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Toward an Understanding of Short Distance Repulsions among Baryons in QCD

—— *NBS Wave Functions and Operator Product Expansion* ——

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We report on our recent attempts to determine the short distance behaviors of general 2-baryon and 3-baryon forces, which are defined from the Nambu-Bethe-Salpeter (NBS) wave function, by using the operator product expansion and a renormalization group analysis in QCD. We have found that the repulsion at short distance increases as the number of valence quarks increases or when the number of different flavors involved decreases. This global tendency suggests a Pauli suppression principle among quark fields at work.

Subject Index: 164, 165, 200, 234

§1. Motivation

The most fundamental quantity in nuclear physics is a force between nucleons, the nuclear force, from which major properties of nuclei are extracted. The phenomenological nuclear potentials,[1)] which well describe nucleon-nucleon (NN) scattering at low energy, exhibit long-to-medium distance attractions, essential for the binding of atomic nuclei, as well as short distance repulsion (a repulsive core), important for the stability of nuclei against collapse. While the former properties have been explained by meson exchanges between nucleons,[2)] the origin of the repulsive core[3)] has not yet been well understood. Considerations of the fundamental degrees of freedom for nucleons, quarks and gluons, and their dynamics, QCD, are required to explain the properties between nucleons at such short distances.

The recent observation of a neutron star as heavy as twice a solar mass[4)] forces us to consider more general short distance repulsions among baryons, not only NN but also 3N, BB and 3B, where B represents an octet baryon which may include one or more strange quarks. This is because although the repulsive core of NN potentials is an important ingredient for the determination of the maximum mass of neutron stars, it alone seems insufficient to sustain the two-solar-mass neutron star. Moreover, an appearance of heavier strange quarks, converted from lighter up and down quarks via the weak interaction in such a high density environment, makes the

equation of state softer in the core of the neutron star, so that the maximum mass of the neutron star is further reduced. It has already been pointed out that general BB potentials alone cannot sustain the two-solar-mass neutron star.[5] It is also argued that the maximum mass of neutron stars can be increased if 3-body forces, becoming important at high density, also have strong short distance repulsions to compensate the reduction due to the appearance of strange quarks. Furthermore it is thought that such repulsions should be universal: they should appear not only in 3NF (3 nucleon forces) but also in 3BF (3 baryon forces).[5] A problem of such an explanation, however, is that both experimental and theoretical determinations of 3-body forces have been known to be very difficult at short distances.

Recently, a new method has been proposed to define and investigate the NN potential from the Nambu-Bethe-Salpeter (NBS) wave function in lattice QCD. This method was employed successfully to calculate NN potentials,[6–8] and it has been widely applied to various hadronic interactions[9–14] including BB potentials at $N_{\rm f}^{\rm v} = 3$,[15,16] and 3N forces at $N_{\rm f}^{\rm v} = 2$,[17] where $N_{\rm f}^{\rm v}$ denotes the number of different quark flavors of the valence quarks of the baryonic systems under consideration.

In this paper, we report our recent attempts to determine the short distance behaviors of general 2-baryon and 3-baryon forces, using the same definition of potentials as employed in the lattice QCD simulations referred to above. In our study we combine the operator product expansion with a renormalization group (RG) analysis of QCD using perturbatively computed RG functions. Our results may give not only useful boundary conditions of 2- and 3-body forces at short distance but also a hint toward an understanding of origins for short distance repulsions. We have observed a tendency toward more short distance repulsions for more nucleons: In the case of the NN potential, the short distance repulsion appears only if the ratio of two matrix elements has a positive sign.[18,19] Although this condition is satisfied at low energy,[18] the short distance repulsion is not universal. On the other hand, a universal short distance repulsion is shown to appear in 3N forces.[20] We also found, for a fixed number of valence quarks, a weaker short distance repulsion for increasing $N_{\rm f}^{\rm v}$. While the NN potentials (and potentials of general BB systems with $N_{\rm f}^{\rm v} = 2$) have the short distance repulsion at low energy, there appear short distance attractions in some channels of BB interactions with $N_{\rm f}^{\rm v} = 3$.[21] This tendency is found to be true also in 3B forces.

In the following, we explain how we obtain the above results and what the above statements mean more precisely. In §2, we first give our definition of potentials in QCD, which has been employed successfully in lattice QCD calculations. We then formulate our analysis methods, involving the operator product expansion and the renormalization group in the framework of perturbative QCD. In §3 we present our results, which include both BB and 3B potentials at both $N_{\rm f}^{\rm v} = 2$ and 3. A conclusion of this paper is given in §4.

§2. Basic idea and methods

Since potentials themselves are scheme-dependent quantities like running couplings of QCD, we first give our definition (or scheme) for them. Let us consider the

equal-time NBS wave function,[22] which in the case of a BB system for example is defined by

$$\varphi_E(\vec{r}) = \langle 0|B(\vec{r},0)B(0)|\mathrm{BB},E\rangle_{\mathrm{in}}\,, \tag{2·1}$$

where $|\mathrm{BB},E\rangle_{\mathrm{in}}$ is a QCD asymptotic in-state with energy E and $B(x)$ is a baryon interpolating operator made of 3 quarks such as $B(x)=\varepsilon^{abc}q^a(x)q^b(x)q^c(x)$ (flavor and spin labels suppressed). Although a choice of $B(x)$ is rather arbitrary as long as it is coupled to a one baryon state, we take our choice to determine a scheme of the NBS wave function (like a choice of the renormalization scheme). It is important to note that the NBS wave function contains the information about the BB scattering phase shift in QCD at energy E.[23),24] Therefore, from this NBS wave function, the non-local but energy-independent BB potential is naturally defined so as to satisfy

$$(e-H_0)\varphi_E(\vec{r}) = \int d^3r'\,U(\vec{r},\vec{r}\,')\varphi_E(\vec{r}\,')\,,\quad e=\frac{k^2}{m_B},\ H_0=-\frac{\nabla^2}{m_B}\,, \tag{2·2}$$

for all $E<E_{\mathrm{th}}$, where $E=2\sqrt{k^2+m_B^2}$ with a baryon mass m_B and the threshold energy $E_{\mathrm{th}}=2m_B+m_M$ with a meson mass m_M. It is easy to show that the non-local but energy-independent $U(\vec{r},\vec{r}\,')$ exists.[8] Indeed

$$U(\vec{r},\vec{r}\,') = \sum_{E,E'<E_{\mathrm{th}}} (e-H_0)\varphi_E(\vec{r})(\eta^{-1})^{EE'}\varphi^*_{E'}(\vec{r}\,') \tag{2·3}$$

satisfies Eq. (2·2) for all $E<E_{\mathrm{th}}$, where $(\eta^{-1})^{EE'}$ is an inverse of

$$\eta_{EE'} = \int d^3r\,\varphi^*_E(\vec{r})\,\varphi_{E'}(\vec{r}) \equiv (\varphi_E,\varphi_{E'}) \tag{2·4}$$

in the restricted space where $E,E'<E_{\mathrm{th}}$. One can construct $(\eta^{-1})^{EE'}$ from eigenvectors corresponding to nonzero eigenvalues of the hermitian operator $\eta_{EE'}$. Note that the non-local but energy-independent potential which satisfies Eq. (2·2) for all $E<E_{\mathrm{th}}$ is not unique, since one can add terms involving wave functions for $E\geq E_{\mathrm{th}}$ which are orthogonal to all wave functions with $E<E_{\mathrm{th}}$.

In practice, the non-local potential $U(\vec{r},\vec{r}\,')$ is expanded in terms of the velocity as $U(\vec{r},\vec{r}\,')=V(\vec{r},\vec{\nabla})\delta(\vec{r}-\vec{r}\,')$, whose lowest few orders for the NN case are given by

$$V(\vec{r},\vec{\nabla}) = V_0(r)+V_\sigma(r)\vec{\sigma}_1\cdot\vec{\sigma}_2+V_T(r)S_{12}+V_{\mathrm{LS}}(r)\vec{L}\cdot\vec{S}+\mathrm{O}(\nabla^2)\,, \tag{2·5}$$

where $r=|\vec{r}|$, $\vec{\sigma}_i$ is the Pauli-matrix acting on the spin index of the i-th nucleon, $\vec{S}=(\vec{\sigma}_1+\vec{\sigma}_2)/2$ is the total spin, $\vec{L}=\vec{r}\times\vec{p}$ is the angular momentum, and S_{12} is the tensor operator $S_{12}=3(\vec{r}\cdot\vec{\sigma}_1)(\vec{r}\cdot\vec{\sigma}_1)/r^2-\vec{\sigma}_1\cdot\vec{\sigma}_2$. This definition of the potential has been employed successfully to calculate NN potentials in lattice QCD for the first time,[6)–8] where the central potentials at the leading order have qualitatively reproduced common features of phenomenological NN potentials: the force is attractive at medium to long distance while it has a short distance repulsion, the

repulsive core. After this success, the method has been widely applied to various hadronic interactions[9)–14)] including BB potentials at $N_{\rm f}^{\rm v}=3$[15),16)] and 3N forces at $N_{\rm f}^{\rm v}=2$.[17)] See Refs. 25) and 26) for reviews of recent activities.

In Ref. 27) we made some toy model studies in order to better understand questions related to the scheme dependence, energy dependence, etc. of the NBS wave functions and potentials. We noticed that in these 2-dimensional integrable models the short distance behavior of the NBS wave function is represented very well by the leading terms appearing in the operator product expansion (OPE). The hope is that the same qualitative feature applies to QCD, for which, thanks to the property of asymptotic freedom, the form of leading short distance behavior of the coefficient functions appearing in the OPE can be computed using perturbation theory also for nucleons (and baryons). In this section we give a very short summary of our method using the example of the NN wave function and potential. Our results for the cases NN,[18)] BB,[21)] 3N[20)] and 3B will be presented in the next section.

The behavior of the wave functions $\varphi_E(\vec{r})$ at short distances ($r=|\vec{r}|\to 0$) is encoded in the operator product expansion (OPE) of the two baryon operators:

$$B(\vec{r}/2,0)\,B(-\vec{r}/2,0)\approx\sum_k D_k(\vec{r})\,\mathcal{O}_k(\vec{0},0)\,, \tag{2·6}$$

where $\{\mathcal{O}_k\}$ is a set of local color singlet 6-quark operators with two-baryon quantum numbers. It is important to note that asymptotically the $\vec{r}$-dependence and energy dependence of the wave function is factorized into

$$\varphi_E(\vec{r})\approx\sum_k D_k(\vec{r})\langle 0|\mathcal{O}_k(\vec{0},0)|{\rm BB},E\rangle_{\rm in}\,. \tag{2·7}$$

Now a standard renormalization group (RG) analysis gives[18)] the leading short distance behavior of the OPE coefficient function as

$$D_k(\vec{r})\approx\lambda(r)^{-\nu_k}\,d_k\,, \tag{2·8}$$

where $\lambda(r)$ is the 2-loop running coupling defined by

$$\frac{1}{\lambda}+\kappa\ln\lambda=\ln\frac{r_*}{r}\,,\quad r\ll r_*\,, \tag{2·9}$$

where (in the case of three light dynamical flavors) $\kappa=\frac{32}{81}$ and r_* is some typical non-perturbative QCD radius (presumably O(0.5 fm) but a precise specification is not needed for our present considerations).

In (2·8) ν_k is related to the 1-loop coefficient of the anomalous dimension of the operator $\mathcal{O}_k$, d_k is the tree-level contribution of $D_k(\vec{0})$. Clearly the operator with largest RG power ν_k dominates the wave function (2·1) at short distances. We will denote the leading (largest) power by ν_1 and the subleading one (second largest) by ν_2.

If neither ν_1 nor d_1 vanishes, this leads to the leading asymptotics of the s-wave potential of the form

$$V(r)\approx-\frac{\nu_1}{m_N\,r^2\left(\ln\frac{r_*}{r}\right)}\,, \tag{2·10}$$

which is attractive for $\nu_1 > 0$ and repulsive for $\nu_1 < 0$. Note that although the running coupling itself is scheme dependent, the above asymptotic form is not (and it is also energy independent). Scheme dependence only affects the sub-leading contributions. Whether (2·10) corresponds to an attractive or repulsive potential is easily seen from the formula but can also be understood intuitively from the short distance behavior of the corresponding wave function. Namely, if $\nu_1 > 0$, the wave function diverges in the origin (attraction), whereas for $\nu_1 < 0$ the wave function vanishes at the origin (repulsion).

If $\nu_1 = 0$, the situation is more complicated. In this case the relative sign of the ratio $R = \langle 0|\mathcal{O}_2(\vec{0},0)|\mathrm{NN},E\rangle_{\mathrm{in}}/\langle 0|\mathcal{O}_1(\vec{0},0)|\mathrm{NN},E\rangle_{\mathrm{in}}$ between the leading contribution and the subleading contribution corresponding to an operator $\mathcal{O}_2$ with $\nu_2 < 0$ is important. As we will see this occurs in the NN case. If R is positive, the potential is repulsive, while it is attractive for negative R. The above intuitive argument seems to work also here, namely, for R positive the wave function decreases approaching the origin (repulsion) and the other way round for negative R.

To determine the asymptotic behavior we thus have to compute the spectrum of the renormalization group power ν_k corresponding to the operators appearing in the BB OPE. The general form of a gauge invariant local 3-quark operator is given by

$$B^F_\Gamma(x) \equiv B^{fgh}_{\alpha\beta\gamma}(x) = \varepsilon^{abc} q^{a,f}_\alpha(x) q^{b,g}_\beta(x) q^{c,h}_\gamma(x)\,, \tag{2·11}$$

where α,β,γ are spinor, f,g,h are flavor, a,b,c are color indices of the (renormalized) quark field q. The color index runs from 1 to $N_c = 3$, the spinor index from 1 to 4, and the flavor index from 1 to $N_{\mathrm{f}}^{\mathrm{v}}$. As indicated in (2·11) we use capital labels for sets of flavors $F = fgh$ and Dirac labels $\Gamma = \alpha\beta\gamma$. Note that $B^{fgh}_{\alpha\beta\gamma}$ is symmetric under any interchange of pairs of indices (e.g. $B^{fgh}_{\alpha\beta\gamma} = B^{gfh}_{\beta\alpha\gamma}$) because the quark fields anticommute. The usual nucleon operator which is employed in lattice simulations is a certain linear combination constructed from the above operators.[18)]

Gauge invariant local 6-quark operators can be written as linear combinations of simple operators $[BB]^{F_1F_2}_{\Gamma_1\Gamma_2} = B^{F_1}_{\Gamma_1} B^{F_2}_{\Gamma_2}$. These local operators mix among themselves under renormalization. The one-loop mixing matrix is given by diagrams corresponding to the exchange of one gluon between all pairs of quark lines. Due to the flavor symmetry (and partially to the chiral symmetry) of massless QCD, only those 6-quark operators (of the form $[BB]^{F_AF_B}_{\Gamma_A\Gamma_B}$) which preserve the set of flavor and Dirac indices as

$$F_1 \cup F_2 = F_A \cup F_B, \quad \Gamma_1 \cup \Gamma_2 = \Gamma_A \cup \Gamma_B\,, \tag{2·12}$$

occur in the mixing problem. Note however that such operators are not all linearly independent. We have the following "gauge" constraints:*)

$$3[BB]^{F_1F_2}_{\Gamma_1\Gamma_2} + \sum_{i,j=1}^{3} [BB]^{(F_1,F_2)[i,j]}_{(\Gamma_1,\Gamma_2)[i,j]} = 0\,. \tag{2·13}$$

*) We call these constraints "gauge" constraints because we used them to show the gauge invariance of the 1-loop anomalous dimensions.

Here $[i,j]$ denotes a simultaneous exchange between the i-th indices of F_1, Γ_1 and the j-th indices of F_2, Γ_2.

The problem is thus reduced to finding the set of independent operators after imposing the gauge constraints and then writing down and diagonalizing the corresponding one-loop mixing matrix. This task can easily be automated and we calculated and checked the results using independent Mathematica and Maple programs. The size of the linear algebra problem turned out quite large in some cases. For example the largest dimensions occur for a certain set of Dirac indices in the BBB case: there are originally 14130 operators before and 1518 after imposing the constraints. We are indebted to Thomas Hahn for providing us with effective linear algebra tools for solving this part of the problem with Mathematica.

§3. Results

In the first three subsections we briefly summarize our results for the leading behavior of NBS were fuctions for 2 nucleons, 2 baryons (involving operators with 3 different flavors) and 3 nucleons; for the details of these computations the reader is requested to consult Refs. 18), 21) and 20) respectively. Our latest results concerning the case of 3 baryon wave functions are presented in the fourth subsection.

In each case the computation consists of three steps. i) a basis of independent gauge invariant operators of engineering dimension equal to that of the operator product appearing in the definition of the NBS wave function is determined for each Dirac structure. ii) The spectrum of 1-loop anomalous dimensions γ_k of the linear combination of these operators which do not mix under renormalization (at 1-loop) is computed. iii) The operators appearing in the OPE at tree level are listed.

For operators which renormalize multiplicatively (without mixing)

$$O^{\rm ren} = Z_O(\epsilon, g) O^{\rm bare}\,, \tag{3·1}$$

the associated anomalous dimensions are, using dimensional regularization in $D = 4 - 2\epsilon$ dimensions, specified by

$$\gamma_O = -\left[\epsilon g + \beta_0 g^3 + \dots\right] \frac{\partial Z_O}{\partial g} = \gamma_{O0} g^2 + \mathrm{O}(g^4)\,, \tag{3·2}$$

where β_0 is the QCD 1-loop beta function coefficient

$$\beta_0 = \frac{1}{16\pi^2}\left[11 - \frac{2}{3}N_{\rm f}\right]\,. \tag{3·3}$$

We will restrict attention to the octet baryon operators appearing in the definition of the NBS wave functions, which have 1-loop anomalous dimension

$$\gamma_{B0} = 24d\,, \qquad d \equiv 1/(96\pi^2)\,. \tag{3·4}$$

It turns out that all 1-loop anomalous dimensions of operators appearing below can be expressed as even integers times $2d$.

3.1. *The NN potentials*

The computationally easiest case is the NBS wave function defined with the product of two nucleons, operators defined with just two (u, d) flavors of quarks. The spectrum of anomalous dimensions in this case has a striking structure:

$$\gamma_{k0} \leq 2\gamma_{B0}\,, \quad \forall k\,. \tag{3·5}$$

It is easy to see that there are NN (also BB) operators with $\gamma_{k0} = 2\gamma_{B0}$ — e.g. those of the form B_+B_- where $B_\pm$ involves only quarks of positive (negative) chirality. Such operators generically appear in the OPE at tree level and hence in the NN case these dominate the short distance behavior of the NBS wave function. However since $D_k = \text{const}$ asymptotically for these operators, their contributions alone do not determine the short distance behavior of the potentials (defined through taking derivatives of the wave function). The leading behavior of the (central) potentials involves also the next-to-largest anomalous dimensions, which are found to be $\gamma^{(0,1)} = 24d\,, \gamma^{(1,0)} = 40d$ in the two spin-isospin channels $(S, I) = (0, 1), (1, 0)$ respectively:

$$V_c^{(S,I)}(r) \sim R^{(S,I)} r^{-2} (-\ln r)^{\beta^{(S,I)}-1}\,, \tag{3·6}$$

with (assuming N_f is such that $\beta_0 > 0$)

$$\beta^{(S,I)} = \frac{\gamma^{(S,I)} - 2\gamma_{B0}}{2\beta_0} < 0\,, \tag{3·7}$$

and the constants $R^{(S,I)}$ appearing above involve the ratio of matrix elements of the leading and subleading operators in the OPE as

$$R^{(S,I)} = \frac{\langle 0|\mathcal{O}_2^{(S,I)}|\mathrm{NN}, E\rangle_\mathrm{in}}{\langle 0|\mathcal{O}_1^{(S,I)}|\mathrm{NN}, E\rangle_\mathrm{in}}. \tag{3·8}$$

Thus although the OPE determines the functional form of the short distance behavior, unfortunately it does not determine the sign in this special case. Information on the sign of $R^{(S,I)}$ requires additional non-perturbative input, which could e.g. be supplied by dedicated numerical simulations. In 18) we have given arguments based on a non-relativistic expansion that the $R^{(S,I)}$ are positive, in which case the potential is repulsive in both channels as indeed observed in the numerical simulations.

3.2. *The BB potentials*

We next consider the operator product of two octet baryon operators where the operators appearing in the OPE can involve 3 different flavors. The tensor product of two octets can be decomposed into 6 channels, according to

$$8 \otimes 8 = (1 \oplus 8 \oplus 27)_s + (8 \oplus 10 \oplus \overline{10})_a\,, \tag{3·9}$$

where subscript $s(a)$ represents the property that the operator is (anti-)symmetric under the interchange of two baryons. The two nucleon states with $(S, I) = (0, 1)$ or $(1, 0)$ considered above belong to the 27_s or $\overline{10}_a$ representations, respectively. It is

Table I. List of channels with anomalous dimensions greater than $2\gamma_{B0}$; others can be obtained by symmetry transforms of the Dirac indices $1 \leftrightarrow 2\,, 3 \leftrightarrow 4$ or $(1,2) \leftrightarrow (3,4)$.

Dirac structure	$\gamma_0^{\mathcal{R}}/(2d)$	$SU(3)$ representation
112334	42	1
111222	36	1
112234	36	1
111234	32	$1 \oplus 8$
112234	32	$1 \oplus 8$

sufficient to consider the situation of composite 6-quark operators with two quarks of each flavor ($uuddss$) since all 6 $SU(3)$ representations appear in this type of operator.

Although most of the operators have 1-loop anomalous dimensions $\leq 2\gamma_{B0}$ we found also operators with anomalous dimensions greater than $2\gamma_{B0}$ corresponding to attractive BS potentials in the three channels $1_s, 8_a, 8_s$; they are listed in Table I. There are no attractive operators in the $27_s, 10_a, \overline{10}_a$ channels, which is consistent with what we found in the two nucleon case. All the attractive operators appear in the OPE of two baryon operators at tree level and therefore the corresponding potentials in these channels (denoted by $\mathcal{R}$) behave asymptotically as in Eq. (2·10) with

$$\nu_1 = \frac{\gamma_0^{\mathcal{R}} - 2\gamma_{B0}}{2\beta_0} > 0\,. \tag{3·10}$$

The larger anomalous dimensions occur in the lower dimensional $SU(3)$ representations. Indeed the tendency of repulsion among the operators with fixed total number of valence quarks is increased if the number of participating valence flavors is decreased; this is a first indication of what we suggestively call a "Pauli suppression principle" at work.

Monte Carlo simulations in 3-flavor lattice QCD[15),16)] have as yet only found attraction in the 1_s channel. The fact that attraction in octet channels has not yet been observed may have many different reasons. Firstly we note that the amplitudes of attractive operators in the 8_s channel vanish in the non-relativistic limit. Secondly, and this comment applies to all our considerations, we do not know at which distance r the asymptotic formulae (2·10) set in. The physical distances of the BS potentials probed in Monte Carlo simulations up to now ($>\sim 0.2$ fm) may not be small enough.

3.3. *The 3N potentials*

For the investigation of the 3 baryon potentials we consider equal-time NBS wave functions involving the product of three baryon operators:

$$\psi_{3\mathrm{B},E}(\vec{r},\vec{\rho}) = \langle 0|B(\vec{x}_1,0)B(\vec{x}_2,0)B(\vec{x}_3,0)|3\mathrm{B},E\rangle_{\mathrm{in}}\,, \tag{3·11}$$

where $|3\mathrm{B},E\rangle_{\mathrm{in}}$ is a 3-baryon asymptotic in-state of energy E. In the argument of $\psi_{3\mathrm{B},E}$ we have introduced Jacobi coordinates $\vec{r} = \vec{x}_1 - \vec{x}_2\,, \vec{\rho} = [\vec{x}_3 - (\vec{x}_1 + \vec{x}_2)/2]\,/\sqrt{3}$. The three baryon potential is defined through the wave functions by

$$\left[\frac{1}{m_B}\left(\nabla_r^2 + \nabla_\rho^2\right) + \mathcal{V}_{3\mathrm{B}}\right]\psi_{3\mathrm{B},E}(\vec{r},\vec{\rho}) = E\psi_{3\mathrm{B},E}(\vec{r},\vec{\rho})\,, \tag{3·12}$$

$$\mathcal{V}_{3B}(\vec{r},\vec{\rho}) = \sum_{1\le i<j\le 3} V_{BB}(\vec{x}_i - \vec{x}_j) + V_{3B}(\vec{r},\vec{\rho})\,, \tag{3·13}$$

where the reduced masses of these Jacobi coordinates are $m_B/2$ and only the leading contributions in the velocity expansion are presented. The V_{BB} are the BB potentials defined through the BB wave functions.

The leading terms in the OPE of 3-baryon operators in $\vec{r},\vec{\rho}\to 0$ limits involve the associated local gauge invariant 9-quark operators (of minimal engineering dimension of 27/2).

For the 3N case the spectrum of anomalous dimensions of the the 9-quark operators (now involving only two flavors) has the simple property:

$$\gamma_{A0} < 3\gamma_{B0}\,, \quad \forall A\,. \tag{3·14}$$

Again it turns out that most of the operators with largest anomalous dimensions appear in the OPE at tree level. From this result, without recourse to additional nonperturbative arguments, we deduce in this case that generically the total central 3N potential $\mathcal{V}_{3N}(\vec{r},\vec{\rho})$ is repulsive at short distances:

$$\mathcal{V}_{3N}(\vec{r},\vec{\rho}) \simeq \frac{-4\beta_A}{m_N s^2(-\ln(s/r_*))}, \tag{3·15}$$

where $s=\sqrt{(\vec{r})^2+(\vec{\rho})^2}$ and

$$\beta_A = \frac{\gamma_{A0}-3\gamma_{B0}}{2\beta_0} < 0\,, \quad \max_A \gamma_{A0} = 16d\,. \tag{3·16}$$

Since the singularity of V_{NN} given in (3·6) is milder than that of $\mathcal{V}_{3N}$ in (3·15), it follows that the 3N potential V_{3N} defined in (3·13) (with B → N) is responsible for the asymptotic repulsion.

We note that increasing the number of quarks, but not the number of flavors, tends to decrease the anomalous dimensions of the operators, thereby increasing the tendency for a repulsive core of the potential, which is a further indication of a "Pauli suppression principle" mentioned before.

3.4. *The 3B potentials*

This problem is very similar to the previous three cases but since the corresponding results have not yet been published we now describe the outcome of our considerations in some detail. However, due to the very large dimension of the problem we refrain from giving full details in the paper. The tensor product of three octets can be decomposed as

$$8\otimes 8\otimes 8 = 64\oplus(35\oplus\overline{35})_2\oplus 27_6\oplus(10\oplus\overline{10})_4\oplus 8_8\oplus 1_2, \tag{3·17}$$

where the subscript indicates the multiplicity of the corresponding irreducible representation.

In the three-baryon case we are considering 9-quark operators with three quarks of each flavor ($uuudddsss$). We find that there are 35 different Dirac structures

Table II. The Dirac case 111111122.

$\gamma_{A0}/(2d)$	$SU(3)$ representation
-52	27
-36	$1, 8$
-22	8
-4	1

(plus their transforms under the symmetries $1 \leftrightarrow 2$, $3 \leftrightarrow 4$ or $(1,2) \leftrightarrow (3,4)$). The simplest Dirac structure is 111111111 and there are 9 such operators but obviously only one of them (corresponding to $\prod_{a=1}^{3}(u_1^a d_1^a s_1^a)$) remains after imposing the gauge constraints. The anomalous dimension of this operator is -36 (in $2d$ units) and is an $SU(3)$ singlet. The next simplest case corresponds to the Dirac structure 111111112 where there are originally 51 operators but only 3 remain independent after imposing the gauge constraints. The remaining three operators correspond to anomalous dimension -36 and form a singlet and the $I = 0$ and $I = 1$ members of an $SU(3)$ octet. Next we consider the Dirac case 111111122. Here the number of operators is reduced from 153 to 9. The results are summarized in Table II. For more complicated Dirac structures the number of operators increases rapidly and for example for 111333224 and 111222334 we have 9960 operators, which are reduced to 1014 by the gauge constraints. The largest case is 111223344. The number of operators here is reduced from 14130 to 1518. The spectrum of these 1518 independent operators is

$$\{-76, -60, -54, -52, -48, -46, -44, -42, -40, -36, -34,$$
$$-30, -28, -24, -22, -20, -18, -16, -12, -10, -8, -6, -4,$$
$$0, 2, 4, 6, 8, 12, 14, 18, 20, 24, 26, 30, 32, 36, 38, 42, 44, 48\}$$

and the dimensions of the $SU(3)$ representations in which these operators transform are

$$\{1, 8, 10, 27, 28, 35, 55, 64, 80, 81\}\,.$$

All eigenvalues and the corresponding $SU(3)$ representations are given in Table IV for this case.

The most important new feature of the spectrum of operators with three-baryon quantum numbers (with respect to the three-nucleon case) is the presence of attractive channels, though the vast majority of eigenvalues correspond to repulsion as before. We give a complete list of such channels (corresponding to eigenvalues $\gamma_{A0} \geq 3\gamma_{B0}$) in Table III, where an asterisk superscript (*) indicates if the operator is already present at tree level in the product of three baryon operators.

As can be seen from the table, we have found tree operators with $\gamma_{A0} > 3\gamma_{B0}$ in the 1 (singlet) and 8 (octet) channels, which give a universal attraction at short distances (attractive cores). In addition, there are tree level operators with $\gamma_{A0} = 3\gamma_{B0}$ in the 8 (octet) and $10, \overline{10}$ (decuplet) channels, which lead to non-universal attraction or repulsion at short distances depending on the sign of the ratio of associated matrix elements. On the other hand, universal repulsion at short distances (repulsive core) appears only in the $64, 35, \overline{35}, 27$ channels.

Note however, that the anomalous dimensions listed in the table give, using

Table III. List of channels with anomalous dimensions greater than or equal to $3\gamma_{B0}$. Operators present at tree level in the OPE are indicated by an asterisk *.

Dirac structure	$\gamma_{A0}/(2d)$	$SU(3)$ representation
111223344	48	$1, 8^*$
	44	$1^*, 8^*$
	42	$1^*, 8^*$
	38	$1, 8$
	36	$8^*, 10^*, \overline{10}^*$
111333224	44	$1^*, 8^*$
	38	$1, 8$
	36	1
111222334	48	8^*
	44	1^*
111133442	42	$1^*, 8^*$
	38	$1, 8$
111122334	44	1^*
	36	1
111133324	38	$1, 8$
111113324	36	1

(3·15) and (3·16), the asymptotic behavior of the "total" 3B potential $\mathcal{V}_{3B}$ and not the "true" 3B potential V_{3B}, which is obtained from the former by subtracting the sum of the 2B potentials. In the 3N case this distinction was not important, since the leading behavior of the 2N potentials is always milder than that of the 3N potential, which is hence the dominant contribution at short distances. The 3B case is made more complicated by the fact that both the "total" 3B potential $\mathcal{V}_{3B}$ and the 2B potential V_{2B} have some attractive channels, competing with each other.

Take for example the 3B configuration with Dirac structure 111223344, which contains the largest anomalous dimension 48 (in $2d$ units). This is in the octet channel and leads to an attractive total force, whose strength is proportional to the numerator of the formula (3·16), 12 in our units. On the other hand, there is a 2B contribution to this total force coming from two baryons with Dirac structure 112334 and forming a singlet state with the highest anomalous dimension 42 in our units (times a free octet baryon). From (3·10) we see that the corresponding 2B force is proportional to 18. So in this example the 2-body attractive potential is actually stronger than the total 3-body attraction hence there must be a repulsive effect coming from V_{3B} as

$$V_{3B}(\vec{r},\vec{\rho}) \simeq \frac{1}{m_B}\left[\frac{-4\beta_{3B}^{\max}}{s^2(-\ln(s/r_*))} + \sum_{1\geq i\geq j\geq 3}\frac{\beta_{BB}^{\max}}{r_{ij}^2(-\ln(r_{ij}/r_*))}\right] \tag{3·18}$$

as $|\vec{r}|, |\vec{\rho}| \to 0$, where m_B is the octet baryon mass,[*)] and (for $N_f = 3$)

$$\beta_{3B}^{\max} = \frac{2}{9}, \quad \beta_{BB}^{\max} = \frac{1}{3}, \quad s = \sqrt{\vec{r}^{\,2} + \vec{\rho}^{\,2}}, \tag{3·19}$$

[*)] We here ignore mass differences among octet baryons caused by the flavor $SU(3)$ breaking.

Table IV. The Dirac case 111223344. Operators present at tree level in the OPE are indicated by an asterisk *.

$\gamma_0^{\mathcal{R}}/(2d)$	$SU(3)$ representation (multiplicity)
48	$1, 8^*$
44	$1^*, 8^*$
42	$1^*, 8^*$
38	$1, 8$
36	$8^*, 10^*$
32	$1^*, 8^*(3), 10^*(2)$
30	$8, 10^*$
26	$1, 8(3), 10(2)$
24	$1, 8^*(4), 10^*(3), 27^*(2)$
20	$1^*(2), 8^*(6), 10^*(4), 27^*(3)$
18	$1, 8^*(4), 10^*(3), 27^*(2)$
14	$1(2), 8(5), 10(3), 27(3)$
12	$1(3), 8^*(7), 10^*(5), 27^*(4), 35^*$
8	$1, 8^*(8), 10^*(9), 27^*(5), 35^*(2)$
6	$1^*(2), 8^*(6), 10^*(6), 27^*(4), 35^*(2)$
4	$1, 8(3), 10(2), 27$
2	$8(6), 10(9), 27(5), 35(3)$
0	$1(3), 8^*(9), 10^*(10), 27^*(9), 35^*(5), 64$
−4	$1^*(4), 8^*(9), 10^*(10), 27^*(9), 35^*(6), 64$
−6	$1(2), 8^*(8), 10^*(9), 27^*(7), 35(4), 64$
−8	$8(2), 10(3), 27, 35$
−10	$1(2), 8(7), 10(9), 27(8), 35(6), 64(2)$
−12	$1, 8(5), 10(8), 27(5), 35(5), 64$
−16	$1(3), 8^*(9), 10^*(11), 27^*(11), 28, 35^*(9), 64(3)$
−18	$1, 8^*(7), 10^*(12), 27^*(9), 28, 35^*(10), 64(3), 81$
−20	$1, 8(4), 10(5), 27(4), 35(3), 64$
−22	$1, 8(5), 10(9), 27(8), 28(2), 35(9), 64(2), 81$
−24	$1, 8^*(4), 10(5), 27^*(7), 28(4), 35^*(9), 64(3), 81$
−28	$1(2), 8(6), 10(7), 27(7), 28(2), 35(9), 64(4), 81(2)$
−30	$1, 8(4), 10(6), 27(7), 28(2), 35(8), 64(3), 81$
−34	$1, 8(5), 10(6), 27(6), 28, 35(7), 64(4), 81(2)$
−36	$8(3), 10(4), 27(4), 28, 35(5), 64(3), 81(2)$
−40	$8, 10(3), 27(3), 28(2), 35(5), 64, 81(2)$
−42	$8(2), 10(3), 27(3), 28, 35(4), 64(2), 81(2)$
−44	$8, 10, 27, 35, 64, 81$
−46	$8, 10(3), 27(3), 28(2), 35(5), 64(2), 80, 81(2)$
−48	$27, 28, 35, 64, 80, 81$
−52	$10, 27(2), 28(2), 35(3), 64(2), 80(2), 81(2)$
−54	$10, 27, 28, 35(2), 64, 80, 81$
−60	$10, 27(2), 28(2), 35(3), 64(2), 80(2), 81(2)$
−76	$35, 55, 64, 80, 81$

$$r_{12} = |\vec{r}|, \quad r_{13} = |\vec{r}/2 - \sqrt{3}\vec{\rho}|, \quad r_{23} = |\vec{r}/2 + \sqrt{3}\vec{\rho}|. \tag{3·20}$$

As another similar example we take the Dirac structure 111133442 in the singlet channel with anomalous dimension 42. A two-baryon operator with Dirac structure 111234 in the octet channel (combined with an extra octet baryon) contributes to

this component of the 3B force and the corresponding numbers are:

$$\beta_{3B}^{\max} = \frac{1}{9}, \qquad \beta_{BB}^{\max} = \frac{4}{27}. \tag{3·21}$$

We do not further attempt to analyze the general case, which is clearly very complex since in addition to the cancellation between a 2B and a 3B force as in the above examples, there are cases where cancellation occurs among the 2B forces.

§4. Conclusion

In this paper, we have presented our recent activities on determinations of short distance behaviors of BB and 3B potentials defined from the NBS wave function in QCD using the operator product expansion and the RG analysis in perturbative QCD. Our results show that the repulsions at short distance (repulsive core) become stronger for more participating valence quarks or less different numbers of flavors N_f^v. Explicitly we have:

1. NN potentials ($N_f^v = 2$) seem to have a repulsive core at low energy, which however is not universal in the sense that the coefficient may depend on the properties of the NBS wave function such as total energy.
2. BB potentials ($N_f^v = 3$) can have not only a repulsive core but also channels with an attractive core, the latter of which is universal (i.e. energy independent). In particular, the attractive core in the flavor singlet potential has indeed been confirmed in lattice QCD simulations.[15),16)]
3. 3N potentials have a universal repulsive core. This is the most unambiguous result in our project.
4. In the 3B case we were able to study the short distance asymptotics of the "total" 3B potential $\mathcal{V}_{3B}$ only with our methods. Although the vast majority of various channels $(64, 35, \overline{35}, 27)$ in 3B potentials have a universal repulsive core, a few channels $(1, 8)$ have a universal attractive core. In addition, there may appear a non-universal repulsive or attractive core in the $10, \overline{10}$ channels.

Roughly speaking, these results suggest that a "Pauli suppression principle" among quarks is at work. Explicit 1-loop calculations are needed, however, to obtain the detailed structure of the anomalous dimensions of BB or 3B operators.

Our results give important information on the short distance behaviors of baryonic interactions, which can be used to constrain baryonic potentials at short distance obtained in lattice QCD,[25),26)] in particular, to constrain 3B potentials.[17)] As mentioned in the introduction, the universal repulsion of 3B potentials seems necessary to explain an existence of two-solar-mass neutron star. Our result shows that the vast majority of 3B channels have a repulsive core in the 3B potential, though a few channels have an attractive core. It is interesting and important to check these qualitative behaviors of 3B potentials at short distances and furthermore to investigate the strength of these cores, employing lattice QCD simulations.

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