DIGITALCOMMONS
@WAYNESTATE

Journal of Modern Applied Statistical Methods

Volume 8 | Issue 1 Article 35

5-2009

On the BLUE of the Population Mean for Location and Scale Parameters of Distributions Based on Moving Extreme Ranked Set Sampling

Walid A. Abu-Dayyeh
Sultan Qaboos University, abudayyehw@yahoo.com

Lana Al-Rousan
Yarmouk University

Journal of Modern Applied Statistical Methods
May 2009, Vol. 8, No. 1, 266-272

1538 – 9472/09/$95.00

On the BLUE of the Population Mean for Location and Scale Parameters of Distributions Based on Moving Extreme Ranked Set Sampling

Walid Abu-Dayyeh
Sultan Qaboos University
Muscat, Oman

Lana Al-Rousan
Yarmouk University
Jordan

The best linear unbiased estimator (BLUE) for the population mean under moving extreme ranked set sampling (MERSS) is derived for general location and scale parameters of distributions which generalizes Al-Odat and Al-Saleh (2001). It is compared with the sample mean of simple random sampling (SRS). The efficient sample size under the MERSS for which the BLUE estimator dominates the usual sample mean under SRS for estimating the population mean is also computed for several distributions.

Key words: Best linear unbiased estimator; location parameter; scale parameter; moving extreme ranked set sampling, simple random sampling.

Introduction

Ranked set sampling (RSS) as introduced by McIntyre (1952) is useful for cases when the variable of interest can be more easily ranked than quantified. The aim of RSS is to increase the efficiency of the sample mean as an estimator for the population mean μ. Takahasi and Wakimoto (1968) established a very important statistical foundation for the theory of RSS. They showed that the mean of the RSS is an unbiased estimator for the population mean and has smaller variance than the mean of SRS. Dell and Clutter (1972) studied the effect of ranking error on the procedure. The RSS has many statistical applications in biological and environmental studies and reliability theory (e.g. Dell & Clutter, 1972; Stokes, 1977, 1980; Mode et al., 1999; Barabesi & El-Sharaawi, 2001; Al-Saleh & Zheng, 2002; & Al-Saleh & Al-Omary, 2002). Sinha, et al., (1996) explored the concept of RSS when the population is partially known using the parameters of normal and exponential distributions. They found that the use of knowledge of the distribution along with RSS provides improvement in estimation over SRS, as well as over nonparametric RSS. Li and Chuiv (1997) discussed the issue of the efficiency of RSS compared to SRS in many parametric estimation problems. They found an improvement in estimation of many common parameters of interest with smaller numbers of measurements compared to SRS.

RSS has been investigated extensively (see for example, Stokes, 1977; Stokes & Sager, 1988; Lam, et al., 1994; Barabesi & El-Sharaawi, 2001). Al-Saleh and Al-Kadiri (2000) introduced Double RSS to increase the efficiency of RSS estimates without increasing the set size m and Al-Saleh and Al-Omary (2002) generalized it to multistage RSS. Samawi, et al., (1996) used extreme ranked set sample (ERSS), which is easier to use than the usual RSS procedure, when the set size is large to estimate the population mean in the case of symmetric distributions. Al-Odat and Al-Saleh (2001) introduced the concept of varied set size RSS, which is coined here as Moving Extreme Ranked Set Sampling (MERSS). They investigated this modification non-parametrically and found that the

Walid Abu-Dayyeh is an associate Professor in the Department of Mathematics and Statistics at Suktan Qaboos University/Sultanate of Oman. Email: abudayyehw@yahoo.com. Lana Al-Rousan is a statistician in the Department of Statistics/Jordan. Email: lanaal211@yahoo.com.

procedure can be more efficient and applicable than the simple random sampling technique (SRS). The MERSS procedure is as follows:

1. Select m random samples of size 1, 2, 3,…, m respectively.
2. Identify the maximum of each set by eye or by some other relatively inexpensive method without actually measuring the characteristic of interest.
3. Measure accurately the selected judgment identified maximum.
4. Repeat steps 1, 2, 3, but for the minimum.
5. Repeat the above steps r times until the desired sample size, $n = 2rm$ is obtained.

Clearly, the procedure of MERSS is easier to use than the usual RSS procedure.

Methodology

The BLUE of the Mean for Distributions with a Location Parameter

Let $\{X^1_{i1}, X^1_{i2}, \cdots X^1_{ii}\}$ and $\{X^2_{i1}, X^2_{i2}, \cdots X^2_{ii}\}$ be simple random samples each of size i. for $i = 1, 2, ..., m$ from a population with distribution function F and a probability density function f. Let μ and σ^2 be the mean and variance of the population respectively. If

$$y_{i1} = Min\{X^1_{i1}, X^1_{i2}, \cdots X^1_{ii}\},$$

and

$$y_{i2} = Max\{X^2_{i1}, X^2_{i2}, \cdots X^2_{ii}\},$$
$$i = 1, 2, ..., m,$$

then

$$\{y_{11}, y_{21}, \cdots y_{m1}, y_{12}, y_{22}, \cdots y_{m2}\}$$

is a MERSS of size *2m*.

The BLUE for μ for a population can be derived with a pdf of the form:

$$f(x-\theta), -\infty < \theta < \infty, \tag{2.1}$$

where f is a pdf.

Result 1

Let $y_1, y_2, \ldots, y_{2m}$ be *2m* independent ordered statistics of simple random samples each of size less than m from an underlying distribution with a pdf as in (2.1). Then the BLUE of the population μ is then given by:

$$\hat{\mu}_{Blue} = \sum_{i=1}^{2m} \frac{1}{2\sigma_i^2 d}\{k - bt + bw\, c_i - tc_i\} y_i \tag{2.2}$$

where

$$k = \sum_{i=1}^{2m} \frac{c_i^2}{\sigma_i^2},\ t = \sum_{i=1}^{2m} \frac{c_i}{\sigma_i^2},\ w = \sum_{i=1}^{2m} \frac{1}{\sigma_i^2},$$

$$d = \sum_{i=1}^{2m} \frac{1}{\sigma_i^2} \sum_{i=1}^{2m} \frac{c_i^2}{\sigma_i^2} - \left(\sum_{i=1}^{2m} \frac{c_i}{\sigma_i^2} \right)^2,$$

and C_i and σ_i^2 are the mean and the variance of Z_i respectively, where $Z_i = y_i - \theta$ and $\mu = E_\theta X = \theta + b$. (Note that $y_1, y_2, \ldots, y_{2m}$ are not necessarily identically distributed.)

Proof

Starting with a class of unbiased linear estimators of μ of the form

$$\hat{\mu} = \sum_{i=1}^{2m} a_i y_i, \tag{2.3}$$

implies that

$$E(\hat{\mu}) = \theta \sum_{i=1}^{2m} a_i + \sum_{i=1}^{2m} a_i c_i = \mu = \theta + b,$$

which, in turn, implies that

$$\sum_{i=1}^{2m} a_i = 1$$

and

$$\sum_{i=1}^{2m} a_i c_i = b . \quad (2.4)$$

Applying the method of the Lagrange multiplier to minimize

$$Var(\hat{\mu}) = \sum_{i=1}^{2m} a_i^2 \sigma_i^2 ,$$

subject to (2.4), results in:

$$a_i^* = \frac{\lambda_1 + \lambda_2 c_i}{2\sigma_i^2}, \ \sum_{i=1}^{2m} a_i^* = 1, \ \sum_{i=1}^{2m} a_i^* c_i = b ,$$

$$\lambda_1 = \frac{\left\{ -\sum_{i=1}^{2m} \frac{c_i^2}{\sigma_i^2} + b \sum_{i=1}^{2m} \frac{c_i}{\sigma_i^2} \right\}}{\left(\sum_{i=1}^{2m} \frac{1}{\sigma_i^2} \sum_{i=1}^{2m} \frac{c_i^2}{\sigma_i^2} - \left\{ \sum_{i=1}^{2m} \frac{c_i}{\sigma_i^2} \right\}^2 \right)},$$

$$\lambda_2 = \frac{\left\{ -b \sum_{i=1}^{2m} \frac{1}{\sigma_i^2} + \sum_{i=1}^{2m} \frac{c_i}{\sigma_i^2} \right\}}{\left(\sum_{i=1}^{2m} \frac{1}{\sigma_i^2} \sum_{i=1}^{2m} \frac{c_i^2}{\sigma_i^2} - \left\{ \sum_{i=1}^{2m} \frac{c_i}{\sigma_i^2} \right\}^2 \right)},$$

where λ_1 and λ_2 are the Lagrange multipliers and

$$a_i^* = \frac{1}{2\sigma_i^2 d} \{ k - bt + c_i bw - c_i t \} . \quad (2.5)$$

Then

$$\hat{\mu}^* = \sum_{i=1}^{2m} \frac{1}{2\sigma_i^2 d} \{ k - bt + c_i bw - c_i t \} y_i , \quad (2.6)$$

is the BLUE of μ with variance

$$Var(\hat{\mu}^*) = \sum_{i=1}^{2m} \left(\frac{1}{2d\sigma_i^2} \{ k - bt + c_i bw - c_i t \} \right)^2 \sigma_i^2 . \quad (2.7)$$

If $y_i = y_{i1}$, for $i = 1, 2, ..., m$ and $y_i = y_{i2}$, for $i = m+1, m+2, ..., 2m$.

If $E(y_i) = E(y_{i1}) = c_{i1} + \theta, \ i = 1,2,..,m$

$$E(y_i) = E(y_{i2}) = c_{i2} + \theta, \ i = m+1, m+2,..,2m$$

$$Var(y_i) = Var(y_{i1}) = \sigma_{i1}^2, \ i = 1,2,..,m$$

and

$$Var(y_i) = Var(y_{i2}) = \sigma_{i2}^2, \ i = m+1m+,2,..,2m,$$

where $c_{i1} = E(u_i)$, $Var(u_i) = \sigma_{i1}^2$, and u_i is the minimum of a SRS of size i, and $c_{i2} = E(w_i)$, $Var(w_i) = \sigma_{i2}^2$ and w_i is the maximum of a SRS of size i, under $\theta = 0$. It then follows that:

$$\hat{\mu}_{MEBlue} = \sum_{i=1}^{m} \frac{1}{2\sigma_{i1}^2 d} \{ k - bt + c_{i1} bw - c_{i1} t \} y_{i1} + \sum_{i=m+1}^{2m} \frac{1}{2\sigma_{i2}^2 d} \{ k - bt + c_{i2} bw - c_{i2} t \} y_{i2} \quad (2.8)$$

and

$$Var(\hat{\mu}_{MEBlue}) = \sum_{i=1}^{m} \frac{1}{4d^2 \sigma_{i1}^2} (k - bt + c_{i1} bw - c_{i1} t)^2 + \sum_{i=m+1}^{2m} \frac{1}{4d^2 \sigma_{i2}^2} (k - bt + c_{i2} bw - c_{i2} t)^2 \quad (2.9)$$

Al-Odat and Al-Saleh (2001) introduced MERSS and studied the linear estimators of the form: $\sum_{i=1}^{m} a_i (y_{i1} + y_{i2})$. They derived the BLUE among such linear combinations for the population mean. The BLUE derived by Al-Odat and Al-Saleh (2001) is not the BLUE estimator based on ($y_{11}, y_{21}, \cdots y_{m1}, y_{12}, y_{22}, \cdots y_{m2}$), but the BLUE based on (k_1, k_2, ... k_m) where $k_i = y_{i1} + y_{i2}$ for $i=1,2,\ldots m$. If the underlying distribution is symmetric about its mean μ, then (2.9) coincides with the results obtained by Al-Odat and Al-Saleh (2001).

The BLUE estimator based on MERSS, obtained with the sample mean based on SRS in case of uniform U(θ,θ +1) and Exp(θ,1) distributions are compared. The first is symmetric about its mean $\theta + \frac{1}{2}$ and the second is skewed to the right with mean $\theta + 1$. Both families are location parameter families of distributions, so the BLUE's are the same as given in (2.8), with b = $\frac{1}{2}$ for U (θ,θ+1) and b=1 for Exp(θ, 1). Balakrishnan and Cohen (1990) computed the variances of the estimators in this case and in the following cases.

The estimators compared are both unbiased for μ. Therefore, they will be compared through their variances. The efficiency between two estimators $\hat{\mu}_1$ and $\hat{\mu}_2$ is defined as:

$$eff\left(\hat{\mu}_2,\hat{\mu}_1\right)=Var\left(\hat{\mu}_1\right)\left[Var\left(\hat{\mu}_2\right)\right]^{-1}.$$

The larger the efficiency, the better the estimator $\hat{\mu}_2$ will be. The efficiency of $\hat{\mu}_{\text{MEBlue}}$ with respect to the sample mean under SRS was computed for both distributions for $m = 2,\ldots,10$. The results are summarized in Tables 1 and 2. From these tables, it may be concluded that the variance of the BLUE decreases as m increases and $eff\left(\hat{\mu}_{MEBlue},\bar{X}_{2m}\right) \geq 1$ for both distributions. Also, the efficiency is more than 2 for $m \geq 4$ in the uniform case and for $m \geq 9$ in the exponential case.

Efficiency of $\hat{\mu}_{\text{MEBlue}}$ with respect to $\bar{X}_{2m}$

Table 1

U (θ,θ+1)

m	$eff\left(\hat{\mu}_{MEBlue},\bar{X}_{2m}\right)$
2	1.333
3	1.765
4	2.200
5	2.863
6	3.150
7	3.683
8	4.288
9	4.932
10	5.620

Table 2

Exp(θ,1)

m	$eff\left(\hat{\mu}_{MEBlue},\bar{X}_{2m}\right)$
2	1.167
3	1.333
4	1.483
5	1.639
6	1.647
7	1.8397
8	1.996
9	2.087
10	2.177

The BLUE of the mean for distributions with a scale parameter

Let $\{y_{11}, y_{21}, \cdots y_{m1}, y_{12}, y_{22}, \cdots y_{m2}\}$ be a MERSS from a population with a pdf of the form:

$$\frac{1}{\theta}f\left(\frac{x}{\theta}\right),\ \theta > 0 \qquad (3.1)$$

where f is a pdf. Then as shown previously, if

$$y_{i1} = \theta \min\left\{\frac{X^1_{i1}}{\theta}, \frac{X^1_{i2}}{\theta}, \ldots \frac{X^1_{ii}}{\theta}\right\},$$

then

$$E\left(y_{i1}\right) = \theta Min\left\{\frac{X^1_{i1}}{\theta}, \frac{X^1_{i2}}{\theta}, \ldots \frac{X^1_{ii}}{\theta}\right\}$$
$$= C_{i1}\theta$$

where $C_{i1} = E\left(U_i\right)$ and U_i is the first order statistic of a SRS of size i from the pdf in (3.1), under θ = 1. Similarly, E (y_{i2}) = $C_{i2}\theta$, for $C_{i2} = E\left(W_i\right)$ where W_i is the maximum order statistic of a SRS of size i from the pdf in (3.1), under $\theta = 1$. Also, Var (y_{i1}) = $\theta^2 \sigma_{i1}^2$ and Var (y_{i2}) = $\theta^2 \sigma_{i2}^2$ where σ_{i1}^2 and σ_{i2}^2 are the variances of u_i and w_i respectively, for *i = 1, 2,…, m*. (The BLUE of the mean of the population with pdf (3.1) proof is similar to that of Result (1) and therefore is omitted.)

Result 2

Let $y_1, y_2, \dots, y_{2m}$ be 2m independent order statistics each of size less than *m* from an underlying distribution with a pdf as in (3.1). Then the BLUE of the population μ is given by:

$$\hat{\mu}_{Blue} = \frac{\sum_{i=1}^{2m} \frac{c_i}{\sigma_i^2} y_i}{\sum_{i=1}^{2m} \frac{c_i^2}{\sigma_i^2}} \tag{3.2}$$

with variance

$$Var\left(\hat{\mu}_{Blue}\right) = \frac{\theta^2}{\sum_{i=1}^{2m} \frac{c_i^2}{\sigma_i^2}} \tag{3.3}$$

where μ = b θ and b = $E_{\theta=1} X$.

The BLUE of μ using MERSS is given by:

$$\hat{\mu}_{MEBlue} = \frac{\sum_{i=1}^{m} \frac{c_{i1}}{\sigma_{i1}^2} y_{i1} + \sum_{i=m+1}^{2m} \frac{c_{i2}}{\sigma_{i2}^2} y_{i2}}{\sum_{i=1}^{m} \frac{c_{i1}^2}{\sigma_{i1}^2} + \sum_{i=m+1}^{2m} \frac{c_{i2}^2}{\sigma_{i2}^2}} \tag{3.4}$$

and

$$Var\left(\hat{\mu}_{MEBlue}\right) = \frac{\theta^2}{\sum_{i=1}^{2m} \frac{c_i^2}{\sigma_i^2}} = \frac{\theta^2}{\left\{\sum_{i=1}^{m} \frac{c_{i1}^2}{\sigma_{i1}^2} + \sum_{i=m+1}^{2m} \frac{c_{i2}^2}{\sigma_{i2}^2}\right\}} \tag{3.5}$$

Comparing the BLUE estimator based on MERSS with the sample mean based on SRS in case of uniform Exp(θ) and U(0, θ) distributions. The first is skewed to the right with mean θ and the second is symmetric about its mean $\frac{\theta}{2}$. So, the BLUE's are the same as given in (3.2). The estimators are unbiased and therefore are compared using their variances for *m = 2… 10*. The results are summarized in Tables (3) and (4). Similar conclusions to those presented for Tables (1) and (2) can be given.

Efficiency of $\hat{\mu}_{MEBlue}$ with respect to $\overline{X}_{2m}$

Table 3	
	Exp(θ)
m	$eff\left(\hat{\mu}_{MEBlue}, \overline{X}_{2m}\right)$
2	1.200
3	1.380
4	1.540
5	1.690
6	1.820
7	1.950
8	2.070
9	2.190
10	2.300

Table 4	
	U(0, θ)
m	$eff\left(\hat{\mu}_{MEBlue}, \overline{X}_{2m}\right)$
2	1.331
3	1.815
4	2.264
5	2.955
6	3.574
7	4.593
8	5.713
9	6.935
10	8.261

Saving by using MERSS to estimate the population mean

Measuring the units of a sample costs money, time, and effort. The previous tables show that the BLUE for estimating the population mean μ under MERSS is more efficient (less variance) than the sample mean of SRS, which is usually used for estimating μ. Therefore, $\hat{\mu}_{MEBlue}$ will be as good as $\overline{X}_{2m}$ by using a smaller number of observations which will result in saving time, money and effort. Table (5) shows the smallest 2m such that the variance of the BLUE under MERSS using 2m observations is smaller than the variance of the sample mean of SRS using a specified sample size in case of the normal, logistic, uniform, and exponential distributions. The first two distributions are location parameter families of distributions while the other two are scale parameter families.

Table (5), shows how the BLUE, under MERSS for estimating the population mean, requires a smaller number of observations than $\overline{X}_{2m}$ based on SRS. This indicates a reduction in the sample size required for estimating the mean. As *m* increases then the savings will be greater for all the cases studied. According to Table (5), the savings in sample sizes range from 0% to 70%. For example, $\hat{\mu}_{\text{MEBlue}}$ based on 12 observations is better than $\overline{X}_{2m}$ based on 40 observations in the case of U $(\theta, \theta + 1)$ for estimating the mean, resulting in saving 70% of the sample size from using the MERSS compared to SRS.

Conclusion

If ordering the data can be done more easily than quantifying it, then the BLUE under MERSS can be used instead of the mean of SRS for estimating the population mean because the BLUE under MERSS provides better results than the mean of SRS with fewer numbers of observations.

References

Al-Odat, M. T., & Al-Saleh, M. F. (2001). A variation of ranked set sampling. *Journal of Applied Statistical Science*, *10*(2), 137-146.

Al-Saleh, M. F., & AL-Hadramy, S. (2003a). Estimation of the mean of the normal distribution using moving extreme ranked set sampling. *Environmetrics*, *14*(7), 651-664.

Al-Saleh, M. F., and AL-Hadramy, S. (2003b). Estimation of the mean of the exponential distribution using moving extreme ranked set sampling. *Statistical Papers*, *44*, 367-382.

Al-Saleh, M. F., & Al-Kadiri, M. (2000). Double ranked set sampling. *Statistics and Probability Letters*, *48*, 205-212.

Al-Saleh, M. F., & Al-Omari, A. I. (2002). Multistage ranked set sampling. *Journal of Statistical Planning and Inferences*, *102*, 273-286.

Al-Saleh, M. F., & Zheng, G. (2002). Estimation of bivariate characteristics using ranked set sampling. *The Australian and New Zealand Journal of* Statistics, *44*(2), 221- 232.

Arnold, B. C, Balakrishnan, N., & Nagaraja, H. N. (1992). A first course in order statistics. *New York: John* Wiley & Sons. Inc.

Balakrishnan, N., & Cohen, A. (1990). Order statistics and inference, estimation *method. New York: Academic* Press, Inc.

Barabesi, L., & El-Sharaawi, A. (2001). The efficiency of ranked set sampling for parameter estimation. *Statistics and Probability Letters*, *53*, 189-199.

Dell, T. R., & Clutter, J. L. (1972). Ranked set sampling theory with order statistics background. *Biometrics*, *28*, 545-555.

Fei, H., Sinha, B. K., & Wu, Z. (1994). Estimation of parameters in two-parameter Weibull and extreme-value distributions using ranked set sampling. *Journal of Statistical Research*, *28*, 149-161.

Lam, K., Sinha, B. K., & Wu, Z. (1994). Estimation of parameters in two-parameter exponential distribution using ranked set sample. *Annals of the Institute of Statistical Mathematics*, *46*, 723-736.

Li, D., & Chuiv, N. (1997). On the efficiency of ranked set sampling strategies in parametric estimation. *Calcutta Statistical Association Bulletin*, *47*, 185-186.

McIntyre, G. A. (1952). A method for unbiased selective sampling using ranked sets. *Australian Journal of Agricultural Research*, *3*, 385-390.

Mode, N., Conquest, L. & Marker, D. (1999). Ranked set sampling for ecological research: Accounting for the total cost of sampling. *Environmetrics*, *10*, 179-194.

Muttlak, H.A. (1997). Median ranked set sampling. *Jornal of Applied Statistical Sciences*, *6*(4), 245-255.

Patil, G., Sinha, A., & Taillie, C. (1999). Ranked set sampling: Bibliography. *Environmental and Ecological Statistics*, *6*, 91-98.

Samawi, H., Ahmed, M., & Abu-Dayyeh, W. (1996). Estimating the population mean using extreme ranked set sampling. *Biometrics*, *30*,577-586.

Sinha, B. K., Sinha, B. K., & Purkayastha, S. (1996). On some aspects of ranked set sampling for estimation of normal and exponential parameters. *Statistics and Decisions*, *14*, 223-240.

Stokes, S. L. (1980). Estimation of variance using judgment ordered ranked set samples. *Biometrics*, *36*, 35-42.

Stokes, S.L.(1977). Ranked set sampling with concomitant variables. Communications in Statistics- Theory and Methods A6, 1207-1211.

Stokes, S. L. (1976). *An investigation of the consequences of ranked set sampling*. Ph.D. Thesis, Department of Statistics, University of North Carolina, Chapel Hill, NC.

Stokes, S. L., & Sager, T. W. (1988). Characterization of ranked set sample with application to estimating distribution functions. *Journal of the American Statistical Association*, *83*, 374-381.

Takahasi, K. (1970). Practical note on estimation of population means based on samples stratified by means of ordering. *Annals of the Institute of Statistical Mathematics*, *22*, 421-428.

Takahasi, K., & Wakimoto, K. (1968). On unbiased estimates of the population mean based on the sample stratified by means of ordering. *Annals of the Institute of Statistical Mathematics*, 20, 1-31.

Zheng, G., & Al-Saleh, M. F. (2002). Modified Maximum Likelihood Estimator based on ranked set sampling. *Annals of the Institute of Statistical Mathematics*, *54*, 641-658.

Table 5: Efficiency of the Smallest Number of Observations for MERSS Compared to the SRS of Size 2m

SRS	MERSS					
2m	$N(\theta,1)$	$L(\theta,1)$	$Exp(\theta,1)$	$Exp(\theta)$	$U(\theta,\theta+1)$	$U(0,\theta)$
2	2	2	2	2	2	2
4	4	4	4	4	4	4
6	6	6	6	6	6	6
8	6	6	8	6	6	6
10	8	8	8	8	6	6
12	8	10	10	8	8	8
14	10	10	10	10	8	8
16	10	12	10	10	8	8
18	12	12	12	12	10	10
20	12	14	12	12	10	10
22	14	14	14	14	10	10
24	14	16	14	14	10	10
26	14	16	16	14	10	10
28	16	18	16	16	10	10
30	16	20	16	16	12	12
32	16	20	18	16	12	12
34	18	21	18	18	12	12
36	18	21	18	18	12	12
38	19	22	20	18	14	12
40	19	22	20	20	14	12