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# Population Aging and Sectoral Employment Shares

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## **Abstract**

This paper theoretically investigates the effect of population aging on the employment share of an elder care service industry. We show that there exists the threshold level of income elasticity of demand for the elder care service, above which population aging spurs economic growth. In a closed economy, the threshold level is unity, whereas it exceeds unity in a small open economy.

**JEL Classification No.:** D91, J14, O41.

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## 1. Introduction

Population aging is a phenomenon which many developed countries have faced recently. As more people live longer and the need for long-term care services increases, it may be natural to consider that more labor inputs will be allocated into the long-term care industry. There may be a concern that such a change slows economic growth because the productivity of the elder care industry is lower than that of other industries (e.g., a manufacturing industry).<sup>1</sup>

Theoretically, van Groezen et al. (2005) showed that the above-mentioned concern does not necessarily hold true in a closed economy. On the other hand, van Groezen et al. (2005) and Hashimoto and Tabata (2010) argued that in a small open economy, the result differs from the result derived in a closed economy. They showed that population aging necessarily increases the employment share of the elder care industry and results in slower economic growth. The important point is that they assume that the utility function is homothetic, or equivalently, the income elasticity of demand for health care is unity.

It is often reported that the income elasticity of demand for the long-term care differs from unity.<sup>2</sup> We examine the case where the income elasticity is not unity, that is, where the preference is nonhomothetic.<sup>3</sup> We show that when the income elasticity is larger than unity, it is theoretically possible that population aging may decrease the employment share of the elder care sector even in a small-open economy.

The remainder of this paper is as follows. Section 2 introduces the basic model and explores the effect of population aging on the employment share in a closed economy. In Section 3, we analyze these issues in a small open economy. Finally, Section 4 concludes the paper.

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<sup>1</sup> Fujisawa and Colombo (2009) argues that the wage level of long-term care workers is lower than the national average wage, especially in Japan, the United Kingdom, and the United States.

<sup>2</sup> For example, in Japan, Ohkusa (2002) showed the elasticity of demand for long-term care exceeds unity. However, there is some controversy. Shimizutani and Noguchi (2004) argued that the level is lower than unity.

<sup>3</sup> Kongsamut et al. (2001) shows a mechanism where a structural change is driven by the non-homothetic preference of consumers. Matsuyama (2008) provides an excellent summary of research in this field.

## 2. The Effect of Population Aging in a Closed Economy

We consider that two sectors of production exist: the commodity sector (labeled  $Y$ ) and the elder care services sector (labeled  $M$ ). The price of  $Y$  is normalized to unity, and  $p_t$  denotes the price of  $M$ . Labor is perfectly mobile across the sectors, so that the wage rate is the same between the sectors.

### 2.1. Firms

Firms act competitively in both sectors. Commodity  $Y$  can be used for either consumption or investment. The production technology is  $Y_t = F(K_t, A_t L_{Yt}) = K_t^\alpha (A_t L_{Yt})^{1-\alpha}$ , where  $K_t$  and  $L_{Yt}$  denote physical capital and labor inputs hired in the commodity sector, respectively, and  $A_t > 0$  denotes a productivity parameter. The elder care sector is a labor-intensive industry, and for simplicity, we assume  $M_t = B L_{Mt}$ , where  $L_{Mt}$  denotes the labor inputs employed in sector  $M$ . Let  $v_t$  denote the employment share of sector  $Y$ ,  $L_{Yt}/(L_{Yt} + L_{Mt})$ .<sup>4</sup>  $A_t$  evolves according to  $A_t = g(v_t)A_{t-1}$ , and each firm considers the level of  $A_t$  as given. A similar formulation is seen in van Groezen et al. (2005). We assume  $g'(v_t) > 0$  and  $g(0) = 1$ . Defining a capital-effective labor ratio as  $k_t \equiv K_t/(A_t L_{Yt})$ , the production of  $Y$  per effective unit of labor is described by  $f(k_t) = k_t^\alpha$ . Through the profit-maximization behavior of firms in sector  $Y$ , the interest rate  $r_t$  and wage rate  $w_t$  are derived, respectively, as follows:

$$r_t = f'(k_t) = \alpha k_t^{\alpha-1}, \quad w_t = A_t(f(k_t) - f'(k_t)k_t) = (1 - \alpha)A_t k_t^\alpha. \quad (1)$$

We assume that physical capital fully depreciates during the production process, so that  $r_t$  represents a gross interest rate. Furthermore, we obtain:

$$p_t = \frac{w_t}{B}, \quad (2)$$

by considering the optimal behavior of firms in sector  $M$ .

### 2.2. Households

We consider an overlapping-generations model where agents live at most two periods, referred to as young and old. The population size of each generation is the same and

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<sup>4</sup> In this paper, the market size of an industry refers to the share of each industry in the labor market.

normalized to unity. Each agent faces uncertainty concerning his or her longevity, and  $q_t$  denotes the probability that agents can be alive in their old period, and it is treated as exogenous. The expected utility function is given by:

$$EU_t = \log c_{1t} + \beta q_t [\delta \log c_{2t+1} + (1 - \delta)h(m_{t+1})], \quad (3)$$

where  $c_{1t}$  and  $c_{2t+1}$  denote the consumption of commodity  $Y$  when young and old, respectively, and  $m_{t+1}$  denotes the consumption of the care services when old.  $\beta$  and  $\delta$  represent the discount rate and an old agent's taste regarding the trade-off between commodity and care services.  $h(m)$  stands for the utility from the elder care, and we assume that  $\sigma_m \equiv -h''(m)m/h'(m) \geq 0$  is constant.<sup>5</sup> When  $\sigma_m = 1$ , the preference is homothetic; otherwise, it is nonhomothetic.

When young, each agent has one unit of labor endowment and works. We assume the existence of the Yaari-type competitive insurance companies. That is, the rate of return on the annuity is  $r_{t+1}/q_t$ . Denoting the purchase of the annuity by  $s_t$ , budget constraints are expressed as:

$$c_{1t} + s_t = w_t, \quad (4)$$

$$c_{2t+1} + p_{t+1}m_{t+1} = \frac{r_{t+1}}{q_t} s_t. \quad (5)$$

Agents maximize (3) subject to (4) and (5) and consequently, the following equations are obtained:

$$c_{2t+1} = \beta \delta r_{t+1} c_{1t}, \quad (6)$$

$$(1 - \delta)h'(m_{t+1}) = \delta \frac{p_{t+1}}{c_{2t+1}}. \quad (7)$$

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<sup>5</sup> A typical functional form is  $h(m) = (h^{1-\sigma_m} - 1)/(1 - \sigma_m)$  when  $\sigma_m \neq 1$ , and  $h(m) = \log h$  when  $\sigma_m = 1$ .

### 2.3. Markets

There are four kinds of markets: commodity, elder care services, capital, and labor markets. The market clearing condition for the elder care services is given by:

$$BL_{Mt} = q_{t-1}m_t. \quad (8)$$

By using the definition of  $k_t$ , the equilibrium condition for the asset market,  $s_t = K_{t+1}$ , is rewritten as:

$$s_t = A_{t+1}L_{Yt+1}k_{t+1}. \quad (9)$$

Noting that  $L_t = 1$  and using  $v_t$ , the labor market clearing conditions are written as:

$$L_{Yt} = v_t, \quad L_{Mt} = 1 - v_t. \quad (10)$$

Walras' law guarantees that the commodity market always clears when (8)–(10) hold.

### 2.4. Equilibrium

Now, we are ready to derive the equilibrium allocation of the economy. Using (1), (4), (9), and (10), (6) is rewritten as:

$$c_{2t+1} = \beta\delta\alpha k_{t+1}^{\alpha-1}[(1-\alpha)A_t k_t^\alpha - A_{t+1}v_{t+1}k_{t+1}]. \quad (11)$$

In addition, applying (1), (2), (8), and (10) to (7) yields the following equation:

$$(1-\delta)h'\left(\frac{B(1-v_{t+1})}{q_t}\right) = \delta \frac{1-\alpha}{B} \frac{A_{t+1}k_{t+1}^\alpha}{c_{2t+1}}. \quad (12)$$

Eq. (5) is rewritten by using (1), (2), and (8)–(10) as follows:

$$c_{2t+1} = \frac{A_{t+1}k_{t+1}^\alpha}{q_t}(v_{t+1} - (1-\alpha)). \quad (13)$$

By substituting (13) for (12), we obtain:

$$H(v_{t+1}, q_t) \equiv (1 - \delta)h' \left( \frac{B(1 - v_{t+1})}{q_t} \right) - \delta \frac{1 - \alpha}{B} \cdot \frac{q_t}{v_{t+1} - (1 - \alpha)} = 0. \quad (14)$$

Here,  $v_{t+1}$  is solved as a function of  $q_t$  from (14);  $v_{t+1} = v(q_t)$ . The effect of the population aging on the employment shares of the  $Y$  and  $M$  sectors is obtained by examining  $v'(q_t)$ .

**Proposition 1:**  $v'(q_t) > 0$  holds if and only if  $\sigma_m < 1$ .

The proof is given in Appendix A. Proposition 1 argues that when  $\sigma_m < 1$ , the employment share of the elder care services sector shrinks, and that of the commodity sector expands, in the process of population aging, which spurs the growth of  $A_t$ . When  $\sigma_m < 1$ , the marginal rate of substitution,  $(\partial EU_t / \partial m_{t+1}) / (\partial EU_t / \partial c_{2t+1})$ , increases along a ray (see the dotted indifference curve and the dotted line from the origin in Fig. 1<sup>6</sup>) and consequently, the income offer curve is depicted as in Fig. 1. This indicates that the income elasticity of demand for the care services exceeds unity. Similarly,  $\sigma_m = 1$  corresponds to the case where the income elasticity is unity.

When expected longevity  $q_t$  increases, the number of people who need  $M$  also increases, which has a positive impact on the aggregate demand for  $M$ . On the other hand, an increase in  $q_t$  decreases the rate of return on annuity, which means that consumption in the old period is more expensive. This price effect has a negative impact on the demand for  $M$ , and the negative impact becomes larger as the income elasticity of demand is greater. Proposition 1 argues that the critical value is unity in a closed economy, and that population aging has no impact on the relative employment share when  $\sigma_m = 1$ . This result is consistent with van Groezen et al. (2005).

We briefly describe a dynamic equilibrium path of the effective capital-labor ratio,  $k_t \equiv K_t / (A_t L_{Yt})$ . From (11), (13), and  $A_{t+1} / A_t = g(v_{t+1})$ , we obtain:

$$k_{t+1} = \Gamma(q_t, v_{t+1}) k_t^\alpha, \quad (15)$$

where  $\Gamma(q_t, v_{t+1})$  is defined as:

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<sup>6</sup> To make presentation simple, we omit a  $c_{1t}$ -axis when Fig. 1 is drawn.

$$\Gamma(q_t, v_{t+1}) \equiv \frac{(1 - \alpha)\alpha\delta\beta q_t}{(1 + \alpha\delta\beta q_t)v_{t+1} - (1 - \alpha)} \cdot \frac{1}{g(v_{t+1})}. \quad (16)$$

Because  $v_{t+1} = v(q_t)$ , at a given level of  $q_t$ , the nontrivial steady state exists uniquely and  $\{k_s\}_{s=t}^{\infty}$  converges to the steady state, as seen in (15). When  $q_t$  increases, two effects are found from (16). First,  $\partial\Gamma(q_t, v_{t+1})/\partial q_t > 0$  indicates that an increase in  $q_t$  induces people to save more to provide for their old age. This is consistent with the result pointed out by Yakita (2001). Second,  $\partial\Gamma(q_t, v_{t+1})/\partial v_{t+1} < 0$  indicates that an increase in  $v_{t+1}$  reduces  $k_{t+1}$ . Thus, when  $v'(q_t) > 0$ , the levels of  $\{k_s\}_{s=t+1}^{\infty}$  becomes smaller as the latter impact is larger.

### 3. The Effect of Population Aging in a Small Open Economy

We consider a small open economy, and assume that only capital and commodity  $Y$  are mobile across the border. The level of  $r_t$  and  $k_t$  are fixed at  $r$  and  $k$ , respectively, and  $r = \alpha k^{\alpha-1}$  holds from (1). The wage rate is expressed as  $w_t = (1 - \alpha)k^\alpha A_t$ . We impose the following assumption.

**Assumption 1:**  $g'(v) < g(v)^2/r$  holds.

The equilibrium system is derived from (2), (4)–(8), and (10). (Note that (9) does not hold in a small open economy.) We obtain the following equations:

$$c_{2t+1} = \beta\delta r c_{1t}, \quad (17)$$

$$(1 - \delta)h'\left(\frac{B(1 - v_{t+1})}{q_t}\right) = \delta \frac{1 - \alpha}{B} k^\alpha A_{t+1} \frac{1}{c_{2t+1}}, \quad (18)$$

$$c_{1t} + \frac{q_t}{r} c_{2t+1} + \frac{1 - \alpha}{\alpha} k A_{t+1} (1 - v_{t+1}) = (1 - \alpha)k^\alpha A_t. \quad (19)$$

Eq. (19) is the lifetime budget constraint derived from (4) and (5), and the third term on the left-hand side is obtained by using (2), (8), and (10). By using  $A_{t+1}/A_t = g(v_{t+1})$  and eliminating  $c_{1t}$  and  $c_{2t+1}$  from (17)–(19), the following equation is obtained:



$$B\beta(1-\delta)h'\left(\frac{B(1-v_{t+1})}{q_t}\right)\frac{1}{g(v_{t+1})} = \frac{1+\delta\beta q_t}{r-g(v_{t+1})(1-v_{t+1})}. \quad (20)$$

$v_{t+1}$  is solved as a function of  $q_t$ ,  $v_{t+1} = \tilde{v}(q_t)$ . The effect of an increase in  $q_t$  on  $v_{t+1}$  is summarized in Proposition 2.

**Proposition 2:**  $\tilde{v}'(q_t) > 0$  holds if and only if  $\sigma_m < \frac{\delta\beta q_t}{1+\delta\beta q_t}$ .

The proof of Proposition 2 is given in Appendix B. We observe that when the income elasticity is unity (that is,  $\sigma_m = 1$ ),  $\tilde{v}'(q_t) < 0$  always holds. This is consistent with the preceding literature. Proposition 2 also shows that it is theoretically possible that an increase in longevity decreases the employment share of the long-term care industry and raises the rate of economic growth.

#### 4. Concluding Remarks

We examined whether population aging induces a sectoral shift towards the long-term care industry. Preceding studies have shown that the result derived in a closed economy differs from the result derived in a small open economy. This paper showed that such a sharp contrast is generated when the homothetic preference is assumed. We revealed that the main difference between the two economies is the threshold level concerning the income elasticity of demand for care services. The threshold level is unity in a closed model, whereas it exceeds unity in a small open economy.

#### Appendix A

Proposition 1 is proved by calculating the signs of  $\partial H(v_{t+1}, q_t)/\partial v_{t+1}$  and  $\partial H(v_{t+1}, q_t)/\partial q_t$ . As regards  $\partial H(v_{t+1}, q_t)/\partial v_{t+1} > 0$ , it is immediately confirmed.  $\partial H(v_{t+1}, q_t)/\partial q_t$  is calculated as follows:

$$\frac{\partial H(v_{t+1}, q_t)}{\partial q_t} = (1-\delta)h''(m_{t+1})\left(-\frac{m_{t+1}}{q_t}\right) - \delta \frac{1-\alpha}{B} \frac{1}{v_{t+1} - (1-\alpha)}.$$

Applying (14) to the second term on the right-hand side yields:

$$\frac{\partial H(v_{t+1}, q_t)}{\partial q_t} = -\frac{1-\delta}{q_t} h''(m_{t+1}) m_{t+1} - \frac{1-\delta}{q_t} h'(m_{t+1}).$$

Furthermore, by using the definition of  $\sigma_m$ , we obtain:

$$\frac{\partial H(v_{t+1}, q_t)}{\partial q_t} = \frac{1-\delta}{q_t} h'(m_{t+1})(\sigma_m - 1).$$

$\partial H(v_{t+1}, q_t)/\partial q_t$  is negative if and only if  $\sigma_m > 1$ . Thus,  $v'(q_t)$  is positive.

## Appendix B

To prove Proposition 2, we take the logarithm of (20):

$$\begin{aligned} \log B\beta(1-\delta) + \log h'\left(\frac{B(1-v_{t+1})}{q_t}\right) - \log g(v_{t+1}) \\ = \log(1 + \delta\beta q_t) - \log(r - g(v_{t+1})(1 - v_{t+1})). \end{aligned}$$

Totally differentiating this equation with respect to  $v_{t+1}$  and  $q_t$  yields the following equation:

$$\begin{aligned} \frac{h''(m_{t+1})}{h'(m_{t+1})} \left[ -\frac{B}{q_t} dv_{t+1} - \frac{B(1-v_{t+1})}{q_t^2} dq_t \right] - \frac{g'(v_{t+1})}{g(v_{t+1})} dv_{t+1} \\ = \frac{\delta\beta}{1 + \delta\beta q_t} dq_t - \frac{g(v_{t+1}) - g'(v_{t+1})(1 - v_{t+1})}{r - g(v_{t+1})(1 - v_{t+1})} dv_{t+1}. \end{aligned}$$

By using  $\sigma_m \equiv -h''(m)m/h'(m)$ , the first term on the left-hand side is written as

$$\frac{h''(m_{t+1})}{h'(m_{t+1})} \left[ -\frac{B}{q_t} dv_{t+1} - \frac{B(1-v_{t+1})}{q_t^2} dq_t \right] = \sigma_m \left[ \frac{dv_{t+1}}{1-v_{t+1}} + \frac{dq_t}{q_t} \right].$$

Consequently, we obtain:

$$\left[ \frac{\sigma_m}{1-v_{t+1}} + \frac{[g(v_{t+1})]^2 - g'(v_{t+1})r}{g(v_{t+1})(r - g(v_{t+1})(1 - v_{t+1}))} \right] dv_{t+1} = \left[ \frac{\delta\beta}{1 + \delta\beta q_t} - \frac{\sigma_m}{q_t} \right] dq_t.$$

Because  $\sigma_m \geq 0$ , the bracket of the left-hand side is positive under Assumption 1 (Note that from (20), the term  $r - g(v_{t+1})(1 - v_{t+1})$  is positive). Thus,  $dv_{t+1}/dq_t > 0$

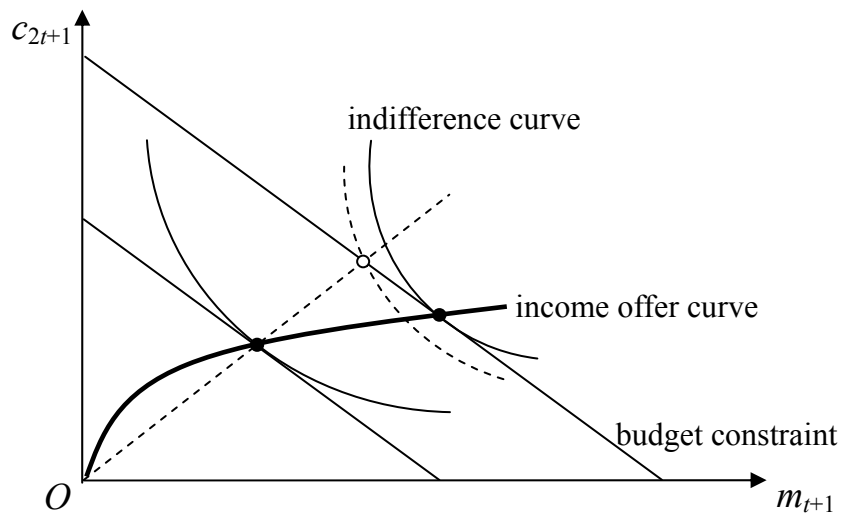
holds if and only if  $\frac{\delta\beta}{1+\delta\beta q_t} - \frac{\sigma_m}{q_t} > 0$ , which is equivalent to  $\sigma_m < \delta\beta q_t/(1 + \delta\beta q_t)$ .

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**Fig.1:** Optimal consumption when  $\sigma_m < 1$ .