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Some existence results of semilinear elliptic equations

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Riassunto: *Il lavoro tratta il problema dell'esistenza di soluzioni positive della equazione* (1) *con la funzione incognita u definita in* $\mathbb{R}^N$ *ed in* $\mathbb{R}^N - \{0\}$. *Si espone una trattazione generale che comprende come casi particolari i problemi studiati in precedenza da altri autori.*

Abstract: *We are concerned with the existence of positive solutions for equations of the form* (1), *defined in* $\mathbb{R}^N$ *and in* $\mathbb{R}^N - \{0\}$. *We give a unified treatment of the problems studied before by others authors.*

1 – Introduction

In this article we study the problem of existence of positive solutions for equations of the form

$$-\Delta u + V(x)u + g(x,u) = 0 \qquad \text{in } \mathbb{R}^N, \tag{1}$$

and in $\mathbb{R}^N$- $\{0\}$. In recent years this problem was studied separately for different potentials $V(x)$. For $V \equiv 0$ and $g(x,u) = K(x)u^q$ with $|K(x)| \le c(1+|x|)^{-2-\epsilon}$, for some positive constants c, ϵ, Ni [7] proved the existence of infinitely many bounded solutions in $\mathbb{R}^N$ with positive lower bounds. Naito [6] improved this result for the case $V \equiv 0$ and

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$g(x,u) = a(x)f(u)$ with conditions on the functions $a(x), f(u)$. Kusano and Swanson [5] proved analogous results for the equation

$$\Delta u - m^2 u + K(x) u^q = 0,$$

in $\mathbb{R}^N$.

In this article, we give a general method in such form that most of the above problems can be studied jointly. This method can be also applied for nonexistence results. Essentially, we transform a radial equation into another and we apply the known results for the last one.

We point out that we are able to prove the existence of positive singular ground states for equation (1) by making use of the obtained results.

In section 1, we generalize existence results given by Ni [7], Naito [6], Kusano and Swanson [5] and others.

In section 2, we give existence results of positive solutions of the equation (1) in all $\mathbb{R}^N$with the potential V and the function $g(x,.)$ not necessarily defined at the origin. In addition we give some examples of the above theorems. Finally, in the last section we prove the existence of positive singular ground states for equation (1).

2 – Existence results

In this section, we begin with the problem of existence of positive solutions of equation (1), defined on the whole space, and with the potential V assumed to be radially symmetric .

We give our main existence result for equation (1) in this section. First, we need some hypotheses on $V(x)$ and $g(x,u)$.

For the potential $V(x)$ we assume that it is radially symmetric, locally Hölder continuous on $[0,\infty)$ and such that the radial linear equation associated to (1), i.e.,

$$u''(r) + \frac{N-1}{r} u'(r) - V(r) u(r) = 0\,, \tag{2}$$

has two linearly independent solutions h_0 and h_1 which are positive on

$(0,\infty)$ and such that

$$\lim_{r\to\infty}\frac{h_0(r)}{h_1(r)}=0\,, \tag{3}$$

$$\lim_{r\to 0}\frac{h_1(r)}{h_0(r)}=0\,. \tag{4}$$

Let us call

$$r^{N-1}(h_1' h_0 - h_0' h_1)=\alpha\,. \tag{5}$$

Since h_0, h_1 are two linearly independent solutions of equation (2), then α is a positive constant. For $V\equiv 0$ we can take $h_0\equiv r^{2-N}$ and $h_1=1$. Then we get $\alpha = N-2$.

As for the function $g(x,u)$ we assume that it is a locally Hölder continuous function which is locally Lipschitz on u. Also we assume the existence of a function $g_1(|x|,u)$ such that $|g(x,u)|\le g_1(|x|,u)$ for all x and for all u nonnegative. The function $g_1(|x|,u)$ is assumed to be nondecreasing on u.

We define

$$p(a)=\int_0^\infty g_1(r, a h_1(r)) h_0(r) r^{N-1} d\,r$$

and the following set

$$P=\Big\{a\in \mathbb{R}^+:\quad p(a)<a\alpha \quad\text{and}\quad \mathrm{p}\Big(\mathrm{a}-\frac{\mathrm{p(a)}}{\alpha}\Big)<\alpha\mathrm{a}-\mathrm{p(a)}\ \Big\}. \tag{6}$$

The next theorem is the main result of this section. The method follows ideas of Naito [6].

Theorem 1. *With the above hypotheses suppose that there exists an $a\in P$.*

Then there exists a positive solution u of equation (1) *such that*

$$\lim_{|x|\to\infty}\frac{u(x)}{h_1(|x|)}=a-\frac{p(a)}{\alpha}.$$

Remark 1. Before get into the proof, let us analyse the case $g_1(r,u) = K_1(r)u^q$, on the above theorem.

If $q = 1$ and $\int_0^\infty K_1(r)h_1^q(r)h_0(r)r^{N-1}\, dr < \alpha$, then $P = \mathbb{R}^+$ and hence for all $c > 0$ there exists a positive solution u of (1) such that $\lim_{|x|\to\infty} u(x)/h_1(|x|) = c$.

If $q > 1$, we define $M = \int_0^\infty K_1(r)h_1^q(r)h_0(r)r^{N-1}\, dr$. In this case $P = \Big\{a \in \mathbb{R}^+ : a < \dfrac{\alpha}{M}^{\frac{1}{q-1}}\Big\}$, then for all $c \in \Big(0, (1-q^{-1})\Big(\dfrac{\alpha}{Mq}\Big)^{\frac{1}{q-1}}\Big)$ there exists a positive solution as in case q=1.

If $0 < q < 1$, Theorem 1 remains valid for all $c > \Big(\dfrac{M}{\alpha}\Big)^{\frac{1}{1-q}}$.

Proof of Theorem 1. Let us first prove Theorem 1 for the case $V \equiv 0$, because the method is similar in both cases.

In this case we can take $h_0(r) = r^{2-N}$ and $h_1(r) = 1$. We are going to construct a solution by using the method of sub and super solutions.

The idea for constructing super and sub solutions consists on the following, if f is a function such that:

$$\int_0^\infty |f(s)|\, ds < \infty \quad \text{and} \quad B =: \int_0^\infty f(s)\, ds\,,$$

then we can take for any constant A, the solution $v(s)$ of the equation

$$\ddot{v}(s) = f(s),$$

$$v(0) = 0,\ \dot{v}(0) = A.$$

Then

$$\lim_{s\to\infty} \dot{v}(s) = A + B.$$

It follows that if $u(r) = v(s)/s$, where $\mathrm{s} = \mathrm{r}^{\mathrm{N}-2}$, is a solution of

$$u''(r) + \frac{N-1}{r}u'(r) = F(r),$$

with $u \simeq A + B$ as $r \to \infty$ and

$$F(r) = (N-2)^2 r^{N-4} f(s).$$

We will see that with an appropiate choice of f and A we can obtain sub and super solutions of

$$-\Delta u + g(x,u) = 0.$$

Let us first construct the super solution. Choose $v_1(s)$ as the solution of

$$\ddot{v}_1 = -\frac{s^{(4-N)/(N-2)}}{(N-2)^2} g_1(r(s),a)\,, \tag{7}$$

with $v_1(0) = 0, \dot{v}_1(0) = a$. In this case

$$\int_0^\infty s^{(4-N)/(N-2)} g_1(r(s),a)ds = (N-2)\int_0^\infty r g_1(r,a)dr < \infty.$$

The function v is positive concave and

$$c \le \dot{v}_1(s) \le a,$$

where $c := a - \dfrac{p(a)}{\alpha}$

Moreover we easily get that $\lim\limits_{s\to\infty} v_1(s)/s = c.$ and

$$cs \le v_1 \le as\,. \tag{8}$$

If we call $u_1(r) = v_1(s)/s$, where $s = r^{N-2}$, then u_1 satisfies

$$\Delta u_1 = -g_1(|x|,a) \le -g_1(|x|,u_1) \le g(x,u_1).$$

Therefore, u_1 is a super solution. Now for the construction of the subsolution we proceed as follows.

Let v_2 be such that

$$\ddot{v}_2 = \frac{s^{(4-N)/(N-2)}}{(N-2)^2} g_1(r(s),c)$$

with $v_2(0) = 0, \dot{v}_2(0) = \beta$, where β is giving by

$$\beta + \frac{1}{(N-2)}\int_0^\infty r g_1(r,c)dr = c.$$

Then, from the definition of β we get that β is positive and

$$\lim_{s\to\infty} \frac{v_2(s)}{s} = \lim_{s\to\infty} \dot{v}_2(s) = c$$

and $v_2 \leq cs$.

Let us now define $u_2(r) = v_2(s)/s$. Then we get that u_2 satisfies

$$\Delta u_2 = g_1(|x|, c) \geq g_1(|x|, u_2) \geq g(x, u_2).$$

Then u_2 is a subsolution, u_1 is a supersolution, and $u_2 \leq u_1$, from (8).

We can then use the method of Ni [7] for constructing a solution u of

$$\Delta u = g(x, u)$$

such that $u_2 \leq u \leq u_1$, and thus

$$\lim_{|x|\to\infty} u(x) = c.$$

Now we prove Theorem 1 for a general radially symmetric potential V. The method is the same as above but now we introduce the following equation instead of (7)

$$\ddot{w}_1 = -\frac{g_1(r(s), ah_1)}{h_0 s'^2}, \qquad s = \frac{h_1}{h_0}$$

with $w_1(0) = 0$ and $\dot{w}_1(0) = a$, where $a \in P$.

Let us define

$$u_1(r) = h_0(r) w_1(s),$$

then $u_1(r)$ is a positive super solution of (1) such that if $c := a - \dfrac{p(a)}{\alpha}$ we have

$$ch_1 \leq u_1 \leq ah_1$$

and

$$\lim_{r\to\infty} \frac{u_1(r)}{h_1(r)} = c.$$

A subsolution u_2 with the appropiate properties can be constructed in the same way, such that

$$u_2 \leq u_1$$

and

$$\lim_{r\to\infty}\frac{u_2(r)}{h_1(r)}=c.$$

Then, as in the previous case $V\equiv 0$, we get the existence of a solution u of equation (1) such that

$$u_2\le u\le u_1$$

$$\lim_{|x|\to\infty}\frac{u(x)}{h_1(|x|)}=c.$$

We end this section with a result concerning a non radial potential V.

THEOREM 2. *Let $g(x,u)$ be a locally Hölder continuous function, locally Lipschitz in u, and $V(x)$ a locally Hölder continuous potential such that*

$$V_1(|x|)\le V(x)\le V_2(|x|),$$

for some locally Hölder continuous radial potentials V_1 and V_2 with V_1 verifying the same properties of V as in the introduction. Let h_0 and h_1 two solutions of

$$h''(r)+\frac{N-1}{r}h'(r)=V_1(r)h(r),$$

satisfying (3) and (4). Assume that

$$\int_0^\infty (V_2(r)-V_1(r))h_0(r)h_1(r)r^{N-1}dr<\infty,$$

$$|g(x,u)|\le K_1(r)u^q,$$

for some $q>1$ and

$$\int_0^\infty r^{N-1}K_1(r)h_1^q(r)h_0(r)dr<\infty.$$

Then there exists infinitely many positive solutions of equation (1). *Moreover, each of these divided by h_1 goes to a positive constant at infinity.*

Before proving this result we need the following lemma

LEMMA 1. *Let V_1 and V_2 be as above. Then there exist y_0 and y_1 two linearly independent positive solutions of*

$$y''(r) + \frac{N-1}{r} y'(r) = V_2(r) y(r),$$

defined for all $r > 0$ such that

$$\lim_{r\to\infty} \frac{h_i(r)}{y_i(r)} = 1, \qquad i = 0, 1,$$

$$\frac{r^{N-1} y_i'}{y_i} = \frac{r^{N-1} h_i'}{h_i} + o\left(\frac{1}{h_0 h_1}\right),$$

for $i = 0, 1$, as r tends to infinity. This is also true as $r \to 0$ but in the origin the above limits are, in general, a constant different from 1. Also $y_1 \le h_1$ and $h_0 \le y_0$.

The proof of this lemma is a consequence of [2] Corollary 6.5 and Theorem 9.1. Next we give the proof of Theorem 2.

PROOF. We can use the method of Theorem 1 to get the existence of a positive supersolution u_1 of

$$-\Delta u_1 + V_1 u_1 + g(x, u_1) \ge 0,$$

moreover $c h_1 \le u_1$ and $\lim\limits_{r\to\infty} u_1(r)/h_1(r) = c$ for some positive constant c. From the above lemma and using the hypothesis on h_1 and h_0 we have $\int\limits_0^\infty r^{N-1} K_1(r) y_1^q(r) y_0(r) dr < \infty$. Then, we construct a subsolution u_2 of

$$-\Delta u_2 + V_2 u_2 + g(x, u_2) \le 0,$$

such that $u_2 \le c y_1$ and $\lim\limits_{r\to\infty} u_2(r)/y_1(r) = c$. Then u_1, u_2 are super and sub solutions respectively of equation (1) and $u_2 \le u_1$. Therefore, there exists a positive solution u of equation (1) such that

$$\lim_{|x|\to\infty} \frac{u(x)}{h_1(|x|)} = c.$$

3 – Existence results with $\mathbf{V(x)}$ and $\mathbf{g(x, u)}$ singular at the origin.

Let us consider the problem of existence of positive solutions of the equation (1) with V and g having a possible singularity at the origin.

In this section the function $g(x,u)$ is a locally Hölder continuous function on (x,u) with x in $\mathbb{R}^N$- $\{0\}$ and u nonnegative, and the potential $V(x)$ is a locally Hölder continuous function on $\mathbb{R}^N$- $\{0\}$ and g(x, u) is locally Lipschitz in u. Let P be given as in (6). Then we have

Theorem 3. *Suppose that there exists a nonnegative function $g_1(r,u)$ such that $|g(x,u)| \leq g_1(|x|,u)$ for all $x \in \mathbb{R}^N - \{0\}$, and u nonnegative. Assume that $h_1 \in L^{N/(N-2)}_{loc}(\mathbb{R}^N)$, Vh_1 and $g_1(x, dh_1(|x|))$ are functions in $L^1_{loc}(\mathbb{R}^N)$, for all constant d positive.*

Assume also the existence of a constant $a \in P$, where P is given by (6).
Then, the conclusion of Theorem 1 remains valid.

Remark 2. We are going to use these results in two ways:
1. The potential V and the function $g(x,u)$ could have a singularity at the origin and we can get existence of positive solutions, as we are going to see in section 3. With the above result, we get existence of positive solutions with a prescribed behaviour at infinity.
2. The method used to study the problem of existence of positive solutions of the equation (1) in $\mathbb{R}^N$- $\{0\}$ consists in changing the problem to all $\mathbb{R}^N$, by means of the Kelvin transform. With this transformation the new potential V and function $g(x,u)$ could have a singularity at 0.

Proof of Theorem 3. Let $n \in N$ and consider

$$G_n = \Big\{x \in \mathbb{R}^N : \frac{1}{n+1} < |x| < n+1\Big\}.$$

It is not difficult to see that in this case we can also define the supersolution u_1 and the subsolution u_2 given on the proof of Theorem 1, by $u_1(r) = h_0(r)w_1(s)$, $u_2(r) = h_0(r)w_2(s)$ where

$$\ddot{w}_1 = -\frac{g_1(r(s), ah_1)}{h_0 s'^2}, \qquad s = \frac{h_1}{h_0}$$

$w_1(0) = 0$ and $\dot{w}_1(0) = a$,

$$\ddot{w}_2 = \frac{g_1(r(s), ch_1)}{h_0 s'^2},$$

$w_2(0) = 0$ and $\dot{w}_2(0) = \beta$, where $a \in P$, $c = a - p(a)/\alpha$ and $\beta = c - p(c)/\alpha$. Observe that they are respectively a supersolution and subsolution of equation (2) in G_n for all n.

Let $f(x, u) = V(x)u + g(x, u)$. We can apply Theorem 3.3 of [10] to get the existence of a solution w_n of

$$-\Delta u + f(x, u) = 0 \quad \text{in} \quad G_n,$$

and such that

$$u_2 \leq w_n \leq u_1 \qquad \text{in} \quad G_n.$$

Now, because $|g(x, w_n)| \leq g_1(|x|, u_1)$, $|Vw_n| \leq |V|u_1$ and $g_1(|x|, u_1)$ and $|V|u_1$ are two functions in $L^1(\mathbb{R}^N)$, we obtain that $\{w_n\}$ has a subsequence $\{w_n^1\}$ which converges weakly in $W^{1,p}(G_1)$, with $1 < p < \dfrac{N}{N-1}$, to a function w^1. Also $\{w_n\}$ strongly converges to w^1 in L^q for some q. Let $\{w_n^i\}$ be a subsequence of $\{w_n^{i-1}\}$ which converges in $W^{1,p}(G_i)$ to a function w^i, $i = 1, 2, \dots$. Define u in G_i by $u(x) = w^i$. This definition is consistent since $w^i = w^{i+1}$ on G_i. Then we can easily get that u is a solution of equation (3) in $D'(\mathbb{R}^N - \{0\})$ and satisfies

$$u_2 \leq u \leq u_1 \quad \text{in } \mathbb{R}^N.$$

Also u is in $L_{loc}^{\frac{N}{N-2}}(\mathbb{R}^N)$ and $f(x, u) \in L^1_{loc}(\mathbb{R}^N)$, hence u is a solution of (1) in $D'(\mathbb{R}^N)$, and the proof is complete.

Examples. Now we give some examples of potential $V's$. For simplicity we assume that $|g(x, u)| \leq K_1(|x|)u^q$ for some $q \geq 1$.

1. $V(x) = 0$ and

$$\int_0^\infty rK_1(r)dr < \infty \qquad \text{for} \quad q \neq 1,$$

$$\int_0^\infty rK_1(r)dr < N - 2 \quad \text{for} \quad q = 1,$$

In this case KAWANO [3] proves the existence of positive entire solutions which converge to positive constants as $|x| \to \infty$.
2. $V(x) = d/r^2$.

Assume that $d > -(N-2)^2/4$. Then, $h_0 = r^{\theta_0}$ and $h_1 = r^{\theta_1}$ with $\theta_{0,1} = (2 - N \mp \sqrt{(N-2)^2 + 4d})/2$. In this case $V(x)h_1^q(x) \in L^1_{loc}(\mathbb{R}^N)$ which follows from the condition on d.

We can similarly study the above case to get existence of positive solutions of the equation

$$\Delta u - \frac{d}{r^2} u + g(x,u) = 0,$$

In this case, there exist positive solutions if

$$\int_0^\infty r^{(q\theta_1 + \theta_0 + N - 1)} K_1(r)dr < \infty \qquad \text{for} \quad q \neq 1,$$

$$\int_0^\infty r^{(\theta_1 + \theta_0 + N - 1)} K_1(r)dr < \theta_1 - \theta_0 \quad \text{for} \quad q = 1,$$

3. $V(x) = -z/|x| + m^2$,with $z \geq 0, m > 0$.

Let us assume that $m > z/(N-1)$. Then it can be proved (see [1], proof of Theorem 3) the existence of h_0 and h_1 two linearly independent positive solutions of

$$h''(r) + \frac{N-1}{r} h'(r) + \frac{z}{r} h(r) - m^2 h(r) = 0,$$

on $(0, \infty)$ and such that

$$\begin{aligned} h_0(r) &\simeq e^{-rm} r^{-k} && \text{as } r \to \infty, \\ h_0(r) &\simeq r^{2-N} && \text{as } r \to 0, \end{aligned}$$

and

$$\begin{aligned} h_1(r) &\simeq e^{rm} r^{k+1-N} && \text{as } r \to \infty, \\ h_1(r) &\simeq c && \text{as } r \to 0, \end{aligned}$$

where $k = \frac{1}{2}((N-1) - z/m)$ and c is a positive constant which depends on V.

In this case and assuming that $q \neq 1$, all of the hypotheses of Theorem 5 are satisfied iff

$$\int^{\infty} K_1(r) r^{(q-1)(k+1-N)} e^{(q-1)rm} dr < \infty,$$

and

$$\int_0 K_1(r) r dr < \infty.$$

If $z = 0$ the above result was proved by Kusano and Swanson [5].

4 – Existence of solutions with a prescribed singularity at 0.

Of concern is the problem of existence of infinitely many solutions of the equation (1) in $\mathbb{R}^N - \{0\}$ with a prescribed singularity at the origin. Such singularity is the one corresponding to the linear part of the equation (1). For instance, if $V \equiv 0$ then we construct positive solutions with a $c|x|^{2-N}$ behaviour at 0.

For it, we use the Kelvin transform and the results of the above sections.

In this section we assume that V and $g(x,u)$ are locally Hölder continuous on $\mathbb{R}^N - \{0\}$ and V also satisfies hypotheses (3) and (4). Suppose that there exists a function g_1 such that $|g(x,u)| \leq g_1(|x|, u)$ for all $x \in \mathbb{R}^N - \{0\}$ and u nonnegative. Also g_1 is nondecreasing in u.

We define

$$\tilde{p}(a) = \int_0^{\infty} g_1(r, a h_0(r)) h_1(r) r^{N-1} d\, r$$

and the following set

$$\tilde{P} = \Big\{ a \in \mathbb{R}^+ : \quad \tilde{p}(a) < a\alpha \quad \text{and} \quad \tilde{\mathrm{p}}\Big(\mathrm{a} - \frac{\tilde{\mathrm{p}}(\mathrm{a})}{\alpha}\Big) < \alpha \mathrm{a} - \tilde{\mathrm{p}}(\mathrm{a}) \quad \Big\} \tag{9}$$

With the above conditions we have the following result

Theorem 4. *Suppose that*

$$\int^{\infty} r^{-1} h_0^{N/(N-2)}(r)dr < \infty,$$

$$\int^{\infty} r|V(r)|h_0(r)dr < \infty,$$

$$\int^{\infty} r g_1(r, \eta h_0(r))dr < \infty, \quad \text{for} \quad \text{all} \quad \eta > 0.$$

Assume that there exists a positive constant $a \in \tilde{P}$.

Then there exists a solution u *of equation* (1) *in* $\mathbb{R}^N - \{0\}$ *such that*

$$\lim_{|x|\to 0} \frac{u(x)}{h_0(|x|)} = a - \frac{\tilde{p}(a)}{\alpha}\,. \tag{10}$$

Also there exist two positive constants B *and* B' *such that*

$$Bh_0(|x|) \le u(x) \le B' h_0(|x|), \qquad \text{for all } \ x \in \mathbb{R}^N - \{0\}.$$

Proof. Let us call z the Kelvin transform of the function u. It is given by

$$z(x) = |x|^{2-N} u(x/|x|^2). \tag{11}$$

The function z verifies the following equation

$$-\Delta z + \tilde{V}(|x|)z + \tilde{g}(x, z) = 0 \quad \text{in } \ \mathbb{R}^N, \tag{12}$$

where

$$\begin{aligned} \tilde{V}(|x|) \ &= \frac{1}{|x|^4} V\Big(\frac{1}{|x|}\Big), \\ \tilde{g}(x,z) \ &= |x|^{-(N+2)} g\Big(\frac{x}{|x|^2}, |x|^{N-2} z\Big), \end{aligned}$$

and

$$\tilde{g}_1(r,z) = |x|^{-(N+2)} g_1\Big(\frac{1}{r}, r^{N-2} z\Big).$$

From the assumptions on h_0 and h_1, we get that $\tilde{h}_0(r) = r^{2-N} h_1(1/r)$ and $\tilde{h}_1(r) = r^{2-N} h_0(1/r)$ are two linearly independent positive solutions of

$$z''(r) + \frac{N-1}{r} z'(r) - \tilde{V}(r) z(r) = 0.$$

Also by (3), (4), (5) and the definition of $\tilde{h}_0$, $\tilde{h}_1$ we have $\lim_{r\to\infty} \tilde{h}_0(r)/\tilde{h}_1(r) = 0$, $\lim_{r\to 0} \tilde{h}_1(r)/\tilde{h}_0(r) = 0$ and $\tilde{h}_1'\tilde{h}_0 - \tilde{h}_0'\tilde{h}_1 = r^{1-N}\alpha$.

Now, it can be easily checked that all the hypotheses of Theorem 3 are satisfied for $\tilde{V}, \tilde{g}$ and $\tilde{g}_1$. Then, there exists a positive solution z of (12) such that

$$\lim_{|x|\to\infty} \frac{z(x)}{\tilde{h}_1(|x|)} = c,$$

and

$$B\tilde{h}_1(|x|) \leq z(x) \leq B'\tilde{h}_1(|x|),$$

for some positive constant B and B'.

It then follows that $u(x)$ given by (11) satisfies the conclusion of this theorem.

Example. Now we give an example to illustrate the above result. Let us consider the problem of existence of positive solutions u of the equation on $\mathbb{R}^N - \{0\}$

$$\Delta u - u + u^q = 0,$$

such that $u \to 0$ at ∞ and $u \to \infty$ at 0. If $q \geq (N+2)/(N-2)$ Ni and Serrin [9] proved the nonexistence of such type of solutions. We can applied Theorem 4 to get the existence of positive solutions u such that $u \to 0$ at ∞ and $u \to \infty$ at 0 for all $q < N/(N-2)$. We also have the behaviour at 0 and at ∞, that is

$$u \simeq c e^{-r} r^{-\frac{N-1}{2}} \quad \text{as} \quad r \to \infty$$

and

$$u \simeq b r^{2-N} \qquad \text{as} \quad r \to 0.$$

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