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Journal of Modern Applied Statistical Methods

Volume 15 | Issue 1 Article 47

5-1-2016

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Recommended Citation

Sundaram, Sampath and Parthasarathy, Deepa S. (2016) "Determination of Optimal Tightened Normal Tightened Plan Using a Genetic Algorithm," *Journal of Modern Applied Statistical Methods*: Vol. 15 : Iss. 1 , Article 47.
DOI: 10.22237/jmasm/1462077960
Available at: http://digitalcommons.wayne.edu/jmasm/vol15/iss1/47

Journal of Modern Applied Statistical Methods
May 2015, Vol. 15, No.1, 893-913.

ISSN 1538 − 9472

JMASM Algorithms and Code

Determination of Optimal Tightened Normal Tightened Plan Using a Genetic Algorithm

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Designing a tightened normal tightened sampling plan requires sample sizes and acceptance number with switching criterion. An evolutionary algorithm, the genetic algorithm, is designed to identify optimal sample sizes and acceptance number of a tightened normal tightened sampling plan for a specified consumer's risk, producer's risk, and switching criterion. Optimal sample sizes and acceptance number are obtained by implementing the genetic algorithm. Tables are reported for various choices of switching criterion, consumer's quality level, and producer's quality level.

Keywords: tightened normal tightened sampling plan, average outgoing quality, switching criterion, genetic algorithm

Introduction

Companies aiming to remain competitive in order to retain a market share in a global economy need to maintain quality standards of highest order. The importance of consumer protection in sectors like the pharmaceutical industry has resulted in the popularity of $c = 0$ attribute sampling plans. It is to be observed that use of any positive acceptance number in a sampling plan results in passing lots which are likely to have defective units in them.

However, in safety and compliance testing, an acceptance number of zero is particularly desirable. In situations involving expensive testing procedures, practitioners often tend to use a single sampling plan with a sample of smaller size and acceptance number zero. But a sampling plan of this kind may result in the rejection of an entire lot based on the presence of even a single non-conforming unit. Apart from this, acceptance probabilities tend to decrease very rapidly for smaller values of p, namely, the fraction nonconforming in the lot.

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This results in an Operating Characteristic (OC) curve with very poor shape. Even though these limitations can be overcome by using a single sampling plan with $c \geq 1$, a double sampling plan, or a multiple sampling plan, these sampling plans require larger sample sizes resulting in prohibitively expensive situations. Hence, to deal with such situations, Calvin (1977) devised a sampling scheme called Tightened Normal Tightened (TNT) sampling scheme.

Soundararajan and Vijayaraghavan (1992) studied TNT schemes with acceptance number $c > 0$ and compared its efficiency over single and double sampling plans. Suresh and Balamurali (1994) developed a Tightened Normal Tightened $TNT(n; 0, 1)$ scheme which has a switching rule between two sampling plans with fixed sample size and two minimum acceptance numbers, namely, $c = 0$ and $c = 1$. Suresh and Ramkumar (1996) studied the selection of single sampling plans indexed through Maximum Allowable Average Outgoing Quality (MAAOQ). Vijayaraghavan and Soundararajan (1996) developed procedures for the selection of $TNT(n; c_1, c_2)$ indexed by (AQL, LQL) and (AQL, AOQL) under the application of a Poisson model. Balamurali (2001) studied the selection of sampling schemes indexed by crossover point for compliance testing. Here, AQL, LQL and AOQL stand for Acceptable Quality Level, Limiting Quality Level and Average Outgoing Quality Level respectively.

Recently, the question of identifying sampling plans based on certain optimality criterion is receiving the attention of researchers. Because most of the times optimality criterion based on quantities like Average Sample Number assume complicated forms identifying optimal sampling plans is not a straightforward job. However, the availability of high speed computers and the evolution of soft computing tools have opened up a new direction in this regard. Sampath and Deepa (2012) developed a genetic algorithm for the determination of optimal sample sizes and acceptance number of double sampling plans under a crisp situation, and Sampath and Deepa (2013) designed a genetic algorithm for the same problem in situations involving both randomness and impreciseness. In this paper, it is proposed to identify optimal sample sizes and acceptance number of a tightened normal tightened plan using a genetic algorithm. Organization of the paper is as follows: A brief description on the tightened normal tightened scheme is given, followed by a description of the various stages involved in the implementation of the genetic algorithm. Finally, computational results are given in the final section.

Tightened Normal Tightened Scheme

The Tightened Normal Tightened (TNT) plan is a sampling plan appropriate for use in compliance sampling as well as in other areas of acceptance sampling. The conditions under which tightened normal tightened scheme can be applied are explained below.

- (i) Production is in a steady state so that results of past, present, and future lots are broadly indicative of a continuing process.
- (ii) Lots are submitted substantially in the order of their production.
- (iii) Inspection is by attributes, with quality defined as p, the fraction nonconforming.

A TNT scheme is specified by tightened sample size n_1 (large), normal sample size n_2 (small), criterion for switching to normal inspection t, and criterion for switching to tightened inspection s. Usually, s is smaller than t. It is carried out starting with tightened inspection.

1. Inspect using tightened inspection, with larger sample size n_1 and acceptance number $c = 0$.
2. Switch to normal inspection when t lots in a row are accepted under tightened inspection.
3. Inspect using normal inspection, with smaller sample size n_2 and acceptance number $c = 0$.
4. Switch to tightened inspection after a rejection if an additional lot is rejected in the next s lots.

The operating procedure for the above scheme, denoted by $TNT(n_1, n_2; 0)$, is based on the switching rule of United States Department of Defense (1963) with $s = 4$ and $t = 5$. One can refer to Dodge (1965), Hald and Thyregod (1965), and Stephens and Larson (1967) for derivation of composite OC function according to United States Department of Defense with the switching parameters $s = 4$ and $t = 5$. Let $P_1(p)$ be the probability of accepting a lot using tightened inspection and $P_2(p)$ be the probability of accepting a lot under normal inspection. The probability of accepting the lot is given by

$$P_a(p) = \frac{\delta P_1(p) + \mu P_2(p)}{\delta + \mu} \tag{1}$$

where

$$\delta(p) = \frac{1 - P_1^5(p)}{\left(1 - P_1(p)\right) \times P_1^5(p)} \tag{2}$$

is the average number of lots inspected on tightened inspection and

$$\mu(p) = \frac{2 - P_2^4(p)}{\left(1 - P_2(p)\right) \times \left(1 - P_2^4(p)\right)} \tag{3}$$

is the average number of lots inspected on normal inspection.

When a Poisson model is implemented,

$$P_1(p) = \sum_{x=0}^{c} \frac{e^{-n_1 p} \left(n_1 p\right)^x}{x!}$$

and

$$P_2(p) = \sum_{x=0}^{c} \frac{e^{-n_2 p} \left(n_2 p\right)^x}{x!}$$

Calvin (1977) devised the OC function of the TNT scheme as

$$P_a(p) = \frac{P_1\left(1 - P_2^s\right)\left(1 - P_1^t\right)\left(1 - P_2\right) + P_2 P_1^t \left(1 - P_1\right)\left(2 - P_2^s\right)}{\left(1 - P_2^s\right)\left(1 - P_1^t\right)\left(1 - P_2\right) + P_1^t \left(1 - P_1\right)\left(2 - P_2^s\right)} \tag{4}$$

The composite OC curve, normal OC curve, and tightened OC curve of the TNT scheme *TNT*(200, 100; 0) for $s = 4$ and $t = 5$ are as described in Figure 1.

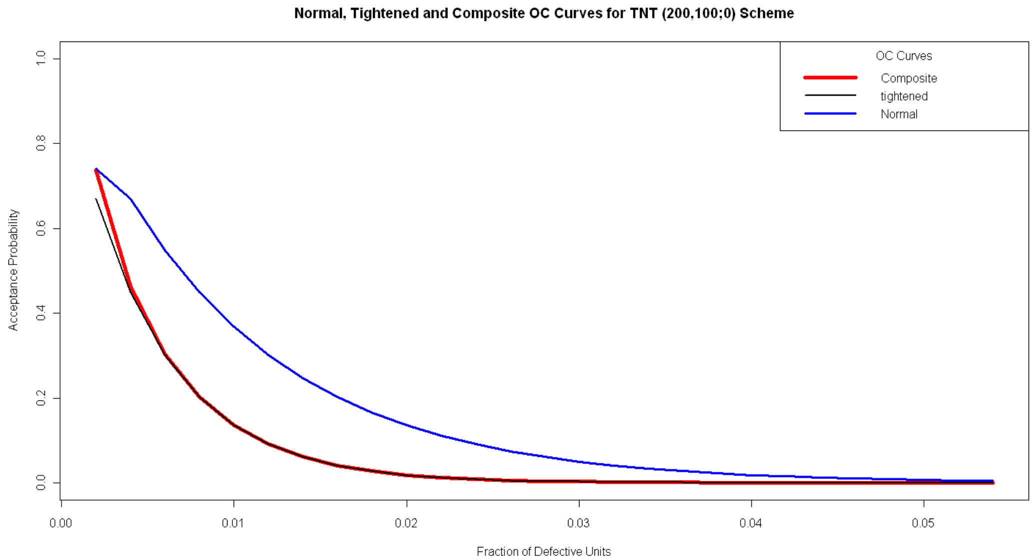

Figure 1. Composite OC curve, normal OC curve, and tightened OC curve of the TNT scheme

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A TNT plan is characterized by three parameters, namely, n_1, n_2, and c, with switching criterion s and t. One can determine the optimal parameters which satisfy the following two conditions for a specified producer's risk α, consumer's risk β, producer's quality level p_0, consumer's quality level p_1, s, and t.

$$\begin{aligned} P_{p_0}(\text{Accept Lot}) &\geq 1-\alpha \\ P_{p_1}(\text{Accept Lot}) &\leq \beta \end{aligned} \tag{5}$$

or, equivalently,

$$\begin{aligned} P_{p_0}(\text{Accept Lot}) &\leq \alpha \\ P_{p_1}(\text{Accept Lot}) &\geq 1-\beta \end{aligned} \tag{6}$$

It may be noted that there exists infinite number of solutions for n_1, n_2, and c satisfying (5) (or (6)). In order to obtain an optimal TNT plan, one has to define a suitable optimality criterion. In acceptance sampling, optimal sampling plans are determined based on measures of performance such as Average Sample Number, Average Outgoing Quality (AOQ), and Average Total Inspection (ATI). In this paper, the problem of developing an optimal tightened normal tightened plan that minimizes the AOQ subject to the condition (5) (or (6)) is considered.

Average Outgoing Quality

In acceptance sampling programs, when the lots are rejected, they require some corrective actions in the form of replacement or elimination through 100 percent inspection. Such programs are known as rectifying inspection programs. AOQ is widely used for the evaluation of rectifying inspection, and represents average value of the lot quality that would be obtained over a long sequence of lots from a process with fraction defective p. AOQ for a TNT plan (Schilling and Neubauer, 2008) is given by

$$AOQ = p_0 P_a\left(p_0\right)\left(\frac{N-\bar{n}}{N}\right) \tag{7}$$

where

$$\bar{n} = \frac{n_1\left(1-P_2^s\right)\left(1-P_1^t\right)\left(1-P_2\right)+n_2 P_1^t\left(1-P_1\right)\left(2-P_2^s\right)}{\left(1-P_2^s\right)\left(1-P_1^t\right)\left(1-P_2\right)+P_1^t\left(1-P_1\right)\left(1-P_2^s\right)}$$

and the lot size N is taken as $(n_1 + n_2)\times 10$ following Naidu et al. (2006).

Note that for a specified producer's risk α, consumer's risk β, producer's quality level p_0, consumer's quality level p_1, s, and t, the expressions for $P_a(p_0)$ and $P_a(p_1)$ are functions of n_1, n_2, and c. Hence solving for these sampling plan parameters such that (5) (or (6)) hold good becomes a complicated process. We therefore intend to make use of an unconventional algorithm like a genetic algorithm. The algorithm looks at a solution for n_1, n_2, and c such that (7) is minimum subject to the condition (5) (or (6)). The various steps associated with a genetic algorithm meant for solving the above problem are given in the following section.

Genetic Algorithm

Genetic algorithms (GAs) are evolutionary algorithms designed using the principle called Survival of the Fittest. These algorithms were first pioneered by Holland (1975). Genetic algorithms find their application in many fields, such as science, engineering, business, and social sciences. Genetic algorithms are a domain independent problem solving approach and are very effective in identifying the optimal solution to a given problem. Details on the mechanism of GAs can be found in Goldberg (1989).

Genetic algorithms randomly search feasible points in a solution space in order to obtain best possible solution. It starts with the definition of what is known as population, which is made up of points representing different regions of the feasible solution space to the maximum extent possible. Each member in the given population is represented in the form of a string called a chromosome, and characters in a string are referred as genes. Defining a chromosome depends on the nature of the given problem. Fitness of a chromosome is determined by evaluating its objective function, namely the function being optimized, which indicates the nature of the solution as well as closeness towards optimality. A genetic algorithm tries to identify the best chromosome by successive breeding of existing chromosomes. Implementation of a genetic algorithm involves five different stages are explained below.

Defining initial population is the first stage of the genetic algorithm. Sets of chromosomes are formed in such a way that each chromosome produces one possible solution for the given optimization problem. Each chromosome defined in the initial population must be distinct in order for the GA to result in better solution. In this study, the initial population consists of 50 randomly generated chromosomes satisfying the probabilistic constraints given in (5) (or (6)). Each chromosome is comprised of nineteen genes. The first eight genes represent the binary encoding of the sample size n_1, the second set of eight genes, i.e. from the ninth to the sixteenth bit position, represents the binary encoding of sample size n_2, and the last three genes, the seventeenth, eighteenth, and nineteenth bit positions, gives the binary encoding of the parameter c. For example, if $n_1 = 130$, $n_2 = 100$, and $c = 2$, then the individual formation of the chromosome is as follows:

$$\overbrace{1\ 0\ 0\ 0\ 0\ 0\ 1\ 0}^{n_1}\overbrace{0\ 1\ 1\ 0\ 0\ 0\ 0\ 0}^{n_2}\overbrace{0\ 1\ 0}^{c}$$

Fitness value evaluation is the second stage of the genetic algorithm. For each chromosome existing in the initial population, the objective function corresponding to the given optimization problem is evaluated. These values are treated as fitness values. In this study, fitness values are computed by making use of the expression given in (7). Chromosomes having minimum AOQ value are treated as fitter.

Selection is the third stage of genetic algorithm. In this stage, chromosomes having high fitness value are selected to enter the mating pool with higher probabilities and a chromosome with lower fitness value is given a lower probability for entering the mating pool. Some of the selection procedures available in the literature are Roulette Wheel selection, Tournament selection, Ranking selection, and Proportional selection. In this paper, the Roulette Wheel selection procedure is used. For details related to selection procedures, one can refer to any standard text book on soft computing, such as Sivanandam and Deepa (2008).

Crossover is the fourth stage of genetic algorithm. In this stage, pair of chromosomes exist in the mating pool are combined to generate new chromosomes, called offspring. Many crossover mechanisms are available in the literature. In this work, a single point crossover mechanism is applied. In single point crossover, a crossover point is selected randomly in the interval [1, l-1] where l is the length of a chromosome. The portions of the chromosome lying to the right of the crossover point are exchanged to produce offspring. For example, if

$$C_1 : 1\quad 0\quad 0\quad 0\quad 1\quad 1\quad 1\quad 0\quad 0\quad 1\quad 1\quad 0\quad 0\quad 1\quad 0\quad 0\quad 0\quad 1\quad 0$$

and

$$C_2 : 0\quad 1\quad 1\quad 0\quad 0\quad 0\quad 1\quad 0\quad 0\quad 1\quad 1\quad 0\quad 0\quad 0\quad 1\quad 1\quad 0\quad 0\quad 1$$

are two chromosomes with l = 14, the resulting offspring are

$$Ch_1 : 1\quad 0\quad 0\quad 0\quad 1\quad 1\quad 1\quad 0\quad 0\quad 1\quad 1\quad 0\quad 0\quad 1\quad 1\quad 1\quad 0\quad 0\quad 1$$

and

$$Ch_2 : 0\quad 1\quad 1\quad 0\quad 0\quad 0\quad 1\quad 0\quad 0\quad 1\quad 1\quad 0\quad 0\quad 0\quad 0\quad 0\quad 0\quad 1\quad 0,$$

respectively.

Mutation is the last stage of the algorithm. Each gene of a chromosome available in the given generation is randomly chosen and a small change is made with the help of mutation operator. In this work, each chromosome undergoes the following changes: A bit position is chosen randomly from the first eight bits (which is an encoding of sample size n_1) and its value is flipped. A second bit position is selected randomly from the ninth to sixteenth bit positions (which is an encoding of sample size n_2) and its value is flipped. Similarly, flipping is done based on the last three bit positions (which is an encoding of acceptance number c). After mutation is over, mutated chromosomes form the new generation of chromosomes.

The five stages of the genetic algorithm described above are repeatedly performed. In this study, the number of times the above algorithm is to be repeated is taken as 50.

Determination of Optimal $TNT(n_1, n_2; c)$ Plan

The optimal TNT sampling plans for a wide range of p_0, p_1, s, and t with producer's risk $\alpha = 0.05$ and consumer's risk $\beta = 0.10$ are determined by implementing the genetic algorithm discussed earlier. The optimal sampling plans are displayed in Tables 1 to 4 in the appendix. The calculations are carried out using macros developed in Microsoft Excel VBA. The Microsoft Excel VBA codes developed in the determination of optimal TNT sampling plan are available from the authors.

Conclusion

A genetic algorithm has been designed and implemented for the determination of optimal $TNT(n_1, n_2; c)$ scheme. Various stages involved in a genetic algorithm are discussed in detail. Tables giving optimal sampling plans are constructed for various choices of s and t. The values are obtained using macros developed in Microsoft Excel VBA. It is observed that, for a specified $\alpha = 0.05$ and $\beta = 0.10$, acceptance number c increases when the producer's quality level p_0 increases. Also, the sample sizes n_1 and n_2 increase with increasing producer's quality level p_0. It is to be noted that an increase in consumer's quality level p_1 decreases the sample sizes n_1 and n_2. Also, the switching criterion s and t have no significant effect in minimum AOQ. That is, various choices of s and t considered in this study have almost the same effect in determining the optimal sampling plans.

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Appendix

Table 1. Optimal TNT plans for s = 1, t = 2 and 3, α =0.05, and β = 0.10.

p_0 = 0.001	s = 1, t = 2				s = 1, t = 3			
p_1	n_1	n_2	c	AOQ	n_1	n_2	c	AOQ
0.020	125	50	0	0.00092347	121	50	0	0.000922529
0.025	96	51	0	0.00091696	94	49	0	0.000919000
0.030	81	50	0	0.00091467	78	50	0	0.000913708
0.035	75	51	0	0.00091161	68	50	0	0.000910685
0.040	70	51	0	0.00091006	65	50	0	0.000909672
0.045	57	50	0	0.00090535	59	50	0	0.000907473
0.050	56	51	0	0.00090494	56	50	0	0.000906278

p_0 = 0.002	s = 1, t = 2				s = 1, t = 3			
p_1	n_1	n_2	c	AOQ	n_1	n_2	c	AOQ
0.020	-	-	-	-	195	177	1	0.001810030
0.025	181	177	1	0.00180668	178	177	1	0.001805900
0.030	181	177	1	0.00180668	181	175	1	0.001809062
0.035	179	176	1	0.00180737	177	175	1	0.001808043
0.040	179	176	1	0.00180737	180	175	1	0.001808809
0.045	179	175	1	0.00180857	179	173	1	0.001810952
0.050	179	173	1	0.00181097	172	169	1	0.001813895

p_0 = 0.003	s = 1, t = 2				s = 1, t = 3			
p_1	n_1	n_2	c	AOQ	n_1	n_2	c	AOQ
0.025	162	118	1	0.00273007	158	118	1	0.002727998
0.030	133	117	1	0.00271945	130	118	1	0.002715045
0.035	123	117	1	0.00271405	120	118	1	0.002709611
0.040	118	117	1	0.00271118	120	118	1	0.002709611
0.045	120	117	1	0.00271234	117	114	1	0.002718657
0.050	117	115	1	0.00271599	114	113	1	0.002719548

Table 1, continued.

$p_0 = 0.004$	s = 1, t = 2				s = 1, t = 3			
p_1	n_1	n_2	c	AOQ	n_1	n_2	c	AOQ
0.025	161	88	1	0.00366635	158	88	1	0.003663347
0.030	200	199	2	0.00362244	198	195	2	0.003631958
0.035	199	194	2	0.00363504	198	195	2	0.003631958
0.040	199	197	2	0.00362723	197	194	2	0.003634081
0.045	89	86	1	0.00362383	94	88	1	0.003619167
0.050	89	88	1	0.00362142	89	86	1	0.003623784

$p_0 = 0.005$	s = 1, t = 2				s = 1, t = 3			
p_1	n_1	n_2	c	AOQ	n_1	n_2	c	AOQ
0.025	-	-	-	-	158	69	1	0.004609512
0.030	179	163	2	0.00452508	178	163	2	0.004524230
0.035	171	163	2	0.00451987	166	163	2	0.004516433
0.040	168	162	2	0.00452204	164	163	2	0.004515071
0.045	162	160	2	0.00452618	163	161	2	0.004522712
0.050	163	160	2	0.00452689	162	161	2	0.004522016

$p_0 = 0.006$	s = 1, t = 2				s = 1, t = 3			
p_1	n_1	n_2	c	AOQ	n_1	n_2	c	AOQ
0.030	180	135	2	0.00546035	180	135	2	0.005459290
0.035	162	136	2	0.00544026	153	135	2	0.005438235
0.040	141	136	2	0.00542138	141	135	2	0.005427310
0.045	137	135	2	0.00542346	135	134	2	0.005427414
0.050	139	136	2	0.00541942	135	134	2	0.005427414

$p_0 = 0.007$	s = 1, t = 2				s = 1, t = 3			
p_1	n_1	n_2	c	AOQ	n_1	n_2	c	AOQ
0.030	182	116	2	0.00639254	180	116	2	0.006388467
0.035	198	195	3	0.00632088	199	195	3	0.006321612
0.040	197	195	3	0.00632008	196	194	3	0.006325072
0.045	122	116	2	0.00633116	195	192	3	0.006335817
0.050	118	116	2	0.00632581	196	192	3	0.006336621

Table 1, continued.

p_0 = 0.008	s = 1, t = 2				s = 1, t = 3			
p_1	n_1	n_2	c	AOQ	n_1	n_2	c	AOQ
0.030	-	-	-	-	179	101	2	0.007325087
0.035	193	170	3	0.00724817	191	170	3	0.007245746
0.040	174	170	3	0.00722943	175	170	3	0.007230354
0.045	171	169	3	0.00723385	170	169	3	0.007232756
0.050	170	169	3	0.00723278	173	169	3	0.007235887

p_0 = 0.009	s = 1, t = 2				s = 1, t = 3			
p_1	n_1	n_2	c	AOQ	n_1	n_2	c	AOQ
0.030	-	-	-	-	179	89	2	0.008268089
0.035	195	151	3	0.00818064	195	151	3	0.008178886
0.040	170	151	3	0.00815343	168	150	3	0.008160119
0.045	159	151	3	0.00813984	157	150	3	0.008146659
0.050	151	150	3	0.00813885	155	150	3	0.008144086

Table 2. Optimal TNT plans for s = 1, t = 4 and 5, α =0.05, and β = 0.10.

p_0 = 0.001	s = 1, t = 2				s = 1, t = 3			
p_1	n_1	n_2	c	AOQ	n_1	n_2	c	AOQ
0.020	118	50	0	0.000921715	116	50	0	0.000921029
0.025	110	50	0	0.000920442	93	50	0	0.000916969
0.030	82	49	0	0.000915993	87	50	0	0.000915651
0.035	67	49	0	0.000911642	80	50	0	0.000913943
0.040	58	49	0	0.000908415	70	50	0	0.000911125
0.045	52	47	0	0.000908707	54	48	0	0.000908146
0.050	49	45	0	0.000910168	49	47	0	0.000907333

Table 2, continued.

p_0 = 0.002	$s = 1, t = 2$				$s = 1, t = 3$			
p_1	n_1	n_2	c	AOQ	n_1	n_2	c	AOQ
0.020	196	177	1	0.001810177	195	177	1	0.001809868
0.025	176	175	1	0.001807781	185	177	1	0.001807581
0.030	176	175	1	0.001807781	184	177	1	0.001807344
0.035	176	175	1	0.001807781	179	177	1	0.001806137
0.040	175	173	1	0.001809911	175	174	1	0.001808714
0.045	175	172	1	0.001811102	175	173	1	0.001809903
0.050	173	172	1	0.001810583	177	172	1	0.001811595

p_0 = 0.003	$s = 1, t = 2$				$s = 1, t = 3$			
p_1	n_1	n_2	c	AOQ	n_1	n_2	c	AOQ
0.025	158	117	1	0.002730294	158	117	1	0.002729820
0.030	134	117	1	0.002719641	133	117	1	0.002718982
0.035	119	117	1	0.002711724	126	117	1	0.002715464
0.040	116	114	1	0.002718050	116	115	1	0.002715363
0.045	115	112	1	0.002722804	115	114	1	0.002717454
0.050	113	111	1	0.002724269	114	113	1	0.002719532

p_0 = 0.004	$s = 1, t = 2$				$s = 1, t = 3$			
p_1	n_1	n_2	c	AOQ	n_1	n_2	c	AOQ
0.025	162	87	1	0.003668336	157	87	1	0.003664266
0.030	200	198	2	0.003625040	200	198	2	0.003625024
0.035	200	198	2	0.003625040	200	198	2	0.003625024
0.040	198	195	2	0.003631937	198	195	2	0.003631915
0.045	198	192	2	0.003639654	198	195	2	0.003631915
0.050	199	197	2	0.003627200	196	194	2	0.003633585

p_0 = 0.005	$s = 1, t = 2$				$s = 1, t = 3$			
p_1	n_1	n_2	c	AOQ	n_1	n_2	c	AOQ
0.025	156	69	1	0.004604499	156	68	1	0.004607669
0.030	182	163	2	0.004526400	180	163	2	0.004524937
0.035	166	163	2	0.004516391	168	162	2	0.004521788
0.040	164	163	2	0.004515057	163	160	2	0.004526771
0.045	161	160	2	0.004525437	161	159	2	0.004529517
0.050	161	160	2	0.004525437	167	159	2	0.004533512

Table 2, continued.

$p_0 = 0.006$	s = 1, t = 2				s = 1, t = 3			
p_1	n_1	n_2	c	AOQ	n_1	n_2	c	AOQ
0.025	156	56	1	0.005554394	156	56	1	0.005543875
0.030	183	135	2	0.005460090	178	135	2	0.005455578
0.035	158	135	2	0.005441970	153	135	2	0.005437430
0.040	136	135	2	0.005422414	135	134	2	0.005427375
0.045	136	135	2	0.005422414	134	133	2	0.005432312
0.050	136	135	2	0.005422414	136	130	2	0.005451835

$p_0 = 0.007$	s = 1, t = 2				s = 1, t = 3			
p1	n_1	n_2	c	AOQ	n_1	n_2	c	AOQ
0.030	181	115	2	0.006394767	185	115	2	0.006393767
0.035	198	194	3	0.006326581	198	195	3	0.006320708
0.040	196	194	3	0.006325035	196	194	3	0.006324995
0.045	194	193	3	0.006329237	196	193	3	0.006330752
0.050	193	192	3	0.006334178	196	193	3	0.006330752

$p_0 = 0.008$	s = 1, t = 2				s = 1, t = 3			
p_1	n_1	n_2	c	AOQ	n_1	n_2	c	AOQ
0.030	183	100	2	0.007333215	180	100	2	0.007324785
0.035	196	170	3	0.007249456	193	170	3	0.007246189
0.040	174	170	3	0.007229235	182	170	3	0.007236643
0.045	172	170	3	0.007227226	177	170	3	0.007232004
0.050	173	169	3	0.007235791	171	169	3	0.007233710

$p_0 = 0.009$	s = 1, t = 2				s = 1, t = 3			
p_1	n_1	n_2	c	AOQ	n_1	n_2	c	AOQ
0.030	178	88	2	0.008272107	179	87	2	0.008275323
0.035	193	151	3	0.008175165	192	150	3	0.008182135
0.040	170	151	3	0.008152119	168	150	3	0.008158866
0.045	162	151	3	0.008142936	150	149	3	0.008146914
0.050	160	151	3	0.008140541	153	149	3	0.008150742

Table 3. Optimal TNT plans for s = 2, t = 3 and 4, α =0.05, and β = 0.10.

p_0 = 0.001	s = 1, t = 2				s = 1, t = 3			
p_1	n_1	n_2	c	AOQ	n_1	n_2	c	AOQ
0.020	117	50	0	0.000921194	117	49	0	0.000921922
0.025	100	50	0	0.000918348	98	49	0	0.000918885
0.030	78	47	0	0.000917536	82	49	0	0.000915505
0.035	71	47	0	0.000915554	66	48	0	0.000912430
0.040	59	47	0	0.000911510	62	48	0	0.000911094
0.045	56	47	0	0.000910345	55	48	0	0.000908488
0.050	54	45	0	0.000912349	48	46	0	0.000908244

p_0 = 0.002	s = 1, t = 2				s = 1, t = 3			
p_1	n_1	n_2	c	AOQ	n_1	n_2	c	AOQ
0.020	195	177	1	0.001809850	195	177	1	0.001809705
0.025	181	177	1	0.001806620	178	176	1	0.001807066
0.030	177	175	1	0.001808025	177	175	1	0.001808010
0.035	176	174	1	0.001808964	177	174	1	0.001809192
0.040	177	174	1	0.001809214	177	173	1	0.001810373
0.045	177	174	1	0.001809214	174	172	1	0.001810819
0.050	177	174	1	0.001809214	177	172	1	0.001811595

p_0 = 0.003	s = 1, t = 2				s = 1, t = 3			
p_1	n_1	n_2	c	AOQ	n_1	n_2	c	AOQ
0.025	163	117	1	0.002731659	159	117	1	0.002729329
0.030	131	117	1	0.002717968	134	117	1	0.002719125
0.035	126	117	1	0.002715442	122	116	1	0.002715917
0.040	118	116	1	0.002713820	119	116	1	0.002714328
0.045	116	114	1	0.002718027	116	115	1	0.002715345
0.050	115	114	1	0.002717451	115	114	1	0.002717435

p_0 = 0.004	s = 1, t = 2				s = 1, t = 3			
p_1	n_1	n_2	c	AOQ	n_1	n_2	c	AOQ
0.025	156	87	1	0.003664103	157	86	1	0.003666564
0.030	200	198	2	0.003625019	199	198	2	0.003624566
0.035	198	197	2	0.003626737	198	195	2	0.003631868
0.040	198	197	2	0.003626737	197	196	2	0.003628867
0.045	199	197	2	0.003627179	197	195	2	0.003631434
0.050	198	196	2	0.003629326	198	196	2	0.003629299

Table 3, continued.

$p_0 = 0.005$	s = 1, t = 2				s = 1, t = 3			
p_1	n_1	n_2	c	AOQ	n_1	n_2	c	AOQ
0.025	158	69	1	0.004602243	156	68	1	0.004602324
0.030	178	163	2	0.004523715	186	163	2	0.004527613
0.035	169	163	2	0.004518243	178	163	2	0.004523300
0.040	167	163	2	0.004516975	178	163	2	0.004523300
0.045	162	161	2	0.004521985	172	162	2	0.004524010
0.050	160	159	2	0.004528829	168	162	2	0.004521611

$p_0 = 0.006$	s = 1, t = 2				s = 1, t = 3			
p_1	n_1	n_2	c	AOQ	n_1	n_2	c	AOQ
0.025	-	-	-	-	160	55	1	0.005547309
0.030	183	135	2	0.005458675	178	135	2	0.005453510
0.035	164	135	2	0.005445872	154	135	2	0.005437400
0.040	137	135	2	0.005423333	139	135	2	0.005425051
0.045	136	134	2	0.005428327	138	134	2	0.005430076
0.050	138	133	2	0.005436135	135	134	2	0.005427335

$p_0 = 0.007$	s = 1, t = 2				s = 1, t = 3			
p_1	n_1	n_2	c	AOQ	n_1	n_2	c	AOQ
0.030	178	115	2	0.006390131	179	114	2	0.006394389
0.035	199	195	3	0.006321429	198	195	3	0.006320584
0.040	196	194	3	0.006324985	196	195	3	0.006319167
0.045	194	193	3	0.006329213	195	193	3	0.006329910
0.050	194	193	3	0.006329213	197	192	3	0.006337049

$p_0 = 0.008$	s = 1, t = 2				s = 1, t = 3			
p_1	n_1	n_2	c	AOQ	n_1	n_2	c	AOQ
0.030	179	100	2	0.007326484	178	99	2	0.007327863
0.035	191	170	3	0.007244411	191	170	3	0.007243322
0.040	175	170	3	0.007230061	171	170	3	0.007226132
0.045	174	170	3	0.007229101	171	170	3	0.007226132
0.050	171	169	3	0.007233695	170	169	3	0.007232657

Table 3, continued.

p_0 = 0.009	s = 1, t = 2				s = 1, t = 3			
p_1	n_1	n_2	c	AOQ	n_1	n_2	c	AOQ
0.030	183	88	2	0.008268077	180	87	2	0.008264219
0.035	193	151	3	0.008173139	195	149	3	0.008190940
0.040	180	151	3	0.008161396	171	149	3	0.008170188
0.045	157	150	3	0.008146156	150	149	3	0.008146852
0.050	153	149	3	0.008150708	150	149	3	0.008146852

Table 4. Optimal TNT plans for $s = 2$, $t = 4$ and 5, α =0.05, and β = 0.10.

p_0 = 0.001	s = 1, t = 2				s = 1, t = 3			
p_1	n_1	n_2	c	AOQ	n_1	n_2	c	AOQ
0.020	118	49	0	0.000921353	125	49	0	0.000922006
0.025	113	49	0	0.000920707	100	49	0	0.000918607
0.030	77	48	0	0.000915360	81	48	0	0.000916232
0.035	66	47	0	0.000913652	70	48	0	0.000913391
0.040	59	47	0	0.000911304	68	48	0	0.000912808
0.045	54	47	0	0.000909407	54	47	0	0.000909380
0.050	50	47	0	0.000907736	52	47	0	0.000908570

p_0 = 0.002	s = 1, t = 2				s = 1, t = 3			
p_1	n_1	n_2	c	AOQ	n_1	n_2	c	AOQ
0.020	195	177	1	0.001809553	195	177	1	0.001809493
0.025	182	177	1	0.001806781	187	177	1	0.001807844
0.030	179	176	1	0.001807280	186	177	1	0.001807631
0.035	178	176	1	0.001807051	180	176	1	0.001807495
0.040	177	174	1	0.001809169	177	175	1	0.001807988
0.045	175	173	1	0.001809871	176	173	1	0.001810098
0.050	174	173	1	0.001809633	173	172	1	0.001810560

Table 4, continued.

$p_0 = 0.003$	s = 1, t = 2				s = 1, t = 3			
p_1	n_1	n_2	c	AOQ	n_1	n_2	c	AOQ
0.025	157	117	1	0.002727763	157	117	1	0.002727495
0.030	133	117	1	0.002718362	140	117	1	0.002721202
0.035	120	117	1	0.002712145	123	117	1	0.002713613
0.040	116	115	1	0.002715328	118	117	1	0.002711102
0.045	116	115	1	0.002715328	116	115	1	0.002715321
0.050	115	114	1	0.002717420	115	114	1	0.002717412

$p_0 = 0.004$	s = 1, t = 2				s = 1, t = 3			
p_1	n_1	n_2	c	AOQ	n_1	n_2	c	AOQ
0.025	157	86	1	0.003663245	157	86	1	0.003662696
0.030	200	198	2	0.003624961	198	197	2	0.003626702
0.035	198	197	2	0.003626709	198	197	2	0.003626702
0.040	198	196	2	0.003629271	195	194	2	0.003633095
0.045	200	196	2	0.003630100	194	192	2	0.003637721
0.050	197	195	2	0.003631407	198	196	2	0.003629259

$p_0 = 0.005$	s = 1, t = 2				s = 1, t = 3			
p_1	n_1	n_2	c	AOQ	n_1	n_2	c	AOQ
0.025	157	66	1	0.004610771	156	67	1	0.004602109
0.030	183	162	2	0.004529595	178	163	2	0.004522695
0.035	163	162	2	0.004518474	165	163	2	0.004515560
0.040	162	161	2	0.004521936	165	163	2	0.004515560
0.045	162	161	2	0.004221936	164	163	2	0.004514975
0.050	162	159	2	0.004530044	163	160	2	0.004526583

$p_0 = 0.006$	s = 1, t = 2				s = 1, t = 3			
p_1	n_1	n_2	c	AOQ	n_1	n_2	c	AOQ
0.025	166	53	1	0.005550880	157	54	1	0.005544301
0.030	183	134	2	0.005459981	185	134	2	0.005460246
0.035	153	134	2	0.005441853	153	134	2	0.005441560
0.040	136	134	2	0.005428185	143	134	2	0.005433987
0.045	135	134	2	0.005427299	135	134	2	0.005427283
0.050	132	131	2	0.005441986	135	134	2	0.005427283

Table 4, continued.

$p_0 = 0.007$	s = 1, t = 2				s = 1, t = 3			
p_1	n_1	n_2	c	AOQ	n_1	n_2	c	AOQ
0.030	179	114	2	0.006388881	180	114	2	0.006388371
0.035	196	195	3	0.006319130	200	195	3	0.006321711
0.040	196	195	3	0.006319130	199	195	3	0.006321071
0.045	195	194	3	0.006324159	196	195	3	0.006319114
0.050	195	194	3	0.006324159	196	194	3	0.006324813

$p_0 = 0.008$	s = 1, t = 2				s = 1, t = 3			
p_1	n_1	n_2	c	AOQ	n_1	n_2	c	AOQ
0.030	179	98	2	0.007328779	178	98	2	0.007328104
0.035	193	170	3	0.007243578	193	170	3	0.007243124
0.040	177	170	3	0.007231282	175	169	3	0.007236929
0.045	174	170	3	0.007228724	173	169	3	0.007235221
0.050	172	170	3	0.007226973	173	169	3	0.007235221

$p_0 = 0.009$	s = 1, t = 2				s = 1, t = 3			
p_1	n_1	n_2	c	AOQ	n_1	n_2	c	AOQ
0.030	186	85	2	0.008271138	188	85	2	0.008272709
0.035	195	150	3	0.008177279	197	150	3	0.008177442
0.040	172	150	3	0.008160109	171	150	3	0.008158701
0.045	152	150	3	0.008139789	156	149	3	0.008153466
0.050	150	149	3	0.008146798	155	148	3	0.008161672