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Estimation of Multi Component Systems Reliability in Stress-Strength Models

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In a system with standby redundancy, there are a number of components only one of which works at a time and the other remain as standbys. When an impact of stress exceeds the strength of the active component, for the first time, it fails and another from standbys, if there is any, is activated and faces the impact of stresses, not necessarily identical as faced by the preceding component and the system fails when all the components have failed. Sriwastav and Kakaty (1981) assumed that the components stress-strengths are similarly distributed. However, in general the stress distributions will be different from the strength distributions not only in parameter values but also in forms, because stresses are independent of strengths and the two are governed by different physical conditions. Assume the components in the system for both stress and strength are independent and follow different probability distributions viz. Exponential, Gamma, Lindley. Different conditions for stress and strength were considered. Under these assumptions the reliabilities of the system have been obtained with the help of the particular forms of density functions of n-standby system when all stress-strengths are random variables. The expressions for the marginal reliabilities R(1), R(2), R(3) etc. have been obtained based on its stress-strength models. Results obtained by J. Gogoi and M. Bohra are particular case presentations.

Keywords: Reliability, standby redundancy, exponential distribution, Lindley distribution.

Introduction

The reliability of a system is the probability that when operating under stated environmental conditions, the system will perform its intended function adequately. For stress-strength models both the strength of the system, *X*, and the stress, *Y*, imposed on it by its operating environments are considered to be random variables.

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The reliability, R, of the system is the probability that the system is strong enough to overcome the stress imposed on it, that is to say $R = \Pr(X > Y)$.

In a standby system, that is, a system with standby redundancy, there are number of components only one of which works at a time and the other remains as standby. When an impact of stress exceeds the strength of the active component, for the first time, it fails and the another component from standbys, if there is any, is activated and faces the impact of stresses, not necessarily identical as faced by the preceding component. The system fails when all the components have failed. This problem has a long history starting with the pioneering work of Birnbaum (1956) and Birnbaum and McCarty (1958). The term stress-strength was first introduced by Church and Harris (1970). Since then significant amount of work has been done both from parametric and non-parametric point of view. A comprehensive treatment of the different stress-strength models till 2001 can be found in the excellent monograph by Kotz et al. (2003). Some of the recent work on the stress-strength model can be obtained in Kundu and Gupta (2005, 2006), Raqab and Kundu (2005), Krishnamoorthy et al. (2007). Gogoi, Borah and Sriwastav (2010) gave an interference model with number of stress a Poisson Process. Gogoi and Borah (2012) obtained the reliability expressions when the stress-strength of the components follow different distributions.

It has been assumed the components stress-strengths are similarly distributed but in general the stress distributions will be different from the strength distributions not only in parameter values but also in forms because stresses are independent of strengths and the two are governed by different physical conditions. It is assumed here that stress-strengths of all the components in the system are independent and obtained the stress-strength parameter $R = \Pr(X > Y)$, when X and Y are independent. Note that the stress-strength parameter plays an important role in the reliability analysis. For example if X is the strength of a system which is subjected to stress Y, then the parameter R measures the system performance and it is very common in the context of mechanical reliability of a system. Moreover, R provides the probability of a system failure, if the system fails whenever the applied stress is greater than its strength.

Model

Consider an n-standby system in which, initially, there are n components, out of which only one is working under impact of stresses and the remaining $(n-1)$ are standby. Whenever the working component fails, one from the standby takes its place and is subjected to impact of stresses and the system works. The system fails when the entire component fails.

Let $X_1, X_2, ..., X_n$ be a set of n independent random variables, representing the strengths of n components arranged in order of activation in the system and let $Y_1, Y_2, ..., Y_n$, be another set of independent random variables representing the stresses on the n components respectively, then the system reliability R_n of the system is given by

$$R_n = R(1) + R(2) + ...R(n) \tag{1}$$

where the marginal reliability $R(r)$ is the contribution to the reliability of the system by the rth component and is defined as

$$R(r) = \Pr\left[X_1 < Y_1, X_2 < Y_2, ..., X_{r-1} < Y_{r-1} \geq Y_r \right]$$

and if $f_i(x)$ and $h_i(y)$ are the probability density functions of X_i and $Y_i, i = 1, 2, 3, ..., n$ respectively then

$$
\begin{aligned}
R(r) &= \left[\int_{-\infty}^{\infty} F_1(y) h_1(y) dy \right]\left[\int_{-\infty}^{\infty} F_2(y) h_2(y) dy \right] \\
&\cdots \left[\int_{-\infty}^{\infty} F_{r-1}(y) h_{r-1}(y) dy \right]\left[\int_{-\infty}^{\infty} \bar{F}_r(y) h_r(y) dy \right]
\end{aligned}
\tag{2}
$$

where $F_i(y)$ is the commutative distribution function of X_i and

$$\bar{F}_i(y) = 1 - F(x) \tag{3}$$

Assume that strength and stress follow different distributions, the following cases are considered.

(I)   One parameter exponential strength and three parameter exponential stress.
(II)   Two parameter Lindley strength and one parameter gamma stress.
(III)   Two parameter Lindley strength and two parameter gamma stress.

One parameter exponential strength and three parameter exponential stress

Let $f_i(x)$ be the one parameter exponential strength with parameter λ_i and $h_i(x)$ be the three parameter exponential stress with parameters α_i, β_i and $\mu_i; i = 1, 2, \ldots, n$, then

$$f_i(x, \lambda) = \begin{cases} \lambda_i e^{-\lambda_i x_i}; x_i \geq 0, \lambda_i \geq 0 \\ 0; \text{ otherwise} \end{cases} \tag{4}$$

and

$$h_i(y, \alpha, \beta, \mu) = \begin{cases} \dfrac{\alpha_i}{\beta_i}\left(1 - e^{-(y_i - \mu_i)/\beta_i}\right); \quad y_i > \mu_i, \alpha_i > 0, \beta_i > 0 \\ 0; \text{ otherwise} \end{cases} \tag{5}$$

Then from equation (2)

$$R(1) = \int_{\mu_1}^{\infty} \bar{F}_1(y) h_1(y) dy = \frac{\alpha_1 e^{-\lambda_1 \mu_1} \Gamma(\alpha_1) \Gamma(\lambda_1 \beta_1 + 1)}{\Gamma(\alpha_1 + \lambda_1 \beta_1 + 1)}$$

$$R(2) = \left[\int_{\mu_1}^{\infty} F_1(y) h_1(y) dy\right]\left[\int_{\mu_2}^{\infty} \bar{F}_2(y) h_2(y) dy\right]$$

$$= \left[1 - \frac{\alpha_1 e^{-\lambda_1 \mu_1} \Gamma(\alpha_1) \Gamma(\lambda_1 \beta_1 + 1)}{\Gamma(\alpha_1 + \lambda_1 \beta_1 + 1)}\right]\left[\frac{\alpha_2 e^{-\lambda_2 \mu_2} \Gamma(\alpha_2) \Gamma(\lambda_2 \beta_2 + 1)}{\Gamma(\alpha_2 + \lambda_2 \beta_2 + 1)}\right]$$

$$R(3) = \left[\int_{\mu_1}^{\infty} F_1(y) h_1(y) dy\right]\left[\int_{\mu_2}^{\infty} F_2(y) h_2(y) dy\right]\left[\int_{\mu_3}^{\infty} \bar{F}_3(y) h_3(y) dy\right]$$

$$= \left[1 - \frac{\alpha_1 e^{-\lambda_1 \mu_1} \Gamma(\alpha_1) \Gamma(\lambda_1 \beta_1 + 1)}{\Gamma(\alpha_1 + \lambda_1 \beta_1 + 1)}\right]\left[1 - \frac{\alpha_2 e^{-\lambda_2 \mu_2} \Gamma(\alpha_2) \Gamma(\lambda_2 \beta_2 + 1)}{\Gamma(\alpha_2 + \lambda_2 \beta_2 + 1)}\right]$$

$$\times \left[\frac{\alpha_3 e^{-\lambda_3 \mu_3} \Gamma(\alpha_3) \Gamma(\lambda_3 \beta_3 + 1)}{\Gamma(\alpha_3 + \lambda_3 \beta_3 + 1)}\right]$$

In general,

$$R(r) = \left[1 - \frac{\alpha_1 e^{-\lambda_1 \mu_1} \Gamma(\alpha_1) \Gamma(\lambda_1 \beta_1 + 1)}{\Gamma(\alpha_1 + \lambda_1 \beta_1 + 1)}\right] \left[1 - \frac{\alpha_2 e^{-\lambda_2 \mu_2} \Gamma(\alpha_2) \Gamma(\lambda_2 \beta_2 + 1)}{\Gamma(\alpha_2 + \lambda_2 \beta_2 + 1)}\right] \cdots$$
$$\times \left[1 - \frac{\alpha_{r-1} e^{-\lambda_{r-1} \mu_{r-1}} \Gamma(\alpha_{r-1}) \Gamma(\lambda_{r-1} \beta_{r-1} + 1)}{\Gamma(\alpha_{r-1} + \lambda_{r-1} \beta_{r-1} + 1)}\right] \left[\frac{\alpha_r e^{-\lambda_r \mu_r} \Gamma(\alpha_r) \Gamma(\lambda_r \beta_r + 1)}{\Gamma(\alpha_r + \lambda_r \beta_r + 1)}\right] \qquad .(6)$$

Particular Case

When in (4) $\alpha_i = 1$, the three parameter exponential distribution becomes two parameter exponential distribution. Then the expression $R(r)$ for one parameter exponential strength and two parameter exponential stress is obtained from (6) by putting $\alpha_i = 1; i = 1, 2, \ldots, r$ and is given by

$$R(r) = \left[1 - \frac{e^{-\lambda_1 \mu_1}}{\lambda_1 \beta_1 + 1}\right] \left[1 - \frac{e^{-\lambda_2 \mu_2}}{\lambda_2 \beta_2 + 1}\right] \cdots \left[1 - \frac{e^{-\lambda_{r-1} \mu_{r-1}}}{\lambda_{r-1} \beta_{r-1} + 1}\right] \left[\frac{e^{-\lambda_r \mu_r}}{\lambda_r \beta_r + 1}\right] \qquad (7)$$

The result in (7) is obtained by Gogoi and Borah (2012)

Two parameter Lindley strength and one parameter gamma stress

Let $f_i(x)$ be the two parameter Lindley strength with parameters θ_i and α_i and $h_i(x)$ be the one parameter gamma stress with parameter $m_i, i = 1, 2, \ldots, n$, the

$$f_i(x, \theta, \alpha) = \begin{cases} \dfrac{\theta_i^2}{\theta_i + \alpha_i} (1 + \theta_i x_i) e^{-\theta_i x_i}; x_i \geq 0, \theta_i > 0, \alpha_i > -\theta_i \\ \\ 0; \text{ otherwise} \end{cases} \qquad (8)$$

and

$$h_i(y, m) = \begin{cases} \dfrac{1}{\Gamma(m_i)} e^{-y_i} y_i^{m_i}; y_i \geq 0, m_i \geq 1 \\ \\ 0; \text{ otherwise} \end{cases} \qquad (9)$$

Then from equation (2)

$$R(1) = \int_0^\infty \bar{F_1}(y)h_1(y)dy = \frac{(\theta_1+\alpha_1)(\theta_1+1)+\alpha_1\theta_1 m_1}{(\theta_1+\alpha_1)(\theta_1+1)^{m_1+1}}$$

$$R(2) = \left[\int_0^\infty F_1(y)h_1(y)dy\right]\left[\int_0^\infty \bar{F_2}(y)h_2(y)dy\right]$$

$$= \left[1-\frac{(\theta_1+\alpha_1)(\theta_1+1)+\alpha_1\theta_1 m_1}{(\theta_1+\alpha_1)(\theta_1+1)^{m_1+1}}\right]\left[\frac{(\theta_2+\alpha_2)(\theta_2+1)+\alpha_2\theta_2 m_2}{(\theta_2+\alpha_2)(\theta_2+1)^{m_2+1}}\right]$$

$$R(3) = \left[\int_0^\infty F_1(y)h_1(y)dy\right]\left[\int_0^\infty F_2(y)h_2(y)dy\right]\left[\int_0^\infty \bar{F_3}(y)h_3(y)dy\right]$$

$$= \left[1-\frac{(\theta_1+\alpha_1)(\theta_1+1)+\alpha_1\theta_1 m_1}{(\theta_1+\alpha_1)(\theta_1+1)^{m_1+1}}\right]\left[1-\frac{(\theta_2+\alpha_2)(\theta_2+1)+\alpha_2\theta_2 m_2}{(\theta_2+\alpha_2)(\theta_2+1)^{m_2+1}}\right]$$

$$\times\left[\frac{(\theta_3+\alpha_3)(\theta_3+1)+\alpha_3\theta_3 m_3}{(\theta_3+\alpha_3)(\theta_3+1)^{m_3+1}}\right]$$

In general

$$R(r) = \left[1-\frac{(\theta_1+\alpha_1)(\theta_1+1)+\alpha_1\theta_1 m_1}{(\theta_1+\alpha_1)(\theta_1+1)^{m_1+1}}\right]\left[1-\frac{(\theta_2+\alpha_2)(\theta_2+1)+\alpha_2\theta_2 m_2}{(\theta_2+\alpha_2)(\theta_2+1)^{m_2+1}}\right]\cdots$$

$$\left[1-\frac{(\theta_{r-1}+\alpha_{r-1})(\theta_{r-1}+1)+\alpha_{r-1}\theta_{r-1} m_{r-1}}{(\theta_{r-1}+\alpha_{r-1})(\theta_{r-1}+1)^{m_{r-1}+1}}\right]\left[\frac{(\theta_r+\alpha_r)(\theta_r+1)+\alpha_r\theta_r m_r}{(\theta_r+\alpha_r)(\theta_r+1)^{m_r+1}}\right]$$

(10)

Particular case

1)   When $\alpha_i=1$ in (8) two parameter Lindley distribution becomes one parameter Lindley distribution. Then the expression $R(r)$ for one parameter Lindley strength and one parameter gamma stress is obtained from (10) by putting $\alpha_i=1; i=1,2,\ldots,r$ and is given by

$$R(r) = \left[1 - \frac{(\theta_1 + 1)(\theta_1 + 1) + \theta_1 m_1}{(\theta_1 + 1)(\theta_1 + 1)^{m_1 + 1}}\right]\left[1 - \frac{(\theta_2 + 1)(\theta_2 + 1) + \theta_2 m_2}{(\theta_2 + 1)(\theta_2 + 1)^{m_2 + 1}}\right]\cdots$$
$$\left[1 - \frac{(\theta_{r-1} + 1)(\theta_{r-1} + 1) + \theta_{r-1} m_{r-1}}{(\theta_{r-1} + 1)(\theta_{r-1} + 1)^{m_{r-1} + 1}}\right]\left[\frac{(\theta_r + 1)(\theta_r + 1) + \theta_r m_r}{(\theta_r + 1)(\theta_r + 1)^{m_r + 1}}\right]$$

(11)

(The result in (11) is obtained by Gogoi and Borah (2012))

2)   When $\alpha_i = 0$ in (8) two parameter Lindley distribution becomes one parameter exponential distribution. Then the expression $R(r)$ for one parameter exponential strength and one parameter gamma stress is obtained from (10) by putting $\alpha_i = 1; i = 1, 2, \ldots, r$ and is given by

$$R(r) = \left[1 - \frac{1}{(\theta_1 + 1)^{m_1}}\right]\left[1 - \frac{1}{(\theta_2 + 1)^{m_2}}\right]\cdots\left[1 - \frac{1}{(\theta_{r-1} + 1)^{m_{r-1}}}\right]\left[\frac{1}{(\theta_r + 1)^{m_r}}\right] \quad (12)$$

Two parameter Lindley strength and two parameter gamma stress

Let $f_i(x)$ be the two parameter Lindley strength with parameters θ_i and α_i and $h_i(x)$ be the two parameter gamma stress with parameter m_i and $\mu_i, i = 1, 2, \ldots, n$, then

$$f_i(x, \theta, \alpha) = \begin{cases} \frac{\theta_i^2}{\theta_i + \alpha_i}(1 + \theta_i x_i) e^{-\theta_i x_i}; x_i \geq 0, \theta_i > 0, \alpha_i > -\theta_i \\ \\ 0; \text{ otherwise} \end{cases} \quad (13)$$

and

$$h_i(y, m, \mu) = \begin{cases} \frac{1}{\mu_i^{m_i} \Gamma(m_i)} e^{\frac{-y_i}{\mu_i}} y_i^{m_i - 1}; y_i, m_i, \mu_i > 0 \\ \\ 0; \text{ otherwise} \end{cases} \quad (14)$$

Then from equation (2)

$$R(1) = \int_0^\infty \bar{F}_1(y) h_1(y) dy = \frac{(\theta_1 + \alpha_1)(\theta_1 \mu_1 + 1) + \alpha_1 \theta_1 m_1 \mu_1}{(\theta_1 + \alpha_1)(\theta_1 \mu_1 + 1)^{m_1 + 1}}$$

$$R(2) = \left[\int_0^\infty F_1(y) h_1(y) dy \right] \left[\int_0^\infty \bar{F}_2(y) h_2(y) dy \right]$$

$$= \left[1 - \frac{(\theta_1 + \alpha_1)(\theta_1 \mu_1 + 1) + \alpha_1 \theta_1 m_1 \mu_1}{(\theta_1 + \alpha_1)(\theta_1 \mu_1 + 1)^{m_1 + 1}} \right] \left[\frac{(\theta_2 + \alpha_2)(\theta_2 \mu_2 + 1) + \alpha_2 \theta_2 m_2 \mu_2}{(\theta_2 + \alpha_2)(\theta_2 \mu_2 + 1)^{m_2 + 1}} \right]$$

$$R(3) = \left[\int_0^\infty F_1(y) h_1(y) dy \right] \left[\int_0^\infty F_2(y) h_2(y) dy \right] \left[\int_0^\infty \bar{F}_3(y) h_3(y) dy \right]$$

$$= \left[1 - \frac{(\theta_1 + \alpha_1)(\theta_1 \mu_1 + 1) + \alpha_1 \theta_1 m_1 \mu_1}{(\theta_1 + \alpha_1)(\theta_1 \mu_1 + 1)^{m_1 + 1}} \right] \left[1 - \frac{(\theta_2 + \alpha_2)(\theta_2 \mu_2 + 1) + \alpha_2 \theta_2 m_2 \mu_2}{(\theta_2 + \alpha_2)(\theta_2 \mu_2 + 1)^{m_2 + 1}} \right]$$

$$\times \left[\frac{(\theta_3 + \alpha_3)(\theta_3 \mu_3 + 1) + \alpha_3 \theta_3 m_3 \mu_3}{(\theta_3 + \alpha_3)(\theta_3 \mu_3 + 1)^{m_3 + 1}} \right]$$

In general,

$$R(r) = \left[1 - \frac{(\theta_1 + \alpha_1)(\theta_1 \mu_1 + 1) + \alpha_1 \theta_1 m_1 \mu_1}{(\theta_1 + \alpha_1)(\theta_1 \mu_1 + 1)^{m_1 + 1}} \right] \left[1 - \frac{(\theta_2 + \alpha_2)(\theta_2 \mu_2 + 1) + \alpha_2 \theta_2 m_2 \mu_2}{(\theta_2 + \alpha_2)(\theta_2 \mu_2 + 1)^{m_2 + 1}} \right] \cdots \quad (15)$$
$$\left[1 - \frac{(\theta_{r-1} + \alpha_{r-1})(\theta_{r-1} \mu_{r-1} + 1) + \alpha_{r-1} \theta_{r-1} m_{r-1} \mu_{r-1}}{(\theta_{r-1} + \alpha_{r-1})(\theta_{r-1} \mu_{r-1} + 1)^{m_{r-1} + 1}} \right] \left[\frac{(\theta_r + \alpha_r)(\theta_r \mu_r + 1) + \alpha_r \theta_r m_r \mu_r}{(\theta_r + \alpha_r)(\theta_r \mu_r + 1)^{m_r + 1}} \right]$$

Particular Case

1)   When $\alpha_i = 1$ in (13) two parameter Lindley distribution becomes one parameter Lindley distribution. Then the expression $R(r)$ for one parameter Lindley strength and two parameter gamma stress is obtained from (15) by putting $\alpha_i = 1; i = 1, 2, \ldots, r$ and is given by

$$R(r) = \left[1 - \frac{(\theta_1+1)(\theta_1\mu_1+1)+\theta_1 m_1\mu_1}{(\theta_1+1)(\theta_1\mu_1+1)^{m_1+1}}\right]\left[1 - \frac{(\theta_2+1)(\theta_2\mu_2+1)+\theta_2 m_2\mu_2}{(\theta_2+1)(\theta_2\mu_2+1)^{m_2+1}}\right]\cdots$$
$$\left[1 - \frac{(\theta_{r-1}+1)(\theta_{r-1}\mu_{r-1}+1)+\theta_{r-1}m_{r-1}\mu_{r-1}}{(\theta_{r-1}+1)(\theta_{r-1}\mu_{r-1}+1)^{m_{r-1}+1}}\right]\left[\frac{(\theta_r+1)(\theta_r\mu_r+1)+\theta_r m_r\mu_r}{(\theta_r+1)(\theta_r\mu_r+1)^{m_r+1}}\right] \tag{16}$$

2)   When $\alpha_i = 0$ in (13) two parameter Lindley distribution becomes one parameter exponential distribution. Then the expression $R(r)$ for one parameter exponential strength and two parameter gamma stress is obtained from (15) by putting $\alpha_i = 1; i = 1, 2, \ldots, r$ and is given by

$$R(r) = \left[1 - \frac{1}{(\theta_1\mu_1+1)^{m_1}}\right]\left[1 - \frac{1}{(\theta_2\mu_2+1)^{m_2}}\right]\cdots\left[1 - \frac{1}{(\theta_{r-1}\mu_{r-1}+1)^{m_{r-1}}}\right]\left[\frac{1}{(\theta_r\mu_r+1)^{m_r}}\right]$$
$$\tag{17}$$

The results in (16) and (17) were obtained by Gogoi and Borah (2012)

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