decisions. Voting when the rule stipulates not voting would only reduce aggregate utility. The analysis of Ali and Lin (2013) suggests that with the introduction of social-image concerns into the ethical voter model of Feddersen and Sandroni, citizens may want to lie about whether they voted.

[5] The view that the social environment is important for understanding voter behavior is not new. More than thirty years ago, Huckfeldt and Sprague (1987) emphasized the importance of personal day-to-day contacts for opinion forming and voting (see also Nickerson, 2008). They empirically show how congruence and dissonance of opinions among members in social groups affect their voting behavior (see also Huckfeldt, 1979; Giles and Dantico, 1982). Furthermore, Kenny (1992), Mutz (2002), and McClurg (2004) report evidence that information exchange in groups influences members' decisions to vote.

[6] See also Alesina and Giuliano (2011) and Pons (2018) for recent articles, emphasizing that the concept of elections is broader than just voting.

[7] Harbaugh (1996) developed a voter model, in which people have a taste for praise and a distaste for lying. The model explains why people vote and lie about non-voting. Ali and Lin (2013) extend the ethical voter model (Feddersen and Sandroni, 2006) with "pragmatic" voters who are concerned about their social esteem.

[8] Equivalently, this concern can be modeled as disutility from being perceived not to have voted.

[9] Excellent surveys of the turnout literature are Mueller (2003), Feddersen (2004), and Geys (2006).

2. The model

The election revolves around policy $x \in \{0,1\}$. The decision about x is made by simple majority rule. The contribution of the project to the payoff of all citizens equals wx, where $w \in \{-1,1\}$ is the state of the world. The probability that $w=1$ equals $\frac{1}{2}$.

The electorate consists of an infinite number of pairs of citizens. We call a pair, consisting of a citizen i, he, and a citizen $-i$ (i's peer), she, "a group". In a group, citizens are identical. Citizens' characteristics, for instance, their education levels, may vary across groups. The model describes information acquisition, opinion formation, turnout decisions, and vote decisions in one group. The description of the model focuses on citizen i. For citizen $-i$ an analogous description holds.

The game starts with an information acquisition stage. Citizen i chooses an effort level $e_i \in \{l,h\}$, with $\frac{1}{2} \leq l < h \leq 1$, and next receives a private signal, $s_i \in \{-1,1\}$ about the state of the world, w. The quality of the signal depends on the chosen effort level, e_i. In particular, the likelihood that citizen i's signal is correct equals e_i, *i.e.*,

$$\Pr\left\{s_i = w|e_i\right\} = e_i$$

Choosing $e_i = l$ is costless. If $e_i = l$, we say that citizen i chooses to be ignorant. If, at cost c^e, citizen i chooses $e_i = h$, we say that he chooses to be knowledgeable.

After citizens have chosen effort levels and have received signals, the communication stage starts. In this stage, citizen i sends a cheap-talk message, $m_i \in \{-1,1\}$, about s_i to his peer, citizen $-i$.[10] By $m_i = 1$ ($m_i = -1$) i wants $-i$ to believe that he has received $s_i = 1$ ($s_i = -1$). Simultaneously, i receives a message m_{-i} from $-i$. After citizens i and $-i$ have exchanged information, citizen i assesses the probability that $w=1$ on the basis of $\left(s_i, m_{-i}\right)$ and conditional on $\left(e_i, e_{-i}\right)$ according to Bayes' rule:[11]

$$z_i = \Pr\left\{w = 1|s_i, m_{-i}\right\}$$

We refer to z_i as citizen i's opinion, and to

$$g_i = \left|z_i - \frac{1}{2}\right| \tag{1}$$

as the strength of i's opinion. We say that i's opinion is weak if z_i is close to $\frac{1}{2}$ and that it is strong if it is close to 0 or 1.

After opinions have been formed, citizen i makes his turnout decision, $t_i \in \{0,1\}$, where $t_i = 1$ denotes that citizen i goes to the polling place, and $t_i = 0$ denotes that he does not. In case $t_i = 1$, i votes for $x = 1$ if $z_i > \frac{1}{2}$, for $x = 0$ if $z_i < \frac{1}{2}$, and tosses a coin if $z_i = \frac{1}{2}$. Thus, conditional on $t_i = 1$, opinions deterministically translate into voting. Let c_i^v denote the direct cost of voting, say, the cost of going to the polling place. When forming an opinion, this cost of voting c_i^v is uncertain. It is distributed over the interval $(0,\bar{c})$, with possibly infinite $\bar{c}$, according to a CDF $F(c)$. We assume that $F(c)$ has a non-decreasing hazard rate (NDHR) $\lambda(c) = \frac{f(c)}{1-F(c)}$, where $f(c) = F'(c)$ is the distribution density function which is assumed continuously differentiable on $(0,\bar{c})$.[12] At the election day, c_i^v is drawn from $F(c)$. When choosing t_i, citizen i observes c_i^v. Throughout we assume that c_i^v and c_{-i}^v are mutually independent.[13] Citizen i's turnout decision, t_i, is made conditional on $\left(s_i, m_i, m_{-i}, c_i^v\right)$.

Citizens' turnout decisions are imperfectly observed by their peers. With probability $k \in [0,1]$ i learns t_{-i}. This parameter k can be interpreted as the probability that after the election citizens talk about their turnout decisions and honestly reveal them.

We model i's utility as follows

$$u_i = wx + \theta_1 \text{Pride}_i - \theta_2 \text{Shame}_i - \frac{e_i - l}{h - l} c^e - t_i c_i^v \tag{2}$$

The first term of the payoff function shows that citizen i cares about the policy outcome. However, he is aware that the probability that his vote affects the policy outcome is negligible. As a result, wx is not decision relevant. Opinion formation and voting behavior are solely driven by social concerns, and the costs of information acquisition and voting.

A key feature of our model is that citizens are subject to two kinds of social image concerns. First, citizen i receives *utility* from being believed to have voted or, equivalently, he receives *disutility* from being believed *not* to have voted. Parameter $\theta_1 \geq 0$ represents the importance of this social image concern. It is driven by pride from being believed to have voted. Let $O_{-i} \in \{0,1,\varnothing\}$ be information that citizen $-i$ has about voting behavior of citizen i at the end of the game, where $O_{-i} = 0$ means $-i$ observes that i has not voted, $O_{-i} = 1$ means that $-i$ observes that i has voted, and $O_{-i} = \varnothing$ means $-i$ does not observes whether i has voted. Knowing $\left(s_{-i}, m_{-i}, m_i, O_{-i}\right)$, citizen $-i$ forms expectation of t_i, *i.e.*, the expected voting probability of i, $\tau\left(s_{-i}, m_{-i}, m_i, O_{-i}\right)$:

$$\tau\left(s_{-i}, m_{-i}, m_i, O_{-i}\right) = \mathbb{E}_{-i}\left[t_i|s_{-i}, m_{-i}, m_i, O_{-i}\right] = \begin{cases} O_{-i}, & \text{if } O_{-i} \in \{0,1\} \\ \mathbb{E}_{-i}\left[t_i|s_{-i}, m_{-i}, m_i\right], & \text{if } O_{-i} = \varnothing \end{cases}$$

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[10] Restricting the message space to two messages does not affect our result qualitatively. Most importantly, we could have added a message "I do not want to talk about politics". This would have made the interpretation of the results easier at the expense of more notation.

[11] For notational simplicity, we omit the conditionality of expectations on $\left(e_i, e_{-i}\right)$. Moreover, posteriors are updated according to Bayes' rule, given voters' equilibrium strategies. To reduce notation, we do not explicitly state equilibrium strategies in the expression of the posteriors.

[12] This assumption holds for various distributions, including (for some parameter values) the gamma distribution, the Weibull distribution, power distributions with finite support, the Pareto distribution, the uniform distribution, and exponential distributions. More on NDHR distributions can be found in Barlow et al. (1963).

[13] In practice c_i^v and c_{-i}^v are dependent. For example, weather conditions are usually similar for peers. We make the assumption that c_i^v and c_{-i}^v are independent to clearly identify other, more interesting reasons why peers' information acquisition decisions and turnout decisions could be dependent.

Pride_i is i's expectation of $\tau\left(s_{-i}, m_{-i}, m_i, O_{-i}\right)$:

$$\text{Pride}_i\left(s_i, m_i, m_{-i}\right) = \mathbb{E}_i\left[\tau\left(s_{-i}, m_{-i}, m_i, O_{-i}\right)|s_i, m_i, m_{-i}\right] = \mathbb{E}_i\left[kt_i + (1-k)\,\mathbb{E}_{-i}\left[t_i|s_{-i}, m_{-i}, m_i\right]|s_i, m_i, m_{-i}\right] \tag{3}$$

Note that Pride_i is modeled as a second-order expectation of i's turnout decision t_i. It is i's expectation of $-i$'s expectation of t_i. In (3), citizen $-i$ learns t_i with probability k. With the remaining probability $(1-k)$ she does not learn it and forms her expectation $\mathbb{E}_{-i}\left[t_i|s_{-i}, m_{-i}, m_i\right]$.

Second, the disutility citizen i receives from being believed not to have voted increases in $-i$'s expectation of i's strength of opinion, g_i. This is the second social image concern, to which we refer as shame. Shame on citizen i equals his opinion strength g_i if he has not voted and zero otherwise. Knowing $\left(s_{-i}, m_{-i}, m_i, O_{-i}\right)$, citizen $-i$ forms expectation $\psi\left(s_{-i}, m_{-i}, m_i, O_{-i}\right)$ of it:

$$\psi\left(s_{-i}, m_{-i}, m_i, O_{-i}\right) = \mathbb{E}_{-i}\left[\left(1-t_i\right)g_i|s_{-i}, m_{-i}, m_i, O_{-i}\right] = \begin{cases} \left(1-O_{-i}\right)\mathbb{E}_{-i}\left[g_i|s_{-i}, m_{-i}, m_i, t_i = O_{-i}\right], & \text{if } O_{-i} \in \{0,1\} \\ \mathbb{E}_{-i}\left[\left(1-t_i\right)g_i|s_{-i}, m_{-i}, m_i\right], & \text{if } O_{-i} = \varnothing \end{cases}$$

Shame_i is i's expectation of $\psi\left(s_{-i}, m_{-i}, m_i, O_{-i}\right)$:

$$\begin{aligned} \text{Shame}_i\left(s_i, m_i, m_{-i}\right) &= \mathbb{E}_i\left[\psi\left(s_{-i}, m_{-i}, m_i, O_{-i}\right)|s_i, m_i, m_{-i}\right] \\ &= k\mathbb{E}_i\left[\left(1-t_i\right)\mathbb{E}_{-i}\left[g_i|s_{-i}, m_{-i}, m_i, t_i = 0\right]|s_i, m_i, m_{-i}\right] + (1-k)\,\mathbb{E}_i\left[\mathbb{E}_{-i}\left[\left(1-t_i\right)g_i|s_{-i}, m_{-i}, m_i\right]|s_i, m_i, m_{-i}\right] \end{aligned} \tag{4}$$

Like Pride_i, Shame_i, is a second-order expectation of $\left(1-t_i\right)g_i$, *i.e.*, of i's strength of opinion in case he has not voted. It is this social image concern that enables citizens to communicate.

To get the idea behind (4), suppose that i believes that $-i$ believes that i has a strong opinion. Equation ((4)) captures that in this situation, citizen i is more embarrassed if $-i$ finds out (or believes) that i did not vote than in a situation where i is believed to be ambivalent regarding the decision on x. Thus, perceived strong opinions raise expectations about one's vote behavior. It is worth emphasizing that we use the labels pride and shame for ease of reference. Specifically, in our model Pride_i captures both utility from being believed to have voted and disutility from being believed to have abstained. This disutility can be seen as shame. A higher disutility from being believed not to have voted when one's opinion is strong seems more connected with shame than pride. Of course, pride for being perceived to have voted may also increase in one's perceived strength of opinion. We analyze this channel in the appendix.[14]

The fourth term in (2) denotes the cost of acquiring information. Finally, the last term in ((2)) denotes the cost of actually going to the polling place.

We solve the game for Symmetric Sequential Equilibria,[15] SSE hereafter, in which citizen i and $-i$ play a symmetric behavioral strategy profile (e^*, m^*, t^*), *i.e.* , i exerts effort $e_i = e^*$, sends a message $m_i = m^*\left(e_i, s_i\right)$, makes turnout decision $t_i = t^*\left(e_i, s_i, m_i, m_{-i}, c_i^v\right)$ and holds beliefs, such that:

1. The turnout strategy t^* maximizes i's expected payoff conditional on $\left(e_i, s_i, m_i, m_{-i}, c_i^v\right)$, given the equilibrium strategy of $-i$;
2. The communication strategy m^* maximizes i's expected payoff conditional on $\left(e_i, s_i\right)$, given the equilibrium strategy of $-i$;
3. The effort level e^* maximizes i's expected payoff, given the equilibrium strategy of $-i$;
4. Citizens beliefs are updated according to Bayes' rule on all information sets.

In games like ours, pooling, or babbling, equilibria always exist. In a pooling equilibrium, m_i does not contain information about s_i and is ignored by citizen $-i$. We use superscript 'P' to denote variables in pooling SSE. For separating SSE we use superscript 'S'.

3. Analysis

We solve the model in two steps. First, in this section we fix citizens' information acquisition decisions and assume

$$e_i = e_{-i} = e$$

We compute citizens' turnout and communication strategies in SSE. We also identify the conditions under which separating SSE exist. Next, in Section 4 we analyze the information acquisition stage, compute equilibrium effort levels, and analyze how those effort levels affect the conditions for the existence of SSE.

[14] In the appendix, we analyze a more general model, in which in ((2))

$$\begin{aligned} \theta_1\text{Pride}_i &= \mathbb{E}_i\left[\mathbb{E}_{-i}\left[t_i\left(a_1 + b_1 g_i\right)|s_{-i}, m_{-i}, m_i, O_{-i}\right]|s_i, m_i, m_{-i}\right] \\ \theta_2\text{Shame}_i &= \mathbb{E}_i\left[\mathbb{E}_{-i}\left[\left(1-t_i\right)\left(a_2 + b_2 g_i\right)|s_{-i}, m_{-i}, m_i, O_{-i}\right]|s_i, m_i, m_{-i}\right] \end{aligned}$$

meaning that citizens with perceived stronger opinions also experience more (if $b_1 > 0$) or less (if $b_1 < 0$) utility from pride.

[15] Weak perfect Bayesian equilibrium (WPBE) is too weak a concept for this game. *E.g.*, after deviating from an equilibrium message m_i, citizen i reaches an off-path information set where beliefs are not restricted in a WPBE, and the relation between signal s_{-i} and message m_{-i} can be arbitrary. In SSE, all beliefs follow Bayes' rule and no additional consistency conditions are required; see Kreps and Wilson (1982).

3.1. The turnout decision

When turnout decisions are made, information has been acquired, discussions have taken place, and opinions and beliefs about opinions have been formed. The difference between citizen i's utility when voting and not voting equals

$$\mathbb{E}_i\left[u_i|t_i=1\right]-\mathbb{E}_i\left[u_i|t_i=0\right]=k\left(\theta_1+\theta_2\hat{g}_i\right)-c_i^v$$

where

$$\hat{g}_i=\mathbb{E}_i\left[\mathbb{E}_{-i}\left[g_i|s_{-i},m_{-i},m_i,t_i=0\right]|s_i,m_i,m_{-i}\right] \tag{5}$$

With probability $(1-k)$, $-i$ does not learn i's turnout decision. Then, social concerns do not affect his turnout decision. Social concerns only matter for i's turnout decision if $-i$ actually learns it. We refer to $\hat{g}_i$ as citizen i's expectation of the strength of his perceived (by $-i$) opinion, *expected opinion strength* in short.

Citizen i follows a threshold strategy. He optimally chooses $t_i=1$ if $c_i^v\le b_i$, and chooses $t_i=0$ otherwise, where the cost threshold b_i is

$$b_i=k\left(\theta_1+\theta_2\hat{g}_i\right) \tag{6}$$

The right-hand side of (6) denotes the benefit of voting. Clearly, the higher is b_i, the wider is the range of c_i^v for which citizen i goes to the ballot place. The *interim* probability that i votes equals

$$\Pr\left\{t_i=1|s_i,m_i,m_{-i}\right\}=\Pr\left[c_i^v\le b_i\right]=F\left(b_i\right)$$

The likelihood that i votes increases with (i) his sensitivities to the two social concerns, θ_1 and θ_2, (ii) his expected opinion strength, and (iii) a higher probability that $-i$ learns i's turnout decision.[16]

The turnout strategy t^* in any SSE is

$$t^*=\begin{cases}1, \text{ if } c_i^v\le b_i\\ 0, \text{ otherwise}\end{cases} \tag{7}$$

3.2. Opinion formation and sharing information

As discussed in Section 2, a pooling SSE, in which citizens' messages do not contain any information about their signals, always exists. Translated into practice, a pooling SSE describes two citizens who do not talk about politics. As a result, opinion formation is a private matter. Citizen i and $-i$ have expected opinion strength $\hat{g}_i=\hat{g}^{\mathrm{P}}$, where

$$\hat{g}^{\mathrm{P}}=e-\frac{1}{2} \tag{8}$$

and vote with the same *ex-ante* probability $F\left(k\left(\theta_1+\theta_2\hat{g}^{\mathrm{P}}\right)\right)$ irrespective of their signals and messages.

We now identify the conditions under which a separating SSE exists. A separating SSE describes two citizens who talk about politics. Opinion-forming is a social matter. In a separating SSE, $m_i=s_i$ and $m_{-i}=s_{-i}$ so that after the communication stage citizens possess the same information about the state and each other. This means that $z_i=z_{-i}$ and, therefore, $g_i=g_{-i}$. Lemma 1 presents citizens' expected opinion strength $\hat{g}_i$ and their beliefs about each other's voting behavior in a separating SSE.

Lemma 1. *Suppose a separating SSE in which citizens' turnout strategies are given by (7). Then,* $\hat{g}_i=\hat{g}^S\left(m_i,m_{-i}\right)$ *where*

$$\hat{g}^S\left(m_i,m_i\right)=\frac{e^2}{e^2+(1-e)^2}-\frac{1}{2}>\hat{g}^S\left(m_i,-m_i\right)=0 \tag{9}$$

The expected strength of opinion $\hat{g}^S\left(m_i,m_i\right)$ *increases in* e. *The turnout cost thresholds are* $b_i=b^S\left(m_i,m_{-i}\right)$, *where* $b^S\left(m_i,m_{-i}\right)=k\left(\theta_1+\theta_2\hat{g}^S\left(m_i,m_{-i}\right)\right)$.

Expression (9) in Lemma 1 shows how the distribution of citizens' signals affects their expected opinion strengths. Since citizens share information, perceived opinions are equal to actual opinions, $\hat{g}_i=g_i$. Clearly, when citizens learn that their signals are opposite, their signals cancel out. In that case, their posteriors about the state are equal to their priors, so that $z_i=\frac{1}{2}$. Opinions are weak. If both signals indicate that $w=1$, opinions are biased towards $x=1$, while if both signals indicate that $w=-1$, opinions are biased towards $x=0$. Their strengths in both cases are the same.

Let us now consider the communication stage. By sending $m_i=-s_i$ rather than $m_i=s_i$ citizen i does not affect the posterior expectations $\hat{g}^S\left(m_i,m_i\right)$ and $\hat{g}^S\left(m_i,-m_i\right)$ that emerge from events $s_{-i}=m_i$ and $s_{-i}=-m_i$. He affects the probabilities that these events occur. Clearly, deviating decreases the probability of signal congruence and increases the probability of signal dissonance. It immediately follows that by deviating, the probability of strong opinions declines. Through citizen i's turnout strategy ((7)), a lower probability of strong opinions makes it less likely that citizen i casts his ballot.

We now show how citizens' sensitivities to not meeting expectations affect the requirements for the existence of a separating SSE.

[16] Hodler et al. (2015) find that lowering voting cost due to the introduction of postal voting increased voter turnout. Moreover, they find that lower voting cost induced citizens with weaker incentives to vote. These findings contain information about the distribution of c_i^v.

Lemma 2. *If $\theta_2 = 0$, communication does not affect turnout decisions.*

In our model, communication and turnout decisions are connected through a citizen's expected opinion strength. When citizens' (dis)utility from social concerns is independent of the strength of opinion, communication does not affect turnout decisions. As a result, if $\theta_2 = 0$, communication is not relevant for turnout decisions. We assume $\theta_2 > 0$ in the rest of the analysis.

Proposition 1. *Suppose $\theta_2 > 0$. Then,*

1. *There exists a threshold level $\underline{\theta_1} \geq 0$, such that a separating SSE does not exist for any $\theta_1 < \underline{\theta_1}$ and any θ_2.*
2. *If $\theta_1 > \underline{\theta_1}$ then a threshold level $\underline{\theta_2}\left(\theta_1\right) \geq 0$ exists such that*
   - *if $\theta_2 \in \left(0, \underline{\theta_2}\right)$ then no separating SSE exists;*
   - *if $\theta_2 \in \left(\underline{\theta_2}, \infty\right)$, then a separating SSE does exist.*
3. *The threshold level $\underline{\theta_2}\left(\theta_1\right)$ is weakly decreasing over $\theta_1 \in \left(\underline{\theta_1}, \infty\right)$, and a threshold level $\bar{\theta}_1$ exists such that $\underline{\theta_2}\left(\overline{\theta_1}\right) = 0$. For $\theta_1 \geq \overline{\theta_1}$, a separating SSE exists for any $\theta_2 > 0$.*
4. *The expected voter turnout in a separating SSE is lower, equal, or higher than in a pooling SSE if the density function $f(c)$ is decreasing, constant, or increasing, respectively.*

Loosely speaking items 1–3 of Proposition 1 show that a separating SSE requires that citizens are sufficiently sensitive to pride. Furthermore, citizens who are more sensitive to pride need to be less concerned about shame to be willing to share information. What is the intuition behind these results? As discussed above, if citizen i deviates from a separating equilibrium by sending $m_i = -s_i$, he lowers the probability of being perceived to have a strong opinion. This has two opposite effects. First, a weaker perceived opinion reduces the probability of voting, and consequently, increases the probability of shame. Second, a weaker perceived opinion reduces the disutility resulting from shame. Which of the two effects dominates depends on the voting probability in equilibrium. If citizen i anticipates that he will vote with a high probability, he also anticipates that he will likely meet $-i$'s expectations. The higher is θ_1, the higher is the probability of voting. As a consequence, for sufficiently high values of θ_1, citizens do not want to lower expectations. By sharing information they try to increase the probability of meeting expectations. If, by contrast, the probability of voting is small, citizens have an incentive to deviate from the separating SSE to lower expectations. As the likelihood of disutility from shame is high, citizens want to reduce the level of this disutility.

Item 4 of Proposition 1 points out that expected voter turnout does not depend on whether citizens share information in an SSE when the cost of voting is uniformly distributed. In a separating SSE, turnout is high in case citizens have the same private opinions and is low otherwise. In a pooling SSE, each citizen votes with the same probability. In expected terms, the probabilities of voting in both equilibria coincide. It is the symmetry of the model (in particular, that conflicting signals cancel out) and the uniform cost distribution that make expected turnout in both types of equilibria equal. Yet we show in Section 4 that once the model incorporates an information acquisition stage, for the uniform distribution expected voter turnout may be higher in an equilibrium where citizens share information than in an equilibrium where citizens do not share information. Item 4 furthermore describes how voter turnout depends on the first derivative of $f(c)$.

Our model describes opinion formation and voting behavior in a group. Proposition 1 gives the requirements for a separating SSE. A pooling SSE always exists. The two equilibria differ in how opinions are formed. In the separating SSE, peers share information, meaning that opinion formation is a social activity. In the pooling SSE, peers do not share information, meaning that opinion formation is a private matter. The natural interpretation of a pooling equilibrium is that citizens simply do not talk about politics.

In the separating SSE, the strengths of opinions depend on what has been exchanged in the communication stage. Sometimes opinions are weak, sometimes strong. One of our predictions is that citizens with stronger opinions are more likely to vote. This prediction is supported by the empirical evidence that political ambivalence decreases the likelihood of voting (Palfrey and Poole, 1987; Mutz, 2002). Similarly, Giles and Dantico (1982) and Grosser and Schram (2006) find that citizens are more likely to vote when surrounded by people who have more similar opinions. This evidence is consistent with our result that turnout is higher in groups where people have the same opinions than in groups where they have different ones.

Remark. Proposition 1 holds not only for NDHR distributions. In the appendix, we show that it holds for any distribution function $F(c)$ which hazard rate $\lambda(c)$ is such that (i) $c\lambda(c)$ is non decreasing, and (ii) $(1-k)\bar{c}\lambda(\bar{c}) > 1$. The monotonicity of $c\lambda(c)$ can be regarded as a single-crossing property, which guarantees the uniqueness of the thresholds in Proposition 1. If $(1-k)\bar{c}\lambda(\bar{c}) < 1$ then the cost effect is always stronger than the pride effect so that $\underline{\theta_1} \to \infty$. In this case, citizens always want to avoid strong opinions in order to avoid very high expected voting costs so that the separating equilibrium never exists.

4. Are voters really rationally ignorant?

In this section, we investigate citizens' incentives to acquire information. For the two equilibria, the pooling one and the separating one presented in Proposition 1, we identify the conditions under which both citizens acquire information, $e_i = h$, and the conditions under which neither of them acquires information, $e_i = l$.

First consider an equilibrium where opinion formation is a private activity (the pooling SSE), that is, m_i does not contain information about s_i. The next proposition describes that when opinion formation is a private activity, citizens choose to be ignorant.

Proposition 2. *A pooling SSE is unique. In this SSE, citizens are rationally ignorant, $e^* = l$.*

The assumption that a citizen's information acquisition decision is not observable is responsible for the result presented in Proposition 2. In our model, social concerns emerge from perceptions by others. As a result, beliefs matter. As in the absence of communication, deviating affects neither beliefs nor posterior probabilities, acquiring costly information never pays. If i's peer believes that i has acquired information, not acquiring information saves c^e. By contrast, if i's peer believes that i has not acquired information, acquiring information has a cost c^e, but brings no benefits.

Now consider equilibria where opinion formation is a social activity, $m_i = s_i$. Proposition 3 presents conditions under which citizens are knowledgeable, $e^* = h$, or ignorant $e^* = l$.

Proposition 3. *There are two effort cost c^e thresholds c^L and c^H, $c^L < c^H$, such that:*

1. *If $c^e > c^H$ and a separating SSE of the game without information acquisition exists for $e = l$, then $e^* = l$ followed by that SSE is the unique separating SSE of the game with information acquisition.*
2. *If $c^e < c^L$ and a separating SSE of the game without information acquisition exists for $e = h$, then $e^* = h$ followed by that equilibrium is the unique separating SSE of the game with information acquisition.*
3. *If $c^e \in \left[c^L, c^H\right]$, then both separating SSE with $e^* = l$ and $e^* = h$ exist.*

Consider a separating SSE in which both citizens choose $e^* = l$. This means that citizens benefit from signal congruence; otherwise, they would not share information. In this equilibrium, the possible strengths of citizen i's opinion are given by (9) with $e = l$. Deviating by choosing $e_i = h$ instead of $e_i = l$, does not affect these opinions. It only affects the probability of signal congruence. By choosing $e_i = h$ citizen i increases the likelihood of signal congruence by

$$\Delta_l = \Pr\left\{s_i = s_{-i} | e_i = h, e_{-i} = l\right\} - \Pr\left\{s_i = s_{-i} | e_i = e_{-i} = l\right\} = (2l-1)(h-l) \tag{10}$$

Citizen i compares the benefit from deviating with the cost of acquiring information, c^e. If c^e is sufficiently small, $c^e < c^L$, choosing $e_i = h$ is a profitable deviation. Thus, the SSE with $e^* = l$ only exists if $c^e > c^L$.

Now suppose a separating SSE in which both citizens choose $e^* = h$. In this equilibrium, the possible strengths of citizen i's opinion are given by (9) with $e = h$. Deviating by choosing $e_i = l$ instead of $e_i = h$ reduces the likelihood of signal congruence by:

$$\Delta_h = \Pr\left\{s_i = s_{-i} | e_i = e_{-i} = h\right\} - \Pr\left\{s_i = s_{-i} | e_i = l, e_{-i} = h\right\} = (2h-1)(h-l) > \Delta_l \tag{11}$$

As information is shared in equilibrium, citizens benefit from signal congruence. A lower probability of signal congruence is, therefore, a cost to i. The benefit of choosing $e_i = l$ instead of $e_i = h$ is not incurring c^e. Hence, only if c^e is sufficiently large, $c^e > c^H$, citizen i has an incentive to deviate. Thus, the SSE with $e^* = h$ only exists if $c^e < c^H$.

There remains to be shown that $c^H > c^L$. This inequality holds for two reasons. First, as discussed in the previous section, the benefit of signal congruence increases in e_i. Furthermore, $\Delta_h > \Delta_l$ means that citizens' effort levels are strategic complements. To accomplish signal congruence, the effort levels reinforce each other. This complementarity is responsible for the existence of multiple separating SSE. If $c^e \in \left[c^L, c^H\right]$, for the same parameters of the model a separating SSE exists in which citizens are rationally ignorant and a separating SSE exists in which they are rationally knowledgeable.

We have identified three equilibria of the voting game with information acquisition. When opinion formation is a private activity social concerns do not provide incentives to acquire information. When opinion formation is a social activity, social concerns may induce citizens to acquire information. The reason for this difference is that social utility revolves around perceptions. When opinion formation is a social activity, citizens prefer signal congruence and report their signals truthfully to increase the probability of signal congruence. Unless c^e is large, deviating by choosing low effort is not profitable because this deviation reduces the probability of signal congruence. When opinion formation is a private activity, deviating by reducing effort is always profitable because whether or not signals are congruent does not affect utility.

Recall that the voting model without information acquisition predicts that expected voter turnout is the same under private opinion formation and social opinion formation for the uniform distribution function. In the voting model with information acquisition, turnout is higher when citizens share information than when they do not, provided that information has been acquired. Sharing information is a necessary condition for higher turnout, not a sufficient one.

Another important insight from the analysis is that information acquisition decisions are strategic complements. By acquiring information a citizen makes it more likely that his signal corresponds with the (unobserved) state. Acquiring information makes it also more likely that his signal is in line with his peer's signal, especially in case his peer also acquired information. As a result, if citizen i prefers signal congruence to signal dissonance, his incentive to acquire information is particularly strong if he anticipates that $-i$ also acquires information. This strategic complementarity may lead to the existence of multiple equilibria. The presence of multiple equilibria suggests that different "political cultures" in groups may explain voter turnout. To put it differently, information acquisition, involvement in political activities, and turnout may vary across otherwise identical groups.

5. Concluding remarks

In this paper, we have taken social image concerns as the point of departure for developing an integrated theory of information acquisition, opinion formation, and voting. We have modeled two social image concerns: first, pride from voting and second, shame from not meeting your peer's expectations which is higher the stronger the perceived strength of your opinion is. We have shown that on election day, when information has been gathered and opinions have been formed, pride and shame both encourage voting. Before the election day, the two social image concerns may have divergent effects. Pride generally encourages people to be politically active, while shame may induce people to shy away from politics.

The key assumptions responsible for almost all of our results are that (i) pride from voting is independent of the perceived opinion strength, and (ii) shame from not meeting expectations is proportional to the expected opinion strength conditional on not voting. To investigate how alternative assumptions on pride and shame affect our results, we have also examined a more general model, in which both pride and shame linearly depend on the expected opinion strength conditional on voting for pride and on not voting for shame. In the appendix, we show that when citizens with stronger opinions experience more (less) utility from voting, the condition for a separating SSE becomes weaker (stronger). The intuition is that if utility depends on the expected opinion strength positively (negatively), citizens have incentives to communicate that their opinions are strong (weak) by reporting their private signals truthfully (not truthfully).

Throughout this paper, we have assumed that voting arouses feelings of pride and not voting arouses feelings of shame. This assumption seems natural for developed democracies. For less developed democracies, where opposition parties are oppressed and silenced, at least for some citizens, it is more natural to assume that not voting arouses feelings of pride and voting arouses feelings of shame.

Our model generates a variety of predictions that are consistent with the empirical literature. Aldashev (2010) uses data from the 2000 American National Election Study to investigate the effect of the sizes of citizens' networks on their information acquisition activities, such as watching TV debates and attention to campaign news. Controlling for several individual characteristics, he finds a positive relationship. In particular, citizens with no social network abstain from acquiring information about politics. This is consistent with our prediction that when opinion formation is a private activity, citizens do not acquire information.[17]

A stylized fact concerning voter turnout is that better educated citizens participate more frequently in elections (Wolfinger and Rosenstone, 1980). Most studies on voter turnout, however, report correlations rather than causal evidence. An exception is Lassen (2005) who was able to estimate the causal effect of being informed on voter turnout by using data from a natural experiment in Copenhagen. Before a referendum on decentralization for all fifteen districts was held, citizens in four districts had experience with decentralization because of pilots. Lassen found that information about the consequences of decentralization led to stronger opinions, and thereby, to higher turnout. Moreover, he found that better educated citizens had stronger opinions. Once having controlled for the effect of education on the strengths of citizens' opinions, no direct effect of education on turnout was obtained. These findings perfectly match with the predictions of our model.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix

Proof of Lemma 1.

Suppose an equilibrium in which citizens share information, $m_i = s_i$. Then, Bayes' rule implies:

$$\Pr\left\{w = 1 | m_i = m_{-i}\right\} = \frac{e^2}{e^2 + (1-e)^2}$$
$$\Pr\left\{w = 1 | m_i = -m_{-i}\right\} = \frac{1}{2}$$

and (9) immediately follows. Plugging (9) into (6) yields the desired expressions for $b^S\left(m_i, m_{-i}\right)$. ■

Proof of Lemma 2 is in the main text.

Proof of Proposition 1.

Suppose a separating SSE exists. Citizens' turnout strategies are given by (7), citizens share information, and posterior expectations are as defined in Lemma 1. In equilibrium, if citizen i with signal s_i reports m_i and receives m_{-i} she gets expected utility

$$\mathbb{E}_i\left[u_i | I_i\right] = wx - \frac{e_i - l}{h - l} c^e - \mathbb{E}_i\left[t_i c_i^v | I_i\right] + \theta_1 \mathbb{E}_i\left[\text{Pride}_i | I_i\right] - \theta_2 \mathbb{E}_i\left[\text{Shame}_i | I_i\right]$$

where we use I_i to denote information available to citizen i after communicating with citizen $-i$. We define I_{-i} similarly:

$$I_i = \left(s_i, m_i, m_{-i}\right) \text{ and } I_{-i} = \left(s_{-i}, m_{-i}, m_i\right)$$

[17] Leeson (2008) reports various correlation between media freedom on the one hand and political knowledge, participation and voter turnout on the other hand. In his view, media freedom leads to lower cost of acquiring information.

To evaluate the expected utility above, we use (3) and (7) to get:

$$\mathbb{E}_i\left[\text{Pride}_i|I_i\right] = k\mathbb{E}_i\left[t_i|I_i\right] + (1-k)\,\mathbb{E}_i\left[\mathbb{E}_{-i}\left[t_i|I_{-i}\right]|I_i\right] = kF\left(b^{\text{S}}\left(m_i,m_{-i}\right)\right) + (1-k)\,\mathbb{E}_i\left[F\left(b^{\text{S}}\left(m_i,m_{-i}\right)\right)|I_i\right] = F\left(b^{\text{S}}\left(m_i,m_{-i}\right)\right)$$

we use (4) and (7) to get

$$\begin{aligned}\mathbb{E}_i\left[\text{Shame}_i|I_i\right] &= k\mathbb{E}_i\left[\left(1-t_i\right)\mathbb{E}_{-i}\left[g_i|I_{-i},t_i=0\right]|I_i\right] + (1-k)\,\mathbb{E}_i\left[\mathbb{E}_{-i}\left[\left(1-t_i\right)g_i|I_{-i}\right]|I_i\right]\\ &= k\mathbb{E}_i\left[\left(1-t_i\right)\widehat{g}^{\text{S}}\left(m_i,m_{-i}\right)|I_i\right] + \ (1-k)\,\mathbb{E}_i\left[\widehat{g}^{\text{S}}\left(m_i,m_{-i}\right)\mathbb{E}_{-i}\left[\left(1-t_i\right)|I_{-i}\right]|s_i,m_i,m_{-i}\right]\\ &= \widehat{g}^{\text{S}}\left(m_i,m_{-i}\right)\left(1-F\left(b^{\text{S}}\left(m_i,m_{-i}\right)\right)\right)\end{aligned}$$

and we use (7) to get

$$\mathbb{E}_i\left[t_i c_i^v|I_i\right] = \int_0^{b^{\text{S}}(m_i,m_{-i})} t\ dF(t)$$

Thus, the expected utility $\mathbb{E}_i\left[u_i|I_i\right]$ can be written as follows:

$$\begin{aligned}\mathbb{E}_i\left[u_i|I_i\right] &= wx - \frac{e_i-l}{h-l}c^e - \int_0^{b^{\text{S}}(m_i,m_{-i})} t\ dF(t) + \theta_1 F\left(b^{\text{S}}\left(m_i,m_{-i}\right)\right) - \theta_2\widehat{g}^{\text{S}}\left(m_i,m_{-i}\right)\left(1-F\left(b^{\text{S}}\left(m_i,m_{-i}\right)\right)\right)\\ &= wx + \theta_1 - \frac{e_i-l}{h-l}c^e - \int_0^{b^{\text{S}}(m_i,m_{-i})} t\ dF(t) - \ \left(\theta_1+\theta_2\widehat{g}^{\text{S}}\left(m_i,m_{-i}\right)\right)\left(1-F\left(b^{\text{S}}\left(m_i,m_{-i}\right)\right)\right)\\ &= wx + \theta_1 - \frac{e_i-l}{h-l}c^e - G\left(b^{\text{S}}\left(m_i,m_{-i}\right)\right) \end{aligned} \tag{12}$$

where function $G(c)$ is defined as follows

$$G(c) = \frac{1}{k}(1-F(c))\,c + \int_0^c t\ dF(t)$$

In equilibrium, citizen i with signal s_i reports $m_i = s_i$. If she deviates and reports $m_i = -s_i$ his net benefit $B\left(s_i,m_{-i}\right)$ from deviation conditional on m_{-i} is:

$$B\left(s_i,m_{-i}\right) = \mathbb{E}_i\left[u_i|s_i,m_i=-s_i,m_{-i}\right] - \mathbb{E}_i\left[u_i|s_i,m_i=s_i,m_{-i}\right] = G\left(b^{\text{S}}\left(s_i,m_{-i}\right)\right) - G\left(b^{\text{S}}\ \left(-s_i,m_{-i}\right)\right) \tag{13}$$

Note that $B\left(s_i,m_{-i}\right)$ is an odd function: $B\left(s_i,-m_{-i}\right) = B\left(-s_i,m_{-i}\right) = -B\left(s_i,m_{-i}\right)$. The unconditional net benefit from this deviation, $\overline{B}\left(s_i\right)$, is

$$\begin{aligned}\overline{B}\left(s_i\right) &= \mathbb{E}_i\left[B\left(s_i,m_{-i}\right)|s_i\right] = \Pr\left\{s_{-i}=s_i|s_i\right\}B\left(s_i,s_i\right) + \Pr\left\{s_{-i}=-s_i|s_i\right\}B\left(s_i,-s_i\right)\\ &= \left(\Pr\left\{s_{-i}=s_i|s_i\right\} - \Pr\left\{s_{-i}=-s_i|s_i\right\}\right)B\left(s_i,s_i\right) = \left(2\Pr\left\{s_{-i}=s_i|s_i\right\}-1\right)B\left(s_i,s_i\right)\end{aligned}$$

Using

$$\Pr\left\{s_{-i}=s_i|s_i\right\} = e^2 + (1-e)^2$$

yields

$$\overline{B}\left(s_i\right) = (2e-1)^2 B\left(s_i,s_i\right)$$

The stipulated separating SSE exists if and only if $B\left(s_i,s_i\right) < 0$, *i.e.*,[18]

$$G\left(b^{\text{S}}\left(s_i,s_{-i}\right)\right) < G\left(b^{\text{S}}\left(-s_i,s_{-i}\right)\right)$$

Suppose $t\lambda(t)$ is non-decreasing in t. This property surely holds for non-decreasing hazard rate (NDHR) distributions. Analyzing the derivative

$$G'(c) = \frac{1-F(c)}{k}\left(1-(1-k)\,c\lambda(c)\right)$$

shows that if $(1-k)\,\bar{c}\lambda(\bar{c}) \le 1$ then $G' > 0$ for all $c \in \left(0,\bar{c}\right)$. In this case, $\overline{B} > 0$ due to $b^{\text{S}}\left(s_i,s_{-i}\right) > b^{\text{S}}\left(-s_i,s_{-i}\right)$ and the monotonicity of G. The separating SSE never exists in such a case. Thus, in the remainder of the proof, we assume

$$(1-k)\,\bar{c}\lambda(\bar{c}) > 1$$

[18] We omit the non-generic case $B\left(s_i,s_i\right) = 0$ where the separating equilibrium exists, it is not strict.

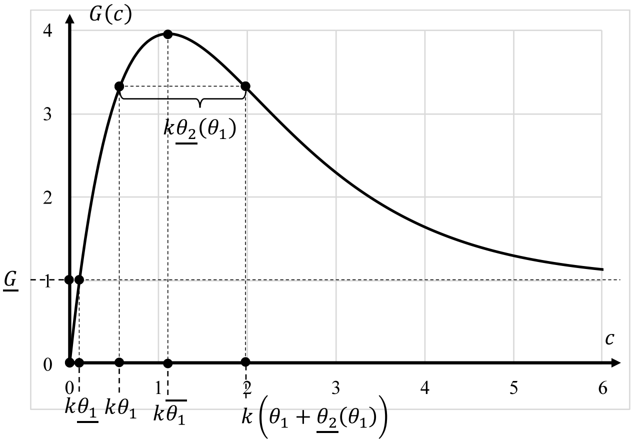

Fig. 1. Function $G(c)$ for the exponential voting cost distribution $F(c) = 1 - \exp(-c)$ and $k = 1$; values $\underline{\theta}_1$, $\overline{\theta}_1$, and $\underline{G}$; construction of $\underline{\theta}_2(\theta_1)$ for $\theta_1 \in \left(\underline{\theta}_1, \overline{\theta}_1\right)$.

Then there exists a unique (due to the assumed monotonicity of $t\lambda(t)$) value $\overline{\theta}_1$ such that $G'\left(k\overline{\theta}_1\right) = 0$. Then, $G'(c) > 0$ for $c < k\overline{\theta}_1$, and $G'(c) < 0$ for $c > k\overline{\theta}_1$. In other words, G is quasi-concave function which increases from $G(0) = 0$ to its maximum $G\left(k\overline{\theta}_1\right)$, and then decreases to $\underline{G} = G(\overline{c})$. By construction, $\underline{G} \in \left(0, G\left(k\overline{\theta}_1\right)\right)$. Therefore, there exists a unique value $\underline{\theta}_1$ such that $G\left(k\underline{\theta}_1\right) = \underline{G}$. Then, $G(c) > G\left(k\underline{\theta}_1\right)$ for all $c > k\underline{\theta}_1$. Hence, for all $\theta_1 < \underline{\theta}_1$ it is the case that

$$G\left(b^{\mathrm{S}}\left(s_i, s_i\right)\right) > G\left(k\underline{\theta}_1\right) > G\left(k\theta_1\right) = G\left(b^{\mathrm{S}}\left(-s_i, s_{-i}\right)\right)$$

because $b^{\mathrm{S}}\left(s_i, s_i\right) > k\underline{\theta}_1 > k\theta_1$. The separating SSE never exists in such a case. This proves item 1 of the proposition.

Now assume $\theta_1 > \underline{\theta}_1$. Then, we define $\underline{\theta}_2\left(\theta_1\right) \geq 0$ as the largest solution θ_2 to the following equation

$$G\left(k\left(\theta_1 + \theta_2 \hat{g}^{\mathrm{S}}\left(m_i, m_i\right)\right)\right) = G\left(k\theta_1\right) \tag{14}$$

Consider two cases.

1. Let $\theta_1 \in \left(\underline{\theta}_1, \overline{\theta}_1\right)$. Then, (14) has two solutions, the trivial one $\theta_2 = 0$, and $\theta_2 = \underline{\theta}_2 > 0$ (this is so because the LHS of (14) monotonically decreases in θ_2 from $G\left(k\overline{\theta}_1\right)$ to $\underline{G}$, with $G\left(k\overline{\theta}_1\right) > G\left(k\theta_1\right) \geq \underline{G}$. By construction,

$$G\left(k\left(\theta_1 + \theta_2 \hat{g}^{\mathrm{S}}\left(m_i, m_i\right)\right)\right) < G\left(k\theta_1\right)$$

   for all $\theta_2 > \underline{\theta}_2$ so that the separating SSE exists. For $\theta_2 < \underline{\theta}_2$, the above inequality fails, and the separating SSE fails to exist. This proves item 2 for this case.
   The LHS of (14) is decreasing in θ_1 whereas the RHS of (14) is increasing in θ_1. Thus, $\underline{\theta}_2$ must be decreasing in θ_1. This proves item 3 of the proposition for this case.
2. Let $\theta_1 > \overline{\theta}_1$. Then, ((14)) has a unique solution, the trivial one $\theta_2 = 0$. Hence,

$$G\left(k\left(\theta_1 + \theta_2 \hat{g}^{\mathrm{S}}\left(m_i, m_i\right)\right)\right) < G\left(k\theta_1\right)$$

   for all $\theta_2 > \underline{\theta}_2 = 0$ so that the separating SSE exists, which ends the proof of items 2 and 3.

Fig. 1 illustrates function $G(c)$ for the exponential voting cost distribution $F(c) = 1 - \exp(-c)$ and $k = 1$. It also shows graphically the relation between $\underline{\theta}_1$, $\overline{\theta}_1$, through k and $G(c)$.

To prove item 4 we note that the expected opinion strength $\mathbb{E}\left[\hat{g}^{\mathrm{S}}\left(m_i, m_{-i}\right)\right]$ in the separating SSE is

$$\mathbb{E}\left[\hat{g}^{\mathrm{S}}\left(m_i, m_{-i}\right)\right] = \Pr\left\{s_{-i} = s_i\right\} \hat{g}^{\mathrm{S}}\left(m_i, m_i\right) + \Pr\left\{s_{-i} = -s_i\right\} \hat{g}^{\mathrm{S}}\left(m_i, -m_i\right) = e - \frac{1}{2} = \hat{g}^{\mathrm{P}}$$

Hence, the expected turnout rate $\mathbb{E}\left[F\left(k\left(\theta_1 + \theta_2 \hat{g}^{\mathrm{S}}\left(m_i, m_{-i}\right)\right)\right)\right]$ in the separating SSE is higher, equal, or lower than the expected turnout rate in the pooling SSE, which we can write as $F\left(k\left(\theta_1 + \theta_2 \mathbb{E}\left[\hat{g}^{\mathrm{S}}\left(m_i, m_{-i}\right)\right]\right)\right)$, when $F(c)$ is convex, linear, or concave correspondingly. This is just Jensen's inequality. This ends the proof. ■

Proof of Proposition 2.

Let e^* be the effort level in a pooling SSE, and let citizen i choose effort level $e_i \in \{l, h\}$. Since this choice is not observable by citizen $-i$, she still believes that $e_i = e^*$. As a result, the expected opinion strength $\hat{g}_i$, given by (5), does not depend on e_i, and neither does the voting cost threshold b_i, given by ((6)). Thus, the voting behavior is unaffected by the deviation from $e_i = e^*$, and the net benefit from the deviation is

$$\mathbb{E}_i\left[u_i|e_i\right] - \mathbb{E}_i\left[u_i|e_i = e^*\right] = \frac{e^* - e_i}{h - l}c^e$$

This implies that the deviation from $e_i = e^* = h$ to $e_i = l$ is always profitable, and the deviation from $e_i = e^* = l$ to $e_i = h$ is never profitable. Consequently, in a pooling SSE, it must be that $e^* = l$, and this equilibrium is unique. This ends the proof. ■

Proof Proposition 3.

Let $e^* = e$ be the effort level in a separating SSE, and let citizen i choose effort level $e_i \in \{l, h\}$. Consider the communication stage. Citizen $-i$ still believes that $e_i = e^*$ so that expected opinion strength $\hat{g}_i$ and the voting cost threshold b_i are as in Lemma 1 and are independent of e_i. Thus, as in the pooling SSE, the voting behavior is unaffected by the deviation from $e_i = e^*$. So is the net benefit from sending an opposite message $m_i = -s_i$ conditional on m_{-i}, $B\left(s_i, m_{-i}\right)$, given by (13). Since

$$\Pr\left\{s_{-i} = s_i|s_i, e_i\right\} = ee_i + (1 - e)\left(1 - e_i\right)$$

the unconditional benefit from the message deviation $\overline{B}\left(s_i\right)$ does depend on e_i:

$$\begin{aligned}\overline{B}\left(s_i\right) &= \Pr\left\{s_{-i} = s_i|s_i, e_i\right\} B\left(s_i, s_i\right) + \Pr\left\{s_{-i} = -s_i|s_i, e_i\right\} B\left(s_i, -s_i\right) = \left(2\Pr\left\{s_{-i} = s_i|s_i, e_i\right\} - 1\right) B\left(s_i, s_i\right)\\ &= (2e - 1)\left(2e_i - 1\right) B\left(s_i, s_i\right)\end{aligned}$$

Thus, the separating SSE existence condition $\overline{B}\left(s_i\right) < 0$ does not depend on e_i. By the proposition assumption, the separating SSE exists so that $B\left(s_i, s_i\right) < 0$.

Using (12), the expected utility of citizen i conditional on $\left(m_i, m_{-i}, s_i, e_i\right)$ is

$$\mathbb{E}_i\left[u_i|\ldots\right] = \mathbb{E}_i\left[wx|\ldots\right] - \frac{e_i - l}{h - l}c^e + \theta_1 - \mathbb{E}_i\left[G\left(b^S\left(m_i, m_{-i}\right)\right)|\ldots\right]$$

where the last term is:

$$\mathbb{E}_i\left[G\left(b^S\right)|m_i, m_{-i}, s_i, e_i\right] = \Pr\left\{s_{-i} = s_i|s_i, e_i\right\} G\left(b^S\left(s_i, s_i\right)\right) + \Pr\left\{s_{-i} = -s_i|s_i, e_i\right\} G\left(b^S\left(s_i, -s_i\right)\right)$$

The net benefit from the effort deviation $e_i \neq e$, which is defined as

$$D\left(e_i, e\right) = \mathbb{E}_i\left[u_i|e_i\right] - \mathbb{E}_i\left[u_i|e_i = e\right] \tag{15}$$

can be written as

$$\begin{aligned}D\left(e_i, e\right) &= \frac{e - e_i}{h - l}c^e + \mathbb{E}_i\left[G\left(b^S\left(m_i, m_{-i}\right)\right)|e_i = e\right] - \mathbb{E}_i\left[G\left(b^S\left(m_i, m_{-i}\right)\right)|e_i\right]\\ &= \frac{e - e_i}{h - l}c^e + \left(\Pr\left\{s_{-i} = s_i|s_i, e_i = e\right\} - \Pr\left\{s_{-i} = s_i|s_i, e_i\right\}\right) G\left(b^S\left(s_i, s_i\right)\right)\\ &\quad + \left(\Pr\left\{s_{-i} = -s_i|s_i, e_i = e\right\} - \Pr\left\{s_{-i} = -s_i|s_i, e_i\right\}\right) G\left(b^S\left(s_i, -s_i\right)\right)\\ &= \frac{e - e_i}{h - l}c^e + \left(\Pr\left\{s_{-i} = s_i|s_i, e_i = e\right\} - \Pr\left\{s_{-i} = s_i|s_i, e_i\right\}\right)\left(G\left(b^S\left(s_i, s_i\right)\right) - G\left(b^S\left(s_i, -s_i\right)\right)\right)\\ &= \frac{e - e_i}{h - l}c^e + (2e - 1)\left(e - e_i\right)\left(G\left(b^S\left(s_i, s_i\right)\right) - G\left(b^S\left(s_i, -s_i\right)\right)\right)\\ &= \frac{e - e_i}{h - l}\left(c^e + (h - l)(2e - 1) B\left(s_i, s_i\right)\right) = \frac{e - e_i}{h - l}\left(c^e - c^*(e)\right)\end{aligned}$$

where

$$c^*(e) = -(h - l)(2e - 1) B\left(s_i, s_i\right) > 0$$

We consider equilibria with $e^* = l$ and $e^* = h$ separately.

1. Let $e^* = l$ so that $e_i = h$. Then, the SSE existence condition $D(h, l) < 0$ becomes $c^e > c^L$, where $c^L = c^*(l)$.
2. Let $e^* = h$ so that $e_i = l$. Then, the SSE existence condition $D(l, h) < 0$ becomes $c^e < c^H$, where $c^H = c^*(h)$.

What remains to be shown is that $c^H > c^L$. The derivative of $c^*(e)$ is

$$\frac{d}{de}c^*(e) = -(h - l)\left(2B\left(s_i, s_i\right) + (2e - 1) G'\left(b^S\left(s_i, s_i\right)\right)\frac{d}{de}b^S\left(s_i, s_i\right)\right)$$

Since $B\left(s_i, s_i\right) < 0$, $G'\left(b^S\left(s_i, s_i\right)\right) < 0$, and

$$\frac{d}{de}b^S\left(s_i, s_i\right) = k\theta_2\frac{2e(1 - e)}{\left(e^2 + (1 - e)^2\right)^2} > 0$$

it follows that $\frac{d}{de}c^*(e) > 0$, $c^*(e)$ is increasing, and $c^L < c^H$. This ends the proof. ■

An alternative (more general) specification of the model

Let pride and shame terms in citizens' utility ((2)) be defined as follows:

$$\theta_1 \text{Pride}_i = \mathbb{E}_i\left[\mathbb{E}_{-i}\left[t_i\left(a_1+b_1 g_i\right)|s_{-i},m_{-i},m_i,k_i\right]|s_i,m_i,m_{-i}\right]$$
$$\theta_2 \text{Shame}_i = \mathbb{E}_i\left[\mathbb{E}_{-i}\left[\left(1-t_i\right)\left(a_2+b_2 g_i\right)|s_{-i},m_{-i},m_i,k_i\right]|s_i,m_i,m_{-i}\right]$$

That is, both pride and shame components of consumer utility depend (linearly) on the opinion strength. With this general specification, our original model assumes $a_1=\theta_1$, $b_2=\theta_2$, and $a_2=b_1=0$.

It can be verified that the voting cost threshold b_i becomes

$$b_i = k\left(\left(a_1+a_2\right)+b_1\hat{g}_i^1+b_2\hat{g}_i^0\right)$$

where the expected opinion strength $\hat{g}_i^t$ is

$$\hat{g}_i^t = \mathbb{E}_i\left[\mathbb{E}_{-i}\left[g_i|s_{-i},m_{-i},m_i,t_i=t\right]|s_i,m_i,m_{-i}\right]$$

In particular, $\hat{g}_i^1$ is the expected opinion strength conditional on citizen i voting whereas $\hat{g}_i^0$ is his expected opinion strength conditional on not voting. For the pooling SSE, not much is affected here. For the separating SSE, to the contrary, the net benefit from deviation $m_i=-s_i$ becomes

$$B\left(s_i,m_{-i}\right)=G^*\left(b^S\left(s_i,m_{-i}\right)\right)-G^*\left(b^S\left(-s_i,m_{-i}\right)\right)$$

where

$$G^*(c)=\frac{1}{k}\left(\frac{b_2}{\left(b_1+b_2\right)}-F(c)\right)c+\int_0^c t\,dF(t)$$

The unconditional benefit from this deviation, $\overline{B}\left(s_i\right)$, is

$$\overline{B}\left(s_i\right)=\mathbb{E}_i\left[B\left(s_i,m_{-i}\right)|s_i\right]=(2e-1)^2 B\left(s_i,s_i\right)$$

Thus, similar to our original model, the separating SSE exists if and only if $B\left(s_i,m_{-i}\right)<0$. Taking the derivative of G^* yields:

$$\frac{dG^*}{dc}(c)=\frac{1}{k}\left((1-F(c))(1-(1-k)c\lambda(c))-\frac{b_1}{\left(b_1+b_2\right)}\right)$$

Proposition 1 provides the analysis for the case $b_1=0$. When $b_1>0$ so that pride is increasing in the opinion strength, the existence conditions for the separating SSE are weaker than in Proposition 1. In particular, when $b_1>b_2=0$, $\frac{dG^*}{dc}<0$ so that the separating SSE always exists, for all parameter values and all distributions. When, to the contrary, $b_1<0$, so that pride is decreasing in the opinion strength, the existence conditions for the separating SSE are stricter.

References

Aldashev, Gani, 2010. Political information acquisition for social exchange. Q. J. Political Sci. 5, 1–25.
Alesina, Alberto, Giuliano, Paola, 2011. Family ties and political participation. J. Eur. Econom. Assoc. 9 (5), 817–839.
Ali, Nageeb S., Lin, Charles, 2013. Why people vote: Ethical motives and social incentives. Am. Econ. J. Microecon. 5 (2), 73–98.
Barlow, Richard, Marshall, Albert, Proschan, Frank, 1963. Properties of probability distributions with monotone hazard rate. Ann. Math. Stat. 34 (2), 375–389.
Belli, Robert F., Traugott, Michael W., Young, Margaret, McGonagle, Katherine A., 1999. Reducing vote overreporting in surveys: Social desirability, memory failure, and source monitoring. Public Opin. Q. 63 (1), 90–108.
Bond, Robert M., Fariss, Christopher J., Jones, Jason J., Kramer, Adam D.I., Marlow, Cameron, Settle, Jaime E., Fowler, James H., 2012. A 61-million-person experiment in social influence and political mobilization. Nature 489, 295–298.
Cantoni, Enrico, Pons, Vincent, 2019, Does context trump individual drivers of voting behavior? Evidence from U.S. movers, Working Paper, 2019.
Coate, Stephen, Conlin, Michael, 2004. A group rule-utilitarian approach to voter turnout: Theory and evidence. Amer. Econ. Rev. 94 (5), 1476–1504.
Dahlgaard, Jens O., 2018. Trickle-up political socialization: The causal effect on turnout of parenting a newly enfranchised voter. Am. Political Sci. Rev. 112 (3), 698–7–5.
DellaVigna, Stefano, List, John A., Malmendier, Ulrike, Rao, Gautam, 2017. Voting to tell others. Rev. Econom. Stud. 84 (1), 143–181.
Downs, Anthony, 1957. An economic theory of political action in a democracy. J. Polit. Econ. 65 (2), 135–150.
Feddersen, Timothy J., 2004. Rational choice theory and the paradox of not voting. J. Econ. Perspect. 18 (1), 99–112.
Feddersen, Timothy J., Sandroni, Alvaro, 2006. A theory of participation in elections. Amer. Econ. Rev. 96 (4), 1271–1282.
Gerber, Alan S., Rogers, Todd, 2009. Descriptive social norms and motivation to vote: Everybody's voting and so should you. J. Politics 71 (1), 178–191.
Geys, Benny, 2006. 'Rational' theories of voter turnout: A review. Political Stud. Rev. 4, 16–35.
Giles, Michael W., Dantico, Marilyn K., 1982. Political participation and neighborhood social context revisited. Am. J. Political Sci. 26, 144–150.
Grosser, Jens, Schram, Arthur, 2006. Neighborhood information exchange and voter participation: An experimental study. Am. Political Sci. Rev. 100 (2), 235–248.
Harbaugh, William T., 1996. If people vote because they like to, then why do so many of them lie? Public Choice 89 (1–2), 63–76.
Harsanyi, John C., 1980. Rule utilitarianism, rights, obligations and the theory of rational behavior. Theory and Decision 12 (2), 115–133.
Hodler, Roland, Luechinger, Simon, Stutzer, Alois, 2015. The effects of voting costs on the democratic process and public finances. Am. Econ. J.: Econ. Policy 7 (1), 141–171.
Huckfeldt, Rober, 1979. Political participation and the neighborhood social context. Am. J. Political Sci. 23 (3), 579–592.
Huckfeldt, Robert, Sprague, John, 1987. Networks in context: The social flow of political information. Am. Political Sci. Rev. 81 (4), 1197–1216.
Kenny, Christopher B., 1992. Political participation and effects from the social environment. Am. J. Political Sci. 36 (1), 259–267.
Kreps, David M., Wilson, Robert, 1982. Sequential equilibria. Econometrica 50 (4), 863–894.
Lassen, David D., 2005. The effect of information on voter turnout: Evidence from a natural experiment. Am. J. Political Sci. 49 (1), 103–118.

Leeson, Peter T., 2008. Media freedom, political knowledge, and participation. J. Econ. Perspect. 22 (2), 155–169.
McClurg, Scott D., 2004. Indirect mobilization: The social consequences of party contacts in an election campaign. Am. Politics Res. 32 (4), 406–443.
Mueller, Dennis C., 2003. Public choice III (book review). Public Choice 118 (3/4), 469–473.
Mutz, Diana C., 2002. The consequences of cross-cutting networks for political participation. Am. J. Political Sci. 46, 838–855.
Nickerson, David W., 2008. Is voting contagious? Evidence from two field experiments. Am. Political Sci. Rev. 102 (1), 49–57.
Palfrey, Thomas R., Poole, Keith T., 1987. The relationship between information, ideology, and voting behavior. Am. J. Political Sci. 31 (3), 511–530.
Palfrey, Thomas R., Rosenthal, Howard, 1985. Voter participation and strategic uncertainty. Am. Political Sci. Rev. 79 (1), 62–78.
Perez-Truglia, Ricardo, 2018. Political conformity. Rev. Econ. Stat. 100 (1), 14–28.
Pons, Vincent, 2018. Will a five minute discussion change your mind? A countrywide experiment on voter choice in France. Amer. Econ. Rev. 108 (6), 1322–1363.
Riker, William H., Ordeshook, Peter C., 1968. A theory of the calculus of voting. Am. Political Sci. Rev. 62 (1), 25–42.
Rogers, Todd, Ternovski, John, Yoeli, Erez, 2016. Potential follow-up increases private contributions to public goods. Proc. Natl. Acad. Sci. USA 113 (19), 5218–5220.
Swank, Otto H., Visser, Bauke, 2019. Committees as Active Audiences: Reputation Concerns and Information Acquisition. Tinbergen Institute, 18-068/VII.
Wolfinger, Raymond E., Rosenstone, Steven J., 1980. Who votes?. Yale University Press, eISBN 978-0-300-16184-7.