



Production, Manufacturing and Logistics

## Markov-modulated analysis of a spare parts system with random lead times and disruption risks

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## ARTICLE INFO

## Article history:

Received 16 December 2016

Accepted 16 February 2018

Available online 28 February 2018

## Keywords:

Supply chain management

Supply risk

Stochastic lead times

Optimal inventory control

Spare parts

## ABSTRACT

Spare parts supply chains are highly dependent on the dynamics of their installed bases. A decreasing number of capital products in use increases the *nonstationary* supply-side risk especially towards the end-of-life of capital products. This supply-side risk appears to present itself through varying lead times coupled with supply disruptions. To model the nonstationary supply-side risk, we consider an exogenous Markov chain that modulates random lead times and disruption probabilities. Assuming that order crossovers do not occur, we prove the optimality of a state-dependent base stock policy. Later, we conduct an impact study to understand the value of considering stochastic lead times and supply disruption risk in spare parts inventory control. Our results indicate that the coupled effect of random lead times and disruptions can be larger than the summation of individual effects even for moderate lead time variances. Also, the effect of nonstationarity on total cost can be as large as the summation of all risk factors combined. In addition to this managerial insight we present a procedure for supply risk mitigation based on an empirical model and our mathematical model. Experiments on a real business case indicate that the procedure is capable of reducing costs while making the inventory system more prepared for disruptions.

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### 1. Introduction

Capital goods usually have a long lifespan. For instance, aircraft can last up to 30 years. After the maturity phase of the life cycle, the number of systems in operation starts to decline, since better systems are on the market. Asset owners yet intend to keep their existing capital goods in operation to maximize their return on investment, and for this they primarily rely on Original Equipment Manufacturers (OEMs) of their capital products. OEMs, however, are troubled with supply-side risk. This risk might be due to changes in technology (Rojo & Roy, 2010), suppliers' financial problems and bankruptcy (Babich, Burnetas, & Ritchken, 2007), or simply parts becoming less profitable for suppliers, which in our experience is the most common cause. After losing a supplier, OEMs of capital products try to restart their spare parts supply process. Depending on the complexity of the manufacturing process and raw material availability this may take up to one year, especially

if the part number needs to be changed and re-certified. The aviation sector typically suffers from long recovery times after supply disruptions. This is because the majority of aircraft spare parts can only be sourced from single suppliers, who possess technical drawings and proprietary rights, and because decreasing demand rates cripple the profitability of manufacturing spare parts for suppliers.

In this paper, we consider supply-side risk for spare parts. We motivate our study in three distinctive steps: First, we cite empirical evidence by Li, Dekker, Heij, and Hekimoğlu (2016), who found that increased lead time variations are followed by supply disruptions in spare parts supply chains. Second, we explain the association of supply disruptions with varying lead times by referring an analytical result stating that suppliers optimally delay spare parts deliveries when they have more important customers with higher demand rates (Duenyas & Neale, 1997). Third, we present a business case, taken from an OEM in the aviation industry, in which lead time variability significantly increases towards the time of supply disruption. The business case is discussed in Section 3.

Empirical evidence from the aviation industry (Hekimoğlu, 2015; Li et al., 2016) indicates that supply disruption risk is coupled with lead time variation. They consider the purchase history of two groups of spare parts. One group comprises parts with *disrupted* supply, the other group consists of parts with *'healthy'*

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supply. Using the proportional hazard model, they find that *increasing* lead time variability is the most important indicator for the risk of losing suppliers for spare parts. That study not only consists of the motivation of our study on coupled random lead times and supply disruptions, we also use survival probabilities estimated by the proportional hazard model in Section 4.7 and 5.5.

In addition to the empirical evidence, an analytical result from queuing theory also explains the link between lead time variability and loss of a supplier: Consider the entire manufacturing processes of a spare part supplier as a single queue with a batch processor. The supplier receives orders from two customers whose orders cannot be processed together in a single batch. If Customer 1 has a higher priority than Customer 2, while having a higher expected order rate, Duenyas and Neale (1997) show that Customer 2's orders are delayed and eventually completely declined as the priority difference gets larger. In our context, Customer 1 may be an OEM that needs components and spare parts for its *in-production* capital products, whereas Customer 2 may be another OEM providing service for aircraft entering their post-maturity phases. Naturally, the part manufacturer will assign a lower priority to Customer 2 due to the difference between expected future profits from the two customers. From the perspective of Customer 2, on the other hand, deliveries of his orders vary and the supply chain is subject to an *increasing* supply disruption risk as presented in the business case in Section 3. This motivates us to consider *nonstationary* supply disruptions together with lead time variability for inventory control of spare parts.

In this study, we analyze the effects of coupling *nonstationary* random lead times and supply disruptions on inventory performance, which is unique in the literature. To this end, we formulate a discounted cost model for the control of spare parts inventory that combines Markov-modulated random lead times with supply disruption risk. A state-dependent base-stock policy is proven to be optimal for the discounted cost model by showing the equivalence of our original multi-state functional equation to a single-state one which is the technical contribution of this paper. Furthermore, we suggest a new queueing system, which generates Markov-modulated random lead times without order crossovers, and propose an algorithm that calculates distribution of state-dependent random lead times. Also we provide the maximum likelihood estimator for the service rate of the queueing mechanism.

We evaluate the coupled effect of random lead time and supply disruptions as well as their individual effects on total cost under different scenarios. The main results are twofold: First, we find that both random lead times and supply disruptions have substantial effects on costs and service level. More importantly, their coupled effect can be between 10 and 30% of the optimal total cost depending on levels of individual supply risks. Second, the effect of nonstationarity on the total cost can be as high as the summation of all risk factors combined. In other words, these risk factors should not be studied in isolation, but should be explicitly modeled together in inventory control of spare parts.

When lead times are driven by a random process and supply disruptions occur occasionally, the status of outstanding orders deserves a specific attention. When a disruptive event arrives to such a system, the fate of outstanding orders takes place in a continuum of possibilities between two extremes: Either the supplier fulfills his commitment for delivery of previous orders but does not accept new ones, or she cancels all outstanding orders. In this paper, we analyze both cases respectively. Our results indicate that the state-dependent base stock policy is optimal for the former case whereas this property does not hold for the latter. Numerical experiments indicate that the state-dependent base stock policy can still be advisable with minor cost deviation when outstanding orders are canceled with disruptions.

In addition, we suggest a procedure for treating nonstationary supply risk for spare parts using our mathematical model and the empirical model by Li et al. (2016) together. Our procedure relies on the idea of using supply risk estimates of the empirical model as a Markovian state and calculate optimum state-dependent base stock policy accordingly. To the best of our knowledge our procedure is the first one combining an empirical supply risk estimator with an optimum inventory control model. The application of this procedure on a real business case, presented in Section 3, indicates that our procedure is capable of cutting costs while making OEMs more prepared for disruptive shocks.

The remainder of this paper is structured as follows: In the next section, we position our paper within the relevant extant literature. In Section 3, we introduce a motivational business case which puts our work into a business context. Next (Section 4), we present our mathematical model and the characterization of the optimal policy and a new queueing system that is used to enumerate Markov-modulated random lead times and disruption risk. Also we present a procedure combining our mathematical model with an empirical model estimating supply disruption probabilities. Section 5 is devoted to our impact analysis of nonstationary supply risk factors on inventory performance together with the application of our model to a real business case. In the final section (Section 6), we discuss and summarize our main findings.

## 2. Literature

Relevant literature for our work consists of two main parts: random lead time and supply disruption studies. In the inventory management literature there is a significant amount of research on both topics. Stochastic lead times have been of interest since the 1950s. Studies in this research stream can be categorized according to their order crossover assumptions. Here we only consider random lead time studies without order crossovers, whereas key studies allowing order crossovers include Bradley and Robinson (2005); Hayya, Bagchi, Kim, and Sun (2008); Robinson, Bradley, and Thomas (2001). Supply disruption studies were rather scarce in the early times of inventory research but the subject has been studied extensively during the last two decades. We review these studies starting from early contributions to the literature.

Supply systems with random lead times under no-order-crossover assumption resemble sequential processors such as queueing systems working under the FIFO principle (Zipkin, 2000). It is known that inventory position constitutes sufficient information for optimal control of such systems (Ehrhardt, 1984; Kaplan, 1970; Song & Zipkin, 1996). Kaplan (1970) shows that the no-crossover assumption allows a multi-state dynamic programming formulation to be reduced to a single state one which considers inventory position. Kaplan's work is extended by Ehrhardt (1984) who utilizes the random position of the outstanding order that is delivered in period  $t$  (Nahmias, 1979). Independence of positions of outstanding orders in successive periods leads to the optimality of base stock policies (Ehrhardt, 1984). Another significant contribution to this research stream is by Song and Zipkin (1996), who consider Markov-modulated random lead times without supply disruptions. This study is extended to a multi-echelon setting by Muharremoğlu and Tsitsiklis (2008). In addition to these studies, Song (1994a, 1994b) are important contributions which deepen our understanding of stochastic lead times and their effects on base stock levels and optimal costs.

Supply disruption studies constitute the second research stream relevant to our work. Disruptions are defined as *temporary* unavailability of supply due to various exogenous reasons. They are characterized by the interarrival times of "up" and "down" states (Tomlin, 2006). In other words, two features of supply disruptions are of interest from an inventory control perspective: length and

frequency. For this problem, Özekici and Parlar (1999), an extension to Parlar, Wang, and Gerchak (1995), consider an exogenous Markov chain which drives supply availability as well as system parameters such as ordering cost and holding cost. Their major assumption is immediate delivery of replenishment orders. Li, Xu, and Hayya (2004) analyze supply disruptions occurring with an alternating renewal process. They find that a base stock policy is optimal if disruptions follow a non-decreasing failure rate distribution. Tomlin (2006) suggests dual sourcing, inventory holding, and acceptance as potential strategies for dealing with supply risk and proves the optimal strategy for deterministic demand. In our paper, we focus on the coupled effect of supply disruptions and random lead times, which are driven by an exogenous Markov chain. Hence, another focus of our study is the nonstationarity in supply-side risk.

Markov chains for modeling dynamic environmental changes were suggested before. Arifoğlu and Özekici (2010), Beyer and Sethi (1997), Cheng and Sethi (1999), Gallego and Hu (2004), Muharremoğlu and Tsitsiklis (2008), Scheller-Wolf and Tayur (1999), Song and Zipkin (1993, 1996) consider Markov chains as a driving mechanism of exogenous factors. Apart from the latter two, all papers assume perfectly observable Markov chains as we choose to do.

To the best of our knowledge, Mohebbi (2003), Song and Zipkin (1996), Tomlin and Snyder (2006) are the closest studies to our work. Mohebbi (2003) assumes an (s,Q) policy and analyzes the system performance numerically in a lost sales setting. In our study, the focus is on the *nonstationary* nature of random lead times as well as supply disruptions. We also develop a dynamic programming formulation considering order movements explicitly. Tomlin and Snyder (2006) consider Markovian supply disruptions with “age-dependent” durations and zero lead times, whereas Song and Zipkin (1996) evaluate Markov-modulated random lead times without disruption risk. Our study extends these two by considering both factors in the same model.

### 3. Motivational example

As an exemplary business case for our problem setting, we consider a spare part taken from an European Original Equipment Manufacturer (OEM) of out-of-production aircraft. To guarantee financial stability, the OEM aims to extend the economic lifetime of its aircraft as long as possible by providing high quality maintenance service. The spare part, which we call Part A, is a strip made of polyurethane on the nose of a certain aircraft family and its main function is to distribute static electricity from the nose towards the body. Despite its simplicity, the part is critical since accumulated static electricity may jeopardize radio communication.

Our communication with the OEM indicated that the supplier announced end of support on October 10, 2011, since the raw material polyurethane was no longer available. Analysis of the purchase history data indicated that lead time fluctuations typically increased towards the disruption (as in Fig. 1). Hence, the OEM had to deal with fluctuating (nonstationary) lead times coupled with supply disruption risk.

The OEM's engineering department discovered another raw material with the same functionality as polyurethane. After the discovery it took two months to develop new technical drawings for using this substitute raw material in production, and the first replenishment order to the new supplier could be placed on December 9, 2011. The two-month disruption resulted in unsatisfied demand and a lower service level for the company.

In this exemplary case the OEM had to deal with fluctuating lead times and disruption risk towards the end-of-life of the spare part. Li et al. (2016) show that this is true for the majority of spare parts empirically. Their results indicate that lead time fluctuations

and increasing supply risk manifest at the same time towards the end-of-life of a spare part. Those empirical findings also confirmed with the procurement department by stating that suppliers tend to delay manufacturing parts that are close to end-of-life, since they give priority to other orders. This reasoning is similar to the results of Duenyas and Neale (1997) as explained in Section 1. Furthermore, it usually takes significant amount of time to restart the supply chain due to technical and administrative constraints once the disruption takes place. Therefore, it is important to consider random lead times and the risk of supply disruption in spare parts inventory management to mitigate the effects of these risk factors.

In this study, we focus on nonstationary disruption risk coupled with lead time fluctuations in spare parts supply chain. Mathematical formulation of our model and its assumptions are presented in the next section.

## 4. Model formulation

This study aims to model and analyze the optimal inventory control policy under nonstationary disruption risk and random lead times. To this end, we consider a Markov chain driving the supply disruption probabilities and lead time distributions over the planning horizon. First, the assumptions and fundamentals of the mathematical model are explained. Later on, the multi-period cost function, the optimal policy and computational issues are discussed, respectively.

### 4.1. Assumptions

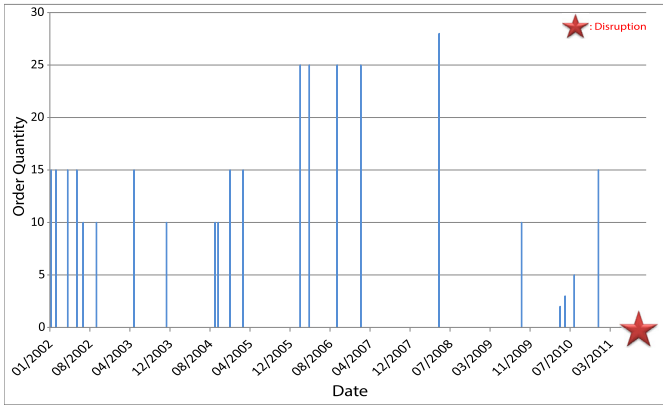
We consider a single-item single-echelon inventory system in discrete time. Replenishment orders are delivered to the inventory after a random number of periods. Due to this complexity, a supply system with random movements of outstanding orders is analytically tractable only under specific assumptions. Kaplan (1970) address this issue by suggesting the no-order-crossover assumption which guarantees that no order can be delivered after the ones placed later. Adapting this contribution, the first assumption of our model is as follows:

**Assumption 1.** Outstanding orders cannot cross each other in the supply system.

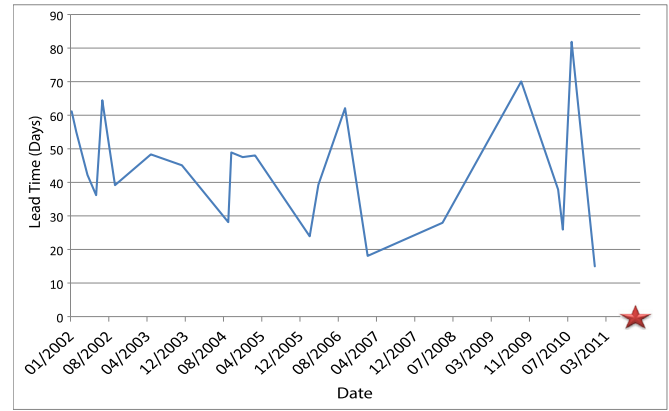
As a consequence, the position of an outstanding order is a random variable that is independent of the position of all prior outstanding orders (Nahmias, 1979).

The no-order-crossover assumption may be too restrictive for problems with high-demand rate such as retail or food supply chains. However, spare parts are mostly characterized by slow, intermittent demand and usually the number of outstanding orders is less-than two at any point in time. According to the OEM for which the research was done, supply disruption risks occur when there is only one supplier and no order has been placed for some time. Hekimoğlu (2015) provided some empirical evidence in the sense that supply disruption risk is associated with long intervals between orders from OEMs to suppliers.

In our context, where we have supply disruptions together with random lead times, outstanding orders after disruption require special attention. In practice, when a disruption occurs, the status of outstanding orders depends on various factors such as the size of the supplier, the commitment level between the two firms, the existence of contractual fines, etc. All possible scenarios for outstanding orders exist between two extreme cases: on one hand, all outstanding orders are preserved after supply disruption, that is, outstanding orders will still be delivered although no new order placements are possible (this is the case when the supplier confirms the order upon acceptance). This scenario is consistent with make-to-stock manufacturing systems and deliveries from overseas



(a) Order Quantities



(b) Lead Time Process

Fig. 1. Purchase history for Part A.

Table 1  
Notation of the mathematical model.

$h$	: Holding cost per unit, per period
$p$	: Shortage cost per unit, per period
$c$	: Acquisition cost per unit, per period
$\alpha$	: Discount rate per period
$D$	: Random demand of a single period
$D_l$	: $l$ -period convolution of random demand $D$ .
$L(i)$	: Discrete random variable for lead time of an order when the supplier is in state $i$
$i_+$	: Random variable indicating the next healthy state after state $i$ .
$d^i$	: Random variable indicating the disruption state of healthy state $i$ .
$d_+^i$	: Random variable indicating an healthy state after the disruption state $d_i$ .
$B$	: Markov chain state space.
$B^h$	: Subset of $B$ including only healthy states of supply.
$q(i)$	: Probability that the supply system stays healthy in the next period, when it is in state $i$ .
$\xi(d^i)$	: Probability that the supply system stays in disruption ( $d^i$ ).
$\mathcal{P}$	: The transition probability matrix.
$p^h$	: Matrix including conditional probabilities of transitions between healthy states.

Table 2  
Experiment factors in scenario analysis.

Supply tendency	Lead time	Supply disruption	Disruption type
Unstable	Random	YES	LID
Stable	Deterministic	NO	SFD

Table 3  
Random lead time parameters for each Markov state ( $b(i)$  for Queue #2).

Parameter set	State 0	State 2	State 3
Set 1	0.6	0.4	0.2
Set 2	0.9	0.85	0.8

manufacturing plants. On the other hand, suppliers might cancel all outstanding orders after disruption. This situation is more consistent with make-to-order systems that manufacture slow-moving and high-value capital products and/or their components. In such a case, the company does not receive previous orders nor can it place new ones. Hence, it has to continue with its existing inventory until the supply system recovers. In this study, we consider the first case which is also consistent with the no-order-crossover assumption (Zipkin, 1986). Another possible scenario between the two extreme cases is that all outstanding orders are preserved but no deliveries occur during a disruption. This case can be analyzed with our model by setting delivery probabilities in disruption periods to zero.

The order of events in each period is as follows: the inventory manager observes the supply system and decides the replenishment order for that period. We assume fixed ordering cost to be zero, hence only the acquisition cost ( $c$ ) is paid at the time of order placement. Later delivery of previous orders and customer demand are realized, and holding and shortage ( $h$  and  $p$ ) costs are incurred. Last, the supplier's state changes. We refer to Table 1 for notation.

#### 4.2. Supply risk driving mechanism

In order to address the nonstationary supply risk, we consider an exogenous, discrete-time Markov chain that drives the supply system. We define  $B$  as the set including all states of the Markov chain which we assume to be known to the decision maker. States in  $B$  consist of two groups: healthy states and disruption states. In healthy states, the inventory manager can place replenishment orders to the supplier considering the known lead time distribution and supply disruption probability of each state. The lead time distribution and disruption probabilities may be different across the healthy states of the Markov chain. In disruption states, no replenishment order is allowed until the system jumps back to a healthy state. We define  $B^h$  as the subset including healthy states of  $B$ .

When the Markov chain is in the healthy state  $i$ , two events are possible at the end of a period: either the supplier stays healthy with probability  $q(i)$  and may stay or jump to another healthy state, or a supply disruption occurs and the system goes to state  $d^i$  with probability  $\bar{q}(i) := 1 - q(i)$ . In state  $d^i$ , either the system jumps to a healthy state with probability  $\xi(d^i)$ , or it stays in the same disruption state (with probability  $\bar{\xi}(d^i) := 1 - \xi(d^i)$ ).



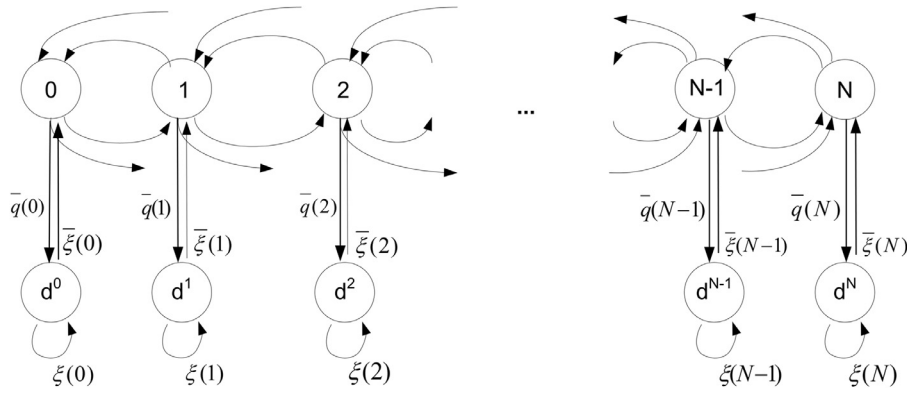


Fig. 2. State space of the Markov chain driving the supply process.

can be interpreted as the probability of finding a solution to the supply problem within one time period. Note that considering a different disruption state for each healthy state ( $i$  and  $d^i$ ), which is a modeling choice rather than a technical requirement, allows us to assign different recovery probabilities for each disruption state. This can be motivated by the fact that solving supply problems may become more difficult as the capital products in use get older. The Markov chain, which is consistent with this description, is given in Fig. 2.

Let the matrix  $P^h$  with elements  $p_{ij}$  (Eq. 2) be the transition probability from healthy state  $i$  to healthy state  $j$  given that the system stays in  $B^h$  in the next period. For convenience, we assume that there is an order between states in  $B^h$ . The transition matrix  $\mathcal{P}$  on the whole state space  $B$  of the Markov chain has the following form:

$$\mathcal{P} = \begin{pmatrix} QP^h & (I - Q) \\ (I - \Xi) & \Xi \end{pmatrix} \quad (1)$$

where

$$Q(i, i) = q(i), Q(i, j) = 0 \quad \forall i \neq j \in B^h,$$

$$\Xi(d^i, d^i) = \xi(d^i), \Xi(d^i, d^j) = 0, \quad \forall d^i \neq d^j \in B - B^h,$$

and

$$P^h = \left\{ p_{ij} : \sum_j p_{ij} = 1, \quad \forall i, j \in B^h \right\}. \quad (2)$$

We locate healthy states to the first  $N$  rows of  $\mathcal{P}$  whereas disruption states are placed to the last  $N$  rows. The disruption probabilities as well as the first two moments of lead time distributions are assumed to be increasing in the indices of the Markov chain states in  $B^h$ . In Fig. 2, for instance, more “problematic” healthy states are positioned to the right-hand side of the Markov chain. Note that placing states of the Markov chain differently does not change the results of our study.

### 4.3. Multi-period cost model

The random outstanding orders in each period require a multi-state recursive equation for the finite-horizon, total discounted cost over the planning horizon. Due to the curse of dimensionality, however, computations are problematic. Therefore, we develop an equivalent single-state cost function (see Appendix A in the online supplement of this paper). The main idea of the state reduction is to combine all future holding and backlog costs with the current period’s acquisition costs (Kaplan 1970). In the remainder of the paper, we continue with the reduced cost function.

Given that the current inventory level is  $x$ , we define holding and backlog cost of  $l$ -periods from now as follows:

$$C^l(x) = \alpha^l \mathbf{E} [h \max(x - D_{l+1}, 0) + p \max(D_{l+1} - x, 0)]. \quad (3)$$

In stationary random lead time models, the cost function in Eq. (3) would be weighted with lead time probabilities to obtain the expected single period costs. In nonstationary systems, however, lead time probabilities should be considered together with Markov transition probabilities until delivery takes place, as they are dependent. Given that the supplier is in state  $i$  and the inventory level is equal to  $x$ , the new single period cost function, due to Song and Zipkin (1996), is as follows:

$$\hat{C}(i, x) = \sum_{l \geq 0} \Pr\{L(i) \leq l \leq L(i_+)\} C^l(x), \quad (4)$$

The single period cost function for a disruption state,  $\hat{C}(d^i, x)$ , can be obtained by replacing  $i$  with  $d^i$  (see Theorem 1 in Appendix A).

If an order is placed to the supplier when it is in a healthy state  $i$ , the probability of this order being delivered within  $l$  periods is  $\Pr\{L(i) \leq l\}$ . The probability of the next period’s order being delivered later than  $l$ -periods is  $\Pr\{L(i_+) > l\}$ . Therefore  $\Pr\{L(i) \leq l \leq L(i_+)\} = \Pr\{L(i) \leq l\} - \Pr\{L(i_+) < l\}$  gives the probability of this period’s order being delivered  $l$ -periods later. From this perspective, the function  $\hat{C}(\cdot)$  is a mere extension of the single-period cost function in Ehrhardt (1984) and Kaplan (1970). The following lemma from Song and Zipkin (1996) is useful for developing further insight into the probability statement  $\Pr\{L(i) \leq l \leq L(i_+)\}$  and cost function  $\hat{C}(i, x)$ .

**Lemma 1.** (Song & Zipkin, 1996) *If the (supply) process is stationary, i.e.  $i = i_+$ , then*

$$\Pr\{L(i) \leq l \leq L(i_+)\} = \Pr\{L(i) = l\}.$$

A  $n$ -period (finite-horizon) discounted cost function,  $\tilde{f}_n(i, x)$ , for a healthy state  $i$  and inventory position before ordering  $x$ , is given in Eq. (5). indicates that this is the transformed total cost function. The original multi-state dynamic programming formulation, whose equivalence to  $\tilde{f}_n(i, x)$  is shown in Appendix A, includes outstanding orders in state space and is very difficult to enumerate due to curse of dimensionality (Kaplan, 1970). Similarly  $n$ -period discounted cost function when the system is in a disruption state  $d^i$  is  $\tilde{g}_n(d^i, x)$ , where  $x$  represents the inventory position when the supply system is disrupted. The formulations of the two cost functions are given below:

$$\begin{aligned} \tilde{f}_n(i, x) = \min_{y \geq x} & \left\{ c(y - x) + q(i)\hat{C}(i, y) + \bar{q}(i)\hat{C}(d^i, y) \right. \\ & \left. + \alpha q(i)\mathbf{E}\tilde{f}_{n-1}(i_+, y - D) \right\} \end{aligned}$$

$$+ \alpha \bar{q}(i) \mathbf{E} \tilde{g}_{n-1}(d^i, y - D) \}, \quad i, i_+ \in B^h, \quad (5)$$

and

$$\tilde{g}_n(d^i, x) = \hat{C}(d^i, x) + \alpha \xi(d^i) \mathbf{E} \tilde{g}_{n-1}(d^i, x - D) + \alpha \bar{\xi}(d^i) \mathbf{E} \tilde{f}_{n-1}(d_+^i, x - D). \quad (6)$$

$\tilde{f}_n(i, x)$  consists of two parts that are associated with two possibilities when the system is in state  $i$ : staying in healthy condition or jumping to the disruption state in the next period. The cost of the first possibility is  $\alpha q(i) \mathbf{E} \tilde{f}_{n-1}(i_+, y - D)$ , whereas the cost of the second possibility is  $\alpha \bar{q}(i) \mathbf{E} \tilde{g}_{n-1}(d^i, y - D)$  where  $y$  stands for the inventory position after the order placement.

After disruption, incoming demand is subtracted from the inventory position  $x$  and no new orders are placed until the system jumps to a healthy state. Hence the only cost comes from the single period holding and backlog costs  $\hat{C}(d^i, x)$  and the two possibilities of the supply system when it is disrupted: staying in the same disruption state, associated cost is  $\alpha \xi(d^i) \mathbf{E} \tilde{g}_{n-1}(d^i, x - D)$ , or jumping to a healthy state whose associated cost is  $\alpha \bar{\xi}(d^i) \mathbf{E} \tilde{f}_{n-1}(d_+^i, x - D)$ . Note that if we set  $d_+^i = i$ , then we obtain the specific Markov chain in Fig. 2, however our model allows consideration of more general cases.

Eqs. 5 and 6 can be interpreted as follows: when the supply system is in a healthy state, the decision maker should place an order considering the supplier's state, its disruption probability, holding, shortage, and acquisition costs. The following section gives a summary of analytic characterization of the optimal policy. An unabridged version of this section is given in Appendix A in the online supplement.

#### 4.4. Optimal policy

To analyze the function  $\tilde{f}_n(i, x)$  and derive the optimal control policy, we utilize the following transformation first introduced by Veinott (1965):  $W_n(i, x) = \tilde{f}_n(i, x) + cx$ . This leads to  $W_n(i, x) = \min_{y \geq x} G_n(i, y)$ , where  $c$  is the acquisition cost and,

$$G_n(i, y) = cy(1 - \alpha q(i)) + q(i) \hat{C}(i, y) + \bar{q}(i) \hat{C}(d^i, y) + \alpha q(i) \mathbf{E} W_{n-1}(i_+, y - D) + \alpha \bar{q}(i) \mathbf{E} \tilde{g}_{n-1}(d^i, y - D). \quad (7)$$

In Eq. (7),  $cy(1 - \alpha q(i))$  stands for the trade-off between purchasing in this period or leaving it to the next one. This trade-off includes the effect of discounting combined with disruption risk.

Theorem 1 states the convexity of Eqs. (4), (6), (7) and the optimal policy.

**Theorem 1.** *The following statements are true:*

- $\hat{C}(i, x)$  is convex in  $x$ .
- $\tilde{g}_n(d^i, x)$ ,  $G_n(i, x)$ ,  $W_n(i, x)$  are convex in  $x$ ,
- a state-dependent base stock policy is optimal.

The statement *a* of the theorem is due to Song and Zipkin (1996) whereas proof of the rest can be found in Appendix A.2. The optimal policy can be characterized with  $S_n(i)$ , which is the optimal inventory position after the replenishment order when there are  $n$  periods ahead and the supply system is in state  $i$ . We analyzed monotonicity conditions for  $S_n(i)$  and derived sufficient conditions for monotone base stock levels over Markov states. Unfortunately, these conditions, which are derived for supply failures, which are defined to be persistent supply disruptions, by Hekimoğlu (2015) are very intricate and it is hard to develop intuition from them. Hence, they are omitted in this paper. Note that in all of our numerical results, that are calculated with the value iteration algorithm, we observe monotonic base stock levels as in Fig. 5. In the next section, we provide a queueing system that generates Markov-modulated random lead times and supply disruptions.

#### 4.5. A queueing system to model random lead time and supply disruptions

We need a stochastic process which (a) is driven by an exogenous Markov chain, (b) is capable of producing state-dependent lead time distributions, and (c) precludes order crossovers. In this part of the paper, we suggest a stochastic system consisting of two semi-dependent queues for modeling a supply system with Markov-modulated random lead times.

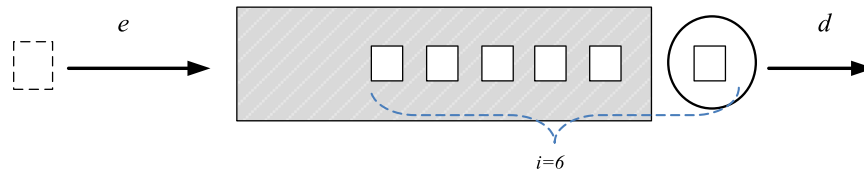
For the exogenous Markov chain (condition *a*), we consider a discrete-time Bernoulli queue, which is dubbed Queue #1 and depicted in Fig. 3. The number of items in this queue defines the healthy states of the Markov chain and increasing number of items in the system indicates the supplier's health gets worse. To include supply disruptions, we modify this queueing system with state-dependent disruption probabilities. Specifically, at the end of each period when there are  $i$  items in the system, the supply process stays healthy with probability  $q(i)$  or a disruption occurs with probability  $1 - q(i)$ . Given that it stays healthy, an item arrives at Queue #1 with probability  $e$ , and an item leaves the queue with probability  $d$ . When a disruption arrives, we assume that neither arrivals nor departures occur until the system jumps back to the associated healthy state, i.e. disruption ends. Then the queueing system continues with the same amount of items. Using this stochastic systems and the abovementioned parameters we calculate the elements of the transition probability matrix  $P^h$  in Eq. (2).

To generate Markov-modulated random lead times without order crossover, we consider another discrete-time queue with *partial-batch bulk* service, with batch size  $K$ , and a finite queue capacity,  $C$ . Additionally we assume  $K = C$ . This queueing system is dubbed Queue #2 in Fig. 3. In each period, an item arrives at Queue #2 with probability  $a$ . This item is associated with that period's replenishment order if there is an available space in the queue. If the number of orders in the queue is equal to the queue capacity, the arriving item is discarded and that period's order is added to the youngest order in the queue. In each period, either the server releases all items in the queue, since  $K = C$ , with probability  $b(i)$ , where  $i$  is the state of the Markov chain, or all items wait. One possible realization of the system is depicted in Fig. 3 when  $i = 6$  and  $C = K = 10$ . Note that this queueing system also satisfies Eq. (4) in Zipkin (1986) which states the conditions of random lead times without order crossovers.

A possible example of such a queueing system is a ferry port, in which the queueing area is equal to the capacity of a single ferry. Every time a ferry leaves the port, all vehicles waiting (in position 2) are taken into the ferry and the port area is emptied. When the ferry arrives its destination, vehicles leave the ferry to position 3. In our supply chain context, the FIFO rule in the queue precludes order crossovers, whereas the partial-batch server provides completely random deliveries independent of previous orders. Another example for Queue #2 is a production manager who makes decisions for consolidating customer demand before opening a production order.

The effect of the Markov chain on the delivery system in Queue #2 is obtained by the process rate of the partial-batch server, which is dependent on the number of items in Queue #1. In our ferry port example, the Markovian state variable may stand for random weather conditions affecting the departure or arrival of ferries, whereas in our context it could be an exogenous factor affecting the consolidation frequency. Using this stochastic system, we were able to generate Markov-modulated random lead times and enumerate the probability statement  $\Pr\{L(i) \leq l \leq L(i_+)\}$ , the summation of which is dubbed *inventory coverage* by (Song & Zipkin, 1996). The algorithm used to calculate  $\Pr\{L(i) \leq l \leq L(i_+)\}$  is provided in the next section.

Queue#1: Markovian Supplier States ( $i=6$ )



Queue#2: State Dependent Deliveries  
(Partial-batch Server with Capacity,  $C=10$ )

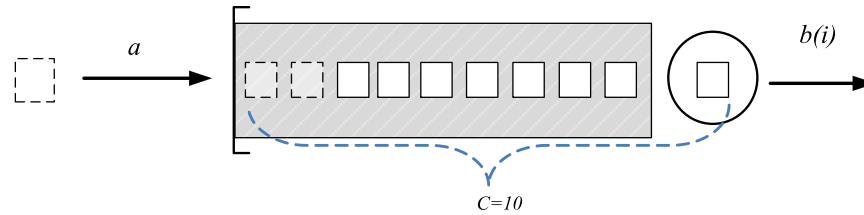


Fig. 3. Queueing systems for impact analysis.

4.6. An algorithm to calculate inventory coverage

The assumption of  $C = K$  implies that the position of an outstanding order in Queue #2 is unimportant as long as it has not yet delivered since the server takes all items at the same time. In addition, we analytically proved that the optimal control of inventory can be achieved using inventory position (summation of outstanding orders plus inventory level) (Theorem 1). Hence it is sufficient to set  $C = 2$  to calculate the probability  $Pr\{L(i) \leq l \leq L(i_+)\}$  which requires explicit modeling of movements of outstanding orders.

In such a system outstanding orders move through three distinct positions: Current period's order enters the supply system from position 1. In the next period, the outstanding order either proceeds to position 2 (or adds up the existing outstanding orders) or it is delivered (position 3) with probabilities  $1 - b(i)$  and  $b(i)$ , respectively. All undelivered, existing outstanding orders are assumed to be in position two. Recall that  $C = K$  implies that orders either stay in their position with probability  $1 - b(i)$ , or all of them are delivered at the same time with probability  $b(i)$ . Movements of an outstanding order can be modeled with a Markov chain  $X_t$  of which the third state (position three, delivery) is an absorbing state. When the health of the supply system, denoted by  $I_t$ , is in state  $i$ , one-step transition diagram,  $A_1^i$ , of  $X_t$  is given as follows:

$$\forall i \in B, A_1^i = \begin{pmatrix} 0 & 1 - b(i) & b(i) \\ 0 & 1 - b(i) & b(i) \\ 0 & 0 & 1 \end{pmatrix}. \tag{8}$$

For the supply system described above  $\forall i \in B, Pr\{L(i) \leq 0 \leq L(i_+)\} = Pr\{L(i) = 0\} = Pr\{X_{t+1} = 3 | X_t = 1, I_t = i\} = b(i)$ . Also we know that  $Pr\{L(i) \leq l \leq L(i_+)\} = Pr\{L(i) \leq l\} - Pr\{L(i_+) < l\}$  (Song & Zipkin, 1996). Hence, we need to calculate  $Pr\{L(i) \leq l, Pr\{L(i_+) < l\}$  to reach the desired probability.

$Pr\{L(i) \leq l\}$  represents the probability of an outstanding order being delivered in less-than-or-equal-to  $l$  periods, i.e.  $Pr\{L(i) \leq l\} = Pr\{X_{t+l+1} = 3 | X_t = 1, I_t = i\}$ . This multi-period probability can be calculated by a first-step analysis as follows:  $Pr\{X_{t+l+1} = 3 | X_t = 1, I_t = i\} = \sum_{k \in B} P_{ik} \sum_{j \neq 3} Pr\{X_{t+l+1} = 3 | X_{t+1} = j, I_{t+1} = k\} Pr\{X_{t+1} = j | X_t = 1, I_t = i\}$ , which is nothing but Chapman-Kolmogorov equations (Ross, 1996). Also the probability statement  $Pr\{X_{t+l+1} = 3 | X_{t+1} = 1, I_{t+1} = i_+\} = Pr\{L(i_+) \leq l - 1\}$ . These two observations are utilized in Algorithm 1.

Algorithm 1 Algorithm to calculate inventory coverage.

- 1:  $A_1^{i+} = \sum_{j \in B} p_{ij} A_1^j$ ,
- 2: **for all**  $l \geq 1$  **do**
- 3:   **for all**  $i \in B$  **do**
- 4:      $A_l^{i+} = \sum_{j \in B} p_{ij} A_l^j$
- 5:      $A_{l+1}^{i+} := A_l^i A_l^{i+}$
- 6:      $Pr\{L(i) \leq l\} = (A_{l+1}^i)_{(1,3)}$
- 7:      $Pr\{L(i_+) < l\} = (A_l^{i+})_{(1,3)}$
- 8:   **end for**
- 9: **end for**
- 10:  $Pr\{L(i) \leq l \leq L(i_+)\} = Pr\{L(i) \leq l\} - Pr\{L(i_+) < l\}$

The algorithm starts with the calculation of  $A_1^{i+}$  for a given state of the supply health using  $\{p_{ij} : p_{ij} \in \mathcal{P}\}$  and recall that  $\mathcal{P}$  is the transition matrix for the supply health in Eq. (1). Multiplication of the two matrices in step 5 follows from the Chapman-Kolmogorov equations.  $(A_{l+1}^i)_{(1,3)}$  stands for the element in the first row and the third column of  $l$ -period transition matrix  $A_{l+1}^i$ , which gives the probability of an outstanding order being delivered within  $l$  periods.

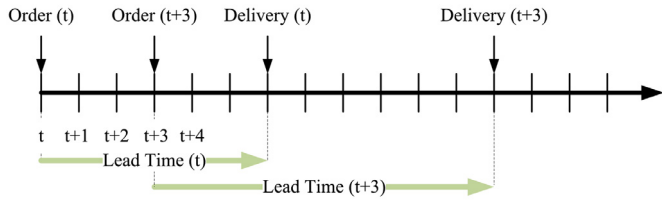
To the best of our knowledge, our queueing system and Algorithm 1 is the first algorithm to calculate  $Pr\{L(i) \leq l \leq L(i_+)\}$ , which is critical for nonstationary random lead time systems. In the rest of this paper, we provide the maximum likelihood estimator for the state-dependent service rate of Queue #2.

4.7. Maximum likelihood estimator of  $b(i)$

Our supply mechanism works with a batch server which delivers  $K$  orders at the same time. And the batch size is assumed to be equal to the queue capacity,  $C = K$ . This means our supply mechanism accepts successive orders and delivers them at the same time rather than sequential deliveries.

To estimate the supply mechanism's parameters from purchase history data, we should model periods with and without deliveries explicitly. Define  $Y_t = 1$  if delivery of outstanding orders take place at time  $t$  and it is equal to 0 otherwise. Then for an order placed

### Overlapping Lead Times



### Nonoverlapping Lead Times

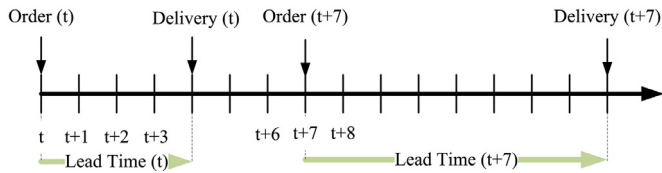


Fig. 4. Overlapping and non-overlapping lead time periods.

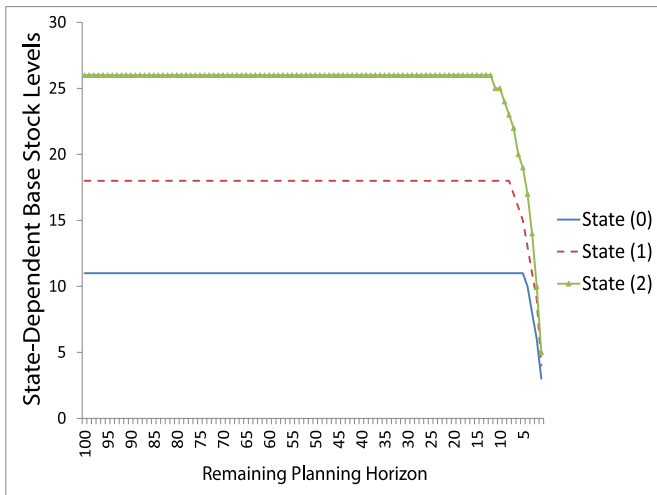


Fig. 5. Base stock levels for the benchmark scenario.

at period  $t$ ,

$$\begin{aligned}
 Pr\{L(i) = l\} &= Pr\{Y_t = 0, I_t = i\}Pr\{Y_{t+1} = 0, I_{t+1} = i_+\} \\
 &\dots Pr\{Y_{t+l-1} = 0, I_{t+l-1} = j\}Pr\{Y_{t+l} = 1, I_{t+l} = j_+\}, \\
 &= Pr\{Y_t = 0|I_t = i\}Pr\{I_t = i\}Pr\{Y_{t+1} = 0|I_{t+1} = i_+\}p_{ii_+} \\
 &\dots Pr\{Y_{t+l} = 1|I_{t+l} = j_+\}p_{jj_+}. \tag{9}
 \end{aligned}$$

Define  $\mathcal{L}(Y, I)$  as the likelihood function for the joint distribution of  $(Y_t, I_t)$ . Then  $\mathcal{L}(Y, I) = \mathcal{L}(Y|I)\mathcal{L}(I)$  due to Eq. (9). In other words, it is sufficient to maximize likelihood function for the parameters of the supply system for a given Markov chain data set.

Define  $m_i$  as the number of periods without delivery and the Markov chain is in state  $i$ , and  $n_i$  is the counter for the periods with delivery in state  $i$ . After counting time periods with and without delivery within lead time data, we can write the likelihood function of lead times using  $m_i$  and  $n_i$ ,  $i = 1, \dots, N$  as follows:

$$\mathcal{L}(Y|I) = (1 - b_1)^{m_1} (1 - b_2)^{m_2} \dots (1 - b_N)^{m_N} b_1^{n_1} b_2^{n_2} \dots b_N^{n_N}. \tag{10}$$

By showing the concavity of the log-likelihood function, we find the maximum likelihood estimator of  $b_i$  is equal to  $\frac{n_i}{n_i + m_i}$ . Note that for orders whose lead time periods do not overlap (Fig. 4), the maximum likelihood estimator is exact. For successive orders whose lead times overlap, the estimator is an heuristic approach as

our supply mechanism precludes different delivery times for overlapping lead times.

Non-overlapping lead times are particularly observed in the supply chain of slow-moving spare parts. Decreasing demand rates towards spare parts' economic life times lead to infrequent orders to suppliers which yield non-overlapping lead time periods as in the business case presented in Fig. 1 (Section 3). Hence, the accuracy of our queueing system increases while parent capital products get older.

In the next section, we suggest a procedure utilizing our mathematical model, the queueing system and empirical model by Li et al. (2016) for mitigation of supply disruption risk.

#### 4.8. A procedure for estimating the nonstationary supply risk

Our model in Section 4.3 assumes that Markov states for supplier health can be observed by the decision maker. Instead of extending the model with hidden or partially-observed Markov states, we use supply risk estimations by an empirical model suggested by Li et al. (2016) who estimated survival probability of suppliers using the proportional hazard model. Their model is capable of estimating survival probability for a given length of time period for each spare part. The main idea of our procedure is using those survival probabilities as the risk indicators and making them compatible with our model by transforming them into Markov states.

Survival probabilities of a spare part supplier are calculated for each historical time point in *rolling horizon* fashion. This means the data available until each time point is used to estimate the survival probability of that time point. Then those probabilities are transformed into states of a Markov chain (driving the supply health) using *threshold levels*. Once the survival probability crosses a threshold level, we accept this as the event of the Markov chain jumping to another state.

By doing so, we calculate the Markov chain state of each time point and use lead time data to estimate parameters of the supply mechanism articulated in Section 4.7. For recovery probabilities of the model, we suggest usage of empirical data that belongs to the part group including the spare part of interest to avoid a potential degrees of freedom problem.

The estimation of parameters of our model is followed by calculation of infinite-horizon base stock levels using the value iteration algorithm. Usage of the state-dependent base stock levels with our procedure allows increasing inventory levels in advance when the disruption risk is increasing. The application of our procedure to the business case in Section 3 is presented in Section 5.5. In the next section, we use the queueing mechanism in Section 4.5 for analyzing the effect of random lead times coupled with disruptions, and present the application of our supply risk mitigation to the real business case in Section 3.

### 5. Impact analysis for nonstationary supply risk factors

To investigate the combined effect of random lead time and supply disruption, we used the stochastic process in Section 4.5 to generate Markovian random lead times and disruption events. We calculate the optimal base stock levels under different risk scenarios (that is, considering only one risk factor, or both, or none). Subsequently, we test the performance of these optimal policies with simulation in the benchmark scenario, which includes both risk factors (Section 5.1). In this way, we analyze the impact of ignoring one or both of the risks in terms of costs and service level.

There are two possible extreme scenarios for a nonstationary supply system. The system stays healthy over the entire planning horizon, or it proceeds to more risky states and eventually fails. In order to evaluate these two scenarios, we run the stochastic process from Section 4.5 with different  $e$  and  $d$  probabilities. Due to



the way that we order Markov chain states (Section 4),  $e < d$  implies the stable supply scenario in which the supplier moves to a healthier state with higher probability than moving to a riskier state. A possible example of this situation is spare parts which are at the beginning of their life cycle. Even if exogenous changes occur, the supply system stays stable for such parts. In the other extreme, we have the unstable scenario, that is  $e > d$ . This situation occurs especially when capital products are in the final phase of their life cycle. Suppliers of spare parts tend to stop manufacturing over time and this tendency is reflected in increasing lead times and higher supply disruption probabilities.

The literature distinguishes two types of disruptions: Long and infrequent disruptions (LID) and short and frequent disruptions (SFD). To evaluate the effect of disruption types, we consider them in our scenario analysis in addition to the stable and unstable supply processes.

In the scenario analysis, we evaluated the effect of each individual risk factor as well as their combined effect on total cost and service levels, expressed in terms of ready rate. Besides the scenarios with random lead times, we consider scenarios with deterministic lead times, while keeping expected lead times equal. Similarly, besides the scenarios with supply disruption risk, we run scenarios without disruption by setting associated probabilities to zero. We conduct each computation for unstable and stable supply scenarios together with LID and SFD scenarios respectively. By considering the factorial design for all variables given in Table 2, we obtained 16 different scenarios in this part of the study.

In addition to these runs, we consider a scenario with a state-independent deterministic lead time without supply disruption. The deterministic lead time is assumed to be equal to the average of expected lead times of all states. Calculated optimal base stock levels of each scenario are used in a simulation model of the benchmark scenario to compare the effect of ignoring both supply risks on the inventory performance. The development of the scenarios and their results are presented in the following sections.

### 5.1. Setup of the computational study

In order to evaluate the deviation of each run from the optimal policy (benchmark), we fed finite-horizon order-up-to levels into a simulation model. This procedure started with selection of parameter values for disruption and supply recovery probabilities as well as random lead time for each state.

For simplicity, we considered a Markov chain consisting of three healthy and three disruption states where each disruption state was assumed to be associated with only one healthy state. Note that considering different numbers of healthy states did not change the qualitative results of the study.

For state-dependent random lead times, we considered two different sets of parameter values for the service rate of Queue #2 ( $b(i)$ ), given in Table 3. Parameter set 1 aimed to examine the effect of significant lead time variations over Markov states. Parameter set 2 aimed to investigate the supply risk factors when the first two moments of lead time distributions are very close to zero. Our goal was to develop a better understanding of the interaction between supply disruption and random lead time.

For the disruption behavior of the model, we calculated disruption and recovery probabilities ( $q(i)$  and  $\xi(i)$  for  $i = 1, 2, 3$ ) that yield the expected number of disruption periods equal to 5, 10, and 15% of the planning horizon under four different supply scenarios: stable-LID, unstable-LID, stable-SFD, and unstable-SFD. Details of these calculations and calculated parameter values are given in Appendix B.

Using the parameter values in Table 3 and Appendix B, we calculated the optimal base stock levels using the value iteration algorithm for 100 periods. The finite-horizon base stock levels for the

benchmark scenario (unstable-LID with both lead time and supply failure risks) are given in Fig. 5. As can be seen, all base stock levels converge to an infinite-horizon base stock level and the end-of-horizon effect appears when there are 10 periods remaining in the planning horizon. Also, there are significant differences between base stock levels of different states.

To evaluate the performance of the optimal policy, we developed a simulation model. This approach is motivated by the fact that calculation of the optimal total cost requires a complete enumeration of a multi-state dynamic programming model which is only feasible for problems with small state spaces. On the other hand, the optimum base stock levels can be calculated using the reduced cost function given in Eq. (5).

In the simulation model, Markov-modulated supply disruptions and random lead times take place randomly. At the beginning of each period, an order is placed according to calculated finite-horizon base stock levels. Each period's acquisition, holding and backlog costs are calculated over the planning horizon. By feeding the optimum policy to the simulation model we obtain desired performance measures of the benchmark scenario.

The performance measures we tracked in our simulation model are total discounted cost, total discounted backlog cost, ready rate (fraction of time with positive stock on hand) and fill rate (fraction of demand that can be satisfied immediately from stock on hand (Axsäter, 2006)). Total cost and total backlog costs are common performance measures in the inventory control literature. Ready rate and fill rate are important in the aviation sector and are utilized in most customer contracts. To determine the number of replications, we first conducted a pilot study consisting of 5000 replications. We used the results of this study to compute the total number of replications, which was set at 50,000. To control the variance, we use common random numbers and used paired-t-tests to compare results of scenarios.

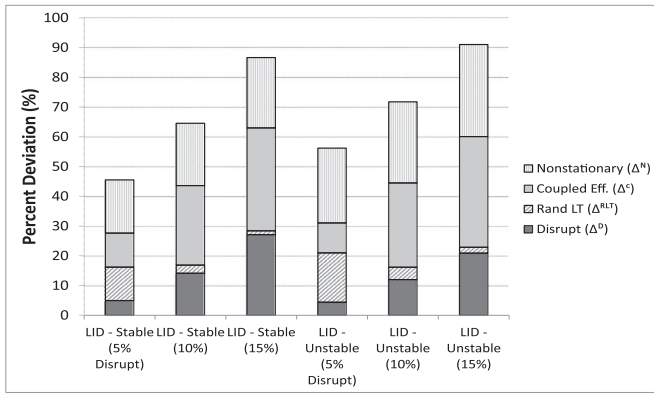
The discount rate per period is set at 0.995, which leads to a 6% annual discount rate over the entire planning horizon, since a period stands for a month in our empirical analysis presented below. Without loss of generality, we set the acquisition cost equal to 2 per item, the holding cost equal to 0.2 and backlog cost is equal to 4 per item per period (0.1 and 2 are taken as holding and backlog cost rate multipliers). Random demand in each period is assumed to follow a Poisson distribution with mean 2.

### 5.2. Coupled effect of random lead time and supply disruption

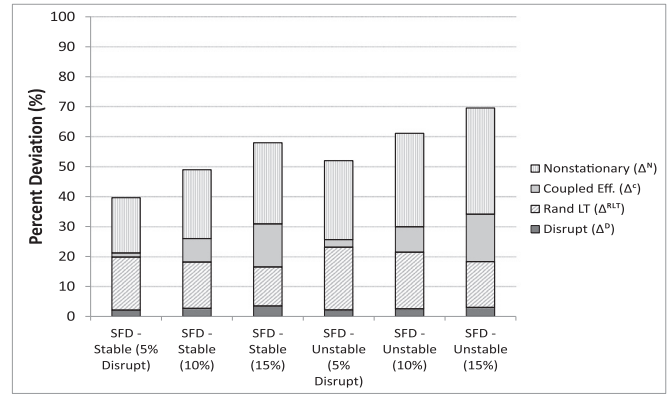
To analyze individual and coupled effects of random lead time and supply disruptions on cost and service level, we present the results of the scenarios given in Table 2 with different expected disruption periods (5, 10, and 15% of the planning horizon) and the two parameter sets in Table 3. In this section we present key findings of our impact analysis. An unabridged version of this section is given in Appendix C.

We define the percent deviation of the scenario where only random lead times are ignored (taken to be deterministic and equal to expectations of each state) as  $\Delta^{RLT}$  and the deviation of the scenario in which disruption risk is ignored from the benchmark is denoted with  $\Delta^D$ .

The coupled effect, denoted by  $\Delta^c$ , is calculated using the scenarios with Markov-modulated deterministic lead time without disruptions. We define the percent deviation of this scenario from the benchmark as  $\Delta^{Nonst}$ . Similarly, we analyze the effect of nonstationarity,  $\Delta^N$ , using a scenario with stationary (*non-Markovian*), deterministic lead time without disruption.  $\Delta^{St}$  denotes the percent deviation of this scenario from the benchmark. The difference between stationary and non-stationary policies is taken as the effect of nonstationarity. Formulations of the coupled effect and the

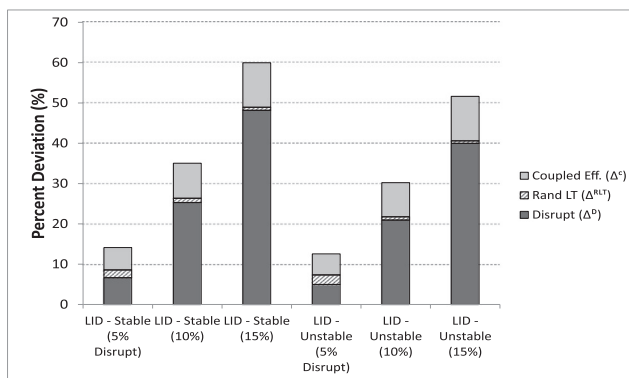


(a) Long and Infrequent Disruptions

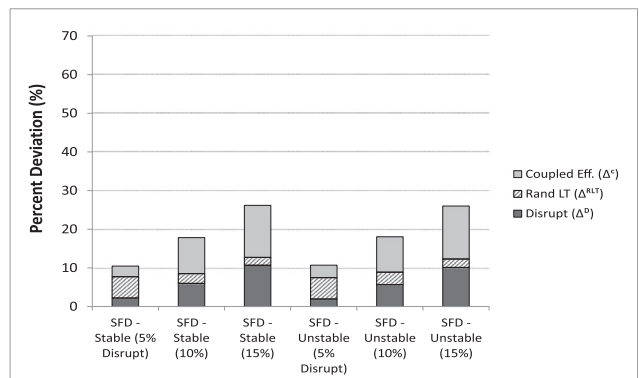


(b) Short and Frequent Disruptions

Fig. 6. Percent deviation from the optimal total cost for different scenarios in lead time set 1.



(a) Long and Infrequent Disruptions



(b) Short and Frequent Disruptions

Fig. 7. Percent deviation from the optimal total cost for different scenarios in lead time set 2.

effect of nonstationarity are given below:

$$\Delta^C = \Delta^{Nonst} - \Delta^{RLT} - \Delta^D,$$

$$\Delta^N = \Delta^{St} - \Delta^{Nonst}.$$

With these formulations we aim to see individual effects of all factors on total cost when they are ignored by the decision maker. Values of these statistics under stable LID and SFD scenarios, which are calculated by comparing appropriate runs and benchmarks, are presented in Fig. 6a and 6b.

The results in Fig. 6a and 6b indicate that increasing levels of disruption risk lead to larger deviations due to disruption ( $\Delta^D$ ) and the coupled effect ( $\Delta^C$ ), whereas it depletes the deviation due to random lead time ( $\Delta^{RLT}$ ). Evidently, increasing disruption risk leads to higher inventory levels, which mitigate the effect of random lead time on inventory performance. Furthermore, the effect of nonstationarity ( $\Delta^N$ ) is larger for unstable supply scenarios compared to stable ones. Expectedly, when the supply health worsens over time, such as in aging aircraft, ignoring supply-side risk creates a larger deviation in total discounted costs.

Another important observation can be done between the two types of disruptions. Our results show that LIDs have a larger effect on system performance compared to SFDs although the expected number of disruption periods are the same. This also holds for the coupled effect. This result is similar to Tomlin (2006) who compared the two types of disruptions in a different context.

Results of the same experiments with lead time parameter set 2 (Table 3) are given in Fig. 7a and 7b. In this run set, the ef-

fect of disruption is 50% and the coupled effect is up to 11% of total optimal cost. Also, we find that the effect of nonstationarity on total cost deviation is almost zero (that's why we did not depict them in Fig. 7a and 7b). This indicates that state-dependent lead time distributions are more important for nonstationarity than state-dependent disruption probabilities. Also note that the deviation due to disruption and random lead time is larger in the stable supply scenarios than in the unstable ones. This observation can be seen as the implicit effect of nonstationarity on individual risk factors.

### 5.3. Sensitivity analysis

In order to analyze the effect of supply risk under different cost parameters, we run a sensitivity analysis in which we consider {0.2, 0.3, 0.4, 0.6} as holding cost rates and {0.9, 0.95, 0.99, 0.995} as target service levels which are used to calculate backlog cost rates using the critical fractile. Calculated holding and backlog cost rates are multiplied with the acquisition cost, 2 per item, to obtain cost parameters of the analysis.

We only present the results of the sensitivity analysis for LID unstable and SFD stable scenarios since these two stand for upper and lower bounds for the effects of supply risk on total cost and ready rates. Our sensitivity analysis indicates that the coupled effect of random lead time and disruptions can be greater than 200% of the total optimal cost for high ready rates in the LID unstable scenario. When all risks are ignored, the total cost deviation can be up to 500% of the optimal cost when the target ready rate is set to 99.5% (Fig. 8). The nature of the deviation is very different

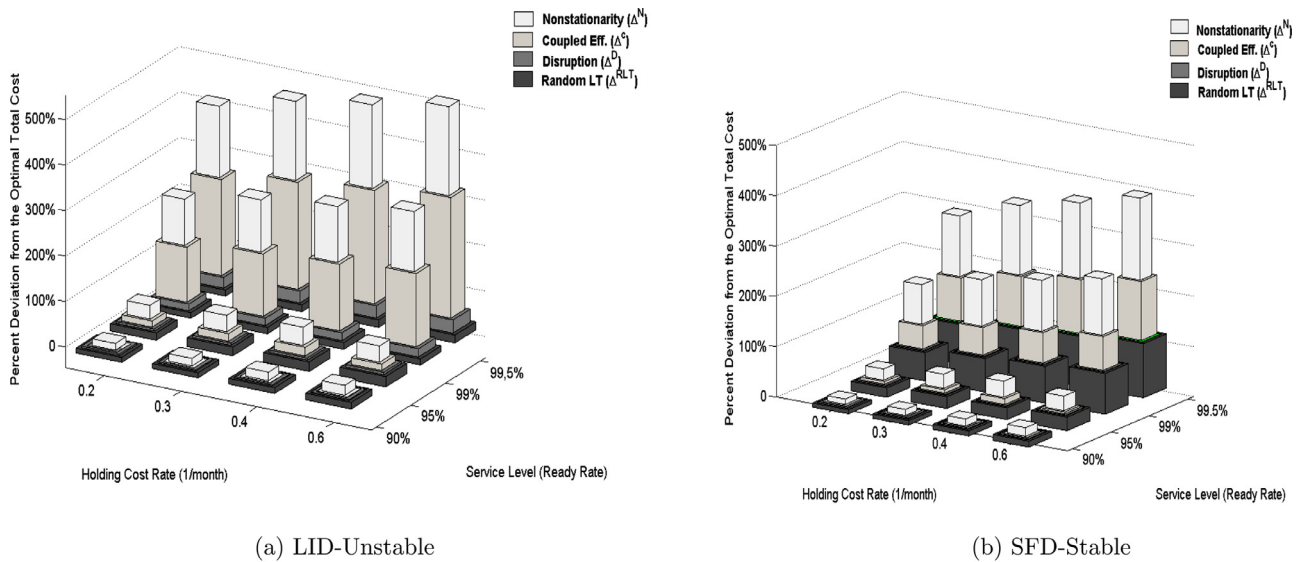


Fig. 8. Percent deviation from total costs for different scenarios with 5% expected disruption.

Table 4

Total canceled outstanding orders as percent of expected total demand.

		Disruption type			
		Long & infreq.		Short & freq.	
Supply tendency		Stable	Unstable	Stable	Unstable
Lead time parameter	Set1	1.86%	1.76%	5.63%	5.33%
	Set2	0.16%	0.14%	0.5%	0.45%

for SFDs, where the effect of random lead time is as high as the coupled effect of the two scenarios. These results indicate the importance of considering both risk factors in a single model to aim for higher service levels with more reasonable inventory costs. In the next section, we consider the case where outstanding orders are canceled with disruption.

5.4. Cancellation of outstanding orders after disruption

In our model, we assume that outstanding orders are preserved and deliveries of previous orders continue during disruption. This assumption leads us to an analytically tractable model and the optimal inventory control policy. Despite these mathematically attractive features, preservation of previous orders during disruption periods does not always hold in practice.

Spare parts suppliers may decide to cancel their support and send a notification to OEMs for failure to delivery. Such cases are modeled as disruption in our model which relies on the idea of carrying additional inventory on account of disruption periods. However, when a supplier cancels previous orders after disruption, this violates the assumption of order preservation (and delivery) of outstanding orders.

In order to analyze the cost deviation due to such violation, we calculate optimum finite-horizon inventory levels with our model and use them in a simulation model in which outstanding orders are canceled after disruption. Total number of orders canceled due to supply disruptions are given as the percentage of total demand in Table 4 and the deviations from the optimum cost due to this falsified model assumption are presented in Table 5.

Results indicate that the total amount of canceled orders are much higher for short and frequent disruptions compared to long and infrequent ones. This can be observed for two lead time parameter sets and both supply tendencies in Tables 4 and 5.

We should also note that cost deviations are almost negligible - compared to other supply risk factors- for random lead time parameter set 2 (smaller lead time variance). Therefore, our method is strongly advisable for such cases. Furthermore, the queueing model and the maximum likelihood estimator are particularly accurate when lead time periods of successive orders are non-overlapping. When this is the case, there might be only a single outstanding order in the system. Hence deviation due to cancellation of outstanding orders are much smaller than articulated here.

5.5. An application of the supply risk mitigation procedure to Part A

To gain further understanding of the practical value of our model and the procedure described in Section 4.8, we evaluated the performance of the optimum policy and infinite-horizon base stock levels on the empirical data that belongs to Part A presented in Section 3. A more detailed version of this section is given in Appendix E while we present the most interesting results in this section.

The procedure starts with the calculation of the survival probabilities of Part A's supplier for each month. To transform survival probabilities to transition probabilities ( $p_{ij}$  in Eq. 2) of a Markov chain with two states (state 0 is healthy; state 1 is unhealthy) we chose 0.75 as a threshold level to move to the unhealthy state. When the survival probability is higher than 0.75, the supplier is assumed to be in state 0, whereas crossing this level represents the Markov chain jumping to state 1. Using this discretization, a time series is obtained from monthly survival probabilities of the supplier.

Then, we used purchase history data for similar parts from the same supplier to calculate the maximum likelihood estimator (MLE) for the geometric distribution, which is the lead time distribution in our queueing system in Section 4.5. Calculated mean

**Table 5**  
Cost deviation due to cancellation of outstanding orders.

		Disruption type			
		Long & infreq.		Short & freq.	
		Stable	Unstable	Stable	Unstable
Lead time parameter	Set1	27.9%	22.3%	103.9%	79.1%
	Set2	2.5%	2.1%	8.3%	7.9%

**Table 6**  
Descriptive statistics of state-dependent lead times (months).

	Healthy	Unhealthy
Mean	1.88	2.12
Std.Dev.	0.606	0.779

and standard deviation of state-dependent lead times are given in Table 6 whereas MLEs are presented in Table 7.

For the disruption probabilities of the model,  $q(i)$ , we used cross-validation results by Li et al. (2016). In their tests, 156 out of 186 parts had a survival probability of less than 0.75 for the horizon of the model, and 21 out of 156 suppliers were disrupted at the time of the analysis. We used this statistic as an estimator for the disruption probability of state 1 (unhealthy state), whereas the disruption probability of state 0 is assumed to be 0. For the disruption recovery probability,  $\xi(d^i)$ , we used the average solution time for disruption cases, which is 4.15 months. We assumed that the recovery probabilities are identical for both Markov states, as there was no indication for different probabilities. All estimated parameters of the model as well as calculated base stock levels are presented in Tables 7 and 8.

Using probability values we calculated infinite-horizon base stock levels. Other model parameters are taken as follows: The acquisition cost of Part A is 158.39€, each period lasts a month, the backlog cost rates are calculated using the critical ratio, and assuming holding cost rate is 0.1 per period (month) with target service level (ready rate) equal to 0.9 and 0.99. Calculated optimal base stock levels for the two different service levels are given in Table 7.

To evaluate our data-driven supply risk mitigation procedure, we compared the optimal inventory levels with the historical inventory levels (Fig. 9). Results indicate that our method not only provides smoother inventory levels, but also gives a better preparation for the disruption, which took place in December 2011. Discounted total costs indicate that our policy, for service levels 0.90 and 0.99, create savings of 8.91 and 23.77% compared to the cost of historical inventory levels called "Business-As-Usual" (BAU). Surprisingly, we find that higher service levels lead to greater cost savings due to lower backlog costs in 2006 (see Fig. 9). Other savings come from lower inventory holding costs during *undisrupted* months of the supply system. Unfortunately, demand during disruption was not captured in our data. Hence, we can only speculate about the savings on the backlog cost during the disruption.

A sensitivity analysis with different threshold levels and holding cost rates are conducted. Results, presented in Table 9, indicate that savings are decreasing in threshold levels since higher threshold value stands for earlier jump to the risky state (wider red zone in Fig. 9). Note that savings decrease sharply in threshold levels for service level of 0.9 whereas the rate of decrease is much smaller for 0.99. The difference stems from the fact that for higher target service levels total cost is dominated by inventory holding cost which are close to each other for different threshold levels. Hence savings are less sensitive to threshold levels.

At this point, we should stress that authors' personal communication with engineers in the service sector reveals that 4.15 months of average solution time for disruption cases is optimistic. It is argued that solutions to disruption problems of spare parts involves engineering departments, which are usually extremely busy with new product development and research processes. Hence, disruptions get lower priority and may last up to three years. A unique feature of the OEM, whom authors have contact with, is that it has a dedicated technical group for the rapid solution of disruptions. Therefore, we postulate that the relevance of our study is even higher than may be reflected in this section. Note that results presented in this section are only calculated for a single spare part mainly for demonstration purpose.

## 6. Summary and discussion

Supply-side risks for spare parts are very important for Original Equipment Manufacturers of capital products. Empirical evidence suggests that towards the end-of-life of capital products spare parts suppliers stop their manufacturing and/or delay deliveries. This behavior creates random lead times coupled with supply disruption risks, which are nonstationary in nature. In order to address the combined effect of these two risks, we consider a supply system driven by an exogenous Markov chain in a finite horizon setting.

Given that order crossovers are not allowed, we prove that the state-dependent base stock policy is optimal. Analysis reveals that intricate sufficient conditions are necessary for establishing the monotonicity of optimal base stock levels. Also we suggest a new queueing system that generates Markov-modulated random lead times (without order crossover) and provide an algorithm to calculate lead time distribution out of this system. Also the maximum likelihood estimator for the queueing system's service rate is derived.

Our impact analyses indicate that random lead times and supply disruptions not only stimulate costs due to their individual effects, their coupled effect also leads total costs to increase and hurt inventory systems' service level. Combined effect of random lead times and disruptions are especially significant when the health of the supplier worsens over the planning horizon. To solve this problem, a supply chain manager should utilize advance signals based on empirical data and inventory control policies that can respond changing supply-side risk levels.

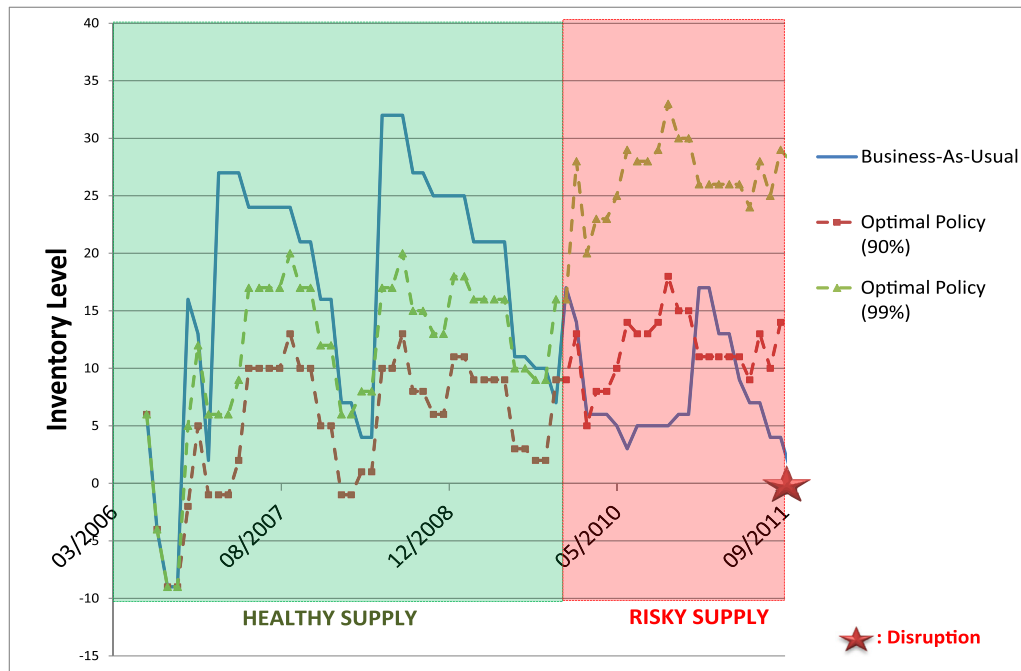
Furthermore, we conducted experiments on the cost deviation due to cancellation of outstanding orders when a supply disruption arrives. Our results indicate that supply systems with long and infrequent disruptions with low variability random lead times are less vulnerable to cancellation of previous replenishment orders.

A heuristic procedure for the application of our model with an empirical supply risk estimator is developed. The application of the procedure to the real business case indicates that recognizing random lead time together with supply disruption risk not only creates savings in total discounted costs, but also makes the company more prepared for supply disruptions.



**Table 7**  
Parameters and the result of the model for Part A.

Markov states	Lead time MLE	$q(i)$	$\xi(d^i)$	Base stock lev.(0.9)	Base stock lev.(0.99)
Healthy	0.53	0	0.2406	<b>9</b>	<b>16</b>
Unhealthy	0.47	0.1346	0.2406	<b>15</b>	<b>30</b>



**Fig. 9.** Inventory levels for Part A.

**Table 8**  
Transition probabilities ( $p_{ij}$ ) for the Markov chain of Part A.

	Healthy	Unhealthy
Healthy	0.929	0.071
Unhealthy	0.032	0.968

**Table 9**  
Savings compared to BAU with different threshold levels, service levels and holding costs.

Holding cost rate	Target ready rate	Threshold levels		
		0.65	0.75	0.85
0.1	0.9	19.73%	8.91%	4.61%
0.1	0.99	22.92%	23.77%	6.94%
0.3	0.9	21.66%	14.81%	14.53%
0.3	0.99	13.01%	13.84%	4.03%

**Supplementary material**

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.ejor.2018.02.040.

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