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# Integrated optimization of capacitated train rescheduling and passenger reassignment under disruptions

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ABSTRACT

During railway operations, unexpected events may influence normal traffic flows. This paper focuses on a train rescheduling problem for handling large disruptions, such as a rolling stock breakdown leading to a cancelled train service, where passenger reassignment strategies have to be considered. A novel mixed-integer linear programming formulation is established with consideration of train retiming, reordering, rerouting, and reservicing (addition of extra stops). The proposed mathematical formulation considers planning extra stops for non-canceled trains in order to transport the disrupted passengers, which were supposed to travel on the canceled train, to their pre-planned destination stations. Other constraints deal with limited seat capacity and track capacity, and mapping train rescheduling with passenger reassignment. A bi-objective function is optimized by a weighted-sum method to maximize the number of disrupted passengers reaching their destination stations and to minimize the weighted total train delay for all non-canceled trains at their destinations. A series of numerical experiments based on a part of the Beijing-Shanghai high-speed railway line is carried out to verify the effectiveness and efficiency of the proposed model and to perform a sensitivity analysis of various performance factors. The results show that an optimal reassignment plan of disrupted passengers is important to achieve real-time efficiency of traffic and re-ticketing. The impact of passenger reassignment on train rescheduling is influenced by the weights for objectives, duration of disruption, allowed additional dwell and running times, and relationship between passenger demand and total available train capacity.

## 1. Introduction

Railway transportation systems provide an efficient and sustainable service for passengers and are strongly competitive with other transportation modes. However, trains do not always arrive or depart on time and are sometimes even canceled. In daily railway operations, some unpredictable external and internal disruptions of the railway system, such as severe weather conditions and rolling stock breakdowns, may lead to a reduction in the capacity of tracks and stations. In such cases, train dispatchers need to make

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appropriate train rescheduling decisions such as retiming, reordering, and rerouting within a short computation time to compensate for the impacted railway operations. During severe disruptions, the stopping pattern of some trains may also be changed to offer more options to the disrupted passengers to travel to their destinations. Other dispatching measures include the cancelation or insertion of train services. Considering this scenario, in a high-density, limited-capacity railway network, real-time train rescheduling becomes extremely complicated and may strongly affect the quality of passenger services and the performance of the overall rail system (Wang et al., 2018).

Under severe disruptions, train rescheduling becomes even more challenging when the railway system has a seat reservation mechanism. Under this mechanism, passengers should buy tickets in advance with a specified departure time from the origin station, arrival time at the destination station, and seat number. If a train service is canceled because of train failure, all the passengers who are supposed to travel on it will be affected. To reach their destination stations, they would need to be accommodated on other trains with limited remaining seats. Therefore, in the case of serious disruptions, passenger reassignment strategies have to be developed together with train rescheduling strategies. Both types of strategies have the general objective of ensuring that the maximum number of passengers arrive at their destinations by trains with the least delay. Under the seat reservation mechanism, the combination of effective train rescheduling and effective passenger reassignment is critical for railway operators in a disrupted high-density network with limited capacity. Additional train stops may be required to serve as many disrupted passengers as possible, although this has the drawback of causing potential delays to the following trains.

This paper deals with the problem of the integration of train rescheduling and passenger reassignment in a railway system with a seat reservation mechanism under severe rolling stock disruptions, i.e., train cancelation because of a breakdown. A mixed-integer programming formulation is proposed for this problem with the following objective function components: the maximization of the number of disrupted passengers reaching their destination stations and the minimization of the weighted total delay of all non-canceled trains. The two objectives are conflicting because if more disrupted passengers are to reach their destinations, extra train stops will be required, which will increase the travel time of trains. Train delay and passenger satisfaction are both important indices for evaluating railway operations. For a busy line, the seat capacity is always limited. If the disrupted passengers cannot reach their destinations, the railway operating company will need to refund the ticket fare to the passengers.

To understand the tradeoff between these two objectives, Pareto optimal solutions are obtained using a weighted-sum method. A series of numerical experiments are performed using a part of the Beijing–Shanghai high-speed railway line as the test bed. This line is one of the most complex and densely occupied lines in China. The efficiency and effectiveness of our method are investigated. In addition, different impact factors such as the weights for objectives, duration of disruption, allowed additional running and dwell times, passenger demand, and total available train capacity are tested to determine their effect on the problem of the integration of capacitated train rescheduling and passenger reassignment.

## 2. Literature review and paper contribution

### 2.1. Literature review

Real-time traffic management has been extensively studied in the past few decades. Advances in scheduling theory make it possible to solve the real-time train scheduling problem, in which train departure/arrival times, train orders, and routes will be determined (D'Ariano et al. (2007), Meng and Zhou (2014), Corman et al. (2010)). Recent surveys by Jespersen-Groth et al. (2009), Hansen (2010), Cacchiani et al. (2014), Fang et al. (2015), and Corman & Meng (2015) summarize the methods and solution techniques for train rescheduling, delay management and disruption management.

A recent research stream is focused on train rescheduling during disruptions from the operation point of view, and with an attention on the passenger flows affected by them. Sato et al. (2013) introduce a timetable rescheduling algorithm when train traffic is disrupted. A MIP formulation is proposed with the objective of minimizing further inconvenience to passengers, which consists of the time on board, the waiting time at platforms and the number of transfers. Louwerse and Huisman (2014) focus on adjusting the timetable of a passenger railway operator in case of partial or complete track blockage. The main objective is to maximize the service level offered to passengers by minimizing the number of cancelled trains and the delays of operated trains, and by distributing the operated trains evenly over time. Niu et al. (2015) formulate a quadratic integer program to minimize the total passenger waiting time at stations by adjusting the train timetable with given time-varying origin-to-destination passenger demand matrices and given skip-stop patterns. Zhan et al. (2015) pay attention to the real-time rescheduling of railway traffic in case of a complete blockage of the railway infrastructure, resulting in trains waiting inside the stations until the disruption is over. A mixed integer programming model is formulated to minimize the total weighted train delay and the number of cancelled trains. Veelenturf et al. (2016) propose a railway timetable rescheduling approach for handling large-scale disruptions, which focuses on timetable rescheduling for passenger train services on a macroscopic (aggregated) modelling level. An integer linear programming formulation is introduced in order to minimize the number of cancelled and delayed train services, by considering infrastructure and rolling stock capacity constraints. Binder et al. (2017) report on the railway timetable rescheduling problem from a macroscopic level of infrastructure representation in case of large disruptions. An integer linear programming formulation is proposed to minimize the passenger dissatisfaction, the operational costs and the deviation from the disrupted timetable. Measures such as cancelling services, delaying or rerouting trains, as well as inserting emergency trains in the schedule are taken into consideration. However, in the above-cited studies, the passenger inconvenience is mostly only considered in the objective and measured by the travel time, or used as an indicator for the effectiveness of train operation, like setting the objective to minimize train delays and the number of cancelled trains. Even when passenger concerns are considered, passengers are often regarded as a group with the same origin–destination demand, and every group is indivisible and takes the same route.

Another field related to disruption management is delay management from the passenger point of view. In Schöbel (2001), a first integer programming formulation for the delay management problem is given. Delay management determines which connections should be maintained in case of a delayed feeder train. Schachtebeck and Schöbel (2010) deal with the delay management problem with consideration of the limited capacity of the track system by priority decisions to determine train orders. An integer programming formulation is proposed that includes headway constraints and several computational tests are given. Dollevoet and Huisman (2014) develop fast heuristics for delay management with passenger rerouting when they miss a connection in case of a delayed feeder train. An integer programming formulation is proposed based on an event-activity network with the objective of minimizing the total passenger delay. Overall, the papers mentioned above formulate the optimization model without considering the capacity of infrastructure or trains.

Some researchers pay attention to train rescheduling and delay management at the same time. Corman et al. (2012) propose a detailed alternative graph model and two heuristic algorithms to solve a bi-objective traffic management problem with consideration of passenger dissatisfaction measured by the number of cancelled connected services. Dollevoet et al. (2014) propose an iterative optimization approach that iteratively solves a macroscopic delay management model and a microscopic train rescheduling model, in order to determine a feasible train schedule inside stations with minimum train delay, and at the same time, minimum travel time for passengers at the network level. Corman et al. (2017) integrate train rescheduling and delay management from the passenger point of view into a single MILP, with the objective to minimize passenger travel time. Passenger flows are modelled based on Origin-Destination description, with the possibility of transfer connections and rerouting. Some fast heuristic algorithms are proposed, based on the iterated resolution of train rescheduling and delay management models. Veelenturf et al. (2017) propose an iterative framework to integrate the rescheduling of the rolling stock and the timetable by taking the changed passenger demand into account. The timetable decisions are limited to make additional stops. However, in the research mentioned above, limited train rescheduling measures are taken into account. In addition, an iterative approach has been applied to solve train rescheduling and passenger rerouting problems.

Table 1 summarizes the relevant studies on the real-time traffic management problem that considers passenger needs. The table provides information in terms of the problem description (i.e., rescheduling measure, passenger flow, infrastructure capacity, and train capacity), mathematical formulation (including the model structure and objective), and solution method.

The table indicates that when more details about train rescheduling are considered, such as detailed infrastructure layouts and different rescheduling measures, the passenger aspects are not included or are included into the objective setting only to consider the negative effects on passengers, such as the minimization of passenger inconvenience measured as the total traveling time. When studies focus more on passengers, some details about train rescheduling are ignored, for example, the capacity of infrastructure or trains and train rescheduling measures. As for the objective function setting, most of the existing studies minimize train delays, passenger delays, or the number of cancelled train services. A few literatures consider to minimize the lost revenue by satisfying unexpected passenger demands.

**Table 1**
Summary of relevant studies on real-time traffic management problem.

| Publication | Rescheduling measure | Passenger aspect | Capacity | Model structure | Objective(s) | Solution method |
|---|---|---|---|---|---|---|
| Schachtebeck and Schöbel (2010) | RT, RO | TC | I | EA + IP | Minimize train delays and the number of missed connections | H |
| Corman et al. (2012) | RT, RO | TC | I | AG + MILP | Minimize train delays and the number of missed connections | B&B, H |
| Sato et al. (2013) | RT, RO, RR | PR | I | MIP | Minimize passenger inconvenience | H |
| Dollevoet and Huisman (2014) | RT, RO | PR | – | EA + IP | Minimize total passenger delay | H, I |
| Dollevoet et al. (2014) | RT, RO | PR | I | EA + IP, AG + MILP | Minimize total passenger delay and train delay | H, I |
| Louwerse and Huisman (2014) | RT, RO, RS | No | I | EA + IP | Maximize service level offered to passengers | CS |
| Niu et al. (2015) | RT, RS | T-D PD | I + T | QIP | Minimize total passenger waiting time | CS |
| Zhan et al. (2015) | RT, RO, RS | No | I | EA + MILP | Minimize total weighted train delay and the number of cancelled trains | CS |
| Veelenturf et al. (2016) | RT, RO, RR, RS | No | I + T | EA + ILP | Minimize the number of cancelled and delayed train services | CS |
| Corman et al. (2017) | RT, RO | PR | I | AG + MILP | Minimize passenger travel time | H, I |
| Binder et al. (2017) | RT, RO, RR, RS | PR | I + T | ILP | Minimize passenger inconvenience, operational cost and deviation from undisrupted timetable | CS |
| Veelenturf et al. (2017) | RS | PR | T | ILP | Minimize rolling stock and timetable rescheduling costs, and passenger service costs | H, I |
| **This study** | **RT, RO, RR, RS** | **PR** | I + T | **MILP** | **Maximize the number of saved passengers; minimize the weighted total train delay** | **CS** |

*Symbol descriptions for Table 1: Retiming (RT); Reordering (RO); Rerouting (RR); Reservicing (RS); Transfer connection (TC); Passenger rerouting (PR); Time-dependent passenger demand (T-D PD); Infrastructure capacity (I); Train capacity (T); Alternative graph (AG); Event-activity network (EA); Integer programming (IP); Integer linear programming (ILP); Mixed integer linear programming (MILP); Quadratic integer programming (QIP); Commercial solver (CS); Heuristics (H); Branch-and-bound (B&B); Iterative solution (I).

In contrast, we consider the problem of train rescheduling in a railway system with a seat reservation mechanism. Not only detailed infrastructure information but also limited seat capacity are taken into account. Trains should be rescheduled by considering both the need for changing their stopping plans to accommodate disrupted passengers and the presence of a disrupted train in the railway network. We formulate different rescheduling measures to minimize the weighted total train delay, such as retiming, reordering, rerouting, and addition of extra stops, for the disrupted passengers while transporting as many of them as possible to their destinations. The disrupted passengers in the same group (passengers who were supposed to travel in the broken train and have the same destination form one passenger group) can be assigned to different specific trains that still have sufficient capacity to take new passengers, which is achieved by the ticket re-allocation. The ticket re-allocation is also concerned with the revenue management to avoid the ticket refund. This kind consideration of the objective is rare in the current literature. We propose an integrated approach to solve the train rescheduling and passenger reassignment problems simultaneously. Assignment optimization of the disrupted passengers is a very complicated task because of the seat reservation mechanism and is similar to that used by airlines (Clausen, 2007; Kohl et al., 2007). At the same time, if a train makes an extra stop, the travel time becomes longer, which would ultimately lead to delays and possibly even cause a domino effect of delay propagation in the overall network during railway operations.

This study considers passenger reassignment as a problem to be addressed during railway operations, thus facilitating the consideration of this problem in collaboration with the train rescheduling process. An optimal solution for trains and passengers must be determined. We thus contribute to the literature on train rescheduling by focusing more on passenger aspects, where the ticket re-allocation is considered. We also advance the research on delay management by combining it with the train rescheduling process.

### 2.2. Statement of contributions

This study focuses on the real-time rescheduling of a train timetable after a disruption (e.g., train failure). The passenger reassignment problem is solved under the assumption that the railway system has a seat reservation mechanism. The contributions of this paper are as follows:

(1) This research focuses on a railway system with a seat reservation mechanism, such as the high-speed railway systems in China. In addition to infrastructure having limited capacity, trains are capacitated with a limited number of seats being available and passengers having booked their seats in a specific train. Considering that a passenger seat reservation system is in place, the passengers of a disrupted train must change their tickets because their booked service has been canceled and they now have to travel on other trains that have free seats available. Accordingly, the following trains will be requested to add extra stops to increase the available capacity, which will enable the disrupted passengers to reach their destinations as early as possible. Real-time re-ticketing will be achieved by resetting the number of available seats in the following trains for the newly planned intermediate stops and final stations in the ticketing system. If the re-ticketing process is not undertaken, the disrupted passengers may have to wait for one or more hours for the available following trains with preplanned stops to arrive at the transfer station and reach the destination stations; in addition, the available seats in the following trains may be limited. However, the extra stops will cause train delay. It is challenging to strike a balance between the number of saved passengers and train delay. This behavior is substantially different from that of a railway system without a seat reservation mechanism, such as the usual ticketing systems used in regional/local trains in most European countries or urban transit systems. In the system without a seat reservation mechanism, passengers with a ticket between an origin station and a destination station can choose their preferred trains when they enter the system, leading to crowding and uncertainty in route choice. In addition, railway operating companies cannot predict how many passengers would take a specific train.

(2) We propose a novel mixed-integer linear programming formulation for the problem of determining a disposition timetable under disruptions while optimizing the following key elements: passenger reassignment (how many passengers of every passenger group travel in which trains), train rescheduling (which trains stop and where; which train times, orders, and routes are adopted in the network), track capacity utilization (adherence to the minimum safety separations between any pair of consecutive trains on rail resources), and combined passenger and train indicators.

(3) Based on real-world infrastructure and operation data from a part of the Beijing–Shanghai high-speed railway line, we perform numerical experiments via a weighted-sum method to obtain Pareto optimal solutions. The large number of experiments helps us to illustrate the tradeoff between the number of saved passengers and the total train delay and validate the efficiency and effectiveness of the proposed method. The experiments also enable us to determine how the impact factors work, such as the weights for objectives, duration of disruption, allowed additional running and dwell times, passenger demand, and total available train capacity. In practice, our method will help dispatchers to determine the settings of parameters (the weights for objectives and allowed additional running and dwell times) according to different actual situations (the duration of disruption, passenger demand, and total available train capacity) and obtain a set of feasible non-dominated schedules. This framework can help decision makers to take a more informed decision on the management of both passenger and train flows.

## 3. Problem description

In railway operations, punctuality is one of the main performance indicators that should be optimized. This indicator can be viewed as the minimization of train delays on a railway network. From the perspective of passengers, each person would like to travel according to the details indicated in his/her ticket, considering that he/she has purchased the ticket in advance according to his/her own preference. However, in the case of a disruption, if the passenger can arrive at his/her destination especially when trains has limited seat capacity, he/she may be satisfied even if he/she has to travel with a revised ticket on another train or with another seat number.

In this study, we focus on the integration problem of train rescheduling and passenger reassignment under severe disruptions in a railway system with a seat reservation mechanism. At each station, the scheduled arrival and departure times of each train are considered for the computation of train delays during railway operations. For each available following train, we consider the possibility of adding station stops for transporting disrupted passengers, including stops at the transfer station for disrupted passengers to board the available following train and stops at the destination stations of disrupted passengers. For each group of passengers in the disrupted train, we need to determine the number of seats that should be reserved and the following trains on which these seats should be reserved to transport the passengers to their destinations. This is determined by considering the available seat capacity in each available following train and the train delay caused by the unplanned stops. We next introduce the input and output information, assumptions and an illustration example considered for this problem.

### 3.1. Input and output

The input includes the following information:

(1) Railway network

We consider a railway network composed of several track segments. The main track and siding tracks in stations will be included. If one train is rescheduled to add an extra stop at one station, it will go through by one siding track with a stop rather than the preplanned route of the main track. We view the railway network as a directed graph $G = (N, A)$ with a set of nodes $N$ and a set of directed arcs $A$. Nodes represent station signals and directed arcs represent track segments. Each arc can be occupied by more than one train at the same time in a given traffic direction and respecting the required minimum headway time (time interval) between consecutive trains. Fig. 1 shows an illustration of a double-track railway network with three stations.

(2) Passenger travel demands

For each passenger group, we consider an amount (demand) of passengers with the same origin and destination stations, and the train assigned to perform their service.

(3) Timetable

The timetable is the planned schedule of all train services, with a detailed description of train timing, ordering and routing, plus arrival/departure times of each train at each station, and the carrying capacity of each train in terms of the available seat capacity.

(4) Disrupted train

We know the location of the train which has a failure, and assume the disruption can be resolved by having the affected train being pulled to the next station along its running direction. We also know when the blocked track is available again.

A rescheduled train timetable and a passenger reassignment plan are obtained as the major outputs. New routes, orders, departure and arrival times for trains, which follows the broken train, will be determined. The available seat capacity on these trains is assigned to the disrupted passengers. The complete timetable information and the number of available seats can be checked in the ticketing system. Disrupted passengers can change their tickets to an available following train of their choice.

### 3.2. Assumptions

We make the following assumptions:

(1) We do not consider the reassignment of each individual passenger. Passengers can change their tickets to any available train by themselves according to the train information shown in the ticketing system. We only reassign the available seat capacity of the following trains to the destination stations of disrupted passengers and then update the train stops and corresponding number of available seats in the ticketing system. Disrupted passengers are divided into different passenger groups; in each of these groups, passengers have the same demand. This means that passengers in the same group have the same destination. The available following trains are rescheduled with extra stops according to the passenger demand. Then, the passengers in every group can change their train tickets according to the ticketing system to a following train that has available seats and that stops at their transfer and destination stations.

(2) We do not consider the reassignment of passengers who are not in the train facing a disruption. In other words, real-time ticket changes only involve disrupted passengers.

(3) We consider only one transfer for passengers on the disrupted train, i.e., from the disrupted train to a following train that can transport them to their destinations.

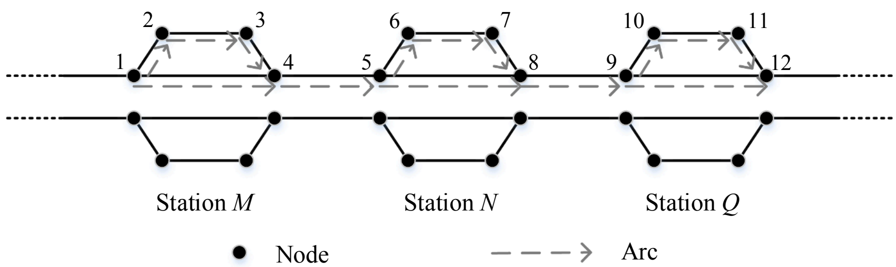


**Fig. 1.** Illustration of railway network.

(4) We do not consider to utilize the back-up rolling stock to serve the disrupted passengers, i.e., inserting one train service. The back-up rolling stock is rare due to their high cost, thus it is not always available at a given time and location.
(5) We use a time discretization of 1 min in this study.

### 3.3. Illustrative example

We next describe a fictitious example to illustrate the studied problem. We compare a passenger reassignment strategy with the traditional strategy without passenger reassignment. In this example, five trains, named Trains G1 to G5, travel from Station *M* to Station *P* with some intermediate stops in the original timetable. There are two passenger groups: grey (2 passengers) and black (2 passengers). Both groups travel on Train G1 but have different destinations (Station *O* for the grey group and Station *P* for the black group). Fig. 2 shows the passenger flows in the original timetable. When Train G1 is at Station *O*, the black-group passengers are still onboard, whereas the grey-group passengers disembark from Train G1.

During railway operations, Train G1 malfunctions on the track section from Station *M* to Station *N* and the train cannot move toward Station *N*. A rescue train is required to move Train G1 to Station *N*. All the passengers on Train G1 must disembark and wait for the available following trains to (eventually) travel to their destinations. Clearly, the disrupted passengers can change their tickets to only those trains with a scheduled stop at Station *N* (so that they can use those trains) and with a scheduled stop at their destinations (so that they can arrive at their destinations without any further need of changing trains). A ticket change is thus needed for the disrupted passengers. This is only possible if there is available seat capacity on the following trains.

As shown in Fig. 3, dispatchers take some train rescheduling measures (such as train retiming) without considering passenger reassignment, i.e., without adding unscheduled stops for the following trains to serve disrupted passengers. This can be viewed as a traditional approach without passenger reassignment, in which the grey-group passengers can choose only Train G5, whereas the black-group passengers can choose only Trains G3 and G5. With regard to the available seat capacity of the following trains, disrupted passengers are aware of the timetable of the following trains and the possibilities to change their ticket in advance from the ticketing system. In this manner, they can be ready to board a following train with sufficient seat capacity and a feasible stopping plan (i.e., a plan satisfying their demand). Not all disrupted passengers can choose the earliest feasible train departing from Station *N* because of insufficient seat capacity. Fig. 3 shows a feasible solution for disrupted passengers. However, not all disrupted passengers can be served (only a single black-group passenger boards Train G3 and a single grey-group passenger boards Train G5, whereas the other two passengers are not served).

Our passenger reassignment strategy works as follows. As shown in Fig. 4, all the disrupted passengers can reach their destination. This is made possible by adding an extra stop for Trains G2 and G4 at Station *N*. However, the number of disrupted passengers who board Trains G2 and G4 depends on the available seat capacity on these trains. The complete timetable information and the number of available seats can be checked in the ticketing system. Disrupted passengers can change their tickets to an available following train that stops at Station *N* and their destinations and has available seats. Therefore, the disrupted passengers can board the following trains of their choice. Here, passenger reassignment refers to the reservation of seats for some passenger groups on the following trains and not the reservation of a seat for a specific disrupted passenger. In the solution shown in Fig. 4, one seat is reserved on both Trains G2 and G5 for grey-group passengers. The decision regarding which passenger in the grey group takes Train G2 and which one takes Train G5 will be decided by the passengers by changing their tickets via the ticketing system. The drawback of serving all the passengers is the need to increase the travel time of the following trains because of the addition of an unscheduled stop for Trains G2 and G4. This increase in the travel time may generate dissatisfaction among the undisrupted passengers if they arrive at their destinations with an additional delay. In a high-density timetable, the addition of unscheduled stops for some trains may generate consecutive delays for the following trains.

In summary, the passenger reassignment strategy requires the addition of extra stops to serve disrupted passengers and requires decision making regarding how many disrupted passengers of every passenger groups should board which of the following trains. This strategy must be combined with the train rescheduling measures taken by dispatchers. The resulting problem is especially challenging and involves a real-time re-ticketing system. Train rescheduling and passenger reassignment affect each other negatively. A compromise decision should be made within a limited time.

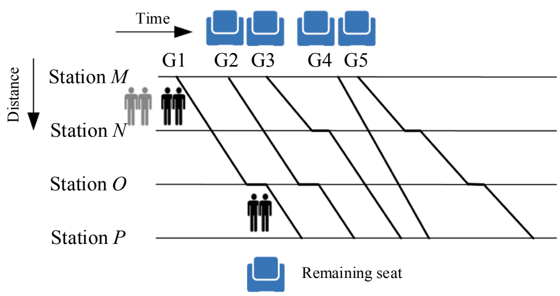


**Fig. 2.** Illustration of original timetable and passenger flows.

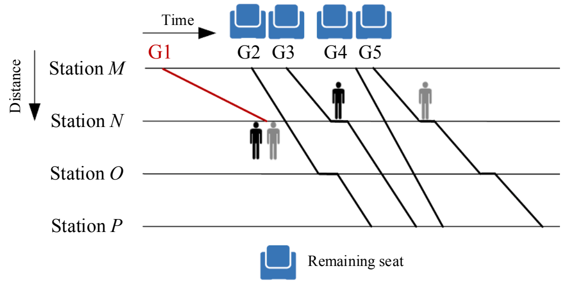


**Fig. 3.** Illustration of disruption and traditional approach without passenger reassignment.

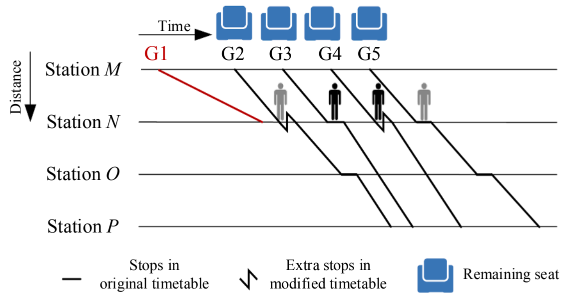


**Fig. 4.** Illustration of feasible passenger reassignment strategy.

## 4. Methodology

This section describes the methodology proposed to solve the studied problem. We first introduce notations used in the mathematical formulation, and then introduce the objective function and the set of constraints required to properly model this problem.

### *4.1. Notations*

We next introduce the general subscripts and the input parameters in Tables 2 and 3, while the decision variables of the mathematical formulation are given in Table 4.

### *4.2. Mathematical formulation*

A bi-objective formulation is proposed in this study with the following components: the maximization of the number of disrupted passengers that are reassigned to other trains in order to reach their destinations ($Z_1$); the minimization of the weighted total train delay ($Z_2$) that considers the positive deviation of all trains with respect to the scheduled arrival time at their destinations. The weight for each following train is calculated by the number of undisrupted passengers on this train divided by 1000, $w_f = NP_f/1000$. As a result, trains with more undisrupted passengers will be given a priority of going through the railway line.

Due to the limited train capacity, whether disrupted passengers can reach their preplanned destinations is the key point. Otherwise, railway operating companies have to refund the ticket fares to disrupted passengers. At the same time, reliable and punctual traffic in the entire railway network is also a main request for the companies. Delays of trains at the destination stations will influence the subsequent train services.

**Table 2**
General subscripts.

| Symbol | Description |
|---|---|
| $i,j,k$ | Node index, $i,j,k \in N$, $N$ is the set of nodes in a railway network |
| $a$ | Arc index, $a \in A$, $A$ is the set of arcs in a railway network |
| $t$ | Time index, $t \in \{1 \cdots T\}$, $T$ is the rescheduling time horizon |
| $f$ | Train index, $f \in F$, $F$ is the set of all trains |
| $p$ | Passenger group index, $p \in P$, $P$ is the set of passenger groups in the broken train |

**Table 3**
Input parameters.

| Symbol | Description |
|---|---|
| $f^*$ | Broken train index, $f^* \in F$ |
| $F_{foll}$ | Set of all following trains of the broken train $f^*$, $F_{foll} \subset F$ |
| $F_{avai}$ | Set of available following trains, which can be rescheduled to serve the disrupted passengers, $F_{avai} \subset F_{foll} \subset F$ |
| $A_f$ | Set of arcs that train $f$ may use, $A_f \subset A$ |
| $\delta_{f,i,j}$ | Running time (scheduled) of train $f$ to travel through arc $(i,j)$ |
| $\vartheta^{min}_{f,i,j}$ | Minimum dwell (waiting) time for train $f$ on arc $(i,j)$ |
| $\vartheta^{max}_{f,i,j}$ | Maximum dwell (waiting) time for train $f$ on arc $(i,j)$ |
| $h_{f_1,f_2,i,j}$ | Safety time interval (headway) between consecutive trains $f_1$ and $f_2$ that are traversing arc $(i,j)$ |
| $o_f$ | Origin node of train $f$ |
| $d_f$ | Destination (sink) node of train $f$ |
| $EST_f$ | Predetermined earliest start time of train $f$ at its origin node |
| $AT_f$ | Predetermined arrival time at the destination node of train $f$ in the original timetable |
| $w_f$ | Weight for the following train $f$, $f \in F_{foll}$ |
| $\rho_f$ | Available passenger carrying capacity (available seats) in train $f$ for the passengers of the broken train $f \in F_{avai}$ |
| $\sigma_{f,i,j,p}$ | Extra running time for train $f$ traveling through arc $(i,j)$ due to an extra stop at a station for passenger group $p$ getting on or off the train |
| $NP_f$ | The number of undisrupted passengers on board for following train $f$, $f \in F_{foll}$ |
| $s$ | Transfer node where passengers need to get off their original train and then (eventually) get on a following train |
| $d_p$ | Destination node for passenger group $p$ |
| $\eta_p$ | Volume of passenger group $p$ |
| $Dst$ | Start time of disruption, $Dst \in \{1 \cdots T\}$ |
| $Det$ | End time of disruption, $Det \in \{1 \cdots T\}$ |
| $b$ | Begin node of disruption, $b \in N$, which means where the train breaks down |
| $e$ | End node of disruption, $e \in N$, which means where the broken train is pulled over |

**Table 4**
Train rescheduling and passenger reassignment decision variables.

| Symbol | Description |
|---|---|
| $a_{f,i,j}$ | Arrival time variable of train $f$ arriving at arc $(i,j)$ |
| $d_{f,i,j}$ | Departure time variable of train $f$ departing from arc $(i,j)$ |
| $x_{f,i,j}$ | 0–1 binary train routing variable: $= 1$, if train $f$ selects arc $(i,j)$; $= 0$, otherwise |
| $order_{f_1,f_2,i,j}$ | 0–1 binary train ordering variable: $= 1$, if train $f_2$ is scheduled after train $f_1$ on arc $(i,j)$; $= 0$, otherwise |
| $y_{f,p,i,j}$ | Passenger reassignment variable, the number of passengers in group $p$ reassigned to the following available train $f$, $f \in F_{avai}, p \in P, (i,j) \in A_f : j = d_p$ |
| $\Delta_{f,p,i,j}$ | 0–1 binary train extra stop variable: $= 1$, if train $f$ needs to perform an unscheduled stop for passengers in passenger group $p$ at arc $(i,j)$; $= 0$, otherwise, $f \in F_{avai}, p \in P, (i,j) \in A_f : j \in \{s, d_p\}$ |
| $TT_{f,i,j}$ | Travel time for train $f$ on arc $(i,j)$ |

$$MaxZ_1 = Max \sum_{f \in F_{avai}} \sum_{p \in P} \sum_{(i,j) \in A_f : j \in \{s, d_p\}} y_{f,p,i,j} \tag{1}$$

$$MinZ_2 = Min \sum_{f \in F_{foll}} w_f \times \left| d_{f,i,d_f} - AT_f \right| \tag{2}$$

Subject to:

Capacity loss constraints:

$$a_{f^*,b,j} = Dst, \forall (b,j) \in A_{f^*} : j \in N \tag{3}$$

$$d_{f^*,i,e} \geqslant Det, \forall (i,e) \in A_{f^*} : i \in N \tag{4}$$

Flow balance constraints at the origin node:

$$\sum_{j:(o_f,j) \in A_f} x_{f,o_f,j} = 1, \ \forall f \in F \tag{5}$$

Flow balance constraints at the intermediate nodes:

$$\sum_{i:(i,j)\in A_f} x_{f,i,j} = \sum_{k:(j,k)\in A_f} x_{f,j,k},\ \forall f\in F, j\in N\backslash\{o_f,d_f\} \tag{6}$$

Flow balance constraints at the destination node:

$$\sum_{i:(i,d_f)\in A_f} x_{f,i,d_f} = 1,\ \forall f\in F \tag{7}$$

Start time constraints at the origin node:

$$a_{f,o_f,j} \geqslant EST_f \times x_{f,o_f,j},\ \forall f\in F, (o_f,j)\in A_f : j\in N \tag{8}$$

Within arc transition constraints:

$$d_{f,i,j} \geqslant a_{f,i,j},\ \ \forall f\in F, (i,j)\in A_f : i,j\in N \tag{9}$$

Arc-to-arc transition constraints:

$$\sum_{i:(i,j)\in A_f} d_{f,i,j} = \sum_{k:(j,k)\in A_f} a_{f,j,k}, \forall f\in F, j\in N\backslash\{o_f,d_f\} \tag{10}$$

Mapping constraints between train routing and timing variables:

$$x_{f,i,j}\times M \geqslant a_{f,i,j}, \forall f\in F, (i,j)\in A_f : i,j\in N \tag{11}$$

$$x_{f,i,j} \leqslant a_{f,i,j} + 1, \forall f\in F, (i,j)\in A_f : i,j\in N \tag{12}$$

$$x_{f,i,j}\times M \geqslant d_{f,i,j}, \forall f\in F, (i,j)\in A_f : i,j\in N \tag{13}$$

$$x_{f,i,j} \leqslant d_{f,i,j} + 1, \forall f\in F, (i,j)\in A_f : i,j\in N \tag{14}$$

Maximum and minimum running time constraints:

$$TT_{f,i,j} = d_{f,i,j} - a_{f,i,j}, \forall f\in F, (i,j)\in A_f : i,j\in N \tag{15}$$

$$TT_{f,i,j} \leqslant \left(\delta_{f,i,j} + \vartheta^{max}_{f,i,j}\right)\times x_{f,i,j}, \forall f\in F, (i,j)\in A_f : i,j\in N \tag{16}$$

$$TT_{f,i,j} \geqslant \left(\delta_{f,i,j} + \vartheta^{min}_{f,i,j}\right)\times x_{f,i,j}, \forall f\in F, (i,j)\in A_f : i,j\in N \tag{17}$$

$$TT_{f,i,j} \geqslant \left(\delta_{f,i,j} + \vartheta^{min}_{f,i,j} + \sigma_{f,i,j,p}\right)\times \Delta_{f,p,i,j}, \forall f\in F_{avai}, p\in P, (i,j)\in A_f : j\in\{s,d_p\} \tag{18}$$

Mapping constraints between order and arc usage:

$$order_{f_1,f_2,i,j} + order_{f_2,f_1,i,j} >= x_{f_1,i,j} + x_{f_2,i,j} - 1, \forall f_1,f_2\in F, (i,j)\in A_{f_1}\cap A_{f_2} : i,j\in N \tag{19}$$

$$order_{f_1,f_2,i,j} + order_{f_2,f_1,i,j} \leqslant 3 - x_{f_1,i,j} - x_{f_2,i,j}, \forall f_1,f_2\in F, (i,j)\in A_{f_1}\cap A_{f_2} : i,j\in N \tag{20}$$

Track capacity constraints:

$$a_{f_2,i,j} + \left(3 - x_{f_1,i,j} - x_{f_2,i,j} - order_{f_1,f_2,i,j}\right)\times M >= a_{f_1,i,j} + h_{f_1,f_2,i,j}, \forall f_1,f_2\in F, (i,j)\in A_{f_1}\cap A_{f_2} : i,j\in N \tag{21}$$

$$d_{f_2,i,j} + \left(3 - x_{f_1,i,j} - x_{f_2,i,j} - order_{f_1,f_2,i,j}\right)\times M >= d_{f_1,i,j} + h_{f_1,f_2,i,j}, \forall f_1,f_2\in F, (i,j)\in A_{f_1}\cap A_{f_2} : i,j\in N \tag{22}$$

$$a_{f_1,i,j} + \left(3 - x_{f_1,i,j} - x_{f_2,i,j} - order_{f_2,f_1,i,j}\right)\times M >= a_{f_2,i,j} + h_{f_2,f_1,i,j}, \forall f_1,f_2\in F, (i,j)\in A_{f_1}\cap A_{f_2} : i,j\in N \tag{23}$$

$$d_{f_1,i,j} + \left(3 - x_{f_1,i,j} - x_{f_2,i,j} - order_{f_2,f_1,i,j}\right)\times M >= d_{f_2,i,j} + h_{f_2,f_1,i,j}, \forall f_1,f_2\in F, (i,j)\in A_{f_1}\cap A_{f_2} : i,j\in N \tag{24}$$

Passenger flow balance constraints:

$$\sum_{f\in F_{avai}} \sum_{(i,j)\in A_f : j=d_p} y_{f,p,i,j} \leqslant \eta_p\ , \forall p\in P \tag{25}$$

Passenger carrying capacity constraints:

$$\sum_{p\in P} \sum_{(i,j)\in A_f, j=d_p} y_{f,p,i,j} \leqslant \rho_f, \forall f\in F_{avai} \tag{26}$$

Mapping constraints between passenger reassignment and train stop plan:

$$\Delta_{f,p,i,d_p} \times \rho_f \geqslant y_{f,p,i,d_p}, \forall f \in F_{avai}, \forall p \in P, \forall (i, d_p) \in A_f : i, d_p \in N \tag{27}$$

$$\Delta_{f,p,i,d_p} \leqslant y_{f,p,i,d_p} \times M, \forall f \in F_{avai}, \forall p \in P, \forall (i, d_p) \in A_f : i, d_p \in N \tag{28}$$

$$\Delta_{f,p,i,s} = \Delta_{f,p,i,d_p}, \forall f \in F_{avai}, \forall p \in P, \forall (i, s), (i, d_p) \in A_f : i, s, d_p \in N \tag{29}$$

Constraints (3) and (4) are capacity loss constraints to ensure that the broken train $f^*$ occupies the arcs during the whole disruption horizon. Constraints (5), (6) and (7) ensure flow balance at the origin node, the intermediate nodes, and the destination node for each train on the network. Constraints (8)–(14) are space–time network constraints. Constraints (8) are needed to make sure that every train will depart from its origin station after its earliest departure time from the original node. Constraints (9) ensure the transition within arcs. Constraints (10) guarantee that if the adjacent arcs $(i,j)$ and $(j,k)$ are both used by train $f$, the departure time from the former arc will be equal to the arrival time at the latter arc. Constraints (11)–(14) are imposed to map train routing variables with train timing variables. Constraints (15) give the calculation method of the travel time on each arc for each train. Constraints (16)–(18) enforce the required maximum and minimum running times. Constraints (16) are needed to make sure that no train can be allowed to wait for any other train always. Constraints (17) ensure the minimum running time due to the limitation of speed. We note that if a train is rescheduled to add an unscheduled stop in one station for disrupted passengers, this train will go through from the siding track (this route composed of several arcs) not the main track (this route composed of only one arc), at this time, the travel time for this train will increase. If a train has preplanned stops at the transfer station or destination stations for the disrupted passengers, and this train is rescheduled to transport some disrupted passengers, the minimum dwell time should be ensured. It can be formalized by introducing extra running (dwell) times at the involved arcs by Constraints (18), like a train stopping at one siding track for 2 min in the original timetable, while the minimum dwell time for disrupted passengers as 3 min, then an additional 1 min dwell time requested.

Constraints (19) and (20) map the train sequencing variables with the train routing variables. Constraints (21)–(24) ensure that the number of trains occupying arc $(i,j)$ is less than the capacity of this arc. We note that there can be more than one train traveling on each track segment as long as the minimum time interval (headway) between trains is satisfied at any time. Constraints (25) require that the total number of passengers in the passenger group $p$ reassigned to the available following trains cannot exceed the total demand of this group. Constraints (26) guarantee that the available seat capacity of every train is not violated. Constraints (27)–(29) make sure that once a following train $f$ is assigned to serve the disrupted passengers of passenger group $p$, train $f$ must stop at the transfer station $s$ and their destination station $d_p$. In this study, the transfer station is the one just after the disruption location, where the disrupted passengers will wait for the following trains.

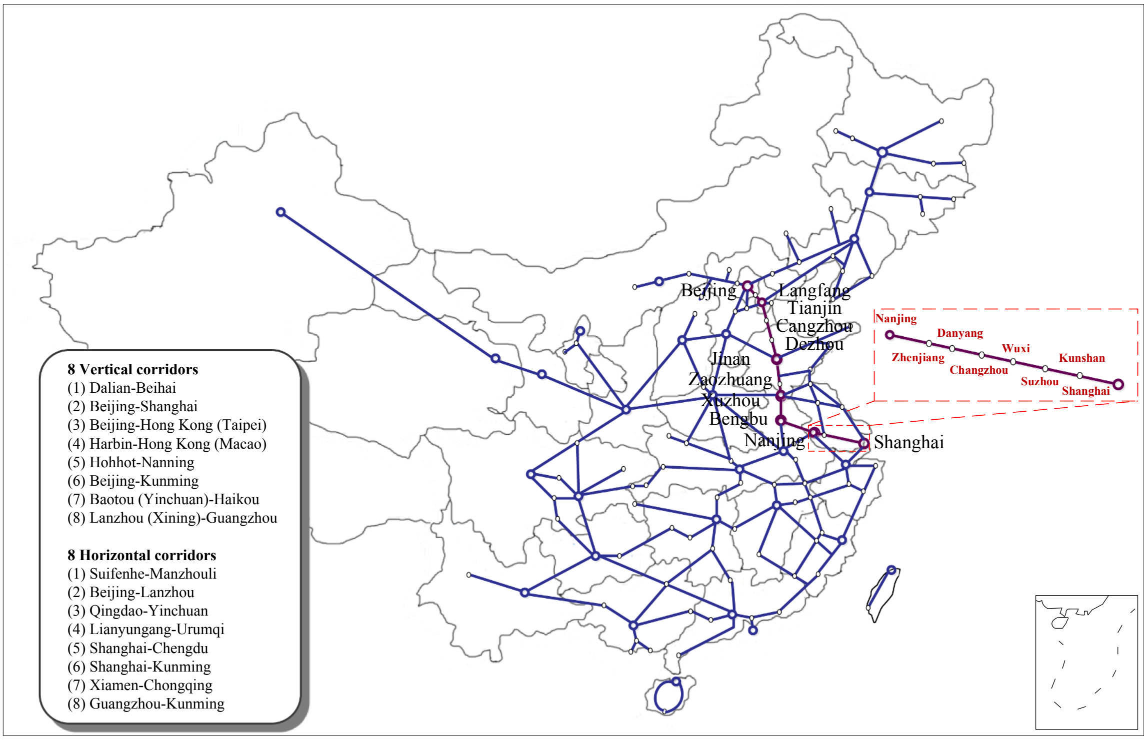


**Fig. 5.** Illustration of network topology.

### 4.3. Weighted-sum formulation for Bi-objective optimization problem

This section presents a weighted-sum method with normalization factors to fairly combine the two objective functions of the considered integrated problem (Zhang et al., 2019; D'Ariano et al., 2019). One input parameter $\alpha$ is defined as the weight for the objective functions. One constraint is considered: $0 \le \alpha \le 1$. The decision maker can set the value of this parameter according to his/her own insights.

The maximization of the number of saved passengers ($Z_1$) and the minimization of the weighted total train delay ($Z_2$) are the objective function components of this problem. We use the symbols $Z_1^*$ and $Z_2^*$ to indicate the optimal solution of the two objective function components. This means that when we focus on only serving as many disrupted passengers as possible by accommodating them on the available following trains, we can solve this problem by using (1), (25), and (26), and the maximum number of saved passengers ($Z_1^*$) can be obtained. If we only reschedule the train timetable without considering passenger reassignment, the problem can be solved by using (2) and (3)–(24), and the minimum weighted total train delay ($Z_2^*$) can be determined. We then set the bi-objective function of the investigated optimization problem as follows:

$$\text{Max Z} = \text{Max}\alpha \times \frac{Z_1}{Z_1^*} - (1 - \alpha) \times \frac{Z_2}{Z_2^*} \tag{30}$$

where $\alpha/Z_1^*$ and $(1-\alpha)/Z_2^*$ can be considered as normalization factors. We consider $Z_1^{'}$ and $Z_2^{'}$ as the values of the performance indicators $Z_1$ and $Z_2$ for the optimal solution of the integrated problem with the objective function Max $Z$ obtained using the weighted-sum method with normalization. The decision maker can vary the value $\alpha$ to search for new non-dominated solutions ($Z_1^{'}$,$Z_2^{'}$), whereby he/she can give greater importance to either the original timetable or the disrupted passengers. The Pareto front will be determined by the evaluation of the above described compromise solutions.

## 5. Numerical experiments

### 5.1. Test case description

The test bed is based on a part of the Beijing–Shanghai high-speed railway line, which is one of the busiest lines in China. The selected part is the section from Nanjing South Station to Shanghai Hongqiao Station, which includes eight stations, as shown in Fig. 5 We focus on the single direction from Nanjing to Shanghai, which means that we consider that opposite trains from Shanghai to Nanjing do not occupy the main and siding tracks used by the trains from Nanjing to Shanghai because in real operations, most of the main and siding tracks are distinguished according to the different operation directions of trains because of the signal system. In addition, the option of turning back trains that are facing in the direction from Shanghai to Nanjing to transport the disrupted passengers is not considered in this study because it will involve the problem of rolling stock circulation.

The timetable used in our experiments is a real one for the year 2018, as shown in Fig. 6, where the wavy lines represent overtaking. We consider that Train G1 suffers from a technical failure after departing from Nanjing South Station at 19:19. A rescue train is needed to pull this train to the forward station, Zhenjiang South Station. During this process, the track section between Nanjing South Station and Zhenjiang South Station will be occupied by the broken rolling stock of Train G1, and no other trains can go through this track section. The duration of disruption depends on the time it takes to pull Train G1 forward. The remaining part of the service of Train G1 after Zhenjiang South Station will be canceled. There are four passenger groups on G1 with the following destinations: Changzhou North Station, Suzhou North Station, Kunshan South Station, and Shanghai Hongqiao Station. All the passengers on this train must disembark and wait for the available following trains to reach their destination stations.

We consider trains operated within an acceptable time horizon that is comparable to the travel time required if passengers take

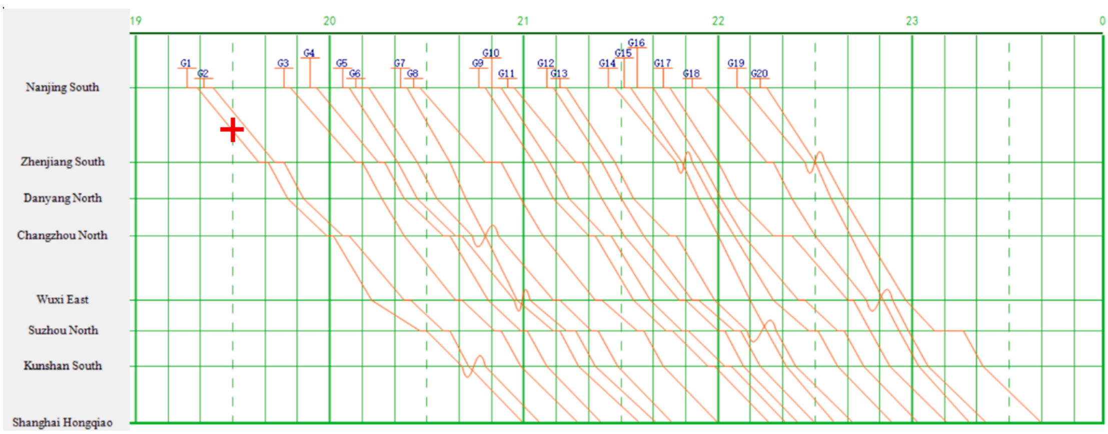


**Fig. 6.** Original timetable.

another transportation mode to reach their destinations. We assume that if passengers are made to wait for an available following train for a longer time than this acceptable time, they will claim a refund on their train tickets and choose another transportation mode. This will also be a major revenue loss component for the involved railway operating companies.

Travel data indicate that if passengers take the bus or the airplane from Nanjing to Shanghai, the travel time will be around 4 h, including the travel time from the train station to the bus station or airport. We consider that if the total travel time taken by disrupted passengers, including the waiting time at Zhenjiang South Station for the available following trains and the onboard time to reach their destinations, is greater than 4 h, they will choose other transportation modes to reach their destinations as soon as possible. This is a reasonable assumption because in reality, most passengers taking the trains on the Beijing–Shanghai high-speed railway line are time-sensitive business travelers. Therefore, in our experiments, 19 trains following Train G1 will be considered; these trains run within the next 4 h from 19:19 to 23:19. Different trains have different stopping plans.

We consider that the trains in this timetable are operated with a high density in the railway line because the Beijing–Shanghai high-speed railway line is one of the lines with the highest demand in China, and many trains barely satisfy the minimum headway time between each other in some sections or stations so that more trains can be operated each day. Therefore, for this timetable, the solution to the problem will be more challenging and interesting. The resulting mathematical formulation is solved by using IBM CPLEX Optimization Studio 12.7 on a computer with Intel(R) Xeon (TM) CPU E5-2660 v4 @ 2.00 GHz and 512 GB memory. The computation time for all experiments is less than 780 s. All the solutions discussed here have been proven to be optimal. We can obtain the optimal solution within 300 s for all the experiments, the remaining seconds are used to prove that the solution is optimal. This satisfies the efficiency requirements in real-time operations.

### 5.2. Experimental setup

We consider different settings for the influence factors such as the weight for the objective to maximize the number of saved passengers ($\alpha$), passenger demand ($D$), total available train capacity ($C$), duration of disruption ($Dur$), and allowed additional running time ($ART$) and dwell time ($ADT$), as listed in Table 5. The meaning of each factor is explained below in detail. Different rescheduling measures including retiming, rerouting, reordering, and addition of extra stops are considered in all the experiments.

We change the weight for the objective function of maximizing the number of saved passengers from 0 to 1 in increments of 0.1 for every step for 11 series. Correspondingly, the weight for the objective function to minimize the weighted total train delay is changed from 1 to 0 in decrements of 0.1 for every step.

The total number of disrupted passengers is considered as the passenger demand ($D$), which is the sum of the demand for each passenger group. Here, we set the demand as 80, 160, 80, and 580 for the passenger groups whose destinations are Changzhou North Station, Suzhou North Station, Kunshan South Station, and Shanghai Hongqiao Station, respectively. The total passenger demand is 900. This setting is based on historical statistical data, and the destination of most passengers traveling on the Beijing–Shanghai high-speed railway line is Shanghai Hongqiao Station.

With regard to the settings of $D$ and $C$, we test only two different conditions—one with $D$ less than $C$ and the other with $D$ greater than $C$—and do not focus on the detailed available capacity settings for each following train. When we aim to transport as many disrupted passengers as possible, under the first condition ($D$ less than $C$), only some of the following trains will be rescheduled with some extra stops and not all the following trains because the total available capacity of some of the following trains will be sufficient for the disrupted passengers. Under the second condition ($D$ greater than $C$), all the trains should stop at Zhenjiang South Station to pick up as many disrupted passengers as possible. Correspondingly, the stopping plans at the next stations of all the trains and the passenger reassignment plans will be determined by the destinations and demands of passengers. Therefore, we consider the relationship between $D$ and $C$ as an important factor to be investigated.

We also consider different durations of disruption ($Dur$) of 35, 45, and 55 min according to the statistical data gathered from practical operations. The allowed additional running time in the section between two adjacent stations ($ART$) and the allowed additional dwell time in stations ($ADT$) are also examined. The running and dwell times for corresponding arcs can be longer than the scheduled times in the original timetable within the range of the allowed additional time. It should be noted that we only consider the $ART$ on the arcs between two consecutive stations, such as arcs (4,5) and (8,9) in Fig. 1, and the $ADT$ on the arcs representing the siding tracks in stations, such as arcs (2,3), (6,7), and (10,11) in Fig. 1. The allowed additional times only have an impact on the traveling process after trains departing from Nanjing South Station. Trains can be delayed for any duration when they depart from Nanjing because of the broken train up ahead in the section from Nanjing South Station to Zhenjiang South Station. Fig. 7(a) illustrates the

**Table 5**
Influence factors.

| Number | Factor | Setting | Total number of possible settings |
|---|---|---|---|
| 1 | Weight for objective to maximize number of saved passengers ($\alpha$) | From 0 to 1 in increments of 0.1 | 11 |
| 2 | Passenger demand ($D$) | 900 | 1 |
| 3 | Total available train capacity ($C$) | 1070, 870 | 2 |
| 4 | Duration of disruption (min) ($Dur$) | 35, 45, 55 | 3 |
| 5 | Allowed additional running time (min) ($ART$) | greater than 0 (=5, infinite), = 0 | 4 |
| 6 | Allowed additional dwell time (min) ($ADT$) | greater than 0 (=1, 10, infinite), = 0 | 3 |
| **Total** | | | 792 |

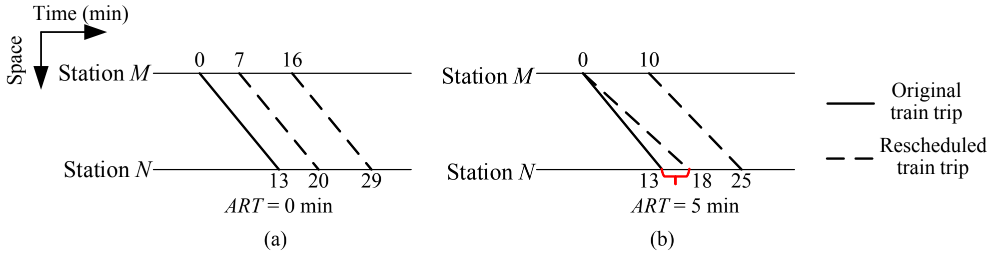


**Fig. 7.** Illustration of settings for allowed additional running time (*ART*).

condition with the *ART* as 0 min, where one train traveling from Station *M* to Station *N* always takes 13 min, which is the same as the running time between these two stations in the original timetable. This train will be delayed from the beginning (the origin station or the first station in the rescheduled area, e.g., Nanjing South Station in our experiments) if needed. In Fig. 7(b), the *ART* is 5 min, i.e., from 13 to 18 min of the running time taken by this train to travel from Station *M* to Station *N*. By setting the *ART* or *ADT* to greater than 0 min, the time taken by trains to travel in sections and dwell at stations can be delayed because of the embarking or disembarking of passengers, overtaking between trains, or limitations of track capacity. There are some preplanned intermediate stops of 2 min in the original timetable, and we set the minimum dwell time for the disrupted passengers as 3 min. These preplanned stops can be rescheduled for an additional 1 min to serve the disrupted passengers by setting the *ADT* as 1 min. The maximum duration of disruption (55 min) is considered to be infinite.

In all the experiments, we consider the bi-objective function and use a weighted-sum method with normalization to get the Pareto front. When we set weights for the number of saved passengers and the weighted total train delay as [1/0], the number of saved passengers is maximized without considering the weighted total train delay. When the weights are set as [0/1], only the weighted total train delay is minimized at one time. It should be noted here that when we consider one objective, the other objective could take any value because no limitation is placed on it. Therefore, we will solve one objective while fixing the other objective to its optimal value, e.g., maximizing $Z_1$ with the additional constraint $Z_2 = Z_2^*$ to obtain the corresponding number of saved passengers under the condition of the minimum weighted total train delay. We can also determine the weighted total train delay when the number of saved passengers is maximized by minimizing $Z_2$ with the additional constraint $Z_1 = Z_1^*$. We combine different settings for these impact factors. In summary, we perform 792 experiments in our computational assessment.

### *5.3. Effectiveness of proposed method*

In actual railway operations, train dispatchers may reschedule the first several following trains to transport all the disrupted passengers. If there are no planned stops at the transfer and destination stations of passengers, the dispatchers may add extra stops at these stations, or if there is no sufficient time at these stations, the dispatchers may extend the dwell time to allow disrupted passengers to embark or disembark. The available train capacity can be occupied by any disrupted passenger of any passenger group provided the passenger has changed his/her ticket in the ticketing system. Because of time limitations, dispatchers choose this simple rescheduling measure.

We propose a method to optimize the train rescheduling and passenger reassignment plans together within an acceptable computation time for this real-time problem. The effectiveness of the proposed method will be illustrated by comparing the minimum total train delay for the actual solutions and our optimized solutions when dispatchers want to save as many disrupted passengers as possible, e.g., for $\alpha = 1$, based on the cases in which the *ART* and *ADT* are all set to infinite.

In our experiments, the passenger demand (*D*) is 900. In the actual operation, under the condition where the total available train capacity (*C*) is 1070, Trains G2–G17 with a total available train capacity of 920 (which is greater than *D*) will be rescheduled to transport all the disrupted passengers. Similarly, for the condition where *C* is 870 (which is less than *D*), all the available following trains will be rescheduled to transport the disrupted passengers. The trains chosen to transport the disrupted passengers will stop at

**Table 6**
Comparison of minimum total train delay between optimized solution and actual solution with maximum number of saved disrupted passengers ($\alpha = 1$) for the cases where *ART* and *ADT* are all infinite.

| | | *Dur* (min) | | |
|---|---|---|---|---|
| | | 35 | 45 | 55 |
| *D* (900) less than *C* (1070) | Minimum total train delay $Z_2^{*'}(Z_1 = 900)$ (actual) | 307 | 323 | 359 |
| | Minimum total train delay $Z_2^{*}(Z_1 = 900)$ (optimized) | 99 | 114 | 152 |
| | Improvement | 67.75% | 64.70% | 57.66% |
| *D* (900) greater than *C* (870) | Minimum total train delay $Z_2^{*'}(Z_1 = 870)$ (actual) | 369 | 385 | **421** |
| | Minimum total train delay $Z_2^{*}(Z_1 = 870)$ (optimized) | 124 | 139 | **177** |
| | Improvement | 66.40% | 63.90% | **57.96%** |

Zhenjiang South, Changzhou North, Suzhou North, Kunshan South, and Shanghai Hongqiao. In our optimized solution, it will be decided as to which of the available following trains will stop at which stations to transport the disrupted passengers of which passenger group. The minimum total train delay with the maximum number of saved passengers for the actual solution is represented by $Z_2^{*'}(Z_1^*)$, which is calculated by (2)–(24), and the predetermined extra stop decision variable $\Delta_{f,p,i,j} = 1$ for Zhenjiang South, Changzhou North, Kunshan South, and Shanghai Hongqiao for the first $n$ trains with sufficient available train capacity. Different durations of disruption (*Dur*) of 35, 45, and 55 min are tested. The detailed results are listed in Table 6. Our proposed method can decrease the total train delay by 57.66%–67.75% compared to the actual operation when the maximum number of disrupted passengers are saved.

Table 7 shows the train delay analysis for the optimized and actual conditions for one of the cases discussed in Table 6, where the duration of disruption (*Dur*) is 55 min, the passenger demand ($D = 900$) is greater than the total available train capacity ($C = 870$). We can recall that the number of saved passengers is 870 for both the conditions, and the minimum total train delay is 172 and 421 min for the optimized and actual conditions, respectively. The primary delay is the departure delay at Nanjing South Station. The secondary delay is the additional delay attributable to the extra stops (7 min delay for each extra stop, including 3 min delay for deceleration, 3 min delay for dwell, and 2 min delay for acceleration), required minimum dwell time for disrupted passengers embarking and disembarking (an additional 1 min dwell time for each preplanned stop of 2 min), and reliving conflicts (the minimum headway time for every two consecutive trains). Total train delay is the sum of the primary delay and secondary delay.

In the optimized solution listed in Table 7, the total primary delay is 65 min, and total secondary delay is 112 min, where the delay of 77 min is caused by extra stops, the delay of 6 min results from more dwell time requested to transport disrupted passengers, and the conflict relief results in 29 min delay. Compared to the actual solution, the total primary delay is 61 min, which is similar to that of the optimized solution. While the delay caused by extra stops and the requirements for the minimum dwell time is 301 and 23 min, respectively. The main reason for the secondary train delay is the inclusion of extra stops. Our study optimizes the stopping plan by considering all the extra stops necessitated by the disrupted passenger reassignment plan. This is why the optimized solution shows a substantial improvement over the actual solution. It can be concluded that our optimization method has a high level of effectiveness.

### 5.4. Cost of passenger reassignment

#### 5.4.1. Negative impact of passenger reassignment on train rescheduling

Here, we analyze how the two objectives of passenger reassignment and train rescheduling react to each other. Fig. 8 shows the Pareto front for the experiment group for which the duration of disruption (*Dur*) is 55 min; the allowed additional running and dwell times are set to 5 and 10 min, respectively; and the passenger demand (*D*) is 900; and the total available train capacity (*C*) is 1070. As more disrupted passengers are saved, the total train delay increases. The increased train delay time is mainly caused by the extra stops. Passenger reassignment has a negative impact on train rescheduling. Hence, there has to be a tradeoff between these two objectives. It should be noted here that there are some same Pareto optimal solutions for different series among the 11 series (obtained by changing the weight $\alpha$ from 0 to 1 in increments of 0.1) for one experiment group, so we will always see less than 11 nodes on the Pareto front. This means that when we change the value for$\alpha$ within a certain range, the same solution can be obtained.

Here, we select two extreme nodes to further explain this aspect. The leftmost node presents the solution when we minimize the weighted total train delay without considering passenger reassignment, where the total train delay and the number of saved passengers are represented by $TD_{npr}$ and $\#P_{npr}$, respectively. The rightmost node presents the solution when we maximize the number of saved passengers, where we use the symbols $TD_{pr}$ and $\#P_{pr}$to indicate the total train delay and the number of saved passengers, respectively. To compare these two extreme nodes, the increased delay rate *DR* and increased passenger rate *PR* are defined. *DR* is calculated as $(TD_{pr} - TD_{npr})/TD_{npr}$, and *PR* is equal to $(\#P_{pr} - \#P_{npr})/\#P_{npr}$. For the abovementioned case, the number of saved passengers increases by 275% with a 78.8% increase in the train delay.

In practice, our method provides a set of non-dominated solutions for dispatchers so that they can make a better decision according to the actual situation. If there are many passengers stranded at stations and saving the passengers is the top priority, the dispatchers can choose a solution with a higher number of saved passengers. If the punctuality of trains is a key point, dispatchers can implement the plan with a lower train delay.

#### 5.4.2. Reduced negative impact of passenger reassignment because of allowing additional running and dwell times

Here, we analyze the effect of the allowed additional running and dwell times on the method of train rescheduling and passenger reassignment by comparing the results for the experiment groups with the same duration of disruption (*Dur*), passenger demand (*D*), and total available train capacity (*C*); the only difference between the groups is the setting for the allowed additional time. The allowed

**Table 7**
Train delay analysis for optimized and actual solutions with maximum number of saved disrupted passengers ($\alpha = 1$) for the case where *ART* and *ADT* are all infinite, *Dur* is 55 min, and passenger demand ($D = 900$) is greater than total available train capacity ($C = 870$).

| Optimized solution | | | | Actual solution | | | |
|---|---|---|---|---|---|---|---|
| **Primary delay (min)** | 65 | | | **Primary delay (min)** | 61 | | |
| **Secondary delay (min)** | 112 | **Delay for extra stops (min)** | 77 | **Secondary delay (min)** | 360 | **Delay for extra stops (min)** | 301 |
| | | **Delay for more dwell time (min)** | 6 | | | **Delay for more dwell time (min)** | 23 |
| | | **Delay for conflict relief (min)** | 29 | | | **Delay for conflict relief (min)** | 36 |
| **Total delay (min)** | 177 | | | **Total delay (min)** | 421 | | |

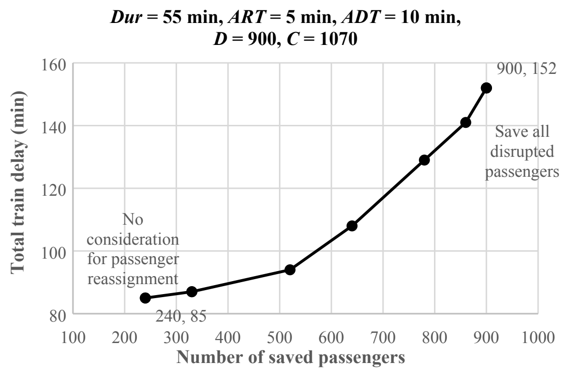


**Fig. 8.** Pareto front for experiments where *Dur* is 55 min, *ART* is 5 min, *ADT* is 10 min, and *D* (900) is less than *C* (1070).

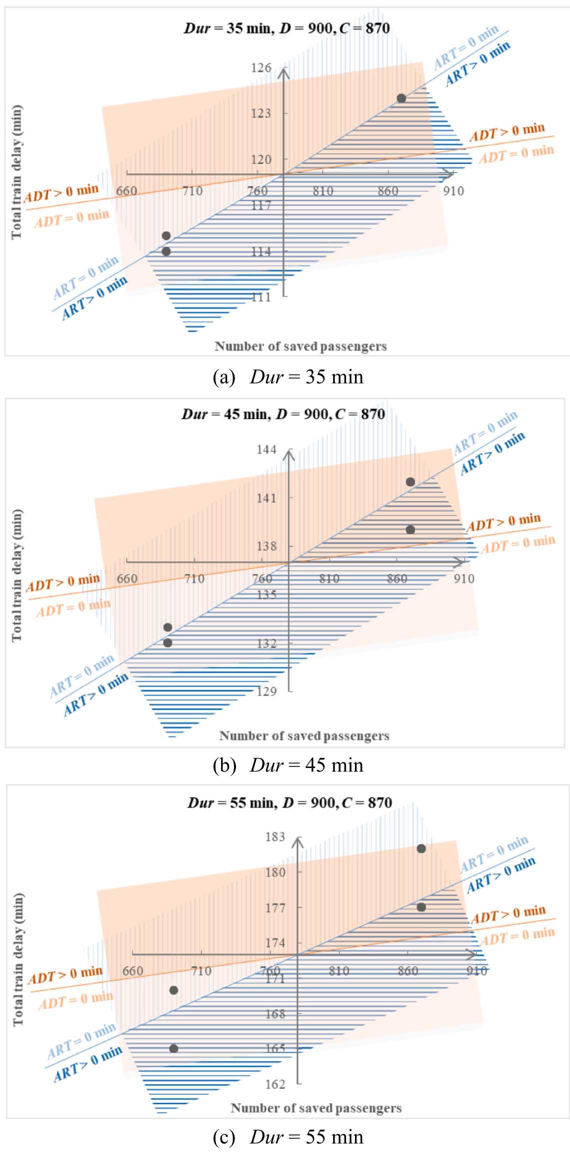


**Fig. 9.** Results of experiments where $\alpha = 1$ and *D* (900) is greater than *C* (870) for different settings of *ART* and *ADT* and *Dur* values of 35, 45, and 55 min.

additional running time (*ART*) is set as 0 min, 5 min, and infinity, whereas the allowed additional dwell time (*ADT*) is set as 0 min, 1 min, 10 min, and infinity. Fig. 9 presents the detailed results of the experiments where the weight for the objective of maximizing the number of saved passengers ($\alpha$) is 1 and *D* is greater than *C*; here, 12 combinations of different settings of the *ART* and *ADT* are considered for *Dur* values of 35, 45, and 55 min. It should be noted here that the solution is the same for some combinations, therefore there are less than 12 nodes shown in Fig. 9(a), (b) or (c).

If the *ART* is set to be greater than 0 min (e.g., the area in dark blue), the total train delay will not exceed that under the condition where the *ART* is set as 0 min (e.g., the area in light blue). Especially under the conditions where *Dur* is 45 and 55 min (Fig. 9 (b) and (c)), the total train delay for the condition with the additional running time will be considerably less than that for the condition without the additional running time. When we compare the conditions with different settings of the *ADT* (0 compared to greater than 0, area in light orange compared to that in dark orange), more disrupted passengers can be served when the *ADT* is greater than 0 (870 compared to 690). This is mainly caused by the necessary extra dwell time for a stop to serve disrupted passengers. If one train has a preplanned stop at the transfer station or passenger destination station for 2 min and the allowed additional dwell time is set to 0, this train cannot be rescheduled to serve disrupted passengers because the minimum dwell time is 3 min. In this case, if we want to transport as many disrupted passengers as possible, extra stops will be added to a limited number of other trains without a preplanned stop at the corresponding stations, resulting in a less number of saved passengers. It can be concluded that the negative impact of passenger reassignment on train rescheduling can be reduced by allowing additional running and dwell times.

#### 5.4.3. Reduction of negative impact of passenger reassignment as duration of disruption increases

We compare three experiment groups with the same settings for the impact factors of the allowed additional running time (*ART*) and dwell time (*ADT*), passenger demand (*D*), and total available train capacity (*C*) for different durations of disruption (*Dur*) of 35, 45, and 55 min. In all these cases, we obtain similar conclusions. Fig. 10 shows the detailed results of the experiment where the *ART* and *ADT* are allowed and *D* is less than *C*. For different durations of disruption, the trend of the Pareto front is similar. The difference between the Pareto fronts for the durations of 45 and 55 min is larger than the difference between the Pareto fronts for the durations of 35 and 45 min.

For every Pareto front, we select three types of nodes to further explain this aspect. Every three nodes on one Pareto front present the objective values for different weight combinations for two objectives. The first and second numbers in the square bracket ([ ]) represent the weights of the number of saved passengers and weighted total train delay, respectively.

A comparison of the three left nodes for the weight [0/1] indicates that for the disruption time of 35 and 45 min, we save the same number of disrupted passengers when we minimize the total train delay. The number of saved passengers is equal to the total available capacity of Trains G2, G8, G14, and G19, which have stops at the transfer station (Zhenjiang South Station) and destination stations of the disrupted passengers. In addition, the dwell time at Zhenjiang South Station is satisfied with the embarking of the disrupted passengers, as shown in Fig. 6. For the disruption time of 55 min, the total number of saved passengers is 240, which is equal to the total available capacity of Trains G2, G4, G8, G14, and G19, because G4 is rescheduled to dwell at Zhenjiang South Station for a dwell time of 3 min for the disrupted passengers in the disposition timetable. A comparison of the three middle nodes for the weight [0.5/0.5] indicates that as the duration of disruption increases, more passengers can be saved even though we set the same importance degree for the number of saved passengers and weighted total train delay. A comparison of the three right nodes for the weight [1/0] indicates that the number of saved passengers is as expected for all the durations of disruption and is equal to the total passenger demand.

A comparison of the right and left nodes for different durations of disruption (Table 8) indicates that the increased train delay rate decreases with an increase in the duration of disruption. Meanwhile, the total number of disrupted passengers who reach their destinations is considerably different between the two extreme conditions, with an increased rate of greater than 275% when we consider the passenger reassignment strategy. The increased passenger rate is always greater than the increased delay rate for the same duration of disruption.

Here, it can be concluded that if we want to save the same number of disrupted passengers under different durations of disruption, the total train delay will be larger under a longer duration of disruption. We can also conclude that the negative impact of the passenger

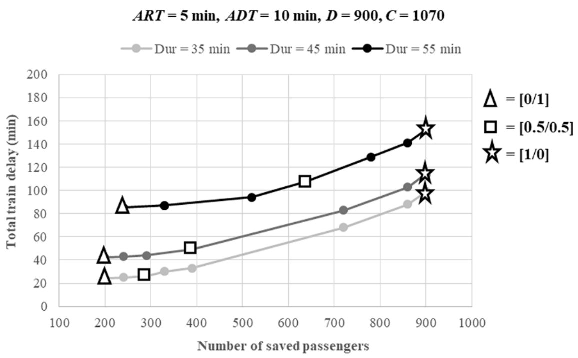


**Fig. 10.** Pareto fronts for experiments where *ART* is 5 min, *ADT* is 10 min, and *D* (900) is less than *C* (1070) with different *Dur* values of 35, 45, and 55 min.

**Table 8**
Increased delay rate *DR* and increased passenger rate *PR* for different durations of disruption.

| *Dur* (min) | $TD_{npr}$ | $\#P_{npr}$ | $TD_{pr}$ | $\#P_{pr}$ | *DR* | *PR* |
|---|---|---|---|---|---|---|
| 35 | 24 | 200 | 99 | 900 | 312.5% | 350% |
| 45 | 42 | 200 | 114 | 900 | 171.4% | 350% |
| 55 | 85 | 240 | 152 | 900 | 78.8% | 275% |

reassignment strategy on the total train delay decreases when the failure duration increases (Table 8). This will help dispatchers to assess the cost of passenger reassignment in advance according to the estimated duration of disruption. If the duration of disruption is relatively long, dispatchers can focus more attention on passenger reassignment because the relative negative impact will be less.

#### 5.4.4. Increased negative impact of passenger reassignment because of limited total available train capacity

Here, we analyze the relationship between the passenger demand (*D*) and total available train capacity (*C*). Fig. 11 presents the detailed results of the experiments where the *ART* and *ADT* are set as 5 and 10 min, respectively. Considering the leftmost nodes on every Pareto front, for different relationships of *D* and *C*, the minimum total train delay is the same for the same duration of disruption, and the difference in the corresponding number of saved passengers is dependent on the available capacity setting for each train. When *D* is less than *C*, the maximum number of saved passengers is consistent with the passenger demand of 900, as indicated by the rightmost nodes represented by gray lines. When *D* is greater than *C*, we can serve a maximum of 870 disrupted passengers because of the limitation of the total available train capacity, as indicated by the rightmost nodes represented by black lines.

A comparison of each pair of gray and black lines with the same mark (triangle, square, or circle) indicates that when the weight of the number of saved passengers is small and the duration of disruption is short, the difference in the results between the two conditions—*D* less than *C* and *D* greater than *C*—will not be too large. When the weight of the number of saved passengers becomes larger, the difference will be bigger. This is because when we want to transport more disrupted passengers and the passenger demand is greater than the total available train capacity, extra stops need to be added to almost all the trains. When the passenger demand is less than the total available train capacity, only some of the trains need to be rescheduled (owing to the addition of extra stops) to pick up disrupted passengers. Even for a duration of disruption of 35 min (*D* greater than *C*) and a larger weight of the number of saved disrupted passengers, the total train delay will exceed that for a disruption time of 45 min (*D* less than *C*) and the same weight of the number of saved disrupted passengers, as represented by the black line with triangles and gray line with squares, respectively.

Next, we compare the computation time for the above experiments, as shown in Fig. 12. When we set the weight as [1/0], the number of saved passengers will be maximized; under this condition, the total train delay will not be limited. Therefore, we do not list the computation time for this condition here. When we compare the computation time for the experiments with the same settings for *ART*, *ADT*, *D*, and *C* under different durations of disruption, as represented by the lines of the same color, we can observe that the longer the duration of disruption and the larger the weight of the number of saved passengers in the bi-objective function, the longer is the computation time. When we compare the computation time for the experiments with the same settings for *ART*, *ADT*, and *Dur* under different relationships between *D* and *C*, as represented by the lines with the same shape but with different colors, we find that for a longer disruption duration and a larger weight for the number of saved passengers, the difference in the computation time for the two conditions—*D* less than *C* and *D* greater than *C*—is more obvious, and the computation time when *D* is less than *C* is longer. The reason for this is that when *D* is less than *C*, the feasible solutions of rescheduling, i.e., extra stops should be added to which trains to transport disrupted passengers, are more and not all trains need to change their stopping plans. When *D* is greater than *C*, if we want to transport more passengers, extra stops should be added to most trains and we should decide as to which trains should take how many passengers of which passenger groups.

Here, it can be concluded that when *D* is greater than *C* and we want to transport as many disrupted passengers as possible, the total train delay will be considerably larger. However, the computation time in this case is shorter than that when *D* is less than *C*. This can help dispatchers to assess the cost of passenger reassignment in relation to the train delay in advance by judging the relationship between the passenger demand and total available train capacity. If the total available train capacity is not sufficient for the disrupted passengers, dispatchers should be cautious to add extra stops for passenger reassignment, which will have a relatively large negative impact on the train traffic.

## 6. Conclusions and future research

This paper proposes a mixed-integer linear programming formulation for the integration of the train rescheduling problem with passenger reassignment under large disruptions (caused by a rolling stock breakdown) in a railway system with a seat reservation mechanism. The remaining part of the train service with the broken rolling stock is canceled. All the impacted passengers should be reassigned to the following trains, which have to be rescheduled with extra stops for offering a new service to the disrupted passengers. However, not all of the disrupted passengers may be served by the following trains, thus forcing the railway operating company to refund the ticket fares and causing a huge loss.

From the results obtained in this work, we make the following observations. First, our method can optimize train rescheduling and passenger reassignment in an efficient and effective manner and thereby facilitate real-time rescheduling and re-ticketing. Second, if additional running and dwell times are allowed, there will be less train delay and more saved passengers. Third, the longer the duration of disruption, the relatively less is the negative impact on train delays. When we want to make a plan with relatively small delays, a

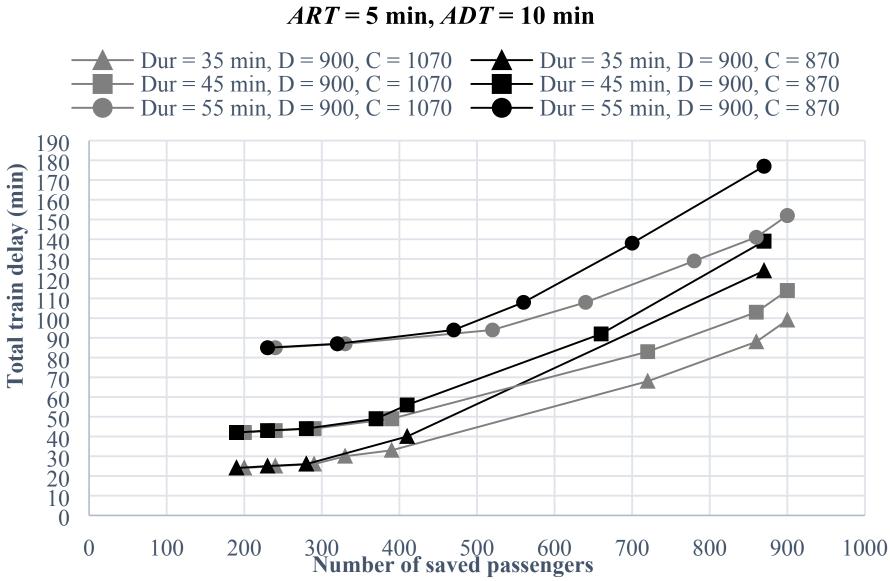


**Fig. 11.** Pareto fronts for experiments where *ART* is 5 min and *ADT* is 10 min for different settings of relationship between *D* and *C* and *Dur* values of 35, 45, and 55 min.

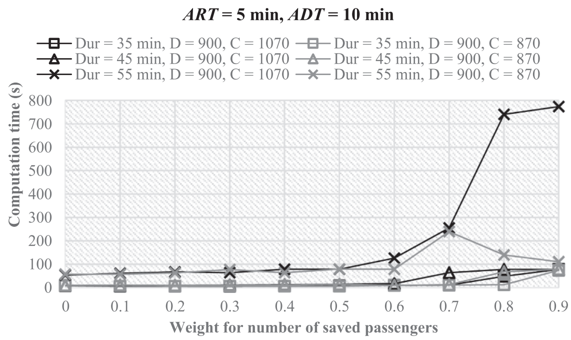


**Fig. 12.** Computation time for experiments where *ART* is 5 min and *ADT* is 10 min for different settings of the relationship between *D* and *C* for *Dur* values of 35, 45, and 55 min.

large weight should be set on the train delay for a long duration of disruption rather than a short duration of disruption. When we want to transport more disrupted passengers, a large weight should be set on the number of saved passengers for a short duration of disruption rather than a long duration of disruption. This is because the solutions for the integrated problem of train rescheduling and passenger reassignment have different sensitivities for the weight settings for the objectives under different durations of disruption. Finally, when the passenger demand is greater than the total available train capacity, passenger reassignment will have a larger negative effect on the train traffic.

In practice, dispatchers can use our proposed method to select the best rescheduling measures. The insights from our analysis results will help them to determine the correct parameters to be included and select the optimal weight settings for the objective functions according to the actual situation, finding a balance between passenger and operational inconvenience. For example, when there are many passengers stranded in stations, transporting the disrupted passengers will be more important, which can be achieved by setting a relatively large weight for the objective of maximizing the number of saved passengers. If the rolling stock units are to be used to handle some important or long-distance services next, a large delay should be avoided by setting a relatively large weight for the objective of minimizing the weighted total train delay.

The proposed methodology can be extended to deal with other rail networks with a similar seat reservation mechanism. A different network can be described as a directed graph with a set of nodes and directed arcs in the same way mentioned in this paper. Trains traveling at different speeds can be considered by setting different input parameters, like the travel time and headway time. The priorities can be set for trains traveling at different speeds in one rail network. Meanwhile, different kinds of disrupted passengers can be distinguished by giving priorities, like the ticket price for passengers traveling on the high-speed trains higher than the regional trains. The disrupted passengers who were supposed to travel on the high-speed train can be given a higher priority to reduce the revenue loss due to the limited available seat capacity.

Meanwhile, the proposed methodology can be utilized to solve a general train rescheduling problem with the objective and constraints relating to the passenger reassignment relaxed. Retiming, reordering and rerouting can be achieved with the consideration to minimize the total train delay.

In future research, considering that some trains will be broken, the optimization of rolling stock circulation could be reorganized to ensure better utilization of the rolling stock by considering the needs of the entire railway network. Further research should also focus on the uncertainty of disruptions because in most cases, we will not have full information regarding the disruptions at the beginning of the disruptions. Some stochastic solutions can be developed in this regard.

## CRediT authorship contribution statement

**Xin Hong:** Conceptualization, Methodology, Software, Writing - original draft. **Lingyun Meng:** Conceptualization, Methodology, Writing - review & editing. **Andrea D'Ariano:** Conceptualization, Methodology, Writing - review & editing. **Lucas P. Veelenturf:** Conceptualization, Methodology, Writing - review & editing. **Sihui Long:** Software, Writing - original draft. **Francesco Corman:** Conceptualization, Methodology, Writing - review & editing.

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