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R. Sampath Kumar
Government Arts College, Coimbatore, India

M. Indra
Muthayammal College of Arts & Science, Rasipuram, India

R. Radhakrishnan
PSG College of Arts & Science, Coimbatore, India

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Selection of Mixed Sampling Plan with QSS-1(n; c_N, c_T) Plan as Attribute Plan Indexed Through MAPD and LQL

R. Sampath Kumar
Government Arts College,
Coimbatore, India

M. Indra
Muthayammal College of
Arts & Science,
Rasipuram, India

R. Radhakrishnan
PSG College of Arts & Science,
Coimbatore, India

A procedure for the construction and selection of the mixed sampling plan using MAPD as a quality standard with the QSS-1 (n; c_N, c_T) plan as an attribute plan is presented. The plans indexed through MAPD and LQL are constructed and compared for efficiency. Tables are provided for selection of an appropriate sampling plan.

Key words: Limiting quality level, maximum allowable percent defective, operating characteristic, tangent intercept.

Introduction

Mixed sampling plans consist of two stages with different natures. During the first stage a given lot is considered as a sample from the respective production process and a criterion by variables is used to check process quality. If process quality is judged to be sufficiently good then the lot is accepted, if not, the second stage of the sampling plan is entered and lot quality is checked directly by means of an attribute sampling plan. There are two types of mixed sampling plans called independent and dependent plans. If the first stage sample results are not utilized in the second stage, then the plan is said to be independent, otherwise it is considered to be dependent. The principal advantage of a mixed sampling plan over a pure attribute sampling plan is a reduction in sample size for a similar amount of protection.

The second stage attribute inspection becomes more important to discriminate the lot if the first stage variable inspection fails to accept the lot. If rejection occurs during the normal inspection, tightened inspection is recommended in the mixed system and vice versa in the second stage. Hence Quick Switching System is imposed in the second stage to sharpen the sampling situation and to insist the producer to manufacture goods within the Limiting Quality Level. Dodge (1967) proposed a sampling system called a 'Quick Switching System' (QSS) consisting of pairs of normal and tightened plans.

Schilling (1967) proposed a method for determining the operating characteristics of mixed variables – attributes sampling plans, single sided specification and standard deviation known using the normal approximation.

R. Sampath Kumar is an Assistant Professor in the Department of Statistics. He has 12 years of teaching experience in theoretical and applied Statistics, has presented 13 papers in both national and international conferences and has published 10 articles in reputed national and international journals. Email him at: rsampath1969@gmail.com. M. Indra is an Assistant Professor in the Department of Statistics. She has been teaching statistics for 7 years. Email: indra_m1980@rediffmail.com. R. Radhakrishnan is an Associate Professor in the Department of Statistics. He has 31 years of experience teaching theoretical and applied statistics, has presented more than 150 papers at national and international conferences and has published more than 100 articles. He is a quality auditor for ISO certification and a certified Six Sigma Black Belt. Email him at: rkrishnan_research@yahoo.com.

Devaarul (2003), Radhakrishnan and Sampath Kumar (2006a, 2006b, 2007a, 2007b, 2009) have investigated mixed sampling plans for the independent case. Radhakrishnan, et al. (2009) studied mixed sampling plan for the dependent case. Quick Switching System (QSS) were originally proposed by Dodge (1967) and have been investigated by Romboski (1969) and Govindaraju (1991). Dodge (1967) proposed a new sampling system consisting of pairs of normal and tightened plans. Romboski (1969) developed a QSS by attributes with a reduction in the sample size required to achieve approximately the same operating characteristic curve.

This study uses the operating procedure of mixed sampling plan with a QSS-1 (n;c_N,c_T) plan as an attribute plan to construct tables for a mixed sampling plan indexed through (i) maximum allowable percent defective (MAPD), and (ii) limiting quality level (LQL). The plan indexed through MAPD is compared to the plan indexed through LQL.

Conditions of Applications of QSS-1-Mixed Sampling Plan

The following assumptions are made with respect to the application conditions of a QSS-1 mixed sampling plan:

- Production is steady so that results regarding current and preceding lots are broadly indicative of a continuing process.
- Lots are submitted substantially in the order of their production.
- Inspection involves costly or destructive tests such that normally only a small number of tests per lot can be justified.

Glossary of Symbols

The symbols used in this article are:

p: submitted quality of lot or process;

P_a (p):probability of acceptance for given quality 'p';

P_2: the submitted quality such that P_a (p_2) = 0.10 (also called LQL);

p_*: maximum allowable percent defective (MAPD);

h_*: relative slope at 'p_*';

n_1: sample size of variable sampling plan;

n_2: sample size of attribute sampling plan;

c_N: acceptance number of normal inspection;

c_T: acceptance number of tightened inspection;

β_j: probability of acceptance for lot quality 'p_j';

β_j': probability of acceptance assigned to first stage for percent defective 'p_j';

β_j'': probability of acceptance assigned to second stage for percent defective 'p_j';

d: observed number of nonconforming units in a sample of n units;

z(j): 'z' value for the j^{th} ordered observation; and

k: variable factor such that a lot is accepted if $\bar{X} \le A = U - k\sigma$.

Operating Procedure of Mixed Sampling Plan with QSS-1(n;cN,cT) as Attribute Plan

Schilling (1967) provided the following procedure for the independent mixed sampling plan with the upper specification limit (U) and known standard deviation (σ).

- Determine the parameters of the mixed sampling plan n_1, n_2, k, c_N and c_T.
- Select a random sample of size n_1 from the lot.
- If a sample average $\bar{X} \le A = U - k\sigma$, accept the lot.
- If a sample average $\bar{X} > A = U - k\sigma$, go to step 1.

Step 1: From a lot, take a random sample of size n_2 at the normal level. Count the number of defectives, d:

- If d ≤ c_N, accept the lot and repeat step 1;

- If $d > c_N$, reject the lot and go to step 2.

Step 2: From the next lot, take a random sample of size n_2 at the tightened level. Count the number of defectives, d:

- If $d \leq c_T$, accept the lot and use step 1 for the next lot;
- If $d > c_T$, reject the lot and repeat step 2 for the next lot.

Construction of Mixed Sampling Plan having QSS-1($n;c_N,c_T$) as Attribute Plan

The operation of mixed sampling plans can be properly assessed by the OC curve for given values of the fraction defective. The development of mixed sampling plans and the subsequent discussions are limited only to the upper specification limit, U. A parallel discussion can be made for lower specification limits.

The procedure for the construction of mixed sampling plans is provided by Schilling (1967) for a given n_1 and a point p_j on the OC curve is:

- Assume that the mixed sampling plan is independent.
- Split the probability of acceptance (β_j) determining the probability of acceptance that will be assigned to the first stage, term this β_j'.
- Determine the sample size n_1 (for variable sampling plan) to be used.
- Calculate the acceptance limit for the variable sampling plan as:

$$\mathrm{A} = \mathrm{U} - \mathrm{k}\sigma = \mathrm{U} - [\mathrm{z}(\mathrm{p_j}) + \{\mathrm{z}(\beta_j{}') / \sqrt{n_1}\}]\sigma$$

where U is the upper specification limit and z(t) is the standard normal variate corresponding to t such that

$$t = \int_{z(t)}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du.$$

- Determine the sample average, $\overline{X}$. If a sample average $\overline{X} >$ A = U −kσ, take a second stage sample of size n_2 using attribute sampling plan.
- Split the probability of acceptance β_j as β_j' and β_j'', such that $\beta_j = \beta_j' + (1 - \beta_j')\beta_j''$. Fix the value of β_j'.
- Determine β_j'', the probability of acceptance assigned to the attributes plan associated with the second stage sample as $\beta_j'' = (\beta_j - \beta_j')/(1 - \beta_j')$.
- Determine the appropriate second stage sample of size n_2 from $Pa(p) = \beta_j''$ for $p = p_j$.

Using this procedure, tables can be constructed to facilitate selection of an appropriate mixed sampling plan with QSS-1($n;c_N,c_T$) plan as an attribute plan indexed through MAPD and LQL.

According to Soundararajan and Arumainayagam (1988), the operating characteristic function of QSS-1 is:

$$P_a(p) = \frac{b}{1 - a + b} \tag{1}$$

where

$$a = \sum_{i=0}^{c_N} \frac{e^{-n_2 p}(n_2 p)^i}{i!} \tag{2}$$

and

$$b = \sum_{j=0}^{c_T} \frac{e^{-n_2 p}(n_2 p)^j}{j!} \tag{3}$$

(for acceptance number tightening).

Construction of Sampling Plans Indexed Through MAPD

MAPD (p_*), introduced by Mayer (1967) and further studied by Soundararajan (1975), is the quality level corresponding to the inflection point of the OC curve. The degree of sharpness of inspection about this quality level p_* is measured by p_t, the point at which the tangent to the OC curve at the inflection point cuts the proportion defective axis. For designing a mixed sampling plan, Soundararajan (1975) proposed a selection procedure indexed with MAPD and $K = p_t/p_*$.

Using the probability mass function of QSS-1 (see expression (1)), the inflection point (p_*) is obtained using

$$\frac{d^2 p_a(p)}{dp^2} = 0$$

and

$$\frac{d^3 p_a(p)}{dp^3} \neq 0 .$$

The relative slope of the OC curve is

$$h_* = \left[\frac{-p}{p_a(p)}\right]\frac{dp_a(p)}{dp}$$

at $p = p_*$. The inflection tangent of the OC curve cuts the p axis at $p_t = p_* + (p_*/h_*)$. The values of n_2p_*, h_*, n_2p_t and $R = p_t/p_*$ are calculated for different values of c_N and c_T for $\beta_*' = 0.04$ using a c++ program (see Table 1).

Selection of the Plan

For the given values of p_* and p_t, the ratio $R = \frac{p_t}{p_*}$ is found and the nearest value of R is located in Table 1.The corresponding value of c_N, c_T and np_* values are noted and the value of n_2 is obtained using $n_2 = \frac{n_2 p_*}{p_*}$.

Example 1

Given $p_* = 0.037$, $p_t = 0.051$ and $\beta_*' = 0.04$, the ratio $R = \frac{p_t}{p_*} = 1.3784$. As shown in Table 1, the nearest R value is 1.3791 which corresponds to $c_N = 5$ and $c_T = 1$. The value $n_2p_* = 3.3452$ is found, hence the value of n_2 is determined to be $n_2 = \frac{n_2 p_*}{p_*} = \frac{3.3452}{0.037} = 90$. Thus $n_2 = 90$, $c_N = 5$ and $c_T = 1$ are the parameters selected for the mixed sampling plan having QSS-1($n;c_N,c_T$) as an attribute plan using the Poisson distribution as a baseline distribution for the given values of $p_* = 0.037$ and $p_t = 0.051$.

Construction of Mixed Sampling Plan indexed through LQL

The described procedure is used to construct the mixed sampling plan indexed through LQL(p_2). Assuming the probability of acceptance of the lot be $\beta_2 = 0.10$ and $\beta_2' = 0.04$, the n_2p_2 values are calculated for different values of c_N and c_T using a c++ program (see Table 1).

Selection of the Plan

Table 1 is used to construct the plans when LQL(p_2), c_N and c_T are given. For any given values of p_2, c_N and c_T one can determine n_2 value using $n_2 = \frac{n_2 p_2}{p_2}$.

Example 2

Given $p_2 = 0.06$, $c_N = 3$ and $c_T = 1$and $\beta_2' = 0.04$. Using Table 1, find $n_2 = \frac{n_2 p_2}{p_2} = \frac{4.8136}{0.06} = 80$. Thus $n_2 = 80$, $c_N = 3$ and $c_T = 1$ are the parameters selected for the mixed sampling plan having QSS-1($n;c_N,c_T$) as attribute plan for a specified $p_2 = 0.06$, $c_N = 3$ and $c_T = 1$.

Selection of the Plan

Table 1 is used to construct the plans when LQL (p_2), 'c_N' and 'c_T' are given. For any given values of p_2, c_N and c_T one can determine n_2 value using $n_2 = \frac{n_2 p_2}{p_2}$.

Comparison of Mixed Sampling Plan Indexed through MAPD and LQL

By fixing parameters c_N, c_T and β_j' for specified values of p_* and p_t and assuming $\beta_*' = 0.04$, the values of c_N, c_T and n_2 indexed through MAPD can be determined. Fixing the values of c_N and c_T, the value of p_2 is found by equating Pa (p) = $\beta_2 = 0.10$. Using $\beta_2' = 0.04$, c_N and c_T the value of n_2 is determined using $n_2 = \frac{n_2 p_2}{p_2}$ from Table 1. Using different combinations ofp_*, p_t, c_N and c_T, the values of n_2 (indexed through MAPD) and n_2 (indexed through LQL) calculated are presented in Table 2.

Table 1: Various Characteristics of the Mixed Sampling Plan when $\beta_*' = \beta_2' = 0.04$ and $\beta_2 = 0.10$

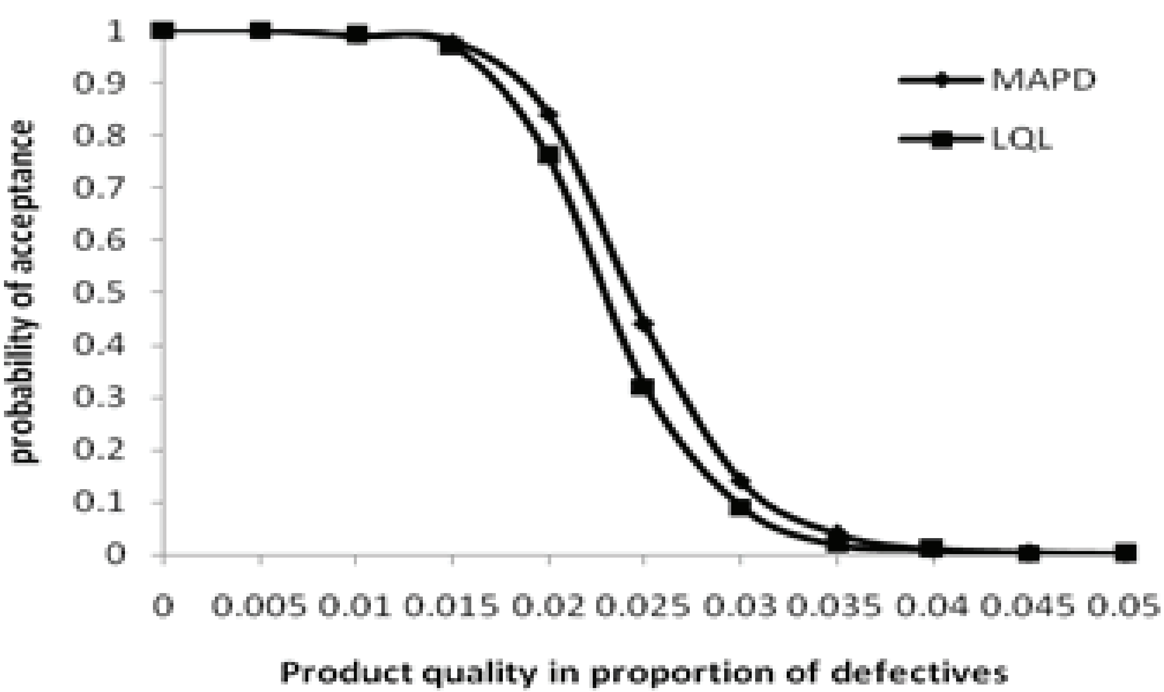
c_N	c_T	n_2p_2	β_*''	n_2p_*	h_*	n_2p_t	$R = p_t/p_*$
1	0	2.9412	0.6690	0.8538	0.7703	1.9622	2.2982
2	1	4.6086	0.6261	1.8158	1.1034	3.4614	1.9063
2	0	3.1894	0.6095	1.3787	1.3370	2.4099	1.7480
3	2	6.1101	0.6044	2.7979	1.3622	4.8519	1.7341
3	1	4.8136	0.5897	2.3815	1.6231	3.8488	1.6161
3	0	3.4844	0.5811	1.8374	1.8735	2.8181	1.5337
6	5	10.2286	0.5734	5.7797	1.9506	8.7427	1.5127
5	3	7.6959	0.5692	4.3733	2.0587	6.4976	1.4857
7	6	11.5328	0.5670	6.7811	2.1146	9.9879	1.4729
6	4	9.0528	0.5628	5.3702	2.2404	7.7672	1.4464
4	0	3.8063	0.5630	2.2695	2.4041	3.2135	1.4160
6	3	7.9097	0.5556	4.9005	2.5342	6.8342	1.3946
5	1	5.3626	0.5554	3.3452	2.6376	4.6135	1.3791
9	7	12.9515	0.5500	8.3665	2.7059	11.4584	1.3696
6	2	6.7933	0.5500	4.3779	2.8356	5.9218	1.3527
5	0	4.1442	0.5508	2.6858	2.9293	3.6027	1.3414
9	6	11.8549	0.5443	7.9073	3.0102	10.5341	1.3322
6	1	5.6790	0.5455	3.7913	3.1439	4.9972	1.3181
8	4	9.4914	0.5425	6.4072	3.1740	8.4259	1.3151
9	5	10.7925	0.5396	7.4154	3.3249	9.6457	1.3008
8	3	8.4444	0.5384	5.8715	3.4952	7.5514	1.2861
7	1	6.0117	0.5379	4.2248	3.6519	5.3817	1.2738
8	2	7.4095	0.5350	5.2914	3.8241	6.6751	1.2615
7	0	4.8464	0.5349	3.4926	3.9742	4.3714	1.2516
8	1	6.3558	0.5329	4.6471	4.1519	5.7664	1.2409
9	2	7.7425	0.5297	5.7313	4.3226	7.0572	1.2313
9	1	6.7080	0.5272	5.0664	4.6691	6.1515	1.2142
9	0	5.5669	0.5243	4.2808	5.0222	5.1332	1.1991
11	2	8.4386	0.5218	6.5894	5.3246	7.8269	1.1878
11	1	7.4283	0.5453	5.8361	5.3854	6.9198	1.1857
12	0	6.6634	0.5217	5.4307	6.4898	6.2675	1.1541

Table 2: Comparison of Plans Indexed Through MAPD and LQL

p_*	p_t	c_N	c_T	Indexed Through	
				MAPD	LQL
				n_2	n_2
0.024*	0.030	7	0	146	155
0.042	0.058	6	3	117	126
0.045	0.062	5	1	74	80
0.048	0.068	4	0	47	51

*OC curves are drawn.

Figure 1: OC Curves for QSS-1(146;7,0) and (155;7,0)



OC Curve Construction

The OC curves for the plans n_2 = 146, c_N= 7, c_T = 0 (indexed through MAPD) and n_2 = 155, c_N = 7, c_T= 0 (indexed through LQL) based on the different values of n_2p_2 and $p_a(p)$ are presented in Figure 1.

Conclusion

This article used the operating procedure of a mixed sampling plan with QSS-1(n; c_N, c_T) as an attribute plan and constructed tables for a mixed sampling plan indexed through parameters MAPD and LQL using the Poisson distribution as a baseline. It may be concluded based on study results that the second stage sample size required for a QSS-1(n; $c_{N,}$ c_T) plan indexed through MAPD is less than that of a second stage sample size of the QSS-1(n; $c_{N,}$ c_T) plan indexed through LQL. Examples were provided for a specified value of $\beta_j' = 0.04$. If engineers know the levels of MAPD or LQL they can reference the tables provided to determine their sampling plans on site at a factory; this provides flexibility to floor engineers in determining appropriate sampling plans. Various plans can also be constructed to make a system user-friendly by changing the first stage probabilities (β_*', β_2') and can also be compared for their efficiency.

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