Closed-loop Control from Data-Driven Open-Loop Optimal Control Trajectories

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***Abstract*— We show how the recent works on data driven open-loop minimum-energy control for linear systems can be exploited to obtain closed-loop piecewise-affine control laws, by employing a state-space partitioning technique which is at the basis of the static relatively optimal control. In addition, we propose a way for employing portions of the experimental input and state trajectories to recover information about the natural movement of the state and dealing with non-zero initial conditions. The same idea can be used for formulating several open-loop control problems entirely based on data, possibly including input and state constraints.**

I. INTRODUCTION

Recently, the control community has witnessed an increased interest in data-driven approaches, that allow to synthesize controllers directly from data, without resorting to system identification. As a matter of fact, as the complexity of the system to be controlled increases, building an accurate model of the system dynamics can be difficult, expensive, and time-consuming [1], [2]. Therefore, finding alternative model-free approaches is desirable. In the case of linear, unconstrained discrete-time systems, for example, explicit formulas for the open-loop minimum energy control, based entirely on experimental data, are provided by [3]. A generalization to a less restricted experimental framework is presented in [4], while some applications on complex systems, such as power-grid networks and brain networks, are reported in [5]. Basically, these are off-line approaches, leading to optimal open-loop input sequences from data-batch collected in preliminary experiments. Besides the mentioned open-loop approaches, the data-driven closed-loop control problem has been studied as well, often employing the *fundamental lemma* [6] stating that, under the assumption of persistence of excitation, a finite set of input-output data is sufficient to describe all possible trajectories of a linear time-invariant system. In [7], [8] authors propose a data-driven formulation of the linear quadratic regulator (LQR) problem with infinite and finite optimization time horizons, respectively. The above results can also be extended to nonlinear systems, if the data collected during the experiment satisfy suitable conditions [9]. An extension to the case when data are corrupted by noise is proposed in [10]. [11], [12], [13] and [14] propose data-driven solutions to address the model predictive control (MPC) problem, again relying on [6]. In [15] authors provide tractable conditions to verify the dissipativity property in non linear systems, without an explicitly identified model. In particular, they present an offline data-based non-parametric characterization of nonlinear functions based on polynomial approximation. For a detailed literature overview, we refer readers to [16].

In the present work, we deal with open-loop optimal control sequences obtained by the sole experimental data, and specifically, by sequences of inputs and the corresponding states. The aim is to exploit such open-loop sequences to get closed-loop control laws, without resorting to the knowledge of the systems matrices; in other words, to obtain a data-driven closed-loop controller. The state-space partitioning technique described in [17], in the context of a static version of the relatively optimal control [18] is suitable to that aim, since it relies on the optimal input and state trajectory only. In particular, it does not require the knowledge of the system dynamics. The remaining of the paper is organized as follows: Section II recalls how the static ROC can be used for synthesizing a closed-loop controller starting from optimal open-loop input and state sequences; Section III provides an explicit, data-driven, formula for the minimum energy control sequence leading the state to zero from an arbitrary initial state and shows how to recover the corresponding state trajectory; Section IV provides an extension that allows to incorporate state and input constraints while computing the optimal input sequence; two numerical examples are provided in Section V, and conclusions are drawn in Section VI.

II. STATIC RELATIVELY OPTIMAL CONTROL

The Relatively Optimal Control (ROC) [18] is a kind of control that, besides being stabilizing, guarantees the optimality of certain trajectories, specifically, those starting from a given (or a set of given) initial conditions. Both linear dynamic [18], and non-linear static [17] implementations of ROC for linear systems have been proposed, as well as a continuous-time solution based on the Youla–Kučera parameterization [19].

Consider the time-invariant discrete-time linear system

$$x(k+1) = Ax(k) + Bu(k), \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, while $x(k) \in \mathbb{R}^n$ and $u(k) \in \mathbb{R}^m$ denote respectively the state and the input at time $k \in \mathbb{N}$. For a given horizon K, and initial state x_{ini}, the following

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open-loop control problem can be formulated:

$$
\begin{aligned}
J_{\text{opt}}(x_{\text{ini}}) &= \min \sum_{k=0}^{K-1} l(x(k), u(k)) \\
\text{subject to: } &(1)\ k = 0 \dots K-1 \\
&x(0) = x_{\text{ini}},\ x(K) = 0,
\end{aligned} \tag{2}
$$

where $l(\cdot,\cdot)$ is a convex function of its arguments, and the decision variables are the control actions $u(0), \dots, u(K-1)$. The main result (Theorem 3) of [17] states that, if the optimal state and input trajectory (solution of the previous optimization problem) is such that the residual cost is strictly decreasing, i.e.,

$$
\begin{gathered}
\sum_{k=\bar{k}}^{K-1} l(x(k),u(k)) < \sum_{k=\bar{k}+1}^{K-1} l(x(k), u(k)), \\
\forall\ \bar{k} = 0, \dots, K-2
\end{gathered} \tag{3}
$$

then a piecewise-affine, globally stabilizing, and relatively optimal control law can be computed based on a suitable partition of the state-space. Such a static state-feedback is referred to as *static ROC*. Furthermore, [17] provides a constructive procedure, to get the mentioned control law, that does not require the knowledge of A and B. The procedure requires the knowledge of the sequence of control actions $u(0), \dots, u(K-1)$, which brings the state from x_{ini} to zero while minimizing the cost $J_{\text{opt}}(x_{\text{ini}})$, and the corresponding state trajectory. The convex hull of states of the optimal trajectory, and their opposite, defines a region in the state space (the gray shaded polytope in the two-dimensional example of Figure 1). Such a region is partitioned into simplices, whose vertices are states of the optimal state sequence (and their opposite).

The remaining part of the state space (i.e., the region outside the convex hull), can be partitioned into cones, centered in origin, and truncated to keep the cone part outside the already defined convex hull (see the dot-dashed lines in Figure 1).

For the detailed partitioning procedure, we refer the reader to [17]. Here, we only recall that, given the simplicial partition of the state space, the local control law associated to each simplex (i.e., the control law to be applied to the system when the current state belongs to a particular simplex) is the convex combination of the control inputs corresponding to the vertices of the simplex. The control law associated with each truncated cone is, as well, a linear combination of a properly chosen subset of the optimal control actions, minimizing the cost $J_{\text{opt}}(x_{\text{ini}})$ (see [17] for details). As a result, a piece-wise affine control law is obtained that is proven to be stabilizing and guaranteeing the optimality of the trajectories starting from x_{ini}.

In the following, we apply the static ROC to obtain a closed-loop control law from data-driven, open-loop, optimal trajectories.

III. DATA-DRIVEN MINIMUM ENERGY CONTROL

The present section is focused on the data-driven minimum energy control, because some of the results we will apply come from the literature on that subject. However, as it will be shown in Section IV, the proposed methodology can be applied to open-loop trajectories other than minimum-energy ones. For a given horizon K, the minimum-energy control problem to zero is that of finding, among the input sequences that drive the state from $x(0) = x_{\text{ini}}$ to $x(K) = 0$, the one of minimum energy, i.e. the one minimizing $\sum_{k=0}^{K-1} \|u(k)\|_2^2$. Clearly, the problem is a special case of (2), corresponding to $l(x(k), u(k)) = \|u(k)\|_2^2$. Let us denote (with slight abuse) by u the sequence $[u(K-1)^\top, \dots, u(0)^\top]^\top$, and by u^* the optimal one. With the same notation, the optimal input sequence u^* can be expressed as the minimum 2-norm solution of the following equation:

$$
0 = A^K x_{\text{ini}} + \underbrace{\begin{bmatrix} B & AB & \dots & A^{K-1}B \end{bmatrix}}_{R_K} u, \tag{4}
$$

where R_K is the K-steps reachability matrix. For A and B (and, thus, R_K) known, the solution of the above problem is well-known to be [20]

$$
u^* = -R_K^\dagger \left(A^K x_{\text{ini}} \right), \tag{5}
$$

where † denotes the Moore-Penrose pseudo-inverse.

Here, we are interested in solving Equation (4) relying on experimental data only. In addition, since the ROC technique described in the previous section requires the optimal open loop *state* trajectory, besides the optimal input sequence, we need to compute from data *the optimal state sequence as well*. The mentioned issues are dealt with in the following subsections. When the optimal input and state sequences have been computed based on data, the ROC technique can be applied, leading to a closed-loop, stabilizing, data-driven control law. An example is reported in Section V.

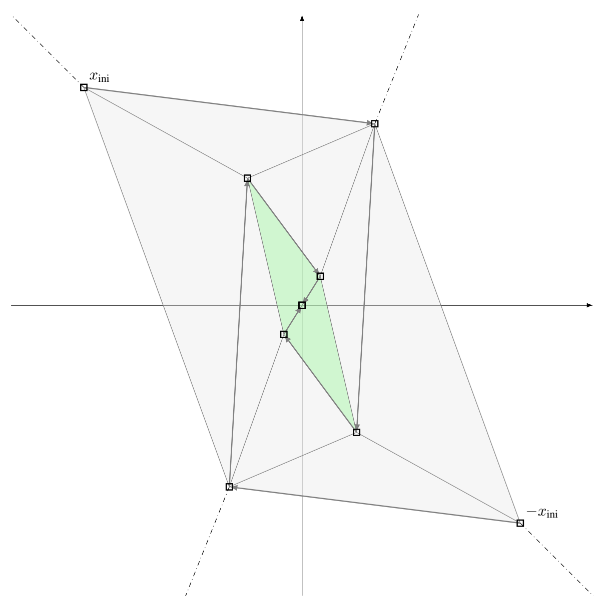

Fig. 1. State-space partition: the gray shaded part represents the convex hull of the optimal trajectory (5-steps long) starting at x_{ini} and its opposite. The green shaded polytope, corresponding to the last n steps of the optimal trajectory, is not partitioned into simplices [17].

A. Optimal input sequence from data

The experimental data employed is similar to that of [3], in which a set of $N \geq n$ experiments is available, each starting from $x_0 = 0$ and lasting K steps. Denoting by u_i the i-th (arbitrary) input sequence, and by x_i the state reached at time K of the i-th experiment, the matrices

$$U = [u_1 \quad \dots \quad u_N], \quad \text{and} \quad X = [x_1 \quad \dots \quad x_N], \tag{6}$$

are constructed. Here, according to [3], we assume that U is a full rank matrix. Clearly, we have:

$$x_i = R_K u_i, \quad i = 1, \dots, N. \tag{7}$$

The previous can be written as $X = R_K U$, and the solution of problem $\min \|X - R_K U\|_F^2$, i.e.,

$$R_K^* = XU^\dagger, \tag{8}$$

is an estimate of the K-steps reachability matrix. Under the assumption of $N = Km$ [3], and $\operatorname{rank}[X^\top | U^\top] = \operatorname{rank}[U^\top]$, R_K^* exactly matches the reachability matrix. Note that the full rank property of the U matrix is a sufficient condition to ensure this match.

In the following we will employ such an estimate in place of the unknown reachability matrix, and we will denote it by R_K.

Due to the term $A^K x_{\text{ini}}$, substituting Equation (8) in Equation (5) is not sufficient to get a solution based on data only. A possibility would be to use the results of [4], that extends [3] to more general problems and less restrictive experimental setups. As a simpler alternative, we propose to collect N sequences of length $2K$:

$$\underbrace{u(2K-1)^\top, \dots, u(K)^\top}_{\hat{u}_i^\top}, \underbrace{u(K-1)^\top, \dots, u(0)^\top}_{u_i^\top}$$

$$x(1), \dots, \underbrace{x(K)}_{x_i}, x(K+1), \dots, \underbrace{x(2K)}_{\hat{x}_i}$$

and construct the following matrices:

$$U = [u_1, \dots, u_i, \dots, u_N], \qquad \hat{U} = [\hat{u}_1, \dots, \hat{u}_i, \dots, \hat{u}_N]$$
$$X = [x_1, \dots, x_i, \dots, x_N], \qquad \hat{X} = [\hat{x}_1, \dots, \hat{x}_i, \dots, \hat{x}_N],$$

where U and X are the same of Equation (6), while $\hat{U}$ and $\hat{X}$ correspond to trajectories of length K, starting (in general) from non-zero states. Then, by construction, $\forall i = 1 \dots N$ we have:

$$\hat{x}_i = A^K x_i + R_K \hat{u}_i,$$

which can be written in compact form as

$$A^K X = \hat{X} - R_K \hat{U}, \tag{9}$$

and, in view of Equation (8), as

$$A^K X = \hat{X} - XU^\dagger \hat{U}. \tag{10}$$

Thus, the right hand side of Equation (10) can be used to compute the term $A^K x_{\text{ini}}$ for any x_{ini} in the column space of X. Specifically, let $\alpha \in \mathbb{R}^N$ be such that

$$x_{\text{ini}} = X\alpha. \tag{11}$$

Then, we have

$$A^K x_{\text{ini}} = A^K X \alpha = \left(\hat{X} - XU^\dagger \hat{U}\right) \alpha.$$

Using the least-norm solution for α in Equation (11), i.e. $\alpha = X^\dagger x_{\text{ini}}$, we get

$$A^K x_{\text{ini}} = \left(\hat{X} - XU^\dagger \hat{U}\right) X^\dagger x_{\text{ini}}. \tag{12}$$

Finally, by substituting in Equation (5), namely $u^* = -R_K^\dagger \left(A^K x_{\text{ini}}\right)$, and recalling that $R_K = XU^\dagger$, we get:

$$u^* = \left(XU^\dagger\right)^\dagger \left(XU^\dagger \hat{U} - \hat{X}\right) X^\dagger x_{\text{ini}}, \tag{13}$$

which gives a data-driven open-loop minimum energy control sequence leading the state to zero in K steps from x_{ini}. The formula provides the optimal sequence when x_{ini} belongs to the column space of X. In particular, if X is rank n, then x_{ini} can be arbitrary.

B. Optimal state trajectory from data

To get the closed-loop control law by means of the partitioning procedure described in Section II, besides the optimal input sequence u^*, given by Equation (13), the corresponding optimal state trajectory is needed. Such trajectory can be recovered from u^* and the data obtained from the same N sequences collected before (i.e., without the need of collecting further data).

It is sufficient to define the matrices U_k, $\hat{U}_k$, X_k, and $\hat{X}_k$, similarly as before, but based on *subsequences* of length $2k$, for $k = 1 \dots K-1$. Let δ_k denote the starting index of the subsequences of length $2k$, and define

$$\begin{aligned} U_k &= \left[{}^k u_1, \dots, {}^k u_i, \dots, {}^k u_N\right], \\ \hat{U}_k &= \left[{}^k \hat{u}_1, \dots, {}^k \hat{u}_i, \dots, {}^k \hat{u}_N\right], \\ X_k &= \left[{}^k x_1, \dots, {}^k x_i, \dots, {}^k x_N\right], \\ \hat{X}_k &= \left[{}^k \hat{x}_1, \dots, {}^k \hat{x}_i, \dots, {}^k \hat{x}_N\right], \end{aligned}$$

where ${}^k u_i = \left[u(\delta_k + k - 1)^\top, \dots, u(\delta_k)^\top\right]^\top$, ${}^k \hat{u}_i = \left[u(\delta_k + 2k - 1)^\top, \dots, u(\delta_k + k)^\top\right]^\top$, ${}^k x_i = x(\delta_k + k)$, and ${}^k \hat{x}_i = x(\delta_k + 2k)$.

Hence, by letting $U_K = U$, $X_K = X$, $\hat{U}_K = \hat{U}$, and $\hat{X}_K = \hat{X}$ we can write

$$A^k X_k = \hat{X}_k - R_k \hat{U}_k, \quad k = 1, \dots, K, \tag{14}$$

where R_k is the k-step reachability matrix, corresponding to the first k columns of R_K:

$$R_k = R_K \begin{bmatrix} I_k \\ 0 \end{bmatrix} = XU^\dagger \begin{bmatrix} I_k \\ 0 \end{bmatrix},$$

where I_k denotes the identity matrix of dimension k. Equations (14) hold irrespective of the choice of the subsequences (i.e., of indices δ_k). However, it is convenient to choose the subsequences in such a way that $\operatorname{rank} X_k = n$, $\forall k$. This is always possible, provided that $\operatorname{rank} X = n$, and can be achieved by taking $\delta_k = K - k$, leading to $X_k = X$, $\forall k$. The full-rank condition on the X_k guarantees that any initial state x_{ini} can be written as a linear combination of the columns of any of the X_k. As a consequence, the optimal state trajectory, in terms of data, is given by:

$$x(k) = \left(\hat{X}_k - R_k \hat{U}_k\right) X_k^\dagger x_{\text{ini}} + R_k u_k^*, \quad k = 1, \dots, K \tag{15}$$

where u_k^* is the vector composed by the first k steps of the optimal input sequence: $u_k^* = \left[u^*(k-1)^\top, \dots, u^*(0)^\top\right]^\top$.

IV. EXTENSIONS

The described approach is not restricted to minimum energy trajectories, but is suitable for any data-driven open-loop optimal trajectory, provided that the residual cost is strictly decreasing. In the following, we show how to obtain data-driven optimal trajectories by solving optimization problems that rely solely on experimental data. It is sufficient to observe that, for a given initial state x_{ini} and K-steps input sequence u, the state at time K can be expressed in terms of data as

$$x(K) = \left(\hat{X} - XU^\dagger \hat{U}\right) X^\dagger x_{\text{ini}} + XU^\dagger u, \tag{16}$$

which is the same as Equation (15) when $k = K$ and the control sequence is generic. As a consequence, the minimum energy problem already discussed, can be stated equivalently in the following quadratic programming (QP) form:

$$\begin{aligned} &\min \|u\|_2^2 \\ &\text{subject to:} \\ &XU^\dagger u = -\left(\hat{X} - XU^\dagger \hat{U}\right) X^\dagger x_{\text{ini}}. \end{aligned} \tag{17}$$

Such a formulation allows the introduction of linear constraints on the input and on the state, while retaining the QP form. The constraints on the input can be directly introduced since u is a decision variable. As far as the constraints on the state are concerned, observe that the state at time $k \leq K$ can be written as

$$x(k) = \left(\hat{X}_k - R_k \hat{U}_k\right) X_k^\dagger x_{\text{ini}} + R_k u_k, \tag{18}$$

where u_k is the vector composed by the first k steps of the input sequence: $u_k = \left[u(k-1)^\top, \dots, u(0)^\top\right]^\top$. Thus, constraints on the state can be added as well. Furthermore, different cost functions can be employed, possibly leading to linear programming (LP) problems.

V. NUMERICAL EXAMPLE

We consider the double integrator:

$$x(k+1) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k). \tag{19}$$

We set $K = 9$ and perform $N = 20$ experiments lasting $2K = 18$ steps, starting from $x_0 = \begin{bmatrix} 0 & 0 \end{bmatrix}^\top$ and applying randomly chosen inputs. The collected states and inputs are then used to construct the U, $\hat{U}$, X and $\hat{X}$ matrices used in Equation (13). We choose $x_{\text{ini}} = \begin{bmatrix} 1 & -5 \end{bmatrix}^\top$, and we solve the minimum-energy control problem by applying Equation (13), thus obtaining the minimum energy input sequence that leads the system to $x(9) = 0$ starting from $x(0) = x_{\text{ini}}$. The obtained optimal (open-loop) control sequence is:

$$\begin{aligned} u^* = [&2.156 \;\; 1.756 \;\; 1.356 \;\; 0.956 \;\; 0.556 \;\; 0.156 \\ &-0.244 \;\; -0.644 \;\; -1.044], \end{aligned} \tag{20}$$

while the resulting optimal trajectory, say x^*, obtained by applying (18), is shown in Figure 2.

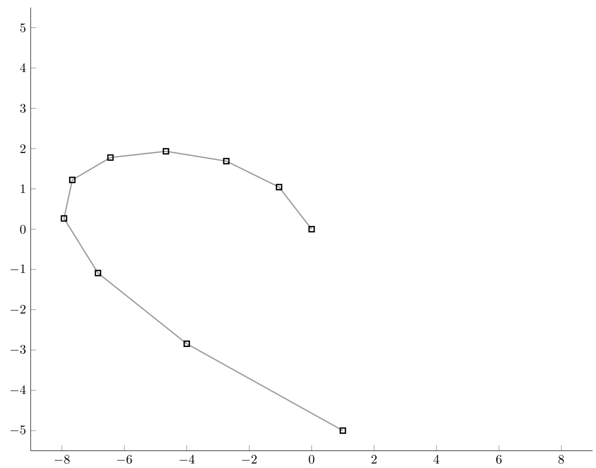

Fig. 2. Optimal trajectory corresponding to the optimal input sequence in (20).

According to the static ROC framework, the optimal control and state trajectories u^* and x^* are then employed to compute the closed-loop control law. The partition of the state-space is shown in Figure 3, where the vertices of the polytopes are the states of the optimal state trajectory and its opposite. In the same figure, the trajectories obtained by applying the static ROC from four non-nominal (i.e., different form x_{ini}) initial conditions are reported in blue.

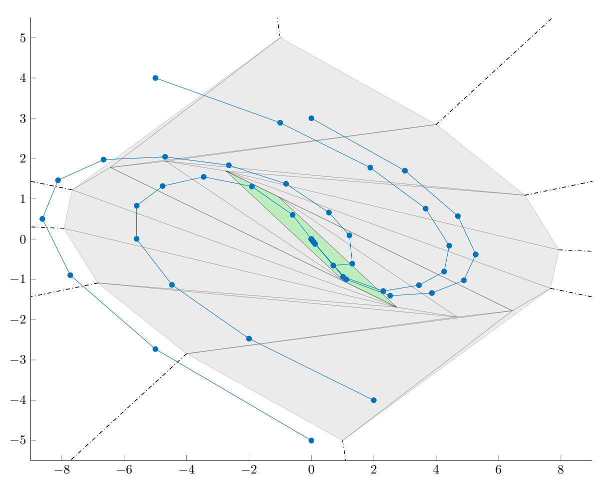

Fig. 3. Triangulation induced by the optimal trajectory and the trajectories from the following non-nominal initial conditions $x1 = [0 \;\; -5]^\top$, $x2 = [-5 \;\; 4]^\top$, $x3 = [2 \;\; -4]^\top$, $x4 = [0 \;\; 3]^\top$.

Finally, we report results obtained by solving the data-driven minimum-energy problem (17), with the following additional state and control constraints:

$$\begin{aligned} |x(k)| &\leq 6.5 \quad \forall k = 1, \dots, K \\ |u(k)| &\leq 3 \quad \forall k = 1, \dots, K. \end{aligned} \tag{21}$$

where the (component-wise) inequalities involving the state have been imposed thanks to the (18). The optimal control sequence, obtained by solving a QP problem, is:

$$\begin{aligned} u^* = [2.831 \;\; 1.839 \;\; 0.849 \;\; 0.582 \;\; 0.315 \;\; 0.047 \\ -0.221 \;\; -0.488 \;\; -0.755], \end{aligned} \tag{22}$$

while the resulting optimal trajectory is shown in Figure 4.

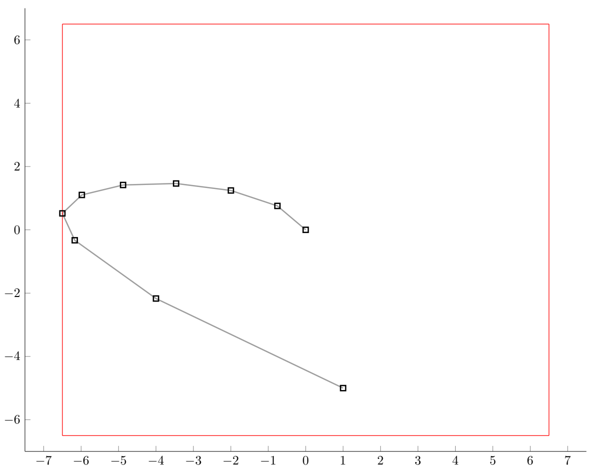

Fig. 4. Optimal trajectory corresponding to the optimal input sequence in (22).

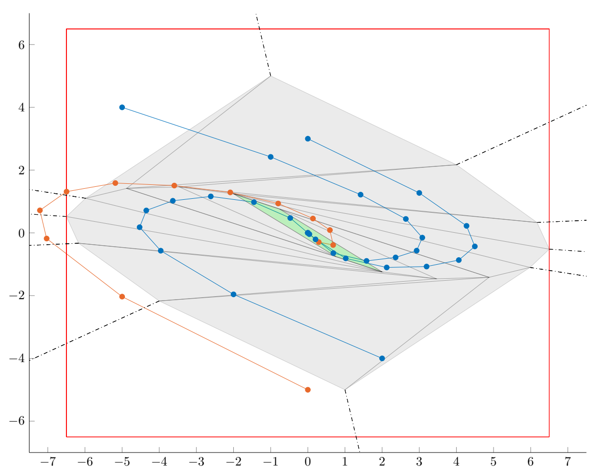

Fig. 5. Triangulation and trajectories induced by the optimal trajectory obtained solving the QP problem (17) from the following non-nominal initial conditions $x1 = [0 \;\; -5]^\top, x2 = [-5 \;\; 4]^\top, x3 = [2 \;\; -4]^\top, x4 = [0 \;\; 3]^\top$. The orange trajectory violates the state constraints.

Figure 5 reports the resulting trajectories from some non-nominal initial conditions. We can observe that all the trajectories starting from initial conditions included in the polytope represented in gray (which is the convex hull of the states of the optimal trajectory and its opposite), satisfy the imposed constraints. Not all the trajectories resulting from initial conditions outside that polytope, however, turn out to satisfy the constraints, (see the orange trajectory of Figure 5). Indeed, the static ROC guarantees the satisfaction of (state and input) constraints from all the initial states belonging to that polytope, while there is no such guarantee for other initial states.

VI. CONCLUSIONS

In this paper, we derived a novel data-driven approach to obtain closed-loop control laws from open-loop data-driven optimal control sequences. The approach is based on the static ROC, which leads to a piece-wise affine static and globally stabilizing control, starting from optimal state and control sequences. It can be applied whenever an open-loop, optimal control sequence is available that leads the system to zero from a given initial state. We reported two numerical examples, based respectively on a minimum energy open loop data-driven control sequence, and on a sequence obtained by solving a constrained, data-driven, minimum energy problem formulated as a quadratic program. Future work includes the experimental assessment on real systems, and the use of the dynamic ROC, instead of the static one. The dynamic version of ROC is not based on the partition of the state space, thus it does not suffer of the well-known computational difficulties associated to state partitioning techniques when applied to high dimensional systems.

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