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REAL HYPERSURFACES OF A NONFLAT COMPLEX SPACE FORM IN TERMS OF THE RICCI TENSOR

By

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Abstract. We know the fact that there are no real hypersurfaces with parallel Ricci tensors in a nonflat complex space form (cf. [5]). In this paper we investigate real hypersurfaces in a nonflat complex space form using some conditions of the Ricci tensor S which are weaker than $\nabla S = 0$. We characterize Hopf hypersurfaces of a nonflat complex space form.

0 Introduction

A Kähler manifold of constant holomorphic sectional curvature c is called a complex space form, which is denoted by $M_n(c)$. A complete and simply connected complex space forms are isometric to a complex projective space $\boldsymbol{C}P_n$, a complex Euclidean space $\boldsymbol{C}^n$ or a complex hyperbolic space $\boldsymbol{C}H_n$ as $c > 0$, $c = 0$ or $c < 0$.

Let M be a real hypersurface of $M_n(c)$. Then M has an almost contact metric structure (ϕ, ξ, η, g) induced from the complex structure J and the Kähler metric of $M_n(c)$ (for details see § 1). The structure vector ξ is said to be principal if $A\xi = \alpha\xi$ is satisfied, where A is the shape operator of M and $\alpha = \eta(A\xi)$. A real hypersurface is said to be a Hopf hypersurface if the structure vector ξ of M is principal.

Typical examples of real hypersurfaces in $\boldsymbol{C}P_n$ are homogeneous ones which are orbits under subgroups of $\mathrm{PU}(n+1)$. The complete classification of them was obtained by Takagi [10] as follows:

THEOREM T [10]. *Let M be a homogeneous real hypersurface of $\boldsymbol{C}P_n$. Then M is a tube of radius r over one of the following Kähler submanifolds*:

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(A_1) *a hyperplane* $\boldsymbol{C}P_{n-1}$, *where* $0 < r < \frac{\pi}{2}$,
(A_2) *a totally geodesic* $\boldsymbol{C}P_k$ $(1 \le k \le n-2)$, *where* $0 < r < \frac{\pi}{2}$,
(B) *a complex quadric* Q_{n-1}, *where* $0 < r < \frac{\pi}{4}$,
(C) $\boldsymbol{C}P_1 \times \boldsymbol{C}P_{(n-1)/2}$, *where* $0 < r < \frac{\pi}{4}$ *and* $n \ge 5$ *is odd,*
(D) *a complex Grassmann* $G_{2,5}\boldsymbol{C}$, *where* $0 < r < \frac{\pi}{4}$ *and* $n = 9$,
(E) *a Hermitian symmetric space* SO(10)/U(5), *where* $0 < r < \frac{\pi}{4}$ *and* $n = 15$.

Also Berndt [1] classified all Hopf real hypersurfaces in $\boldsymbol{C}H_n$ with constant principal curvatures as follows:

THEOREM B [1]. *Let M be a real hypersurface of* $\boldsymbol{C}H_n$. *Then M has constant principal curvatures and* ξ *is principal if and only if M is locally congruent to one of the following*:

(A_0) *a self-tube, that is, a horosphere,*
(A_1) *a geodesic hypersphere, or a tube over a hyperplane* $\boldsymbol{C}H_{n-1}$,
(A_2) *a tube over a totally geodesic* $\boldsymbol{C}H_k$ $(1 \le k \le n-2)$,
(B) *a tube over a totally real hyperbolic space* $\boldsymbol{R}H_n$.

Let ∇ and S be the Levi-Civita connection and the Ricci tensor of M, respectively. There are many studies about Ricci tensors of real hypersurfaces (cf. [2], [3], [4], [5], [6], [7], [8], [9]). Very important fact is that there are no real hypersurfaces with parallel Ricci tensors S (that is, $\nabla_X S = 0$ for each vector field X tangent to M) in $M_n(c)$, $c \neq 0$, $n \ge 3$ (cf. [5]). Especially, there exist no Einstein real hypersurfaces M in $M_n(c)$, $c \neq 0$, $n \ge 3$. So, it is natural to investigate real hypersurfaces M by using some conditions (on the derivatives of S) which are weaker than $\nabla S = 0$.

Recently, the first author, Hwang and Kim proved the following theorem:

THEOREM 0.1. *Let M be a real hypersurface in a nonflat complex space form. If the Ricci tensor S of M satisfies* $\nabla_\xi S = 0$, $\nabla_{\phi\nabla_\xi\xi} S = 0$ *and* $S\xi = g(S\xi, \xi)\xi$, *then M is locally congruent to one of the homogeneous real hypersurfaces of Theorem T and Theorem B.*

In this paper we pay particular attention to the fact that for each Hopf hypersurface M in $M_n(c)$, $c \neq 0$ the characteristic vector ξ of M is an eigenvector of the Ricci tensor S of M. So it is natural to consider a problem that if the vector ξ

is an eigenvector of the Ricci tensor S of a real hypersurface M in $M_n(c)$, $c \neq 0$, is M a Hopf hypersurface?

The purpose of this paper is to eatablish the following theorem which gives a partial answer to this problem:

THEOREM 4.1. *Let M be a real hypersurface in $M_n(c)$, $c > 0$. If it satisfies $\nabla_{\phi\nabla_\xi\xi}S = 0$ and at the same time satisfies $S\xi = \sigma\xi$ for some constant σ, then M is a Hopf hypersurface.*

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1 Preliminaries

Let M be a real hypersurface immersed in a complex space form $(M_n(c), G)$ with almost complex structure J and the Kähler metric G of constant holomorphic sectional curvature c, and let C be a unit normal vector field on M. The Riemannian connection $\tilde{\nabla}$ in $M_n(c)$ and ∇ in M are related by the following formulas for any vector fields X and Y on M:

$$\tilde{\nabla}_Y X = \nabla_Y X + g(AY, X)C, \tag{1.1}$$

$$\tilde{\nabla}_X C = -AX, \tag{1.2}$$

where g denotes the Riemannian metric on M induced from that G of $M_n(c)$ and A is the shape operator of M in $M_n(c)$. An eigenvector X of the shape operator A is called a principal curvature vector. Also an eigenvalue λ of A is called a principal curvature. It is known that M has an almost contact metric structure induced from the almost complex structure J on $M_n(c)$, that is, we define a tensor field ϕ of type $(1,1)$, a vector field ξ, an 1-form η on M by $g(\phi X, Y) = G(JX, Y)$ and $g(\xi, X) = \eta(X) = G(JX, C)$. Then we have

$$\phi^2 X = -X + \eta(X)\xi, \quad g(\xi, \xi) = 1, \quad \phi\xi = 0. \tag{1.3}$$

From (1.1) we see that

$$(\nabla_X \phi)Y = \eta(Y)AX - g(AX, Y)\xi, \tag{1.4}$$

$$\nabla_X \xi = \phi AX. \tag{1.5}$$

Since the ambient space is of constant holomorphic sectional curvature c, equations of the Gauss and Codazzi are respectively given by

$$R(X, Y)Z = \frac{c}{4}\{g(Y, Z)X - g(X, Z)Y + g(\phi Y, Z)\phi X - g(\phi X, Z)\phi Y - 2g(\phi X, Y)\phi Z\} + g(AY, Z)AX - g(AX, Z)AY, \tag{1.6}$$

$$(\nabla_X A)Y - (\nabla_Y A)X = \frac{c}{4}\{\eta(X)\phi Y - \eta(Y)\phi X - 2g(\phi X, Y)\xi\} \tag{1.7}$$

for any vector fields X, Y and Z on M, where R denotes the Riemannian curvature tensor of M. We shall denote the Ricci tensor of type $(1,1)$ by S. Then it follows from (1.6) that

$$SX = \frac{c}{4}\{(2n+1)X - 3\eta(X)\xi\} + hAX - A^2X, \tag{1.8}$$

where $h = \operatorname{trace} A$. Further, using (1.5), we obtain

$$(\nabla_X S)Y = -\frac{3}{4}c\{g(\phi AX, Y)\xi + \eta(Y)\phi AX\} + (Xh)AY + (hI - A)(\nabla_X A)Y - (\nabla_X A)AY, \tag{1.9}$$

where I is the identity map.

To write our formulas in convention forms, we denote $\alpha = \eta(A\xi)$, $\beta = \eta(A^2\xi)$, $\mu^2 = \beta - \alpha^2$ and ∇f by the gradient vector field of a function f on M. In the following, we use the same terminology and notation as above unless otherwise stated.

If we put $U = \nabla_\xi \xi$, then U is orthogonal to the structure vector field ξ. Then it is, using (1.3) and (1.5), seen that

$$\phi U = -A\xi + \alpha\xi, \tag{1.10}$$

which shows that $g(U, U) = \beta - \alpha^2$. By the definition of U, (1.3) and (1.5) it is verified that

$$g(\nabla_X \xi, U) = g(A^2\xi, X) - \alpha g(A\xi, X). \tag{1.11}$$

Now, differentiating (1.10) covariantly along M and using (1.4) and (1.5), we find

$$\eta(X)g(AU + \nabla\alpha, Y) + g(\phi X, \nabla_Y U) = g((\nabla_Y A)X, \xi) - g(A\phi AX, Y) + \alpha g(A\phi X, Y), \tag{1.12}$$

which enables us to obtain

$$(\nabla_\xi A)\xi = 2AU + \nabla\alpha \tag{1.13}$$

because of (1.7). From (1.12) we also have

$$\nabla_\xi U = 3\phi A U + \alpha A\xi - \beta\xi + \phi\nabla\alpha, \tag{1.14}$$

where we have used (1.3), (1.5) and (1.11).

If $A\xi - g(A\xi,\xi)\xi \neq 0$, then we can put

$$A\xi = \alpha\xi + \mu W, \tag{1.15}$$

where W is a unit vector field orthogonal to ξ. Then from (1.10) it is seen that $U = \mu\phi W$ and hence $g(U,U) = \mu^2$, and W is also orthogonal to U. Thus, we see, making use of (1.5), that

$$\mu g(\nabla_X W, \xi) = g(AU, X). \tag{1.16}$$

2 Real Hypersurfaces Satisfying $S\xi = g(S\xi,\xi)\xi$

Let M be a real hypersurface of a nonflat complex space form $M_n(c)$. If it satisfies

$$S\xi = g(S\xi,\xi)\xi, \tag{2.1}$$

then we have by (1.8)

$$A^2\xi = hA\xi + (\beta - h\alpha)\xi, \tag{2.2}$$

where we have put $g(S\xi,\xi) = \sigma$,

$$\beta - h\alpha = \frac{c}{2}(n-1) - \sigma. \tag{2.3}$$

In what follows we assume that $\mu \neq 0$ on M, that is, ξ is not a principal curvature vector field and we put $\Omega = \{p \in M \mid \mu(p) \neq 0\}$. Then Ω is an open subset of M, and from now on we discuss our arguments on Ω.

From (1.15) and (2.2), we see that

$$AW = \mu\xi + (h-\alpha)W \tag{2.4}$$

and hence

$$A^2W = hAW + (\beta - h\alpha)W \tag{2.5}$$

because of $\mu \neq 0$.

Now, differentiating (2.4) covariantly along Ω, we find

$$(\nabla_X A)W + A\nabla_X W = (X\mu)\xi + \mu\nabla_X\xi + X(h-\alpha)W + (h-\alpha)\nabla_X W. \tag{2.6}$$

By taking the inner product with W in the last equation, we obtain

$$g((\nabla_X A)W, W) = -2g(AX, U) + Xh - X\alpha \tag{2.7}$$

since W is a unit vector field orthogonal to ξ. We also have by applying ξ to (2.6)

$$\mu g((\nabla_X A)W, \xi) = (h - 2\alpha)g(AU, X) + \mu(X\mu), \tag{2.8}$$

where we have used (1.16), which together with the Codazzi equation (1.7) gives

$$\mu(\nabla_W A)\xi = (h - 2\alpha)AU - \frac{c}{2}U + \mu\nabla\mu, \tag{2.9}$$

$$\mu(\nabla_\xi A)W = (h - 2\alpha)AU - \frac{c}{4}U + \mu\nabla\mu. \tag{2.10}$$

Replacing X by ξ in (2.6) and taking account of (2.10), we find

$$(h - 2\alpha)AU - \frac{c}{4}U + \mu\nabla\mu + \mu\{A\nabla_\xi W - (h - \alpha)\nabla_\xi W\}$$
$$= \mu(\xi\mu)\xi + \mu^2 U + \mu(\xi h - \xi\alpha)W. \tag{2.11}$$

By the way, from $\phi U = -\mu W$ we have

$$g(AU, X)\xi - \phi\nabla_X U = (X\mu)W + \mu\nabla_X W.$$

Replacing X by ξ in this and using (1.10) and (1.14), we get

$$\mu\nabla_\xi W = 3AU - \alpha U + \nabla\alpha - (\xi\alpha)\xi - (\xi\mu)W, \tag{2.12}$$

which implies

$$W\alpha = \xi\mu. \tag{2.13}$$

From the last equations, it follows that

$$3A^2U - 2hAU + A\nabla\alpha + \frac{1}{2}\nabla\beta - h\nabla\alpha + \left(\alpha h - \beta - \frac{c}{4}\right)U$$
$$= 2\mu(W\alpha)\xi + \mu(\xi h)W - (h - 2\alpha)(\xi\alpha)\xi, \tag{2.14}$$

which enables us to obtain

$$\xi\beta = 2\alpha(\xi\alpha) + 2\mu(W\alpha). \tag{2.15}$$

Differentiating (2.2) covariantly and making use of (1.5), we get

$$(\nabla_X A)A\xi + A(\nabla_X A)\xi + A^2\phi AX - hA\phi AX$$
$$= (Xh)A\xi + h(\nabla_X A)\xi + X(\beta - h\alpha)\xi + (\beta - h\alpha)\phi AX, \tag{2.16}$$

which together with (1.7) implies that

$$\begin{aligned}
&\frac{c}{4}\{u(Y)\eta(X)-u(X)\eta(Y)\}+\frac{c}{2}(h-\alpha)g(\phi Y,X)-g(A^2\phi AX,Y)\\
&\quad+g(A^2\phi AY,X)+2hg(\phi AX,AY)-(\beta-h\alpha)\{g(\phi AY,X)-g(\phi AX,Y)\}\\
&=g(AY,(\nabla_X A)\xi)-g(AX,(\nabla_Y A)\xi)+(Yh)g(A\xi,X)-(Xh)g(A\xi,Y)\\
&\quad+Y(\beta-h\alpha)\eta(X)-X(\beta-h\alpha)\eta(Y),
\end{aligned}\tag{2.17}$$

where we have defined an 1-form u by $u(X)=g(U,X)$ for any vector field X. If we replace X by μW to the both sides of (2.17) and take account of (1.13), (2.4), (2.5), (2.8) and (2.9), then we obtain

$$\begin{aligned}
&(3\alpha-2h)A^2U+2\left(h^2+\beta-2h\alpha+\frac{c}{4}\right)AU+(h-\alpha)\left(\beta-h\alpha-\frac{c}{2}\right)U\\
&\quad=\mu A\nabla\mu+(\alpha h-\beta)\nabla\alpha-\frac{1}{2}(h-\alpha)\nabla\beta+\mu^2\nabla h\\
&\qquad-\mu(Wh)A\xi-\mu W(\beta-h\alpha)\xi.
\end{aligned}\tag{2.18}$$

Using (1.15), the equation (2.16) can be written as

$$\begin{aligned}
&A(\nabla_X A)\xi+(\alpha-h)(\nabla_X A)\xi+\mu(\nabla_X A)W\\
&\quad=(Xh)A\xi+X(\beta-h\alpha)\xi+(\beta-h\alpha)\phi AX+hA\phi AX-A^2\phi AX.
\end{aligned}$$

Thus, replacing X by $\alpha\xi+\mu W$ in this and making use of (1.5), (1.13), (1.15) and (2.7)–(2.9), we find

$$\begin{aligned}
&2hA^2U+2\left(\alpha h-\beta-h^2-\frac{c}{4}\right)AU+\left(h^2\alpha-h\beta+\frac{c}{2}h-\frac{3}{4}c\alpha\right)U\\
&\quad=g(A\xi,\nabla h)A\xi-\frac{1}{2}A\nabla\beta+\frac{1}{2}(h-2\alpha)\nabla\beta+\beta\nabla\alpha\\
&\qquad-\mu^2\nabla h+g(A\xi,\nabla(\beta-h\alpha))\xi.
\end{aligned}\tag{2.19}$$

3 Real Hypersurfaces Satisfying $\nabla_{\phi\nabla_\xi\xi}S=0$ and $S\xi=g(S\xi,\xi)\xi$

We continue now, our arguments under the same hypothesis $S\xi=g(S\xi,\xi)\xi$ as in section 2. Furthermore, suppose that $\nabla_{\phi\nabla_\xi\xi}S=0$, that is, $\nabla_W S=0$ since we now suppose that $\mu\neq 0$.

Then, by replacing X by W, we have from (1.9)

$$-\frac{3}{4}c(h-\alpha)(u(Y)\xi+\eta(Y)U)+\mu(Wh)AY+\mu h(\nabla_W A)Y$$

$$=\mu A(\nabla_W A)Y-\mu(\nabla_W A)AY, \tag{3.1}$$

where we have used (1.5) and (2.4). If we replace Y by W and make use of (2.7) and (2.9), then we find

$$(Wh)AW=hAU-\frac{c}{2}U-2A^2U+\frac{1}{2}\nabla\beta-\alpha\nabla h+A\nabla h-A\nabla\alpha \tag{3.2}$$

because of $\mu\neq 0$.

In the following we assume that σ is constant on M and then $\beta-h\alpha=$ constant. In this case we notice here that the following fact:

REMARK 3.1. *$h-\alpha\neq 0$ on Ω.*

In fact, if not, then we have $h=\alpha$ and hence $\beta-\alpha^2=$ constant, because $\sigma=$ constant. Thus (3.2) implies $Wh=W\alpha=0$ and hence

$$2A^2U=\alpha AU-\frac{c}{2}U. \tag{3.3}$$

Further, (2.14) and (2.18) turns out respectively to

$$2A^2U-2\alpha AU+\left(\alpha^2-\beta-\frac{c}{4}\right)U=-A\nabla\alpha+(\xi\alpha)A\xi, \tag{3.4}$$

$$\alpha A^2U+2\left(\beta-\alpha^2+\frac{c}{4}\right)AU=0. \tag{3.5}$$

It is, using (3.3)–(3.5), verified that $\alpha\neq 0$ on this set.

Combining (3.3) with (3.5), we see that

$$\alpha AU=2\left(\alpha^2-\beta-\frac{c}{4}\right)U \tag{3.6}$$

and thus $AU=\nu U$ because of $\alpha\neq 0$, where we have put

$$\alpha\nu=2\left(\alpha^2-\beta-\frac{c}{4}\right). \tag{3.7}$$

From this and (3.3), we obtain

$$\nu^2 + \beta - \alpha^2 + \frac{c}{2} = 0. \tag{3.8}$$

Therefore $\nu = \text{constant} \neq 0$ because of (3.3). Hence it is, using (3.7), seen that $\alpha = \text{constant}$ and thus

$$3\nu^2 - 2\alpha\nu + \alpha^2 - \beta - \frac{c}{4} = 0,$$

which together with (3.7) and (3.8), produces a contradiction. Consequently $h - \alpha \neq 0$ on Ω is proved. In what follows we assume that $h - \alpha \neq 0$ is satisfied everywhere.

Differentiating (2.1) covariantly, we find

$$(\nabla_X S)\xi + S\nabla_X \xi = \sigma \nabla_X \xi$$

because $\sigma = \text{constant}$ is assumed, which together with hypothesis $\nabla_W S = 0$ yields

$$S\nabla_W \xi = \sigma \nabla_W \xi. \tag{3.9}$$

By the way we have $\mu\nabla_W \xi = (h - \alpha)U$ with the aid of (1.5) and (2.4), (3.9) implies $SU = \sigma U$ because of Remark 3.1. Hence (1.8) leads to

$$A^2 U = hAU + \left(\beta - h\alpha + \frac{3}{4}c\right)U. \tag{3.10}$$

From (2.3) we have

$$\nabla\beta = \alpha\nabla h + h\nabla\alpha. \tag{3.11}$$

Thus (2.15) is reduced to

$$2\mu(W\alpha) = (h - 2\alpha)(\xi\alpha) + \alpha(\xi h). \tag{3.12}$$

Using (1.15), (3.10) and (3.12), the equation (2.14) turns out to be

$$hAU + 2(\beta - h\alpha + c)U = (\xi h)A\xi - A\nabla\alpha + h\nabla\alpha - \frac{1}{2}\nabla\beta. \tag{3.13}$$

From (2.19) and (2.10), we also find

$$\left(2\beta - 2h\alpha + \frac{c}{2}\right)AU + \left\{h(h\alpha - \beta) + \frac{c}{4}(3\alpha - 8h)\right\}U + g(A\xi, \nabla h)A\xi$$
$$= \frac{1}{2}A\nabla\beta - \beta\nabla\alpha + \left(\alpha - \frac{1}{2}h\right)\nabla\beta + \mu^2\nabla h. \tag{3.14}$$

Because of (3.2) and (3.10), we see that

$$(Wh)AW = -hAU - 2(\beta - h\alpha + c)U + A\nabla h - A\nabla\alpha + \frac{1}{2}\nabla\beta - \alpha\nabla h,$$

which together with (3.10) and (3.11) gives

$$A\nabla h = (Wh)AW + (\xi h)A\xi. \tag{3.15}$$

Making use of (3.13) and (3.15), we have from (3.14)

$$\begin{aligned}(4\beta - 4h\alpha + h^2 + c)AU + \left(\frac{3}{2}c\alpha - 2ch\right)U \\ = \alpha(Wh)AW - \{(\alpha - h)(\xi h) + 2\mu(Wh)\}A\xi \\ + \left(2\alpha h - 2\beta - \frac{1}{2}h^2\right)\nabla\alpha + \left(2\beta - \frac{3}{2}h\alpha\right)\nabla h.\end{aligned} \tag{3.16}$$

If we use (2.2), (2.5) and (3.10), then above equation implies

$$\begin{aligned}\frac{3}{4}c\left\{(4\beta - 4h\alpha + h^2 + c)AU + \left(\frac{3}{2}c\alpha - 2ch\right)U\right\} \\ = \left(2\alpha h - 2\beta - \frac{1}{2}h^2\right)\{A^2\nabla\alpha - hA\nabla\alpha - (\beta - h\alpha)\nabla\alpha\} \\ + \left(2\beta - \frac{3}{2}h\alpha\right)\{A^2\nabla h - hA\nabla h - (\beta - h\alpha)\nabla h\},\end{aligned}$$

which together with (3.15) yields

$$\begin{aligned}\frac{3}{4}c\left\{(4\beta - 4h\alpha + h^2 + c)AU + \frac{c}{2}(3\alpha - 4h)U\right\} \\ = \left(2\alpha h - 2\beta - \frac{1}{2}h^2\right)\{A^2\nabla\alpha - hA\nabla\alpha - (\beta - h\alpha)\nabla\alpha\} \\ + \left(2\beta - \frac{3}{2}h\alpha\right)(\beta - h\alpha)\{(Wh)W + (\xi h)\xi - \nabla h\}.\end{aligned} \tag{3.17}$$

On the other hand, we have from (3.13)

$$\begin{aligned}A^2\nabla\alpha - hA\nabla\alpha + (h^2 + 2\beta - 2h\alpha + 2c)AU + h\left(\beta - h\alpha + \frac{3}{4}c\right)U \\ = (\xi h)A^2\xi - \frac{1}{2}A\nabla\beta,\end{aligned}$$

where we have used (3.10), or using (3.11) and (3.14),

$$\begin{aligned}
&A^2\nabla\alpha - hA\nabla\alpha + (\beta - h\alpha)\nabla\alpha \\
&\quad = \left(4h\alpha - 4\beta - h^2 - \frac{5}{2}c\right)AU + \frac{c}{4}(5h - 3\alpha)U \\
&\qquad - \frac{1}{2}h^2\nabla\alpha + \left(\beta - \frac{1}{2}h\alpha\right)\nabla h + (\xi h)A^2\xi - g(A\xi, \nabla h)A\xi. \qquad (3.18)
\end{aligned}$$

If we take the inner product ξ with this and make use of (1.15) and (2.2), then we obtain

$$\mu\alpha(Wh) = \left(2h\alpha - 2\beta - \frac{1}{2}h^2\right)(\xi\alpha) + \left(2\beta - \frac{1}{2}h\alpha - \alpha^2\right)(\xi h). \qquad (3.19)$$

Substituting (3.18) into (3.17) and taking account of (3.16), we find

$$\begin{aligned}
&\frac{3}{2}c\left\{cAU + \frac{c}{2}(3\alpha - 4h)U + (h - \alpha)\left(2\alpha h - 2\beta - \frac{1}{2}h^2\right)U\right\} \\
&\quad = h(h - \alpha)(\beta - h\alpha)\{\nabla h - (\xi h)\xi - (Wh)W\}. \qquad (3.20)
\end{aligned}$$

Applying A to both sides of this and using (3.10) and (3.15), we have

$$\left\{\frac{c}{2}(3\alpha - 2h) + (h - \alpha)\left(2\alpha h - 2\beta - \frac{1}{2}h^2\right)\right\}AU + c\left(\beta - h\alpha + \frac{3}{4}c\right)U = 0. \qquad (3.21)$$

Lemma 3.1. *Let M be a real hypersurface of $M_n(c)$ $(c \neq 0)$. If it satisfies $\nabla_W S = 0$ and $S\xi = \sigma\xi$ for some constant σ, then we have*

$$AU = \lambda U \qquad (3.22)$$

on Ω, where $\mu^2\lambda = g(AU, U)$.

Proof. Let Ω_0 be a set of points in M such that $\|AU - \lambda U\| \neq 0$ on Ω and suppose that Ω_0 be nonempty. If $\beta - h\alpha + \frac{3}{4}c \neq 0$, then we have from (3.21)

$$\frac{c}{2}(3\alpha - 2h) + (h - \alpha)\left(2\alpha h - 2\beta - \frac{1}{2}h^2\right) \neq 0$$

and hence (3.22) is valid. Thus it is, using (3.21), seen that

$$\beta - h\alpha + \frac{3}{4}c = 0 \qquad (3.23)$$

and therefore $h(h^2 - \alpha h - c) = 0$ on Ω_0. So we have

$$h^2 - \alpha h - c = 0 \tag{3.24}$$

on Ω_0. In fact, if not, then we have $h = 0$. Thus (3.10) and (3.23) are respectively to

$$AU = 0, \quad \beta + \frac{3}{4}c = 0.$$

Hence (3.13) becomes $2(\beta + c)U + A\nabla\alpha = 0$. But, by (3.14) we have $\nabla\alpha = \alpha U$. Combining the last two equations, we obtain $\beta + c = 0$, a contradiction. Thus (3.24) is accomplished.

Differentiating (3.24), and using (3.23), we find

$$2h\nabla h = \alpha\nabla h + h\nabla\alpha = \nabla\beta. \tag{3.25}$$

From this and (3.15) we obtain

$$A\nabla\beta = 2h\{(Wh)AW + (\xi h)A\xi\}. \tag{3.26}$$

If we take account of (3.23)–(3.26), then (3.14) turns out to be

$$-cAU + \frac{c}{4}(3\alpha - 5h)U = (h - \alpha)(\xi h)A\xi - \mu(Wh)A\xi + h(Wh)AW$$
$$+ (\mu^2 + \alpha h - c)\nabla h - \beta\nabla\alpha. \tag{3.27}$$

On the other hand, we have from (3.13)

$$h^2AU + \frac{c}{2}hU = (\alpha - h)(\xi h)A\xi + (\alpha - 2h)(Wh)AW + c\nabla h$$

because of (3.24)–(3.26). Comparing with the last two equations, it follows that

$$(h^2 - c)AU + \frac{3}{4}c(\alpha - h)U$$
$$= (\alpha - h)(Wh)AW - \mu(Wh)A\xi + (\beta - \alpha^2 + \alpha h)\nabla h - \beta\nabla\alpha.$$

Applying this by hA and making use of (2.2), (2.5) and (3.23), we find

$$\left\{h^2(h^2 - c) + \frac{3}{4}ch(\alpha - h)\right\}AU$$
$$= h(\alpha - h)(Wh)\left\{hAW - \frac{3}{4}cW\right\} - \mu h(Wh)\left(hA\xi - \frac{3}{4}c\xi\right)$$
$$+ h(\beta - \alpha^2 + \alpha h)A\nabla h - \beta hA\nabla\alpha,$$

which together with (3.15) and (3.23)–(3.25) implies that

$$\left\{(\alpha h+c)\alpha h-\frac{3}{4}c^2\right\}AU$$

$$=h(\alpha-h)(Wh)\left(hAW-\frac{3}{4}cW\right)-\mu h(Wh)\left(hA\xi-\frac{3}{4}c\xi\right)$$

$$+\frac{3}{4}c(h-\alpha)\{(Wh)AW+(\xi h)A\xi\}. \tag{3.28}$$

Furthermore, using (2.2) and (2.5), we have from (3.28)

$$\left\{(\alpha h+c)\alpha h-\frac{3}{4}c^2\right\}AU=0$$

because U is orthogonal to ξ and W. Hence we have

$$(\alpha^3+c\alpha)h+c\alpha^2-\frac{3}{4}c^2=0$$

on Ω_0. Since $c\neq 0$, it follows that

$$h=\frac{\frac{3}{4}c^2-c\alpha^2}{\alpha(\alpha^2+c)}. \tag{3.29}$$

From this and (3.24) we have $12\alpha^4+52c\alpha^2-9c^2=0$ on Ω_0. So we see that $\nabla\alpha=0$ and hence $\nabla h=0$ because of (3.29). Thus (3.27) becomes $AU=\frac{1}{4}(3\alpha-5h)U$ on Ω_0. Therefore Ω_0 is void. This completes the proof. ■

Lemma 3.2. *Under the same assumptions as those stated in Lemma* 3.1, *we have* $\xi\alpha=0$, $W\alpha=0$, $\xi h=0$ *and* $Wh=0$ *on* Ω.

Proof. As in the proof of Lemma 3.1, it is sufficient to show that the following two cases:

Case 1. $\beta-h\alpha+\frac{3}{4}c=0$ and $h^2-h\alpha-c=0$,

Case 2. $\frac{c}{2}(3\alpha-2h)+(h-\alpha)\left(2\alpha h-2\beta-\frac{1}{2}h^2\right)\neq 0$.

Case 1: By taking the inner product with ξ in (3.14), we obtain

$$\mu(h-\alpha)(Wh)=\left(2\alpha^2-3h\alpha+\frac{7}{4}c\right)(\xi h)+\left(h\alpha-\frac{3}{4}c\right)(\xi\alpha). \tag{3.30}$$

From (3.19) we have

$$\mu\alpha(Wh) = -\frac{1}{2}(h\alpha - 2c)(\xi\alpha) + \frac{1}{2}(3h\alpha - 2\alpha^2 - 3c)(\xi h). \tag{3.31}$$

Using (3.24), (3.30) and (3.31), we are led to

$$\{(\xi h)^2 + (\xi\alpha)^2\}(25h\alpha + 14c - 3\alpha^2) = 0. \tag{3.32}$$

So, on the set of points satisfying $25h\alpha + 14c - 3\alpha^2 \neq 0$,

$$\xi h = \xi\alpha = 0.$$

On account of Remark 3.1 and (3.30), we deduce that

$$Wh = 0.$$

Further, from (3.12), we get $W\alpha = 0$ since $\mu \neq 0$.

If $2h\alpha + 14c - 3\alpha^2 \equiv 0$, then $\alpha \neq 0$ since $c \neq 0$. So, we have

$$h = \frac{3\alpha^2 - 14c}{25\alpha}. \tag{3.33}$$

Combining this with (3.24), we see that

$$(3\alpha^2 - 14c)^2 - 25\alpha^2(3\alpha^2 - 14c) - 625c\alpha^2 \equiv 0.$$

Therefore we have $\nabla\alpha = 0$. So we have $\nabla h = 0$ by (3.33).

Case 2: Putting $\beta - h\alpha + \frac{3}{4}c = c'$, (3.21) is reduced to

$$\left\{\frac{c}{2}(3\alpha - 2h) + (h - \alpha)\left(\frac{3}{2}c - 2c' - \frac{1}{2}h^2\right)\right\}AU + cc'U = 0.$$

From this we have

$$AU = \lambda U, \quad \lambda = \frac{-2cc'}{c(3\alpha - 2h) + (h - \alpha)(3c - 4c' - h^2)}.$$

Therefore we are led to the following equation by (3.10):

$$(4c' + h^2)\{(4c' + h^2)\alpha^2 - 2h(4c' + h^2)\alpha + h^2(4c' + h^2) - c^2\} = 0. \tag{3.34}$$

If $4c' + h^2 \equiv 0$, then $h =$ constant. So, using (3.19), we are led to $\xi\alpha = 0$ since $c \neq 0$. Furthermore, from (3.12), we have $W\alpha = 0$.

If $4c'+h^2\neq 0$, then from (3.34) we have

$$(4c'+h^2)\alpha^2-2h(4c'+h^2)\alpha+h^2(4c'+h^2)-c^2=0. \tag{3.35}$$

Differentiating both sides of (3.35), we obtain

$$(\alpha-h)(4c'+h^2)\nabla\alpha+\{h\alpha^2-(4c'+h^2)\alpha+2h(2c'+h^2)\}\nabla h=0. \tag{3.36}$$

By taking the inner products with ξ in (3.14), we obtain

$$\mu\alpha(Wh)-\mu h(W\alpha)=\left(-\alpha^2+h\alpha+2c'-\frac{3}{2}c\right)(\xi h)$$
$$+\left(h\alpha-2c'+\frac{3}{2}c-h^2\right)(\xi\alpha). \tag{3.37}$$

By our assumption (3.19) is reduced to

$$\mu\alpha(Wh)=\left(\frac{3}{2}h\alpha-\alpha^2+2c'-\frac{3}{2}c\right)(\xi h)-\left(\frac{1}{2}h^2+2c'-\frac{3}{2}c\right)(\xi\alpha). \tag{3.38}$$

Using (3.36) and (3.37), we obtain

$$2\mu(h^2+2c')(\alpha-h)(Wh)$$
$$=\left\{-2h(h^2+4c')\alpha+(h^2+4c')\left(h^2+h\alpha+2c'-\frac{3}{2}c\right)-c^2\right\}(\xi h)$$
$$+(h^2+4c')\left(h\alpha-2c'+\frac{3}{2}c-h^2\right)(\xi\alpha). \tag{3.39}$$

Making use of (3.35), we have from (3.38) and (3.39)

$$\left[-2(h^2+2c')\alpha^3+2h(3h^2+7c')\alpha^2+\left\{-4h^4-\left(8c'+\frac{3}{2}c\right)h^2+c^2\right\}\alpha\right.$$
$$\left.+(3c-4c')h(h^2+2c')\right](\xi h)-\left\{h^2\left(2h^2-\frac{3}{2}c+8c'\right)\alpha\right.$$
$$\left.+h(c^2-10c'h^2-2h^4+3ch^2-8c'^2+6cc')\right\}(\xi\alpha)=0. \tag{3.40}$$

From (3.36) we have

$$(\alpha-h)(h^2+4c')(\xi\alpha)+\{h\alpha^2-(4c'+3h^2)\alpha+2h(2c'+h^2)\}(\xi h)=0. \tag{3.41}$$

From (3.40) and (3.41) we obtain

$$\{(\xi h)^2 + (\xi\alpha)^2\}$$

$$\times\left[(\alpha - h)(h^2 + 4c')\left\{-2(h^2 + 2c')\alpha^3 + 2h(3h^2 + 7c')\alpha^2\right.\right.$$

$$\left. - 4h^4\alpha - \left(8c' + \frac{3}{2}c\right)h^2\alpha + c^2\alpha + (3c - 4c')h(h^2 + 2c')\right\}$$

$$+ \{h\alpha^2 - (4c' + 3h^2)\alpha + 2h(2c' + h^2)\}\left\{h^2\left(2h^2 - \frac{3}{2}c + 8c'\right)\alpha\right.$$

$$\left.\left. + h(c^2 - 10c'h^2 - 2h^4 + 3ch^2 - 8c'^2 + 6c'c)\right\}\right] = 0. \tag{3.42}$$

If $(\xi h)^2 + (\xi\alpha)^2 \neq 0$, then from (3.42) we have

$$(-12h^2c' - 2h^4 - 16c'^2)\alpha^4$$

$$+ \left(-\frac{3}{2}h^3c + 72hc'^2 + 58h^3c' + 10h^5\right)\alpha^3$$

$$+ \left(2h^2c^2 + 3h^4c + \frac{9}{2}h^2c + 4c'c^2 - 88c'h^4\right.$$

$$\left. - 6h^4 - 14h^6 - 24c'h^2 - 128c'^2h^2 + 6c'ch^2\right)\alpha^2$$

$$+ \left(-18c'ch - 8c'c^2h + 6h^5 + 62c'h^5 - 3c^2h - 2h^3c^2\right.$$

$$\left. + 24c'^2h + 10h^7 + 88c'h^3 - 9ch^3 - \frac{3}{2}ch^5 + 30c'h^3\right)\alpha$$

$$+ 6c'ch^4 + 4c'c^2h^2 - 4h^8 - 24c'h^6 - 32c'h^4 + 2c^2h^4 + 3ch^6 = 0. \tag{3.43}$$

Using Sylvester's elimination method to (3.35) and (3.43), we deduce that

$$(-24cc' - 7c^2 + 16c'^2)h^{20} + (-576c'^2c + 72c'c + 384c'^3 - 48c'^2$$

$$+ 21c^2 + 36c^3 - 120c'c^2)h^{18} + f(h) = 0, \tag{3.44}$$

where $f(h)$ is the polynomial of h of degree ≤ 16. (We use a computer to calculate this.)

We can check that the coefficients of h^{20} and h^{18} does not vanish simulteneously since $c \neq 0$. (We use a computer to check this.)

By the above argument, we know that (3.44) is a non-trivial algebraic equation of h. So, we arrive at $h =$ constant. From (3.41), we have $\xi\alpha = 0$. These are contradictions. So, we have $\xi\alpha = \xi h = 0$. Furthermore, using (3.12) and (3.39), we arrive at $W\alpha = Wh = 0$. We have thus proved the lemma. ■

4 Proof of the Theorem

We continue our discussion under the same assumption of § 3. First, we prove the following two lemmas:

LEMMA 4.1. *Let λ be a principal curvature corresponding to U. Then λ does not vanish identically on $\Omega = \{p \in M \mid \mu(p) \neq 0\}$.*

PROOF. From Lemma 3.1 and (3.10) the following equation holds on Ω:

$$\lambda^2 = \lambda h + \beta - h\alpha + \frac{3}{4}c. \tag{4.1}$$

By Lemma 3.2, (3.15) becomes

$$A\nabla h = 0, \quad \lambda(Uh) = 0. \tag{4.2}$$

Because of Lemma 3.1 and Lemma 3.2, (3.13) and (3.16) are reduced respectively to

$$\{h\lambda + 2(\beta - h\alpha + c)\}U = -A\nabla\alpha + \frac{1}{2}(h\nabla\alpha - \alpha\nabla h), \tag{4.3}$$

$$\theta U = \left(2\alpha h - 2\beta - \frac{1}{2}h^2\right)\nabla\alpha + \left(2\beta - \frac{3}{2}h\alpha\right)\nabla h, \tag{4.4}$$

where we define θ by $\theta = (4\beta - 4h\alpha + h^2 + c)\lambda + \frac{3}{2}c\alpha - 2ch$.

From (3.11) and Lemma 3.2, we have $\xi\beta = 0$. Therefore it is seen, using Lemma 3.2, that

$$\xi\theta = 0.$$

From this and Lemma 3.1, we see, making use of (4.4), that

$$\theta\, du(\xi, X) = 0 \tag{4.5}$$

for any vector fields X on Ω, where u is defined by $u(X) = g(U, X)$, and exterior derivation du of u is given by

$$du(\xi, X) = \frac{1}{2}\{\xi u(X) - Xu(\xi) - u([\xi, X])\}.$$

On the other hand, using (1.15) and $AU = \lambda U$, the equation (1.14) turns out to be

$$\nabla_\xi U = \mu(\alpha - 3\lambda)W - \mu^2\xi + \phi\nabla\alpha,$$

which together with (1.11) and (2.2) implies that

$$du(\xi, X) = (h - 3\lambda)\mu w(X) + g(\phi\nabla\alpha, X), \tag{4.6}$$

where $w(X) = g(W, X)$.

If $\lambda = 0$, then by (3.1) we have

$$\beta - h\alpha = -\frac{3}{4}c. \tag{4.7}$$

Thus (4.3) and (4.4) becomes respectively

$$cU = -2A\nabla\alpha + h\nabla\alpha - \alpha\nabla h, \tag{4.8}$$

$$(3c\alpha - 4ch)U = (3c - h^2)\nabla\alpha - (3c - h\alpha)\nabla h. \tag{4.9}$$

Because of Lemma 3.1 and (4.2), we see, using (4.9), that

$$(3c - h^2)A\nabla\alpha = 0. \tag{4.10}$$

If the set of points satisfying $A\nabla\alpha \neq 0$ is not empty, then on that set we have

$$h = \text{constant}$$

because of (4.10). So, from (4.9), we are led to

$$\nabla\alpha = 0.$$

This is a contradiction. So, we obtain

$$A\nabla\alpha = 0 \quad \text{on } \Omega. \tag{4.11}$$

Thus (4.7) becomes

$$cU = h\nabla\alpha - \alpha\nabla h.$$

So, we have

$$du(\xi, X) = 0$$

because of Lemma 3.2. Therefore (4.6) means that

$$\phi\nabla\alpha = \mu(h - 3\lambda)W.$$

Since $\xi\alpha = 0$, it follows that

$$\nabla\alpha = hU. \tag{4.12}$$

So, from (4.8), we have

$$\alpha\nabla h = (h^2 - c)U. \tag{4.13}$$

Combining last two equations with (3.2) and (3.11), we obtain

$$A\nabla\beta = 0, \quad A\nabla\mu = 0.$$

Thus (2.18) with $AU = 0$ and (4.7) implies

$$\begin{aligned} -\frac{5}{4}c(h-\alpha)U = \frac{3}{4}c\nabla\alpha - \frac{1}{2}(h-\alpha)\{\alpha\nabla h + h\nabla\alpha\} \\ + \left(h\alpha - \frac{3}{4}c - \alpha^2\right)\nabla h. \end{aligned} \tag{4.14}$$

Substituting (4.12) and (4.13) in the right-hand side of (4.14), we are led to

$$(h-\alpha)^2 = c. \tag{4.15}$$

Combining this with (4.12) and (4.13), we have

$$\alpha(h-\alpha) = 0.$$

Since $h - \alpha \neq 0$, we have

$$\alpha = 0. \tag{4.16}$$

So, (4.12) implies that $h = 0$. These are contradictions. We have thus proved the lemma. ■

LEMMA 4.2. *$\theta = 0$ on Ω.*

PROOF. If not, then from (4.5) we have

$$du(\xi, X) = 0.$$

By (4.6), we obtain

$$\nabla\alpha = (h - 3\lambda)U. \tag{4.17}$$

Hence (4.3) is reduced to

$$\alpha\nabla h = \{h^2 - 7\lambda h + 6\lambda^2 - 4(\beta - h\alpha + c)\}U. \tag{4.18}$$

Applying A to both sides of (4.18), we have

$$4(\beta - h\alpha) = h^2 - 7h\lambda + 6\lambda^2 - 4c \tag{4.19}$$

since $A\nabla h = 0$ and $\lambda \neq 0$ on Ω.

Combining (4.19) with (4.1), we are led to

$$2\lambda^2 - 3\lambda h + h^2 - c = 0. \tag{4.20}$$

Differentiating both sides of (4.20), we obtain

$$(4\lambda - 3h)\nabla\lambda + (2h - 3\lambda)\nabla h = 0. \tag{4.21}$$

On the other hand, from (4.1) we have

$$(2\lambda - h)\nabla\lambda = \lambda\nabla h. \tag{4.22}$$

Combining (4.22) with (4.21), we are led to

$$(h - \lambda)^2\nabla\lambda = 0.$$

Furthermore, we have

$$\nabla\lambda = 0$$

since $h \neq \lambda$ by (4.20) and $c \neq 0$. So, from (4.22) we obtain

$$\nabla h = 0 \tag{4.23}$$

since $\lambda \neq 0$ by Lemma 4.1. Thus (4.4) becomes

$$(4\beta - 4h\alpha + h^2 + c)\lambda + \frac{3}{2}c\alpha - 2ch = (h - 3\lambda)\left(2\alpha h - 2\beta - \frac{1}{2}h^2\right). \tag{4.24}$$

Differentiating both sides of (4.24), we have

$$\nabla\alpha = 0 \tag{4.25}$$

since $c \neq 0$.

From (4.4), (4.23) and (4.25), we are led to

$$\theta = 0.$$

This is a contradiction. We have thus proved the lemma. ■

Finally, we prove

THEOREM 4.1. *Let M be a real hypersurface in $M_n(c)$, $c > 0$. If it satisfies $\nabla_{\phi\nabla_\xi\xi}S = 0$ and at the same time satisfies $S\xi = \sigma\xi$ for some constant σ, then M is a Hopf hypersurface.*

PROOF. By Lemma 4.2 and (4.1), we have

$$\lambda(4\lambda^2 - 4h\lambda + h^2 - 2c) = \frac{c}{2}(4h - 3\alpha), \tag{4.26}$$

$$\left(2\alpha h - 2\beta - \frac{1}{2}h^2\right)\nabla\alpha + \left(2\beta - \frac{3}{2}h\alpha\right)\nabla h = 0. \tag{4.27}$$

Applying A to both sides of (4.27) and using (4.2), we obtain

$$\left(2\alpha h - 2\beta - \frac{1}{2}h^2\right)A\nabla\alpha = 0.$$

Now, suppose that $A\nabla\alpha \neq 0$, then we have

$$2\alpha h - 2\beta - \frac{1}{2}h^2 = 0.$$

From this and our auumption $\sigma = \text{constant}$, we have

$$\nabla h = 0. \tag{4.28}$$

Differentiating both sides of (4.1), we obtain

$$(h - 2\lambda)\nabla\lambda = 0. \tag{4.29}$$

From (4.28) and (4.29), we are led to

$$\nabla\lambda = 0. \tag{4.30}$$

Thus from (4.26) we see that

$$\nabla\alpha = 0.$$

This contradicts to $A\nabla\alpha = 0$. So, we have

$$A\nabla\alpha = 0, \quad U\alpha = 0 \tag{4.31}$$

since $\lambda \neq 0$.

Using (4.2) and (4.31) and applying U to both sides of (4.3), we have

$$h\lambda + 2(\beta - h\alpha + c) = 0. \tag{4.32}$$

From (4.1) and (4.32), we obtain

$$\lambda^2 = \frac{1}{2}h\lambda - \frac{1}{4}c. \tag{4.33}$$

Substituting (4.33) to both sides of (4.26), we are led to

$$\alpha = h + 2\lambda \tag{4.34}$$

since $c \neq 0$.

Combining (4.34) with (4.32), we have

$$g(U, U) = \beta - \alpha^2 = -7\lambda^2 - \frac{9}{4}c < 0.$$

This is a contradiction. The theorem is now proved by all the above arguments. ■

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