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OPTIMIZATION OF STRATEGIC PLANNING PROCESSES FOR CONFIGURABLE PRODUCTS: CONSIDERATIONS FOR MARKETING, SUPPLY CHAIN, AND SUSTAINABILITY

by

EDWARD LAWRENCE UMPFENBACH

DISSERTATION

Submitted to the Graduate School

of Wayne State University,

Detroit, Michigan

in partial fulfillment of the requirements

for the degree of

DOCTOR OF PHILOSOPHY

2013

MAJOR: INDUSTRIAL ENGINEERING

Approved By:

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Advisor Date

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DEDICATION

Dedicated to my friends and family for all their support

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I would like to thank my advisor, Professor Ratna Babu Chinnam, for the incredible amount of dedication and support during my time as a student of his. He has inspired me to be a better student and person. I would also like to thank Dr. Ekrem Alper Murat and Dr. Evrim Dalkiran for invaluable advice when my progress stalled out. It was greatly appreciated. Next, I would like to thank Gintaras Puskorius and Dean Pichette for providing great insight into the automotive industry and helping to keep my work practically significant. I would like to thank the remaining members of my dissertation committee, Dr. Leslie Monplaisir and Dr. Yinlun Huang, for their support and recommendations throughout this journey. Last but not least, I would like to thank my mom, dad, and two sisters for always being there for me whenever I needed them. I could not have done this without you all.

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Chapter 1: Introduction

There are a few key questions that businesses need to consider when developing a strategic plan defining how they will compete and be successful in today's economic climate. These broad questions include (not exclusively) "what products will the firm provide to our customers to reach an appropriate level of customer satisfaction?" and "how can the firm supply and price these products to customers in a way that leaves costs low enough to provide adequate profit for its shareholders?" Despite being simple questions, the answers can be incredibly complex in today's environment of rapid technological changes and rising consumer expectations. Shrinking product life-cycles and a proliferation of products has subjected companies to tremendous pressure in planning, developing, and offering their product to fragmenting segments. Companies that deal with highly engineered, configurable products such as automobiles can face especially difficult and complex decisions. Henry Ford's famous statement "Any customer can have a car painted any color he wants so long as it is black" (Ford and Crowther 1924) may have been the idea that jump started the Ford Motor Company down the path to becoming one of the largest companies in the world, but it clearly would not work in today's automotive sales environment.

Recent trade articles in the automotive industry have presented evidence to suggest that in the absence of objective and comprehensive strategic planning models, companies seem to be struggling with decisions regarding their products, in particular, the size and variety of their product configuration assortments. A product assortment is the set of orderable configurations for the product and can be rather large depending on the number of core options (e.g., number of engine, transmission, and seating choices available for an automotive model) and optional content (e.g., navigation system and moon roof) available for the product. Some companies have dramatically reduced the size of their product assortments, while others are offering increasing product configuration complexity. In 2007, Volvo introduced the C30, a vehicle expected to sell approximately 10,000 units, with the slogan “The car you can customize 5 million ways to be just like you.” At nearly the same time, Ford Motor Company announced a reduction in the ordering complexity of the F-150 truck by more than 90%, with most car lines offering fewer than 1,000 combinations (Wilson 2008). In order to achieve these goals, Ford has taken such actions as reducing the number of global seat frames from twenty-eight to two and the number of six-cylinder engine families from eight to two (Wilson 2008). We believe that the auto industry is lacking in models to make the tradeoff between supply chain complexity (we utilize the definition of supply chain from Chopra and Meindl (2007) – “all parties involved, directly or indirectly, in fulfilling a customer request,” – not simply logistics) and assortment satisfaction, instead relying on managerial experience and heuristics to expand or decrease the number of configurations they offer to their customers in an attempt to increase profitability.

1.1 Research Objective

The objective of this research is to develop assortment planning models that holistically address the needs of companies producing configurable products by explicitly accounting for both supply and demand considerations. We believe that these models can significantly improve profitability over existing heuristics, such as a sequential planning process. In a sequential planning process, a marketing group might first make a projection of demand without knowledge of the effect that the assortment might have on the complexity and cost of the supply chain. A supply chain group then takes that demand projection as input, and works to develop a supply chain that meets the projection, with some limited feedback between the two groups. By taking the decisions jointly, we aim to find a more profitable solution for the firm as a whole. The models must scale to the levels required by a producer of configurable goods – sometimes tens of thousands of configurations. In addition to the fundamental idea of this research that integrated strategic planning will improve profitability, we extend our model to apply it to two more important decisions in the automotive industry today – 1) The effect these decisions have on the sustainability of the product, both during its production and life cycle and 2) The effect that packaging has on both demand and the supply chain (we use the term packaging instead of the popular term "bundling" by choice). Throughout the dissertation, we mostly focus on configurable automotive products. However, the models and tools developed can be relevant for other configurable products (e.g., computers, tablets, printers, cell phones, copiers, televisions, play sets). The models mostly assume an assemble-to-stock production environment and would need significant modification to support a mostly assemble-to-order environment.

1.2 Dissertation Organization

In the second chapter, we introduce our model to integrate assortment planning and supply chain management decisions. We utilize a multinomial logit (MNL) demand model, which through the Charnes-Cooper transformation can be transformed into a linear programming problem under certain conditions. Through the MNL model we model assortment based substitution (substitution occurring from whether a product is offered or not) but cannot handle stock-out based substitution (where a product is offered, but is temporarily out of stock). On the supply chain side of the problem, we explicitly model the costs associated with developing components, manufacturing them, assembling them into finished configurations, and the associated complexity cost in the supply chain (defined in detail in chapter 2). We model a centralized supply chain, where the goal is to maximize the total profit across all tiers (i.e., all supplier plants are owned by the configurable product assembler and work for a total combined profit). Throughout the dissertation, select data elements will be presented where appropriate, with full data explanation left for the appendices. The effect of packaging on the assortment and supply chain of a configurable product manufacturer is the subject of Chapter 3. Chapter 4 studies how sustainability can be incorporated into our model and the effects that it has on the assortment and supply chain. Chapter 5 concludes the dissertation and offers recommendations for future research.

Chapter 2: Integrated Automotive Assortment Planning Model

2.1 Introduction

Assortment planning is a field of research where decisions regarding the set of products to offer to customers are made. It is a highly strategic decision and impacts nearly every subsequent decision a firm will make. Demand is often treated endogenously, and the objective is to either maximize revenue or profit with consideration for substitution effects subject to some constraints, usually a fixed cost for stocking a product or a limit on the available shelf space.

We make a set of broad claims about the current process of strategic planning in large automakers. These claims might apply more strongly to some automakers than others, but we believe that all face the same problem in some form – Assortment planning decisions are usually relegated to a marketing group. Although top level leadership tries to set incentives and motivate marketing professionals to act in a way to maximize profitability of the firm as a whole, their success is often based on traditional marketing metrics – sales, revenue, conquest sales, and market share. Thus, marketing professionals often have incentives that push them to expand the number of product offerings to the customer. In addition, even if they are cognizant of the effects that assortment decisions have on manufacturing and supply chain costs, they may lack the fidelity of information to understand anything more than the directional impact their decisions have (e.g., more product variety leads to increased complexity costs, but how much?).

The operational impacts of product variety were well known to Henry Ford in the days of the Model-T. Academic literature supports that product variety increases complexity cost, decreases economies of scale, and causes more volatility in demand forecasts (Fisher, Ramdas et al. 1999), (Pil and Holweg 2004). The field of mass customization has developed a set of tools to reduce the negative impacts of assortment complexity by reducing lead times and increasing manufacturing agility (Fogliatto, da Silveira et al. 2012), however, it cannot fully mitigate the pervasive effects of assortment complexity on operational cost in a supply chain. In addition, analogous to the claims about a lack of supply chain knowledge in marketing groups, supply chain groups often lack knowledge about customer demands and market trends. Thus, they lack the ability to independently and objectively decide on the product assortment “sweet spot” between customer satisfaction and cost of complexity.

It is in this scenario that we believe there is a huge opportunity for an integrated approach to product variety. Empirical evidence of the European auto market suggests that there is not a strong correlation between product variety and sales, suggesting that other market factors play key roles in determining sales (Pil and Holweg 2004). Bowman and Kogut (1995) claim that whenever confronted with a decision regarding product variety, the U.S. Big-3 automakers have chosen to reduce complexity, which they attribute to a focus to catch Japanese automakers in terms of productivity and quality. This heuristic may have been successful in the 1990s and beyond due to excessive levels of complexity to begin with, but logic dictates that eventually automakers will reach a point where reducing complexity is no longer a dominant decision (they may have already). A limited number of articles have been published that connect assortment and supply chain complexity. Many assortment planning models: 1) have overly simplistic supply cost assumptions; 2) neglect the cost of complexity in their decisions; and 3) do not scale to problem sizes required by automakers.

We believe we have developed a model to overcome these shortcomings. We jointly model a two-tier supply chain. However, unlike conventional supply chain models where demand is treated as an exogenous parameter that constrains the supply chain, we view it as an endogenous variable to be decided on. We utilize a multinomial logit (MNL) demand model, with a single, deterministic decision to be made on the assortment and the supply chain configuration to meet the assortment-conditional demand.

The rest of the chapter is organized as follows: section 2 reviews the literature relevant to the problem. In section 3 we explicitly define the assumptions made in our modeling approach. In section 4 we present the model mathematical formulation. In section 5 we discuss the experiment we will use to demonstrate the model, results, and sensitivity analysis. In section 6 we conclude and comment on future research avenues.

2.2 Literature Review

For a more thorough review of assortment planning literature than this chapter could possibly give, see Kök and Fisher et al. (2009). We beginning by summarizing the information from this literature review most relevant to our research, then proceed to discuss the contributions of individual research papers.

The literature on assortment planning can be broadly divided into two categories based on the type of substitution effects they model. Assortment based (or “static”) substitution neglect the effects that stockouts have on a customer’s purchase, while stock-out based (or “dynamic”) substitution models include the effects of stockouts. Assortment based substitution models can be thought of as a special class of stockout-based models in which inventory levels are either zero or the product never goes out of stock. In general, stock-out based models tend to be more complex and more difficult to solve. Due to an intractability of the problem with an unlimited number of substitution attempts, usually the assumption is made that a customer who doesn’t find their first choice makes one attempt to substitute. If the first substitute is unavailable, the customer will choose to walk away without purchasing.

Another broad division of assortment planning models is based on the type of the type of demand model used – utility based or an exogenous demand model. Utility based models utilize well known econometric models such as those in the logit family. Exogenous demand models explicitly define the demand for each product and how customers react to stockouts. Exogenous models have greater degrees of freedom, but as a consequence, also have more parameters to estimate.

The functional format of the MNL model will be introduced later in our model. Van Ryzin and Mahajan (1999) studies assortment planning under the MNL demand model and static substitution. They model identical selling prices and costs across all products, with costs assumed to have a concave structure at different levels of production. Thus, the problem becomes one of trading off the cannibalization of products currently in the assortment and lost demand customers walking away. It can be proven under this situation that the optimal set of products is also the popular set. Mahajan and Van Ryzin (2001) extend their previous model to include stockout-based substitution. Using a stochastic gradient algorithm, they conclude that more of the more popular and less of the less popular products should be stocked than a traditional newsvendor analysis would suggest. Maddah and Bish (2007) extend the model of Van Ryzin and Mahajan (1999) to include pricing decisions. They discuss structural properties of optimal solutions, such as that the optimal assortment contains the set of products with the highest reservation price when costs are identical (the case of horizontally differentiated products). Cachon and Terwiesch et al. (2005) extend the static MNL problem to the case where customers are unsure of the assortments that are carried by retailers. As they search, they become more confident in their ability to choose. Schon (2010) presents a method to overcome the nonconvexity of the assortment problem when selling price is an endogenous variable. By treating attractiveness as the decision variable instead of price and utility, a convex MINLP can be formulated. The method works for several types of utility based models, the MNL model included. Schon (2010) extends the work of Chen and Hausman (2000) to include personalized pricing of products. Both articles utilize the fractional programming structure of many utility based models to create mixed-integer models with relatively few to zero binary variables from problems that would generally be intractable.

In the locational choice model, products are defined by a set of characteristics which corresponds to an ideal point in the characteristic space. A function represents the distance of a substitute product from the customer's ideal point. In contrast to a logit based model, substitution in locational choice models occurs to products with similar characteristics to the original first choice, as opposed to possible substitution to any product. Fisher and Vaidyanathan (2008) present a demand model formulation similar to a locational choice model. They develop maximum likelihood estimators for the demand parameters of their problem and describe a greedy heuristic to solve the assortment problem. They demonstrate their model on examples of snack food and tire retailers. Gaur and Honhon (2005) discuss properties of an optimal assortment under a locational choice model with assortment based substitution. They extend their model to the case of stockout based substitution by approximating it with the case where the retailer can control the substitution activity of a customer.

Smith and Agrawal (2000) present an exogenous demand model with a newsvendor problem definition. They develop bounds on the effect of stockout-based substitution and show that for high service levels, the effect of stockout based substitution is small enough that it can be ignored. Kok and Fisher (2007) use exogenous demand to model sales at a chain of supermarkets in the Netherlands. They determine demand under stockout substitution using a simulation model, and develop a greedy heuristic to optimize the assortment.

A topic closely related to assortment planning is the subject of demand model data estimation. A model's real-world usefulness is closely tied to the data available as input. A significant amount of marketing research has tackled the problem of estimating customer utility. The work of Guadagni and Little (1983) demonstrates how to fit an MNL model to panel data, such as data from a customer loyalty-rewards program. Anupindi and Dada et al. (1998) developed maximum likelihood estimators from periodic (stockout times are unobserved) and perpetual (stockout times are observed) inventory systems with Poisson demand. In Agrawal and Smith (1996) the authors make a case for the negative binomial demand distribution being a better fit in some cases than normal or Poisson demand, and present a method to estimate its parameters from sales data.

The opposite side of the coin focuses on complexity and its effect on profitability of a supply chain. Yunes and Napolitano et al. (2007) estimate customer rankings of configurations and solve a mixed integer program to maximize profit for John Deere Company. The specifics of their measure of complexity costs are kept proprietary but they define the costs as "the costs associated with frequent changeovers, reduced efficiency, and the overhead of carrying extra configurations." Kusiak and Smith (2007) developed a clustering approach to managing complexity of configurable products. They first identify a subset of "prime configurations" with attributes similar to customer demands through clustering, and then find the minimum number of prime configurations needed to satisfy customer service levels. Huang and Su (2010) investigate the impact of complexity on the reverse loop of a closed supply chain. They use a queuing model approach to find the optimal batch sizes for products under various conditions. Chen and

Plambeck (2008) derive maximum likelihood estimates of demand and substitution rates for a capacitated production-inventory system under continuous review (such as a customer accepting a rain check for an unavailable product). They find that it is sometimes optimal to reduce inventory levels to learn substitution parameter to prevent overstocking in later periods. Lamothe and Hadj-Hamou et al. (2006) discuss the problem of optimizing product families and supply chains of modular products. Their main tradeoff is whether to over-satisfy demand with higher level products or to introduce many products at different levels. Kim and Chhajed (2000) model the optimal amount of modularity for a product, considering modularity reduces supply chain costs but also reduces customers' ability to distinguish high quality products from low quality ones. ElMaraghy and Mahmoudi (2009) consider the supply chain configuration of a modular product and focus on the decision whether to outsource the modules.

There are numerous articles that deal specifically with the complexity in the auto industry. MacDuffie and Sethuraman et al. (1996) present a practical look at product variety in the automotive industry and tools to overcome it's negative effects, such as process flexibility and common parts sharing. Fisher and Ittner (1999) conduct empirical analysis into the effects of product variety on the efficiency of automotive plants. The study found that variability in the day-to-day option content of vehicles, but not the mean number of options per vehicle, had a negative impact on labor hours per car, assembly line downtime, rework, inventory, and overhead hours per car. Cachon and Olivares (2010) find that differences in inventory between Toyota and the Big-3 domestic automakers can be attributed to differences in the number of dealers and the amount of plant flexibility. Ro and Liker et al. (2007) discuss modularity in the auto industry, which is closely related to product variety. The authors discuss factors pushing for modularity and the organizational effects it has on an automaker.

2.3 Assumptions

We explicitly define the assumptions behind our model, as well as discuss the reasoning for making the assumptions.

1) Demand follows a multinomial logit model.

The MNL model is a probabilistic model with a closed form solution for the probability of choosing an individual product, which makes it an attractive model for the purposes of optimization. Utilizing the notation of Kok and Fisher et al. (2009), let S represent the set of choices presented to the customer and 0 represent the option of not purchasing from the assortment. A customer associates each option j with a utility value, $U_j, j \in S \cup \{0\}$. Each utility value is separated into a deterministic portion of utility, u_j, and a random portion of utility, ε_j, $U_j = u_j + \varepsilon_j$. The error term is assumed to follow a Gumbel distributed random variable. The probability of purchasing a product j is given in equation (**1**). The Gumbel distribution is closed under maximization, leading to the expression in equation (**2)** for the probability a customer chooses a product j from the assortment. See Anderson et al. (1992) for a proof of these claims.

$$P_j(S) = P(U_j = \max_{k \in S \cup \{0\}} U_k) \tag{1}$$

$$P_j(S) = \frac{e^{u_j}}{\sum_{k \in S \cup \{0\}} e^{u_k}} \tag{2}$$

The MNL model is not without its drawbacks, however. The most widely recognized of which is the Independence of Irrelevant Alternatives (IIA) assumption, stating that the ratio of choice probabilities of two alternatives is independent of the other alternatives in the choice process (Kök, Fisher et al. 2009). This can be explained in our case through an example. Consider two vehicles a customer is considering, one blue and one red, with equal interest (i.e., P(blue) = P(red) = ½). Now let us suppose a new model is added to the customer's consideration, a blue vehicle with floor mats (which were previously not offered in the blue vehicle). Assuming floor mats have very little to no impact on a customer's choice, we would expect that P(red) = ½ and P(blue) = P(blue with floor mats) = ¼ . However, because of the IIA assumption, we would see P(red) = P(blue) = P(blue with floor mats) = ⅓. In reality, the option of a blue car with floor mats was not irrelevant, as it is a near perfect substitute for the other blue vehicle.

The nested multinomial logit is an attempt to overcome the IIA property in which customers take a hierarchical approach to product selection (i.e., they might first select a vehicle model, then choose between the color options) but it is a nonlinear model that does not feature the mathematical properties that make the MNL model capable of handling the large assortments in the automotive industry. Beyond the logit family of demand models, there are a multitude of other demand models that could be utilized (e.g. preference set, locational choice, exogenous, etc.). Very few possess the mathematical properties of the MNL model that make it attractive to problems of very large size, however.

We believe that the MNL model is sufficient for modeling demand in our case for several reasons: 1) Our model is designed to be used in the strategic planning phase (2+ years in advance of production for automotive products) when future customer demands are relatively uncertain. Our model is designed to strike a good balance between supply chain complexity and product variety, not to define on an operational level how many of specific configurations will be supplied to dealers when production begins, which would require a more sophisticated treatment of demand modeling. 2) The negative effect of the IIA assumption can be mitigated by modeling components that are reasonably close to each other in terms of price and customer importance (not irrelevant options such as floor mats) and 3) Our model is valuable in providing a canvas to start discussion between managers in industry leading to a better understanding of the problem, not a single run that provides a blueprint for the product assortment.

2) The supply chain is centralized (i.e., all costs are summed to form a single objective function)

Automakers today typically manufacture far fewer of the parts that are assembled into cars than they did in the past (Ro, Liker et al. 2007). Often only a few key components are produced in house (engine, transmission, body, etc.) with the rest pushed onto their supplier base. In our model, we consider facility location and production costs for all components, as if they were all owned by the automaker (even some parts that would typically never be manufactured by an automaker, such as a radio). This could easily be changed to reflect a scenario where the automaker only incurs a fixed price for purchased parts (possibly with economies of scale) and the only decision is the quantity purchased.

3) Selling prices for options are exogenous and fixed (excluding any effects of packaging in Chapter 3), and thus, utilities are exogenous and fixed for each customer segment.

Assumption 3 is based on several ancillary assumptions. From industry web sites, we can see that automakers frequently divide their customers for a vehicle model into segments that are targeted with a series. A series is usually differentiated according to the amount of optional content available as well as the customer perceived value associated with the standard options included in the series (leather seats over cloth, for example). We assume that prices will be set in a way that will push customers towards a series based on their price sensitivity. Thus, we assume that the prices for options, the utilities for options at those prices, and the number of customers that will consider purchasing from each series are known a priori.

4) Demand is deterministic and the manufacturer is required to meet the assortment-conditional demand. Thus, there is no need for supply replenishment.

These assumptions are made to keep the initial problem simpler. Stochastic demand with/without supply replenishment is left as a future extension.

5) We do not model costs on a platform level.

We model demand for a single mid-sized passenger car program (such as a “Ford Fusion” or a “Chevy Malibu”). However, it is very common in today’s automotive industry to have vehicles built on common platforms within a single automaker, which share many common components and are built on the same assembly lines. We assume that we are able to separate a vehicle from its platform and define each vehicle program’s share of the costs, capacities, etc. For example, we assume that there is already known demand for a certain type of radio, and we model our economies of scale as cost savings if the vehicle uses a certain number of components on top of what will be used by the others. Our model could be adapted so that production planning can be done on a platform basis and sales occurred on vehicle model basis across several models, provided adequate computational power to handle the larger problem. It was not attempted here due to the amount of data needed for multiple vehicle programs.

6) We assume that the product is available under several series (e.g., a base-series, a mid-series, a high-end series, and a sport-series), as is typical in the automotive industry.
7) The base-series (denoted S) is constrained so that it always has at least one configuration chosen for the assortment, with few total option choices in the series.

From industry input, we know that the base-series typically has very small supplied inventory and can sometimes serve as a loss leader. Its purpose is to set the base manufacturer’s suggested retail price (MSRP) for the vehicle line as low as possible for marketing purposes (e.g., generating lot of traffic to the dealership). Thus, we constrain the assortment in the base-series to include only two powertrain choices, a single choice for radio and seat variant, and no binary options are allowed.

8) Each demand region can be thought of as a single store where all customers purchase from or walk away.

In reality, automakers face strong channel inventory constraints that limit the amount of vehicles they can store on dealer lots. Some transshipment (exchange of vehicles) occurs between dealers. The majority of customers buy vehicles off a nearby dealer's lot but some custom order the vehicles they prefer to own. Our model cannot model this complex ordering system, so we assume no stockout substitution and neglect the effects of a dealer ordering process. That level of detail is left for operational level decisions regarding assortments that come during the vehicle production phase.

2.4 Model Formulation

Please Note: The model is first presented in its fractional, binary form and then it is transformed into the form used in the experiments. The nonlinear form of the model is not utilized in practice.

2.4.1 MNL Demand Model

Let $I = \{1, \dots, n_i\}$ be the set of configurations, $i, j \in I$, $K = \{1, \dots, n_k\}$ be the set of series, $k \in K$, and $L = \{1, \dots, n_l\}$ be the set of demand regions, $l \in L$. Each series and demand region combination represents a customer segment. Let a_{ikl} and $a_{\emptyset kl}$ be the attractiveness of configuration i and the combined attractiveness of all outside options, respectively, in a certain customer segment. Let ω_{kl} be the size of a customer segment. p_{ikl} denotes the selling price of configuration i in each segment. We can define (inefficiently) a binary variable X_{ikl} representing whether configuration i is chosen (with a variable $X_{\emptyset kl}$ representing the inclusion of the no purchase option). A simple mathematical program can be formulated to maximize revenue using the multinomial demand model in equations 3-4.

$$Revenue = \sum_{i=1}^{n_i}\sum_{k=1}^{n_k}\sum_{l=1}^{n_l}\frac{\omega_{kl}p_{ikl}a_{ikl}X_{ikl}}{\sum_{j=1}^{n_1} a_{jkl}X_{jkl} + a_{\emptyset kl}X_{\emptyset kl}} \tag{3}$$

$$X_{ikl} \leq X_{\emptyset kl} \qquad \forall i \in I, k \in K, l \in L \tag{4}$$

$$X_{ikl}, X_{\emptyset kl} \in \{0,1\}$$

2.4.2 Facility Decisions

We assume that each configuration is a combination of platform and series specific "common" components (which are not modeled), and "configurable" components (which are modeled). For simplicity, "components" refers to the latter unless otherwise noted. Let $M = \{1, \dots, n_m\}$ denote the set of components, $m \in M$.

Let $S = \{1, \dots, n_s\}$ be the set of component supplier facilities, $s \in S$ and $T = \{1, \dots, n_t\}$ be the set of assembly plant facilities, $t \in T$. Component suppliers often produce different "variants" from a technology group. For example, an engine plant might have the capability to produce a 2.0L engine, a 2.5L engine, etc. Some costs are incurred regardless of the number of variants that are produced in a facility (we term these "facility costs") and some are incremental based on whether a variant is chosen (we term these "tooling costs"). Let $U = \{1, \dots, n_u\}$ be the set of technology groups, $u \in U$. Let c_{su} be the costs to build component supplier facility s for technology group u, represented by decision variable φ_{su}. Let c_{ms} be the costs to install tooling for component m at component supplier facility s, represented by decision variable φ_{ms}. Let c_t be the costs to build component assembly facility t, represented by decision variable φ_t. Total facility cost is presented in equation 5. Constraint 6 is required to ensure that tooling is not installed unless a facility is built to house it.

$$Total\ Facilities\ Cost = \sum_{s=1}^{n_s}\sum_{u=1}^{n_u} c_{su}\varphi_{su} + \sum_{m=1}^{n_m}\sum_{s=1}^{n_s} c_{ms}\varphi_{ms} + \sum_{t=1}^{n_t} c_t\varphi_t \qquad (5)$$

$$\varphi_{su} \geq \varphi_{ms} \qquad \forall m \in M, s \in S, u \in U \qquad (6)$$

$$\varphi_{su}, \varphi_{ms}, \varphi_t \in \{0,1\}$$

2.4.3 Product Flow Decisions

Let $V = \{1, \dots, n_v\}$ be the set of economies of scale levels for components, $v \in V$. Let c_{msvt} be the cost to build component m at supplier facility s and economy of scale level v shipped to assembly plant t, represented by decision variable ρ_{msvt}. Let c_{iklt} be the cost to build configuration i for series k and demand region l at assembly plant t, represented by decision variable ρ_{iklt}. Let b_{im} be an indicator of whether configuration i requires component m. Let $d^{\downarrow}_{msv}$ and $d^{\uparrow}_{msv}$ represent the lower and upper break points for building component m at supplier s at economy of scale level v (the final break point, $d^{\uparrow}_{msn_v}$, is the facility capacity). Let e_t represent the capacity of assembly facility t. Let ρ_{msv} be a binary decision variable representing whether economy of scale level v is chosen for component m at supplier s. Finally, we model an all unit discount for all programs that use a component *m*. Even though our decisions do not affect the volumes of the other programs, the decisions made on the volumes of component *m* by the studied program affect the unit price of component *m* for all programs. Let ϑ_{msv} represent the savings across all other programs that will be realized if economy of scale level v is chosen for product m at component facility s. Equation 7 represents the cost of product flow through the supply chain. Constraints 8-12 ensure adequate production to meet demand requirements.

$$Total\ Flow\ Cost$$

$$= \sum_{m=1}^{n_m}\sum_{s=1}^{n_s}\sum_{v=1}^{n_v}\sum_{t=1}^{n_t} c_{msvt}\rho_{msvt} + \sum_{i=1}^{n_i}\sum_{k=1}^{n_k}\sum_{l=1}^{n_l}\sum_{t=1}^{n_t} c_{iklt}\rho_{iklt} - \sum_{m=1}^{n_m}\sum_{s=1}^{n_s}\sum_{v=1}^{n_v} \vartheta_{msv}\rho_{msv} \tag{7}$$

$$\frac{\omega_{kl}a_{ikl}X_{ikl}}{\sum_{j=1}^{n_1} a_{jkl}X_{jkl} + a_{\emptyset kl}X_{\emptyset kl}} = \sum_{t=1}^{n_t}\rho_{iklt} \qquad \forall i \in I, k \in K, l \in L \tag{8}$$

$$\sum_{v=1}^{n_v}\rho_{msv} = 1 \qquad \forall m \in M, s \in S \tag{9}$$

$$d_{msv}^{\downarrow}\rho_{msv} \le \sum_{t=1}^{n_t}\rho_{msvt} \le d_{msv}^{\uparrow}\rho_{msv} \qquad \forall m \in M, s \in S, v \in V \tag{10}$$

$$\sum_{i=1}^{n_i}\sum_{k=1}^{n_k}\sum_{l=1}^{n_l} b_{im}\rho_{iklt} = \sum_{s=1}^{n_s}\sum_{v=1}^{n_v}\rho_{msvt} \qquad \forall m \in M, t \in T \tag{11}$$

$$\sum_{i=1}^{n_i}\sum_{k=1}^{n_k}\sum_{l=1}^{n_l}\rho_{iklt} \le e_t\varphi_t \qquad \forall t \in T \tag{12}$$

$$\rho_{msvt}, \rho_{iklt} \ge 0, \rho_{msv} \in \{0,1\}$$

2.4.4 Complexity Considerations

Let c_m be the engineering and integration costs if component m is chosen for the program, represented by decision variable γ_m. Let W_u be the set of integers up to the maximum number of components in technology group u. Let c_{wu} be the complexity cost resulting from offering w

components in technology group u, represented by decision variable γ_{wu}. Equation 13 represents the cost of complexity in the model. Constraints 14-15 link the complexity variables to the decisions in the supply chain.

$$Total\ Complexity\ Cost = \sum_{m=1}^{n_m} c_m \gamma_m + \sum_{w=1}^{W_u} \sum_{u=1}^{n_u} c_{wu} \gamma_{wu} \tag{13}$$

$$\sum_{w=1}^{W_u} \gamma_{wu} = 1 \qquad \forall u \in U \tag{14}$$

$$\sum_{m=1}^{n_u} \gamma_m = \sum_{w=1}^{n_u} w \times \gamma_{wu} \qquad \forall u \in U \tag{15}$$

$$\varphi_{ms} \leq \gamma_m \qquad \forall m \in M, s \in S \tag{16}$$

$$\gamma_m, \gamma_{wu} \in \{0,1\}$$

2.4.5 Final Objective Function

We introduce another cost component titled "Overhead". Overhead captures the fixed costs of the system that are incurred regardless of the number of configurations produced. These include supplying, engineering, and integration of common parts, marketing costs, managerial salaries, etc. The final objective function is given in equation 17.

$$\begin{aligned} max\ \{&Revenue - Total\ Facilities\ Cost - Total\ Flow\ Cost \\ &- Total\ Complexity\ Cost - Overhead\} \end{aligned} \tag{17}$$

2.4.6 Structural Properties of the Solution

Chen and Hausman (2000) show that the assortment planning problem using an MNL model for a single segment with only unit costs has a totally unimodular (TU) constraint matrix, and thus a solution to the relaxed problem is a solution to the integral problem. Schon (2010) extends the claim to show that given a similar modeling framework in the presence of fixed costs, although the root node of a search tree may not possess a TU constraint matrix, at each leaf node in a search tree the problem will be separable across multiple segments and possess the TU property. We will discuss similarities and differences of our model to those presented in these papers.

In equations (**18)**-(**19)**, we define to functions $f(i,k,l,m,s,v,t)$ and $h(m,s,v,t,u)$ that represent respectively the contribution to the objective function of the continuous and binary variables in the model.

$$\begin{aligned} f(i,k,l,m,s,v,t) \\ = \sum_{i=1}^{n_i}\sum_{k=1}^{n_k}\sum_{l=1}^{n_l}\frac{\omega_{kl}p_{ikl}a_{ikl}X_{ikl}}{\sum_{j=1}^{n_1} a_{jkl}X_{jkl} + a_{\emptyset kl}X_{\emptyset kl}} - \sum_{m=1}^{n_m}\sum_{s=1}^{n_s}\sum_{v=1}^{n_v}\sum_{t=1}^{n_t} c_{msvt}\rho_{msvt} \\ - \sum_{i=1}^{n_i}\sum_{k=1}^{n_k}\sum_{l=1}^{n_l}\sum_{t=1}^{n_t} c_{iklt}\rho_{iklt} \end{aligned} \tag{18}$$

$$\begin{aligned} h(m,s,v,t,u) = \sum_{s=1}^{n_s}\sum_{u=1}^{n_u} c_{su}\varphi_{su} + \sum_{m=1}^{n_m}\sum_{s=1}^{n_s} c_{ms}\varphi_{ms} + \sum_{t=1}^{n_t} c_t\varphi_t + \sum_{m=1}^{n_m} c_m\gamma_m \\ + \sum_{w=1}^{n_u}\sum_{u=1}^{n_u} c_{wu}\gamma_{wu} \end{aligned} \tag{19}$$

Also, let $g(i,k,l,m,s,v,t,u)$ represent the model constraints (not repeated for conciseness). We redefine our objective as $max\{\phi(m,s,v,t,u): m,s,v,t,u \in \{0,1\}\}$ where $\phi(m,s,v,t,u) \coloneqq max\{f(i,k,l,m,s,v,t): g(i,k,l,m,s,v,t,u), X_{ikl} \in \{0,1\}, \rho_{msvt} \geq 0, \rho_{iklt} \geq 0, \forall i \in I, j \in J, k \in K, m \in M, s \in S, v \in V, t \in T, u \in U\} - h(m,s,v,t,u)$. The constraint set of the subproblem at a leaf node of the search tree (i.e. all binary decisions are fixed), $g(i,k,l,m,s,v,t,u)$, is presented in its expanded form in equations (**20)**-(**24)**.

$$\frac{\omega_{kl} a_{ikl} X_{ikl}}{\sum_{j=1}^{n_1} a_{jkl} X_{jkl} + a_{\emptyset kl} X_{\emptyset kl}} = \sum_{t=1}^{n_t} \rho_{iklt} \qquad \forall i \in I, k \in K, l \in L \qquad (20)$$

$$d_{msv}^{\downarrow} \rho_{msv} \leq \sum_{t=1}^{n_t} \rho_{msvt} \leq d_{msv}^{\uparrow} \rho_{msv} \qquad \forall m \in M, s \in S, v \in V \qquad (21)$$

$$\sum_{i=1}^{n_i} \sum_{k=1}^{n_k} \sum_{l=1}^{n_l} b_{im} \rho_{iklt} = \sum_{s=1}^{n_s} \sum_{v=1}^{n_v} \rho_{msvt} \qquad \forall m \in M, t \in T \qquad (22)$$

$$\sum_{i=1}^{n_i} \sum_{k=1}^{n_k} \sum_{l=1}^{n_l} \rho_{iklt} \leq e_t \varphi_t \qquad \forall t \in T \qquad (23)$$

$$X_{ikl} \leq X_{\emptyset kl} \qquad \forall i \in I, k \in K, l \in L \qquad (24)$$

$X_{ikl} \in \{0,1\}$, $\rho_{iklt} \geq 0$, $\rho_{msvt} \geq 0$

In this form, our model differs from those of Schon (2010) and Chen and Hausman (2000). We will further explore the cases where we can utilize the Charnes-Cooper and be guaranteed an integral solution, as well as present some comments on the types of assortments that will be chosen even when integrality cannot be guaranteed. To begin, assume that there are no binding capacity constraints in the supply chain.

Lemma 1: Consider the optimal solution to any leaf node subproblem $\phi(m, s, v, t, u)$ within the search tree. The cardinality of the set of variables ρ_{msvt} will be 0 or 1 $\forall m \in M, t \in T$.

Explanation: There are several possibilities regarding the channels that supply a component *m* to an assembly plant *t*:

1) The realized values of the binary variables at the subproblem preclude the component *m* from being supplied.

2) Production at assembly plant t does not require component m based on assortment decisions.
3) Component *m* is utilized at assembly plant *t*. Only the minimum cost open supply channel will be utilized. This can be proved (trivially) through contradiction.

Lemma 2: Consider the optimal solution to any subproblem $\phi(m, s, v, t, u)$. The cardinality of the set of variables ρ_{iklt} will be 0 or 1 $\forall i \in I, k \in K, l \in L$.

Explanation: There are several possibilities regarding the channels that supply a configuration *i* to a series *k* and a demand region *l*:

1) The realized values of the binary variables at the subproblem preclude the configuration *i* from being supplied.
2) Production at assembly plant *t* does not include configuration *i* based on assortment decisions in series *k* and demand region *l*.
3) Configuration *i* is included in the assortment in series *k* and demand region *l*. Only the minimum cost open supply channel will be utilized (taking into account the costs of both the components and configurations). This can also be proved (trivially) through contradiction.

Let $c_{mt}^* = min(c_{msvt} | \varphi_{ms} = 1, \ \varphi_t = 1, \rho_{msv} = 1) \ \forall m \in M, t \in T$. Let ρ_{mt}^* represent the respective decision variables associated with c_{mt}^*. Following from Lemma 1, all variables in the set $\rho_{msvt} \neq \rho_{mt}^*$ at a leaf node subproblem are redundant and can be removed from the formulation without affecting the optimal solution. Thus, the subproblem formulation can be simplified in equations **(25)**-30.

$$max \sum_{i=1}^{n_i}\sum_{k=1}^{n_k}\sum_{l=1}^{n_l}\frac{\omega_{kl}p_{ikl}a_{ikl}X_{ikl}}{\sum_{j=1}^{n_1} a_{jkl}X_{jkl}+a_{\emptyset kl}X_{\emptyset kl}} - \sum_{m=1}^{n_m}\sum_{t=1}^{n_t} c_{mt}^{*}\rho_{mt}^{*} - \sum_{i=1}^{n_i}\sum_{k=1}^{n_k}\sum_{l=1}^{n_l}\sum_{t=1}^{n_t} c_{iklt}\rho_{iklt} \quad (25)$$

Subject To:

$$\frac{\omega_{kl}a_{ikl}X_{ikl}}{\sum_{j=1}^{n_1} a_{jkl}X_{jkl}+a_{\emptyset kl}X_{\emptyset kl}} = \sum_{t=1}^{n_t}\rho_{iklt} \qquad \forall i \in I, k \in K, l \in L \quad (26)$$

$$d_{msv}^{\downarrow}\rho_{msv} \leq \sum_{t=1}^{n_t}\rho_{mt}^{*} \leq d_{msv}^{\uparrow}\rho_{msv} \qquad \forall m \in M \quad (27)$$

$$\sum_{i=1}^{n_i}\sum_{k=1}^{n_k}\sum_{l=1}^{n_l} b_{im}\rho_{iklt} = \rho_{mt}^{*} \qquad \forall m \in M, t \in T \quad (28)$$

$$\sum_{i=1}^{n_i}\sum_{k=1}^{n_k}\sum_{l=1}^{n_l} \rho_{iklt} \leq e_t\varphi_t \qquad \forall t \in T \quad (29)$$

$$X_{ikl} \leq X_{\emptyset kl} \qquad \forall i \in I, k \in K, l \in L \quad (30)$$

$X_{ikl} \in \{0,1\}$, $\rho_{iklt} \geq 0$, $\rho_{mt}^{*} \geq 0$

We can substitute $\sum_{i=1}^{n_i}\sum_{k=1}^{n_k}\sum_{l=1}^{n_l} b_{im}^2\rho_{iklt}$ in place of ρ_{mt}^{*} to obtain equations (**31)**-(**35)**.

$$max \sum_{i=1}^{n_i}\sum_{k=1}^{n_k}\sum_{l=1}^{n_l}\frac{\omega_{kl}p_{ikl}a_{ikl}X_{ikl}}{\sum_{j=1}^{n_1} a_{jkl}X_{jkl}+a_{\emptyset kl}X_{\emptyset kl}} - \sum_{i=1}^{n_i}\sum_{k=1}^{n_k}\sum_{l=1}^{n_l}\sum_{t=1}^{n_t}\left(c_{iklt}+\sum_{m=1}^{n_m} b_{im}c_{mt}^{*}\right)\rho_{iklt} \quad (31)$$

Subject To:

$$\frac{\omega_{kl}a_{ikl}X_{ikl}}{\sum_{j=1}^{n_1} a_{jkl}X_{jkl}+a_{\emptyset kl}X_{\emptyset kl}} = \sum_{t=1}^{n_t}\rho_{iklt} \qquad \forall i \in I, k \in K, l \in L \quad (32)$$

$$d_{msv}^{\downarrow}\rho_{msv} \leq \sum_{t=1}^{n_t}\sum_{i=1}^{n_i}\sum_{k=1}^{n_k}\sum_{l=1}^{n_l} b_{im}\rho_{iklt} \leq d_{msv}^{\uparrow}\rho_{msv} \qquad \forall m \in M \tag{33}$$

$$\sum_{i=1}^{n_i}\sum_{k=1}^{n_k}\sum_{l=1}^{n_l}\rho_{iklt} \leq e_t\varphi_t \qquad \forall t \in T \tag{34}$$

$$X_{ikl} \leq X_{\emptyset kl} \qquad \forall i \in I, k \in K, l \in L \tag{35}$$

$X_{ikl} \in \{0,1\}$, $\rho_{iklt} \geq 0$

Let $c_{ikl}^{*} = min\left(c_{iklt} + \sum_{m=1}^{n_m} b_{im}^{2} c_{mt}^{*} \mid \varphi_{ms} = 1,\ \varphi_t = 1, \rho_{msv} = 1\right) \forall i \in I, k \in K, l \in L$. Let ρ_{ikl}^{*} represent the respective decision variables associated with c_{ikl}^{*}. Following from Lemma 2, all variables in the set $\rho_{iklt} \neq \rho_{ikl}^{*}$ at a leaf node subproblem are redundant and can be removed from the formulation without affecting the optimal solution. Thus, the subproblem formulation can be simplified to equations (**36**)-(**40**).

$$max \sum_{i=1}^{n_i}\sum_{k=1}^{n_k}\sum_{l=1}^{n_l} \frac{\omega_{kl}p_{ikl}a_{ikl}X_{ikl}}{\sum_{j=1}^{n_1} a_{jkl}X_{jkl} + a_{\emptyset kl}X_{\emptyset kl}} - \sum_{i=1}^{n_i}\sum_{k=1}^{n_k}\sum_{l=1}^{n_l} c_{ikl}^{*}\rho_{ikl}^{*} \tag{36}$$

Subject To:

$$\frac{\omega_{kl}a_{ikl}X_{ikl}}{\sum_{j=1}^{n_1} a_{jkl}X_{jkl} + a_{\emptyset kl}X_{\emptyset kl}} = \rho_{ikl}^{*} \qquad \forall i \in I, k \in K, l \in L \tag{37}$$

$$d_{msv}^{\downarrow}\rho_{msv} \leq \sum_{i=1}^{n_i}\sum_{k=1}^{n_k}\sum_{l=1}^{n_l} b_{im}\rho_{ikl}^{*} \leq d_{msv}^{\uparrow}\rho_{msv} \qquad \forall m \in M \tag{38}$$

$$\sum_{i=1}^{n_i}\sum_{k=1}^{n_k}\sum_{l=1}^{n_l}\rho_{ikl}^{*} \leq e_t\varphi_t \qquad \forall t \in T \tag{39}$$

$$X_{ikl} \leq X_{\emptyset kl} \qquad \forall i \in I, k \in K, l \in L \tag{40}$$

$$X_{ikl} \in \{0,1\}, \rho^*_{ikl} > 0$$

Finally, we can substitute $\frac{\omega_{kl} a_{ikl} X_{ikl}}{\sum_{j=1}^{n_1} a_{jkl} X_{jkl} + a_{\emptyset kl} X_{\emptyset kl}}$ in place of ρ^*_{ikl} to obtain equations (**41)**-(**44)**.

$$max \sum_{i=1}^{n_i} \sum_{k=1}^{n_k} \sum_{l=1}^{n_l} \frac{\omega_{kl}(p_{ikl} - c^*_{ikl}) a_{ikl} X_{ikl}}{\sum_{j=1}^{n_1} a_{jkl} X_{jkl} + a_{\emptyset kl} X_{\emptyset kl}} \tag{41}$$

Subject To:

$$d^{\downarrow}_{msv} \rho_{msv} \leq \sum_{i=1}^{n_i} \sum_{k=1}^{n_k} \sum_{l=1}^{n_l} \frac{b_{im} \omega_{kl} a_{ikl} X_{ikl}}{\sum_{j=1}^{n_1} a_{jkl} X_{jkl} + a_{\emptyset kl} X_{\emptyset kl}} \leq d^{\uparrow}_{msv} \rho_{msv} \qquad \forall m \in M \tag{42}$$

$$\sum_{i=1}^{n_i} \sum_{k=1}^{n_k} \sum_{l=1}^{n_l} \frac{\omega_{kl} a_{ikl} X_{ikl}}{\sum_{j=1}^{n_1} a_{jkl} X_{jkl} + a_{\emptyset kl} X_{\emptyset kl}} \leq e_t \varphi_t \qquad \forall t \in T \tag{43}$$

$$X_{ikl} \leq X_{\emptyset kl} \qquad \forall i \in I, k \in K, l \in L \tag{44}$$

$$X_{ikl} \in \{0,1\}$$

With no economies of scale in component production costs and infinite capacity at both component and assembly production facilities, we have a model equivalent to that of Schon (2010) at leaf nodes in the search tree. The subproblems are separable in all series *k* and demand region *l* with a TU constraint matrix, and can thus utilize the Charnes-Cooper transformation to create an equivalent binary linear programming formulation to the original problem which can be solved significantly quicker than a nonlinear program.

Lemma 3: At any leaf node subproblem $\phi(m,s,v,t,u)$ with integrality of the assortment variables X_{ikl} relaxed and no capacity constraints, the optimal assortment will consist of the set of products with the highest profit margins.

Explanation: Let $p^*_{ikl} = p_{ikl} - c^*_{ikl}\ \forall i \in I, k \in K, l \in L$ and for any configuration i that is precluded at a leaf node of the search tree by the fixed supply chain decisions $p^*_{ikl} = 0\ \forall k \in K, l \in L$. Let X^*_{ikl} represent the optimal assortment decision for each configuration *i*, series *k*, and demand region *l*. Assume, without loss of generality, that the configurations can be ordered in terms of profitability, $p^*_{1kl} > p^*_{2kl} > \cdots > p^*_{n_i kl}$. Let $X_{\sim kl}$ be a configuration not in X^*_{ikl}. Assume for simplicity that $X_{\emptyset kl} = 1$. A necessary condition for $X_{\sim kl}$ to be part of the optimal solution is given in equation **(45)** (the subscripts for series and demand region are removed for simplicity, as the problem is separable in all *k* and *l*). This equation is simplified in equation **(46)**-**(48)** until arriving on the condition in equation **(49)**.

$$\sum_{i=1}^{n_i} \frac{\omega p^*_i a_i X^*_i}{\sum_{j=1}^{n_1} a_j X^*_j + a_\emptyset} < \sum_{i=1}^{n_i} \left(\frac{\omega p^*_i a_i X^*_i}{\sum_{j=1}^{n_1} a_j X_j + a_\emptyset + a_\sim} + \frac{\omega p^*_\sim a_\sim}{\sum_{j=1}^{n_1} a_j X_j + a_\emptyset + a_\sim} \right) \tag{45}$$

$$\sum_{i=1}^{n_i} \frac{p^*_i a_i X^*_i}{\sum_{j=1}^{n_1} a_j X^*_j + a_\emptyset} < \sum_{i=1}^{n_i} \frac{p^*_i a_i X^*_i + p^*_\sim a_\sim}{\sum_{j=1}^{n_1} a_j X_j + a_\emptyset + a_\sim} \tag{46}$$

$$\sum_{i=1}^{n_i} p^*_i a_i X^*_i \left(\sum_{j=1}^{n_1} a_j X_j + a_\emptyset + a_\sim \right) < \sum_{i=1}^{n_i} \left((p^*_i a_i X^*_i + p^*_\sim a_\sim) \left(\sum_{j=1}^{n_1} a_j X^*_j + a_\emptyset \right) \right) \tag{47}$$

$$\sum_{i=1}^{n_i} p^*_i a_i X^*_i < \sum_{i=1}^{n_i} p^*_\sim \left(\sum_{j=1}^{n_1} a_j X^*_j + a_\emptyset \right) \tag{48}$$

$$\sum_{i=1}^{n_i} p_i^* a_i X_i^* < p_{\sim}^* \sum_{j=1}^{n_1} \left(a_j X_j^* + a_\emptyset\right) \tag{49}$$

It is clear from inspection of equation (**49)** that any product included in the assortment must be in the set of products with the largest values of p_{ikl}^* and that the decision of whether to include a product in the assortment depends on only on the product's margin, and not on its attractiveness. If the condition is not satisfied, it is equivalent to saying the cannibalization of the products in the assortment with higher margins outweighs the increase in demand from adding the product to the assortment.

Lemma 4: Each capacity constraint has the potential to introduce a single nonintegral assortment variable X_{ikl} in the optimal solution.

Explanation: A subproblem $\phi(m, s, v, t, u)$ with a binding capacity constraint forms a variation of a knapsack problem, where the items "weight" is equivalent to the amount of facility capacity it utilizes (i.e. its demand, as only demand exhausts capacity) and the "value" is the revenue it generates. When solving the problem with the integrality of the assortment variables X_{ikl} relaxed, the problem becomes the fractional knapsack problem. It has been proven that in the case of the fractional knapsack problem, a greedy algorithm in which items are introduced in decreasing order of $\frac{value}{weight}$ until exhausting the capacity of the knapsack will provide an optimal solution (Cormen, Leiserson et al. 2001). For the model above, $\frac{value}{weight} = \frac{revenue}{demand} =$ profit margin. Consider an assortment problem formulation with a single capacity constraint. The optimal solution can be found by greedily including the products with the highest margins into the assortment until: 1) exhausting the capacity available; or 2) the incremental revenue obtained from adding the configuration with the next highest margin is negative (i.e. the configuration fails condition (**49)**). Terminating on condition 2 will always lead to an integral solution as it is only checked before a configuration is added to the assortment. Terminating on condition 1 may cause the final configuration added to the assortment to be fractional (analogous to the final item added to a fractional knapsack possibly being nonintegral). The case of multiple capacity constraints is merely the extension of the single dimensional knapsack to the multidimensional knapsack problem, which is similarly guaranteed to limit the number of nonintegral values to the number of capacity constraints (Puchinger, Raidl et al. 2010).

Lemma 5: The optimality gap (GAP) of a solution where fractional assortment variables are rounded down can be bounded: $0 \leq GAP \leq \sum_{i=1}^{n_i} \sum_{k=1}^{n_k} \sum_{l=1}^{n_l} \frac{\omega_{kl} p_{ikl}^{*} a_{ikl} \tilde{X}_{ikl}}{\sum_{j=1}^{n_1} a_{jkl} X_{jkl} + a_{\emptyset kl} X_{\emptyset kl}}$, where $\tilde{X}_{ikl}$ is any variable that is nonintegral in the relaxation chosen as the optimal node of the search tree due to capacity constraints.

Explanation: Following from the previous lemmas, a nonintegral variable $\tilde{X}_{ikl}$ will be the lowest profit margin product that utilizes a capacitated resource and all products not in the assortment will have a lower margin than $\tilde{X}_{ikl}$. Thus, assume that there exists a configuration whose profit margin is $p_{ikl}^{*} - \epsilon$, where epsilon is a very small, positive number yet whose a_{ikl} value is a "perfect" fit (i.e. setting its value of $X_{ikl} = 1$ exactly exhausts the capacity constraint). This product would provide the essentially the same objective value through an integral assortment and the optimality gap would be maximized. Another case, is that the assortment with the variables rounded down is optimal (either no configuration can utilize the capacity without creating a nonintegral variable or every suitable configuration for inclusion in the assortment fails condition (**49)**), resulting in a GAP of 0.

Lemma 6: Assuming no capacity constraints in the component facilities (i.e. $d_{msn_v}^{\uparrow} = \infty$), then the presence of economies of scale in purchased components will not create a nonintegral solution at the optimal subproblem.

Explanation: Consider two subproblems, $\phi(m, s, v_1, t, u)$ and $\phi(m, s, v_2, t, u)$, differing only in the chosen level of economy of scale, v_1 or v_2, for a component m at a component facility s. Economies of scale in any realistic problem will form concave costs, thus $\rho_{msv_2t} < \rho_{msv_1t}$ and thus $p^*_{ikl}(v_2) > p^*_{ikl}(v_1)$. There are no fixed costs associated with achieving a higher level economy of scale level. For a product to be included in the assortment at economy of scale level v_1, it must have already fulfilled the necessary condition in equation (**49)** and will only achieve a higher margin at a level v_2. Thus, $\phi(m, s, v_2, t, u)$ will include at least the same set of products (if not more) at a lower unit cost as $\phi(m, s, v_1, t, u)$, which will lead to a better solution. Thus, although economies of scale have the same propensity to create nonintegral variables that capacity constraints do, any subproblem with a binding economy of scale constraint that is not the capacity of the component facility will be suboptimal and not be chosen as the optimal solution to the entire problem.

From a practical point of view, assortment nonintegrality due to capacity constraints are a minor concern for two reasons: 1) The model is designed to be used during strategic planning so there should be relatively few truly binding capacity constraints; and 2) The model is designed to be used for configurable products, where the chosen assortment might be thousands of configurations or more. Thus, an assortment with thousands of integral assortment decisions and a few fractional ones is likely "close enough" for most practical uses.

2.4.7 Transformed MNL Model

In the previous section, we presented comments on when we can expect our model to produce integral assortments and in the cases where the assortment cannot be guaranteed to be integral, the limit on the number of variables that can be fractional. We now transform our model into an equivalent binary linear programming problem using the Charnes – Cooper transformation

(Bazaraa, Sherali et al. 2006). Let $\tau_{kl} = \frac{1}{\sum_{j=1}^{n_1} a_{jkl}X_{jkl} + a_{\emptyset kl}X_{\emptyset kl}}$ and $\hat{X}_{ikl} = \tau_{kl}X_{ikl}$. Equations 3-4 can now be expressed with the transformed terms with equations 50-52. Equation (53) replaces equation (8).

$$Revenue = \sum_{i=1}^{n_i}\sum_{k=1}^{n_k}\sum_{l=1}^{n_l} \omega_{kl} p_{ikl} a_{ikl} \hat{X}_{ikl} \tag{50}$$

$$\sum_{i=1}^{n_i} a_{ikl}\hat{X}_{ikl} + a_{\emptyset kl}\hat{X}_{\emptyset kl} = 1 \tag{51}$$

$$\hat{X}_{ikl} \leq \hat{X}_{\emptyset kl} \qquad \forall i \in I, k \in K, l \in L \tag{52}$$

$$\omega_{kl} a_{ikl}\hat{X}_{ikl} = \sum_{t=1}^{n_t} \rho_{iklt} \qquad \forall i \in I, k \in K, l \in L \tag{53}$$

$$0 \leq \hat{X}_{ikl}, \hat{X}_{\emptyset kl} \leq 1 \qquad \forall i \in I, k \in K, l \in L$$

2.5 Experimental Results

2.5.1 Preliminary Comments

All experiments were completed on the Wayne State University Computing Grid[1] on a 64 processor 2.6 Ghz AMD system. The solver was IBM Ilog-Cplex 12.4. We used a data set created by us, but a large amount of effort was exerted to make it representative of a real world vehicle program, through both internet research and consultation of subject matter experts (SMEs) from a large OEM. These models require a lot of data, which is to be expected from a model making as many decisions as ours does. To cope with this, we make some simplifying

[1] www.grid.wayne.edu

assumptions, such as that configuration utility is always the sum of the individual option utilities. In depth treatment of the data is left for the appendix.

Also, a great amount of time was spent trying to improve the search speed of the problems through advanced MIP techniques – Bender's Decomposition and Branch and Price. These attempts were not fruitful, though, and are left out of this chapter. State-of-the-art MIP solvers are incredibly advanced and designed to solve general mixed integer problems. We were not able to find any special structure to the model to exploit when solving.

2.5.2 Base Case

Our model has cast a wide scope in the amount of decisions that it makes, and it is no surprise in that case that it requires a large amount of data. The majority of the data is left to the appendix. However, we do define the sets we use in the model to give a sense of the problem formulation we have created. Some of the sets may need clarification. We model "powertrains" as a single component that is built in a single facility. A powertrain is a combination of an engine of a certain size and a transmission (either automatic or manual) as well as the drive mode (front-wheel drive or all-wheel drive). We model four series (S, SE, SEL, and Titanium) which correlate to demand for configurations of low, mid-low, mid-high, and high quality. We model two demand regions (the U.S. and Canada), but could easily incorporate demand regions on a regional level (i.e., Midwest U.S., Northeast U.S., etc.) if data was available. We identify all component suppliers and assembly plants by their location of production, with a single supplier and assembly plant in each country. We model economies of scale for components only.

Sets

Powertrains (15) – (2.0L, Auto / Manual, FWD / AWD) | (2.5L, Auto / Manual, FWD / AWD) | (3.0L, Auto, FWD / AWD) | (3.5L, Auto, FWD / AWD) | Hybrid | Plug-In Hybrid | Fully Electric

Radios (3) – Low | Mid | High

Seats (6) – (Cloth / 60-40) | (Leather / 60-40) | (Premium Leather / 60-40)

Binaries (2) – Moon Roof | Navigation System

Configurations (1080) – Identified by number 1..1080

Series (4) – S | SE | SEL | Titanium

Demand Regions (2) – United States | Canada

Component Suppliers (3) – China | Mexico | US

Assembly Plants (2) – Mexico | US

Economy of Scale Levels (2) – Low | High

One piece of data is presented here in detail. We want to demonstrate that the option utilities can be estimated reasonably easily. Because we have vertically differentiated products with fixed prices, we assume that product quality can be tied to price using sigmoid functions. Also, we assume that the automaker will logically price options so that each series has options that appeal to customers with different reservation prices (grouped by series). We chose single sigmoids for some of the series (lowest and highest) and double sigmoids (difference of two sigmoids to be more precise) for others (the middle series). Single sigmoids require two parameters – a slope parameter and a threshold parameter. Double sigmoids require four parameters – two of each. The formulas for both are given in equations 54-55.

$$\text{Single Sigmoid: } utility = \frac{10}{1+e^{-slope^1(selling\ price - threshold^1)}} \tag{54}$$

$$\text{Two Sigmoids: } utility = \frac{10}{1+e^{-slope^1(selling\ price - threshold^1)}} - \frac{10}{1+e^{-slope^2(selling\ price - threshold^2)}} \tag{55}$$

A marketing group should have a sense of its customer's desires and be able to generally find the parameters that produce the correct shape. We make two more adjustments to the utility estimates: 1) We set a threshold value for utility and set any utility less than that value to zero. Setting very low utilities to zero helps the solver to converge faster and prevents low utility, low sales configurations from being present in the assortment. This cannot be achieved through a penalty cost on the number of configurations offered in a series, since the transformed MNL model no longer has an indicator variable for whether a configuration is present or not; 2) The above formulas produce utilities on a scale of 0-10. We believe this is the easiest scale for defining the utilities from customer input, but clearly a powertrain and a moon roof will not contribute equal importance in determining a configuration's total utility. Thus, we scale the utilities by a factor based on the average price of the option (a different factor for each group of powertrains, radios, binaries, etc.). Figure 1 shows the result for standard powertrains in our base case.

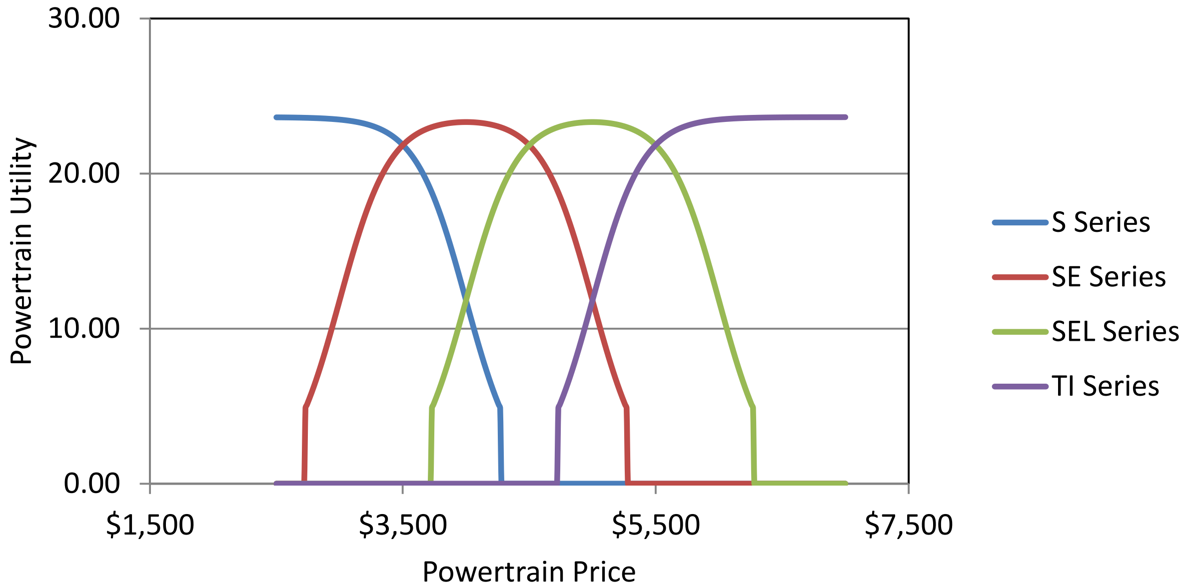

Figure 1: Graph of Scaled Sigmoid Functions for Powertrains in the Base Case

<u>Base Case Results</u>

Effort was put in to present the data in a way similar to the way that an automaker's annual report would look. Figure 2 shows a sample of how Ford Motor Company reports financial data.

Table **1** contains the data from our base case run. Figure 3 displays the costs incurred during the base case in the form of a pie chart.

OPERATING HIGHLIGHTS

Revenues (a)	2011	2010
Worldwide wholesale unit volumes by automotive segment (in thousands)		
Ford North America	**2,686**	2,413
Ford South America	**506**	489
Ford Europe	**1,602**	1,573
Ford Asia Pacific Africa	**901**	838
Volvo	**–**	211
Total	**5,695**	5,524
Revenues (in millions)		
Automotive	**$ 128,168**	$ 119,280
Financial Services	**8,096**	9,674
Total	**$ 136,264**	$ 128,954
Financial Results (a)		
Income/(loss) before income taxes (in millions)		
Automotive	**$ 6,250**	$ 4,146
Financial Services	**2,431**	3,003
Total	**$ 8,681**	$ 7,149
Amounts Attributable to Ford Motor Company (a)		
Net income/(loss) (in millions)	**$ 20,213**	$ 6,561
Diluted net income/(loss) per share of Common and Class B Stock	**$ 4.94**	$ 1.66
Cash and Spending (a)		
Automotive capital expenditures		
Amount (in billions)	**$ 4.3**	$ 4.1
As a percentage of Automotive sales	**3.3%**	3.4%
Automotive cash at year end (in billions)		
Automotive gross cash (b)	**$ 22.9**	$ 20.5
– Cash net of Automotive debt	**9.8**	1.4
Shareholder Value		
Dividends declared per share	**$ 0.05**	$ -
Total shareholder returns % (c)	**(36)%**	68%

Figure 2: Sample from Ford 2011 Annual Report

Table 1: Base Case Results

	Value	Additional Info
REVENUES (in millions)	**$7,003**	

Estimated Program Wholesale Unit Volume (in thousands)	300	
Ford U.S. Total	270	
S-Series	25	
SE-Series	106	
Standard	106	
Electrified	0	
SEL-Series	88	
TI-Series	51	
Ford Canada Total	30	
S-Series	3	
SE-Series	12	
Standard	12	
Electrified	0	
SEL-Series	10	
TI-Series	6	
COSTS (in millions)	**$6,256**	
Administrative, Marketing, and Common Parts Engineering - Program share of annualized costs	$1,500	
Design, Engineering, and Development Costs - Program share of annualized costs		
Powertrains	$144	
Standard	$144	
Electrified	$0	
Radios	$9	
Seats	$13	
Moon Roof	$4	
Nav System	$5	
Facility Costs - Program share of annualized costs		
OEM Plants (Body, Paint, Final Assembly)	$250	Mexico
Powertrain Plants	$110	Mexico
Radio Plants	$6	Mexico
Seat Plants	$17	Mexico
Moon Roof Plants	$28	Mexico

Nav System Plants	$25	China
Tooling Cost (in millions) - Program share of annualized costs		
Powertrains	$17	3 Models - 2.0L Auto FWD | 3.0L Auto FWD | 3.5L Auto FWD
Radios	$2	3 Models - Low Radio | Mid Radio | High Radio
Seats	$10	6 Models - Cloth | Cloth 60/40 | Leather | Leather 60/40 | Premium Leather | Premium Leather 60/40
Manufacturing Cost (in millions) - Labor, Materials, Energy, Consumables		
OEM	$3,284	
Suppliers	$731	
Powertrains	$500	
Standard	$500	
Electrified	$0	
Radios	$43	
Seats	$110	
Moon Roof	$31	
Nav System	$55	
Transportation cost (in millions)		
Finished Product Distribution	$74	
Component/Sub-Assembly Shipping	$2	
Component Duties	$7	
Complexity Cost (in millions) - Additional costs from variant complexity		
Powertrains	$11	
Radios	$1	
Seats	$9	
NET PROFIT BEFORE TAXES (in millions)	**$747**	

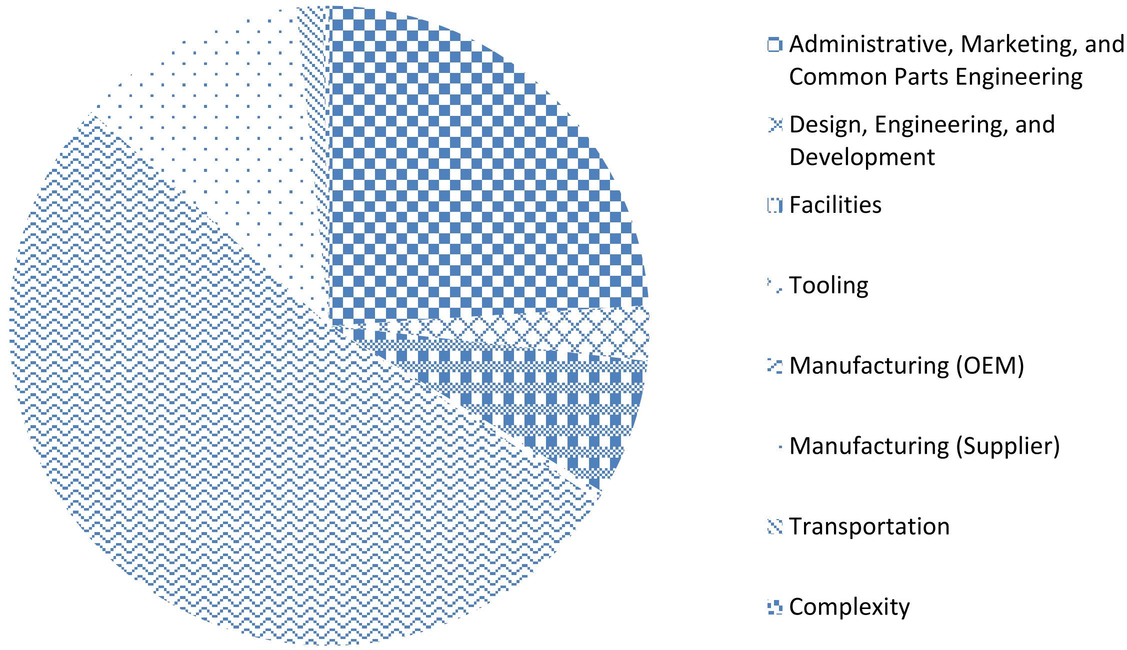

Figure 3: Pie Chart Representing the Costs Incurred in the Base Case

Comments on Base Case

- All assembly operations take place in Mexico, due to cheaper unit costs than production in the U.S. The capacity of the Mexican assembly plant is a binding constraint (300k). However, there is not enough total demand leftover (400k total demand) to justify opening a U.S. assembly plant as well.
- Most of the production of components also takes place in Mexico. This is reasonably intuitive, since most of the products are heavy and not easily shipped (engines, seats, etc). The navigation system is produced in China. Its small size makes it worthwhile to incur lower unit manufacturing costs at the cost of increased transportation costs.
- It is not profitable to produce any electrified vehicles without a constraint on carbon or fuel economy in our base case.

- Assembly plant unit costs make up the majority of the total cost. This is intuitive, in that there are thousands of parts of an automobile that are not part of our “studied” components. Their costs are accounted for in the assembly plant unit costs.
- Complexity costs make up a small portion of the total costs. Even with the full set of radios and seats chosen, their respective complexity costs sum up to approximately $10 million, less than 1% of the cost. Powertrain complexity has potential to be significantly more expensive. The model chooses only 3 of the 15 powertrains, however, resulting in $11 million dollars of powertrain complexity cost.

2.5.3 Sensitivity Analysis

We believe our model is more valuable as a tool to conduct sensitivity analysis to uncover managerial insight, rather than a single run model that defines how decisions should be made. Sensitivity analysis is a way to overcome epistemic uncertainties that a real-world company faces. However, our model includes a relatively large number of uncertain parameters and it is impractical to run sensitivity analysis on them all (demand parameters and complexity costs would likely be the most difficult to quantify). We use our judgment to present what we believe to be the most interesting cases.

2.5.3A Sensitivity Analysis of the Level of Outside Competition

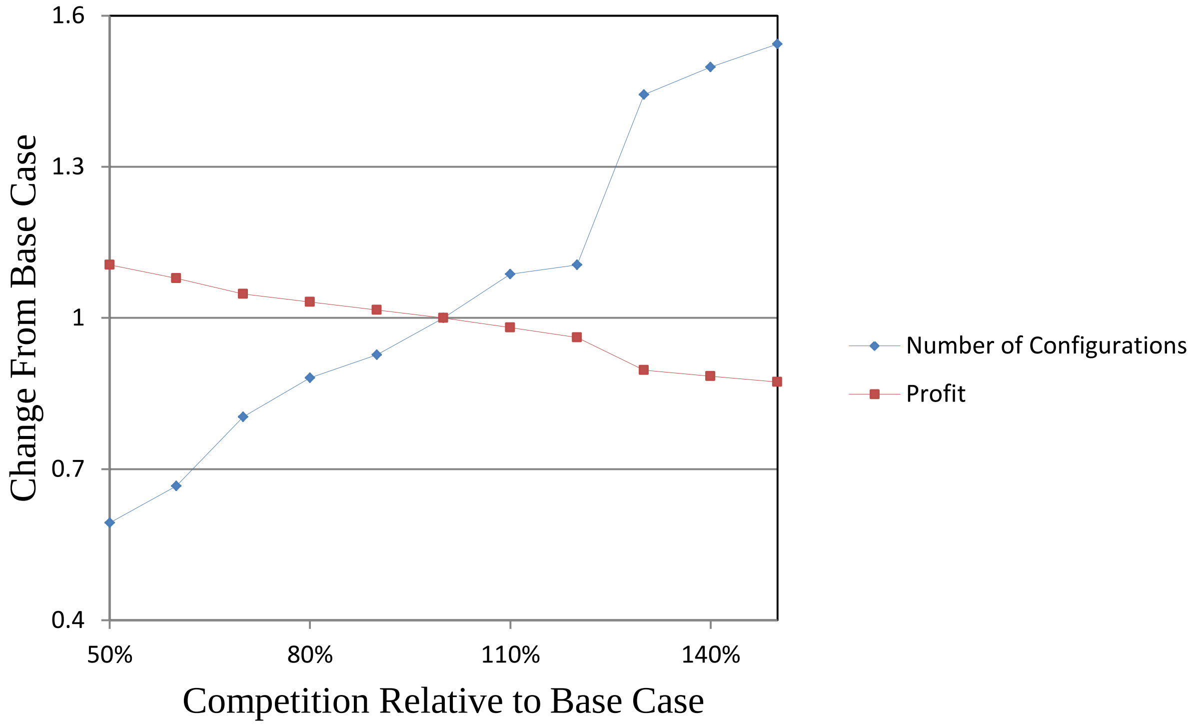

Figure 4: Effects of Changing the Level of Outside Competition

Base Case: Profit = $746.5 million, Number of Configurations = 219.

Figure 4 shows the effect of changing the level of outside competition on profit and the number of configurations offered. The capacity constraint is a binding constraint in almost all instances, meaning that sales are 300k across the ranges. The single exception is when the competition level is set to 1.2, when the model chooses to target 294k in sales instead. However, as expected, the number of configurations that must be supplied to reach the desired amount of sales increases with external competition, and supplying higher numbers of configurations diminishes profit. The large jump in the number of configurations supplied when competition is 30% higher than the base case can be attributed to the optimal solution including 4 powertrains instead of 3, which allows many more configurations to be produced.

2.5.3B Reduce Cost / Capacity of U.S. Assembly Plant

One interesting sensitivity run is to reduce both the capacity and fixed cost of the U.S. assembly plant to 10% of their values in the base case. This scenario is meant to mimic the case of a small amount of extra capacity available on an assembly line dedicated to another vehicle, but capable of being utilized by the vehicle program. To make the presentation easy for the reader, we focus only on the differences between the base case and the reduced cost / capacity case:

- The U.S. assembly plant is chosen.
- Profit increases to $832.2 million (+$85.7 million).
- Sales of 327.8k (chosen capacity of 330k).
- A 3.0L Auto AWD powertrain is added to the powertrains chosen in the base case to allow the extra sales to be captured.

2.6 Conclusion and Future Research

We have demonstrated a model that can be used to jointly plan the assortment and supply chain configuration of a configurable product. This model can be used to identify and explore tradeoffs between lost sales and supply chain costs resulting from product complexity. The model scales to the problem sizes that are found in real world companies manufacturing configurable products. We believe through integrated planning of the assortment and supply chain, companies can come to solutions that lead to better solutions than a process in which the decisions are made sequentially by managerial groups with differing incentives.

Even though the model scales well, that is not to say that computational performance could not be improved. Our attempts at customized algorithms built around decomposition techniques did not prove fruitful over ILOG CPLEX’s out-of-the-box algorithms. There could, however, be a customized algorithm that could improve computational performance. Also, the fidelity of the data used could be improved through a closer partnership with industry. Finally, a stochastic programming approach could be useful to overcome uncertainties in the most critical parameters of the model such as the total number of customers willing to purchase the product. This approach will help ensure a robust solution, rather than one that is optimal for a single case but poor over a range of possible business scenarios.

2.7 Appendix

Key Data Parameters

Administrative, Marketing, and Common Parts Engineering costs – $1.5b
MSRP Discount – Actual selling price is 90% of MSRP
Base Price of Vehicle – S $16k, SE $18k, SEL $20k, TI $22k
Option Selling Prices – 2.0L $2.5k, 2.5L $3k, 3.0L $4k, 4.0L $5k, AWD +$9h, Auto +$5h, Hybrid $10k, Plug-In Hybrid $17k, Fully Electric $23k, Low Radio $1h, Mid Radio $3h, High Radio $8h, Cloth $3h, Leather $1k, Premium Leather $1.2k, 60/40 folding +$1h, Moon Roof $6h, Navigation System $1k
Total Customers – US S 36k, US SE 144k, US SEL 108k, US TI 72k, Canada S 4k, Canada SE 16k, Canada SEL 12k, Canada TI 8k
Walk Utility (US and Canada) – S 7, SE 400, SEL 400, TI 400
Assembly Plant Fixed Costs – Mexico $250m, US $300m
Engine Facility Costs – China $100m, Mexico $110m, US $120m
Radio Facility Costs – China $5m, Mexico $5.5m, US $6m
Seat Facility Costs – China $15m, Mexico $16.5m, US $18m
Unit Cost to Produce a Configuration as a Percentage of US Selling Price – Mexico 50%, US 60%
Unit Cost to Produce a Component as a Percentage of Selling Price – China 35%, Mexico 40%, US 50%
Duties – Components from China incur 5% duty cost
Capacity of All Facilities – 300k units
Economies of Scale Level – Production in the high level of economy of scale reduce unit costs by 10%
Price to Move a Shipping Container One Mile – $2
Number of Configurations Per Shipping Container – 10
Distance from Assembly Plant to Center of Demand Region – Mexico to US 1200 miles, Mexico to Canada 1600 miles, US to US 600 miles, US to Canada 800 miles
Fixed Cost For Each Standard Powertrain Variant Tooling – China $5m, Mexico $5.5m, US $6m
Fixed Cost For Hybrid Powertrain Variant Tooling – China $2.5m, Mexico $3.0m, US $3.5m
Fixed Cost For Plug-In Powertrain Variant Tooling – China $3m, Mexico $3.5m, US $4m
Fixed Cost For Fully Electric Powertrain Variant Tooling – China $4m, Mexico $4.5m, US $5m
Fixed Cost For Each Radio Variant Tooling – China $500k, Mexico $550k, US $600k
Fixed Cost For Each Seat Variant Tooling – China $1.5m, Mexico $1.65m, US $1.8m
Moon Roof Cost For Facility and Tooling – China $25m, Mexico $27.5m, US $30m
Navigation System Cost For Facility and Tooling – China $25m, Mexico $27.5m, US $30m
Component Supplier to Assembly Plant Distances – China to Mexico 7700 miles, China to US 7450 miles, Mexico to Mexico 0 miles, Mexico to US 1800 miles, US to Mexico 1800 miles, US to US 0 miles
Break Points – 100k for all components

Number of Components Per Container – Powertrains 36, Radios 1000, Seats 18, Moon Roofs 120, Navigation Systems 1000

Cost of Powertrain Complexity (cost of total number of powertrains) -- $0, $7m, $10.5m, $15.7m, $23.6m, $35.4m, $53.1m, $79.7m, $119.6m, $179.4m, $269.1m, $403.6m, $908m, $1.3b, $2.0b

Cost of Radio Complexity (cost of total number of radios) -- $0, $700k, $1.1m

Cost of Seat Complexity (cost of total number of seats) -- $0, $1.7m, $2.6m, $3.9m, $5.9m, $8.8m

Engineering and Development Costs:

2.0L Manual FWD	$30m
2.0L Manual AWD	$36m
2.0L Auto FWD	$36m
2.0L Auto AWD	$42m
2.5L Manual FWD	$36m
2.5L Manual AWD	$42m
2.5L Auto FWD	$42m
2.5L Auto AWD	$48m
3.0L Auto FWD	$48m
3.0L Auto AWD	$54m
3.5L Auto FWD	$60m
3.5L Auto AWD	$66m
Hybrid	$130m
Plug-In Hybrid	$300m
Fully Electric	$300m
Low Radio	$1.5m
Mid Radio	$2.7m
High Radio	$4.5m
Cloth	$1.5m
Cloth 60/40	$2.1m
Leather	$1.8m
Leather 60/40	$2.4m
Premium Leather	$2.1m
Premium Leather 60/40	$2.7m
Moon Roof	$3.6m
Nav System	$5.4m

Key Assumptions

Please note – We present these assumptions in slightly different terms than the mathematical model. It is made to be easily read and understandable.

Configuration Selling Price = MSRP Discount * (Series Base Price + Option Prices)

Component Utility = Sum of Option Utilities
Configuration Unit Cost = Configuration Unit Cost Percentage * Selling Price + Price Per Mile * Assembly Plant to Demand Region Distance / Configurations Per Shipping Container
Component Unit Cost = Economy of Scale Multiplier * (Duty Cost + Component Unit Cost Percentage * Selling Price + Component to Assembly Distance * Price Per Mile / Components Per Container)

Chapter 3: Packaging and Assortment Planning Implications

3.1 Introduction

The optimal packaging of components in configurable products is an incredibly complex problem, and could be the entire subject of many dissertations. There are many important questions to be answered surrounding packaging of configurable components that require an analysis of packaging's effects on a product's supply chain, ordering complexity, and the customer's valuation of the product / package. Venkatesh and Mahajan (2009) define some of the motivations for packaging of all products from both a supply and demand perspective. From a supply perspective, packaging leads to lower inventory costs and greater economies of scale as total variety is decreased. Also, packaging can lead to lower sorting costs, as in a grocer selling a bag of apples versus individual ones. From a demand perspective, packaging can be used as a way to implement price discrimination, as in the packaging of season tickets for a sporting event for a lower per game cost. Also, it can increase sales through complementarity, such as the offering of ski lessons with ski rental.

Packaging is without question important to firms in the automotive industry. From industry experience, we know that upwards of 90% of mass market vehicle sales occur on dealer lots, which have limited room to store inventory. Some transshipment occurs between dealers, but overall, customers choose from a small set of vehicles in nearby lots. Packaging is a necessity in these conditions to ensure that the few configurations that are chosen as "buildable" in the product order guide (a guide distributed to the dealer by the OEM) provide a good representation of vehicles that are attractive to diverse customers, in addition to the effects described above (limit supply chain complexity and upselling on options).

It should be noted that in order to study and implement packaging in the real world, a manufacturer must first have a strong understanding for customer choice. Some have assumed, as in Kusiak and Smith et al. (2007), availability of knowledge regarding true demand in the form of sales records from custom orders. In the auto industry, however, the ordering process that customers and dealers partake in can be fairly rigid (only a few independent choices). Vehicles that sell fast are ordered again, creating a self-fulfilling prophecy. There is significant data sparsity, as only a small percentage of the total possible configurations may be built for a vehicle program. Lost customers are not recorded and laws in the U.S. prevent OEMs from learning the actual selling price of a vehicle. Thus, the problem of determining customer demand from vehicle lot sales data is a significantly complex one and is left to be the subject of other research.

For reasons outlined above, we do not focus on the problem of characterizing primary demand or identifying high potential packages. This does not mean, however, that our model cannot contribute to the problem of optimal packaging in the automotive industry. We focus on the problem of package selection, not identifying potential packages. We assume that marketing groups have expertise in identifying a set of high potential packages that we can leverage. We also assume that we can mathematically express the effect that a package has on the utility of the components (and thus, the product as a whole) if a package is selected. Thus, our goal is to decide which subset of packages to include in the assortment. We do this through a novel approach to packaging, which allows us to mathematically parameterize the effect of packaging on customer utility which affects the accompanying supply chain, a subject where there has been very little academic work published on. We demonstrate the effectiveness of our method on a case modeled after the automotive industry, and discuss the model's inability to scale to large scale problems. A simple heuristic is presented that scales significantly better.

The rest of this chapter is organized as follows: Section 2 presents a review of literature on packaging. Section 3 discusses the packaging model. Section 4 presents the results of our packaging experiments. Section 5 presents our conclusions on the effect of packaging on automotive assortments.

3.2 Literature Review

Common notation throughout bundling literature defines 3 types of sales techniques: 1) A pure components approach, in which products (or components in our case) are sold separately; 2) A pure bundling approach, in which products are offered only as a bundle; and 3) A mixed bundling strategy, in which both the individual products and the bundle are sold. Our model would fall under the category of mixed bundling. Schmalensee (1984) studied the cases where each strategy is optimal when buyers reservation prices are normally distributed. Prasad and Venkatesh et al. (2010) consider market conditions where each type of bundling is optimal in the presence of network externality (such as a data network, where clearly more users is desirable to current users).

Bitran and Ferrer (2007) develop an approach to find a single optimal bundle and price to offer to a market to compete against competitor's bundles under MNL demand. They develop a method to find the pareto frontier of bundles (removing "dominated" packages – packages with a higher cost to produce and lower utility than another product). Ferrer and Mora et al. (2010) use dynamic programming to find the optimal pricing of either one or two bundles offered to customers on a subscription basis. Bakos and Brynjolfsson (1999) study the bundling of information goods, claiming that producers are able to better estimate a consumer's valuation of a bundle of goods than a single good as the law of large numbers averages out very low and high valuations. This approach could not be applied to physical goods, however, as the value in increased predictability of customer valuations is negated by increased production cost. Lv and Rouskas (2011) use dynamic programming to find optimal bundling of tiered network services, such as internet data and phone services. Kar and Singh (2012) present a neural network based approach to the bundling of mobile data services.

Some packaging literature focuses on experimentally (as opposed to analytically) describing the effects of packaging. Brough and Chernev (2012) show that in the case of a high quality product and a low quality product, the low quality product can actually have a subtractive effect on the valuation of the high quality one. Kim and Bojanic et al. (2009) study the effects of bundling travel products as a plane ticket and hotel room. They find that customers do receive monetary savings by purchasing bundles from online travel agents, as opposed to purchasing them directly from the company providing the service. Hennessy and Haynes (2000) examine the bundling strategies of players in the genetically engineered seed market, using their actions to reveal information about their beliefs regarding market dynamics and where their company is positioned.

3.3 Packaging Model

We assume the manufacturer has a predetermined set of packages that can be chosen. If a package is chosen and all of the components in the package are present in a configuration, the configuration's attractiveness increases at a cost of a lower selling price. We assume that packaging decisions are at a series level (i.e., packaging decisions are consistent across demand regions). Let $O = \{1, \dots, n_o\}$ denote the set of packages, $o, q \in O$. Let Δa_{iok} and Δp_{iok} represent the changes to a configuration's attractiveness and selling price, respectively, if package o is selected. Let Y_{ok}^1 be a binary variable representing the decision to choose package o in series k. Thus, the objective function of our mathematical program with packaging effects is presented in equation 56. Expanding equation 56, we get equation 57.

$$\text{maximize} \sum_{i=1}^{n_i}\sum_{k=1}^{n_k}\sum_{l=1}^{n_l} \omega_{kl}(p_{ikl} + \sum_{o=1}^{n_o} \Delta p_{iok} Y_{ok}^1)(a_{ikl} + \sum_{o=1}^{n_o} \Delta a_{iok} Y_{ok}^1)\hat{X}_{ikl} \tag{56}$$

$$\text{maximize} \sum_{i=1}^{n_i}\sum_{k=1}^{n_k}\sum_{l=1}^{n_l} \omega_{kl}\hat{X}_{ikl} \times \begin{bmatrix} p_{ikl}\left(a_{ikl} + \Delta a_{i1k}\, Y_{1k}^1 + \cdots + \Delta a_{in_5k}\, Y_{n_5k}^1\right) \\ +\Delta p_{i1k}\left(a_{ikl}Y_{1k}^1 + \Delta a_{i1k}Y_{1k}^1Y_{1k}^1 + \Delta a_{in_5k}Y_{1k}^1Y_{n_5k}^1\right) \\ \dots \\ +\Delta p_{in_5k}\left(a_{ikl}Y_{n_5k}^1 + \Delta a_{i1k}Y_{n_5k}^1Y_{1k}^1 + \Delta a_{in_5k}Y_{n_5k}^1Y_{n_5k}^1\right) \end{bmatrix} \tag{57}$$

Terms $Y_{ok}^1 Y_{ok}^1$ can be replaced with simply Y_{ok}^1. We term each combination $Y_{ok}^1 Y_{qk}^1$ as a "second order interaction". Let $R = \{1, \dots, n_r\}$ be the set of second order interactions, $r \in R$. We define a new variable Y_{rk}^2 representing whether a certain bilinear term $Y_{ok}^1 Y_{qk}^1$ evaluates to 0 or 1. Let b_{or}^1 be an indicator of whether package o is a part of second order indicator r. Constraints 58-59 enforce the relationship between packages and second order interactions.

$$Y_{rk}^2 \geq \sum_{o=1}^{n_o} b_{or}^1 Y_{ok}^1 - 1 \qquad \forall r \in R, k \in K \tag{58}$$

$$Y_{rk}^2 \leq 1 - b_{or}^1 + b_{or}^1 Y_{ok}^1 \qquad \forall r \in R, o \in O, q \in O, k \tag{59}$$

To eliminate bilinear terms of the form $\hat{X}_{ikl} Y_{ok}^1$, we substitute a new variable $\hat{Z}_{iokl}^1$. To eliminate bilinear terms of the form $\hat{X}_{ikl} Y_{rk}^2$, we substitute a new variable $\hat{Z}_{irkl}^2$. Constraints 60-66 are necessary to ensure the feasible region is unchanged. Constraint 67 ensures that final assembly production meets the demands of the assortment after packaging effects.

$$\hat{Z}^1_{iokl} \leq Y^1_{ok} \qquad \forall i \in I, k \in K, l \in L, o \in O \qquad (60)$$

$$\hat{Z}^1_{iokl} \leq \hat{X}_{ikl} + 1 - Y^1_{ok} \qquad \forall i \in I, k \in K, l \in L, o \in O \qquad (61)$$

$$\hat{Z}^1_{iokl} \geq \hat{X}_{ikl} + 1 - Y^1_{ok} \qquad \forall i \in I, k \in K, l \in L, o \in O \qquad (62)$$

$$\hat{Z}^2_{irkl} \leq Y^2_{rk} \qquad \forall i \in I, k \in K, l \in L, o \in O \qquad (63)$$

$$\hat{Z}^2_{irkl} \leq \hat{X}_{ikl} + 1 - Y^2_{rk} \qquad \forall i \in I, k \in K, l \in L, r \in R \qquad (64)$$

$$\hat{Z}^2_{irkl} \geq \hat{X}_{ikl} + 1 - Y^2_{rk} \qquad \forall i \in I, k \in K, l \in L, r \in R \qquad (65)$$

$$\sum_{i=1}^{n_i} \left[a_{ikl}\hat{X}_{ikl} + \sum_{o=1}^{n_o} \Delta a_{iok}\hat{Z}^1_{iokl} \right] + a_{\emptyset kl}\hat{X}_{\emptyset kl} = 1 \qquad \forall k \in K, l \in L \qquad (66)$$

$$\omega_{kl}a_{ikl}\hat{X}_{ikl} + \sum_{o=1}^{n_o} \omega_{kl}\Delta a_{iok}\hat{Z}^1_{iokl} = \sum_{t=1}^{n_t} \rho^2_{iklt} \qquad \forall i \in I, k \in K, l \in L \qquad (67)$$

Also, we introduce an intermediate parameter $\Delta ap_{irk} = \Delta a_{iok}\Delta p_{iqk} + \Delta a_{iqk}\Delta p_{iok}$ for packages $o \in O$ and $q \in O$ associated with a second order interaction $r \in R$. We now express revenue as a linear function with binary terms in equation 68 accounting for the effects of packaging. Our cost functions are the same as presented in Chapter 2.

$$\text{revenue} = \sum_{i=1}^{n_i}\sum_{k=1}^{n_k}\sum_{l=1}^{n_l}\omega_{kl}a_{ikl}p_{ikl}\hat{X}_{ikl} + \sum_{i=1}^{n_i}\sum_{o=1}^{n_o}\sum_{k=1}^{n_k}\sum_{l=1}^{n_l}\omega_{kl}p_{ikl}\Delta a_{iok}\hat{Z}^1_{iokl} + \sum_{i=1}^{n_i}\sum_{o=1}^{n_o}\sum_{k=1}^{n_k}\sum_{l=1}^{n_l}\omega_{kl}\Delta p_{iol}a_{ikl}\hat{Z}^1_{iokl} + \sum_{i=1}^{n_i}\sum_{o=1}^{n_o}\sum_{k=1}^{n_k}\sum_{l=1}^{n_l}\omega_{kl}\Delta p_{iok}\Delta a_{iok}\hat{Z}^1_{iokl} + \sum_{i=1}^{n_i}\sum_{r=1}^{n_r}\sum_{k=1}^{n_k}\sum_{l=1}^{n_l}\omega_{kl}\Delta ap_{irk}\hat{Z}^2_{irkl} \quad (68)$$

3.4 Packaging Examples

3.4.1 Single Run With and Without Packages

Unless otherwise noted, we utilize the assortment planning and supply chain model, in its base case form, as presented in chapter 2. The model is extended to incorporate packaging aspects in the following manner:

- In the base case from Chapter 2, the optimal solution is to choose three powertrains, which is the lowest number that can be chosen while still providing popular vehicles to customers in all series. This creates a situation where packaging is not likely to make an impact on the supply chain, since the number of powertrains is low and while the full set of radios and seats are chosen, the related complexity associated with radios and seats is not large enough to be a factor in the decisions made. Thus, we increase the walk utility in the SE, SEL, and Titanium series from 400 to 800 to create a situation where the manufacturer must offer more configurations than under previous settings to achieve a high level of demand coverage. Earlier comments on the S series still apply to the packaging runs (its main purpose is to create a low base MSRP and it will make up a low portion of total vehicle program sales, thus it is not modeled with packages and its walk utility is left constant).

- Assembly plant capacity is increased from 300k to 400k to remove any capacity constraints from having an effect on packages chosen.
- In the SE series, we introduce the following 4 packages – low radio / moon roof, low radio / navigation system, mid radio / moon roof, mid radio / navigation system. In the SEL and Titanium series, we introduce these packages -- mid radio / moon roof, mid radio / navigation system, high radio / moon roof, high radio / navigation system. This is consistent with the current actions of automakers, who often create packages from electronics and binary options.
- We make the following assumption regarding the way that customers value packages – If a package is chosen for a series and a configuration contains all the components within that package, the prices for those options are discounted 5% while the utility associated with those options increases by 40%. Utility values for radios, moon roofs, and navigation systems in our model vary between 1 and 7. The largest utility boost any configuration could receive under these packages would be approximately 4, whereas a very popular engine in a series could have utility approaching 30. Packaging effects in this case will not dominate the need to provide a sufficient variety of popular vehicles.
- One drawback of the MNL model is that provided the walk utility used is greater than zero and there are no binding capacity constraints, adding configurations to an assortment will always lead to increased sales. This is not representative of the real world, however, where firms have a limit to the amount of market share they can obtain through the introduction of configurations. Thus, we constrain our packaging models to have no

higher market share than the same model solved without packages available. In this way, we can focus on the effects of packaging on supply chain complexity, not the effects of packaging on optimal pricing.

After making these changes, we can see the effect of packaging on our new packaging base case. These results are presented in Table 2 and Table 3.

Table 2: Results of Modified Base Case Without Packaging

	Value	Additional Info
REVENUES (in millions)	$7,417	
Estimated Program Wholesale Unit Volume (in thousands)	325.0616	
Ford U.S. Total	293	
S-Series	30	
SE-Series	120	
Standard	120	
Electrified	0	
SEL-Series	92	
TI-Series	51	
Ford Canada Total	33	
S-Series	3	
SE-Series	13	
Standard	13	
Electrified	0	
SEL-Series	10	
TI-Series	6	
COSTS (in millions)	$6,761	
Administrative, Marketing, and Common Parts Engineering - Program share of annualized costs	$1,500	
Design, Engineering, and Development Costs - Program share of annualized costs		
Engines	$330	
Standard	$330	

Electrified	$0	
Radios	$9	
Seats	$13	
Moon Roof	$4	
Nav System	$5	
Facility Costs - Program share of annualized costs		
OEM Plants (Body, Paint, Final Assembly)	$250	Mexico
Engine Plants	$110	Mexico
Radio Plants	$6	Mexico
Seat Plants	$17	Mexico
Moon Roof Plants	$28	Mexico
Nav System Plants	$25	China
Tooling Cost (in millions) - Program share of annualized costs		
Engines	$39	7 Models - 2.0L Auto FWD | 2.0L Auto AWD | 2.5L Manual AWD | 2.5L Auto AWD | 3.0L Auto FWD | 3.0L Auto AWD | 3.5L Auto FWD
Radios	$2	3 Models - Low Radio | Mid Radio | High Radio
Seats	$10	6 Models - Cloth | Cloth 60/40 | Leather | Leather 60/40 | Premium Leather | Premium Leather 60/40 -
Manufacturing Cost (in millions) - Labor, Materials, Energy, Consumables		
OEM	$3,479	
Suppliers	$784	
Engines	$541	
Standard	$541	
Electrified	$0	
Radios	$45	
Seats	$117	
Moon Roof	$33	
Nav System	$57	
Transportation cost (in millions)		
Finished Product Distribution	$81	
Component/Sub-Assembly	$2	

Shipping		
Component Duties	$7	
Complexity Cost (in millions) - Additional costs from variant complexity		
Engines	$53	
Radios	$1	
Seats	$9	
NET PROFIT BEFORE TAXES (in millions)	**$656**	

Table 3: Results of Modified Base Case with Packaging

	Value	Additional Info
REVENUES (in millions)	**$7,301**	
Estimated Program Wholesale Unit Volume (in thousands)	319.368	
Ford U.S. Total	287	
S-Series	30	
SE-Series	115	
Standard	115	
Electrified	0	
SEL-Series	91	
TI-Series	51	
Ford Canada Total	32	
S-Series	3	
SE-Series	13	
Standard	13	
Electrified	0	
SEL-Series	10	
TI-Series	6	
COSTS (in millions)	**$6,644**	
Administrative, Marketing, and Common Parts Engineering - Program share of annualized costs	$1,500	
Design, Engineering, and Development Costs - Program share of annualized costs		
Engines	$288	

Standard	$288	
Electrified	$0	
Radios	$9	
Seats	$13	
Moon Roof	$4	
Nav System	$5	
Facility Costs - Program share of annualized costs		
OEM Plants (Body, Paint, Final Assembly)	$250	Mexico
Engine Plants	$110	Mexico
Radio Plants	$6	Mexico
Seat Plants	$17	Mexico
Moon Roof Plants	$28	Mexico
Nav System Plants	$25	China
Tooling Cost (in millions) - Program share of annualized costs		
Engines	$33	6 Models - 2.0L Auto FWD | 2.5L Manual AWD | 2.5L Auto AWD | 3.0L Auto FWD | 3.0L Auto AWD | 3.5L Auto FWD
Radios	$2	3 Models - Low Radio | Mid Radio | High Radio
Seats	$10	6 Models - Cloth | Cloth 60/40 | Leather | Leather 60/40 | Premium Leather | Premium Leather 60/40
Manufacturing Cost (in millions) - Labor, Materials, Energy, Consumables		
OEM	$3,432	
Suppliers	$782	
Engines	$540	
Standard	$540	
Electrified	$0	
Radios	$45	
Seats	$116	
Moon Roof	$32	
Nav System	$57	
Transportation cost (in millions)		
Finished Product Distribution	$79	

Component/Sub-Assembly Shipping	$2	
Component Duties	$7	
Complexity Cost (in millions) - Additional costs from variant complexity		
Engines	$35	
Radios	$1	
Seats	$9	
NET PROFIT BEFORE TAXES (in millions)	**$657**	

To summarize, the optimal solution is to choose all of the packages in the SE and SEL series, none in the Titanium series. This allows seven powertrains to be chosen instead of six, reducing complexity costs by $18 million, $42 million in development costs for the 2.0L Auto AWD, and $5.5 million in tooling costs ($65.5 million total). Without packaging, 76.9% of customers in the SE and SEL series combined purchase a vehicle with the respective components that make up the packages under consideration. With packaging, this number rises to 78.0% as a result of vehicles with packages having higher total utility. The total number of configurations under packaging is 530 configurations across the four series, while 626 configurations are supplied without packaging. Total sales between the SE and SEL drop from 212k to 206k vehicles and total revenue drops by $116 million. However, the combined cost of the fixed cost for the 7th powertrain and the unit costs to produce another 6k vehicles, create a situation where even though $116 million in revenue is lost, total profitability increases by $1.3 million. The model without packages included takes 42 seconds to solve on the WSU Grid with 64 cores using the IBM ILOG CPLEX solver. With the four packages added, the model takes 1321 seconds to solve on the same system.

3.4.2 Packaging Examined Under a Channel Inventory Constraint

We model each demand region as if it were a single location where customers travel to purchase vehicles. Clearly, this is not the case in the real world. Real world sales take place on dealer lots, which have finite capacity to hold finished inventory. Packaging can be especially useful in a situation such as this, when the number of finished products that can be stocked is limited. Packaging, as we model it, funnels consumer demand into a smaller set of finished products while increasing the total amount of utility that a customer obtains from purchasing a product with the packaged components. This allows the retailer to increase the amount of total utility in the market from its products, therefore increasing sales.

Because of the transformation used to keep the model linear, it is not possible to explicitly penalize or constrain the number of configurations within the mathematical model without nonlinear equations. Instead, we must constrain the decisions on which components to develop and utilize to build the assortment. In the models used in our experiments, there are four series. Thus, the set of components used to build a single configuration could actually be used to build four configurations if each series / set of components defines a unique configuration. Some of the components have zero utility in some of the series (such as a small powertrain in a high series), however, so by constraining the combinations of components to be offered we see the number of configurations offered somewhere between each configuration offered in one series and each configuration offered in all series. With our data, it is usually much closer to the former, but the exact number is problem specific.

Figure 5 shows the effect that packaging has under a channel inventory constraint versus the case where the same channel inventory constraint is present without packages. In every case, whether packages were present or not, the supply chain configuration and the assortment are the same between the two cases. This is intuitive. Since there is no penalty cost for adding configurations in our model, it will add as many configurations as possible under the channel inventory constraint in both cases. We see the impact on profit of packaging increase as the channel inventory constraint becomes tighter. Initially, the difference in profit between the two cases is under 1%. Under the tightest channel inventory constraint, the difference in profitability grows to approximately 5% with packages versus without.

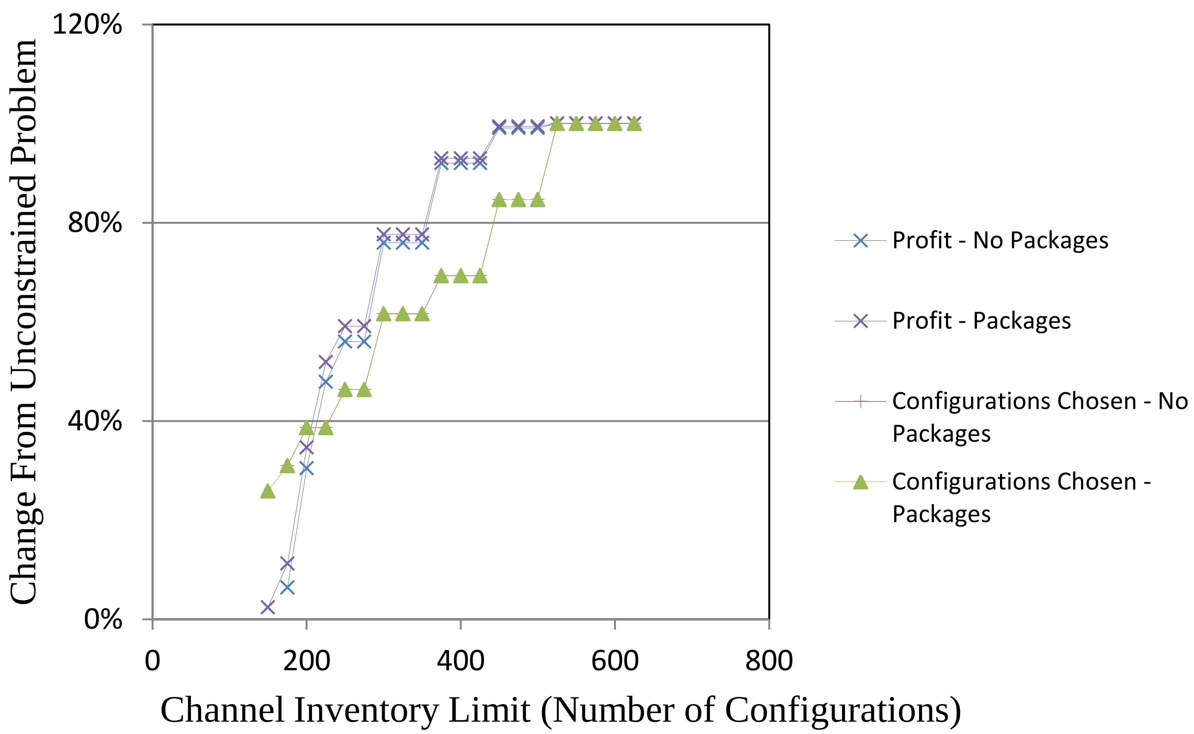

Figure 5: Effect of Packaging on Profit under a Channel Inventory Constraint

3.5 Packaging Heuristic

3.5.1 Packaging Heuristic Introduction

As the illustrative example in section 3.3 displays, the addition of packages can significantly slow down the search speed. Packaging adds a significant number of variables and constraints to the problem through transformations to preserve linearity. The exact number is problem specific based on the number of packages present for consideration as well as the number of configurations that contain the components in each package. In addition, in our experiments we usually have found that solutions where packages are chosen are usually only a couple percent more profitable than the same problem without packages. Thus, nodes within the search tree are not easily prunable, as they would be if packaging had a very large positive or negative impact on profitability.

We present a simple heuristic method that can help to deal with the intractability of our assortment planning problem with packaging. The idea is centered around the idea that while the number of packages grows linearly in the model, the number of second order interactions grows at a rate of $\binom{n_o}{2}$, where n_o is the number of packages available to be chosen. For example, while increasing the number of packages in a series from 9 to 10 increases the number of $\hat{Z}^1_{iokl}$variables by the number of configurations in that series, it increases the number of $\hat{Z}^2_{irkl}$variables by 9 times the number of configurations. The packaging heuristic can be summarized in two points:

1. Instruct CPLEX to make a deep initial dive through the search tree and find the optimal solution of the problem if all packages were set to zero. This gives CPLEX a good initial solution to use in pruning nodes in the rest of the search tree.
2. Remove the second order interactions from the problem. This will overestimate the benefits of packaging, as we are solving a maximization problem and Δap_{irk} will always be negative. However, it provides good solutions to the packaging problem while removing the dimensionality problems regarding the second order interactions.

3.5.2 Packaging Heuristic Performance

Table 4 displays the performance of the heuristic compared to solving the complete problem over a set of randomized problems of equal size (8 packages per series). We believe eight packages per series provides the right number of decisions (i.e., with fewer packages the solver does not make enough decisions to fairly judge the optimality gap) while providing that the complete model is still solvable in a reasonable time. To create randomized packaging effects, we drew a random number from [0, 0.6] to represent the percentage increase in the utility of the components that would be achieved if a package was chosen. The price decrease of the components in the package was kept constant at 5% of the component's selling price. The packages consisted of popular radios, seats, and binary options in the series. As the table clearly shows, the heuristic method results in a small deviation from optimality (less than 1%) while providing a very large time savings.

Table 4: Summary of Heuristic Performance over Random, Equally Sized Problems

		Heuristic		Full Problem		Difference	
Packages per Series	% Utility Increase	Profit (mill)	Solution Time (sec)	Profit (mill)	Solution Time (sec)	Profit (%)	Solution Time (%)
8	rand(0,0.6)	$678.44	4818.9	$678.75	40447.5	99.95%	11.91%
8	rand(0,0.6)	$686.61	3774.1	$686.94	39039.7	99.95%	9.67%
8	rand(0,0.6)	$687.05	5568.7	$687.13	34750.1	99.99%	16.02%
8	rand(0,0.6)	$674.61	4659.3	$674.61	35145.8	100.00%	13.26%
8	rand(0,0.6)	$671.87	2981.8	$671.87	22285.1	100.00%	13.38%
8	rand(0,0.6)	$672.68	5912.2	$672.71	58914.1	100.00%	10.04%
8	rand(0,0.6)	$672.65	5407.6	$672.65	43709.8	100.00%	12.37%
8	rand(0,0.6)	$672.55	6219.4	$672.90	38159.6	99.95%	16.30%

8	rand(0,0.6)	$675.35	3883.2	$675.37	22477.2	100.00%	17.28%
8	rand(0,0.6)	$671.53	4918.0	$671.53	106344.7	100.00%	4.62%
Average						99.98%	12.48%

We also look at the heuristic's ability to scale to larger problems. We do not solve the full problem to compare optimality in these cases. The results are presented in Table 5, showing that the model can scale to 40 packages while still solving in a reasonable time. We solve 5 problems with randomly generated utility increases for each package. These results are also highly problem specific. With a large number of packages, there are a large number of possible combinations of packages that make up an individual solution. With many solutions that could potentially be nearly optimal (i.e., within the optimality tolerance gap of the solver), the model may find a solution that is within the optimality tolerance without having to explore the same percentage of search tree nodes as it would with a smaller number of packages.

Table 5: Heuristic Time Performance over Five Differently Sized Problems

Packages per Series	Number of Problems Solved	% Utility Increase	Average Solution Time (sec)
4	5	rand(0,0.6)	691.9
6	5	rand(0,0.6)	1027.9
8	5	rand(0,0.6)	1366.5
10	5	rand(0,0.6)	1927.6
20	5	rand(0,0.6)	1922.5
40	5	rand(0,0.6)	1929.3

3.6 Conclusions and Future Work

Optimal packaging in configurable products is a very complex problem, both in terms of formulating the problem in a model and in terms of solving. We believe we have introduced a novel approach to modeling packaging that incorporates mixed bundling to create an objective function that is practical for optimization. This is in contrast to much of the academic research, which studies highly stylized models of packaging. We have shown examples where packaging decisions are made with our models. We find relatively small increases in profit due to packaging (on the order of a single percent) but given the scale of the problems modeled, this can amount to significant savings. Under different problem settings, the relative effects of packaging might be magnified as well. The presence of stockout based substitution could also significantly impact the effect on the assortment of packaging.

Due to the transformations made in our approach to keep the model linear, the problem does not scale well with large numbers of packages. We present a simple heuristic that can be used to mitigate the computational inefficiency of the full problem and allow larger problems to be solved. There is an opportunity to create a specialized algorithm that could further improve performance. There are two ideas that could contribute to a successful custom algorithm: 1) When solving the full formulation, large subproblems need to be solved. Based on branching decisions within the tree, however, many of the constraints and variables may be of no use after branching down on a packaging variable; and 2) The heuristic solution always provides an upper bound on the profitability of an assortment under packaging. The heuristic could be used as a tool to prune the search tree of unprofitable nodes, leading to a quicker solution of the full formulation. An algorithm that made use of this structure to improve computational performance would be useful and is left as a future extension.

Chapter 4: Sustainability of Configurable Product Assortments

4.1 Introduction

The sustainability of their products and operations is without a doubt, a top concern for firms producing configurable products in today's business climate. The Brundtland report, a seminal work on the subject, defines sustainable development as development that meets the needs of the current generation without compromising the ability of future generations to meet their own needs (Brundtland 1987). Many companies are today publishing sustainability reports through standardized reporting venues, such as the Global Reporting Initiative (GRI)[2]. These reports have embraced the concept of a triple bottom line – that sustainable companies are able to balance economic prosperity, environmental preservation, and be socially responsible, while meeting the needs of current and future generations.

For the automotive industry with its large, complex supply chains producing greenhouse gas (GHG) emitting vehicles, the environmental leg of the triple bottom line has received increased attention since the Brundtland Report. Corporate Average Fuel Economy (CAFE) standards produced by the National Highway Traffic Safety Administration (NHTSA) and the Environmental Protection Agency (EPA) restrict the average fleet fuel economy for a company's vehicles that are sold in the U.S., with penalties for missing its guidelines. GHG are not the sole inspiration for the introduction of CAFE legislation, but they are a contributing factor. The penalties are dependent on total volume, but can quickly add up to many millions of dollars for a company that misses their mark by a single mile per gallon. The unit of measurement is typically Co2e, which translates the potential global warming effects of all greenhouse gases into the

[2] www.globalreporting.org

equivalent amount of Co2[3]. Other government actions to force environmental sustainability into the automotive industry include taxation of fuel sources (Parry and Small 2005) and rebates on fuel efficient vehicles (de Haan, Peters et al. 2007).

In this chapter, we utilize the model presented in Chapter 2 to explore aspects of environmental sustainability through our joint assortment planning and supply chain model. In chapter 2 we claimed that assortment planning work has typically been retail-focused and neglected a treatment of the effect of the assortment decisions on supply chain complexity. We extend this claim to add that to the best of our knowledge, there has been no published work that combines assortment planning with sustainable supply chain management. We look at how the assortment and supply chain of an automobile reacts to constraints on: 1) vehicle fuel economy; 2) product use GHG; and 3) supply chain GHG emissions. Supply chain emissions are frequently broken down into different "scopes" when reported[4]. Scope 1 emissions are produced by the company's directly owned equipment and activities. Scope 2 emissions are from purchased electricity or other energy. Scope 3 emissions would be any other emissions that are associated with the product from 3rd parties, such as suppliers. We also look at the impact of assortment decisions on the various scopes of supply chain carbon emissions.

The rest of this chapter proceeds as follows – in section 2 we review the relevant literature. In chapter 3, we discuss important details to the design of our experiments, including the environmentally-related data. In section 4, we present and draw conclusions from our results of the individual experiments. In section 5, we conclude and discuss future work.

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[3] http://www.epa.gov/nsr/fs20090930action.html

[4] http://www.thecarbonreport.com/carbon-footprint-definition/

4.2 Literature Review

Section 2.2 has a literature review on assortment planning. This section can be viewed as an extension of that one.

Sustainable assortment planning may be a field with little research but sustainable supply chain management has been a rapidly growing field of study in recent years. Kajikawa and Ohno et al. (2007) show the annual number of publications containing variations of the word "sustainable" in the title increased from almost zero in 1990 to nearly 4000 articles a year by the mid-2000s. A great number of online tools have also been published. Padgett and Steinemann et al. (2008) analyze ten different carbon calculators for the carbon footprint of Americans. They find that there are large differences in the estimates even for common input parameters, and little visibility into the inner-workings of the model to explain the differences.

One popular method of determining carbon emissions for a given amount of activity is an economic input-output life cycle analysis. We use the Carnegie-Mellon developed tool[5] in our analysis. A detailed description of the method can be found in Hendrickson and Horvath et al. (1998) and the idea is an extension of work done in Leontief (1970). The method can be summarized as follows – A matrix is created that equates the amount of output from an industry j to create one dollar of output from industry i. There may be a second tier supplier k, with the amount of output known to create one dollar of output from industry j. Through matrix algebra, it is possible to express the amount of output from all suppliers required to produce one dollar in industry i. With knowledge of the impacts of each industry on a national level (i.e., units of carbon produced, water used, etc. per dollar of work in each industry) all the pieces are available to equate dollars spent in a single industry to its impact across its entire supply chain.

[5] www.eiolca.net

Many researchers have been focusing on life cycle analysis of electrified vehicles. Notter and Gauch et al. (2010) conduct a life cycle analysis on the environmental impact of lithium ion battery powered vehicles. They find that the environmental impacts of the vehicle operation phase dominate the impacts during production, with operation impacts of electrified vehicles being heavily dependent on the source of fuel used in producing the electricity. They found the greatest environmental burden from the production of batteries comes from the production of copper and aluminum, used in battery itself and connecting wires. Samaras and Meisterling (2008) study the GHG emissions of plug-in hybrid electric vehicles and find a 32% decrease in life cycle greenhouse emissions compared to a traditional gasoline engine, but small improvements over a traditional hybrid. Also, they report that 2-5% of life cycle emissions for a plug-in hybrid can be attributed to battery manufacturing. Ma and Balthasar et al. (2012) conduct a life cycle comparison of electrified vehicles and conventional vehicles under different conditions, accounting for factors such as driving behavior and electrical grid load. They find that due to the mixes of fuel sources used at varying electrical grid loads, electrified vehicles have lower life cycle emissions when batteries are charged at times when electrical demand is lowest. Also, electrified vehicles perform best compared to conventional vehicles when driven at low speeds, with low weights, and low demands for auxiliary power (such as air conditioning). Graedel and Allenby et al. (1995) present a scoring method of life cycle analysis and use it to compare the environmental effects of automobiles from cradle-to-grave in the 1950s and 1990s. The analysis is at a very high level, providing both ease of modeling but also a lack of detailed information.

4.3 Mathematical Modifications for Sustainability Modeling

We create an addendum to the model presented in Chapter 2 to allow its use for sensitivity analysis of the environmental impact of the decisions. As in Chapter 2, ρ^1_{msvt} and ρ^2_{iklt} represent the unit production at supplier and assembly facilities. Let F be an exogenous limit on Co2e emissions from the supply chain per vehicle produced. Let g^1_{msvt} be the unit Co2e emissions produced if component m is produced at supplier s at economy of scale level v for assembly plant t and g^2_{iklt} be the unit Co2e emissions produced if configuration i is produced for series k and demand region l at assembly plant t. Let G be an exogenous limit on the amount of Co2e emissions from product use per vehicle produced. Let g^3_{ikl} be the average Co2e emitted over the product use cycle of configuration i for series k in demand region l. Let H be an exogenous limit on the average fuel economy for a vehicle program. Let g^4_i be the fuel economy of a configuration i. Thus, the constraints for product use Co2e, the MPG for the program, and supply chain Co2e are given in constraints 69-71.

$$\sum_{m=1}^{n_m}\sum_{s=1}^{n_s}\sum_{v=1}^{n_v}\sum_{t=1}^{n_t} g^1_{msvt}\rho^1_{msvt} + \sum_{i=1}^{n_i}\sum_{k=1}^{n_k}\sum_{l=1}^{n_l}\sum_{t=1}^{n_t} g^2_{iklt}\rho^2_{iklt} \leq F\sum_{i=1}^{n_i}\sum_{k=1}^{n_k}\sum_{l=1}^{n_l}\sum_{t=1}^{n_t}\rho^2_{iklt} \tag{69}$$

$$\sum_{i=1}^{n_i}\sum_{k=1}^{n_k}\sum_{l=1}^{n_l}\sum_{t=1}^{n_t} g^3_{ikl}\rho^2_{iklt} \leq G\sum_{i=1}^{n_i}\sum_{k=1}^{n_k}\sum_{l=1}^{n_l}\sum_{t=1}^{n_t}\rho^2_{iklt} \tag{70}$$

$$\sum_{i=1}^{n_i}\sum_{k=1}^{n_k}\sum_{l=1}^{n_l}\sum_{t=1}^{n_t} g^3_{i}\rho^2_{iklt} \leq H\sum_{i=1}^{n_i}\sum_{k=1}^{n_k}\sum_{l=1}^{n_l}\sum_{t=1}^{n_t}\rho^2_{iklt} \tag{71}$$

4.4 Data Considerations for Sustainability Modeling

4.4.1 MPG Requirements and Product Use Emissions

To estimate the amount of product use emissions for each of the powertrain variants, we need a few key parameters. We assume that only the powertrain variant determines the amount of product use Co2e that will be emitted, not the radio, seats, or binary options. This is a simplifying assumption made for our purposes. In reality, while the physical weight of components such as electronics may be negligible, their impact on the power needs of the system as a whole is not (Bjelkengren 2008). Accounting for the effect of components on the vehicle as a system (rather than the sum of individual components) is left as an extension through future research.

From SMEs, we assume that cars will average 125k miles of life. For each gallon of gasoline that is combusted, 20 pounds of Co2e are emitted[6]. Electric vehicle fuel efficiency is typically rated on a measure called MPGe (MPG equivalent). 34 kilowatt hours of electricity are used to produce each MPGe[7] and 1 kilowatt hour of energy produces 1 pound Co2e[8]. Table 6 displays the fuel efficiency of each of the powertrains, taken from the Ford website (with some interpolation and expansion, as their variant set is not the same as ours). These are the inputs required for an analysis of a fuel efficiency requirement. The fuel efficiencies combined with the data on carbon emissions per unit of fuel are the building blocks needed to determine the product use carbon emissions for vehicles, presented in Table 7.

Table 6: Fuel Efficiency of Powertrain Variants

Powertrain Variant	**Fuel Efficiency**
2.0L Manual FWD	27 MPG

[6] http://www.peoplesworld.org/a-gallon-of-gas-makes-20-pounds-of-co2/
[7] http://www.csmonitor.com/Innovation/2011/1225/Electric-cars-101-What-does-MPGe-mean-exactly
[8] http://www.stewartmarion.com/carbon-footprint/html/carbon-footprint-kilowatt-hour.html

2.0L Manual AWD	26 MPG
2.0L Auto FWD	28 MPG
2.0L Auto AWD	27 MPG
2.5L Manual FWD	22 MPG
2.5L Manual AWD	21 MPG
2.5L Auto FWD	23 MPG
2.5L Auto AWD	22 MPG
3.0L Auto FWD	23 MPG
3.0L Auto AWD	22 MPG
3.5L Auto FWD	21 MPG
3.5L Auto AWD	20 MPG
Hybrid	47 MPG
Plug-In Hybrid	100 MPGe
Fully Electric	110 MPGe

Table 7: Product Use Co2e for Powertrain Variants

Powertrain Variant	**Product Use Co2e (1000s of pounds)**
2.0L Manual FWD	92.59
2.0L Manual AWD	96.15
2.0L Auto FWD	89.29
2.0L Auto AWD	92.59
2.5L Manual FWD	104.17
2.5L Manual AWD	108.70
2.5L Auto FWD	100.00
2.5L Auto AWD	104.17
3.0L Auto FWD	108.70
3.0L Auto AWD	113.64
3.5L Auto FWD	119.05
3.5L Auto AWD	125.00
Hybrid	53.19
Plug-In Hybrid	42.50
Fully Electric	38.64

4.4.2 Supply Chain Emissions

We rely heavily on the Carnegie Mellon Economic Input-Output Life Cycle Analysis tool in our supply chain analysis.[9] This serves several purposes: 1) It provides a summary of the emissions by production in the automotive industry based on national-level economic data, providing a good estimate of emissions data without a need for a deep dive into an individual companies processes, 2) It provides a way to measure carbon impact from raw material collection to final vehicle assembly, and 3) It provides the ability to separate the emissions by scope. We model a cradle-to-gate supply chain using the model. Our demand model is not capable of incorporating the level of detail required to model demand on a dealership level, thus final transportation of the products to dealers is left out of the scope of supply chain emissions.

Information from EIOLCA.net in this sections is presented as such [Co2e emissions (metric tons per million dollars of activity) | scope 1 % | scope 2 % | scope 3 %]. From the website, we can gather the following information directly:

United States – "Automotive Manufacturing" – [563 | 2% | 32% | 66%]

United States – "Motor Vehicle Parts Manufacturing" – [757 | 3% | 31% | 66%]

United States – "Broadcast and Wireless Communications Equipment" – [322 | 3% | 43% | 54%]

China – "Motor Vehicles" – [2240 | 4% | 40% | 55%]

China – "Communication Equipment" – [1870 | 1% | 44% | 55%]

There are few things that need to be clarified. Both the U.S. and China models on the website were created using producer economic and environmental data from 2002. It is very likely emission rates have been lowered since then, due to 10 years of effort in the industry to improve sustainability. In addition, the models use different categories. We use the category most related to the product we are trying to study (i.e., "Broadcast and Wireless Communications Equipment"

[9] www.eiolca.net

in the more detailed U.S. model and "Communication Equipment" in the China model are used for the same electronic parts). We assume that the three radio variants and navigation would fall into the category of electronic components, while the rest fall under the title of standard "Motor Vehicle Parts Manufacturing". In addition, because we model some of the parts separately and add them in to find the total carbon throughout the supply chain and the above emissions per dollar for automotive assembly is for an *entire* car, we multiply the assembly numbers by a factor of .6, as we assume that 60% of that activity is attributed to activities not related to our studied components.

We are still missing some required data – standard parts manufacturing in China and all the categories for Mexico. For standard parts manufacturing in China we assume the same ratio between the U.S. final assembly and U.S. parts manufacturing can be applied to China. We do not have access to a life cycle analysis of the country of Mexico. However, we do know that the country's ratio of fossil fuels to renewable energy and nuclear (nuclear produces very little carbon also) is similar to the U.S., due to large amounts of geothermal energy sources[10], a key factor in GHG efficiency. We believe production in Mexico and the U.S. probably produce similar amounts of GHG emissions. Due to lack of data, we assume that Co2e emissions per dollar are 10% higher than in the U.S., with scope percentages that fall between the U.S. and China. A life cycle analysis of Mexico would allow us to be significantly more accurate. The indirectly estimated data is thus:

China – "Motor Vehicle Parts Manufacturing" – [3011 | 7% | 38% | 55%]

Mexico – "Automotive Manufacturing" – [619 | 3% | 36% | 61%]

Mexico – "Motor Vehicle Parts Manufacturing" – [832 | 5% | 35% | 60%]

Mexico – "Broadcast and Wireless Communications Equipment" – [354 | 2% | 43% | 55%]

[10] http://en.wikipedia.org/wiki/Electricity_sector_in_Mexico

4.5 Experimental Results

4.5.1 Program MPG Target

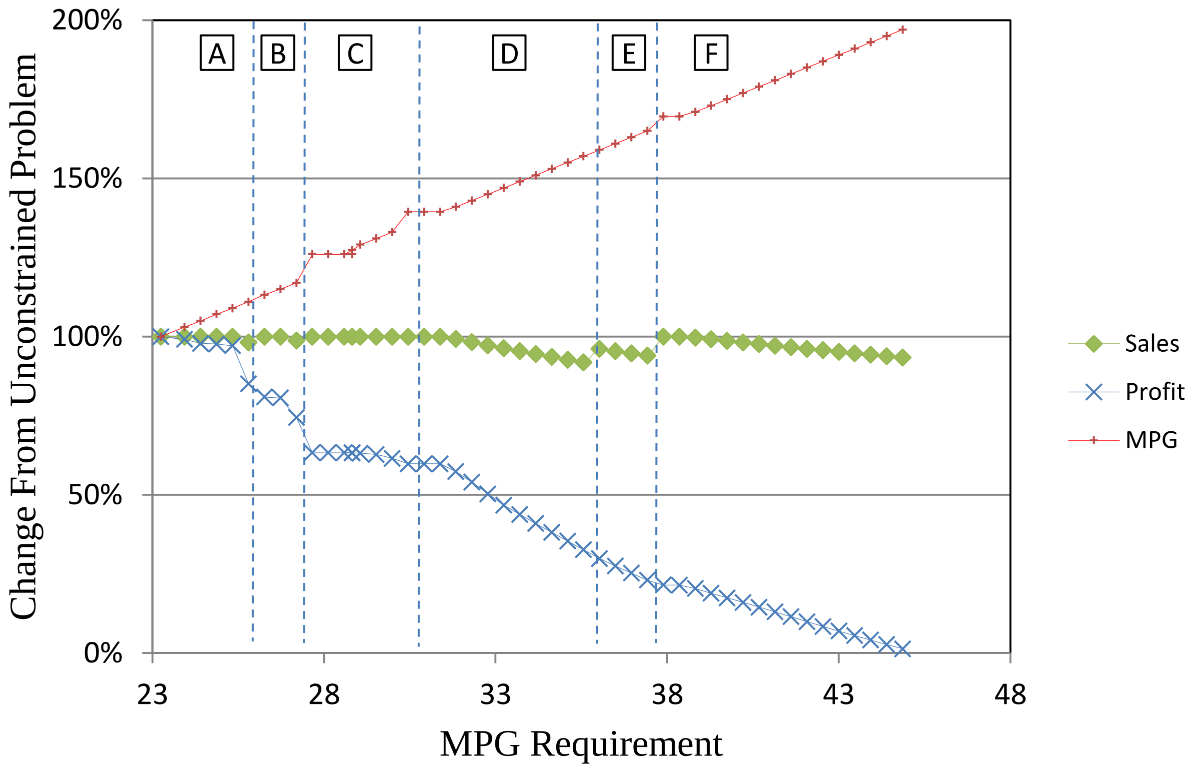

Figure 6: Effects of an MPG Requirement on Profit, Sales, and MPG

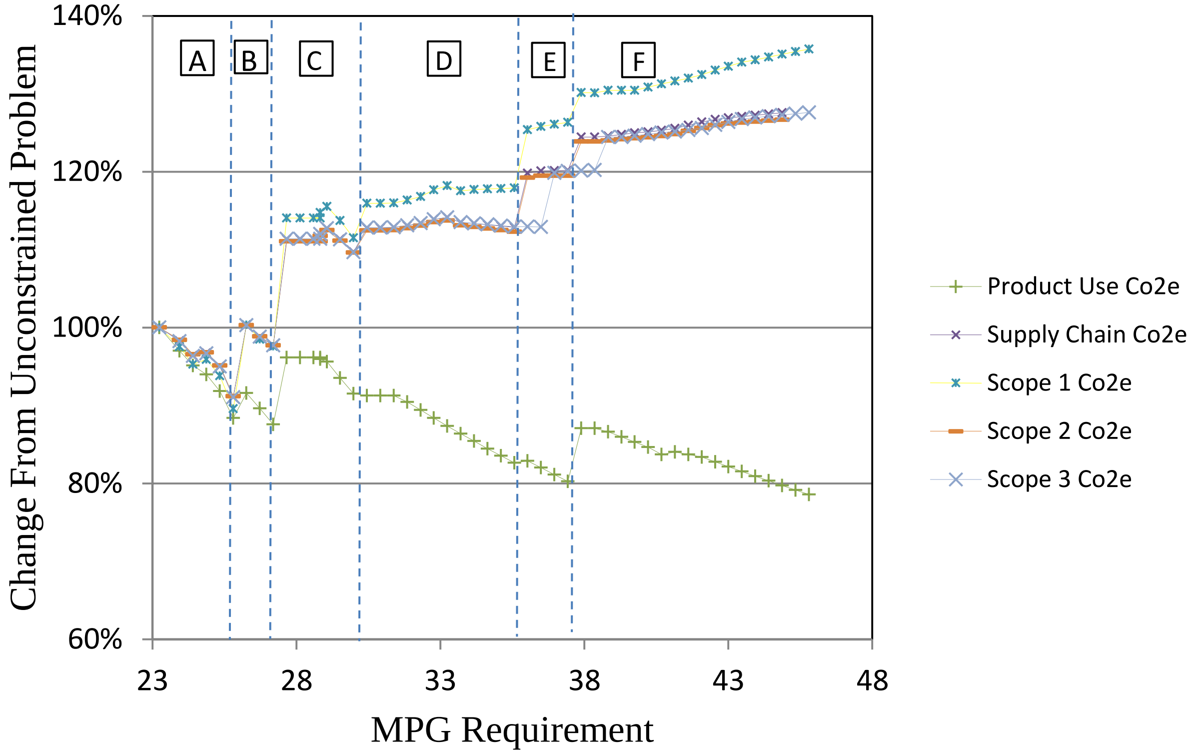

Figure 7: Effects of an MPG Requirement on All Co2e Emissions

Unconstrained Program MPG Case: Profit = $746.5 million, MPG = 23.25 MPG, Sales = 300k vehicles, Product Use Co2e = 108.6k pounds per vehicle, Supply Chain Co2e = 34k pounds per vehicle, Scope 1 Co2e = 1.1k pounds per vehicle, Scope 2 Co2e = 11.6k pounds per vehicle, Scope 3 Co2e = 19.3k pounds per vehicle.

Program MPG Range A

- Powertrain sizes move towards the smallest options that are still popular in each series (such as the 3.0L Auto AWD powertrain in the Titanium series instead of the originally chosen 3.5L Auto FWD powertrain).
- The number of configurations offered in the SE series increases from 36 to 48 from the beginning of the range to the end, while the number offered in the SEL series decreases

from 48 to 39. The number of Titanium configurations stays constant, as the model protects the highest margin vehicles.

Program MPG Range B

- The hybrid powertrain is selected for production, creating a jump in program MPG and a nearly flat profit graph immediately after.
- Initially, the addition of the hybrid creates enough slack between the MPG requirement and the realized MPG to allow assortments in the SEL and Titanium to reset to the choices that were present in the base case (more configurations and larger engines).
- Supply chain Co2e emissions rise as sales increase slightly and hybrid is produced, which creates more supply chain emissions than the standard powertrains.
- Product use Co2e emissions increase. However, with some SE customers substituting from standard engines to the lower product use emission hybrid, total product emissions are lower than the base case.
- As the MPG requirement is tightened within this range, the same changes that occurred in range A repeat (more configurations in the SE series and smaller engines throughout).

Program MPG Range C

- The fully electric powertrain is now selected for production. This instantly raises the program MPG from about 27 to 30. As the MPG requirement is increased but still under 30, no changes occur.
- When the MPG requirement reaches approximately 30 MPG, configurations with standard engines start to be eliminated from the SE series. This results in lost sales in the SE series, but pushes SE customers remaining towards the fully electric powertrain.

Conversion rates in the SEL and Titanium series increase to keep the total sales near 300K, but a 3.0L Auto AWD replaces the 3.5L Auto FWD, creating the dip in supply chain Co2e.

- The separation in the scope 1 emissions from the other scopes can be explained as follows – When the model is unconstrained, it will sometimes neglect to offer configurations that do not include a moon roof and/or navigation system, as they have lower selling prices and compete with other higher priced configurations the manufacturer can offer. During the experiments, it becomes important for the manufacturer to offer the full set of configurations created from certain engines to meet MPG requirements, including configurations without a navigation system. Because the navigation system is an electronic component, it has a lower percentage of its emissions attributed to scope 1 than standard components. Thus, with fewer navigation systems being produced, there is a higher percentage of scope 1 emissions. Because scope 1 emissions are on a much smaller scale than scope 2 and 3 emissions, the effect is magnified in comparison.
- Total supply chain emissions increase over the base case, as electrified powertrains create more supply chain carbon than standard powertrains.

Program MPG Range D

- At the beginning of the section, the SE assortment changes from having 2 standard powertrains plus the fully electric to just having a single standard powertrain and the fully electric. This increases the sales of the fully electric from approximately 25k to 30k. However, total sales in the SE series between the US and Canada fall from 112k to 102k.

- With the single standard powertrain in the SE series combined with a fully electric powertrain, it is now profitable to add a second engine to the Titanium series (both a 3.0L Auto AWD and a 3.5L Auto AWD are now offered), increasing demand coverage in that series from 72% to 80%.
- As the CAFE requirement is tightened throughout section D, fewer and fewer standard powertrain configurations are offered in the SE series, resulting in fewer overall sales in the series but a higher proportion of sales having a fully electric powertrain.

Program MPG Range E

- Supply chain carbon increases as the manufacturer now chooses to supply the hybrid powertrain in addition to the fully electric.
- The model continues to protect the sales of the SEL and Titanium series (approximately 80% conversion rate) at the cost of the SE series (approximately 50% conversion rate). Added MPG is achieved by offering fewer configurations in the SE series with standard powertrains, forcing customers to either choose electrified powertrains or walk.

Program MPG Range F

- Section F is essentially a repeat of section E, but the plug-in hybrid is chosen instead of the standard hybrid.
- The model protects the SEL and Titanium series as long as it can, but eventually it is under too much stress (approximately 46 MPG) and the optimal solution becomes not to produce anything.

Conclusions

- The first derivative of the profit function of a manufacturer can change significantly with respect to the program MPG requirement, as offering electrified vehicles can raise the program MPG beyond the requirement at certain levels.
- We see total supply chain carbon emissions increase with the introduction of electric vehicles, which is consistent with the literature.
- Our model is somewhat rigid in that only customers in the SE series consider electrified vehicles and customers do not switch between series. However, we still believe we have shown that in some cases it may be optimal for automakers to: 1) Use electrified vehicles as loss leaders to enable the continued selling of vehicles with large standard powertrains and 2) Limit the number of conventional powertrains offered to customer segments that are likely to consider electric powertrains, pushing customers towards electric powertrains by reducing possible substitutes.

4.5.2 Program Product Use Co2e Target

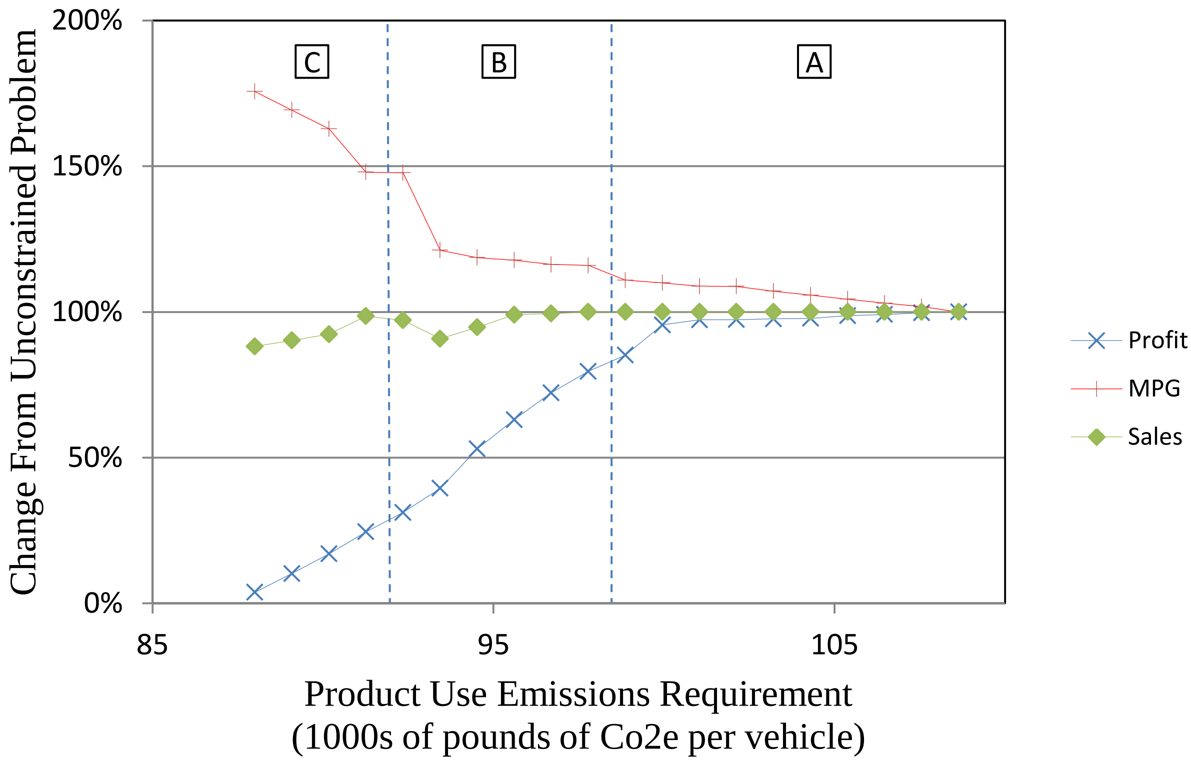

Figure 8: Effect of a Product Use Co2e Requirement on Profit, Sales, and MPG

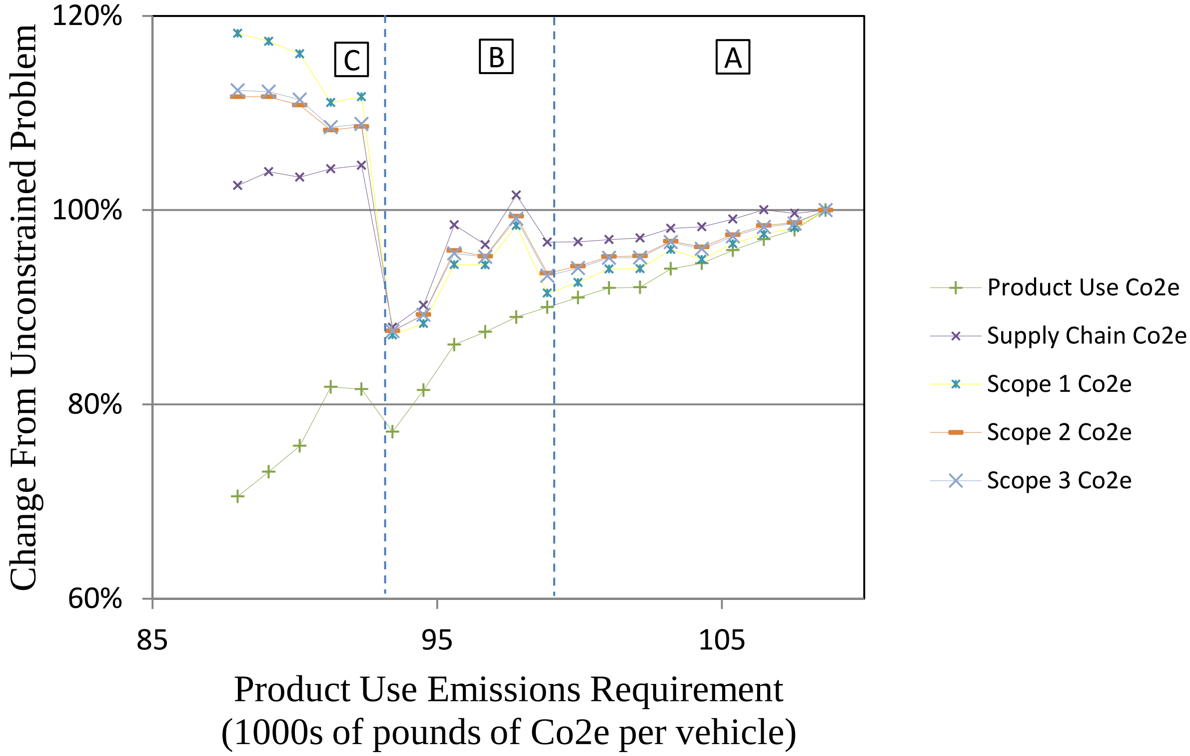

Figure 9: Effects of a Life Cycle Co2e Requirement on All Co2e Emissions

Unconstrained Program MPG Case: Profit = $746.5 million, MPG = 23.25 MPG, Sales = 300k vehicles, Product Use Co2e = 108.6k pounds per vehicle, Supply Chain Co2e = 34k pounds per vehicle, Scope 1 Co2e = 1.1k pounds per vehicle, Scope 2 Co2e = 11.6k pounds per vehicle, Scope 3 Co2e = 19.3k pounds per vehicle.

<u>Please note:</u> We graph the above figures with the absolute life cycle Co2e requirement on the horizontal axis. It is easier to explain the changes incrementally from the base case to the point where the life cycle Co2e requirement is too stringent to allow the company to be profitable. Thus, we analyze the graphs from right to left.

<u>Product Use Co2e Range A</u>

- As in the program MPG requirement experiment, the least detrimental change that the model can make (in terms of overall profit) to improve the product use Co2e emissions is to substitute smaller, but still popular powertrains in place of larger ones. The S series continuously offers only the 2.0L Auto FWD powertrain. From the unconstrained case to the end of range A, the SE series substitutes a 2.0L Auto AWD for a 3.0L Auto FWD. The SEL series substitutes from an assortment containing the 3.0L Auto FWD and 3.5L Auto FWD to one containing the 2.0L Auto FWD, 2.5L Auto FWD, and 3.0L Auto AWD. The Titanium series initially chooses a 3.5L Auto FWD but at the end of the range chooses a 3.0L Auto AWD.
- Despite all these changes, the sales conversion rate remains between 70% and 80%, depending on the combination of options chosen. Sales do not uniformly increase or decrease over the range, however.

Product Use Co2e Range B

- The S and Titanium series assortment remains constant throughout range B.
- The hybrid powertrain is chosen for the SE series throughout the range.
- The peaks and valleys of the supply chain Co2e can be explained through differences in the SE and SEL series. The SEL series starts with a 2.0L Auto AWD and a 3.0L Auto AWD. When the product use Co2e requirement reaches 95 thousand pounds per vehicle, a 2.5 Auto AWD is substituted for the 3.0L Auto AWD. At 94 thousand pounds per vehicle, only a 2.0 Auto AWD is offered to the SEL series, and sales begin to fall. Depending on the amount of sales in the SE series, the model has to offer a certain number of hybrids to keep the average product use Co2e under the requirement, causing supply chain carbon to rise when more hybrids are produced.

Product Use Co2e Range C

- The model offers a constant assortment to the S, SEL, and Titanium series.
- The hybrid and fully electric powertrains are offered to the SE series, in addition to a 2.0L Auto AWD.
- As the product use emissions constraint is tightened, the model offers fewer and fewer SE vehicles with standard powertrains. At the start of range C, the conversion rate for the SE series is 70%. By the end, it has been reduced to 51%. The model is restricting the supply of standard SE vehicles to push SE customers towards electrified vehicles, which enables it to make profit off of vehicles in the other series and still meet the product use emission requirement.
- We see higher scope 1 emissions relative to the base case than scope 2 or 3 under a strong product use emission requirement, as in the program MPG case and for the same reason (a lower percentage of the vehicles containing the navigation system).
- Any product use requirement that is stricter than 88 thousand pounds per vehicle will cause the automaker to be unable to create a profit.

Conclusions

- As in the program MPG requirement case, we see the automaker restricting the supply of possible substitutes for electrified vehicles in order to push customers towards buying them, while trying to recoup lost sales in the SE series in the SEL and Titanium series.
- Restricting product use emissions can lead to higher relative supply chain emissions, as electric vehicles produce more carbon in the supply chain phase. Due to the scales of the two emissions being different (product use is much larger than supply chain Co2e, even

under a cradle-to-gate analysis), the net is a reduction in overall greenhouse gas emissions.

- We model electrified vehicles gaining power from the current power grid in place in the US and Canada. If these power sources were to generate electricity using more renewable energy and fewer fossil fuels, the graphs would look different and the manufacturer could produce under more stringent product use emission requirements and still be profitable.

4.5.3 Program Supply Chain Co2e Target

There are two ways to consider a supply chain Co2e target – either the assortment is restricted to the assortment chosen in the base case or the assortment is free to be changed. We present both. Due to the great amount of similarity in the shapes of the graphs for scope 1, scope 2, and scope 3 Co2e, we do not do sensitivity runs on them.

4.5.3A Program Supply Chain Co2e Target (Assortment Fixed)

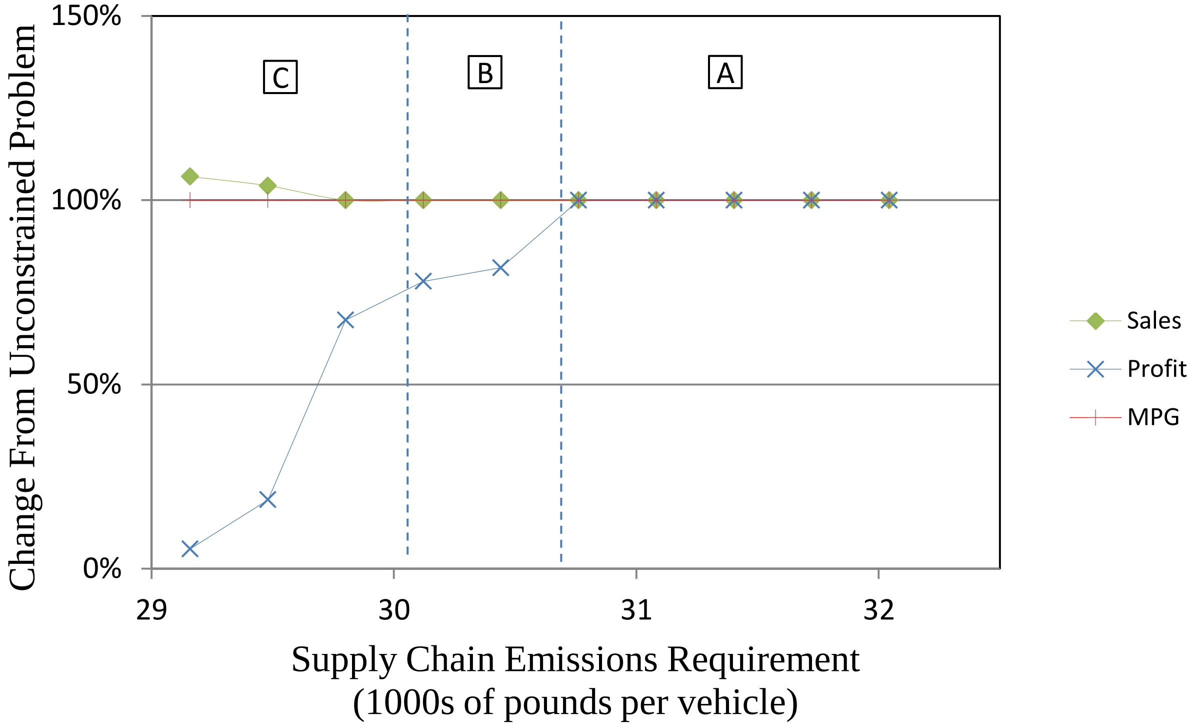

Figure 10: Effects of a Supply Chain Co2e Requirement on Profit (Assortment Fixed)

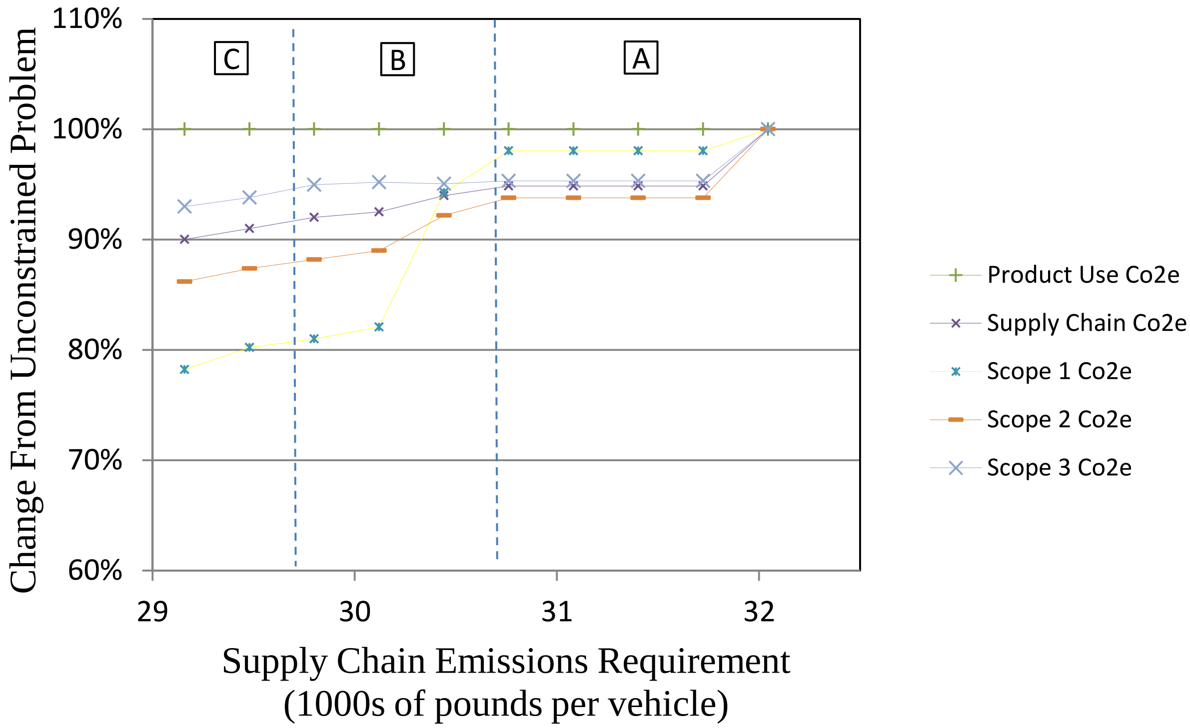

Figure 11: Effects of a Supply Chain Co2e Requirement on All Co2e Emissions (Assortment Fixed)

Unconstrained Program MPG Case: Profit = $746.5 million, MPG = 23.25 MPG, Sales = 300k vehicles, Product Use Co2e = 108.6k pounds per vehicle, Supply Chain Co2e = 34k pounds per vehicle, Scope 1 Co2e = 1.1k pounds per vehicle, Scope 2 Co2e = 11.6k pounds per vehicle, Scope 3 Co2e = 19.3k pounds per vehicle.

Supply Chain Co2e (Assortment Fixed) Range A

- Production of the navigation system moves from China to Mexico. Profit is reduced by approximately $500k. No further supply chain changes are required within range A to meet the supply chain emissions requirement.

Supply Chain Co2e (Assortment Fixed) Range B

- In the case of the first data point (reading right to left), production of all radios, seats, and binary options moves to the US. Production of the powertrains and final assembly remain in Mexico.
- In the case of the second data point, the production of all seats, radios, and binary options as well as final assembly takes place in Mexico. Powertrain production takes place in the US.

Supply Chain Co2e (Assortment Fixed) Range C

- In the case of the first data point, production of the binaries, radios, and powertrains takes place in the US. Final assembly and seat production takes place in Mexico.
- In the case of the second data point, the US assembly plant is opened in addition to the Mexican assembly plant. Powertrain production takes place in the US but all other components are produced in Mexico.
- In the case of the final data point, more of the production is shifted from the Mexican assembly plant to the US assembly plant. Powertrain and moon roof production takes place in the US. Radio, seat, and navigation system production takes place in Mexico.

Conclusions

- As expected, when the assortment is fixed, to meet a tighter supply chain Co2e requirement the model moves production either from China to the Mexico or from Mexico to the US, where fewer greenhouse gases are produced per dollar spent manufacturing.

4.5.3B Program Supply Chain Co2e Target (Assortment Free)

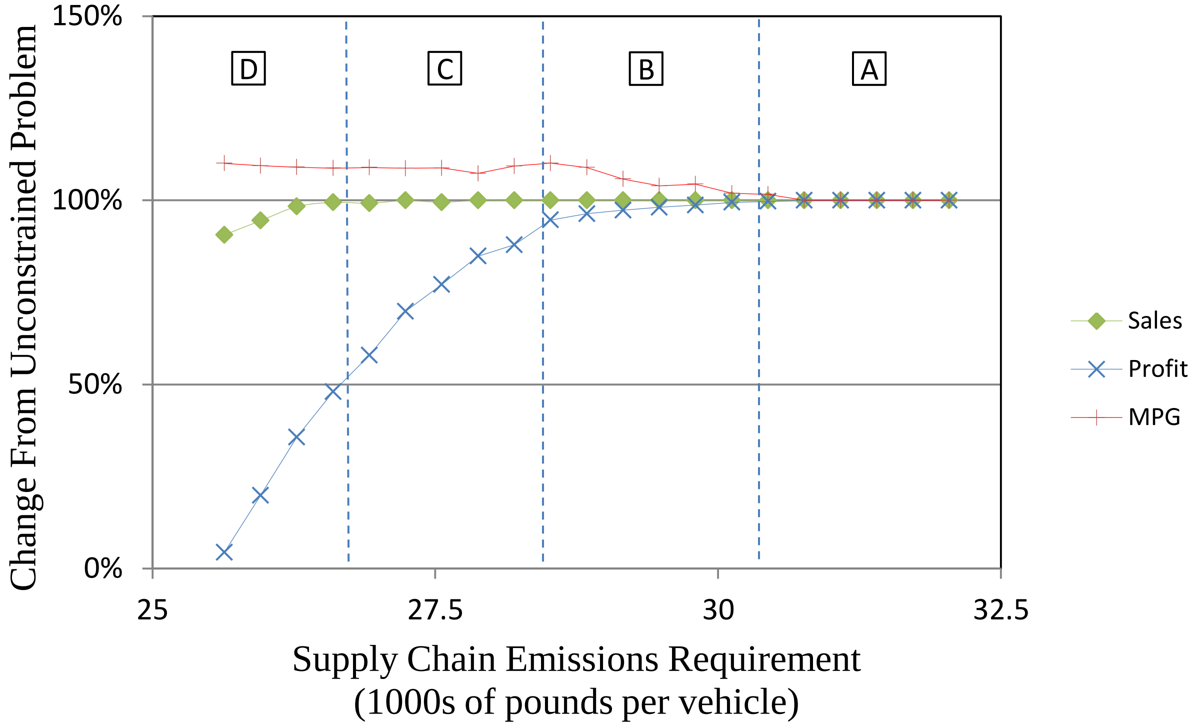

Figure 12: Effects of a Supply Chain Emissions Requirement on Profit and MPG (Assortment Free)

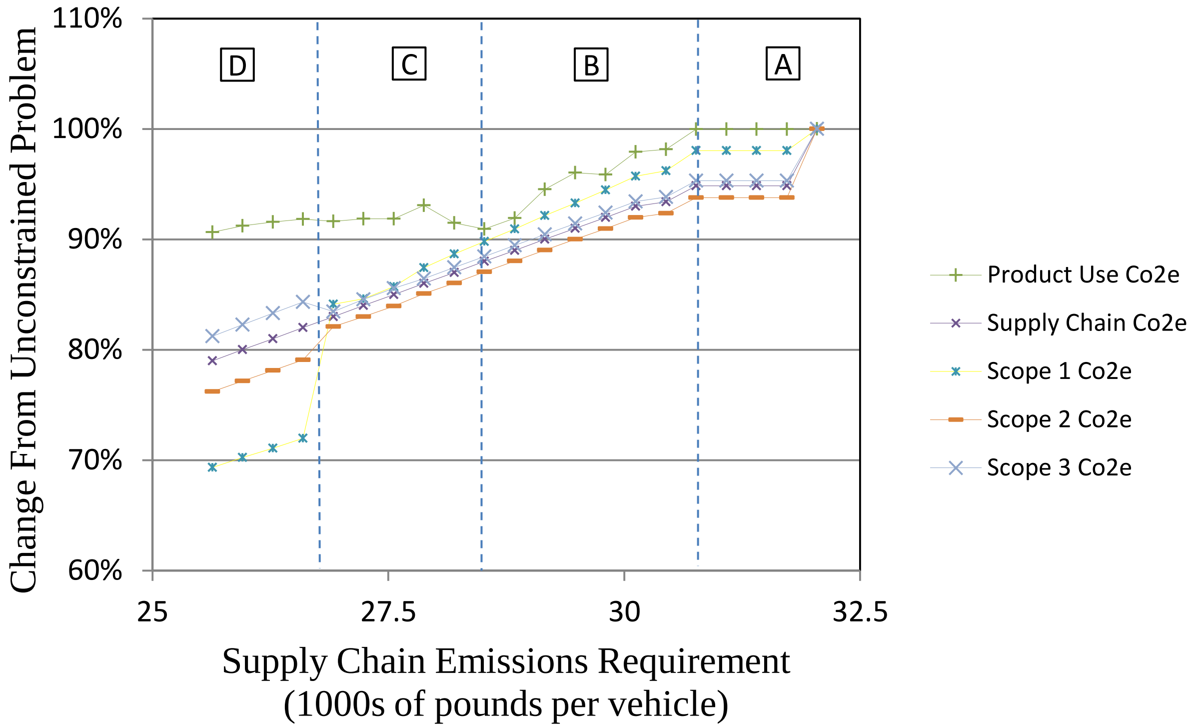

Figure 13: Effects of a Supply Chain Emissions Requirement on All Co2e Emissions (Assortment Free)

Unconstrained Program MPG Case: Profit = $746.5 million, MPG = 23.25 MPG, Sales = 300k vehicles, Product Use Co2e = 108.6k pounds per vehicle, Supply Chain Co2e = 34k pounds per vehicle, Scope 1 Co2e = 1.1k pounds per vehicle, Scope 2 Co2e = 11.6k pounds per vehicle, Scope 3 Co2e = 19.3k pounds per vehicle.

Supply Chain Co2e (Assortment Free) Range A

- The navigation system production is moved from China to Mexico. This reduces profitability by $500k and is the only change necessary until to meet the supply chain emissions requirements in range A.

Supply Chain Co2e (Assortment Fixed) Range B

- Smaller powertrains produced less Co2e in their manufacturing. In range B, the automaker substitutes smaller, but still popular, powertrains for larger ones. The SE series adds a 2.0L Auto AWD in place of a 3.0L Auto FWD. The SEL series adds a 2.0L Auto AWD in place of a 3.0L Auto FWD. The Titanium series substitutes a 3.0L Auto AWD for a 3.5L Auto FWD.
- The number of configurations supplied in the SE series increases from 37 to 48 (with conversion rate increasing from 74% to 78%), while the number of configurations supplied in the SEL series decreases from 48 to 39 (with conversion rate decreasing from 82% to 72%).
- Total sales remains constant but the sale of lower margin, smaller powertrains in place of larger ones causes profits to decrease.

Supply Chain Co2e (Assortment Free) Range C

- The model continues to offer more configurations to the SE series, but in range C, it reduces the conversion rate in the Titanium series to create capacity for extra vehicles in the SE series as well as removing more large powertrains from the assortment. Throughout range C, conversion rate in the Titanium series decreases from 70% to 56%. The model adds a 2.0L Manual AWD and a 2.5L Manual AWD to the assortment in the SE series to raise its conversion rate from 78% to 83%.
- Near the end of range C, a 2.0L Manual FWD powertrain is added to the S series, raising the S series conversion rate from 84% to 94%. This

Supply Chain Co2e (Assortment Free) Range D

- The model moves production of the powertrains and the moon roof from Mexico to the US. Radio, seat, navigation system, and final assembly activity remain in Mexico. This allows some series to offer more configurations. For example, the conversion rate in Titanium series increases from 55% to 59%. However, the model is still extremely constrained by the supply chain emission requirement and the changes to conversion rates are relatively minor.
- Further tightening of the supply chain emissions requirement within range D leads to further reductions in the conversion rates in the SEL and Titanium series. When the requirement reaches 25,600 pounds of Co2e emitted in the supply chain per vehicle, the conversion rate in the Titanium series has been reduced to 33%. Any further tightening of the requirement, and the automaker can no longer be profitable.

Conclusions

- Electric vehicles are not used as a tool by the model to limit supply chain Co2e emissions. This is intuitive, since electrified powertrains have higher supply chain emissions standard powertrains.
- An automaker can stay popular under stricter supply chain greenhouse gas emission requirements by manipulating its assortment. When the assortment was fixed, production was no longer profitable when the requirement reached approximately 29k pounds of Co2e per vehicle. When the assortment was free, however, production was profitable until the requirement reached approximately 26k pounds of Co2e per vehicle.

4.6 Conclusions and Future Work

We have demonstrated our model's ability to accommodate the needs of a producer of a configurable product who needs to increase the environmental sustainability of its products, either by legislation or corporate image. In the case of an automaker, we have shown the general trends that occur in the assortment and supply chain as a manufacturer is faced with a requirement on the average fuel economy, average supply chain emissions, and average lifecycle emissions of its products. In all cases, these three sustainability metrics can be improved through: 1) substituting smaller powertrains for larger ones (hopefully powertrains that are still popular within the series they are being targeted) and 2) modify the assortment to increase the conversion rate of customers in the lower series relative to the higher series, again, to increase the ratio of smaller powertrains that are sold to large ones. In addition, for our dataset, we have identified the circumstances under which it is profitable to invest in electrified vehicles and the effects that they have on the assortment of conventional vehicles offered.

Our method of analysis is relatively straightforward – tighten one requirement at a time while maximizing profit to identify a pareto optimal solution with respect to the constraint. More sophisticated techniques such as multiobjective optimization could be used to find solutions that are optimal with respect to all of the requirements simultaneously. However, we believe that work could be better served to increase the fidelity of the data, not the sophistication of the solving technique. This requires working with SMEs who can provide better estimates of emissions data than our currently used methods.

Chapter 5: Conclusions and Future Research

In chapter two, we introduced a model that can be utilized to by producers of configurable products such as automobiles, computers, or cell phones to jointly optimize their assortment and supply chain. The goal of the model is to find the "sweet spot" between offering too many configurations (incurring a high cost of complexity and low economies of scale) and offering too few configurations (causing unsatisfied customers to be lost to competitor's products). We introduced the model's mathematical formulation and demonstrated its efficacy through a realistic case problem modeled after today's automotive industry. We fill a literature gap in that existing models either: 1) do not scale to the needs of companies producing configurable products; or 2) do not adequately capture the supply chain cost considerations.

Despite large amounts of effort invested, we were unable to develop a custom algorithm that could outperform CPLEX's base algorithms in terms of convergence time. The development of such an algorithm could further increase the size of problem that is able to be solved in a realistic amount of time. In addition, we understand that decisions of the complexity level as the ones we make are subject to a large amount of uncertainty regarding the business climate. Our model is not intended to be a stand-alone model that determines exactly the decisions a producer of configurable products should make, but more a tool that can be used to start discussion and identify managerial insights. The tools presented from the field of stochastic programming could be useful to rigorously explore for solutions that perform well over a large range of business climates. Finally, we put forth a large amount of effort to make our data realistic, but it would be preferable to use real data directly from an industrial partner.

In chapter 3, we extend the model presented in chapter 2 to include considerations for the effect of packaging on the assortment and supply chain of a producer of configurable products.

Packaging is an incredibly complex subject, touching customer demand satisfaction, pricing, and supply chain considerations. We focus on the problem of determining from a predetermined set of packages the subset of optimal packages to be chosen. We present a mathematical formulation that captures the effect that packaging has on the assortment and supply chain, something there is very little practical research on. Although the model is able to be solved for relatively small numbers of packages (on the order of 10 or so), it does not scale well with the addition of many more packages. Thus, we introduce a heuristic that we show to have very little impact on the optimal solution while reducing the convergence time of CPLEX many times over.

One direction for future research would be to study the identification of packages from customer sales data. In the case of the automotive industry, this is very difficult due to the dynamics of the ordering system and the complexity of the products. In the least case, a marketing group could identify packages through managerial instinct and knowledge of the customer base. A more rigorous, data driven approach to package identification would help to make the model more complete. Also, there is ample opportunity to reduce the computational efficiency of the problem with packaging. One possibility would be to use the heuristic within the branch and bound tree to prune nodes in the process of finding an optimal solution to the full problem. While the heuristic on its own finds very good solutions in our case, the scale of our objective function is also in the billions of dollars – a fraction of a percent is still a relatively large amount of lost profit. Also, both the heuristic and full problem contain special structure that can be exploited to improve speed – many variables and constraints can be eliminated if a packaging variable is branched down on. The current version of CPLEX does not provide this capability with little upfront investment, but it is possible through other algorithmic means.

In chapter 4, we explore the effect of sustainability metrics on the assortment and supply chain of our case developed in chapter 2. A company's performance with regards to environmental sustainability has become increasingly important over recent years, as a result of increased governmental regulation and consumer expectations that companies take steps to mitigate the environmental impacts of their products. This is especially true in the automotive industry, whose products by nature contribute large amounts of greenhouse gases to the atmosphere. We consider 3 types of metrics that a producer of automobiles might consider and try to improve upon: 1) vehicle program average miles per gallon; 2) product use phase average greenhouse gas emissions; and 3) supply chain average greenhouse gas emissions. Under each of these scenarios, we constrain the metric and require the automotive producer to meet it, while trying to optimize profitability. We analyze the results in depth over the range from no environmental requirements to the point where the requirements are so stringent that it is no longer profitable to produce, discussing trends in the assortment and supply chain. Through this analysis, we hope to provide managerial insight into the ways in which profitability is effected by environmental requirements and what changes are required to be made in order to stay profitable.

Chapter 4 does not contain any material that is algorithmically complex. It would be interesting to apply multiobjective techniques to the problem in order to find solutions that are good across all metrics at once, rather than sequentially. However, we believe that since the goal of the section is to provide managerial insight, the best application of future effort would be to improve the fidelity of data. We use the EIOLCA method not because it is the best and most accurate way to model greenhouse gas emissions from the actions a manufacturer takes, but because we do not have detailed process level data from a manufacturer that could be utilized to model their processes more precisely. Given the amount of effort that automotive manufacturers have exerted to report and improve on their environmental sustainability, they almost assuredly have data that could be used to better model the impact of environmental requirements on the assortment and supply chain. We would like to adapt to the data available and integrate it into our research, to better justify the conclusions we draw from our experiments.

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ABSTRACT

The assortment planning problem is to decide on the set of products that a retailer or manufacturer will offer to its customers to maximize profitability. While assortment planning research has been expanding in recent years, the current models are inadequate for the needs of a configurable product manufacturer. In particular, we address assortment planning for an automobile manufacturer. We develop models to integrate assortment planning and supply chain management, designed for use by a large automaker in its strategic planning phase. Our model utilizes a multinomial logit model transformed into a mixed integer linear program through the Charnes-Cooper transformation. It is able to scale to problems that contain thousands of configurations to possibly be offered, a necessity given the number of possible configurations an automaker can build. In addition, most research in assortment planning contains simplified costs associated with product complexity. We model a full supply chain and give a rich treatment of the complexity associated with product complexity. We believe that our model can significantly aid automotive manufacturers to balance their product complexity with supply chain complexity, thus increasing profitability.

In addition, we study the effect of packaging on the assortment and supply chain of an automaker. We develop a new model for mathematically expressing the effect that packaging has on the way in which customers choose products. Packaging significantly complicates the search space of the assortment planning problem. We introduce a heuristic method based on our packaging model that speeds up the solve times of the models while finding reasonably good solutions.

Finally, we extend our initial model to study the effects of sustainability requirements on an automaker’s assortment and supply chain. We introduce constraints on the vehicle program

average fuel economy, greenhouse gas emissions in the supply chain, and greenhouse gas emissions in the product use phase. We dive deep into each case to glean insights about how automakers can change their decision-making process to balance making their companies more sustainable with profit maximization. While all the examples discussed are from the automotive industry, the models developed can be adapted to address assortment planning for other types of configurable products (e.g., computers, printers, phones).

AUTOBIOGRAPHICAL STATEMENT

Edward Lawrence Umpfenbach received his B.S., M.S. and PhD in Industrial Engineering from Wayne State University over the period from 2007-2013. He has experience working in the automotive industry through several internships and worked full time for General Motors as a supervisor in an engine plant in 2008. He is a recipient of both a STIET Fellowship and a SMART Fellowship, and will work for the U.S. Army following graduation as part of a SMART Fellowship service commitment.

His research interests are in assortment planning and operations research more broadly. He has studied assortment planning in the automotive industry, with implications towards demand modeling, supply chain management, and sustainability. Edward has made several technical presentations at conferences such as the Midwest Optimization Meeting and the INFORMS Annual meeting.