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Non-Parametric Quantile Selection for Extreme Distributions

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The objective is to select the best non-parametric quantile estimation method for extreme distributions. This serves as a starting point for further research in quantile application such as in parameter estimation using LQ-moments method. Thirteen methods of non-parametric quantile estimation were applied on six types of extreme distributions and their efficiencies compared. Monte Carlo methods were used to generate the results, which showed that the method of Weighted Kernel estimator of Type 1 was more efficient than the other methods in many cases.

Keywords: Order statistics, sample quantiles, kernel quantile estimators, weighted kernel quantile estimators, HD quantile, weighted HD quantile, LQ-moments, IMSE.

Introduction

In model fitting, one of the key steps is finding the accurate estimates of parameters based on the data in-hand. Several well-known methods include the maximum-likelihood method (ML), method of moments (MOM) and Probability Weighted Moments (PWM). An extension of PWM, termed L-moments, was introduced by Sillitto (1951) for increased accuracy and ease of use of PWM-based analysis.

Mudholkar & Hutson (1998) introduced LQ-moments, an extension of L-moments that was found to be more robust. LQ-moments are constructed using a series of robust linear location measures in place of expectations of order statistics in the L-moments. The r-th LQ-moment, ξ_r of X is defined as:

$$\xi_r = r^{-1}\sum(-1)^k\binom{r-1}{k}\tau_{p,\alpha}(X_{r-k:r}), \quad r=1,2,\ldots \tag{1}$$

where $0 \le \alpha \le 1/2$, $0 \le p \le 1/2$, and

$$\begin{aligned}\tau_{p,\alpha}(X_{r-k:r}) &= pQ_{X_{r-k:r}}(\alpha)+(1-2p)Q_{X_{r-k:r}}(1/2) + pQ_{X_{r-k:r}}(1-\alpha) \\ &= pQ_X\left(B^{-1}_{r-k:r}(\alpha)\right)+(1-2p)Q_X\left(B^{-1}_{r-k:r}\left(\frac{1}{2}\right)\right) + pQ_X\left(B^{-1}_{r-k:r}(1-\alpha)\right)\end{aligned} \tag{2}$$

is the linear combination of symmetric quantiles of the order statistics $X_{r-k:r}$ with $Q_X(\cdot)=F_X^{-1}(\cdot)$ as the quantile function of the random variable X, and $B^{-1}_{r-k:r}(\alpha)$ denotes the corresponding α -th quantile of a beta random variable with parameters $r-k$ and $k+1$. From (2) it can be concluded that proper selection of quantile estimators is crucial to obtain the most accurate parameter estimation based on LQ-moments. As there are many non-parametric quantile estimation methods available, selection is based on statistical ground to propose the most efficient method in many cases.

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Methodology

The quantile function estimators

Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed random variables with common continuous distribution function (cdf) $F(x), x \in \Re$. Let $X_{1:n} \le X_{2:n} \le \ldots \le X_{n:n}$ denote the corresponding order statistics. The population quantile function, $Q(u)$ of a distribution is defined as:

$$Q(u) = \inf\{x : F(x) \ge u\}, \qquad 0 < u < 1. \quad (3)$$

A traditional estimator of Q(u) is the *u*-th sample quantile given by

$$SQ_u = X_{([nu]+1):n} \quad (4)$$

where $[nu]$ denotes the integral part of nu (David, 2003). However, this estimator suffers a drawback in efficiency, caused by the variability of individual order statistics (Huang, 2001). In their article on LQ-moments, Mudholkar & Hutson (1998) employed the linear interpolation-based quantile (LIQ) estimator, defined as,

$$\hat{Q}_X(u) = (1-\varepsilon)X_{[n'u]:n} + \varepsilon X_{[n'u]+1:n}, \quad (5)$$

where $\varepsilon = n'u - [n'u]$ and $n' = n+1$. This is the simplest estimator, and is available in most statistical software packages. It was used as the base for efficiency study in this research.

To overcome the drawback in efficiency of (4), many authors use *L* statistics to reduce the variability. A popular class of kernel quantile estimators has been applied for improving the efficiency of sample quantiles, using an appropriate weight function to average over the order statistics (Sheather & Marron, 1990). Parzen (1979) provided the formula

$$KQ_u = \sum_{i=1}^{n}\left(\int_{(i-1)/n}^{i/n} K_h(t-u)\,dt\right)X_{i:n}, \quad (6)$$

where *K* is a density function symmetric about 0, $h \to 0$ as $n \to \infty$ and $K_h(\bullet) = \left(\frac{1}{h}\right)K\left(\frac{\bullet}{h}\right)$.

Using the classical empirical distribution function S_n, given by

$$S_n(x) = \frac{1}{n}\sum_{i=1}^{n} I(X_i \le x), x \in \Re, \quad (7)$$

where I_A is the indicator function of set *A*, the following are various approximation forms of KQ_u which are often used for practical reasons:

$$KQ_{u.1} = \sum_{i=1}^{n}\left(n^{-1}K_h\left(\frac{i}{n} - u\right)\right)X_{i:n}, \quad (8)$$

$$KQ_{u.2} = \sum_{i=1}^{n}\left(n^{-1}K_h\left(\frac{i-\frac{1}{2}}{n} - u\right)\right)X_{i:n}, \quad (9)$$

$$KQ_{u.3} = \sum_{i=1}^{n}\left(n^{-1}K_h\left(\frac{i}{n+1} - u\right)\right)X_{i:n}, \quad (10)$$

$$KQ_{u.4} = \frac{\sum_{i=1}^{n}\left(K_h\left(\frac{i-\frac{1}{2}}{n} - u\right)\right)X_{i:n}}{\sum_{i=1}^{n}\left(K_h\left(\frac{i-\frac{1}{2}}{n} - u\right)\right)} \quad (11)$$

Huang & Brill (1995, 1996) introduced a level crossing empirical distribution function

$$F_n(x) = \sum_{i=1}^{n} I(X_{i:n} \le x)w_{i,n}, \quad (12)$$

where the data point weights are

$$w_{i,n} = \begin{cases} \frac{1}{2}\left(1 - \frac{n-2}{\sqrt{n(n-1)}}\right), & i = 1, n \\ \frac{1}{\sqrt{n(n-1)}}, & i = 2,3,\ldots,n-1 \end{cases} \quad (13)$$

From (12) and (13), they obtained the following level crossing u-th sample quantile to estimate $Q(u)$, namely,

$$SQ_{u(lc)} = X_{([b]+2)}, \quad (14)$$

where

$$b = \sqrt{n(n-1)}\left(u - \frac{1}{2}\left(1 - \frac{n-2}{\sqrt{n(n-1)}}\right)\right) \quad (15)$$

Huang & Brill (1999) then introduced the level crossing u-th sample kernel quantile given by,

$$WKQ_{u(lc)} = \sum_{i=1}^{n}\left(\int_{q_{i-1,n}}^{q_{i,n}} K_h(t-u)\,dt\right) X_{i:n}, \quad (16)$$

where $q_{i,n} = \sum_{j=1}^{i} w_{j,n}$ and $w_{i,n}$ is given in (13).

The approximation forms of $WKQ_{u(lc)}$ corresponding to (8)-(11) are as below:

$$WKQ_{u.1(lc)} = \sum_{i=1}^{n}\left(n^{-1}K_h\left(\sum_{j=1}^{i} w_{j,n} - u\right)\right) X_{i:n}, \quad (17)$$

$$WKQ_{u.2(lc)} = \sum_{i=1}^{n}\left(n^{-1}K_h\left(\sum_{j=1}^{i-1} w_{j,n} + \frac{1}{2}w_{i,n} - u\right)\right) X_{i:n}, \quad (18)$$

$$WKQ_{u.3(lc)} = \sum_{i=1}^{n}\left(n^{-1}K_h\left(\sum_{j=1}^{i} w_{j,n}\frac{n}{n+1} - u\right)\right) X_{i:n}, \quad (19)$$

$$WKQ_{u.4(lc)} = \frac{\sum_{i=1}^{n}\left(K_h\left(\sum_{j=1}^{i-1} w_{j,n} + \frac{1}{2}w_{i,n} - u\right)\right) X_{i:n}}{\sum_{i=1}^{n}\left(K_h\left(\sum_{j=1}^{i-1} w_{j,n} + \frac{1}{2}w_{i,n} - u\right)\right)}. \quad (20)$$

In the study, Huang & Brill investigated the relative efficiency of the u-th sample level crossing quantile, $SQ_{u(lc)}$ in (14) relative to the u-th sample quantile SQ_u in (4) and the relative efficiency of the level crossing quantile estimator $KQ_{u(lc)}$ in (16) relative to the ordinary kernel quantile estimator KQ_u in (6). From both theoretical and computational points of view, they showed that the proposed level crossing estimations were more efficient in many cases, especially for the tails of the distribution and for small sample sizes. Their simulation used the exponential and three types of generalized lambda distribution with small sample sizes (n=10 and n=20).

The selection of kernel or bandwidth of the kernel estimators has always been a sensitive problem. To overcome this, Harrell & Davis (1982) proposed an L-quantile estimator of $Q(u)$, defined by,

$$HD_u = \sum_{i=1}^{n}\left(\int_{\frac{(i-1)}{n}}^{\frac{i}{n}} \frac{1}{\beta(a,b)} u^{a-1}(1-u)^{b-1}\,du\right) X_{i:n}, \quad (21)$$

where $a = (n+1)u$, $b=(n+1)v$, $v=1-u$ and $\beta(a,b)$ is the beta function with parameters a and b.

Huang (2001) proposed a level-crossing HD quantile estimator based on (12) and (21) as follows:

$$WHD_u = \sum_{i=1}^{n}\left(\int_{q_{i-1,n}}^{q_{i,n}} \frac{1}{\beta\{a,b\}} u^{a-1}(1-u)^{b-1}\,dy\right) X_{i:n}, \quad (22)$$

where $q_{i,n} = \sum_{j=1}^{i} w_{j,n}$, $q_{0:n} = 0$ and $w_{i,n}$ is given in (13).

Similar to previous research, Huang investigated the relative efficiency of the level crossing quantile estimator $HD_{u(lc)}$ in (22) relative to the ordinary quantile estimator HD_u in (21). From both theoretical and computational points of view, the result proved that the proposed level crossing estimations are more efficient in many cases, especially for the tails of the distribution and for small sample sizes. In their simulation, the exponential and three types of generalized lambda distribution with small sample sizes (n=10 and 20) were used.

Thirteen quantile estimation methods are used: (4), (5), (8) - (11), (14), (17) - (20), (21) and (22). An efficiency study is conducted based on integrated mean square error (IMSE) to determine the most efficient quantile estimation methods for several extreme values distributions. LIQ is used as the base because it is the simplest, is easily available in most statistical packages and is most often used quantile estimation method. The relative efficiency results are compared; the method with the lowest IMSE relative efficiency was considered the best and was recommended.

Extreme values distributions

In this research, six common extreme-values distributions were investigated, namely, the Generalized Extreme Value (GEV), Generalized Pareto Distribution (GPD), Generalized Logistic Distribution (GLD), the three-parameter Lognormal (LN3) and Pearson (PE3) distributions and the five-parameter extreme events such as extreme rainfall and flood.

Table 1 provides the list of the extreme value distributions, their corresponding quantile functions and the associated parameters to be tested. The parameters are ε, the position parameter, α, the scale parameter and κ is the Wakeby distribution (WAK5). These six

Table 1: Extreme Value Distributions

Distribution	Quantile Function, Q(u)	Parameters				
		ε	α	κ		
1. Generalized Extreme Value (GEV)	$\varepsilon + \frac{\alpha}{\kappa}\left[1-(-\ln u)^{\kappa}\right]$	0	1	-0.3, -0.2, -0.1, 0.1, 0.2, 0.3		
2. Generalized Pareto Distribution (GPD)	$\varepsilon + \frac{\alpha}{\kappa}\left[1-(1-u)^{\kappa}\right]$	0	1	-0.3, -0.2, -0.1, 0.1, 0.2, 0.3		
3. Generalized Logistic Distribution (GLD)	$\varepsilon + \frac{\alpha}{\kappa}\left[1-\left\{\left(\frac{1-u}{u}\right)\right\}^{\kappa}\right]$	0	1	-0.1, -0.2, -0.3, -0.4, -0.5, -0.6		
4. The three-parameter Lognormal distribution (LN3)	$\varepsilon + \frac{\alpha}{\kappa}\left[1-e^{-\kappa Z}\right]$	0	1	0.2(0.2)1.2		
5. The three-parameter Pearson distribution (PE3)	$\frac{2}{\gamma}\left(1+\frac{\gamma z_u}{6}-\frac{\gamma^2}{36}\right)^3-\frac{2}{\gamma}$ (by Wilson-Hilferty transformation and Z_u is the *u*-th quantile of the standard normal distribution)	0	1	1, 2, 3, 4, 6, 8		
6. The five-parameter Wakeby distribution (WAK5)	$\varepsilon + \alpha\left[1-(1-u)^{\beta}\right] - \gamma\left[1-(1-u)^{-\delta}\right]$	0	1	β	γ	δ
				16	4	0.2
				7.5	5	0.12
				1	5	0.12
				16	10	0.04
				1	10	0.04
				2.5	10	0.02

distributions are commonly applied in regional frequency analysis to model many situations of shape parameter unless stated otherwise. The distributions are studied at various shape parameters, κ while fixing the position, ε and scale parameters, α at 0 and 1 respectively, except for Wakeby distribution. The parameters selected were based on previous studies (e.g. for Wakeby) the parameters were proposed by Landwher, et al. (1980). Ani & Aziz (2007) studied and compared the efficiency of (5), (17) and (22) quantile estimators based on this distribution. They performed simulation on GEV based on LQ-moments and the results showed that $WKQ_{u.1(lc)}$ (17) was the most efficient quantile estimator.

Simulation Study

Several Monte Carlo simulation experiments were conducted to determine the best quantile estimators corresponding to different extreme values distributions. The data with small sample sizes, *n*=10(5)30 were generated from respective distribution quantile functions at various values of *u* = 0.01, 0.25, 0.33, 0.50, 0.66, 0.75, 0.90 and replicated (*m*) 5,000 times each.

For the kernel and weighted kernel quantile estimators, the Gaussian Kernel was used $K(u)=(2\pi)^{-\frac{1}{2}}\exp\left(-\frac{1}{2}u^2\right)$ with an optimal bandwidth $h_{opt}=\left(\frac{uv}{n}\right)^{\frac{1}{2}}$ where *v=1-u*, as proposed by Sheather & Marron (1990).

The expected values obtained from the quantile estimators, $\hat{Q}_i(u)$ were compared with the distribution actual (population) *u*-th quantile value, $Q(u)$, that is bias

$$\text{Bias} = \frac{1}{m}\sum_{i=1}^{m}\left(\hat{Q}_i(u)-Q(u)\right).$$

From this value the mean square value was calculated

$$\text{MSE} = \frac{1}{m}\sum_{i=1}^{m}\left(\hat{Q}_i(u)-Q(u)\right)^2 \quad (4.2)$$

along with the integrated mean square errors (IMSE), which is defined as the sum of Mean Square Error across all defined *u* values. The IMSE from all other methods was divided by the IMSE from LIQ to gain the relative efficiency. The estimator which gave the lowest relative IMSE was selected as the best estimator.

The computational results comparing various quantile estimation methods for various distributions are shown in Tables 2-7 for the six extreme distributions respectively. Note that bold font indicates the smallest IMSE value; the most efficient at each respective group.

Results

Table 2 shows the relative efficiency values for six types of Generalized Extreme Value (GEV) distribution. The selection of best quantile estimation method changes when the shape parameter changes from negative to positive.

Table 2 also shows that when the shape parameter is negative, as in GEV types 1,2 and 3, the method suggested was the Weighted Kernel Quantile estimator of Type 1, $WKQ_{u.1(lc)}$, as in (17). However, when the shape parameter is positive, as in GEV types 4, 5 and 6, the most efficient method was the Kernel Quantile estimator Type 4 as in (11) for GEV types 5 and 6. The result is similar in the case of *n*=10 for GEV type 4, but for this type, the more efficient estimator was the Weighted Kernel Quantile estimator Type 4, $WKQ_{u.4(lc)}$, as in (20) followed by the Kernel Quantile estimator Type 4. Hence, we suggest that – in the case of GEV distribution – when analyzing data which is lower-bounded (κ<0), as in most hydrological data, the best estimator would be Weighted Kernel Quantile estimator Type 1, $WKQ_{u.1(lc)}$, and for data that is upper-bounded (κ>0), the Kernel Quantile estimator Type 4, $KQ_{u.4}$, would be the best choice.

The IMSE relative efficiency for six types of Generalized Pareto distribution (GPD) is shown in Table 3. Similar to the GEV case, the selection of best quantile estimation method changes when the shape parameter changes from negative to positive. From Table 3, in almost all cases, the best estimator was the Weighted Kernel Quantile estimator Type 1, $WKQ_{u.1(lc)}$,

Table 2: Generalized Extreme Value (GEV) Distribution

Parameters: Position, ε = 0; Scale, α = 1; Shape, κ = -0.3 (GEV1)

n	SQP1	SQP2	KQ1	KQ2	KQ3	KQ4	WKQ1	WKQ2	WKQ3	WKQ4	HDQ	WHDQ
10	1.430	0.764	0.378	0.487	0.546	0.635	**0.373**	0.481	0.534	0.466	0.838	0.703
15	1.183	1.176	0.338	0.453	0.529	0.588	**0.333**	0.446	0.528	0.431	0.894	0.695
20	1.102	**0.158**	0.171	0.245	0.294	0.316	0.168	0.236	0.305	0.229	0.530	0.389
25	0.200	0.200	0.186	0.273	0.354	0.346	**0.182**	0.257	0.367	0.252	0.602	0.418
30	0.422	0.361	0.279	0.395	0.526	0.485	**0.270**	0.366	0.547	0.363	0.770	0.529

Parameters: Position, ε = 0; Scale, α = 1; Shape, κ = -0.2 (GEV2)

n	SQP1	SQP2	KQ1	KQ2	KQ3	KQ4	WKQ1	WKQ2	WKQ3	WKQ4	HDQ	WHDQ
10	1.286	0.810	0.475	0.559	0.667	0.658	**0.457**	0.538	0.635	0.524	0.811	0.705
15	1.133	1.121	0.455	0.542	0.653	0.636	**0.437**	0.524	0.628	0.508	0.885	0.714
20	1.100	0.289	0.262	0.320	0.379	0.384	**0.255**	0.312	0.377	0.303	0.564	0.439
25	0.377	0.377	0.308	0.375	0.455	0.444	**0.299**	0.362	0.460	0.355	0.647	0.489
30	0.586	0.514	0.366	0.448	0.550	0.527	**0.355**	0.426	0.564	0.421	0.743	0.558

Parameters: Position, ε = 0; Scale, α = 1; Shape, κ = -0.1 (GEV3)

n	SQP1	SQP2	KQ1	KQ2	KQ3	KQ4	WKQ1	WKQ2	WKQ3	WKQ4	HDQ	WHDQ
10	1.055	0.700	0.522	0.570	0.741	0.588	**0.486**	0.528	0.688	0.516	0.686	0.616
15	1.143	1.128	0.595	0.650	0.799	0.692	**0.562**	0.621	0.737	0.599	0.874	0.746
20	1.101	0.465	0.394	0.432	0.515	0.469	**0.377**	0.418	0.486	0.404	0.608	0.504
25	0.536	0.536	0.434	0.478	0.562	0.525	**0.418**	0.463	0.545	0.451	0.678	0.553
30	0.720	0.643	0.493	0.541	0.633	0.595	**0.478**	0.524	0.625	0.514	0.753	0.613

Parameters: Position, ε = 0; Scale, α = 1; Shape, κ = 0.1 (GEV4)

n	SQP1	SQP2	KQ1	KQ2	KQ3	KQ4	WKQ1	WKQ2	WKQ3	WKQ4	HDQ	WHDQ
10	0.797	0.691	0.659	0.645	0.889	**0.560**	0.590	0.587	0.793	**0.560**	0.584	0.567
15	1.067	1.051	0.879	0.865	1.112	0.767	0.804	0.820	0.958	**0.758**	0.820	0.778
20	1.074	0.806	0.692	0.686	0.833	0.631	0.645	0.664	0.723	**0.615**	0.688	0.640
25	0.836	0.836	0.724	0.716	0.825	0.681	0.690	0.707	0.736	**0.663**	0.740	0.688
30	0.915	0.876	0.724	0.718	0.805	0.705	0.699	0.715	0.742	**0.679**	0.770	0.711

Parameters: Position, ε = 0; Scale, α = 1; Shape, κ =0. 2 (GEV5)

n	SQP1	SQP2	KQ1	KQ2	KQ3	KQ4	WKQ1	WKQ2	WKQ3	WKQ4	HDQ	WHDQ
10	0.734	0.690	0.710	0.666	0.921	**0.555**	0.628	0.615	0.812	0.575	0.562	0.558
15	1.033	1.022	0.957	0.915	1.182	**0.779**	0.867	0.886	1.005	0.798	0.800	0.781
20	1.041	0.931	0.815	0.785	0.954	**0.694**	0.757	0.780	0.814	0.703	0.721	0.696
25	0.924	0.924	0.916	0.794	0.862	**0.734**	0.790	0.810	0.812	0.741	0.764	0.736
30	0.975	0.949	0.821	0.795	0.888	**0.749**	0.790	0.804	0.796	0.750	0.782	0.751

Parameters: Position, ε = 0; Scale, α = 1; Shape, κ = 0.3 (GEV6)

n	SQP1	SQP2	KQ1	KQ2	KQ3	KQ4	WKQ1	WKQ2	WKQ3	WKQ4	HDQ	WHDQ
10	0.702	0.704	0.757	0.682	0.938	**0.564**	0.664	0.649	0.821	0.595	0.565	0.566
15	1.004	1.006	1.005	0.930	1.198	**0.779**	0.905	0.930	1.013	0.815	0.788	0.780
20	0.979	0.969	0.868	0.817	0.993	**0.712**	0.802	0.834	0.840	0.737	0.729	0.714
25	0.951	0.951	0.880	0.839	0.967	**0.754**	0.832	0.860	0.836	0.777	0.774	0.756
30	0.979	0.973	0.863	0.828	0.923	**0.766**	0.828	0.848	0.817	0.783	0.788	0.768

Table 3: Generalized Pareto Distribution (GPD)

Parameters: Position, ε = 0; Scale, α = 1; Shape, κ = -0.3 (GPD1)

n	SQP1	SQP2	KQ1	KQ2	KQ3	KQ4	WKQ1	WKQ2	WKQ3	WKQ4	HDQ	WHDQ
10	1.437	0.719	0.331	0.457	0.521	0.603	**0.325**	0.438	0.508	0.429	0.819	0.677
15	1.215	1.209	0.305	0.434	0.506	0.576	**0.301**	0.421	0.505	0.409	0.903	0.695
20	1.103	**0.145**	0.165	0.250	0.316	0.325	0.162	0.236	0.323	0.232	0.556	0.397
25	0.169	0.169	0.166	0.257	0.339	0.328	**0.163**	0.239	0.351	0.236	0.594	0.406
30	0.390	0.331	0.257	0.378	0.513	0.470	**0.249**	0.347	0.534	0.346	0.771	0.519

Parameters: Position, ε = 0; Scale, α = 1; Shape, κ = -0.2 (GPD2)

n	SQP1	SQP2	KQ1	KQ2	KQ3	KQ4	WKQ1	WKQ2	WKQ3	WKQ4	HDQ	WHDQ
10	1.250	0.721	0.404	0.510	0.636	0.604	**0.386**	0.468	0.603	0.469	0.764	0.653
15	1.241	1.230	0.399	0.510	0.612	0.622	**0.385**	0.480	0.587	0.475	0.893	0.717
20	1.103	0.273	0.248	0.315	0.386	0.383	**0.239**	0.298	0.381	0.296	0.573	0.437
25	0.337	0.337	0.265	0.341	0.422	0.416	**0.257**	0.322	0.429	0.321	0.632	0.467
30	0.547	0.462	0.339	0.433	0.549	0.519	**0.327**	0.403	0.563	0.404	0.755	0.551

Parameters: Position, ε = 0; Scale, α = 1; Shape, κ = -0.1 (GPD3)

n	SQP1	SQP2	KQ1	KQ2	KQ3	KQ4	WKQ1	WKQ2	WKQ3	WKQ4	HDQ	WHDQ
10	1.049	0.675	0.458	0.551	0.749	0.570	**0.425**	0.470	0.689	0.484	0.677	0.598
15	1.154	1.136	0.522	0.619	0.786	0.674	**0.492**	0.553	0.718	0.559	0.884	0.735
20	1.102	0.391	0.333	0.390	0.476	0.435	**0.317**	0.358	0.448	0.360	0.593	0.477
25	0.465	0.465	0.370	0.427	0.517	0.482	**0.356**	0.397	0.500	0.400	0.658	0.517
30	0.660	0.566	0.420	0.490	0.593	0.557	**0.405**	0.455	0.588	0.459	0.742	0.580

Parameters: Position, ε = 0; Scale, α = 1; Shape, κ = 0.1 (GPD4)

n	SQP1	SQP2	KQ1	KQ2	KQ3	KQ4	WKQ1	WKQ2	WKQ3	WKQ4	HDQ	WHDQ
10	0.760	0.603	0.498	0.593	0.897	0.499	0.445	**0.442**	0.788	0.475	0.538	0.507
15	1.055	1.032	0.734	0.835	1.161	0.727	0.666	**0.651**	0.964	0.683	0.812	0.744
20	1.106	0.708	0.580	0.634	0.814	0.577	0.539	**0.523**	0.678	0.540	0.655	0.591
25	0.754	0.754	0.623	0.669	0.818	0.637	0.588	**0.576**	0.702	0.590	0.721	0.645
30	0.891	0.812	0.645	0.681	0.801	0.672	0.618	**0.607**	0.715	0.618	0.757	0.676

Parameters: Position, ε = 0; Scale, α = 1; Shape, κ = 0.2 (GPD5)

n	SQP1	SQP2	KQ1	KQ2	KQ3	KQ4	WKQ1	WKQ2	WKQ3	WKQ4	HDQ	WHDQ
10	0.656	0.571	0.497	0.601	0.948	0.462	0.436	**0.417**	0.815	0.458	0.478	0.464
15	0.984	0.958	0.800	0.918	1.326	0.727	0.715	**0.669**	1.062	0.714	0.762	0.728
20	1.109	0.864	0.716	0.787	1.047	0.663	0.659	**0.618**	0.829	0.644	0.704	0.665
25	0.882	0.882	0.770	0.818	1.014	0.714	0.724	**0.675**	0.821	0.695	0.755	0.715
30	0.975	0.907	0.762	0.795	0.943	0.726	0.728	**0.685**	0.794	0.700	0.772	0.727

Parameters: Position, ε = 0; Scale, α = 1; Shape, κ = 0.3 (GPD6)

n	SQP1	SQP2	KQ1	KQ2	KQ3	KQ4	WKQ1	WKQ2	WKQ3	WKQ4	HDQ	WHDQ
10	0.584	0.553	0.503	0.615	0.992	0.441	0.437	**0.405**	0.841	0.454	0.441	0.441
15	0.936	0.912	0.842	0.986	1.472	0.722	0.746	**0.677**	1.140	0.733	0.724	0.717
20	1.114	1.036	0.865	0.962	1.315	0.738	0.788	**0.710**	0.998	0.749	0.743	0.734
25	0.993	0.993	0.897	0.962	1.223	0.780	0.838	**0.759**	0.942	0.786	0.787	0.777
30	1.045	1.005	0.868	0.908	1.093	0.778	0.827	**0.758**	0.876	0.777	0.789	0.777

as in (2.15) and in all cases for positive shape parameter, the most efficient estimator was Weighted Kernel Quantile estimator Type 2 $WKQ_{u.2(lc)}$, as in (18). Hence, it is suggested that in the case of GPD, when analyzing data which is lower-bounded (κ<0), as in most hydrological data, the best estimator would be the Weighted Kernel Quantile estimator Type 1, $WKQ_{u.1(lc)}$, and for data that is upper-bounded (κ>0), the Weighted Kernel Quantile estimator Type 2 $WKQ_{u.2(lc)}$, would be the best choice.

For Generalized Logistic Distribution (GLD), the IMSE relative efficiency values point to several selections, as show in Table 4. Compared to the other two previous distributions, no one obvious estimator can be considered the most efficient for all types of GLD included in this study. The frequently quoted choices are Weighted Kernel Quantile estimator Type 1, ($WKQ_{u.1(lc)}$), SQP1 and 2, and Kernel Quantile estimator Type 4, ($KQ_{u.4}$). However, using Weighted Kernel Quantile estimator Type 1, ($WKQ_{u.1(lc)}$), is recommended since this estimator is frequently quoted as the most efficient compared to the others, and for ease of further analysis in its future application.

The IMSE relative efficiency values for Lognormal Type 3 (LN3) distributions are displayed in Table 5. This table shows that, for LN3 Types 1 and 2, the suggested estimator is the Weighted Kernel Quantile estimator Type 4, $WKQ_{u.4(lc)}$, for LN3 types 3, 4 and 5 it was the Weighted Kernel Quantile estimator Type 1, $WKQ_{u.1(lc)}$, and there was no obvious choice for LN3 Type 6. Hence, $WKQ_{u.1(lc)}$ is recommended for this distribution because it is the best estimator for 3 types of LN3 (LN3 Types 3, 4 and 5) in this study, however, further analysis of the IMSE relative efficiency values for LN3 Types 2 and 6 showed that this method gave the second smallest IMSE.

Table 6 shows the IMSE relative efficiency values for the Pearson Type 3 (PE3) distribution. In general, for PE3 Types 1 and 2, the recommended estimator was $WKQ_{u.4(lc)}$ for PE3 Types 3 and 4, was $WKQ_{u.1(lc)}$ for PE3 Type 5, and for Type 6 was $WKQ_{u.2(lc)}$. Because only one type of estimator from the thirteen choices available needs to be chosen, although the simulation results showed three different methods, $WKQ_{u.1(lc)}$ is recommended to use in other distributions. Another possible alternative would be to use $WKQ_{u.4(lc)}$ and $WKQ_{u.2(lc)}$ as quantile estimators.

Finally, Table 7 shows the IMSE relative efficiency values for the Wakeby Type 5 (WAK5) distribution. Although the most efficient quantile estimator for WAK5 Types 1 and 2 was $WKQ_{u.1(lc)}$, the $WKQ_{u.2(lc)}$ is often recommended for WAK5 Types 3, 4, 5, and 6. Hence, for WAK5, $WKQ_{u.2(lc)}$ is recommended as the quantile estimation method, with $WKQ_{u.1(lc)}$ as another alternative.

Conclusion

Table 8 summarizes the two most efficient quantile estimation methods (in sequence) with respect to the six extreme distributions.

Table 8: The Top Two Most Efficient Quantile Estimation Methods

Distribution	Most Efficient	2nd Most Efficient
GEV	$WKQ_{u.1(lc)}$	$WKQ_{u.4(lc)}$
GPD	$WKQ_{u.1(lc)}$	$WKQ_{u.2(lc)}$
GLD	$WKQ_{u.1(lc)}$	$WKQ_{u.4(lc)}$
LN3	$WKQ_{u.1(lc)}$	$KQ_{u.1(lc)}$
PE3	$WKQ_{u.1(lc)}$	$WKQ_{u.4(lc)}$
WAK5	$WKQ_{u.2(lc)}$	$WKQ_{u.1(lc)}$

The IMSE relative efficiency of level crossing estimators was compared to the ordinary quantile estimator and the number of times the result showed that the level crossing estimators are better than the ordinary quantile

Table 4: Generalized Logistic Distribution (GLD)

Parameters: Position, ε = 0; Scale, α = 1; Shape, κ =-0.1 (GLD1)

n	SQP1	SQP2	KQ1	KQ2	KQ3	KQ4	WKQ1	WKQ2	WKQ3	WKQ4	HDQ	WHDQ
10	0.852	0.772	0.882	0.784	0.939	**0.667**	0.808	0.815	0.869	0.692	0.676	0.674
15	1.064	1.073	1.113	1.011	1.128	**0.856**	1.047	1.078	1.039	0.899	0.877	0.870
20	1.155	1.084	0.977	0.897	0.945	**0.762**	0.943	0.970	0.879	0.815	0.768	0.775
25	1.088	1.088	1.072	0.991	0.995	0.853	1.058	1.076	0.940	0.921	**0.846**	0.870
30	1.117	1.210	1.138	1.051	1.016	0.912	1.143	1.150	0.974	0.997	**0.896**	0.937

Parameters: Position, ε = 0; Scale, α = 1; Shape, κ =-0.2 (GLD2)

n	SQP1	SQP2	KQ1	KQ2	KQ3	KQ4	WKQ1	WKQ2	WKQ3	WKQ4	HDQ	WHDQ
10	1.027	0.797	0.753	0.720	0.839	0.668	0.702	0.722	0.790	**0.647**	0.728	0.692
15	1.109	1.114	0.893	0.856	0.953	0.778	0.849	0.869	0.892	**0.773**	0.876	0.819
20	1.125	0.800	0.718	0.684	0.731	**0.616**	0.697	0.703	0.689	0.628	0.691	0.646
25	0.843	0.843	0.823	0.778	0.805	**0.701**	0.812	0.805	0.766	0.728	0.772	0.732
30	0.944	1.020	0.937	0.876	0.873	**0.795**	0.941	0.918	0.844	0.838	0.853	0.828

Parameters: Position, ε = 0; Scale, α = 1; Shape, κ =-0.3 (GLD3)

n	SQP1	SQP2	KQ1	KQ2	KQ3	KQ4	WKQ1	WKQ2	WKQ3	WKQ4	HDQ	WHDQ
10	1.150	0.672	0.570	0.594	0.681	0.599	**0.541**	0.577	0.653	0.541	0.716	0.644
15	1.132	1.135	0.644	0.676	0.754	0.683	0.621	0.663	0.723	**0.620**	0.884	0.760
20	1.105	0.506	0.461	0.476	0.523	0.472	0.450	0.470	0.506	**0.441**	0.616	0.521
25	0.554	0.554	0.536	0.543	0.594	0.529	0.530	0.538	0.577	**0.508**	0.688	0.577
30	0.688	0.738	0.651	0.652	0.704	0.636	0.651	0.652	0.691	**0.620**	0.813	0.685

Parameters: Position, ε = 0; Scale, αa = 1; Shape, κ =-0.4(GLD4)

n	SQP1	SQP2	KQ1	KQ2	KQ3	KQ4	WKQ1	WKQ2	WKQ3	WKQ4	HDQ	WHDQ
10	1.468	0.893	0.479	0.579	0.609	0.694	**0.464**	0.559	0.595	0.533	0.894	0.769
15	1.130	1.131	0.378	0.485	0.559	0.565	**0.369**	0.459	0.550	0.439	0.887	0.687
20	1.104	**0.200**	0.213	0.271	0.320	0.309	0.210	0.256	0.318	0.246	0.517	0.388
25	0.196	**0.196**	0.224	0.294	0.372	0.329	0.221	0.273	0.370	0.265	0.591	0.415
30	**0.329**	0.341	0.343	0.428	0.547	0.465	0.342	0.401	0.547	0.391	0.804	0.547

Parameters: Position, ε = 0; Scale, α = 1; Shape, κ =-0.5 (GLD5)

n	SQP1	SQP2	KQ1	KQ2	KQ3	KQ4	WKQ1	WKQ2	WKQ3	WKQ4	HDQ	WHDQ
10	1.616	0.891	0.374	0.513	0.510	0.683	**0.370**	0.499	0.507	0.477	0.931	0.777
15	1.297	1.297	0.248	0.390	0.436	0.514	**0.246**	0.364	0.436	0.348	0.872	0.666
20	1.105	**0.060**	0.121	0.217	0.293	0.274	0.120	0.194	0.295	0.188	0.538	0.362
25	0.163	**0.163**	0.196	0.288	0.387	0.337	0.194	0.262	0.391	0.257	0.612	0.411
30	**0.299**	0.306	0.338	0.532	0.831	0.590	0.335	0.468	0.834	0.465	0.907	0.551

Parameters: Position, ε = 0; Scale, α = 1; Shape, κ =-0.6 (GLD6)

n	SQP1	SQP2	KQ1	KQ2	KQ3	KQ4	WKQ1	WKQ2	WKQ3	WKQ4	HDQ	WHDQ
10	1.513	0.510	0.198	0.334	0.340	0.474	**0.197**	0.316	0.340	0.302	0.740	0.583
15	1.257	1.257	0.208	0.362	0.415	0.491	**0.207**	0.333	0.417	0.319	0.864	0.647
20	1.106	**0.065**	0.116	0.205	0.272	0.262	0.115	0.184	0.275	0.179	0.514	0.351
25	0.074	**0.074**	0.119	0.218	0.312	0.271	0.118	0.192	0.316	0.188	0.570	0.366
30	0.126	**0.124**	0.187	0.344	0.536	0.408	0.185	0.297	0.540	0.294	0.831	0.488

Table 5: Log-Normal Type 3 Distribution (LN3)

Parameters: Position, ε = 0; Scale, α = 1; Shape, κ =-0.2 (LN3_1)

n	SQP1	SQP2	KQ1	KQ2	KQ3	KQ4	WKQ1	WKQ2	WKQ3	WKQ4	HDQ	WHDQ
10	0.846	0.730	0.761	0.658	0.876	0.591	0.674	0.692	0.783	**0.588**	0.619	0.599
15	1.067	1.055	0.934	0.842	1.054	0.764	0.845	0.907	0.929	**0.752**	0.826	0.779
20	1.077	0.801	0.723	0.669	0.795	0.627	0.670	0.724	0.710	**0.611**	0.686	0.637
25	0.796	0.796	0.719	0.683	0.781	0.663	0.681	0.735	0.718	**0.639**	0.733	0.674
30	0.896	0.844	0.716	0.693	0.774	0.690	0.689	0.738	0.730	**0.659**	0.764	0.697

Parameters: Position, ε = 0; Scale, α = 1; Shape, κ =-0.4 (LN3_2)

n	SQP1	SQP2	KQ1	KQ2	KQ3	KQ4	WKQ1	WKQ2	WKQ3	WKQ4	HDQ	WHDQ
10	0.980	0.729	0.641	0.614	0.794	0.598	0.585	0.618	0.729	**0.555**	0.664	0.617
15	1.106	1.092	0.762	0.745	0.916	0.732	0.705	0.758	0.832	**0.676**	0.851	0.763
20	1.100	0.602	0.527	0.525	0.620	0.531	0.498	0.538	0.573	**0.485**	0.638	0.556
25	0.641	0.641	0.544	0.553	0.638	0.577	0.521	0.568	0.608	**0.521**	0.697	0.598
30	0.786	0.713	0.570	0.590	0.676	0.625	**0.552**	0.598	0.658	0.560	0.752	0.640

Parameters: Position, ε = 0; Scale, α = 1; Shape, κ =-0.6 (LN3_3)

n	SQP1	SQP2	KQ1	KQ2	KQ3	KQ4	WKQ1	WKQ2	WKQ3	WKQ4	HDQ	WHDQ
10	1.163	0.764	0.545	0.577	0.709	0.631	**0.515**	0.573	0.670	0.537	0.748	0.665
15	1.141	1.128	0.568	0.613	0.732	0.682	**0.542**	0.615	0.695	0.576	0.883	0.741
20	1.103	0.446	0.384	0.418	0.493	0.465	**0.368**	0.416	0.477	0.394	0.611	0.500
25	0.509	0.509	0.414	0.457	0.540	0.509	**0.400**	0.449	0.531	0.432	0.671	0.540
30	0.693	0.609	0.461	0.513	0.604	0.577	**0.448**	0.502	0.608	0.487	0.748	0.598

Parameters: Position, ε = 0; Scale, α = 1; Shape, κ =-0.8 (LN3_4)

n	SQP1	SQP2	KQ1	KQ2	KQ3	KQ4	WKQ1	WKQ2	WKQ3	WKQ4	HDQ	WHDQ
10	1.313	0.763	0.457	0.532	0.625	0.640	**0.444**	0.527	0.604	0.506	0.804	0.688
15	1.193	1.182	0.439	0.523	0.607	0.635	**0.427**	0.520	0.596	0.498	0.891	0.719
20	1.105	0.284	0.303	0.316	0.414	0.377	**0.274**	0.301	0.394	0.294	0.559	0.431
25	0.372	0.372	0.291	0.359	0.436	0.433	**0.283**	0.348	0.447	0.340	0.639	0.479
30	0.590	0.505	0.364	0.447	0.552	0.528	**0.352**	0.425	0.569	0.419	0.745	0.557

Parameters: Position, ε = 0; Scale, α = 1; Shape, κ =-1.0 (LN3_5)

n	SQP1	SQP2	KQ1	KQ2	KQ3	KQ4	WKQ1	WKQ2	WKQ3	WKQ4	HDQ	WHDQ
10	1.502	0.859	0.404	0.521	0.561	0.692	**0.402**	0.522	0.555	0.502	0.906	0.761
15	1.208	1.200	0.338	0.451	0.514	0.592	**0.335**	0.449	0.519	0.432	0.897	0.699
20	1.102	0.206	0.203	0.274	0.326	0.350	**0.200**	0.268	0.337	0.260	0.552	0.413
25	0.287	0.287	0.231	0.313	0.395	0.389	**0.225**	0.297	0.409	0.292	0.623	0.446
30	0.444	0.376	0.275	0.387	0.510	0.474	**0.266**	0.357	0.531	0.354	0.755	0.521

Parameters: Position, ε = 0; Scale, α = 1; Shape, κ =-1.2 (LN3_6)

n	SQP1	SQP2	KQ1	KQ2	KQ3	KQ4	WKQ1	WKQ2	WKQ3	WKQ4	HDQ	WHDQ
10	1.600	0.878	**0.363**	0.504	0.524	0.706	0.367	0.508	0.526	0.488	0.948	0.783
15	1.237	1.232	**0.265**	0.401	0.451	0.552	0.266	0.393	0.462	0.378	0.893	0.682
20	1.104	**0.123**	0.145	0.225	0.278	0.300	0.143	0.215	0.289	0.209	0.522	0.376
25	0.196	0.196	0.175	0.271	0.356	0.350	**0.171**	0.254	0.373	0.249	0.610	0.419
30	0.347	0.294	0.233	0.364	0.503	0.454	**0.226**	0.331	0.525	0.328	0.772	0.510

Table 6: Pearson Type 3 Distribution (PE3)

Parameters: Position, ε = 0; Scale, α = 1; Shape, κ = 1 (PE3_1)

n	SQP1	SQP2	KQ1	KQ2	KQ3	KQ4	WKQ1	WKQ2	WKQ3	WKQ4	HDQ	WHDQ
10	0.916	0.741	0.747	0.647	0.844	0.599	0.668	0.697	0.763	**0.579**	0.642	0.611
15	1.086	1.072	0.873	0.788	0.973	0.745	0.797	0.862	0.874	**0.710**	0.833	0.768
20	1.103	0.727	0.659	0.611	0.720	0.592	0.615	0.667	0.658	**0.561**	0.669	0.607
25	0.735	0.735	0.662	0.632	0.723	0.630	0.629	0.682	0.678	**0.591**	0.718	0.644
30	0.863	0.808	0.683	0.665	0.748	0.672	0.658	0.705	0.716	**0.628**	0.763	0.681

Parameters: Position, ε = 0; Scale, α = 1; Shape, κ = 2 (PE3_2)

n	SQP1	SQP2	KQ1	KQ2	KQ3	KQ4	WKQ1	WKQ2	WKQ3	WKQ4	HDQ	WHDQ
10	1.119	0.788	0.634	0.617	0.747	0.654	0.599	0.646	0.707	**0.580**	0.748	0.678
15	1.120	1.099	0.673	0.672	0.794	0.715	0.639	0.697	0.752	**0.630**	0.873	0.755
20	1.106	0.535	0.464	0.469	0.543	0.503	0.445	0.482	0.522	**0.443**	0.625	0.529
25	0.582	0.582	0.474	0.493	0.565	0.542	**0.460**	0.503	0.560	0.470	0.681	0.566
30	0.754	0.667	0.510	0.541	0.628	0.593	**0.495**	0.539	0.626	0.511	0.738	0.608

Parameters: Position, ε = 0; Scale, α = 1; Shape, κ = 3 (PE3_3)

n	SQP1	SQP2	KQ1	KQ2	KQ3	KQ4	WKQ1	WKQ2	WKQ3	WKQ4	HDQ	WHDQ
10	1.286	0.804	0.539	0.576	0.648	0.685	**0.534**	0.601	0.637	0.568	0.826	0.721
15	1.160	1.141	0.530	0.582	0.660	0.684	**0.520**	0.592	0.653	0.565	0.896	0.743
20	1.099	0.383	0.321	0.362	0.414	0.431	**0.315**	0.364	0.419	0.351	0.589	0.470
25	0.473	0.473	0.367	0.418	0.491	0.487	**0.359**	0.409	0.500	0.400	0.661	0.518
30	0.666	0.571	0.406	0.471	0.563	0.546	**0.396**	0.452	0.577	0.445	0.733	0.570

Parameters: Position, ε = 0; Scale, α = 1; Shape, κ = 4 (PE3_4)

n	SQP1	SQP2	KQ1	KQ2	KQ3	KQ4	WKQ1	WKQ2	WKQ3	WKQ4	HDQ	WHDQ
10	1.398	0.819	**0.463**	0.541	0.575	0.698	0.469	0.556	0.577	0.545	0.883	0.750
15	1.181	1.167	**0.420**	0.502	0.555	0.644	0.421	0.508	0.567	0.499	0.904	0.721
20	1.062	0.285	0.251	0.304	0.351	0.382	**0.248**	0.299	0.363	0.297	0.564	0.433
25	0.396	0.396	0.298	0.364	0.436	0.445	**0.293**	0.348	0.452	0.348	0.645	0.485
30	0.596	0.509	0.358	0.439	0.542	0.524	**0.349**	0.409	0.559	0.412	0.741	0.552

Parameters: Position, ε = 0; Scale, α = 1; Shape, κ = 6 (PE3_5)

n	SQP1	SQP2	KQ1	KQ2	KQ3	KQ4	WKQ1	WKQ2	WKQ3	WKQ4	HDQ	WHDQ
10	1.129	0.751	0.415	0.463	0.477	0.650	0.417	**0.400**	0.479	0.529	0.888	0.728
15	1.193	1.191	0.387	0.428	0.476	0.585	0.383	**0.366**	0.478	0.452	0.898	0.693
20	0.631	**0.192**	0.247	0.264	0.316	0.346	0.237	0.218	0.314	0.266	0.556	0.409
25	0.275	0.275	0.293	0.304	0.387	0.388	0.273	**0.247**	0.378	0.296	0.626	0.444
30	0.433	0.391	0.383	0.383	0.512	0.474	0.349	**0.302**	0.490	0.363	0.748	0.517

Parameters: Position, ε = 0; Scale, α = 1; Shape, κ = 8 (PE3_6)

n	SQP1	SQP2	KQ1	KQ2	KQ3	KQ4	WKQ1	WKQ2	WKQ3	WKQ4	HDQ	WHDQ
10	0.870	0.802	0.568	0.515	0.566	0.674	0.532	**0.442**	0.540	0.614	0.875	0.735
15	1.184	1.184	0.530	0.481	0.548	0.617	0.501	**0.413**	0.523	0.533	0.894	0.708
20	0.357	0.257	0.335	0.296	0.352	0.372	0.312	**0.253**	0.331	0.315	0.562	0.425
25	0.337	0.337	0.386	0.333	0.412	0.414	0.355	**0.273**	0.382	0.347	0.632	0.462
30	0.461	0.455	0.509	0.426	0.545	0.513	0.458	**0.351**	0.495	0.436	0.751	0.543

Table 7: Wakeby 5-parameter Distribution (WAK5)

Parameters: Position, ε = 0; Scale, α = 1 and γ = 4; Shape, β = 16 and δ = 0.20 (WAK5_1)

n	SQP1	SQP2	KQ1	KQ2	KQ3	KQ4	WKQ1	WKQ2	WKQ3	WKQ4	HDQ	WHDQ
10	1.097	0.631	0.375	0.490	0.692	0.532	**0.342**	0.407	0.622	0.422	0.671	0.577
15	1.157	1.150	0.416	0.543	0.709	0.626	**0.392**	0.476	0.640	0.491	0.895	0.716
20	1.107	0.312	0.266	0.337	0.424	0.395	**0.254**	0.303	0.395	0.315	0.576	0.447
25	0.369	0.369	0.291	0.366	0.455	0.433	**0.281**	0.333	0.441	0.349	0.639	0.482
30	0.555	0.500	0.354	0.444	0.549	0.526	**0.345**	0.406	0.549	0.426	0.755	0.562

Parameters: Position, ε = 0; Scale, α = 1 and γ = 5; Shape, β = 7.5 and δ = 0.12 (WAK5_2)

n	SQP1	SQP2	KQ1	KQ2	KQ3	KQ4	WKQ1	WKQ2	WKQ3	WKQ4	HDQ	WHDQ
10	0.868	0.587	0.394	0.540	0.816	0.503	**0.353**	0.372	0.711	0.452	0.577	0.526
15	1.111	1.108	0.563	0.738	1.003	0.721	**0.521**	0.538	0.840	0.653	0.878	0.770
20	1.159	0.555	0.398	0.500	0.618	0.516	**0.383**	0.394	0.531	0.472	0.640	0.550
25	0.622	0.622	0.443	0.540	0.623	0.569	**0.435**	0.446	0.560	0.525	0.700	0.601
30	0.768	0.734	0.509	0.610	0.677	0.655	**0.509**	0.525	0.639	0.607	0.791	0.679

Parameters: Position, ε = 0; Scale, α = 1 and g = 5; Shape, β = 1 and δ = 0.12 (WAK5_3)

n	SQP1	SQP2	KQ1	KQ2	KQ3	KQ4	WKQ1	WKQ2	WKQ3	WKQ4	HDQ	WHDQ
10	0.782	0.636	0.532	0.667	0.940	0.540	0.463	**0.457**	0.832	0.528	0.573	0.552
15	1.082	1.082	0.757	0.892	1.135	0.751	0.685	**0.658**	0.965	0.731	0.846	0.789
20	1.128	0.803	0.670	0.748	0.888	0.624	0.622	**0.578**	0.749	0.627	0.694	0.653
25	0.840	0.840	0.771	0.828	0.927	0.709	0.736	**0.679**	0.798	0.720	0.777	0.736
30	0.926	0.980	0.862	0.898	0.963	0.785	0.839	**0.771**	0.850	0.804	0.852	0.814

Parameters: Position, ε = 0; Scale, α = 1 and γ = 10; Shape, β = 16 and δ = 0.04 WAK5_4)

n	SQP1	SQP2	KQ1	KQ2	KQ3	KQ4	WKQ1	WKQ2	WKQ3	WKQ4	HDQ	WHDQ
10	0.583	0.450	0.384	0.504	0.939	0.385	**0.311**	0.313	0.750	0.360	0.427	0.396
15	1.011	1.004	0.654	0.842	1.370	0.705	0.557	**0.534**	1.005	0.664	0.810	0.731
20	1.146	0.668	0.485	0.602	0.855	0.564	0.440	**0.418**	0.629	0.534	0.655	0.584
25	0.766	0.766	0.531	0.642	0.820	0.638	0.503	**0.475**	0.632	0.613	0.726	0.652
30	0.910	0.868	0.580	0.697	0.821	0.715	0.566	**0.533**	0.672	0.695	0.790	0.723

Parameters: Position, ε = 0; Scale, α = 1 and γ = 10; Shape, β = 1 and δ = 0.04 (WAK5_5)

n	SQP1	SQP2	KQ1	KQ2	KQ3	KQ4	WKQ1	WKQ2	WKQ3	WKQ4	HDQ	WHDQ
10	0.606	0.594	0.557	0.732	1.061	0.515	0.468	**0.436**	0.916	0.543	0.501	0.515
15	0.905	0.909	0.967	1.172	1.531	0.828	0.848	**0.755**	1.244	0.888	0.787	0.824
20	1.161	1.339	1.139	1.302	1.560	0.954	1.040	0.918	1.252	1.032	**0.900**	0.955
25	1.258	1.258	1.230	1.342	1.496	1.017	1.157	1.021	1.216	1.108	**0.957**	1.027
30	1.192	1.313	1.244	1.311	1.378	1.034	1.203	1.068	1.151	1.130	**0.976**	1.055

Parameters: Position, ε = 0; Scale, α = 1 and γ = 10; Shape, β = 2.5 and δ = 0.02 (WAK5_6)

n	SQP1	SQP2	KQ1	KQ2	KQ3	KQ4	WKQ1	WKQ2	WKQ3	WKQ4	HDQ	WHDQ
10	0.565	0.568	0.432	0.692	1.064	0.484	0.374	**0.333**	0.881	0.520	0.453	0.482
15	0.860	0.878	0.800	1.171	1.603	0.831	0.717	**0.637**	1.229	0.904	0.763	0.826
20	1.386	1.513	1.041	1.408	1.732	1.042	0.971	**0.876**	1.312	1.135	0.955	1.047
25	1.365	1.365	1.111	1.415	1.594	1.092	1.071	**0.980**	1.237	1.187	1.003	1.108
30	1.293	1.387	1.114	1.355	1.427	1.083	1.102	1.020	1.149	1.174	**1.004**	1.111

estimators was calculated; these findings are summarized in the Table 9.

Table 9: IMSE Relative Efficiency when Comparing Between Level Crossing and Ordinary Quantile Estimators

$\frac{WKQ_{u.1(lc)}}{KQ_{u.1}}$	$\frac{WKQ_{u.2(lc)}}{KQ_{u.2}}$	$\frac{WKQ_{u.3(lc)}}{KQ_{u.3}}$	$\frac{WKQ_{u.4(lc)}}{KQ_{u.4}}$	$\frac{WHD_u}{HD_u}$
95%	82%	83%	80%	92%

Hence, it can be concluded that the level crossing estimators are better than the ordinary quantile estimators as shown in our analysis most of the time.

Analysis on the most efficient method among the ordinary quantile estimators family showed that the $KQ_{u.1}$ quantile estimation method is the most efficient.

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