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On A Comparison between Two Measures of Spatial Association

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Two measures of spatial association between two variables were used by many researchers. These are the Wartenberg (1985) and Lee (2001) measures. Based on simulation for lattice data, the sensitivity of both measures was studied and compared with different choices of spatial structures, spatial weights and sample sizes using bias and mean square error. Different scenarios are used in terms of assumed numbers and sample sizes. Moran's I is used to examine the spatial autocorrelation of such a variable with itself. Both the Wartenberg and Lee measures are found to be sensitive, however, Wartenberg's measure is found to be somewhat better than Lee's measure because it is slightly more sensitive when sample size is small.

Key words: Wartenberg and Lee measures, simulation study, spatial association, sensitivity, spatial structures, weights.

Introduction

It is argued that lattice data are spatially correlated. The Wartenberg (1985) and Lee (2001) are two measures used for investigating the spatial association between two or more variables taking into account neighboring information. Lee criticized Wartenberg's measure and suggested two criteria for developing a measure for bivariate spatial association. First, the measure should conform to Pearson's r between two variables in terms of direction and magnitude. Second, a bivariate spatial association measure should reflect the degrees of spatial autocorrelation for both variables under investigation. Lee developed an index, L, that combines Pearson's bivariate correlation with Moran's spatial autocorrelation measures, to measure spatial association. Lee stated that Wartenberg's measure is vulnerable to a reverse of the direction of spatial association. Also, Lee's measure has the spatial lags of two variables while Wartenberg's measure has the spatial lag for one variable. Thus, this study makes a comparison between these measures in terms of their sensitivity. The observations for each particular sub-area can be either univariate or multivariate data. When the data are univariate, Moran's I statistic can be used to describe the spatial autocorrelation of such a variable. If the observations are multivariate then the Wartenberg and Lee measures can be used.

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Methodology

Real data are important for the development of statistical methods and ideally their analysis also stimulates research in statistical theory. Simulated data is also important and has a different role. This role is particularly valuable when several competing methods are available but little or no theory exists to indicate which is superior. Simulating spatial data is important because statistical inference for spatial data often relies on randomization tests. The ability to simulate realization of a hypothesized process quickly and efficiently is important to allow a sufficient number of realizations to be produced (Schabenberger & Gotway, 2005). The performance of the Wartenberg and Lee measures is evaluated based on simulated data.

The spatial association measures of Wartenberg and Lee can be given respectively as

$$M_{X,Y} = \frac{N}{S_0} \times \frac{\sum_{i=1}^{N}\sum_{j=1}^{N} w_{ij}(x_i - \bar{x})(y_j - \bar{y})}{\sqrt{\sum_{i=1}^{N}(x_i - \bar{x})^2}\sqrt{\sum_{i=1}^{N}(y_i - \bar{y})^2}}$$

$$L_{X,Y} = \frac{N}{\sum_{i=1}^{N}\left(\sum_{j=1}^{N} w_{ij}\right)^2} \times \frac{\sum_{i=1}^{N}\left[\left(\sum_{j=1}^{N} w_{ij}(x_j - \bar{x})\right)\times\left(\sum_{j=1}^{N} w_{ij}(y_j - \bar{y})\right)\right]}{\sqrt{\sum_{i=1}^{N}(x_i - \bar{x})^2}\sqrt{\sum_{i=1}^{N}(y_i - \bar{y})^2}}$$

where $S_0 = \sum_{i=1}^{N}\sum_{j=1}^{N} w_{ij}, i \neq j$, N is the sample size and w_{ij} is the binary spatial weight (1, 0). The univariate statistic for spatial association or autocorrelation of Moran's I is defined as (Cliff & Ord, 1981)

$$I = \frac{N}{S_0} \times \frac{\sum_{i=1}^{N}\sum_{j=1}^{N} w_{ij}(x_i - \bar{x})(x_j - \bar{x})}{\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

Because the neighbor structure is the basic structure for the covariance model of lattice data, a careful definition of spatial neighbors is a crucial analysis step (Kaluzny, et al., 1998). Neighbors may be defined as locations which border each other or as locations within a certain distance of each other. If neighbors are defined as locations bordering each other, then there are several types of spatial neighbors. For example, the first order method (the rook pattern) identifies neighbors as those to left, to the right, or above or below each location, that is, the rook makes links in four cardinal directions. The diagonal method (the bishop pattern) makes only diagonal links. The second order method (the queen pattern) includes the first-order neighbors and those diagonally linked, that is, the queen makes links in all eight directions. Figure 1 shows these three types of spatial connectivity.

The sensitivity to the choice or the definition of spatial structures of neighbors was studied for both Wartenberg and Lee measures. The simulation study was based on six spatial structures: sharing boundary (rook), sharing boundary (bishop), sharing boundary (queen), distance apart (1.5), distance apart (2.25) and distance apart (3).

If the spatial structure was made based on distance apart, the distances between location i and all its surrounding neighbors will be calculated. These distances were calculated in the SPLUS program based on such distance measures, for example, Euclidian. If the calculated distances were found within for example, distance apart (1.5), the surrounded locations will be considered as neighbors to the location i.

Kaluzny, et al., (1998) stated the choice of spatial weights between such ith location and its neighbors is a crucial step. They recommended that several choices of spatial weights be tried so that the sensitivity of the results can be determined. However, three different spatial weights w_{ij} were used ($w_{ij} = 1, w_{ij} = 1/d_{ij}$ and $w_{ij} = 1/d_{ij}^2$), where d_{ij} is the distance between location i and location j and when d_{ij} is large, the w_{ij} will be less. This means that w_{ij} for the nearest neighbors will be higher than that for the farthest neighbors.

The $bias$ and mean square error (MSE) were used to decide which statistic is better. Let θ be the parameter of interest, then the MSE of $\hat{\theta}$ is defined as follows (Garthwaitw, et al., 1995)

$$MSE(\hat{\theta}) = E\left[\left(\hat{\theta} - \theta\right)^2\right]$$

and the estimated value is calculated using

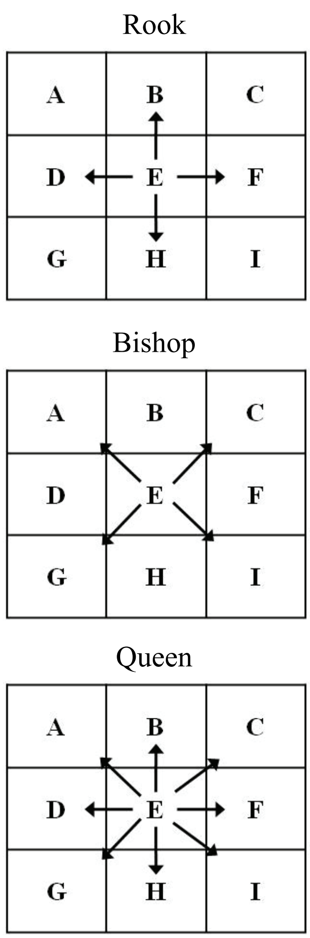
$$\widehat{MSE}(\hat{\theta}) = \widehat{Var}(\hat{\theta}) + [\widehat{bias}(\hat{\theta})]^2$$

where

$$\widehat{Var}(\hat{\theta}) = \frac{\sum_{i=1}^{s}\left(\hat{\theta}_i - \theta\right)^2}{s},$$

and $\hat{\theta}_i$ is the estimated bivariate spatial association measure based on simulated data, θ is the actual value of bivariate spatial association measure based on Wartenberg's or Lee's measure and s is the total number of runs (in this study, $s = 10,000$).

Figure 1: Three Different Types of Sharing Boundary Connectivity



The estimate $\hat{\theta}$ is an unbiased estimator for θ if $E(\hat{\theta}) = \theta$; otherwise it is biased. The $bias$ of $\hat{\theta}$ is defined to be (Garthwaitw, et al., 1995)

$$bias(\hat{\theta}) = E(\hat{\theta}) - \theta$$

and the estimated value of $E(\hat{\theta})$ is given by

$$\bar{\hat{\theta}} = \frac{\sum_{i=1}^{s}\hat{\theta}_i}{s},$$

then the estimated value of $bias(\hat{\theta})$ is given by

$$\widehat{bias}(\hat{\theta}) = \bar{\hat{\theta}} - \theta.$$

Simulation Process

The process of the simulation study included several steps which were considered somewhat complicated. The complication arose from allowing three kinds of spatial correlations before starting the spatial analysis and because the simulation must be made under a randomness assumption. The spatial correlations were: the bivariate spatial correlation between two variables and spatial autocorrelation for each variable.

The simulation study was carried out using SPLUS programming and accomplished in four steps. First, the original samples for two variables were designated to act as the population for sampling purposes. In the second step, the original samples were re-sampled a specified number of times (up to several thousands) to generate a large number of new samples, where each sample was a random subset of the original sample. In the third step, the bivariate spatial measures of Wartenberg and Lee were estimated for each new sample. In the last step, the estimated values of $bias$ and MSE were calculated using the computed spatial measures found in step 3.

During the process of generating new samples, the simulation program may change certain characteristics of the sample to meet the researcher's objectives. For example, the degree of correlation between variables may be varied across the generated samples in some systematic manner. The simulation process was run using 10000 runs, where Wartenberg's measure, W and Lee's measure L, were each estimated 5,000 times. The $bias$ and MSE were then calculated for each measure. The mechanism proposed herein contains certain assumed form of univariate spatial correlation (autocorrelation) for each variable as shown from the distribution of assumed observed values, and hence there is also a bivariate spatial correlation between these two autocorrelated variables based on the actual value of Wartenberg and Lee measures.

Results

To assess the performance of both the Wartenberg and Lee measures, a series of simulations were conducted. Several scenarios were studied to investigate the sensitivity of both the Wartenberg and Lee measures based on different choices of spatial structures and spatial weights using $bias$ and MSE. The values of two variables, X and Y, were generated based on their assumed true means and standard deviation one for each observation. Autocorrelation values of each X and Y variables based on Moran's I were found positive, negative, high or low because different choices of spatial structures and spatial weights were used; the nine resulting scenarios follow.

Scenario 1: Sample Size ($4 \times 4 = 16$)

Figures 3a and 3b show two study areas with their assumed true means. Table 1, shows the autocorrelation values for both variables X and Y based on global Moran's I statistic. Table 2 shows the actual values of the Wartenberg and Lee measures and their $bias$ and MSE using different choices of spatial structures.

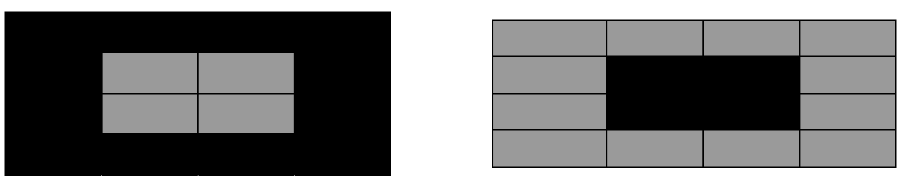
Figure 3 Proposed Area Divided into 16 Quadrates in a 4×4 Lattice

3a

3.6	3.6	3.6	3.6
3.6	1.2	1.2	3.6
3.6	1.2	1.2	3.6
3.6	3.6	3.6	3.6

3b

2.6	2.6	2.6	2.6
2.6	5.2	5.2	2.6
2.6	5.2	5.2	2.6
2.6	2.6	2.6	2.6



The numbers in the quadrates are the assumed true means for two different variables X and Y, where is a gradient patch from low values (in the center) to high values (in the edges) for X variable, and (b) is the opposite direction of (a) for Y variable.

Table 1: Autocorrelation for Variables X and Y Based on Moran's I Using Different Types of Spatial Structures

Type of Spatial Structure	Autocorrelation X	Autocorrelation Y
Sharing Boundary (Rook)	0.31	0.31
Sharing Boundary (Bishop)	-0.24	-0.24
Sharing Boundary (Queen)	0.07	0.07
Distance Apart (1.5)	0.07	0.07
Distance Apart (2.25)	-0.18	-0.18
Distance Apart (3)	-0.19	-0.19

Table 2: The *bias* and *MSE* of Wartenberg's Measure (W) and Lee's Measure (L) with Actual Values Using Different Types of Spatial Structures

Type of Spatial Structure	W			L		
	Actual	*bias*	*MSE*	Actual	*bias*	*MSE*
Sharing Boundary (Rook)	-0.31	0.14	0.03	-0.17	0.07	0.01
Sharing Boundary (Bishop)	0.24	-0.11	0.02	-0.86	0.38	0.16
Sharing Boundary (Queen)	-0.07	0.03	0.00	-0.08	0.04	0.00
Distance Apart (1.5)	-0.07	0.03	0.00	-0.08	0.04	0.00
Distance Apart (2.25)	0.18	-0.08	0.01	-0.10	0.04	0.00
Distance Apart (3)	0.19	-0.08	0.01	-0.11	0.05	0.00

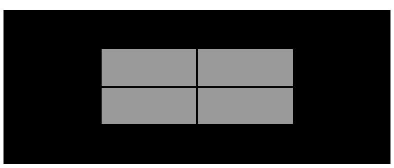
Scenario 2: Sample Size ($4 \times 4 = 16$)

Figures 4a and 4b show two study areas with their assumed true means. Table 3 shows the autocorrelation values for both variables X and Y based on Moran's I statistic. Table 4 shows the actual values of Wartenberg and Lee measures and their *bias* and *MSE* using different choices of spatial structures.

Figure 4: Proposed Area Divided into 16 Quadrates in a 4×4 Lattice

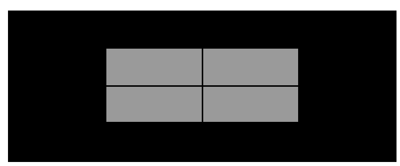
4a

3.6	3.6	3.6	3.6
3.6	1.2	1.2	3.6
3.6	1.2	1.2	3.6
3.6	3.6	3.6	3.6



4b

2.6	2.6	2.6	2.6
2.6	1.8	1.8	2.6
2.6	1.8	1.8	2.6
2.6	2.6	2.6	2.6



The numbers in the quadrates are the assumed true means for two different variables X and Y, where both (a) and (b) are a gradient patches from low values (in the center) to high values (in the edges) for X and Y variables respectively.

Table 3: Autocorrelation for Variables X and Y Based on Moran's I Using Different Types of Spatial Structures

Type of Spatial Structure	Autocorrelation	
	X	Y
Sharing Boundary (Rook)	0.31	0.31
Sharing Boundary (Bishop)	-0.24	-0.24
Sharing Boundary (Queen)	0.07	0.07
Distance Apart (1.5)	0.07	0.07
Distance Apart (2.25)	-0.18	-0.18
Distance Apart (3)	-0.19	-0.19

Table 4: The *bias* and *MSE* of Wartenberg's Measure (W) and Lee's Measure (L) with Actual Values Using Different Types of Spatial Structures

Type of Spatial Structure	W			L		
	Actual	*bias*	*MSE*	Actual	*bias*	*MSE*
Sharing Boundary (Rook)	0.31	-0.24	0.07	0.17	-0.13	0.02
Sharing Boundary (Bishop)	-0.24	0.18	0.04	0.86	-0.64	0.45
Sharing Boundary (Queen)	0.07	-0.06	0.01	0.08	-0.06	0.01
Distance Apart (1.5)	0.07	-0.06	0.01	0.08	-0.06	0.01
Distance Apart (2.25)	-0.18	0.13	0.02	0.10	-0.07	0.01
Distance Apart (3)	-0.19	0.14	0.02	0.11	-0.09	0.01

Scenario 3: Sample Size ($4 \times 4 = 16$)

Figures 5a and 5b show two study areas with their assumed true means. Table 5 shows the autocorrelation values for both variables X and Y based on Moran's I statistic. Table 6 shows the actual values of Wartenberg and Lee measures and their *bias* and *MSE* using different choices of spatial structures.

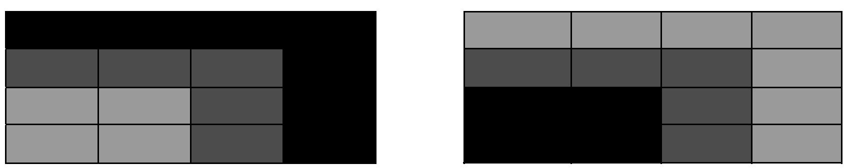
Figure 5: Proposed Area Divided into 16 Quadrates in a 4×4 Lattice

5a

3.6	3.6	3.6	3.6
2.4	2.4	2.4	3.6
1.5	1.5	2.4	3.6
1.5	1.5	2.4	3.6

5b

1.3	1.3	1.3	1.3
2.2	2.2	2.2	1.3
3.8	3.8	2.2	1.3
3.8	3.8	2.2	1.3



The numbers in the quadrates are the assumed true means for two different variables X and Y, where (a) is a gradient patch from low values (the bottom left side) to high values (the upper right side) for X variable, and (b) is the opposite direction of (a) for Y variable.

Table 5: Autocorrelation for Variables X and Y Based on Moran's I Using Different Types of Spatial Structures

Type of Spatial Structure	Autocorrelation	
	X	Y
Sharing Boundary (Rook)	0.57	0.59
Sharing Boundary (Bishop)	0.22	0.31
Sharing Boundary (Queen)	0.42	0.47
Distance Apart (1.5)	0.42	0.47
Distance Apart (2.25)	0.14	0.13
Distance Apart (3)	0.09	0.08

Table 6: The *bias* and *MSE* of Wartenberg's Measure (W) and Lee's Measure (L) with Actual Values Using Different Types of Spatial Structures

Type of Spatial Structure	W			L		
	Actual	*bias*	*MSE*	Actual	*bias*	*MSE*
Sharing Boundary (Rook)	-0.58	0.30	0.10	-0.48	0.24	0.07
Sharing Boundary (Bishop)	-0.28	0.14	0.03	-0.26	0.13	0.03
Sharing Boundary (Queen)	-0.45	0.23	0.06	-0.35	0.18	0.04
Distance Apart (1.5)	-0.45	0.23	0.06	-0.35	0.18	0.04
Distance Apart (2.25)	-0.14	0.07	0.01	-0.09	0.05	0.00
Distance Apart (3)	-0.09	0.05	0.00	-0.06	0.03	0.00

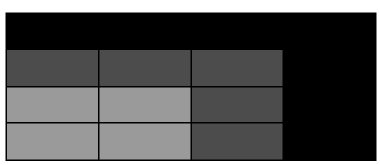
Scenario 4: Sample Size ($4 \times 4 = 16$)

Figures 6a and 6b show two study areas with their assumed true means. Table 7 shows the autocorrelation values for both variables X and Y based on Moran's I statistic. Table 8 shows the actual values of Wartenberg and Lee measures and their *bias* and *MSE* using different choices of spatial structures.

Figure 6: Proposed Area Divided into 16 Quadrates in a 4×4 Lattice

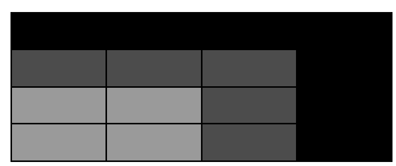
6a

3.4	3.4	3.4	3.4
2.4	2.4	2.4	3.4
1.2	1.2	2.4	3.4
1.2	1.2	2.4	3.4



6b

3.9	3.9	3.9	3.9
2.7	2.7	2.7	3.9
1.8	1.8	2.7	3.9
1.8	1.8	2.7	3.9



The numbers in the quadrates are the assumed true means for two different variables X and Y, where both (a) and (b) are a gradient patches from low values (the bottom left side) to higher values (the upper right side) for X and Y variables respectively.

Table 7: Autocorrelation for Variables X and Y Based on Moran's I Using Different Types of Spatial Structures

Type of Spatial Structure	Autocorrelation	
	X	Y
Sharing Boundary (Rook)	0.59	0.57
Sharing Boundary (Bishop)	0.28	0.22
Sharing Boundary (Queen)	0.46	0.42
Distance Apart (1.5)	0.46	0.42
Distance Apart (2.25)	0.14	0.14
Distance Apart (3)	0.09	0.09

Table 8: The *bias* and *MSE* of Wartenberg's Measure (W) and Lee's Measure (L) with Actual Values Using Different Types of Spatial Structures

Type of Spatial Structure	W			L		
	Actual	*bias*	*MSE*	Actual	*bias*	*MSE*
Sharing Boundary (Rook)	0.58	-0.31	0.11	0.48	-0.26	0.08
Sharing Boundary (Bishop)	0.25	-0.14	0.03	0.27	-0.14	0.04
Sharing Boundary (Queen)	0.44	-0.24	0.06	0.34	-0.18	0.04
Distance Apart (1.5)	0.44	-0.24	0.06	0.34	-0.18	0.04
Distance Apart (2.25)	0.14	-0.08	0.01	0.09	-0.05	0.00
Distance Apart (3)	0.09	-0.05	0.00	0.06	-0.03	0.00

Scenario 5: Sample Size ($6 \times 6 = 36$)

Figures 7a and 7b show two study areas with their assumed true means. Table 9 shows the autocorrelation values for both variables X and Y based on Moran's I statistic. Table 10 shows the actual values of Wartenberg and Lee measures and their *bias* and *MSE* using different choices of spatial structures.

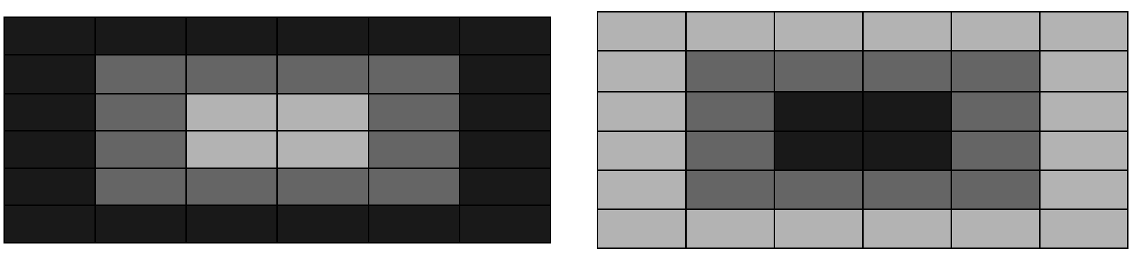
Figure 7: Proposed Area Divided into 36 Quadrates in a 6×6 Lattice

7a

5.4	5.4	5.4	5.4	5.4	5.4
5.4	3.6	3.6	3.6	3.6	5.4
5.4	3.6	1.2	1.2	3.6	5.4
5.4	3.6	1.2	1.2	3.6	5.4
5.4	3.6	3.6	3.6	3.6	5.4
5.4	5.4	5.4	5.4	5.4	5.4

7b

1.3	1.3	1.3	1.3	1.3	1.3
1.3	2.6	2.6	2.6	2.6	1.3
1.3	2.6	5.2	5.2	2.6	1.3
1.3	2.6	5.2	5.2	2.6	1.3
1.3	2.6	2.6	2.6	2.6	1.3
1.3	1.3	1.3	1.3	1.3	1.3



The numbers in the quadrates are the assumed true means for two different variables X and Y, where (a) is a gradient patch from low values (in the center) to high values (in the edges) for X variable, and (b) is the opposite direction of (a) for Y variable.

Table 9: Autocorrelation for Variables X and Y Based on Moran's I Using Different Types of Spatial Structures

Type of Spatial Structure	Autocorrelation	
	X	Y
Sharing Boundary (Rook)	0.63	0.63
Sharing Boundary (Bishop)	0.33	0.35
Sharing Boundary (Queen)	0.49	0.51
Distance Apart (1.5)	0.49	0.51
Distance Apart (2.25)	0.18	0.16
Distance Apart (3)	0.10	0.08

Table 10: The $bias$ and MSE of Wartenberg's Measure (W) and Lee's Measure (L) with Actual Values Using Different Types of Spatial Structures

Type of Spatial Structure	W			L		
	Actual	$bias$	MSE	Actual	$bias$	MSE
Sharing Boundary (Rook)	-0.63	0.23	0.06	-0.41	0.15	0.02
Sharing Boundary (Bishop)	-0.35	0.12	0.02	-0.18	0.06	0.01
Sharing Boundary (Queen)	-0.50	0.18	0.03	-0.23	0.08	0.01
Distance Apart (1.5)	-0.50	0.18	0.03	-0.23	0.08	0.01
Distance Apart (2.25)	-0.18	0.06	0.00	-0.08	0.03	0.00
Distance Apart (3)	-0.10	0.03	0.00	-0.08	0.03	0.00

Scenario 6: Sample Size ($8\times 8 = 64$)

Figures 8a and 8b show two study areas with their assumed true means. Table 11 shows the autocorrelation values for both variables X and Y based on Moran's I statistic. Table 12 shows the actual values of Wartenberg and Lee measures and their $bias$ and MSE using different choices of spatial structures.

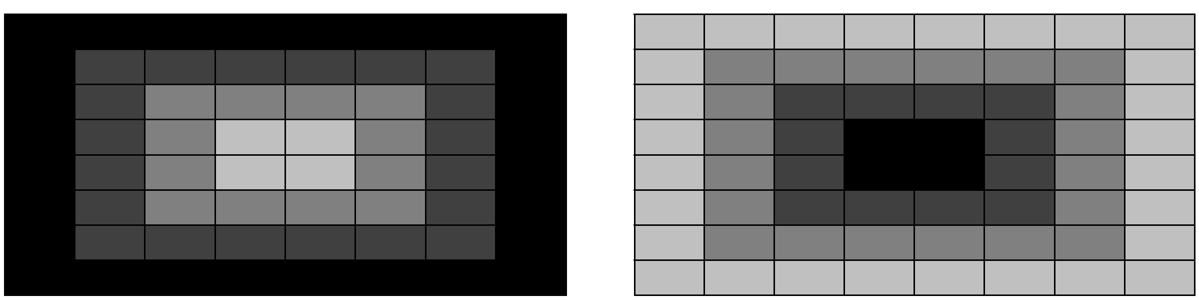
Figure 8: Proposed Area Divided into 64 Quadrates in a 8×8 Lattice

8a

7.9	7.9	7.9	7.9	7.9	7.9	7.9	7.9
7.9	5.4	5.4	5.4	5.4	5.4	5.4	7.9
7.9	5.4	3.6	3.6	3.6	3.6	5.4	7.9
7.9	5.4	3.6	1.2	1.2	3.6	5.4	7.9
7.9	5.4	3.6	1.2	1.2	3.6	5.4	7.9
7.9	5.4	3.6	3.6	3.6	3.6	5.4	7.9
7.9	5.4	5.4	5.4	5.4	5.4	5.4	7.9
7.9	7.9	7.9	7.9	7.9	7.9	7.9	7.9

8b

2.3	2.3	2.3	2.3	2.3	2.3	2.3	2.3
2.3	4.2	4.2	4.2	4.2	4.2	4.2	2.3
2.3	4.2	6.7	6.7	6.7	6.7	4.2	2.3
2.3	4.2	6.7	8.2	8.2	6.7	4.2	2.3
2.3	4.2	6.7	8.2	8.2	6.7	4.2	2.3
2.3	4.2	6.7	6.7	6.7	6.7	4.2	2.3
2.3	4.2	4.2	4.2	4.2	4.2	4.2	2.3
2.3	2.3	2.3	2.3	2.3	2.3	2.3	2.3



The numbers in the quadrates are the assumed true means for two different variables X and Y, where (a) is a gradient patch from low values (in the center) to high values (in the edges) for X variable, and (b) is the opposite direction of (a) for Y variable.

Table 11: Autocorrelation for Variables X and Y Based on Moran's I Using Different Types of Spatial Structures

Type of Spatial Structure	Autocorrelation	
	X	Y
Sharing Boundary (Rook)	0.73	0.76
Sharing Boundary (Bishop)	0.48	0.54
Sharing Boundary (Queen)	0.61	0.66
Distance Apart (1.5)	0.61	0.66
Distance Apart (2.25)	0.42	0.44
Distance Apart (3)	0.36	0.36

Table 12: The $bias$ and MSE of Wartenberg's Measure (W) and Lee's Measure (L) with Actual Values Using Different Types of Spatial Structures

Type of Spatial Structure	W			L		
	Actual	$bias$	MSE	Actual	$bias$	MSE
Sharing Boundary (Rook)	-0.75	0.15	0.02	-0.59	0.12	0.01
Sharing Boundary (Bishop)	-0.52	0.10	0.01	-0.31	0.06	0.00
Sharing Boundary (Queen)	-0.64	0.13	0.02	-0.41	0.08	0.01
Distance Apart (1.5)	-0.64	0.13	0.02	-0.41	0.08	0.01
Distance Apart (2.25)	-0.43	0.09	0.01	-0.19	0.04	0.00
Distance Apart (3)	-0.36	0.07	0.01	-0.14	0.03	0.00

Scenario 7: Sample Size ($4 \times 4 = 16$)

Figures 9a and 9b show two study areas with their assumed true means. The spatial structure is defined using the distance apart (1.5). Table 13 shows the autocorrelation values for both variables X and Y based on Moran's I statistic. Table 14 shows the actual values of Wartenberg and Lee measures and their $bias$ and MSE using different choices of spatial weights.

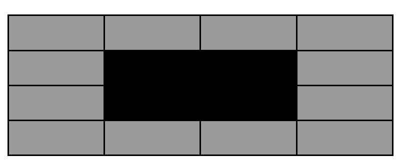
Figure 9: Proposed Area Divided into 16 Quadrates in a 4×4 Lattice

9a

3.6	3.6	3.6	3.6
3.6	1.2	1.2	3.6
3.6	1.2	1.2	3.6
3.6	3.6	3.6	3.6

9b

2.6	2.6	2.6	2.6
2.6	5.2	5.2	2.6
2.6	5.2	5.2	2.6
2.6	2.6	2.6	2.6



The numbers in the quadrates are the assumed true means for two different variables X and Y, where (a) is a gradient patch from low values (in the center) to high values (in the edges) for X variable, and (b) is the opposite direction of (a) for Y variable.

Table 13: Autocorrelation for Variables X and Y Based on Moran's I Using Different Types of Spatial Weights

Type of Spatial Weight	Autocorrelation X	Autocorrelation Y
$w_{ij} = 1$	0.07	0.07
$w_{ij} = 1/d_{ij}$	0.12	0.12
$w_{ij} = 1/d_{ij}^2$	0.16	0.16

Table 14: The $bias$ and MSE of Wartenberg's Measure (W) and Lee's Measure (L) with Actual Values Using Different Types of Spatial Weights

Type of Spatial Weight	W			L		
	Actual	$bias$	MSE	Actual	$bias$	MSE
$w_{ij} = 1$	-0.07	0.03	0.00	-0.08	0.04	0.00
$w_{ij} = 1/d_{ij}$	-0.12	0.05	0.01	-0.06	0.02	0.00
$w_{ij} = 1/d_{ij}^2$	-0.16	0.07	0.01	-0.05	0.02	0.00

Scenario 8: Sample Size ($6 \times 6 = 36$)

Figures 10a and 10b show two study areas with their assumed true means. The spatial structure is defined using the distance apart (1.5). Table 15 shows the autocorrelation values for both variables X and Y based on Moran's I statistic. Table 16 shows the actual values of Wartenberg and Lee measures and their $bias$ and MSE using different choices of spatial weights.

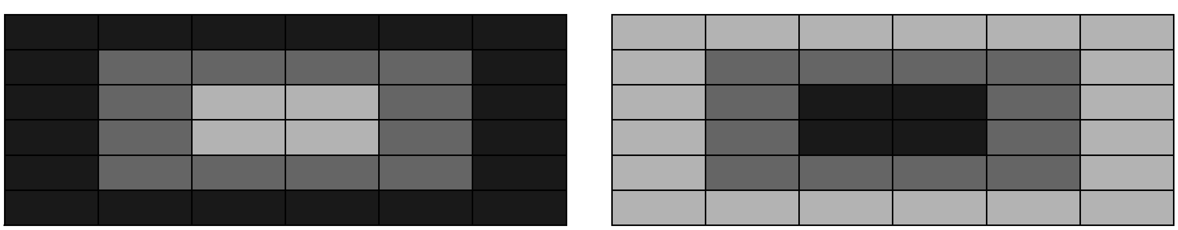
Figure 10: Proposed Area Divided into 36 Quadrates in a 6×6 Lattice

10a

5.4	5.4	5.4	5.4	5.4	5.4
5.4	3.6	3.6	3.6	3.6	5.4
5.4	3.6	1.2	1.2	3.6	5.4
5.4	3.6	1.2	1.2	3.6	5.4
5.4	3.6	3.6	3.6	3.6	5.4
5.4	5.4	5.4	5.4	5.4	5.4

10b

1.3	1.3	1.3	1.3	1.3	1.3
1.3	2.6	2.6	2.6	2.6	1.3
1.3	2.6	5.2	5.2	2.6	1.3
1.3	2.6	5.2	5.2	2.6	1.3
1.3	2.6	2.6	2.6	2.6	1.3
1.3	1.3	1.3	1.3	1.3	1.3



The numbers in the quadrates are the assumed true means for two different variables X and Y, where (a) is a gradient patch from low values (in the center) to high values (in the edges) for X variable, and (b) is the opposite direction of (a) for Y variable.

Table 15: Autocorrelation for Variables X and Y Based on Moran's I Using Different Spatial Weights

Type of Spatial Weight	Autocorrelation	
	X	Y
$w_{ij} = 1$	0.49	0.51
$w_{ij} = 1/d_{ij}$	0.52	0.53
$w_{ij} = 1/d_{ij}^2$	0.54	0.55

Table 16: The $bias$ and MSE of Wartenberg's Measure (W) and Lee's Measure (L) with Actual Values Using Different Types of Spatial Weights

Type of Spatial Weight	W			L		
	Actual	$bias$	MSE	Actual	$bias$	MSE
$w_{ij} = 1$	-0.50	0.18	0.03	-0.23	0.08	0.01
$w_{ij} = 1/d_{ij}$	-0.53	0.19	0.04	-0.19	0.07	0.00
$w_{ij} = 1/d_{ij}^2$	-0.55	0.20	0.04	-0.16	0.06	0.00

Scenario 9: Sample Size ($8 \times 8 = 64$)

Figures 11a and 11b show two study areas with their assumed true means. The spatial structure is defined using the distance apart (1.5). Table 17 shows the autocorrelation values for both variables X and Y based on Moran's I statistic. Table 18 shows the actual values of Wartenberg and Lee measures and their $bias$ and MSE using different choices of spatial weights.

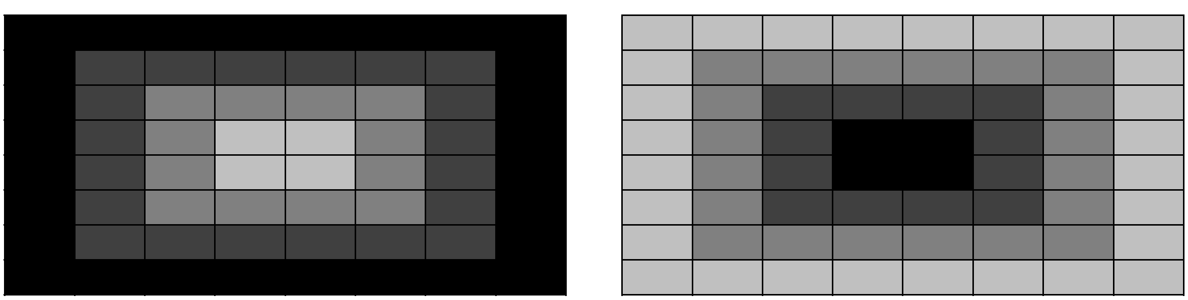
Figure 11: Proposed Area Divided into 64 Quadrates in a 8×8 Lattice

11a

7.9	7.9	7.9	7.9	7.9	7.9	7.9	7.9
7.9	5.4	5.4	5.4	5.4	5.4	5.4	7.9
7.9	5.4	3.6	3.6	3.6	3.6	5.4	7.9
7.9	5.4	3.6	1.2	1.2	3.6	5.4	7.9
7.9	5.4	3.6	1.2	1.2	3.6	5.4	7.9
7.9	5.4	3.6	3.6	3.6	3.6	5.4	7.9
7.9	5.4	5.4	5.4	5.4	5.4	5.4	7.9
7.9	7.9	7.9	7.9	7.9	7.9	7.9	7.9

11b

2.3	2.3	2.3	2.3	2.3	2.3	2.3	2.3
2.3	4.2	4.2	4.2	4.2	4.2	4.2	2.3
2.3	4.2	6.7	6.7	6.7	6.7	4.2	2.3
2.3	4.2	6.7	8.2	8.2	6.7	4.2	2.3
2.3	4.2	6.7	8.2	8.2	6.7	4.2	2.3
2.3	4.2	6.7	6.7	6.7	6.7	4.2	2.3
2.3	4.2	4.2	4.2	4.2	4.2	4.2	2.3
2.3	2.3	2.3	2.3	2.3	2.3	2.3	2.3



The numbers in the quadrates are the assumed true means for two different variables X and Y, where (a) is a gradient patch from low values (in the center) to high values (in the edges) for X variable, and (b) is the opposite direction of (a) for Y variable.

Table 17: Autocorrelation for Variables X and Y Based on Moran's I Using Different Spatial Weights

Type of Spatial Weight	Autocorrelation	
	X	Y
$w_{ij} = 1$	0.61	0.66
$w_{ij} = 1/d_{ij}$	0.63	0.68
$w_{ij} = 1/d_{ij}^2$	0.65	0.70

Table 18: The $bias$ and MSE of Wartenberg's Measure (W) and Lee's Measure (L) with Actual Values Using Different Types of Spatial Weights

Type of Spatial Weight	W			L		
	Actual	$bias$	MSE	Actual	$bias$	MSE
$w_{ij} = 1$	-0.64	0.13	0.02	-0.41	0.08	0.01
$w_{ij} = 1/d_{ij}$	-0.66	0.13	0.02	-0.33	0.07	0.00
$w_{ij} = 1/d_{ij}^2$	-0.68	0.13	0.02	-0.27	0.05	0.00

Conclusion

The results from these scenarios show that the Wartenberg and Lee measures differ slightly in terms of their sensitivity to different choices of spatial structures and spatial weights. Results show that Wartenberg's measure is somewhat more sensitive than Lee's measure to the different choices of spatial structures and spatial weights when the sample size is small; for the large sample sizes the results of both measures are approximately the same. Several techniques in statistics are sensitive - meaning they sometimes provide inaccurate results when a small sample size is used - because the information in a small sample is less than that of a large sample size.

Wartenberg's equation is vulnerable to a reverse in direction of association as stated by Lee. This reverse in direction was found in scenarios 1 and 2 as shown in the column of actual value of Wartenberg's measure in Tables 2 and 4 respectively.

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