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One of the bases for assessment of wind energy potential for a specified region is the probability distribution of wind speed. Thus, appropriate and adequate specification of the probability distribution of wind speed becomes increasingly important. Several distributions have been proposed for describing wind distribution. Among the most popular distributions is the Weibull whose choice is due to its flexibility. An exponentiated Weibull distribution is proposed as an alternative to model wind speed data with a view to comparing it with the existing Weibull distribution. Results indicate that the proposed distribution outperforms the existing Weibull distribution for modeling wind speed data in terms of minimum Akaike information criterion (AIC) and likelihood function. Thus, the exponentiated Weibull can be used as an alternative distribution that adequately describe the wind speed and thereby provide better representation of the potentials of wind energy.

Keywords: Wind power, Weibull, exponentiated Weibull, model selection criteria, maximum likelihood estimation

Introduction

Energy demand increases proportionally as world population grows rapidly. Governments and societies become interested to renewable energies. Wind energy is considered the most attractive as it ensures high output power compared to other renewable energies. Nevertheless, the assessment of the wind energy potential is complicated since the wind speed availability is probabilistic. Several statistical distributions have been used for the description of the wind speed distribution. The two-parameter Weibull distribution function has been commonly

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used in many fields including wind energy assessment (Rehman et al., 1994; Bivona et al., 2003).

Silva and Cordeiro (2012) were among the first among researchers to use compound distributions to model wind speed. They showed that Burr type XII distribution outperformed the commonly used Weibull distribution. Therefore, this article received its motivation from this and attempts to model wind speed using exponentiated Weibull distribution, which is a generalization of the Weibull distribution for increased and improved modeling potential.

Weibull Distribution

The Weibull distribution is characterized by two parameters *K* and *S*, the shape and scale respectively. A random variable *V* (wind speed) is distributed as Weibull if it satisfies the following probability density function.

$$f_{(V)} = \frac{K}{C}\left(\frac{V}{C}\right)^{K-1} \exp\left[-\left(\frac{V}{C}\right)^{K}\right]. \tag{1}$$

The corresponding distribution function is

$$F_{(V)} = 1 - \exp\left[-\left(\frac{V}{C}\right)^{K}\right]. \tag{2}$$

If *V* denotes the wind speed, then the average wind speed is expressed as

$$E_{(V)} = \int_0^\infty Vf\left(V\right)dv = \int_0^\infty V\frac{K}{C}\left(\frac{V}{C}\right)^{K-1} \exp\left[-\left(\frac{V}{C}\right)^{K}\right]dv \tag{3}$$

giving rise to

$$E_{(V)} = \bar{V} = C\Gamma\left(1+\frac{1}{K}\right). \tag{4}$$

The variance of *V* is

$$Var\left(V\right) = \int_0^\infty \left(V-\bar{V}\right)^2 f\left(v\right)\ dv \tag{5}$$

which on simplification gives

$$\sigma^2 = Var(V) = C^2\left[\Gamma\left(1+\frac{2}{K}\right) - \Gamma^2\left(1+\frac{1}{K}\right)\right] \tag{6}$$

from which

$$\sigma = C\left[\Gamma\left(1+\frac{2}{K}\right) - \Gamma^2\left(1+\frac{1}{K}\right)\right]^{\frac{1}{2}}. \tag{7}$$

Method of Estimating the Weibull Parameters

Commonly used methods known as graphical and maximum likelihood methods are now considered.

Graphical Method

From (2)

$$\frac{1}{1-F_{(V)}} = \exp\left[\left(\frac{V}{C}\right)^K\right] \tag{8}$$

Introducing *In* to both sides results in

$$In\left[\frac{1}{1-F_{(V)}}\right] = \left(\frac{V}{C}\right)^K \tag{9}$$

and further introduction of *In* results in

$$In\ In\left|\frac{1}{1-F_{(V)}}\right| = K|nV - K|nC \tag{10}$$

Equation (10) can be expressed as $Y = aX + b$ where

$$Y = In\ In\left[\frac{1}{1-F_{(V)}}\right],\ X = Inv, a = K \text{ and } b = -K\ InC$$

Maximum Likelihood Method

Harter and Moore (1965) were the earliest statisticians to use the maximum likelihood procedure because of its desirable characteristics. Given a random sample of size *n* wind speed drawn from a probability density function in (1), then the likelihood function will be

$$L\left(V_1, V_2, ...V_n,\ K, C\right) = \prod_{i=1}^{n}\left(\frac{K}{C}\right)\left(\frac{V_i}{C}\right)^{k-1} \exp\left[-\left(\frac{V_i}{C}\right)^{K}\right] \tag{11}$$

The logarithm of (11) becomes

$$l = n\log\frac{K}{C} + \left(K-1\right)\sum_{i=1}^{n}\log\left(\frac{V}{C}\right) - \sum_{i=1}^{n}\left(\frac{V_i}{C}\right)^{K} \tag{12}$$

by differentiating (12) with respect to *K* and *C* in turn and equating to zero, the following are obtained

$$\frac{\delta l}{\delta C} \Rightarrow \frac{-nK}{C} + \frac{K}{C}\sum_{i=1}^{n}\left(\frac{V_i}{C}\right)^{K} = 0 \tag{13}$$

$$\frac{\delta l}{\delta K} = \frac{n}{K} + \sum_{i=1}^{n}\log\left(\frac{V_i}{C}\right)^{K} - \log\left(\frac{V_i}{C}\right) = 0 \tag{14}$$

Equations (13) and (14) are termed normal equations and can be solved numerically to obtain the maximum likelihood estimates of *K* and *C*.

Exponentiated Weibull Distributions

According to Mudhokar, et al., (1995), the exponentiated Weibull density function is defined as

$$g(V)=\frac{K\delta}{C}\left[1-\exp\left(-\frac{V}{C}\right)^{K}\right]^{\delta-1}\left(\frac{V}{C}\right)^{K-1}\exp\left(-\frac{V}{C}\right)^{K} \tag{15}$$

where $K, C\,and\;d>0 \quad V_i>0$.

This distribution is proposed to model wind speed for the first time. For adequate determination of wind speed, the parameters in equation (15) need to be estimated. For this, we adopt the use of maximum likelihood method.

As before if $V_1,V_2,...;V_n$ is a random sample of size *n* wind speed drawn from the density function in (15), then the likelihood function is

$$L(V_1,V_2,...,V_n,K,C,\delta)=\frac{K^n\delta^n}{C^n}\prod_{i=1}^{n}\left[1-\exp\left(-\frac{V}{C}\right)^{K}\right]^{\delta-1}\prod_{i=1}^{n}\left(\frac{V_i}{C}\right)^{K-1}\exp\left(-\frac{V_i}{C}\right)^{K}. \tag{16}$$

The corresponding log-likelihood function is obtained by finding the logarithm of (16) is

$$\begin{aligned} l &= n\log K+n\log\delta-n\log C \\ &+(\delta-1)\sum_{i=1}^{n}\log\left(1-\exp\left(\frac{V_i}{C}\right)^{K}\right)+(K-1)\sum_{i=1}^{n}\log\left(\frac{V_i}{C}\right)-\sum_{i=1}^{n}\log\left(\frac{V_i}{C}\right)^{K}. \end{aligned} \tag{17}$$

Taking the derivative of (17) with respect to *K*, *C* and δ, results in

$$\begin{aligned} \frac{\delta l}{\delta K} &= \frac{n}{K}+(\delta-1)\sum_{i=1}^{n}\left[\frac{\left(\frac{V_i}{C}\right)^{K}\exp\left(-\frac{V_i}{C}\right)^{K}\log\left(\frac{V_i}{C}\right)}{\left(1-\exp\left(-\frac{V_i}{C}\right)^{K}\right)}\right] \\ &+\sum_{i=1}^{n}\log\left(\frac{V_i}{C}\right)^{K}-\log\left(\frac{V_i}{C}\right)=0 \end{aligned} \tag{18}$$

$$\frac{\delta l}{\delta C} = \frac{-n}{C} + \frac{K(\delta-1)}{C}\sum_{i=1}^{n}\left[\frac{\left(\frac{V_i}{C}\right)^K \exp\left(-\frac{V_i}{C}\right)^K}{\left(1-\exp\left(-\frac{V_i}{C}\right)^K\right)}\right] - \frac{n(K-1)}{C} - \frac{K}{C}\sum_{i=1}^{n}\left(\frac{V_i}{C}\right)^K \quad (19)$$

$$\frac{\delta l}{\delta d} = \frac{n}{d} + \sum_{i=1}^{n}\log\left(1-\exp\left(-\frac{V_i}{C}\right)^K\right). \quad (20)$$

Equations (18), (19) and (20) are solved iteratively to obtain the maximum likelihood estimates of the parameters *K*, *C* and *d*.

Moments of the Exponentiated Weibull Distribution

Following the density function in (15), its r^{th} moment can be obtained as:

$$\mu_r' = E\left(X^r\right) = \int_0^\infty \frac{Kd}{C} V^r \left(1-\exp\left(\frac{-V_i}{C}\right)^K\right)^{d-1} \left(\frac{V_i}{C}\right)^{K-1} \exp\left(\frac{-V_i}{C}\right)^K dy$$

If $y = \left(\frac{V_i}{C}\right)^K \Rightarrow V = y^{\frac{1}{K}C}$ *and* $dv = \frac{C^K}{KV^{K-1}} dy$,

which reduces to

$$\mu_r' = E\left(X^r\right) = \int \left(y^{1/K}C\right)^r \exp(-y)\left(1-\exp(-y)\right)^{d-1} dy. \quad (21)$$

Note from binomial series expansion that

$$(1-m)^b = \sum_{j=0}^{\infty}(-1)^j \binom{b}{j} m^j, \text{ then } \left(1-\exp(-y)\right)^{d-1} = \sum_{j=0}^{\infty}(-1)^j \binom{d-1}{j}\exp\left(-y_j\right)$$

thus, equation (21) becomes

$$\mu_r^{'} = C^r d \sum_{j=0}^{\infty} (-1)^j \binom{d-1}{j} \int_0^{\infty} y^{r/K} \exp\left(-y(1+j)\right) dy . \tag{22}$$

If $P = y(1+j) \Rightarrow y = \dfrac{P}{1+j}$ and $dy = \dfrac{dP}{1+j}$, then

$$\mu_r^{'} = E\left(X^r\right) = C^r d \sum_{j=0}^{\infty} \frac{(-1)^j \binom{d-1}{j}}{(1+j)^{r/K+1}} \int_0^{\infty} P^{r/K} \exp(-P) dy , \tag{23}$$

therefore, the r^{th} moment of the exponentiated Weibull distribution is

$$E\left(X^r\right) = C^r d \sum_{j=0}^{\infty} \frac{(-1)^j \binom{d-1}{j}}{(1+j)^{r/K+1}} \Gamma\left(\frac{r}{K}+1\right). \tag{24}$$

For simplicity let $w_j = \dfrac{(-1)^j \binom{d-1}{j}}{(1+j)^{r/K+1}}$

$$E\left(X^r\right) = C^r d \sum_{j=0}^{\infty} w_j \ \Gamma\left(\frac{r}{K}+1\right) \tag{25}$$

If $r = 1$ and $d = 1 \Rightarrow \sum w_j = 1$, then this reduces to the mean of the Weibull distribution and the moments, such as the Mean, Variance, Skewness and Kurtosis, can be obtained from (24).

The mean and variance are respectively

$$E\left(V_i\right) = C\, d \Gamma\left(\frac{1}{K}+1\right) \sum_{j=0}^{\infty} \frac{(-1)^j \binom{d-1}{j}}{(1+j)^{\frac{1}{K}+1}}$$

$$Var(V_i) = C^2\, d\Gamma\left(\frac{2}{K}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j\binom{d-1}{j}}{(1+j)^{\frac{1}{K}+1}} - C^2\, d^2\,\Gamma^2\left(\frac{1}{K}+1\right)\left(\sum_{j=0}^{\infty}\frac{(-1)^j\binom{d-1}{j}}{(1+j)^{\frac{1}{K}+1}}\right)^2$$

Application

The fitting of monthly wind data collected across regions in the south western part of Nigeria was considered using data from the period between 1992 and 2012. Using the R-Package, the following results were obtained.

Estimates and Goodness-of-Fit for the Wind Speed Data

January

		MLE			
Distributions	**K**	**C**	δ	**–2log l**	**AIC**
Weibull	0.2276	0.0002	1	19.3754	23.3754
Exponentiated Weibull	0.6678	1	10.2721	0.9916	4.9916

February

		MLE			
Distributions	**K**	**C**	δ	**–2log l**	**AIC**
Weibull	0.2168	0.0001	1	21.32016	25.32016
Exponentiated Weibull	0.6598	1	13.5975	3.15927	7.15927

March

		MLE			
Distributions	**K**	**C**	δ	**–2log l**	**AIC**
Weibull	0.20399	0.00006	1	23.75956	27.75956
Exponentiated Weibull	0.641638	1	15.335852	6.31854	10.31854

April

		MLE			
Distributions	**K**	**C**	δ	**–2log l**	**AIC**
Weibull	0.19996	0.00005	1	24.55724	28.55724
Exponentiated Weibull	0.6896499	1	41.4258026	15.38302	19.38302

May

		MLE			
Distributions	**K**	**C**	δ	**–2log l**	**AIC**
Weibull	0.209261	0.000072	1	22.73786	26.73786
Exponentiated Weibull	0.6710878	1	20.7852047	8.297559	12.297589

June

		MLE			
Distributions	**K**	**C**	δ	**–2log l**	**AIC**
Weibull	0.2096060	0.000073	1	22.67258	26.67258
Exponentiated Weibull	0.6915926	1	25.247869	10.20779	14.20779

July

		MLE			
Distributions	**K**	**C**	δ	**–2log l**	**AIC**
Weibull	0.21815	0.000106	1	21.0735	25.0735
Exponentiated Weibull	0.677590	1	15.3195704	5.094789	9.094789

August

		MLE			
Distributions	**K**	**C**	δ	**–2log l**	**AIC**
Weibull	0.211048	0.0000782	1	22.39852	26.39852
Exponentiated Weibull	0.692516	1	23.699390	9.502349	13.502349

September

	MLE				
Distributions	**K**	**C**	δ	**–2log l**	**AIC**
Weibull	0.2077786	0.00067	1	23.0227	27.0227
Exponentiated Weibull	0.6699132	1	21.8338828	8.774993	12.774993

October

	MLE				
Distributions	**K**	**C**	δ	**–2log l**	**AIC**
Weibull	0.222508	0.000127	1	20.28256	24.28256
Exponentiated Weibull	0.6600494	1	11.9804517	2.943342	6.943342

November

	MLE				
Distributions	**K**	**C**	δ	**–2log l**	**AIC**
Weibull	0.2262799	0.0001477	1	19.61016	23.61016
Exponentiated Weibull	0.5949411	1	4.3888947	21.41094	25.41094

December

	MLE				
Distributions	**K**	**C**	δ	**–2log l**	**AIC**
Weibull	0.2426764	0.0002682	1	16.81126	20.81126
Exponentiated Weibull	0.6716818	1	7.055548	5.285842	9.285842

Summary Statistics

Min	1st Quarter	Median	Mean	3rd Quarter	Mae
1.54	2.94	4.06	4.578	5.88	9.81

Note: Kurtosis = 2.502187, Skewness = 0.6333066

Conclusion

The performance of Exponentiated Weibull and Weibull distribution functions to model wind energy was systematically compared. It was observed that the log likelihood values and the Akaike information criterion (AIC) for the Exponentiated Weibull was always smaller for the Weibull distribution for each month except the month of November. This indicates that the proposed Exponentiated Weibull distribution outperformed the existing Weibull distribution for wind speed data in terms of minimum AIC and likelihood function over the months of the years under review. Thus, the exponentiated Weibull can be used as an alternative distribution that adequately describes wind speed, and may provide better representation of the potentials of wind energy.

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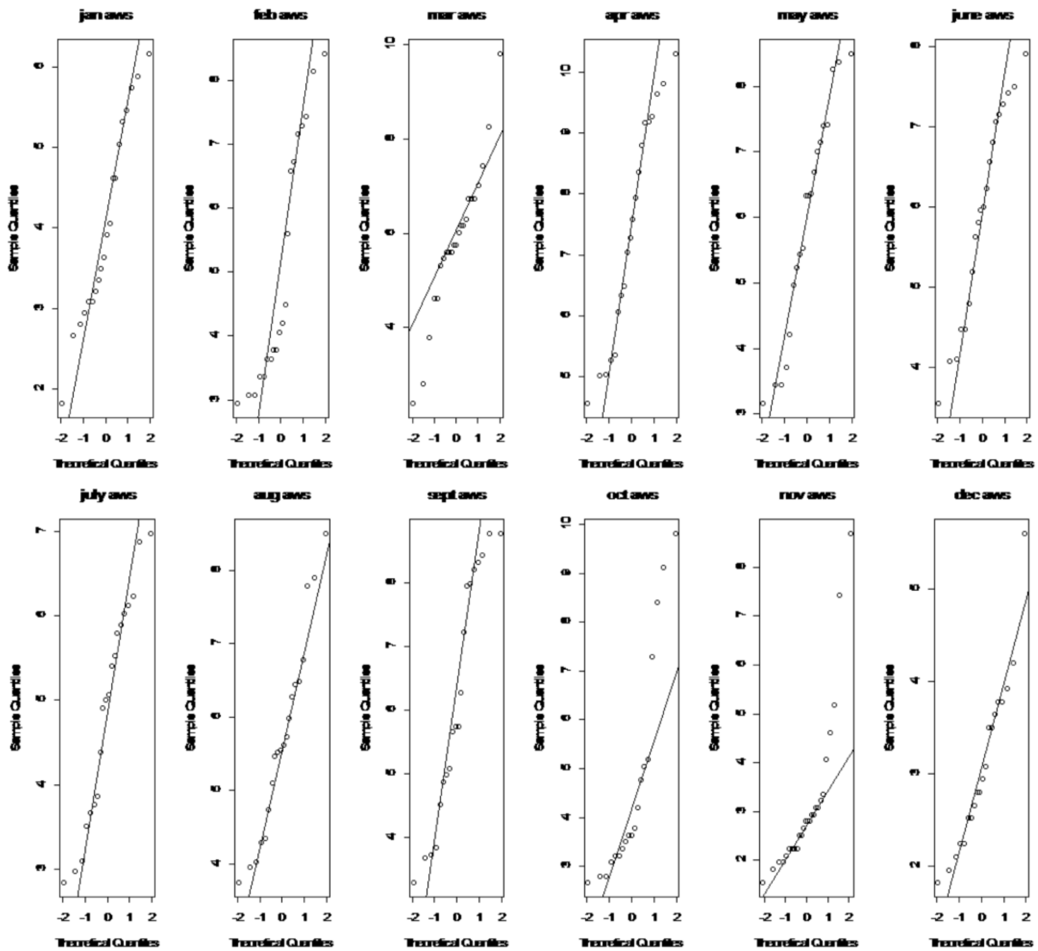
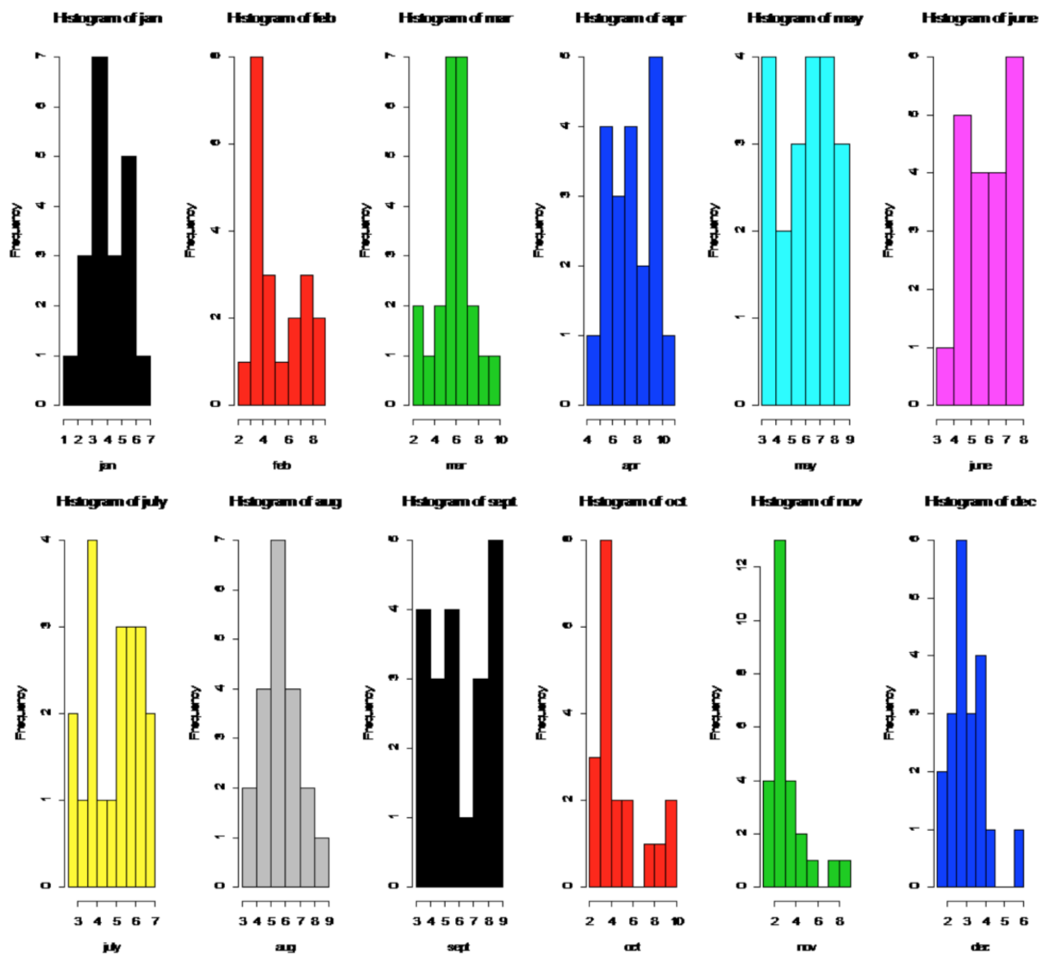
Appendix

QQ Plots



Histograms


Histogram of jan
Histogram of feb
Histogram of mar
Histogram of apr
Histogram of may
Histogram of june
Histogram of july
Histogram of aug
Histogram of sept
Histogram of oct
Histogram of nov
Histogram of dec