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Thrust Measurement of a Multi-Cycle Partially Filled Pulse Detonation Rocket Engine

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Abstract

In the present research, we experimentally verified the partial-fill effect in a multi-cycle pulse detonation rocket engine (PDRE). Intermittent thrust of a PDRE was measured by using a spring-damper mechanism that smoothes this intermittent thrust in the time direction. The intermittent mass flow rates were assessed by gas cylinder pressure or mass difference measurement. The maximum specific impulse was 305 ± 9 seconds at an ethylene and oxygen propellant fill fraction of 0.130 ± 0.004 . When the fill fraction was greater than 0.130, the specific impulse was increased as the partial fill fraction was decreased. When the fill fraction was less than 0.130, the specific impulse was sharply decreased as the partial fill fraction was decreased. This decrease was due to diffusion between propellant and purge gases and for the short length of the transition from deflagration to detonation. The multi-cycle pulse detonation rocket engine has a partial-fill effect that may be mainly due to the suctioned air and is consistent with the single-cycle partial-fill model of Endo et al. [Endo, T., Yatsufusa, T., Taki,

S., Matsuo, A., Inaba, K., and Kasahara, J., “Homogeneous-Dilution Model of Partially-Fueled Simplified Pulse Detonation Engines,” Journal of Propulsion and Power, Vol. 23, No. 5, 2007, pp. 1033–1041.]

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Nomenclature

A_e	= control-volume surface area through which an exhaust jet flows
c	= attenuation coefficient of a damper
F_f	= force acting on the PDE tube by the rail in the x direction
F_{kc}	= force acting on the PDE tube by the spring-damper system in the x direction
F_{lc}	= force acting on the spring-damper system by the load cell in the x direction
F_p	= force acting on the PDE tube by the fluid in the x direction
F_t	= force acting on the PDE tube by the supplying tubes in the x direction
F_0	= plateau thrust
$f_{\text{propellant}}$	=propellant fill fraction
f_{purge}	=purge-gas fill fraction
$f_{\text{air, est}}$	=estimated suctioned-air fill fraction

g = gravitational acceleration

I = impulse acting on the PDE tube by the fluid in the x direction

$I_{\text{total}, x}$ = total impulse acting on the PDE tube by the fluid in the $-x$ direction during all cycles

I_{total} = absolute value of the total impulse acting on the PDE tube by the fluid in the $-x$ direction during all cycles

I_{sp} = specific impulse

k = spring constant

M_c = molecular weight in the cylinder

$m_{\text{propellant}}$ = propellant mass during all of the cycles

m_{purge} = purge-gas mass during all of the cycles

m_d = mass of weight

m_t = PDE tube mass

p_a = ambient pressure around the control volume

p_e = exhausted-jet pressure on the control surface

R_c = gas constant of the gas in the cylinder

T_c = gas temperature in the cylinder

t = time

- t_{total} = time during all of the cycles
- t_{cyc} = period of one cycle
- t_{pl} = time during which the plateau thrust is maintained
- u_e = fluid velocity component in the x direction on the control surface
- u_x = fluid velocity component in the x direction
- V = volume
- V_c = volume of the cylinder
- v_w = velocity of the weight
- v_s = horizontal velocity of a solid element
- x = horizontal axis fixed in the control volume
- x_p = PDE tube position in the x axis, of which the origin is the natural length point
of a spring
- Y = mass fraction that equals the propellant mass divided by the total mass of both
the propellant and the purge gases
- Δm_c = mass difference between the initial and final masses of the gas in the cylinder
- Δp_c = pressure difference between the initial and final pressures of the gas in the
cylinder
- ρ_g = density of the gas

ρ_s = density of the solid

ϕ = equivalence ratio

Subscripts

cs = control surface

cv = control volume

I. Introduction

A pulse detonation engine (PDE) obtains thrust by generating detonation waves intermittently [1, 2]. PDE review papers have been published by Kailasanath [3, 4], Bazhenova and Golub [5], and Roy et al. [6]. The PDE for aerospace propulsion obtains thrust by blowing the detonated high-pressure gas down. The typical shape of a PDE is a straight tube, one end of which is closed while the other is open. After the tube is filled with propellant gas, the gas is ignited at the closed end of the tube and the detonation is initiated. A self-sustained detonation wave, which compresses the propellant by shock waves, propagates toward the open end of the tube, and the engine can be operated even if there are no compression mechanisms such as compressors or pistons. Sufficient specific impulse (200-350 sec) can be obtained even with the same fill pressure as that of ambient air

(Zitoun and Desbordes [7], Morris [8], Endo and Fujiwara [9], Endo et al. [10], Wintenberger et al. [11], Cooper et al. [12]), and thus a rocket engine with an extremely low combustor fill pressure (pulse detonation rocket engine, PDRE) becomes possible [8]. The flow as in a simplified PDE [9, 10] can be achieved in the combustor of a PDRE, because there is no interaction with the air-breathing mechanism (intake and compressor) installed before the combustor. The air-breathing PDE has been studied by many researchers, including Tally and Coy [13], Wintenberger and Shepherd [14], Harris et al. [15], Ma et al. [16], and Kojima and Kobayashi [17]. Moreover, it is easy to increase the specific impulse by using a mechanism (an extension tube for partial filling [18], an ejector [19, 20], or a nozzle [8]) installed after the combustor. In the PDRE, the specific impulse can be increased by partially filling the detonation tube with propellant. With a partial fill, the burned gas generated during the previous cycle remains in the tube and purge gas is added in front of this propellant in the detonation tube. The burned gas and purge gas are called inertia gas in this paper. As the detonation wave propagates in the propellant gas with which the detonation tube is partially filled, the burned high-pressure gas is expanded and accelerated toward the open end of the tube. Since the burned gas pressure force acts on the inertia gas, the burned gas obtains reaction force from the inertia gas. The closed-end thrust wall of the detonation tube obtains a larger specific impulse in the partially filled tube than it would obtain in a fully filled one. The specific impulse generally increases with the decrease of the fill fraction. By this increase of specific impulse in the

partially filled tube, the intermittent thrust engine like a PDE is distinguished from other aerospace steady-state-flow engines. In the PDE system, an ambient-air inertia can be available for increasing thrust by using a simple straight tube, without a ducted fan. Many experimental, computational, and analytical works have been performed to examine the effects of partial filling on single-tube and single-cycle PDEs. Endo et al. [9] analyzed the performance of the straight-tube PDE fully filled with propellant and formulated the pressure history at the thrust wall of the closed end of the PDE tube with no empirical parameters. In a subsequent study [10] they formulated more accurately the pressure decay portion of this thrust-wall pressure history.

Recently, Endo et al. [21] proposed a single-cycle partially filled PDE performance model based on the experimental results of Zhdan et al. [22] showing that the impulse generated in the PDE tube during one-cycle operation did not depend on the detonation initiation points. Endo et al. assumed that the thrust of the partially filled PDE tube is identical to the thrust of the fully filled PDE tube with propellant gas homogeneously mixed with inertia gas. Their model [22] is in good agreement with the results of experiments and computational fluid dynamics (CFD) calculations to within 15%. Sato et al. [18] obtained the specific impulse of a partially filled PDE tube with a hydrogen-oxygen mixture by using CFD, and they proposed the following semi-empirical formula for the specific impulse:

$$\frac{I_{sp,p}}{I_{sp,p}|_{ff=1}} = \frac{1}{\sqrt{Y}}.$$

This equation allows one to definitely account for the effect of partial filling. Specifically, the specific impulse was in inverse proportion to the square root of the ratio of the mass of the propellant to the mass of all gas. As a result, the thrust also increases. This empirical equation gives the same results as those of the model of Endo et al. [21] in almost all of the fill fraction ($\phi \geq 0.01$). Although the specific impulse increases infinitely as the fill fraction approaches zero, such a phenomenon never occurs physically. Accordingly, this equation cannot be applied if the fill fraction is extremely small.

In a recent study, Kasahara et al. [23] showed the specific impulse upper limit in the small fill fraction region by using a partially filled shock tube with an open-ended extension tube. In their experiment, high-pressure inert gas in the shock tube expanded instantaneously into the ambient-inertia air in the extension tube. This specific impulse upper limit could be theoretically explained by the bubble model (Cooper [24], Cooper et al. [25]). Kasahara et al. [23] showed that the partial-fill effect is essentially related to the interaction between the pressure wave (shock and expansion waves) propagated in the gas and the contact discontinuity surface. Although the single-cycle PDE gas dynamics in a partially filled tube has been well clarified, the multi-cycle PDE gas dynamics in a partially filled tube has not been sufficiently verified, because the inertia gas in the multi-cycle PDE operation is not only an inert gas but also a burned gas generated during a previous cycle, and the negative gauge pressure in the exhaust phase can cause the suction of ambient air at

the open end of the tube.

Schauer et al. [26] reported that a multi-cycle PDE had its specific impulse increase when the fill fraction decreased. Cooper et al. [25] compared their single-cycle model with the multi-cycle data of Schauer et al. [26] by assuming that multicycle operation is equivalent to a series of ideal single cycles. Schauer et al. [26] did accurate measurements of thrust, fill fraction, and specific impulse in the multi-cycle PDEs, but they only described the thrust and mass-flow-rate measurement setup briefly in their paper, and they did not discuss the fraction between the burned gas and the purge gas (air) in the tube. In this experimental study we introduced a spring-damper mechanism that smoothes the impulsive thrust generated by the multi-cycle PDE operation. We show that this mechanism is effective for obtaining smoothed thrust from a single-tube PDE. In the present study, we achieved multi-cycle PDE tube operation while varying the fill fraction and verified the increase in specific impulse by comparing the present experimental results with the partial-fill model of Endo et al. [21], which takes into account the purge gas, the previous-cycle burned gas, and the suctioned air.

II. Experimental Equipment and Thrust Measurements

We used ethylene-oxygen mixtures as propellant. These mixtures have very high detonability and have been used in a number of experimental works [27-31] due to reliable detonation initiations in

each cycle in the multi-cycle PDE operation. In the present experiment, the impulsive thrust in the multi-cycle PDE operation was approximately 8000 N lasting 10 ms in each cycle. Such an impulsive thrust cannot be directly measured based on the typical load cells because of the measurement limits of force and time resolution. The spring-damper mechanism of our device was placed between the load cell and the impulsive-thrust wall of the PDE tube. This mechanism can smooth the impulsive thrust in the direction of time. Mechanical model calculation was performed to set an appropriate spring constant and damping coefficient. The propellant mass flow of the PDE was intermittent due to its propellant supply system typically having high-frequency valves. In the present experiment, we employed the measurement method of one-cycle mass flow, which is obtained from the directly weighted total mass decrease in the propellant cylinders divided by the total cycle number (total mass weighted method). The other method we employed was to measure the cylinder-pressure differences before and after experiments and calculate mass differences from these pressure differences and the equation of state. However, the latter method was calibrated by the former total mass-weighted method, in which a one-liter gas cylinder and electrobalance were used.

A. Experimental Devices

A schematic of the experimental apparatus is shown in Fig. 1, and Fig. 2 is a photograph of the experimental devices. The four ball-bearing wheels of the PDE tube (a cylindrical tube connected to

a tapered tube) are attached to two parallel guide rails. The thrust generated at the tapered closed end of the PDE tube is measured by the load cell through the spring-damper mechanism. The PDE tube can slide on the rail before and after detonation in the x direction with little friction. The propellant gases are supplied to the PDE tube through the top portion of the tapered closed-end tube. The ethylene and oxygen gases as propellant are mixed at and supplied through the cross joint directly connecting to the tapered closed-end tube. The helium gas as a purge gas is also supplied to the PDE tube through the cross joint. This cross joint is attached to the three tubes with solenoid valves, as shown in Fig. 1. These valves' mass flow rate and operation frequency are controlled by a programmable controller. One solenoid valve is used for ethylene gas, and one is used for helium gas. Two parallel solenoid valves are used for oxygen gas (Figs. 1 and 3). These solenoid valves are connected to the propellant and purge gas cylinders through flexible tubes. The igniter is supplied with power by the igniter controller and is controlled by a programmable controller with a 1-ms time resolution sequence. The igniter is an automobile spark plug (BOSCH, FR8DPIX). The voltage in the electrode gap of the spark plug is approximately 7 kV. This spark plug gap voltage can be monitored during the PDE operation. The length of the cylindrical tube is 2000 mm, and the inner diameter of the cylindrical tube is 100 mm. The material of the tube is stainless steel (SUS 316). The open end of the PDE tube is exposed to the ambient air. The solenoid valves attached to the tapered tubes are shown in Fig. 3. The tapered tube has a half-opened angle of 15 degrees and a volume of

0.47 liter. If a cylindrical tube is replaced by a tapered tube with the same cross-section area, the tapered tube length is 60 mm. If the PDE tube is replaced by a 100-mm constant-inner-diameter cylindrical tube, the reduced total length of this cylindrical tube is 2060 mm. The ignition point is on the inside of the tapered tube. Although the tapered tube has 6 ports for igniters, we used only one port, 157 mm from the open end of the tapered tube (2159 mm from the open end of the PDE tube), as shown in Fig. 3. The tapered tube is preheated to 100 degree Celsius or hotter. The solenoid valves are directly driven two-port valves (CKD, AB-41-02-1, Normally Closed), as shown in Fig. 3. The orifice diameter, Cv value, and maximum primary pressure of this solenoid valve are 1.5 mm, 0.1, and 5 MPa, respectively.

B. Experimental Conditions

Figure 4 shows the operation sequence when the operation cycle is 200 ms. When the propellant (ethylene and oxygen) valves are opened, the purge gas (helium) valve is closed, and vice versa. The plug igniter is sparked just after the control signal falls below a trigger level. Figure 4 shows sequence A, in which the time during which the propellant is supplied is identical to the time for purging by helium gas. Stable multi-cycle operation is achieved when the operation frequency is comparatively low. At such a low frequency, the volume filled with propellant or purge gas is large, and therefore the length identical to the purge gas volume divided by the PDE tube cross-section

area becomes large as well [31, 32]. If the operation frequency increases over these criteria, the purging becomes not sufficient due to decreased helium purging time. With this type of high-frequency operation, a stable multi-cycle operation cannot be achieved due to improperly timed ignition caused by diffusing and mixing of the previous-cycle high-temperature burned gas with the unburned gas. To prevent this improperly timed ignition, when the time for supplying propellant is below 60 ms, we employed sequence B (Fig. 5), in which the time for supplying propellant is half as long as for purging. By using sequence B, multi-cycle operations can be performed in the conditions of higher frequency and lower fill fraction. In the present experiment, the total operation time of one cycle's time multiplied by the cycle number does not exceed 3000 ms. The experimental conditions are shown in Table 1. The primary pressure of all the valves is 3 MPa. The time for supplying is varied from 200 ms to 40 ms in 11 steps. The multi-cycle operation is performed in sequence A with shots 1-7 and in sequence B with shots 8-11.

C. Thrust Measurements

In the present study, we used the load cell (Aikoh Engineering, DUD-100K) to measure the thrust during multi-cycle operation. The maximum load of this load cell is ± 1000 N. The oscilloscope receives and records input from this load cell through an amplifier (Unipulse, F304A). The mechanical model for measuring the thrust generated by the PDE tube is shown in Fig. 6. The

control volume is defined by the broken line shown in Fig. 6. The motion of the PDE tube in the x direction is derived using the following equation:

$$F_p + F_{kc} + F_f + F_t = m_t \ddot{x} \quad (1).$$

The motion of the load cell in the x direction is determined using the following equation:

$$-F_{lc} + F_{kc} = 0 \quad (2).$$

The motion of the total control volume in the x direction can be shown by

$$F_{lc} + F_f + F_t + A_e p_a - A_e p_e = \int_{cs} \rho \mathbf{u}_x (\mathbf{u} \cdot \mathbf{n}) dA + \frac{d}{dt} \int_{cv} \rho \mathbf{u} dV + \frac{d}{dt} \int_{cv} \rho_s v_s dV \quad (3).$$

The total impulse of the PDE tube is calculated as follows:

$$I_{total, x} = \int_0^{t_{total}} F_p dt \quad (4).$$

From Equations 1, 2, and 4, we can obtain the following equation:

$$I_{total, x} = - \int_0^{t_{total}} (F_{lc} + F_f + F_t) dt + m_t \int_0^{t_{total}} \frac{dv}{dt} dt ,$$

in which the condition of $\int_0^{t_{total}} \frac{dv}{dt} dt = 0$ is satisfied due to the initial and final PDE tube velocities

being zero. Therefore, we obtain

$$I_{total, x} = - \int_0^{t_{total}} (F_{lc} + F_f + F_t) dt \quad (5).$$

Since the right-moving direction in Fig. 6 is positive in the x direction, the value of $I_{total, x}$ is usually negative. In Equation 5, the force generated by the supplying tube, F_t , is neglected due to its magnitude being assumed to be smaller than those of other forces. The PDE tube can slide on the rail in the positive or negative direction of x with little friction on the bearings, wheels, or rails. We

obtain this static friction force by measuring the difference between the load on the PDE tube in the x direction by the calibrated weight and the load cell output. This result is shown in Fig. 7. From this figure, we found the static friction was approximately 20 N in the load range from 0 to 200 N. We also verified the high linearity of the load cell. In the multi-cycle PDE operation, the dynamic friction force of the rails and bearings exists, but this force is less than the force of the static friction, and its vector periodically varies in the positive and negative directions of x . Since the temporally averaged dynamic force is much less than the other forces, we neglect the dynamic friction force, F_f .

We finally obtained the following expression regarding total impulse:

$$I_{\text{total},x} = -\int_0^{t_{\text{total}}} F_{\text{lc}} dt \quad (6).$$

We used Equation (6) to obtain the temporally integrated impulse acting on the PDE tube, $I_{\text{total},x}$.

D. Smoothing of Pulsed Thrust by Using the Spring-Damper Mechanism

As the plateau pressure near the thrust wall of the PDE tube is maintained at approximately 1 MPa for a few milliseconds, the impulsive thrust on the wall is approximately 8000 N. As shown in Fig. 8, this impulsive thrust on the PDE tube is smoothed by the damping system. In this experiment, the additional H-beam weight of 45 kg is attached to the PDE tube. The total mass of the PDE tube, m_t , including this additional mass is 100 kg. To design this mechanism, we had to determine two

characteristic parameters, the viscous damping coefficient, c , and the spring constant, k . To do so, we calculated the impulsive force history on the load cell using the following model. The impulsive thrust of the PDE tube is approximately rectangular, as shown and defined in Fig. 9, which can be expanded to a Fourier series, as shown in the following equations. This rectangular thrust is defined by the operation cycle, t_{cyc} , plateau thrust, F_0 , and time for maintaining the plateau thrust, t_{pl} . The position of the multi-cycled PDE tube, $x_p(t)$, can be expressed in the following form:

$$x_p(t) = a_0 + \frac{1}{\pi} \sum_{n=1}^{\infty} (a(n) \cos(n\omega t - \phi(n)) + b(n) \sin(n\omega t - \phi(n)))$$

$$a(n) = \frac{\omega_n^2}{\sqrt{(\omega_n^2 - n^2\omega^2) + (2\xi\omega_n\omega n)^2}} \delta_{\text{st}} \frac{\sin(n\alpha)}{n}$$

$$b(n) = \frac{\omega_n^2}{\sqrt{(\omega_n^2 - n^2\omega^2) + (2\xi\omega_n\omega n)^2}} \delta_{\text{st}} \frac{1 - \cos(n\alpha)}{n}$$

where $\alpha = \frac{2\pi t_{\text{pl}}}{T}$, $\omega = \frac{2\pi}{t_{\text{cyc}}}$, $\delta_{\text{st}} = \frac{F_0}{k}$, $\omega_n = \sqrt{\frac{k}{m_p}}$, $\xi = \frac{c}{2\sqrt{k m_p}}$, and $a_0 = \delta_{\text{st}} \frac{\alpha}{2\pi}$.

By using this PDE-tube position $x_p(t)$, the thrust on the load cell, $-F_{\text{lc}} = -F_{\text{kc}} = c\dot{x}_p + kx_p$, can be obtained from Equation 2. To determine the design parameters, t_{cyc} is set at 170 ms as a typical experimental condition, in which sequence A is employed and the propellant and purging times are 85 ms. The plateau thrust, F_0 , is set at 7850 N due to the following calculation, in which the inner radius of the cylindrical PDE tube is 50 mm and the plateau pressure is 1 MPa:

$$F_0 = 0.05 \times 0.05 \times 3.14 \times 10^6 = 7850 \text{ [N]}.$$

The plateau pressure time is set as $t_p=1$ [ms] according to the condition in which the typical total impulse of the PDE tube is identical to $t_p F_0$. The PDE tube mass and the spring constant are given as $m_t = 100$ [kg] and $k = 10000$ [N/m] $\times 2 = 20000$ [N/m], respectively. The transient and damped oscillations of this system are calculated by using the second-order differential equation $\ddot{x}_p = -\frac{c}{m_t}\dot{x}_p - \frac{k}{m_t}x_p$. We obtained the solutions of the piston position, x_p , of which the typical case is plotted in Figs. 10 and 11. In these calculations, the initial velocity of the PDE tube is zero, and the initial position is the maximum displacement position in the multi-cycle operation. This system is also required to shorten the transient oscillation time. In the present experiment, the total operation time of the multi-cycle operation is 3 sec. We employed the damper attenuation coefficient, which enables the initial amplitude to decrease to 10% of its initial value within 1.5 seconds. From the calculations we find that this condition is satisfied when $c \geq 250$ Ns/m. Therefore, the experimental conditions are $c=250$ Ns/m and $k=20000$ N/m. Figures 10 and 11 show the cyclic and transient oscillations in this condition. The damper used in this experiment is shown in Fig. 12. We created this damper for this experiment so that we could modify the orifice of the piston and the fluid in the cylinder. The damper has two orifices, 3.5 mm and 5.5 mm in diameter, as shown in Fig. 12. The working fluid is automobile oil (SAE viscosity 5W-30, 3.8 mm²/s and more). The attenuation coefficient of the damper is determined by the following preliminary experiment. The axis of the damper is set vertically, and the piston is below the cylinder part. A weight with a mass of 0.7134 kg

is suspended from the piston of the damper and descends. The descent velocity of the weight is measured using a video camera. Since the derivation of the descent velocity is negligible, the attenuation coefficient of the damper can be determined by the equation $c = m_d g / v_w$. The attenuation coefficient of the damper depending on the piston position is shown in Fig. 13. The initial position of the piston is zero on the horizontal axis of Fig. 13, when the piston is attached to the bottom of the cylinder and the total length of the damper is at its shortest. Figure 13 shows that the attenuation coefficient increases as the piston position increases. The designed attenuation coefficient of 250 Ns/m is satisfied at the piston position of 30 mm. Thus, to realize simulated damping, the total damper length is adjusted to the length at which the piston oscillates around 30 mm all the time.

E. Measurements of the Mass Flow Rate of Propellant and the Filling Rate

To obtain specific impulses of the PDE tube, we had to properly measure the mass flow rates of the propellant. In the present study, the mass flow rates of the propellant are measured using two methods. In the first method, the mass flow rate is obtained by measuring the pressure difference between the before and after operations. If in the constant-volume cylinders the gas temperatures are constant, we obtain

$$\Delta m_c = \Delta p_c \frac{M_c V_c}{RT_c}. \quad (7)$$

Therefore, if we obtain Δp_c , then Δm_c and the mass flow rate can be calculated. In the present experiment, the mass flow rate of the ethylene supply system is calibrated using oxygen for safety reasons. We assumed that the molar flow ratio of ethylene was equivalent to that of oxygen in this system.

The mass flow rate can also be obtained by the direct measurement of cylinder-mass differences before and after operations using an electrobalance. In this method, the mass flow rate data can be obtained in each experimental shot. If we employ this method, we cannot neglect the mass of the gas remaining in the flexible tubes connected between the cylinders and the electromagnetic valves. As shown in Fig. 14, the two stop valves are attached to the separation part between a regulator and a tube. These two valves minimize the mass loss in this separation before the cylinder-weight measurement by the balance. Figure 15 shows measurement results of both methods, which are almost equal to one another. We noted that the ethylene mass flow rate determined by the pressure difference method in which the oxygen molar flow rate is assumed to be equivalent to that of ethylene is identical to that determined by the mass difference method.

In the following description, the results of the mass flow rate determined by the mass difference method will be used. The propellant fill fraction of the PDE tube is calculated by using the measured mass flow rate. The volumes of the tapered tube and the cylindrical tube are 0.47 and 15.71 liters, respectively. The PDE tube, which is the sum of the tapered and cylindrical tubes, has a volume of

16.18 liters. The propellant fill fraction in each cycle is calculated by the following procedure. The temperature and pressure in the PDE tube are assumed to be 290 K and 1 atm, respectively. The volume flow rate can be obtained from the mass flow rate and the equation of state of each gas. The fill fraction can be obtained from the total volumes of ethylene and oxygen in one cycle divided by the PDE tube volume. This fill fraction is plotted against the time for supplying propellant shown in Fig. 16.

III. Results and Discussion

A. Thrust and Impulse

The typical thrust history measured by the load cell is shown in Fig. 17. The load cell data is recorded after processing by the low-pass filter (below 60 Hz) to reduce the electrical noises caused by the power source. In every shot, the thrust in the first cycle is larger than that in the second and later cycles. We considered this to be due to the effect of the high-density gases remaining in the flexible tubes before the start of the operation. Before the first cycle only, these tubes are filled with high-pressure gases just before the electromagnetic valves are activated. Therefore, the gases are injected at higher pressure and fill the PDE tube at a larger volume in the first cycle than in the second and later cycles. The total impulse in multi-cycle operation can be defined as follows:

$$I_{\text{total}} = |I_{\text{total},x}| = \left| - \int_0^{t_{\text{total}}} F_{\text{ic}} dt \right|. \quad (8)$$

The impulse per cycle can be obtained using the following equation:

$$I_{\text{cyc}} = \frac{I_{\text{total}}}{N_{\text{cyc}}}. \quad (9)$$

The impulse per cycle, I_{cyc} , for each cycle is plotted against the time for supplying propellant in Fig. 18. In this figure we see that I_{cyc} decreases abruptly as the time for supplying propellant decreases to 50 ms or less. As mentioned above, when the time for supplying propellant is below 60 ms, sequence B is employed to stabilize the PDE operation. Figure 19 (shot no. 11) shows the typical thrust history in such a sequence B. An approximately 1-Hz small-amplitude oscillation is superimposed on the oscillated thrust history. As Fig. 19 shows, in sequence B the multi-cycle operation is not considered to be perfectly stabilized. These oscillation phenomena probably occur because as the propellant volume becomes much smaller than the purge gas and/or previous-cycle combusted gas, the propellant density and length become smaller and shorter by diffusion, respectively, and are not sufficient to initiate a detonation wave in the PDE tube. By the additional measurement using a shadow-graph optical system and a high-speed video camera, we confirmed that, in the premixed stoichiometric ethylene-oxygen 0.7-atm 290-K mixture in the square-cross-section smooth-wall tube without an obstacle, the detonation initiation length is larger than the square tube height (50 mm). Assuming this detonation-initiation-length requirement in the PDE tube shown in Fig. 1, the condition, $f \gg 0.05$, is necessary for detonation initiation and this

condition is consistent with the result of Fig. 19 (shot No. 11, $\bar{f}=0.075$). Specific impulses can be calculated from the measured total impulse and the measured total mass ($\text{C}_2\text{H}_4\text{-O}_2\text{-He}$) using the following equation:

$$I_{\text{sp}} = \frac{I_{\text{total}}}{(m_{\text{propellant}} + m_{\text{purge}}) g} \quad (10)$$

The specific impulse of every shot is listed in Table 2.

B. Specific Impulse

Fig. 20 shows the specific impulses obtained from this multi-cycle experiment and from single-cycle calculation using the partial-fill model of Endo et al. [21], which are plotted against the propellant ($\text{C}_2\text{H}_4\text{-O}_2$) fill fraction at 1 atm and 290 K, $\bar{f}_{\text{propellant}}$, in the PDE tube. In the figure, the specific impulse increases as the partial fill fraction decreases in the region $\bar{f}_{\text{propellant}} \geq 0.130$. From this figure, we can verify that the partial-fill effect exists in the multi-cycle operation. When the propellant fill fraction is 0.130 ± 0.004 , the specific impulse achieves a maximum value of 305 ± 9 sec. The specific impulse sharply decreases as the fill fraction decreases in the region $\bar{f}_{\text{propellant}} < 0.130$.

Both the partial-fill model of Endo et al. [21] and the bubble model of Cooper et al. [23, 25], which can estimate the maximum specific impulse in the case of an extremely small fill fraction, operate under the assumption that the fuel and oxidizer are perfectly reacted. In these models, in most of the cases, specific impulses do not decrease as the fill fractions decrease to zero. The inert-gas open-shock-tube experiment by Kasahara et al. [23] showed the specific impulse upper

limit and also no decrease as the fill fraction decreased in the small fill fraction region. Recently, Kasahara et al. [31] showed that for stable operation of a multi-cycle ethylene-oxygen propellant PDE the sufficient purge gas thickness divided by the tube diameter ($L/D \geq 3.12$) was necessary. In the present experiment, in the region $f_{\text{propellant}}^* < 0.087$, the non-dimensional purge gas thickness, L/D , was 2.27 ($f_{\text{propellant}}^* = 0.075$), 2.61 ($f_{\text{propellant}}^* = 0.083$), and 3.55 ($f_{\text{propellant}}^* = 0.087$), and might not be sufficient for stable operation. From these analytical and experimental results, we conclude that this sharp decrease is due to diffusion between the fresh mixture and the helium purge gas and an imperfect transition from a deflagration to a detonation wave.

We next compare our experimental results with the partial-fill model calculation results of Endo et al [21]. The specific impulses for Endo's partial-fill model are shown by open symbols in Fig. 20. Figure 21 is a schematic picture of the initial gas distribution in the PDE tube in Endo's model. The filling gases are a propellant and a purge gas, and the remaining gases are burned gas and air. The propellant is an ethylene-oxygen mixture at 1 atm and 290 K, and the equivalence ratio of the propellant mixture is used in Table 2. The purge gas is helium at 1 atm and 290 K. The burned gas is an isentropically expanded propellant (in the previous cycle) from the Chapman-Jouguet (C-J) state to 1 atm with an equilibrium composition, which was obtained by the chemical-thermodynamics software STANJAN [33]. The suction air, which is assumed to be suctioned from the space around the open end of the PDE tube, has a state of 1 atm and 290 K. The experimental specific impulses

are between the calculated results of a half and 70% remaining gas as burned gases when the partial fill fraction was above 0.1 in Fig. 20. Therefore, assuming that Endo's single-cycle partial-fill calculation model can correctly predict the multi-cycle partially-fueled PDE's specific impulse, we cannot assume that only burned gas remained inside the PDE tube. As the burned gas density is between 0.093 and 0.097 kg/m³, which is much lower than that of the propellant (1.28 kg/m³ at 1 atm and 300 K) or air (1.22 kg/m³ at 1 atm and 300 K), the partial-fill effect by the burned gas inertia may be negligible. The partial-fill effect observed in the present experiment may be mainly due to the air existing near the open end of the PDE tube, which is suctioned by the negative gauge pressure during the exhaust phase of the PDE cycle.

Based on the following equation, we can estimate the fill fraction of the suctioned-air to the PDE tube, $\mathit{ff}_{\text{air, est}}$:

$$\mathit{ff}_{\text{air, est}} = (1 - \mathit{ff}_{\text{propellant}} - \mathit{ff}_{\text{purge}}) \left(0.5 - 0.2 \frac{I_{\text{sp, BG50\% \& Air50\%}} - I_{\text{sp}}}{I_{\text{sp, BG50\% \& Air50\%}} - I_{\text{sp, BG70\% \& Air30\%}}} \right), \quad (11)$$

where $\mathit{ff}_{\text{purge}}$ is the purge gas volume fraction at 290 K and 1 atm, and $I_{\text{sp, BG50\% \& Air50\%}}$ and $I_{\text{sp, BG70\% \& Air30\%}}$ are the calculated specific impulses by Endo's partial-fill model. Figure 22 shows the estimated suctioned-air fill fraction, $\mathit{ff}_{\text{air, est}}$. From this figure, in the present experiment, we see that $0.175 \leq \mathit{ff}_{\text{air, est}} \leq 0.327$. It is confirmed that air having almost the same volume order is suctioned in the region $\mathit{ff}_{\text{propellant}} \geq 0.1$. It can be concluded that the multi-cycle pulse detonation rocket engine has a partial fill effect and that this effect may be mainly due to the suction air, which is consistent

with the single-cycle partial fill model of Endo et al.

If the operation frequency becomes too high, the suctioned air volume may be small because exhaust time duration becomes small. In such a condition the partial-fill effect may be very small. Contrastively, we believe we will be able to obtain larger impulse from the ejector or other devices that can make use of the inertia of the air around the PDE tube more effectively.

IV. Conclusions

Multi-cycle PDE operation was achieved. In the thrust measurements using a load cell, the PDE's pulsed thrust was successfully smoothed and measured by a spring-damper-mass mechanism. The mass flow rate obtained by the direct cylinder-mass difference measurement method was almost identical with that obtained by the cylinder-pressure difference measurement method. As experimental results, specific impulses increased when the fill fraction was decreased. When the propellant partial fill fraction was 0.130 ± 0.004 , the specific impulse achieved a maximum value of 305 ± 9 sec. The specific impulse sharply decreases as the fill fraction decreases below 0.130. This sharp decrease is considered to be due to the propellant gas densities becoming too small by diffusion and due to the propellant gas fill length becoming too short to initiate detonation waves in the PDE tube. Our calculations show that the multi-cycle pulse detonation rocket engine has a

partial-fill effect and that this effect may be mainly due to the suction air, which is consistent with the single-cycle partial fill model of Endo et al.

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Table 1 Experimental conditions

shot number	supplying pressure	fueling duration	purging duration	frequency	operating duration	propellant fill fraction
	[MPa]	[ms]	[ms]	[Hz]	[ms]	[-]
1	3	200	200	2.5	2800	0.264
2	3	175	175	2.9	2800	0.252
3	3	150	150	3.3	3000	0.223
4	3	125	125	4.0	3000	0.203
5	3	100	100	5.0	3000	0.155
6	3	85	85	5.9	2890	0.130
7	3	70	70	7.1	2940	0.117
8	3	60	120	5.6	2880	0.106
9	3	50	100	6.7	3000	0.087
10	3	45	90	7.4	2970	0.083
11	3	40	80	8.3	3000	0.075

Table 2 Experimental results

shot No.	number of cycles	impulse	impulse per cycle	propellant mass flow per cycle	helium mass flow per cycle	total mass flow per cycle	equivalence ratio	volume of propellant at 290 K and 1 atm	fill fraction	specific impulse
N	I_{all}	I_{all}/N	$m_{\text{propellant}}/N$	m_{purge}/N	$(m_{\text{propellant}} + m_{\text{purge}})/N$	η	V	ff	I_{sp}	
	[Ns]	[Ns/cycle]	[g/cycle]	[g/cycle]	[g/cycle]	[-]	[L/cycle]	[-]	[s]	
1	7	99 ± 4	14 ± 1	5.5 ± 0.2	0.657 ± 0.02	6.2 ± 0.2	1.93	4.3 ± 0.2	0.26 ± 0.01	240 ± 10
2	8	111 ± 4	13.8 ± 0.4	5.22 ± 0.16	0.700 ± 0.02	5.92 ± 0.16	1.75	4.07 ± 0.12	0.252 ± 0.008	238 ± 7
3	10	121 ± 4	12.1 ± 0.4	4.63 ± 0.14	0.540 ± 0.02	5.17 ± 0.14	1.85	3.61 ± 0.11	0.223 ± 0.007	239 ± 7
4	12	143 ± 5	11.9 ± 0.4	4.21 ± 0.13	0.625 ± 0.02	4.83 ± 0.13	1.90	3.29 ± 0.10	0.203 ± 0.006	251 ± 8
5	15	148 ± 5	9.85 ± 0.30	3.22 ± 0.10	0.407 ± 0.01	3.63 ± 0.10	1.83	2.51 ± 0.08	0.155 ± 0.005	277 ± 8
6	17	153 ± 5	8.99 ± 0.27	2.69 ± 0.08	0.312 ± 0.01	3.00 ± 0.08	1.79	2.10 ± 0.06	0.130 ± 0.004	305 ± 9
7	21	151 ± 5	7.20 ± 0.22	2.43 ± 0.07	0.305 ± 0.01	2.73 ± 0.07	2.02	1.90 ± 0.06	0.117 ± 0.004	269 ± 8
8	16	113 ± 4	7.04 ± 0.21	2.19 ± 0.07	0.425 ± 0.01	2.62 ± 0.07	1.90	1.71 ± 0.05	0.106 ± 0.003	274 ± 8
9	20	77 ± 3	3.9 ± 0.1	1.8 ± 0.1	0.500 ± 0.02	2.3 ± 0.1	1.94	1.4 ± 0.1	0.087 ± 0.003	170 ± 10
10	22	89 ± 3	4.0 ± 0.1	1.7 ± 0.1	0.368 ± 0.01	2.1 ± 0.1	1.97	1.3 ± 0.1	0.083 ± 0.002	200 ± 10
11	25	88 ± 3	3.5 ± 0.1	1.6 ± 0.1	0.320 ± 0.01	1.9 ± 0.1	1.84	1.2 ± 0.1	0.075 ± 0.002	190 ± 10

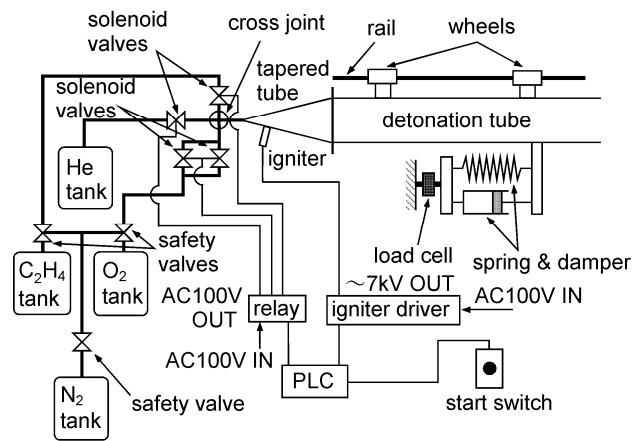


Fig.1 Schematic diagram of the experimental device

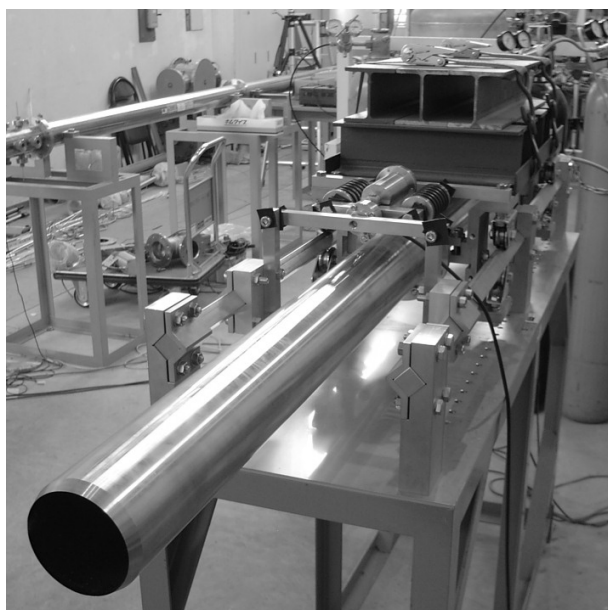


Fig.2 Photograph of the experimental device

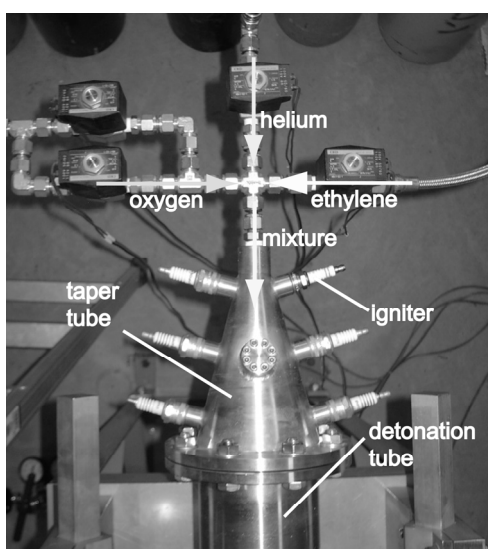


Fig. 3 Tapered tube and solenoid valves

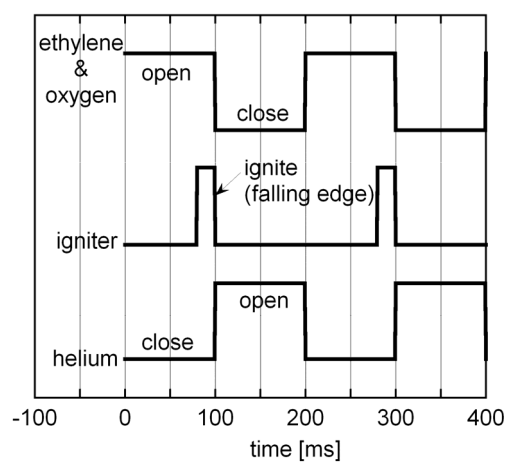


Fig. 4 Sequence A (mixture : helium = 1 : 1)

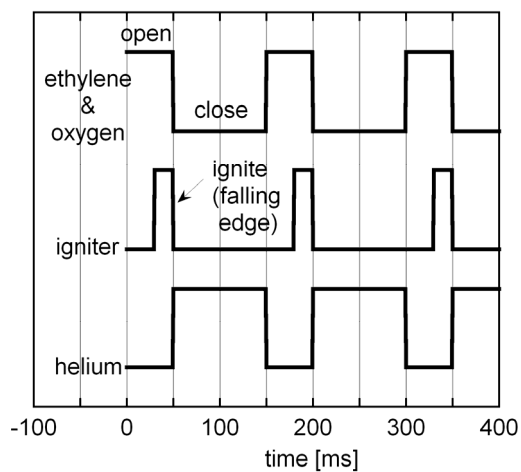


Fig. 5 Sequence B (mixture : helium = 1 : 2)

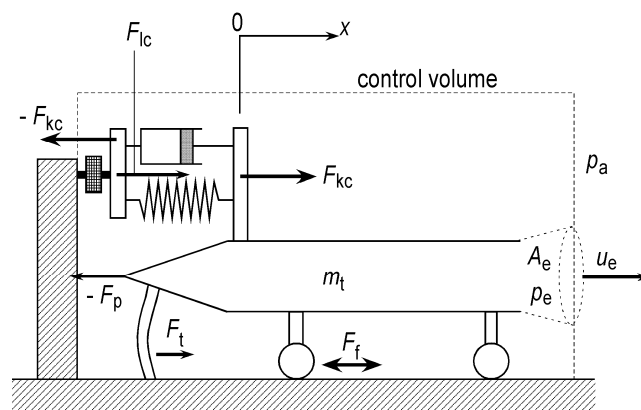


Fig. 6 Model for thrust measurement

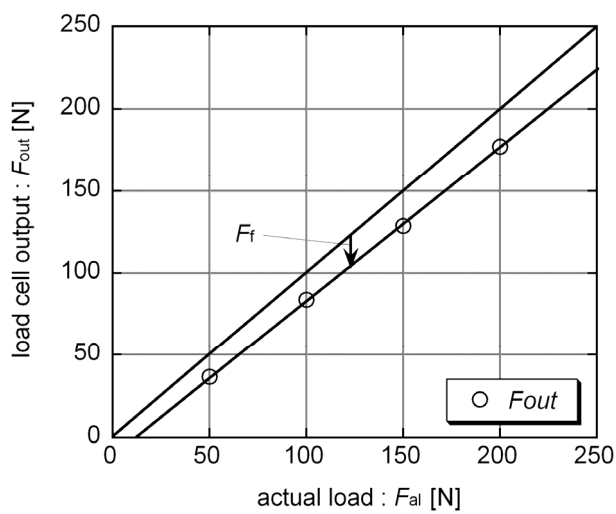


Fig. 7 Measurement of static friction

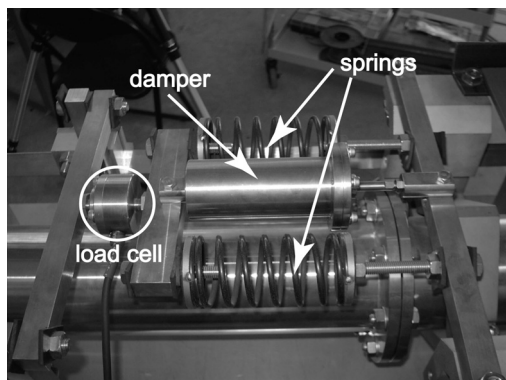


Fig. 8 Damping system

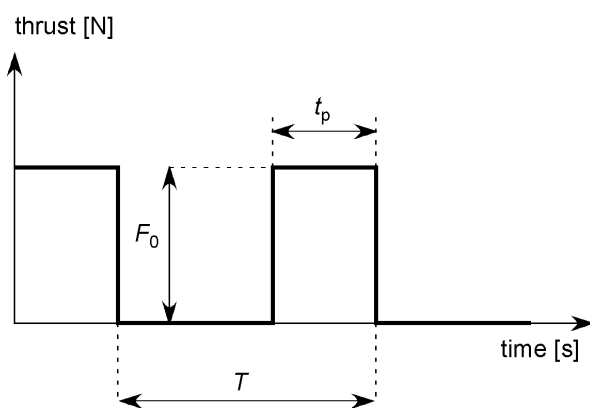


Fig. 9 Pulse wave for simulation

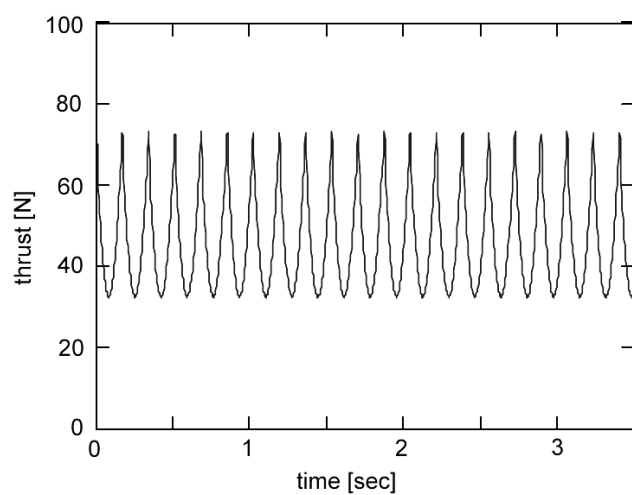


Fig. 10 Thrust history ($c=250$ Ns/m)

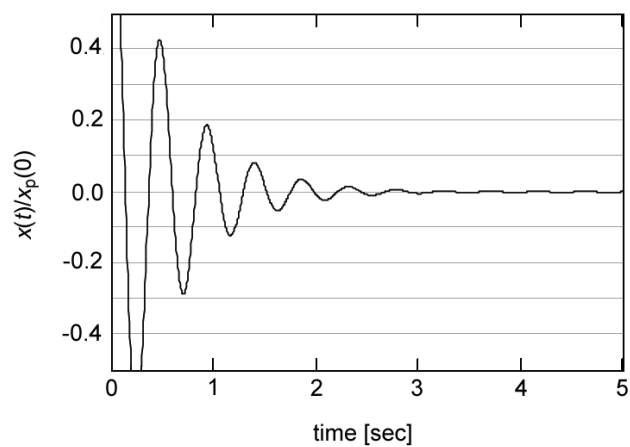


Fig. 11 Damping oscillation ($c=250$ Ns/m)

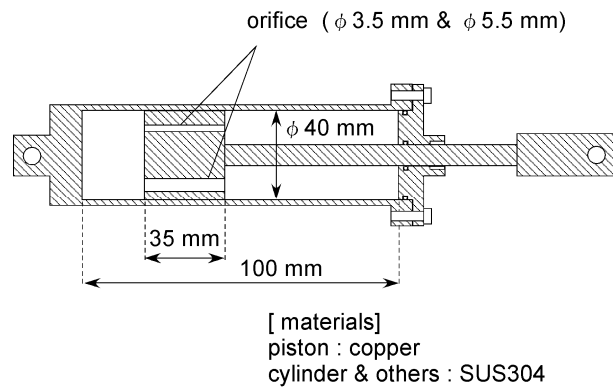


Fig. 12 Cross-section diagram of damper

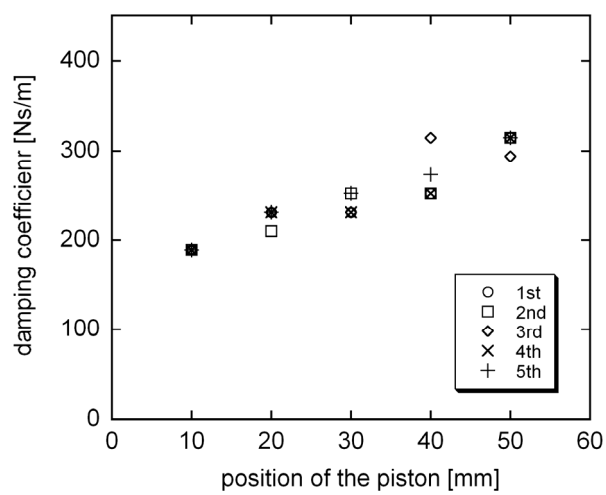


Fig. 13 Damping coefficient versus position of piston

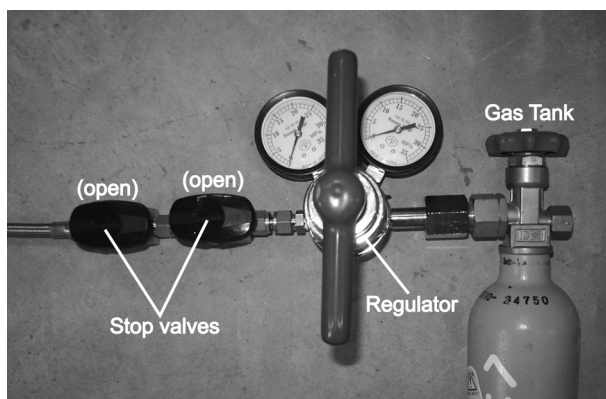


Fig. 14 Stop valves (connected)

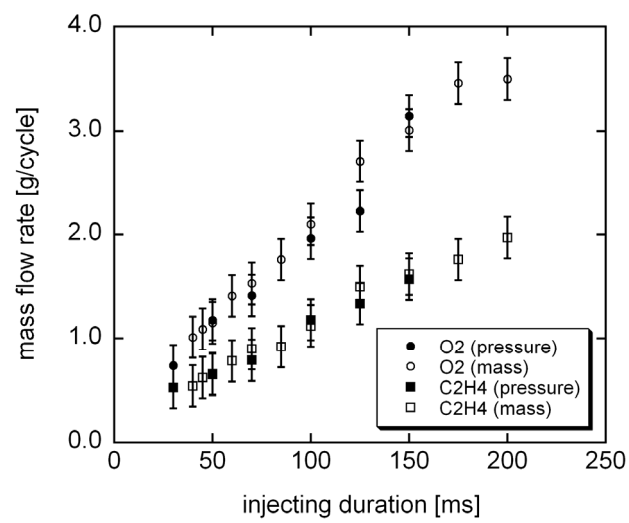


Fig. 15 Comparison of mass flow rates between pressure and mass measurements

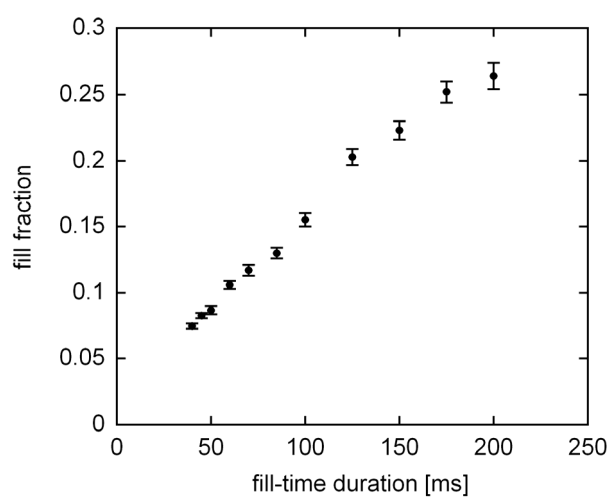


Fig. 16 Fill fraction of mixture versus fueling duration

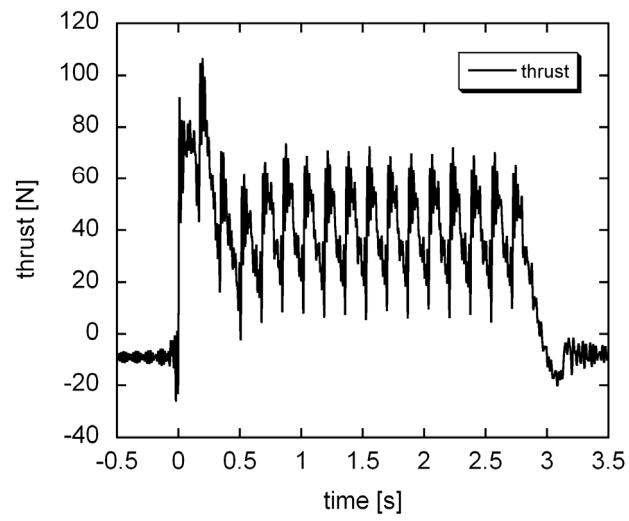


Fig. 17 Thrust history of 85-ms fueling and
85-ms purging operation
(shot no. 6, 5.9 Hz)

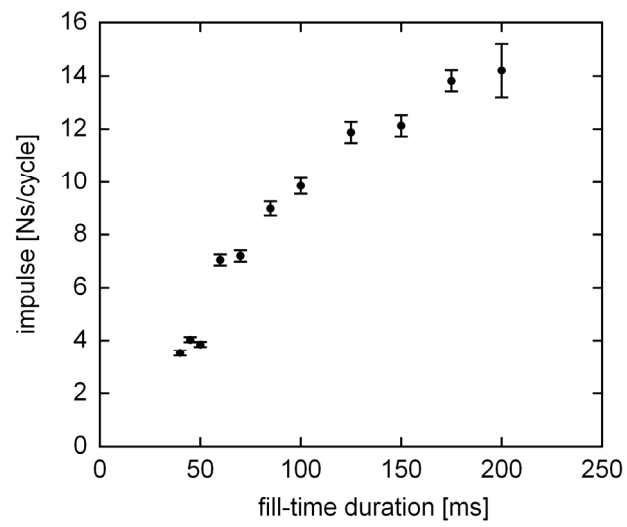


Fig. 18 Impulse versus fueling duration

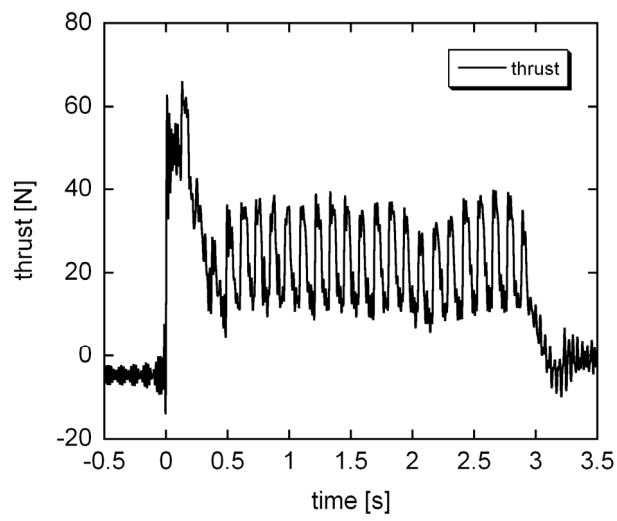


Fig. 19 Thrust history of 40-ms fueling and 80-ms purging operation (shot no. 11, 8.3 Hz)

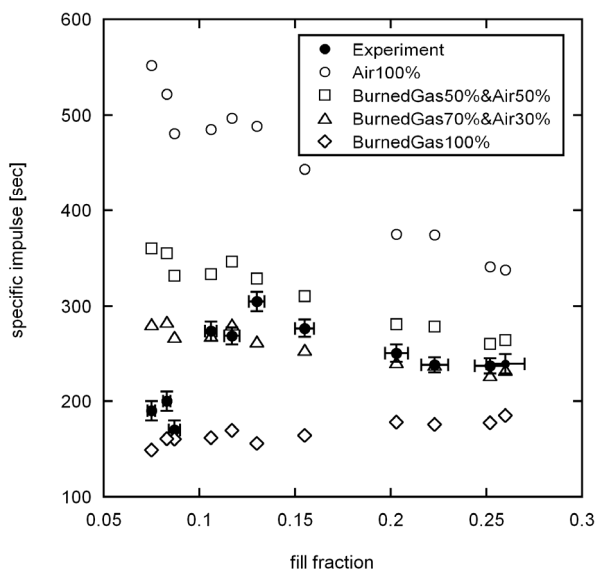


Fig. 20 Comparison of specific impulses between experimental results and Endo's partial-fill model calculation. The open symbols show model calculation results. "Air100%" or "BurnedGas100%" means that the remaining gas is air only or burned gas only, respectively. "BurnedGas50%&Air50%" or "BurnedGas70%&Air30%" means that a half or 70% of the remaining gas is burned gas and the other remaining gas is air, respectively.

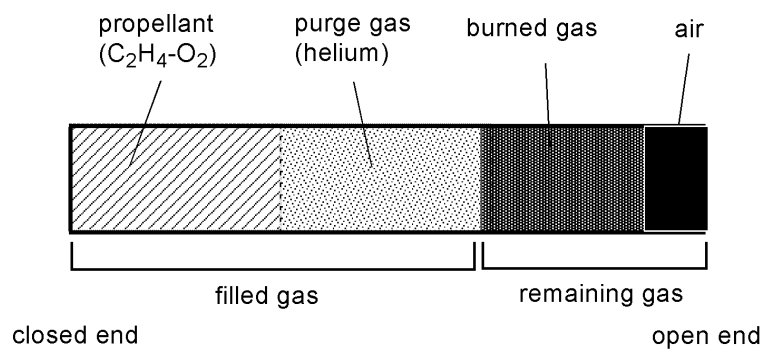


Fig. 21 Initial gas condition used in Endo's partial-fill model.

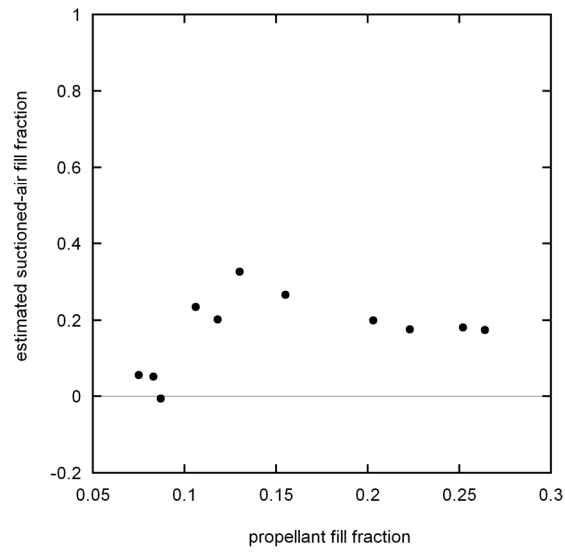


Fig. 22 Estimated volume fraction of air