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Comparison of the effects of traditional and innovative tie-rods in reducing the seismic vulnerability of church façades: the case of San Francesco in Mirandola (Italy)

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Abstract

This contribution discusses the effect of steel tie-rods installed in a church façade subjected to 60 recorded earthquakes. The church façade is firstly analysed as a two-sided and a one-sided rocking SDOF system. For the one-sided rocking, sidewalls are modelled through either elastic or rigid contact to compare the two modelling techniques. Secondly, two configurations of tie-rods are investigated: (i) traditional tie-rods with a specific elasto-plastic constitutive law and (ii) innovative tie-rods with a dissipative component. The chosen case study is the San Francesco Church located in Mirandola, hit by the 2012 Emilia Romagna earthquake. The results of nonlinear dynamic analyses are presented in terms of median and standard deviation of maximum normalised displacements for each of the 60 seismic inputs. The results show the great benefit introduced by both traditional and dissipative tie-rods, with remarkable reductions of maximum rotations up to an order of magnitude with respect to the unrestrained façade. Moreover, the increment of damping coefficient is associated to a reduction of the standard deviation of the amplitude peaks, which is a positive aspect for the reliability of the response of the damped wall. Finally, the two models, rigid and elastic contact, of sidewalls provide results in excellent agreement in terms of median and standard deviation of the maximum normalised rotations.

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Keywords: out-of-plane; dissipation of energy; one-sided rocking; rigid contact; elastic contact; sidewalls

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1. Introduction

Masonry buildings may develop out-of-plane (OOP) failure modes (Abrams et al. 2017; Casapulla et al. 2021), especially in the case of churches with an irregular plan and poor connections between walls and between walls and floors/roof (Alecci and De Stefano 2019; Lagomarsino 2015)). In fact, churches often consist of few vertical walls (façade, side walls, etc.), which are characterised by the presence of thrusting elements (e.g., arches, vaults) and the absence of widespread systems of bracings or shear walls, with a single horizontal diaphragm at the top and, often, a pitched roof, usually not designed to redistribute seismic actions. Because of this structural configuration, churches have limited capacity to distribute and counteract horizontal actions and are prone to develop instability phenomena or to activate OOP mechanisms, among which the rocking façade is the most recurring one (Sorrentino et al. 2014).

If a monolithic behaviour can be assured for OOP loaded walls, they can be regarded as rigid blocks, and their seismic response can be treated through the two fundamental approaches of rocking dynamics and kinematic analysis. The kinematic methods include static force-based and displacement-based approaches of limit analysis, while dynamic effects are more appropriately considered by means of the dynamic approach, since it also accounts for the energy dissipation in the motion. However, rocking models are extremely sensitive to small variations of many parameters, such as the coefficient of restitution, boundary conditions and particularly those of the input motion, at least for near collapse configurations. Comprehensive reviews of the issues involved in classical and non-classical theories are addressed in the literature (Abrams et al. 2017; Casapulla et al. 2017).

On the other hand, to prevent such a structural failure, strengthening interventions on wall connections with traditional or innovative strategies can play a significant role since allowing to improve the box-like behaviour of the structure. The most common retrofitting strategies for limiting the risk of OOP failure modes are traditional metal tie-rods (Pugi and Galassi 2013; Tempesta and Galassi 2019; Walsh et al. 2014), recently designed considering sustainability aspects (Giresini et al. 2021a), while the use of pultruded fibre reinforced plastic (FRP) bars as strengthening systems based on grouted anchors is gaining increasing attention, thanks to several interesting features of FRP materials such as their resistance to corrosion, a low weight-to-strength ratio, and the easy installation and transport (Ceroni and Prota 2009).

Nevertheless, common tie-rods (AlShawa et al. 2019; Prajapati et al. 2022) or even innovative grouted anchors (Melatti and D'Ayala 2021) perform as rigid links and in general are not able to dissipate energy coming from earthquakes. Recently, dissipative systems were proposed to overcome such a limitation: among them, a friction-based dissipative device (Melatti and D'Ayala 2021) and a dissipative tie-rod coupling the traditional earthquake resistant device with a shock absorber, specifically designed and experimentally tested by Giresini et al. (2021b). The influence of the latter strategy on the rocking motion has been analysed through nonlinear dynamic analyses also in terms of fragility curves (Giresini 2022; Giresini et al. 2022).

This paper presents a further contribution on modelling OOP rocking of masonry façades, applying elastic and rigid contact models for simulating the impact with the sidewalls. The case study of the San Francesco Church hit by the 2012 Emilia Romagna earthquake is chosen as a reference case. Seven models are adopted and compared each other: two-sided rocking, one-sided rocking with either elastic or rigid contact. The boundary conditions are traditional tie-rods and dissipative tie-rods with different damping properties, while 30 recorded spectrum compatible earthquakes are considered as seismic input.

2. Methodology and analysis models

The basic model considered in this study to simulate the OOP performance of a monumental masonry façade is a rigid-like block. The sidewalls against which the rocking wall impacts are modelled through either rigid or elastic unilateral contact, in a one-sided (1S) motion. By contrast, motion is said "two-sided (2S)" when sidewalls are neglected.

A total of seven models are analysed: firstly, a two-sided (2S) rocking is considered, whereas the other six regard the model of the façade interacting with sidewalls. Three of them have unilateral rigid contact, whilst the others have unilateral elastic contact (Fig. 1). In each of the two groups in which the 1S rocking is analysed, the models are:

- One-sided free from any restraint (1S-F);
- One-sided with traditional tie-rods (1S-T);

- One-sided with dissipative tie-rods (1S-DT).

The 2S model is compared to the 1S models to understand the role of sidewalls in the rocking motion and their different modelling.

The equation of motion associated with the dynamics of the free rocking block is the following (AlShawa et al. 2019):

$$\ddot{\theta} + \frac{p^2}{g}\left[gU(\theta, \alpha, \alpha_i, \Delta_1, \Delta_2) - \ddot{x}_g cos(\alpha_i - \theta) + \frac{\chi(\varepsilon_t)F_y R_t}{m\,R_i} cos(\alpha_t + \theta)\right] = 0 \qquad (1)$$

where θ = wall rotation, $p = \sqrt{mR_i g/I_O}$ = frequency parameter, m = mass of the wall, R_i = distance between centroid G and indented hinge O (Fig. 1), R_t = distance between wall anchor and indented hinge O, α_t = angle between Rt and vertical line, g = gravity acceleration, I_O = polar moment of inertia of the wall with respect to O, $\ddot{x}_g$ = horizontal ground motion acceleration, U = non-dimensional wall-self weight restoring moment parameter equal to:

$$U = \begin{cases} \frac{\theta}{\Delta_1 \alpha} sin(\alpha_i - \Delta_2 \alpha) & \theta \le \Delta_1 \alpha \\ sin(\alpha_i - \Delta_2 \alpha) & \Delta_1 \alpha < \theta \le \Delta_2 \alpha \\ sin(\alpha_i - \theta) & \theta > \Delta_2 \alpha \end{cases} \qquad (2)$$

where α = arctan (B/H), see Fig. 1, and Δ_1 and Δ_2 non-dimensional parameters defining the three-branch law as reported in (AlShawa et al. 2019). The hinge O is indented with respect to the geometric corner of the wall by a quantity, u, depending on the masonry design compressive strength, $f_{m,d}$, equal to:

$$u = \frac{m\,g}{2 \cdot 0.85\, f_{m,d} L_h} \qquad (3)$$

where L_h = hinge length, coincident with the wall length if no openings are present. In Eq. (3) a stress block distribution of amplitude 0.85 $f_{m,d}$ is assumed. Considering the indentation of the hinge expressed by Eq. (3), the following relation holds:

$$\alpha_i = arctan\left(\frac{B - 2u}{H}\right) \qquad (4)$$

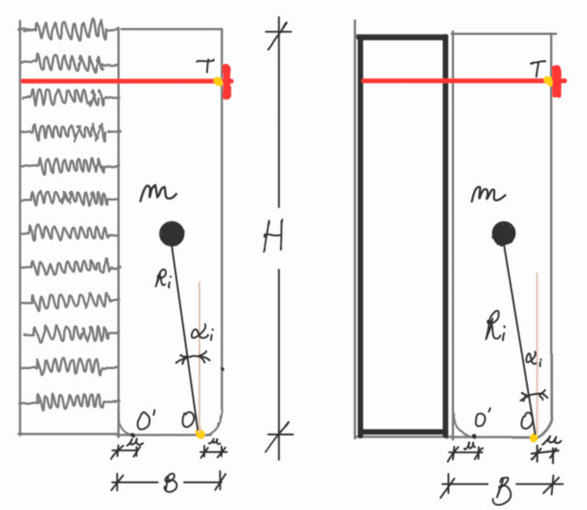

Fig. 1 One-sided rocking façade with elastic (left) and rigid (right) contact simulating lateral walls and horizontal restraints (1S-T or 1S-DT).

The tie is assumed to keep itself horizontal over rocking, although its wall anchor position T (Fig. 1) is updated during the analysis, accounting for finite displacements. The tie contribution is determined by the following parameters: F_y = yield force of the tie, R_t = distance between wall anchor and O (Fig. 1), χ = tie non-dimensional force, as function of the axial deformation of the tie-rod as illustrated in AlShawa et al. (2019) .

When the wall hits the base and the sidewalls, energy dissipation occurs; that is modelled by means of a negative velocity reduction coefficient (model 1S with rigid contact). The minus sign involves a rebound, hence the rotations can only be positive. The value of the velocity reduction coefficient, also known as 'coefficient of restitution', is assumed equal to (Sorrentino et al. 2011):

$$e_{an,1s} = 1.05\left(1 - 2\frac{mR_i^{\,2}}{I_O}sin^2\alpha_i\right)^2\left(1 - 2\frac{mR_i^{\,2}}{I_O}cos^2\alpha_i\right) \quad (5)$$

Another type of modelling the sidewalls is the elastic contact, for which the sidewalls are simulated through a spring bed model with equivalent unitary stiffness, as reported in Giresini et al. (2022). Also in this case, when the façade impacts the sidewalls, the coefficient of restitution is assumed equal to Eq. (5).

The equation of motion for the wall with the dissipative component is enriched with the term depending on the damping coefficient c as reported in Giresini et al. (2022):

$$\frac{T_D}{I_O} = \frac{c\,R_t^2\,cos^2(\alpha_1 - sgn(\vartheta)\vartheta)\,\dot{\vartheta}}{I_O} \quad (6)$$

This term must be added to the first term of Eq. (1) when the damping element is considered in the 1S motion.

3. Case study

Damages observed after strong earthquakes, e.g., 2009 in L'Aquila (M_W = 6.3) (Decanini et al. 2012), 2012 in Emilia (M_W = 6.1 on May 20, M_W = 5.8 on May 29) (Iervolino et al. 2012), and 2016-2017 Central Italy earthquakes (the strongest event with M_W = 6.5, on October 30) (Mazzoni et al. 2018) have shown that the seismic behavior of existing masonry buildings is often determined by the response of local mechanisms (Casapulla et al. 2021). Such an OOP dynamic behavior can be modelled assuming rigid-body mechanisms, provided that no crumbling phenomena occur. Experimental tests performed on shaking tables have confirmed such a behavior (Shawa et al. 2012; Costa et al. 2013). In recent years, dynamic analyses of rigid bodies were applied by several authors to simulate the seismic behavior of existing masonry buildings damaged by earthquakes (De Lorenzis et al. 2007; Derakhshan et al. 2013; Doherty et al. 2002).

Such a response was also observed in the façade and in the belfry of the San Francesco Church in Mirandola during the 2012 Emilia Romagna earthquake.

3.1. San Francesco Church

The Church of San Francesco is located in Mirandola in the Province of Modena (Italy). Almost destroyed by the 2012 Emilia earthquake, the building composed of unreinforced clay brick masonry has three naves, with a basilica cross-section and the central nave taller than the lateral ones (Fig. 2a).

Mirandola is also the near-field location for which strong ground motion records are available for both the events of the considered 2012 earthquake (May 20 and May 29).

The bell tower (39.5 m high) crashed down onto the church, almost completely destroying it, while only the façade, limit damaged after the 2012 May 20 event, was able to survive the 2012 May 29. Therefore, it could be of some interest to study the dynamic behaviour of this structural element triggered by the close-by recorded accelerograms.

The pictures of the Church of San Francesco before the 2012 seismic sequence showed cracks on the longitudinal walls close to the façade, thus allowing the study of the façade as a one-sided rocking body.

The façade is modelled as closely as possible, accounting for its actual shape, including pitched top side, openings and buttresses (Fig. 2b). As input, 30 natural accelerograms spectrum-compatible with Mirandola seismicity are used (Fig. 2c). The code REXEL (Iervolino et al. 2010) is used for the records selection assuming site class C, topographic category T1, return period 475 years, magnitude M_W in the interval of 5.5, 6.5 and source-distance R up to 50 km. Given the asymmetry of the mechanism due to the presence of transversal structures, the records are considered with both positive and negative polarity, thus obtaining 60 records.

a) b) c)

Fig. 2 (a) Church of San Francesco, Mirandola(Sorrentino et al. 2014) ; (b) façade before the 2012 earthquakes; (c) Elastic response spectra compatible with the seismicity of Mirandola.

3.2. Assumptions of the analysis

The model of the church façade is assumed as a rigid-like block with dimensions 17.9 x 19.9 x 0.8 (width x height x thickness) m^3. The volume of the façade considering its actual configuration with openings is 264 m^3. The model 1S-T (one-sided with tie-rods) is characterised by two tie-rods with diameter ϕ = 27 mm and length L = 11 m, installed at 15.4 m from the ground and with a resulting axial stiffness of $EA/L = 2.1 \cdot 10^4$ kN/m.

The equivalent stiffness of the spring bed model (elastic contact model) simulating the sidewalls is computed according to Giresini et al. (2022) and equals to 466 MPa. The parameter adopted in the model 1S-DT (one-sided with dissipative tie-rods) is the equivalent damping coefficient (entering in Eq. (6)) which is equal to c = 2.5 kNs/m, 5.0 kNs/m, 7.5 kNs/m, 10.0 kNs/m. These values are chosen referring to dissipative tie-rods recently developed for mitigating the seismic vulnerability of OOP modes in masonry buildings (Giresini et al. 2021).

4. Results and discussion

The results are shown in terms of time-histories for normalised rotations and of median and standard deviation for each configuration (free façade and façade restrained by either "traditional", namely steel tie-rods, or dissipative tie-rods).

More in detail, Fig. 3 shows the most severe rotation time-histories of the church façade, modelled through rigid contact with the sidewalls, under some selected records of the 2012 Emilia Romagna earthquake. These responses occur for stations MDN and MRN, respectively. As expected, the maximum rotation occurs for the 2S façade (maximum normalised rotation 89%) followed by 1S façade free from any restraint, which attains an OOP normalised rotation by about 45%. Even with the traditional (non-dissipative) tie-rod (1S-T, red dashed curve in Fig. 3a), such a rotation becomes negligible, being less than 5% (median 3.5%, maximum value 15%). The dissipative component with low damping coefficient ($c = 2.5$ kNs/m) furtherly improves such a response, smoothing the motion, halving the median value recorded for 1S-T (median 1.2%), and reducing the maximum value by an order of magnitude (1.9%).

Fig. 3b shows the time-histories detailed for the model 1S-DT with all the damping coefficients adopted as analysis parameters. It is easy to observe that there is not a linear trend of reduction with the increase of damping coefficient, namely even a low damping coefficient is beneficial to remarkably reduce the amplitude of motion. Such an aspect reveals the possibility of selecting even low values of damping coefficients; this would also be justified by technical limitation of the dissipative tie-rod, for which a too high damping coefficient would cause a damper too stiff to re-open in short time and hence it might become inactive (Giresini et al. 2021). As already discussed by Giresini et al. (2021) and Giresini et al. (2022), the addition of tie-rods increases the rocking frequency with respect to the unrestrained façade. No variation of rocking frequency is observed passing from 1S-T to 1S-D.

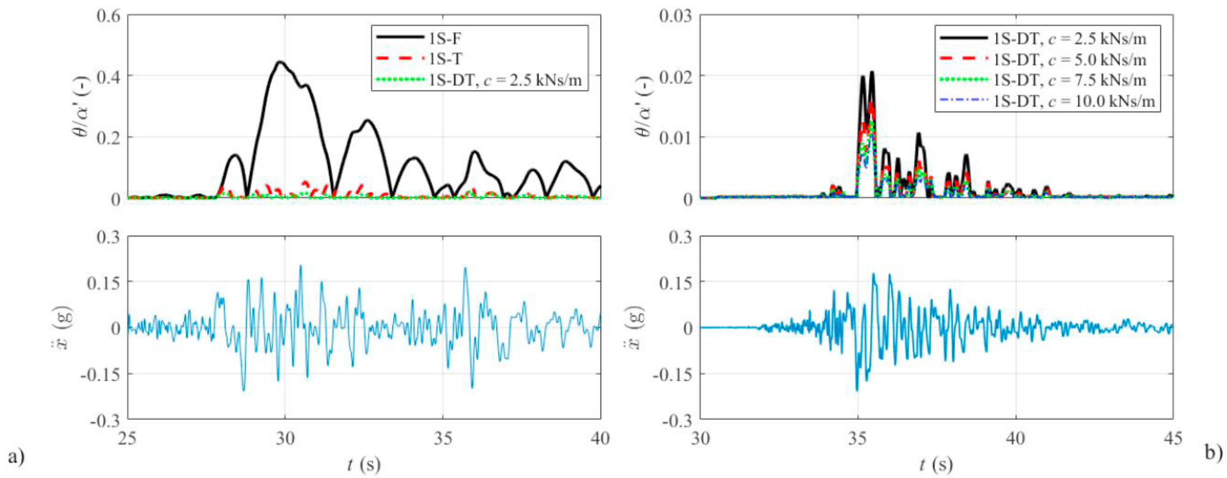

Fig. 3 Time history due to (a) Emilia_1st_shock, Station MDN, N record SF = 6.239; (b) Emilia_2nd_shock, Station MRN, E record SF = 0.934.

Fig. 4 shows the median and the standard deviation for each unrestrained and restrained configuration; this curve is valid for the rigid unilateral contact; the curves of the elastic contact models overlap the first ones, showing the very relevant agreement between the two models. It is possible to observe that higher values of damping coefficient, even though not remarkably reducing the maximum normalised rotation, are associated to a reduction of the standard deviation, which is a positive aspect for the reliability of the response.

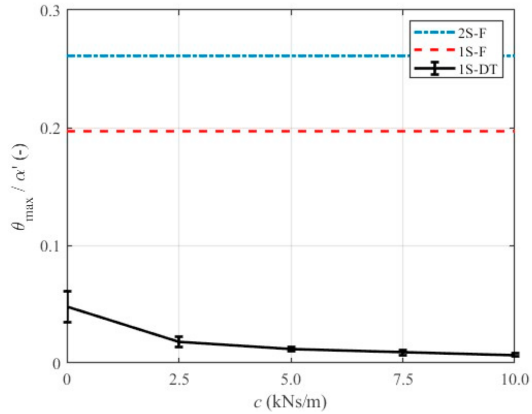

Fig. 4 Median maximum normalised absolute rotation, θ_{max}/α' in function of the equivalent damping coefficient c.

5. Conclusions

This paper provided a contribution on the study of the effect of traditional and dissipative tie-rods in masonry church façades. It also compared two methods for modelling the sidewalls in the out-of-plane (OOP) rocking of masonry façades, applying elastic and rigid contact models to the San Francesco Church hit by the 2012 Emilia Romagna earthquake. Seven models are investigated: two-sided rocking and one-sided rocking with either elastic or rigid contact for simulating the impact with the sidewalls. The boundary conditions are traditional tie-rods and dissipative tie-rods, the latter with different damping properties. The model is excited with 60 recorded spectrum compatible earthquakes and maximum, median and standard deviation values are registered. As expected, the maximum rotation occurs for the 2S façade (maximum normalised rotation 89%), followed by 1S façade free from any restraint, which attains an OOP normalised rotation by about 45%. Even with the non-dissipative tie-rods, such a rotation becomes negligible, being less than 5%. The dissipative component with low damping coefficient (c = 2.5 kNs/m) furtherly improves such a response, smoothing the motion and halving the median value recorded for 1S-T. The dissipative component also reduces the maximum value registered for 1S-T by an order of magnitude. Moreover, the analyses show that even a low damping coefficient is beneficial to remarkably reduce the amplitude of motion. The increment of damping coefficient, even though not sensitively lowering the maximum normalised rotation with respect to lower values of it, is associated to a reduction of standard deviation, which is a positive aspect for the reliability of the response of the damped wall. Finally, the results for the elastic and the rigid contact models overlap in terms of median and standard deviation, showing the very relevant agreement between the two models.

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