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Least Absolute Value vs. Least Squares Estimation and Inference Procedures in Regression Models with Asymmetric Error Distributions

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A Monte Carlo simulation is used to compare estimation and inference procedures in least absolute value (LAV) and least squares (LS) regression models with asymmetric error distributions. Mean square errors (MSE) of coefficient estimates are used to assess the relative efficiency of the estimators. Hypothesis tests for coefficients are compared on the basis of empirical level of significance and power.

Key words: L_1 regression, least absolute deviations, robust regression, simulation.

Introduction

The use of regression analysis relies on the choice of a criterion in order to estimate the coefficients of the explanatory variables. Traditionally, the least squares (LS) criterion has been the method of choice; however, the least absolute value (LAV) criterion provides an alternative. LAV regression coefficients are chosen to minimize the sum of the absolute values of the residuals. By minimizing sums of absolute values rather than sums of squares, the effect of outliers on the coefficient estimates is diminished.

In most previous studies comparing the performance of LAV and LS estimation, the distributions examined have been symmetric. Fat-tailed distributions that introduce outliers have been used, but these have typically been symmetric fat-tailed distributions (Laplace, Cauchy, etc). This study examined the performance of LAV and LS coefficient estimators when the regression disturbances come from asymmetric distributions.

Also, hypothesis tests for coefficient significance are examined. For the LAV regression, the tests compared include the likelihood ratio (LR) test, the Lagrange multiplier (LM) test suggested by Koenker and Bassett (1982) and a bootstrap test. The tests are compared in terms of both observed significance level and empirical power. Four alternative variance estimates are considered for the LR and bootstrap tests. The LAV tests are also compared with the traditional t-test for LS regression.

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Methodology

Least Absolute Value Estimation and Testing

The model considered in this article is the linear regression model:

$$y_i = \beta_0 + \sum_{k=1}^{K} \beta_k x_{ik} + \varepsilon_i$$

$$i = 1,2,\ldots,n \qquad (1)$$

where y_i is the ith observation on the dependent variable, x_{ik} is the ith observation on the kth explanatory variable, and ε_i is a random disturbance for the ith observation. The distribution of the disturbances may not be normal or even symmetric in this examination. The parameters β_0, β_1, β_2,…, β_K are unknown and must be estimated. For a discussion of algorithms to produce LAV coefficient estimates, see Dielman (1992, 2005).

In matrix notation, the model in (1) can be written

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \qquad (2)$$

where $\mathbf{Y}$ is an n x 1 vector of values of the dependent variable, $\mathbf{X}$ is an n x (K+1) matrix of values of the explanatory variables, including a column of ones for the constant, $\boldsymbol{\beta}$ is a (K+1) x 1 vector of the regression coefficients to be estimated and $\boldsymbol{\varepsilon}$ is an n x 1 vector of disturbances. Bassett and Koenker (1978) showed that, under reasonable conditions, the LAV coefficient estimator has an asymptotic distribution that converges to $N(\boldsymbol{\beta}, \lambda^2(\mathbf{X}'\mathbf{X})^{-1})$ where $\frac{\lambda^2}{n}$ is the asymptotic variance of the sample median for a sample of size n from the disturbance distribution.

Equation (2) can be rewritten in the following form:

$$\mathbf{Y} = \mathbf{X_1}\boldsymbol{\beta_1} + \mathbf{X_2}\boldsymbol{\beta_2} + \boldsymbol{\varepsilon} \qquad (3)$$

The coefficient vector $\boldsymbol{\beta}$ and the data matrix $\mathbf{X}$ from equation (2) have been partitioned: $\boldsymbol{\beta_1}$ is a k_1 x 1 vector of coefficients to remain in the model and $\mathbf{X_1}$ is the associated part of the original data matrix, $\mathbf{X}$; $\boldsymbol{\beta_2}$ represents the k_2 x 1 vector of coefficients to be included in a hypothesis test, and $\mathbf{X_2}$ is the associated part of the original data matrix, $\mathbf{X}$. The test considered is the basic test for coefficient significance, i.e., H_0: $\boldsymbol{\beta_2} = \mathbf{0}$. In the simulation $\boldsymbol{\beta_2}$ consists of a single coefficient.

Koenker and Bassett (1982) proposed three procedures for conducting hypothesis tests on the LAV regression model coefficients. The three tests are based on Wald, likelihood ratio (LR), and Lagrange multiplier (LM) test statistics, each of which has the same limiting Chi-square distribution. The LR and LM statistics will be examined in the Monte Carlo simulation. In previous studies, the Wald test has been shown to be inferior to the LR and LM statistics in small samples, so it is not included in this study (See, for example, Dielman and Pfaffenberger, 1988, 1990, 1992; Dielman, 2006).

The Lagrange Multiplier (LM) test statistic for the test of the null hypothesis H_0: $\boldsymbol{\beta_2} = \mathbf{0}$ is given by

$$\text{LM} = \mathbf{g_2'}\mathbf{D}\mathbf{g_2}, \qquad (4)$$

where $\mathbf{g_2}$ is the appropriate portion of the normalized gradient of the unrestricted LAV objective function, evaluated at the restricted estimate, and $\mathbf{D}$ is the appropriate block of the $(\mathbf{X}'\mathbf{X})^{-1}$ matrix to be used in the test.

The Likelihood Ratio (LR) test statistic (assuming the disturbances follow a Laplace distribution) is

$$\text{LR} = \frac{2(SAD_1 - SAD_2)}{\lambda} \qquad (5)$$

where SAD_1 is the sum of the absolute deviations of the residuals in the restricted or reduced model (i.e., $\boldsymbol{\beta_2} = \mathbf{0}$) and SAD_2 is the sum of the absolute deviations of the residuals in the unrestricted model.

The LR test statistic requires the estimation of the scale parameter λ, whereas the LM test statistic does not. One often-suggested estimator for λ can be computed as follows:

$$\hat{\lambda} = \frac{\sqrt{n'}\,[e_{(n'-m-1)} - e_{(m)}]}{z_{\alpha/2}},$$

where,

$$m = \frac{n'+1}{2} - z_{\alpha/2}\sqrt{\frac{n'}{4}} \qquad (6)$$

where the $e_{(.)}$ are ordered residuals from the LAV-fitted model, and $n' = n - r$ where r is the number of zero residuals. A value of $\alpha = 0.05$ is usually suggested. This estimator will be referred to as the SECI estimator. See McKean and Schrader (1984), McKean and Schrader (1987), Sheather (1987), Dielman and Pfaffenberger (1990, 1992) and Dielman and Rose (1995, 1996) for discussions and uses of this estimator.

When computing the variance of the slope coefficient in a LAV regression, the estimator of λ in equation (6) will be used. However, four different options in constructing this estimator will be considered. These options are as follows:

SECI1: $\hat{\lambda}_1$ uses $z = 1.96$ ($\alpha = 0.05$ value) and n' = total number of observations (n).

SECI2: $\hat{\lambda}_2$ uses $t_{0.025}$ with n degrees of freedom rather than the z value and n' = total number of observations (n).

SECI3: $\hat{\lambda}_3$ uses $z = 1.96$ ($\alpha = 0.05$ value) and n' = $n - r$ where r is the number of zero residuals.

SECI4: $\hat{\lambda}_4$ uses $t_{0.025}$ with $n - r$ degrees of freedom rather than the z value and $n' = n - r$ where r is the number of zero residuals.

The notation L1, L2, L3 and L4 will be used to indicate the LR test using variance estimator 1, 2, 3, or 4. Much of the literature in this area recommends using the estimator SECI3. However, Dielman (2006) performed a simulation study that suggested using SECI2. These results were for symmetric distributions only. Results for asymmetric distributions will be examined in this paper. In addition, the bootstrap tests were not included in the previous study.

The bootstrapping methodology provides an alternative to the LR and LM tests. In a LAV simple regression, for example, a bootstrap test statistic for H_0: $\beta_1 = 0$ can be computed in several ways (see Li & Maddala, 1996). The following procedure will be used in this study: The model shown as equation (1) is estimated (when K = 1 for simple regression) using LAV estimation procedures and residuals are obtained. The test statistic, $\frac{|\hat{\beta}_1 - 0|}{se(\hat{\beta}_1)}$, is computed from the regression on the original data, where $se(\hat{\beta}_1)$ represents the standard error of the coefficient estimate. The residuals, e_i ($i = 1,2,\ldots,n$), from this regression are saved, centered, and resampled (with replacement, excluding zero residuals), to obtain a new sample of disturbances, e_i^*. The e_i^* values are used to create pseudo-data as follows:

$$y_i^* = \hat{\beta}_0 + \hat{\beta}_1 x_i + e_i^* \qquad (7)$$

where $\hat{\beta}_0$ and $\hat{\beta}_1$ are the initial LAV estimates of the intercept and slope. The coefficients in equation (7) are then re-estimated to obtain new parameter estimates, $\hat{\beta}_1^*$ and $\hat{\beta}_0^*$, and the test statistic $T = \frac{|\hat{\beta}_1^* - \hat{\beta}_1|}{se(\hat{\beta}_1^*)}$ is computed and saved. The process of computing T is repeated a large number of times. For a test to be performed at a particular level of significance, α, the critical value is the $(1 - \alpha)^{th}$ percentile from the ordered test statistic values. If the original test statistic is larger than this critical value, then the null hypothesis that $\beta_1 = 0$ is rejected. The extension to a single coefficient in a multiple regression is easily accomplished.

Although Li and Maddala (1996) suggested that the pseudo-data generating process can proceed in other ways, the method outlined here is fairly typical. Research by van Giersbergen and Kiviet (2002) and Dielman and Rose (2002) suggest that the aspect of primary importance is that the resampling scheme should mimic the null distribution of the test statistic to be bootstrapped. This suggestion is followed in the bootstrap approach used in this paper. Results from the traditional LS t-test are compared to those from the LAV-based tests.

Description of the Simulation Experiment

The simulation is based on the model in equation (1). The sample sizes used are $n = 20$, 30, 40 and 100. The disturbances are generated using stable distributions with the following combinations of characteristic exponent (alpha) and skewness parameter (beta):

Beta = 0.0, 0.4 and 0.8 with Alpha = 1.2

Beta = 0.0, 0.4 and 0.8 with Alpha = 1.8

In addition the normal (beta = 0.0 with alpha = 1.2) and Cauchy distributions (beta = 0.0 with alpha = 1.0) were used. The normal and Cauchy distributions serve as extremes. Stable distributions are infinite variance distributions when the characteristic exponent is less than 2.0, so the LAV estimator would be expected to outperform LS in these cases. When the characteristic exponent equals 2.0 (and beta is zero), the distribution is normal and LS will be optimal. For a characteristic exponent close to 2.0 (and a symmetric distribution), we would expect LS to perform relatively better than for an exponent near 1.0 (Cauchy disturbances). As alpha approaches 1.0, LAV is expected to perform better than LS.

The independent variables are generated as independent standard normal random variables, independent of the disturbances. Bootstrap tests used 199 bootstrap replications. The value of β_0 is set equal to zero (without loss of generality). In the simple regression, the value of β_1 is set equal to 0.0 to assess the level of significance and is set equal to 0.2, 0.4, 0.6, 0.8, 1.0 and 2.0 to examine power. In the multiple regressions, all slope coefficients are set equal to zero (without loss of generality), except for one coefficient which is set equal to 0.0 to assess the level of significance and is set equal to 0.2, 0.4, 0.6, 0,8, 1.0 and 2.0 to examine power. For each factor combination in the experimental design, 5,000 Monte Carlo simulations are used, and the number of rejections of the null hypothesis of whether the selected slope coefficient is equal to zero is counted for each setting. All testing is done using a nominal 5% level of significance.

Results

Estimation

Table 1 contains ratios of mean square errors (MSEs) for estimates of the intercept and slope coefficients in simple regressions (K = 1) and for the intercept and one of the coefficients in the multiple regressions (K = 3 and 5) for sample size n = 20 in Panel A, n = 30 in Panel B, n = 40 in Panel C, and n = 100 in Panel D. The extremes of alpha = 0.0 (Cauchy) and alpha = 2.0 (normal) show the range of possibilities when distributions are symmetric. LS is always preferred to LAV when disturbances are normal. The ratio of MSEs is consistently 0.8 except when n = 20 and K = 5, in which case the preference for LS is even stronger.

The LAV estimator is preferred over LS for alpha of 1.8 and 1.2, although the advantage decreases as alpha approaches two (normal distribution) as would be expected. The only exception to this rule is when n = 20 and K = 5 when LS is preferred for beta = 0.0 or 0.4, that is, when the skewness is less extreme. LAV is preferred in all cases when beta = 0.8.

When alpha = 1.8, the preference for LAV over LS increases in all cases as skewness increases. When alpha = 1.2 and K = 1, the preference for LAV over LS decreases (although LAV is still better than LS by a wide margin). With alpha = 1.2 and K = 3 or 5, the results are mixed in terms of the increase or decrease of the preference for LAV over LS based on skewness. This may be a result of looking at an efficiency measure for only a single coefficient. Regardless, LAV is still preferable to LS by a wide margin when alpha = 1.2.

Hypothesis Tests

Tables 2 through 5 contain the median percentage of trials in which H_0: coefficient = 0 is rejected for various combinations of test and coefficient values for n = 20, 30, 40 and 100, respectively, when K = 1. The medians are taken over the disturbance distributions. Thus, the results for the symmetric distributions (beta = 0.0) include Stable distributions with alpha = 1.0 (Cauchy) 1.2, 1.8 and 2.0 (normal). The asymmetric distributions include Stable with alpha = 1.2 and 1.8 when beta is either 0.4 or 0.8. When the coefficient value is zero, the empirical significance levels can be assessed; when it is non-zero, power for the tests can be compared. Tables 6 through 9 contain the same information for K = 3 while tables 10 through 13 contain results for K = 5.

The empirical level of significance for the LS t-test never exceeds 0.06 in any of the experimental settings (nominal level = 0.05). However, the test lacks power when compared to the LAV tests. For example, consider Table 5 with K = 1 and n = 100. All tests have empirical level of significance 0.05, but LST has considerably lower power.

There is little difference in performance for skewed and symmetric error distributions. When LAV is preferred to LS, the preference is due to the presence of outliers from the fat-tailed distribution rather than from any lack of symmetry in the distributions.

Among the LAV tests, the bootstrap tests and the LM test tend to maintain a median

Table 1: Ratios of mean square error of estimates of intercept and slope (or one of the slope coefficients if K = 3 or 5): LS/LAV. Numbers greater than one favor LAV, numbers less than one favor LS. Alpha is the characteristic exponent of the Stable distribution; beta is the skewness parameter. (Alpha = 2.0 is the normal distribution, Alpha = 0.0 is the Cauchy).

Panel A: Intercept (n = 20)

		Beta		
Alpha	K	0.0	0.4	0.8
0.0	1	102.0		
	3	55.9		
	5	82.3		
1.2	1	83.1	68.8	38.1
	3	17.8	57.0	21.4
	5	25.6	22.2	14.7
1.8	1	1.3	1.3	1.4
	3	1.2	1.2	1.2
	5	0.7	0.7	2.5
2.0	1	0.8		
	3	0.8		
	5	0.4		

Panel A: Slope (n = 20)

		Beta		
Alpha	K	0.0	0.4	0.8
0.0	1	69.5		
	3	46.3		
	5	28.6		
1.2	1	99.1	83.9	52.8
	3	13.7	27.6	11.9
	5	10.0	10.7	12.8
1.8	1	1.2	1.2	1.2
	3	1.1	1.1	1.1
	5	0.7	0.7	1.4
2.0	1	0.8		
	3	0.8		
	5	0.5		

Panel B: Intercept (n = 30)

		Beta		
Alpha	K	0.0	0.4	0.8
0.0	1	130.8		
	3	104.7		
	5	1016.7		
1.2	1	76.8	67.2	37.0
	3	64.6	53.3	29.8
	5	44.9	37.7	25.6
1.8	1	1.3	1.3	1.4
	3	1.2	1.3	1.4
	5	1.1	1.2	1.3
2.0	1	0.8		
	3	0.8		
	5	0.8		

Panel B: Slope (n = 30)

		Beta		
Alpha	K	0.0	0.4	0.8
0.0	1	90.4		
	3	56.0		
	5	933.9		
1.2	1	71.4	83.4	56.1
	3	50.4	43.2	29.7
	5	58.0	51.1	38.8
1.8	1	1.3	1.4	1.5
	3	1.2	1.3	1.3
	5	1.3	1.3	1.4
2.0	1	0.8		
	3	0.8		
	5	0.8		

Table 1: continued

Panel C: Intercept (*n* = 40)				
		Beta		
Alpha	K	0.0	0.4	0.8
0.0	1	176.2		
	3	136.3		
	5	130.0		
1.2	1	53.7	46.0	29.2
	3	39.0	35.3	26.2
	5	41.2	36.8	26.3
1.8	1	1.4	1.5	1.6
	3	1.4	1.5	1.6
	5	1.4	1.5	1.6
2.0	1	0.8		
	3	0.8		
	5	0.8		

Panel C: Slope (*n* = 40)				
		Beta		
Alpha	K	0.0	0.4	0.8
0.0	1	123.8		
	3	74.3		
	5	109.7		
1.2	1	25.7	24.0	21.0
	3	28.8	28.9	29.7
	5	22.6	23.0	24.0
1.8	1	1.2	1.3	1.3
	3	1.4	1.4	1.5
	5	1.3	1.4	1.4
2.0	1	0.8		
	3	0.8		
	5	0.8		

Panel D: Intercept (*n* = 100)				
		Beta		
Alpha	K	0.0	0.4	0.8
0.0	1	1467.5		
	3	1555.1		
	5	1513.6		
1.2	1	96.4	96.4	76.1
	3	119.2	121.6	96.4
	5	117.9	121.1	95.4
1.8	1	2.0	2.3	2.6
	3	2.4	2.8	3.0
	5	2.4	2.8	3.1
2.0	1	0.8		
	3	0.8		
	5	0.8		

Panel D: Slope (*n* = 100)				
		Beta		
Alpha	K	0.0	0.4	0.8
0.0	1	1017.6		
	3	751.2		
	5	278.5		
1.2	1	57.3	50.7	38.5
	3	117.3	132.0	149.6
	5	99.4	107.1	115.9
1.8	1	1.2	1.3	1.3
	3	2.5	2.9	3.3
	5	2.3	2.6	3.0
2.0	1	0.8		
	3	0.8		
	5	0.8		

significance level close to nominal. The LR tests often deviate considerably from nominal. However, LR2 has median significance level closer to nominal than the other LR tests in most cases. Performance is similar for the LR tests for skewed and symmetric distributions. Among the bootstrap tests, there is little difference in performance for any of the experimental settings.

In choosing among the LAV tests, it appears that the LR2 test maintains relatively high power - even when the level of significance is lower compared to the other tests. Also, the LM test is consistently lower in power. This negates some of the advantage the LM test might have due to the fact that it does not need an estimate of the nuisance parameter. As noted, the bootstrap tests have levels of significance that tend to be close to the nominal level. Power for the bootstrap tests can be slightly lower than that for LR2, even when the level of significance is equal or lower for LR2. Increasing the number of bootstrap iterations might improve the power of these tests. When sample size is large (n = 100), there is little difference among any of the LAV based tests. These tests still improve on the LS t-test even in large samples.

The variance estimate used to obtain LR2 uses n in the computations rather than $n- r$ (where r is the number of zero residuals). This adjustment for zero residuals does not appear to be necessary. The variance estimates used to obtain LR1 and LR2 differ in that LR1 uses the z value while LR2 uses the appropriate t value in the computations. This provides some improvement in test performance for LR2 in small samples but the advantage vanishes for a sample size of 100.

Conclusion

Previous research examining small sample performance of some of the test statistics discussed in this article based on symmetric error distributions include Dielman (2006), Dielman and Pfaffenberger (1988, 1990, 1992), Dielman and Rose (1995, 1996, 2002), Koenker (1987) and Stangenhaus (1987). The results of these studies suggest that, in small samples, the LR and LM tests generally outperform the Wald test (not considered in the present study) in terms of both power and observed significance level.

The LR and LM tests differ in that the LR test requires an estimate of the λ parameter discussed previously, while the LM test does not. However, using a fairly simple estimate of this scale parameter, the LR test has generally performed as well as, or better than, the LM test. In addition to the Wald, LR, and LM tests, bootstrap approaches have also been examined for inference in LAV regression. Dielman and Pfaffenberger (1988) used a bootstrap approach to estimate the scale parameter, λ, but the significance tests based on these bootstrap estimates did not perform particularly well.

Dielman and Rose (1995) compared a true bootstrap test statistic with the LR and LM tests, and found that the bootstrap performed well in small samples. Dielman and Rose (2002) compared the LR, LM and three versions of the bootstrap suggested by Li and Maddala (1996) along with the LS t-test. They found that the LR test performed at least as well as, and often better than, the competing tests. Prior results for symmetric error distributions are consistent with the results from this study for both symmetric and asymmetric error distributions.

If error distributions are suspected to be fat-tailed, improvements in estimation and inference are possible using LAV estimation rather than LS. This is true regardless of whether the distributions are symmetric or skewed. When choosing a test procedure for LAV estimated models, the bootstrap approaches perform reasonably well for all cases examined here. If a likelihood ratio test is to be used, LR2 seems to perform better than the other choices examined here. In addition, the LM test performs reasonably well in most settings examined although the power may be somewhat lower than the LR2 test. Differences in performance between the LAV based tests are small once the sample size reaches 100.

References

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Table 2: Median percentages of rejections of H_0: $\beta_1 = 0$, normal explanatory variable, n = 20, K = 1 for symmetric (stable with beta = 0.0 for alpha = 1.0, 1.2, 1.8, and 2.0) and skewed (stable with beta = 0.4 and 0.8 for alpha = 1.2 and 1.8) distributions.

Panel A: Symmetric Distributions										
Beta	LR1	LR2	LR3	LR4	B1	B2	B3	B4	LM	LST
0.0	0.06	0.03	0.07	0.08	0.06	0.06	0.06	0.06	0.06	0.04
0.2	0.10	0.06	0.10	0.12	0.08	0.08	0.08	0.08	0.09	0.07
0.4	0.20	0.14	0.21	0.23	0.15	0.16	0.15	0.15	0.18	0.15
0.6	0.37	0.28	0.38	0.41	0.28	0.29	0.27	0.27	0.31	0.27
0.8	0.56	0.47	0.57	0.60	0.43	0.46	0.43	0.43	0.47	0.41
1.0	0.72	0.64	0.73	0.75	0.59	0.62	0.59	0.59	0.61	0.54
2.0	0.99	0.98	0.99	0.99	0.97	0.98	0.97	0.97	0.94	0.81

Panel B: Skewed Distributions										
Beta	LR1	LR2	LR3	LR4	B1	B2	B3	B4	LM	LST
0.0	0.04	0.03	0.07	0.05	0.05	0.06	0.05	0.05	0.06	0.04
0.2	0.07	0.06	0.10	0.08	0.08	0.08	0.07	0.07	0.09	0.07
0.4	0.16	0.13	0.20	0.18	0.15	0.15	0.14	0.15	0.17	0.13
0.6	0.29	0.26	0.36	0.32	0.27	0.28	0.24	0.25	0.29	0.25
0.8	0.46	0.44	0.54	0.50	0.41	0.42	0.38	0.40	0.43	0.38
1.0	0.63	0.60	0.70	0.66	0.57	0.58	0.52	0.55	0.56	0.53
2.0	0.98	0.98	0.99	0.98	0.97	0.97	0.95	0.96	0.92	0.82

Table 3: Median percentages of rejections of H_0: $\beta_1 = 0$, normal explanatory variable, n = 30, K = 1 for symmetric (with beta = 0.0 for alpha = 1.0, 1.2, 1.8, and 2.0) and skewed (stable with beta = 0.4 and 0.8 for alpha = 1.2 and 1.8) distributions.

Panel A: Symmetric Distributions										
Beta	LR1	LR2	LR3	LR4	B1	B2	B3	B4	LM	LST
0.0	0.06	0.04	0.07	0.07	0.06	0.06	0.06	0.06	0.05	0.05
0.2	0.11	0.08	0.12	0.13	0.09	0.09	0.10	0.10	0.10	0.09
0.4	0.26	0.20	0.27	0.28	0.21	0.21	0.21	0.21	0.22	0.21
0.6	0.48	0.40	0.49	0.50	0.37	0.39	0.38	0.38	0.40	0.41
0.8	0.69	0.62	0.70	0.71	0.57	0.59	0.57	0.57	0.58	0.61
1.0	0.84	0.79	0.84	0.85	0.74	0.76	0.74	0.74	0.73	0.77
2.0	1.00	0.99	1.00	1.00	0.99	1.00	1.00	1.00	0.97	0.97

Panel B: Skewed Distributions										
Beta	LR1	LR2	LR3	LR4	B1	B2	B3	B4	LM	LST
0.0	0.06	0.04	0.06	0.07	0.06	0.06	0.06	0.06	0.05	0.05
0.2	0.11	0.08	0.11	0.12	0.09	0.09	0.09	0.09	0.09	0.08
0.4	0.25	0.20	0.26	0.27	0.19	0.20	0.19	0.19	0.21	0.18
0.6	0.47	0.40	0.48	0.49	0.36	0.37	0.36	0.36	0.41	0.33
0.8	0.69	0.62	0.69	0.71	0.55	0.58	0.56	0.56	0.60	0.49
1.0	0.84	0.80	0.85	0.86	0.73	0.75	0.73	0.73	0.76	0.60
2.0	1.00	1.00	1.00	1.00	0.99	1.00	0.99	0.99	0.98	0.83

Table 4: Median percentages of rejections of H_0: $\beta_1 = 0$, normal explanatory variable, n = 40, K = 1 for symmetric (stable with beta = 0.0 for alpha = 1.0, 1.2, 1.8, and 2.0) and skewed (stable with beta = 0.4 and 0.8 for alpha = 1.2 and 1.8) distributions.

Panel A: Symmetric Distributions										
Beta	LR1	LR2	LR3	LR4	B1	B2	B3	B4	LM	LST
0.0	0.05	0.06	0.06	0.06	0.05	0.05	0.05	0.05	0.05	0.05
0.2	0.13	0.14	0.14	0.14	0.11	0.11	0.11	0.11	0.12	0.09
0.4	0.37	0.38	0.37	0.38	0.30	0.30	0.30	0.30	0.32	0.25
0.6	0.65	0.66	0.66	0.67	0.56	0.56	0.56	0.56	0.58	0.44
0.8	0.85	0.86	0.86	0.86	0.78	0.78	0.78	0.78	0.78	0.59
1.0	0.96	0.96	0.96	0.96	0.92	0.92	0.92	0.92	0.90	0.68
2.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.86

Panel B: Skewed Distributions										
Beta	LR1	LR2	LR3	LR4	B1	B2	B3	B4	LM	LST
0.0	0.06	0.06	0.06	0.06	0.05	0.05	0.05	0.05	0.05	0.05
0.2	0.13	0.14	0.13	0.14	0.11	0.11	0.11	0.11	0.12	0.09
0.4	0.35	0.36	0.36	0.37	0.28	0.28	0.27	0.27	0.32	0.25
0.6	0.64	0.65	0.65	0.66	0.53	0.53	0.53	0.53	0.56	0.44
0.8	0.85	0.86	0.85	0.86	0.76	0.76	0.77	0.77	0.77	0.59
1.0	0.95	0.96	0.95	0.96	0.91	0.91	0.91	0.91	0.90	0.68
2.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.86

Table 5: Median percentages of rejections of H_0: $\beta_1 = 0$, normal explanatory variable, n = 100, K = 1 for symmetric (stable with beta = 0.0 for alpha = 1.0, 1.2, 1.8, and 2.0) and skewed (stable with beta = 0.4 and 0.8 for alpha = 1.2 and 1.8) distributions.

Panel A: Symmetric Distributions										
Beta	LR1	LR2	LR3	LR4	B1	B2	B3	B4	LM	LST
0.0	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
0.2	0.24	0.24	0.24	0.25	0.22	0.22	0.22	0.22	0.23	0.14
0.4	0.68	0.68	0.68	0.69	0.63	0.63	0.63	0.63	0.64	0.39
0.6	0.94	0.94	0.94	0.94	0.92	0.92	0.91	0.91	0.92	0.57
0.8	0.99	1.00	1.00	1.00	0.99	0.99	0.99	0.99	0.99	0.67
1.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.73
2.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.88

Panel B: Skewed Distributions										
Beta	LR1	LR2	LR3	LR4	B1	B2	B3	B4	LM	LST
0.0	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
0.2	0.22	0.22	0.22	0.22	0.20	0.20	0.19	0.19	0.20	0.14
0.4	0.63	0.64	0.64	0.64	0.57	0.57	0.56	0.56	0.62	0.39
0.6	0.94	0.94	0.94	0.94	0.88	0.88	0.88	0.88	0.91	0.57
0.8	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.67
1.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.73
2.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.89

Table 6: Median percentages of rejections of H_0: coefficient = 0, normal explanatory variable, n = 20, K = 3 for symmetric (stable with beta = 0.0 for alpha = 1.0, 1.2, 1.8, and 2.0) and skewed (stable with beta = 0.4 and 0.8 for alpha = 1.2 and 1.8) distributions.

Panel A: Symmetric Distributions										
Beta	LR1	LR2	LR3	LR4	B1	B2	B3	B4	LM	LST
0.0	0.09	0.04	0.09	0.11	0.06	0.06	0.06	0.06	0.06	0.04
0.2	0.14	0.08	0.14	0.15	0.08	0.08	0.08	0.08	0.11	0.07
0.4	0.27	0.18	0.27	0.29	0.15	0.16	0.15	0.15	0.20	0.16
0.6	0.45	0.33	0.45	0.47	0.27	0.29	0.27	0.27	0.34	0.29
0.8	0.63	0.52	0.64	0.66	0.41	0.45	0.41	0.41	0.49	0.44
1.0	0.78	0.69	0.78	0.80	0.56	0.62	0.56	0.56	0.62	0.56
2.0	0.99	0.98	0.99	0.99	0.96	0.97	0.96	0.96	0.92	0.82

Panel B: Skewed Distributions										
Beta	LR1	LR2	LR3	LR4	B1	B2	B3	B4	LM	LST
0.0	0.09	0.05	0.08	0.10	0.06	0.06	0.05	0.05	0.06	0.04
0.2	0.14	0.08	0.13	0.14	0.08	0.08	0.07	0.07	0.10	0.07
0.4	0.26	0.17	0.25	0.28	0.15	0.15	0.14	0.14	0.20	0.16
0.6	0.44	0.33	0.42	0.45	0.26	0.29	0.25	0.25	0.33	0.29
0.8	0.62	0.51	0.61	0.63	0.41	0.44	0.39	0.39	0.48	0.43
1.0	0.77	0.68	0.76	0.78	0.55	0.61	0.54	0.54	0.62	0.56
2.0	0.99	0.98	0.99	0.99	0.95	0.97	0.95	0.95	0.93	0.82

Table 7: Median percentages of rejections of H_0: coefficient = 0, normal explanatory variable, n = 30, K = 3 for symmetric (stable with beta = 0.0 for alpha = 1.0, 1.2, 1.8, and 2.0) and skewed (stable with beta = 0.4 and 0.8 for alpha = 1.2 and 1.8) distributions.

Panel A: Symmetric Distributions										
Beta	LR1	LR2	LR3	LR4	B1	B2	B3	B4	LM	LST
0.0	0.09	0.06	0.10	0.10	0.06	0.06	0.06	0.06	0.07	0.05
0.2	0.14	0.09	0.14	0.15	0.08	0.09	0.08	0.08	0.11	0.08
0.4	0.26	0.20	0.27	0.29	0.16	0.16	0.16	0.16	0.22	0.15
0.6	0.44	0.36	0.45	0.47	0.28	0.30	0.28	0.28	0.36	0.27
0.8	0.63	0.55	0.65	0.66	0.43	0.47	0.44	0.44	0.53	0.42
1.0	0.79	0.72	0.80	0.81	0.59	0.63	0.60	0.60	0.67	0.54
2.0	0.99	0.99	0.99	0.99	0.97	0.98	0.98	0.98	0.96	0.80

Panel B: Skewed Distributions										
Beta	LR1	LR2	LR3	LR4	B1	B2	B3	B4	LM	LST
0.0	0.09	0.06	0.10	0.10	0.06	0.06	0.05	0.05	0.07	0.05
0.2	0.13	0.09	0.14	0.15	0.08	0.08	0.08	0.08	0.11	0.08
0.4	0.26	0.20	0.26	0.27	0.15	0.17	0.15	0.15	0.22	0.15
0.6	0.44	0.36	0.44	0.46	0.27	0.29	0.27	0.27	0.36	0.27
0.8	0.63	0.55	0.63	0.65	0.43	0.46	0.42	0.42	0.52	0.42
1.0	0.80	0.72	0.79	0.80	0.59	0.62	0.59	0.59	0.66	0.54
2.0	0.99	0.99	0.99	0.99	0.97	0.98	0.97	0.97	0.96	0.80

Table 8: Median percentages of rejections of H_0: coefficient = 0, normal explanatory variable, n = 40, K = 3 for symmetric (stable with beta = 0.0 for alpha = 1.0, 1.2, 1.8, and 2.0) and skewed (stable with beta = 0.4 and 0.8 for alpha = 1.2 and 1.8) distributions.

Panel A: Symmetric Distributions										
Beta	LR1	LR2	LR3	LR4	B1	B2	B3	B4	LM	LST
0.0	0.08	0.08	0.08	0.09	0.05	0.05	0.05	0.05	0.06	0.05
0.2	0.14	0.14	0.14	0.15	0.09	0.09	0.08	0.08	0.12	0.08
0.4	0.31	0.32	0.32	0.33	0.19	0.19	0.19	0.19	0.26	0.19
0.6	0.54	0.55	0.55	0.56	0.36	0.36	0.37	0.37	0.45	0.34
0.8	0.75	0.76	0.76	0.77	0.57	0.57	0.57	0.57	0.65	0.49
1.0	0.88	0.89	0.89	0.89	0.75	0.75	0.75	0.75	0.79	0.60
2.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.82

Panel B: Skewed Distributions										
Beta	LR1	LR2	LR3	LR4	B1	B2	B3	B4	LM	LST
0.0	0.08	0.09	0.08	0.09	0.05	0.05	0.05	0.05	0.06	0.05
0.2	0.13	0.14	0.14	0.14	0.08	0.08	0.08	0.08	0.12	0.08
0.4	0.30	0.31	0.31	0.32	0.19	0.19	0.19	0.19	0.25	0.18
0.6	0.53	0.54	0.54	0.55	0.36	0.36	0.36	0.36	0.44	0.34
0.8	0.75	0.76	0.75	0.76	0.57	0.57	0.56	0.56	0.63	0.49
1.0	0.88	0.89	0.88	0.89	0.74	0.74	0.74	0.74	0.78	0.60
2.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.82

Table 9: Median percentages of rejections of H_0: coefficient = 0, normal explanatory variable, n = 100, K = 3 for symmetric (stable with beta = 0.0 for alpha = 1.0, 1.2, 1.8, and 2.0) and skewed (stable with beta = 0.4 and 0.8 for alpha = 1.2 and 1.8) distributions.

Panel A: Symmetric Distributions										
Beta	LR1	LR2	LR3	LR4	B1	B2	B3	B4	LM	LST
0.0	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.05	0.05
0.2	0.25	0.26	0.26	0.27	0.21	0.21	0.21	0.21	0.23	0.14
0.4	0.68	0.68	0.69	0.69	0.60	0.60	0.60	0.60	0.63	0.38
0.6	0.94	0.94	0.94	0.94	0.90	0.90	0.90	0.90	0.91	0.57
0.8	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.67
1.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.73
2.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.88

Panel B: Skewed Distributions										
Beta	LR1	LR2	LR3	LR4	B1	B2	B3	B4	LM	LST
0.0	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.06	0.05	0.05
0.2	0.24	0.24	0.24	0.25	0.20	0.20	0.20	0.20	0.22	0.14
0.4	0.67	0.67	0.67	0.68	0.55	0.55	0.56	0.56	0.62	0.38
0.6	0.93	0.93	0.93	0.93	0.87	0.87	0.88	0.88	0.88	0.57
0.8	0.99	0.99	0.99	0.99	0.98	0.98	0.98	0.98	0.99	0.67
1.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.73
2.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.88

Table 10: Median percentages of rejections of H_0: coefficient = 0, normal explanatory variable, n = 20, K = 5 for symmetric (stable with beta = 0.0 for alpha = 1.0, 1.2, 1.8, and 2.0) and skewed (stable with beta = 0.4 and 0.8 for alpha = 1.2 and 1.8) distributions.

Panel A: Symmetric Distributions										
Beta	LR1	LR2	LR3	LR4	B1	B2	B3	B4	LM	LST
0.0	0.18	0.09	0.11	0.13	0.05	0.05	0.05	0.05	0.15	0.06
0.2	0.21	0.10	0.13	0.15	0.06	0.06	0.05	0.05	0.17	0.07
0.4	0.27	0.14	0.18	0.20	0.07	0.08	0.07	0.07	0.20	0.09
0.6	0.35	0.21	0.25	0.28	0.10	0.11	0.09	0.09	0.24	0.14
0.8	0.45	0.31	0.34	0.37	0.13	0.15	0.14	0.14	0.29	0.20
1.0	0.56	0.40	0.44	0.47	0.18	0.21	0.19	0.19	0.33	0.28
2.0	0.90	0.83	0.84	0.86	0.51	0.63	0.56	0.56	0.53	0.65

Panel B: Skewed Distributions										
Beta	LR1	LR2	LR3	LR4	B1	B2	B3	B4	LM	LST
0.0	0.19	0.09	0.10	0.12	0.05	0.05	0.04	0.04	0.15	0.06
0.2	0.21	0.10	0.12	0.14	0.05	0.05	0.05	0.05	0.17	0.07
0.4	0.27	0.14	0.16	0.18	0.07	0.07	0.06	0.06	0.20	0.09
0.6	0.35	0.21	0.23	0.26	0.09	0.10	0.09	0.09	0.23	0.14
0.8	0.44	0.30	0.32	0.34	0.13	0.15	0.13	0.13	0.28	0.20
1.0	0.55	0.40	0.41	0.44	0.18	0.20	0.17	0.17	0.32	0.28
2.0	0.89	0.82	0.82	0.84	0.51	0.62	0.53	0.53	0.51	0.65

Table 11: Median percentages of rejections of H_0: coefficient = 0, normal explanatory variable, n = 30, K = 5 for symmetric (stable with beta = 0.0 for alpha = 1.0, 1.2, 1.8, and 2.0) and skewed (stable with beta = 0.4 and 0.8 for alpha = 1.2 and 1.8) distributions.

Panel A: Symmetric Distributions										
Beta	LR1	LR2	LR3	LR4	B1	B2	B3	B4	LM	LST
0.0	0.14	0.08	0.13	0.14	0.05	0.05	0.05	0.05	0.08	0.04
0.2	0.20	0.13	0.18	0.20	0.08	0.08	0.07	0.07	0.11	0.07
0.4	0.35	0.26	0.34	0.36	0.15	0.16	0.15	0.15	0.19	0.16
0.6	0.55	0.46	0.55	0.57	0.26	0.30	0.28	0.28	0.34	0.30
0.8	0.74	0.66	0.74	0.75	0.42	0.47	0.45	0.45	0.50	0.45
1.0	0.86	0.81	0.86	0.87	0.58	0.64	0.61	0.61	0.64	0.58
2.0	1.00	0.99	1.00	1.00	0.96	0.98	0.97	0.97	0.95	0.82

Panel B: Skewed Distributions										
Beta	LR1	LR2	LR3	LR4	B1	B2	B3	B4	LM	LST
0.0	0.14	0.08	0.12	0.13	0.05	0.05	0.05	0.05	0.07	0.04
0.2	0.19	0.13	0.17	0.19	0.07	0.08	0.08	0.08	0.10	0.07
0.4	0.35	0.26	0.33	0.34	0.15	0.16	0.15	0.15	0.20	0.16
0.6	0.55	0.46	0.53	0.55	0.27	0.30	0.27	0.27	0.34	0.30
0.8	0.73	0.65	0.72	0.73	0.41	0.46	0.43	0.43	0.50	0.45
1.0	0.86	0.81	0.85	0.86	0.57	0.63	0.59	0.59	0.64	0.58
2.0	1.00	0.99	1.00	1.00	0.96	0.98	0.97	0.97	0.95	0.82

Table 12: Median percentages of rejections of H_0: coefficient = 0, normal explanatory variable, n = 40, K = 5 for symmetric (stable with beta = 0.0 for alpha = 1.0, 1.2, 1.8, and 2.0) and skewed (stable with beta = 0.4 and 0.8 for alpha = 1.2 and 1.8) distributions.

Panel A: Symmetric Distributions										
Beta	LR1	LR2	LR3	LR4	B1	B2	B3	B4	LM	LST
0.0	0.11	0.12	0.12	0.13	0.06	0.06	0.06	0.06	0.07	0.05
0.2	0.20	0.21	0.21	0.21	0.10	0.10	0.10	0.10	0.13	0.09
0.4	0.44	0.45	0.44	0.45	0.24	0.24	0.24	0.24	0.30	0.23
0.6	0.69	0.70	0.70	0.71	0.45	0.45	0.45	0.45	0.53	0.42
0.8	0.87	0.87	0.88	0.88	0.66	0.66	0.67	0.67	0.73	0.57
1.0	0.95	0.96	0.96	0.96	0.82	0.82	0.83	0.83	0.86	0.67
2.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.85

Panel B: Skewed Distributions										
Beta	LR1	LR2	LR3	LR4	B1	B2	B3	B4	LM	LST
0.0	0.12	0.12	0.11	0.12	0.06	0.06	0.06	0.06	0.06	0.05
0.2	0.20	0.21	0.20	0.21	0.10	0.10	0.10	0.10	0.13	0.09
0.4	0.43	0.44	0.43	0.44	0.23	0.23	0.23	0.23	0.30	0.23
0.6	0.69	0.70	0.69	0.70	0.44	0.44	0.44	0.44	0.52	0.42
0.8	0.87	0.87	0.87	0.88	0.65	0.65	0.65	0.65	0.72	0.57
1.0	0.95	0.96	0.96	0.96	0.82	0.82	0.82	0.82	0.85	0.67
2.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.85

Table 13: Median percentages of rejections of H_0: coefficient = 0, normal explanatory variable, n = 100, K = 5 for symmetric (stable with beta = 0.0 for alpha = 1.0, 1.2, 1.8, and 2.0) and skewed (stable with beta = 0.4 and 0.8 for alpha = 1.2 and 1.8) distributions.

Panel A: Symmetric Distributions										
Beta	LR1	LR2	LR3	LR4	B1	B2	B3	B4	LM	LST
0.0	0.08	0.08	0.08	0.08	0.05	0.05	0.05	0.05	0.05	0.05
0.2	0.28	0.28	0.29	0.29	0.19	0.19	0.19	0.19	0.21	0.14
0.4	0.72	0.72	0.72	0.73	0.58	0.58	0.58	0.58	0.61	0.39
0.6	0.95	0.95	0.95	0.95	0.88	0.88	0.88	0.88	0.89	0.57
0.8	0.99	0.99	1.00	1.00	0.98	0.98	0.98	0.98	0.98	0.67
1.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.73
2.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.88

Panel B: Skewed Distributions										
Beta	LR1	LR2	LR3	LR4	B1	B2	B3	B4	LM	LST
0.0	0.08	0.08	0.08	0.08	0.05	0.05	0.05	0.05	0.05	0.06
0.2	0.27	0.27	0.27	0.28	0.18	0.18	0.19	0.19	0.20	0.15
0.4	0.70	0.70	0.70	0.71	0.55	0.55	0.56	0.56	0.61	0.39
0.6	0.94	0.94	0.94	0.95	0.87	0.87	0.87	0.87	0.89	0.58
0.8	1.00	1.00	0.99	0.99	0.98	0.98	0.98	0.98	0.99	0.67
1.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.73
2.0	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.88

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