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Flavor physics beyond the standard model

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FLAVOR PHYSICS BEYOND THE STANDARD MODEL

by

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DEDICATION

To my family from the loving son, husband, father, brother, nephew and cousin.

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Chapter 1 Introduction

The term ”flavor” in particle physics stands for describing the types of quarks and leptons. Presently, six types or flavors of quarks - up, down, strange, charm, bottom and top, three types or flavors of charged leptons - electron, muon, and τ-lepton, and three neutrino flavors - electron, muon and tau neutrinos, are known to exist experimentally. Among the quarks, the up and down are the ones that make up protons and neutrons and hence the matter that surrounds us. The other quark flavors, along with the up and down, are also confined in compound states, mesons and baryons, which are, however, unstable.

Flavor physics incorporates studies of such properties of quarks and leptons as transitions between different flavors in weak decays, flavor-antiflavor oscillations, origin of quark and lepton masses and flavor mixing, etc. These studies play an important role in understanding the origin of the Universe and its fundamental structure. Except when the top quark is involved, the transitions between flavors (weak decays, oscillations) occur at a few GeV or lower energy scales. Yet in spite of this, these transitions serve as one of the most powerful tools in searching for physics that may occur at energies as high as 100 GeV and even higher up to a 10^4 TeV scale [1].

Presently, the world of elementary particles that involves quarks and leptons is described by the so-called Standard Model (SM) [2] - a theory that incorporates the description of phenomena occurring due to electromagnetic, strong and weak interactions of elementary particles. However, there are strong reasons to believe that at energy scales higher than 100 GeV the Standard Model should be replaced by a more fundamental theory. Arguments in favor of such a hypothesis are based on experimental observations of neutrino oscillations [3, 4, 5, 6, 7] (and hence masses), evidence for Dark Matter (DM) [8], baryon and lepton asymmetries of the Universe that the SM is unable to explain. Besides, the SM has theoretical inconsistencies that are related to quadratic divergence of the loop corrections to the Higgs particle mass, to renormalize which one needs to tune two quantities to an unnatural accuracy of 10^{-34}; unexplained hierarchies of the quark and lepton masses and mixing; large number of free parameters, etc [2, 9].

Presently, there are several extensions of the Standard Model that propose different solutions of the above-mentioned problems. Those incorporate supersymmetric theories [9, 10, 11], Left-Right Symmetric models [12, 13], models with a quark/lepton family symmetry [14, 15], models with extra generations of quarks and leptons [16, 17, 18], dynamical electroweak symmetry breaking models [19, 20, 21], models with extra dimensions [22, 23] and many others. This great variety of the SM extensions is being tested at the Tevatron and LHC now, at the center-of-mass energies of 1.96 TeV and 7 TeV respectively. Presently there is no signal for New Physics (NP) beyond the Standard Model, however significant progress has been made in placing limits on hypothetical particles masses, their interaction coupling constants and their production cross sections (see [24] and references therein).

In this work we examine possible impact of New Physics on heavy meson decays and meson-antimeson oscillations within some of the SM extensions, mentioned above. Also, we propose a version of the simplest SM extension with two Higgs doublets that can explain the existing hierarchy of quark and lepton masses.

Studies of heavy (with masses $\sim$ few GeV) meson decays and meson-antimeson oscillations represent ways to search for New Physics beyond the Standard Model. It may be a direct search for a weakly-coupled hypothetical particle with a mass of order of a few GeV or less [25, 26, 27, 28]. Or it may be a search for a decay forbidden within the Standard Model, but allowed in some of the SM extensions. However, in most of the cases this is a study of the processes where the NP effects may enter due to exchange of virtual heavy particles that are predicted by the SM extensions, whereas both the initial and the final states consist of the SM particles. In the Standard Model such processes occur due to exchange of $W^{\pm}$ and Z bosons with masses $\sim$ 100 GeV. The New Physics contribution may in principle be comparable, if replacing $W^{\pm}$ and/or Z by another relevant heavy particles - the NP contribution is suppressed by the same power of heavy mass as the SM contribution.

Moreover, certain flavor-changing processes occur within the Standard Model only at loop level and hence are suppressed by a certain loop factor. They may also be suppressed by a power of a light-to-heavy mass ratio and/or be additionally suppressed if occurring due to quark generation mixing. In contrast, certain extensions of the SM predict that the same processes occur at the tree level or even if being loop-induced, may have no other suppression factors present within the SM. In that case the NP contribution is, in general, essential or even dominant. Such processes are especially valuable: an experimental evidence for them may lead to a New Physics signal even earlier than that from the LHC. Alternatively, if the experimental data turn out to be in accord with the SM predictions, such processes may be used to put constraints on the relevant NP parameters. These constraints are in general much more severe than those from the Tevatron and LHC direct searches for New Physics, as we will see in Chapters 3 and 4.

In this work we will be concentrating on meson-antimeson oscillations as a primary example. Within the Standard Model, these oscillations are loop-induced and occur via quark generation mixing (see the next chapter). In addition some of the oscillation amplitudes contain light-to-heavy mass ratios. As mentioned above, a New Physics contribution may be essential for these processes.

Study of an NP contribution to meson-antimeson oscillations involves two possibilities. First, one may examine if a given oscillation may be dominated by NP effects or at least if the NP contribution may be comparable with that of the SM. Here $D^0 - \overline{D}^0$ oscillations are of the primary interest, as the SM predictions for this process are still uncertain [29]. It is still not excluded that $D^0 - \overline{D}^0$ is dominated by New Physics effects. A large NP contribution, comparable with that of the SM, is also possible in $B_q - \overline{B}_q$ mixing (q = s, d), provided that there is a large CP-violating phase beyond the Standard Model [30].

Another possibility is related to the use of the existing experimental data for meson-antimeson oscillations, to place constraints on the relevant NP parameters. One may then transform these constraints into those on the NP contribution to heavy meson decays.

In this work we will consider $D^0 - \overline{D}^0$ and $B_s - \overline{B}_s$ oscillations. We will first examine a possibility of a sizable New Physics contribution to the lifetime difference in $D^0 - \overline{D}^0$ mixing within specific SM extensions.

We proceed then to the $B_s - \overline{B}_s$ mixing and examine possible correlations between the NP contribution to B_s mass difference and the leptonic decay $B_s \to \mu^+\mu^-$. These correlations provide in general more powerful constraints on the NP contribution to $B_s \to \mu^+\mu^-$ than the existing experimental limit on the branching ratio for this process.

Since the experimental bound on $B_s \to \mu^+\mu^-$ decay rate [24] is an order of magnitude greater than the SM prediction, one believes that there is still room to search for a New Physics signal in this process. Yet, we show that bounds on the NP parameters from the study of B_s mass difference tend to drive the NP contribution to $B_s \to \mu^+\mu^-$ decay rate below the SM value.

Study of quark and lepton flavors beyond the Standard Model enables one also to explain the existing mass pattern of quarks and charged leptons. Within the Standard

Model the quark and lepton mass hierarchy is simply related to the unexplained hierarchy of the fermion Yukawa interaction couplings with the Higgs doublet. In this work we illustrate that the existing quark and lepton mass pattern may be explained within the simplest extension of the Standard Model with two Higgs doublets, without imposing a hierarchy on the fermion Yukawa couplings.

The work is organized as follows. In Chapter 2 we provide a theoretical background that may be useful to understand the further discussion. We provide a brief introduction to the Standard Model, then we discuss in detail the quark (and lepton) masses generation mechanism within the Standard Model, the quark CKM mixing [31, 32] and the related issues such as Glashow-Iliopoulos-Maiani (GIM) mechanism [33] and the flavor problem [34]. These issues are going to play an important role in our further analysis. We finish Chapter 2 with a brief introduction of the meson-antimeson mixing formalism.

Chapter 3 is devoted to discussion of New Physics searches in the charm sector and, in particular, in $D^0 - \overline{D}^0$ oscillations as a primary example. We examine the lifetime difference in $D^0 - \overline{D}^0$ mixing within R-parity violating supersymmetric models and within the Left-Right Symmetric models. It is shown that within R-parity violating supersymmetric models the experimental value of the lifetime difference in $D^0 - \overline{D}^0$ mixing may be due to destructive interference between the SM and NP contributions. Otherwise, if the NP contribution is small, it implies rigorous bounds on the relevant R-parity violation couplings and/or charged slepton masses. In principle, diagrams with large NP contribution to the lifetime difference in $D^0 - \overline{D}^0$ mixing may also occur within other SM extensions, however their contribution in sum may be negligible due to the GIM cancelation mechanism. This is the case, as we show, within the non-manifest Left-Right Symmetric Model.

In Chapter 4 we consider possible correlations between the NP contributions to $B_s - \overline{B}_s$ mixing and $B_s \to \mu^+\mu^-$ decays. We show that the experimental constraints on the NP contribution to the mass difference in B_s mixing lead to severe constraints on the NP contribution to the $B_s \to \mu^+\mu^-$ decay rate within many SM extensions.

As mentioned above, study of quark and lepton flavors beyond the Standard Model enables one also to explain the existing mass pattern of quarks and charged leptons. In Chapter 5 we propose an explanation of this pattern within a general two-Higgs doublet extension of the Standard Model, without assuming a hierarchy in the quark and lepton Yukawa couplings. The desired values of the quark and lepton mass ratios are reached imposing the quark/lepton basis invariant conditions on the quark/lepton Yukawa matrices and assuming that the ratio of the Higgs doublets vacuum expectation values (vev's) is sufficiently large. We make concluding remarks in Chapter 6. Some of derivations and useful formulae are placed in the Appendices.

Chapter 2 Theoretical Background

This chapter is organized as follows. We make first a brief introduction to the Standard Model in Section 2.1. Then in Section 2.2 we discuss in detail the quark (and lepton) mass generation mechanism and the quark CKM mixing. We also discuss the related issues such as the GIM mechanism and the flavor problem. Finally Section 2.3 is devoted to a brief introduction of the meson-antimeson mixing formalism.

2.1 The Standard Model

The Standard Model of electroweak and strong interactions consists of three generations of quarks and leptons,

$$\begin{pmatrix} u \\ d \end{pmatrix}, \quad \begin{pmatrix} c \\ s \end{pmatrix}, \quad \begin{pmatrix} t \\ b \end{pmatrix},$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}, \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix},$$

the gauge bosons of their interactions: photon, $W^\pm$, Z bosons, gluon, and the Higgs doublet - it is needed to generate quark, lepton and $W^\pm$, Z boson masses via the Higgs mechanism [35].

The Standard Model Lagrangian may be presented as follows:

$$\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_F + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa} \tag{2.1.1}$$

In other words, it may be broken into the parts that consist of

- gauge fields kinetic and self-interaction terms, $\mathcal{L}_{gauge}$;
- fermion kinetic terms and gauge interaction terms, $\mathcal{L}_F$;
- the Higgs doublet kinetic, gauge interaction and self-interaction terms, $\mathcal{L}_{Higgs}$;
- fermion and the Higgs doublet Yukawa interaction terms, $\mathcal{L}_{Yukawa}$.

The gauge fields kinetic and self-interaction terms may be presented as

$$\mathcal{L}_{gauge} = -\frac{1}{4}G^{a\mu\nu}G^a_{\mu\nu} - \frac{1}{4}W^{a\mu\nu}W^a_{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} \tag{2.1.2}$$

where superscript a runs from 1 to 8 in the first term and from 1 to 3 in the second term of (2.1.2);

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f^{abc} G^b_\mu G^c_\nu \tag{2.1.3}$$

is the color SU(3) non-Abelian gauge gluon octet field tensor, G^a_μ is a gluon field and g_s is the QCD coupling constant;

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\varepsilon^{abc} W^b_\mu W^c_\nu \tag{2.1.4}$$

is the weak left isospin SU(2) non-Abelian gauge triplet field tensor, W^a_μ is a weak isospin triplet gauge field, g is the weak coupling constant;

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \tag{2.1.5}$$

is the hypercharge U(1) Abelian gauge field tensor, B_μ is the the hypercharge gauge field and g' is the hypercharge coupling constant. The relationships between the fields W_μ^a, $a = 1, 2, 3$, B_μ and $W^\pm$ and Z bosons and the photon are given below.

The fermion kinetic terms and gauge interaction terms may be presented as

$$\mathcal{L}_F = \sum_{f=1}^{3} \Big[\bar{Q}_L^f \gamma^\mu i D_\mu Q_L^f + \bar{L}^f \gamma^\mu i D_\mu L^f + \bar{u}_R^f \gamma^\mu i D_\mu u_R^f + \bar{d}_R^f \gamma^\mu i D_\mu d_R^f + \\ + \bar{\ell}_R^f \gamma^\mu i D_\mu \ell_R^f \Big] \quad (2.1.6)$$

where f stands for a generation number, Q_L^f and L^f are left-handed quark and lepton weak isospin doublets,

$$Q_L^f = \begin{pmatrix} u^f \\ d_f \end{pmatrix}_L , \quad L^f = \begin{pmatrix} \nu^f \\ \ell_f \end{pmatrix}_L$$

and u_R^f ($= u_R$, c_R, t_R for f = 1, 2, 3 respectively), d_R^f ($= d_R$, s_R, b_R for f = 1, 2, 3 respectively), ℓ_R^f ($= e_R$, μ_R, τ_R for f = 1, 2, 3 respectively) are right-handed quark and lepton isospin singlets. Note that the quark fields u_f and d_f are color triplets in the $SU_c(3)$ space. The covariant derivative is given by

$$D_\mu = \partial_\mu - i g_s T_c^a G_\mu^a - i g T^a W_\mu^a - i g' \frac{Y'}{2} B_\mu \quad (2.1.7)$$

where T_c^a, $a = 1, .., 8$, are the $SU_c(3)$ group generators,

$$T_c^a u^f = \frac{\lambda^a}{2} u^f, \quad T_c^a d^f = \frac{\lambda^a}{2} d^f, \quad T_c^a \ell^f = T_c^a \nu^f = 0$$

with λ^a being Gell-Mann matrices;

T^a, $a = 1, 2, 3$ are the weak left isospin operator components $(SU(2)_L$ group generators),

$$T^a \Psi_L^f \equiv T^a \begin{pmatrix} \psi_1^f \\ \\ \psi_2^f \end{pmatrix}_L = \frac{\tau^a}{2} \begin{pmatrix} \psi_1^f \\ \\ \psi_2^f \end{pmatrix}_L , \qquad T^a \psi_R^f = 0, \qquad \psi^f = u^f, d^f, \ell^f, \nu^f$$

with τ^a being Pauli matrices;

Y' is the hypercharge operator and is related to the electromagnetic charge and the third component of the isospin operator as

$$\frac{Y'}{2} = Q - T_3 \tag{2.1.8}$$

The Higgs sector of the SM Lagrangian has the following form:

$$\mathcal{L}_{Higgs} = (D^\mu \Phi)^\dagger (D_\mu \Phi) - V(\Phi) \tag{2.1.9}$$

where

$$\Phi = \begin{pmatrix} \Phi^{(+)} \\ \\ \Phi^0 \end{pmatrix} \tag{2.1.10}$$

$$D_\mu \Phi = \left(\partial_\mu - ig\frac{\tau^a}{2} W_\mu^a - \frac{ig'}{2} B_\mu \right) \Phi \tag{2.1.11}$$

and

$$V(\Phi) = \frac{\lambda}{2} \left(\Phi^\dagger \Phi \right)^2 + \mu^2 \left(\Phi^\dagger \Phi \right) \tag{2.1.12}$$

Note that one must have $\lambda > 0$ to assure vacuum stability of the Higgs potential (for $\lambda < 0$, the potential becomes unbound from below as $|\Phi|^2 \to \infty$). The sign of the mass parameter μ^2 may be arbitrary.

Finally the Yukawa term of (2.1.1) may be presented as

$$\mathcal{L}_{Yukawa} = -\sum_{f,f'} Y_{u_{ff'}} \bar{Q}_L^f \; \tilde{\Phi} \; u_R^{f'} + Y_{d_{ff'}} \bar{Q}_L^f \; \Phi \; d_R^{f'} + Y_{\ell_{ff'}} \; \bar{L}^f \; \Phi \; \ell_R^{f'} \tag{2.1.13}$$

where $\tilde{\Phi} = i\sigma_2 \Phi^\star$. Here we behold the original version of the SM where there are no right handed neutrinos and no Yukawa terms for the neutrino sector.

The SM Lagrangian given by (2.1.1)-(2.1.13) is symmetric under $SU(3)_c \times SU(2)_L \times U(1)_{Y'}$ group transformations, both local and global. Note that the fields entering the SM Lagrangian are all massless. The masses of fermions, weak interaction gauge bosons and the physical Higgs state are generated due to spontaneous breaking of the Lagrangian $SU_L(2) \times U(1)_{Y'}$ symmetry by the Higgs doublet non-zero vacuum expectation value (vev).

Possibility of having a non-zero Higgs vacuum state or a non-trivial minimum for the Higgs potential is related to the sign of the parameter μ^2 of the Higgs potential. For $\mu^2 > 0$ it is straightforward to see that the Higgs potential minimum $V(\langle\Phi\rangle_0) \equiv V_0 = 0$ is reached for $\langle\Phi\rangle_0 = 0$. On the other hand if $\mu^2 < 0$, the minimum condition for the Higgs potential has a non-trivial solution, given by

$$|\langle\Phi\rangle_0| = \sqrt{\frac{-\mu^2}{\lambda}} \tag{2.1.14}$$

and $V_0 = -\mu^4/(2\lambda)$.

Further on we will consider $\mu^2 < 0$ and the non-trivial solution for the Higgs potential minimum. The Higgs vacuum state may be presented then in a following form:

$$\langle\Phi\rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \tag{2.1.15}$$

with $v > 0$ (if v is complex or negative, one may redefine the Higgs doublet phase to make v real and positive). According to (2.1.14) and (2.1.15),

$$v = \sqrt{\frac{-2\mu^2}{\lambda}} \tag{2.1.16}$$

Note that the Higgs vacuum state given by Eq. (2.1.14) is not uniquely defined. Instead of (2.1.15), one might choose the upper component of $\langle\Phi\rangle_0$ to be non-vanishing, or even both components of $\langle\Phi\rangle_0$ to be non-vanishing. All these possibilities are related by $SU(2)_L \times U(1)_{Y'}$ transformations, or they are mathematically equivalent in light of $SU(2)_L \times U(1)_{Y'}$ symmetry of the SM Lagrangian.

However, each possible configuration for the Higgs vev specifies a certain direction in the $SU(2)_L \times U(1)_{Y'}$ space or *spontaneously breaks* $SU(2)_L \times U(1)_{Y'}$ symmetry of the SM Lagrangian. The Higgs vev configuration given by Eq. (2.1.15) corresponds to the physical situation when the electromagnetic charge is conserved and the photon remains massless. In other words, $SU(2)_L \times U(1)_{Y'}$ symmetry of the SM Lagrangian is spontaneously broken to $U(1)_{EM}$ symmetry of the electromagnetic interactions by non-zero vacuum expectation value of the Higgs doublet.

Spontaneous breakdown of $SU(2)_L \times U(1)_{Y'}$ symmetry provides an elegant mechanism to generate the SM particles masses. The details can be found e.g. in [2], in this chapter we will discuss only the quark (and lepton) mass generation mechanism because of its crucial importance. This is done in the next section, as for here, we briefly point out the main consequences of the spontaneous breaking of $SU(2)_L \times U(1)_{Y'}$ symmetry:

- After the global $SU(2)_L \times U(1)_{Y'}$ symmetry is spontaneously broken, the Higgs doublet may be presented in a following form:

$$\Phi(x) = \begin{pmatrix} G^+(x) \\ \frac{1}{\sqrt{2}}\left[v + h(x) + iG^0(x)\right] \end{pmatrix} \tag{2.1.17}$$

where $G^{\pm}(x)$, $G^0(x)$ and h(x) are excitations (quantum fluctuations) about the Higgs doublet vacuum state. Three of these states - the Goldstone bosons $G^{\pm}$ and G^0, remain massless whereas h acquires a mass $m_h = \sqrt{\lambda}v$.

- The Goldstone modes may be removed from the Higgs doublet by some $SU(2)_L \times U(1)_{Y'}$ gauge transformation, so that

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \tag{2.1.18}$$

The particular gauge, where the Higgs doublet may be presented in this form, is called unitary gauge. The remaining physical state, $h(x)$, is not detected experimentally yet. Presently there is only a lower bound on its mass [24], $m_h > 114.4$ GeV, and the mass range of 162 $GeV \le m_h \le 166$ GeV is ruled out. [36].

- The same gauge transformation affects the gauge fields W_μ^a and B_μ as well: the Goldstone bosons re-appear as longitudinal components of these fields. Recall that massless vector bosons have transverse degrees of freedom only, the appearance of three additional longitudinal degrees of freedom means that three of the four $SU(2)_L \times U(1)_{Y'}$ gauge fields acquire masses. Thus, after the local (gauge) $SU(2)_L \times U(1)_{Y'}$ symmetry is spontaneously broken, the Goldstone modes are "eaten" by the gauge bosons, so that three linear combinations of them,

$$W_\mu^{\pm}(x) = \frac{1}{\sqrt{2}} \left(W_\mu^1(x) \mp W_\mu^2(x) \right) \tag{2.1.19}$$

and

$$Z_\mu(x) = W_\mu^3(x) \cos\theta_W - B_\mu(x) \sin\theta_W \tag{2.1.20}$$

become massive. The fourth linear combination, the photon,

$$A_\mu(x) = W_\mu^3(x)\sin\theta_W + B_\mu(x)\cos\theta_W \tag{2.1.21}$$

remains massless. Thus, $U(1)_{EM}$ symmetry remains unbroken and electric charge is conserved.

- The weak mixing angle θ_W in (2.1.20) and (2.1.21) (also called the Weinberg angle) is given by $\tan\theta_W = g'/g$, experimentally [24] $\sin^2\theta_W = 0.231$. Also, $g\sin\theta_W = g'\cos\theta_W = e$. With the use of this relationship between the couplings and (2.1.20) and (2.1.21), one may derive the electromagnetic interaction Lagrangian from the relevant terms in (2.1.6).

- $W^\pm$ and Z bosons acquire masses due to the interaction with the Higgs vev. The masses are related to the vev as

$$M_W^2 = \frac{g^2 v^2}{4}, \qquad M_Z^2 = \frac{(g^2 + g'^2)\, v^2}{8} \tag{2.1.22}$$

Experimentally, $M_W = 80.4$ GeV and $M_Z = 91.2$ GeV [24], they exceed the quark and lepton masses (except for the top quark) by orders of magnitude.

- The quarks and leptons acquire masses due to Yukawa interactions with the Higgs vev. More details on how this occurs are presented in the next section.

- The interactions of fermions with $W^\pm$ and Z bosons are given by

$$\begin{aligned} \mathcal{L}_{weak} &= \frac{g}{2\sqrt{2}} \sum_f \overline{\Psi}_f \gamma^\mu (1-\gamma_5)\left(\tau^+ W_\mu^+ + \tau^- W_\mu^-\right)\Psi_f \\ &+ \frac{g}{2\cos\theta_W} \sum_f \overline{\Psi}_f \gamma^\mu \left(g_V^f - g_A^f \gamma_5\right)\Psi_f Z_\mu \end{aligned} \tag{2.1.23}$$

where

$$\tau^{\pm} = \frac{1}{2}\left(\tau_1 \pm i\tau_2\right), \quad g_V^f = T_{3L}^f - 2Q^f \sin^2\theta_W, \quad g_A^f = T_{3L}^f$$

with $T_{3L}^f = 1/2$ for u_f and ν_f, and $T_{3L}^f = -1/2$ for d_f and ℓ_f. The fermion fields Ψ_f are given in the weak isospin basis. The relationship between the weak isospin and mass bases and quark (and lepton) generation mixing are discussed in the next section.

- In many processes the momentum flowing through $W^{\pm}$ or Z boson propagators is much less than $W^{\pm}$ or Z boson mass, $p^2 \ll M_W^2, M_Z^2$. Such a momentum may be neglected in the propagators, or heavy $W^{\pm}$ or Z dynamical degrees of freedom may be integrated out with the weak interaction Lagrangian (2.1.23) being replaced by a low-energy effective Lagrangian, with an effective coupling proportional to the inverse heavy mass squared. For example, if having a $W^{\pm}$ propagator, one would get

$$\frac{g^2}{8(M_W^2 - p^2)} \approx \frac{g^2}{8M_W^2} \equiv \frac{G_F}{\sqrt{2}} \tag{2.1.24}$$

The relevant low-energy effective Lagrangian would be that containing four-fermion interactions:

$$\mathcal{L}_{eff} = -\frac{G_F}{\sqrt{2}} \sum_{i,j,k,l} \bar{\psi}_i \gamma^{\mu}(1-\gamma_5)\psi_j \bar{\psi}_k \gamma_{\mu}(1-\gamma_5)\psi_l \tag{2.1.25}$$

- The magnitude of the Fermi coupling constant, $G_F = 1.166 \times 10^{-5} GeV^{-2}$ [24], may be found experimentally, using muon decay. Also, using (2.1.22) and (2.1.24), one may relate G_F with the Higgs vacuum expectation value as

$$v = (\sqrt{2}G_F)^{-1/2} \tag{2.1.26}$$

which in its turn yields $v = 246$ GeV.

- Spontaneous electroweak symmetry breaking is also used in other extensions of the Standard Model to generate particles masses. Also, beyond the Standard Model an extended Higgs sector is used, for example consisting of two Higgs doublets. This Higgs sector for a general two-Higgs doublet model is described in Appendix C.1.

As it was mentioned above, in this work charm and bottom meson-antimeson oscillations and weak decays are of the primary interest. For these kinds of processes that occur due to weak interactions and at energy scales much lower than $W^\pm$ and Z boson mass scale, it is often more convenient to use a low-energy effective Lagrangian instead of (2.1.23). Note that if $W^\pm$, Z boson and top quark propagators are running in a loop and/or QCD corrections are taken into account, the set of the relevant low-energy effective operators is much richer than that in (2.1.25). Low-energy effective theory is also used to take into account an NP contribution that occurs due to exchange of NP particles with masses ~ 100 GeV or larger. The set of relevant low-energy effective operators depends on a particular process, as we will see in the next chapters.

2.2 Flavor Problem, Quark Masses and Mixing and GIM Mechanism

As mentioned above, quark masses are generated due to Yukawa interactions of the quarks with the Higgs doublet vev. The relevant terms of the SM Lagrangian are, using (2.1.13) and (2.1.15),

$$-\mathcal{L}_{QM} = \sum_{f,f'} \left[Y_{u_{ff'}} \bar{u}_L^f \; u_R^{f'} + Y_{d_{ff'}} \bar{d}_L^f \; d_R^{f'} + h.c. \right] \frac{v}{\sqrt{2}} \tag{2.2.1}$$

or, in the matrix form,

$$-\mathcal{L}_{QM} = (\bar{u},\bar{c},\bar{t})_L \, \frac{Y_u \; v}{\sqrt{2}} \begin{pmatrix} u \\ c \\ t \end{pmatrix}_R + \left(\bar{d},\bar{s},\bar{b}\right)_L \frac{Y_d \; v}{\sqrt{2}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_R + h.c. \tag{2.2.2}$$

with

$$\frac{Y_u \; v}{\sqrt{2}} \equiv \hat{M}_u, \qquad \frac{Y_d \; v}{\sqrt{2}} \equiv \hat{M}_d$$

being mass matrices for the up- and down-type quarks respectively. Note that $\hat{M}_u$ and $\hat{M}_d$ are in general non-diagonal, thus the quark weak isospin or gauge basis is different from the quark mass basis.

Let V_{u_L}, V_{d_L} and V_{u_R}, V_{d_R} be respectively left and right unitary transformation matrices that diagonalize matrices Y_u and Y_d:

$$V_{u_L} Y_u V_{u_R}^\dagger = Y_u^m = \begin{pmatrix} y_u & 0 & 0 \\ 0 & y_c & 0 \\ 0 & 0 & y_t \end{pmatrix}, \quad V_{d_L} Y_d V_{d_R}^\dagger = Y_d^m = \begin{pmatrix} y_d & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix} \tag{2.2.3}$$

where superscript m stands for the quark mass basis. The quark states are transformed subsequently as

$$V_{u_{L,R}} \begin{pmatrix} u \\ c \\ t \end{pmatrix}_{L,R} = \begin{pmatrix} u^m \\ c^m \\ t^m \end{pmatrix}_{L,R}, \quad V_{d_{L,R}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{L,R} = \begin{pmatrix} d^m \\ s^m \\ b^m \end{pmatrix}_{L,R} \tag{2.2.4}$$

or

$$(V_{q_{L,R}})_{ff'} \; q_{L,R_{f'}} = q^m_{L,R_f} \qquad q_f = u_f, d_f \tag{2.2.5}$$

The quark masses can be read off from Eqs. (2.2.2) and (2.2.3) - they are given by

$$m_q = \frac{y_q v}{\sqrt{2}}, \quad q = u, d, s, c, b, t, \tag{2.2.6}$$

or quark masses are given by the product of their Yukawa couplings with the Higgs vev.

Thus, the spontaneous breaking of the electroweak symmetry provides us with a rather simple mechanism of generation of quark masses. Yet, the quark states transformation from the weak into the mass bases plays a crucial role in understanding flavor phenomenology. To start with, note that in the weak interaction Lagrangian given by Eq. (2.1.23) in terms of weak isospin fermion states, there are no quark generation mixing terms. In other words, the only allowed hadronic currents are flavor-conserving ones and those involving quark flavor transitions within the same quark family.

On the other hand, in terms of the quark mass eigenstates, the weak interaction Lagrangian has the following form:

$$\begin{aligned}\mathcal{L}_{weak} = & \frac{g}{2\sqrt{2}} \sum_{f,f'} \bar{u}_f^m \gamma^\mu (1-\gamma_5) W_\mu^+ V_{ff'} d_{f'}^m + \bar{d}_f^m \gamma^\mu (1-\gamma_5) W_\mu^- V_{ff'}^\star u_{f'}^m + \\ & + \frac{g}{2\cos\theta_W} \sum_f \bar{q}_f^m \gamma^\mu \left(g_V^f - g_A^f \gamma_5 \right) q_f^m Z_\mu, \quad q_f^m = u_f^m, d_f^m \end{aligned} \tag{2.2.7}$$

The second term in (2.2.7) - the neutral current interactions with the Z boson, is still flavor conserving (obviously, this will be true also for the electromagnetic and strong interaction currents in the mass basis). This is a consequence of the quark transformation matrices unitarity:

$$(V_{q_{L,R}})_{ff'} (V^\star_{q_{L,R}})_{f''f'} = \delta_{ff''} \tag{2.2.8}$$

Thus, the Standard Model Lagrangian contains no Flavor Changing Neutral Currents (FCNC's) - those may be induced only via loop diagrams. This is in accord with the experimental data - the processes occurring due to FCNC's are greatly suppressed (see [24] and references therein).

However, in the first term in (2.2.7) - with charged currents interacting with $W^{\pm}$, three u_f quarks are transformed into three $d_{f'}$ quarks (or vice versa), and one may have $f \neq f'$. In other words, the quark generations (in the mass basis) are mixed in weak interactions of charged quark currents. The 3×3 mixing matrix, $V_{ff'}$, is related to the quark rotation matrices $V_{u_{L,R}}$ and $V_{d_{L,R}}$ as

$$V = V_{u_L} V_{d_L}^{\dagger} \tag{2.2.9}$$

It is also called the Cabibbo-Kobayashi-Maskawa matrix or "CKM matrix" [31, 32] (it is also denoted as V_{CKM}), and the quark generation mixing in weak interactions is called "CKM mixing". The elements of the CKM matrix are determined experimentally in processes that occur via quark generation mixing.

It is important to stress that the CKM matrix is unitary (which stems from unitarity of V_{u_L} and V_{d_L}). Thus, measured values of the CKM matrix elements must satisfy the relevant unitarity conditions - any deviation from these conditions would imply a New Physics contribution to the process that is used to measure a given matrix element.

Another consequence of the CKM matrix unitarity is the so-called GIM (Glashow-Iliopoulos-Maiani) mechanism [33] responsible for cancelations between different diagram contributions to a quark flavor transition that occurs due to CKM mixing. Suppose that in such a transition a quark propagator, e.g. a down-type one, is exchanged. Then the amplitude (or its relevant part) will have a form

$$M = V_{u_1 d} V^{\star}_{u_2 d} A(p, m_{u_1}, m_{u_2}, m_d) + V_{u_1 s} V^{\star}_{u_2 s} A(p, m_{u_1}, m_{u_2}, m_s) + \\ + V_{u_1 b} V^{\star}_{u_2 b} A(p, m_{u_1}, m_{u_2}, m_b) \qquad (2.2.10)$$

where u_1 and u_2 are the external up-type quark lines, with $u_1 \neq u_2$, and p is the external momentum (or the set of external momenta). The explicit form of function $A(p, m_{u_1}, m_{u_2}, m)$ depends on a particular process, but apparently does not depend on which down-type quark propagator is exchanged, as the strong, electromagnetic and weak interaction couplings are the same for all the down-type quarks.

Consider now the part of M, M', that does not depend on the masses of down-type quarks exchanged as propagators. E.g. it may be the ultraviolet (UV) divergent terms of the diagrams contributing to M. Or the energy-momentum at which the process occurs is much greater than the masses of down-type quarks, $p \gg m_d, m_s, m_b$, in which case one may rewrite M as $M \approx M' + O(m_b/p)$. Obviously, M' is derived from Eq.(2.2.10) by setting $m_d = m_s = m_b = 0$, or

$$M' = \left[V_{u_1 d} V^{\star}_{u_2 d} + V_{u_1 s} V^{\star}_{u_2 s} + V_{u_1 b} V^{\star}_{u_2 b} \right] A(p, m_{u_1}, m_{u_2}, 0) \qquad (2.2.11)$$

Yet, as

$$V_{u_1 d} V^{\star}_{u_2 d} + V_{u_1 s} V^{\star}_{u_2 s} + V_{u_1 b} V^{\star}_{u_2 b} = 0$$

due to the unitarity of the CKM matrix, M' vanishes! In other words, the contributions of particular diagrams are canceled out in sum in the limit of vanishing (internal) quark masses, or in the limit of the exact U(3) quark family symmetry. Thus, the GIM cancelation mechanism, stemming from the unitarity of the CKM matrix, is also a manifestation of the underlying symmetry of the electroweak and strong interactions with respect to transformations in the quark family space.

If the up-type quarks are exchanged as propagators, the same mechanism is applied due to

$$V_{ud_1}V^{\star}_{ud_2} + V_{cd_1}V^{\star}_{cd_2} + V_{td_1}V^{\star}_{td_2} = 0 \qquad (2.2.12)$$

where d_1 and d_2 are the external down-type quark lines with $d_1 \neq d_2$, and (2.2.12) stems again from the unitarity of the CKM matrix.

Of course, the quark family symmetry is badly broken by the SM Yukawa sector (or by the quark masses), and often the quarks exchanged as propagators have masses much greater than the external momenta. The transition amplitudes discussed above are distinctly different from zero in reality. Nevertheless, the GIM cancelation mechanism plays a crucial role in flavor physics. It suffices to note that it accounts for cancelations of ultraviolet divergences in loop-induced quark FCNC processes and thus assures the renormalizibility of the Standard Model.

GIM cancelation effects are especially important in the charmed hadron involved processes, in particular in the charmed meson-antimeson oscillations. As the strange and down quarks are much lighter than the charm quark, the limit of vanishing down- and strange quarks masses, or of the exact flavor U-spin symmetry is relevant if s- and d-quark propagators are exchanged [37]. Often the limit of the exact flavor SU(3) symmetry is considered rather than its U-spin subgroup, as long as with the down and strange masses, the up quark mass is also set to be zero.

If CP-violation is neglected, one may use the two quark generation mixing approximation in studying charm decays and charmed meson oscillations. This approximation is based on the fact that mixing of the third generation with the first two generations of quarks is suppressed as compared to the first two generations mixing. Within this approximation, the quark mixing matrix V is a 2×2 complex unitary matrix, it is also called the Cabibbo matrix, V_C [31]. The unitarity condition relevant to the GIM cancelation mechanism in the charm sector is now

$$V_{ud}V_{cd}^{\star} + V_{us}V_{cs}^{\star} = 0 \tag{2.2.13}$$

Of course, flavor SU(3) is broken as well by the up, down and strange masses. Yet, as mentioned above, these masses are much less than the charm mass and hence the energy-momentum of the processes of interest. As a consequence, the charm decay and oscillation amplitudes, affected by the GIM mechanism, are suppressed in powers of the strange-to-charm mass ratio, m_s/m_c. This makes physics of the charm sector different from that of bottom and strange flavors. We will discuss this in more detail in the next section and in Chapter 3.

The GIM mechanism is perhaps the unique property of the quark generation mixing that is independent of the explicit form and structure of the CKM matrix. More generally, flavor physics depends on the CKM matrix structure in a crucial way, so let us discuss this structure in more detail.

It is instructive to start with the case of two-generation mixing (called Cabibbo mixing as mentioned above), when the quark mixing matrix is a 2×2 complex unitary matrix. Such a matrix has four independent parameters: a rotation angle in the two-dimensional quark family space and three phases. Yet, we have a freedom in redefining four quark flavor phases. One of these phase transformations, an equal phase rotation for all the four flavors, cancels out in (2.2.7) and has no impact on the Cabibbo matrix. However, the other three phase rotations may be used to eliminate the three phases in the Cabibbo matrix. Thus, in the two-generation mixing scenario, the mixing matrix V_C depends on only one parameter - the rotation angle in the two-dimensional quark family space, or on Cabibbo angle θ_C:

$$V_C = \begin{pmatrix} \cos\theta_C, & \sin\theta_C \\ -\sin\theta_C, & \cos\theta_C \end{pmatrix} \tag{2.2.14}$$

where $\sin\theta_C = 0.225$ [24].

It is worth noting that for two generations the mixing matrix is real. Thus, for two quark families the weak interaction Lagrangian is CP-invariant.

For three quark generations, the 3×3 unitary mixing matrix has nine independent parameters - three $O(3)$ rotation angles and six phases. Again, one may redefine six quark flavor phases, to eliminate the phases of the CKM matrix. Like in the two-generation case, one of these phase transformations, an equal phase rotation for all the six flavors, cancels out in (2.2.7) and has no impact on the CKM matrix. Thus, only five out of six phases in the CKM matrix may be eliminated. In what follows, we are left with four independent parameters in the CKM matrix - three rotation angles in the quark family space and a phase that accounts for CP-violation.

There are several ways to parameterize the CKM matrix [38, 24]. In this work we will use the so-called Wolfenstein parametrization [39] based on the hierarchy of the quark generations mixing:

$$V_{CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2}, & \lambda, & A\lambda^3(\rho - i\eta) \\ -\lambda, & 1-\frac{\lambda^2}{2}, & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta), & -A\lambda^2, & 1 \end{pmatrix} + O(\lambda^4) \tag{2.2.15}$$

where $\lambda \simeq \sin\theta_C = 0.225$. For the other parameters the experimental fits give [24]

$$A = 0.808^{+0.022}_{-0.015}, \quad \bar{\rho} = 0.132^{+0.022}_{-0.014}, \quad \bar{\eta} = 0.341 \pm 0.013 \tag{2.2.16}$$

where $\bar{\rho} \equiv \rho\,(1-\lambda^2/2)$, $\bar{\eta} \equiv \eta\,(1-\lambda^2/2)$.

Note that the CKM matrix is nearly diagonal. While its diagonal elements are of order of unity, the non-diagonal ones are suppressed in powers of the Wolfenstein parameter λ. Nevertheless, CKM mixing plays a crucial role in the particle phenomenology. It suffices to mention that for many hadrons the dominant (weak) decay mode is one occurring via CKM mixing (see [24] and references therein).

The discussed mass generation mechanism is readily extended to the leptonic sector: for the charged leptons,

$$m_\ell = \frac{y_\ell v}{\sqrt{2}} \tag{2.2.17}$$

The neutrinos are massless within the SM, the neutrino flavors ν_e, ν_μ and ν_τ are *defined* to be the states that are transformed respectively into electron, muon and tau-lepton when emitting or absorbing W-boson. There is no mixing between the lepton generations within the Standard Model, the lepton number is conserved for each generation separately.

Presently there is compelling evidence from several experiments [3, 4, 5, 6, 7] for the neutrino flavor oscillations, which implies that neutrinos have masses and that the mass eigenstates are different from the flavor states. The matrix that relates the neutrino mass and flavor eigenstates is called the MNSP (Maki-Nakagawa-Sakata-Pontecorvo) matrix [40] - this is the analogue of CKM matrix for the leptonic sector. Yet, unlike the CKM, the non-diagonal elements of MNSP matrix are of order unity [24]. To parameterize the MNSP matrix, one often uses the empirically well-supported tri-bi-maximal mixing approximation. The explicit form of the MNSP matrix in this approximation may be found e.g. in [41].

Note that in general one needs right-handed neutrinos to generate the neutrino mass terms. Within the Standard Model there are no right handed neutrinos: they are singlets under all three SM gauge groups. Thus, one should invoke some New Physics beyond the Standard Model, to introduce right-handed neutrinos[1]. The study of neutrino masses and their generation mechanism goes beyond the scope of the present work, however the problem of neutrino masses illustrates that in spite of providing an elegant mechanism to generate the fermion masses and flavor mixing, there are certain related issues that the SM is unable to explain.

Another issue of that kind is the so-called flavor puzzle or flavor problem [34]. It may be formulated by the following set of questions:

- Are there only three generations of quarks and leptons?
- Where does the hierarchy of the quark and lepton masses and that of CKM mixing come from?
- Why are flavor changing neutral currents suppressed?

Presently, there is no well-motivated theoretical explanation for limiting the quark and lepton generation number to three. Viable models with four generations of fermions still exist [16, 17, 18].

Nor is the Standard Model able to explain the existing hierarchy of the quark and lepton masses as well as that of CKM mixing. It just provides a mechanism to generate the masses and the mixing and attributes the mass hierarchy to that of Yukawa couplings, and the CKM matrix elements hierarchy to suppression in powers of the Wolfenstein parameter λ.

The Standard Model does explain why the FCNC's are suppressed - there are no flavor changing neutral currents in the SM Lagrangian. Yet, any FCNC process has room for some New Physics contribution, either due to some experimental uncertainties or due to some theoretical uncertainties of the SM prediction. Unlike the SM,

[1] Alternatively, if one assumes that neutrinos are Majorana fermions and uses solely the left handed Majorana neutrino mass terms, one should still invoke some New Physics to explain where such mass terms come from.

many of its extensions have FCNC terms in the Lagrangian. To fit the existing experimental data, either the New Physics FCNC couplings should be suppressed ($\sim 10^{-7}$), or if not, the New Physics scale should be $\sim 10^4$ TeV [1]. There is no explanation why the NP FCNC couplings are so suppressed or why the NP scale should be so heavy.

Solving the flavor problem represents one of the challenging tasks of flavor physics. An attempt is made in this work as well: in Chapter 5 we try to explain the quark and charged lepton mass hierarchy within the simplest SM extension with two Higgs doublets.

As for here, we continue with providing some theoretical background needed for the further discussion. We proceed now to introducing some basic inputs of the meson-antimeson oscillation formalism.

2.3 Meson - Antimeson Oscillations

Meson-antimeson oscillations are a manifestation of transformations of matter into antimatter and vice versa in the Nature. Because of their pure quantum nature, these oscillations may be used to understand fundamental properties of elementary particles. The first evidence for CP-violation, observed in neutral Kaon decays [42], has been interpreted properly by using the $K^0-\overline{K}^0$ oscillation mechanism [2, 38]. It is also believed that in the presence of a source of large CP-violation (which may occur beyond the Standard Model) the meson-antimeson mixing mechanism may provide an explanation of the existing baryon asymmetry of the Universe. More generally, due to its quantum nature and occurrence via exchange of heavy virtual particles (~ 100 GeV or heavier), meson-antimeson mixing is invaluable as a source of information on the physics that occurs at high energy scales.

The time evolution of an oscillating meson-antimeson system, e.g the $D^0 - \overline{D}^0$

system, is given by the Schrodinger equation:

$$i\frac{\partial}{\partial t}\begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix} = \left(M - \frac{i}{2}\Gamma\right)\begin{pmatrix} D^0(t) \\ \overline{D}^0(t) \end{pmatrix} \tag{2.3.1}$$

where the mass matrix M and the width matrix Γ are Hermitian, and CPT invariance requires that $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$. The oscillation is parameterized by the off-diagonal elements, $M_{12} = M^\star_{21}$ and $\Gamma_{12} = \Gamma^\star_{21}$. M_{12} corresponds to the dispersive part and Γ_{12} corresponds to the absorptive part of the oscillation amplitude [43]:

$$M_{12} - \frac{i}{2}\Gamma_{12} = \frac{1}{2M_D}\langle\overline{D}^0|H_W^{\Delta C=2}|D^0\rangle + \frac{1}{2M_D}\sum_n \frac{\langle\overline{D}^0|H_W^{\Delta C=1}|n\rangle\langle n|H_W^{\Delta C=1}|D^0\rangle}{M_D - E_n + i\epsilon} \tag{2.3.2}$$

where M_D is the meson average mass, $|n\rangle$ is an intermediate state and H_W is a weak interaction low-energy effective Hamiltonian.

The mass eigenstates, D_1 and D_2, are related to D^0 and $\overline{D}^0$ as

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\overline{D}^0\rangle \tag{2.3.3}$$

where

$$\left(\frac{q}{p}\right)^2 = \frac{M^\star_{12} - \frac{i}{2}\Gamma^\star_{12}}{M_{12} - \frac{i}{2}\Gamma_{12}} \tag{2.3.4}$$

The physical observables used to describe meson-antimeson mixing are the eigenstates mass difference and the eigenstates width difference or lifetime difference[2]:

$$\Delta M - \frac{i}{2}\Delta\Gamma = 2\sqrt{\left(M_{12} - \frac{i}{2}\Gamma_{12}\right)\left(M^\star_{12} - \frac{i}{2}\Gamma^\star_{12}\right)} \tag{2.3.5}$$

[2]Rigorously speaking, width difference and lifetime difference are equivalent when using dimensionless quantities: normalized width difference, $\Delta\Gamma/\Gamma$, and normalized lifetime difference, $\Delta\tau/\tau$, where Γ and τ are the average width and lifetime respectively.

In the limit of CP conservation M_{12} and Γ_{12} are real, thus $|q| = |p|$ and

$$\Delta M \equiv M_+ - M_- = 2M_{12} \tag{2.3.6}$$

$$\Delta\Gamma \equiv \Gamma_+ - \Gamma_- = 2\Gamma_{12} \tag{2.3.7}$$

where "+" and "-" are the CP-even and CP-odd eigenstates respectively. This limit is well-justified when considering $D^0 - \overline{D}^0$ mixing, where $\Delta M \sim \Delta\Gamma$ and no CP-violation has been observed so far [44].

For the other mode of interest, $B_s - \overline{B}_s$, according to the experimental data [24], one has $\Delta M \gg \Delta\Gamma$. As one can see after doing some algebra, $|M_{12}| \gg |\Gamma_{12}|$ then. In this limit equation (2.3.5) may be rewritten as

$$\Delta M \equiv M_H - M_L \simeq 2|M_{12}| \tag{2.3.8}$$

$$\Delta\Gamma \equiv \Gamma_L - \Gamma_H \simeq 2|\Gamma_{12}| \cos\Phi \tag{2.3.9}$$

where $\Phi = \arg(-M_{12}/\Gamma_{12})$, and "H" and "L" denote heavy and light eigenstates respectively.

As if follows from Eq. (2.3.4), for $M_{12} \gg \Gamma_{12}$ the ratio $|q/p|$ is close to one - CP-violation is small in $B_s - \overline{B_s}$ mixing regardless of the phases of M_{12} and Γ_{12}. However, these phases are non-negligible for the B_s width difference, as follows from Eq. (2.3.9). In the case when M_{12} has two or more components (e.g. the SM and NP contribution or different NP contributions), the relative phases of the components may be of importance as well [30] .

Within the Standard Model meson-antimeson oscillations occur due to Cabibbo-Kobayashi-Maskawa mixing of quark generations in weak interactions. To the lowest order in perturbation theory these oscillations occur at one-loop via the box diagrams with $W^{\pm}$ bosons and quarks running in the loops, as depicted in Figure 2.1. For $B_s -$

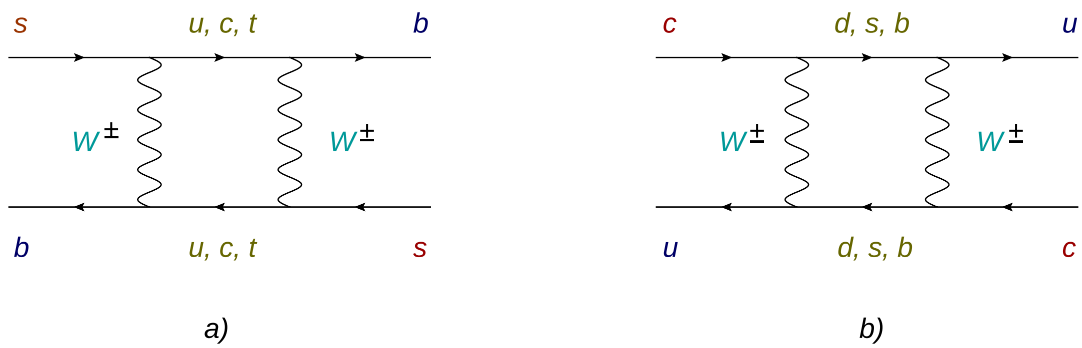

Figure 2.1: Box diagrams within the SM a) for $B_s - \overline{B_s}$, b) for $D^0 - \overline{D}^0$.

$\overline{B_s}$ meson oscillations, the box diagrams give the dominant contribution to the process amplitude - this is a short-distance contribution dominated process (see e.g. [45]).

In contrast, for $D^0 - \overline{D}^0$ mixing the total contribution of the SM box diagrams is suppressed [46] due to GIM cancelation effects. As discussed in the previous section, GIM cancelation in charm decays and/or oscillations occurs, if the flavor transition is due to CKM mixing and is mediated by a down-type quark propagator. In the diagram in Fig. 2.1 (b) two down-type quark propagators are present, hence *double* GIM cancelation occurs. Furthermore, studying the behavior of the oscillation amplitude under the flavor U-spin transformations, one can show [37] that in terms of the strange-to-charm mass ratio the amplitude is suppressed as $(m_s^2/m_c^2)^2 = m_s^4/m_c^4$ rather than just $(m_s/m_c)^2$ (as could be naively expected when having double GIM cancelation). As a result, box diagram contribution to $D^0 - \overline{D}^0$ mixing is negligible in the SM [43].

Within the SM $D^0 - \overline{D}^0$ oscillations are dominated by long-distance effects [29] - due to exchange of charmless mesonic states. A large short-distance contribution to $D^0 - \overline{D}^0$ may however occur due to New Physics interactions (if no GIM cancelations occur).

As mentioned above, the NP contribution compatible with the SM one or even exceeding it comes from diagrams with $W^{\pm}$ bosons being replaced by other heavy particles that may exist beyond the Standard Model. In some of the SM extensions the quarks running in the box loops are replaced along with W-s by heavy degrees of freedom as well.

Also, if Flavor Changing Neutral Currents (FCNC) are present in the model, meson-antimeson oscillations (or at least some of them) may occur at tree level. Unless it contradicts the experimental data, the NP contribution may exceed the SM one by orders of magnitude.

In general, New Physics may have impact both on the mass difference and the lifetime difference in meson-antimeson oscillations. Yet, a sizable NP contribution to the lifetime difference occurs within specific SM extensions only, as we discuss in the next two chapters. Therefore an NP contribution to the mass difference is studied more frequently. Study of the lifetime difference in meson-antimeson oscillations within the SM extensions has however its own advantage: it allows one to get an information on a specific model that is independent or weakly dependent on assumptions made about this model. We discuss this in more detail in the next chapter.

Chapter 3 New Physics Searches in the Charm Sector

We will examine here one the most prominent charmed particle involving phenomena, the $D^0 - \overline{D}^0$ mixing. As discussed above, meson-antimeson mixing is an important vehicle for indirect studies of New Physics. Due to the absence of tree-level flavor-changing neutral current transitions in the Standard Model (SM), it can only occur via quantum effects associated with the SM and NP particles. In fact, the existence of both charm and top quark were inferred from the kaon and beauty mixing amplitudes [38]. The estimates of masses of those particles were later found to be in agreement with direct observations. This motivates indirect searches for NP particles in a meson-antimeson mixing.

Recently, there has been a considerable interest in the only available meson-antimeson mixing in the up-quark sector, the $D^0 - \overline{D}^0$ mixing [46]. The fact that the search is indirect and complimentary to existing constraints from the bottom-quark sector actually provides parameter space constraints for a large variety of NP models [47, 48].

A flurry of recent experimental activity in that field led to the observation of $D^0 - \overline{D}^0$ mixing from several different experiments such as BaBar [49], Belle [50] and CDF [51]. These results have been combined by the Flavor Averaging Group (HFAG) [44] to yield

$$y_{\rm D}^{exp} = (8.0 \pm 1.3) \cdot 10^{-3} \tag{3.1}$$

$$x_{\rm D}^{exp} = (5.9 \pm 2.0) \cdot 10^{-3}, \tag{3.2}$$

where x_{D} and y_{D} are defined as

$$x_{\mathrm{D}} \equiv \frac{\Delta M_{\mathrm{D}}}{\Gamma_{\mathrm{D}}}, \qquad \text{and} \qquad y_{\mathrm{D}} \equiv \frac{\Delta \Gamma_{\mathrm{D}}}{2\Gamma_{\mathrm{D}}} \quad , \tag{3.3}$$

Γ_{D} is the average width of the two neutral D meson mass eigenstates, and ΔM_{D}, $\Delta\Gamma_{\mathrm{D}}$ are the mass and width differences of the neutral D-meson mass eigenstates. In the limit of CP-conservation, $\Delta\Gamma_D \equiv \Gamma_+ - \Gamma_-$, where "+" and "-" are CP-even and CP-odd D-meson eigenstates respectively.

One can also write y_D as an absorptive part of the $D^0 - \overline{D}^0$ mixing matrix [52],

$$y_D = \frac{1}{\Gamma_{\mathrm{D}}} \sum_n \rho_n \langle \overline{D}^0 | \mathcal{H}_w^{\Delta C=1} | n \rangle \langle n | \mathcal{H}_w^{\Delta C=1} | D^0 \rangle, \tag{3.4}$$

where ρ_n is a phase space function that corresponds to a charmless intermediate state n. This relation shows that $\Delta\Gamma_{\mathrm{D}}$ is driven by transitions $D^0, \overline{D}^0 \to n$, *i.e.* physics of the $\Delta C = 1$ sector.

Eqs. (3.1) and (3.2) imply one-sigma window for the HFAG values of x_D and y_{D},

$$3.9 \cdot 10^{-3} < x_D < 7.9 \cdot 10^{-3} \qquad (\text{one} - \text{sigma window}) \tag{3.5}$$

$$6.7 \cdot 10^{-3} < y_{\mathrm{D}} < 9.3 \cdot 10^{-3} \qquad (\text{one} - \text{sigma window}) \tag{3.6}$$

In principle, these results can be used to constrain parameters of NP models with the anticipated improved accuracy for the future D-mixing measurements. In reality, those results can only provide the ballpark estimate to be used for constraining NP models. The reason is that the SM estimate for the parameters x_{D} and y_{D} is rather uncertain, as it is dominated by long-distance QCD effects [29, 52, 53]. It was nevertheless shown that even this estimate provides rather stringent constraints on the NP parameter space for many models affecting the mass difference x_{D} [47], [54]-[59].

It was recently shown [48] that $D^0 - \overline{D}^0$ mixing is a rather unique system, where

$y_{\rm D}$ can also be used to constrain the models of New Physics[1]. This stems from the fact that there is a well-defined theoretical limit (the flavor $SU(3)$-limit) where the SM contribution vanishes and the lifetime difference is dominated by the NP $\Delta C = 1$ contributions. In the real world, flavor $SU(3)$ is, of course, broken, so the SM contribution is proportional to a (second) power of m_s/Λ, which is a rather small number. If the NP contribution to $y_{\rm D}$ is non-zero in the flavor $SU(3)$-limit, it can provide a large contribution to the mixing amplitude.

To see this, consider a D^0 decay amplitude which includes a small NP contribution, $A[D^0 \to n] = A_n^{\rm (SM)} + A_n^{\rm (NP)}$. Experimental data for D-meson decays are known to be in decent agreement with the SM estimates [61, 62]. Thus, $A_n^{\rm (NP)}$ should be smaller than (in sum) the current theoretical and experimental uncertainties in predictions for these decays.

One may rewrite equation (3.4) in the form (neglecting the effects of CP-violation)

$$y_D = \sum_n \frac{\rho_n}{\Gamma_{\rm D}} A_n^{\rm (SM)} \bar{A}_n^{\rm (SM)} + 2\sum_n \frac{\rho_n}{\Gamma_{\rm D}} A_n^{\rm (NP)} \bar{A}_n^{\rm (SM)} + \sum_n \frac{\rho_n}{\Gamma_{\rm D}} A_n^{\rm (NP)} \bar{A}_n^{\rm (NP)} \quad . \tag{3.7}$$

The first term in this equation corresponds to the SM contribution, which vanishes in the $SU(3)$ limit. In ref. [48] the last term in (3.7) has been neglected, thus the NP contribution to $y_{\rm D}$ comes there solely from the second term, due to interference of $A_n^{\rm (SM)}$ and $A_n^{\rm (NP)}$. While this contribution is in general non-zero in the flavor $SU(3)$ limit, in a large class of (popular) models it actually is [48, 63]. Then, in this limit, $y_{\rm D}$ is completely dominated by pure $A_n^{\rm (NP)}$ contribution given by the last term in eq. (3.7)! It is clear that the last term in equation (3.7) needs more detailed and careful studies, at least within some of the NP models.

Indeed, in reality, flavor $SU(3)$ symmetry is broken, so the first term in Eq. (3.7) is not zero. It has been argued [29] that in fact the SM $SU(3)$-violating contributions could be at a percent level, dominating the experimental result. The SM predictions

[1] A similar effect is possible in the bottom-quark sector [60].

of y_D, stemming from evaluations of long-distance hadronic contributions, are rather uncertain. While this precludes us from placing explicit constraints on parameters of NP models, it has been argued that, even in this situation, an upper bound on the NP contributions can be placed [47] by displaying the NP contribution only, i.e. as if there were no SM contribution at all. This procedure is similar to what was traditionally done in the studies of NP contributions to $K^0 - \overline{K}^0$ mixing, so we shall employ it here too.

In this chapter we revisit the problem of the NP contribution to $y_{\rm D}$ and provide constraints on R-parity-violating supersymmetric (SUSY) models as a primary example. It has been recently argued in [64] that within $\not{R}$- SUSY models, the New Physics contribution to $y_{\rm D}$ is rather small, mainly because of stringent constraints on the relevant pair products of RPV coupling constants. However, this result has been derived neglecting the transformation of these couplings from the weak isospin basis to the quark mass basis. This approach seems to be quite reasonable for the scenarios with baryonic number violation. However, in the scenarios with leptonic number violation, transformation of the RPV couplings from the weak eigenbasis to the quark mass eigenbasis turns out to be crucial, when applying the existing phenomenological constraints on these couplings.

We show here that within R-parity-breaking supersymmetric models with leptonic number violation, the New Physics contribution to the lifetime difference in $D^0 - \overline{D}^0$ mixing may be large, due to the last term in eq. (3.7). When being large, it is negative (if neglecting CP-violation), i.e. opposite in sign to what is implied by the recent experimental evidence for $D^0 - \overline{D}^0$ mixing.

Of course, diagrams with a large NP contribution to y_D are possible also within other SM extensions. Moreover, some NP diagrams, even though vanishing in the exact flavor SU(3) limit, are proportional to the first power of m_s/m_c and hence may give a sizable contribution to the lifetime difference in $D^0 - \overline{D}^0$ mixing. This is in particular the case within the non-manifest Left-Right Symmetric Model, where such diagrams contribution to y_D may be $\sim 10^{-3}$ and hence compatible with the experimental data [65].

Yet, the contribution of these types of diagrams should be considered in sum, in light of possible cancelations due to the GIM mechanism. We show that the diagrams considered in ref. [65] have negligible contribution in sum due to the GIM cancelation effects. Thus, the NP contribution to y_D within the non-manifest Left-Right Symmetric Model is negligible.

This chapter is organized as follows. In Section 3.1 we discuss the R-parity violating interactions that, in particular, contribute to $D^0 - \overline{D}^0$ lifetime difference. We confront the form of these interactions in the weak isospin basis to that in the quark mass basis, emphasizing the important differences. In Section 3.2 we re-derive formulae for the RPV SUSY contribution to $y_{\rm D}$. Unlike ref. [64], transformation of the RPV coupling constants from the weak to the quark mass eigenbasis is taken into account. Also the behavior of different $\not{R}$- SUSY contributions in the limit of the flavor SU(3) symmetry is discussed in detail. In Section 3.3 we examine the existing phenomenological constraints on the RPV coupling constants. The importance of taking into account the transformation of these couplings from the weak to the mass eigenbasis is emphasized again. We present our numerical results within the RPV SUSY model in Section 3.4. Finally in Section 3.5 we discuss briefly the New Physics contribution to $D^0 - \overline{D}^0$ lifetime difference within the non-manifest Left-Right Symmetric Model. Some details of the derivation of bounds on the pair products of RPV couplings, relevant for our analysis, are contained in Appendix A. The results presented in this chapter are based on those published in [66] and [63].

3.1 R-Parity Breaking Interactions: Weak vs Mass Eigenbases

We consider a general low-energy supersymmetric scenario with no assumptions made on a SUSY breaking mechanism at the unification scales ($\sim\ (10^{16} - 10^{18}) GeV$). The most general Yukawa superpotential for an explicitly broken R-parity supersymmetric theory is given by

$$W_{\not R} = \sum_{i,j,k} \left[\frac{1}{2} \lambda_{ijk} L_i L_j E^c_k + \lambda'_{ijk} L_i Q_j D^c_k + \frac{1}{2} \lambda''_{ijk} U^c_i D^c_j D^c_k \right] \qquad (3.1.1)$$

where L_i, Q_j are $SU(2)_L$ weak isodoublet lepton and quark superfields, respectively; E^c_i, U^c_i, D^c_i are SU(2) singlet charged lepton, up- and down-quark superfields, respectively; λ_{ijk} and λ'_{ijk} are lepton number violating Yukawa couplings, and λ''_{ijk} is a baryon number violating Yukawa coupling; $\lambda_{ijk} = -\lambda_{jik}$, $\lambda''_{ijk} = -\lambda''_{ikj}$. To avoid rapid proton decay, we assume that $\lambda''_{ijk} = 0$ and work with a lepton number violating $\not R$- SUSY model.

For meson-to-antimeson oscillation processes, to the lowest order in perturbation theory, only the second term of (3.1.1) is of importance. The relevant R-parity breaking part of the Lagrangian is the following:

$$\mathcal{L}_{\not R} = \sum_{i,j,k} \lambda'_{ijk} \Big[-\widetilde{e}_{i_L} \bar{d}_{k_R} u_{j_L} - \widetilde{u}_{j_L} \bar{d}_{k_R} e_{i_L} - \widetilde{d}_{k_R} \bar{e}^c_{i_R} u_{j_L} + \widetilde{\nu}_{i_L} \bar{d}_{k_R} d_{j_L} +$$
$$+ \widetilde{d}_{j_L} \bar{d}_{k_R} \nu_{i_L} + \widetilde{d}_{k_R} \bar{\nu}^c_{i_R} d_{j_L} \Big] + h.c. \qquad (3.1.2)$$

The quark and squark states in (3.1.2) are weak isospin eigenstates. The weak and mass quark eigenstates are related by the unitary transformations (2.2.4), (2.2.5).

Generally speaking, squark transformation matrices from the weak to the mass eigenstates are different from those for quarks. Nevertheless, we choose for squarks to be rotated by the same matrices $V_{u_{L,R}}$ and $V_{d_{L,R}}$ that make quark mass matrices diagonal, i.e.

$$\begin{aligned}
\widetilde{u}^m_{j_L} &= V_{u_{L_{jn}}} \widetilde{u}_{n_L}, \quad \widetilde{u}^m_{j_R} = V_{u_{R_{jn}}} \widetilde{u}_{n_R} \\
\widetilde{d}^m_{k_L} &= V_{d_{L_{kp}}} \widetilde{d}_{p_L}, \quad \widetilde{d}^m_{k_R} = V_{d_{R_{kp}}} \widetilde{d}_{p_R}
\end{aligned} \tag{3.1.3}$$

This is a super-CKM basis, in which the squark mass matrices are non-diagonal and result in mass insertions that change the squark flavors [9, 34, 67, 68]. This source of flavor violation is very important in the pure MSSM sector. In particular, it plays a crucial role in examining the MSSM contribution to $D^0 - \bar{D}^0$ mass difference [47].

In the R-parity breaking part of the SUSY Lagrangian, flavor changing neutral currents are present *a priori*. In order to simplify our analysis, we put all the squark masses to be nearly equal. Then the squark mass matrix is proportional to the identity matrix, i.e. it is diagonal in any basis.

In the quark mass mass basis one may rewrite (3.1.2) as[2]

$$\begin{aligned}
\mathcal{L}_{\not R} = &- \sum_{i,j,k} \widetilde{\lambda}'_{ijk} \left[\widetilde{e}_{i_L} \bar{d}_{k_R} u_{j_L} + \widetilde{u}_{j_L} \bar{d}_{k_R} e_{i_L} + \widetilde{d}^*_{k_R} \bar{e}^c_{i_R} u_{j_L} \right] + \\
&+ \sum_{i,j,k} \lambda'_{ijk} \left[\widetilde{\nu}_{i_L} \bar{d}_{k_R} d_{j_L} + \widetilde{d}_{j_L} \bar{d}_{k_R} \nu_{i_L} + \widetilde{d}^*_{k_R} \bar{\nu}^c_{i_R} d_{j_L} \right] + h.c.
\end{aligned} \tag{3.1.4}$$

where $\widetilde{\lambda}'_{ijk} = V^*_{nj}\ \lambda'_{ink}$, and we redefine the couplings λ' to absorb the relevant elements of matrices $V_{d_{L,R}}$. Such a redefinition of λ' is also equivalent to choosing the weak and mass eigenbases for down-quarks being the same, while for up-quarks they are related by CKM matrix[3]. As it follows from (3.1.4), (s)down-down-(s)neutrino vertices have the weak eigenbasis couplings λ', while charged (s)lepton-(s)down-(s)up

[2]Hereafter in this chapter, since all the formulae are given in the quark mass basis, we will drop the superscript m for simplicity of notation.

[3]This redefinition of λ' is not unique. For example, Allanach et al. [69] used the up-quark weak and mass eigenbases to be the same, relating the bases for down-quarks by the CKM matrix. Another possibility is to redefine λ' in such a way that (s)up-(s)down-charged (s)lepton vertices have the couplings λ' while (s)down-down-(s)neutrino vertices have the couplings $\lambda' \cdot V_{CKM}$ [70]. Clearly all these approaches are equivalent.

vertices have the up quark mass eigenbasis couplings $\tilde{\lambda}'$.

Very often in the literature (see e.g. [48], [64], [71]-[73]) one neglects the difference between λ' and $\widetilde{\lambda}'$, based on the fact that the diagonal elements of the CKM matrix dominate over non-diagonal ones, i.e.

$$V_{jn} = \delta_{jn} + O(\lambda) \qquad \text{so} \qquad \widetilde{\lambda}_{ijk} \approx \lambda'_{ijk} + O(\lambda) \tag{3.1.5}$$

where $\lambda = \sin\theta_C$ is the Wolfenstein parameter.

Notice that relation Eq. (3.1.5) is valid if only there is *no hierarchy* in couplings λ'. On the other hand, the existing strong bounds on pair products $\lambda' \times \lambda'$ (or $\widetilde{\lambda}' \times \widetilde{\lambda}'$) [69, 71, 72] and relatively loose bounds on individual couplings λ' [69] suggest that such a hierarchy may exist. As we will see in Section 4, pair products $\widetilde{\lambda}' \times \widetilde{\lambda}'$ may be orders of magnitude greater than corresponding products $\lambda' \times \lambda'$.

To the end of this section, we explicitly write down the terms of the R-parity breaking part of the Lagrangian that contribute to $D^0 - \bar{D}^0$ lifetime difference:

$$\begin{aligned}
\mathcal{L}^{D^0-\bar{D}^0} = & -\sum_i \Bigg[\widetilde{\lambda}'_{i21} \tilde{e}_{i_L} \bar{d} \left(\frac{1-\gamma_5}{2}\right) c + \widetilde{\lambda}'_{i22} \tilde{e}_{i_L} \bar{s} \left(\frac{1-\gamma_5}{2}\right) c + \\
& + \widetilde{\lambda}'^{*}_{i11} \tilde{e}^{*}_{i_L} \bar{u} \left(\frac{1+\gamma_5}{2}\right) d + \widetilde{\lambda}'^{*}_{i12} \tilde{e}^{*}_{i_L} \bar{u} \left(\frac{1+\gamma_5}{2}\right) s \Bigg] - \\
& - \sum_k \Bigg[\widetilde{\lambda}'_{12k} \tilde{d}^{*}_{k_R} \bar{e}^c \left(\frac{1-\gamma_5}{2}\right) c + \widetilde{\lambda}'_{22k} \tilde{d}^{*}_{k_R}\, \bar{\mu}^c \left(\frac{1-\gamma_5}{2}\right) c + \\
& + \widetilde{\lambda}'^{*}_{11k} \tilde{d}_{k_R} \bar{u} \left(\frac{1+\gamma_5}{2}\right) e^c + \widetilde{\lambda}'^{*}_{21k} \tilde{d}_{k_R} \bar{u} \left(\frac{1+\gamma_5}{2}\right) \mu^c \Bigg]
\end{aligned} \tag{3.1.6}$$

In the next section we will integrate out heavy degrees of freedom in (3.1.6), thus finding the $\not{R}$-SUSY part of the $\Delta C = 1$ effective Hamiltonian. Then we will compute the R-parity breaking SUSY contribution to $\Delta\Gamma_D$.

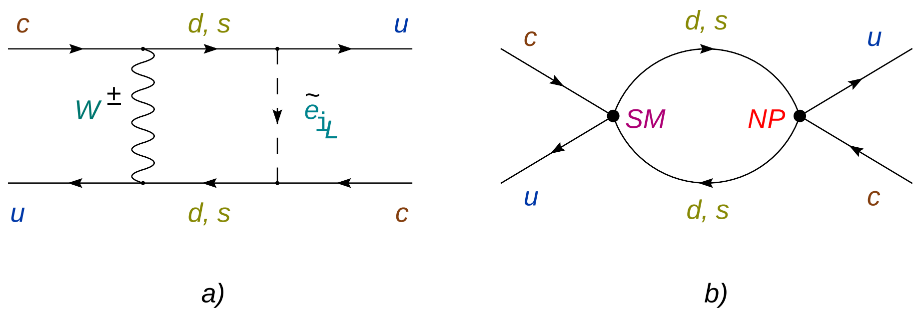

Figure 3.1: $D^0 - \bar{D}^0$ diagrams with a slepton exchange due to RPV interactions.

3.2 $D^0 - \bar{D}^0$ Lifetime Difference Within RPV SUSY

Assuming CP-conservation, the normalized $D^0 - \bar{D}^0$ lifetime difference is given by

$$y_D = \frac{1}{2m_D\Gamma_D} Im \left[\langle \bar{D}^0 | i \int d^4x \; T \left\{ H_W^{\Delta C=1}(x) H_W^{\Delta C=1}(0) \right\} | D^0 \rangle \right], \tag{3.2.1}$$

where $H_W^{\Delta C=1}$ is an effective Hamiltonian including both SM and NP parts. To the lowest order in perturbation theory, the $\not{R}$-SUSY contribution to $D^0 - \bar{D}^0$ mixing comes from the one-loop graphs with

- $W^\pm$ boson, charged slepton and two down-type quarks (Fig. 3.1a);
- two charged sleptons and two down-type quarks (Fig. 3.2a);
- two down-type squarks and two charged leptons[4] (Fig. 3.3a) .

Within the low-energy effective theory, $D^0 - \bar{D}^0$ lifetime difference occurs as a result of a bi-local transition with two $\Delta C = 1$ effective vertices. The relevant low-energy diagrams in Fig.'s 3.1b) - 3.3b) are derived by integrating out heavy $W^\pm$ boson, charged slepton and down-type squark degrees of freedom.

For R-parity-violating SUSY models one can therefore write

$$H_W^{\Delta C=1} = H_{W_{SM}}^{\Delta C=1} + H_{W_{\tilde{\ell}}}^{\Delta C=1} + H_{W_{\tilde{q}}}^{\Delta C=1} \tag{3.2.2}$$

[4] As it follows from (3.1.6), lepton propagators in Fig. 3.3 must be constructed by contractions of charge conjugates of the electron and/or muon field operators.

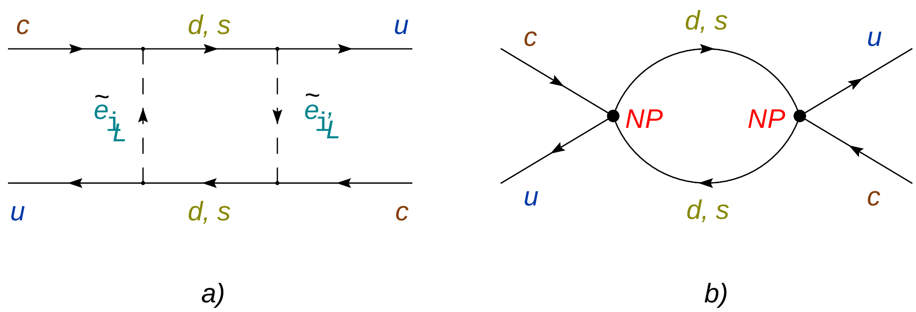

Figure 3.2: Same as in Fig. 3.1, but due to two charged slepton exchange.

The first term in the r.h.s of (3.2.2) is the Standard Model contribution, whereas the second term comes from $\Delta C = 1$ transitions with a slepton exchange and the last term comes from $\Delta C = 1$ transitions with a squark exchange. The Standard model part of $\Delta C = 1$ effective Hamiltonian is given by

$$H_{W_{SM}}^{\Delta C=1} = \frac{G_F}{\sqrt{2}} \Big[C_1(\mu_c)\ \delta^{a_1 a_4}\ \delta^{a_3 a_2} + C_2(\mu_c)\ \delta^{a_1 a_2}\ \delta^{a_3 a_4} \Big] \times \sum_{q_1,\ q_2} V_{uq_1} V_{cq_2}^*\ \bar{u}^{a_1}(x)\gamma^{\mu}(1-\gamma_5) q_1^{a_2}(x)\ \bar{q}_2^{a_3}(x)\gamma_{\mu}(1-\gamma_5)c^{a_4}(x) \quad (3.2.3)$$

where $q_1 = s, d$, $q_2 = s, d$, a_i are the color indices, and C_1 and C_2 are the operator Wilson coefficients. The Wilson coefficients are to be evaluated at a low-energy scale μ_c, which we choose here as $\mu_c = m_c$.

To simplify the following calculations, let us assume that all the sleptons and all squarks are nearly degenerate, i.e.

$$m_{\tilde{e}_i} = m_{\tilde{\nu}_i} = m_{\tilde{\ell}}, \qquad \text{and} \qquad m_{\tilde{d}_k} = m_{\tilde{u}_k} = m_{\tilde{q}}. \quad (3.2.4)$$

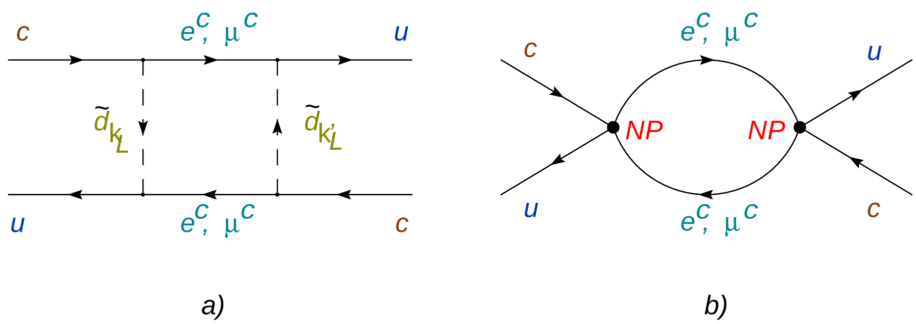

Figure 3.3: Same as in Fig.'s 3.1, 3.2, but due to down-type squark exchange.

With this assumption, the low energy effective Hamiltonian for the R-parity-violating interactions are given by

$$H_{W_{\tilde{\ell}}}^{\Delta C=1} = -\left[\widetilde{C}_1(\mu_c)\ \delta^{a_1a_4}\ \delta^{a_3a_2} + \widetilde{C}_2(\mu_c)\ \delta^{a_1a_2}\ \delta^{a_3a_4}\right] \times \sum_{q_1,\ q_2} \frac{\lambda_{q_1q_2}}{4m_{\tilde{\ell}}^2}\ \bar{u}^{a_1}(x)(1+\gamma_5)q_1^{a_2}(x)\ \bar{q}_2^{a_3}(x)(1-\gamma_5)c^{a_4}(x), \qquad (3.2.5)$$

and

$$H_{W_{\tilde{q}}}^{\Delta C=1} = -\sum_{\ell_1,\ \ell_2} \frac{\lambda_{\ell_1\ell_2}}{4m_{\tilde{q}}^2}\ \bar{u}^a(x)(1+\gamma_5)\ell_1^c(x)\ \bar{\ell}_2^c(x)(1-\gamma_5)c^a(x) \qquad (3.2.6)$$

where $q_1 = s, d,\ q_2 = s, d,\ \ell_1 = e, \mu,$ and $\ell_2 = e, \mu$. The superscript $''c''$ stands for charge conjugation. Also,

$$\lambda_{q_1q_2} \equiv \sum_i \widetilde{\lambda}'^*_{i1q_1}\ \widetilde{\lambda}'_{i2q_2} \qquad \text{and} \qquad \lambda_{\ell_1\ell_2} \equiv \sum_k \widetilde{\lambda}'^*_{\ell_11k}\ \widetilde{\lambda}'_{\ell_22k} \qquad (3.2.7)$$

We assume that $\lambda_{q_1q_2}$ and λ_{ℓ_1,ℓ_2} are real.

The insertions of Hamiltonians of eqs. (3.2.3), (3.2.5), and (3.2.6) can lead to the lifetime difference in the $D^0 - \overline{D}^0$ system. Let us write it as

$$y_D = y_{SM} + y_{SM,NP} + y_{\tilde{\ell}\tilde{\ell}} + y_{\tilde{q}\tilde{q}}, \qquad (3.2.8)$$

where

$$y_{SM,NP} = \frac{1}{2m_D\Gamma_D}\; Im\left[\langle \bar{D}^0|i\int d^4x\; T\left\{H^{\Delta C=1}_{W_{SM}}(x)H^{\Delta C=1}_{W_{\tilde{\ell}}}(0) + \right.\right.$$

$$\left.\left. + H^{\Delta C=1}_{W_{\tilde{\ell}}}(x)H^{\Delta C=1}_{W_{SM}}(0)\right\}|D^0\rangle\right] \qquad (3.2.9)$$

is the term coming form the interference of the SM and NP contributions to $H^{\Delta C=1}_{W}$, and

$$y_{\tilde{\ell}\tilde{\ell}} = \frac{1}{2m_D\Gamma_D}\; Im\left[\langle \bar{D}^0|i\int d^4x\; T\left\{H^{\Delta C=1}_{W_{\tilde{\ell}}}(x)H^{\Delta C=1}_{W_{\tilde{\ell}}}(0)\right\}|D^0\rangle\right], \qquad (3.2.10)$$

$$y_{\tilde{q}\tilde{q}} = \frac{1}{2m_D\Gamma_D}\; Im\left[\langle \bar{D}^0|i\int d^4x\; T\left\{H^{\Delta C=1}_{W_{\tilde{q}}}(x)H^{\Delta C=1}_{W_{\tilde{q}}}(0)\right\}|D^0\rangle\right] \qquad (3.2.11)$$

are coming from two insertions of the NP vertices.

It might seem unreasonable to include double insertions of the NP Hamiltonian to compute y_D, as each insertion generates a contribution that is suppressed by some NP scale M_{NP}, which in general is greater than the electroweak scale set here by M_W. Yet, as the Standard Model contribution is zero in the flavor SU(3) limit (i.e. suppressed by powers of strange quark mass), New Physics contributions can be large [48]. Also, as can be seen from refs. [48] and [64], $y_{SM,NP}$ resulting from the single insertion of the NP Hamiltonian is forbidden in the $SU(3)$ flavor symmetry limit. Thus, double insertion of the NP Hamiltonian can be important, especially if this contribution does not vanish in the $SU(3)$ limit! This construction can give numerically large contribution to y_D if $(M_W/M_{NP})^2 > (m_s/m_c)^2$.

Note that contribution to $\Delta\Gamma_D$ is nonzero if the intermediate states are the on-mass-shell real physical states. It is therefore easy to see from energy-momentum conservation that diagrams like those in Fig.'s 3.1-3.3 but with b-quarks, $\tau\tau$, $\tau\mu$ pairs running in a loop, are irrelevant for our analysis. While the diagrams with a τe

pair running in a loop do give nonzero contribution to $\Delta\Gamma_D$, their contributions are suppressed by the available phase space. Thus, we shall not consider them too.

It is known that the correlation function in (3.2.1) (as well as those in (3.2.9)-(3.2.11)) may be presented as a sum of local $\Delta C = 2$ operators, which corresponds to a $1/m_c$ power expansion of (3.2.1) (or (3.2.9) - (3.2.11)). Here we are interested in the lowest order terms in this expansion. Keeping only the leading terms in $x_s \equiv m_s^2/m_c^2$ and $x_d \equiv m_d^2/m_c^2$, we get

$$\begin{aligned} y_{SM,NP} = & -\frac{G_F}{\sqrt{2}} \frac{(K_1+K_2)}{4\pi m_D \Gamma_D} \left(\frac{m_c^2}{m_{\tilde{\ell}}^2}\right) \Bigg[\lambda_{sd}\sqrt{x_s x_d} \; + \\ & + \lambda\left(\lambda_{ss}x_s - \lambda_{dd}x_d\right) - \lambda^2 \lambda_{ds}\sqrt{x_s x_d} \; \Bigg] \langle Q \rangle \end{aligned} \tag{3.2.12}$$

and

$$\begin{aligned} y_{\tilde{\ell}\tilde{\ell}} = & \frac{m_c^2\left(\lambda_{ss}^2+\lambda_{dd}^2+2\lambda_{sd}\lambda_{ds}\right)}{192\pi m_D \Gamma_D m_{\tilde{\ell}}^4} \Bigg\{ - \left[\frac{\widetilde{K}_2}{2}+\widetilde{K}_1\right]\langle Q\rangle + \\ & + \left[\widetilde{K}_2 - \widetilde{K}_1\right]\langle Q_S \rangle \Bigg\} \end{aligned} \tag{3.2.13}$$

where $\lambda = \sin\theta_C$ is the Wolfenstein parameter, and

$$\langle Q \rangle \equiv \langle \bar{D}^0 | \; \bar{u}^{a_1}(0)\gamma^\mu \left(\frac{1-\gamma_5}{2}\right) c^{a_1}(0) \; \bar{u}^{a_2}(0)\gamma_\mu \left(\frac{1-\gamma_5}{2}\right) c^{a_2}(0) \; | D^0 \rangle \tag{3.2.14}$$

$$\langle Q_S \rangle \equiv \langle \bar{D}^0 | \; \bar{u}^{a_1}(0) \left(\frac{1+\gamma_5}{2}\right) c^{a_1}(0) \; \bar{u}^{a_2}(0) \left(\frac{1+\gamma_5}{2}\right) \; c^{a_2}(0) \; | D^0 \rangle \tag{3.2.15}$$

are the matrix elements of the effective low energy $\Delta C = 2$ operators and

$$K_1 = 3\, C_1\, \widetilde{C}_1 + C_1\, \widetilde{C}_2 + C_2\, \widetilde{C}_1, \qquad K_2 = C_2\, \widetilde{C}_2 \tag{3.2.16}$$

$$\widetilde{K}_1 = 3\, \widetilde{C}_1^2 + 2\, \widetilde{C}_1\, \widetilde{C}_2, \qquad \widetilde{K}_2 = \widetilde{C}_2^2 \tag{3.2.17}$$

are the Wilson coefficients. It is important to stress that $y_{SM,NP}$, just like a Standard Model contribution, vanishes in the limit of exact flavor $SU(3)$ symmetry - it is proportional to light quark masses via x_s, x_d and $\sqrt{x_s x_d}$. On the contrary, $y_{\tilde{\ell}\tilde{\ell}}$ is nonzero even in the limit of exact flavor $SU(3)$ symmetry! Therefore, as we shall see in Section 5, $y_{\tilde{\ell}\tilde{\ell}}$ dominates over $y_{SM,NP}$ if R-parity breaking coupling products λ_{ss} and/or λ_{dd} approach their boundaries. In other words, contribution of diagrams in Fig. 2 with both $\Delta C = 1$ vertices generated by New Physics interactions, dominates over the contribution of diagrams in Fig. 1, with one of the $\Delta C = 1$ vertices coming from the Standard Model and the other one coming from New Physics.

Similarly, keeping only the leading order terms in $x_e \equiv m_e^2/m_c^2$, $x_\mu \equiv m_\mu^2/m_c^2$, one gets

$$y_{\tilde{q}\tilde{q}} = \frac{-m_c^2 \left(\lambda_{\mu\mu}^2 + \lambda_{ee}^2 + 2\ \lambda_{\mu e}\lambda_{e\mu}\right)}{192\pi m_D \Gamma_D\ m_{\tilde{q}}^4}\ \left[\langle Q \rangle\ +\ \langle Q_S \rangle\right]. \tag{3.2.18}$$

As one can see from (3.2.18), $y_{\tilde{q}\tilde{q}}$ is non-vanishing in the limit of exact flavor $SU(3)$ symmetry as well.

As usual, we parameterize matrix elements $\langle Q \rangle$ and $\langle Q_s \rangle$ in terms of B-factors [47], i.e.

$$\langle Q \rangle = \frac{2}{3}\ f_D^2\ m_D^2\ B_D, \quad \langle Q_S \rangle = -\frac{5}{12}\ f_D^2\ m_D^2\ \bar{B}_D^S \tag{3.2.19}$$

where

$$\bar{B}_D^S \equiv B_D^S\ \frac{m_D^2}{m_c^2} \tag{3.2.20}$$

We shall follow the approach of ref. [48] and neglect QCD running of the local $\Delta C = 1$ operators generated by NP interactions. Thus, $\widetilde{C}_1 = 0$ and $\widetilde{C}_2 = 1$, or

$$K_1 = C_1(m_c), \quad K_2 = C_2(m_c), \quad \widetilde{K}_1 = 0, \quad \widetilde{K}_2 = 1. \tag{3.2.21}$$

Using (3.2.19) and (3.2.21), one may rewrite (3.2.12), (3.2.13) and (3.2.18) in a following form:

$$y_{SM,NP} = \frac{-\,G_F}{\sqrt{2}}\ \frac{f_D^2 B_D m_D}{6\pi\Gamma_D}\ \left(\frac{m_c^2}{m_{\tilde{\ell}}^2}\right)\ [C_1(m_c)+C_2(m_c)]\left[\lambda_{sd}\sqrt{x_s x_d}\,+\right.$$
$$\left. +\,\lambda\,(\lambda_{ss}x_s-\lambda_{dd}x_d)-\lambda^2\lambda_{ds}\sqrt{x_s x_d}\,\right] \tag{3.2.22}$$

$$y_{\tilde{\ell}\tilde{\ell}} = \frac{-\,m_c^2\ f_D^2 B_D m_D}{288\pi\Gamma_D\ m_{\tilde{\ell}}^4}\left[\frac{1}{2}+\frac{5}{8}\frac{\bar{B}_D^S}{B_D}\right]\left[\ \lambda_{ss}^2+\lambda_{dd}^2+2\ \lambda_{sd}\lambda_{ds}\ \right] \tag{3.2.23}$$

$$y_{\tilde{q}\tilde{q}} = \frac{m_c^2\ f_D^2 B_D m_D}{288\pi\Gamma_D\ m_{\tilde{q}}^4}\left[\frac{5}{8}\frac{\bar{B}_D^S}{B_D}-1\right]\left[\ \lambda_{\mu\mu}^2+\lambda_{ee}^2+2\ \lambda_{\mu e}\lambda_{e\mu}\ \right] \tag{3.2.24}$$

Formulae (3.2.22)-(3.2.24) involve only the lowest order short-distance (perturbative) contribution to the $D^0-\overline{D}^0$ lifetime difference. Yet, it has been mentioned already that long-distance effects play a very important role in $D^0-\overline{D}^0$ oscillations. In particular, in the Standard Model, where the short-distance contribution to y_{D} has a suppressing factor $\sim m_s^4/m_c^4$ [43], the long distance contribution to $D^0-\overline{D}^0$ lifetime difference dominates [29]. However, within $\not{R}$-SUSY models we have a different situation. As it is mentioned above, New Physics contribution to y_{D} is non-vanishing in the exact flavor SU(3) limit, thus there is no suppression in powers of m_s/m_c in the dominant short-distance NP terms. In what follows, long distance effects, which may be interpreted as Λ_{DCD}/m_c power corrections, are subdominant. Thus, they may be neglected to the leading-order approximation that is used here.

Further analysis depends on bounds on R-parity breaking coupling constants, so in the next section we discuss the existing constraints on these couplings.

3.3 Present Bounds on R-parity Breaking Coupling Constants

Bounds on R-parity violating couplings λ' have been widely discussed in the literature [69] - [86]. Summary of bounds on λ'_{ijk} may be found e.g. in [69]. More recent (updated) bounds on some $\lambda' \times \lambda'$ pair products, coming from the studies of $K^0 - \bar{K}^0$ and $B^0 - \bar{B}^0$ mixing and $K^+ \to \pi^+ \nu \bar{\nu}$ decays, are presented in [71, 73] and [74] respectively.

It is interesting to note that bounds on RPV couplings coming from $K^0 - \bar{K}^0$ and $B^0 - \bar{B}^0$ mixing and empirical individual bounds on couplings λ'_{ijk} are derived *neglecting* the difference between λ' and $\tilde{\lambda}'$. While for the individual bounds it is a self-consistent approach, for the constraints on RPV coupling pair products such an approach in general is questionable.

Empirical individual bounds on RPV couplings are derived, assuming that only one coupling λ'_{ijk} is nonzero at a time. If such an assumption is made, then it is easy to see that

$$\widetilde{\lambda}'_{ijk} = \lambda'_{ijk} \times \left(1 + O(\lambda^2 = \sin^2\theta_C)\right), \tag{3.3.1}$$

$$\widetilde{\lambda}'_{ink} = O(\lambda) \times \lambda'_{ijk} \tag{3.3.2}$$

if $n \neq j$, and

$$\widetilde{\lambda}'_{rnm} = 0 \tag{3.3.3}$$

if $r \neq i$ or $m \neq k$.

Thus, as it follows from (3.3.1)-(3.3.3), when deriving an individual bound on λ'_{ijk} by studying a given process, there is no essential difference whether the $\not{R}$-SUSY diagram for this process contains λ'_{ijk} or it contains $\widetilde{\lambda}'_{ijk}$ at the vertices.

Of course, in the realistic $\not{R}$-SUSY scenarios several λ' couplings are in general non-zero. As it has been pointed out in [69], even if at the unification scales ($\sim$ $(10^{16} - 10^{18})$GeV) one has only one non-zero RPV coupling, other non-zero RPV couplings appear when evolving down from the unification scales to the electroweak breaking scale. However, the individual bounds on λ' couplings are still approximately valid, if one assumes that one RPV coupling dominates over all other ones. If several couplings dominate, individual bounds may still be used, if they are not correlated or weakly correlated with each other.

The situation with the constraints on the RPV coupling pair products is more complicated. As we will see, bounds on $\tilde{\lambda}' \times \tilde{\lambda}'$ and the corresponding $\lambda' \times \lambda'$ products may be different by several orders of magnitude. One must therefore be careful when using the bounds given in the literature and specify whether these bounds are on $\lambda' \times \lambda'$ product or they are on $\tilde{\lambda}' \times \tilde{\lambda}'$. This may be easily done, using the following "rule of thumb":

- If the process that is used to put constraints on the RPV coupling products is described by diagram(s) with down-down-sneutrino or down-sdown-neutrino vertices, bounds are derived on a $\lambda' \times \lambda'$ product.
- If such a process is described by diagram(s) with up-down-charged slepton, up-sdown-charged lepton or sup-down-charged lepton vertices, bounds are derived on a $\tilde{\lambda}' \times \tilde{\lambda}'$ product.
- If both types of vertices are present, bounds are derived on some admixture of $\lambda' \times \lambda'$ and $\tilde{\lambda}' \times \tilde{\lambda}'$ products.

In addition to the individual bounds, we use here constraints on the RPV coupling pair products that are derived from study of $K^+ \to \pi^+ \nu \bar{\nu}$ decay and Δm_{K^0}. An R-parity breaking SUSY contribution to $K^+ \to \pi^+ \nu \bar{\nu}$ is described by tree-level diagrams with a down-type squark exchange and quark-squark-neutrino interaction vertices [74, 75, 70]. Thus, this decay gives bounds on $\lambda' \times \lambda'$ products.

The situation with $K^0 - \bar{K}^0$ mixing is more involved: there are several sets of $\not{R}$-SUSY diagrams that contribute to this process. In order to get bounds on the RPV couplings, one assumes that only a given RPV coupling product or a given sum of RPV coupling products is nonzero. Possible bounds on the RPV coupling pair products have been originally listed in [72]. Recently these bounds have been improved in [71]. Bounds that are relevant for our analysis are presented in Appendix A.1. We also specify which of them are for $\lambda' \times \lambda'$ pair products and which of them are for $\tilde{\lambda}' \times \tilde{\lambda}'$.

Keeping in mind everything that has been said above, let us consider the RPV coupling products, which are present in formulae (3.2.22)-(3.2.24). We start with

$$\lambda_{ss} \equiv \sum_i \widetilde{\lambda}'^*_{i12} \widetilde{\lambda}'_{i22} = \sum_{i,j,n} V_{1n} V^*_{2j} \, \lambda'^*_{in2} \lambda'_{ij2}. \tag{3.3.4}$$

Using the Wolfenstein parametrization for the CKM matrix, keeping for each $\lambda' \times \lambda'^*$ product only the leading order term in $\lambda = \sin\theta_C$, and assuming that all $\lambda' \times \lambda'^*$ products are real (no new source of CP-violation), we rewrite (3.3.4) in a following form:

$$\begin{aligned} \lambda_{ss} \equiv \sum_i \widetilde{\lambda}'^*_{i12} \, \widetilde{\lambda}'_{i22} = \sum_i \lambda'^*_{i12} \, \lambda'_{i22} + \lambda \Big[\sum_i |\lambda'_{i22}|^2 - \sum_i |\lambda'_{i12}|^2 \Big] \\ + A\lambda^2 \sum_i \lambda'^*_{i12} \, \lambda'_{i32} + A\lambda^3 (1 + \rho - i\eta) \sum_i \lambda'^*_{i32} \, \lambda'_{i22} \\ + A^2 \lambda^5 (\rho - i\eta) \sum_i |\lambda'_{i32}|^2 \end{aligned} \tag{3.3.5}$$

There is a strong bound on the Cabibbo-favored term in the r.h.s. of (3.3.5) from the $K^+ \to \pi^+ \nu \bar{\nu}$ decay. Assuming that $\lambda'^*_{i1k} \, \lambda'_{i2k} \neq 0$ only for k=2, one gets [74]

$$|\lambda'^*_{i12} \, \lambda'_{i22}| \leq 6.3 \cdot 10^{-5} \left(\frac{m_{\tilde{q}}}{300 GeV} \right)^2 \tag{3.3.6}$$

We have rescaled the bound of ref. [74] to the units of $m_{\tilde{q}}/300$ GeV. Values of the squark masses less than 300 GeV are disfavored by many experiments (see [24] for more details). For this reason, we follow ref. [71] assuming that $m_{\tilde{q}} \geq 300$ GeV.

If squarks happen to be superheavy[5], there is still a strong bound on the Cabibbo favored term in (3.3.5) from $K^0 - \bar{K}^0$ mixing. As it follows from our discussion in Appendix A.1,

$$|\sum_i \lambda'^*_{i12} \, \lambda'_{i22}| \leq 2.7 \times 10^{-3} \left(\frac{m_{\tilde{\ell}}}{100 GeV} \right)^2 \tag{3.3.7}$$

[5]We thank X. Tata for discussion of this scenario.

Thus, the Cabibbo favored term in (3.3.5) is strongly suppressed, if one assumes that only $\lambda'_{i12} \neq 0.$ and $\lambda'_{i22} \neq 0.$ On the other hand, even under such an assumption, one still has

$$\lambda_{ss} \equiv \widetilde{\lambda}'^{*}_{i12}\ \widetilde{\lambda}'_{i22} \neq \lambda'^{*}_{i12}\ \lambda'_{i22}$$

due to the first order Cabibbo suppressed terms in (3.3.5). Furthermore, constraints (3.3.6) or (3.3.7) may in particular be satisfied, when $|\lambda'_{i22}|$ is close to its boundary value whereas $|\lambda'_{i12}| \to 0$, and vice versa. Taking into account that individual bounds are, in general, orders of magnitude looser than (3.3.6) or (3.3.7), it is not hard to see that λ_{ss} is dominated by the first order Cabibbo suppressed term in (3.3.5).

Further on we will very often deal with a situation, when expanding $\widetilde{\lambda}' \times \widetilde{\lambda}'$ products in a basis of λ' couplings, the Cabibbo favored term is negligible whereas the first order Cabibbo suppressed term dominates, and the only possible constraints on the first order Cabibbo suppressed term are the individual bounds on λ' couplings. In order to use these bounds we assume hereafter that only one coupling λ'_{ijk} dominates at a time.

After making such an assumption, it is easy to see that

$$\begin{aligned} -0.025\left(\frac{m_{\tilde{q}}}{300GeV}\right)^2 \leq \lambda_{ss} \leq 0.29, \quad \text{if} \quad m_{\tilde{q}} \leq 1TeV, \\ -0.29 \leq \lambda_{ss} \leq 0.29, \quad \text{if} \quad m_{\tilde{q}} \geq 1TeV \end{aligned} \qquad (3.3.8)$$

The upper bound on λ_{ss} is derived when one of λ'_{i22} couplings dominates. Individual bounds on λ'_{i22} are the loosest for $i = 3$ [69]. For $m_{\tilde{q}} \geq 300$GeV, $|\lambda_{322}| \leq 1.12$ - this is the perturbativity bound on λ_{322}. The lower bound on λ_{ss} is derived when one of the λ'_{i12} couplings dominates. Individual bounds on λ'_{i12} are the loosest for i=3 again: $|\lambda'_{312}| \leq 0.33(m_{\tilde{q}}/300GeV)$, if $m_{\tilde{q}} \leq 1TeV$ and $|\lambda_{312}| \leq 1.12$ - the perturbativity bound, if $m_{\tilde{q}} \geq 1TeV$.

It is important to stress that, in general, as it follows from (3.3.6), (3.3.7), (3.3.8),

$$\lambda_{ss} \equiv \sum_i \widetilde{\lambda}'^*_{i12} \widetilde{\lambda}'_{i22} \gg \sum_i \lambda'^*_{i12} \lambda'_{i22} \tag{3.3.9}$$

Thus, as it has been already pointed out in the beginning of this section, bounds on $\tilde{\lambda}' \times \tilde{\lambda}'$ products differ by several orders of magnitude from those on corresponding $\lambda' \times \lambda'$ products. In the considered case, the $\tilde{\lambda}' \times \tilde{\lambda}'$ product is restricted by a much weaker bound than the corresponding $\lambda' \times \lambda'$ product.

Relation (3.3.9) plays a crucial role in our analysis. We will see in the next section that, as a consequence of this relation, the R-parity breaking SUSY contribution to $\Delta\Gamma_D$ is quite large.

For λ_{dd}, analysis is performed in exactly the same way and yields

$$\begin{aligned} -0.025 \left(\frac{m_{\tilde{q}}}{300GeV}\right)^2 \leq \lambda_{dd} \leq 0.29, \quad if \quad m_{\tilde{q}} \leq 1TeV, \\ -0.29 \leq \lambda_{dd} \leq 0.29, \quad if \quad m_{\tilde{q}} \geq 1TeV \end{aligned} \tag{3.3.10}$$

Also, the relation similar to (3.3.9) is obtained:

$$\lambda_{dd} \equiv \sum_i \widetilde{\lambda}'^*_{i11} \widetilde{\lambda}'_{i21} \gg \sum_i \lambda'^*_{i11} \lambda'_{i21} \tag{3.3.11}$$

and relation (3.3.11) is as crucial as (3.3.9). It is also useful to transform (3.3.8) and (3.3.10) into restrictions on λ^2_{ss} and λ^2_{dd}:

$$\lambda^2_{ss} \approx \lambda^2 \Big[\sum_i |\lambda'_{i22}|^2 - \sum_i |\lambda'_{i12}|^2 \Big]^2 \leq 0.0841 \tag{3.3.12}$$

$$\lambda^2_{dd} \approx \lambda^2 \Big[\sum_i |\lambda'_{i21}|^2 - \sum_i |\lambda'_{i11}|^2 \Big]^2 \leq 0.0841 \tag{3.3.13}$$

Bounds on λ_{ds} and λ_{sd} are derived using the experimental data for Δm_{K^0}. As it follows from formula (A.1.1) in Appendix A.1,

$$|\lambda_{ds}| \equiv \left| \sum_i \widetilde{\lambda}'^*_{i11} \widetilde{\lambda}'_{i22} \right| \leq 1.7 \cdot 10^{-6} \left(\frac{m_{\tilde{\ell}}}{100GeV} \right)^2 \tag{3.3.14}$$

In order to derive constraints on λ_{sd}, one must write it in the following form (using $\lambda'_{ijk} = V_{nj} \widetilde{\lambda}'_{ink}$):

$$\lambda_{sd} \equiv \sum_i \widetilde{\lambda}'^*_{i12} \widetilde{\lambda}'_{i21} = (V^*_{11} V_{22})^{-1} \left[\sum_i \lambda'^*_{i12} \lambda'_{i21} - \sum_{j,n}{}' \, V^*_{j1} V_{n2} \Big(\sum_i \widetilde{\lambda}'^*_{ij2} \widetilde{\lambda}'_{in1} \Big) \right] \tag{3.3.15}$$

where prime indicates that the sum over j and n does not contain the term with $j = 1$ and $n = 2$. Bounds on the terms present in the r.h.s. of (3.3.15) are given in Appendix A.1. Using these bounds, one can see that

$$\lambda_{sd} < \text{few} \times 10^{-7} \left(\frac{m_{\tilde{\ell}}}{100GeV} \right)^2 \tag{3.3.16}$$

It is interesting to note that such strong constraints on λ_{ds} and on λ_{sd} are derived assuming that only one $\tilde{\lambda}' \times \tilde{\lambda}'$ or $\lambda' \times \lambda'$ product is nonzero. It is also assumed that the pure MSSM sector gives a negligible contribution to Δm_{K^0} [71]. These two assumptions are not necessarily true. If one gives up these assumption, then destructive interference of the pure MSSM and $\not{R}$-SUSY diagrams or the one of different $\not{R}$-SUSY diagrams will somehow distort bounds (3.3.15), (3.3.16). However, unless there is a fine-tuning or an exact cancelation between two (or more) diagram contributions, it is very unlikely for the distortion of these bounds to be such that λ_{ds} and/or λ_{sd} be $\sim 10^{-1}$ or $\sim 10^{-2}$. Therefore in our numerical calculations we will use the following relations:

$$\lambda_{ds} \ll \lambda_{ss}, \lambda_{dd} \tag{3.3.17}$$

$$\lambda_{sd} \ll \lambda_{ss}, \lambda_{dd} \tag{3.3.18}$$

For the remaining four coupling products - λ_{ee}, $\lambda_{\mu\mu}$, $\lambda_{\mu e}$ and $\lambda_{e\mu}$ - that are contained in the expression (3.26) for $y_{\tilde{q}\tilde{q}}$, the analysis is similar to that for λ_{ss} and λ_{dd}. For the details and subtleties of the analysis, we refer the reader to Appendix A.2. Here we only point out that bounds on λ_{ee}, $\lambda_{\mu\mu}$ are the following:

$$
\begin{aligned}
-0.91\cdot 10^{-3}\left(\frac{m_{\tilde{q}}}{300GeV}\right)^2 &\leq \lambda_{ee} \leq 3.83\cdot 10^{-3}\left(\frac{m_{\tilde{q}}}{300GeV}\right)^2 \\
-0.0072\left(\frac{m_{\tilde{q}}}{300GeV}\right)^2 &\leq \lambda_{\mu\mu} \leq 0.091\left(\frac{m_{\tilde{q}}}{300GeV}\right)^2, \text{ if } \quad m_{\tilde{q}} \leq 530 \text{ GeV}, \quad (3.3.19) \\
-0.0072\left(\frac{m_{\tilde{q}}}{300GeV}\right)^2 &\leq \lambda_{\mu\mu} \leq 0.29, \qquad \text{if } \quad m_{\tilde{q}} \geq 530 \text{ GeV}. \quad (3.3.20)
\end{aligned}
$$

Also, for two other couplings we get

$$
\begin{aligned}
&|\lambda_{\mu e}| \leq 0.019\left(\frac{m_{\tilde{q}}}{300GeV}\right)^2, \quad |\lambda_{e\mu}| \leq 0.019\left(\frac{m_{\tilde{q}}}{300GeV}\right)^2, \\
&\quad \text{if} \quad m_{\tilde{q}} \leq 530 \text{ GeV} \\
&|\lambda_{\mu e}| \leq 0.033\left(\frac{m_{\tilde{q}}}{300GeV}\right), \quad |\lambda_{e\mu}| \leq 0.033\left(\frac{m_{\tilde{q}}}{300GeV}\right), \qquad (3.3.21) \\
&\quad \text{if} \quad m_{\tilde{q}} \geq 530 \text{ GeV}
\end{aligned}
$$

We also obtain that

$$\lambda_{\mu e} \approx \lambda_{e\mu} \qquad (3.3.22)$$

As $m_{\tilde{q}}$ increases, squark mass dependent empirical bounds on the RPV couplings are replaced by squark mass independent perturbativity bounds. In formulae (3.3.19)-(3.3.22), we indicate the change in the behavior of the bounds with the squark mass, if it occurs for $m_{\tilde{q}} \leq 1$TeV.

When transforming (3.3.19)-(3.3.22) into the restrictions on λ_{ee}^2, $\lambda_{\mu\mu}^2$, $\lambda_{\mu e}\lambda_{e\mu}$, one can see that these restrictions are much weaker than the relevant constraints listed in ref. [64]. This is because in the present work we do not neglect the transformations of RPV couplings from the weak eigenbasis to the quark mass eigenbasis. More precisely, we do not neglect the difference between $\widetilde{\lambda}' \times \widetilde{\lambda}'$ and $\lambda' \times \lambda'$ pair products.

From (3.3.19)-(3.3.22), one can also see that generally speaking,

$$\lambda_{\mu\mu}^2 \gg \lambda_{\mu e}\lambda_{e\mu} \gg \lambda_{ee}^2 \tag{3.3.23}$$

It is worth mentioning here that additional bounds on λ_{ee}, $\lambda_{\mu\mu}$, $\lambda_{\mu e}$, $\lambda_{e\mu}$ may be derived from studying rare D-meson decays, such as $D \to X\ell^+\ell^-$, $D^0 \to \ell^+\ell^-$, etc [62]. As it follows from the analysis performed in ref. [62], bounds derived in this way may be even stronger than those given by (3.3.19) -(3.3.22). Bounds coming from the rare D-meson decays are however still to be elaborated in detail, taking into account new experimental data, as well as possible impact of the long-distance SM and (short-distance) pure MSSM contributions. Such an elaboration is beyond the scope of this work, in particular because $y_{\tilde{q}\tilde{q}}$ turns out to be a (numerically) subdominant part of the New Physics contribution to the $D^0 - \bar{D}^0$ lifetime difference, even if we use constraints on λ_{ee}, $\lambda_{\mu\mu}$, $\lambda_{\mu e}$, $\lambda_{e\mu}$ given by (3.3.19)-(3.3.22) (see the next section).

Having obtained constraints on all RPV coupling products in (3.2.22)-(3.2.24), we may proceed to computation of $y_{SM,NP}$, $y_{\tilde{\ell}\tilde{\ell}}$, $y_{\tilde{q}\tilde{q}}$.

3.4 RPV SUSY Contribution to y_D: Numerical Analysis

In our numerical calculations we use [24, 87] $G_F = 1.166 \cdot 10^{-5}$ GeV^{-2}, $\lambda \approx 0.23$, $\Gamma_D \approx 1.6 \cdot 10^{-12}$ GeV, $m_D \approx 1.865$ GeV; $m_c \equiv m_c(m_c) \approx 1.25$ GeV, $m_s(2GeV) \approx$ 95 MeV,

$$m_s(m_c) \approx m_s(2GeV)\left(\frac{\alpha_s(m_c)}{\alpha_s(2GeV)}\right)^{12/25} \approx 105 \text{ MeV}, \quad x_s \equiv \frac{m_s^2(m_c)}{m_c^2(m_c)} \approx 0.007;$$

$C_1(m_c) = -0.411$, $C_2(m_c) \approx 1.208$ [43], $B_D \approx 0.8$ [43, 88], $f_D \approx 0.22$ [89].

While the value of B_D is known from the lattice QCD calculations, there is no theoretical or experimental prediction on B_D^S. Here we follow the approach of ref.

[43], assuming that

$$B_D^S = B_D, \quad B_D^S = 0.8B_D, \quad B_D^S = 1.2B_D. \qquad (3.4.1)$$

Let us first determine the sign of $y_{SM,NP}$, $y_{\tilde{\ell}\tilde{\ell}}$, $y_{\tilde{q}\tilde{q}}$. Using relations (3.3.17), (3.3.18), (3.3.23), one may rewrite equations (3.2.22)-(3.2.24) in a much simpler form,

$$y_{SM,NP} \approx \frac{-G_F}{\sqrt{2}} \frac{f_D^2 B_D m_D}{6\pi\Gamma_D} \left(\frac{m_c^2}{m_{\tilde{\ell}}^2}\right) \Big[C_1(m_c) + C_2(m_c)\Big] \lambda \, \lambda_{ss} \, x_s \qquad (3.4.2)$$

$$y_{\tilde{\ell}\tilde{\ell}} \approx \frac{-m_c^2 \, f_D^2 B_D m_D}{288\pi\Gamma_D \, m_{\tilde{\ell}}^4} \left[\frac{1}{2} + \frac{5}{8}\frac{\bar{B}_D^S}{B_D}\right] [\, \lambda_{ss}^2 + \lambda_{dd}^2] \qquad (3.4.3)$$

$$y_{\tilde{q}\tilde{q}} \approx \frac{m_c^2 \, f_D^2 B_D m_D}{288\pi\Gamma_D \, m_{\tilde{q}}^4} \left[\frac{5}{8}\frac{\bar{B}_D^S}{B_D} - 1\right] \lambda_{\mu\mu}^2 \qquad (3.4.4)$$

It follows from (3.4.2), (3.4.3) that the sign of $y_{SM,NP}$ is opposite to that of λ_{ss} and $y_{\tilde{\ell}\tilde{\ell}} < 0$.

One can see from (3.4.4) that the sign of $y_{\tilde{q}\tilde{q}}$ is determined by the factor $\left[\frac{5}{8}\frac{\bar{B}_D^S}{B_D} - 1\right]$. As it follows from (3.2.20) and (3.4.1), for $m_c \equiv m_c(m_c) \approx 1.25$GeV, this factor is positive, hence

$$y_{\tilde{q}\tilde{q}} > 0.$$

On the other hand, $\left[\frac{5}{8}\frac{\bar{B}_D^S}{B_D} - 1\right]$ and hence $y_{\tilde{q}\tilde{q}}$ flips its sign when using the charm quark pole mass [6], $m_c^{pole} \approx 1.65$ GeV.

In general, such an ambiguity in sign of $y_{\tilde{q}\tilde{q}}$ may cause trouble in numerical evaluation of the results, signaling the need for next-to-leading order evaluation of the appropriate contributions, where the scheme ambiguity cancels out. Here we disregard this sign ambiguity, as $y_{\tilde{q}\tilde{q}}$ turns to be a (numerically) subdominant part of the

[6]To derive the proper value of m_c^{pole}, the two-loop relation between the pole and $\overline{MS}$ quark masses must be used. This is because the $\overline{MS}$ value of the c-quark mass has been extracted using the perturbative QCD analysis up to the order α_s^2 [24]. One can check that the use of the three loop relation between the pole and $\overline{MS}$ quark masses [90] leads to the physically meaningless result $m_c^{pole} \approx 1.93$ GeV $> m_D$.

New Physics contribution to $D^0 - \bar{D}^0$ lifetime difference. In our opinion, the use of the $\overline{MS}$ charm mass, $m_c(m_c) = 1.25$ GeV, is more appropriate in this calculation. Then $y_{\tilde{q}\tilde{q}}$ has positive sign.

Let us proceed to our results. It is convenient to start with $y_{\tilde{q}\tilde{q}}$. Using the listed numerical values of parameters present in (3.4.4), we get

$$\begin{aligned} B_D^S = 0.8 B_D: &\quad y_{\tilde{q}\tilde{q}} \approx 0.0011\ \lambda_{\mu\mu}^2 \left(\frac{300GeV}{m_{\tilde{q}}}\right)^4 \\ B_D^S = B_D: &\quad y_{\tilde{q}\tilde{q}} \approx 0.0038\ \lambda_{\mu\mu}^2 \left(\frac{300GeV}{m_{\tilde{q}}}\right)^4 \\ B_D^S = 1.2 B_D: &\quad y_{\tilde{q}\tilde{q}} \approx 0.0064\ \lambda_{\mu\mu}^2 \left(\frac{300GeV}{m_{\tilde{q}}}\right)^4 \end{aligned} \tag{3.4.5}$$

As it follows from (3.4.5), to the lowest order in perturbation theory, $y_{\tilde{q}\tilde{q}}$ is highly sensitive to the choice of parameters B_D^S and B_D. Moreover, if one uses the approach of ref. [64], choosing $\bar{B}_D^S = B_D$ or $B_D^S = (m_c^2/m_D^2) B_D \approx 0.45 B_D$, $y_{\tilde{q}\tilde{q}}$ flips sign[7].

Using the bounds on $\lambda_{\mu\mu}$ given by (3.3.20) yields

$$\begin{aligned} B_D^S = 0.8 B_D: &\quad y_{\tilde{q}\tilde{q}} \leq 0.9 \cdot 10^{-5} \\ B_D^S = B_D: &\quad y_{\tilde{q}\tilde{q}} \leq 3.12 \cdot 10^{-5} \\ B_D^S = 1.2 B_D: &\quad y_{\tilde{q}\tilde{q}} \leq 5.34 \cdot 10^{-5} \end{aligned} \tag{3.4.6}$$

for $m_{\tilde{q}} \leq 530$ GeV and

$$\begin{aligned} B_D^S = 0.8 B_D: &\quad y_{\tilde{q}\tilde{q}} \leq 0.9 \cdot 10^{-5} \left(\frac{530GeV}{m_{\tilde{q}}}\right)^4 \\ B_D^S = B_D: &\quad y_{\tilde{q}\tilde{q}} \leq 3.12 \cdot 10^{-5} \left(\frac{530GeV}{m_{\tilde{q}}}\right)^4 \\ B_D^S = 1.2 B_D: &\quad y_{\tilde{q}\tilde{q}} \leq 5.34 \cdot 10^{-5} \left(\frac{530GeV}{m_{\tilde{q}}}\right)^4 \end{aligned} \tag{3.4.7}$$

[7] $y_{\tilde{q}\tilde{q}}$ is equivalent to $-y_{(RPV-RPV,l)}$ in the notations of [64].

for $m_{\tilde{q}} \geq 530$ GeV.

Thus, if using bounds on λ_{ee}, $\lambda_{\mu\mu}$, $\lambda_{\mu e}$, $\lambda_{e\mu}$, given by (3.3.19) - (3.3.22), one obtains that $y_{\tilde{q}\tilde{q}}$ is at least two orders of magnitude less than the experimental value of y_{D}. As it was mentioned above, constraints on λ_{ee}, $\lambda_{\mu\mu}$, $\lambda_{\mu e}$, $\lambda_{e\mu}$ and hence on $y_{\tilde{q}\tilde{q}}$ may become even stronger if one elaborates the constraints on RPV couplings coming from the rare D-meson decays. Further on we simply ignore $y_{\tilde{q}\tilde{q}}$ because of its smallness. This way we also avoid the problems related to the dependence of the obtained results on the choice of the renormalization scheme and B_D-factors.

Consider $y_{SM,NP}$ now. For this quantity one gets

$$y_{SM,NP} \approx 0.0040\ \lambda_{ss} \left(\frac{100GeV}{m_{\tilde{\ell}}}\right)^2 \tag{3.4.8}$$

which after using (3.3.8) yields

$$-0.0011 \left(\frac{100GeV}{m_{\tilde{\ell}}}\right)^2 \leq y_{SM,NP} \leq 0.99 \cdot 10^{-4} \left(\frac{m_{\tilde{q}}}{300GeV}\right)^2 \left(\frac{100GeV}{m_{\tilde{\ell}}}\right)^2 \tag{3.4.9}$$

for $m_{\tilde{q}} \leq 1$ TeV and

$$-0.0011 \left(\frac{100GeV}{m_{\tilde{\ell}}}\right)^2 \leq y_{SM,NP} \leq 0.0011 \left(\frac{100GeV}{m_{\tilde{\ell}}}\right)^2 \tag{3.4.10}$$

for $m_{\tilde{q}} \geq 1$ TeV.

As it follows from (3.4.9), (3.4.10), $|y_{SM,NP}|$ may be by an order of magnitude greater than quoted in [64][8]. This is because the analysis in ref. [64] has been restricted by consideration of $m_{\tilde{q}} = 100$ GeV only. On the other hand, as it follows from Table I of ref. [69] and our analysis in Section 4, bounds on RPV couplings and hence on λ_{ss} become weaker for the greater values of squark masses. Else, unlike ref.'s [48, 64], we obtain that $y_{SM,NP}$ can be both positive and negative. This is because,

[8] $y_{SM,NP} = -y_{(SM-RPV)}$ in the notations of [64].

as one can see from equation (3.3.5) and the following it discussion, λ_{ss} may have both of signs even if one assumes that all RPV couplings are real and positive.

Finally, consider $y_{\tilde{\ell}\tilde{\ell}}$. Using the numerical values of the parameters present in (3.4.3), one gets

$$\begin{aligned} B_D^S = 0.8 B_D: &\quad y_{\tilde{\ell}\tilde{\ell}} \approx -1.25\left[\lambda_{ss}^2 + \lambda_{dd}^2\right]\left(\frac{100GeV}{m_{\tilde{\ell}}}\right)^4 \\ B_D^S = B_D: &\quad y_{\tilde{\ell}\tilde{\ell}} \approx -1.47\left[\lambda_{ss}^2 + \lambda_{dd}^2\right]\left(\frac{100GeV}{m_{\tilde{\ell}}}\right)^4 \\ B_D^S = 1.2 B_D: &\quad y_{\tilde{\ell}\tilde{\ell}} \approx -1.69\left[\lambda_{ss}^2 + \lambda_{dd}^2\right]\left(\frac{100GeV}{m_{\tilde{\ell}}}\right)^4 \end{aligned} \tag{3.4.11}$$

As one can see from (3.4.11), varying the ratio B_D^S/B_D from 0.8 to 1.2, one gets about 15% uncertainty in the predictions for $y_{\tilde{\ell}\tilde{\ell}}$. Thus, $y_{\tilde{\ell}\tilde{\ell}}$ is only weakly sensitive to the choice of the parameter B_D^S. As we are interested in the order of the effect only, we may for simplicity assume $B_D^S = B_D$ hereafter.

To be consistent with a one dominant coupling approximation, we will assume that only one of the coupling products λ_{ss} or λ_{dd} is at its boundary at a time. Notice however that if we allow both λ_{ss} and λ_{dd} to be simultaneously large, our results will change at most by a factor two, which is unimportant, if one is interested in the order-of-magnitude of the effect only.

Using the bounds on λ_{ss}^2 and λ_{dd}^2 given by (3.3.12) and (3.3.13) we obtain

$$-0.12\left(\frac{100GeV}{m_{\tilde{\ell}}}\right)^4 \leq y_{\tilde{\ell}\tilde{\ell}} < 0 \tag{3.4.12}$$

It is important to stress that $|y_{\tilde{\ell}\tilde{\ell}}|$ may be $\sim 10^{-1}$, if $m_{\tilde{\ell}} = 100$ GeV.

This result is in contradiction with the one of ref. [64]: $y_{RPV-PRV,q} = -y_{\tilde{\ell}\tilde{\ell}} \leq 2.5 \cdot 10^{-11}$, for $m_{\tilde{\ell}} = 100$GeV. This contradiction is related to the fact that authors of ref. [64], following other papers on the meson-antimeson mixing phenomenon, have neglected the transformation of the RPV couplings from the weak eigenbasis to the quark mass eigenbasis. This allowed them to impose very stringent constraints on λ^2_{ss} and λ^2_{dd} from $K^+ \to \pi^+ \nu \bar{\nu}$ decay. As it follows from our discussion in Section 4, this approach is not always appropriate[9].

We are now able to compute the total New Physics contribution to $D^0 - \bar{D}^0$ lifetime difference,

$$y_{new} = y_{SM,NP} + y_{\tilde{\ell}\tilde{\ell}} + y_{\tilde{q}\tilde{q}}.$$

As it is mentioned above, we neglect $y_{\tilde{q}\tilde{q}}$ because of its smallness. Also, as it follows from (3.4.8) and (3.4.11), $y_{\tilde{\ell}\tilde{\ell}} \gg y_{SM,NP}$ unless $\lambda_{dd} \to 0$ and the ratio $\lambda_{ss}/m^2_{\tilde{\ell}}$ is small enough. It is not very hard to see after doing some algebra that

$$-0.12 \left(\frac{100GeV}{m_{\tilde{\ell}}}\right)^4 \leq y_{\tilde{\ell}\tilde{\ell}} + y_{SM,NP} \leq 2.72 \cdot 10^{-6} \qquad (3.4.13)$$

The (negative) lower bound in (3.4.13) is derived neglecting $y_{SM,NP}$ as compared to $y_{\tilde{\ell}\tilde{\ell}}$. The (positive) upper bound in (3.4.13) is derived for $\lambda_{dd} = 0$ and $\lambda_{ss} = -0.00136\,(m_{\tilde{\ell}}/100GeV)^2$, when $y_{SM,NP} = -2y_{\tilde{\ell}\tilde{\ell}}$. As it follows from (3.4.6) and (3.4.13), y_{new} is negligible, if positive, and may be as large as $\sim 10^{-1}$, if negative.

Thus, within the R-parity breaking supersymmetric models with lepton number violation, the New Physics contribution to $D^0 - \bar{D}^0$ lifetime difference is *predominantly negative* and may exceed in absolute value the experimentally allowed interval. In order to avoid a contradiction with experiment, one must either have a large positive contribution from the Standard Model, or place severe restrictions on the values of RPV couplings. As follows from [29], y_{SM} may be as large as $\sim 1\%$. In what follows, $|y_{new}|$ must be $\sim 1\%$ or smaller as well. If $|y_{new}| \sim 1\%$, one may neglect $y_{SM,NP}$ as compared to $y_{\tilde{\ell}\tilde{\ell}}$. Then, imposing the condition

$$-0.01 \leq y_{new} \approx y_{\tilde{\ell}\tilde{\ell}} \qquad (3.4.14)$$

[9]Unless one imposes the conditions $\lambda'_{i22} \sim \lambda'_{i12}$ and $\lambda'_{i21} \sim \lambda'_{i11}$.

one obtains that either $m_{\tilde{\ell}} > 185$ GeV, or if $m_{\tilde{\ell}} \leq 185$ GeV, condition (3.4.14) implies new bounds on λ_{ss} and λ_{dd}:

$$|\lambda_{ss}| \leq 0.082 \left(\frac{m_{\tilde{\ell}}}{100GeV}\right)^2 \tag{3.4.15}$$

$$|\lambda_{dd}| \leq 0.082 \left(\frac{m_{\tilde{\ell}}}{100GeV}\right)^2 \tag{3.4.16}$$

Note that bounds (3.4.15) and (3.4.16) may not be saturated simultaneously. (3.4.15) is saturated if $\lambda_{dd} = 0$. Subsequently, (3.4.16) is saturated if $\lambda_{ss} = 0$. For the opposite limiting case, $\lambda_{ss} = \lambda_{dd}$, one gets $\sqrt{2}$ times stronger restrictions:

$$|\lambda_{ss}| \leq 0.058 \left(\frac{m_{\tilde{\ell}}}{100GeV}\right)^2, \qquad |\lambda_{dd}| \leq 0.058 \left(\frac{m_{\tilde{\ell}}}{100GeV}\right)^2 \tag{3.4.17}$$

It is interesting to compare the restrictions on λ_{ss} and λ_{dd}, given by (3.4.15)-(3.4.17), with those derived in [47] from study of $D^0 - \bar{D}^0$ mass difference. Translated to our notation, we may rewrite the relevant constraints of ref. [47] in the following form:

$$\lambda_{ss} \leq 0.085\sqrt{x_{exp}} \left(\frac{m_{\tilde{q}}}{500GeV}\right), \qquad \lambda_{dd} \leq 0.085\sqrt{x_{exp}} \left(\frac{m_{\tilde{q}}}{500GeV}\right) \tag{3.4.18}$$

This constraint has been derived assuming that $m_{\tilde{q}} = m_{\tilde{\ell}}$. If $m_{\tilde{q}} \neq m_{\tilde{\ell}}$, bounds in (3.4.18) must be divided by the factor $\frac{1}{2}\sqrt{1 + m_{\tilde{q}}^2/m_{\tilde{\ell}}^2}$, as it follows from formulae (130)-(134) of ref. [47]. Assuming for simplicity that $m_{\tilde{q}}^2 \gg m_{\tilde{\ell}}^2$ and inserting $x_{exp} = 0.0117$ into (3.4.18), one gets

$$\lambda_{ss} \leq 0.0037 \left(\frac{m_{\tilde{\ell}}}{100GeV}\right), \qquad \lambda_{dd} \leq 0.0037 \left(\frac{m_{\tilde{\ell}}}{100GeV}\right) \tag{3.4.19}$$

Thus, bounds of [47] on λ_{ss} and λ_{dd} are about 20 times stronger than our ones. On the other hand, constraints of ref. [47] on the RPV coupling products are derived in the limit when the pure MSSM contribution to Δm_D is negligible. Generally speaking, the MSSM contribution to $D^0 - \bar{D}^0$ mass difference is significant even for the squark masses of about 2 TeV. In what follows, destructive interference of the pure MSSM and $\not{R}$-SUSY contributions may distort bounds (3.4.19), making them inessential as compared to (3.4.15)-(3.4.17) or even to (3.3.8), (3.3.10).

Contrary to this, pure MSSM contributes to $\Delta\Gamma_D$ only in the next-to-leading order via two-loop dipenguin diagrams. Naturally, this contribution is expected to be small. In what follows, unlike those of ref. [47], our constraints on the RPV coupling products λ_{ss} and λ_{dd}, given by (3.4.15)-(3.4.17), seem to be insensitive or weakly sensitive to assumptions on the pure MSSM sector of the theory.

Thus, our main result is that within R-parity breaking supersymmetric theories with leptonic number violation, the New Physics contribution to $\Delta\Gamma_D$ may be quite large and is predominantly negative.

For simplicity we assumed that all sleptons have nearly the same mass and all squarks have nearly the same mass. It is easy to see that taking into account the difference between the slepton masses does not affect our main results. There are however subtleties concerning the squark masses. First, recall that our analysis has been performed for $m_{\tilde{q}} \geq 300$ GeV. While this constraint is quite reasonable for $\tilde{d}$ and $\tilde{s}$, the bottom squark is still allowed experimentally to be about 100 GeV [24]. On the other hand, we have seen that bounds on $y_{SM,NP}$ and $y_{\tilde{\ell}\tilde{\ell}}$ either grow or are insensitive to the squark masses. As for the bound on $y_{\tilde{q}\tilde{q}}$, it is insensitive to $m_{\tilde{q}}$ for low values of the squark masses. Thus, no new effect is going to be observed, if one takes the squark masses to be about 100 GeV.

Another point to be made, is that the squark mass matrix is in general non-diagonal in the super-CKM basis, if one takes the squark masses to be different. It has been already mentioned in Section 2, that no new flavor violation effects are obtained, however this may somehow weaken bounds (3.3.19) - (3.3.22) on λ_{ee}, $\lambda_{\mu\mu}$, $\lambda_{\mu e}$ $\lambda_{e\mu}$, when applying arguments analogous to those used in Section 4. However, as it was mentioned above, λ_{ee}, $\lambda_{\mu\mu}$, $\lambda_{\mu e}$ $\lambda_{e\mu}$ are expected to get additional strong constraints from the analysis of the rare D-meson decays, so that one may expect for $y_{\tilde{q}\tilde{q}}$ to be in any case restricted by an even more stringent bound than (3.4.5). In other words, giving up the assumption of nearly equal squark masses leads to complication of the analysis without observation of any new effect. If large, the RPV SUSY contribution to the lifetime difference in $D^0 - \overline{D}^0$ mixing still may have only negative sign.

When studying the lifetime difference in $D^0 - \overline{D}^0$ mixing within the Standard Model and beyond, one usually assumes that CP-violating effects are negligible [48, 29, 43, 63, 64]. Following this strategy, we have chosen for the RPV coupling products that contribute to $D^0 - \overline{D}^0$ mixing amplitude to be real. The natural question arises if our results may be affected by possible complex phases of these coupling products. Clearly, $|y_{new}|$ still may be large, however the complex phases may possibly affect its sign. One may suggest - because of no evidence of CP-violation in the $D^0 - \overline{D}^0$ system [49, 50] - that the phases of the relevant RPV coupling products are small. In this case, the contribution to $D^0 - \overline{D}^0$ lifetime difference, proportional to the imaginary parts of the RPV coupling products, is subdominant and cannot affect the sign of y_{new}: if large in absolute value, y_{new} is negative . Yet, it may happen that RPV coupling products that contribute to $D^0 - \overline{D}^0$ mixing have large phases, and no evidence of CP-violation in $D^0 - \overline{D}^0$ system is related to the fact that - unlike the $D^0 - \overline{D}^0$ oscillations - the $\not{R}$-SUSY contribution to D^0 meson decays is rather small. In that case the formalism, used here, is not valid anymore. A more general and involved approach should be used, taking into account possible correlations in the values of $D^0 - \overline{D}^0$ mass and lifetime differences as well as possible correlations in the SM, pure MSSM and RPV sector contributions. Thus, to clarify if the RPV couplings complex phases may affect the sign of the NP contribution to $D^0 - \overline{D}^0$ lifetime difference, thorough and detailed study of the case, when the relevant phases are large, is needed.

3.5 Non-Manifest Left-Right Model Contribution to y_D.

In this section we will briefly discuss the NP contribution to $D^0 - \overline{D}^0$ lifetime difference within $SU(2)_R \times SU(2)_L \times U(1)$ models or Left-Right (LR) Symmetric Models. Within these models, along with the left weak isospin gauge triplet W_L^a, a=1, 2, 3, one has a right weak isospin gauge triplet, W_R^a. The right-handed fermions are isodoublets and the left-handed fermions are isosinglets with respect to the $SU(2)_R$ group.

The fermion and gauge boson masses are generated in two steps. First, $SU(2)_R \times SU(2)_L \times U(1)$ symmetry is spontaneously broken to $SU(2)_L \times U(1)_{Y'}$ at some scale $M_R \gg 100$ GeV, to assure that non-SM gauge bosons, $W_2^{\pm}$, Z', are too heavy to be seen experimentally. Next, $SU(2)_L \times U(1)_{Y'}$ is spontaneously broken to $U(1)_{EM}$, as discussed in Chapter 2. To implement such a two-step spontaneous symmetry breaking scenario, an involved Higgs sector is needed. More about the Left-Right Symmetry Models and their phenomenology may be found elsewhere else [91, 92, 93]. Here only the basic aspects of the model, relevant for our analysis, are pointed out.

The lightest charged W-boson, $W_1^{\pm}$ (with $M_{W_1} = 80.4$ GeV), is predominantly the Standard Model W-boson, $W_L^{\pm}$, yet it also contains a small admixture of $W_R^{\pm}$:

$$W_1^{\pm} = W_L^{\pm} \cos\zeta + W_R^{\pm} \sin\zeta \approx W_L^{\pm} + \zeta W_R^{\pm} \tag{3.5.1}$$

$\zeta \sim M_{W_1}^2/M_{W_2}^2 \ll 1$. The heaviest charged W-boson, $W_2^{\pm}$ (with a mass $\sim M_R$) is predominantly $W_R^{\pm}$:

$$W_2^{\pm} \approx W_R^{\pm} - \zeta W_L^{\pm} \tag{3.5.2}$$

The quark charged current interactions with exchange of $W_1^{\pm}$ consist of both the

SM and the NP interaction terms now:

$$\mathcal{L}_W = \frac{1}{\sqrt{2}} \sum_{f,f'} \left[\bar{u}_f W_1^+ \left(g_L V^L_{ff'} P_L + \zeta g_R V^R_{ff'} P_R \right) d_{f'} + h.c \right] \tag{3.5.3}$$

where $P_L = (1-\gamma_5)/2$, $P_R = (1+\gamma_5)/2$, g_L and g_R are respectively the SM and NP weak coupling constants, $V^L = V_{u_L} V^\dagger_{d_L}$ is the SM (left-handed quark) CKM matrix and $V^R = V_{u_R} V^\dagger_{d_R}$ is the right-handed quark CKM matrix. Depending on assumptions on V^R, two types of Left-Right Symmetric Models are considered:

Pseudo-manifest Left-Right Symmetric Model, where $|V^R_{ff'}| = |V^L_{ff'}|$.

Non-manifest Left-Right Symmetric Model, where V^R is arbitrary.

It has been shown in [48] that within the pseudo-manifest Left-Right Symmetric Model, the New Physics contribution to the lifetime difference in $D^0 - \overline{D}^0$ mixing is negligible. Yet, as it has been argued in [65], within the non-manifest Left-Right Symmetric Model, the New Physics contribution to the lifetime difference in $D^0 - \bar{D}^0$ mixing may be significant:

$$|y_{LR}| \equiv \frac{|\Delta\Gamma_{D_{LR}}|}{2\Gamma_D} \leq 1.4 \times 10^{-3}, \tag{3.5.4}$$

which means that y_{LR} may be of the same order as the experimental value of y_D. This result has been derived by considering the box diagrams with one of $\Delta C = 1$ transitions being generated by a new physics (NP) interaction and mediated by a propagator with $W_L - W_R$ mixing (Fig. 3.4), or equivalently mediated by a $W_1^\pm$ propagator with one of the vertices being the one with the SM interaction and the other one being that with the NP interaction.

Note that W_R part of the propagator couples with the u-quark, which (assuming that $V_{R_{us}} \sim 1$) allows one to remove a power of the suppression in terms of $\lambda = \sin\theta_C \approx 0.23$.

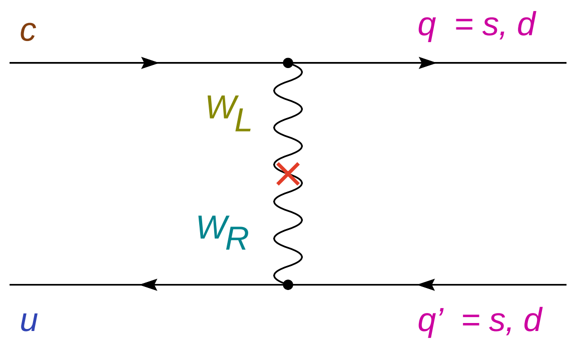

Figure 3.4: $\Delta C = 1$ transition mediated by a propagator with $W_L - W_R$ mixing.

In this section we revisit the contribution of the box diagrams with the new physics generated $\Delta C = 1$ transition, presented in Fig 3.4. While the analysis of ref. [65] is restricted by considering only the diagrams with the intermediate s-quark states, i.e. $q = s$ and $q' = s$, we include also the diagrams with $q = d$ and/or $q' = d$. We will see that diagrams with the intermediate d-quark states may not be neglected, in spite of $m_d \ll m_s$. Moreover, they play a crucial role in properly taking into account GIM cancelation effects.

We show that box diagrams with the new physics generated $\Delta C = 1$ transition, presented in Fig 3.4, are negligible *in sum* due to GIM cancelation. Thus, one must replace the bound on y_{LR}, given by equation (3.5.4), by

$$|y_{LR}| \leq 8.8 \times 10^{-5} \tag{3.5.5}$$

This constraint on y_{LR} has been derived in [48], neglecting the $\Delta C = 1$ transition presented in Fig. 3.4.

For the $\Delta C = 1$ interaction in Fig. 3.4, the relevant part of the low-energy effective

Hamiltonian has the following form:

$$H_{W_{L-R}}^{\Delta C=1} = -\frac{4G_F\xi_g}{\sqrt{2}}\sum_{q,q'} V_{cq}^{*L}V_{uq'}^{R}\Big[\bar{C}_1(m_c)Q_1 + \\ +\bar{C}_2(m_c)Q_2\Big] \tag{3.5.6}$$

$$Q_1 = \bar{u}_i\gamma^\nu P_R q'_j \bar{q}_j \gamma_\nu P_L c_i, \quad Q_2 = \bar{u}_i\gamma^\nu P_R q'_i \bar{q}_j \gamma_\nu P_L c_j$$

where i, j stand for color indices and $\xi_g = \zeta g_R/g_L$. If only one $\Delta C = 1$ transition in the box diagrams is generated by an NP interaction, the approach described in ref. [48] may be used. For the new physics $\Delta C = 1$ effective Hamiltonian given by equation (3.5.6), only the term $I_4(x_q, x_{q'}) \langle \bar{D}^0 \mid O_4^{ijkl} \mid D^0 \rangle$ in equation (7) of [48] contributes. Basically, this result is in agreement with that of ref. [65], however there is an essential difference. While $q = q' = s$ in [65], we take here $q = s, d$ and $q' = s, d$. If one denotes by $y_{LR}^{(1)}$ the contribution to the lifetime difference in $D^0 - \bar{D}^0$ mixing considered here, then, using Eqs. (7), (9), (10) in ref. [48] (setting there $D_{qq'} = -\left(G_F/\sqrt{2}\right)\xi_g V_{cq}^{L*}V_{uq'}^{R}$, $\bar{\Gamma}_1 = \gamma^\nu P_R$, $\bar{\Gamma}_2 = \gamma_\nu P_L$), it is straightforward to show after doing some algebra that

$$y_{LR}^{(1)} = \sum_{q,\ q'} C_{LR}^{qq'}\ V_{cq'}^{L^*}V_{uq'}^{R}\ \left[K_2\langle Q'\rangle + K_1\langle\tilde{Q}'\rangle\right] \tag{3.5.7}$$

where

$$C_{LR}^{qq'} = \frac{G_F^2 m_c^2\ \xi_g}{2\pi m_D \Gamma_D}\ V_{cq}^{L^*}V_{uq}^{L}\ \sqrt{x_{q'}}\ \Big[(1-x_{q'})^2 - \\ -2x_q x_{q'} - x_q^2\Big] \tag{3.5.8}$$

and the notations in (3.5.7) and (3.5.8) are the same as in [65].

Formulae (3.5.7) and (3.5.8) are generalization of formulae (3) and (4) of ref. [65] for the case when both s- and d-quark intermediate states are considered, thus C_{LR}

of [65] is replaced here by $C_{LR}^{qq'}$ and the sum over q, q' is implemented. Furthermore, in order to take properly into account GIM cancelation effects, we keep in equation (3.5.8) higher order terms in the expansion in powers of $x_q \equiv m_q^2/m_c^2$ and $x_{q'} \equiv m_{q'}^2/m_c^2$.

It is worth noting that dependence on x_q appears only in the next-to-next-to-leading order terms of this expansion. The difference in the behavior of $y_{LR}^{(1)}$ with x_q and with $x_{q'}$ is related to different chiralities of the light quarks q and q' in (3.5.6). More detailed discussion of the behavior of the $D^0 - \bar{D}^0$ mixing amplitude with the light quark masses, depending on these quarks chiralities, may be found in refs. [37, 94, 95]. Discussion for a particular case of the width difference is also available in [29, 96].

If one takes the limit $x_d \equiv m_d^2/m_c^2 = 0$, $C_{LR}^{qq'} = 0$ for $q' = d$. Thus, formula (3.5.7) is significantly simplified:

$$y_{LR}^{(1)} = \left[C_{LR}^{ss} + C_{LR}^{ds}\right] \; V_{cs}^{L^*} V_{us}^{R} \; \left[K_2 \langle Q' \rangle + K_1 \langle \tilde{Q}' \rangle\right] \tag{3.5.9}$$

where

$$C_{LR}^{ss} = \frac{G_F^2 m_c^2 \, \xi_g}{2\pi m_D \Gamma_D} \; V_{cs}^{L^*} V_{us}^{L} \; \sqrt{x_s} \; \left[(1 - x_s)^2 - 3x_s^2\right] \tag{3.5.10}$$

$$C_{LR}^{ds} = \frac{G_F^2 m_c^2 \, \xi_g}{2\pi m_D \Gamma_D} \; V_{cd}^{L^*} V_{ud}^{L} \; \sqrt{x_s} \; (1 - x_s)^2 \tag{3.5.11}$$

As it follows from (3.5.9) - (3.5.11), in the limit $m_d = 0$ there is an additional contribution - as compared to that of ref. [65] - from the diagram in Fig. 3.4 when $q = d$ and $q' = s$: $C_{LR}^{ds} \neq 0$. Moreover, using the fact that $V_{cs}^{L^*} V_{us}^{L} \approx -V_{cd}^{L^*} V_{ud}^{L} + O(\lambda^5)$, it is not hard to see that $C_{LR}^{ss} \approx -C_{LR}^{ds}$ with accuracy of the terms $\sim \lambda^5$ or $\sim x_s^{5/2}$. Thus, the sum of C_{LR}^{ss} and C_{LR}^{ds} is much less in absolute value than these quantities by themselves. This is a manifestation of (approximate) GIM cancelation that makes

$y^{(1)}_{LR}$ negligible.

Using the unitarity condition,

$$V^{L^*}_{cs}V^L_{us} + V^{L^*}_{cd}V^L_{ud} + V^{L^*}_{cb}V^L_{ub} = 0 \tag{3.5.12}$$

one gets after doing some algebra

$$\begin{aligned} C^{ss}_{LR} + C^{ds}_{LR} \;&=\; \frac{G_F^2 m_c^2\ \xi_g}{2\pi m_D \Gamma_D}\ \sqrt{x_s}\ \Big[-Re\left(V^{L^*}_{cb}V^L_{ub}\right)(1- \\ &\qquad -x_s)^2 -\ 3\ V^{L^*}_{cs}V^L_{us}\ x_s^2 \Big] \end{aligned} \tag{3.5.13}$$

Note that unlike the CKM products in (3.5.7) - (3.5.11), $V^{L^*}_{cb}V^L_{ub}$ has a non-negligible phase [24], thus one must explicitly indicate that the real part of this product is only relevant. It is assumed no new source of CP-violation [48] (V_R is real and no spontaneous CP-violation). In this case, the impact of CP-violating effects on $\Delta\Gamma_D$ is negligible.

As it was mentioned above, when studying $D^0 - \bar{D}^0$ oscillations, one puts $V^{L^*}_{cb}V^L_{ub} \approx 0$, as $|V^{L^*}_{cb}V^L_{ub}| \ll V^{L^*}_{cs}V^L_{us}$, thus using the two quark generation mixing approximation. However, in the considered case this approximation is not valid. Indeed, using $Re\left(V^{L^*}_{cb}V^L_{ub}\right) \approx A^2\lambda^5\rho$ and $V^{L^*}_{cs}V^L_{us} \approx \lambda$, it is not hard to see that the first term in the square brackets in (3.5.13) dominates over the last one, for $A \approx 0.81$, $\lambda \approx 0.23$, $\rho \approx 0.13$ [24] and $x_s \equiv m_s^2(m_c)/m_c^2(m_c) \approx 0.007$ [66].

To the lowest order in perturbation theory, one gets a rough estimate of the effect rather than a precise numerical evaluation. In what follows, one may to a good approximation disregard the subdominant terms in (3.5.13). Then, one may rewrite equation (3.5.9) in a more compact form:

$$y^{(1)}_{LR} \;= -\bar{C}_{LR}\ V^{L^*}_{cs}V^R_{us}\ \left[K_2\langle Q'\rangle + K_1\langle \tilde{Q}'\rangle\right] \tag{3.5.14}$$

where

$$\bar{C}_{LR} = \frac{G_F^2 m_c^2 \, \xi_g}{2\pi m_D \Gamma_D} \, Re\left(V_{cb}^{L^*} V_{ub}^{L}\right) \, \sqrt{x_s} \tag{3.5.15}$$

We parameterize $\langle Q' \rangle$ and $\langle \tilde{Q}' \rangle$, using the moderate vacuum saturation approach [47]:

$$\begin{aligned} \langle Q' \rangle &\equiv \langle \bar{D}^0 | \bar{u}_i \gamma^\mu P_L c_i \bar{u}_j \gamma_\mu P_R c_j | D^0 \rangle = \\ &= -\frac{1}{2} f_D^2 m_D^2 B_D - \frac{1}{3} f_D^2 m_D^2 \left(\frac{m_D}{m_c}\right)^2 B_D^S \end{aligned} \tag{3.5.16}$$

$$\begin{aligned} \langle \tilde{Q}' \rangle &\equiv \langle \bar{D}^0 | \bar{u}_i \gamma^\mu P_L c_j \bar{u}_j \gamma_\mu P_R c_i | D^0 \rangle = \\ &= -\frac{1}{6} f_D^2 m_D^2 B_D - f_D^2 m_D^2 \left(\frac{m_D}{m_c}\right)^2 B_D^S \end{aligned} \tag{3.5.17}$$

where $f_D \approx 0.22$GeV [89], $B_D \approx 0.8$ [43, 88], and we choose $B_D^S \approx B_D$. Then, using $G_F = 1.166 \times 10^{-5} GeV^{-2}$, $\Gamma_D \approx 1.6 \times 10^{-12}$GeV, $m_D \approx 1.865$GeV, $m_c \equiv m_c(m_c) \approx 1.25$GeV [24, 87], $K_1 \equiv 3C_1\tilde{C}_1 + C_1\tilde{C}_2 + C_2\tilde{C}_1 \approx 3C_1^2 + 2C_1C_2$, $K_2 \equiv C_2\tilde{C}_2 \approx C_2^2$, $C_1(m_c) = -0.411$, $C_2(m_c) = 1.208$ [43], $V_{cs}^L \approx 1 - \lambda^2/2$, $V_{us}^R \approx 1$ and [65, 97] $\xi_g \leq 0.033$, one gets

$$y_{LR}^{(1)} \leq 1.4 \times 10^{-7} \tag{3.5.18}$$

Thus, due to GIM cancelation, box diagrams with the new physics generated $\Delta C = 1$ transition, presented in Fig. 3.4, give *in sum* negligible contribution to the lifetime difference in $D^0 - \bar{D}^0$ mixing.

It is left for the reader to verify that one gets a negligible contribution to y_{LR} also in the case when the $W_L - W_R$ propagator in Fig. 3.4 is flipped so that W_R couples with the charm quark.

In what follows, one should use the result of ref. [48] that has been derived neglecting the $\Delta C = 1$ transition in Fig. 3.4. In other words, one should use the bound on y_{LR}, given by equation (3.5.5). Thus, within the non-manifest Left-Right Symmetric Model, the New Physics contribution to the lifetime difference in $D^0 - \bar{D}^0$

mixing is rather small.

In is worth noting here that this result has been derived considering the diagrams with only one $\Delta C = 1$ transition generated by a New Physics interaction. There are also box diagrams with both $\Delta C = 1$ transitions occurring due to NP interactions. These diagrams have not been considered so far, as within the Left-Right Symmetric Models they are estimated to have a small contribution to $\Delta\Gamma_D$. On the other hand, it is still possible that within the non-manifest version of the LR model, there are some corners of the parameter space with M_{W_R} below 1 TeV [98], where such diagrams are perhaps non-negligible. Study of this possibility requires detailed and careful scanning of the parameter space of the theory, taking into account all possible constraints, coming from $K_L - K_S$ and $B^0 - \bar{B}^0$ mass differences, as well as other phenomenological constraints. Such a detailed analysis is beyond the scope of this work.

Chapter 4 NP Searches in B_s Decays and Oscillations

Study of the bottom mesons is often more advantageous that the charmed ones, as the long-distance effects in B meson decays and oscillations are rather small. Study of B_s meson is of special interest, as there is still room for New Physics in B_s meson decays and oscillation.

In this chapter we examine possible correlations between the NP contribution to the mass difference in $B_s - \overline{B}_s$ mixing and that to $B_s \to \mu^+\mu^-$ leptonic decay. The Standard Model (SM) prediction for $B_s \to \mu^+\mu^-$ is currently smaller than the experimental branching fraction limit [24] of $\mathcal{B}^{(\text{expt})}_{B_s\to\mu^+\mu^-}$ by about a factor of 15. This presents a window of opportunity for observing New Physics (NP) effects in this mode.

This topic is particularly timely in view of experimental indications of NP effects in both the exclusive decay $B_s \to J/\Psi + \phi$ [99] (for recent CDF results, also see Ref. [100]) as well as the inclusive like-sign dimuon asymmetry observed in $p\bar{p} \to \mu\mu + X$ [101]. Moreover, future work at LHC, e^+e^- Super B-factories and ongoing CDF & D0 measurements at Fermilab (see the discussion following Eq. (4.6)) is expected to markedly improve the current branching fraction bound.

Our strategy in this chapter is somewhat reminiscent of the recent studies in [102] noting that in some NP models the D^0 mixing and $D^0 \to \mu^+\mu^-$ decay amplitudes have a common dependence on the NP parameters. If so, one can predict the $D^0 \to \mu^+\mu^-$ branching fraction in terms of the observed ΔM_D provided that much or all of the mixing is attributed to NP. This is a viable possibility for D^0 mixing: as discussed above, the Standard Model (SM) signal has large theoretical uncertainties and many NP models can produce the observed mixing [47].

For ΔM_{B_s} the situation is very different. Here, the SM prediction is in accord with the observed value (*e.g.* see Refs. [103, 45] and papers cited therein). In fact, the analysis described below (*cf.* see Eqs. (4.12),(4.13)) gives $|\Delta M_{B_s}^{(\mathrm{NP})}/\Delta M_{B_s}^{(\mathrm{SM})}| \leq 0.20$, which demonstrates just how well the SM prediction agrees with the experimental value of ΔM_{B_s}. In view of this, our SM expression for ΔM_{B_s} will be given at NLO [105, 106] whereas LO results will suffice for NP models. As regards the corresponding width difference $\Delta\Gamma_{B_s}$, the experimental and theoretical uncertainties are still rather significant (*viz* Sect. 4.1-C).

In those NP models where mixing and $B_s \to \mu^+\mu^-$ arise from a common set of parameters, the severe constraint on any NP signal to B_s mixing places strong bounds on its contribution to $\mathcal{B}_{B_s\to\mu^+\mu^-}$.[1] In fact, we shall find the constraint can be so strong that for some NP models the predicted $B_s \to \mu^+\mu^-$ branching fraction lies well below the SM prediction.

The first step in our study (*cf* Section 4.1) will be to revisit the SM predictions for mixing in the b-quark system by using up-to-date inputs. We carry this out for the two mixing quantities ΔM_{B_s} and $\Delta\Gamma_{B_s}/\Delta M_{B_s}$. The former in turn yields phenomenological bounds on NP mixing contributions which in certain models can be used to bound the magnitude of the $B_s \to \mu^+\mu^-$ decay mode. We also update the SM branching fraction for $B_s \to \mu^+\mu^-$ by using the observed B_s mixing as input. Then, in Section 4.2 we discuss general properties of NP models with tree-level amplitudes. In Section 4.3, we explore various NP models such as extra Z' bosons, family symmetry, R-parity violating supersymmetry, flavor-changing Higgs models, and models with the fourth sequential generation. Some technical details are relegated to Appendix B.

[1] In particular, Ref. [103] considers the possibility, not covered here, of effects of so-called minimal flavor violation which affect the quark mixing-matrix elements.

The results presented in this chapter are based on those published in [104].

4.1 Update of B_s Mixing and $B_s \to \mu^+\mu^-$ in the Standard Model

We begin by considering the SM predictions for B_s mixing. This step is crucial to obtaining bounds on NP contributions. We also use the B_s mixing signal as input to a determination of the branching fraction for $B_s \to \mu^+\mu^-$.

4.1.1 Inputs to the Analysis

The work in this Section takes advantage of recent progress made in determining several quantities used in the analysis. We summarize our numerical inputs in Table 4.1, along with corresponding references. Included in Table 4.1 is an updated determination of the top quark pole mass [107] $m_t^{(\text{pole})}$ which in turn is used to determine the corresponding running mass $\bar{m}_t(\bar{m}_t)$ [90] along with several decay constants and B-factors as evaluated in lattice QCD. For definiteness, we have used values appearing in Ref. [109]. This area is, however, constantly evolving and one anticipates further developments in the near future [110]. Our values for the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements $|V_{ts}|$ and $|V_{tb}|$ are taken from Ref. [24]. Similar values occur for the global fits cited elsewhere (*e.g.* Refs. [111, 112]).

$M_{B_s} = 5366.3 \pm 0.6$ MeV [24]	$\tau_{B_s} = (1.425 \pm 0.041) \times 10^{-12}$ s [24]
$\Delta M_{B_s} = (117.0 \pm 0.8) \times 10^{-13}$ GeV	$\Delta\Gamma_{B_s}/\Gamma_{B_s} = 0.092^{+0.051}_{-0.054}$ [24]
$x_{B_d} = 0.776 \pm 0.008$ [24]	$x_{B_s} = 26.2 \pm 0.5$ [24]
$m_t^{(\text{pole})} = 173.1 \pm 1.3$ [107]	$\alpha_s(M_Z) = 0.1184 \pm 0.0007$ [108]
$f_{B_s} = 0.2388 \pm 0.0095$ GeV [109]	$f_{B_s}\sqrt{\hat{B}_{B_s}} = 275 \pm 13$ MeV [109]
$\lvert V_{ts}\rvert = 0.0403^{+0.0011}_{-0.0007}$ [24]	$\lvert V_{tb}\rvert = 0.999152^{+0.000030}_{-0.000045}$ [24]

Table 4.1: List of Input Parameters.

4.1.2 ΔM_{B_s}

The pdg2010 value for ΔM_{B_s},

$$\Delta M_{B_s}^{(\mathrm{expt})} = (117.0 \pm 0.8) \times 10^{-13}\ \mathrm{GeV} \quad , \tag{4.1}$$

is a very accurate one – the uncertainty amounts to about 0.7%. The NLO SM formula,

$$\Delta M_{B_s}^{(\mathrm{SM})} = 2\frac{G_{\mathrm{F}}^2 M_{\mathrm{W}}^2 M_{B_s} f_{B_s}^2 \hat{B}_{B_s}}{12\pi^2} |V_{\mathrm{ts}}^* V_{\mathrm{tb}}|^2 \eta_{B_s} S_0(\bar{x}_t) \quad , \tag{4.2}$$

is arrived at from an operator product expansion of the mixing hamiltonian. The short-distance dependence in the Wilson coefficient appears in the scale-insensitive combination $\eta_{B_s} S_0(\bar{x}_t)$, where the factor $S_0(\bar{x}_t)$ is an Inami-Lin function [113] (with $\bar{x}_t \equiv \bar{m}_t^2(\bar{m}_t)/M_{\mathrm{W}}^2$) and $\bar{m}_t(\bar{m}_t)$ is the running top-quark mass parameter in $\overline{\mathrm{MS}}$ renormalization. In particular, we have $\bar{m}_t(\bar{m}_t) = (163.4 \pm 1.2)$ GeV which leads to $S_0(\bar{x}_t) = 2.319 \pm 0.028$. Using the same matching scale, we obtain $\eta_{B_s} = 0.5525 \pm 0.0007$ for the NLO QCD factor.

Our evaluation for $\Delta M_{B_s}^{(\mathrm{SM})}$ then gives

$$\Delta M_{B_s}^{(\mathrm{SM})} = \left(117.1^{+17.2}_{-16.4}\right) \times 10^{-13}\ \mathrm{GeV} \quad , \tag{4.3}$$

which is in accord with the experimental value of Eq. (4.1). The theoretical uncertainty in the SM prediction of Eq. (4.3) is roughly a factor of twenty larger than the experimental uncertainty of Eq. (4.2). The largest source of error occurs in the non-perturbative factor $\hat{B}_{B_s} f_{B_s}^2$, followed by that in the CKM matrix element V_{ts}. The asymmetry in the upper and lower uncertainties in $\Delta M_{B_s}^{(\mathrm{SM})}$ arises from the corresponding asymmetry in the value of V_{ts} cited in Ref. [24].

Finally, we note in passing that for the ratio $\Delta M_{B_d}/\Delta M_{B_s}$ the experimental value is 0.02852 ± 0.00034 whereas the SM determination gives 0.02714 ± 0.00193. This good agreement is not surprising since the ratio $\Delta M_{B_d}/\Delta M_{B_s}$ contains less theoretical uncertainty than ΔM_{B_d} or ΔM_{B_s} separately.

4.1.3 The Ratio $\Delta\Gamma_{B_s}/\Delta M_{B_s}$

The above discussion of $\Delta M_{B_s}^{(\mathrm{SM})}$ sets the stage for analyzing NP contributions to $B_s \to \mu^+\mu^-$. There is, in principle, a second approach which instead utilizes $\Delta\Gamma_{B_s}$. The pdg2010 value for the B_s width difference is $\Delta\Gamma_{B_s}^{(\mathrm{expt})} = 0.062^{+0.034}_{-0.037} \times 10^{12}\mathrm{s}^{-1}$. Together with Eq. (4.1), this gives[2]

$$r^{(\mathrm{expt})} \equiv \frac{\Delta\Gamma_{B_s}^{(\mathrm{expt})}}{\Delta M_{B_s}^{(\mathrm{expt})}} = \frac{0.062^{+0.034}_{-0.037} \times 10^{12}\ \mathrm{s}^{-1}}{(17.77 \pm 0.12) \times 10^{12}\ \mathrm{s}^{-1}} = (34.9 \pm 20.0) \times 10^{-4} \quad . \tag{4.4}$$

whereas the corresponding SM prediction from Ref. [45] is $r^{(\mathrm{SM})} = (49.7 \pm 9.4) \times 10^{-4}$. In contrast to the mass splitting ΔM_{B_s}, the theoretical uncertainty in the ratio $\Delta\Gamma_{B_s}/\Delta M_{B_s}$ is much smaller than in the current experimental determination. Nonetheless, this situation is expected to change once LHCb gathers sufficient data. As such, we would expect a highly accurate value of $\Delta\Gamma_{B_s}^{(\mathrm{expt})}$ to eventually become available. We propose that it could be applied to the kind of analysis used in this chapter as follows. We define a kind of mass difference $\mathcal{D}M_{B_s}$ as

$$\mathcal{D}M_{B_s} \equiv \frac{\Delta M_{B_s}^{(\mathrm{thy})}}{\Delta\Gamma_{B_s}^{(\mathrm{thy})}}\ \Delta\Gamma_{B_s}^{(\mathrm{expt})} \quad . \tag{4.5}$$

The point is that if NP contributions are neglected in $\Delta B = 1$ transitions, then $\Delta\Gamma_{B_s}^{(\mathrm{thy})}$ is purely a SM effect. In addition, the ratio $\Delta M_{B_s}^{(\mathrm{SM})}/\Delta\Gamma_{B_s}^{(\mathrm{SM})}$ will be less dependent on hadronic parameters than either factor separately.

This quantity is also important in the scenarios where NP contributes a significant CP-violating phase to ΔM_{B_s}. In this situation, $\Delta\Gamma_{B_s}^{(\mathrm{expt})}$ will be reduced compared

[2] Using instead the recent CDF evaluation $\Delta\Gamma_{B_s}^{(\mathrm{CDF})} = 0.075 \pm 0.035 \pm 0.01 \times 10^{12}\ \mathrm{s}^{-1}$ implies $r^{(\mathrm{expt})} = (42.2 \pm 20.5) \times 10^{-4}$, consistent with the value in Eq. (4.4).

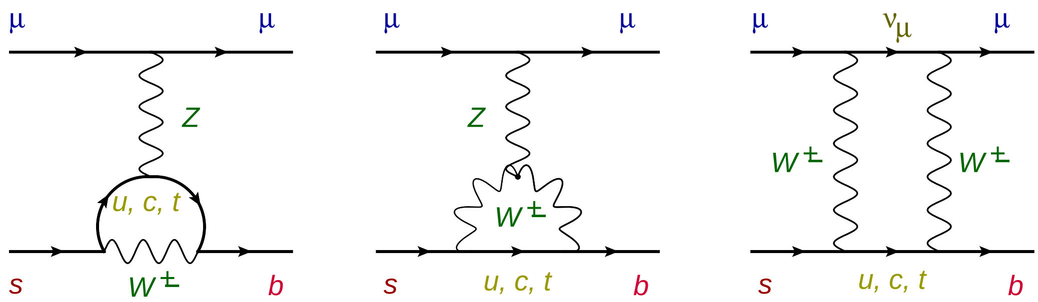

Figure 4.1: SM diagrams for $B_s \to \mu^+\mu^-$.

to its SM value $\Delta\Gamma_{B_s}^{(\mathrm{SM})}$ by a factor of $\cos 2\xi$, where ξ is related to the relative phase between the SM and NP contributions to ΔM_{B_s} [114].

At the very least, the relation in Eq. (4.5) would be of interest to analyze the NP issue using both quantities ΔM_{B_s} and the above $\mathcal{D}M_{B_s}$.

4.1.4 $B_s \to \mu^+\mu^-$

pdg2010 entries for $\mathcal{B}_{B_s\to\ell^+\ell^-}$ are

$$\mathcal{B}^{(\mathrm{expt})}_{B_s\to\mu^+\mu^-} < 4.7 \times 10^{-8} \qquad \text{and} \qquad \mathcal{B}^{(\mathrm{expt})}_{B_s\to e^+e^-} < 5.4 \times 10^{-5} \quad , \tag{4.6}$$

with no experimental limit currently for the $B_s \to \tau^+\tau^-$ transition. Data collected by the D0 and CDF collaborations will improve the above branching fraction limit. For example, the D0 collaboration reports $\mathcal{B}^{(\mathrm{D0})}_{B_s\to\mu^+\mu^-} < 5.1 \times 10^{-8}$, with an anticipated limit of eleven times the SM prediction and similarly for the CDF collaboration [115].

To the lowest order in perturbation theory, the SM diagrams for the $B_s \to \mu^+\mu^-$ are depicted in Fig. 4.1. Since the LD estimate for the branching fraction of $B_s \to \mu^+\mu^-$ in the SM gives $\mathcal{B}^{(\mathrm{LD})}_{B_s\to\mu^+\mu^-} \sim 6 \times 10^{-11}$, we consider only the SD component in the following. Using Eq. (4.2) as input to the SD-dominated $B_s \to \mu^+\mu^-$ transition (see also Ref. [103]) we arrive at

$$\mathcal{B}^{(\mathrm{SM})}_{B_s\to\mu^+\mu^-} = \Delta M_{B_s}\ \tau_{B_s}\frac{3G_{\mathrm{F}}^2 M_{\mathrm{W}}^2 m_\mu^2}{4\eta_{B_s}\hat{B}_{B_s}\pi^3}\left[1-4\frac{m_\mu^2}{M_{B_s}^2}\right]^{1/2}\frac{Y^2(\bar{x}_t)}{S_0(\bar{x}_t)}\quad , \tag{4.7}$$

where $Y(\bar{x}_t)$ is another Inami-Lin function [113]. Expressing $\mathcal{B}^{(\mathrm{SM})}_{B_s\to\mu^+\mu^-}$ in this manner serves to remove some of the inherent model dependence. Numerical evaluation gives

$$\mathcal{B}^{(\mathrm{SM})}_{B_s\to\mu^+\mu^-} \simeq 3.3\times 10^{-9}\quad . \tag{4.8}$$

4.2 Study of New Physics Models

In this section, we first obtain a numerical (1σ) bound on any possible New Physics contribution to ΔM_{B_s}. We then use this to constrain couplings in a variety of NP models and thereby learn something about the $B_s\to\mu^+\mu^-$ transition.

4.2.1 Constraints on NP Models from B_s Mixing

As shown in Ref. [60], New Physics in $\Delta B = 1$ interactions can *in principle* markedly affect $\Delta\Gamma_s$. The logic is similar to that used in Ref. [48] regarding the possible impact of NP on $\Delta\Gamma_D$. Since, however, in B_s mixing such models are not easy to come up with, one can simply assume that $\Delta B = 1$ processes are dominated by the SM interactions. Thus we can write

$$\Delta M_{B_s} = \Delta M_{B_s}^{(\mathrm{SM})} + \Delta M_{B_s}^{(\mathrm{NP})}\cos\phi\quad , \tag{4.9}$$

If the $\Delta B = 1$ sector were to contain significant NP contributions, then the above relation would no longer be valid due to interference between the SM and NP components.

As can be seen from Eq. (4.9), interference between the SM and NP components may also occur in the presence of a CP-violating phase ϕ in the NP part of the mixing amplitude [30]. This large NP phase could markedly affect $\Delta\Gamma_{B_s}^{(\rm expt)}$ even in the absence of a NP contribution to the on-shell $\Delta B = 1$ transitions (recall that $\Delta\Gamma_{B_s}^{(\rm expt)}$ depends explicitly on the cosine of the CP-violating phase ξ [45, 114]; the explicit relation between ϕ and ξ can be found in [114]). It is therefore more reasonable to use $\Delta\Gamma_{B_s}^{(\rm expt)}$ in studying those scenarios with a large NP phase. The appropriate strategy here would be to use $\Delta\Gamma_{B_s}^{(\rm expt)}$ and $\Delta\Gamma_{B_s}^{(\rm SM)}$ to extract the phase ξ, eliminate $\cos\phi$ from Eq. (4.9), and then extract $\Delta M_{B_s}^{(\rm NP)}$ in order to relate it to the rare leptonic decay rate. To do so, however, will require a significant reduction in the experimental uncertainty of $\Delta\Gamma_{B_s}^{(\rm expt)}$. Alternatively, CP-violating phases could be extracted at LHCb from the studies of $B_s \to J/\psi\phi$ transition [30]. We shall defer those studies to a future publication [172]. Here we shall assume that the phase in the NP component of ΔM_{B_s} is sufficiently small (although not necessarily negligible),

$$\Delta M_{B_s} = \Delta M_{B_s}^{(\rm SM)} + \Delta M_{B_s}^{(\rm NP)} \quad . \tag{4.10}$$

Accounting for NP as an additive contribution,

$$\Delta M_{B_s}^{(\rm expt)} = \Delta M_{B_s}^{(\rm SM)} + \Delta M_{B_s}^{(\rm NP)} \quad , \tag{4.11}$$

we have from Eqs. (4.1),(4.3),

$$\Delta M_{B_s}^{(\rm NP)} = \left(-0.1^{+16.4}_{-17.2}\right) \times 10^{-13}\ \text{GeV} \quad . \tag{4.12}$$

The error in $\Delta M_s^{(\rm expt)}$ has been included, but it is so small compared to the theoretical error in $\Delta M_s^{(\rm SM)}$ as to be negligible. The 1σ range for the NP contribution is thus

$$\Delta M_{B_s}^{(\rm NP)} = (-17.3 \to +16.5) \times 10^{-13}\ \text{GeV} \quad . \tag{4.13}$$

To proceed further without ambiguity, we would need to know the relative phase between the SM and NP components. Lacking this, we employ the absolute value of the largest possible number,

$$|\Delta M_{B_s}^{(\mathrm{NP})}| \leq 17.3 \times 10^{-13}\ \mathrm{GeV} \quad , \tag{4.14}$$

to constrain the NP parameters.

4.2.2 Generic NP Models with tree-level amplitudes

New Physics can affect both B_s mixing and rare decays like $B_s \to \mu^+\mu^-$ by engaging in these two transitions at tree level. In this section we will, for generality, consider a generic spin-1 boson V or a spin-0 boson S with flavor-changing and flavor-conserving neutral current interactions that couple both to quarks and leptons. The bosons V and S can be of either parity. This situation is frequently realized, as in the interactions of a heavy Z' boson or in multi-Higgs doublet models without natural flavor conservation.

Spin-1 Boson V: Assuming that the spin-1 particle V has flavor-changing couplings, the most general Lagrangian can be written as

$$\mathcal{H}_V = g'_{V1}\overline{\ell}'_L\gamma_\mu \ell_L V^\mu + g'_{V2}\overline{\ell}'_R\gamma_\mu \ell_R V^\mu + g_{V1}\overline{b}_L\gamma_\mu s_L V^\mu + g_{V2}\overline{b}_R\gamma_\mu s_R V^\mu + \mathrm{h.c.} \tag{4.15}$$

Here V_μ is the vector field and the flavor of the lepton ℓ' might or might not coincide with ℓ. It is not important whether the field V_μ corresponds to an abelian or non-abelian gauge symmetry group. Using methods similar to those in Ref. [102], we obtain

$$\Delta M_{\mathrm{B}_s}^{(\mathrm{V})} = \frac{f_{B_s}^2 M_{B_s}}{3M_V^2}\,\mathcal{R}e\left[C_1(\mu)B_1 + C_6(\mu)B_6 - \frac{5}{4}C_2(\mu)B_2 + \frac{7}{8}C_3(\mu)B_3\right] \quad , \tag{4.16}$$

where the superscript on $\Delta M_{\mathrm{B}_s}^{(\mathrm{V})}$ denotes propagation of a vector boson in the tree amplitude. The Wilson coefficients evaluated at a scale μ are related to the couplings g_{V1} and g_{V2} as

$$\mathrm{C}_1(\mu) = r(\mu, M_V)\ g_{V1}^2\ , \qquad \mathrm{C}_3(\mu) = \tfrac{4}{3}\left[r(\mu, M_V)^{1/2} - r(\mu, M_V)^{-4}\right] g_{V1} g_{V2}\ ,$$
$$\mathrm{C}_2(\mu) = 2\ r(\mu, M_V)^{1/2} g_{V1} g_{V2}\ , \qquad \mathrm{C}_6(\mu) = r(\mu, M_V)\ g_{V2}^2\ \ ,$$

where (presuming that $M > m_t$ and $\mu \geq m_b$),

$$r(\mu, M) = \left(\frac{\alpha_s(M)}{\alpha_s(m_t)}\right)^{2/7} \left(\frac{\alpha_s(m_t)}{\alpha_s(\mu)}\right)^{6/23} \quad . \tag{4.17}$$

Similar calculations can be performed for the $B_s^0 \to \ell^+\ell^-$ decay. The effective Hamiltonian in this case is

$$\mathcal{H}_{b\to q\ell^+\ell^-}^{(\mathrm{V})} = \frac{1}{M_V^2}\left[g_{V1} g'_{V1} \widetilde{Q}_1 + g_{V1} g'_{V2} \widetilde{Q}_7 + g'_{V1} g_{V2} \widetilde{Q}_2 + g_{V2} g'_{V2} \widetilde{Q}_6\right]\ , \tag{4.18}$$

where the operators $\{\widetilde{Q}_i\}$ can be read off from those in Ref. [102] with the label changes $c \to s$ and $u \to b$. This leads to the branching fraction,

$$\mathcal{B}_{B_s^0\to\ell^+\ell^-}^{(\mathrm{V})} = \frac{f_{B_s}^2 m_\ell^2 M_{B_s}}{32\pi M_V^4 \Gamma_{B_s}} \sqrt{1 - \frac{4m_\ell^2}{M_{B_s}^2}}\ |g_{V1} - g_{V2}|^2 |g'_{V1} - g'_{V2}|^2\ . \tag{4.19}$$

Clearly, Eqs. (4.16),(4.19) can be related to each other only for a specific set of NP models.

Spin-0 Boson S: Analogous procedures can be followed if now the FCNC is generated by quarks interacting with spin-0 particles. Again, the most general Hamiltonian can be written as

$$\mathcal{H}_S = g'_{S1}\overline{\ell}_L \ell_R S + g'_{S2}\overline{\ell}_R \ell_L S + g_{S1}\overline{b}_L s_R S + g_{S2}\overline{b}_R s_L S + \mathrm{h.c.}\ \ . \tag{4.20}$$

Evaluation of $\Delta M_{\mathrm{B}_s}^{(\mathrm{S})}$ at scale $\mu = m_b$ gives

$$\Delta M_{\mathrm{B}_s}^{(\mathrm{S})} = \frac{5 f_{B_s}^2 M_{B_s}}{24 M_S^2} \mathcal{R}e\left[\frac{7}{5} C_3(\mu) B_3 - (C_4(\mu) B_4 + C_7(\mu) B_7) + \frac{12}{5}\left(C_5(\mu) B_5 + C_8(\mu) B_8\right)\right] \tag{4.21}$$

with the Wilson coefficients defined as

$$\begin{aligned}
C_3(\mu) &= -2r(\mu, M_S)^{-4}\ g_{S1} g_{S2} \equiv \overline{C}_3(\mu)\ g_{S1} g_{S2} \\
C_4(\mu) &= -\left[\left(\frac{1}{2} - \frac{8}{\sqrt{241}}\right) r_+(\mu, M_S) + \left(\frac{1}{2} + \frac{8}{\sqrt{241}}\right) r_-(\mu, M_S)\right] g_{S2}^2 \equiv \overline{C}_4(\mu)\ g_{S2}^2 \\
C_5(\mu) &= \frac{1}{8\sqrt{241}} \left[r_+(\mu, M_S) - r_-(\mu, M_S)\right] g_{S2}^2 \equiv \overline{C}_5(\mu)\ g_{S2}^2 \\
C_7(\mu) &= -\left[\left(\frac{1}{2} - \frac{8}{\sqrt{241}}\right) r_+(\mu, M_S) + \left(\frac{1}{2} + \frac{8}{\sqrt{241}}\right) r_-(\mu, M_S)\right] g_{S1}^2 \equiv \overline{C}_7(\mu)\ g_{S1}^2 \\
C_8(\mu) &= \frac{1}{8\sqrt{241}} \left[r_+(\mu, M_S) - r_-(\mu, M_S)\right] g_{S1}^2 \equiv \overline{C}_8(\mu)\ g_{S1}^2 \quad ,
\end{aligned} \tag{4.22}$$

where for notational simplicity we have defined $r_\pm \equiv r^{(1\pm\sqrt{241})/6}$. Note that Eq. (4.21) is true only for the real spin-0 field S. If S is a complex field, then only operator Q_3 will contribute to Eq. (4.21).

The effective Hamiltonian for the $B_s^0 \to \ell^+\ell^-$ decay via a heavy scalar S with FCNC interactions is then

$$\mathcal{H}_{b\to s\ell^+\ell^-}^{(\mathrm{S})} = -\frac{1}{M_S^2}\left[g_{S1} g'_{S1} \widetilde{Q}_9 + g_{S1} g'_{S2} \widetilde{Q}_8 + g'_{S1} g_{S2} \widetilde{Q}_3 + g_{S2} g'_{S2} \widetilde{Q}_4\right] , \tag{4.23}$$

and from this, it follows that the branching fraction is

$$\mathcal{B}_{B_s^0\to\ell^+\ell^-}^{(\mathrm{S})} = \frac{f_B^2 M_{B_s}^5}{128\pi m_b^2 M_S^4 \Gamma_{B_s}} \sqrt{1 - \frac{4m_\ell^2}{M_{B_s}^2}}\ |g_{S1} - g_{S2}|^2 \times \left[|g'_{S1} + g'_{S2}|^2 \left(1 - \frac{4m_\ell^2}{M_{B_s}^2}\right) + |g'_{S1} - g'_{S2}|^2\right]. \tag{4.24}$$

Note that if the spin-0 particle S only has *scalar* FCNC couplings, *i.e.* $g_{S1} = g_{S2}$, no contribution to $B_s^0 \to \ell^+\ell^-$ branching ratio is generated at tree level; the non-zero contribution to rare decays is instead produced at one-loop level. This follows from the *pseudoscalar* nature of the B_s-meson.

Let us now consider specific models where the correlations between the $B_s - \overline{B_s}$ mixing rates and (in particular) the $B_s \to \mu^+\mu^-$ rare decay can be found.

4.2.3 Z' Boson

B_s Mixing: The B_s mixing arising from the Z' pole diagram has the same form as in D^0 mixing [47],

$$\Delta M_{B_s}^{(\mathrm{Z'})} = \frac{M_{B_s} f_{B_s}^2 B_{B_s} r_1(m_b, M_{Z'})}{3} \cdot \frac{g_{Z's\bar{b}}^2}{M_{Z'}^2} \quad , \tag{4.25}$$

where $r_1(m_b, M_{Z'})$ is a QCD factor which we take to be

$$r_1(m_b, M_{Z'}) \simeq 0.79 \quad . \tag{4.26}$$

This is a compromise between $r_1(m_b, 1\ \mathrm{TeV}) = 0.798$ and $r_1(m_b, 2\ \mathrm{TeV}) = 0.783$. Solving for the Z' parameters, we have

$$\frac{g_{Z's\bar{b}}^2}{M_{Z'}^2} = \frac{3|\Delta M_{B_s}^{(\mathrm{NP})}|}{M_{B_s} f_{B_s}^2 B_{B_s} r_1(m_b, M_{Z'})} \leq 2.47 \times 10^{-11}\ \mathrm{GeV}^{-2} \tag{4.27}$$

upon using the constraint from B_s mixing.

$B_s \to \mu^+\mu^-$ Decay: This has already been calculated for $D^0 \to \mu^+\mu^-$ decay in Ref. [102]. Inserting obvious modifications for $D^0 \to B_s$, we have from the branching fraction relation Eq. (39) of Ref. [102],

$$\mathcal{B}_{B_s \to \mu^+\mu^-}^{(\mathrm{Z'})} = \frac{G_F f_{B_s}^2 m_\mu^2 M_{B_s}}{16\sqrt{2}\pi \Gamma_{B_s}} \sqrt{1 - \frac{4m_\mu^2}{M_{B_s}^2}}\ \frac{g_{Z's\bar{b}}^2}{M_{Z'}^2} \cdot \frac{M_Z^2}{M_{Z'}^2} \quad . \tag{4.28}$$

Upon inserting numbers, we obtain

$$\mathcal{B}^{(Z')}_{B_s\to\mu^+\mu^-} \;\leq\; 0.25\times10^{-9}\;\cdot\left(\frac{1\ \text{TeV}}{M_{Z'}}\right)^2 \quad . \tag{4.29}$$

This value is already below the corresponding SM prediction ($\mathcal{B}^{(SM)}_{B_s\to\mu^+\mu^-} = 3.3\times10^{-9}$) even if we take a Z' mass as light as $M_{Z'} \simeq 1$ TeV.

4.2.4 R Parity Violating Supersymmetry

One of the models of New Physics that has a rich flavor phenomenology is R-parity violating (RPV) SUSY. The crucial difference between studies of RPV SUSY contributions to phenomenology of the up-quark (see [102]) and down-type quark sectors is the possibility of tree-level diagrams contributing to B_s-mixing[3] and $B_s \to \ell^+\ell^-$ decays [71, 116, 117, 118] Like in studies of $D^0-\overline{D}^0$ oscillations in the previous chapter, we shall require baryon number symmetry by setting λ'' to zero in the superpotential (3.1.1) Also, we will assume CP-conservation, so all couplings λ_{ijk} and λ'_{ijk} are treated as real.

$B_s^0-\overline{B}_s^0$ *Mixing*: Neglecting the baryon-number violating contribution, the Lagrangian describing the RPV SUSY contribution to $B_s^0-\overline{B}_s^0$ mixing can be written as

$$\mathcal{L}_{\not R} = -\lambda'_{i23}\widetilde{\nu}_{i_L}\overline{b}_R s_L - \lambda'_{i32}\widetilde{\nu}_{i_L}\overline{s}_R b_L + h.c. \quad , \tag{4.30}$$

where $i = 1, 2, 3$ is a generational index for the sneutrino. Matching to Eq. (4.20) implies that the only non-zero contribution comes from the operator Q_3. Taking into account renormalization group running, we obtain for ΔM_s from the R-parity

[3] We assume in this subsection that there is no strong hierarchy between the RPV SUSY couplings that favors possible box diagrams.

violating terms,

$$\Delta M_{\mathrm{B_s}}^{(\not R)} = \frac{5}{24} f_{B_s}^2 M_{B_s} F(C_3, B_3) \sum_i \frac{\lambda'_{i23} \lambda'^*_{i32}}{M_{\tilde{\nu}_i}^2}, \tag{4.31}$$

where $M_{\tilde{\nu}_i}$ denotes the mass of the sneutrino of ith generation and the function

$$F(C_3, B_3) = \frac{7}{5} \overline{C}_3(\mu, M_{\tilde{\nu}_i}) B_3, \tag{4.32}$$

is defined in terms of reduced Wilson coefficient of Eq. (4.22) and the B-factor is defined in Table B.1 of the Appendix B.

$B_s \to \mu^+\mu^-$ *Decay*: In RPV-SUSY, the underlying transition for $B_s \to \mu^+\mu^-$ is $s + \bar{b} \to \mu^+ + \mu^-$ via tree-level u-squark or sneutrino exchange. In order to relate the rare decay to the mass difference contribution from RPV SUSY $\Delta M_{\mathrm{B_s}}^{(\not R)}$, we need to assume that the up-squark contribution is negligible. This can be achieved in models where sneutrinos are much lighter than the up-type squarks, which are phenomenologically viable. Employing this assumption leads to the predicted branching fraction

$$\begin{aligned} \mathcal{B}_{B_s \to \mu^+\mu^-}^{(\not R)} = & \frac{f_{B_s}^2 M_{B_s}^3}{64\ \pi\ \Gamma_{B_s}} \left(\frac{M_{B_s}}{m_b} \right)^2 \left(1 - \frac{2m_\mu^2}{M_{B_s}^2} \right) \sqrt{1 - \frac{4m_\mu^2}{M_{B_s}^2}} \\ & \times \left(\left| \sum_i \frac{\lambda^*_{i22} \lambda'_{i32}}{M_{\tilde{\nu}_i}^2} \right|^2 + \left| \sum_i \frac{\lambda_{i22} \lambda'^*_{i23}}{M_{\tilde{\nu}_i}^2} \right|^2 \right). \end{aligned} \tag{4.33}$$

In order to relate $B_s \to \mu^+\mu^-$ to ΔM_s in the framework of RPV SUSY, we need to make additional assumptions. In particular, we shall assume that the sum is dominated by a single sneutrino state, which we shall denote by $\tilde{\nu}_k$. In addition, we will assume that $\lambda'_{k23} = \lambda'_{k32}$, which will reduce the number of unknown parameters. This assumption is not needed, however, if one wishes to set a bound on a combination of coupling constants directly from the experimental bound on $\mathcal{B}_{B_s \to \mu^+\mu^-}$. Then,

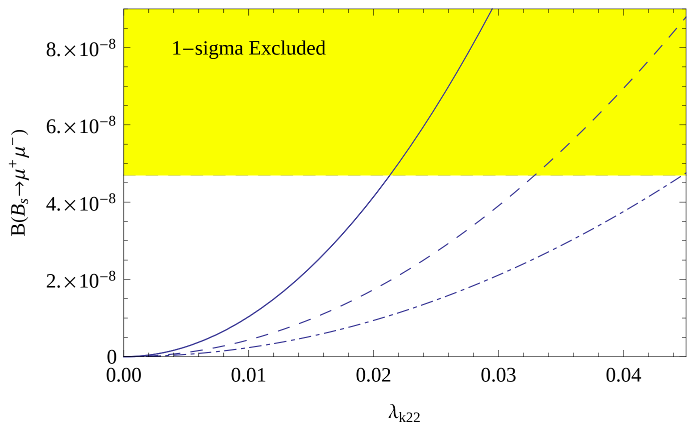

Figure 4.2: $\mathcal{B}_{B_s^0\to\mu^+\mu^-}$ as a function of λ_{k22}.

neglecting CP-violation,

$$\mathcal{B}^{(\not R)}_{B_s\to\mu^+\mu^-} = k\frac{f_{B_s}^2 M_{B_s}^3}{64\pi\ \Gamma_{B_s}}\left(\frac{\lambda_{i22}\lambda'_{i32}}{M_{\tilde\nu_i}^2}\right)^2\left(\frac{M_{B_s}}{m_b}\right)^2\left(1-\frac{2m_\mu^2}{M_{B_s}^2}\right)\sqrt{1-\frac{4m_\mu^2}{M_{B_s}^2}}, \quad (4.34)$$

where $k = 2$ if an assumption that $\lambda'_{k23} = \lambda'_{k32}$ is made, and $k = 1$ otherwise.

Since no $B_s \to \mu^+\mu^-$ signal has yet been seen, we can use the experimental bound to obtain an updated constraint on the RPV couplings,

$$\lambda_{k22}\lambda'_{k32} \le 5.5\times 10^{-6}\left(\frac{M_{\tilde\nu_k}}{100\text{ GeV}}\right)^2 \quad . \quad (4.35)$$

Now, assuming $\lambda'_{k23} = \lambda'_{k32}$, one can relate the branching ratio $\mathcal{B}_{B_s\to\mu^+\mu^-}$ to $x^{(\not R)}_{B_s}$,

$$\mathcal{B}^{(\not R)}_{B_s\to\mu^+\mu^-} = \frac{3}{20\pi}\frac{M_{B_s}^2}{F(C_3,B_3)}\left(\frac{M_{B_s}}{m_b}\right)^2\left(1-\frac{2m_\mu^2}{M_{B_s}^2}\right)\sqrt{1-\frac{4m_\mu^2}{M_{B_s}^2}}\ x^{(\not R)}_{B_s}\ \frac{\lambda_{k22}^2}{M_{\tilde\nu_i}^2}. \quad (4.36)$$

It is possible to plot the dependence of $\mathcal{B}_{B_s\to\mu^+\mu^-}$ on λ_{k22} for different values of $M_{\tilde\nu_i}$, which we present in Fig. 4.2 for $M_{\tilde\nu_i} = 100$ GeV, 150 GeV and 200 GeV.

4.2.5 Family (Horizontal) Symmetries

The gauge sector in the Standard Model has a large global symmetry which is broken by the Higgs interaction [119]. By enlarging the Higgs sector, some subgroup of this symmetry can be imposed on the full SM Lagrangian and the symmetry can be broken spontaneously. This family symmetry can be global [14] as well as gauged [15]. If the new gauge couplings are very weak or the gauge boson masses are large, the difference between a gauged or global symmetry is rather difficult to distinguish in practice [120]. In general there would be FCNC effects from both the gauge and scalar sectors. Here we study the gauge contribution. Consider the family gauge symmetry group $SU(3)_G$ acting on the three left-handed families. Spontaneous symmetry breaking renders all the gauge bosons massive. If the SU(3) symmetry is broken first to SU(2) before being completely broken, we may have an effective 'low' energy symmetry $SU(2)_G$. This means that the gauge bosons $\mathbf{G} \equiv \{G_i\}$ $(i = 1, \ldots, 3)$ are much lighter than the $\{G_k\}$ $(k = 4, \ldots, 8)$. For simplicity we assume that after symmetry breaking the gauge boson mass matrix is diagonal to a good approximation. If so, the light gauge bosons $\mathbf{G}$ are mass eigenstates with negligible mixing.

The LH doublets

$$\begin{pmatrix} u^0 \\ d^0 \end{pmatrix}_L , \quad \begin{pmatrix} c^0 \\ s^0 \end{pmatrix}_L , \quad \begin{pmatrix} t^0 \\ b^0 \end{pmatrix}_L , \tag{4.37}$$

transform as $I_G = 1/2$ under $SU(2)_G$, as do the lepton doublets

$$\begin{pmatrix} \nu_e^0 \\ e^0 \end{pmatrix}_L , \quad \begin{pmatrix} \nu_\mu^0 \\ \mu^0 \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau^0 \\ \tau^0 \end{pmatrix}_L . \tag{4.38}$$

and the right-handed fermions are singlets under $SU(2)_G$. In the above, the superscript 'o' refers to the fact that these are weak eigenstates and not mass eigenstates. The couplings of fermions to the light family gauge bosons $\mathbf{G}$ is given by

$$L = f\left[\bar{\psi}_{d^0,L}\gamma_\mu\boldsymbol{\tau}\cdot\mathbf{G}^\mu\psi_{d^0,L} + \bar{\psi}_{u^0,L}\gamma_\mu\boldsymbol{\tau}\cdot\mathbf{G}^\mu\psi_{u^0,L} + \bar{\psi}_{\ell^0,L}\gamma_\mu\boldsymbol{\tau}\cdot\mathbf{G}^\mu\psi_{\ell^0,L}\right] \quad , \qquad (4.39)$$

where f denotes the coupling strength and $\boldsymbol{\tau}$ are the generators of $SU(2)_G$

The fermion mass eigenstates are given by, first for quarks,

$$\begin{pmatrix} d \\ s \\ b \end{pmatrix}_L = U_d \begin{pmatrix} d^0 \\ s^0 \\ b^0 \end{pmatrix}_L \qquad and \qquad \begin{pmatrix} u \\ c \\ t \end{pmatrix}_L = U_u \begin{pmatrix} u^0 \\ c^0 \\ t^0 \end{pmatrix}_L \qquad (4.40)$$

and then for leptons,

$$\begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_L = U_\ell \begin{pmatrix} u^0 \\ \mu^0 \\ \tau^0 \end{pmatrix}_L \qquad \text{and} \qquad \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L = U_\nu \begin{pmatrix} \nu_e^0 \\ \nu_\mu^0 \\ \nu_\tau^0 \end{pmatrix}_L \quad . \qquad (4.41)$$

The four matrices U_d, U_u, U_ℓ and U_ν are unknown, except for

$$U_u U_d^\dagger = V_{\text{CKM}} \qquad \text{and} \qquad U_\nu^\dagger U_\ell = V_{\text{MNSP}} \quad . \qquad (4.42)$$

where V_{MNSP} is the Maki-Nakagawa-Sakata-Pontcorvo lepton mixing matrix. The couplings of the gauge bosons relevant for the B_s system in the mass basis are:

$$
\begin{aligned}
L = \ & f\Big[G_1^\mu . \Big(U_{b1}U_{s2}^*\bar{b}_L\gamma_\mu s_L \ + \ U_{s1}U_{b2}^*\bar{s}_L\gamma_\mu b_L \\
& +U_{b2}U_{s1}^*\bar{b}_L\gamma_\mu s_L + U_{s2}U_{b1}^*\bar{s}_L\gamma_\mu b_L\Big) \\
& +iG_2^\mu\Big(-U_{b1}U_{s2}^*\bar{b}_L\gamma_\mu s_L \ - \ U_{s1}U_{b2}^*\bar{s}_L\gamma_\mu b_L \\
& + U_{b2}U_{s1}^*\bar{b}_L\gamma_\mu s_L \ + \ U_{s2} \ U_{b1}^*\bar{s}_L\gamma_\mu b_L\Big) \\
& +G_3^\mu\Big(U_{b1}U_{s1}^*\bar{b}_L\gamma_\mu s_L \ + \ U_{s1}U_{b1}^*\bar{s}_L\gamma_\mu b_L \\
& - \ U_{b2}U_{s2}^*\bar{b}_L\gamma_\mu \bar{s}_L \ - \ U_{s2}U_{b2}^*\bar{s}_L \ \gamma_\mu \ b_L\Big)\Big]
\end{aligned}
\tag{4.43}
$$

The contribution to $B_s^0 - \bar{B}_s^0$ mixing is given by

$$
\Delta M_{B_s}^{(\mathrm{FS})} = \frac{2M_{B_s}f_{B_s}^2 B_{B_s} r(m_{B_s,M})}{3} \ f^2\left[\frac{A}{m_1^2} \ + \frac{C}{m_3^2} + \frac{B}{m_2^2}\right] \tag{4.44}
$$

where

$$
\begin{aligned}
A &= \mathrm{Re}\left[(U_{b1} \ U_{s2}^* + U_{b2} \ U_{s1}^*)^2\right] \\
B &= -\mathrm{Re}\left[(U_{b1} \ U_{s2}^* - \ U_{b2} \ U_{s1}^*)^2\right] \\
C &= \mathrm{Re}\left[(U_{b1}U_{s1}^* - U_{b2}U_{s2}^*)^2\right]
\end{aligned}
\tag{4.45}
$$

In a simple scheme of symmetry breaking [121], one obtains $m_1 = m_3$ and the square bracket in Eq. (4.44) becomes

$$
\left[\frac{A+C}{m_1^2} \ + \frac{B}{m_2^2}\right] \quad . \tag{4.46}
$$

Although the matrices U_i $(i = d, u, \ell)$ in principle are unknown, it has been argued that a reasonable ansatz [122], which is incorporated in many models is $U_u = I, \ U_d^\dagger =$

$V_{\rm CKM}$. In this case[4] one can simplify A, B and C further:

$$A, B \ll C \simeq 1.6 \times 10^{-3} \quad . \tag{4.47}$$

Thus the B_s mixing becomes

$$\Delta M_{B_s}^{\rm (FS)} \simeq \frac{2M_{B_S} f_{B_s}^2 B_{B_s} r(m_b, M)}{3} \; \frac{f^2}{m_1^2} \; 1.6 \times 10^{-3} \quad , \tag{4.48}$$

so that, substituting the experimental bound $\Delta M_{B_s}^{\rm (FS)} = \Delta M_{B_s}^{\rm (NP)}$,

$$\frac{f^2}{m_1^2} \leq \frac{3|\Delta M_{B_s}^{\rm (NP)}|}{2M_{B_S} f_{B_s}^2 B_{B_s} r(m_b, M) 1.6 \times 10^{-3}} \quad . \tag{4.49}$$

The same above ansatz also implies that $U_\ell^\dagger = U_{\rm MNSP}$ and $U_\nu = 1$. Then the coupling of the gauge bosons to muon pairs is given by

$$\begin{aligned} \mathcal{L}_{{\rm G}\mu^+\mu^-} = f \Big[& \left(U_{\mu 1}^* U_{\mu 2} + U_{\mu 1} \; U_{\mu 2}^* \right) G_1^\lambda \\ & + i \left(-U_{\mu 1} \; U_{\mu 2}^* + U_{\mu 1}^* U_{\mu 2} \right) G_2^\lambda \; + \left(U_{\mu 1} \; U_{\mu 1}^* - U_{\mu 2} \; U_{\mu 2}^* \right) G_3^\lambda \Big] \bar{\mu}_L \gamma_\lambda \mu_L \quad . \end{aligned} \tag{4.50}$$

The branching ratio for $B_s \to \mu^+\mu^-$ is given by

$$\begin{aligned} \mathcal{B}_{B_s \to \mu^+\mu^-} = \; & \frac{M_{B_S} f_{B_s}^2 \; m_\mu^2}{32\pi \Gamma_{B_s}} \; f^4 \Bigg| \frac{\left(U_{b1} U_{s2}^* \; + U_{b2} U_{s1}^* \right) \left(U_{\mu 1} U_{\mu 2}^* + U_{\mu 1}^* \; U_{\mu 2} \right)}{m_1^2} \\ & - \frac{\left(U_{b1} U_{s2}^* - U_{b2} U_{s1}^* \right) \left(U_{\mu 1} U_{\mu 2}^* - U_{\mu 2} U_{\mu 1}^* \right)}{m_2^2} \\ & + \frac{\left(U_{b1} U_{s1}^* - U_{b2} U_{s2}^* \right) \left(U_{\mu 1} U_{\mu 1}^* - U_{\mu 2} U_{\mu 2}^* \right)}{m_3^2} \Bigg|^2 \end{aligned} \tag{4.51}$$

Next we employ the approximation (well-supported empirically) that $U_{\rm MNSP} \simeq U_{\rm TBM}$,

[4] Here, we use values listed in Ref. [24].

where $U_{\rm TBM}$ is the tri-bi-maximal matrix [41]. Then Eq. (4.50) becomes

$$\mathcal{L}_{{\rm G}\mu^+\mu^-} = -f\left[\frac{\sqrt{2}}{3}\ G_1^\mu + \frac{1}{6}\ G_3^\mu\right]\bar{\mu}_L\gamma_\mu\mu_L \quad . \tag{4.52}$$

With this, the contribution to the branching ratio for $B_s \to \mu^+\mu^-$ becomes

$$\begin{aligned} B_{B_s\to\mu^+\mu^-} &= \frac{M_{B_s}f_{B_s}^2 m_\mu^2 f^4}{32\pi\Gamma_{B_s}}\left[\frac{\sqrt{2}}{3}\left(1.1\times 10^{-2}\right)\ +\ \frac{1}{6}\times 0.04\right]^2\frac{1}{m_1^4} \\ &\simeq \frac{M_{B_s}f_{B_s}^2 m_\mu^2 f^4}{32\pi\Gamma_{B_s}}\ \frac{1.4\times 10^{-4}}{m_1^4} \quad . \end{aligned} \tag{4.53}$$

The dependence on unknown factors in Eq. (4.53) (*i.e.* $(f/m_1)^4$) can be entirely removed by using the bound in Eq. (4.49) to yield

$$\mathcal{B}^{\rm (FS)}_{B_s\to\mu^+\mu^-} \leq \frac{3.85 m_\mu^2}{\pi M_{B_S} f_{B_s}^2\Gamma_{B_s}B_{B_s}^2 r^2(m_b,m_1)}\ |\Delta M_{B_s}^{\rm (NP)}|^2 \quad . \tag{4.54}$$

From the bounds of Eqs. (4.12),(4.13), we obtain

$$\mathcal{B}^{\rm (FS)}_{B_s\to\mu^+\mu^-} \leq 0.5\times 10^{-12} \quad . \tag{4.55}$$

4.2.6 FCNC Higgs interactions

Many extensions of the Standard Model contain multiple scalar doublets, which increases the possibility of FCNC mediated by flavor non-diagonal interactions of neutral components. While many ideas exist on how to suppress those interactions (see, *e.g.* [124, 125, 126]), the ultimate test of those ideas would involve direct observation of scalar-mediated FCNC.

Consider a generic Yukawa interaction consisting of a set of N Higgs doublets H_n $(n = 2, .., N)$ with SM fermions,

$$\mathcal{H}_Y = \lambda_{ijn}^U\overline{Q}_{Li}U_{Rj}\widetilde{H}_n + \lambda_{ijn}^D\overline{Q}_{Li}D_{Rj}H_n + \lambda_{ijn}^E\overline{L}_{Li}E_{Rj}H_n + {\rm h.c.} \quad , \tag{4.56}$$

where $\widetilde{H}_n = i\sigma_2 H_n^*$ and Q_{Li} (L_{Li}) are respectively the left-handed weak doublets of an ith-generation of quarks (leptons). Restricting the discussion to B_s Mixing and $B_s \to \mu^+\mu^-$ decay, we find that Eq. (4.56) reduces to

$$\mathcal{H}_Y^H = \lambda_{23n}^D \overline{s}_L b_R \Phi_n^0 \; + \lambda_{32n}^D \overline{b}_L s_R \Phi_n^0 + \lambda_{22n}^E \overline{\mu}_L \mu_R \Phi_n^0 + \text{h.c.}, \tag{4.57}$$

where $\Phi_n^0 \equiv \left(\phi_n^0 + i a_n^0\right)/\sqrt{2}$. Bringing this to the form of Eq. (4.20) and confining the discussion only to the contribution of the lightest ϕ_n^0 and a_n^0 states, we obtain

$$\begin{aligned} \mathcal{H}_Y^H \; = \; & \frac{\lambda_{23}^{D\dagger}}{\sqrt{2}} \, \overline{b}_R s_L \phi^0 \; + \frac{\lambda_{32}^D}{\sqrt{2}} \, \overline{b}_L s_R \phi^0 + \frac{\lambda_{22}^E}{\sqrt{2}} \, \overline{\mu}_L \mu_R \phi^0 \\ & - i \frac{\lambda_{23}^{D\dagger}}{\sqrt{2}} \, \overline{b}_R s_L a^0 \; + i \frac{\lambda_{32}^D}{\sqrt{2}} \, \overline{b}_L s_R a^0 + \; i \frac{\lambda_{22}^E}{\sqrt{2}} \, \overline{\mu}_L \mu_R a^0 + \ldots \; + \text{h.c.} \;\; , \end{aligned} \tag{4.58}$$

where ellipses stand for the terms containing heavier ϕ_n^0 and a_n^0 states whose contributions to ΔM_{B_s} and $\mathcal{B}_{B_s \to \mu^+\mu^-}$ will be suppressed.

If the matrix of coupling constants in Eq. (4.58) is Hermitian, *e.g.* $\lambda_{23}^{D\dagger} = \lambda_{32}^D$, then we can identify the couplings of Eq. (4.20) as

$$g_{S_1} = g_{S_2} = \frac{\lambda_{32}^D}{\sqrt{2}}, \qquad g'_{S_1} = g'_{S_2} = \frac{\lambda_{22}^E}{\sqrt{2}} \tag{4.59}$$

for *scalar* interactions and

$$g_{S_1} = -g_{S_2} = \frac{i\lambda_{32}^D}{\sqrt{2}}, \quad g'_{S_1} = -g'_{S_2} = \frac{i\lambda_{22}^E}{\sqrt{2}} \tag{4.60}$$

for *pseudoscalar* interactions.

To proceed, we need to separate two cases: (i) the lightest FCNC Higgs particle is a scalar, and (ii) the lightest FCNC Higgs particle is pseudoscalar.

Light scalar FCNC Higgs

The case of relatively light scalar Higgs state is quite common, arising most often in Type-III two-Higgs doublet models (models without natural flavor conservation) [127, 128, 130].

B_s^0-$\bar{B}_s^0$ *Mixing*: Given the general formulas of Eq. (4.21), it is easy to compute the contribution to $\Delta M_{\mathrm{B}_s}^{(\phi)}$ of an intermediate scalar (ϕ) with FCNC couplings,

$$\Delta M_{\mathrm{B}_s}^{(\phi)} = \frac{5 f_{B_s}^2 M_{B_s} f_\phi(\overline{C}_i, m_b)}{48} \left(\frac{\lambda_{32}^D}{M_\phi}\right)^2 \quad , \tag{4.61}$$

$$f_\phi(\overline{C}_i, m_b) \equiv \frac{7}{5}\overline{C}_3(m_b)B_3 - \left(\overline{C}_4(m_b)B_4 + \overline{C}_7(m_b)B_7\right) + \frac{12}{5}\left(\overline{C}_5(m_b)B_5 + \overline{C}_8(m_b)B_8\right) \ ,$$

with 'reduced' Wilson coefficients $\{\overline{C}_i(\mu)\}$ given in Eq. (4.22).

$B_s^0 \to \mu^+\mu^-$ *Decay*: Comparing Eq. (4.59) to Eq. (4.24), we can easily see that the branching fraction for the rare decay $B_s^0 \to \ell^+\ell^-$ is zero for the intermediate scalar Higgs,

$$\mathcal{B}_{B_s^0 \to \ell^+\ell^-}^{(\phi)} = 0 \quad . \tag{4.62}$$

This is consistent with what was already discussed in Sec. 4.2.2 and implies that the FCNC Higgs model does not produce a contribution to $B_s^0 \to \mu^+\mu^-$ at tree level. The non-zero contribution to $B_s^0 \to \mu^+\mu^-$ decay is produced at one-loop level [129].

Light pseudoscalar FCNC Higgs

The case of a lightest *pseudoscalar* Higgs state can occur in the non-minimal supersymmetric standard model (NMSSM) [131, 132, 134, 135], or related models [133]. In the NMSSM, a complex singlet Higgs is introduced to dynamically solve the μ problem. The resulting pseudoscalar can be as light as tens of GeV. This does not mean, however, that it necessarily gives the dominant contribution to both $B_s^0 - \overline{B}_s^0$ mixing and the $B_s^0 \to \mu^+\mu^-$ decay rate since there can be loop contributions from other Higgs states. In the following, we shall work in the region of the parameter space where it does.

B_s^0-$\bar{B}_s^0$ Mixing: The contribution to $\Delta M_{\mathrm{B}_s}^{(\mathrm{a})}$ due to intermediate pseudoscalar with flavor-changing couplings can be computed using the general formula in Eq. (4.21) along with the identification given in Eq. (4.60),

$$\begin{aligned}
\Delta M_{\mathrm{B}_s}^{(\mathrm{a})} &= \frac{5 f_{B_s}^2 M_{B_s} f_a(\overline{C}_i, m_b)}{48} \left(\frac{\lambda_{32}^D}{M_a}\right)^2 , \qquad (4.63)\\
f_a(\overline{C}_i, m_b) &= \left[\frac{7}{5}\overline{C}_3(m_b)B_3 + \left(\overline{C}_4(m_b)B_4 + \overline{C}_7(m_b)B_7\right) - \frac{12}{5}\left(\overline{C}_5(m_b)B_5 + \overline{C}_8(m_b)B_8\right)\right]
\end{aligned}$$

with 'reduced' Wilson coefficients $\overline{C}_i(\mu)$ again being defined in Eq. (4.22).

$B_s^0 \to \mu^+\mu^-$ Decay: The branching ratio for rare decay can be computed with the help of the general formula of Eq. (4.24),

$$\mathcal{B}_{B_s^0\to\ell^+\ell^-}^{(\mathrm{a})} = \frac{1}{32\pi}\frac{f_B^2 M_{B_s}^5}{m_b^2 \Gamma_{B_s}}\left(1 - \frac{4m_\ell^2}{M_{B_s}^2}\right)^{1/2}\left(\frac{\lambda_{32}^D\,\lambda_{22}^E}{M_a^2}\right)^2 . \qquad (4.64)$$

We can now eliminate one of the three unknown parameters (λ_{32}^D, λ_{22}^E, and M_a) which appear in Eqs.(4.63) and (4.64). We choose to eliminate λ_{32}^D, so

$$\mathcal{B}_{B_s^0\to\ell^+\ell^-}^{(\mathrm{a})} = \frac{3}{10\pi}\cdot\frac{M_{B_s}^4 x_s^{(\mathrm{a})}}{m_b^2 f_a(\overline{C}_i, m_b)}\left(1 - \frac{4m_\ell^2}{M_{B_s}^2}\right)^{1/2}\left(\frac{\lambda_{22}^E}{M_a}\right)^2 , \qquad (4.65)$$

where $x_s^{(\mathrm{a})} = \Delta M_{\mathrm{B}_s}^{(\mathrm{a})}/\Gamma_{B_s}$. As one can see, the unknown factors enter Eq. (4.65) in the combination λ_{22}^E/M_a. It is, however, more convenient to plot the dependence on M_a for different values of λ_{22}^E, which we present in Fig. 4.3 for $\lambda_{22}^E = 1, 0.5, 0.1$ (left) and $\lambda_{22}^E = 0.1, 0.05, 0.01$ (right).

It must be emphasized that the discussion above assumed the absence of large destructive interference of the NP and SM contributions to $B_s^0 - \overline{B}_s^0$ mixing. Concrete models where such interference is present (and thus the New Physics contribution is

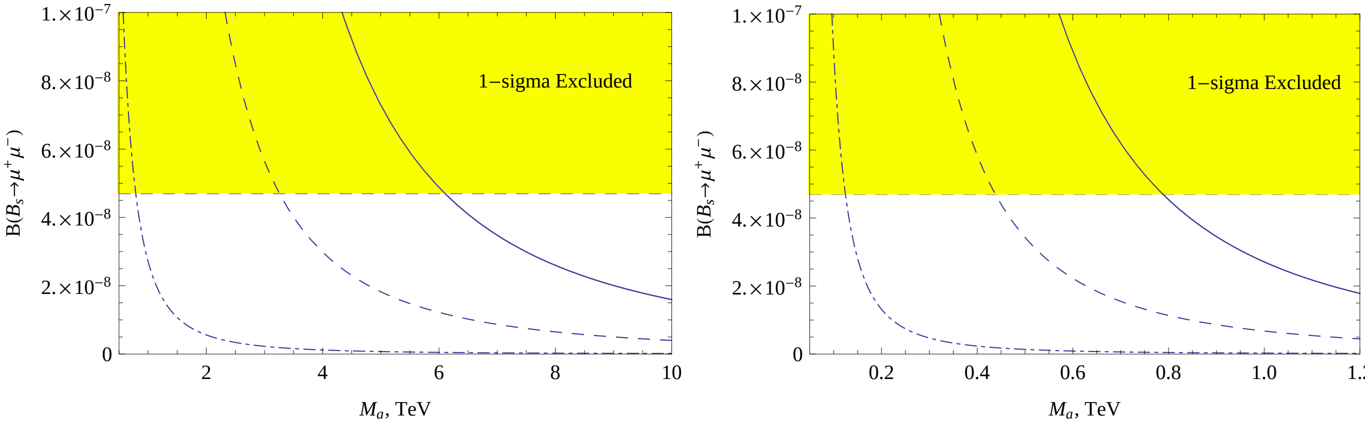

Figure 4.3: $\mathcal{B}_{B_s^0 \to \mu^+\mu^-}$ as a function of M_a.

larger than the SM one) can be constructed [136]. In such models possible contribution to $B_s \to \mu^+\mu^-$ could be large.

4.2.7 Fourth generation models

One of the simplest extensions of the Standard Model involves addition of the sequential fourth generation of chiral quarks [16, 17, 18], denoted for the lack of the better names by t' and b'. The addition of the sequential fourth generation of quarks leads to a 4×4 CKM quark mixing matrix [137]. This implies that the parametrization of this matrix requires six real parameters and three phases. Besides providing new sources of CP-violation, the two additional phases can affect the branching ratios considered in this chapter due to interference effects [138].

There are many existing constraints on the parameters related to the fourth generation of quarks. In particular, a fit of precision electroweak data (S and T parameters) [139, 140, 141] implies that the masses of the new quarks are strongly constrained to be [142]

$$m_{t'} - m_{b'} \simeq \left(1 + \frac{1}{5}\frac{m_H}{(115\ \text{GeV})}\right) \times 50\ \text{GeV}, \qquad (4.66)$$

with $m_{t'} > 400$ GeV. Here m_H is the SM Higgs mass, which we take for simplicity to be 120 GeV. We also used updated constraints on CKM matrix elements [143].

The relationship between ΔM_{B_s} and $\mathcal{B}_{B_s \to \mu^+\mu^-}$ in the model with four generations of quarks has been previously studied in detail in [144]. Here we update their result. The branching ratio of $B_s \to \mu^+\mu^-$ can be related to the experimentally-measured[5] x_{B_s} as [144]

$$\mathcal{B}_{B_s \to \mu^+\mu^-} = \frac{3\alpha^2 m_\mu^2 x_{B_s}}{8\pi \hat{B}_{B_s} M_W^2} \sqrt{1 - \frac{4m_\mu^2}{m_{B_s}^2}} \frac{|C_{10}^{tot}|^2}{|\Delta'|}, \tag{4.67}$$

where the parameter Δ' is a B_s-mixing loop parameter [144],

$$\Delta' = \eta_t S_0(x_t) + \eta_{t'} R_{t't}^2 S_0(x_{t'}) + 2\eta_{t'} R_{t't} S_0(x_t, x_{t'}), \tag{4.68}$$

and $R_{t't} = V_{t's} V_{t'b}^* / V_{ts} V_{tb}^*$. $\hat{B}_{B_s}$ can be obtained from Table 4.1. The definition of the function $S_0(x_t, x_{t'})$ can be found in Ref. [144]. The Wilson coefficient C_{10}^{tot} is defined as

$$C_{10}^{tot}(\mu) = C_{10}(\mu) + R_{t't} C_{10}^{t'}(\mu) \tag{4.69}$$

with $C_{10}^{t'}$ obtained by substituting $m_{t'}$ into the SM expression for C_{10} [145]. The results can be found in Fig. 4.4, where we plot the branching ratio of $\mathcal{B}_{B_s^0 \to \mu^+\mu^-}$ as a function of the top-prime mass $m_{t'}$ for different values of the phase $\phi_{t's} = 0, \pi/2, \pi$ (solid, dashed, dash-dotted lines) and $\lambda_{bs}^{t'} = |V_{t's} V_{t'b}^*| \simeq 10^{-4}$ [143], [146], and as a function of the CKM parameter combination $\lambda_{bs}^{t'}$ with $\phi_{t's} = 0$ and different values of $m_{t'} = 400$ GeV (solid), 500 GeV (dashed), and 600 GeV (dash-dotted). As one can see, the resulting branching ratios are still lower than the current experimental bound of Eq. (4.6), but for the values of the four-generation CKM matrix $\lambda_{bs}^{t'} = |V_{t's} V_{t'b}^*|$ of about 0.01, disfavored by [143], but still favored by [146], can be quite close to it.

[5] Here we use ΔM_{B_s} from Table 4.1, as the separation of NP and SM contributions used in the rest of this chapter, $x_{B_s} = x_{SM3} + x_{SM4}$, is not possible due to loops with both t' and t, c, or u quarks.

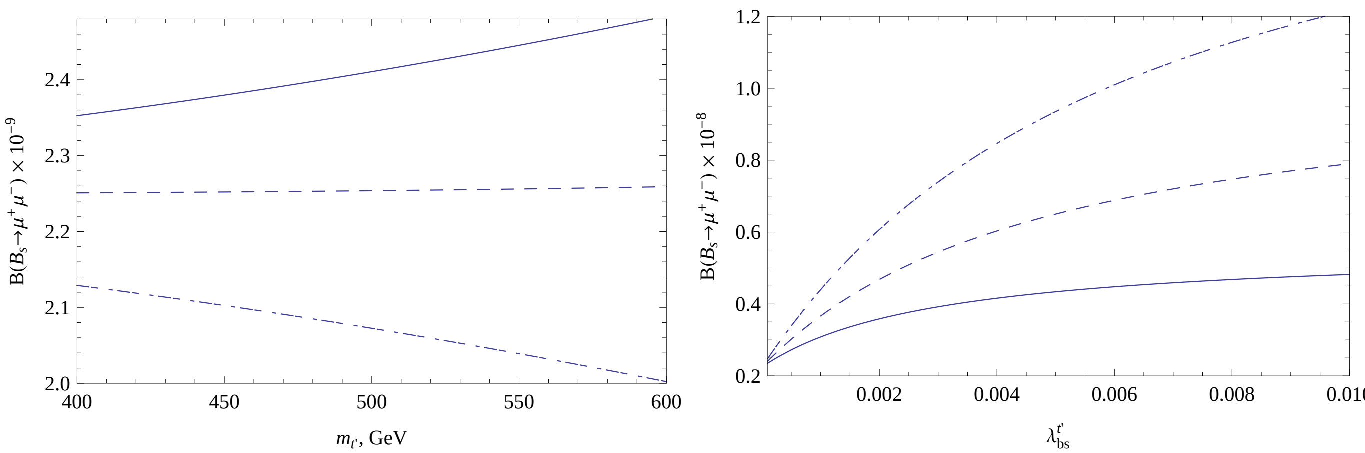

Figure 4.4: $\mathcal{B}_{B_s^0 \to \mu^+\mu^-}$ as a function $m_{t'}$ (left) and $\lambda_{bs}^{t'}$ (right).

Chapter 5 The Flavor Puzzle in Multi-Higgs Models

So far we were examining the New Physics impact on the charm and bottom flavor oscillations and decays and possibility of detecting a New Physics signal in these processes. As discussed previously, study of quark and lepton flavors within the Standard Model extension also enables one to resolve some of the particle physics puzzles that the Standard Model is unable to explain. As it was mentioned above, one of these puzzles is the existing pattern of the quark and lepton masses. The Standard Model provides a way to generate masses of quarks and leptons, however it does not explain the apparent hierarchal structure of flavor parameters such as fermion masses and mixing parameters [147]. The ratios of the quark and lepton masses are known experimentally, for the central values [148],

$$\begin{array}{ll} \dfrac{m_t}{m_c} \simeq 267 \ , & \dfrac{m_c}{m_u} \simeq 431 \ , \\ \dfrac{m_b}{m_s} \simeq \ 47.5 \ , & \dfrac{m_s}{m_d} \simeq \ 21 \ , \\ \dfrac{m_\tau}{m_\mu} \simeq \ 17 \ , & \dfrac{m_\mu}{m_e} \simeq 207 \ . \end{array} \tag{5.1}$$

Here we use the four loop $\overline{MS}$ masses evaluated at $\mu = m_t$ for the quark masses as defined in [149]. In addition, the Cabibbo-Kobayashi-Maskawa (CKM) quark matrix elements have a clear hierarchal structure, as the elements further away from the main diagonal tend to get smaller and smaller, *e.g.*, $V_{ud} \sim 1$, $V_{us} \sim 0.2$, $V_{cb} \sim 0.04$, and $V_{ub} \sim 0.004$. To add to the puzzle, the neutrino mixing matrix has a completely different structure. In comparison, gauge couplings do not exhibit such an apparent hierarchy.

All quark and lepton masses are generated in the SM via Higgs Yukawa interactions. For a single fermion field ψ interacting with a single scalar field ϕ,

$$\mathcal{L}_1 = -y_\psi \bar{\psi}_L \psi_R \phi + h.c. \to -\frac{y_\psi v}{\sqrt{2}} \left(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L \right), \tag{5.2}$$

the mass $m_\psi = y_\psi v/\sqrt{2}$ is set by the value of the Yukawa coupling, y_ψ, if the scalar vacuum expectation value (vev) $v = \langle \phi \rangle$ is fixed. This is so in the SM, where the Higgs vev $v = 246$ GeV is fixed by the electroweak measurements, leaving a strong hierarchy in the dimensionless Yukawa coupling sector for different quarks and leptons,

$$\begin{aligned} &y_u \sim 10^{-5}, \quad y_c \sim 10^{-2}, \quad y_t \sim 1, \\ &y_d \sim 10^{-5}, \quad y_s \sim 10^{-3}, \quad y_b \sim 10^{-2}, \\ &y_e \sim 10^{-6}, \quad y_\mu \sim 10^{-3}, \quad y_\tau \sim 10^{-2}. \end{aligned} \tag{5.3}$$

The reason for this hierarchy is the essence of the SM flavor problem.

One can observe that since the value of the fermion mass is given by the product of the Higgs vev and the Yukawa coupling, the problem of the strong hierarchy of Yukawa couplings can be made less prominent in models with several scalar fields. For example, a hierarchy of masses of two fermions, ψ and χ, can be arranged by tuning both the ratio of vev's of the scalar fields and Yukawas. Limiting the scalar sector to two scalar fields, this can be done in several ways. For example, each scalar can interact only with one fermion at a time,

$$\mathcal{L}_2 = -y_\psi \bar{\psi}_L \psi_R \phi_1 - y_\chi \bar{\chi}_L \chi_R \phi_2 + \text{h.c.} \tag{5.4}$$

In this case, $m_\psi = y_\psi v_1/\sqrt{2}$ and $m_\chi = y_\chi v_2/\sqrt{2}$, where $\langle \phi_1 \rangle = v_1$ and $\langle \phi_2 \rangle = v_2$. Here the mass hierarchy

$$\frac{m_\chi}{m_\psi} = \frac{y_\chi}{y_\psi}\frac{v_2}{v_1} = \frac{y_\chi}{y_\psi}\tan\beta \gg 1 \ , \tag{5.5}$$

can be arranged if either $y_\chi/y_\psi \gg 1$ or $\tan\beta \equiv v_2/v_1 \gg 1$ or both. Alternatively, one scalar can couple to both fermions, while the other to only one,

$$\mathcal{L}_2' = -y_\psi \bar{\psi}_L \psi_R \phi_1 - y_\chi \bar{\chi}_L \chi_R \phi_1 - y_\chi' \bar{\chi}_L \chi_R \phi_2 + \text{h.c.}, \tag{5.6}$$

in which case the fermion masses are given by

$$\begin{aligned} m_\psi &= y_\psi v_1/\sqrt{2}, \quad m_\chi = y_\chi v_1/\sqrt{2}\left(1 + \frac{y_\chi'}{y_\chi}\tan\beta\right), \text{ and} \\ \frac{m_\chi}{m_\psi} &= \frac{y_\chi}{y_\psi}\left(1 + \frac{y_\chi'}{y_\chi}\tan\beta\right). \end{aligned} \tag{5.7}$$

Clearly, both (5.5) and (5.7) can ameliorate the fermion mass hierarchy problem by tuning additional parameters, such as $\tan\beta$. Models along the lines of (5.4) and (5.6) have been considered in [150, 151]. However, the situation is somewhat more complicated than what one would naively expect from this simplified picture. In general, these models are actually the same up to field redefinitions to a model with a single Higgs field getting a vacuum expectation value (vev) [152, 153]. Therefore, if one wishes to build a model with the flavor structure leading to (5.5) or (5.7), one must supplement the above Lagrangians with additional conditions that fix which combination of Higgs fields generate a vacuum expectation value (vev). Only after this additional constraint is specified do parameters such as $\tan\beta$ take on a physical meaning. In models such as the minimal supersymmetric standard model (MSSM) [11] supersymmetry is sufficient to fix a basis for the Higgs fields; in general, however, this is an added requirement. In this work, we find suitable conditions by imposing constraints on the Yukawa matrices. This fixes a special "Higgs basis" [154, 155] which can be used to define $\tan\beta$.

Another complication of the SM over the above models comes from the flavor structure: while the couplings of Higgs fields to fermions are defined in the gauge basis, the mass parameters are measured in the mass basis. In this chapter we analyze models with an extended Higgs sector that can be built to naturally generate the mass hierarchy. We find basis-independent conditions on the Yukawa matrices that ensure the hierarchy remains after rotations of fermion basis.

We consider a class of models with two Higgs doublets,

$$\Phi_i = \begin{pmatrix} \phi_i^+ \\ \phi_i^0 \end{pmatrix} \qquad i = 1, 2 \ . \tag{5.8}$$

each of which can couple to both up-type and down-type quarks and leptons. These models are sometimes referred to as Type-III two-Higgs doublet models [125, 128, 156]. The vacuum expectation values of the Higgs states can be defined as

$$\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix} \ . \tag{5.9}$$

We assume that $v_{1,2} > 0$ and real. These Higgs fields then have couplings to the SM fermions

$$-\mathcal{L}_Y = \sum_{i=1,2} \left(\bar{Q}_L [Y_u^{(i)}] u_R \tilde{\Phi}_i + \bar{Q}_L [Y_d^{(i)}] d_R \Phi_i + \bar{L}_L [Y_\ell^{(i)}] \ell_R \Phi_i \right) + \text{h.c.} \ . \tag{5.10}$$

where $\tilde{\Phi}_i = i\sigma_2 \Phi_i^\star$ and $Y_{u,d,\ell}^{(1,2)}$ are complex generally non-Hermitian Yukawa matrices.

This chapter is organized as follows. We consider two toy versions of the Standard Model with two generations in Section 5.1: first to generate the hierarchy between the first and second generation, and then the first and third generation. We then consider the realistic scenario of all three generations in Section 5.2. Some phenomenological implications are discussed in Section 5.3. Finally, we discuss our results in Section 5.4. The Higgs sector of the Type-III two-Higgs doublet model is reviewed in Appendix C.1. Finally, several formulae are collected in Appendix C.2 and C.3 for future reference. The results presented in this Chapter are published in [130].

5.1 Quark mass hierarchy: two generation case

5.1.1 $\tan\beta$ hierarchy in the 1–2 generation

We start the quark mass hierarchy analysis by considering a toy model with two quark generations:

$$\begin{pmatrix} u \\ d \end{pmatrix}, \quad \begin{pmatrix} c \\ s \end{pmatrix} .$$

In the most general case the Lagrangian mass terms in (5.10) may be written (in the weak isospin basis) as

$$(\bar{q}_{1_L}, \bar{q}_{2_L}) \left[Y^{(1)} + Y^{(2)} \tan\beta \right] \begin{pmatrix} q_{1_R} \\ q_{2_R} \end{pmatrix} v \cos\beta + \text{h.c.} \; , \tag{5.1.1}$$

where $q_1 = u, d$; $q_2 = c, s$; $\tan\beta = v_2/v_1$; and we assume throughout this chapter that $\tan\beta \gg 1$. $Y^{(1)}$ and $Y^{(2)}$ are 2×2 complex non-Hermitian Yukawa matrices of the quark interactions with the Higgs doublets Φ_1 and Φ_2 respectively. It is also convenient to define the total Yukawa matrix,

$$Y = Y^{(1)} + Y^{(2)} \tan\beta \; , \tag{5.1.2}$$

which is diagonalized by the rotation

$$V_L Y V_R^\dagger = \begin{pmatrix} y_1 & 0 \\ 0 & y_2 \end{pmatrix} , \tag{5.1.3}$$

with the quark masses related to the eigenvalues as

$$m_{q_{1,2}} = |y_{1,2}| v \cos\beta , \tag{5.1.4}$$

and[1] $V_{u_L} V_{d_L}^\dagger = V_{CKM}$. Our aim is to find some $U(2)$ invariant conditions on the Yukawa matrices that assure having a hierarchy in the eigenvalues y_1 and y_2 and hence in the quark masses.

For 2×2 matrices the $U(2)$ invariants are related to traces and determinants of those matrices. Rigorously speaking, only the traces and determinants of Hermitian matrices are invariant under $U(2)$ rotations: for instance, the traces and determinants of $YY^\dagger$ and $Y^\dagger Y$. Note that

$$V_L Y Y^\dagger V_L^\dagger = \begin{pmatrix} |y_1|^2 & 0 \\ 0 & |y_2|^2 \end{pmatrix} , \tag{5.1.5}$$

$$V_R Y^\dagger Y V_R^\dagger = \begin{pmatrix} |y_1|^2 & 0 \\ 0 & |y_2|^2 \end{pmatrix} . \tag{5.1.6}$$

Yet, dealing with the products $YY^\dagger$ and $Y^\dagger Y$ would make our analysis too involved. For the two generation case, it is more instructive to generate the quark mass hierarchy, studying the matrices Y, $Y^{(1)}$, $Y^{(2)}$ by themselves. We will however discuss briefly what the conditions imposed on Y, $Y^{(1)}$ and/or $Y^{(2)}$ invariants imply on $YY^\dagger$

[1] In the two generation case, this matrix is just the Cabibbo matrix, but the generalization to CKM is clear.

and its components. This is going to be useful for the realistic scenario with three quark (or lepton) generations.

As the matrices Y, $Y^{(1)}$ and $Y^{(2)}$ are non-Hermitian, one must be careful when dealing with the traces and determinants. Notice first that the traces of Y, $Y^{(1)}$ and $Y^{(2)}$ are not invariant under $U(2)$ rotations. For instance, the diagonal elements of Y in the weak isospin basis are related to that in the quark mass basis by (no sum over i)

$$Y_{ii}^{m} = V_{L_{ij}} Y_{jk} V^{\star}_{R_{ik}} \ . \tag{5.1.7}$$

So Tr $Y^m \neq$ Tr Y as $\sum_i V^{\star}_{R_{ik}} V_{L_{ij}} \neq \delta_{kj}$.

On the other hand, for the determinants we have

$$\begin{aligned} \det Y^{m} &= e^{i(\Phi_L - \Phi_R)} \det Y \ , \\ \det Y^{(1)m} &= e^{i(\Phi_L - \Phi_R)} \det Y^{(1)} \ , \\ \det Y^{(2)m} &= e^{i(\Phi_L - \Phi_R)} \det Y^{(2)} \ , \end{aligned} \tag{5.1.8}$$

where $e^{i\Phi_L} = \det V_L$ and $e^{i\Phi_R} = \det V_R$. In other words, the determinants of Y, $Y^{(1)}$ and $Y^{(2)}$ are only multiplied by some phase factor under $U(2)$ rotations. Thus the absolute values of the determinants are rotational invariants. This allows one to use Y, $Y^{(1)}$ and $Y^{(2)}$ determinants to impose some $U(2)$ rotational invariant conditions on the Yukawa matrices and generate the desired quark mass hierarchy.

Here we impose the condition[2]

$$\det Y^{(2)} = 0 \ . \tag{5.1.9}$$

Certainly, this condition is invariant under $U(2)$ rotations. By imposing this condition, one generates the hierarchy $y_2 \sim y_1 \tan\beta$. To see this, consider the eigenvalue

[2]Up to this point, $\tan\beta$ is not a physical parameter (see the discussion in Appendix C.1, but once we impose this constraint on the Yukawa matrices, this ambiguity is lost.

equation for the total matrix in Equation (5.1.2)

$$y^2 - (\mathrm{Tr}\ Y)\ y + \det Y = 0\ . \tag{5.1.10}$$

Generally speaking, Tr Y, $\det Y$ and hence y_1, y_2 are complex. Yet, in the quark mass basis one redefines quark phases so that $y_1 > 0$ and $y_2 > 0$ with both real. As q_2 corresponds to heavier quark states c and s, we will choose $y_2 > y_1$.

As

$$y_1 + y_2 = \mathrm{Tr}\ Y = \mathrm{Tr}\ [Y^{(1)} + Y^{(2)} \tan\beta] \sim O(Y^{(2)} \tan\beta)\ , \tag{5.1.11}$$

one infers that

$$y_2 \sim O(Y^{(2)} \tan\beta)\ . \tag{5.1.12}$$

On the other hand

$$\begin{aligned} y_1 y_2 = \det Y = \det Y^{(1)} + \varepsilon_{ij}\varepsilon_{kl} \left(Y^{(1)}_{ik} Y^{(2)}_{jl} + Y^{(2)}_{ik} Y^{(1)}_{jl} \right) \tan\beta \\ + \det Y^{(2)} \tan^2\beta \end{aligned} \tag{5.1.13}$$

Condition (5.1.9) on the $Y^{(2)}$ determinant assures that $O(\tan^2\beta)$ terms on the r.h.s. of (5.1.13) vanish. Thus,

$$y_1 y_2 \sim O(Y^{(1)} Y^{(2)} \tan\beta)\ . \tag{5.1.14}$$

Hence, combining (5.1.12) and (5.1.14) one gets

$$y_1 \sim O(Y^{(1)})\ , \tag{5.1.15}$$

where $O(Y^{(1)})$ denotes the order of the $Y^{(1)}$ matrix elements – during our analysis we assume that this matrix elements are of the same order (at least the diagonal ones).

Thus, as it follows from (5.1.12) and (5.1.15),

$$y_2 \sim y_1 \tan\beta \ , \tag{5.1.16}$$

provided that there is no hierarchy in the elements of the matrices $Y^{(1)}$ and $Y^{(2)}$.

The exact solutions of the eigenvalue equation (5.1.10) may be written as

$$\begin{aligned} y_{1,2} = \frac{1}{2}\Bigg\{ & Y_{11}^{(1)} + Y_{22}^{(1)} + \left(Y_{11}^{(2)} + Y_{22}^{(2)}\right)\tan\beta \\ \mp \Bigg[& \left(Y_{11}^{(1)} + Y_{22}^{(1)} + \left(Y_{11}^{(2)} + Y_{22}^{(2)}\right)\tan\beta\right)^2 - 4\left(Y_{11}^{(1)}Y_{22}^{(1)} - Y_{12}^{(1)}Y_{21}^{(1)}\right) \\ & -4\left(Y_{11}^{(1)}Y_{22}^{(2)} + Y_{11}^{(2)}Y_{22}^{(1)} - Y_{12}^{(1)}Y_{21}^{(2)} - Y_{12}^{(2)}Y_{21}^{(1)}\right)\tan\beta\Bigg]^{1/2}\Bigg\} \ . \end{aligned} \tag{5.1.17}$$

Expanding (5.1.17) in terms of $1/\tan\beta$ power series, one gets

$$y_1 \approx \frac{Y_{11}^{(1)}Y_{22}^{(2)} + Y_{11}^{(2)}Y_{22}^{(1)} - Y_{12}^{(1)}Y_{21}^{(2)} - Y_{12}^{(2)}Y_{21}^{(1)}}{Y_{11}^{(2)} + Y_{22}^{(2)}} \ , \tag{5.1.18}$$

$$\begin{aligned} y_2 \approx & \left(Y_{11}^{(2)} + Y_{22}^{(2)}\right)\tan\beta + Y_{11}^{(1)} + Y_{22}^{(1)} \\ & - \frac{Y_{11}^{(1)}Y_{22}^{(2)} + Y_{11}^{(2)}Y_{22}^{(1)} - Y_{12}^{(1)}Y_{21}^{(2)} - Y_{12}^{(2)}Y_{21}^{(1)}}{Y_{11}^{(2)} + Y_{22}^{(2)}} \ . \end{aligned} \tag{5.1.19}$$

The $O(\tan\beta)$ hierarchy in the values of y_1 and y_2 is apparent. Also, in terms of the mass ratios one gets

$$\frac{m_{q_2}}{m_{q_1}} \approx \frac{\left|Y_{11}^{(2)} + Y_{22}^{(2)}\right|^2 \tan\beta}{|Y_{11}^{(1)}Y_{22}^{(2)} + Y_{11}^{(2)}Y_{22}^{(1)} - Y_{12}^{(1)}Y_{21}^{(2)} - Y_{12}^{(2)}Y_{21}^{(1)}|} \ . \tag{5.1.20}$$

Note that $O(\tan\beta)$ hierarchy alone is insufficient to reproduce quark mass ratios for both types of quarks (as well as charged leptons). Recall that for the central values of the fermion masses one has

$$\frac{m_s(m_t)}{m_d(m_t)} \simeq 21, \quad \frac{m_c(m_t)}{m_u(m_t)} \simeq 431, \quad \frac{m_\mu}{m_e} \simeq 207 \; .$$

Choosing e.g. $\tan\beta = 20$, one can reproduce the strange to down quark mass ratio. Yet, to reproduce the other ratios, an additional reduction of the denominator in (5.1.20) is necessary, by imposing some conditions on the relevant Yukawa couplings. The simplest way to do it is to assume that $(Y_u^{(1)})_{ij} \sim 0.05\mathrm{Tr}\; Y_u^{(2)}$ and $(Y_\ell^{(1)})_{ij} \sim 0.1\mathrm{Tr}\; Y_\ell^{(2)}$. There is nothing technically unnatural in imposing such conditions, and this small tuning is drastically reduced from the usual SM Yukawas. Moreover, as it follows from our analysis, we have an expansion in terms of $\frac{Y^{(1)}}{Y^{(2)}\tan\beta}$ rather than of $1/\tan\beta$. In what follows, these assumptions on the up-quark and charged lepton Yukawa matrices do not spoil our derivations.

Thus, imposing the rotationally invariant condition (5.1.9) on the $Y^{(2)}$ determinant, one is able to reproduce the first and second generation quark and lepton mass ratios, without assuming a large family hierarchy in the couplings with the Higgs doublets.

To see what the imposed condition on the $Y^{(2)}$ determinant implies on the quark interactions with the Higgs doublets, note that in addition to the mass and weak isospin bases, two additional quark bases exist that are relevant:

- basis (a) where the matrix $Y^{(1)}$ is diagonal; this basis is related to the weak isospin basis as

$$\begin{pmatrix} q_1^{(a)} \\ q_2^{(a)} \end{pmatrix}_{L,R} = V_{L,R}^{(a)} \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}_{L,R} \quad V_L^{(a)} Y^{(1)} V_R^{(a)\dagger} \equiv Y^{(1)a} = \begin{pmatrix} y_1^{(1)} & 0 \\ 0 & y_2^{(1)} \end{pmatrix} \tag{5.1.21}$$

- basis (b) where the matrix $Y^{(2)}$ is diagonal; this basis is related to the weak isospin basis as

$$\begin{pmatrix} q_1^{(b)} \\ \\ q_2^{(b)} \end{pmatrix}_{L,R} = V_{L,R}^{(b)} \begin{pmatrix} q_1 \\ \\ q_2 \end{pmatrix}_{L,R} \qquad V_L^{(b)} Y^{(2)} V_R^{(b)\dagger} \equiv Y^{(2)b} = \begin{pmatrix} y_1^{(2)} & 0 \\ \\ 0 & y_2^{(2)} \end{pmatrix} \tag{5.1.22}$$

As the condition is imposed on $Y^{(2)}$ determinant, it is natural to consider the quark interactions with the Higgs doublets in basis (b). In that basis, condition (5.1.9) implies

$$Y^{(2)b} = \begin{pmatrix} 0 & 0 \\ \\ 0 & y_2^{(2)} \end{pmatrix} \quad \text{or} \quad Y^b = \begin{pmatrix} Y_{11}^{(1)b} & Y_{12}^{(1)b} \\ \\ Y_{21}^{(1)b} & Y_{22}^{(1)b} + y_2^{(2)} \tan\beta \end{pmatrix} . \tag{5.1.23}$$

In other words, in basis (b) the second Higgs doublet interacts with the second generation quarks only. The first generation quarks interact with each other and with the second generation quarks solely due to exchange of Φ_1. This interaction scheme is depicted below.

$$\begin{array}{ccc} \begin{pmatrix} u^{(b)} \\ \\ d^{(b)} \end{pmatrix} & & \begin{pmatrix} c^{(b)} \\ \\ s^{(b)} \end{pmatrix} \\ \uparrow & \nearrow & \uparrow \\ \Phi_1 & & \Phi_2 \end{array}$$

This scheme is very similar in spirit to "texture" models in [157, 158, 159]. The big difference between these models and ours is that they assume this structure in the gauge basis, whereas we impose the basis independent condition (5.1.9) and *derive* this scenario. However, as we see below, basis (b) is generally distinct from the gauge basis, and this will have important consequences in what follows.

It is also worth mentioning that in terms of the Yukawa matrix elements in basis (b), the formula for the quark mass ratios looks like

$$\frac{m_{q_2}}{m_{q_1}} \approx \frac{|Y_{22}^{(2)b}|\tan\beta}{|Y_{11}^{(1)b}|} = \frac{|y_2^{(2)}|\tan\beta}{|Y_{11}^{(1)b}|}\ . \tag{5.1.24}$$

A similar interaction scheme and formula for the mass ratio may be derived in basis (b) for the charged lepton families as well.

One may choose basis (b) to coincide with the weak isospin basis, by assuming that $V_{d_L}^{(b)} = V_{d_R}^{(b)} = V_{u_L}^{(b)} = V_{u_R}^{(b)} = V^{(b)}$ and redefining the isospin basis as

$$\begin{pmatrix} d \\ s \end{pmatrix} \to V^{(b)} \begin{pmatrix} d \\ s \end{pmatrix}, \qquad \begin{pmatrix} u \\ c \end{pmatrix} \to V^{(b)} \begin{pmatrix} u \\ c \end{pmatrix}.$$

However, such a scenario does not seem to be realistic. It is not hard to infer from (5.1.23) and (5.1.24) that basis (b) is transformed to the quark mass basis by means of rotation angles $\sim m_{q_1}/m_{q_2} \ll \theta_C$, where θ_C is the Cabibbo angle with $\sin\theta_C \approx 0.2259$. Thus, generating the Cabibbo mixing properly within a scenario with coinciding weak isospin basis and basis (b) is very unlikely. One should rather have the weak isospin basis distinctly different from basis (b) and with Φ_2 interacting (in the isospin basis) with both the first and second quark generations, however with the Yukawa couplings being constrained by condition (5.1.9).

On the other hand, basis (b) differs only slightly from the quark mass basis: as discussed, these two bases are related by small rotations ($\sim m_d/m_s \sim 0.05$ and $\sim m_u/m_c \sim 0.002$ for the down and up sectors respectively; also if extending our analysis to the charged lepton sector, $\sim m_e/m_\mu \sim 0.005$). Thus, the interaction scheme within basis (b) presented above in (5.1.24), is *nearly true* in the mass basis as well. Namely, one has $Y_{11}^{(2)m}, Y_{12}^{(2)m}, Y_{21}^{(2)m} \sim (m_{q_1}/m_{q_2})\, Y_{22}^{(2)m}$ and $Y_{11}^{(2)m}, Y_{12}^{(2)m}, Y_{21}^{(2)m} \sim Y_{11}^{(1)m}/\tan\beta, Y_{22}^{(1)m}/\tan\beta$, since we assumed $Y_{11}^{(1)m}, Y_{22}^{(1)m} \sim (m_{q_1}\tan\beta/m_{q_2})\, Y_{22}^{(2)m}$, as discussed above. In other words, within the quark mass basis, the interaction of Φ_2 with the first generation quarks is greatly suppressed as compared both to that of Φ_2 with the second generation quarks and to that of the other doublet, Φ_1, with both generations of quarks.

Thus, we conclude that imposing the rotationally invariant condition (5.1.9) on the $Y^{(2)}$ matrix determinant for $\tan\beta \gg 1$ gives the desired quark mass hierarchy, as well as an interaction scheme where, within the quark mass basis, the Higgs doublet Φ_2 interacts predominantly with the second generation quarks, while the other Higgs doublet Φ_1 interacts equally with both quark generations. Extending this picture for the charged lepton generations is also straightforward.

To conclude this subsection, we discuss what condition (5.1.9) implies when considering the Hermitian product $(YY^\dagger)$; we will need this when switching to the three-generation case as well as in the next subsection. Note that in addition to the constraints $\det(Y^{(2)}Y^{(2)\dagger}) = \det(Y^{(2)}Y^{(1)\dagger}) = \det(Y^{(1)}Y^{(2)\dagger}) = 0$, condition (5.1.9) also implies

$$\begin{aligned}\det\left[\left(Y^{(1)}Y^{(2)\dagger} + Y^{(2)}Y^{(1)\dagger}\right)\tan\beta + Y^{(2)}Y^{(2)\dagger}\tan^2\beta\right]& \\ = \det\left[Y^{(1)}Y^{(2)\dagger} + Y^{(2)}Y^{(1)\dagger}\right]\tan^2\beta\ ,& \qquad (5.1.25)\end{aligned}$$

which is easily proven in basis (b). The product $YY^\dagger$ may be presented as

$$YY^\dagger = Y^{(1)}Y^{(1)\dagger} + Y^{(1)}Y^{(2)\dagger}\tan\beta + Y^{(2)}Y^{(1)\dagger}\tan\beta + Y^{(2)}Y^{(2)\dagger}\tan^2\beta\ . \qquad (5.1.26)$$

Generally, for large $\tan\beta$, $\det(YY^\dagger) \sim O(\tan^4\beta)$, however as condition (5.1.25) is imposed, one gets

$$\det(YY^\dagger) \sim O(\tan^2\beta)\ . \qquad (5.1.27)$$

5.1.2 $\tan^2\beta$ hierarchy in the 1–3 generation

Having just one scheme for generating the fermion mass hierarchy is insufficient to reproduce all three quark and charged lepton masses. In order to reproduce properly the first and second and the first and third family mass ratios, at least two mechanisms for generating the mass hierarchy are needed. The first mechanism has been discussed in the previous subsection. The natural candidate for the second mechanism is the one that generates an $O(\tan^2\beta)$ hierarchy. Indeed, the quark mass ratios may be presented as:

$$A \equiv \frac{m_s(m_t)}{m_d(m_t)} \simeq 21 \ , \qquad \frac{m_b(m_t)}{m_d(m_t)} \simeq 2.26 \times A^2 \ , \tag{5.1.28}$$

$$B \equiv \frac{m_c(m_t)}{m_u(m_t)} \simeq 431 \ , \qquad \frac{m_t(m_t)}{m_u(m_t)} \simeq 0.62 \times B^2 \ . \tag{5.1.29}$$

Thus, the third to first generation mass ratios may be presented as the second to first generation mass ratios squared multiplied by some $\mathcal{O}(1)$ factors. These factors may easily be generated by appropriately choosing the values of the Yukawa matrix elements without imposing any family hierarchy on the Yukawa couplings.

In this subsection we continue to study the toy model with two quark generations, however we now look for a $U(2)$ invariant condition that generates an $O(\tan^2\beta)$ hierarchy in the total Yukawa matrix eigenvalues and hence in the quark masses. Subsequently, q_2 now denotes t or b quark states.

An $O(\tan^2\beta)$ hierarchy in the quark masses may be generated by imposing the rotationally invariant condition

$$|\det Y| = |\det Y^{(1)}| \ . \tag{5.1.30}$$

This condition assures that

$$y_1 y_2 = \det Y \sim O\left((Y^{(1)})^2\right) \ , \tag{5.1.31}$$

which, combined with $y_2 \sim O(Y^{(2)} \tan\beta)$ as shown in Equation (5.1.12), yields

$$y_1 \sim O\left(\frac{\left(Y^{(1)}\right)^2}{Y^{(2)} \tan\beta}\right) \ , \tag{5.1.32}$$

and subsequently,

$$\frac{y_2}{y_1} \sim \tan^2\beta \ . \tag{5.1.33}$$

The exact solutions of the eigenvalue equation (5.1.10) is now

$$\begin{aligned} y_{1,2} = \frac{1}{2}\Bigg\{ & Y_{11}^{(1)} + Y_{22}^{(1)} + \left(Y_{11}^{(2)} + Y_{22}^{(2)}\right)\tan\beta \\ & \mp \left[\left(Y_{11}^{(1)} + Y_{22}^{(1)} + \left(Y_{11}^{(2)} + Y_{22}^{(2)}\right)\tan\beta\right)^2 - 4\det Y\right]^{1/2}\Bigg\} \ , \end{aligned} \tag{5.1.34}$$

which, after expansion in powers of $1/\tan\beta$, may be rewritten as

$$y_1 \approx \frac{\det Y}{\left(Y_{11}^{(2)} + Y_{22}^{(2)}\right)\tan\beta} + \mathcal{O}(\tan^{-2}\beta) \ , \tag{5.1.35}$$

$$\begin{aligned} y_2 \approx & \left(Y_{11}^{(2)} + Y_{22}^{(2)}\right)\tan\beta + Y_{11}^{(1)} + Y_{22}^{(1)} \\ & - \frac{\det Y}{\left(Y_{11}^{(2)} + Y_{22}^{(2)}\right)\tan\beta} + \mathcal{O}(\tan^{-2}\beta) \ . \end{aligned} \tag{5.1.36}$$

In general, there is an ambiguity in solutions (5.1.35) and (5.1.36) because of an unknown phase in

$$\det Y = e^{i\phi} \det Y^{(1)} = e^{i\Phi}\left(Y_{11}^{(1)} Y_{22}^{(1)} - Y_{12}^{(1)} Y_{21}^{(1)}\right) \ .$$

Yet, in the mass basis where $y_1 > 0$, $y_2 > 0$ and hence $\det Y = |\det Y^{(1)}| > 0$, this ambiguity is removed. More generally, for large $\tan\beta$, the last term in the expression for y_2 may be neglected, and for y_1 this problem is avoided by considering the absolute values of the eigenvalues, as only the absolute values have physical meaning. Then

$$|y_1| \approx \frac{\left|Y_{11}^{(1)}Y_{22}^{(1)} - Y_{12}^{(1)}Y_{21}^{(1)}\right|}{\left|Y_{11}^{(2)} + Y_{22}^{(2)}\right| \tan\beta} , \tag{5.1.37}$$

$$|y_2| \approx \left|Y_{11}^{(2)} + Y_{22}^{(2)}\right| \tan\beta . \tag{5.1.38}$$

Subsequently,

$$\frac{m_{q_2}}{m_{q_1}} \approx \frac{\left|Y_{11}^{(2)} + Y_{22}^{(2)}\right|^2 \tan^2\beta}{\left|Y_{11}^{(1)}Y_{22}^{(1)} - Y_{12}^{(1)}Y_{21}^{(1)}\right|} . \tag{5.1.39}$$

Thus, imposing condition (5.1.30) on $|\det Y|$, one gets the desired $O(\tan^2\beta)$ hierarchy in the total Yukawa matrix eigenvalues and subsequently on the quark mass ratios.

To see what this condition on $|\det Y|$ implies on the quark interactions with the Higgs doublets, it is convenient to rewrite (5.1.30) in the following form:

$$\det(YY^\dagger) = \det\left(Y^{(1)}Y^{(1)\dagger}\right) . \tag{5.1.40}$$

Comparing to (5.1.26), $\tan\beta$ dependent terms in the expression for $\det(YY^\dagger)$ must vanish to satisfy condition (5.1.40). In general, this may occur in different ways. Yet, for $\tan\beta \gg 1$, the natural way to satisfy (5.1.40) is to demand for the $\tan\beta$-dependent terms to vanish to all orders in $\tan\beta$.

It has already been discussed in the previous subsection that the vanishing of $O(\tan^4\beta)$ and $O(\tan^3\beta)$ terms in $\det(YY^\dagger)$ may be assured by imposing condition (5.1.9) on $\det Y^{(2)}$. This means that we have again the interaction scheme where Φ_2 interacts with the heaviest family of quarks – exactly in basis (b) and predominantly in the mass basis.

Yet, as condition (5.1.30) or equivalently (5.1.40) on $\det Y$ is much stronger than (5.1.9), one may expect that the interaction scheme corresponding to $O(\tan^2\beta)$ quark mass hierarchy is more constrained than that discussed in the previous subsection. To see this, one may rewrite the Hermitian product $YY^\dagger$ in basis (b) in the following form (provided that $\det Y^{(2)} = 0$):

$$Y^b Y^{b\dagger} =$$
$$\begin{pmatrix} |Y_{11}^{(1)b}|^2 + |Y_{12}^{(1)b}|^2, & Y_{21}^{(1)b\star} Y_{11}^{(1)b} + Y_{22}^{(1)b\star} Y_{12}^{(1)b} + y_2^{(2)\star} Y_{12}^{(1)b} \tan\beta \\ Y_{21}^{(1)b} Y_{11}^{(1)b\star} + Y_{22}^{(1)b} Y_{12}^{(1)b\star} + y_2^{(2)} Y_{12}^{(1)b\star} \tan\beta, & |Y_{21}^{(1)b}|^2 + |Y_{22}^{(1)b}|^2 + 2Re\left[y_2^{(2)} Y_{22}^{(1)b\star}\right] \tan\beta + |y_2^{(2)}|^2 \tan^2\beta \end{pmatrix} \tag{5.1.41}$$

The conditions for $O(\tan^2\beta)$ and $O(\tan\beta)$ terms in $\det(YY^\dagger)$ to vanish in the rotational invariant form are respectively (provided that $\det Y^{(2)} = 0$)

$$\det\left(Y^{(1)}Y^{(2)\dagger} + Y^{(2)}Y^{(1)\dagger}\right)\tan^2\beta + \det\left(Y^{(1)}Y^{(1)\dagger} + Y^{(2)}Y^{(2)\dagger}\tan^2\beta\right) - \det\left(Y^{(1)}Y^{(1)\dagger}\right) = 0\ , \tag{5.1.42}$$

$$\det\left[Y^{(1)}Y^{(1)\dagger} + \left(Y^{(1)}Y^{(2)\dagger} + Y^{(2)}Y^{(1)\dagger}\right)\tan\beta\right] - \det\left(Y^{(1)}Y^{(2)\dagger} + Y^{(2)}Y^{(1)\dagger}\right)\tan^2\beta - \det\left(Y^{(1)}Y^{(1)\dagger}\right) = 0\ . \tag{5.1.43}$$

It is a matter of algebra to show that these two conditions in basis (b) become

$$Y_{11}^{(1)b} = 0\ . \tag{5.1.44}$$

In other words, the rotationally invariant condition (5.1.30) not only leads to an $O(\tan^2\beta)$ hierarchy in the quark (and charged lepton) masses, but also implies that in basis (b) the lightest generation quarks do not interact with the doublet Φ_2 and interact with the doublet Φ_1 only via transitions to the heavier generation quarks. This scheme is also nearly true in the quark mass basis, since as before, basis (b) differs from the mass basis by small rotation angles ($\sim m_d/m_b \sim 0.001$; $\sim m_u/m_t \sim 10^{-5}$; $\sim m_e/m_\mu \sim 0.0005$).

5.2 Quark mass hierarchy: three generation case

5.2.1 Conditions on Yukawa Matrices

Having the mass hierarchy generation mechanisms at hand, we may now turn to the realistic three generation model. For the three generation case, the mass terms in the Lagrangian may be written as

$$(\bar{q}_{1_L}, \bar{q}_{2_L}, \bar{q}_{3_L}) \left[Y^{(1)} + Y^{(2)} \tan\beta\right] \begin{pmatrix} q_{1_R} \\ q_{2_R} \\ q_{3_R} \end{pmatrix} v \cos\beta + \text{h.c.} \;, \tag{5.2.1}$$

where $Y^{(1)}$ and $Y^{(2)}$ are now 3×3 complex generally non-Hermitian matrices. The total Yukawa matrix is still given by (5.1.2), and

$$V_L Y V_R^\dagger = \begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{pmatrix} \;, \tag{5.2.2}$$

with the quark masses related to the eigenvalues as

$$m_{q_i} = |y_i| v \cos\beta, \quad i = 1, 2, 3 \;. \tag{5.2.3}$$

The eigenvalue equation is now

$$y^3 - (\text{Tr } Y)\, y^2 + (\det{}_2 Y)\, y - \det Y = 0 \;, \tag{5.2.4}$$

where

$$\det{}_2 Y = \sum_{i<j} (Y_{ii}Y_{jj} - Y_{ij}Y_{ji}) \ , \tag{5.2.5}$$

is the sum of all the second order diagonal minors of Y. In the mass basis, one may choose real $y_1 > 0$, $y_2 > 0$ and $y_3 > 0$, by redefining the quark phases. As $q_1 = u, d$; $q_2 = c, s$; $q_3 = t, b$; we assume $y_3 > y_2 > y_1$.

If no condition is imposed on the Yukawa matrices, one gets

$$\begin{aligned}
&y_1 + y_2 + y_3 = \mathrm{Tr}\ Y = \mathrm{Tr}\ \left(Y^{(1)} + Y^{(2)} \tan\beta\right) \sim O(Y^{(2)} \tan\beta) \ , \\
&y_1 y_2 + y_1 y_3 + y_2 y_3 = \det{}_2 Y = \det{}_2 \left(Y^{(1)} + Y^{(2)} \tan\beta\right) \sim O\left((Y^{(2)})^2 \tan^2\beta\right) \ , \\
&y_1 y_2 y_3 = \det Y = \det \left(Y^{(1)} + Y^{(2)} \tan\beta\right) \sim O\left((Y^{(2)})^3 \tan^3\beta\right) \ ,
\end{aligned}$$

and subsequently

$$y_3 \sim y_2 \sim y_1 \sim O(Y^{(2)} \tan\beta) \ .$$

Yet our aim is to find $U(3)$ invariant constraints on the matrix elements that yield

$$\det{}_2 Y = \det{}_2 \left(Y^{(1)} + Y^{(2)} \tan\beta\right) \sim O\left(Y^{(1)} Y^{(2)} \tan\beta\right) \ , \tag{5.2.6}$$

$$\det Y = \det \left(Y^{(1)} + Y^{(2)} \tan\beta\right) \sim O\left((Y^{(1)})^3\right) \ , \tag{5.2.7}$$

and thus

$$y_3 \sim O(Y^{(2)} \tan\beta) \ , \tag{5.2.8}$$

$$y_2 \sim O(Y^{(1)}) \ , \tag{5.2.9}$$

$$y_1 \sim O\left(\frac{Y^{(1)}}{Y^{(2)} \tan\beta}\right) \ . \tag{5.2.10}$$

The relevant condition on $\det Y$ is still given by (5.1.30) or, equivalently, by (5.1.40). However, there is a problem with imposing conditions on $\det_2 Y$, $\det_2 Y^{(1)}$ or

$\det_2 Y^{(2)}$: these quantities are not invariant under $U(3)$ rotations. Thus, at this point we cannot use matrices Y, $Y^{(1)}$ and $Y^{(2)}$ anymore. Rather we have to proceed to the Hermitian product $YY^\dagger$ (or $Y^\dagger Y$) and its components.

For $YY^\dagger$ we have

$$V_L Y Y^\dagger V_L^\dagger = \begin{pmatrix} |y_1|^2 & 0 & 0 \\ 0 & |y_2|^2 & 0 \\ 0 & 0 & |y_3|^2 \end{pmatrix} , \tag{5.2.11}$$

and the eigenvalue equation is now

$$|y|^6 - (\mathrm{Tr}\ \left(YY^\dagger\right))\ |y|^4 + (\det_2\left(YY^\dagger\right))\ |y|^2 - \det\left(YY^\dagger\right) = 0\ , \tag{5.2.12}$$

and thus

$$\begin{aligned} |y_1|^2 + |y_2|^2 + |y_3|^2 = \mathrm{Tr}\ \left(YY^\dagger\right) = \mathrm{Tr}\ \Big[& Y^{(1)}Y^{(1)\dagger} + \left(Y^{(1)}Y^{(2)\dagger} + Y^{(2)}Y^{(1)\dagger}\right)\tan\beta \\ & + Y^{(2)}Y^{(2)\dagger}\tan^2\beta \Big] \sim O\left(|Y^{(2)}|^2\tan^2\beta\right) , \end{aligned} \tag{5.2.13}$$

and (with the use of condition (5.1.40))

$$|y_1|^2|y_2|^2|y_3|^2 = \det\left(YY^\dagger\right) = \det\left(Y^{(1)}Y^{(1)\dagger}\right) \sim O\left(|Y^{(1)}|^6\right)\ . \tag{5.2.14}$$

Note that for 3×3 Hermitian matrices the sum of the second order diagonal minors *is* invariant under $U(3)$ rotations and therefore may be used to derive the missing condition that leads to the desired hierarchy of the eigenvalues. This condition is

$$\det_2\left(Y^{(2)}Y^{(2)\dagger}\right) = 0\ . \tag{5.2.15}$$

Apart from the fact that this condition implies $\det\left(Y^{(2)}Y^{(2)\dagger}\right) = 0$, one also gets

$$|y_1|^2|y_2|^2 + |y_1|^2|y_3|^2 + |y_2|^2|y_3|^2 = \det_2\left(YY^\dagger\right) \sim O\left(|Y^{(1)}|^2|Y^{(2)}|^2\tan^2\beta\right) \ . \quad (5.2.16)$$

As before, one can show this working in basis (b), where the matrix $Y^{(2)}$ is diagonal. With condition (5.2.15), one has

$$Y^{(2)b}Y^{(2)\dagger b} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & |y_3^{(2)}|^2 \end{pmatrix} \quad \Rightarrow \quad Y^{(2)b} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_3^{(2)} \end{pmatrix} \ . \quad (5.2.17)$$

Because of the importance for our analysis, we also present explicitly the total Yukawa matrix Y and $YY^\dagger$ in basis (b) in Appendix C.2. With the use of (5.2.17) and the formulae in the Appendix, proving that $\det_2\left(YY^\dagger\right) \sim O\left(|Y^{(1)}|^2|Y^{(2)}|^2\tan^2\beta\right)$ is straightforward.

One infers from Eqs. (5.2.13), (5.2.14), (5.2.16), for $|y_3|^2 > |y_2|^2 > |y_1|^2$,

$$|y_3|^2 \sim O\left(|Y^{(2)}|^2\tan^2\beta\right) \ , \quad (5.2.18)$$

$$|y_2|^2|y_3|^2 \sim O\left(|Y^{(1)}|^2|Y^{(2)}|^2\tan^2\beta\right) \ , \quad (5.2.19)$$

$$|y_1|^2|y_2|^2|y_3|^2 \sim O\left(|Y^{(1)}|^6\right) \ , \quad (5.2.20)$$

or

$$|y_3|^2 \sim O\left(|Y^{(2)}|^2\tan^2\beta\right) \ , \quad (5.2.21)$$

$$|y_2|^2 \sim O\left(|Y^{(1)}|^2\right) \ , \quad (5.2.22)$$

$$|y_1|^2 \sim O\left(\frac{|Y^{(1)}|^4}{|Y^{(2)}|^2\tan^2\beta}\right) \ . \quad (5.2.23)$$

This is the desired hierarchy in the values of y_1, y_2 and y_3.

Formulae (5.2.21)-(5.2.23) determine only the order of magnitude of $|y_1|$, $|y_2|$ and $|y_3|$ qualitatively. Finding the most general solution of the cubic eigenvalue equation (5.2.12) is not easy. However, if $|y_3|^2 \gg |y_2|^2 \gg |y_1|^2$, as it follows from Eqs. (5.2.21)-(5.2.23), one gets

$$|y_3|^2 \approx \mathrm{Tr}\ \left(YY^\dagger\right)\ , \tag{5.2.24}$$

$$|y_2|^2 \approx \frac{\det_2\left(YY^\dagger\right)}{\mathrm{Tr}\ \left(YY^\dagger\right)}\ , \tag{5.2.25}$$

$$|y_1|^2 \approx \frac{\det\left(YY^\dagger\right)}{\det_2\left(YY^\dagger\right)}\ , \tag{5.2.26}$$

where $\det\left(YY^\dagger\right)$ is given by (5.1.40) and, following the accuracy of the approach, one should leave only the leading-order in $\tan\beta$ terms in the expressions for Tr $\left(YY^\dagger\right)$ and $\det_2\left(YY^\dagger\right)$. The resulting formulae for the $|y_i|^2$ and the subsequent mass ratios are given in Appendix C.3.

For $\tan\beta = 20$, the down-type quark mass ratios

$$\frac{m_s(m_t)}{m_d(m_t)} \simeq 1.05\tan\beta, \qquad \frac{m_b(m_t)}{m_s(m_t)} \simeq 2.38\tan\beta, \qquad \frac{m_b(m_t)}{m_d(m_t)} \simeq 2.5\tan^2\beta, \tag{5.2.27}$$

may be reproduced by choosing the elements of matrices $Y_d^{(1)}$ and $Y_d^{(2)}$ to be of the same order while satisfying the imposed rotationally invariant conditions. Numerically, the elements of these matrices must be chosen appropriately to reproduce the finite factors in front of $\tan\beta$ and $\tan^2\beta$ in (5.2.27), however no family hierarchy in the down-quark Yukawa interactions is needed.

To reproduce the up-type quark mass ratios,

$$\frac{m_c(m_t)}{m_u(m_t)} \simeq 21.6\tan\beta, \qquad \frac{m_t(m_t)}{m_c(m_t)} \simeq 13.4\tan\beta, \qquad \frac{m_t(m_t)}{m_u(m_t)} \simeq 290\tan^2\beta, \tag{5.2.28}$$

some weak tuning must be imposed on the denominators of (C.3.4)-(C.3.6). Like in the toy models with two generations, the easiest way to do this is to assume

$$|(Y_u^{(1)})_{ij}|^2 \sim 0.01 \text{ Tr } \left(Y_u^{(2)} Y_u^{(2)\dagger}\right) \ .$$

As discussed, this condition does not spoil our derivations: in fact our expansion is in powers of $\frac{Y^{(1)}}{Y^{(2)} \tan\beta}$ rather than in powers of $1/\tan\beta$. Again, no large family hierarchy in the Yukawa interactions is needed.

Thus, imposing condition (5.1.40) on the $YY^\dagger$ determinant and condition (5.2.15) on the sum of the $Y^{(2)}Y^{(2)\dagger}$ second order diagonal minors, one is able to reproduce the actual ratios of the quark masses, without imposing a large family hierarchy on the Yukawa interactions of the quarks with the Higgs doublets.

While no family hierarchy in the quark Yukawa interactions is assumed in our model, the imposed rotational invariant conditions (5.1.40) and (5.2.15) certainly have an impact on interactions, as discussed in the previous section. As before, it is convenient to examine this impact in basis (b) where the matrix $Y^{(2)}$ is diagonal. In this basis, as it follows from Eq. (5.2.17), only the third generation quarks interact with Φ_2, as depicted in the scheme below.

$$\begin{array}{ccccc} \begin{pmatrix} u^{(b)} \\ d^{(b)} \end{pmatrix} & & \begin{pmatrix} c^{(b)} \\ s^{(b)} \end{pmatrix} & & \begin{pmatrix} t^{(b)} \\ b^{(b)} \end{pmatrix} \\ & \nwarrow & \uparrow & \nearrow & \uparrow \\ & & \Phi_1 & & \Phi_2 \end{array}$$

This interaction scheme remains nearly true in the mass basis too, as

$$q_3^{(b)} \approx q_3^{(m)} \ , \qquad (5.2.29)$$

with accuracy of $O(m_{q_2}/m_{q_3}) \sim O\left(\frac{Y^{(1)}}{Y^{(2)}\tan\beta}\right)$ terms. This stems from the fact that

$$(Y^bY^{b\dagger})_{33} \approx |y_3^{(2)}|^2\tan^2\beta \gg (Y^bY^{b\dagger})_{13}, (Y^bY^{b\dagger})_{23} \sim O\left(Y^{(1)}y_3^{(2)}\tan\beta\right)$$
$$\gg (Y^bY^{b\dagger})_{11}, (Y^bY^{b\dagger})_{12}, (Y^bY^{b\dagger})_{22} \sim O\left((Y^{(1)})^2\right) \quad (5.2.30)$$

So far the analysis has been conducted along the same lines as within the previous section for the toy two generation models. Yet, as the three generation case is more involved in general, it is natural to expect that some differences in the analysis still may occur. One of them is related to the constraints on the light quark interactions with Φ_1, due to condition (5.1.40) on det $\left(YY^\dagger\right)$. For the two-generation case condition (5.1.40) gives (5.1.44) or equivalently that the lightest generation quarks interact in basis (b) with Φ_1 only via transitions to the heaviest generation quarks; this remains nearly true in the mass basis as well. For the three generation case condition (5.1.40) places constraints on combinations of the Yukawa couplings rather than on only one of them. For instance, one gets

$$Y_{11}^{(1)b}Y_{22}^{(1)b} - Y_{12}^{(1)b}Y_{21}^{(1)b} = 0 \ . \quad (5.2.31)$$

The scenario where for example $Y_{11}^{(1)b} = Y_{12}^{(1)b} = 0$, *i.e.:* the first generation quarks in basis (b) interact with Φ_1 only via transitions to heavier generation quarks, is only one particular scenario that satisfies (5.2.31). More generally, (5.2.31) may be satisfied in any scenario with $Y_{ij}^{(1)b}$ tuned appropriately.

Most importantly, any condition expressed in terms of $Y^{(1)}$ matrix elements in basis (b) changes drastically when rotating to the mass basis. This is because unlike the toy models of the previous section, in the three-generation case *basis (b) and the mass basis are not related by small rotations* as far as the first two generation mixing angles are concerned. In other words, if neglecting the third generation mixing with two others, one has

$$q_1^{(m)} \approx q_1^{(b)} \cos\theta_{12}^{(b\to m)} + q_2^{(b)} \sin\theta_{12}^{(b\to m)} \ , \tag{5.2.32}$$

$$q_2^{(m)} \approx -q_1^{(b)} \sin\theta_{12}^{(b\to m)} + q_2^{(b)} \cos\theta_{12}^{(b\to m)} \ , \tag{5.2.33}$$

where $\theta_{12}^{(b\to m)}$ is not small in general. This stems from the fact that the elements of the 2×2 upper sub-matrix of the matrix $Y^b Y^{b\dagger}$ are in general the same order, as it follows from formula (C.2.2) in the Appendix C.2. Thus, $\theta_{12}^{(b\to m)}$ should not be small in general for the hierarchy in the values of m_{q_2} and m_{q_1} to be generated.

One may in principle have $\theta_{12}^{(b\to m)} \sim \theta_C$ if one assumes a slight hierarchy, $Y_{11}^{(1)b} \sim 0.25 Y_{22}^{(1)b}$. The advantage of allowing such a hierarchy is that unlike the two generation toy models, basis (b) may naturally coincide with the weak isospin basis; the necessary conditions for this to occur have been discussed in the previous section. In that case, the interaction scheme depicted above (5.2.29) is valid both in the mass basis and in the isospin basis.

In summary, when imposing the rotationally invariant condition (5.1.40) on the $YY^\dagger$ determinant and (5.2.15) on the sum of the $Y^{(2)}Y^{(2)\dagger}$ second order diagonal minors, in addition to reproducing the actual ratios of the quark masses, one derives a quark-to-Higgs interacting scheme where in basis (b) the Higgs doublet Φ_2 interacts only with the third generation of quarks. This scheme remains nearly true in the mass basis as well. Also, if one allows a slight hierarchy in the elements of the upper 2×2 sub-matrix of the matrix $Y^{(1)}$, one may choose basis (b) to coincide with the weak isospin basis. In that case the derived interaction scheme is the one both within the isospin basis (precisely) and within the mass basis (approximately). Notice also that the imposed rotationally invariant conditions imply some conditions on (rather complicated) combinations of the $Y^{(1)}$ matrix elements.

We complete this section by considering the charged lepton mass problem. One may proceed in the same way as for the quarks. For $\tan\beta = 20$,

$$\frac{m_\mu}{m_e} \simeq 10.4 \tan\beta, \qquad \frac{m_\tau}{m_\mu} \simeq 0.85 \tan\beta, \qquad \frac{m_\tau}{m_e} \simeq 8.8 \tan^2\beta. \tag{5.2.34}$$

The $\mathcal{O}(1)$ coefficient in front of $\tan\beta$ for the ratio $\frac{m_\tau}{m_\mu}$ indicates that the elements of the matrices $Y_\ell^{(1)}$ and $Y_\ell^{(2)}$ must be of the same order, as one can infer from Eq. (C.3.5). Yet, to reproduce the coefficient 10.4 in front of $\tan\beta$ for the ratio $\frac{m_\mu}{m_e}$, the elements of the matrix $Y_\ell^{(1)}$ must be tuned appropriately for $\det\left(Y_\ell^{(1)} Y_\ell^{(1)\dagger}\right)$ to be suppressed, as it follows from (C.3.4).

5.2.2 More on Basis (b)

Because of its crucial importance, basis (b) and its physical meaning, as well as the meaning of condition (5.2.15), deserve more detailed discussion. If one assumes for the Higgs masses $m_{A^0}, m_{H^+}, m_{H^0} \gg m_{h^0}$, so that flavor changing neutral currents (FCNCs) are suppressed, then for the CP-even Higgs rotation angles defined in Appendix C.1, one has $\alpha \approx \beta - \pi/2$. If $\tan\beta \gg 1$, (C.1.7) and (C.1.8) (ignoring Goldstone modes) may be approximated by

$$\Phi_1 \approx \begin{pmatrix} -H^+ \\ \frac{1}{\sqrt{2}}\left[v_1 + H^0 - iA^0\right] \end{pmatrix}, \tag{5.2.35}$$

$$\Phi_2 \approx \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}\left[v + h^0\right] \end{pmatrix}. \tag{5.2.36}$$

To this approximation, Φ_2 is the SM Higgs doublet, while Φ_1 is new physics (NP). Thus, basis (b) is the basis where the SM Yukawa matrix is diagonal.

In our model, the family symmetry is broken in two steps. Quark interactions with the SM Higgs doublet Φ_2 break $U(3)$ quark family symmetry down to $U(2)$. If only Φ_2 gets a vev, then only the top and bottom quarks would acquire masses, while other quarks would remain massless. Yet interactions of the NP Higgs doublet Φ_1 with quarks break the family symmetry completely and generate both the first two generation quark masses and the CKM mixing. Thus, in the scenario considered here, the up, down, strange and charm quark interactions with the Higgs particles as well as the CKM mixing are predominantly beyond the Standard Model physics. Yet, the Yukawa interactions of the first two generation quarks with the Higgs doublets are still suppressed, due to the NP Higgs masses being at TeV or even higher scales.

This interpretation of the model assumes that the weak isospin basis coincides with basis (b). On the other hand, if this model is an effective theory originating from a more fundamental theory at TeV or higher scales, then the weak isospin basis may be different from basis (b). Note that our results based on the rotationally invariant conditions are independent of how these two bases are related to each other.

There are strong reasons to believe that the two-Higgs doublet model discussed here is an effective theory that originates from a more fundamental theory that occurs at TeV or higher scales. For instance, having the NP Higgs masses at TeV or higher scales requires the mass parameters μ_1, μ_2 and μ_3 of the Higgs potential to have magnitudes of the order of TeV or higher scales as well. A possible explanation of the scale of these parameters may be the existence of a gauge singlet scalar field S, with interactions

$$\mathcal{L}_S \supset \lambda_1^S |S|^2 \left(\Phi_1 \Phi_1^\dagger\right) + \lambda_2^S |S|^2 \left(\Phi_2 \Phi_2^\dagger\right) + \left(\lambda_3^S S^2 \left(\Phi_1 \Phi_2^\dagger\right) + \text{h.c.}\right) \ , \tag{5.2.37}$$

with

$$\mu_1^2 = \lambda_1^S \langle S \rangle^2, \quad \mu_2^2 = \lambda_2^S \langle S \rangle^2, \quad \mu_3^2 = \lambda_3^S \langle S \rangle^2 \ , \tag{5.2.38}$$

and $\langle S \rangle \gg v = 246\ \text{GeV}$.

Another reason to believe there is a more fundamental theory at higher scales is that presently we are able to clearly interpret only condition (5.2.15) on the sum of the $Y^{(2)}Y^{(2)\dagger}$ second order diagonal minors through the importance of basis (b). The meaning of the other condition, (5.1.40) on the $YY^\dagger$ determinant and the resulting constraints on the $Y^{(1)}$ matrix elements remain obscure.

5.3 Phenomenological Implications: FCNC Processes and $K-\bar{K}$

Let us now consider flavor changing processes. As mentioned in Appendix C.1, in the limit that $m_A \gg v$, these are naturally suppressed, but we would like to see this explicitly. To do that, we write out the Yukawa interactions in a very suggestive way:

$$
\begin{aligned}
-\mathcal{L}_Y = {} & \bar{Q}_L[Y_u]u_R\tilde{\Phi}_1 + \bar{Q}_L[Y_u^{(2)}]u_R\tilde{\Psi} \\
& + \bar{Q}_L[Y_d]d_R\Phi_1 + \bar{Q}_L[Y_d^{(2)}]d_R\Psi + \text{h.c.}\ . \qquad (5.3.1)
\end{aligned}
$$

where $\tilde{\Phi}_1 = i\sigma_2\Phi_1^\star$, $\tilde{\Psi} = i\sigma_2\Psi^\star$ and $Y_{u,d}$ are the total Yukawa matrices for the up-type and down-type quarks, defined in (5.1.2). We have also defined the linear combination of Higgs fields

$$
\Psi = \Phi_2 - \Phi_1 \tan\beta\ , \qquad (5.3.2)
$$

and we are only considering the physical Higgs fields ((C.1.7) and (C.1.8) minus the vev's). It should be clear that this is the same as our original Yukawa interactions, but the first term in each line is proportional to the mass matrices and is therefore flavor diagonal in the mass basis by construction. Therefore all the tree level flavor-changing processes in the Higgs sector couple to the $\Psi-$combination of Higgs fields and appear in the second term on each line. Also note that all FCNCs are coming from $Y^{(2)}$, whose off diagonal elements in the mass basis are naturally small due to (5.2.15). Notice that this is consistent with the interpretation of Section 5.2.2.

With FCNC's at tree level, we can apply constraints from various flavor standard candles, such as meson mixing and electric dipole measurements [160]. Since we have already shown that we can suppress FCNCs in various regions of parameter space, we will only consider $K - \bar{K}$ mixing here (which is typically the strongest constraint), and leave the other flavor observables for future research [161].

To study $K - \bar{K}$ mixing, we consider the effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \frac{1}{m_{h^0}^2} \sum_i C_i \mathcal{O}_i \ ;$$

we will use the operator basis of [162], where they define the four-quark operators (i, j are color indices):

$$\begin{aligned}
\mathcal{O}_1 &= (\bar{d}_L^i \gamma^\mu s_L^i)(\bar{d}_L^j \gamma_\mu s_L^j) \ , & \widetilde{\mathcal{O}}_1 &= (\bar{d}_R^i \gamma^\mu s_R^i)(\bar{d}_R^j \gamma_\mu s_R^j) \ , \\
\mathcal{O}_2 &= (\bar{d}_R^i s_L^i)(\bar{d}_R^j s_L^j) \ , & \mathcal{O}_3 &= (\bar{d}_R^i s_L^j)(\bar{d}_R^j s_L^i) \ , \\
\widetilde{\mathcal{O}}_2 &= (\bar{d}_L^i s_R^i)(\bar{d}_L^j s_R^j) \ , & \widetilde{\mathcal{O}}_3 &= (\bar{d}_L^i s_R^j)(\bar{d}_L^j s_R^i) \ , \\
\mathcal{O}_4 &= (\bar{d}_R^i s_L^i)(\bar{d}_L^j s_R^j) \ , & \mathcal{O}_5 &= (\bar{d}_R^i s_L^j)(\bar{d}_L^j s_R^i) \ .
\end{aligned} \tag{5.3.3}$$

There are also dipole operators, but these are irrelevant at tree level. For $K - \bar{K}$ mixing there are three Higgs exchange diagrams at tree level that give

$$\mathcal{M}_1 = \frac{i}{2}(Y_{d21}^{(2)*})^2 \langle \Psi^{0\star}\Psi^{0\star} \rangle \langle K^0 | \mathcal{O}_2 | \bar{K}^0 \rangle \ , \tag{5.3.4}$$

$$\mathcal{M}_2 = \frac{i}{2}(Y_{d12}^{(2)})^2 \langle \Psi^{0}\Psi^{0} \rangle \langle K^0 | \widetilde{\mathcal{O}}_2 | \bar{K}^0 \rangle \ , \tag{5.3.5}$$

$$\mathcal{M}_3 = i \ (Y_{d12}^{(2)} Y_{d21}^{(2)*}) \langle \Psi^{0}\Psi^{0\star} \rangle \langle K^0 | \mathcal{O}_4 | \bar{K}^0 \rangle \ , \tag{5.3.6}$$

where the Ψ^0 propagators are for the neutral Higgs states (that is, the lower component of the doublet). It is a straightforward exercise to expand out the Higgs propagators using the mass basis defined in Appendix C.1 and this allows us to write down the tree level Higgs contributions to the matching conditions at the Higgs mass scale[3] μ_h:

$$C_2(x,y;\mu_h) = -\frac{1}{4}(Y^{(2)*}_{d21})^2 \Bigg[(\cos\alpha + \sin\alpha\tan\beta)^2 + \frac{(\sin\alpha - \cos\alpha\tan\beta)^2}{x} - \frac{\sec^2\beta}{y}\Bigg] , \tag{5.3.7}$$

$$C_4(x,y;\mu_h) = -\frac{1}{2}(Y^{(2)}_{d12}Y^{(2)*}_{d21}) \Bigg[(\cos\alpha + \sin\alpha\tan\beta)^2 + \frac{(\sin\alpha - \cos\alpha\tan\beta)^2}{x} + \frac{\sec^2\beta}{y}\Bigg] , \tag{5.3.8}$$

where $x \equiv m^2_{H^0}/m^2_{h^0}$ and $y \equiv m^2_{A^0}/m^2_{h^0}$; $\widetilde{C}_2$ is the same as C_2 with $Y^{(2)*}_{d21} \to Y^{(2)}_{d12}$ and

$$C_1(\mu_h) = \widetilde{C}_1(\mu_h) = C_3(\mu_h) = \widetilde{C}_3(\mu_h) = C_5(\mu_h) = 0 \ . \tag{5.3.9}$$

Notice that in the limit $m_{A^0} \to \infty$, the heavy Higgs contributions vanish[4]. Furthermore, in the same limit, $\alpha \simeq \beta - \pi/2$ and a little trigonometry shows that the light Higgs contribution also vanishes. Therefore, there are no contributions to $K - \bar{K}$ mixing in this limit, as expected.

Yet, in an actual scenario, the masses of the A^0, H^0 fields should be set at some reasonable scale. Also, the CP-even mixing angle α deviates somehow from the saturation limit. To get insight into model constraints from $K-\bar{K}$ mixing, we consider the simplified scenario where $m_{A^0} \gg m_{h^0}$ and $Y^{(2)}_{d12} = 0$; in this case, $\widetilde{C}_2 = C_4 = 0$. As we are close to the decoupling limit, we write $\alpha = \beta - \pi/2 + \epsilon$, where $\epsilon \ll 1$, and we may keep only the first term in (5.3.7) due to a cancelation between the H^0 and A^0 contributions. This approximation is valid up to a $\mathcal{O}(1)$ factor, and should be sufficient for our purposes. In this limit, the nonvanishing matching conditions

[3]Here we will chose $\mu_h = m_{h^0}$ and ignore the errors of order $\log\left(\frac{m_{\rm heavy}}{m_{\rm light}}\right)$, but for the sake of generality we keep μ_h arbitrary in these formulae.

[4]Recall the Heavy CP-even Higgs field mass also grows with m_{A^0} from (C.1.10).

become

$$C_2(m_{h^0}) = -\frac{1}{4}(Y_{d21}^{(2)*})^2 \left(\frac{\epsilon}{\cos\beta}\right)^2 + \mathcal{O}(\epsilon^3) \ , \tag{5.3.10}$$

To get the final answers, we must run down to the hadronic scale to resume QCD logarithms and match operator matrix elements to the expressions with bag factors, as described in [162], for instance. Using their equations (14-15), we find:

$$\begin{aligned} C_2(\mu_{\text{had}}) &= \eta_{22} C_2(m_{h^0}) \ , \\ C_3(\mu_{\text{had}}) &= \eta_{32} C_2(m_{h^0}) \ , \end{aligned} \tag{5.3.11}$$

and all others zero, where

$$\begin{aligned} \eta_{22} &= 0.983\eta^{-2.42} + 0.017\eta^{2.75} \ , \\ \eta_{32} &= -0.064\eta^{-2.42} + 0.064\eta^{2.75} \ , \end{aligned} \tag{5.3.12}$$

and

$$\eta = \left(\frac{\alpha_s(m_c)}{\alpha_s(\mu_{\text{had}})}\right)^{6/27} \cdot \left(\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right)^{6/25} \cdot \left(\frac{\alpha_s(m_{h^0})}{\alpha_s(m_b)}\right)^{6/23} \ . \tag{5.3.13}$$

We choose μ_{had} to be where $\alpha_s(\mu_{\text{had}}) = 1$ and defining nonperturbative matrix elements at this scale

$$\begin{aligned} \langle K|\mathcal{O}_2|\bar{K}\rangle\big|_{\mu_{\text{had}}} &= -\frac{5}{24}\left(\frac{m_K}{m_d + m_s}\right)^2 m_K f_K^2 B_2 \ , \\ \langle K|\mathcal{O}_3|\bar{K}\rangle\big|_{\mu_{\text{had}}} &= \frac{1}{24}\left(\frac{m_K}{m_d + m_s}\right)^2 m_K f_K^2 B_3 \ , \end{aligned} \tag{5.3.14}$$

we can put constraints on the size of $Y_{d21}^{(2)*}$ and ϵ given m_{h^0}. Here B_i are the bag factors; in what follows, we set $B_i = 1$, the "vacuum saturation approximation," which is sufficient at this level of accuracy.

For example, we can assume $m_{h^0} = 120$ GeV, as suggested by the EW fits and direct searches, and apply constraints on Δm_K

$$\Delta m_K = 2\mathrm{Re}\big(\langle K|\mathcal{L}_{\text{eff}}|\bar{K}\rangle\big) < 3.48 \times 10^{-12}\ \text{MeV}\ . \tag{5.3.15}$$

For simplicity, we let the Yukawa phases vanish[5]. To satisfy (5.3.15) we require that $|\epsilon| < 10^{-5}$ for $\mathcal{O}(1)$ or slightly smaller values of the off-diagonal Yukawas.

To understand the meaning of this constraint, one can use (C.1.14) and a bit of mathematical analysis to find

$$\epsilon \sim \sin(4\beta) m_{h^0}^2 / m_{A^0}^2\ . \tag{5.3.16}$$

For $\tan\beta = 20$ and $m_{h^0} = 120$ GeV, this means that the heavy Higgses should have masses around 10 TeV or higher. Yet, due to condition (5.2.15) $Y_{d21}^{(2)}$ is driven to be significantly less than one. Then the bound on ϵ may be about two orders of magnitude weaker ($\epsilon \lesssim 10^{-3}$), or the Heavy Higgses may have masses around 1 TeV.

Of course, these bounds should be taken with an appropriate grain of salt, since we should also include the $1/m_{A^0}^2$ terms in the matching conditions, as well as perform a more careful scan over the full parameter space. However, this simplified analysis gives us a good place to start, and a more careful analysis is reserved for future work [161].

[5]The introduction of phases would naively weaken the bounds by allowing for destructive interference, so by setting phases to zero gives us the most conservative bound.

5.4 Discussion

In this chapter we have attempted to explain the flavor hierarchy by appealing to the two Higgs doublet model. We have found that we can explain the fermion masses quite easily with little or no hierarchies in the dimensionless Yukawa couplings so long as our Yukawa matrices satisfy two flavor basis independent conditions

$$\det{}_2(Y^{(2)}Y^{(2)\dagger}) = 0 \ , \tag{5.4.1}$$

$$|\det(Y)| = |\det(Y^{(1)})| \ , \tag{5.4.2}$$

where Y is given by (5.1.2). With these conditions, the Yukawa couplings need at most a 10% tuning, as opposed to a tuning of one part in 10^6 in the usual SM. Furthermore, we have shown that although this model has tree level flavor changing neutral currents, they are all proportional to $Y^{(2)}$ matrix elements in the mass basis which are naturally small in this setup. The first condition implies that this matrix has (at least) two vanishing eigenvalues, and this motivated us to define a basis where only the 33 component of this matrix was nonzero, which we call "basis (b)." This basis may or may not be related to the gauge basis, which is relevant for deriving the CKM matrix, but the conditions we impose are basis independent and therefore will hold everywhere, including the physical mass basis.

We have taken these conditions as axioms of the flavor sector, but it is certainly within the realm of possibility [163, 164, 165, 166, 167, 168, 169, 170] that there is a dynamical explanation for this Yukawa pattern. For example, one might imagine that the Yukawa matrices are actually vev's of fields that are charged under some larger flavor symmetry which is spontaneously broken at some high scale. Then this pattern can come from minimizing some as yet unknown effective potential, and technical naturalness of the couplings will protect the pattern as we run to lower scales.

Typically the most important flavor changing standard candle is $K-\bar{K}$ mixing due to the high precision of the measurements. We considered the simple case of the near-decoupling limit in the vacuum saturation approximation, where only the light Higgs boson contributes appreciatively to the mixing parameters. We estimate that as long as the heavy Higgs states are around a TeV or higher, there are no significant contributions to this observable. Since we remain agnostic on what mechanism stabilizes the Higgs masses, we do not view this as a problem from the flavor puzzle point of view. Generalizing this to other points in Higgs parameter space is straightforward and will be considered in more detail in future [161]. In addition, it is a straightforward exercise to repeat the analysis for $D-\bar{D}$ [47, 1] and $B-\bar{B}$ [171] mixing as well. Each of these are sensitive to different $Y_{ij}^{(2)}$, and together, along with the above condition, can be used to test the full validity of this model. For the lepton sector, $\mu-e$ conversion, as well as rare μ and τ decays can also be used.

One can also imagine solving the larger Higgs fine tuning problem with some extended model such as supersymmetry. If one wishes to incorporate this model into the MSSM, we would require four Higgs doublets. Then there would be a basis analogous to our basis (b) where two of these Higgs doublets only coupled to the heavier generations, and the other pair of Higgs doublets coupled to all three generations, where each pair would have an up-type and a down-type Higgs.

Chapter 6 Conclusions

We examined possible New Physics contributions to $D^0 - \overline{D}^0$ and $B_s - \overline{B}_s$ oscillations as well as $B_s \to \mu^+\mu^-$ leptonic decay. We also considered a possibility of explaining quark and lepton mass hierarchies within a general two-Higgs doublet extension of the Standard Model.

We computed first a possible contribution from R-parity-violating SUSY models to the lifetime difference in $D^0 - \overline{D}^0$ mixing. The contribution from RPV SUSY models with leptonic number violation is found to be negative, i.e. opposite in sign to what is implied by recent experimental evidence, and possibly quite large, which implies stronger constraints on the size of relevant RPV couplings.

We discussed currently available constraints on those couplings (especially on their products), available from kaon mixing and rare kaon decay experiments. We emphasize that the use of these data in charm mixing has to be done carefully separating the constraints on RPV couplings taken in the mass and weak eigenbases, given the gauge and CKM structure of $D^0 - \overline{D}^0$ mixing amplitudes.

Diagrams with a large New Physics contribution to the lifetime difference in $D^0 - \bar{D}^0$ mixing may be present within other Standard Model extensions as well, however contribution of such diagrams is often negligible in sum. In particular this is the case within the non-manifest Left-Right Symmetric Model. It has been shown that, due to GIM cancelation effects, new physics contribution to the lifetime difference in $D^0 - \bar{D}^0$ mixing within this model is rather small, as compared to the experimental value.

We studied next an experimentally allowed New Physics contribution to the other meson-antimeson oscillation mode, $B_s - \overline{B}_s$, and the possible correlations with the New Physics contribution to $B_s \to \mu^+\mu^-$ leptonic decay.

Experiment has determined ΔM_{B_s} exceedingly well. The Standard Model determination provides a consistent value, although with a markedly greater uncertainty (due mainly to the dependence on the nonperturbative quantity $f_{B_s}^2 \hat{B}_{B_s}$ and to a lesser extent on the CKM mixing element V_{ts}). We have argued that this fact can be used to constrain NP predictions for other processes, such as the $B_s \to \mu^+\mu^-$ transition considered here.

We expect this kind of correlation to be a rather general feature of New Physics models, provided there is an overlap between the NP parameters which describe ΔM_{B_s} and (for our purposes here) $B_s \to \mu^+\mu^-$. However, given the abundance of New Physics scenarios, each with its particular structure, it is not reasonable to expect any *universal* correlation between B_s-mixing and $B_s \to \mu^+\mu^-$. Instead, what we have done in this work is to analyze several NP models in detail. In each case, we have first determined the set of unknown NP parameters and then, using dynamical assumptions, have been able to reduce (or entirely eliminate) the arbitrariness. Analyzing specific NP models this way has two purposes: to serve as an instructive example for further study and to see what kinds of numerical predictions these particular models yield.

Not surprisingly, the simplest model (with a single Z' boson) provides a strong correlation between ΔM_{B_s} and $B_s \to \mu^+\mu^-$ in which the latter is determined in terms of $M_{Z'}$. An even stronger prediction occurs in the particular version of the Family Symmetry model discussed earlier, where a clean determination of $B_s \to \mu^+\mu^-$ is obtained. In this instance, a set of reasonable assumptions allows for the initial presence of unknown parameters to be totally overcome. A similar, but not quite as fortunate, situation occurs for R-parity violating supersymmetry, wherein a reasonable assumption partially reduces the NP parameter set. In this case, $B_s \to \mu^+\mu^-$ can be expressed in terms of a ratio of a coupling constant and sneutrino mass $M_{\tilde{\nu}}$. The flavor-changing Higgs model turns out to be less accommodating in that no set of assumptions known to us can reduce the original set of three unknown parameters. Thus, the constraint from B_s mixing still leaves one with two unknowns (see Fig. 4.3). We also updated constraints on the models with a fourth sequential generation of quarks.

Finally, we have attempted to explain the flavor hierarchy by appealing to the two Higgs doublet model. We have found that we can explain the fermion masses quite easily with little or no hierarchies in the dimensionless Yukawa couplings so long as our Yukawa matrices satisfy some flavor basis independent conditions. With these conditions, the Yukawa couplings need at most a 10% tuning, as opposed to a tuning of one part in 10^6 in the usual SM. We have shown that although this model has tree level flavor changing neutral currents, they are all proportional to non-diagonal Yukawa matrix elements which are naturally small in this setup.

So, from our analysis one may conclude that study of the charm and bottom flavor-antiflavor oscillations may serve as a powerful tool in searching possible indirect signals for New Physics or, alternatively, in placing rigorous constraints on the considered NP models parameter space. Also, study of the quark and lepton flavors within the SM extensions may lead to understanding the origin of existing quark and lepton mass pattern and perhaps the other puzzles that particle physics pushes forward.

Of course, these topics may be explored further. Future projects in particular include study of possible NP contribution to CP-violation in $D^0-\overline{D}^0$ mixing and correlations between the NP contributions to $D^0-\overline{D}^0$ mass difference and CP-violation observables.

Also, as discussed in Sect. 4.2, it would be of interest to address the impact of NP CP-violating contributions to B_s mixing. Indeed, we plan do so in a future project, but first await more accurate data on $\Delta\Gamma_s$ or studies of $B_s \to J/\psi\phi$ transition at LHCb.

When exploring correlations between B_s mixing and $B_s \to \mu^+\mu^-$, additional NP models are available for study, *e.g.* R-parity conserving supersymmetry [172], and work proceeds on these.

$D^0 - \overline{D}^0$ and $B_q - \overline{B}_q$ oscillations may also be used to test the model that we proposed to explain the quark and charged lepton masses hierarchy.

As, mentioned in Sect. 5.4, one could also try to construct such a model within supersymmetric scenarios. It would be interesting to see what analogous constraints we would have to put on the corresponding Yukawa matrix elements in such a model.

Another interesting task would be to test how our model works for the neutrino sector, provided that neutrino masses or their ratios (rather than mass differences) are known, and all the neutrino mass terms (beyond the Yukawa sector) are specified.

Finally, there are other phenomenological questions we can ask in this model of the Higgs sector. For example, the important decay $h \to \gamma\gamma$ is typically dominated by top and W/Z particles in a loop. But with the possibility of changing the Yukawa couplings, this can have strong effects on this decay and possibly change the expectations for discovery at LHC.

These questions lay an important framework for future analysis of problems discussed in this thesis.

APPENDIX A

Bounds on the RPV Couplings

A.1 Bounds on the RPV coupling pair products from Δm_{K^0}

R-parity breaking part of SUSY contributes to $K^0 - \bar{K}^0$ mixing by the tree-level diagram with a sneutrino exchange, by the so-called L2 type of box diagrams with $W^{\pm}$ boson and a charged slepton exchange and by the so-called L4 type of box diagrams with all four vertices being new physics generated vertices [71]. Bounds on the RPV coupling products are derived assuming that only a given pair product or a given sum of pair products is non-zero.

Here we list the bounds, derived in [71], that are relevant for our analysis. We consider only the case when the pair products are real. We specify which of constraints are for $\lambda' \times \lambda'$ products and which of them are for $\tilde{\lambda}' \times \tilde{\lambda}'$:

$$|\lambda_{ds}| \equiv \Big|\sum_i \widetilde{\lambda}'^*_{i11}\widetilde{\lambda}'_{i22}\Big| \leq 1.7\cdot 10^{-6}\left(\frac{m_{\tilde{\ell}}}{100GeV}\right)^2 \tag{A.1.1}$$

$$\Big|\sum_i \widetilde{\lambda}'^*_{i32}\widetilde{\lambda}'_{i11}\Big| \leq 2.2\cdot 10^{-6}\left(\frac{m_{\tilde{\ell}}}{100GeV}\right)^2 \tag{A.1.2}$$

$$\Big|\sum_i \widetilde{\lambda}'^*_{i32}\widetilde{\lambda}'_{i21}\Big| \leq 5.1\cdot 10^{-7}\left(\frac{m_{\tilde{\ell}}}{100GeV}\right)^2 \tag{A.1.3}$$

$$\Big|\sum_i \widetilde{\lambda}'^*_{i12}\widetilde{\lambda}'_{i31}\Big| \leq 7.5\cdot 10^{-6}\left(\frac{m_{\tilde{\ell}}}{100GeV}\right)^2 \tag{A.1.4}$$

$$\Big|\sum_i \widetilde{\lambda}'^*_{i22}\widetilde{\lambda}'_{i31}\Big| \leq 3.3\cdot 10^{-5}\left(\frac{m_{\tilde{\ell}}}{100GeV}\right)^2 \tag{A.1.5}$$

$$\Big|\sum_i \lambda'^*_{i12}\lambda'_{i21}\Big| \leq 9.8\cdot 10^{-8}\left(\frac{m_{\tilde{\ell}}}{100GeV}\right)^2 \tag{A.1.6}$$

$$\Big|\sum_{i,k} \lambda'^*_{i1k}\lambda'_{i2k}\Big| \leq 2.7\cdot 10^{-3}\ for\ m_{\tilde{\ell}} = 100GeV,\ m_{\tilde{q}} = 300GeV \tag{A.1.7}$$

If one assumes that the RPV coupling products are non-zero only for a given i and a given k, one may apply them to each term in the above sums.

Bounds (A.1.1) - (A.1.5) are derived from charged slepton mediated L2 diagrams and (A.1.6) is derived from a tree level sneutrino mediated diagram. Naturally these bounds scale with the slepton mass squared. Contrary to this, to derive (A.1.7), both sneutrino mediated and squark mediated L4 diagrams are used. Thus, it is not easy to scale this bound. However for $m_{\tilde{\ell}} = 100GeV$ and $m_{\tilde{q}} = 300GeV$, the squark mediated diagrams contribution is about 10% of that of the slepton mediated ones [71]. In what follows, (A.1.7) is also approximately valid if $m_{\tilde{q}} \gg m_{\tilde{\ell}}$. Then this bound may be scaled with the slepton mass squared as well. Assuming that $\lambda'^*_{i1k}\lambda'_{i2k} \neq 0$ only for a given value of k, one gets

$$\Big|\sum_i \lambda'^*_{i1k}\lambda'_{i2k}\Big| \leq 2.7\cdot 10^{-3}\left(\frac{m_{\tilde{\ell}}}{100GeV}\right)^2 \tag{A.1.8}$$

We do not use bounds of [71] for $ij2\times ij1$ combination products. Using our "rule of thumb" one can see that these are bounds on some admixture of $\lambda'^*_{ij2}\lambda'_{ij1}$ and $\widetilde{\lambda}'^*_{ij2}\widetilde{\lambda}'_{ij1}$. We use instead earlier bounds of ref. [72]. These bounds are derived using L2 diagrams only, neglecting L4 ones. These diagrams vertices contain $\tilde{\lambda}'$ couplings, but not λ'. Thus one has

$$\Big|\sum_i \widetilde{\lambda}'^*_{i12}\widetilde{\lambda}'_{i11}\Big| \leq 1.4 \cdot 10^{-6}\left(\frac{m_{\tilde{\ell}}}{100GeV}\right)^2 \tag{A.1.9}$$

$$\Big|\sum_i \widetilde{\lambda}'^*_{i22}\widetilde{\lambda}'_{i21}\Big| \leq 1.4 \cdot 10^{-6}\left(\frac{m_{\tilde{\ell}}}{100GeV}\right)^2 \tag{A.1.10}$$

$$\Big|\sum_i \widetilde{\lambda}'^*_{i32}\widetilde{\lambda}'_{i31}\Big| \leq 7.7 \cdot 10^{-4}\left(\frac{m_{\tilde{\ell}}}{100GeV}\right)^2 \tag{A.1.11}$$

A.2 Bounds on λ_{ee}, $\lambda_{\mu\mu}$, $\lambda_{e\mu}$, $\lambda_{\mu e}$

We may present λ_{ee}, $\lambda_{\mu\mu}$, $\lambda_{\mu e}$, $\lambda_{e\mu}$ in a following form:

$$\lambda_{ee} \equiv \sum_k \widetilde{\lambda}'^*_{11k}\widetilde{\lambda}'_{12k} = \sum_k \lambda'^*_{11k}\lambda'_{12k} + \lambda\left[\sum_k |\lambda'_{12k}|^2 - \sum_k |\lambda'_{11k}|^2\right] + O(\lambda^2) \tag{A.2.1}$$

$$\lambda_{\mu\mu} \equiv \sum_k \widetilde{\lambda}'^*_{21k}\widetilde{\lambda}'_{22k} = \sum_k \lambda'^*_{21k}\lambda'_{22k} + \lambda\left[\sum_k |\lambda'_{22k}|^2 - \sum_k |\lambda'_{21k}|^2\right] + O(\lambda^2) \tag{A.2.2}$$

$$\lambda_{\mu e} \equiv \sum_k \widetilde{\lambda}'^*_{11k}\widetilde{\lambda}'_{22k} = \sum_k \lambda'^*_{11k}\lambda'_{22k} + \lambda\left[\sum_k \lambda'^*_{12k}\lambda'_{22k} - \sum_k \lambda'^*_{11k}\lambda'_{21k}\right] + O(\lambda^2) \tag{A.2.3}$$

$$\lambda_{e\mu} \equiv \sum_k \widetilde{\lambda}'^*_{21k}\widetilde{\lambda}'_{12k} = \sum_k \lambda'^*_{21k}\lambda'_{12k} + \lambda\left[\sum_k \lambda'^*_{22k}\lambda'_{12k} - \sum_k \lambda'^*_{21k}\lambda'_{11k}\right] + O(\lambda^2) \tag{A.2.4}$$

The Cabibbo favored terms in (A.2.1)-(A.2.4) have severe constraints e.g. from study of $K^+ \to \pi^+\nu\bar{\nu}$ decay [74]:

$$\sum_k \lambda'^*_{i1k}\lambda'_{i'2k} \leq 4.75 \times 10^{-5}\left(\frac{m_{\tilde{q}}}{300GeV}\right)^2 \tag{A.2.5}$$

for $i \neq i'$, and

$$\sum_k \lambda'^*_{i1k}\lambda'_{i2k} \leq 6.3 \times 10^{-5}\left(\frac{m_{\tilde{q}}}{300GeV}\right)^2 \tag{A.2.6}$$

For $i = i'$, bounds are about 30% weaker because of the impact of the SM and pure MSSM contributions [74].

It turns out that because of the stringent bounds on the Cabibbo favored terms, r.h.s. of (A.2.1)-(A.2.4) are dominated by the first order Cabibbo suppressed terms.

The analysis for λ_{ee} and $\lambda_{\mu\mu}$ is very similar to that for λ_{ss} and λ_{dd}. Assuming that one of the couplings λ_{12k} or λ_{11k} dominates (say for k=3), one gets

$$-0.91 \cdot 10^{-3} \left(\frac{m_{\tilde{q}}}{300GeV}\right)^2 \leq \lambda_{ee} \leq 3.83 \cdot 10^{-3} \left(\frac{m_{\tilde{q}}}{300GeV}\right)^2 \tag{A.2.7}$$

In analogous way, assuming that one of the couplings λ_{22k} or λ_{21k} dominates, one gets

$$\begin{aligned} &-0.0072 \left(\frac{m_{\tilde{q}}}{300GeV}\right)^2 \leq \lambda_{\mu\mu} \leq 0.091 \left(\frac{m_{\tilde{q}}}{300GeV}\right)^2, \quad if \quad m_{\tilde{q}} \leq 530GeV, \\ &-0.0072 \left(\frac{m_{\tilde{q}}}{300GeV}\right)^2 \leq \lambda_{\mu\mu} \leq 0.29, \quad if \quad m_{\tilde{q}} \geq 530GeV \end{aligned} \tag{A.2.8}$$

The upper bound in the second line of (A.2.8) comes from the perturbativity bound on λ'_{22k} for k=2,3 [69]: $\lambda'_{22k} \leq 1.12$. We indicate the perturbativity bound saturation if only it occurs for $m_{\tilde{q}} \leq 1TeV$.

The analysis for $\lambda_{\mu e}$ and $\lambda_{e\mu}$ is more subtle: instead of individual couplings squared in absolute value, the first order Cabibbo suppressed terms contain RPV coupling pair products now. On our knowledge, there is no bounds on pair products[1] $\lambda'_{12k}\lambda'^*_{22k}$ and $\lambda'_{11k}\lambda'^*_{21k}$. Thus, we must use individual bounds on these four couplings. As we deal with a pair product, we may not anymore assume that only one RPV coupling dominates. We must now allow for two RPV couplings to be at their boundaries at a time. There is however one subtlety: one may do this, if only there is no correlations between the constraints on λ'_{22k} and λ'_{12k} or between those on λ'_{21k} and λ'_{11k}.

One can check that constraints on λ'_{22k} and λ'_{12k} are indeed independent of each

[1]One can meet some bounds in the literature on $\lambda'_{1mk}\lambda'^*_{2mk}$ from study $\mu \to e\gamma$ decay (see [86] and references therein). However, using our "rule of thumb", it is easy to see that these are bounds on $\widetilde{\lambda}'_{12k}\widetilde{\lambda}'^*_{22k}$, thus they may not be used here.

other and constraints on λ'_{11k} are independent of the values of λ'_{21k}. The sources of these constraints and references to the relevant literature are given in [69]. At first glance, the situation with λ'_{21k} seems to be more complicated: bounds on λ'_{21k} are derived from $R_\pi \equiv \Gamma(\pi \to e\nu)/\Gamma(\pi \to \mu\nu)$, assuming that [76]

$$|\lambda'_{11k}|^2 \ll |\lambda'_{21k}|^2 \tag{A.2.9}$$

On the other hand, one can see from Table I in ref. [69] that

$$\max\left[|\lambda'_{11k}|^2\right] \leq 0.13 \max\left[|\lambda'_{21k}|^2\right] \tag{A.2.10}$$

Thus, condition (A.2.9) is satisfied to a good extent, when λ'_{11k} and λ'_{21k} are at their boundaries.

In what follows, one may use individual bounds on couplings λ'_{11k}, λ'_{21k}, λ'_{12k}, λ'_{22k} presented in ref. [69], to get constraints on the pair products $\lambda'^*_{11k}\lambda'_{21k}$ and $\lambda'^*_{12k}\lambda'_{22k}$. Using these constraints and assuming that only one of these pairs is non-zero (dominant) and only for a given k (say k=3), one gets

$$\begin{array}{lll} |\lambda_{\mu e}| \leq 0.019 \left(\frac{m_{\tilde{q}}}{300GeV}\right)^2, & |\lambda_{e\mu}| \leq 0.019 \left(\frac{m_{\tilde{q}}}{300GeV}\right)^2, & if \quad m_{\tilde{q}} \leq 530GeV \\ |\lambda_{\mu e}| \leq 0.033 \left(\frac{m_{\tilde{q}}}{300GeV}\right), & |\lambda_{e\mu}| \leq 0.033 \left(\frac{m_{\tilde{q}}}{300GeV}\right), & if \quad m_{\tilde{q}} \geq 530GeV \end{array} \tag{A.2.11}$$

In deriving (A.2.11), one must take into account that products $\lambda'^*_{11k}\lambda'_{21k}$ and $\lambda'^*_{12k}\lambda'_{22k}$ may be both positive and negative.

Coincidence of bounds on $\lambda_{\mu e}$ and $\lambda_{e\mu}$ is not accidental: the first order Cabibbo suppressed terms in equations (A.2.3) and (A.2.4) are complex conjugates of each other. Thus, $\lambda_{\mu e} \approx \lambda^*_{e\mu}$ or because we assume that RPV coupling products relevant for our analysis are real, one has

$$\lambda_{\mu e} \approx \lambda_{e\mu} \tag{A.2.12}$$

When deriving (A.2.11) and (A.2.12), we neglected $O(\lambda^2)$ Cabibbo suppressed terms in the expressions for $\lambda_{e\mu}$ and $\lambda_{\mu e}$. If one assumes that two RPV couplings dominate at a time, one should take into account these terms as well. We leave for the reader to verify that $O(\lambda^2)$ terms in the expressions for $\lambda_{e\mu}$ and $\lambda_{\mu e}$ have at least several times stronger bounds than the first order Cabibbo suppressed terms.

APPENDIX B

B_s Mixing Matrix Elements

There are eight $\Delta b = 2$ effective operators that can contribute to B_s-mixing. The operator basis we shall employ is

$$
\begin{aligned}
Q_1 &= (\bar{b}_L \gamma_\mu s_L)\,(\bar{b}_L \gamma^\mu s_L)\,, & Q_5 &= (\bar{b}_R \sigma_{\mu\nu} s_L)\,(\bar{b}_R \sigma^{\mu\nu} s_L)\,,\\
Q_2 &= (\bar{b}_L \gamma_\mu s_L)\,(\bar{b}_R \gamma^\mu s_R)\,, & Q_6 &= (\bar{b}_R \gamma_\mu s_R)\,(\bar{b}_R \gamma^\mu s_R)\,,\\
Q_3 &= (\bar{b}_L s_R)\,(\bar{b}_R s_L)\,, & Q_7 &= (\bar{b}_L s_R)\,(\bar{b}_L s_R)\,,\\
Q_4 &= (\bar{b}_R s_L)\,(\bar{b}_R s_L)\,, & Q_8 &= (\bar{b}_L \sigma_{\mu\nu} s_R)\,(\bar{b}_L \sigma^{\mu\nu} s_R)\,,
\end{aligned}
\tag{B.1.1}
$$

where quantities enclosed in parentheses are color singlets, *e.g.* $(\bar{b}_L \gamma_\mu s_L) \equiv \bar{b}_{L,i} \gamma_\mu s_{L,i}$. These operators are generated at a scale M where the NP is integrated out. A non-trivial operator mixing then occurs via renormalization group running of these operators between the heavy scale M and the light scale μ at which hadronic matrix elements are computed.

We need to evaluate the B_s^0-to-$\overline{B}_s^0$ matrix elements of these eight dimension-six basis operators. This introduces eight *non-perturbative* B-parameters $\{B_i\}$ that require evaluation by means of QCD sum rules or QCD-lattice simulation. We express these in the form

$$\begin{aligned}
\langle Q_1\rangle &= \tfrac{2}{3} f^2_{\mathrm{B}_s} M^2_{\mathrm{B}_s} B_1 \ , & \langle Q_5\rangle &= f^2_{\mathrm{B}_s} M^2_{\mathrm{B}_s} B_5 \ , \\
\langle Q_2\rangle &= -\tfrac{5}{6} f^2_{\mathrm{B}_s} M^2_{\mathrm{B}_s} B_2 \ , & \langle Q_6\rangle &= \tfrac{2}{3} f^2_{\mathrm{B}_s} M^2_{\mathrm{B}_s} B_6 \ , \\
\langle Q_3\rangle &= \tfrac{7}{12} f^2_{\mathrm{B}_s} M^2_{\mathrm{B}_s} B_3 \ , & \langle Q_7\rangle &= -\tfrac{5}{12} f^2_{\mathrm{B}_s} M^2_{\mathrm{B}_s} B_7 \ , \\
\langle Q_4\rangle &= -\tfrac{5}{12} f^2_{\mathrm{B}_s} M^2_{\mathrm{B}_s} B_4 \ , & \langle Q_8\rangle &= f^2_{\mathrm{B}_s} M^2_{\mathrm{B}_s} B_8 \ ,
\end{aligned} \tag{B.1.2}$$

where f_{B_s} is the B_s meson decay constant and $\langle Q_i\rangle \equiv \langle \bar{B}^0_s|Q_i|B^0_s\rangle$.

Ref. [123] has performed a QCD-lattice determination (quenched approximation) of the B-parameters in an operator basis $\{O_i\}$ which is distinct from the $\{Q_i\}$ of Eq. (B.1.1),

$$\begin{aligned}
O_1 &= \overline{b}^i \gamma_\mu (1+\gamma_5) s^i \ \overline{b}^j \gamma^\mu (1+\gamma_5) s^j \ , & & \\
O_2 &= \overline{b}^i (1+\gamma_5) s^i \ \overline{b}^j (1+\gamma_5) s^j \ , & O_4 &= \overline{b}^i (1+\gamma_5) s^i \ \overline{b}^j (1-\gamma_5) s^j \ , \\
O_3 &= \overline{b}^i (1+\gamma_5) s^j \ \overline{b}^j (1+\gamma_5) s^i \ , & O_5 &= \overline{b}^i (1+\gamma_5) s^j \ \overline{b}^j (1-\gamma_5) s^i \ .
\end{aligned} \tag{B.1.3}$$

Three more operators O_i $(i = 6, 7, 8)$ can be obtained by substituting right-handed chiral projection operators with the left-handed ones O_i $(i = 1, 2, 3)$ in Eq. (B.1.3). The B^0_s-to-$\overline{B}^0_s$ matrix elements of these operators have been parameterized in Ref. [123] as

$$\begin{aligned}
\langle O_1\rangle &= \tfrac{8}{3} f^2_{\mathrm{B}_s} M^2_{\mathrm{B}_s} \widetilde{B}_1 \ , & & \\
\langle O_2\rangle &= -\tfrac{5}{3} R_s^2 f^2_{\mathrm{B}_s} M^2_{\mathrm{B}_s} \widetilde{B}_2 \ , & \langle O_4\rangle &= 2 R_s^2 f^2_{\mathrm{B}_s} M^2_{\mathrm{B}_s} \widetilde{B}_4 \ , \\
\langle O_3\rangle &= \tfrac{1}{3} R_s^2 f^2_{\mathrm{B}_s} M^2_{\mathrm{B}_s} \widetilde{B}_3 \ , & \langle O_5\rangle &= \tfrac{2}{3} R_s^2 f^2_{\mathrm{B}_s} M^2_{\mathrm{B}_s} \widetilde{B}_5 \ .
\end{aligned} \tag{B.1.4}$$

Also, the chiral structure of QCD requires that $\langle O_6\rangle = \langle O_1\rangle$, $\langle O_7\rangle = \langle O_2\rangle$, and $\langle O_8\rangle = \langle O_3\rangle$.

Several of the quantities introduced above are scale dependent, *i.e.* $\{B_i(\mu)\}$, $\{\widetilde{B}_i(\mu)\}$ and $R_s^2(\mu)$. Throughout this paper, we shall understand all these quantities to be renormalized at a common scale $\mu = m_b$ and to simplify notation, we shall denote them simply as $\{B_i\}$, $\{\widetilde{B}_i\}$ and R_s^2. In particular, our evaluation at scale $\mu = m_b$ of the quantity $R_s(\mu) \equiv M_{B_s}/(m_b(\mu) + m_s(\mu))$ yields

$$R_s^2 = M_{B_s}^2 / \left(\bar{m}_b(\bar{m}_b) + \bar{m}_s(\bar{m}_b)\right)^2 = 1.57^{+0.04}_{-0.10} \quad , \tag{B.1.5}$$

where we have used the input values $\bar{m}_b(\bar{m}_b) = 4.2^{+0.17}_{-0.07}$ GeV [24] and $\bar{m}_s(\bar{m}_b) = 0.085 \pm 0.017$ GeV [45].

The two bases $\{Q_i\}$ and $\{O_i\}$ can be related via Fierz rearrangement,

$$\begin{aligned} O_1 &= 4\ Q_1 \ , & O_4 &= 4\ Q_3 \ , \\ O_2 &= 4\ Q_4 \ , & O_5 &= -2\ Q_2 \ . \\ O_3 &= -2\ Q_4 - \tfrac{1}{2}\ Q_5 \ , \end{aligned} \tag{B.1.6}$$

from which we find

$$\begin{aligned} B_1 &= \widetilde{B}_1 \ , & B_5 &= -\tfrac{1}{3} R_s^2 \left(2\widetilde{B}_3 - 5\widetilde{B}_2\right) \ , \\ B_2 &= \tfrac{2}{5}\widetilde{B}_5 R_s^2 \ , & B_6 &= \widetilde{B}_1 \ , \\ B_3 &= \tfrac{6}{7}\widetilde{B}_4 R_s^2 \ , & B_7 &= \tfrac{6}{7}\widetilde{B}_4 R_s^2 \ , \\ B_4 &= \widetilde{B}_2 R_s^2 \ , & B_8 &= -\tfrac{1}{3} R_s^2 \left(2\widetilde{B}_3 - 5\widetilde{B}_2\right) \ . \end{aligned} \tag{B.1.7}$$

Alternatively, the B-parameters can be estimated using the 'modified vacuum saturation' (MVS) approach, wherein all matrix elements in Eq. (B.1.2) are written in terms of (known) matrix elements of $(V-A)\times(V-A)$ and $(S-P)\times(S+P)$ matrix

List of $\{B_i\}$ (in $\{Q_i\}$ Basis)	$\{B_i\}$ from lattice QCD (from Ref. [123])	B_i in MVS (from Eq. (B.1.8))
$B_1 = B_6$	0.87	0.87
B_2	$0.70 R_s^2$	$0.87\left[\frac{3}{5} + \frac{2}{5} R_s^2\right]$
B_3	$0.99 R_s^2$	$0.87\left[\frac{1}{7} + \frac{6}{7} R_s^2\right]$
$B_4 = B_7$	$0.80 R_s^2$	$0.87 R_s^2$
$B_5 = B_8$	$0.71 R_s^2$	$0.87 R_s^2$

Table B.1: Numerical Estimates of the B-parameters.

elements B_{B} and $B_{\mathrm{B}}^{(\mathrm{S})}$,

$$\begin{aligned}
\langle Q_1 \rangle &= \frac{2}{3} f_{\mathrm{B}_s}^2 M_{\mathrm{B}_s}^2 B_{B_s} \ , & \langle Q_5 \rangle &= \frac{3}{N_c} f_{\mathrm{B}_s}^2 M_{\mathrm{B}_s}^2 B_{B_s}\ \eta \ , \\
\langle Q_2 \rangle &= f_{\mathrm{B}_s}^2 M_{\mathrm{B}_s}^2 B_{B_s} \left[-\frac{1}{2} - \frac{\eta}{N_c} \right] \ , & \langle Q_6 \rangle &= \langle Q_1 \rangle \ , \\
\langle Q_3 \rangle &= f_{\mathrm{B}_s}^2 M_{\mathrm{B}_s}^2 B_{B_s} \left[\frac{1}{4N_c} + \frac{\eta}{2} \right] \ , & \langle Q_7 \rangle &= \langle Q_4 \rangle \ , \\
\langle Q_4 \rangle &= -\frac{2N_c - 1}{4N_c} f_{\mathrm{B}_s}^2 M_{\mathrm{B}_s}^2 B_{B_s}\ \eta \ , & \langle Q_8 \rangle &= \langle Q_5 \rangle \ ,
\end{aligned} \tag{B.1.8}$$

where we take $N_c = 3$ as the number of colors and define

$$\eta \equiv \frac{B_{\mathrm{B}_s}^{(\mathrm{S})}}{B_{B_s}} \cdot \frac{M_{\mathrm{B}_s}^2}{\left(\bar{m}_b(\bar{m}_b) + \bar{m}_s(\bar{m}_b)\right)^2} \to R_s^2 \quad \text{for} \quad B_{\mathrm{B}_s}^{(\mathrm{S})} = B_{B_s} \quad . \tag{B.1.9}$$

It is instructive to compare how well the MVS approximation estimates the recent lattice results. We provide such a comparison in Table B.1.

APPENDIX C

2HDM Higgses, Basis (b) and the Mass Ratios

C.3 The Higgs sector

In this appendix we review the structure of the Higgs sector. We have two Higgs doublets:

$$\Phi_i = \begin{pmatrix} \phi_i^+ \\ \phi_i^0 \end{pmatrix} \qquad i = 1, 2\ . \tag{C.1.1}$$

We can write a generic potential for the these fields:

$$\begin{aligned}\mathcal{V} &= \mu_1^2 \Phi_1^\dagger \Phi_1 + \mu_2^2 \Phi_2^\dagger \Phi_2 + \mu_3^2 (\Phi_1^\dagger \Phi_2 + \text{h.c.}) + \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 \\ &+ \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \left(\frac{\lambda_5}{2} (\Phi_1^\dagger \Phi_2)^2 + \text{h.c.} \right) \\ &+ \Big(\lambda_6 (\Phi_1^\dagger \Phi_1)(\Phi_1^\dagger \Phi_2) + \lambda_7 (\Phi_2^\dagger \Phi_2)(\Phi_1^\dagger \Phi_2) + \text{h.c.} \Big)\ .\end{aligned} \tag{C.1.2}$$

One can easily check that the $\lambda_{6,7}$ terms introduce no essential change in the analysis [173], thus they may be neglected for simplicity. We also assume that λ_5 and μ_3^2 are real: thus there is no explicit CP-violation in the Higgs potential. Also, no spontaneous CP-violation is assumed, thus the Higgs doublet vacuum expectation values are taken to be real.

The Higgs doublet vacuum states may be presented in the following form:

$$\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \\ v_1 \end{pmatrix} , \quad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \\ v_2 \end{pmatrix} , \tag{C.1.3}$$

with $v_{1,2} > 0$ and real. The Higgs potential minimum conditions,

$$\frac{\partial \mathcal{V}}{\partial v_1} = \frac{\partial \mathcal{V}}{\partial v_2} = 0 \ , \tag{C.1.4}$$

may be written as

$$\lambda_1 v_1^2 + \tilde{\lambda} v_2^2 + 2\mu_1^2 + 2\mu_3^2 v_2/v_1 = 0 \ , \tag{C.1.5}$$

$$\lambda_2 v_2^2 + \tilde{\lambda} v_1^2 + 2\mu_2^2 + 2\mu_3^2 v_1/v_2 = 0 \ , \tag{C.1.6}$$

where $\tilde{\lambda} = \lambda_3 + \lambda_4 + \lambda_5$.

The Higgs doublet vacuum expectation values must satisfy the following condition: $v_1^2 + v_2^2 = v^2 = (246 \text{ GeV})^2$. Constraints on the coupling constants λ_i may be derived from the analysis of their renormalization group equations [173, 174]. Two of the mass parameters of the Higgs potential, say μ_1^2 and μ_2^2, may be eliminated from minimum conditions (C.1.5) and (C.1.6). The parameter μ_3^2 however remains arbitrary.

It should be mentioned at this point that in a general Type-III two-Higgs doublet model, v_1 and v_2 are not well defined [152, 153]. In fact, since $\Phi_{1,2}$ have the same quantum numbers, any linear combination of them can get a vev, and one can always perform a field redefinition that changes the value of $v_{1,2}$ while keeping the value of $v^2 = v_1^2 + v_2^2$ fixed. However, when we discuss Higgs couplings to the fermions in Sections 5.1 and 5.2, in particular conditions (5.1.9), (5.1.40) and (5.2.15), this ambiguity is removed, and so we will proceed as if these vevs have a physical meaning.

One may express Φ_1 and Φ_2 in terms of the excited Higgs states in the following from:

$$\Phi_1 = \begin{pmatrix} G^+ \cos\beta - H^+ \sin\beta \\ \frac{1}{\sqrt{2}} \left[v_1 + h_1 + i \left(G^0 \cos\beta - A^0 \sin\beta \right) \right] \end{pmatrix} , \tag{C.1.7}$$

$$\Phi_2 = \begin{pmatrix} G^+ \sin\beta + H^+ \cos\beta \\ \frac{1}{\sqrt{2}} \left[v_2 + h_2 + i \left(G^0 \sin\beta + A^0 \cos\beta \right) \right] \end{pmatrix} , \tag{C.1.8}$$

where $\tan\beta = v_2/v_1$, G^0, $G^\pm$ are the Goldstone modes, h_1, h_2 are CP-even, A^0 is CP-odd and $H^\pm$ is the charged physical Higgs states. It is straightforward to check, using minimum conditions (C.1.5) and (C.1.6), that the Higgs potential contains no terms linear in the physical Higgs fields.

Without any CP violation, the CP-even and -odd Higgs states will not mix, and can be considered separately. The mass of the CP-odd Higgs boson is given by

$$m_{A^0}^2 = \frac{-2\mu_3^2}{\sin 2\beta} - \lambda_5 v_2^2 = \mu_1^2 + \mu_2^2 + \frac{1}{2} \left(\lambda_1 \cos^2\beta + \lambda_2 \sin^2\beta + \lambda' \right) v^2 , \tag{C.1.9}$$

where $\lambda' = \lambda_3 + \lambda_4 - \lambda_5$. The CP-odd mass may be chosen to be a free parameter of the theory. Then the charged Higgs mass is given by

$$m_{H^\pm}^2 = m_{A^0}^2 - \frac{(\lambda_4 - \lambda_5) v^2}{2} . \tag{C.1.10}$$

The 2×2 mass matrix for the CP-even Higgs fields h_1 and h_2 is the following:

$$M^2 = \begin{pmatrix} \left(\lambda_1 \cos^2\beta + \lambda_5 \sin^2\beta \right) v^2 + m_{A^0}^2 \sin^2\beta & \left((\lambda_3 + \lambda_4) v^2 - m_{A^0}^2 \right) \sin\beta\cos\beta \\ \left((\lambda_3 + \lambda_4) v^2 - m_{A^0}^2 \right) \sin\beta\cos\beta & \left(\lambda_2 \sin^2\beta + \lambda_5 \cos^2\beta \right) v^2 + m_{A^0}^2 \cos^2\beta \end{pmatrix} . \tag{C.1.11}$$

The CP-even Higgs eigenstates, h^0, H^0, are related to h_1 and h_2 as

$$H^0 = h_1 \cos\alpha + h_2 \sin\alpha \ , \tag{C.1.12}$$

$$h^0 = -h_1 \sin\alpha + h_2 \cos\alpha \ , \tag{C.1.13}$$

where

$$\tan 2\alpha = \frac{2M_{12}^2}{M_{11}^2 - M_{22}^2} \ , \tag{C.1.14}$$

and

$$m_{h^0,H^0}^2 = \frac{1}{2}\left[M_{11}^2 + M_{22}^2 \mp \sqrt{(M_{11}^2 - M_{22}^2)^2 + 4(M_{12}^2)^2}\right] \ . \tag{C.1.15}$$

Writing explicitly the matrix elements in (C.1.14)-(C.1.15) would make these formulae rather complicated – due to large number of independent couplings the predictive power of the general two-Higgs doublet model is rather weak. Nevertheless, one can derive an upper bound on the lightest CP-even Higgs mass

$$m_{h^0}^2 \leq \left(\lambda_1 \cos^4\beta + \lambda_2 \sin^4\beta + 2\tilde{\lambda}\sin^2\beta\cos^2\beta\right) v^2 \ , \tag{C.1.16}$$

which is saturated as $m_{A^0}^2 \to \infty$; this state is usually identified with the "Standard Model Higgs." In the same limit, $m_{H^0}^2 \approx m_{H^\pm}^2 \approx m_{A^0}^2$, that is to say all the other Higgs particles may be arbitrarily heavy. Also, at this limit the mixing angle is given by $\alpha \approx \beta - \pi/2$.

Note that for $m_{A^0}^2 \gg v^2$, the problem of flavor changing neutral currents is avoided in a natural way. The FCNCs are suppressed when A^0 or H^0 is exchanged. One can also show that for $\alpha = \beta - \pi/2$, no FCNCs occur when quarks interact with the exchange of the lightest Higgs boson h^0. This result is intuitive, since in this limit we effectively only have one Higgs doublet as in the usual SM, and there are no FCNCs coming from the SM Higgs sector.

C.4 Y and $YY^\dagger$ in basis (b)

For the three generation case, in basis (b) the total Yukawa matrix is given by

$$Y^b = \begin{pmatrix} Y^{(1)}_{11} & Y^{(1)}_{12} & Y^{(1)}_{13} \\ \\ Y^{(1)}_{21} & Y^{(1)}_{22} & Y^{(1)}_{23} \\ \\ Y^{(1)}_{31} & Y^{(1)}_{32} & Y^{(1)}_{33} + y^{(2)}_3 \tan\beta \end{pmatrix} \tag{C.2.1}$$

The elements of the Hermittean matrix $YY^\dagger$ in the same basis are

$$\begin{aligned}
\left(Y^bY^{b\dagger}\right)_{11} &= |Y^{(1)b}_{11}|^2 + |Y^{(1)b}_{12}|^2 + |Y^{(1)b}_{13}|^2 \\
\left(Y^bY^{b\dagger}\right)_{21} &= \left(Y^bY^{b\dagger}\right)^\star_{12} = Y^{(1)b}_{21}Y^{(1)b\star}_{11} + Y^{(1)b}_{22}Y^{(1)b\star}_{12} + Y^{(1)b}_{23}Y^{(1)b\star}_{13} \\
\left(Y^bY^{b\dagger}\right)_{31} &= \left(Y^bY^{b\dagger}\right)^\star_{13} = Y^{(1)b}_{31}Y^{(1)b\star}_{11} + Y^{(1)b}_{32}Y^{(1)b\star}_{12} + \left(Y^{(1)b}_{33} + y^{(2)}_3 \tan\beta\right) Y^{(1)b\star}_{13} \\
\left(Y^bY^{b\dagger}\right)_{22} &= |Y^{(1)b}_{21}|^2 + |Y^{(1)b}_{22}|^2 + |Y^{(1)b}_{23}|^2 \qquad \text{(C.2.2)} \\
\left(Y^bY^{b\dagger}\right)_{32} &= \left(Y^bY^{b\dagger}\right)^\star_{23} = Y^{(1)b}_{31}Y^{(1)b\star}_{21} + Y^{(1)b}_{32}Y^{(1)b\star}_{22} + \left(Y^{(1)b}_{33} + y^{(2)}_3 \tan\beta\right) Y^{(1)b\star}_{23} \\
\left(Y^bY^{b\dagger}\right)_{33} &= |Y^{(1)b}_{31}|^2 + |Y^{(1)b}_{32}|^2 + |Y^{(1)b}_{33}|^2 + 2Re\left[y^{(2)}_3 Y^{(1)b\star}_{33}\right] \tan\beta + |y^{(2)}_3|^2 \tan^2\beta
\end{aligned}$$

C.5 Mass eigenvalues and ratios in terms of Isospin basis Yukawa couplings

The mass matrix eigenvalues can be written in terms of the Yukawa couplings. To leading order in $\tan\beta$ the results are:

$$|y_3|^2 \approx \mathrm{Tr}\ \left(Y^{(2)}Y^{(2)\dagger}\right) \tan^2\beta \tag{C.3.1}$$

$$|y_2|^2 \approx \frac{\det_2\left(Y^{(1)}Y^{(2)\dagger} + Y^{(2)}Y^{(1)\dagger}\right) + \sum_{i\neq j}\left[\left(Y^{(1)}Y^{(1)\dagger}\right)_{ii}\left(Y^{(2)}Y^{(2)\dagger}\right)_{jj} - \left(Y^{(1)}Y^{(1)\dagger}\right)_{ij}\left(Y^{(2)}Y^{(2)\dagger}\right)_{ji}\right]}{\mathrm{Tr}\ \left(Y^{(2)}Y^{(2)\dagger}\right)} \tag{C.3.2}$$

$$|y_1|^2 \approx \frac{\det\left(Y^{(1)}Y^{(1)\dagger}\right)\tan^{-2}\beta}{\det_2\left(Y^{(1)}Y^{(2)\dagger}+Y^{(2)}Y^{(1)\dagger}\right)+\sum_{i\neq j}\left[\left(Y^{(1)}Y^{(1)\dagger}\right)_{ii}\left(Y^{(2)}Y^{(2)\dagger}\right)_{jj}-\left(Y^{(1)}Y^{(1)\dagger}\right)_{ij}\left(Y^{(2)}Y^{(2)\dagger}\right)_{ji}\right]} \tag{C.3.3}$$

Subsequently, for the mass ratios, $m_{q_i}/m_{q_j} = |y_i|/|y_j|$, one gets

$$\frac{m_{q_2}}{m_{q_1}} \approx \frac{\tan\beta}{\sqrt{\mathrm{Tr}\ \left(Y^{(2)}Y^{(2)\dagger}\right)\ \det\left(Y^{(1)}Y^{(1)\dagger}\right)}} \times \left[\det_2\left(Y^{(1)}Y^{(2)\dagger}+Y^{(2)}Y^{(1)\dagger}\right)+\sum_{i\neq j}\left[\left(Y^{(1)}Y^{(1)\dagger}\right)_{ii}\left(Y^{(2)}Y^{(2)\dagger}\right)_{jj}-\left(Y^{(1)}Y^{(1)\dagger}\right)_{ij}\left(Y^{(2)}Y^{(2)\dagger}\right)_{ji}\right]\right] \tag{C.3.4}$$

$$\frac{m_{q_3}}{m_{q_2}} \approx \frac{\mathrm{Tr}\ \left(Y^{(2)}Y^{(2)\dagger}\right)\tan\beta}{\sqrt{\det_2\left(Y^{(1)}Y^{(2)\dagger}+Y^{(2)}Y^{(1)\dagger}\right)+\sum_{i\neq j}\left[\left(Y^{(1)}Y^{(1)\dagger}\right)_{ii}\left(Y^{(2)}Y^{(2)\dagger}\right)_{jj}-\left(Y^{(1)}Y^{(1)\dagger}\right)_{ij}\left(Y^{(2)}Y^{(2)\dagger}\right)_{ji}\right]}} \tag{C.3.5}$$

$$\frac{m_{q_3}}{m_{q_1}} \approx \sqrt{\frac{\mathrm{Tr}\ \left(Y^{(2)}Y^{(2)\dagger}\right)}{\det\left(Y^{(1)}Y^{(1)\dagger}\right)}}\tan^2\beta \times \sqrt{\det_2\left(Y^{(1)}Y^{(2)\dagger}+Y^{(2)}Y^{(1)\dagger}\right)+\sum_{i\neq j}\left[\left(Y^{(1)}Y^{(1)\dagger}\right)_{ii}\left(Y^{(2)}Y^{(2)\dagger}\right)_{jj}-\left(Y^{(1)}Y^{(1)\dagger}\right)_{ij}\left(Y^{(2)}Y^{(2)\dagger}\right)_{ji}\right]} \tag{C.3.6}$$

BIBLIOGRAPHY

[1] Oram Gedalia, Yuval Grossman, Yosef Nir, Gilad Perez, Phys. Rev. D 80, 05502 (2009).

[2] For an introduction to the SM see e.g. M. B. Voloshin, K. A. Ter-Martirosian, "The Theory of Gauge Interactions of Elementary Particles", Moscow (1984);
J. F. Donoghue, E. Golowich, B.R. Holstein "Dynamics of the Standard Model", Cambridge University Press (1994);
M. E.Peskin, D. V. Schroeder, "An Introduction to Quantum Field Theory", Addison-Wesley (1995).

[3] Q.R. Ahmad et al., [SNO Collab.], Phys. Rev. Lett. 87, 071301 (2001);
Q.R. Ahmad et al., [SNO Collab.], Phys. Rev. Lett. 89, 011301 (2002).

[4] B. Aharmim et al., Phys. Rev. C72, 055502 (2005);
B. Aharmim et al., Phys. Rev. Lett. 101, 111301 (2008).

[5] K. Eguchi et al., [KamLAND Collab.], Phys. Rev. Lett. 90, 021802 (2003);
T. Araki et al., [KamLAND Collab.], Phys. Rev. Lett. 94, 081801 (2005);
S. Abe et al., Phys. Rev. Lett. 100, 221803 (2008).

[6] Y. Fukuda et al., [Super-Kamiokande Collab.], Phys. Rev. Lett. 81, 1562 (1998).

[7] M.H. Ahn et al., [K2K Collab.], Phys. Rev. D74, 072003 (2006).

[8] G. Bertone, D. Hooper, J. Silk, Phys. Rep. 405, 279 (2005).

[9] H. P. Nilles, Phys. Rep. **110**, 1 (1984).

[10] P. Nath, R. Arnowitt, A. H. Chamseddine, "Applied N=1 Supergravity", World Scientific, Singapore (1984).

[11] S. P. Martin, in *Perspectives on Supersymmetry*, G. L. Kane (ed), arXiv:hep-ph/9709356.

[12] J. C. Pati and A. Salam, Phys. Rev. D 10, 275 (1974).

[13] R. N. Mohapatra and J. C. Pati, Phys. Rev. D 11, 566 (1975); Phys. Rev. D 11, 2558 (1975).

[14] S. Pakvasa and H. Sugawara, Phys. Lett. B **73**, 61 (1978).

[15] T. Maehara and T. Yanagida, Lett. Nuovo Cim. **19**, 424 (1977); M. A. B. Beg and A. Sirlin, Phys. Rev. Lett. **38**, 1113 (1977); C. L. Ong, Phys. Rev. D **19**, 2738 (1979); F. Wilczek and A. Zee, Phys. Rev. Lett. **42**, 421 (1979); A. Davidson, M. Koca and K. C. Wali, Phys. Rev. D **20**, 1195 (1979), Phys. Rev. Lett. **43**, 92 (1979).

[16] B. Holdom, W. S. Hou, T. Hurth *et al.*, PMC Phys. **A3**, 4 (2009) [arXiv:0904.4698 [hep-ph]].

[17] A. J. Buras, B. Duling, T. Feldmann *et al.*, JHEP **1009**, 106 (2010) [arXiv:1002.2126 [hep-ph]].

[18] W. -S. Hou, C. -Y. Ma, Phys. Rev. **D82**, 036002 (2010) [arXiv:1004.2186 [hep-ph]].

[19] K. Lane, arXiv:hep-ph/0202255.

[20] C. T. Hill, E. H. Simmons Phys. Rep. 381, 235 (2003).

[21] R. Shrock, arXiv:hep-ph/0703050.

[22] N. Arkani-Hamed, S. Dimopoulos, G. Dvali, Phys. Lett. B 429, 263 (1998).

[23] N. Arkani-Hamed, S. Dimopoulos, G. Dvali, Phys.Rev. D 59 (1999) 086004.

[24] K. Nakamura et al. (Particle Data Group), J. Phys. G 37, 075021 (2010).

[25] R. Dermisek, J. F. Gunion, B. McElrath, Phys. Rev. D 76, 051105 (2007); R. Dermisek, J. F. Gunion, Phys. Rev. D 75, 075019 (2007); R. Dermisek, J. F. Gunion, Phys. Rev. D73, 111701 (2006).

[26] J. F. Gunion, D. Hooper, B. McElrath, Phys. Rev. D73, 015011 (2006); J. Cao, H. E. Logan, J. M. Yang, Phys. Rev. D 79, 091701 (2009).

[27] C. Boehm, P. Fayet, Nucl. Phys. B 683, 219 (2004); P. Fayet, Phys. Rev. D 75, 115017 (2007); Phys.Rev. D 81, 054025 (2010).

[28] G. K. Yeghiyan, Phys. Rev. D 80, 115019 (2009).

[29] A. F. Falk, Y. Grossman, Z. Ligeti, Y. Nir and A. A. Petrov, Phys.Rev. **D 69**, 114021 (2004);
A. F. Falk, Y. Grossman, Z. Ligeti and A. A. Petrov, Phys. Rev. **D 65**, 054034 (2002).

[30] Ball and R. Fleischer, Eur. Phys. J. C48, 413 (2006).

[31] N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963).

[32] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).

[33] S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970).

[34] H. Georgi, Phys. Lett. **B 169**, 231 (1986).

[35] P. W. Higgs Phys. Lett. 12, 132 (1964); F. Englert, R. Brout, Phys. Rev. Lett. 13, 321 (1964); G. S. Guralnik, C. R. Hagen, T. W. B. Kibble, Phys. Rev. Lett. 13, 585 (1964).

[36] T. Aaltonen et al. (CDF and D0 Collaborations), Phys. Rev. Lett. 104, 061802 (2010).

[37] H. Georgi, Phys. Lett. **B 297** (1992) 353.

[38] See e.g. L. B. Okun' "Leptony i Kvarki" (Leptons and Quarks), Moscow: Nauka, (1981) [Traslated into English, Amsterdam: North-Holland, (1984)].

[39] L. Wolfenstein, Phys. Rev. Lett. 51, 1945 (1983).

[40] Z. Maki, M. Nakagawa, S. Sakata, Prog. Theor. Phys. **28** (1962) 870; B. Pontecorvo, Zh. Eksp. Theor. Fiz. **53** (1967) 1717 [Sov. Phys. JETP **26** (1968) 984].

[41] P. F. Harrison, D. H. Perkins and W. G. Scott, Phys. Lett. B **530**, 167 (2002).

[42] J. H. Christenson et al., Phys. Rev. Lett, 13, 138 (1964).

[43] E. Golowich and A. A. Petrov, Phys. Lett. **B 625**, 53 (2005).

[44] Heavy Flavor Averaging Group, http://www.slac.stanford.edu/xorg/hfag/charm/index.html

[45] A. Lenz and U. Nierste, JHEP **0706**, 072 (2007) [arXiv:hep-ph/0612167].

[46] A. Datta and D. Kumbhakar, Z. Phys. **C 27**, 515 (1985).

[47] E. Golowich, J. Hewett, S. Pakvasa and A. A. Petrov, Phys. Rev. D **76**, 095009 (2007) [arXiv:0705.3650 [hep-ph]].

[48] E. Golowich, S. Pakvasa and A. A. Petrov, Phys. Rev. Lett. **98**, 181801 (2007).

[49] B. Aubert *et al.* [The BaBaR Colaboration], Phys. Rev. **D 76**, 014018 (2007) [arXiv:hep-ex/0705.0704];

B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **98**, 211802 (2007) [arXiv:hep-ex/0703020];

B. Aubert et al. (BaBar Collab.), Phys. Rev. Lett. 103, 211801 (2009).

[50] L. M. Zhang, *et al.* [Belle Collaboration], Phys. Rev. Lett. **99**, 131803 (2007). [arXiv:hep-ex/0704.1000];

M. Staric *et al.* [Belle Collaboration], Phys. Rev. Lett. **98**, 211803 (2007). [arXiv:hep-ex/0703036];

U. Bitenc et al. (Belle Collab.), Phys. Rev. D 72, 071101 (2005).

[51] T. Aaltonen et al. (CDF Collab.), Phys. Rev. Lett. 100, 121802 (2008).

[52] A. A. Petrov, *In the Proceedings of Flavor Physics and CP Violation (FPCP 2003), Paris, France, 3-6 Jun 2003, pp MEC05* [arXiv:hep-ph/0311371].

[53] A. A. Petrov, Phys. Rev. **D 56**, 1685 (1997).

[54] M. Ciuchini *et al.*, Phys. Lett. B 655, 162 (2007).

[55] M. Blanke *et al.*, Phys. Lett. B 657, 81 (2007).

[56] X. G. He, G. Valencia, Phys. Lett. B 651, 135 (2007).

[57] Ch. H. Chen, Ch. Q. Geng, T. Ch. Yuan, Phys. Lett. **B** 655, 50 (2007).

[58] X. Q. Li, Z. T. Wei, Phys. Lett. **B** 651, 380 (2007).

[59] B. Dutta, Y. Mimura, arXiv:0708.3080 [hep-ph].

[60] A. Badin, F. Gabbiani and A. A. Petrov, Phys. Lett. B **653**, 230 (2007) [arXiv:0707.0294 [hep-ph]]; see also A. Badin, F. Gabbiani and A. A. Petrov, arXiv:0909.4897 [hep-ph].

[61] F. Buccella *et al.*, Phys. Rev. **D 51**, 3478 (1995).

[62] G. Burdman *et al.*, Phys. Rev. **D 66**, 014009 (2002).

[63] G. K. Yeghiyan, Phys. Rev. **D 76**, 117701 (2007).

[64] S. L. Chen, X. G. He, A. Hovhannisyan and H. C. Tsai, JHEP **09**, 044 (2007) [arXiv:hep-ph/0706.1100].

[65] Chuan-Hung Chen, Chao-Qiang Geng, Soo-Hyeon Nam, Phys. Rev. Lett. **99** (2007) 019101.

[66] A. A. Petrov, G. K. Yeghiyan, Phys. Rev. D 77, 034018 (2008).

[67] J. Ellis, D. Nanopoulos, Phys. Lett. **B 110**, 44 (1982).

[68] L. J. Hall *et al.*, Nucl. Phys. **B 267**, 415 (1986).

[69] B. C. Allanach, A. Dedes, H. K. Dreiner, Phys. Rev. **D 60**, 075014 (1999).

[70] K. Agashe, M. Graesser, Phys. Rev. **D 54**, 4445 (1996).

[71] A. Kundu, J. P. Saha, Phys. Rev, **D 70**, 096002 (2004).

[72] G. Bhattacharyya, A. Raychaudhuri, Phys. Rev. **D 57**, R3837 (1998).

[73] S. Nandi, J. P. Saha, Phys.Rev. **D 74**, 095007 (2006).

[74] A. Deandrea, J. Welzel, M. Oertel, JHEP **0410**, 038 (2004).

[75] N. G. Deshpande, D. K. Ghosh, X. G. He, Phys. Rev. **D 70**, 093003 (2004).

[76] V. Barger, G. F. Giudice, T. Han, Phys. Rev. **D 40**, 2987 (1989).

[77] S. C. Bennett, C. E. Wieman, Phys. Rev. Lett **82**, 2484 (1999);
C. S. Wood *et al.*, Science **275**, 1759 (1997).

[78] J. Ellis *et al.*, Mod. PHys. Lett. **A 10**, 1583 (1995).

[79] R. M. Godbole, R. P. Roy, X. Tata, Nucl. Phys. **B 401**, 67 (1993).

[80] R. N. Mohapatra, Phys. Rev. **D 34**, 3457 (1986);
M. Hirsch *et al.*, Phys. Rev. Lett. **75**, 17 (1995); Phys. Rev. **D53**, 1329 (1996).

[81] J. E. Kim, P. Ko, D. G. Lee, Phys. Rev. **D 56**, 100 (1997); K. Huitu, J. Maalampi, M. Raidal, A. Santamaria, Phys. Lett. **B 430**, 355 (1998).

[82] S. Abel, Phys. Lett. **B 410** 173 (1997).

[83] B. C. Allanach, A. Dedes, H. K. Dreiner, Phys. Rev. **D 69**, 115002 (2004).

[84] H. Baer, X. Tata, "Weal Scale Supersymmetry: from Superfields to Scattering Events", Camridge University Press, 2006.

[85] M. Chemtob, Prog. Part. Nucl. Phys. **54**, 71 (2005).

[86] R. Barbier *et al.*, Phys. Reports **420**, 1 (2005).

[87] W. M. Yao *et al.*, Journal of Phys. **G 33**, 1 (2006).

[88] R. Gupta, T. Bhattacharya, S. R. Sharpe, Phys. Rev. **D 55**, 4036 (1997).

[89] D. Asner, Contribution to WG2 Report on Flavor in the ERA of the LHC, CERN, March 26-28, 2007.

[90] K. Melnikov, T. van Ritbergen, Phys. Lett. **B 482**, 99 (2000).

[91] H. M. Asatryan and A. N. Ioannissyan, Mod. Phys. Lett. A 5, 1089 (1990).

[92] K. S. Babu, K. Fujikawa and A.Yamada, Phys. Lett. B 333, 196 (1994).

[93] P.Cho and M. Misiak, Phys. Rev. D 49, 5894 (1994).

[94] T Ohl et al., Nucl. Phys. **B 403** (1993) 605.

[95] I. I. Bigi, N. G. Uraltsev, Nucl. Phys. **B 592** (2001) 92.

[96] A. A. Petrov, Proceedings of 4th Workshop on Continuous Advances in QCD, Minneapolis, Minnesota, 12-14 May 2000, hep-ph/0009160.

[97] S. H. Nam, Phys. Rev. **D 68** (2003)115006.

[98] P. Langacker, S. U. Sankar, Phys. Rev. **D 40** (1989) 1569.

[99] M. Bona *et al.* [UTfit Collaboration], PMC Phys. A **3**, 6 (2009) [arXiv:0803.0659 [hep-ph]].

[100] 'Measurement of β_s at CDF', talk by Louise Oakes at Flavor Physics and CP Violation COnference 2010, Turin, Italy (May 25-29, 2010); 'New Measurement of B_s Mixing Phase at CDF', talk by Gavril Giurgiu talk delivered at ICHEP 2010 (Paris, France).

[101] V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. D **82**, 032001 (2010) [arXiv:1005.2757 [hep-ex]].

[102] E. Golowich, J. Hewett, S. Pakvasa and A. A. Petrov, Phys. Rev. D **79**, 114030 (2009) [arXiv:0903.2830 [hep-ph]].

[103] A. J. Buras, Phys. Lett. B **566**, 115 (2003) [arXiv:hep-ph/0303060].

[104] E. Golowich, J. Hewett, S. Pakvasa, A. A. Petrov and G. K. Yeghiyan, Phys. Rev. D 83, 114017 (2011), arXiv: 1102.0009 [hep-ph].

[105] A. J. Buras, M. Jamin and P. H. Weisz, Nucl. Phys. B **347**, 491 (1990).

[106] J. Urban, F. Krauss, U. Jentschura and G. Soff, Nucl. Phys. B **523**, 40 (1998) [arXiv:hep-ph/9710245].

[107] "Top Quark Physics", plenary talk by Mousumi Datta at the 2009 meeting of the Division of Particles and Fields of the American Physical Society, Wayne State University Detroit, MI (7/26/09- 7/31/09).

[108] S. Bethke, Eur. Phys. J. C **64**, 689 (2009) [arXiv:0908.1135 [hep-ph]].

[109] J. Laiho, E. Lunghi and R. S. Van de Water, Phys. Rev. D **81**, 034503 (2010) [arXiv:0910.2928 [hep-ph]].

[110] Aida El-Khadra, private communication.

[111] M. Antonelli *et al.*, Phys. Rept. **494**, 197 (2010) [arXiv:0907.5386 [hep-ph]]; see also the web sites for CKMfitter and UTfit [112].

[112] See also the latest results in the CKMfitter web site *http://ckmfitter.in2p3.fr* as well as those for UTfit at *http://www.utfit.org*.

[113] T. Inami and C.S. Lim, Prog. Theor. Phys. **65**, 297 (1981) [Erratum-ibid. **65**, 1772 (1981)].

[114] Y. Grossman, Phys. Lett. B **380**, 99 (1996) [arXiv:hep-ph/9603244].

[115] B.C.K. Casey, *New upper limit on the decay* $B_s \to \mu^+\mu^-$ *from D0*, talk delivered at ICHEP 2010 (Paris, France); see also V. M. Abazov *et al.* [D0 Collaboration], Phys. Lett. B **693**, 539 (2010) [arXiv:1006.3469 [hep-ex]].

[116] Y. Kao and T. Takeuchi, arXiv:0910.4980 [hep-ph].

[117] H. K. Dreiner, M. Kramer and B. O'Leary, Phys. Rev. D **75**, 114016 (2007) [arXiv:hep-ph/0612278].

[118] J. P. Saha and A. Kundu, Phys. Rev. D **66**, 054021 (2002) [arXiv:hep-ph/0205046]; we concur with a conclusion of Ref. [117] regarding the missing factor of 4 in this paper.

[119] M. Sher, Phys. Rept. **179**, 273 (1989).

[120] S. Weinberg, UTTG-05-91, Proceedings *High Energy Physics and Cosmology* (Islamabad, Pakistan), M.A.B. Beg Memorial Volume.

[121] V. A. Monich, B. V. Struminsky and G. G. Volkov, Phys. Lett. B **104**, 382 (1981) [JETP Lett. **34**, 213.1981 ZETFA,34,222 (1981 ZETFA,34,222-225.1981)].

[122] J.D. Bjorken, S. Pakvasa and S.F. Tuan, Phys. Rev. D **66**, 053008 (2002) [arXiv:hep-ph/0206116].

[123] D. Becirevic, V. Gimenez, G. Martinelli, M. Papinutto and J. Reyes, JHEP **0204**, 025 (2002) [arXiv:hep-lat/0110091].

[124] L. J. Hall and S. Weinberg, Phys. Rev. D **48**, 979 (1993) [arXiv:hep-ph/9303241].

[125] T. P. Cheng and M. Sher, Phys. Rev. D **35**, 3484 (1987).

[126] A. Pich and P. Tuzon, Phys. Rev. D **80**, 091702 (2009) [arXiv:0908.1554 [hep-ph]].

[127] V. D. Barger, J. L. Hewett and R. J. N. Phillips, Phys. Rev. D **41**, 3421 (1990).

[128] D. Atwood, L. Reina and A. Soni, Phys. Rev. D **55**, 3156 (1997) [arXiv:hep-ph/9609279].

[129] R. A. Diaz, R. Martinez and C. E. Sandoval, Eur. Phys. J. C **41**, 305 (2005) [arXiv:hep-ph/0406265]; M. J. Savage, Phys. Lett. B **266**, 135 (1991); M. E. Luke and M. J. Savage, Phys. Lett. B **307**, 387 (1993) [arXiv:hep-ph/9303249].

[130] A. E. Blechman, A. A. Petrov and G. Yeghiyan, JHEP **1011**, 075 (2010) [arXiv:1009.1612 [hep-ph]].

[131] H. P. Nilles, M. Srednicki and D. Wyler, Phys. Lett. B **120**, 346 (1983); J. M. Frere, D. R. T. Jones and S. Raby, Nucl. Phys. B **222**, 11 (1983); J. P. Derendinger and C. A. Savoy, Nucl. Phys. B **237**, 307 (1984).

[132] J. R. Ellis, J. F. Gunion, H. E. Haber, L. Roszkowski and F. Zwirner, Phys. Rev. D **39**, 844 (1989).

[133] B. A. Dobrescu, Phys. Rev. D **63**, 015004 (2001) [arXiv:hep-ph/9908391].

[134] U. Ellwanger, M. Rausch de Traubenberg and C. A. Savoy, Nucl. Phys. B **492**, 21 (1997) [arXiv:hep-ph/9611251].

[135] G. Hiller, Phys. Rev. D **70**, 034018 (2004) [arXiv:hep-ph/0404220].

[136] M. Endo and N. Yokozaki, arXiv:1012.5501 [hep-ph].

[137] M. S. Chanowitz, Phys. Rev. **D79**, 113008 (2009) [arXiv:0904.3570 [hep-ph]].

[138] M. Bobrowski, A. Lenz, J. Riedl and J. Rohrwild, Phys. Rev. D **79**, 113006 (2009) [arXiv:0902.4883 [hep-ph]].

[139] V. A. Novikov, L. B. Okun, A. N. Rozanov *et al.*, Mod. Phys. Lett. **A10**, 1915-1922 (1995).

[140] V. A. Novikov, L. B. Okun, A. N. Rozanov *et al.*, Phys. Lett. **B529**, 111-116 (2002). [hep-ph/0111028].

[141] J. Erler and P. Langacker, Phys. Rev. Lett. **105**, 031801 (2010) [arXiv:1003.3211 [hep-ph]].

[142] G. D. Kribs, T. Plehn, M. Spannowsky and T. M. P. Tait, Phys. Rev. D **76**, 075016 (2007) [arXiv:0706.3718 [hep-ph]].

[143] A. K. Alok, A. Dighe and D. London, arXiv:1011.2634 [hep-ph].

[144] A. Soni, A. K. Alok, A. Giri, R. Mohanta and S. Nandi, Phys. Rev. D **82**, 033009 (2010) [arXiv:1002.0595 [hep-ph]].

[145] A. J. Buras, M. Munz, Phys. Rev. **D52**, 186-195 (1995). [hep-ph/9501281].

[146] S. Nandi and A. Soni, arXiv:1011.6091 [hep-ph].

[147] For a review, see, *e.g.* Y. Nir, arXiv:0708.1872 [hep-ph]; O. Gedalia and G. Perez, arXiv:1005.3106 [hep-ph].

[148] C. Amsler et al. Phys. Lett. B 667, 1 (2008).

[149] K. Chetyrkin, *Phys. Lett.* **B404**, 161 (1997).

[150] A. Das, C. Kao, *Phys. Lett.* **B372**, 106 (1996).

[151] R.-M. Xu, *Phys. Rev.* **D44**, R590 (1991).

[152] S. Davidson, H. E. Haber, *Phys. Rev.* **D72**, 035004 (2005).

[153] H. E. Haber, D. O'Neil, *Phys. Rev.* **D74**, 015018 (2006).

[154] F.J. Botella, J. P. Silva, Phys. Rev. D 51, 3870-3875 (1995).

[155] G. C. Branco, L. Lavoura, J. P. Silva, Int. Ser. Monogr. Phys. 103, 1, (1999).

[156] J. F. Gunion, H. E. Haber, G. Kane, S. Dawson, *The Higgs Hunter's Guide*, Addison-Wesley Publishing Company, 1990.

[157] H. Fritzsch and Z. Z. Xing, Phys. Lett. B **353**, 114 (1995); H. Fritzsch and Z. Z. Xing, Phys. Lett. B **413**, 396 (1997); H. Fritzsch and Z. Z. Xing, Phys. Lett. B **372**, 265 (1996).

[158] H. Fritzsch and Z. Z. Xing, Prog. Part. Nucl. Phys. **45**, 1 (2000).

[159] J.L. Diaz-Cruz, R. Noriega-Papaqui and A. Rosado, Phys. Rev. D69 (2004) 095002.

[160] G. Isidori, Y. Nir and G. Perez, arXiv:1002.0900 [hep-ph].

[161] A. E. Blechman, A. A. Petrov, G. Yeghiyan, to be published.

[162] J. A. Bagger, K. T. Matchev, R.-J. Zhang, *Phys. Lett.* **B412**, 77 (1997).

[163] C. D. Froggatt, H. B. Nielsen, Nucl. Phys. B147 277 (1979).

[164] L. Lavoura and E. Ma, Mod. Phys. Lett. A 20, 1217 (2005).

[165] S. L. Chen and E. Ma, Phys. Lett. B 620, 151 (2005).

[166] K. S. Babu and J. Kubo, Phys. Rev. D 71, 056006 (2005).

[167] E. Ma, H. Sawanaka and M. Tanimoto, Phys. Lett. B 641, 301 (2006).

[168] T. Araki and J. Kubo, Int. J. Mod. Phys. A 24, 5831 (2009).

[169] R. Friedgerg and T. D. Lee, Annal Phys. 323, 1087 (2008).

[170] T. Araki, C. Q. Geng, Phys. Rev. D81, 076009 (2010).

[171] Y. Grossman, Z. Ligeti and Y. Nir, Prog. Theor. Phys. **122**, 125 (2009); M. Bona *et al.* [UTfit Collaboration], JHEP **0803**, 049 (2008).

[172] E. Golowich, J. Hewett, S. Pakvasa, A. A. Petrov, and G. K. Yeghiyan, to be published.

[173] H. Asatryan, A. Ioannissyan, S. Matinyan, *Z. Phys.* **C61**, 265 (1994).

[174] G. M. Asatrian, A. N. Ioannisian, S. G. Matinyan, Sov. J. Nucl. Phys. 53, 371, (1991), Yad. Fiz. 53, 592, (1991).

ABSTRACT

FLAVOR PHYSICS BEYOND THE STANDARD MODEL

by

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We examine possible New Physics impact on certain heavy quark flavor involving processes, such as charm and bottom strange meson-antimeson oscillations and leptonic meson decays. Also, we consider a possibility of explaining within a two-Higgs doublet extension of the Standard Model the quark and charged lepton mass hierarchy. We show that the experimental value of the lifetime difference in $D^0 - \overline{D}^0$ mixing may be due to destructive interference of the Standard Model and New Physics contributions. We examine next possible correlations between the New Physics contribution to $B_s - \overline{B}_s$ mass difference and $B_s \to \mu^+\mu^-$ leptonic decay. We show that these correlations tend to rule out possible large New Physics contribution to $B_s \to \mu^+\mu^-$. We propose also to explain within a general two-Higgs doublet extension of the Standard Model the quark and lepton mass hierarchy by imposing some basis invariant conditions on the quark and lepton Yukawa matrices with no or little hierarchy in Yukawa couplings.

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