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Abstract

Attempts to describe language competition and extinction in a mathematical way have enjoyed increased popularity recently. In this paper I review recent modeling approaches and, based on these findings, propose a model of reaction-diffusion type. I analyze the dynamics of interactions of a population with two monolingual groups and a group that is bilingual in these two languages. The results show that demographic factors, such as population growth or population dispersal, play an important role in the competition dynamic. Furthermore, I consider the impact of two strategies for language maintenance: adjusting the status of the endangered language and adjusting the availability of monolingual and bilingual educational resources.

Keywords

language competition, language extinction, language maintenance, population growth, population dispersal, reaction dispersal competition model, agent-based models.

Cover Page Footnote

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Demography and Language Competition

ANNE KANDLER[1]

Abstract Attempts to describe language competition and extinction in a mathematical way have enjoyed increased popularity recently. In this paper I review recent modeling approaches and, based on these findings, propose a model of reaction-diffusion type. I analyze the dynamics of interactions of a population with two monolingual groups and a group that is bilingual in these two languages. The results show that demographic factors, such as population growth or population dispersal, play an important role in the competition dynamic. Furthermore, I consider the impact of two strategies for language maintenance: adjusting the status of the endangered language and adjusting the availability of monolingual and bilingual educational resources.

Language competition and death is a phenomenon that can be observed worldwide. Linguists estimate that there are 5,000–6,700 languages in the world today, but because of an explosive spread of a few dominant languages (e.g., English or Chinese), at least half of them will become extinct in the 21st century (Krauss 1992). The processes that lead to the disappearance of languages have greatly accelerated over the past 200 years, and this worrying rate of extinction is probably unique to our time (Grenoble and Whaley 2006). A number of different socioeconomic, political, and cultural factors can be identified as driving this decline of linguistic diversity. In the course of globalization and of recent trends for urbanization and long-distance economic migration, interactions between groups that speak different languages have increased and so has the need for a common language of communication. Some languages (e.g., English) have come to fill that role for historical, economic, and hegemonic reasons and, as a consequence, have risen in importance in official and nonofficial matters. Their lexicons have consequently expanded to represent all the paraphernalia of modernization, further enhancing their competitive advantage. In contrast, minority languages are particularly subject to pressure and are at risk of extinction, mainly because speakers perceive an economic gain from shifting (Mufwene 2002). A number of prominent linguists have called for an ecological approach to this global linguistic "extinction crisis"

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KEY WORDS: LANGUAGE COMPETITION, LANGUAGE EXTINCTION, LANGUAGE MAINTENANCE, POPULATION GROWTH, POPULATION DISPERSAL, REACTION-DISPERSAL COMPETITION MODEL, AGENT-BASED MODELS.

(however, they differ in what they mean by that term) and for the development of a therapeutic understanding for the process of extinction (Fishman 2001). Language extinction may be caused by the death of the population speaking the language or by language shift (Tsunoda 2004).

In this paper I focus on language shift, which is defined as the process by which members of a community in which more than one language is spoken abandon their original language in favor of another (Tsunoda 2004). The reasons for language shift are complex, and Fishman (1964: 49) has stated that "it is currently impossible to specify in advance an invariant list of psychological, social, and cultural processes or variables that might be of universal importance for an understanding of language maintenance or language shift." According to Crawford (1996), there seems to be no established and comprehensive theory of language shift, especially in terms of causes and varying conditions that might prevent them. Abrams and Strogatz (2003) addressed the problem of language shift and extinction from a different perspective by characterizing and modeling the dynamics of language competition in a mathematical way. They presented a two-language competition model to explain historical data on the decline of endangered languages. The mathematical simplicity and, despite some obvious unrealistic assumptions, convincing fits of their model to empirical data have generated a burst of attempts to model the dynamics of language competition.

I start with an overview of recent mathematical modeling approaches in the field of language competition (I try to use, as far as possible, a common notation throughout the description of the different models to make them comparable). However, I do not claim that this is a complete overview.

The reviewed models still lack linguistic reality and are often criticized by linguists. I propose and analyze a reaction-dispersal competition system. Nevertheless, the proposed model does not solve all the problems, and I address some critiques. I analyze the resulting competition dynamic and am interested in certain characteristics, such as extinction time. In addition, I demonstrate the effects of demographic factors, such as initial abundance, population growth, and dispersal, on the language competition and investigate the crucial aspect of language maintenance.

Recent Modeling Approaches

Abrams and Strogatz (2003) generated a burst of modeling attempts into the dynamic of language competition. However, in 1990 Baggs and Freedman had already published a (mainly overlooked) model based on the predator-prey paradigm for describing a situation in which a bilingual population group interacts with a monolingual population group. The main idea is that the dynamics of the growth of the bilingual and monolingual populations is determined by bounded birth-death processes with separate carrying capacities for each population group, a "conversion" mechanism, and emigration processes. Baggs and Freedman (1990) proposed the following model:

$$\frac{\partial n_A}{\partial t} = \left(B_A - D_A - H_A\right) n_A - L_A n_A^2 - \frac{\alpha\, n_A n_C}{1 + n_A} + P_A B_C n_C, \tag{1a}$$

$$\frac{\partial n_C}{\partial t} = \left(B_C - D_C - H_C\right) n_C - L_C n_C^2 + \frac{\alpha\, n_A n_C}{1 + n_A} - P_A B_C n_C, \tag{1b}$$

where n_A is the frequency of monolingual individuals speaking language A and n_C is the frequency of bilingual individuals in the population. The coefficients B_i, D_i, and H_i describe the birth, death, and emigration rates of each group, respectively. The parameter P_A defines the rate at which children of bilingual parents are raised monolingual [denoted as the infant language acquisition parameter by Wyburn and Hayward (2008)], and α is the rate at which monolingual individuals convert to being bilingual, that is, the rate at which they learn the second language [denoted as the noninfant language acquisition parameter by Wyburn and Hayward (2008)].

The model in Eqs. (1) suggests that coexistence between the population groups is possible if the conversion rate α and the rate at which children of bilingual parents are raised monolingual P_A are both moderate. Only extreme parameter values lead to the extinction of one of the language groups. Furthermore, Baggs and Freedman (1990) found that small emigration rates do not alter the competition dynamics; however, if either group experiences large emigration, then it is possible that this population group will go extinct.

Baggs and Freedman (1993) generalized their model and considered the dynamics of the interactions of a population with two monolingual groups (in which one language is assumed to be the high-status language) and a group that is bilingual in these two languages in a closed environment. They concluded that different environmental conditions favor different dynamics. They found that, besides the extinction of the monolingual group speaking the low-status language and of the bilingual group, coexistence between all three population groups and coexistence between the monolingual group speaking the high-status language and the bilingual group were possible. El-Owaidy and Ismail (2002) generalized this model by incorporating a third monolingual population group.

Recently, Wyburn and Hayward (2008) revisited Baggs and Freedman's (1993) model. They analyzed the outcomes of the Baggs-Freedman model for varying conversion rates α and defined four different scenarios (bilingual death, bilingual prestige, bilingual maintenance, bilingual shift) in the long-term future of the bilingual population. Furthermore, they applied the Baggs-Freedman model to various real-life situations (e.g., the English and Welsh-English competition in Wales). After estimating the model parameter from external sources, Wyburn and Hayward (2008) predicted the state of the bilingual population group, that is, which of the four long-term scenarios would be most likely. In most cases the prediction coincided with the observed situation. In addition, Wyburn and Hoyward (2009) applied a model based on operational research methodology to the problem of language competition and language planning.

Abrams and Strogatz (2003) developed a simple deterministic model for describing the changes in the patterns of language usage within a population in which two languages compete. Their aim was to explain historical data on the decline of endangered languages and to quantify indicators of language endangerment so that useful language-preservation programs could be set up at an early stage. They defined language competition as competition for speakers; language itself was assumed to be fixed. The dynamics of the competition is described by the temporal change of the proportion of speakers of both languages, which results in

$$\frac{dn_A}{dt} = n_B P_{BA}(n_A, s_A) - n_A P_{AB}(n_B, s_B). \qquad (2)$$

The terms n_A and n_B stand for the frequencies of speakers of language A and B, respectively, and it is assumed that $n_A + n_B = 1$. This condition implies a fixed population size. The term P_{BA} models the rate at which an individual shifts from language B to language A per time unit. Abrams and Strogatz (2003) assumed that this shift rate is determined by the attractiveness of language A, the target of shifting. They defined attractiveness by the proportion of speakers of language A and its social status s_A and described the shift rate P_{BA} with the power law

$$P_{BA}(n_A, s_A) = c n_A^a s_A. \qquad (3)$$

The status parameter s_A reflects the social or economic opportunities afforded to the speakers of language A relative to language B (Abrams and Strogatz 2003). The parameter s_B is defined in the same way, and it yields $s_A + s_B = 1$. Thus the higher the proportion of speakers of language A and the higher its social status, the higher its attractiveness and therefore the higher the likelihood that speakers of language B will shift to language A. The exponent a models how the attractiveness of language A scales with the proportion of speakers of language A, and the coefficient c indicates the peak rate at which speakers of language B shift to language A. The opposed shift rate P_{AB} is defined analogously by

$$P_{AB} = c n_B^a s_B. \qquad (4)$$

The analysis of dynamic systems such as model (2) encompasses the determination of the stable equilibria expressing the frequencies, which do not change over time anymore and are robust against small perturbations. The Abrams-Strogatz model predicts that one language (depending on the social status parameters s_A and s_B and on the distribution of the initial frequencies) will always acquire all speakers in the population, causing the language with which it competes to become extinct. To evaluate the significance of their approach, Abrams and Strogatz (2003) fitted the model to diachronic data collected for Scottish Gaelic, Welsh, and Quechua and found a good coincidence between the predicted and observed declines of the language usage. Last, they suggested that language maintenance can be achieved by controlling for the social status of the endangered language.

On the basis of these results, the Abrams-Strogatz model appears to be an appropriate approach for modeling the dynamics of language competition. However, it includes the following idealistic assumptions, which may limit the explanatory value of the model: (1) Languages are assumed to be fixed; (2) the population is highly connected with no spatial or social structure; (3) all speakers are monolingual; (4) the population size is assumed to be constant; and (5) there is no distinction between different uses of language in different social contexts. Subsequent modeling approaches generalized the Abrams-Strogatz model by addressing one or more of these shortcomings. These approaches can be categorized into two groups: those concerned with a more realistic modeling of the demography of the population and those concerned with a more realistic modeling of the process of language shift.

The important role of demography in language competition can be powerfully demonstrated with the example of the farming-language hypothesis (cf., e.g., Diamond and Bellwood 2003; Renfrew 1987). In the period known as the Neolithic revolution, agriculture quickly spread to cover large parts of the world. Archaeological and linguistic evidence (Bellwood and Renfrew 2003) suggests that the dispersal of farming was accompanied by the dispersal of the Indo-European language. However, it should be mentioned that this hypothesis is not without controversy [for a discussion on this topic see, for example, Shouse (2001)]. As soon as the advantageous technology farming was established in its centers in the Near East, Asia, and Africa, it spread out in a traveling wavelike pattern. Whether the concept of farming was an advantageous technology in the Neolithic is still subject to debate, because nutritional standards of Neolithic populations were generally inferior to those of hunter-gatherers and life expectancy may have been shorter. Supported by a much higher fertility and an ability to sustain higher population densities, farmers replaced hunter-gatherers. As farming spread out from its centers, language was pushed out with it and the Neolithic became the first period of linguistic homogenization (Nettle and Romaine 2000). Furthermore, farming and consequently the spread of language were stopped only where the environment was not suitable or where geographic boundaries made it impossible. This example shows how the spread and therefore competition of languages can be determined by the demographic characteristics of the competing subpopulations. In this context Ackland et al. (2007) developed a model of cultural hitchhiking that can serve as a null model for explaining the spread of the Indo-European language.

Patriarca and Leppänen (2004) accounted for the fact that the spread of a language is influenced by the dispersal of its speakers, and they generalized the Abrams-Strogatz model by introducing spatial dependence. This results in a reaction-diffusion system of the form

$$\frac{\partial u_A}{\partial t} = d_A \Delta u_A + u_B P_{BA}(n_A, s_A) - u_A P_{AB}(n_A, s_B), \tag{5a}$$

$$\frac{\partial u_B}{\partial t} = d_B \Delta u_B + u_A P_{AB}(n_A, s_B) - u_B P_{BA}(n_A, s_A) \tag{5b}$$

where u_A and u_B are the space- and time-dependent frequencies of both languages and P_{AB} and P_{BA} describe the shift rates (which are still determined by the overall frequencies n_A and n_B, respectively), as in the Abrams-Strogatz model. The diffusion components $d_A \Delta u_A$ and $d_B \Delta u_B$ (Δ is the Laplace operator) model the spatial dispersal of the speakers based on the random walk hypothesis. The overall population size is assumed to be constant.

The analysis of the model in Eqs. (5) shows that again only the extinction states are stable equilibria. Furthermore, Patriarca and Leppänen (2004) found that if the considered domain is divided into two distinct zones and if the shift rates P_{AB} and P_{BA} differ from zone to zone depending on the local population frequency, then two languages with different social statuses can coexist if languages acquire speakers in different locations.

Pinasco and Romanelli (2006) developed an ecological model of the Lotka-Volterra type to explain cases in which both languages survive in only one zone of competition:

$$\frac{\partial n_A}{\partial t} = r_A n_A \left(1 - \frac{n_A}{K_A}\right) + c n_A n_B, \tag{6a}$$

$$\frac{\partial n_B}{\partial t} = r_B n_B \left(1 - \frac{n_B}{K_B}\right) - c n_A n_B. \tag{6b}$$

Again n_A and n_B are the time-dependent frequencies of both languages. This model introduces an additional logistic growth term, whereby new speakers of each language are recruited not just by shifting but also by reproduction. To maintain finite population sizes, reproduction is modeled as a logistic process with the carrying capacities K_A and K_B, respectively. The assumed language shift dynamic is similar to the Abrams-Strogatz model. Pinasco and Romanelli (2006) found a stable equilibrium in which both languages coexist.

Kandler and Steele (2008) incorporated both demographic factors (population growth and spatial dispersal) into a reaction-diffusion system and questioned the assumption of the Pinasco-Romanelli approach that there are two separate carrying capacities K_A and K_B for speakers of languages A and B. They assume only one carrying capacity K to describe the maximum population size that can be supported by a given environment without reducing its ability to support the same population in the future (Ehrlich 1994), regardless of the language spoken. That means that the time- and space-dependent frequencies u_A and u_B of both languages have to fulfill the condition

$$u_A + u_B \leq K, \tag{7}$$

where K is for the common carrying capacity. This consideration leads to a reaction-diffusion system of the form

$$\frac{\partial u_A}{\partial t} = d_A \Delta u_A + r_A u_A \left(1 - \frac{u_A}{K - u_B}\right) + c u_A u_B, \tag{8a}$$

$$\frac{\partial u_B}{\partial t} = d_B \Delta u_B + r_B u_B \left(1 - \frac{u_B}{K - u_A}\right) - c u_A u_B. \tag{8b}$$

Analogous to the Patriarca-Leppänen approach, the diffusion terms $d_i \Delta u_i$ model the spatial dispersal, whereas the terms $r_i u_i \{1 - [u_i/(K - u_j)]\}$ model the intrinsic growth behavior of each subpopulation. The competition coefficient $c > 0$ reflects the social status differences of both languages.

The analysis of the model in Eqs. (8) shows that coexistence, as in the Pinasco-Romanelli approach, is no longer possible. Language B cannot resist the permanent conversion of speakers to language A, and the extinction of the lower status language B is predicted. However, the extinction times and the temporal and spatial course of the extinction process depend crucially on the demographic parameters d_A, d_B, r_A, r_B, and c. Furthermore, in adapting Patriarca and Leppänen's results, Kandler and Steele (2008) showed that spatial heterogeneity (in the form of spatially varying competition behavior) can affect the interaction dynamic. Kandler and Steele found that coexistence is possible in situations in which the attractiveness or dominance of languages changes between regions. The reasons for dominance can relate to political, social, and/or economic factors. An example is the situation in the Basque Country, which enjoys substantial cultural and political autonomy and where the Basque language is protected by laws—and by the regulatory activity of an academy set up to maintain it. The Basque language is an isolate, unrelated to the Indo-European languages also spoken in this and adjacent regions. It is plausible that this makes it more difficult for speakers in one group to learn the other group's language and that this has also impeded rates of shifting [although Kandler and Steele (2008) did not explicitly consider such factors].

Recently, Patriarca and Heinsalu (2009) published a generalization of the Patriarca-Leppänen model by adding a logistic growth term (as in the other models described here) and an advection term. Their aim was to examine the influence of geography on language competition based on human dispersal models. They showed that factors that are not related to the cultural transmission process, such as initial population distribution or geographic boundaries and inhomogeneities, can strongly affect the dynamics of language competition. They obtained situations of coexistence (assuming a fixed population size) in which a geographic boundary (e.g., a mountain chain) divides the area into two regions and for historical reasons the subpopulations speaking languages A and B are initially localized on opposite sides of the barrier.

We have seen that the incorporation of demographic aspects of language competition can change the result of the original Abrams-Strogatz model significantly. Accounting for spatial population dispersal even in its simplest mathematical description as a diffusion process can reverse the competition result, whereas spatial heterogeneity (whether in the form of heterogeneous growth behavior or spatially varying attractiveness of languages) can lead to coexistence. Nevertheless, the reviewed models fail to address crucial social and linguistic aspects. Wang and Minett (2005) made the criticism that these dynamic models do not account for bilingualism, sociolinguistic factors such as social structure, multiple registers of speech, and the effect of heterogeneous strategies, both at the level of individuals and at the level of policymakers. Following this critique, the second group of generalizations of the Abrams-Strogatz model dealt with a more realistic modeling of the process of language shift, in particular, the incorporation of bilingualism.

Mira and Paredes (2005) suggested a model in which the two languages that compete for speakers are partly mutually intelligible. When two languages possess a certain degree of similarity, monolingual speakers of one language can sometimes communicate effectively with monolingual speakers of the competing language, which allows monolingual speakers to become bilingual. Mira and Paredes assumed that, apart from the proportion of speakers of the competing language, the rate at which monolingual speakers shift to the bilingual subpopulation depends on the degree of similarity between the languages. The model shows that for a sufficiently large degree of similarity, a stable equilibrium is obtained in which both bilingual speakers and monolingual speakers of the more prestigious language survive. Mira and Paredes fitted their model to diachronic data collected for Castilian Spanish and Galician and found a good coincidence.

Minett and Wang (2008) questioned the applicability of the Abrams-Strogatz model to general situations of language competition, because competing languages are often mutually unintelligible. Minett and Wang extended the Abrams-Strogatz model by including bilingualism explicitly. This approach encompasses three subpopulations, two monolingual groups (denoted A and B) and one bilingual group (denoted C). Minett and Wang assumed that language is transmitted vertically or horizontally. To determine which individuals followed vertical transmission and which followed horizontal transmission, they introduced a mortality rate μ at which adults are replaced by children. These assumptions led to the following system of differential equations:

$$\frac{dn_A}{dt} = \mu c_{CA}(1-n_A-n_B)P_{BA}(n_A,s_A)-(1-\mu)c_{AC}n_A P_{AB}(n_B,s_B), \tag{9a}$$

$$\frac{dn_B}{dt} = \mu c_{CB}(1-n_A-n_B)P_{AB}(n_B,s_B)-(1-\mu)c_{BC}n_B P_{BA}(n_A,s_A). \tag{9b}$$

The terms n_A and n_B describe the time-dependent frequencies of languages A and B, respectively, and $n_A + n_B + n_C = 1$, where n_C is the frequency of the bilingual population. The shift rates P_{AB} and P_{BA} are the same as in the Abrams-Strogatz model. This means that in this model too the attractiveness of a language determines the strength of vertical and horizontal transmission.

The model in Eqs. (9) predicts the extinction of one language, regardless of the initial conditions. Bilingualism is not able to produce coexistence. Inspired by the result of Abrams and Strogatz (2003) that an endangered language can be preserved by controlling for its social status, Minett and Wang (2008) turned to the important aspect of language maintenance. They found that increasing the social status of the endangered language and modifying the parameters c_{CA}, c_{CB}, c_{AC}, and c_{BC} (which can be associated with different intervention strategies) can lead to stable coexistence of the two monolingual subpopulations.

Summarizing, these generalizations of the Abrams-Strogatz model point out clearly that both demographic factors and linguistic and social aspects can play an important role in explaining language competition and death. Neglecting one of them can change the competition dynamics drastically. Nevertheless, in situations in which every individual experiences the same homogeneous (cultural) environment and is exposed to both languages at some point, most of the models indicate that one language will go extinct. But the process of extinction and especially the extinction time depend crucially on the chosen model setup. Coexistence is achieved under consideration of population or environmental heterogeneity or external interventions.

Having these facts in mind, I believe that demographic aspects and realistic linguistic and social transmission processes should not be considered separately. Therefore in the next section I propose a reaction-dispersal competition model that incorporates bilingualism, vertical and horizontal language transmission [as suggested by Minett and Wang (2008)], social structure, and demographic factors such as population dispersal and population growth.

Besides these dynamic models, which describe the competition dynamics deterministically using coupled differential equations, another group of powerful approaches based on agent-based simulations has been developed. Whereas the previously reviewed approaches act on the population level and deal with the expected general pattern of the language competition, agent-based simulations act on the individual level and model the actions and interactions of agents in a network, with a view to assessing their effects on the system as a whole. Agent-based models focus on a realistic modeling of the contact situation to find appropriate social structures through modeling the underlying interaction network, incorporating bilingualism, and modeling language evolution.

Much work on agent-based modeling of language competition was done by Stauffer and Schulze [for a review of their models, see Schulze et al. (2008) and Stauffer and Schulze (2005)]. They used a bit-string approach in which languages are described by strings of F bits. All their models exhibit the following common mechanisms (cf. Schulze et al. 2008). A language is characterized by F independent features (which are identified with an independent grammatical element), where each feature can take one of Q different values. In each iteration each of the F features is changed with the probability p. With probability q, this change is deterministic and the value of the feature is simply transferred from another individual in the population; on the contrary, with probability $(1 - q)$, the change is chosen randomly. Language shift is determined by the density-dependent probability $(1 - x)^2 r$, where x is the proportion of the population speaking the individual's native language and r is the shift rate. Stauffer, Schulze, and colleagues analyzed this basic model under different assumptions for the population structure, geography, migration behavior, or population growth but were mainly interested in the competition of many languages and the distribution of the world's languages. One of the findings was that the simulated distribution of the number of languages spoken by s individuals could be described by a parabola in a log-log plot and matched empirical estimates.

In the context of the language size distribution, de Oliveira et al. (2006) developed a simulation model based on the idea that the fitness of a language is proportional to the number of its speakers and that the mutation probability is inversely proportional to the language's fitness. They found that the number of languages spoken in an area A varied as $A^{0.4}$. This result coincided with empirical data. The competition of many languages was also studied numerically by Nettle (1999). He hypothesized that the temporal evolution of the number of language groups L can be described by the equation $dL/dt = 70/t - L/20$.

However, Stauffer and Schulze (2005) applied their model to the situation studied by Patriarca and Leppänen (2004). Initially, half of the domain is occupied by language A and half of the domain is occupied by the opposite language B. There is no status difference between the two languages, but Stauffer and Schulze introduced a local conformist bias to the competition dynamic. They obtained coexistence of the two dominant languages A and B in which both languages were spatially separated and interacted only in a small transition zone. The initial separation of both languages is a crucial assumption for that result.

Schulze and Stauffer (2006) studied the survival of a minority language that has no status disadvantage. They concluded that coexistence between a majority language and a minority language can be achieved if speakers of the minority language refuse to shift their language as soon as the frequency of the majority language exceeds a certain threshold.

Kosmidis et al. (2005) developed a similar agent-based approach for the competition of two languages. Every agent has the capacity to speak two languages. Here languages are characterized by a vocabulary of 10 words, and each time an agent interacts with another, the agent can learn a word from the other. It is assumed that the agent's fitness (defined by the agent's reproductive success) increases by learning words from the other language. This model leads mainly to coexistence situations. Under the assumption of a finite population size with no birth and death, the population will speak on average five words of each language. The inclusion of birth and death processes results in a situation in which nearly everyone is bilingual (that means everyone speaks all 10 words of each language). Kosmidis and co-workers found that one language goes extinct if there are disadvantages in the initial frequency and fitness level of its speakers or if demographic stochasticity is added.

Schwämmle (2005) also used a bit-string approach but was interested in the effect that biological aging has on language competition. Aging was incorporated through the Penna model (Penna 1995), and the model approved the fact that languages are learned more easily in youth than in old age. This approach is seen as a bridge between the language learning and language competition literature (Schulze et al. 2008).

In direct relation to the basic Abrams-Strogatz model, Stauffer et al. (2007) developed a microscopic version and analyzed this system on a fully connected network and on a d-dimensional lattice. A fully connected network is a system in which all nodes are connected to each other. That means that agents can interact directly with all others. On the contrary, in a d-dimensional lattice, only the $2d$ nearest neighbors are connected and thus agents interact only locally. Stauffer and co-workers assumed that in each iteration one agent is chosen at random and will change its language according to the frequency-dependent probabilities P_{AB} and P_{BA} defined by the Abrams-Strogatz approach. Note that under the assumption of a *d*-dimensional lattice, P_{AB} and P_{BA} are determined by using local frequencies. Stauffer et al. (2007) showed that this microscopic version coincides with the original Abrams-Strogatz model for situations in which both languages possess different social statuses ($s_A \neq s_B \neq 0.5$). If the competing languages have the same status, then the differential equation formulation results in the unstable coexistence state (0.5, 0.5), whereas the agent-based model leads to the more realistic situation in which language A acquires all speakers in half of the simulations and language B acquires all speakers in the other half of the simulations.

The Stauffer et al. (2007) simulation model was extended by Castelló et al. (2007) by introducing bilingualism and social structure. Castelló's agents were allowed to speak either one of the languages A or B or to speak both, and besides *d*-dimensional networks, there were small-world networks, which accounted for short- and long-range interactions. In small-world networks most nodes are not neighbors of one another, but most nodes can be reached from every other node by a small number of steps (cf., e.g., Watts and Strogatz 1998). In each iteration one agent *i* was chosen at random and the local frequencies $n_A(i)$ and $n_B(i)$ of both languages around that agent were determined. Then the agent changed its state (A denotes the state of speaking language A, B of speaking language B, and C of being bilingual) according to the frequency-dependent probabilities

$$P_{AC,i} = 0.5 n_B(i), \quad (10a)$$

$$P_{BC,i} = 0.5 n_A(i), \quad (10b)$$

$$P_{CA,i} = 0.5[1 - n_B(i)], \quad (10c)$$

$$P_{CB,i} = 0.5[1 - n_A(i)]. \quad (10d)$$

Note that an agent can only change from being monolingual to being bilingual, and vice versa. The shift rates are symmetric, which implies that both languages possess the same social status.

The analysis of the Castelló et al. (2007) model shows that neither bilingualism nor social structure is able to produce coexistence between the two equal-status languages. Further, Castelló and co-workers concluded that bilingualism is not a stable strategy. Bilingual agents place themselves at the boundaries between the monolingual spatial domains to favor communication between them. Social structure in the form of small-world networks acts as a way of accelerating the extinction process.

Minett and Wang (2008) developed a microscopic analogue to their differential equation approach. Their aim was to extend the results from the continuous approach by analyzing the range of behaviors of language competition that can result from specific intervention mechanisms and initial conditions. Similar to the Castelló et al. (2007) approach, Minett and Wang assumed that each agent had the possibility of being monolingual in language A or B or of being bilingual. Agents underwent vertical transmission with probability μ; otherwise they underwent horizontal transmission. In each iteration each agent sampled its neighborhood to determine the shift probabilities according to

$$P_{AC} = c_{AC} s_B n_B^a, \tag{11a}$$

$$P_{BC} = c_{BC} s_A n_A^a, \tag{11b}$$

$$P_{CA} = c_{CA} s_A n_A^a, \tag{11c}$$

$$P_{CB} = c_{CB} s_B n_B^a. \tag{11d}$$

The state of the agent was then changed randomly. Minett and Wang (2008) analyzed the model on a fully connected network and on a local-world network to account for social structure. In contrast to small-world networks, a local-world network is an evolving network with local rather than global connections. It reflects the assumption that individuals have local rather than global knowledge of the language usage patterns of other speakers in the population and only interact with a fraction of the other speakers making up the population (Minett and Wang 2008).

Minett and Wang found that the more efficiently a community is able to increase the social status of the endangered language, the later such interventions have to take place. Further, they concluded that in a system without intervention, the social structure does not influence the competition dynamic. However, interventions are more likely to be successful in a population whose social structure can be described by a fully connected network.

Most of the mathematical approaches used to describe language competition are based on either differential equations or agent-based simulations. However, the focus of both groups is different. As already mentioned, the continuous systems of differential equations describe the general expected behavior of the competition on the population level, whereas agent-based simulations model the interactions between agents and their influences on the whole system on an individual level. Differential-equation-based approaches are often criticized for their deterministic nature, negligence of a finite population size, and inability to capture all possible behaviors occurring in language competition. Minett and Wang (2008) stated that if maintenance of endangered languages is considered, speakers might live in small, relatively isolated communities or form cliques within larger communities with which they have comparatively little interaction, and it might not be possible to model these effects with systems based on differential equations. That is undoubtedly true for basic models (as it would be for basic agent-based simulations), but it has already been shown in other fields that with appropriate adjustments many of these shortcomings can be addressed. (The effects of different population sizes cannot be modeled with systems of differential equations.) However, the effects of small population sizes cannot be modeled with systems of differential equations. So there is no "right" model to describe language competition. Both groups have proved to have powerful tools (with their specific shortcomings), and we should see them not as competing but as complementary approaches that can benefit from the respectively gained insights, as was demonstrated powerfully by Minett and Wang (2008).

A Reaction-Diffusion Competition Approach

The Model. In this section I model the dynamic of language competition in a temporally and spatially homogeneous environment using a reaction-diffusion competition model. The use of this model type was encouraged by earlier studies of language competition (e.g., Kandler and Steele 2008; Patriarca and Heinsalu 2009; Pinasco and Romanelli 2006), cultural hitchhiking (Ackland et al. 2007), or prestige bias (Ihara 2008) that exploit similar methods. Reaction-diffusion competition models include growth, dispersal, and competition components, which collectively are well suited to capturing aspects of the spread and competition of languages in a population. A mathematical detailed description of these approaches can be found in, for example, Freedman (1980) and Murray (1996). I propose the following model:

$$\frac{\partial u_A}{\partial t} = d_A \Delta u_A + r_A u_A \left(1 - \frac{u_A}{K - u_B - u_C}\right) - c_{AB} u_A u_B + c_{AC} u_A u_C, \tag{12a}$$

$$\frac{\partial u_B}{\partial t} = d_B \Delta u_B + r_B u_B \left(1 - \frac{u_B}{K - u_A - u_C}\right) - c_{BA} u_A u_B + c_{BC} u_B u_C, \tag{12b}$$

$$\frac{\partial u_C}{\partial t} = d_C \Delta u_C + r_C u_C \left(1 - \frac{u_C}{K - u_A - u_B}\right) - (c_{AC} u_A + c_{BC} u_B) u_C + (c_{AB} + c_{BA}) u_A u_B, \tag{12c}$$

with the boundary conditions $\partial u_i / \partial n = 0$ and $x \in \partial D$. The expression $\partial / \partial n$ describes the outer normal derivation. These boundary conditions model situations in which no diffusion beyond the boundary ∂D of the domain D is possible. The time- and space-dependent variables u_A and u_B stand for the frequencies of speakers of languages A and B, respectively, and u_C describes the frequency of bilingual speakers. The terms $\partial u_i / \partial t$ indicate the temporal change of the frequencies. In addition, the spatial dispersal behavior of the subpopulations is described by the diffusion components $d_i \Delta u_i$ (Δ denotes the Laplace operator). That means that spatial dispersal has only a local dimension. I analyze the effects of nonlocal dispersal behavior by replacing the diffusion components with an integral formulation that allows for short and long-range dispersal.

The reaction terms

$$r_A u_A \left(\frac{1 - u_A}{K - u_B - u_C}\right) - c_{AB} u_A u_B + c_{AC} u_A u_C, \tag{13a}$$

$$r_B u_B \left(\frac{1 - u_B}{K - u_A - u_C}\right) - c_{BA} u_A u_B + c_{BC} u_B u_C, \tag{13b}$$

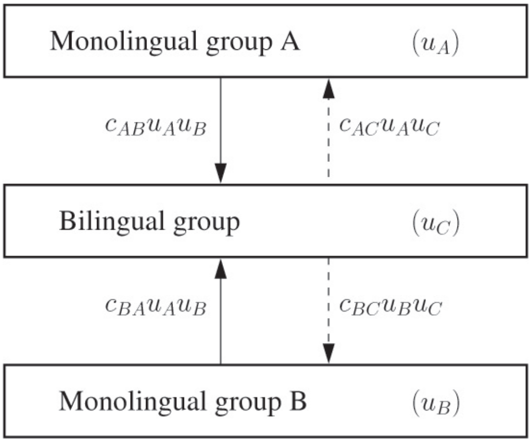


Figure 1. Schema of the assumed shift mechanisms in the proposed model [Eqs. (12)].

$$r_C u_C\left(\frac{1-u_C}{K-u_A-u_B}\right)-(c_{AC}u_A-c_{BC}u_B)u_C+(c_{AB}+c_{BA})u_Au_B, \tag{13c}$$

describe the growth behavior of the three subpopulations for which two main factors are involved. The first factor in Eqs. (13) is an intrinsic growth component that models coupled biological and cultural reproduction within each subpopulation. This component has the form

$$r_i u_i\left(\frac{1-u_i}{K-u_j-u_k}\right), \tag{14}$$

which denotes a bounded logistic growth process with an intrinsic natural rate of increase r_i defined as the difference between birth and death rates. The growth of the whole population (i.e., of $u_A + u_B + u_C$) is restricted to the common carrying capacity K, which defines an upper limit of the population size (cf. Kandler and Steele 2008).

The second factor in Eqs. (13) captures the dynamic of language shift and is illustrated in Figure 1. I assume that language shift must involve a bilingual transition state and that the loss of individuals of one subpopulation is the gain of another subpopulation. The solid arrows indicate fundamental shift rates from the two monolingual subpopulations to the bilingual stage, and the dashed arrows represent the rate of loss of bilingual speakers to each of the monolingual subpopulations. In more detail, language shift is described by the density-dependent conversion terms $c_{ij}u_iu_j$. The coefficients c_{AB} and c_{BA} describe the rate at which monolingual individuals become bilingual. In total, a proportion of $(c_{AB} + c_{BA})u_Au_B$ of the monolingual population shifts to the bilingual population. On the contrary, the bilingual subpopulation loses a proportion of $c_{AC}u_Au_C + c_{BC}u_Bu_C$ to the monolingual population. Following Abrams and Strogatz (2003) the rates c_{AC}, c_{BC}, c_{AB}, and c_{BA} reflect the prestige or socioeconomic advantage that accrue to speakers of language A and language B, respectively. Further, I introduce two variables s_A and s_B that quantify the status or prestige differences between the two languages. For simplification I set $s_A + s_B = 1$ (Abrams and Strogatz 2003) and substitute s_A with $1 - s_B$. I assume

$$c_{AC} = \tilde{c}_{AC}(1-s), \tag{15a}$$

$$c_{BC} = \tilde{c}_{BC}s, \tag{15b}$$

$$c_{AB} = \tilde{c}_{AB}s, \tag{15c}$$

$$c_{BA} = \tilde{c}_{BA}(1-s). \tag{15d}$$

The higher the social status of a language, the lower the loss rate to the bilingual subpopulation and the higher the gain rate from the bilingual group. Following Minett and Wang (2008), I introduce coefficients $\tilde{c}_{AC}$, $\tilde{c}_{BC}$, $\tilde{c}_{AB}$, and $\tilde{c}_{BA}$, which allow us to model the peak attractiveness of a language. I define the overall balance of competitive advantage to speaking each language on the basis of these conversion rates: For example, language A can be assumed to be more advantageous if it holds that $c_{AB} < c_{BA}$ and $c_{AC} > c_{BC}$. This implies that monolinguals who speak language A are less likely to become bilingual and that bilinguals are more likely to shift to language A.

In more detail, language transmission occurs vertically or horizontally, and I explain in the following discussion how this is incorporated into the model in Eqs. (12). In general, vertical transmission describes the passing of cultural traits from generation to generation. We assume that children of monolingual parents will be raised monolingual. In these cases biological and cultural reproductions coincide. However, bilingual parents may choose to raise their children monolingual or bilingual. Vertical transmission is modeled in Eqs. (12) using the logistic growth processes and the frequency-dependent conversion terms $c_{AC}u_A u_C$ and $c_{BC}u_B u_C$. The loss of bilingual offspring to language A occurs at a rate $c_{AC}u_A$, and the loss to language B occurs at a rate $c_{BC}u_C$.

Horizontal transmission (the spread of cultural traits between individuals of the same generation) is caused by the need for a common communication base when speakers of different languages are present in the same domain. Then monolingual speakers are encouraged to learn the other language and therefore to become bilingual. Horizontal transmission is incorporated into Eqs. (12) by the frequency-dependent conversion terms $c_{AB}u_A u_B$ and $c_{BA}u_A u_B$. As for vertical transmission, the rate at which speakers choose to learn the second language depends crucially on the subpopulation's attitude toward learning a foreign language and, again, the attractiveness of the languages. The effect of horizontal transmission is to swell the bilingual subpopulation, as monolinguals learn a second language. I do not account for extended diglossia, therefore assuming that individuals use the languages in all social contexts equally.

Results

In the following discussion I analyze the competition dynamic that is modeled by Eqs. (12) for situations in which one language has a status advantage. I assume a spatially and temporally homogeneous environment, which is reflected by constant parameters in the considered model. To carry out the analysis, I implemented the model in Eqs. (12) in C++ and solved it numerically using finite element methods.

The only stable equilibrium states that are obtained are the extinction states $(K, 0, 0)$ and $(0, 0, K)$. The proposed model predicts that, depending on the attractiveness of both languages, the demographic parameters of the subpopulations, and their initial distributions, one language will acquire all speakers over time. Interestingly, that does not have to be the high-status language.

For the sake of illustration, Figure 2 shows an example for the temporal and spatial competition dynamic. Assume that the parameter constellation is $K = 1$, $r_A = 0.06$, $r_B = 0.03$, $r_C = 0.05$, $c_{AC} = 0.05$, $c_{BC} = 0.02$, $c_{AB} = 0.03$, and $c_{BA} = 0.05$, which models a situation in which speakers of language A have an intrinsic advantage. Furthermore, assume that initially language B is the language with the most speakers in the considered domain D but that the high-status language A has entered the population in a small region. Figure 2 shows the spatial distribution of the subpopulation frequencies for $d_A = d_B = d_C = 10^{-4}$ (straight lines) and for $d_A = 10^{-2}$, $d_B = d_C = 10^{-5}$ (dashed lines) at two different time points. For the sake of simplification and better illustration, Figures 2, 6, and 7 show cuts through the two-dimensional rectangular domain $D = [0, 1] \times [0, 1]$ at $x_2 = 0.5$. When all subpopulations show the same dispersal behavior, it is obvious that, because of its competitive advantage, the subpopulation speaking language A grows in the center of the domain and local diffusion causes a steady expansion of the zone in which language A was found at high enough frequencies for it to prevail. With time the contact and mixing zone is shifted toward the edges of the domain with extinction of language B and then of bilingualism as the long-term outcome. However, for $d_A = 10^{-2}$ and $d_B = d_C = 10^{-5}$ (dashed lines in Figure 2), the result of the competition is reversed. In this situation the relatively greater diffusivity ($d_A = 10^{-2}$) of the speakers of language A causes dramatic dilution of the initial concentration of its speakers in the center of the domain. The intrinsic growth rate r_A is not able to compensate for this and the density-dependent dynamics predominate, leading to the extinction of language A.

These findings raise the question of under which conditions the initially mainly spoken but low-status language B is able to resist the presence of the high-status language. I explore this question by analyzing the competition outcome for different parameter values for d_A (a measure of the mobility of speakers of language A), s (the social status of language B), and its initial distribution. Assume again that language A has entered the population in a small area and that the frequency at which language A is present in this area initially is varied. Figure 3 shows the interface that separates the area of attraction of the extinction

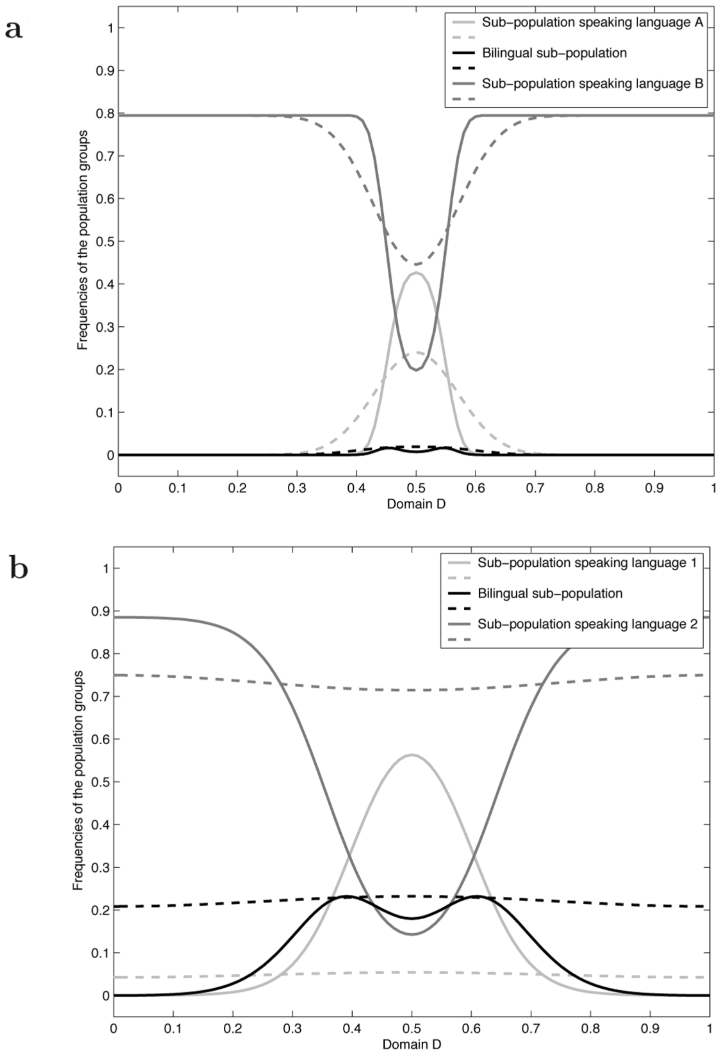


Figure 2. Spatial frequency distributions of the three subpopulations at different times. Solid lines represent the competition behavior for $d_A = d_B = d_C = 10^{-4}$, and dashed lines describe the behavior for $d_A = 10^{-2}$, $d_B = d_C = 10^{-5}$. At the beginning the high-status language A is present only in a small area ($D' = [0.45, 0.55]$) and at a low frequency ($u_A = 0.3$ for $x \in D'$). The social status variable s is assumed to be 0.3.

states (K, 0, 0) (above the surface) and (0, 0, K) (below the surface). We observe a nonlinear relationship between the three parameters. A larger mobility of the subpopulation of speakers of language A can be balanced by a higher social status and/or a higher initial concentration. Summarizing, besides the social status difference, the dispersal behavior and the initial distribution of the subpopulations

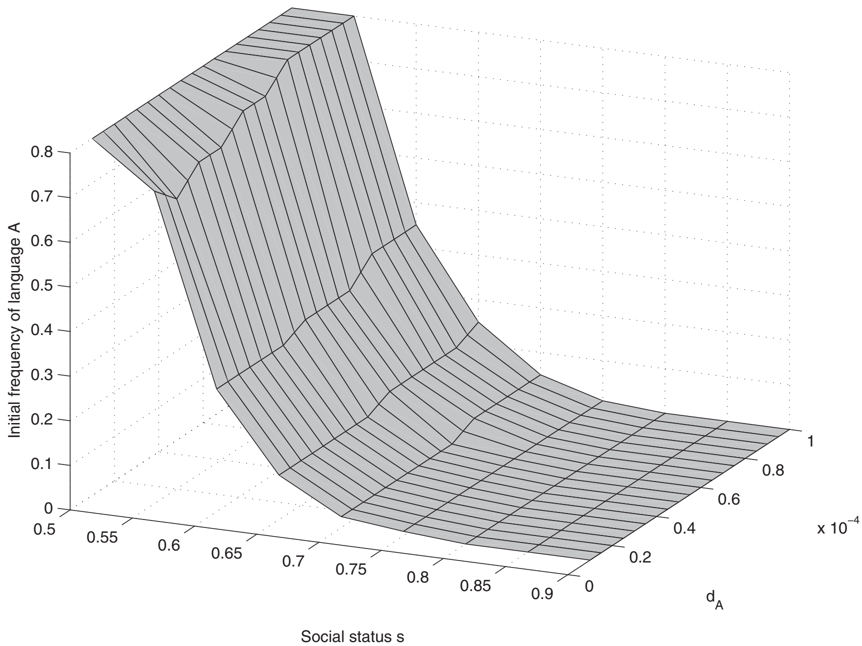
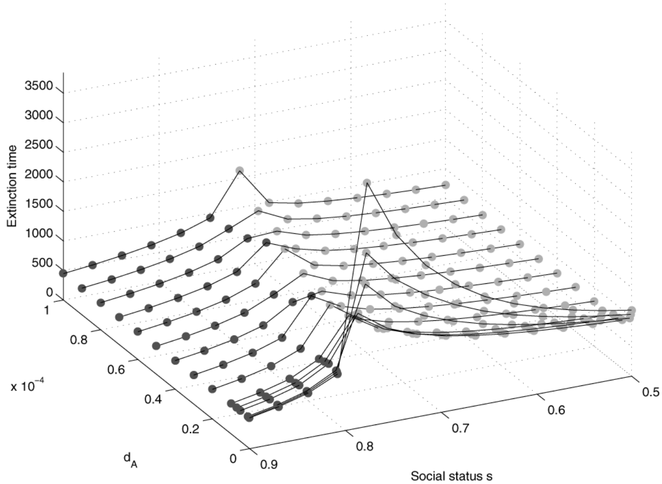


Figure 3. Surface that separates the areas of attraction of the extinction states (*K*, 0, 0) (area above the surface) and (0, 0, *K*) (area below the surface).

play an important role in the competition behavior. If the low-status language is sufficiently established in the population, it can prevail in competition with a more intrinsically advantageous.

We are able to predict which language will acquire all speakers in the long run. However, the time until the extinction state is reached may vary greatly. Figure 4 shows the extinction times for different values of the parameters s and d_A and an initial frequency of speakers of language A of 0.1 (Figure 4a) and 0.4 (Figure 4b). It is obvious that the peak extinction time is reached for the parameter constellation where language A outcompetes language B for the first time. This effect is caused by the chosen initial distribution. Language A has to "conquer" a larger area than language B because it is present only in a small area at the beginning. Then a further decrease of s (the relative social status of language B) accelerates the extinction process. In contrast, when the low-status but initially more abundant language B wins the competition, we observe an increase in the extinction time if language A's status is increased. A comparison of Figures 4a and 4b shows the influence of the initial condition. The higher the initial abundance of language A, the lower the social status s or the dispersal rate d_A has to be to outcompete language B.

Role of Bilingualism. Language shift is said to presuppose the existence of a transitional stage of bilingualism, involving receding language and the replacing a



b

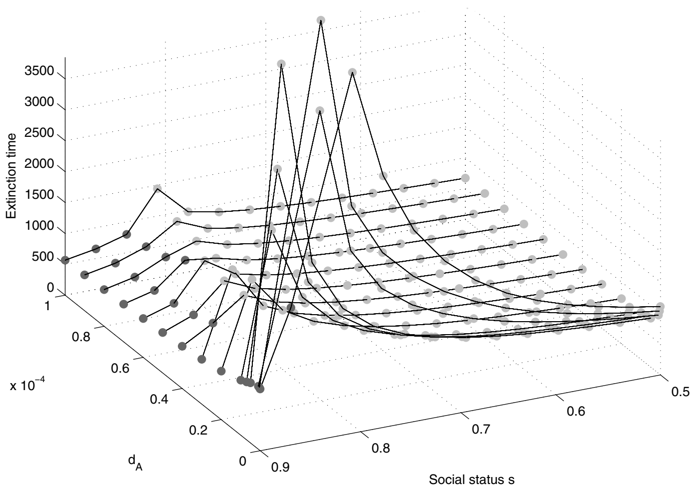


Figure 4. Extinction times depending on the social status and the diffusivity d_A of the subpopulation speaking language A for an initial frequency of (a) 0.1 and (b) 0.4. The light gray circles indicate the situations in which language A will acquire all speakers over time, whereas the dark gray circles indicate situations in which the low-status language B will acquire all speakers.

one (e.g., Campbell 1994). In this context Tsunoda (2004) asked whether it is possible for language shift to happen without a transitional stage of bilingualism. It appears that there is no incidence reported (Tsunoda 2004). It is often stated that it takes three generations for language shift to be completed (Brenzinger 1997). However, there are examples where an even shorter period [e.g., shift from Welsh to English among emigrants from South Wales (Giles et al. 1977)] or a longer period (e.g., shift of third-generation Chinese Americans to English among Chinatown residents) of language shift is found.

The model in Eqs. (12) assumes that it takes at least two generations to complete a language shift, and we have seen that bilingualism does not change the qualitative outcome of the competition dynamic. Analogous to the Abrams-Strogatz model (which does not account for bilingualism), the model proposed here predicts the extinction of one language in the long run. So in this section I explore the role of bilingualism in the competition dynamic.

We have observed that the extinction of a language is followed by the extinction of the bilingual subpopulation. This means that the concept of bilingualism cannot be maintained in a homogeneous environment. It serves the function of favoring the communication between two monolingual, spatially separated domains, and if there is no need for this, then function bilingualism will vanish. I stress that this result holds only if extended diglossia is not considered. However, bilingualism has a large effect on the extinction time. By comparing the extinction times of the three-population (bilingual) model [Eqs. (12)] with the two-population (monolingual) model [described by Kandler and Steele (2008)], which is obtained by setting $u_C = 0$, I found that the process can be slowed down significantly. This effect results from the different shift mechanisms. Equations (12) assume that individuals do not shift languages directly; they must pass the intermediate state of bilingualism, and then their children might become bilingual. So the process of language shift needs at least two generations, whereas in models that do not account for bilingualism, individuals can shift their language within their lifetime. Furthermore, the more equal both languages are in terms of their competitive abilities, the greater the influence of the bilingual component on the duration of the extinction process.

However, bilingualism can reverse the competition dynamic. My results show that in the proposed model the language that is less attractive to shifters can outcompete its rival if it is, for example, already well established in the domain. This is a different result from that obtained from the two-population (monolingual) model, where regardless of the initial frequency distribution, the language that is intrinsically more advantageous to shifters always prevails (Kandler and Steele 2008).

I conclude that the concept of bilingualism does not lead to coexistence of two languages with different social statuses in a homogeneous environment. Nevertheless, bilingualism can influence the extinction process significantly by reversing the dynamic or prolonging the extinction time.

Role of Social Structure. The previous considerations are based on the assumption that human dispersal can be described as a diffusion process on the basis of the locally acting Laplace operator Δ. This implies that individuals interact only within their neighborhood and therefore that the standard diffusion cannot replicate fast demic expansion or long-range dispersal with plausible values for human mobility and reproduction. To explore the effects of social structure in

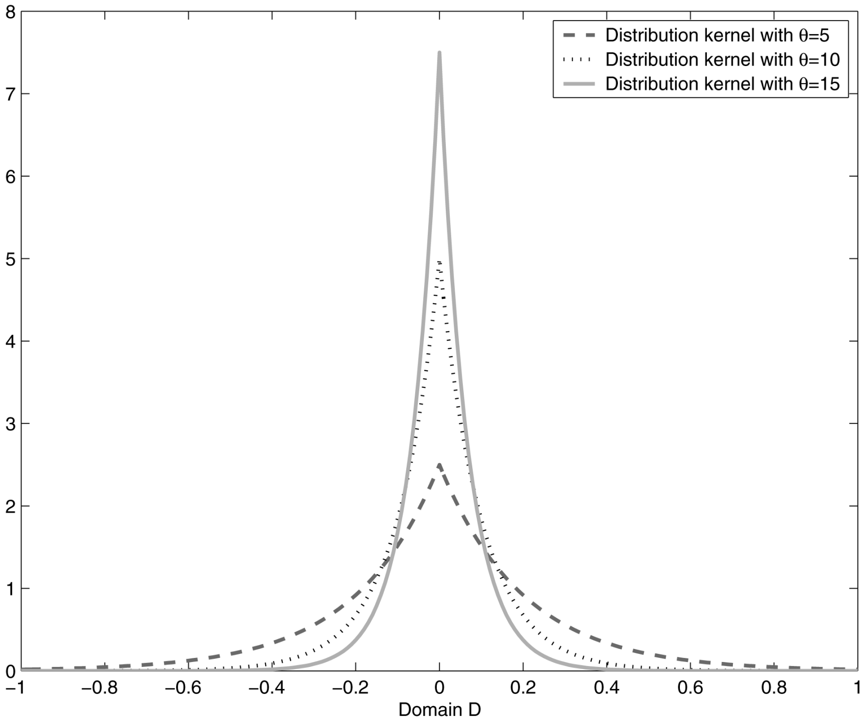


Figure 5. Shapes of the leptokurtic Laplace distribution kernels defined by $\phi(\delta) = \theta/2e^{-\theta|\delta|}$ for different values of θ.

particular of long-range dispersal on the competition dynamic, I replace in system (12) the diffusion term $d_i \Delta u_i$ with the integral formulation

$$\lambda \left[\int_D u(t, x+\delta)\phi(\delta)d\delta - u(t,x) \right] \tag{16}$$

The variable u represents again the space- and time-dependent frequency of a subpopulation. The kernel function $\phi(\delta)$ defines the probability distribution of the dispersal lengths δ. Figure 5 shows an example of such a kernel. It is obvious that large dispersal lengths δ are rare but occur with positive probabilities. The coefficient λ can be interpreted as a measure of the dispersal rate. A detailed mathematical review of such dispersal models can be found in, for example, Mendez et al. (2002) and Fedotov (2001). Network-based models have addressed this problem by constructing networks that reflect the underlying social structure of the population appropriately. An example is the small-world networks (Watts and Strogatz 1998) that account for local and long-range interactions, and the integral formulation (16) can be seen as its continuous analogue.

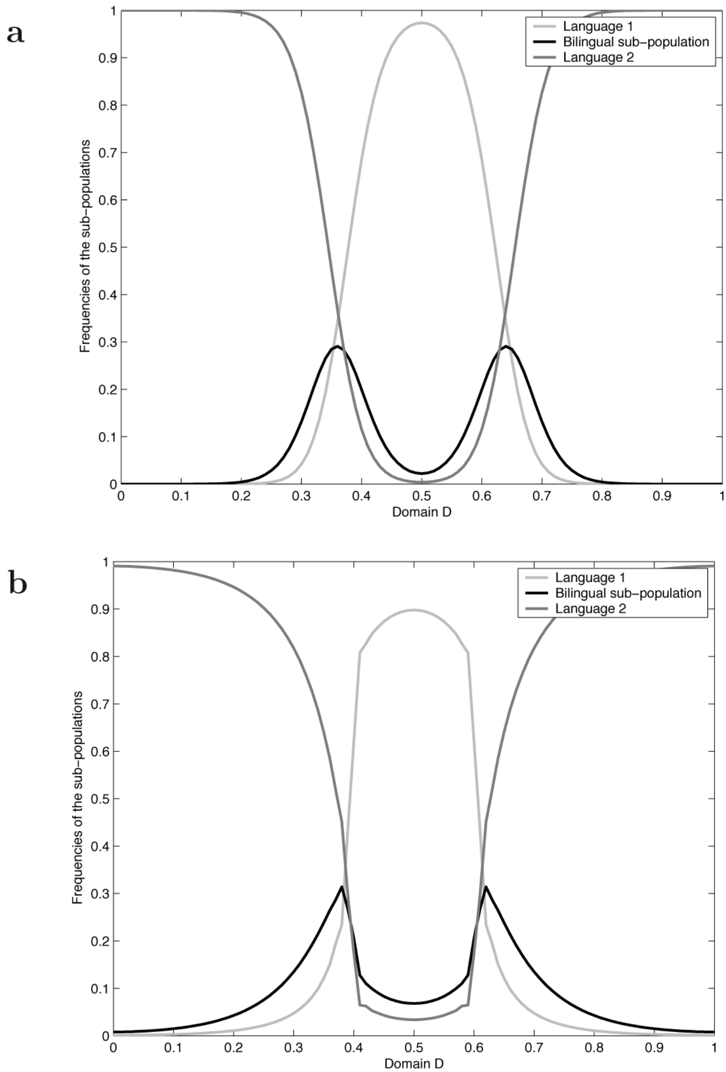


Figure 6. Spatial frequency distributions of the three subpopulations at the same time but under different dispersal hypotheses. (a) Spatial dispersal is modeled by a diffusion process. (b) Spatial dispersal is modeled by the distribution kernel of Figure 5 ($\theta = 15$). Note that both situations lead to the same extinction time.

To illustrate the effect of long-range dispersal, consider the example shown in Figure 6. The spatial distribution of the subpopulation frequencies for system (12) (Figure 6a) and expression (16) (Figure 6b) are shown at the same time. Both situations lead to the extinction of language B and, importantly, to the same extinction time. However, the competition dynamics are obviously different. In Figure 6a language A is already present in a larger area and the local dispersal is

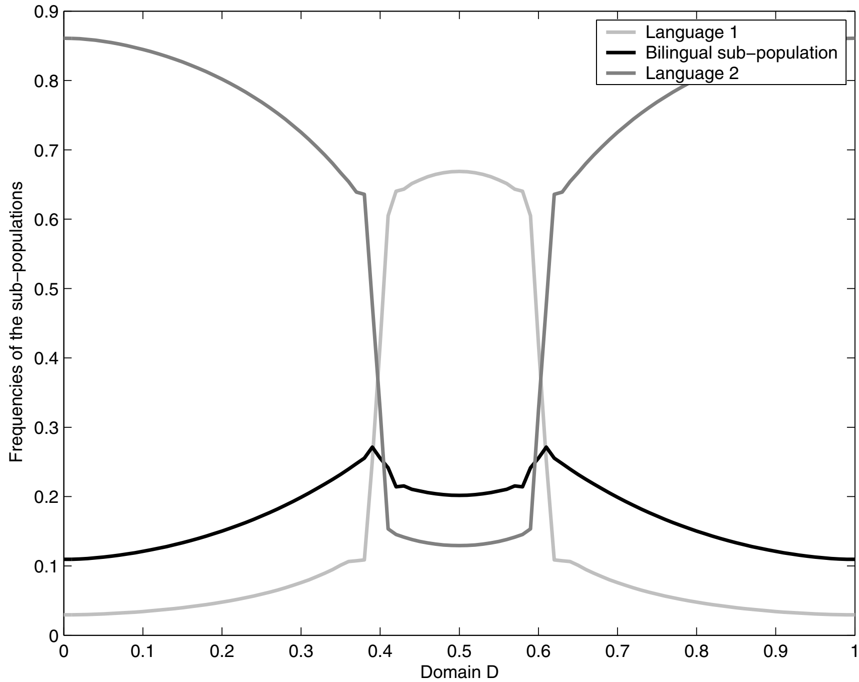


Figure 7. Spatial frequency distributions of the three subpopulations at the same time as in Figure 6. Spatial dispersal is modeled by the distribution kernel of Figure 5 ($\theta = 5$).

not as strong as in the purely diffusive situation. The subpopulation speaking language A is still clustered around the initial condition. These effects are even more obvious when the likelihood of long-range dispersal is increased.

In Figure 7 I assume a dispersal distribution as given by the dashed line in Figure 5. Now language A is already present over the whole domain and the clustering effect around the initial condition caused by a weaker local dispersal is more pronounced. Furthermore, the extinction time is significantly shorter than in the first example. Social structure changes the course of language competition—in particular, the duration of the extinction process—significantly.

Maintenance. At present, there is a drastic decline in linguistic diversity, and the general reasons for that process are well known in linguistics. Individuals do not change their languages, which are often part of their ethnic identity, without good reasons; rather, language shift is an adaptive answer to changing social economic or political reasons (Mufwene 2002). It can be seen as a survival strategy. For various reasons linguists are concerned about today's rate of language extinction, and a lot of effort is devoted to the maintenance of endangered languages. According to Nettle and Romaine (2000), the key to language maintenance lies in the youngest generation. Nettle and Romaine stated that languages are at risk when they are no longer transmitted naturally to children. The strength of this vertical transmission is used as a benchmark for whether the language will maintain its vitality (Grenoble and Whaley 2006). In that light, language planners have developed action plans that will help to revitalize endangered languages. For example, Crystal (2000) identified six main mechanisms of intervention; these mechanisms are aimed at increasing the attractiveness of the endangered language on the one hand and at creating an environment where the language can be used on the other.

Following these findings, I explore two maintenance strategies based on the proposed model [Eqs. (12)]. Inspired by Minett and Wang (2008), I generalize the model by incorporating actions that are able to increase the vertical transmission and the presence of the endangered language in the educational system. However, I do not suggest which specific intervention should be taken because the relation between interventions and the model parameter is only poorly understood.

In the proposed model of Eqs. (12), vertical transmission of the endangered language (assumed to be language B) is determined by the reproduction of the subpopulation speaking language B and the proportion of children of bilingual parents who are raised in the endangered language. I do not attempt to increase the reproduction rate. The aim is to strengthen the vertical transmission regarding the endangered language of the bilingual population. That means that the transmission rate $c_{BC}u_B$ must increase and that the transmission rate $c_{AC}u_A$ must decrease. In this way the education of children of bilingual parents in the endangered language is enhanced. This can be achieved by making interventions that increase the status of the endangered language [as already mentioned by Abrams and Strogatz (2003) and Minett and Wang (2008)] and/or of the coefficient $\tilde{c}_{BC}$ (respectively, decrease the coefficient $\tilde{c}_{AC}$). In the following I consider the first strategy and define a frequency-dependent social status variable (Figure 8). If vertical transmission of the endangered language (determined by $x = r_B u_B\{1 - [u_B/(K - u_A - u_C)] + c_{BC}u_B u_C\}$ falls below a certain threshold, interventions are taken to increase the social status of the endangered language. The slope and upper limit of the increase are determined by the specific interventions. If maintenance is achieved, the intensity of interventions can be weakened.

Another strategy could be to increase the presence of the endangered language in the educational system. The outcome of this intervention is that a certain percentage of pupils become bilingual. We can model this strategy by adding another shift term epu_A (from the u_B and u_C independent shift terms), where p is the percentage of pupils in the subpopulation speaking language A and e defines the strength of the education program. Obviously, this intervention results in the coexistence of the bilingual subpopulation and the subpopulation of language A. Language B goes extinct nevertheless, but it is maintained in the bilingual population. A negative aspect of this intervention is that coexistence depends entirely on the education policy. Furthermore, if the endangered language cannot be used in everyday life, the ability of the pupils to speak the endangered language will be weakened after leaving school.

Figure 9 shows the ratios of language A (Figure 9a) and B (Figure 9b) at equilibrium for different critical thresholds th and strengths of the interventions

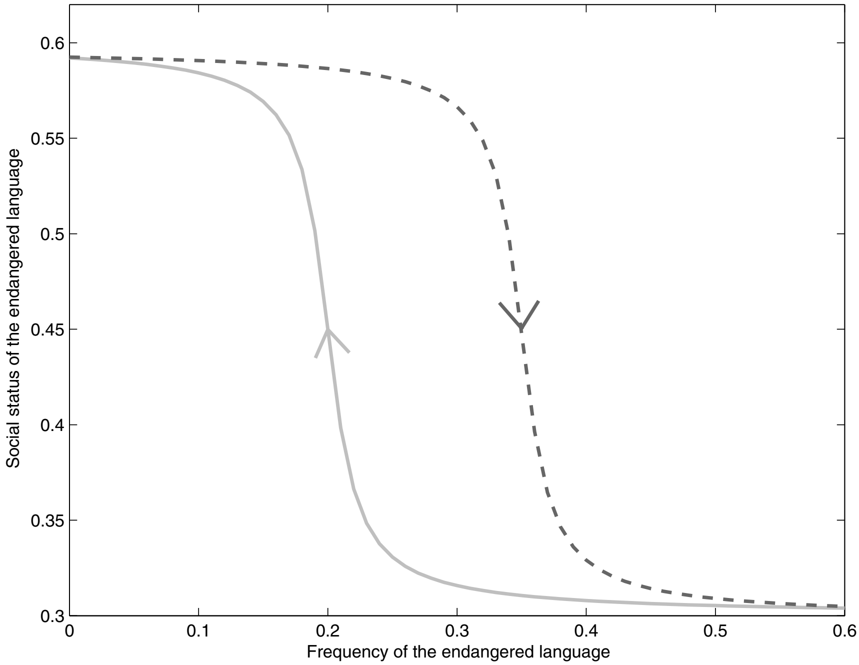


Figure 8. Shape of the increase of the social status of the endangered language if its frequency falls below a certain threshold *th* (solid line). If maintenance is achieved, intensity of interventions is weakened and the social status decreases again (dashed line).

(defined by the increase of the social status s_{max} of the endangered language). The endangered language can be maintained in the population. However, this figure visualizes a crucial aspect of language maintenance too. There exists only a certain timeframe in which interventions can lead to maintenance of the subpopulation speaking the endangered language only. If a language is detected as endangered too late, the generation-to-generation transmission cannot be revitalized. This finding highlights that reliable detection techniques of endangered languages are essential for maintenance. In addition, we see the impacts of an appropriate education policy. Even if the subpopulation that is monolingual in language B vanished, language B would still be maintained in the bilingual population, having a proportion of 10% of the whole population.

Summary

The model described by Eqs. (12) describes the dynamics of the language competition between two monolingual and a bilingual subpopulation and accounts for a number of demographic and social-linguistic aspects. I considered

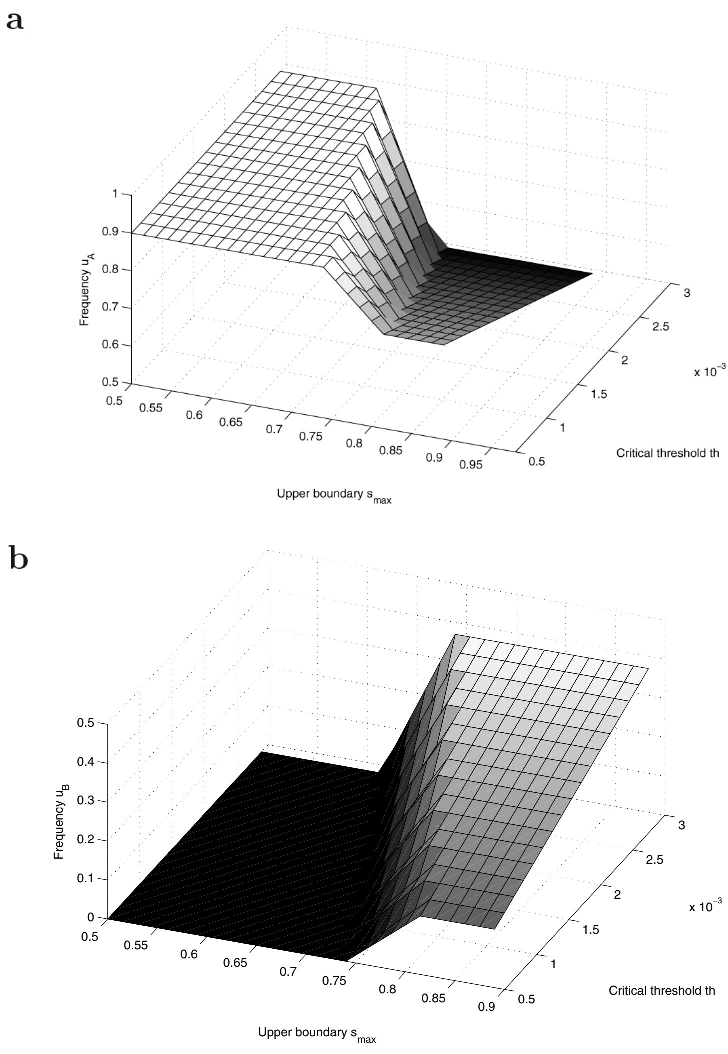


Figure 9. Ratios of the frequencies of (a) the subpopulation speaking language A and (b) the subpopulation speaking language B. Note that language A never acquires all speakers; the difference to 1 is the frequency of the bilingual subpopulation.

population growth and dispersal, bilingualism, and social structure. With an appropriate choice of the involved parameters, the model can be adjusted to specific competition situations. This will be shown in a forthcoming publication. I have found that in a homogeneous nonchanging environment, the extinction of one monolingual subpopulation and subsequently of the bilingual subpopulation is inevitable. However, not always the high-prestige language acquires all speakers over time. Such demographic factors as initial abundance of speakers, growth, and dispersal of the language groups influence the outcome significantly. I have shown that bilingualism is not a negligible intermediate state, as assumed by Abrams and Strogatz (2003). Bilingualism can influence the extinction process significantly by reversing the dynamics or prolonging the extinction time. The same applies if we allow for the more realistic assumption of long-range dispersal of individuals.

Creating an environment in which the endangered languages can be used and transmitted from generation to generation without experiencing social or economic disadvantages may change the course of the competition and allow for stable coexistence of all three subpopulations. I have shown that maintenance of both languages can be achieved by increasing the status of the endangered language and controlling its presence in the educational system. Interventions have to be taken within a certain time window and with certain strength. Otherwise, the viability of the endangered language cannot be enhanced.

Importantly, the considered model does not account for extended diglossia. Extended diglossia is a situation in which, in a given population, there are two languages, one of high prestige, which is generally used in public matters (at work, in education, in government, etc.), and one of low prestige, which is usually the spoken vernacular tongue and often used in private matters (at home, with friends, etc.) (Myers-Scotton 2006). The incorporation of extended diglossia can lead to stable coexistence, which I will discuss in a forthcoming publication.

Future Research

Recently, the field of language competition and death has been broadened by approaches that mathematically describe the changes in the pattern of language use in time and space. These approaches get more and more complex, yet it still remains to be demonstrated that the dynamics of these more complex models fit empirical data better than those of other simpler models. Only Abrams and Strogatz (2003) and Mira and Paredes (2005) have fitted their models to data and obtained a convincing coincidence. This task can be highlighted as one of the necessary future steps in this field.

In addition, most of the models reviewed here still oversimplify the linguistic components of language shift. For example, as mentioned, the incorporation of extended diglossia can change the coexistence results. A more accurate model setup might be achieved by closer cooperation between mathematicians and linguists. Also, the result that extinction of one language is inevitable was obtained under the assumption of a spatially and temporally homogeneous environment. However, political, social, economic, and geographic influences can easily lead to spatially and temporally varying (language) environments. Patriarca and Leppänen (2004), Patriarca and Heinsalu (2009), and Kandler and Steele (2008) have already shown that spatially varying shift mechanisms can lead to coexistence. Future models should account for possible heterogeneity of the environment where language competition takes place, because this has the potential to change the dynamics of language competition significantly.

As already pointed out by Castelló et al. (2007), most of the modeling approaches lack important mechanisms that act in the dynamics of language competition. One of them involves the emergence of new linguistic varieties resulting from, for example, code switching. Minett and Wang (2008) stated that code switching and language shift might be incorporated into the model by treating the languages as consisting of multiple components (e.g., the lexicon and the syntax), each having its own status and attractiveness and each being learned independently by each speaker. Here interdisciplinary collaboration with linguists would be highly beneficial because it would help to achieve a closer approximation of the real world.

Nevertheless, within their still simplifying frameworks, mathematical approaches contribute to a better understanding of the dynamics of language competition. If adjusted to specific situations, they can serve as prognostic tools. Furthermore, models can help to optimize the time point and specific kind of interventions that are needed for maintenance of an endangered language. In this context more research has to be done to discover the precise quantitative relationships among the various maintenance mechanisms and the model parameters.

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