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Examining Cronbach Alpha, Theta, Omega Reliability Coefficients According to the Sample Size

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Differentiations according to the sample size of different reliability coefficients are examined. It is concluded that the estimates obtained by Cronbach alpha and teta coefficients are not related with the sample size, even the estimates obtained from the small samples can represent the population parameter. However, the Omega coefficient requires large sample sizes.

Key words: Cronbach alpha, theta, omega, reliability, scale, sample size.

Introduction

A scale is needed to measure and that scale must be reliable and valid. The scale's reliability does not matter in the case of measuring the concrete characteristics. But, it is an important problem in the case of measuring the abstract characteristics. So, it is necessary to analyze the reliability of the scales using some statistical

methods. In making a reliability analysis, the reliability coefficients that are suitable in obtaining the reliability of the scale and the structure of the empirical study must be examined. Sample size is also important to determine the reliability level of the scale. Thus, one of the dimensions that must be examined is the changes in Cronbach alpha, theta, and omega coefficients according to the sample size.

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Reliability

The scale, used to get some information on a defined subject, must have some properties. Reliability, a property that a scale must have, is an indicator of consistency of measurement values obtained from the measurements repeated under the same circumstances (Gay, 1985; Carmines & Zeller, 1982; Arkin & Colton, 1970; O'Connor, 1993; Carey, 1988).

The reliability of the scale can be examined by different ways. The reliability of the scale can be examined by applying the scale once, applying the scale twice or applying the equivalent scales once. In case of applying the scale once, the reliability of internal consistency is examined. The reliability coefficient ranges between 0 and 1.

Methods of Internal Consistency

If the reliability can be estimated by applying the scale once, the error in reliability estimation will be less than the other reliability estimation methods. In this kind of reliability estimation, wrong management, scoring, temporary changes in personal performance affect the internal consistency, the leading affect will be the content sampling (O'Connor, 1993).

Another method, split-half, denotes the homogeneity indices of the items in the scales. It pertains to the relationship level between the responses of the items and the total scale score (Oncu, 1994). An increase in homogeneity in the set of items increases this reliability estimate (O'Connor, 1993). The idea that the internal consistency methods depend upon is that every measurement tool is constructed to realize an objective and those have known equal weights (Karasar, 2000). The internal consistency methods are preferred because they are economical and easy to apply (Oncu, 1994).

Cronbach Alpha

The Alpha coefficient method (Cronbach, 1951), is a suitable method that can be used for likert scale items (e.g., 1-3, 1-4, 1-5). Thus, it is not limited to the true-false or correct-incorrect format (Oncu, 1994).

Cronbach alpha coefficient is weighted standard variations mean, obtained by dividing the total of the k items in the scale, to the general variance (Thorndike et al., 1991).

$$\alpha = \frac{n}{(n-1)} \left[1 - \frac{\sum_{i=1}^n \sigma_{Y_i}^2}{\sigma_x^2} \right] \quad (2.1)$$

n : Number of the items

σ_{Y_i} : ith item's standard deviation

σ_x : General standard deviation

If the items are standardized, coefficient is calculated by using the items' correlation mean or variance-covariances' mean (Carmines & Zeller, 1982; Ozdamar, 1999a; SPSS, 1991; SPSS, 1999).

Calculation of alpha coefficient due to the correlation mean,

$$\alpha = \frac{n \bar{\rho}}{1 + (n-1)\bar{\rho}} \quad (2.2)$$

Calculation of alpha coefficient due to the variance-covariance mean,

$$\alpha = \frac{n \overline{\sigma_{XX}} / \overline{\sigma_X}}{1 + (n-1) \overline{\sigma_{XX}} / \overline{\sigma_X}} \quad (2.3)$$

When the formula for calculating Cronbach alpha using the correlation means between items is examined, it can be seen that it is proportionally related with the number of the items and the mean of the correlation between items (Carmines & Zeller, 1982). If the correlation between the items is negative, alpha coefficient will also be negative. Because this situation will spoil the scale's additive property, it also causes a spoil in the reliability model and the scale is no more additive (Ozdamar, 1999a). The coefficient is equal to the mean of all probable coefficients using split-half method (Carmines & Zeller, 1982; Gursakal, 2001).

Theta Coefficient

The Theta coefficient depends on the principal components analysis. In principal components analysis, the components are in descending order due to the variances of each of the constructions (Carmines & Zeller, 1982). The first component is the linear component with the maximum variance. The second component is the linear component with the second maximum variance. Components can be explained by the component variances defined by the percentage values to explain the variance of the original data set in order (Ozdamar, 1999b). Theta coefficient depends on that property. The Theta coefficient, takes into account the eigenvalue that maximum explains the event, is calculated as follows:

$$\theta = (N / N - 1)(1 - 1 / \lambda_i)$$

N : Number of items

λ_i : The largest eigenvalue (the first eigenvalue)

$$(2.4)$$

Omega Coefficient

Another coefficient for linear dependencies is the Omega coefficient proposed by Heise and Bohrnstedt (1970). It depends on the factor analysis model. Omega coefficient is modeled on factor analysis. In this type of modeling, in calculating the coefficient, before factoring “1” values on diagonal in the correlation matrix are replaced with the communality values. The Omega coefficient can be calculated with two ways, using variance-covariance matrix and correlation matrix (Carmines & Zeller, 1982).

When studied with variance-covariance matrix,

$$\Omega = 1 - \left(\sum \sigma_i^2 - \sum \sigma_i^2 h_i^2 \right) / \left(\sum \sum \sigma_{x_i x_j} \right)$$

h_i^2 : Communality of the i^{th} item

$$(2.5)$$

When studied with correlation matrix,

$$\Omega = 1 - \left(a - \sum h_i^2 \right) / (a + 2b)$$

a: Number of items

b: Sum of the correlations among items

$$(2.6)$$

There are some differences between the Theta and Omega coefficients. They depend on different factor-analytic models. The Theta coefficient depends on principal components model, whereas the Omega coefficient depends on factor analysis model. Therefore, in calculating the eigenvalues for Theta coefficients, the diagonal 1.0 values are used, but in calculating the Omega coefficients,

communality values that are not related with 1.0 values are used (Carmines & Zeller, 1982).

There is a relationship between Alpha, Theta, and Omega coefficients. If the items take parallel values, three coefficients are equal each other and will be 1.0. Otherwise, the relationship of magnitude for the coefficients will be $\alpha < \theta < \Omega$. Among these internal consistency coefficients, α gives the lower bound of the reliability coefficient and Ω gives the upper bound of the reliability coefficient (Carmines & Zeller, 1982).

Methodology

To compare the Alpha, Theta and Omega coefficients, a data set has been used from an instrument developed by Ercan et al. (2004) to measure patient satisfaction in the secondary health-care units. To obtain the effects of different number of items and different sample sizes, 3 different scales are constructed with 39, 34, and 30 items by subtracting some items from the scale with 43 items. Because all the subjects did not answer all the items, the subject numbers in the scales are also different. There are 170 subjects answered all of the 43 items, 240 subjects answered all of the 39 items, 230 subjects answered all of the 34 items, and 320 subjects answered all of the 30 items.

After giving a number to each of the subjects, samples are constructed by producing random numbers using MINITAB 13.2 beginning from 10 and increasing 10 units each of those random numbers. The same procedure was repeated 10 times and for each of the samples Cronbach alpha, Theta and Omega reliability coefficients are calculated.

SPSS 13.0 was used for these analyses. Statistical comparisons are performed in order to determine if alpha, theta and omega coefficients change or not according to the sample size and in order to determine the sample size that the reliability coefficients begin to get stable. Before the between group comparisons, the homogeneity of variances is tested using the Levene statistic. If the variances are found to be homogeneous, then analysis of variance

Table-4.1: The homogeneity test results for the scale with 30 items

	Levene Statistic	Degree of Freedom 1	Degree of Freedom 2	Significance level (p)
α	5.631	31	288	<0.001
θ	5.578	31	288	<0.001
Ω	1.531	31	288	0.040

Table-4.2: Significance level in comparison of α , θ and Ω reliability coefficients according to different sample sizes using Kruskal-Wallis test for the scale with 30 items

	α	θ	Ω
χ^2	23.706	46.720	259.636
Degree of freedom	31	31	31
Significance level (p)	0.822	0.035	<0.001

Bonferroni correction: $\alpha^* = 1 - (1 - \alpha)^{1/k}$

$$\alpha^* = 1 - (1 - 0.05)^{1/32} = 0.0016$$

and Tukey HSD post-hoc comparison test are applied. If the variances are heterogeneous, Kruskal-Wallis and Mann-Whitney U tests are applied to make reliability comparisons according to sample size. The level of significance in multiple comparisons is determined after Bonferroni correction ($\alpha^* = 1 - (1 - \alpha)^{1/k}$ k: number of groups).

Results

The results of comparisons α , θ and Ω coefficients according to different sample sizes are given in Table 4.1, 4.2, 4.3, 4.4 for the scale with 30 items.

The results of comparisons α , θ and Ω coefficients according to different sample sizes are given in Table 4.5, 4.6, 4.7 for the scale with 34 items.

Table-4.5: The homogeneity test results for the scale with 34 items

	Levene Statistic	Degree of freedom 1	Degree of freedom 2	Significance level (p)
α	11.003	22	207	<0.001
θ	10.477	22	207	<0.001
Ω	3.238	22	207	<0.001

Table-4.6: Significance level in comparison of α , θ and Ω reliability coefficients according to different sample sizes using Kruskal-Wallis test for the scale with 34 items

	α	θ	Ω
χ^2	6.329	8.960	176.741
Degree of freedom	22	22	22
Significance level (p)	1.000	0.994	<0.001

Bonferroni correction: $\alpha^* = 1 - (1 - \alpha)^{1/k}$

$$\alpha^* = 1 - (1 - 0.05)^{1/23} = 0.0022$$

The results of comparisons α , θ and Ω coefficients according to different sample sizes

are given in Table 4.8, 4.9, 4.10, 4.11 for the scale with 39 items.

Table-4.8 : The homogeneity test results for the scale with 39 items

	Levene Statistic	Degree of freedom 1	Degree of freedom 2	Significance level (p)
α	10.692	23	216	<0.001
θ	12.048	23	216	<0.001
Ω	1.418	23	216	0.104

Table-4.9: Significance level in comparison of α and θ reliability coefficients according to different sample sizes using Kruskal-Wallis test for the scale with 39 items

	α	θ
χ^2	7.206	8.702
Degree of freedom	23	23
Significance level (p)	0.999	0.997

Table-4.10: Significance level in comparison of Ω reliability coefficients according to different sample sizes by analysis of variance for the scale with 39 items

	Sum of Squares	Degrees of freedom	Sum of Squares	F	Significance level (p)
Between groups	0.00536	23	0.0002329	142.881	<0.001
Within groups	0.000352	216	0.00000163		
Total	0.00571	239			

Bonferroni correction: $\alpha^* = 1 - (1 - \alpha)^{1/k}$

$$\alpha^* = 1 - (1 - 0.05)^{1/24} = 0.0021$$

The results of comparisons α , θ and Ω coefficients according to different sample sizes

are given in Table 4.12, 4.13, 4.14 for the scale with 43 items.

Table-4.12: The homogeneity test results for the scale with 43 items

	Levene Statistic	Degree of freedom 1	Degree of freedom 2	Significance level (p)
α	6.313	16	153	<0.001
θ	7.654	16	153	<0.001
Ω	2.463	16	153	0.002

Table-4.13: Significance level in comparison of α , θ and Ω reliability coefficients according to different sample sizes using Kruskal-Wallis test for the scale with 43 items

	α	θ	Ω
χ^2	11.248	7.026	141.750
Degree of freedom	16	16	16
Significance level (p)	0.794	0.973	<0.001

Bonferroni correction: $\alpha^* = 1 - (1 - \alpha)^{1/k}$

$$\alpha^* = 1 - (1 - 0.05)^{1/17} = 0.003$$

Table-4.14: Significance level (p values $\times 10^{-3}$) in comparison of Ω reliability coefficients according to different sample sizes using Mann-Whitney U test for the scale with 39 items ($\alpha^*=0.003$)

n	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170
10		000	000	000	000	000	000	000	000	000	000	000	000	000	000	000	000
20			000	000	000	000	000	000	000	000	000	000	000	000	000	000	000
30				143	000	000	000	000	000	000	000	000	000	000	000	000	000
40					001	000	000	000	000	000	000	000	000	000	000	000	000
50						353	015	002	000	000	000	000	000	000	000	000	000
60							123	011	003	000	000	000	000	000	000	000	000
70								218	089	009	003	000	000	000	000	000	000
80									353	105	105	005	001	000	000	000	000
90										393	315	035	015	009	002	001	002
100											912	280	190	089	035	029	023
110												280	165	105	035	029	009
120													631	481	190	075	023
130														912	579	123	043
140															436	218	063
150																481	165
160																	481
170																	

Conclusion

The answer to the question of sample size in this context is important. The accuracy of reliability coefficients changes according to the sample size. There is high positive correlation between number of items and reliability coefficient as mentioned in Carmines and Zeller (1982). Also, the difference in number of items must be taken into account.

Significant differences are not observed due to the sample size in the commonly used Cronbach Alpha, and with the Theta coefficient which is based on principal components. However, with the Omega coefficient, based on factor analysis, large differences were observed due to the sample size. With an increase in item numbers, however, the Omega coefficient is stabilized even for smaller sample sizes.

Ozdamar (1999a) mentioned that the sample size should be more than 50 in reliability

analysis applications. According to the results of this study, that sample size is not important for the Cronbach alpha or theta coefficients, and is stable even for a small number of items (although of course an increase in the number of items will increase the magnitude.) However, in order to estimate the population parameter with Omega coefficient, the item number is important. With an increase in item number, either the consistency of estimation or the reliability level increases.

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