Method to Quantify Modal Interactions Between Converter Interfaced Generators and Synchronous Generators

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***Abstract*—The aim of this paper is to quantify the interactions between the oscillation modes of power systems within the realm of small-signal stability. This paper focuses on the interactions between Converter Interfaced Generator (CIG) oscillation modes and electromechanical oscillation modes of Synchronous Generators (SGs). It is difficult to determine the modal interactions using the existing analysis such as mode sensitivity, mode loci and participation factors. This paper proposes an extension to eigenvalue sensitivity analysis in order to determine the interaction between modes and impact of the interaction on system stability. Interaction coefficients are proposed to quantify the interaction between the modes. A modified IEEE 39-bus system with CIGs is considered to carry out the proposed analysis. The analysis is carried out to investigate the impact of PLL parameters on the interaction among the oscillation modes. The analysis is also carried out considering renewable energy penetration levels of 50-70%. It is observed that the interaction between CIG and electromechanical modes of SG results in increased participation of SGs' states in CIG modes. This increased participation of SG states in CIG modes results in reduced damping of oscillations in SG states.**

***Index Terms*—Interaction, eigenvalues, renewable generations.**

I. INTRODUCTION

The interaction among different components of power systems is important to understand the dynamic behaviour of system [1]. The increased penetration of Converter Interfaced Generations (CIGs) into power systems could lead to interactions among Synchronous Generators (SGs) and CIGs. The control structures of CIGs differ from SG controls significantly. The control parameters have high impact on the dynamics of CIG generator. The aim of this paper is to quantify the interactions between the oscillation modes of power systems within the realm of small-signal stability. Two new categories of modes (Type 1 and Type 2) associated with CIGs are observed in power systems with CIGs.

- Type 1 modes are dominated only by the CIGs.
- Type 2 modes have participation of both CIGs and SGs.

It is found that there is an overlap in the frequency ranges of SG and CIG modes [1]. The overlap in frequency ranges could lead to interaction among the CIG and SG oscillation modes. The interaction among modes may hamper the damping of otherwise well damped states of the power system [2]–[5] resulting in system instability. So, analyzing the interaction among oscillation modes can help in understanding the interaction among the CIGs and SGs. The interaction analysis should capture the following features:

- *Feature-1:* Influence of a set of modes on damping and frequency of a desired oscillation mode
- *Feature-2:* Resonance between the oscillation modes
- *Feature-3:* Change in mode shapes due to interaction

The interaction among generators in the power systems is analyzed in some of the existing literature. The works [6-15] highlight the importance of analyzing the interaction among modes in power system. Literature [6] discusses about the impact of interactions among SG and CIGs on power system stability. The work in [7] presents that the interaction among full converter-based wind power generation (FCWG) and SG will result in new category of oscillation modes called quasi-electromechanical modes. A method to classify the electromechanical modes from quasi-electromechanical modes is proposed in [7] based on participation factors of FCWG states and SG states. Literature [8] discusses about the impact of Phase Locked Loop (PLL) tracking ability on stability of power system with doubly fed induction generator (DFIG) incorporating virtual inertia control. The study in [8] is based on coupling matrices and participation factors. In [9], using participation factors, it is observed that interaction of PLL and automatic current control in wind turbines may lead to grid instability during grid faults. Literature [10] discusses about the impact of PLL and voltage source converter parameters on interaction between AC and Multiple Terminal DC (MTDC) systems. The impact of converter control parameters on interactions among grid connected wind generators is discussed in [11], [12]. The works in [6]- [12] have studied interaction by analyzing the shift in modes. However, the modes which are responsible for the shift in modes can not be identified through the works [6]- [12]. So, the works [6]- [12] can not capture the Feature-1 and Feature-3 interactions. The impact of open loop PLL oscillation modes of a wind generator on nearby electromechanical oscillation modes is discussed in [13] based on mode loci. In [13], [14], it is identified that the the open loop modal resonance between PLL and the rest of power system is one of the reasons behind interaction among the oscillation modes. The interactions between full converter-based wind power generation and rest of the power system is investigated in [15]. The interaction between electromechanical modes and permanent magnet synchronous generator (PMSG) modes

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is carried out in [15] by employing modal shift evaluation method. The works in [13]- [15] have studied interaction by analyzing the resonance between the modes. However, the works [13]- [15] can not capture the Feature-1 and Feature-3 interactions. A resonance index has been proposed in [15] and the index is used for controller tuning. This research is in the same direction to that of [15]. However, the goal of this paper is to develop a more generalized interaction coefficients which can capture various features of interaction. The work in [16] discusses about mode coupling in the system which results in mode shapes of one operating point influencing the mode shapes at another operating point. The work [16] focuses on the feature 3 of interaction. However, analytical proof is not presented in [16] for analyzing feature 3. In addition, feature 1 and 2 are not discussed in [16]. The work presented in [17] presents a method to calculate sensitivity of modes and mode shapes towards a system parameter but does not discuss about interaction between modes in particular. In summary, there is a lack of literature on methods to quantify all the three features of interactions.

This paper proposes a method to quantify modal interactions (Feature-1,2 and 3) between CIGs and SGs. The contributions of this paper are:

- An extension to eigenvalue sensitivity analysis is proposed to quantify interactions among oscillation modes of CIGs and SGs. Interaction coefficients are proposed to estimate the interaction among oscillation modes that can occur due to the change in system parameters.
- The proposed method can be used to investigate interaction between any two modes of the system. In this study, the proposed method is used to analyze the impact of PLL parameters on the interaction among CIG and SG oscillation modes. This analysis can be used to design SG controls to minimize the effect of critical CIGs on damping of SG states.
- It is identified that the change in PLL parameters does not directly influence the electromechanical modes. However, participation of SG states in CIG modes is affected due to the interaction among CIG and electromechanical modes.

The proposed method can be used in following applications. These applications highlight the practical value of the method.

- *Application-1:* This study can be used to tune CIG parameters (such as PLL parameters of CIGs) to maximise the damping of CIG modes and minimise the participation of SG states in critical CIG modes.
- *Application-2:* Proposed interaction coefficients can be used to study the impact of RE penetration level on small signal stability.
- *Application-3:* Oscillation source identification methods can be developed from the proposed interaction index for power systems with both SGs and CIGs.
- *Application-4:* The interaction coefficients can be used in controlled islanding algorithms to determine whether the proposed islands would lead to adverse interactions.
- *Application-5:* Proposed interaction coefficients can be used in new applications related to RE integration as a measure of interactions.

II. Proposed Extension to Eigenvalue Sensitivity Analysis

Power system with penetration from CIG generations is considered in this paper to investigate the interaction among modes. Small signal analysis is carried out on the system to analyze the modal interaction [18]. Linear model of power system is derived by linearizing the system around an operating point. The SGs in the system are considered to be equipped with AVR and PSS. The state space model for SGs is formulated using the 1.1 model of SG [19]. CIGs are modeled as in [20] and [21]. Dynamic equations of CIGs are linearized to form the state equation as shown in (1), where A_{CIG} and B_{CIG} denote the state matrix and the control matrix of CIG; X_{CIG} is the vector of CIG states; ΔX_{CIG} is the change in X_{CIG}; $V_{dq_{term}}$ is the vector of q and d components of terminal voltages of CIGs i.e. in dq-frame.

$$\Delta \dot{X}_{CIG} = A_{CIG}\ \Delta X_{CIG} + B_{CIG}\ \Delta V_{dq_{term}} \tag{1}$$

The state equations of SGs are coupled with CIG state equations through transmission network equations. The network dynamic equations are used to combine the CIG and SG dynamic equations to formulate the equations representing the whole power system. Dynamic equations of the whole power system can be written as (2), where A denotes the state matrix of the power system. The matrix A is a square matrix with n rows and n columns. The size n depends on the components of power system. Formulation of (2) is given in Appendix A. The influence of a system parameter on the interaction of eigenvalues (also termed as modes) of power system is explored in the following section.

$$\Delta \dot{X} = A\ \Delta X \tag{2}$$

To analyze the interaction of modes, a perturbation in system parameter, k_{sys} is considered here. Considering the perturbation Δk_{sys}, (2) can be rewritten as (3). As given in (3), Δk_{sys} gives rise to an additional component of state matrix i.e. $\Delta A_{k_{sys}}$.

$$\Delta \dot{X} = (A + \Delta A_{k_{sys}})\ \Delta X \tag{3}$$

The derivation to analyze the impact of change in operating point on the interaction of modes can be obtained in the similar lines. When the operating point changes, the additional component can be denoted as ΔA_{x_0}. Symbolically, the equation representing change in system parameters is same as that of change in operating point. Hence, the proposed derivation for interaction analysis is valid for change in operating point as well.

A. Limitation of the Existing Eigenvalue Sensitivity Analysis in Calculating Interaction Among CIG and SG Oscillation Modes

The term 'interaction' in this paper refers to the interaction between the modes. Any perturbation – either a change in operating point or system parameters – shifts the modes (eigenvalues) and alters the corresponding mode shapes (eigenvectors). The proposed approach quantifies how this change in modes and mode shapes due to a perturbation is influenced by the interaction between the modes.

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The interaction among the generations (SG and CIG) in power system is analyzed in existing works by tracking the eigenvalues when a system parameter is changed. In addition, to investigate the impact of system parameters on the eigenvalues, eigenvalue sensitivity analysis has been widely used in literature [22]–[24]. Eigenvalue sensitivity analysis essentially quantifies the change in eigenvalues as a parameter of interest is changed, as shown in (4). Participation factor is also derived from sensitivity analysis as shown in (5). Participation factor (5) signifies the participation of a state variable of the system in an eigenvalue of the tsystem.

$$\frac{\partial \lambda_i}{\partial para} = \frac{w_i^T(\partial A/\partial para)v_i}{w_i^T v_i} \tag{4}$$

$$\frac{\partial \lambda_i}{\partial a_{kk}} = w_i(k)v_i(k) \tag{5}$$

where, λ_i denotes an eigenvalue of A; w_i, v_i denote the left and right eigenvectors of λ_i; $para$ is the parameter of interest; $(\partial A/\partial para)$ denotes the change in system matrix A with change in $para$; a_{kk} denotes the element of A in k^{th} row and k^{th} column; $w_i(k)$, and $v_i(k)$ denote k^{th} element of w_i and v_i respectively.

It can be seen that the sensitivity and participation factors provide information on amount of change in modes. However, these indexes do not quantify the interaction among modes of the system. In addition, the modes which participate in the interaction can not be calculated through the available methods.

So, the existing methods are not sufficient to quantify the interaction among SG and CIG modes of a power system. An extension to the sensitivity analysis is proposed in the following sections to quantify the interaction among modes. Method to calculate the interaction coefficients (for example, $\alpha_{1,1}$ and $\alpha_{2,1}$) is derived in the following sections. The proposed method is used to analyze the interaction among SG and CIG modes.

B. Proposed Method to Quantify Interaction of Modes

To analyze the interaction among modes in power system, all possible scenarios of eigenvalues for LTI system are considered in this work. The small signal/linear model of power system will typically have multiple distinct (non-repetitive) eigenvalues corresponding to dynamic components of generators and different control loops of generators. However, there could be repeated eigenvalues in power system, for instance, this could be due to the lack of uniqueness of absolute rotor angles (referred to a common reference frame without an infinite bus) and the absence of the governor loop. So, for completeness of the interaction analysis method the following three cases are considered.

- Case 1: All eigenvalues of A are distinct from one another. As all the eigenvalues are distinct, all the eigenvectors of A are linearly independent. So, the matrix A can be diagonalized.
- Case 2: The matrix A has repetitive eigenvalues, with linearly independent eigenvectors. This case arises if the geometric multiplicity is equal to algebraic multiplicity [25] for all eigenvalues. As all the eigenvectors are linearly independent, the matrix A can be diagonalized.
- Case 3: The matrix A has repetitive eigenvalues, but the eigenvectors of repeated eigenvalues are not linearly independent. This case arises if the geometric multiplicity is less than the algebraic multiplicity for repeated eigenvalues [25]. As the eigenvectors of A are not linearly independent, the matrix A can not be diagonalized. In this case, the generalized eigenvectors are used to obtain Jordan canonical form (a block diagonal form) of A [25].

It can be seen that, cases 2 and 3 deal with repetitive eigenvalues. So, these cases reflect the modal resonance present in the system, if any.

Case 1: Distinct eigenvalues: For the system described in (2) and with distinct eigenvalues of A, the eigenvectors and eigenvalues of A satisfy (6).

$$\begin{aligned} A\ V(:,i) &= \lambda_i\ V(:,i) \\ W(:,i)^T\ A &= \lambda_i\ W(:,i)^T \end{aligned} \tag{6}$$

where, W^T and V are the left and right eigenvector matrices of A respectively; $V(:,i)$ represents i^{th} column of matrix V; $W(:,i)$ represents i^{th} column of matrix W and λ_i is the i^{th} eigenvalue of A; i takes values $1, 2, 3, ..., n$.

Using the transformation $\Delta X = V\Delta Z$ on (3),

$$\dot{\Delta Z} = W^T(A + \Delta A_{k_{sys}})V\ \Delta Z \tag{7}$$

$$\begin{aligned} \dot{\Delta Z} &= (\Lambda + \Delta A_1)\ \Delta Z \\ \Delta A_1 &\equiv W^T \Delta A_{k_{sys}} V \end{aligned} \tag{8}$$

where, Λ is the diagonal matrix with eigenvalues of A along the diagonal, ΔA_1 is the modified representation of $\Delta A_{k_{sys}}$.

The matrix ΔA_1 being non-zero signifies the influence of k_{sys} on the modes of system. So, eigenvalues of $\Lambda + \Delta A_1$ are not same as Λ. Main interest of this study is to evaluate the interaction among eigenvalues of Λ which result in new set of eigenvalues of $\Lambda + \Delta A_1$.

The matrix $\Lambda + \Delta A_1$ is diagonalized in order to investigate it's eigenvalues. For the system (8), eigenvectors and eigenvalues satisfy (9).

$$\begin{aligned} (\Lambda + \Delta A_1)\ V_z(:,i) &= \Xi_i\ V_z(:,i) \\ W_z(:,i)^T\ (\Lambda + \Delta A_1) &= \Xi_i\ W_z(:,i)^T \end{aligned} \tag{9}$$

where, W_z^T and V_z are the left and right eigenvector matrices of $\Lambda + \Delta A_1$ respectively; Ξ_i denotes i^{th} eigenvalue of $\Lambda + \Delta A_1$. Using the transformation $\Delta Z = V_z\ \Delta Y$ on (8),

$$\dot{\Delta Y} = W_z^T(\Lambda + \Delta A_1)V_z\ \Delta Y \tag{10}$$

$$W_z^T\ (\Lambda + \Delta A_1)\ V_z \equiv \begin{bmatrix} W_{z,1}^T(\Lambda + \Delta A_1)V_{z,1} & W_{z,1}^T(\Lambda + \Delta A_1)V_{z,2} \cdots & W_{z,1}^T(\Lambda + \Delta A_1)V_{z,n} \\ W_{z,2}^T(\Lambda + \Delta A_1)V_{z,1} & W_{z,2}^T(\Lambda + \Delta A_1)V_{z,2} \cdots & W_{z,2}^T(\Lambda + \Delta A_1)V_{z,n} \\ \vdots & \vdots \quad \ddots & \vdots \\ W_{z,n}^T(\Lambda + \Delta A_1)V_{z,1} & W_{z,n}^T(\Lambda + \Delta A_1)V_{z,2} \cdots & W_{z,n}^T(\Lambda + \Delta A_1)V_{z,n} \end{bmatrix} \tag{11}$$

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where, $W_{z,i}^T,\ V_{z,i},\ i \in 1,2,...n$ are the left and right eigenvectors of i^{th} eigenvalue of $\Lambda + \Delta A_1$ respectively. As $W_{z,i}^T,\ V_{z,j}$ are orthogonal for $i \neq j$, the off-diagonal elements in (11) are zero. Equation (10) can be written as (12).

$$\dot{\Delta Y} = \begin{bmatrix} \Xi_1 & 0 & \cdots & 0 \\ 0 & \Xi_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Xi_n \end{bmatrix} \Delta Y \tag{12}$$

Comparing (12) and (11),

$$\Xi_i = W_{z,i}^T (\Lambda + \Delta A_1) V_{z,i}$$

$$\implies \Xi_i = \sum_{j=1}^{n} W_{z,i}(j)\ \lambda_j\ V_{z,i}(j)\ + f(\text{elements of } \Delta A_1) \tag{13}$$

where, $W_{z,i}(j)$ denote j^{th} element of left eigenvector of i^{th} eigenvalue; $V_{z,i}(j)$ denote j^{th} element of right eigenvector of i^{th} eigenvalue; f is a linear function; $f(\text{elements of}\Delta A_1) = W_{z,i}^T(\Delta A_1)V_{z,i}$. The factor $W_{z,i}(j) * V_{z,i}(j)$ determines the contribution of j^{th} eigenvalue in i^{th} eigenvalue. The coefficients of (13) determines the interaction among the eigenvalues, λ_i.

It is to be noted that the modes that participate in the interaction depends on the matrix ΔA_1. If all the non-diagonal elements of ΔA_1 are zero, there will not be any interaction among the modes. For a i^{th} eigenvalue (Ξ_i), the coefficients of (13) will be zero except for λ_i. In this scenario, the shift in the eigenvalues do not have any contribution from the interaction. On the other hand, if the non-diagonal elements of ΔA_1 are non-zero, then the coefficients of (13) will be non-zero. The coefficients determine the contribution of old eigenvalues towards the new eigenvalues. So, the interaction among modes is dictated by the elements of matrix ΔA_1.

Case 2: Repetitive eigenvalues, geometric multiplicity is equal to algebraic multiplicity: The derivation for $Case2$ is presented here considering that A has one repeating eigenvalue without loss of generality. The derivation holds for any number of repeating eigenvalues. Consider the matrix A with one repeating eigenvalue with an algebraic and geometric multiplicity of n_{rep}. The repeating eigenvalue and corresponding eigenvectors of A satisfy (14).

$$\begin{aligned} A\ V(:,i) &= \lambda\ V(:,i) \\ W(:,i)^T\ A &= \lambda\ W(:,i)^T \end{aligned} \tag{14}$$

where, W^T and V are the left and right eigenvector matrices of A respectively; $V(:,i)$ represents i^{th} column of matrix V; $W(:,i)^T$ represents i^{th} column of matrix W^T and λ is the repeating eigenvalue of A; i takes values $1,2,3,...,n_{rep}$.

The distinct eigenvalues and corresponding eigenvectors of A satisfy (6) as stated above. As the geometric multiplicity of eigenvalues is equal to algebraic multiplicity, the eigenvectors of A constitute a set of linearly independent vectors. So, using the transformation $\Delta X = V\Delta Z$ on (3) will result in the system of the form (8). The rest of the derivation is exactly the same steps as mentioned in $Case1$.

Case 3: Repetitive eigenvalues, geometric multiplicity is less than algebraic multiplicity: The derivation for $Case3$ is presented here considering that A has one repeating eigenvalue. The derivation holds for any number of repeating eigenvalues. Consider the matrix A with one repeating eigenvalue with an algebraic of $n_{rep,alg}$ and geometric multiplicity of $n_{rep,geo}$. The repeating eigenvalue and corresponding eigenvectors of A satisfy (15). The repeating eigenvalue and corresponding generalized eigenvectors of A satisfy (16).

$$\begin{gathered} A\ V(:,i) = \lambda\ V(:,i) \\ W(:,i)^T\ A = \lambda\ W(:,i)^T \\ i = 1,2,3,...,n_{rep,geo} \end{gathered} \tag{15}$$

$$\begin{gathered} (A-\lambda I)^{k+1}\ V(:,n_{rep,geo}+k) = 0 \\ (A-\lambda I)^{k}\ V(:,n_{rep,geo}+k) \neq 0 \\ k = 1,2,3,...,n_{rep,alg}-n_{rep,geo} \end{gathered} \tag{16}$$

where, λ is the repeating eigenvalue of A. It is to be noted that $V(:,1)$ to $V(:,n_{rep,geo})$ are the normal eigenvectors of λ; $V(:,n_{rep,geo}+k)$ denotes the generalized eigenvectors of λ of rank $k+1$, $k = 1,2,3,...,n_{rep,alg} - n_{rep,geo}$. The generalized eigenvectors being linearly independent set of vectors, the matrix V is a full rank matrix. The matrix W^T is obtained as $W^T = V^{-1}$.

The distinct eigenvalues and corresponding eigenvectors of A satisfy (6) as stated in $Case1$. The index of distinct eigenvalues will range from $n_{rep,alg}+1$ to n. Using the transformation $\Delta X = V\Delta Z$ on (3),

$$\dot{\Delta Z} = W^T(A + \Delta A_{k_{sys}})V\ \Delta Z \tag{17}$$

$$\begin{aligned} \dot{\Delta Z} &= (J + \Delta A_1)\ \Delta Z \\ \Delta A_1 &\equiv W^T \Delta A_{k_{sys}} V \end{aligned} \tag{18}$$

where, J is the Jordan canonical form of A, ΔA_1 is the modified representation of $\Delta A_{k_{sys}}$. J is shown in (19) where $m = n_{rep,alg}+1$.

$$J = \begin{bmatrix} \begin{matrix} \lambda & 0 & \cdots & 0 & 0 \\ 0 & \lambda & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \lambda & 0 \end{matrix} & \cdots & \begin{matrix} \cdots & 0 \\ & 0 \\ & \vdots \end{matrix} \\ \begin{matrix} 0 & 0 & \cdots & 0 \\ & \vdots & & \end{matrix} & \begin{matrix} \lambda & 1 & 0 & 0 & 0 \\ 0 & \lambda & 1 & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \lambda & 1 \\ 0 & \cdots & 0 & 0 & \lambda \end{matrix} & \vdots \\ \begin{matrix} \vdots \\ 0 \quad 0 \quad \cdots \end{matrix} & \begin{matrix} \cdots \\ \cdots \end{matrix} & \begin{matrix} \lambda_m & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_n \end{matrix} \end{bmatrix} \tag{19}$$

In the similar fashion the transforming matrices W_z and V_z for $J + \Delta A_1$ can be obtained from generalized eigenvectors, if needed. Using the transformation $\Delta Z = V_z\ \Delta Y$ on (18):

$$\dot{\Delta Y} = W_z^T(J + \Delta A_1)V_z\ \Delta Y \tag{20}$$

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Due to the orthogonality properties of eigenvectors and generalized eigenvectors, (20) can be written as shown in (21). It is to be noted that the number of distinct and repeating eigenvalues of the matrix $J+\Delta A_1$ may not be same as that of A. In case of repeating eigenvalues of $J+\Delta A_1$, the algebraic multiplicity and geometric multiplicity may differ from that repeating eigenvalues of A. In (21), the second block matrix with 0 and 1 along the off diagonals and Ξ along the diagonal indicates that Ξ has geometric multiplicity less than algebraic multiplicity. The third block diagonal matrix indicates the distinct eigenvalues of $J+\Delta A_1$.

$$\dot{\Delta Y} = \begin{bmatrix} \begin{matrix}\Xi & 0 & \cdots & 0 & 0\\ 0 & \Xi & \cdots & 0 & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ 0 & 0 & \cdots & \Xi & 0\end{matrix} & \cdots & \cdots & \begin{matrix}0\\ 0\\ \vdots\end{matrix} \\ \begin{matrix}0 & 0 & \cdots & 0\\ & \vdots & & \end{matrix} & \begin{matrix}\Xi & 1 & 0 & 0\\ \vdots & \ddots & \ddots & \vdots\\ 0 & \cdots & \Xi & 1\\ 0 & \cdots & 0 & \Xi\end{matrix} & & \vdots \\ \vdots & \cdots & & \begin{matrix}\Xi_{n'} & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & \Xi_n\end{matrix} \\ 0 \quad 0 \quad \cdots & \cdots & & \end{bmatrix} \Delta Y \tag{21}$$

By expanding W_z^T and V_z of (20) into rows and columns, comparing with (21) the new eigenvalues can be expressed as (22).

$$\Xi \equiv W_{z,l}^T(J+\Delta A_1)V_{z,l};\ \Xi_i \equiv W_{z,i}^T(J+\Delta A_1)V_{z,i} \tag{22}$$

where, $W_{z,l}^T$ is the l^{th} row of W_z^T; $V_{z,l}$ is the l^{th} column of V_z; l takes values from 1 to algebraic multiplicity of repeating eigenvalue of $J+\Delta A_1$; i takes values from algebraic multiplicity +1 to n. Expanding (22),

$$\begin{aligned} \Xi = & \sum_{j=1}^{n_{rep,alg}} W_{z,l}(j)\ \lambda\ V_{z,l}(j)\ + \\ & \sum_{j=n_{rep,alg}+1}^{n} W_{z,l}(j)\ \lambda_j\ V_{z,l}(j)\ + f_1(\text{elements of } \Delta A_1); \\ \Xi_i = & \sum_{j=1}^{n_{rep,alg}} W_{z,i}(j)\ \lambda\ V_{z,i}(j)\ + \\ & \sum_{j=n_{rep,alg}+1}^{n} W_{z,i}(j)\ \lambda_j\ V_{z,i}(j)\ + f_2(\text{elements of } \Delta A_1) \end{aligned} \tag{23}$$

where, $W_{z,l}(j)$ denotes j^{th} element of $W_{z,l}^T$; $V_{z,l}(j)$ denotes j^{th} element of $V_{z,l}$; f_1 and f_2 are linear functions; $f_1(\text{elements of } \Delta A_1) = W_{z,l}^T \Delta A_1 V_{z,l}$ and $f_2(\text{elements of } \Delta A_1) = W_{z,i}^T \Delta A_1 V_{z,i}$.

It is to be noted that the relationship between the modes prior to and after the perturbation is similar in all the three cases. From (13) and (23) it can be conclude that the modes of the power system with new value of k_{sys}, Ξ and Ξ_i have contributions from modes of system prior to the perturbation, λ_i and elements of ΔA_1. The factor $W_i(j) * V_i(j)$ determines the contribution of j^{th} mode in i^{th} mode. The coefficients of (13) and (23) determines the interaction among the modes, λ_i. If there is no impact of k_{sys} on the interaction among the modes, then W_z^T, V_z will be identity matrices. W_z^T, V_z being identity matrices results in zero contribution of j^{th} mode in i^{th} mode for $i \neq j$.

Sensitivity analysis shows that eigenvectors of A holds the information about the variation in modes of a system. From the above derivation, it can be concluded that the eigenvectors of $\Lambda + \Delta A$ or $J + \Delta A$ holds the information about the interaction of modes of a system.

C. Influence of Interaction Among Modes due to Change in System Parameters

1) Influence on oscillation modes: The influence of this interaction on small signal stability of the system can be observed from the shift in modes of the system i.e. damping and frequency. From (13) and (23) it is identified that, the interaction between the modes also contribute to shift in modes of the system. So, the change in damping and frequency of oscillation modes (due to change in system parameters) is influenced by interaction between modes.

Consider a power system with n_e electromechanical modes, λ_{e_i}, n_{c1} type-1 CIG modes, λ_{c1_i}, and n_{c2} type-2 CIG modes, λ_{c2_i}. After changing the values of system parameters, the new set of modes can be written as (24). From (24), $\alpha_{e_i}, \alpha_{c1_i}$ and α_{c2_i} determine the interaction between the CIG and electromechanical modes of SG. This interaction results in change in participation of SG states in CIG modes.

$$\begin{aligned} \Xi_j = & \sum_{1}^{n_e} \alpha_{e_i}\lambda_{e_i} + \sum_{1}^{n_{c1}} \alpha_{c1_i}\lambda_{c1_i} \\ & + \sum_{1}^{n_{c2}} \alpha_{c2_i}\lambda_{c2_i} + f(\text{elements of } \Delta A_1) \end{aligned} \tag{24}$$

where, $\alpha_{e_i}, \alpha_{c1_i}$ and α_{c2_i} are the interaction coefficients; $\lambda_{e_i}, \lambda_{c1_i}$ and λ_{c2_i} are the interacting modes. In (24), the contribution of mode j in mode i is given by $\alpha_{j,i} = W_i(j) * V_i(j)$

The interaction coefficients in (24) quantifies the influence of system oscillation modes (λ_i) before perturbation on the system oscillation modes (Ξ_i) after the perturbation. So, the Feature 1 interaction discussed in section I can be captured by (24). In addition, in case of resonance in the system, the interaction coefficients of the modes participating in the resonance will have much higher value compared to the rest of modes. So, the Feature 2 interaction discussed in section I can also be captured by (24).

The sequence of eigenvalues Ξ_j depends on the eigenvectors chosen for diagonalizing the matrix $(\Lambda + \Delta A_1)$ or $(J + \Delta A_1)$. It is to be noted that the eigenvectors of a matrix are not unique. So, the sequence of the eigenvalues after diagonalizing $(\Lambda + \Delta A_1)$ or $(J + \Delta A_1)$ will vary depending on the eigenvectors. However, the sequence of the eigenvalues doesn't tamper with the inferences about interaction among modes

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of the system. It is also to be noted that, as the proposed method is based on a linearized model of the power system, the coefficients need to be recalculated when the operating point changes.

2) Influence on degree of participation of states in modes: From (7), (10) and (12), it can be seen that the new right eigenvector matrix of the system (ν) can be written as (25). It is noted from (26) that, the right eigenvector of any mode of system after considering change in k_{sys} can be written as linear combination of eigenvectors of the system before the change. As mentioned earlier, if there had been no interaction about modes, V_z will be an identity matrix resulting in no interaction in eigenvectors. The right eigenvector of a mode signifies the participation of states in the mode. This shows that the participation of states of the system in modes are affected by system parameters.

$$\nu = V * V_z \tag{25}$$

$$\implies [\nu(:,1)\ \nu(:,2)\dots\nu(:,m)] = [V(:,1)\ V(:,2)\dots V(:,m)] * V_z$$

$$\implies \nu(:,i) = \sum_j V(:,j) * V_z(j,i) \tag{26}$$

where, $V_z(j,i)$ is the element in j^{th} row and i^{th} column of matrix V_z.

The relation presented in (26) quantifies the influence of system mode shapes (V_i) before perturbation on the system mode shapes(ν_i) after the perturbation. The change in mode shapes of a power system can be analyzed by computing the V_z matrix as discussed in section II-B. So, the Feature 3 interaction discussed in section I can be captured by (26).

So, the amount of interaction can be determined by the coefficients of (24) which signifies the contribution of old modes in formation of new modes. High value of coefficients indicates strong interaction between modes. One more factor that determines the amount of interaction is the eigenvector relation derived in the paper. High value of coefficients $V_z(j,i)$ indicates strong interaction between the modes.

III. Results

In section I various applications of the proposed interaction coefficients are identified. Due to space constraints, in this section the results are presented for Application-1. The proposed coefficient (24) is used for quantifying the impact of PLL parameters on the interaction between electromechanical and CIG modes. The focus of this work is interaction among modes of the system, where modes are closely related to the frequency domain analysis. So, this paper mainly presents mathematical analysis and frequency domain results of the system to determine the validity of the proposed method. Time domain simulation results are also presented to further validate the proposed method.

To validate the proposed analysis technique, a modified IEEE-39 bus system is considered for simulation studies. Some of the SGs of IEEE-39 bus system are replaced by PV generators to represent the base case as shown in Fig. 1. In addition, one type-4 WG has also been added to demonstrate that the proposed method is also applicable for type-4 WG. Generation and load scenario of the system is presented in TABLE I. The SGs in the power system are equipped with a static AVR and PSS [26]. SGs are represented by model 1.1 [19] and the magnetic saturation in neglected. Parameters of SGs, AVR and PSS are taken from [26]. Two types of PV generators are considered in the present study, utility scale PV (PV-UT) and residential rooftop PV (PV-RT) generators. PV-UT generators can supply active and reactive power to the system and PV-RT generators can supply only active power [20]. PV-UT generators are equipped with PQ control strategy and modeled as in [20] and the parameters are employed from [27] with proper scaling. PV-RT generator represents an aggregated model of the roof-top PV generations in the area. PV-RT is modeled as constant negative PQ load, with reactive power equal to zero [27]. WGs considered in the system are type-4 wind generations (equipped with PQ control strategy) and are modeled as per [21]. One-mass drive train model of wind turbine in [21] is replaced by two-mass drive train model in [28]. Converter and controller parameters are adopted from [21] with appropriate scaling. All loads are modeled as constant PQ loads as given in [19]. Parameters of transmission lines are taken from [26]. The modified system is modeled as (2). MATLAB software is used for analysis of the system.

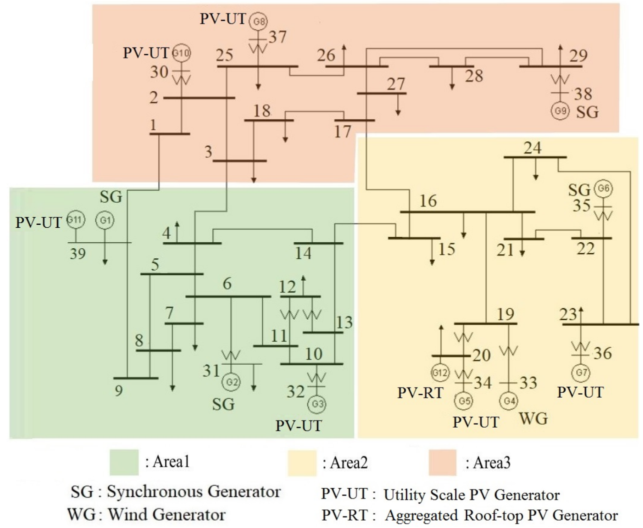


Fig. 1. Modified IEEE 39 bus system

TABLE I
Load-Generation Scenario

	Real Power(MW)	Percentage contribution
PV	2810	45.8
WG	830	13.5
SG	2500	40.7
Loads	6097	-
Losses	43	-

TABLE II
CRITICAL OSCILLATION MODES

With SRF-PLL			
S.No	Oscillation modes	Frequency of oscillation	Damping (%)
1	-2.04±j11.582	1.84	17.35
2	-2.164±j12.593	2.01	16.94
3	-2.219 ±j12.027	1.92	18.14
4	-2.284±j12.579	2.00	17.86
5	-2.41 ±j12.794	2.04	18.51
6	-0.137±j4.359	0.69	3.14
7	-0.326±j2.248	0.36	14.36
8	-2.58 ±j12.96	2.06	19.53
With MRF-PLL			
1	-1.751 ±j12.796	2.04	13.56
2	-1.699 ±j11.747	1.87	14.31
3	-1.867 ±j12.790	2.04	14.45
4	-1.836 ±j12.224	1.95	14.85
5	-1.971±j13.022	2.07	14.97
6	-2.135±j13.202	2.10	15.96
7	-0.137±j4.359	0.69	3.14

A. PLL and Critical Oscillation Modes

PLL is an integral block in most of the VSC based generations. The existing literature points out that the PLL has significant impact on the interactions in the power systems. So, the interaction analysis is carried out here considering perturbation in PLL parameters.

The parameters of PI blocks of PLL are chosen as $k_{p,pll} = 5$ and $k_{i,pll} = 170$ (bandwidth=3.14 Hz). The parameter w_p of MRF-PLL is chosen as 180 as per [29]. Equations of PLLs are linearized and combined with equations of PVs, WGs and SGs to formulate the state matrix of entire power system. Oscillation modes of the power system with percentage damping less than 20%, referred to as critical oscillation modes, are presented in TABLE II.

B. Interaction Among SG and PV Modes

As per analysis discussed in Section II, interaction among oscillation modes due to perturbation in $k_{p,pll}$ and $k_{i,pll}$ is calculated. The modes which change due perturbation of the PLL parameters are listed in TABLE III. As seen in Section II, the interaction among the modes is a factor contributing to the change in modes of the system. It is to be noted that, most of the modes which interact on perturbing $k_{p,pll}$ and $k_{i,pll}$ are type-2 CIG modes. Modes that contribute to change in (interact with) the 3^{rd} oscillation mode of TABLE III SRF-PLL case, are presented in Table IV (under column $With\ SRF-PLL$). The 3^{rd} oscillation mode is chosen for presenting results because the 3^{rd} mode is found to be interacting with all the categories of modes (Type-1, Type-2 and electromechanical). Interaction coefficients of the modes listed in Table III are calculated as per (24). The oscillation modes are designated as inter area and local modes (in Table III) by dividing the modified IEEE 39 bus system into three areas as given in [30]. $G1, G2, G3, G11$ are considered to be in $Area1$; $G4, G5, G6, G7, G12$ are considered to be in $Area2$; $G9, G8, G10$ are considered to be in $Area3$. The areas are demarcated using different colors in Fig. 1.

TABLE III
INTERACTING MODES

With SRF-PLL			
S.No	Oscillation mode	Damping (%)	Type of oscillation mode
1	-2.0402 ±j11.5819	17.35	Type 2
2	-2.1640 ±j12.5930	16.94	Type 2
3	-2.2192 ±j12.0277	18.14	Type 2
4	-2.2839 ±j12.5796	17.86	Type 2
5	-2.4105 ±j12.7940	18.51	Type 2
6	-2.58 ±j12.96	19.53	Type 1
With MRF-PLL			
1	-1.751 ±j12.796	13.56	Type 2
2	-1.699 ±j11.747	14.31	Type 2
3	-1.867 ±j12.790	14.45	Type 2
4	-1.836 ±j12.224	14.85	Type 2
5	-1.971±j13.022	14.97	Type 2
6	-2.135±j13.202	15.96	Type 2

TABLE IV
MODES THAT CONTRIBUTED TO CHANGE IN 3^{rd} MODE OF TABLE III

With SRF-PLL		
Oscillation mode	Type of oscillation mode	Interaction Coefficient
-2.2839 ±j12.5796	Type 2	1.022
-2.4105 ±j12.7940	Type 2	0.0136
-2.219 ±j 12.027	Type 2	0.0048
-14.599 ±j 6.425	Electromechanical	0.0013
-2.0402 ±j11.5819	Type 2	0.0010
-17.4517 ±j 4.3384	Type 1	0.0005
-5.678 ±j 5.441	Type 2	0.0003
With MRF-PLL		
Oscillation mode	Type of oscillation mode	Interaction Coefficient
-1.867 ±j12.790	Type 2	1.026
-1.751 ±j12.796	Type 2	0.0228
-1.836 ±j12.224	Type 2	0.004
-14.604 ±j 6.419	Electromechanical	0.0012
-1.699 ±j11.747	Type 2	0.001
-17.464 ±j 4.354	Type 1	0.0005

From Table III, it is to be noted that electromechanical modes of SG are not affected due to perturbation in $k_{p,pll}$ and $k_{i,pll}$. But Table IV shows that 3^{rd} oscillation mode interacts with an electromechanical mode. As discussed in Section II-C2, this interaction results in changing the participation of SG states in the 3^{rd} oscillation mode. In addition to this, interacting with type-2 modes may also result in change of participation from SG states as type-2 modes have participation from SG states. Similar analysis is carried out for the oscillation modes presented in Table III (under the column $With\ MRF-PLL$). The results for the 3^{rd} oscillation mode of Table III are presented in Table IV (under the column $With\ MRF-PLL$).

In TABLE IV, it can be seen that the interaction coefficient of the electromechanical mode is smaller than that of type 2 modes. However, it is to be noted that, presence of interaction among type 2 and electromechanical mode will increase the participation of SG states in the type 2 mode. This is because the electromechanical modes are dominated by SG states. So, the presence of interaction will influence the mode shape of

TABLE V
CHANGE IN THE DEGREE OF PARTICIPATION OF SG STATES IN THE INTERACTING MODES AS $k_{p,pll}$ IS VARIED

With SRF-PLL			With MRF-PLL		
	K_p=5 to 25			K_p=5 to 25	
S.No	No. of SG states	ΔSG PI	S.No	No. of SG states	ΔSG PI
1	24	0.0223	1	24	0.0133
2	27	0.2764	2	27	0.2271
3	31	0.4972	3	31	0.6968
4	20	0.4833	4	28	1.2623
5	11	-0.0043	5	10	-0.0053
6	2	0.0008	6	9	0.0846

TABLE VI
PARTICIPATION OF SG STATES IN THE MODES AS $k_{i,pll}$ IS VARIED

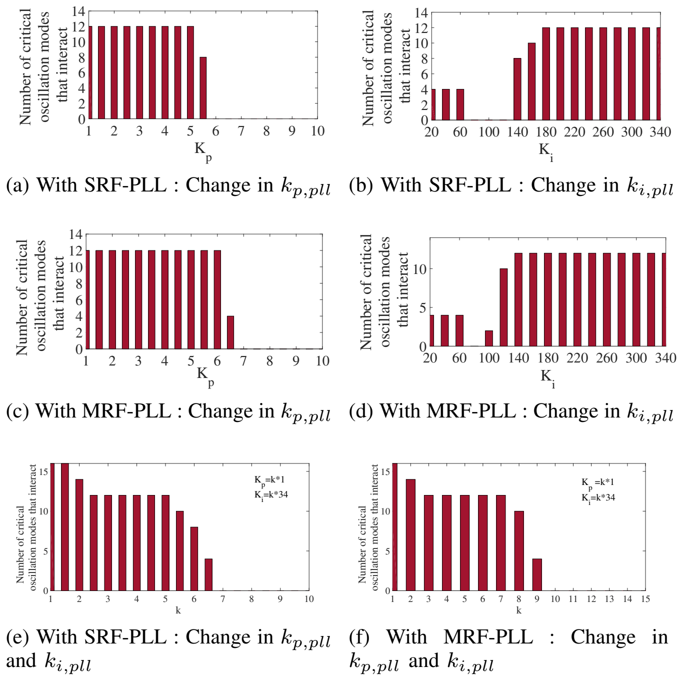
With SRF-PLL			With MRF-PLL		
	K_i=170 to 1700			K_i=170 to 1700	
S.No	No. of SG states	ΔSG PI	S.No	No. of SG states	ΔSG PI
1	29	0.005	1	29	0.0104
2	10	-0.0115	2	10	-0.0108
3	22	-0.028	3	22	-0.0261
4	20	-0.024	4	20	-0.0218
5	10	-0.005	5	9	-0.0049
6	2	-0.0009	6	2	-0.0009

type 2 oscillation mode (especially the elements corresponding to SG states in mode shape) even if the value is relatively small. When the mode shape of the studied type 2 mode is calculated for a shift from $k_{p,pll} = 5; k_{i,pll} = 170$ to $k_{p,pll} = 10; k_{i,pll} = 340$, it is identified that the elements corresponding to SG states in the mode shape have undergone a change of 20%. This implies that even the smaller value of interaction coefficient between type 2 mode and electromechanical mode can influence the mode shape of interacting type 2 mode. In addition, the operating frequency range of CIGs may overlap with stator transients and subsynchronous resonance frequencies [1]. When such overlap exists, the interaction coefficients corresponding to SG modes can be of the same order of interaction coefficients for CIG modes.

The results representing the increase in degree of participation of SG states in the modes impacted due to change in PLL parameters are presented here. The change in participation of SG states in the modes listed in Table III, when $k_{p,pll}$ is changed from 5 to 25 is presented in Table V. The change in degree of participation of SG states is also presented in Table V using $\Delta SG\ PI$ (Synchronous Generator Participation Index). The index $\Delta SG\ PI$ is calculated for i^{th} mode as given in (27).

$$\Delta SG\ PI_i = \sum^{SG} p_{ki}^{new} - \sum^{SG} p_{ki} \tag{27}$$

where, $\Delta SG\ PI_i$ denotes the change in participation of all SG



(a) With SRF-PLL : Change in $k_{p,pll}$ (b) With SRF-PLL : Change in $k_{i,pll}$
(c) With MRF-PLL : Change in $k_{p,pll}$ (d) With MRF-PLL : Change in $k_{i,pll}$
(e) With SRF-PLL : Change in $k_{p,pll}$ and $k_{i,pll}$ (f) With MRF-PLL : Change in $k_{p,pll}$ and $k_{i,pll}$

Fig. 2. Number of critical modes that interact

states in i^{th} mode; p_{ki}^{new} denotes the normalized participation factor of k^{th} state in i^{th} mode after the change in PLL parameters; p_{ki} denotes the normalized participation factor of k^{th} state in i^{th} mode of the system before the change in PLL parameters; $\sum^{SG} p_{ki}$ denotes the sum of p_{ki} of the states of all the SG units in the power system. The results obtained when $k_{i,pll}$ is changed from 170 to 1700 are presented in Table VI. From Table V and Table VI, it can be inferred that interaction between CIG modes and electromechanical modes will change the participation of SG states in CIG modes.

Comparing the columns $With\ SRF-PLL$ and $With\ MRF-PLL$ of Table V and Table VI, it is observed that SRF and MRF-PLLs have similar impact on interaction of modes. For both the PLLs, the proportional gain $k_{p,pll}$ has more impact on interaction than the integral gain $k_{i,pll}$. It is identified that for both the PLLs the change in participation factor is prominent in states corresponding to rotor frequency, field winding and exciter control of SGs.

C. Interaction of Critical Modes with Change in PLL Gain

As seen in Section III-B and Section III-D, participation of SG states in the interacting CIG modes is affected due to interaction of CIG modes with electromechanical modes of SG. Lesser the damping of CIG modes that interact with electromechanical modes of SG, larger will be the time taken for SG states to reach their steady state value, thus resulting in detrimental impact on system performance. Thus, the less damping ratio of CIG modes and their interaction with electromechanical modes of SG and type-2 modes cumulatively result in degraded system performance. This can be avoided by ensuring that the oscillation modes which interact with other modes are well damped.

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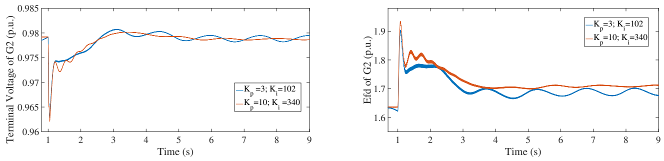

(a) Voltage at terminals of G2 (b) Efd of G2

Fig. 3. Time domain simulations

TABLE VII
RE PENETRATION LEVEL

RE generation	Synchronous generation	Description
60%	40%	Base case
70%	30%	High RE
50%	50%	Low RE

The percentage is with respect to total generation in the system

TABLE VIII
INTERACTING MODES AT 70% AND 50% RE PENETRATION LEVEL

With SRF-PLL		
70% RE penetration		
S.No	Oscillation mode	Type of oscillation mode
1	-1.8183 ±j11.6493	Type 2
2	-2.1197 ±j12.5547	Type 2
3	-2.2127 ±j12.6225	Type 2
4	-2.2827 ±j12.1770	Type 2
5	-2.4169 ±j12.7744	Type 2
6	-2.5816 ±j12.9711	Type 2
50% RE penetration		
1	-2.1813 ±j11.8104	Type 2
2	-2.2683 ±j12.1043	Type 2
3	-2.2975 ±j12.6724	Type 2
4	-2.3686 ±j12.6207	Type 2
5	-2.4265 ±j12.8269	Type 2
6	-2.5800 ±j12.9658	Type 2

Number of critical oscillation modes that interact with other oscillation modes are calculated for different values of $k_{p,pll}$ with $k_{i,pll} = 170$ for SRF-PLL (bandwidth varied from 1.4 Hz to 3.14 Hz), is shown in Fig. 2 (a). The result for different values of $k_{i,pll}$ with $k_{p,pll} = 5$ for SRF-PLL (bandwidth varied from 3.14 Hz to 10.2 Hz), is shown in Fig. 2 (b). Similar results for MRF-PLL, are presented in Fig. 2 (c) and Fig. 2 (d). The results obtained by varying $k_{p,pll}$ and $k_{i,pll}$ together, are shown in Fig. 2 (e) and 2 (f). From these Fig. 2, it is observed that for higher values of $k_{p,pll}$ and $k_{i,pll}$, there are no critical oscillation modes that interact with electromechanical or type-2 CIG modes. So, it is preferred to ensure that PLL parameters are tuned to values higher than that of dictated from Fig. 2.

D. Time Domain Simulations

Interaction of CIG modes with electromechanical modes of SG and participation factors of all states in the modes that are affected by PLL parameters are calculated. It is found that G2 states has increased participation in the critical CIG modes for lower values of $k_{p,pll}$ and $k_{i,pll}$. Time domain simulations are performed to validate the increased participation of SG states at low values of $k_{p,pll}$ and $k_{i,pll}$. Simulations are carried out on the modified IEEE-39 bus system considering SRF-PLL with $k_{p,pll} = 3$; $k_{i,pll} = 102$ and $k_{p,pll} = 10$; $k_{i,pll} = 340$. At $t = 1\ s$, load at bus 7 is suddenly increased and the observed responses are shown in Fig. 3. Terminal voltage of G2, a SG, is shown Fig. 3(a) and output signal of the exciter (Efd) of G2 is shown in Fig. 3(b). The analysis performed to calculate change in participation of SG states due to interaction of modes has pointed that the states of G2 corresponding to the field winding undergo prominent change in degree of participation. So, the time domain simulations for terminal voltage and Efd are presented in Fig. 3. The decreased damping of low frequency oscillations (frequency around $1\ Hz$) in figures, at $k_{p,pll} = 3$; $k_{i,pll} = 102$, validate the poor damping of the SG states at lower values of PLL parameters. As discussed in section III-B, due to interaction, it is observed that the participation of SG states in the CIG modes is relatively higher at lower values of PLL parameters. This proves that the interaction among modes can hamper the damping of otherwise well damped states of the power system.

The change in participation of field winding states in the CIG modes when $k_{p,pll}$ is changed from 3 to 10 and $k_{i,pll}$ is changed from 102 to 340 are obtained as 0.269, 0.089, 0.099, 0.214, 0.168 and 0.217 (in the order presented in TABLE III). As the CIG modes in TABLE III has damping less than 20%, it is expected that participating SG states also have less damping. To crosscheck the claim that the increased participation of SG states in the CIG modes results in reduced damping, time domain simulations are carried out. The obtained time domain simulation results for terminal voltage and E_{fd} of G2 are presented in Fig. 3. The decreased damping for $k_{p,pll} = 3; k_{i,pll} = 102$ compared to $k_{p,pll} = 10; k_{i,pll} = 340$ validate the poor damping of the SG states at higher participation in CIG modes.

E. Impact of Variation in PV Penetration Levels

To understand the impact of seasonal variation of PV on the impact of PLL, different RE penetration levels are investigated as shown in Table VII. The change in RE penetration is achieved by changing the PV generation level in the system. The analysis presented in section II is carried at different penetration levels of PV to investigate the impact of PLL on interaction of modes. Interaction between the modes is calculated as per proposed technique and the modes that interact are presented in Table VIII. It is noted from Table VIII and Table III that the number of modes which interact remain unaltered with change in penetration level of PV. The results obtained for 60% penetration level are compared with the results in section III-B (i.e. for 50% PV, referred to as base case) to analyze the impact of PV.

It is identified that the mode investigated in section III-B, $-2.2192 \pm j12.0277$ of Table III has changed to $-2.2827 \pm$

TABLE IX
MODES THAT CONTRIBUTED TO CHANGE IN 4^{th} MODE OF TABLE VIII

With SRF-PLL : 70% RE penetration		
Oscillation mode	Type of mode	Interaction Coefficient
-2.2127 ±j12.6225	Type 2	1.026
-2.283 ±j12.1770	Type 2	0.0612
-2.417 ±j12.7744	Type 2	0.0252
-1.818 ±j 11.649	Type 2	0.0072
-14.643 ±j 5.128	Electromechanical	0.0021
-17.637 ±j 4.3877	Type 1	0.0013
-4.824 ±j 0.861	Type 2	0.0010
-18.0574	Electromechanical	0.0010
-0.7828 ±j 6.1282	Electromechanical	0.0005
-61.95±j62.10	Type 2	0.0004
-0.4895 ±j 5.2715	Electromechanical	0.0003
-55.377	Type 2	0.0001

$j12.177$ (S.No 4 in Table VIII) for 60% penetration level due to change in PV penetration level. To determine the impact of PLL on interaction between modes at different PV penetration level, analysis (presented in Section II) is carried out for 4^{th} oscillation mode of Table VIII. The results of 4^{th} oscillation mode of system, (given in Table VIII and with 60% PV penetration level) are presented in Table IX. From Table IV and Table IX, it can be noticed that there is an increase in number of modes with which the mode $-2.2827 \pm j12.177$ of Table VIII interacts compared to that of base case mode $-2.2192 \pm j12.0277$. It can also be identified that the interaction coefficients also have increased. It is identified that the participation of SG state variables E_q', E_{fd} and ω in CIG modes increase with PV penetration. In addition, the participation of PV state variables δ_{pll}, x_{pll} and i_q in electromechanical modes increase with PV penetration

F. Comparison with existing methods

In this section, the performance of the proposed method is compared against two existing methods available in the literature. First method is the conventional mode sensitivity analysis approach and the second method is modal shift evaluation method presented in [15].

- *Conventional approach*: In conventional eigenvalue sensitivity analysis, the eigenvalue sensitivity is calculated from eigenvectors as shown in (4) and (5). However, the modes which are participating in the interaction can not be known from conventional approach. So, the quantification of interaction can not be done using conventional approach. In addition, the change in mode shapes can not be calculated from the conventional approach.
  In addition, the conventional eigenvalue sensitivity analysis considers a change in one element of the state matrix at a time. The proposed approach can analyze the impact of change in multiple system parameters at a time on the interaction among modes. The results obtained by changing two parameters viz., $k_{p_{pll}}$ and $k_{i_{pll}}$, are presented in TABLE III - IV and Fig. 2 (e-f).
- *Modal shift evaluation method presented in [15]*: In the modal shift evaluation method [15], the resonance index (RI) between electromechanical oscillation modes (EOMs) and PMSG oscillation modes (POMs) are calculated as shown in (28).

$$RI = \frac{|\lambda_{CEOM} - \lambda_{CPOM}|}{|\lambda_{EOM} - \lambda_{POM}|} \tag{28}$$

  where $|\lambda_{CEOM} - \lambda_{CPOM}|$ is the distance between closed-loop EOM and POM and $|\lambda_{EOM} - \lambda_{POM}|$ is the distance between open-loop EOM and POM. From the method in [15], the resonance between EOM and POMs can be calculated. However, the interaction among oscillation modes which are not in resonance can not be captured. In addition, the change in mode shapes can not be analyzed from the method in [15].

The proposed method can effectively determine the modes participating in the interaction and also quantify the interaction as derived in section IIB.

IV. DISCUSSIONS

In this paper, the proposed extension to eigenvalue sensitivity analysis is used to study the interaction between SG and CIG modes of a power system. With the growing penetration of power electronics into power systems, oscillation modes of the system are expected to be influenced by numerous factors and parameters. The proposed extension to eigenvalue sensitivity analysis can also be used to determine the impact of any parameter on the interaction between modes.

The present study has identified the presence of interaction between the CIG modes and electromechanical modes in PV dominated power systems. The studies presented in this paper identify that the poor damping observed in SG states at lower values of PLL parameters is due to the interaction between the modes of the system. It is observed that at lower values of PLL parameters, the participation of SG states is increased in poorly damped CIG modes. Hence the damping of oscillations in SG states is reduced at lower values of PLL parameters. The present study indicates that interaction between the modes should also be considered while tuning the PLL parameters. From the studies, it is observed that even though there is no direct influence of PLL parameters on electromechanical modes, the interaction results in change of participation of SG states in CIG modes. The interaction of type-2 modes (modes that have noticeable participation from CIG and SG states) also will result in change in participation of SG states. This interaction is problematic for system stability when the participation of SG states increases in CIG modes with critical damping. The oscillations in SG states (due to the participation) may result in activation of unnecessary controls which can be avoided if PLL is tuned appropriately.

Two different types of PLLs: SRF and MRF PLL are investigated in this study. It is observed that SRF and MRF PLLs have similar impact on interaction of modes. For both the PLLs, the proportional gain $k_{p,pll}$ has more impact on interaction than the integral gain $k_{i,pll}$. It is identified that for both the PLLs the change in participation factor is prominent in states corresponding to rotor frequency, field winding and exciter control of SGs.

As the proposed method is based on the small signal model of power system, the nonlinear interactions can not be captured by this method. However, the nonlinear modal analysis techniques face scalability and computation capability issues which makes it difficult to apply for power systems [18]. Hence in this work, linear modal analysis is chosen to analyze interaction among modes.

V. Conclusion

An extension to eigenvalue sensitivity analysis is proposed in order to quantify the interaction between any two modes in system and the impact of interactions on system stability. The impact of the interaction on degree of participation of state variables (states) in the oscillation modes is analyzed. The proposed analysis is performed on a modified IEEE-39 bus system. The impact of PLL dynamics on interaction of CIG modes and electromechanical modes of SG is studied. The analysis is carried out for two different types of PLLs viz., SRF-PLL and MRF-PLL. It is identified that for both the PLLs the interaction has major impact on SG states corresponding to rotor frequency, field winding and exciter control of SGs. It is also identified that the interaction among electromechanical and CIG modes is higher for lower bandwidth of PLL. The analysis is also carried out considering 40-60% penetration levels of PV. It is identified that PV penetration levels mainly impact the SG state variables E_q', E_{fd} and ω and the PV state variables δ_{pll}, x_{pll} and i_q.

Appendix A

The equations of SGs, CIGs and transmission lines are linearized to formulate state space equations as (29). The components with state space representation are grouped under category 1 in this work. The equations of constant PQ loads are linearized to formulate algebraic equations as (30).

$$\begin{aligned} \Delta \dot{X}_{cat1_n} &= A_{cat1_n}\ \Delta X_{cat1_n} + B_{cat1_n}\ \Delta V_{cat1_n}\ ; \\ \Delta I_{cat1_n} &= C_{cat1_n}\ \Delta X_{cat1_n} + D_{cat1_n}\ \Delta V_{cat1_n} \end{aligned} \tag{29}$$

where X_{cat1_n} denotes states; $A_{cat1_n},\ B_{cat1_n},\ C_{cat1_n},\ D_{cat1_n}$ denotes the state, control, output and feed-forward matrices respectively of the n^{th} category 1 component.

$$\Delta I_{PQl_n} = Y_{PQl_n}\ \Delta V_{PQl_n} \tag{30}$$

where Y_{PQl_n} denotes the admittance matrix of the n^{th} load; V_{cat1_n}, V_{PQl_n} are the vectors with Q and D components of terminal voltage and I_{cat1_n}, I_{PQl_n} are the vectors with Q and D components of current injection of the n^{th} category 1 component and constant PQ load respectively.

Equations of all category 1 components and loads are concatenated to formulate (31) and (32).

$$\Delta \dot{X}_{sys} = A_1\ \Delta X_{sys} + B_1\ \Delta V_{cat1}\ ; \tag{31a}$$

$$\Delta I_{cat1} = C_1\ \Delta X_{sys} + D_1\ \Delta V_{cat1} \tag{31b}$$

$$\Delta I_{PQl} = Y_{PQl}\ \Delta V_{PQl} \tag{32}$$

Current injections to the buses from all the components of power system can be written as (33). ΔI_{bus} denotes the vector of bus current injections. Substituting (31b) and (32) in (33), voltages of all buses in network are expressed in terms of X_{sys} of the form (34). X_{sys} is the vector of all state variables and V_{sys} denotes the vector of bus voltages of the power system. Substituting (34) in (31a) will result in the state equation of whole power system as shown in (35).

$$\Delta I_{bus} = \sum \Delta I_{cat1} + \sum \Delta I_{PQl} \tag{33}$$

$$\Delta V_{sys} = A_2\ \Delta X_{sys} \tag{34}$$

$$\Delta \dot{X}_{sys} = A\ \Delta X_{sys} \tag{35}$$

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